Royal Institute of Technology

Politecnico di Milano
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Corso di Laurea Specialistica in Ingegneria Energetica

Dynamic Modelling of Mechanical Heat Pumps for Comfort Heating

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Academic Year 2009-2010

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Acknowledgments

The authors would like to thank:
Prof. Ing. Joppolo, Dr. Kassem, Miss Susanne, our fellow students Marco, Andrea, Massimo, Giovanni, Pedro and Steve, our international friends Sven, Patrick, Sascha, Philipp, André, Claudia, Nicole, Linn and Jessica, Sweden and the Northern Lights we could not see, our friends Jim, Jack and Umbala, Mr. Brown, SOF and HCSS.

Co-author Stefano Badiali would like to thank:
My parents, my brothers and my sister Francesca, my girlfriend Alessia, Fabio, Matteo, Marco and all my friends for supporting me in good and bad times.

Co-author Stefano Colombo would like to thank:
All my family and all my friends for supporting me during the writing of this work.

Abstract

Heat pumps devices are energy-efficient, in particular, for climates with moderate heating and cooling needs. They find wide applications in residential and office building with the objective to reduce energy consumption. The main function is to pump heat from low temperature heat sources to high temperature heat sinks, thus providing both comfort heating and cooling. Conventional heat pump (HP) includes basic items such as an evaporator, a compressor, a condenser and an expansion (throttling) valve. This components have different dimensions and possess different operational characteristics, particularly under transient conditions. It is conceivable, however, that the transient behaviour of the integrated HP system will be quite different from those of the single components. Therefore, the objective of the project is to develop an integrated dynamic model of a HP and to study its transient responses, when subject to changes in operating parameters. The methodology applied includes computer modelling and simulations of the HP system using commercial software.

Sommario

Le pompe di calore sono macchine operatrici in grado di trasferire calore da una sorgente a bassa temperatura a un pozzo ad alta temperatura, attraverso un apporto di energia elettrica/meccanica. Una pompa di calore è costituita da diversi componenti, tra i quali un evaporatore, un compressore, un condensatore e una valvola di espansione. Questi componenti presentano diverse caratteristiche geometriche e diverse caratteristiche operative, soprattutto in regime transitorio. Tuttavia, il comportamento dell'intero sistema in regime transitorio è differente da quello dei singoli componenti. Di conseguenza, l'obiettivo di questo lavoro è quello di studiare dei modelli fisici per ogni componente e quindi di sviluppare un modello integrato per analizzare le caratteristiche dinamiche dell'intero sistema.
Il compressore studiato è di tipo volumetrico alternativo a pistoni; trascurando la dinamica delle valvole, il modello permette il calcolo della pressione media all'interno della camera di lavoro e delle portate elaborate dal compressore. Il condensatore considerato è di tipo shell and tubes; il modello è stato sviluppato con un approccio a volumi finiti e parametri distribuiti, dividendo il condensatore in tre zone. Il modello prevede l'analisi del comportamento in regime transitorio sia del fluido di lavoro che del fluido secondario. Per quanto riguarda la valvola di espansione, è stato considerato un modello puramente statico, dal momento che la dinamica del componente è di gran lunga più veloce rispetto a quella degli scambiatori. L'evaporatore considerato è di tipo shell and tubes; a differenza del condensatore, il modello è stato sviluppato con un approccio a parametri concentrati e considerando una singola zona; la dinamica del fluido secondario non è stata presa in analisi.
La metodologia utilizzata include l'utilizzo di software commerciale (Matlab Simulink) per l'analisi e la simulazione del modello presentato.

Nomenclature

$\dot{m}$	Mass flow (kg/s)
$\dot{V}$	Volumetric flow (m^3/s)
$\dot{Q}$	Heat flow (W)
W	Compressor work (W)
ε	Volumetric compressor work (J/m^3)
η_c	Carnot efficiency
η_k	Compressor volumetric efficiency
η_{is}	Compressor isentopic efficiency
ω	Compressor running frequency (rad/s)
N	Compressor revolutions per minute (rpm)
μ	Compressor clearance volume by stroke volume
β	Compressor pressure ratio
γ	Adiabatic index
n	Polytropic index
m	Refrigerant mass aspired per cycle (kg/cycle)
C	Valve non-ideality coefficient
S	Valve cross sectional area (m^2)
D	Condenser shell diameter (m)
d	Condenser tube diameter (m)
A	Refrigerant flow sectional area (m^2)
z	Condenser axial coordinate (m)
V	Volume (m^3)
v	Velocity (m/s) (Refrigerant velocity with no subscript)
τ	Time constant (s)
α	Heat transfer coefficient (W/m^2K) (Refrigerant HTC with no subscript)
h	Refrigerant enthalpy (J/kg)
u	Refrigerant internal energy (J/kg)
P	Refrigerant pressure (Pa)
$\mathcal{v}$	Refrigerant specific volume (m^3/kg)
ρ	Density (kg/m^3) (Refrigerant density with no subscript)
T	Temperature (K)
c_p	Specific heat capacity at constant pressure (J/kgK)
c_v	Specific heat capacity at constant volume (J/kgK)
k	Thermal conductivity (W/mK)
μ	Dinamic viscosity (kg/ms)
ν	Cinematic viscosity (m^2/s)
Nu	Nusselt number
Re	Reynolds number
Pr	Prandtl number

Subscripts

a	evaporator inlet
b	compressor inlet
c	condenser inlet
d	expansion valve inlet
c,is	isentropic compression outlet
1	before aspiration
2	after aspiration
cle	clearance
cyl	cylinder
dis	discharge
dv	discharge valve
e	evaporator
i	ideal
il	inlet-liquid
ov	outlet-vapour
r	refrigerant
str	stroke
suc	suction
sv	suction valve
th	theoretical
v	valve
w	water
ρ	density

1 General review

1.1 Introduction

As shown from the experience, the second law of thermodynamics states that heat spontaneously flows from higher temperature bodies to lower temperature ones. With heat pumps, however, it is possible to force the heat flow in the other direction, i.e. from a source at a lower temperature to a sink at a higher temperature using a relatively small quantity of mechanical work. In heating mode, the heat pump transfers heat from the ground or the outdoor air to a building or an industrial application. Theoretically, the total heat delivered by the heat pump is equal to the heat extracted from the heat source, plus the amount of drive work (Figure 1). It is important to notice that the heat extracted from the source is renewable energy in the form of low-temperature heat.

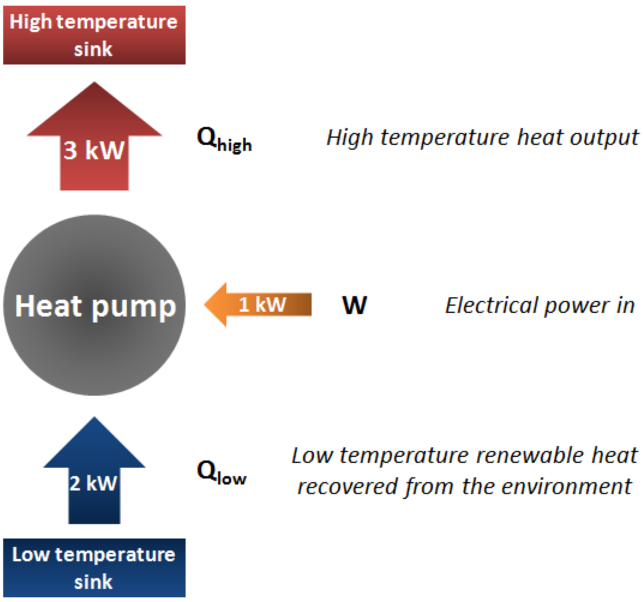

Figure 1 Energy balance in a heat pump.

1.2 Operating principles

The great majority of heat pumps work on the principle of the vapour compression cycle. The main components in such a heat pump system are the compressor, the expansion valve and two heat exchangers referred to as evaporator and condenser. The components are connected to form a closed circuit which the working fluid or refrigerant circulates in, as shown in Figure 2.

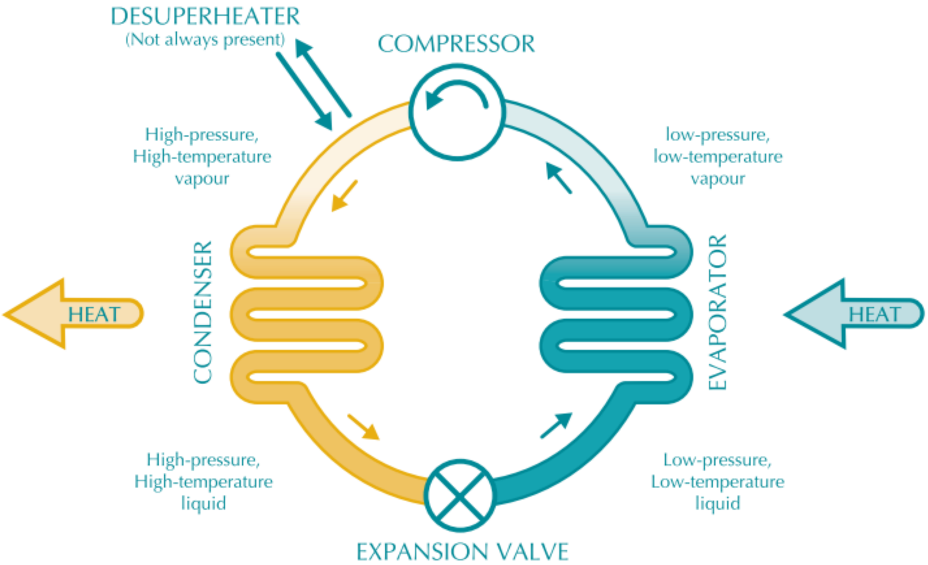

Figure 2 Vapour compression cycle of a typical heat pump.

At the entrance of the evaporator, the refrigerant is in a liquid state and it is characterized by a low pressure and a low temperature, compared to the temperature of the heat source. For this reason inside the evaporator the heat flows from the heat source to the working fluid, causing the liquid refrigerant to evaporate. Its temperature does not increase so much. This low pressure and low temperature vapour then passes into an electrically-driven compressor. This raises the refrigerant's pressure and, as a consequence, its temperature. The hot output of the compressor is fed into the condenser. As the sink temperature is lower than the refrigerant temperature, the heat flows from the refrigerant to the secondary fluid (sink), causing the gaseous refrigerant to condense. Finally, the high-pressure working fluid is expanded to the evaporator pressure and temperature in the expansion valve. The working fluid is returned to its original state and once again enters the evaporator.

A desuperheater provides domestic hot water when the compressor is operating. The desuperheater is a small auxiliary heat exchanger at the compressor outlet. It transfers excess heat from the compressed gas to water that circulates to a hot water tank.

The heat pump cycle is fully reversible. In fact, heat pumps normally include a reversing valve and optimized heat exchangers so that the direction of the heat flow may be reversed. By changing the role of the two coils heat pumps can be used both for a heating and a cooling purpose.

In the cooling mode the indoor coil acts as the evaporator. The liquid

refrigerant absorbs heat and evaporates, becoming a low temperature vapour. Then it passes through the reversing valve to the compressor. The hot output passes through the reversing valve to the outdoor coil, which acts as the condenser. The hot gas releases heat and condenses. This high-pressure liquid passes through the expansion device and the cycle is repeated.
Refrigerators and air conditioners are examples of heat pumps operating only in the cooling mode. A refrigerator can be intended as an insulated box connected to a heat pump system: heat is absorbed from the evaporation coil located inside the freezer compartment and it is transferred outside, usually behind the unit, where the condenser coil is located.

1.3 Thermodynamics and efficiency

The thermodynamics of the vapour compression cycle can be analyzed on a pressure-enthalpy diagram as shown in Figure 3. The ideal refrigeration cycle is made of 4 transformations:

- isobaric low pressure heat exchange inside the evaporator (a-b);
- isentropic compression starting from saturated vapour (b-c_{is});
- isobaric high pressure heat exchange inside the condenser (c_{is}-d);
- isenthalpic expansion through the expansion device (d-a).

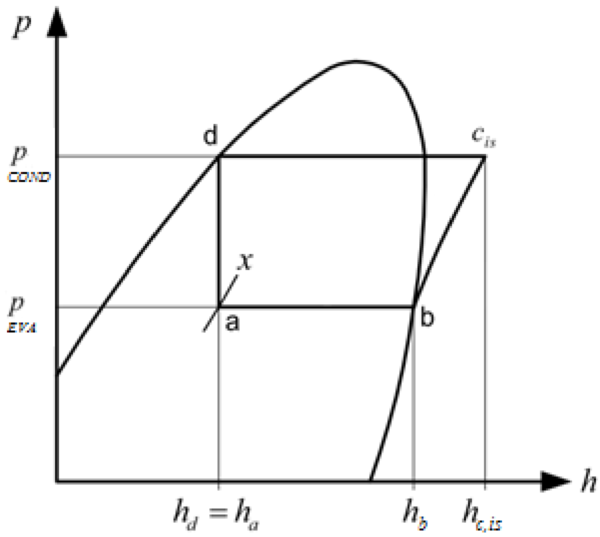

Figure 3 Diagram of the ideal vapour compression cycle.

From the diagram it is possible to define two main parameters that are the evaporator heat flow ($\dot{Q}_{EVA}$) and the condenser heat flow ($\dot{Q}_{COND}$), referred to the refrigerant mass flow and its enthalpy variation:

$$\dot{Q}_{EVA} = \dot{m} * (h_b - h_a) \quad (1.1)$$

$$\dot{Q}_{COND} = \dot{m} * \left(h_{c,is} - h_d\right) \quad (1.2)$$

Eq. 1.1 represents the useful cooling energy when the system is working in cooling mode; eq. 1.2, instead, the thermal energy generated when the system is working as heat pump.
It is important to remember that the ideal refrigeration cycle is different from the inverse Carnot cycle, which is fully reversible and it is characterized by the presence of a turbine in spite of the throttling device and by the presence of two phases after the isenthalpic evaporation.
As shown in Figure 4, the real refrigeration cycle differs both from the ideal cycle and the Carnot cycle for the following reasons:

- the compression phase is not isentropic because of fluid dynamic losses during the transformation;
- subcooling and superheating phases are generally utilized to increase the cycle efficiency by improving the energy outcome of systems working in heating or cooling mode. Moreover in practice wet vapour is quite difficult to compress;
- evaporation and condensation phases are characterized by pressure drops due to certain frictional losses inside the heat exchangers.

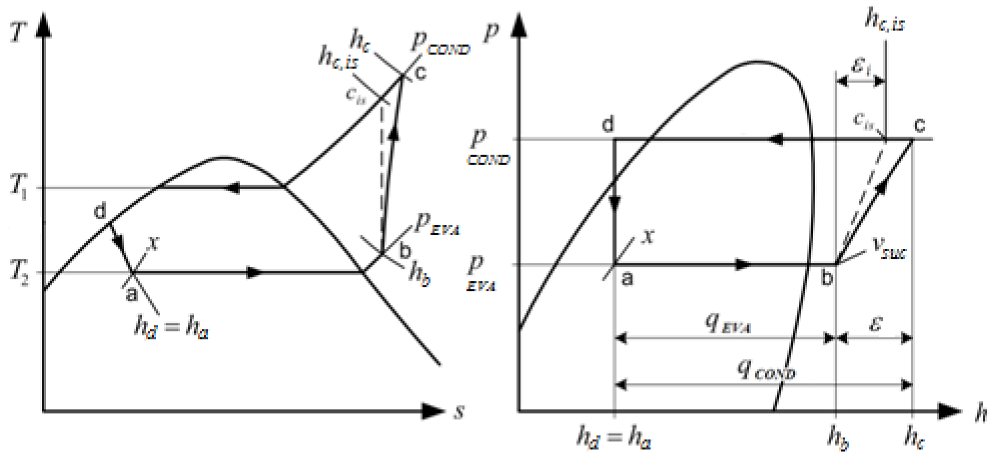

Figure 4 Diagrams of the real vapour compression cycle.

About the compression phase is useful to introduce two kinds of compression efficiency. The first one is called volumetric efficiency η_k. It is defined as the rate between the actual volumetric flow $\dot{V}$ and the theoretical one $\dot{V}_{th}$:

$$\eta_k = \frac{\dot{V}}{\dot{V}_{th}} \quad (1.3)$$

As mentioned above the compression is not isentropic. Therefore the compressor is characterized by an isentropic efficiency defined as the rate between the ideal energy supplied to the compressor for an isentropic compression and the real one:

$$\eta_{is} = \frac{h_{c,is} - h_b}{h_c - h_b} \quad (1.4)$$

Moreover it is possible to define the ideal volumetric compressor work ε_i (eq. 1.6) as the rate between the ideal compressor work W_i (eq. 1.5) and the volumetric rate at compressor inlet $\dot{V}$:

$$W_i = \dot{m} * (h_{c,is} - h_b) \quad (1.5)$$

$$\varepsilon_i = \frac{W_i}{\dot{V}} = \frac{h_{c,is} - h_b}{v_{suc}} \quad (1.6)$$

Now remembering that:

$$\dot{m} = \frac{\dot{V}}{v_{suc}} = \frac{\eta_k * \dot{V}_{th}}{v_{suc}} \quad (1.7)$$

the real compressor work W can be related to the ideal volumetric compressor work and both the volumetric and isentropic efficiency through eq. 1.5, eq. 1.6 and eq. 1.7:

$$W = \frac{W_i}{\eta_{is}} = \frac{\dot{m} * (h_{c,is} - h_b)}{\eta_{is}} = \frac{\eta_k * \dot{V}_{th} * (h_{c,is} - h_b)}{\eta_{is} * v_{suc}} = \frac{\eta_k}{\eta_{is}} * \dot{V}_{th} * \varepsilon_i \quad (1.8)$$

The steady-state performance of an electric compression heat pump at a given set of temperature conditions can be referred to two different performance parameters. They are called coefficient of performance COP_1 (here referred as COP) and COP_2. COP is related to the heating mode and it is determined by dividing the energy output of the heat pump by the electrical energy needed to run it at a specific temperature:

$$COP = \frac{\dot{Q}_{COND}}{W} \quad (1.9)$$

The COP of a heat pump is closely related to the temperature lift, i.e. the difference between the temperature of the heat source and the output condenser temperature.
In Figure 5 it is shown the qualitative variation of the cycle shape with regard to a decrease in the evaporation temperature (ΔT_{EVA}). The specific heat flow on the evaporator decreases, but this implies that the specific input work on the compressor increases. For this reason, the COP becomes lower. Moreover, the volumetric efficiency and the suction gas density both decrease: as consequence the refrigerant mass flow rate in the system and the condenser heat flow decrease as well.
The same reasoning can be applied when a rise in the condensation temperature (ΔT_{COND}) would occur. The compressor work increases while the energy output of the condenser decreases, so that the system efficiency declines very quickly.

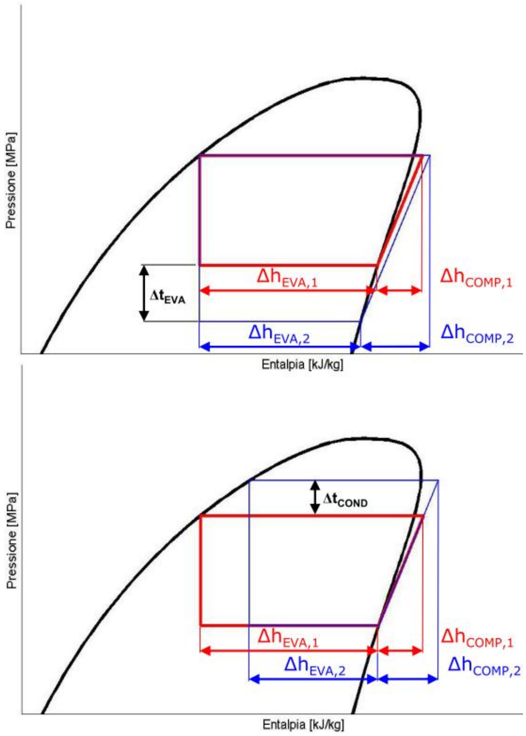

Figure 5 Influence of evaporation and condensation temperatures over efficiency.

The COP of a Carnot cycle heat pump is determined solely by the temperature lift and it is given by the rate between the condensation temperature and the temperature lift:

$$\mathrm{COP} = \frac{\mathrm{T_{COND}}}{\mathrm{T_{EVA}} - \mathrm{T_{COND}}} \qquad (1.10)$$

Figure 6 shows the COP for an ideal heat pump as a function of temperature lift, where the temperature of the heat source is 0°C. Also shown is the range of actual COPs for various types and sizes of real heat pumps at different temperature lifts. The ratio of the actual COP of a heat pump and the ideal COP is defined as the Carnot-efficiency (η_c).

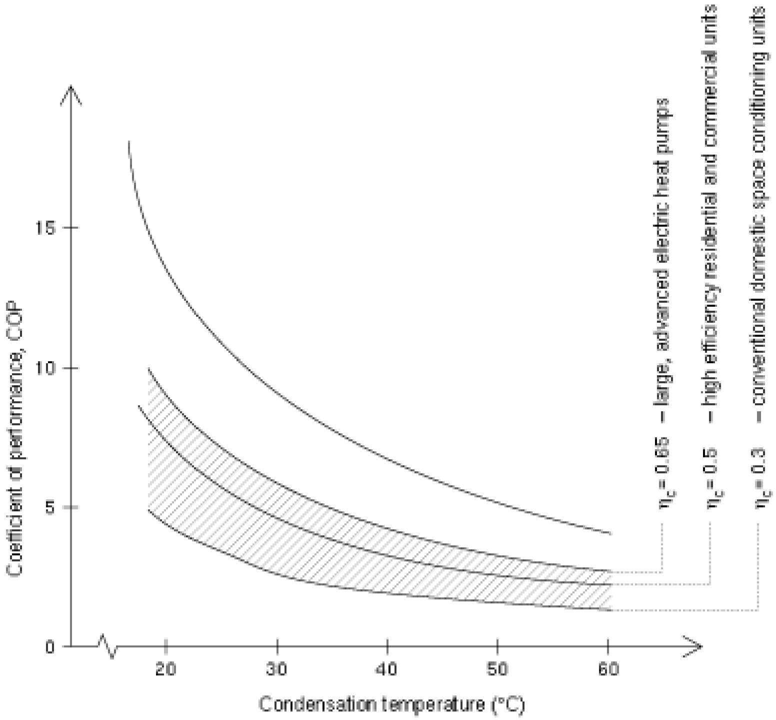

Figure 6 COP as a function of the temperature lift.

COP_2 is instead referred to the cooling mode, therefore is determined by dividing the cooling capacity of the heat pump by the electrical energy input:

$$\mathrm{COP_2} = \frac{\dot{Q}_{EVA}}{W} \quad (1.11)$$

Remembering that theoretically the condenser heat flow is given by the sum of the evaporator heat flow and the electrical energy input, these two parameters can be related in the following way:

$$\mathrm{COP_1} = 1 + \mathrm{COP_2} \quad (1.12)$$

In relation to these coefficients is possible to define two other important parameters: the heating seasonal performance factor (HSPF) and the seasonal energy efficiency ratio (SEER), which measure, respectively, the entire heating efficiency of the heat pump during the heating period and the cooling efficiency during the cooling period. They take into account the variable heating and/or cooling demands, the variable heat source and sink temperatures over the year. The performance of a heat pump is affected by a large number of factors such as:

- the climate, which affects the annual heating and cooling demand and determines the maximum peak loads;
- the temperatures of the heat source and of the sink;
- the auxiliary energy consumption (pumps, fans, etc.);
- the technical standard of the heat pump;
- the sizing of the heat pump in relation to the heat demand and the operating characteristics of the heat pump;
- the heat pump control system.

1.4 Why heat pumps

Heat pumps offer the most energy-efficient way to provide heating and cooling in many applications, as they can use renewable heat sources such as air, ground and water, at temperatures that are usually considered low. Similarly, heat pumps can also use waste heat sources, such as from industrial processes, cooling equipment or ventilation air extracted from buildings. By applying a little more energy, a heat pump can raise the temperature of this heat to the level needed. For example, with 1 kWh of electric energy input, a typical heat pump can recover 2 kWh of low temperature renewable energy from the environment and give back 3 kWh of high temperature heat energy as output. Electrically-driven heat pumps for heating buildings typically supply 100 kWh of heat with just 20-40 kWh of electricity. Many industrial heat pumps can achieve even higher performance, and supply the same amount of heat with only 3-10 kWh of electricity.
In order to make a comparison between a heat pump and a conventional heating system, the valuable electrical energy input of a heat pump must be converted into primary energy through the average fossil fuel to electrical energy conversion factor (0,4). Therefore, a typical heat pump for 100 kWh heat output needs 30 kWh electricity input, which corresponds to 75 kWh of primary energy. So as to have the same 100 kWh output, a modern condensing boiling, assuming a 100% efficiency, needs 100 kWh primary energy input. This means that the heat pump system allows to save 25 kWh of primary energy and thus it is more convenient than a conventional one. To be more accurate, a heat pump with a COP higher than 2,5 is sufficient to ensure a primary energy saving.

Because heat pumps consume less primary energy they are an important technology for reducing gas emissions that harm the environment, such as carbon dioxide (CO_2), sulphur dioxide (SO_2) and nitrogen oxides (NO_x). However, the overall environmental impact of electric heat pumps depends very much on how the electricity is produced. Heat pumps driven by electricity from, for instance, hydropower or renewable energy reduce emissions more significantly than if the electricity is generated by coal, oil or gas-fired power plants.

1.5 Working fluids

It is known that the most suitable working fluids for heat pumps vapour compression cycles are chlorofluorocarbons such as R-11, R-12, R-114, R-500 and R-502. However, due to their chlorine content and chemical stability, they have both a high ozone depletion potential (ODP) and a global warming potential (GWP). Therefore, the use of CFCs in new plants is now banned although they are still permitted in existing plants. Even today, alternative working fluids with the same performances of traditional CFCs have not been found yet. The characteristics of an ideal working fluid are the following:

- non-toxic;
- non-flammable;
- usable at low pressures;
- with a high vapour density;
- easily transportable and recyclable;
- economic;
- compatible to normal lubricants and common construction materials;
- compatible with the environment (ozone depletion or global warming).

Moreover, an ideal working fluid should have the following thermodynamic properties:

- high volumetric specific heat (to reduce the flow);
- high thermal conductivity and high phase transition enthalpy (to optimize the heat exchange);
- wide range of acceptable temperatures;
- suitable relationship between pressure and temperature.

In addition to finding new working fluids, it is also important to modify or redesign the heat pumps. Generally speaking, the energy efficiency of a heat pump system depends more on the heat pump and system design

than on the working fluid.
HCFC-22 has been the first alternative to CFCs. Hydrochlorofluorocarbons have much lower ODP and GWP than CFCs, but the hydrogen only partially substitutes the chlorine present in the molecule, so they are considered transitional refrigerants. HCFCs also include R-123, R-401, R-402, R-403, R-408 and R-409. HCFCs should be phased out for industrialized countries by the year 2020. However, some countries in Europe such as Sweden and Germany have already banned the use of R-22 in new systems.
HCFCs elimination is in favor of hydrofluorocarbons, which do not contain chlorine and have no impact on the ozone. However, they do still contribute to global warming and may present flammability characteristics (Figure 7). Special attention must be given to the use of lubricants, because these refrigerants are not miscible with mineral oils. Mineral oils come from the distillation of petroleum and they can be classified in paraffinic oils (based on n-alkanes), naphthenic oils (based on cycloalkanes) and aromatic oils (based on aromatic hydrocarbons). Normally only ester-based lubricant oils should be used. HFCs are considered long-term alternative refrigerants and include R-134a, R-152a, R-32 and R-125.
R-134a is quite similar to R-12 in thermophysical properties. The coefficient of performance is generally comparable to R-22 one, except at low evaporating temperatures and/or high temperature lifts.
R-152a has been successfully applied in a number of small heat pumps and domestic refrigerators. Because of its flammability, it should only be used as pure working fluid in small systems with low fluid charge.
R-32 and R-125 are mainly applied as components in non-flammable ternary mixtures replacing R-502 and R-22. R-32 has a GWP close to zero instead R-125 has a GWP about three times as high as that of R-134a.

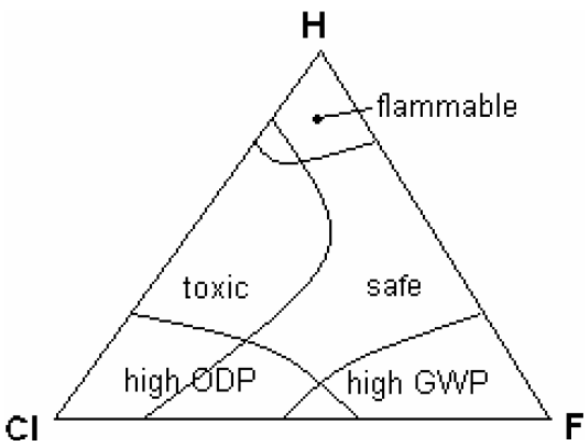

Figure 7 Effect of the fluid composition over its properties.

Blends represent an important possibility for replacement of CFCs. They are a mixture of two or more pure working fluids and can be zeotropic, azeotropic or near-azeotropic. Azeotropic mixtures evaporate and condense at a constant temperature, the others over a certain temperature range (temperature glide, Figure 8). The temperature glide can be utilized to enhance performance, but this requires equipment modification.
The advantage of blends is that they can be custom-made to fit particular needs. They can be used to:

- modify the compressor input vapour density;
- modify the compressor output vapour temperature;
- obtain a certain grade of compatibility with a lubricant oil;
- produce a non-flammable working fluid.

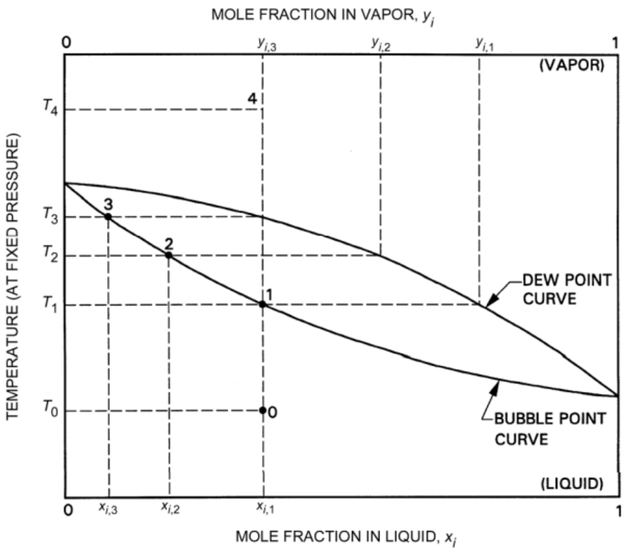

Figure 8 Phase transition chart for a zeotropic mixture.

Early blends for replacement of R-12 all contained R-22 and are therefore considered as transitional or medium-term working fluids.
The new generation of blends are chlorine-free, and are mainly be made from HFCs and hydrocarbons (e.g. propane). Two of the most promising alternative working fluids for eventually replacing R-22 in heat pumping applications are the blends R-407c and R-410a.
R-407c is a non-flammable mixture of R-134a, R-125 and R-32 (Figure 9). It is a zeotropic blend with a 7,2°C temperature glide. Due to its zeotropic characteristic, a leak in the system may cause a variation in the refrigerant composition. The properties and operating conditions of R-407c are close to those of R-22 even if its efficiency is a bit worse. Its working pressure is similar to that of R-22, therefore it implies a limited revision of the system.

On the other hand its heat exchange coefficient is lower and it requires the use of polyolester (POE) oil.

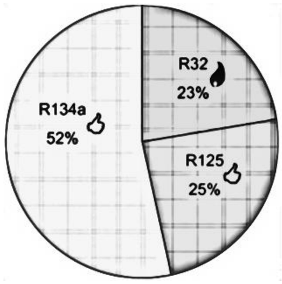

Figure 9 R-407c composition.

R-410a is a non-flammable mixture of R-32 and R-125 (Figure 10). It is a near-azeotropic blend so that a leak in the system does not cause a variation in the refrigerant composition. Research has shown that the use of R-410a can result in an improved COP compared to R-22. Using R-410a means that overall cost reductions can be achieved, because the system components, particularly the compressor, can be significantly downsized since it has a higher volumetric capacity. The main disadvantage is the higher operating pressure compared to R-22, which indicates that the pressure-proof design of most components should be reviewed. R-410a is very popular, mainly in the US and Japan, for packaged heat pumps and air-conditioning units. Commercial R-410a components for small- and medium-sized refrigeration systems are either already available or under development.

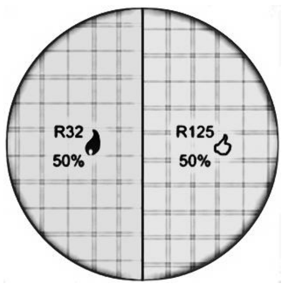

Figure 10 R-410a composition.

In alternative to artificial refrigerants, natural working fluids such as water, ammonia, hydrocarbons and CO_2 can be used.
Water (R-718) is obviously neither flammable nor toxic and it is an excellent working fluid for high-temperature industrial heat pumps. Typical temperatures are in the range from 80°C to 150°C even if 300°C has been achieved in a test plant in Japan. The main problem of water is its low volumetric capacity which implies large and expensive compressors.

Ammonia (R-717) is considered one of the most efficient natural refrigerants. It has no impact on the ozone neither on the global warming but it is very toxic and quite flammable. Therefore, regulations and legislation have been developed to ensure adequate security considerations. Ammonia is in many countries the leading working fluid in medium- and large refrigeration and cold storage plants. The two main advantages of ammonia are its high heat exchange coefficient and critical temperature. Therefore heat exchangers can be rather smaller and the system can work with high efficiency at higher temperatures. However it is not yet used in high-temperature industrial heat pumps because there are currently no suitable high-pressure compressors available. The main problem of ammonia is its non-compatibility with copper and its alloys, so that the piping and the heat exchangers must be made of steel. Ammonia can also be considered in small systems, the largest part of the heat pump market. In small systems the safety aspects can be handled by using equipment with low working fluid charge.

Hydrocarbons such as propane (R-290), butane (R-600), and propylene (R-1270) are well known flammable working fluids with favorable thermodynamic properties and material compatibility. HCs are widely used in the petroleum industry, sporadically applied in transport refrigeration, domestic refrigerators/freezers and residential heat pumps (notably in Europe). Due to the high flammability, hydrocarbons should only be applied in systems with low working fluid charge and particular safety measures.

CO_2 (R-744) is a potentially strong refrigerant that is attracting growing attention from all over the world. CO_2 is non-toxic, non-flammable and is compatible to normal lubricants and common construction materials. The volumetric refrigeration capacity is high and the pressure ratio is greatly reduced. However, CO_2 heat pumps work on the principle of the transcritical cycle when the upper sink temperature is over 20-22°. In these cases the condenser must be replaced by a gas cooler. The efficiency of CO_2 heat pump systems is rather poor and effective application of this fluid depends on the development of suitable methods to achieve competitively low power consumption during operation near and above the critical point.

1.6 Heat pumps types

1.6.1 Air-Source Heat Pumps

As mentioned before heat pump cycle is fully reversible, so heat pumps can provide year round climate control: heating during the winter and cooling and dehumidifying during the summer. Since the ground and air outside always contain some heat, a heat pump can supply heating to a

building even on cold winter days. In fact air at -18 °C contains 87% of the heat it contains at 21°C (the heat variation can be calculated as $\dot{m} c_p \Delta T$; the percentage variation is given by $\Delta T/T$, where the temperatures are in Kelvin degrees: in this case we have $(294 - 255)/294 * 100 = 13\%$). In relation to this statement it is possible to exploit outside air even in winter time through air-source heat pumps.
If the outdoor temperature falls to near or below freezing when the heat pump is operating in the heating mode, moisture in the air passing over the outdoor coil will condense and freeze on it. The built up frost decreases the efficiency of the coil by reducing its ability to transfer heat to the working fluid. Nowadays, through a particular defrosting mode, it is possible to prevent this kind of trouble. The presence of this kind of system is a fundamental parameter to evaluate both technical and economical performances of heat pumps. Since air-source heat pumps are not difficult to be realized, they are probably the best solution in the next future.
There are two different types of air-source heat pumps: air to water systems and air to air systems. The first of them are called air to water because they are used in buildings with heated water distribution system. It means that the working fluid, for example in the heating period, flows from the condenser through the heating elements network. On the other hand, in air to air heat pumps the refrigerant is in direct contact with air both on evaporator and condenser side. They extract heat from outside air and then transfer it either to the inside or outside from the building, depending on the season, through a forced air system. In Figure 11 is represented the heating cycle for a typical air-air heat pump. It draws heat from the outside and releases it in the indoor air thanks to a forced air distribution system.

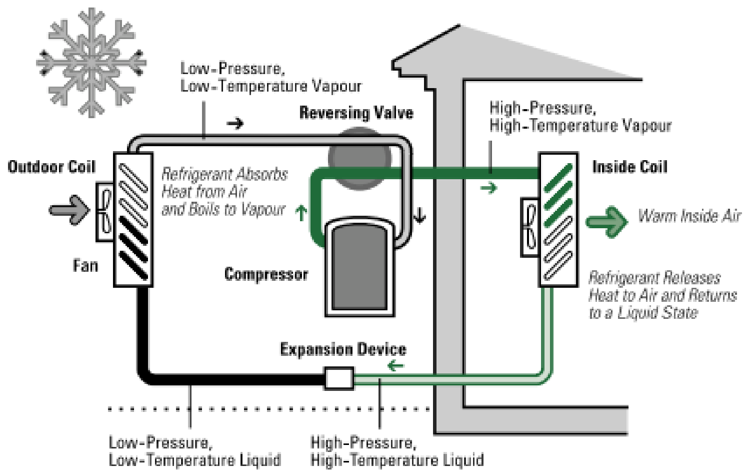

Figure 11 Air to air heat pump, heating cycle.

However, during the summer season, the cycle is reversed whereby the unit extracts the heat from indoor air and rejects it outdoor (Figure 12).

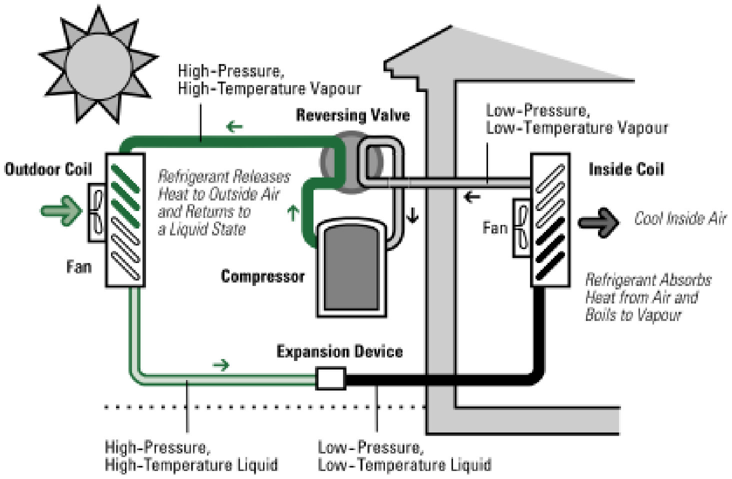

Figure 12 Air to air heat pump, cooling cycle.

1.6.2 Ground-Source Heat Pumps

Considering that a large amount of solar energy is absorbed by earth, an interesting option is to use this large quantity of energy both to heat and cool a building. Main advantages are that earth energy is available on-site so it does not need to be transported over long distances and that it is available in massive amounts. The ground has a high heat storage capacity therefore temperature changes very slowly, on order of months or even years. As shown in Figure 13, while air temperature trend is really changeable, earth and groundwater temperature trend is quite constant along the year. Because of this low thermal conductivity, heating absorbed during the summer can be used during the winter.

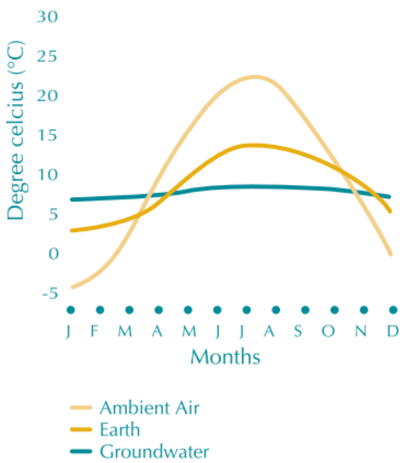

Figure 13 Typical average monthly temperatures for air, earth and groundwater.

The ground is a great insulator: the mean soil temperature varies depending on depth, but just few meters of ground are enough to keep the heart temperature almost constant during the year (Figure 14).

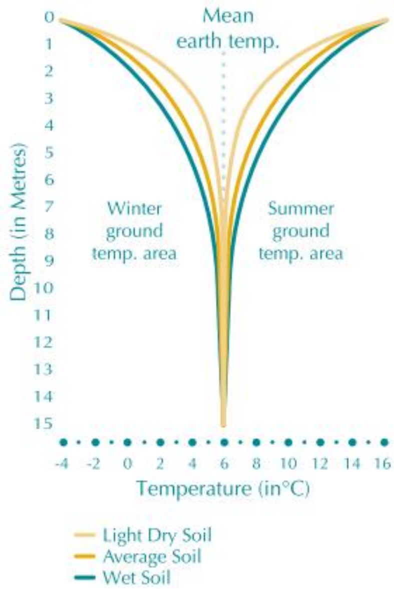

Figure 14 Typical ground temperature variation in function of depth.

A ground-source heat pump uses warm earth or groundwater, or both, as the source of heat in the winter and as the sink for heat removed from the home in summer. For this reason ground-source heat pump systems have come to be known as earth-energy system (EESs).There are several kinds of ground-source heat pumps.

Ground water Ground water is available with stable temperatures in many regions. Open or closed systems can be utilized. In open systems ground water is pumped up, cooled, trough the working fluid, and then rejected in a separate well or returned to surface water. Open systems should be carefully designed to avoid problems such as freezing, corrosion and fouling. Closed systems are direct expansion systems with the working fluid evaporating in underground heat exchanger pipes. The major disadvantage of ground water heat pumps is the installation cost. Additionally, local regulations may impose severe constraints regarding interference with the water table and the possibility of soil pollution.

Ground source systems Ground systems have similar advantages as water-source systems, i.e. they have relatively high annual temperatures because of the high heat store capacity of the ground and because of its thermal insulation. In ground source systems heat is extracted from pipes laid horizontally or vertically in the soil. There are two main different

ground source heat pumps types: direct exchange systems and closed loop systems. The direct exchange geothermal heat pump is the simplest and easiest to understand. The ground-coupling is achieved through a single loop circulating refrigerant in direct thermal contact with the ground. The name "direct exchange" refers to heat transfer between the refrigerant and the ground without the use of an intermediate fluid. Direct exchange systems are significantly more efficient and have potentially lower installation costs (depending on copper price) than closed loop water systems. The main reasons for the higher efficiency are the elimination of the water circuit pump (which consumes electric energy) and the elimination of the intermediate water-refrigerant heat exchanger (which implies a degradation of heat quality). The copper loop must be protected from corrosion in acidic soil through the use of an anodic or cathodic protection. However, in some European countries direct expansion systems are not allowed because of their strongly negative environmental impact. Moreover, annual system check-ups might be required in order to ensure that there are no leakages in the ground loop.
On the other side, closed loop systems use a mixture of anti-freeze (propylene glycol, denatured alcohol or methanol) and water and present two loops on the ground side: the primary refrigerant loop is contained in the appliance cabinet where it exchanges heat with a secondary water loop that is buried underground. The primary loop is made by copper while the secondary one is typically made of high-density polyethylene. The secondary loop is placed below the frost line where the temperature is more stable or preferably submerged in a body of water if available. Systems in wet ground or in water are generally more efficient than dry ground loops since less work is required to transfer heat in and out of water than solids in sand or soil. Closed loop systems need a heat exchanger between the refrigerant loop and the water loop, and pumps in both loops. Closed loop systems have lower efficiency than direct exchange systems, so they require longer and larger pipe to be placed in the ground, increasing excavation costs. Closed loop tubing can be installed horizontally as a loop field in trenches or vertically as a series of long U-shapes in wells.

Rock (geothermal heat) Underground rocks can be used in regions with no or negligible occurrence of ground water. Typical bore hole depth ranges from 100 to 200 meters. This type of heat pump is always connected to a brine system with welded plastic pipes extracting heat from the rock. Because of the relatively high cost of the drilling operation, rock is seldom economically attractive for domestic use.

Some ground source heat pumps exploit lake, sea and river water or even waste water and effluent, but they are used especially for large heat pump

system. This kind of heat pumps can be used both with a forced air and a water heating system.
Ground source heat pumps permit to provide a passive cooling. According to the second law of thermodynamics heat flows spontaneously from hotter to colder matter and as stated above ground has a high heat storage capacity as well as is a good thermal insulator. Therefore its temperature during the year is more or less constant around 5°C. During the summer season occurs that the ground’s temperature is lower than the indoor temperature, so that heat can flow in the desire direction by itself. However the heat pump still operates in order to ensure that the rate of heat flow is adequate. This rate is related to the temperature difference between the heat pump and the earth connection: during cooling, the higher the temperature of the building, the better the rate of transfer with the earth connection would be.
As said for air source heat pumps, the technical and economic performance of a heat pump is closely related to the characteristics of the heat source. An ideal heat source is abundantly available, is not corrosive or polluted, has favourable thermophysical properties, and its utilisation requires low investment and operational costs.

1.7 Components review

Heat pumps are relatively complicated technical devices which consist of a number of construction parts. As with another complicated device, inspection and reliability of the items are important because the only one item, though not important from the functional point of view, may shut down the whole system. The main construction items of a heat pump are the following:

- a compressor;
- a condenser;
- an expansion valve;
- an evaporator.

1.7.1 Compressors

The compressor is the most important and expensive part of the heat pump. Its task is to suck saturated or superheated vapour at low pressure and to discharge it at higher pressure and temperature. Compressors are often described as either open, hermetic, or semi-hermetic in order to describe how the compressor and motor drive interacts with the compressed vapour. In hermetic and most semi-hermetic compressors, the compressor and its related motor are integrated, and operate within the system. The motor is designed to be cooled by the compressed

vapour. The cooling is effective but the obvious disadvantage of hermetic compressors is that the vapour temperature at compressor inlet increase therefore working compression increases and the efficiency of the system declines. Moreover motor drive cannot be maintained in situ, and the entire compressor must be removed if a motor fails. A further disadvantage is that burnt out windings can contaminate associated subsystems requiring the system to be entirely pumped down and the gas replaced. On the other hand, an open compressor has a motor which is outside of the system and provides drive to the compressor, by means of an input shaft with suitable gland seals. Open compressors driven by an input shaft can be powered by various sources such as an electric motor, or internal combustion engine or turbine, or any other engine.
Apart from this classification, compressors can be also divided in relation to their operating principle, as shown in Figure 15.

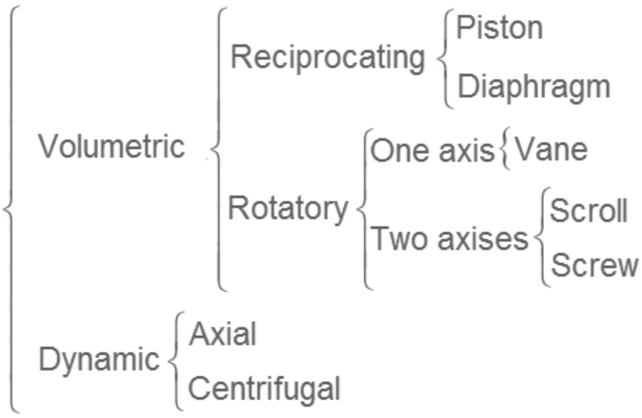

Figure 15 Different kinds of compressors.

Reciprocating compressor It is part of the volumetric compression group, it means that the pressure increase of the vapour is given through the reduction of the volume occupied by the gas. A reciprocating compressor is formed by these main components: the piston, the crankshaft, the cylinder and the discharge and suction valves (Figure 16). The piston is driven by a crankshaft in a alternative action inside the cylinder (working chamber) reducing the volume of the vapour inside it in order to increase the gas pressure from the intake value to the discharge one. The compression cycle is constituted from four stages which are compression, discharge, expansion and suction (Figure 17).

At the conclusion of the previous cycle, the piston is fully retreated within the cylinder in the position 2 and the cylinder is filled with vapour at the suction condition. The suction and discharge valves are all closed. As the piston advances, the volume within the cylinder decreases. This causes the vapour pressure and temperature to increase until the pressure reaches the discharge value. This part of the cycle is called compression (from 2 to 3). The compression ends at the top dead centre because between the head of the piston and the cylinder must be a vacant volume, known as clearance volume, in order to prevent the contact between the head of the piston and the cylinder. Afterwards, in the discharge stage (from 3 to 4), the discharge valve is automatically opened allowing the compressed vapour to go to the receiver.

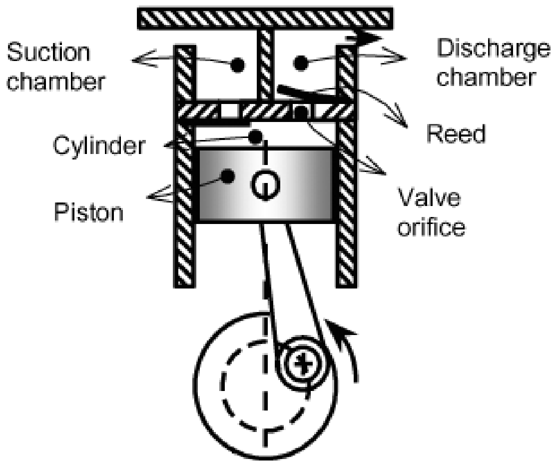

Figure 16 Main components of a reciprocating compressor.

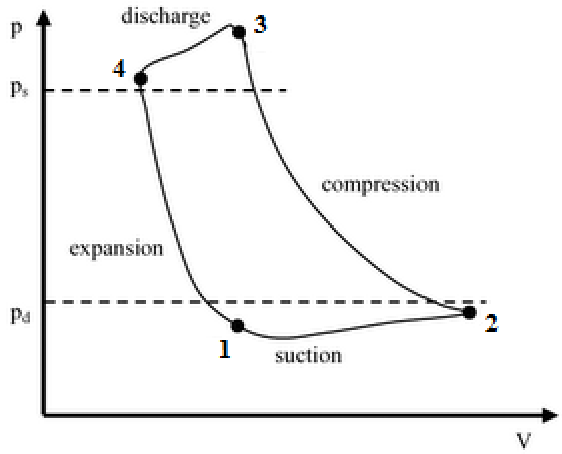

Figure 17 Pressure - volume diagram of a compression cycle.

From the top dead centre position the piston starts moving towards the downward direction. The next step of the cycle is the expansion (from 4 to

1) where the compressed vapour in the clearance volume is expanded until a pressure a bit lower than the suction one that permit the intake valve to open. Finally there is the last step called suction (from 1 to 2) where the refrigerant from the suction pipeline is taken inside the cylinder of the compressor through the suction valve. As the piston moves downwards, the amount of the refrigerant taken inside the cylinder increases. When the piston reaches the bottom dead centre position the maximum amount of the refrigerant is sucked by the compressor.

For these kind of compressors the volumetric efficiency is function of different parameters as pressure ratio, pressure losses in suction and discharge, type of refrigerant used and clearance volume.

Rotatory scroll compressor They use two interleaved spiral-like vanes to compress vapour. They present different kind of geometry and they operate more smoothly, quietly, and reliably than other types of compressors in the lower volume range. As shown in Figure 18, one of the scrolls is fixed, while the other circulates eccentrically without rotating, thereby trapping and compressing pockets of fluid or gas between the scrolls.

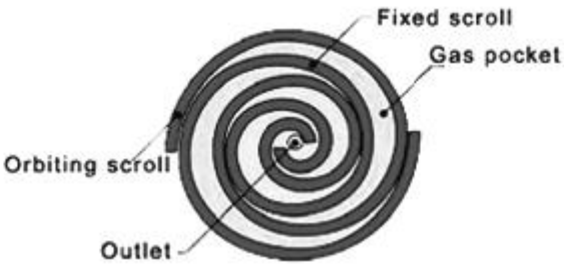

Figure 18 Rotatory scroll compressor.

Scroll compressors are characterized by:

- high volumetric efficiency;
- high isentropic efficiency due to limited movements between of spirals;
- absence of suction and delivery valves;
- low mechanic frictions.

Scroll compressors are oiled by a centrifugal pump located at the bottom of the component. The oil pressure and flow vary in relation to the rpm. A shortcoming of this type of compressors is the possibility that the lubricating oil does not reach all moving parts at low rpm.
Their isentropic and volumetric efficiency decrease when the pressure ratio increases as shown in Figure 19.

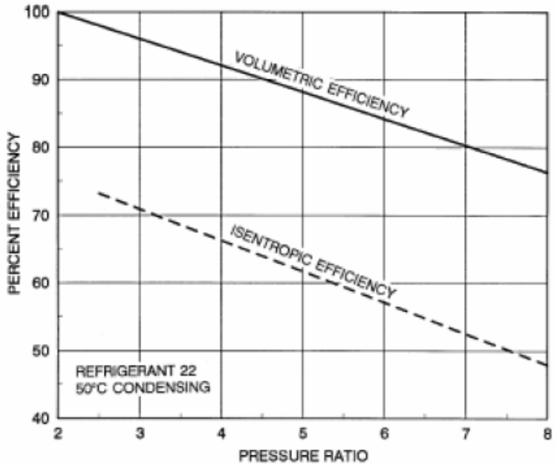

Figure 19 Isentropic and volumetric efficiency of scroll compressors.

Rotatory vane compressor It consists of a rotor with a certain number of blades. This one is mounted offset in a larger housing which can be circular or a more complex shape. As the rotor turns, so do the blades keeping contact with the wall of the cylinder. Vapour enters at the largest opening and exits at the smallest, reducing its volume and increasing its pressure (Figure 20).

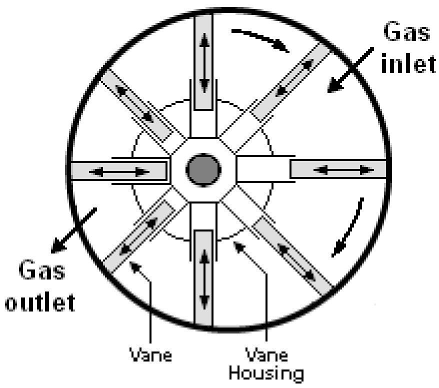

Figure 20 Rotatory vane compressor.

They are suitable for relatively low pressure systems and they are characterized by high isentropic and volumetric efficiency and by an easy lubrication. They are usually managed by an inverter.

Rotatory screw compressor It uses two meshed rotating helical screws to force the gas into a smaller space. As the screws rotate, gas is drawn into the inlet port and fills up the space between the screws. The volume available between the compressor body and these two threads is then progressively decreased during rotation. Finally the compressed volume is ejected from the delivery outlet (Figure 21).

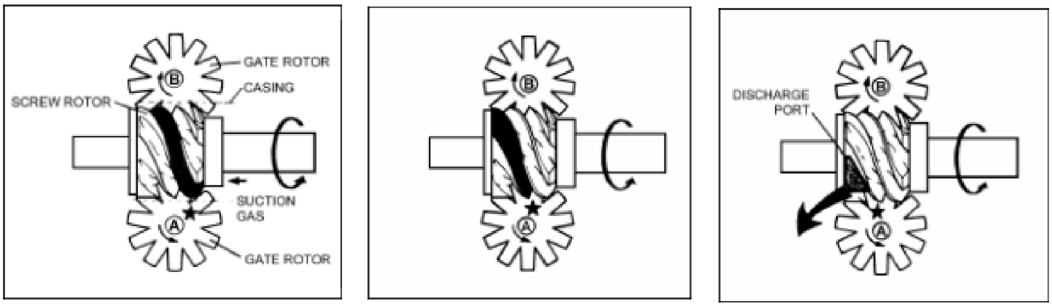

Figure 21 Rotatory screw compressor.

Its application can be from 2 kW to over 890 kW and from low pressure to moderately high pressure. It means that it is suitable for lots of applications because of its wide operational range. In summary it is characterized by:

- high volumetric efficiency;
- non pulsating gas flow;
- high fluid dynamic frictions;
- managed by an inverter or by a slide valve;
- suitable for high pressure and power.

Dynamic centrifugal compressor It is a kind of dynamic compressor which means that it does not increase the pressure through the reduction of the gas volume, but it creates a pressure drop through the interaction between the gas and a rotating disk. Thus centrifugal compressor uses a rotating impeller in a shaped housing to force the gas to the rim of the impeller, increasing the velocity of the gas. A diffuser section converts the velocity energy to pressure energy. Because of they don't reduce the volume of the gas it is unsuitable to speak about volumetric efficiency for these kind of compressors. It is important to avoid the possibility of occurring events such as surge or chocking since they can significantly degrade the performance. They can be managed in different ways as through an inverter, through the variation of rpm or regulating the blades angle in the most suitable way.

Dynamic axial compressor It consists of rotating airfoils (rotors) which rotate between a similar number of stationary airfoil rows (stators) attached to a stationary tubular casing. Rotors impart energy into the fluid, and the stators convert the increased rotational kinetic energy into increased static pressure at the delivery valve. A pair of rotating and stationary airfoils is called a stage. The cross-sectional area between rotor drum and casing is reduced in the flow direction to maintain axial velocity as the fluid is compressed. They are relatively expensive, requiring a large number of components, tight tolerances and high quality materials.

At this point it is convenient to relate compressor presented above to the main causes of efficiency losses. In fact actual compressors performances, as known, deviate from ideal performances because of various losses, with a resulting decrease in capacity and increase in power input. Compressor efficiency losses are caused mainly by the following factors.

Mechanical frictions Usually they are not considered as a major cause of degrade efficiency. They are caused by the contact of the parts in movement of the compressor. In relation of the compressor model mechanical frictions can be more or less significant. For example the reciprocating compressor, from this point of view, is disadvantaged because it has large parts in movement which create not negligible mechanic resistances. Also the screw compressor presents lots of mechanic frictions caused by the lubricating oil used to seal up escape ways between the two screws. It is also important to highlight that the loss of mechanical efficiency per mass unit is almost constant, but the ideal work supplied to vapour decreases with a decreasing pressure rate. Therefore mechanical frictions are more incisive with low pressure rates.

Fluid frictions Fluid friction normally occurs in reciprocating compressors especially in the suction and discharge plenum due to the increased pressure drop caused by the valves. This problem is reduced in rotary compressor since no valves are involved.

Heat transfer It could happens that hot and cold vapour be in direct or indirect contact during the compression phase. This happening reduces the vapour density at the compressor inlet, therefore volumetric efficiency and energy efficiency decrease. This kind of efficiency loss depends on the compressor model. In the reciprocating compressor this heat transfer does not develop directly between the two different temperature fluids but thanks to the compressor metallic parts, as the cylinder chamber, which on one hand release heat to the sucked vapour during the suction phase and the at beginning of the compression phase, while on the other hand they receive heat from hot vapour at the end of the compression and at the discharge. Therefore the inlet vapour density decreases reducing volumetric and energy efficiency. Screw compressor presents a similar situation because its screws are comparable to the cylinder of the reciprocating compressor, so it receives and releases heat from and to the vapour. Anyway screw compressor take advantages from the refrigerant action given by the lubricating oil, although friction increases cause of the oil action. However rotary compressors, from this point of view, are preferred because, thanks to their geometry, their metallic surfaces are in contact with a constant temperature fluid.

Leakages The working chamber is not completely sealed against suction and delivery side, even at fully closed valves. Losses due to leakages increase with the pressure drop between the suction and the discharge. They are typical in rotatory compressors where they can be reduced introducing additional chambers at an intermediate pressure compared to neighbouring chambers pressures. They are not significant in reciprocating compressors. Leakages of the working chambers cause a reduction in mass flow and thereby to an increase of the specific work (leakage losses).

Re-expansion It occurs when after the compression an amount of compressed vapour re-expands during the suction phase and in this way it limits the quantity of new vapour that can be carried inside the compressor chamber. This kind of problem is typical for reciprocating compressors.

Compression excess/lack The compression phase presents a typical pressure value called ejection value and it is the most suitable value to discharge the vapour. If this value is not reached there is a compression excess or a compression lack. A compression excess appears when pressure inside the compression chamber reaches the ejection value before getting to the discharge point. Instead, a compression lack takes place when the vapour does not attain the right pressure value at the end of compression phase. It is typical for rotatory compressors.

1.7.2 Heat exchangers

Around 95% of the heat pumps installed in Italy use air as cold source. In particular, around 85% of them are air to air systems. The most common configuration of these systems implies the use of a mini-split air conditioning unit. In a split system, the condenser coil is located inside the building, while the compressor and the evaporator coil are located in a separate external package with the two units connected via two refrigerant pipes. If multiple rooms need to be cooled, several indoor components can be installed to one outdoor component. During the summer season, the heat pump turns into a cooling unit; the indoor coil becomes the evaporator, while the outdoor coil becomes the condenser. The outdoor evaporator coil is made of copper internally finned tubes, with the refrigerant flowing inside. The air flow is forced by an electrical fan. Aluminium fins with a complex design can be utilized to improve the heat transfer between the air and the refrigerant (Figure 22). Similarly, indoor condenser unit is made of copper finned tubes in which the hot refrigerant condenses and warms the air flow passing over. The indoor unit is responsible for the air handling inside the room. In order to adapt the system to any necessity, several types of indoor units are available in the market, such as wall, ceiling and false ceiling units.

In larger direct expansion units, the principle remains the same but one important consideration must be made. Upon exiting from the expansion valve, the refrigerant stream is mostly liquid, but the vapour occupies more space. This causes the two fluids to travel at different rates and separate, with the gas at the top and the liquid at the bottom of the stream. For this reason evaporator coils generally come with distributors, to avoid that some coil circuits receive mostly the gas, and other circuits, the liquid. This would degrade the heat transfer capabilities of the evaporator coil.

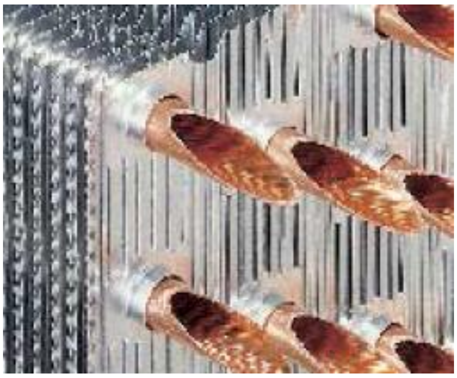

Figure 22 Direct expansion heat exchanger.

When water has to be managed, the characteristics of the heat exchangers are completely different. Shell and tube, spiral or plate heat exchangers must be used in relation to radiant or fan coil heating systems and water-source heat pumps.

The shell and tube heat exchanger (Figure 23) is one of the most common type of heat exchanger for several different applications. As its name implies, this type of heat exchanger consists of a shell with a bundle of tubes inside it. One fluid runs through the tubes, and another fluid flows over the tubes (through the shell) to transfer heat between the two fluids. Shell and tube heat exchanger can be either dry type or flooded type. In the dry type the refrigerant flows through the tubes while in the flooded type the refrigerant is in the shell. Tubes are the main component for this kind of heat exchanger providing the heat transfer surface between the fluid running inside the tubes and the fluid flowing through the shell. Tubes may be seamless or welded and most commonly made by copper or steel alloys. Their number can be less than 50 to several thousands and the length varies in a wide range. The set of tubes is called a tube bundle, and may be composed by several types of tubes with different geometry (plain, longitudinally finned, etc) in order to increase the heat transfer between the two fluids. The shell is simply the container of the shell-fluid and normally it has a circular cross section and commonly is made by rolling a metal plate. This kind of heat exchanger can be single pass or multipass.

In multipass type the liquid changes direction in the heads of the shell.

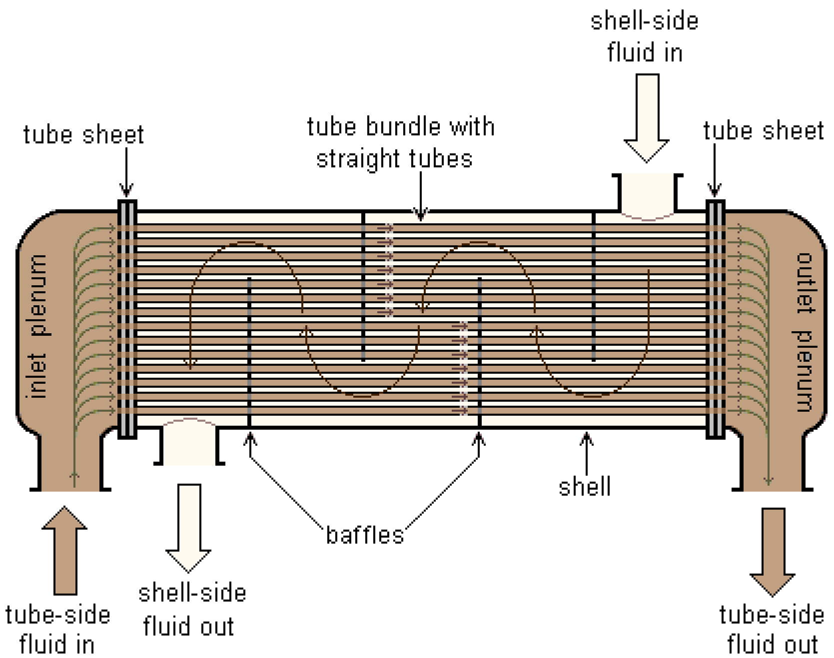

Figure 23 Shell and tube heat exchanger.

The concept of a spiral heat exchanger (SHE) (Figure 24) is as simple as it is sophisticated. Two long metal strips, onto which spacer studs are welded, are wound around a core, thus creating two equally spaced single passage channels. The refrigerant passes through one channel while water flows in the other channel. Heat transfer takes place through the channel walls. The concentric shape of the flow-passages and the studs yield turbulence already at low Reynolds numbers. By optimizing the flow pattern heat transfer is enhanced, whilst fouling is reduced. This yields a compact and space saving construction that can be readily integrated in any system. Because of the all welded and robust design and the low fouling properties, maintenance costs are reduced to a minimum.

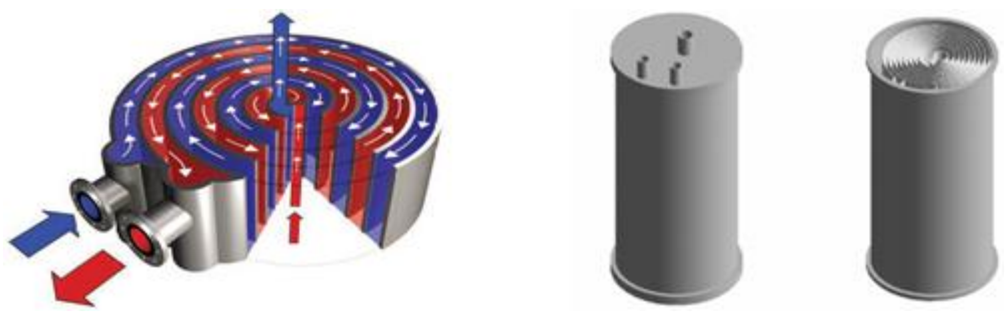

Figure 24 Spiral heat exchanger (SHE).

Plate heat exchangers (PHE) (Figure 25) are composed of multiple, thin, slightly-separated plates that have very large surface areas and narrow fluid flow passages. Advances in gasket and brazing technology have made the plate-type heat exchanger increasingly practical. Stainless steel and titanium are two commonly used metals for the plates because of their ability to withstand high temperatures, their strength, and their corrosion resistance. Similarly to the spiral heat exchangers, these units are generally very compact due to their high heat transfer efficiency associated with the narrow passages and the corrugated surfaces. Moreover, plate heat exchangers are very appreciated thanks to their modularity characteristic and their ease of cleaning. However, they are very exposed to fouling and the pressure drop that occurs through a plate heat exchanger is relatively high and sometimes spiral heat exchangers are preferred.

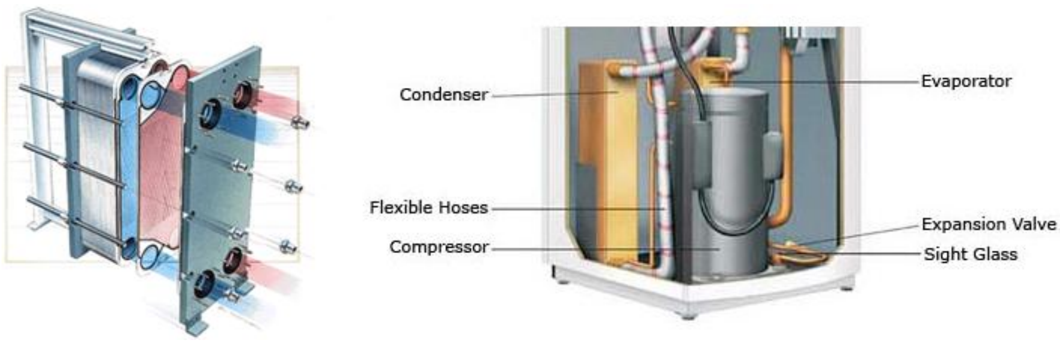

Figure 25 Plate heat exchanger (PHE).

1.7.3 Expansion valves

The expansion valve is the pressure reducing device. When the high pressure refrigerant enters the expansion valve, after the condensation phase, its pressure reduces suddenly and along with it its temperature also becomes very low. In heat pumps application there are different designs for expansion devices.

Capillary tube expansion valve These kind of valves are characterized by tubes with variable length (between 1 and 6 meters) and by diameters in the range of 5 to 2 mm. This narrow diameter creates a constant throttle on the working fluid which decreases its pressure, and its temperature, from the condensation pressure to evaporation pressure. Capillary tube expansion valve is a static device and it is very reliable, however it does not have adjustment capacity and it can be used just for small system.

Manual controlled expansion valve This is a very simple device. It provides a constant pressure loss and it is possible to set the wanted pressure value. They cannot adjust themselves in relation to the thermal load, therefore their main application is to provide protection to components, as compressor, and act as a safety devices.

Thermostatic expansion valve It controls the amount of refrigerant flow into the evaporator thereby controlling the superheat at the outlet of the evaporator. This is accomplished by use of a temperature sensing bulb filled with a similar gas as in the system that causes the valve to close against the spring pressure in the valve body as the temperature on the bulb increases. As temperature in the evaporator decreases, so the pressure in the bulb does, hence causing the valve to open. The logical sequence is the following: the temperature sensor senses the evaporator temperature and changes the pressure on the top of valve diaphragm. From the bottom there are two forces applied to the diaphragm, P2 is the evaporator pressure which acts in a closing direction below the diaphragm and P3 is the pressure from the valve spring; they both cause the valve to close. The result force is related, to the refrigerant superheat temperature. The scheme of this kind of valve is shown in Figure 26.

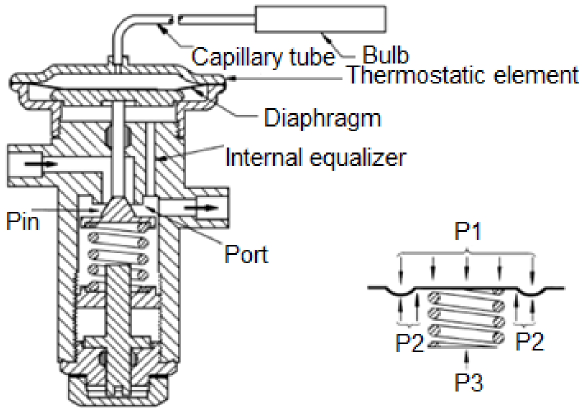

Figure 26 Thermostatic expansion valve.

Electronically controlled expansion valve (EEV) It operates with a much more sophisticated design. EEVs control the flow of refrigerant entering in the evaporator. They do this in response to signals sent to them by an electronic controller. The electronic signals sent by the controller to the EEV are usually provided by a pressure sensor and by a temperature sensor, both of them connected to the evaporator outlet. EEVs use a step motor to open and close the valve port. Step motors do not rotate continuously. They are controlled by the electronic controller and rotate a fraction of a revolution for each signal sent to them. The step motor is driven by a gear train, which positions a pin in a port in which refrigerant flows (Figure 27). Because of their sophisticated design, EEVs can operate in a wide operating range and they have no limits on the condensation pressure.

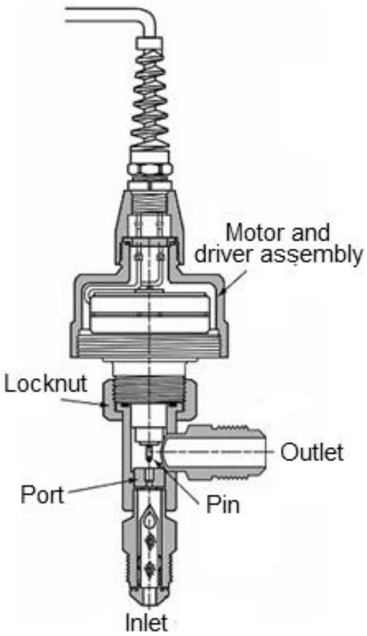

Figure 27 Electronically controlled expansion valve.

1.7.4 Accessory components

Finally it is helpful to present some accessory components which permit to improve and safeguard the functioning of the heat pump.

Reversing valve this component change the direction of refrigerant flow. In this way the heat pumps is able to change its cycle from heating to cooling or vice versa. This allows a residence to be heated by a single system, so by the same equipment and the same hardware. Reversing valves can be driven directly by the heat pump through a control board or a by a thermostat.

Filters In a heat pump system are used different kind of filters in order to prevent damages to the system itself. At the condenser outlet is placed a filter with the aim to absorb the excessive moisture contained in the working fluid because moisture is dangerous for other components, as valves, and lubricating oil. Moreover, still at the condenser outlet another filter is placed. Its task is to filter the refrigerant from impurities.

Control valves They are both manual and solenoid type and they are used to permit the refrigerant to bypass pipes to allow maintenance operations. The first of them are auctioned by refrigerant pressure and in this way they avoid undesired refrigerant inversion, while the second one is usually driven by the compressor.

Accumulator It is located at the compressor inlet to adsorb residual liquid which can degrade the compressor performance.

2 Problem definition and approach

2.1 Problem definition

Building heating load varies with occupancy and climate changes so that heat pump systems experience unsteady time varying disturbances from time to time and operate under part load conditions most of the time. A heat pump system can regulate its capacity by modulating compressor speed step by step or continuously with input frequency control. Furthermore, the thermostatic expansion valve can also be adjusted to regulate the cooling capacity. Therefore, it is important to clearly identify and evaluate the dynamic performance of refrigeration systems subject to these changes so that better control strategies can be designed and system operation could be optimized.

To optimize the operation of vapour compression systems, mathematical models constitute one of the best tools both for analysis and control. Researchers are mainly using two types of strategies: statistical models based on mathematical routines that obtain the formulation of the system from experimental data, or physical-based models, which approximate the behaviour of the system by using physical relations. This latter group is the one that provides more information about the system behaviour, since all the operating variables are considered.

The heat exchangers physical equations generally utilized are derived from the basic mass, energy and momentum conservation equations. The application of few assumptions allows to neglect the momentum equation. The form of the various terms in these equations depends on the level of detail required in the final model. In terms of approaches to modelling heat exchangers presented in the literature, the most important differences are in the way the refrigerant in the heat exchangers is treated. The two predominant approaches are the moving-boundary (MB) and finite-volume (FV) methods. In the moving-boundary method, each heat exchanger is divided into control volumes that have moving boundaries that occur at the interfaces where the refrigerant transitions between single- and two-phase flow. A lumped parameter approach is considered. The mass and energy equations are integrated for each zone over the total length of the heat exchanger between arbitrary time-varying spatial limits. With the finite-volume method, each heat exchanger is divided into a number of fixed and equal-sized control volumes. A distributed parameter approach is considered. The transient conservation equations are discretized over these volumes and result in a system of ordinary differential equations.

Generally, the moving-boundary model requires significantly less computation than the finite-volume approach because it utilizes fewer control volumes. However, this approach utilizes lumped characteristics for each of the control volumes, such as a single void fraction for the entire two-phase section, which could lead to lower accuracy. The moving-boundary approach also has difficulty in handling start-up and shut- down because it employs assumptions about the location of phase regions within a condenser and evaporator that do not apply when the unit is off and heat exchangers come to equilibrium.

The type of the compressor mathematical model depends on the study objective. The steady state model is the simplest one, and can be adopted neglecting the effect of the compressor shell on system performance and assuming that compressor reaches operating speed instantaneously. Once the calculation method with the semi-empirical parameters for polytropic exponent, mass-flow rate coefficient and motor efficiency is determined, the calculation of the compressor performance becomes explicit and very fast.

The steady state compressor model is certainly suitable for a steady simulation of a heat pump system, but it can also be used to simulate the mass-flow rate of refrigerant through compressor in a dynamic simulation because the time constant of refrigerant flow rate variation is very small compared to that of the heat exchangers. However, for the start-up process of the compressor when the rotating speed of the compressor varies from 0 to its full rotation speed, the steady state model cannot well predict the mass-flow rate and power, and cannot simulate the temperature variation inside the compressor either. When using a hermetic compressor the effect of the compressor shell should not be neglected and a dynamic model should be preferred.

As well as the compressor, the approach to the expansion valve model depends on the desired level of accuracy. It is know that the operation of the expansion device plays a crucial role in refrigeration system performance: inaccurate superheat temperature prediction will make the TEV give poor regulation.

With a thermostatic expansion valve, the sensing bulb responds to the refrigerant temperature at evaporator exit and disturbs the force balance in the valve. The inertia of the sensing element is the source of the valve thermal dynamic. The mechanical dynamic is an order of magnitude faster than the thermal dynamic, and the latter is of the same scale as the overall system dynamic. The valve model can be therefore simplified to include only the thermal inertia of the sensing element. However, most of the studies present in literature model the superheat section of evaporator by neglecting the thermal capacity (i.e. assuming that, during normal operating conditions, the dynamic of the expansion valve is much faster

than that of the two heat exchangers), or by using either empirical method or a lumped parameter approach.
In an electronic expansion valve the sensitive element is a thermistor located at the evaporator outlet to maintain constant the superheated temperature. The response of the thermistor to a temperature variation is much faster than the bulb of the thermostatic expansion valve. It implies that also the thermal dynamic of the EEv is faster and can be, in general, neglected.

2.2 Literature review

The first studies on the problem started in late ‘70s. The most important works, which the modern studies are based on, are reported here.
Wedekind et al. (1978) [1] were among the earliest to study transient behaviour with their work on modelling of two-phase flow dynamics in heat exchangers. A significant achievement of this work is that the complete two-phase region can be treated in adequate detail even in a lumped form, while avoiding the necessity of handling the transient form of the momentum equation.
Dhar and Soedel (1979) [2] present one of the first models of a complete vapour compression refrigeration system. This model of a window air conditioner is based on a moving- boundary approach with the two phases of refrigerant in the heat exchangers treated as a coupled pair of lumps.
Chi and Didion’s (1982) [3] model is among the few that works with the transient form of the momentum equation. Their model of an air-to-air heat pump system is built on a moving-boundary lumped parameter formulation.
Yasuda et al. (1983) [4] built a complete system model, on lines similar to Dhar and Soedel (1979) [2], except that the condenser modelled is a shell-and-tube construction instead of air-cooled. The object of this model is to capture what are termed small transients as caused by feedback control and instabilities triggered by poor valve setting. For this narrowly defined purpose, broad assumptions such as constant sub-cooling and uniform two-phase condition in the entire condenser are found to work without serious consequence.
MacArthur (1984) [5] presents one of the earliest of models that used a fully distributed formulation. This, along with MacArthur and Grald (1987) [6] and Rasmussen et al. (1987) [7], constitutes a body of work using similar formulations for the system components. The space–time-dependent conservation equations are simplified by assuming one-dimensional flow in both heat exchangers. The two-phase region in the condenser is treated as homogenous while in the evaporator the liquid and vapour phases are modelled separately.

He et al. (1997) [8] developed a system model for a basic vapour compression refrigeration system, using the moving-boundary lumped parameter formulation with the system mean void-fraction method of Wedekind et al. (1978) [1], for the purpose of studying the effect of multivariable feedback control. This model was then used to study a multi-input–multi-output (MIMO) control method developed by He et al. (1998) [9].

During the years, several models have been developed. Nowadays, the most complex, complete and accurate are the works by Bendapudi et al. [10] and Zhang and Zhang [11].

Bendapudi et al. (2005) [10] presented model development and validation for a centrifugal chiller with flooded heat exchangers. The model employs the finite-volume approach and studies the impact of number of control volumes and time step on model accuracy and execution speed.

Zhang and Zhang (2006) [11] developed a generalized model based on the moving-boundary approach to describe the transient behaviour of dry-expansion evaporators in the vapour-compression refrigeration system. To improve the robustness of the traditional moving-boundary model under larger disturbances, the time-variant mean void fraction is employed instead of the constant.

More recently, Bendapudi et al. (2008) [12] presented a comparative study of moving-boundaries and finite-volume formulations. They found out that both models yielded comparable accuracy in predicting system-level transients and steady-state behaviour of a centrifugal chiller. The FV formulation was found to be more robust through start-up and all load-change transients. However, the moving-boundary system model executed about three times faster than the finite-volume while maintaining nearly identical accuracy in the predictions of system steady-state and transient-performance.

In this work, the following models have been mainly used.

Srinivas and Padmanabhan (2001) [13] developed a computationally efficient steady state model for refrigeration compressor gas dynamics. The compression process is modelled as a one-dimensional gas dynamics equation. Valve dynamic models, based on a single vibration mode approximation, are coupled with the gas dynamics equation and acoustic plenum models. The steady state solution of the resultant coupled non-linear equations are posed as a boundary value problem and solved using Warner’s algorithm.

Llopis et al. (2007) [14] proposed a dynamic model of a shell-and-tube condenser operating in a vapour compression refrigeration plant. The model is formulated as a combination of control volumes that represents all the refrigerant states in the condenser and it is based on mass continuity, energy conservation and heat transfer physical fundamentals by using a lumped-parameter approach. The model is capable of representing the quick dynamics of the condenser without using complicated formulation.

2.3 Simulink

The modelling, analysis and simulation of the dynamic model are developed by using Simulink. Simulink provides an interactive graphical environment and a customizable set of block libraries that let design, simulate, implement, and test a variety of time-varying systems. Simulink is integrated with MATLAB, providing immediate access to an extensive range of tools that let develop algorithms, customize the modelling environment and define signal, parameter, and test data. With Simulink, it is possible to build models by dragging and dropping blocks from the library browser onto the graphical editor and connecting them with lines that establish mathematical relationships between the blocks. As represented in Figure 28, a block diagram model of a dynamic system graphically simply consists of blocks and lines (signals).

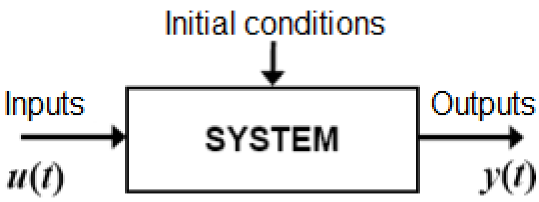

Figure 28 Simulink block diagram.

Simulink updates the state of each block of the model at each time step, starting from the first value and ending to the final value of the time span. We decided to use this approach because it is easier and immediately to understand how the model is organized and how its part interact. In order to create the model using Simulink we followed next steps:

- define the system;
- identify system components;
- build the Simulink block diagram;
- run the simulation.

3 Modelling

3.1 Compressor model

In this model a single cylinder reciprocating compressor has been considered. The piston has reciprocating action through the crank shaft arrangement and no leakage is assumed. A clearance volume of 10% of the stroke volume is assumed. We are not interested in capturing the pressure variation of the compressor cylinder spatially, rather, only the average pressure as the reciprocation takes place. This facilitates the use of the overall mass balance (eq. 3.2), which becomes (eq. 3.4):

$$m_{cyl} = \rho_{cyl} V_{cyl} \quad (3.1)$$

$$\frac{dm_{cyl}}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (3.2)$$

$$\frac{d\rho_{cyl} V_{cyl}}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (3.3)$$

$$\rho_{cyl} \frac{dV_{cyl}}{dt} + V_{cyl} \frac{d\rho_{cyl}}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (3.4)$$

where V_{cyl} is the cylinder volume varying with time according to a sinusoidal law (eq. 3.5):

$$V_{cyl} = V_{cle} + \frac{V_{str}}{2}\{1 - \cos(\omega t)\} \quad (3.5)$$

$$\frac{dV_{cyl}}{dt} = \frac{V_{str}}{2}\frac{d}{dt}\{-\cos(\omega t)\} = \frac{V_{str}}{2}\sin(\omega t) \quad (3.6)$$

$$\omega = 2\pi \frac{N}{60} \quad (3.7)$$

Assuming that the process is quasistatic and the values of heat capacities are almost constant (they are actually functions of temperature, but are nearly constant within small changes of temperature), we can consider the compression to be polytropic. A polytropic transformation is valid both for ideal and real gasses and the relation which describes the process is the following:

$$Pv^n = C \quad (3.8)$$

where n is the polytropic index and C is a constant. In case of adiabatic process (no heat transferred) n becomes equal to $\gamma = \frac{c_p}{c_v}$. The polytropic index in a quasi-adiabatic process such as the considered compression goes from 1 (isothermal process, constant temperature) to γ. From this law (eq. 3.8) it is possible to find a relation between cylinder gas density and pressure (eq. 3.10) along with their time derivatives (3.12):

$$\frac{P_{cyl}}{{\rho_{cyl}}^n} = C \quad (3.9)$$

$$P_{cyl} = C{\rho_{cyl}}^n \quad (3.10)$$

$$\frac{dP_{cyl}}{dt} = C\, n\, \rho_{cyl}^{n-1}\, \frac{d\rho_{cyl}}{dt} = \frac{P_{cyl}}{{\rho_{cyl}}^n}\, n\, \rho_{cyl}^{n-1}\, \frac{d\rho_{cyl}}{dt} = \frac{nP_{cyl}}{\rho_{cyl}} \frac{d\rho_{cyl}}{dt} \quad (3.11)$$

$$\frac{d\rho_{cyl}}{dt} = \frac{\rho_{cyl}}{nP_{cyl}} \frac{dP_{cyl}}{dt} \quad (3.12)$$

Eq. 3.12 can be substituted in the overall mass balance (eq. 3.4) in order to obtain the governing equation (eq. 3.14):

$$\rho_{cyl} \frac{dV_{cyl}}{dt} + V_{cyl} \frac{\rho_{cyl}}{nP_{cyl}} \frac{dP_{cyl}}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (3.13)$$

$$\frac{dP_{cyl}}{dt} = -\frac{nP_{cyl}}{V_{cyl}} \frac{dV_{cyl}}{dt} + \frac{nP_{cyl}}{V_{cyl}\rho_{cyl}} (\dot{m}_{in} - \dot{m}_{out}) \quad (3.14)$$

where cylinder gas density and mass flow rates through the suction and discharge valves can be written as a function of P_{cyl}, the gas pressure inside the cylinder. Since suction density and pressure are known, cylinder density can be calculated by eq. 3.17:

$$\frac{P_{cyl}}{{\rho_{cyl}}^n} = C = \frac{P_{suc}}{{\rho_{suc}}^n} \quad (3.15)$$

$${\rho_{cyl}}^n = {\rho_{suc}}^n \frac{P_{cyl}}{P_{suc}} \quad (3.16)$$

$$\rho_{cyl} = \rho_{suc}\left(\frac{P_{cyl}}{P_{suc}}\right)^{\frac{1}{n}} = \frac{\rho_{suc}}{P_{suc}^{\frac{1}{n}}} P_{cyl}^{\frac{1}{n}} \quad (3.17)$$

As per the mass flow rates, valves dynamic have been neglected (no gas leakage, valves shut down instantaneously as soon as unfavourable pressure difference is created). For this reason inlet and outlet mass flow rates can be written in the following forms based on flow through an orifice equations:

$$\dot{m}_{in} = \rho_{suc} C_{sv} S_{sv} \sqrt{\frac{2(P_{suc} - P_{cyl})}{\rho_{suc}}} \quad (3.18)$$

$$\dot{m}_{out} = \rho_{cyl} C_{dv} S_{dv} \sqrt{\frac{2(P_{cyl} - P_{dis})}{\rho_{cyl}}} \quad (3.19)$$

where suction and discharge pressures correspond to evaporator and condenser pressures and suction density along with orifice coefficients and valves flow areas have to be known.
In order to estimate the input work we have to calculate the mass rate treated by the compressor per cycle. In an ideal cycle (Figure 29) the mass input to the compressor is given by:

$$m = \rho_{suc}(V_2 - V_1) \quad (3.20)$$

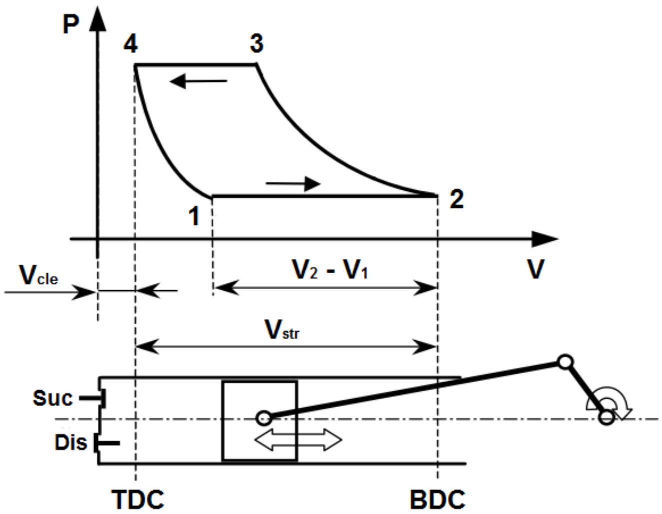

Figure 29 Pressure - volume diagram of an ideal compression cycle.

The mass theoretically treatable by a compressor, instead, is greater and it is equal to $\rho_{suc}V_{str}$. The difference between the two values is due to the presence of the clearance volume: it is impossible to fill entirely the cylinder with fresh charge because a certain amount of volume is reserved to the expansion of the clearance mass. Therefore, even in ideal cycles, the volumetric efficiency, already seen in par. 1.3 and defined by eq. 3.21, is lower than one:

$$\eta_k = \frac{m}{m_{th}} = \frac{\rho_{suc}(V_2 - V_1)}{\rho_{suc}V_{str}} = \frac{V_2 - V_1}{V_{str}} \quad (3.21)$$

where V_2 and V_1 are given by eq. 3.22 and eq. 3.23. By substituting them in eq. 3.21 it is possible to write a simple relation (eq. 3.26) between the volumetric efficiency η_k and the compression rate β:

$$V_2 = (1 + \mu)V_{str} \quad (3.22)$$

$$V_1 = \mu V_{str}\beta^{\frac{1}{n}} \quad (3.23)$$

$$\mu = \frac{V_{cle}}{V_{str}} \quad (3.24)$$

$$\beta = \frac{P_{dis}}{P_{suc}} \quad (3.25)$$

$$\eta_k = \frac{V_2 - V_1}{V_{str}} = 1 + \mu - \mu\beta^{\frac{1}{n}} = 1 - \mu\left(\beta^{\frac{1}{n}} - 1\right) \quad (3.26)$$

According to eq. 3.26, with increasing clearance volume and compression ratio the volumetric efficiency will decrease. Now it is possible to define the real mass per cycle treated by the compressor in this way:

$$m = \eta_k\rho_{suc}V_{str} \quad (3.27)$$

The amount of input work can be calculated through the following equation:

$$W = m\frac{N}{60}(h_c - h_b) \quad (3.28)$$

where h_c is the refrigerant enthalpy at the compressor outlet.

3.2 Condenser model

The dominant dynamic of the system is due to the heat exchangers, however the study of the heat transfer in a heat exchanger can be very difficult, sometimes impossible. Accuracy and execution speed of system models are highly dependent on the modelling approach. In this work, a finite volume method with a small number of control volumes has been considered. This approach provides a relative simple and accurate model with a low computational load.
A shell and tube flooded condenser has been taken into account. The refrigerant (R134a) flows through the shell and the water inside the tubes. While the water flow inside the tubes is clearly one dimensional, the refrigerant flow is truly three dimensional. In order to have manageable equations, this model takes into account a one-dimensional refrigerant flow simplifying assumption. Since there is a dominant flow direction between the refrigerant inlet and outlet, this is a reasonable assumption. For this reason, it is possible to consider a pure counter flow between the water and the refrigerant.
The conductivity of the tube is supposed to be high so the resistance to radial heat transfer between the refrigerant and the water is negligible. Moreover, by considering a small tube thickness, its capacitance can be neglected too: because of this assumption, this model does not take into account the tube energy balance.
Other assumptions considered in the model are the following:

- Negligible pressure drops. Water side pressure drops have been neglected because they have a very small impact on the heat transfer rate since water properties have a weak correlation with pressure; on the refrigerant side, for a flooded heat exchanger, the flow path is fairly short and the available areas in the shell are large. This results in small pressure drops that can be neglected.
- Negligible axial conduction. It is also neglected axial heat transfer because the cross section area available for the axial heat transfer is very small.
- Negligible shell capacitance. The refrigerant transfers heat to the shell and some energy is stored therein. Even if the shell capacitance is not negligible, the amount of heat transferred to it is very small because the area available for the heat transfer from the shell side is relatively smaller than the area available for the heat transfer from the water tube side.

Input parameters to the condenser model are refrigerant inlet mass flow, pressure and enthalpy, which are the outputs of the compressor model. As per the water, inlet temperature and mass flow must be fixed.

A distributed approach has been considered, which means that the transient equations are discretized over each control volume and result in a system of ordinary differential equations. In the present work, the condenser has been divided in three zones, to better capture the refrigerant properties in the different regions:

- in the first zone $\Delta z'$ the refrigerant is liquid;
- in the second zone $\Delta z''$ the refrigerant is a vapour/liquid mixture;
- in the third zone $\Delta z'''$ the refrigerant is vapour.

Water enters the first zone at the initial temperature $T_w(0)$ and exits the third zone at the maximum temperature $T_w(3)$. Figure 30 represents a schematic overview of the considered condenser model.

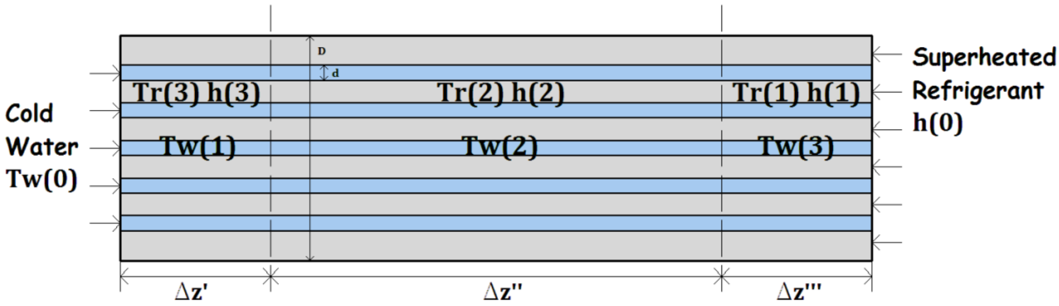

Figure 30 Scheme of the condenser model.

3.2.1 Water side equations

If we consider a single element Δz, it is possible to write the following energy balance, neglecting the wall equation:

$$\frac{\partial}{\partial t}\left(\frac{\pi d^2}{4}\Delta z \rho_w c_{pw} T_w\right) = \pi d \Delta z \alpha (T_r - T_w) - \frac{\pi d^2}{4} v_w \rho_w c_{pw} (T_w|_{z+\Delta z} - T_w|_z) \quad (3.29)$$

Dividing eq. 3.29 by Δz and setting $\Delta z \rightarrow 0$ we can obtain the water side governing equation (eq. 3.32):

$$\frac{\pi d^2}{4}\rho_w c_{pw}\frac{\partial T_w}{\partial t} = \pi D \alpha (T_r - T_w) - \frac{\pi d^2}{4} v_w \rho_w c_{pw}\frac{\partial T_w}{\partial z} \quad (3.30)$$

$$\frac{\partial T_w}{\partial t} = -v_w \frac{\partial T_w}{\partial z} + \frac{4\alpha}{d \rho_w c_{pw}}(T_r - T_w) \quad (3.31)$$

$$\frac{\partial T_w}{\partial t} = -v_w \frac{\partial T_w}{\partial z} + \frac{1}{\tau_w}(T_r - T_w) \quad (3.32)$$

where:

$$\tau_w = \frac{d\rho_w c_{pw}}{4\alpha} \quad (3.33)$$

The heat transfer coefficient α present in eq. 3.33 depends both on water and refrigerant conditions. Water side coefficient can be calculated through the Dittus-Boelter equation (eq. 3.34) for forced convection inside tubes in a heating process:

$$Nu = \frac{\alpha d}{k} = 0{,}023 Re^{0{,}8} Pr^{0{,}4} \quad (3.34)$$

where:

$$Re = \frac{\rho v d}{\mu} = \frac{vd}{\nu} \quad (3.35)$$

$$Pr = \frac{c_p \mu}{k} \quad (3.36)$$

Discretizing the partial derivative respect to space according to eq. 3.37:

$$\frac{\partial T_w}{\partial z} \approx \frac{T_w(j) - T_w(j-1)}{\Delta z} \quad (3.37)$$

the equation 3.32 for the jth node becomes eq. 3.39:

$$\frac{dT_w(j)}{dt} = -v_w \frac{T_w(j) - T_w(j-1)}{\Delta z} + \frac{1}{\tau_w}[T_r(j) - T_w(j)] \quad (3.38)$$

$$\frac{dT_w(j)}{dt} = \frac{v_w}{\Delta z} T_w(j-1) - \left(\frac{v_w}{\Delta z} + \frac{1}{\tau_w}\right) T_w(j) + \frac{1}{\tau_w} T_r(j) \quad (3.39)$$

The water inlet temperature is fixed so the boundary condition at $z = 0$ is $T_w(0)$. The condenser has been divided into three zones: for each node (discretization point) it is possible to write a governing equation. This results into three equation which must be coupled with refrigerant side equations in order to solve the condenser model. The water side variables are the water average temperatures in each zone. The three equations are the following (eq. 3.40, 3.41, 3.42):

$$\frac{dT_w(1)}{dt} = \frac{v_w}{\Delta z'} T_w(0) - \left(\frac{v_w}{\Delta z'} + \frac{1}{\tau_w}\right) T_w(1) + \frac{1}{\tau_w} T_r(2) \quad (3.40)$$

$$\frac{dT_w(2)}{dt} = \frac{v_w}{\Delta z''}T_w(1) - \left(\frac{v_w}{\Delta z''} + \frac{1}{\tau_w}\right)T_w(2) + \frac{1}{\tau_w}T_r(1) \quad (3.40)$$

$$\frac{dT_w(3)}{dt} = \frac{v_w}{\Delta z'''}T_w(2) - \left(\frac{v_w}{\Delta z'''} + \frac{1}{\tau_w}\right)T_w(3) + \frac{1}{\tau_w}T_r(0) \quad (3.42)$$

3.2.2 Refrigerant side equations

The refrigerant is flowing through the shell and we need to take into account the mass conservation equation and the energy balance. The first one is rather simple and it can be written in this way:

$$\frac{d}{dt}(A\Delta z\rho) = \dot{m}_z - \dot{m}_{z+\Delta z} \quad (3.43)$$

where:

$$A = \frac{\pi D^2}{4} - N_{tubes}\frac{\pi d^2}{4} \quad (3.44)$$

Dividing by Δz and setting $\Delta z \to 0$ the mass balance assumes the following form:

$$\frac{d\rho}{dt} = -\frac{1}{A}\frac{\partial \dot{m}}{\partial z} \quad (3.45)$$

As per the energy balance, it is quite similar to the water one and it is given by:

$$\frac{d}{dt}(A\Delta z\rho u) = Av\rho(h_z - h_{z+\Delta z}) - \dot{Q}_{COND} \quad (3.46)$$

where the heat transferred between the water and the refrigerant can be calculated according to the Newton’s law:

$$\dot{Q}_{COND} = \pi d\Delta z\alpha(T_r - T_w) \quad (3.47)$$

For each zone, the amount of heat exchanged between the water and the refrigerant is the same. Remembering the definition of the specific internal energy:

$$u = h - {}^{P}/_{\rho} \quad (3.48)$$

the energy balance equation (eq. 3.46) becomes eq. 3.50:

$$A\Delta zu\frac{d\rho}{dt} + A\Delta z\rho\frac{du}{dt} = Av\rho(h_z - h_{z+\Delta z}) - \pi d\Delta z\alpha(T_r - T_w) \quad (3.49)$$

$$A\Delta z\rho\frac{d\left(h - {}^{P}/_{\rho}\right)}{dt} = -A\Delta z(h - {}^{P}/_{\rho})\frac{d\rho}{dt} + Av\rho(h_z - h_{z+\Delta z}) - \pi d\Delta z\alpha(T_r - T_w) \quad (3.50)$$

which combined with the mass balance equation (3.45) leads to eq. 3.52:

$$\frac{d\left(h - {}^{P}/_{\rho}\right)}{dt} = \frac{\left(h - {}^{P}/_{\rho}\right)}{\rho A}\frac{\partial \dot{m}}{\partial z} - v\frac{\partial h}{\partial z} - \frac{\pi d\alpha}{\rho A}(T_r - T_w) \quad (3.51)$$

$$\frac{dh}{dt} = \frac{h}{\rho A}\frac{\partial \dot{m}}{\partial z} - v\frac{\partial h}{\partial z} - \frac{\pi d\alpha}{\rho A}(T_r - T_w) + \frac{d(P/\rho)}{dt} - \frac{(P/\rho)}{\rho A}\frac{\partial \dot{m}}{\partial z} \quad (3.52)$$

Discretizing the partial derivatives of enthalpy and mass flow respect to space (eq. 3.53 and 3.54), and dividing enthalpy and pressure terms, it is possible to obtain the two refrigerant side governing equations (eq. 3.55 and 3.56):

$$\frac{\partial h}{\partial z} = \frac{h(j) - h(j-1)}{\Delta z} \quad (3.53)$$

$$\frac{\partial \dot{m}}{\partial z} = \frac{\dot{m}(j) - \dot{m}(j-1)}{\Delta z} \quad (3.54)$$

$$\frac{dh(j)}{dt} = \frac{h(j)}{\rho(j)A\Delta z}[\dot{m}(j) - \dot{m}(j-1)] - \frac{v}{\Delta z}[h(j) - h(j-1)] - \frac{\pi d\alpha(j)}{\rho(j)A}[T_r(j) - T_w(j)] \quad (3.55)$$

$$\frac{dP_\rho(j)}{dt} = \frac{P_\rho(j)}{\rho(j)A\Delta z}[\dot{m}(j) - \dot{m}(j-1)] \quad (3.56)$$

where:

$$P_\rho = \frac{P_c}{\rho} \quad (3.57)$$

As done with the water, it is possible to write the two governing equations for each node. The six refrigerant variables are the average enthalpies and pressures in each zone:

$$\frac{dh(1)}{dt} = \frac{h(1)}{\rho(1)A\Delta z'''}[\dot{m}(1) - \dot{m}(0)] - \frac{v}{\Delta z'''}[h(1) - h(0)] - \frac{\pi d\alpha(1)}{\rho(1)A}[T_r(1) - T_w(2)] \quad (3.58)$$

$$\frac{dh(2)}{dt} = \frac{h(2)}{\rho(2)A\Delta z''}[\dot{m}(2) - \dot{m}(1)] - \frac{v}{\Delta z''}[h(2) - h(1)] - \frac{\pi d\alpha(2)}{\rho(2)A}[T_r(2) - T_w(1)] \quad (3.59)$$

$$\frac{dh(3)}{dt} = \frac{h(3)}{\rho(3)A\Delta z'}[\dot{m}(3) - \dot{m}(2)] - \frac{v}{\Delta z'}[h(3) - h(2)] - \frac{\pi d\alpha(3)}{\rho(3)A}[T_r(3) - T_w(0)] \quad (3.60)$$

$$\frac{dP_\rho(1)}{dt} = \frac{P_\rho(1)}{\rho(1)A\Delta z'''}[\dot{m}(1) - \dot{m}(0)] \quad (3.61)$$

$$\frac{dP_\rho(2)}{dt} = \frac{P_\rho(2)}{\rho(2)A\Delta z''}[\dot{m}(2) - \dot{m}(1)] \quad (3.62)$$

$$\frac{dP_\rho(3)}{dt} = \frac{P_\rho(3)}{\rho(3)A\Delta z'}[\dot{m}(3) - \dot{m}(2)] \quad (3.63)$$

where $h(0)$ and $h(3)$ are respectively the inlet and outlet refrigerant enthalpy, i.e. h_c and h_d; $\dot{m}(0)$ is the pulsating mass flow rate coming from the compressor; $\dot{m}(3)$ is the condenser outlet mass flow rate, which corresponds to the evaporator inlet; $\dot{m}(1)$, $\dot{m}(2)$ and $\dot{m}(3)$ are averaged and delayed values of $\dot{m}(0)$ and they have been calculated with increasing delay and mean interval. Eq. from 3.58 to 3.63, combined with eq. 3.40, 3.41 and 3.42, give a set of nine equations with nine variables which have to be solved simultaneously.

3.3 Evaporator model

In this work, a shell and tube flooded evaporator has been considered. Since we are not interest in capturing the secondary fluid dynamic, the model of the evaporator is rather simple. Mass conservation and energy balance equations have been written for a single zone model. Since the input of the compressor is saturated vapour, the refrigerant can be assumed to be a vapour/liquid mixture all along the evaporator.
The input parameters are refrigerant inlet and outlet mass flow rates and enthalpies. Starting from inlet mass flow rate and enthalpy, it is possible to calculate the heat flow rate that must be recovered from the secondary fluid. By assuming no dynamic in the secondary fluid, the model gives as output the average values of enthalpy and pressure as function of time. Other assumption that have been considered are: negligible tubes conductivity and capacitance, negligible pressure drops, negligible axial conduction, negligible shell capacitance.
Mass conservation and energy balance applied to the refrigerant side are given by eq. 3.64 and 3.65:

$$\frac{d(\rho V_e)}{dt} = \dot{m}_{il} - \dot{m}_{ov} \quad (3.64)$$

$$\frac{d(\rho V_e u)}{dt} = \dot{m}_{il} h_a - \dot{m}_{ov} h_b + \dot{Q}_e \quad (3.65)$$

Where $\dot{m}_{il}$ is the evaporator inlet-liquid mass flow rate coming from the from the condenser and $\dot{m}_{ov}$ is the evaporator outlet-vapour mass flow rate, which corresponds to the mass flow rate aspired by the compressor. Remembering the definition of the internal energy (eq. 3.48) and rearranging eq. 3.65 together with eq. 3.64, the energy balance becomes eq. 3.71:

$$\rho V_e \frac{du}{dt} + u \frac{d(\rho V_e)}{dt} = \dot{m}_{il} h_a - \dot{m}_{ov} h_b + \dot{Q}_e \quad (3.66)$$

$$\rho V_e \frac{d\left(h - {}^{P}/_{\rho}\right)}{dt} = \dot{m}_{il} h_a - \dot{m}_{ov} h_b + \dot{Q}_e - \left(h - {}^{P}/_{\rho}\right) \frac{d(\rho V_e)}{dt} \quad (3.67)$$

$$\rho V_e \frac{d\left(h - {}^{P}/_{\rho}\right)}{dt} = \dot{m}_{il} h_a - \dot{m}_{ov} h_b + \dot{Q}_e - \left(h - {}^{P}/_{\rho}\right)(\dot{m}_{il} - \dot{m}_{ov}) \quad (3.68)$$

$$\rho V_e \frac{dh_e}{dt} - \rho V_e \frac{d\left({}^{P}/_{\rho}\right)}{dt} = \dot{m}_{il}(h_a - h_e) - \dot{m}_{ov}(h_b - h_e) + \dot{Q}_e + \frac{P}{\rho}(\dot{m}_{il} - \dot{m}_{ov}) \quad (3.69)$$

$$\rho V_e \frac{dh_e}{dt} = \dot{m}_{il}(h_a - h_e) - \dot{m}_{ov}(h_b - h_e) + \dot{Q}_e + \frac{P}{\rho}(\dot{m}_{il} - \dot{m}_{ov}) + \rho V_e \frac{d\left({}^{P}/_{\rho}\right)}{dt} \quad (3.70)$$

$$\rho V_e \frac{dh_e}{dt} + \dot{m}_{il}(h_e - h_a) + \dot{m}_{ov}(h_b - h_e) - \dot{Q}_e = \frac{P}{\rho}(\dot{m}_{il} - \dot{m}_{ov}) + V_e \frac{dP}{dt} \quad (3.71)$$

As seen in the condenser model, enthalpy and pressure terms can be divided (eigenvalues problem) in order to obtain the two evaporator governing equations:

$$\rho V_e \frac{dh_e}{dt} + \dot{m}_{il}(h_e - h_a) + \dot{m}_{ov}(h_b - h_e) - \dot{Q}_e = 0 \quad (3.72)$$

$$V_e \frac{dP}{dt} + \frac{P}{\rho}(\dot{m}_{il} - \dot{m}_{ov}) = 0 \quad (3.73)$$

which give as output the refrigerant average enthalpy and pressure.

3.4 Expansion valve model

The assumption made in this model is that the valve acts as a static element with negligible inertia effects. This assumption is based on the fact that the dynamic response of the valve is much faster than the two heat exchangers. Therefore the steady-state equation for the mass flow rate passing through the expansion device can be written in this way:

$$\dot{m} = \rho C_v S_v \sqrt{\frac{2(P_{cond} - P_{eva})}{\rho}} \quad (3.74)$$

where C_v is the orifice flow coefficient and S_v is the cross sectional area of the orifice hole.
The expansion process in the expansion valve is considered to be isenthalpic. As a result the energy balance equation is written as:

$$h_d = h_a \quad (3.75)$$

where h_d and h_a are respectively the enthalpy at the expansion valve inlet and outlet.

4 Results and discussion

4.1 System overview

As result of the models adopted, the heat pump system considered for simulation can be represented by the scheme in Figure 31. The main inputs of the heat pump model (along with the geometrical parameters which characterize compressor, condenser and evaporator), are the following:

- condensation pressure,
- evaporation pressure,
- compressor speed,
- refrigerant conditions at the compressor inlet,
- water mass flow rate,
- water temperature at the condenser inlet.

The most interesting output parameters are:

- water temperature at the condenser outlet,
- refrigerant enthalpy in the defined condenser nodes,
- condenser heat flow and COP,
- evaporator heat flow,
- refrigerant average enthalpy in the evaporator.

In this work a heat pump for comfort heating has been simulated. About 14 kW condenser and 4 kW compressor have been considered in the model. The condenser works with 1000 l/h of water at 30°C. The compressor is described by a steady state model, which means that it operates under constant conditions: the rotation speed doesn't vary from its starting value of 3000 rpm. However, compressor inlet and outlet mass flow rates are not constant and they depend on the pressure inside the cylinder according to eq. 3.18 and 3.19.

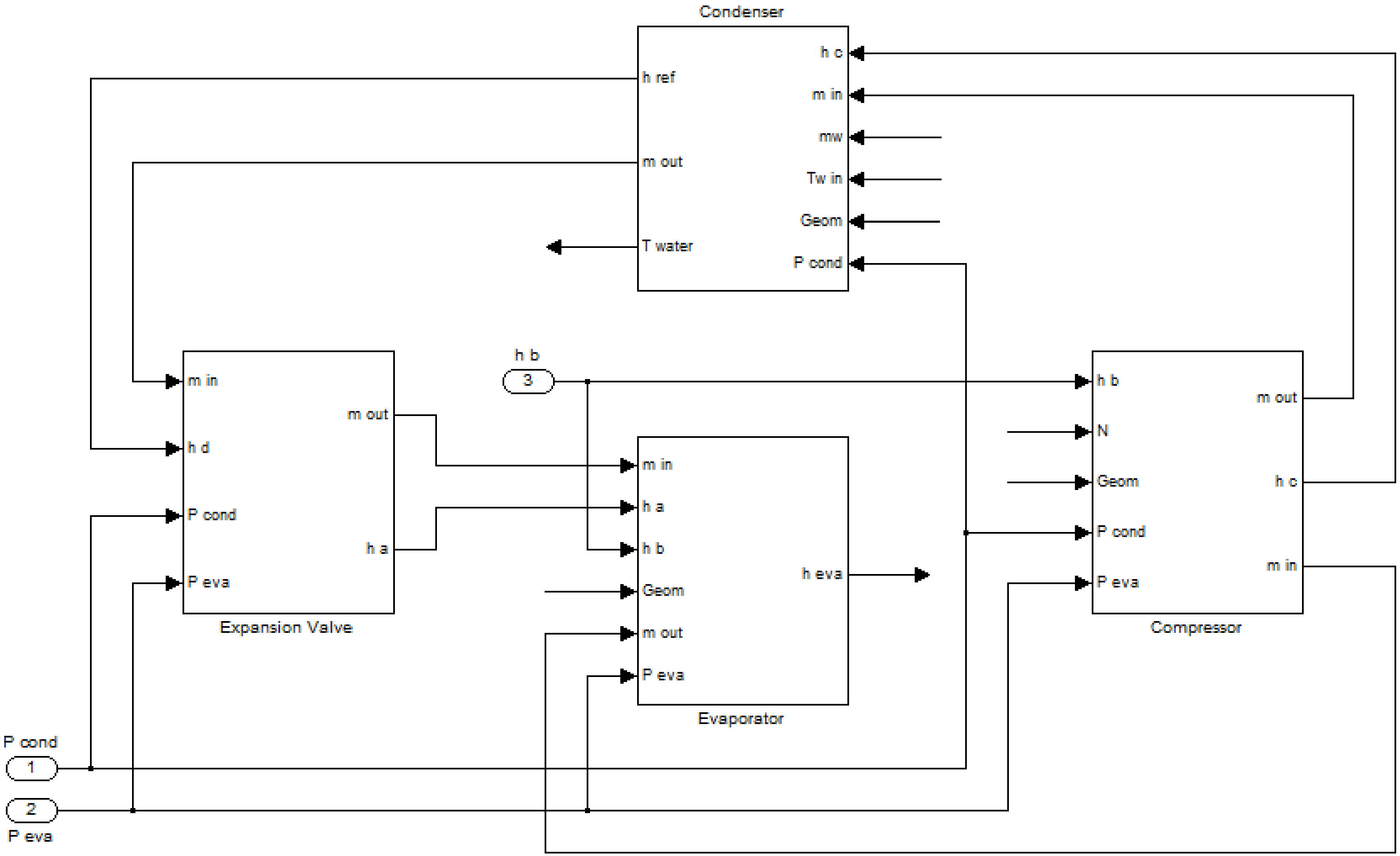

Figure 31 Heat pump Simulink scheme.

4.2 System specifications and results

The detailed list of the parameters utilized to run the simulation is the following:

- $N = 3000$ rpm
- $V_{str} = 0.0002\ m^3 = 200\ cm^3$
- $V_{cle} = 10\%\ V_{str} = 0.00002\ m^3$
- $P_{cond} = 12$ bar
- $P_{eva} = 3$ bar
- $h_b = 400$ kJ/kg
- $\rho_{suc} = 14.7\ kg/m^3$
- $n = 1.05$
- $C_{sv} = 0.8$
- $A_{sv} = 0.0003\ m^2 = 300\ mm^2$
- $C_{dv} = 0.8$
- $A_{dv} = 0.0003\ m^2 = 300\ mm^2$
- $m_w = 0.278$ kg/s = 1000 l/h
- $T_w(0) = 30°C$
- $D = 100$ mm
- $d = 10$ mm
- $N_{tubes} = 50$
- $Dz' = 40$ cm
- $Dz'' = 80$ cm
- $Dz'' = 30$ cm
- $\alpha(1) = 500\ W/m^2K$
- $\alpha(2) = 1500\ W/m^2K$
- $\alpha(3) = 300\ W/m^2K$
- $\rho_w = 995\ kg/m^3$
- $cp_w = 4178$ J/kgK
- $k_w = 0.62$ W/mK
- $\mu_w = 0.000769$ kg/ms
- $v_w = 0.0000007\ m^2/s$
- $V_e = 0.0035\ m^3$

Figure 32 represents the saturation curve of the refrigerant R134a. Its properties can be read as function of pressure and enthalpy. The compression starts from saturated vapour conditions in order to limit the superheating of the refrigerant. Figures from Figure 33 to Figure 39 represent the results obtained by the simulation presented above.

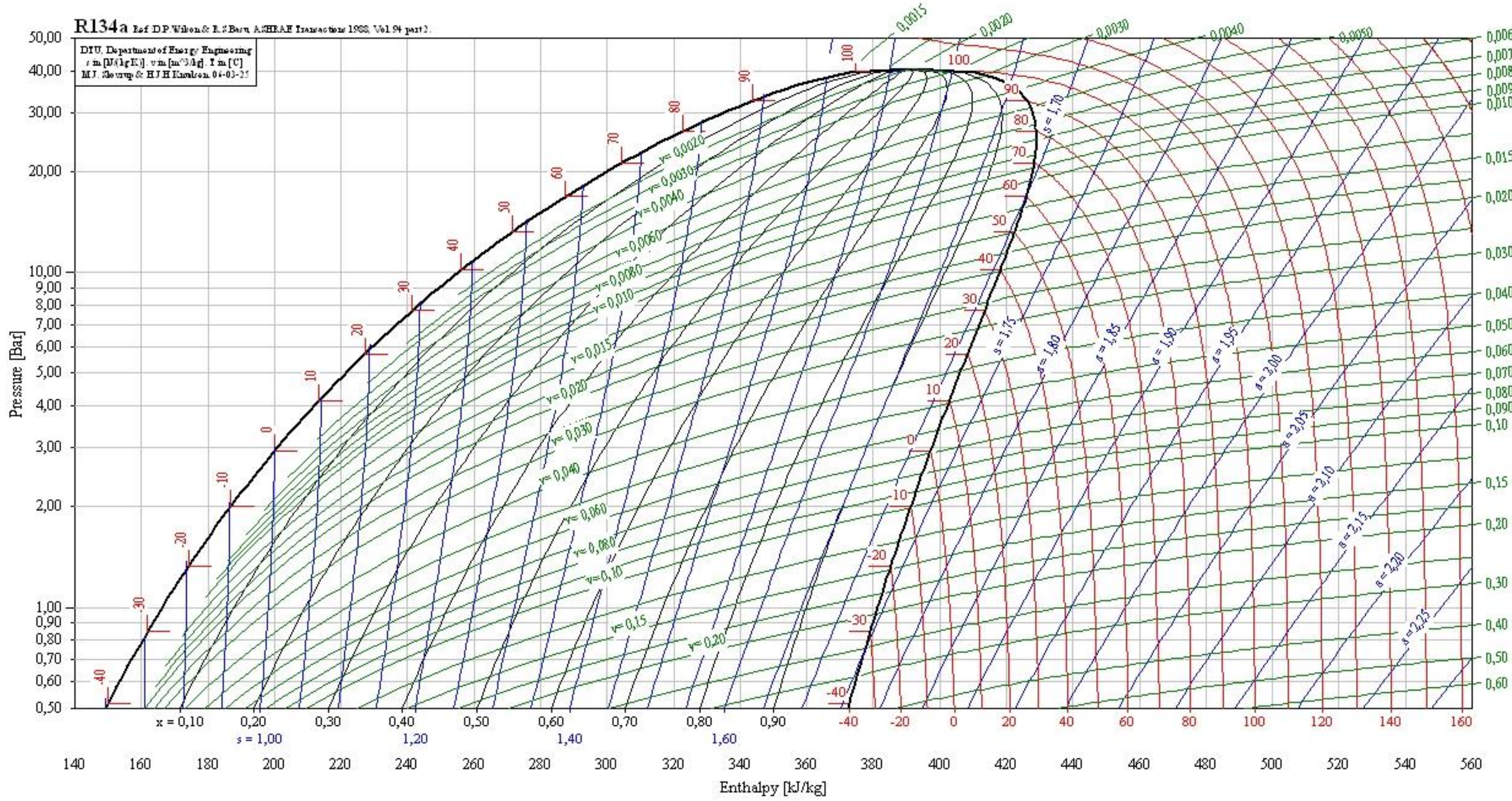

Figure 32 R134a properties chart.

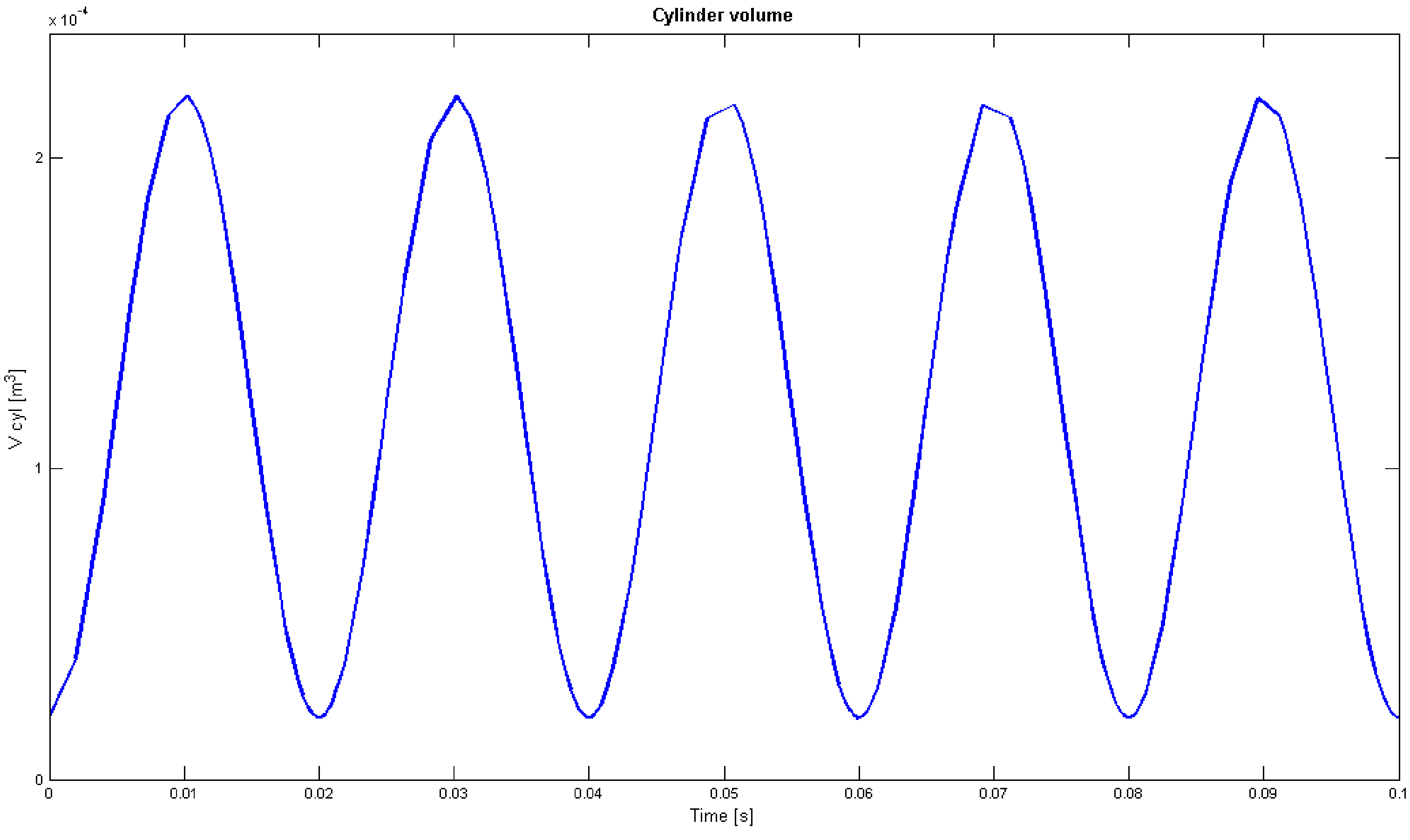

Figure 33 Cylinder volume as function of time.

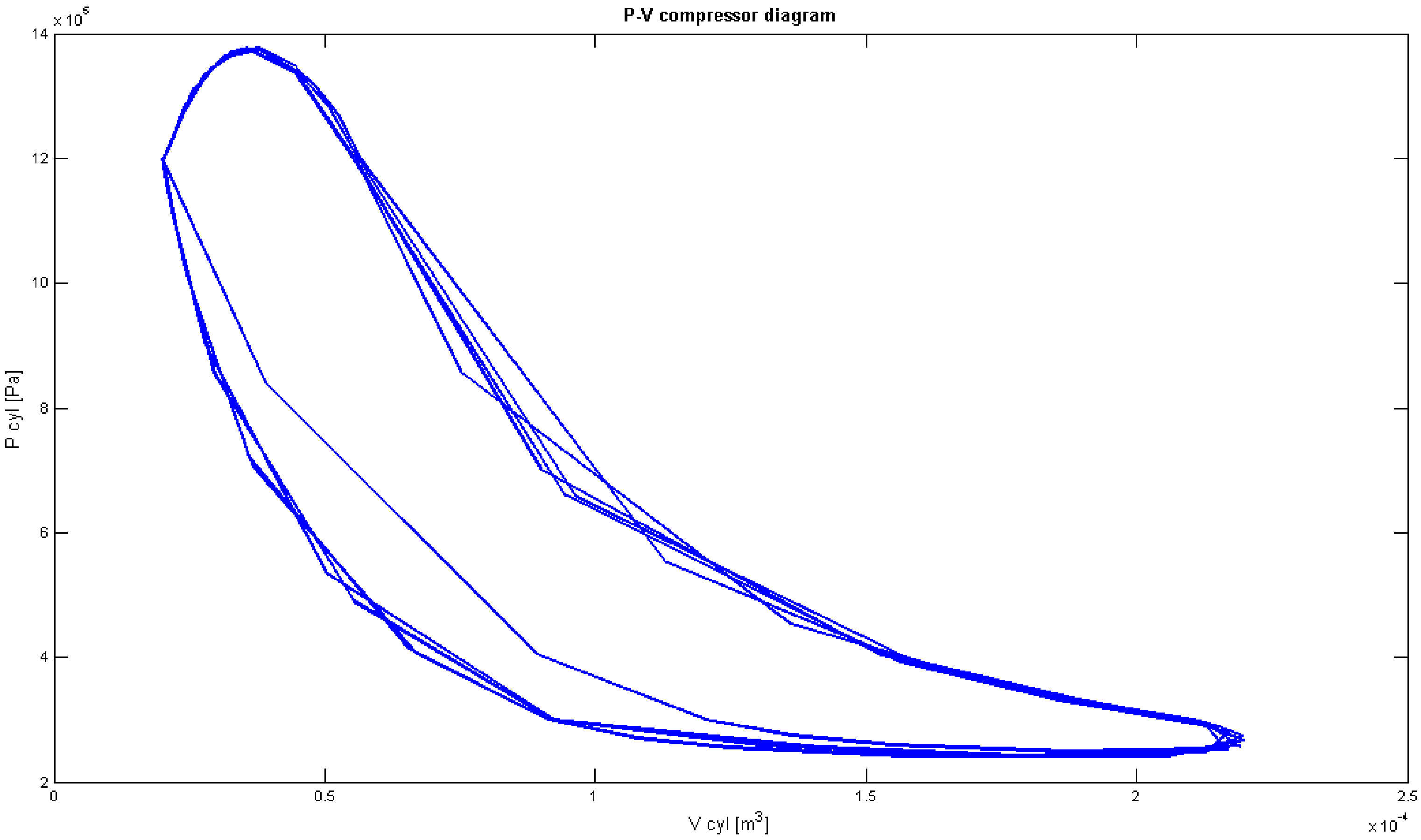

Figure 34 Cylinder pressure as function of cylinder volume.

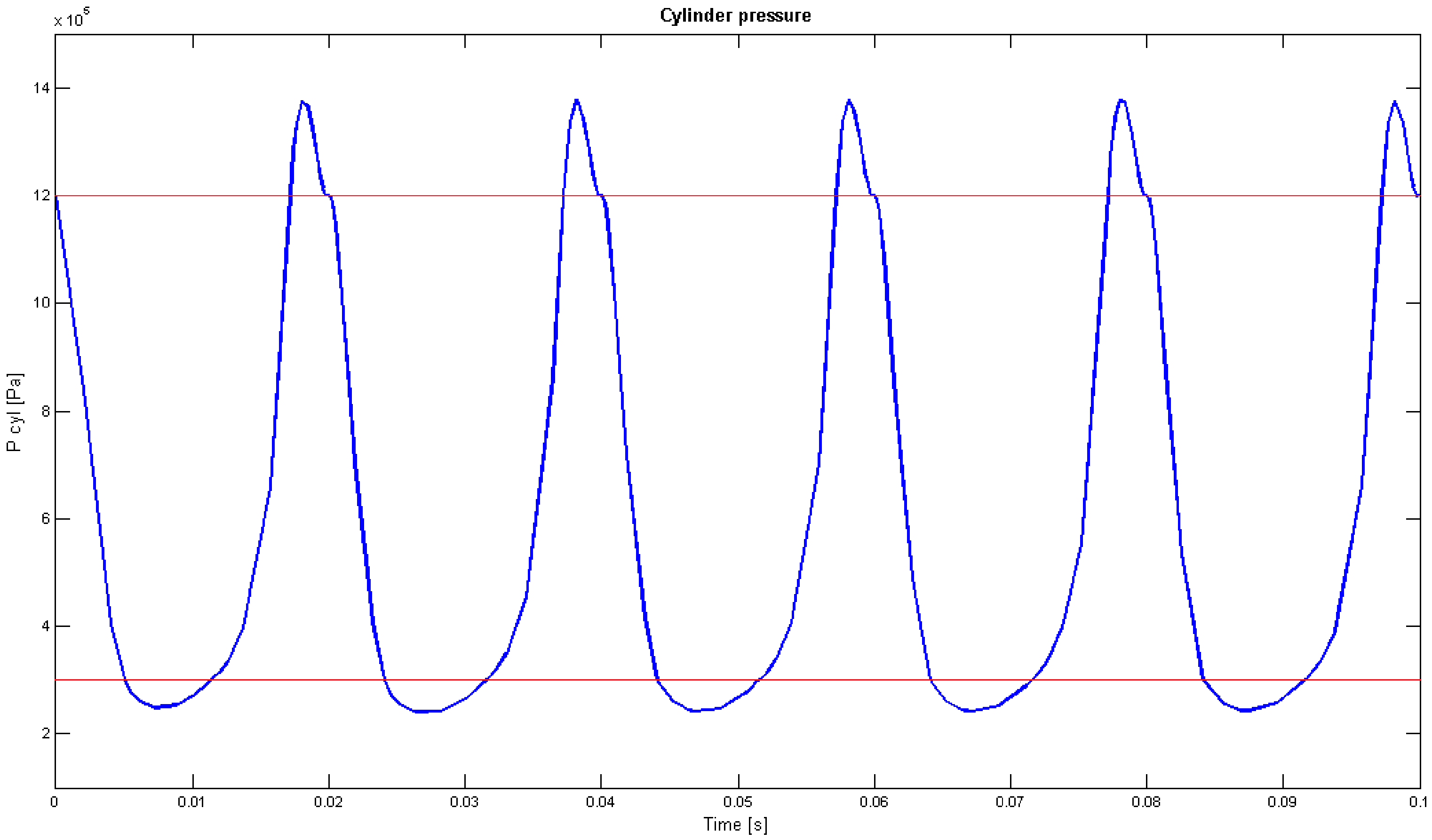

Figure 35 Cylinder pressure as function of time.

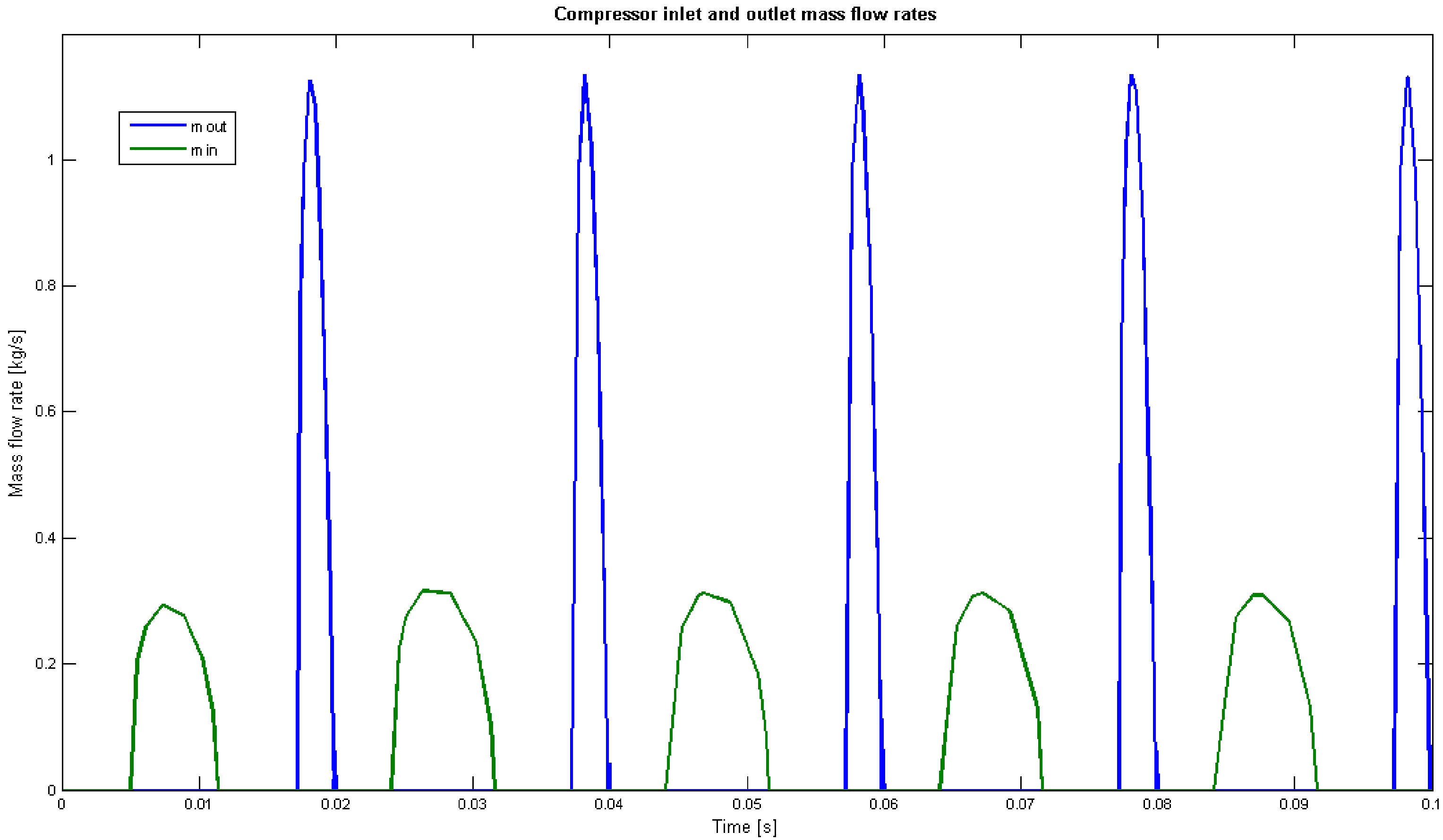

Figure 36 Compressor inlet and outlet mass flow rates as function of time.

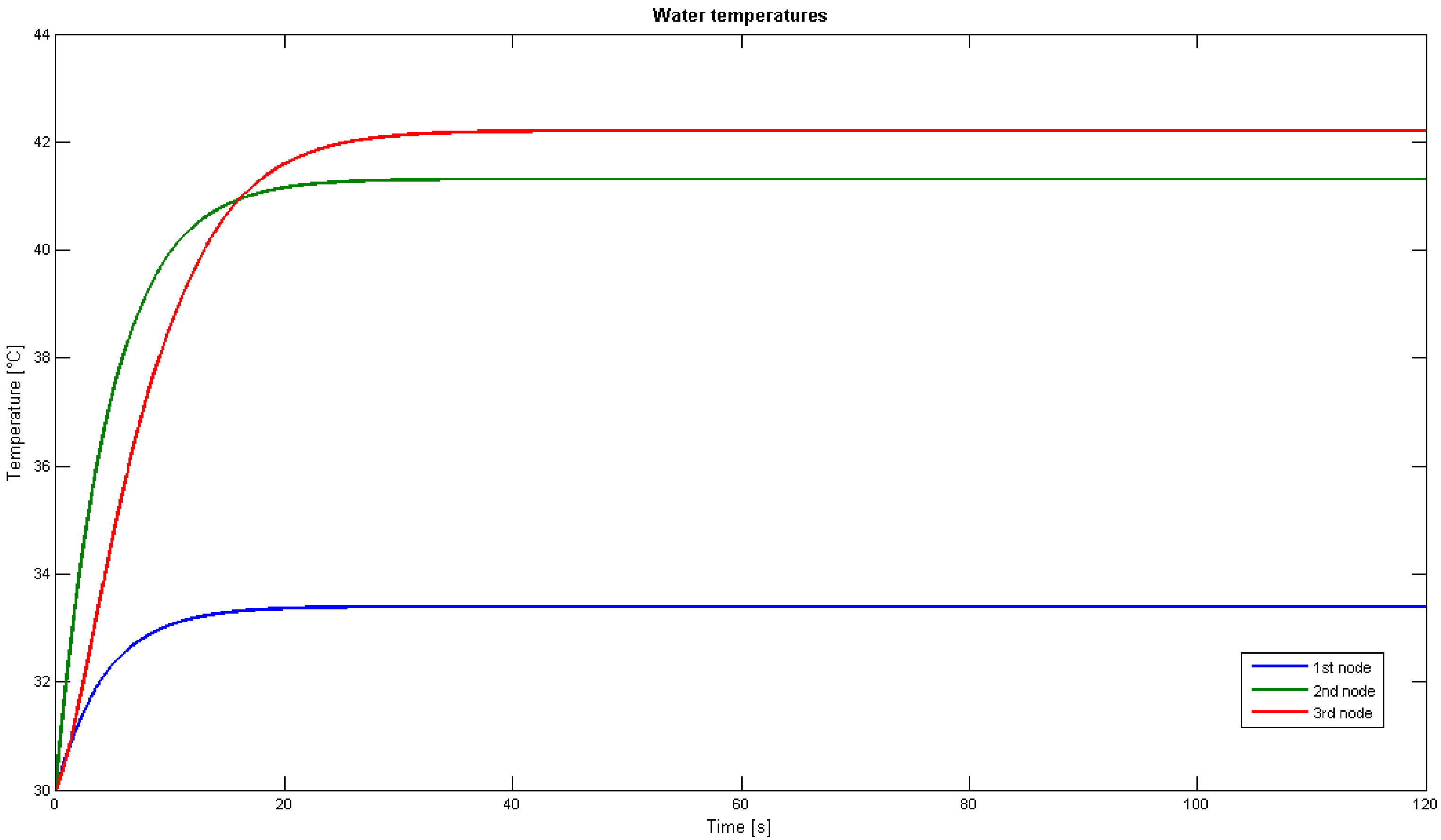

Figure 37 Water temperatures in the condenser as function of time.

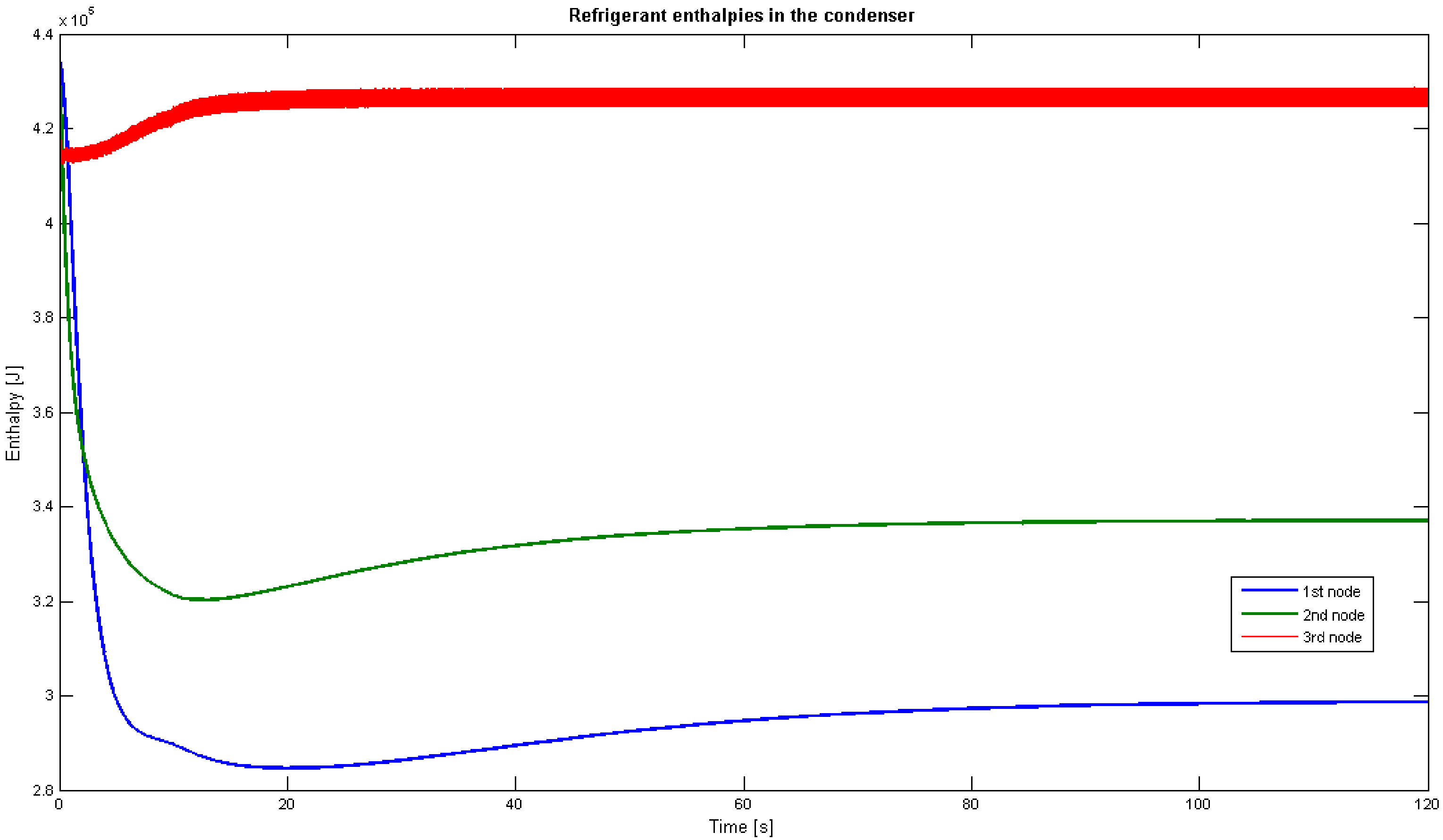

Figure 38 Refrigerant enthalpies in the condenser as function of time.

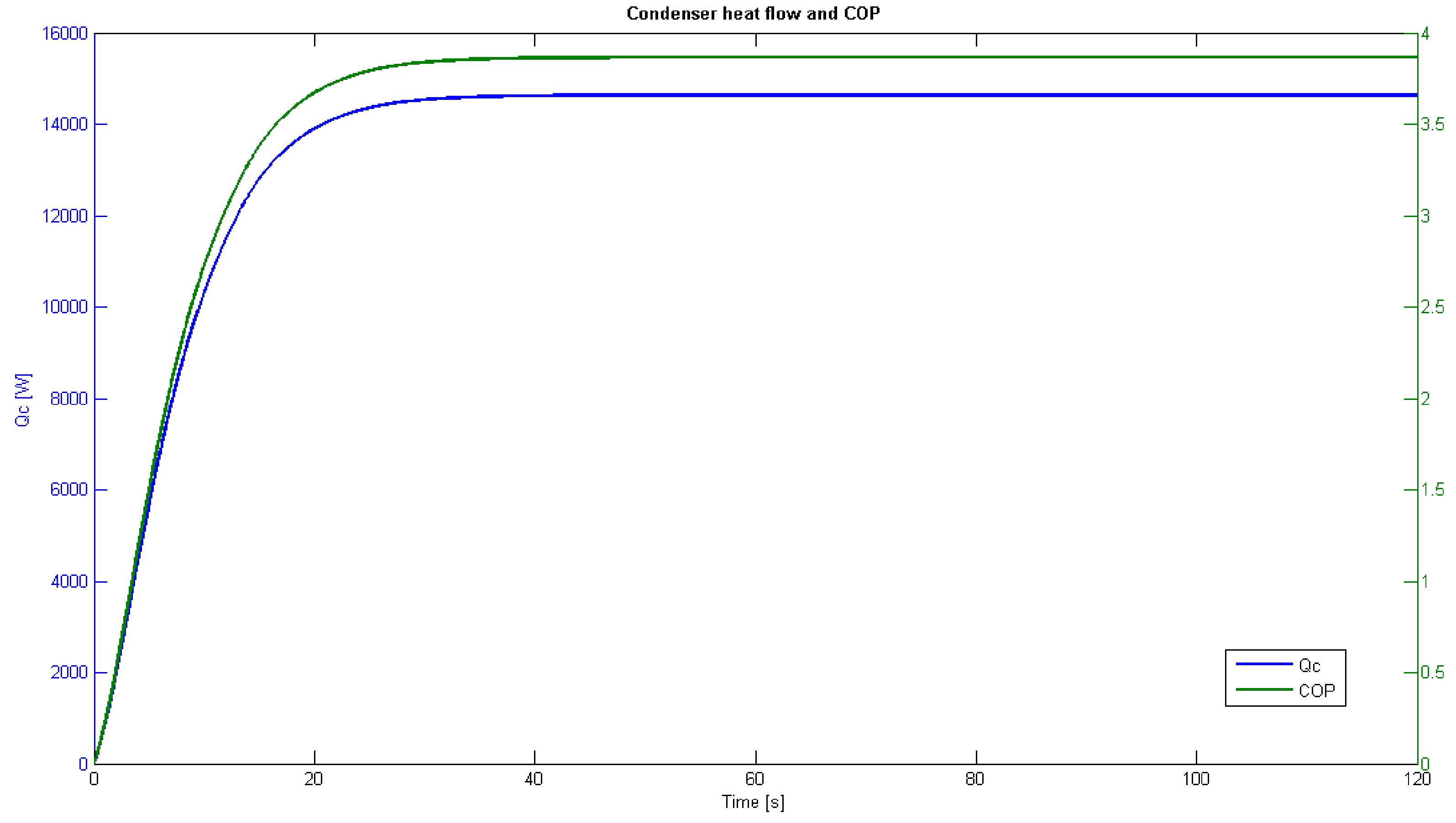

Figure 39 Condenser heat flow and COP as functions of time.

4.3 Comments

As shown in Figure 33, cylinder volume varies as function of time according to a sinusoidal law. The minimum value is given by the clearance volume, instead the maximum one is equal to the sum of the clearance volume and the stroke volume.

Figure 34 represents the pressure-volume compressor diagram, arranged by the values of the cylinder pressure as function of the cylinder volume. In this figure the different compression phases and cycles can be individuated. The area delimited by the diagram represents the compression input work.

In Figure 35 cylinder pressure is represented as function of time instead as function of cylinder volume. Evaporator and condenser pressure are highlighted in the chart. When cylinder pressure goes below evaporator pressure, the refrigerant is aspired in the working chamber. This is the suction phase. On the contrary, when cylinder pressure overtakes condenser pressure, discharge valve opens and superheated refrigerant exits the compressor.

The length of one cycle is related to the compressor speed and it is equal to 60/N (0.02 sec for chosen parameters). In every cycle, one suction and one discharge take place. Figure 36 shows compressor inlet and outlet mass flow rates as function of time. Discharge phase is shorter than the suction one. For this reason, it is characterized by higher instantaneous values. However, the area delimited by suction curve is equal to the area delimited by the discharge one. This means that the refrigerant mass aspired by the compressor is equal to the mass discharged.

Referring to the condenser, it is interesting to study the dynamic behaviour of water and refrigerant in the different nodes, as functions of time. As shown in Figure 37, water enters initially the condenser at 30°C. For this reason, water temperatures along the condenser start from the specified state. The heat transfer between water and refrigerant is not uniform in the three zones because of the different refrigerant conditions. The maximum amount of heat is transferred in the central section, where the refrigerant is a vapour/liquid mixture condensing over the water tubes. Here, once transient period is over, water temperature increases from 33°C to 41°C, before exiting the condenser at 42°C. Water transient period lasts about 40 sec. It is important to remember that it has been assumed a compressor steady state model: by considering a gradual increasing of compressor speed in start-up operations, the condenser transient period can result a little longer.

As per the refrigerant, in Figure 38 it is possible to figure out the trends of the average enthalpies inside the condenser. The initial condition is given by compressor output condition. Similarly to the water, the greatest part of the heat transfer is concentrated in the condensation region. The enthalpy of the refrigerant in the third zone (desuperheating region) is very oscillating due to the pulsation of the incoming mass flow rate discharged by the compressor. Refrigerant transient period lasts about 120 sec.

Figure 39 represents condenser heat flow and coefficient of performance. Since the compressor input work is constant (3.8 kW) along system operation, they are characterized by the same trend. Condenser heat flow starts from zero and reaches its maximum value of 14.6 kW after about 40 sec.

5 Conclusions and future development

The study of the dynamic behaviour of a heat pump system is a very actual and interesting topic but very complex at the same time. In literature it is possible to find several works related to this subject but they are generally very articulated and require years of studies. As consequence of this situation it is very hard to create a model both simple and accurate. In this work a new kind of approach has been investigated in order to create a model easily manageable in which both water and refrigerant dynamic can be studied. The result is a model with a low computational load.
Even if the compressor is not the most important part from a dynamic point of view, this works presents a relatively complicated model which provides precise information about the component.
As per the condenser, the model presented in this work reveals to be suitable for its purpose. Both water and refrigerant dynamics can be studied in a practical way. Water temperatures and refrigerant enthalpies trends are in accordance with starting expectations.
The model presented in this work does not aim to solve all the problems concerning the dynamic behaviour of heat pump systems, but it is a good starting point for future works.
The model needs to be validated with existing heat pump systems or by particular experimental data. After that, the model can be further developed by using different strategies. First of all it would be useful to link the model to a refrigerant properties program such as EES or RefProp in order to obtain more flexible simulations. Moreover, a deeper study of physical relations and heat transfer characteristics could lead to more accurate and detailed results.

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