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Digital predictive controller for power converters with variable output voltage

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Riassunto

La maggior parte dei convertitori di potenza è controllata per mezzo di regolatori analogici, tuttavia negli ultimi anni, grazie alla forte riduzione del prezzo dei componenti elettronici, si va affermando anche in questo campo l'uso di controllori digitali. Questi portano alcuni vantaggi rispetto ai comuni controllori analogici tra cui si può menzionare una maggiore flessibilità d'impiego, implementazione di tecniche di controllo avanzate, maggiore integrazione, minor sensibilità a variazioni di temperatura e possibilità di comunicazione remota. Per questi motivi i controllori digitali rappresentano un'importante attrattiva futura nel controllo di convertitori di potenza ed è in questo campo di studi che è incentrato questo lavoro.

In particolare il lavoro mira alla realizzazione di un controllo lineare per convertitori Buck, in grado di trarre vantaggio dall'informazione futura del riferimento di tensione di uscita per migliorarne l'inseguimento. Nella maggior parte dei casi questa informazione non è disponibile ma vi sono un gruppo ristretto di applicazioni in cui le variazioni del riferimento sono note in anticipo. Esempi sono gli amplificatori di potenza per trasmettitori di radio frequenza e microprocessori. Partendo da questa idea, mi sono proposto di realizzare un controllo capace di utilizzare con efficacia tali informazioni basato sulla tecnica di controllo predittivo.

Utilizzando un modello a tempo discreto del convertitore si ottiene una predizione dello stato del sistema in un finito numero di istanti successivi. La predizione del controllo viene confrontata con i futuri valori del riferimento e l'errore viene propriamente pesato per decidere l'attuale valore della variabile di controllo, in questo caso il tempo di chiusura degli interruttori.

La definizione della variabile di controllo non dipende solo dall'errore tra tensione di uscita e il suo riferimento, ma si è utilizzato un approccio leggermente differente. Si è considerato anche l'errore tra un appropriato valore di riferimento e le altre variabili di stato, che in questo caso sono la corrente dell'induttore del Buck e la variabile di stato associata all'azione integrale. Il problema che si è dovuto affrontare in tale approccio è come definire un riferimento per tutte le variabili di stato conoscendo il valore di riferimento della sola tensione di uscita. La soluzione proposta consiste nell'implementazione di una diversa descrizione del modello in variabili di stato differenziali ottenute tramite la trasformata delta.

Un secondo problema riguarda la realizzabilità del controllo e l'instabilità del sistema a fronte di ritardi del microcontrollore. Questo è stato risolto introducendo un compensatore del ritardo, basato sulla tecnica di predizione alla Kalman.

In seguito si è affrontato la scelta dei parametri di controllo, che nella teoria classica del controllo predittivo, sono scelti manualmente basandosi sulla risposta in anello chiuso del sistema in fase di simulazione. Tale metodo è apparso insufficiente per le difficoltà legate all'uso di valori futuri del riferimento di tensione. E' risultato difficile incontrare il giusto set di parametri in modo che il sistema sia stabile e la variabile di controllo non anticipi troppo la sua azione. Si è quindi utilizzato un metodo di ricerca basato sulla tecnica di algoritmo genetico per la selezione dei parametri.

Infine si è implemento lo schema di controllo a un passo di predizione, in un microcontrollore, ottenendo dati sperimentali sulla stabilità e realizzabilità del sistema.

Abstract

Nowadays most of the power converters are controlled by analog regulators but the consistent reduction of the price of electronic components over the past years is favoring the use of digital controllers this area. Among the advantages of digital controllers we can mention greater flexibility of use, the implementation of advanced control techniques, more integration, less sensitivity to temperature changes and the possibility of remote communication. For these reasons, digital controllers have become a major focus of research for power converters controls.

Following these research trends, my work is an attempt to implement a linear control for Buck converters to take advantage of future information of the output voltage reference and to improve its tracking. In most cases this information is not available but there are applications in which variations in the reference are known in advance. Examples include power amplifiers for radio frequency transmitters and microprocessors. Starting from this idea, I set out to create a control that can make effective use of such information based on the predictive control technique.

A discrete-time model of the converter is used to obtain a prediction of the system state in a finite number of moments later. The prediction is compared with the future values of the reference and the error is

properly weighed to determine the current value of the control variable, in this case, the closing time of the switches.

The definition of control variable does not depend only on the error between output voltage and its reference, but we use a slightly different approach. We considered also the error between a proper reference and the other state variables, which are, in this case of the Buck converter, the inductor current and the state variable associated with the integral action. We had to solve the problem of how to define a reference for all state variables from the reference value of the single output voltage. The proposed solution is based on a different description model in differential state variables obtained from the delta transform.

A second problem concerns the feasibility of control and the instability of the overall system due to delays in the microcontroller. This was solved by introducing a delay compensator, based on the technique of the Kalman prediction.

Then we faced the choice of control parameters, which, in the classical theory of predictive control, are selected manually depending on the response of the closed-loop system in simulation. This method appeared to be insufficient for the difficulties associated with the use of future values of the voltage reference. It was difficult to find the right set of parameters to make the system stable and the control variable not too much in action. Therefore, we used a search method based on the genetic algorithm for the selection of parameters.

Finally, the control scheme is implemented with one step of prediction in a microcontroller, obtaining experimental data on the stability and feasibility of the system.

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List of abbreviations

ADC	Analog Digital Converter
CCM	Continues Conduction Mode
CPU	Central Processing Unit
DCM	Discontinuous Conduction Mode
DSP	Digital Signal Processor
DVS	Dynamic Voltage Scaling
EA	Evolutionary Algorithm
EER	Envelope Elimination and Restoration
FPGA	Field Programmable Gate Array
GA	Genetic Algorithm
MOSFET	Metal Oxide Semiconductor Field Effect Transistor
MPC	Model Predictive Control
PWM	Pulse Width Modulation
RF	Radio Frequency
SMPS	Switch Mode Power Supply
SISO	Single-input single-output

Introduction

Thanks to a growing sensibility towards the so-called *smart power* concept, much research has been addressed to reduce the economic and environmental impact of electronic components. In this respect a developing field of research in power electronics is working on the integration of control circuits and power devices on the same semiconductor chip in order to minimize the cost and size of all electronic components. From this point of view digital controllers can play a really significant role in Switch Mode Power Supplies (SMPS), they may produce relevant results in terms of efficiency due to high level of integration and the implementation of advanced control techniques [1].

The possibility to implement sophisticated control law techniques is probably the main advantage of digital controllers compared to the analog ones. As other benefits they allow largely automated design flow to reduce development time and less sensitivity to tolerances and temperature variations, which are of major concern for analog controllers. All these advantages are well known in power electronics for a long time, but due to the high cost and relatively low processing speed, digital controllers have been applied only on high cost or high power (low switching frequency) applications, e.g. high power three-phase converters.

However, the price of digital controllers has been constantly decreasing in the past years, as shown in Fig.1 and their performances have increased. For these reasons digital controllers have become a functional and convenient option to analog controllers in many applications.

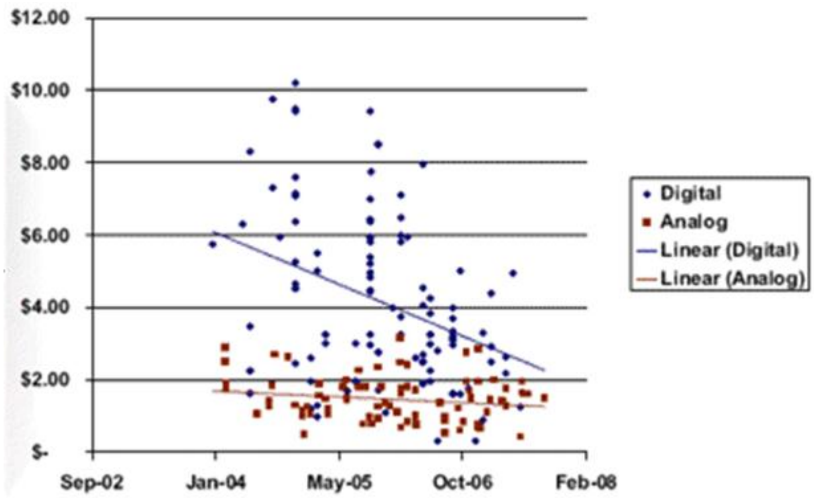

Fig.1 Comparison between analog and digital controllers cost

Among digital control I focused my studies on discrete-time Model Predictive Control (MPC) and on the possibility to exploit predicted information of the reference.

In some application advanced knowledge of voltage references area available and may be used in order to improve reference tracking. An example is given by and the Envelope Elimination and Restoration (EER) technique, used in Radio Frequency (RF) transmitters to cut down the losses in the amplifier [2].

Fig.2 shows the power stage of a RF transmitter. The input signal is decomposed by a Digital Signal Processor (DSP) or a Field Programmable Gate Array (FPGA) in the envelope component that contains the information of the amplitude and the phase modulation that contains the information of frequency components. The envelope is the reference voltage for the DC/DC converter supplying the amplifier with a voltage that follows the envelope of the input signal. In the low side the local oscillator adds a high frequency component (carrier) to the low frequency phase modulation in order to increase the transmission distance of the message signal.

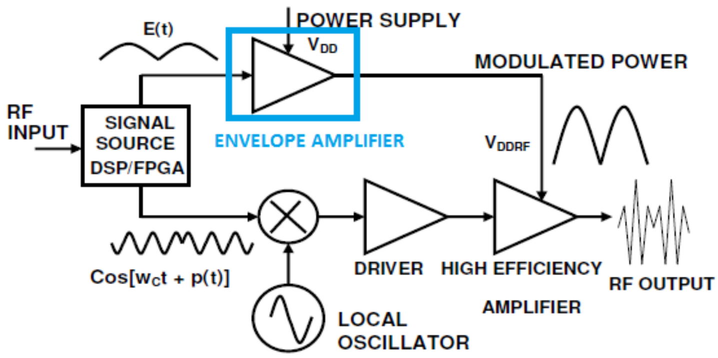

Fig.2 Power amplifier RF transmitter

Adjusting the amplifier voltage to follow the envelope signal as shown Fig.3 we can reduce the losses in the amplifier, where these are proportional to the difference between the amplifier voltage and the envelope signal as shown in Fig.4.

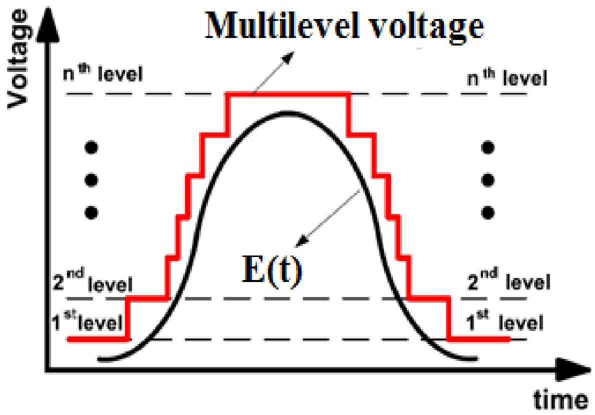

Fig.3 Multilevel voltage

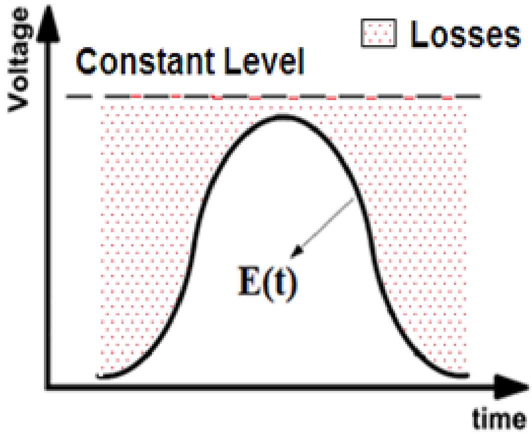

Fig.4 Amplifier power losses

With the signal source we can add some delay in the phase modulation, and using this time to evaluate the best way to track the envelope signal, since it is important that the envelope and the phase modulation are in phase. Delaying the phase modulation respect to the envelope is equivalent to provide the regulator of the DC/DC converter with future information of the reference. This possibility prompted me to consider MPC for this application because of its ability to handle predictive future information of the reference and the system.

Control systems that try to use future information, like the *repetitive control* [3], still present performance problems and in particular low efficiency of the buck converter at high frequency. The predictive control might present a different solution and it should effectively use the predicted information to improve the tracking and to reduce the total losses by decreasing the switching frequency.

The future values of the voltage reference are compared with a prediction of the output voltage. The prediction, performed by the MPC, is based on the discrete-time model of the converter and it is evaluated in open loop. Actual and future errors between reference and predicted output voltage are properly weighted to evaluate the value of the control variable, the duty cycle (closing time on switching period) of the switches. Knowing the future error the controller will try to anticipate its action, in this way we expect some improvement in the reference tracking. In particular, the controller should track the reference even when its frequency is comparable with the switching frequency of the converter. As another benefit the control should be able to follow high frequency variation of the reference without increasing the switching frequency. Since the power consumption of the converter increases at higher switching frequency we expect that the proposed solution should be able to reach the same performance in terms of tracking than a conventional regulator with lower power losses in the converter.

Another possible application could be Dynamic Voltage Scaling (DVS) [4] where the supply voltage of microprocessors is modulated depending on the demand of calculation power. A microprocessor needs a high voltage in order to process the information faster, but if it is not working at full power, a lower voltage will reduce considerably dynamic losses. In this case the reference is set by a prediction algorithm, which, in order to execute a certain number of instructions, evaluates the required future reference voltage.

Also in this second example we expect that by the use of MPC we are able to exploit the future reference information to improve the tracking and reduce the switching frequency of the power supply.

After this introduction which explained briefly the motivation and the objective of this work, the next two chapters provide some background.

The first chapter outlines the systematic way to obtain a time invariant linear model of a SMPS and subsequently the discretization method is presented.

The second chapter introduces some basic knowledge of MPC: cost function, open loop prediction and receding horizon. Finally the used method to integrate an integral action is presented and its two possible formulations are discussed.

In the third chapter the proposed approach is outlined, highlighting the difference with other similar control schemes. The problematic of its feasibility will be discussed and the delay compensator for the stability of the overall system is explained.

The fourth chapter will describe the design method based on Genetic Algorithm (GA). The simulation results are compared with a conventional regulator.

The fifth chapter describes the system used and the experimental results obtained with one predictive step are shown.

Finally we present the conclusions of these work and possible future developments.

Chapter 1:

Modeling a SMPS

Depending on its purpose, a model can be more or less complex and reproduce with higher or lower precision the behavior of the system. So, for simulation purpose the model has high level of detail in order to have an accurate representation of the plant, while for control design the model is simpler, but still able to reproduce the dominant behavior of system.

In this chapter I will describe the simulation model of a converter and, subsequently, I will outline the method to obtain a control-oriented model, on the particular case of a Buck Converter but valid for a general SMPS. Finally I will derive the discrete time model that I will use in the regulator design.

1.1. Synchronous Buck converter

The method for deriving a time-discrete control model will be described on the example of a Buck converter [7] with synchronous rectification whose circuital model is shown in Fig.1.1. However, this is a general method that can be applied to any SMPS.

The synchronous rectification refers to the substitution of the low side diode with a Metal Oxide Semiconductor Field Effect Transistors (MOSFETs). This solution is particularly indicated for low power applications since the power consumption of a MOSFET depends mainly on its resistance that can be decreased by parallelizing more MOSFETs. Besides, the synchronous rectification avoids working in Discontinuous Conduction Mode (DCM) [3].

In the example, the output filter has an inductor of 27 uH a capacitor of 10 uF and a nominal load of 2.7Ω. The voltage supply is at 12V. With a nominal voltage of 3.3 V it determines a nominal duty cycle of 27.5%.

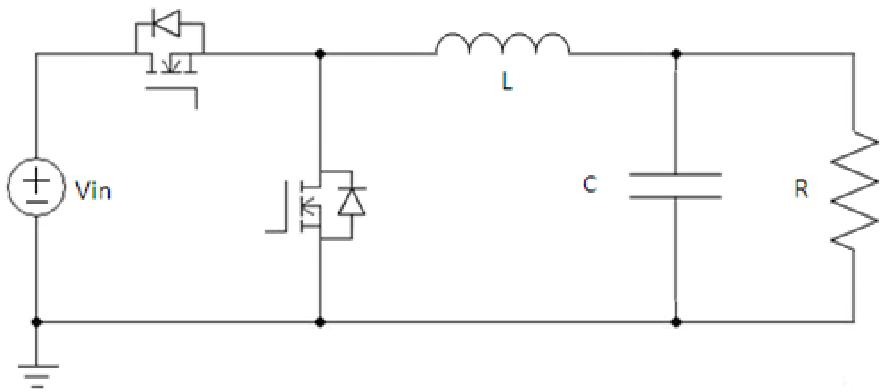

Fig.1.1 Synchronous Buck Converter

The MOSFETs are driven with complementary signal at the switching frequency of 100 kHz. At nominal condition, the averaged value of the inductor current is 1.22A, with a ripple of the 50% that determines a ripple output voltage of0.153V.

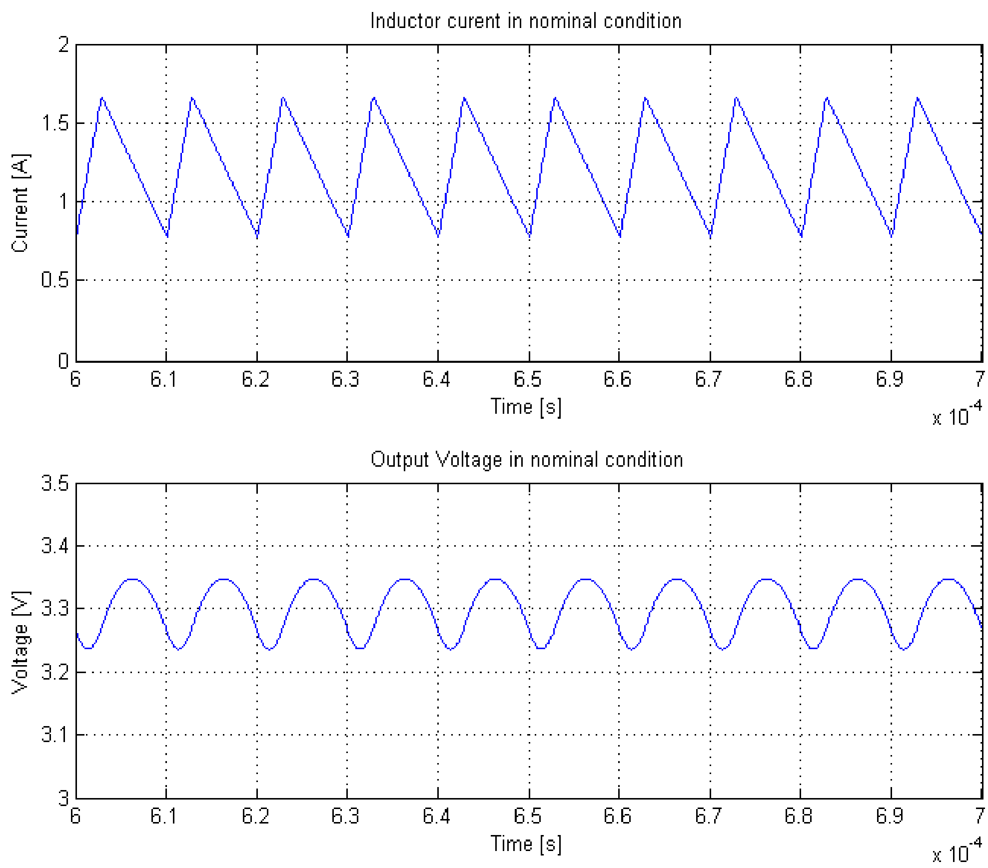

Fig.1.2 Steady-state waveforms

1.2. Switching Model and Averaged model

Changing the status of the switches, a converter changes its configuration over time, resulting in a characteristic of time variance typical in SMPS. The main purpose of the Switching Model is to represent this feature, but also to consider parasitic elements, usually neglected in the control-oriented model. These elements, such as the inductor resistance or the

MOSFET on resistance, are useful in order to have a better estimation of the system global behavior.

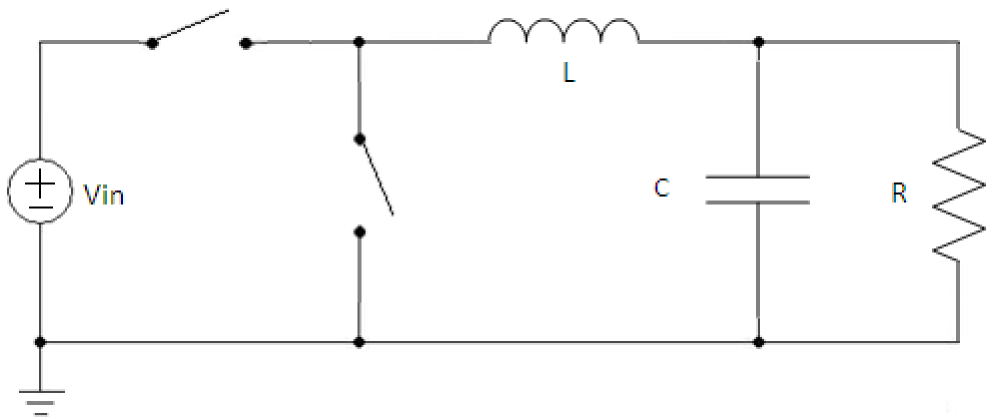

Fig.1.3 Switching model of the Buck Converter

The Switching Model, shown in the Fig.1.3, is sufficiently detailed to simulate properly the operation of the Buck converter. On the other hand it is too complicated to synthesize a regulator. In order to achieve a control-oriented model, it is necessary, first, to eliminate the characteristic of time variance of the converter.

In the case of the Buck, the switches determine two different configurations during a switching period, as it is possible to see in figure 1.4. The on-off state is referred to the high-side switch, while the low-side one is complementary.

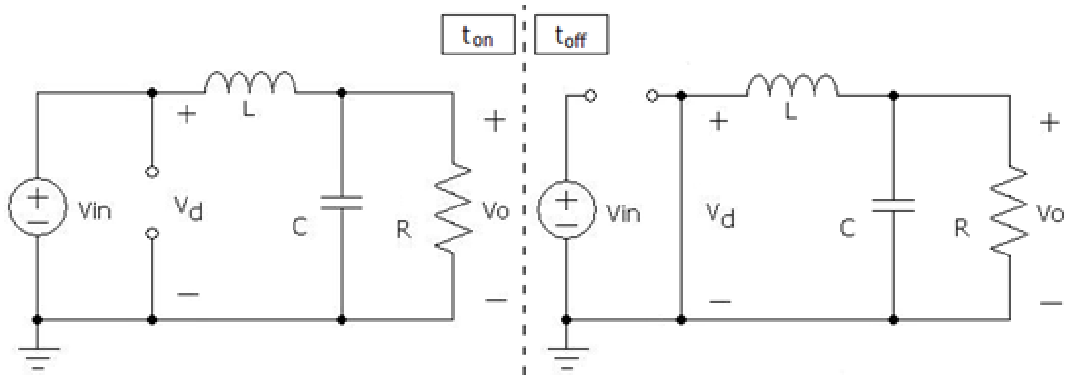

Fig.1.4 Configurations of Buck in CCM

This behavior determines the presence of small ripples on the output current and output voltage waveforms, if the converter is working in Continues Conduction Mode (CCM) and if the values of capacitance and inductance are chosen properly. Under these assumptions, we can obtain a simpler model that reproduces the dominant behavior of the system by eliminating the ripples. This is equivalent to average over one switching period the signals of the converter, obtaining the averaged model of the converter that is able to reproduce the large signal behavior.

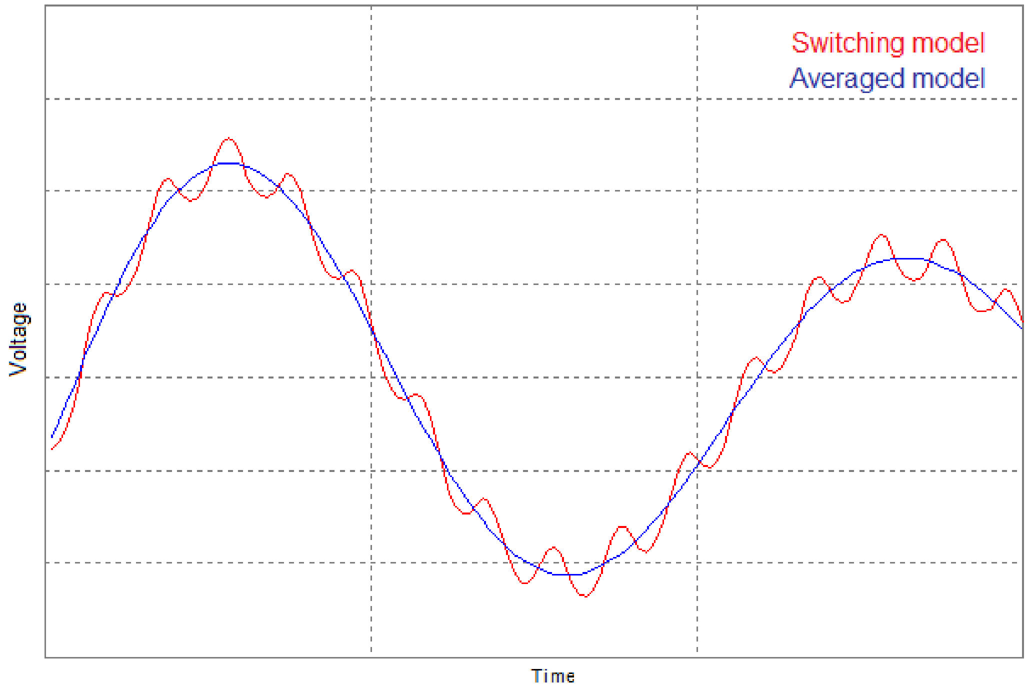

Fig.1.5 Switching model and Averaged model waveforms

A systematic way [4][5] to derive a time-invariant model from a switching circuit is to replace each switch for a dependent current or voltage source, where its average value is calculated, in each interval, as function of the average quantities of the circuit. In this we should take into account some rules, because not all the possible configurations of dependent sources are suitable: an inductive branch cannot be driven by dependent current sources, as well as a capacitive branch by a voltage source. Applying this method on the Buck Converter, we obtain the electronic circuit shown in fig.1.6. Also another configuration with the voltage source on the high side and the current one on the low side would be correct. However, the chosen configuration allows simplifying more the circuit by eliminating the current dependent source in the high side.

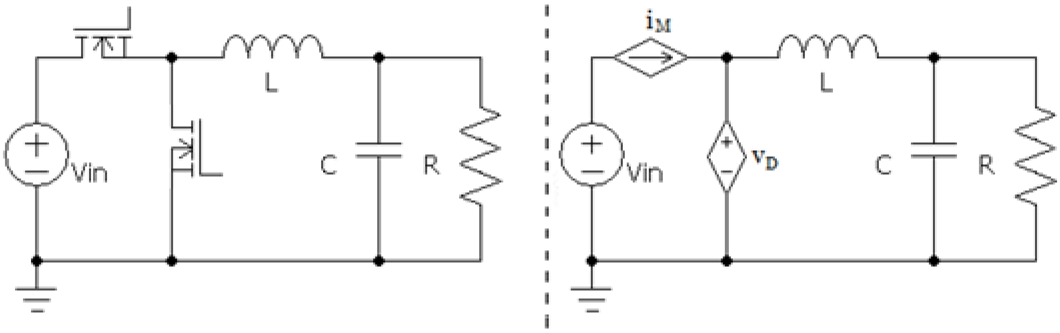

Fig.1.6 General method to derive the Averaged Model

The circuit obtained, shown in fig.1.7, is equivalent to the previous one. In fact, once the voltage on the low side is known, the state variable of the circuit and the entire behavior of the system is determined, also if the information regarding the input current is lost.

After this, the averaged value of the sources needs to be evaluated in each configuration as function of the independent variables of the system. As said above, in the Buck there are two configurations. When high side MOSFET is conducting (t_{on}), the voltage on the other MOSFET is equal to the input. Meanwhile, when the main switch is open (t_{off}), the voltage on the auxiliary MOSFET is zero because it is conducting. This draws the waveform shown in figure 1.7 and the expression of the Averaged voltage and the large signal model in (1.2).

$$\begin{cases} \frac{d}{dt} v_c(t) = \frac{1}{C}\left(i_l(t) - \frac{v_c(t)}{R}\right) \\ \frac{d}{dt} i_l(t) \ = \frac{1}{L}(v_D(t) - v_c(t)) \end{cases}, \quad v_D = \frac{v_{in} t_{on}}{T_{sw}} = v_{in}(t) d(t) \qquad (1.2)$$

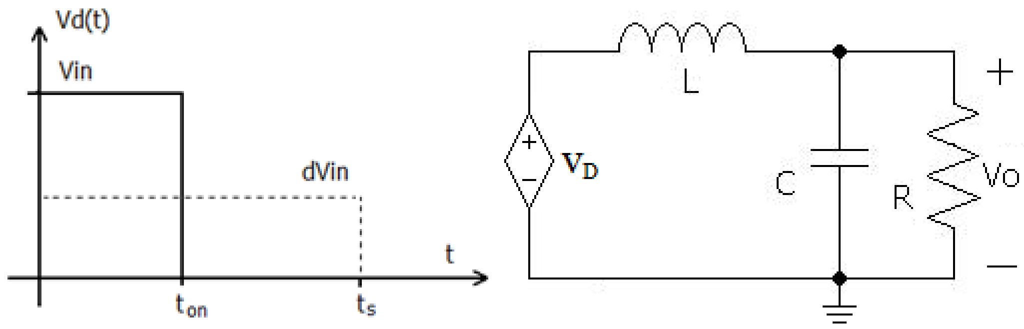

Fig.1.7 Averaged Model and averaged waveform

We should notice that the voltage on the MOSFET depends on the input voltage (vin) and on the duty cycle (d) that is the control variable. If the input is considered variable in time, the system is non-linear. Therefore, the next step in the modeling of the converter is to linearize the large signal model around the operation point, in order to arrive to a control-oriented description. Before this, we should validate the Averaged Model.

1.3. Validation of the Averaged Model

In order to be sure that the large signal model is correct, it is necessary to compare it with the Switching model in simulation. It means to evaluate the responses of the state variables under load variation and variation in the duty cycle. The figure 1.8 shows the output waveforms of the switching model and the averaged model when the duty cycle is increased from its nominal to 37% of the switching period. We can notice that the large signal model depreciates the output ripple debt to the switching frequency, but shows the same large dynamic.

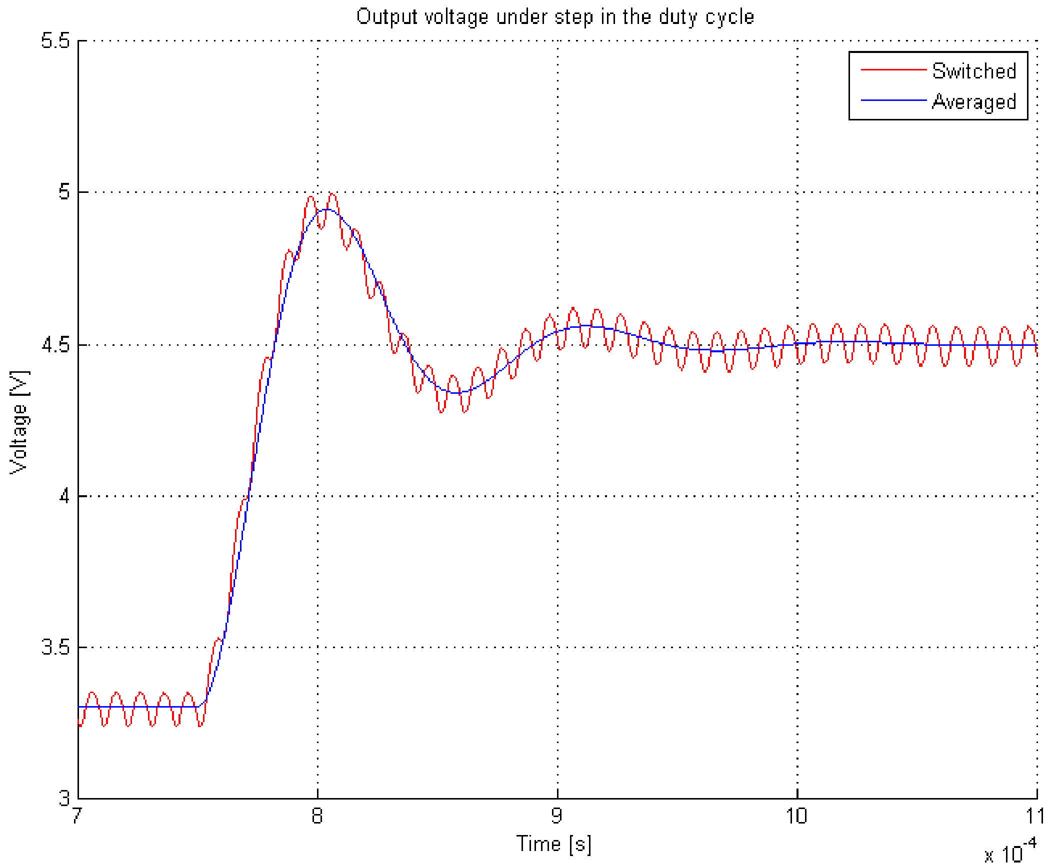

Fig.1.8 Output Voltage response under duty cycle step

Figure1.9 shows the behavior of the inductor currents under the same condition. Also in this case, the averaged model reproduces precisely the dominant behavior. Once we have validated all the system state variables we are sure that all other variables of the system are well described. Fig.1.10 and fig.1.11 show again the inductor current and capacitor voltage waveforms, under 21% of load variation, changed from 2.7Ω to 2.13Ω.

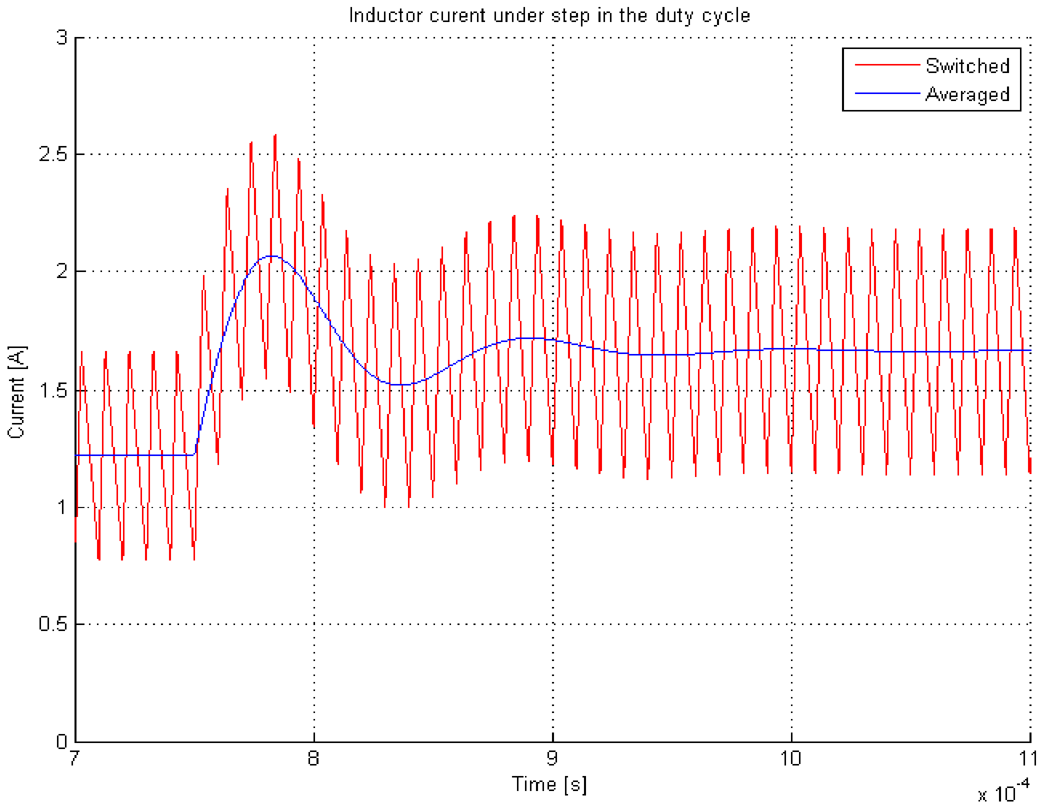

Fig.1.9 Output Current response under duty cycle step

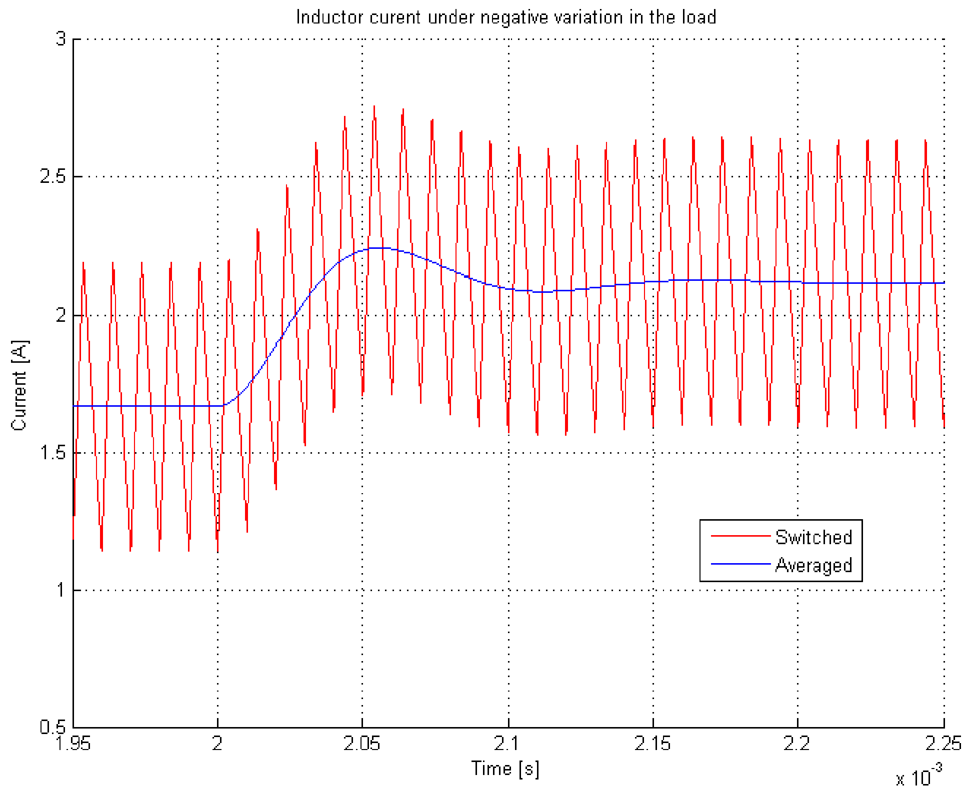

Fig.1.10Validation under load variation

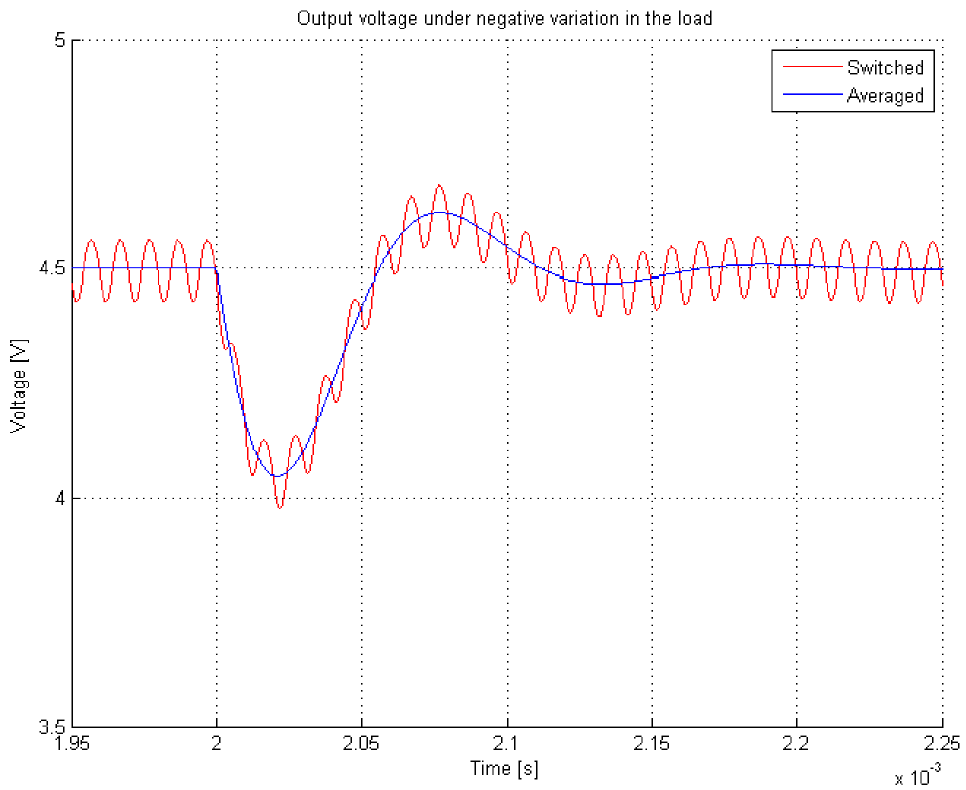

Fig.1.11 Output Validation under load variation load

1.4. Discrete time model

If, at first approximation, we consider the voltage supply constant, the resulting system is not only time-invariant but also linear. Because we are interested on the effect of possible variations of the input voltage, we consider the input voltage as a variable signal; therefore the system needs to be linearized.

$$\begin{cases} 0 = \left(i_l - \dfrac{v_c}{R}\right) \\ 0 = (DV_{in} - v_c) \end{cases} \tag{1.3}$$

The operation point is defined by nominal values of duty cycle (D) and the input voltage (V_{in}). The system of linear equations (1.3) defines the nominal value of inductor current and capacitor voltage.

$$\begin{cases} L\dfrac{d\tilde{\imath}_l}{dt} = \left(-\tilde{v}_c + V_{in}\tilde{d} + D\tilde{v}_{in}\right) \\ C\dfrac{d\tilde{v}_c}{dt} = \left(\tilde{\imath}_l - \dfrac{\tilde{v}_c}{R}\right) \end{cases} \tag{1.4}$$

The large signal system is linearized around this point, to derive the small signal description in (1.4), that can be also described in its matrix form as in (1.5), where the value of the matrix elements are evaluated as in (1.6).

$$\begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t) \\ y(t) = C\underline{x}(t) \end{cases} \tag{1.5}$$

$$A = \begin{bmatrix} -\dfrac{1}{RC} & \dfrac{1}{C} \\ -\dfrac{1}{L} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ \dfrac{V_{in}}{L} & \dfrac{D}{L} \end{bmatrix} \tag{1.6}$$
$$C = [\,1 \quad 0\,]$$

Once obtained the linearized model, it is possible to evaluate the discrete-time model. There are different methods to obtain a discretized model, each one with its advantages and drawbacks. Most of them are based on substituting the variable s of the corresponding transfer function with a proper function in the transformer z. All of them are good approximation of continues time model. However, there is another method that describes correctly the evolution of the system without approximation. This consists of applying matrix exponential and zero order hold to the state space system in continues time [8], yielding to the equivalent system of difference equations.

$$\begin{cases} \underline{x}(k+1) = A_d\underline{x}(k) + B_d\underline{u}(k) \\ \qquad y(k) = C_d\underline{x}(k) \end{cases} \tag{1.7}$$

The value of the matrices obtained by applying the discretization to the state-space continues system is shown in (1.8), where T_s is the sampling time.

$$A_d = e^{AT_s} B_d = \int_0^{T_s} e^{A\sigma} B \, d\sigma \quad C_d = C \tag{1.8}$$

The absence of approximation of this method is demonstrated by the simulation results in fig1.2, in which the voltage waveform of the discrete and continues time models under step reference of 3.3V, show a perfect matching.

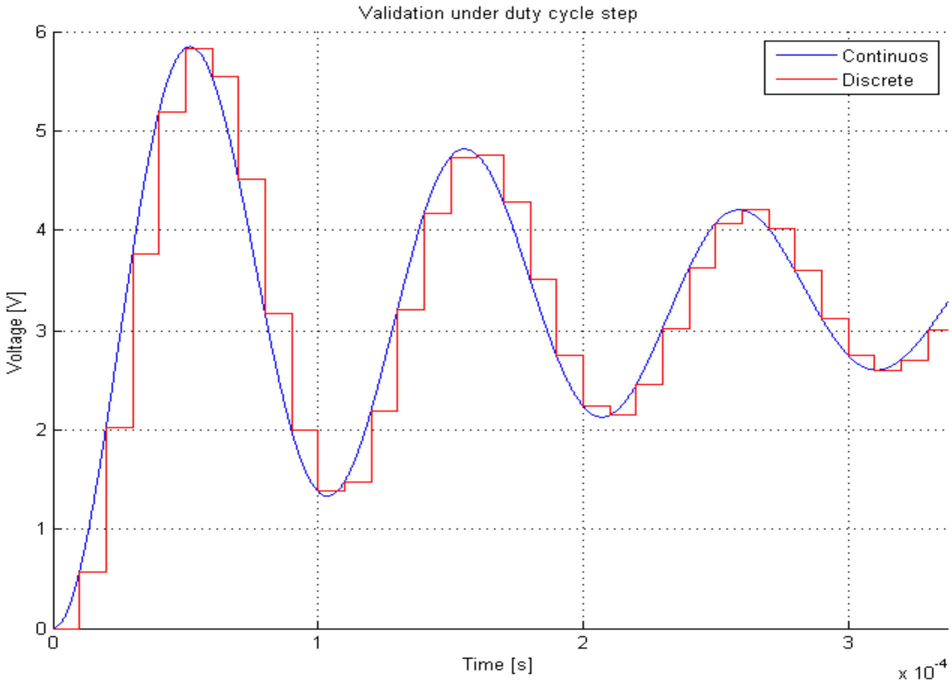

Fig.1.12Discrete time validation

Chapter 2:

Discrete–time MPC

There are different approaches to the MPC, but they all have some common features, as the definition of a cost function the open loop prediction and the receding horizon. This chapter will introduce the MPC starting from some basic knowledge, and eventually two methods to add the integral action and the associated anti wind-up schemes.

2.1. Model Predictive Control

The MPC [9][10][11] is a control strategy to optimize future control inputs on the basis of foreseen plant responses, where these are predicted, generally in open-loop, by a discrete time control model.

The idea consists of predicting the open loop response of the system, by the knowledge of its model, and knowing the future reference values it is possible to determine the future control action in order to decrease the error between reference and predicted output, as shown in fig.2.1. Therefore, the control action is evaluated as the optimum sequence of values that minimizes a function cost depending on the error, whose form can be expressed in different way.

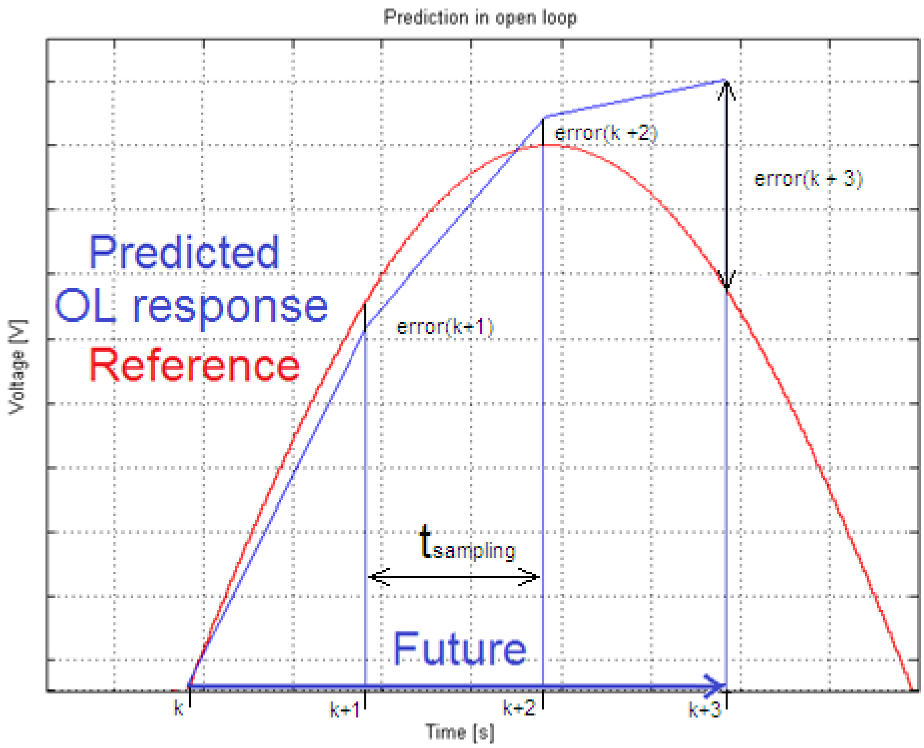

Fig.2.1Idea of MPC

Choosing the function is a key decision because it defines the values of the control variable, depending on what we would like to minimize. It is really interesting because allows us taking into account different terms apart from the error, as for instance, losses or control effort.

For problems of reference tracking, the cost function usually includes a quadratic term of the error between reference and output and a quadratic term of the control action, as shown in (2.1).

$$J(e(k),u(\cdot),k) = \sum_{i=0}^{N} [e(k+i)^T Q_i e(k+i) + u(k+i)^T R_i u(k+i)] \qquad (2.1)$$

$$e(k+i) = y^{\circ}(k+i) - y(k+i)$$

The minimization of this function finds the optimum value for the input variables that reduce the error, keeping limited the control input. It is important to underline that the optimum solution will depend on the weight coefficient Q and R, and on the prediction horizon N; these parameters are a degree of freedom. The control will tend to be faster if the value of Q is higher than R, but this will cause higher stress on the control variable with the risk of entering often in saturation. Finally the right value to assign to these two parameters will be a designer decision, based on the behavior of inputs and outputs during the simulation of the closed loop.

MPC gives the possibility of creating nonlinear controls, taking into account constraints on the variables, as for instance the saturation. But in order realize a non-linear regulator it is necessary to perform the minimization of the cost function in line. However, this work is concentrated more on the possibilities given by using the anticipated knowledge of the future reference, assuming this known a priori, without considering constraints of the system. Usually the future values of the reference are considered to be equal to its current value, due to the fact that in general these value are not supposed to be known, but for the particular applications discussed in the introduction, this can be the case.

In this case, the key point of the regulator is represented by its ability to predict the future response of the system by the knowledge of its model.

2.2. Open loop Prediction

The second common feature of all the versions of the predictive control is the evaluation of the future response of the plant through the discrete time model of the system.

$$\begin{cases} x(k+1) = A\,x(k) + Bu(k) \\ y(k) = C\,x(k) \end{cases} \tag{2.2}$$

Considering, for instance, the problems of reference tracking, we can derive from the discrete time model the system in (2.3), in order to evaluate the future value of the state vector. In the system the future values of the output can be evaluated knowing the current value of the state vector and the future values of the inputs.

$$Y(k+1) = A_C x(k) + B_C U(k) \tag{2.3}$$

$$\begin{bmatrix} y(k+1) \\ \vdots \\ y(k+N) \end{bmatrix} = \begin{bmatrix} CA \\ \vdots \\ CA^n \end{bmatrix} x(k) + \begin{bmatrix} CB & 0 & 0 \\ \vdots & \ddots & \vdots \\ CA^{n-1}B & \dots & CB \end{bmatrix} \begin{bmatrix} u(k) \\ \vdots \\ u(k+N-1) \end{bmatrix} \tag{2.3b}$$

In this method the future values of the outputs are evaluated only by the model of the plant that, as we know, is affected by uncertainty. In principle there is no feedback that corrects the prediction error, but certain robustness can be reached through the receding horizon technique explained in paragraph 2.3.

Ignoring the model constraints, the minimization of the cost function leads to a linear control law. The cost function is rewritten in a matrix form (2.4), in order to substitute the equation of the predicted state vector.

$$\overline{J}\left(x(k),U(\cdot),k\right)=\left(Y^{\circ}(k+1)-Y(k+1)\right)^{T}Q_{s}\left(Y^{\circ}(k+1)-Y(k+1)\right)+U(k)^{T}R_{s}U(k) \tag{2.4}$$

$$Q_s=\begin{bmatrix} Q_1 & 0 & \dots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & Q_{N-1} & 0 \\ 0 & \dots & 0 & S \end{bmatrix} \qquad R_s=\begin{bmatrix} R_0 & 0 & \dots & 0 \\ 0 & R_1 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & R_{N-1} \end{bmatrix}$$

The element Q_0 has disappeared because the current value of the output does not depend on the regulator, as the system is strictly proper. The new formulation of the cost function in (2.5) is obtained by substituting the expression of the predicted outputs (2.3) in (2.4). We can minimize the cost function respect to the control variable, by equating to zero (2.5b) the derivative of (2.5) respect to U.

$$\overline{J}\left(x(k),U(\cdot),k\right)=U(k)^T(B_c'Q_cB_c+R_c)_sU(k)) - 2\left(Y°(k+1)-Y(k+1)\right)^TQ_cB_cU(k \tag{2.5}$$

$$U°(k)=(B_c'Q_cB_c+R_c)^{-1}B_c^TQ_c(Y°(k+1)-Y(k+1)) \tag{2.5b}$$

Without considering the constraints of the system, it leads to the closed form expression of the control law in (2.6).

$$U°(k)=K_{mpc}\left(Y°(k+1)-A_Cx(k)\right)(\qquad K_{mpc}=(B_c'Q_cB_c+R_c)^{-1}B_c'Q_c \tag{2.6}$$

2.3. Receding Horizon

This technique is another important feature presented in all the MPC approaches. As we said, the input is evaluated by minimizing the cost function on an adequately defined prediction horizon. The receding horizon technique consists on shifting the prediction window at each step after that the first value of the input vector has been applied to the system (2.7).

$$u(k)=[1\,0\ \dots\ 0]K_{mpc}\left(Y°(k+1)-A_Cx(k)\right) \tag{2.7}$$

Hence, the output is measured again, and its prediction is performed based on the new information acquired. This allows a kind of feedback on the prediction because at each step the future values are predicted based on the last measure of the outputs.
Eventually the Receding Horizon introduces a feedback into the MPC law, thus providing a degree of robustness, and it also compensate its limit by continually shifting over time.

2.4. Integral action

Taking a look at the control law obtained (2.5), it is clear that the MPC in its classical formulation is not suitable to guarantee a steady-state error to zero.

A solution to this problem is to explicitly add an integral action in the regulator, considering it as part of the plant, even if there are other solutions to consider the integration of the error as an additional term of the cost function.

There are two different way to perform integration in discrete time for the first approach: using a strictly proper integrator or a proper one. Both have some advantages and disadvantages, as examined below, in order to be able to choose which one is more appropriate for the purpose of this work.

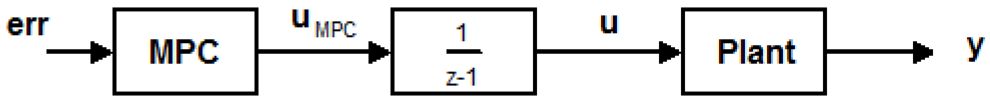

Fig.2.2 Strictly proper integral action

The most important advantage of the strictly proper integrator is its feasibility; the control variable depends only on the previous value of the integral action (d) and of the input (u_{MPC}) (2.8). It means we have an entire switching period in order to evaluate the next value of the integral action.

$$u(k+1) = u(k) + u_{MPC}(k) \tag{2.8}$$

On the other hand there are some pitfalls. As said, this technique of inserting the integral action in the MPC control law yields to consider the integrator as part of the system. Therefore the integral control law has to be added in the equations of the system, obtaining its new expression as in (2.9 and 2.9b).

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ u(k+1) = \ u(k) + \ u_{MPC}(k) \end{cases} \tag{2.9}$$

$$\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_{MPC}(k) \tag{2.9b}$$

Reminding that the prediction is performed in open loop, it is possible to see that the expression of the integral action adds some more uncertainty in the prediction. The value of the control variable is supposed to be known at each time, because it is assigned by the regulator, but clearly the real value of the input that acts on the plant might be different, due to possible disturbance. This method is not robust against disturbances on the control variable.

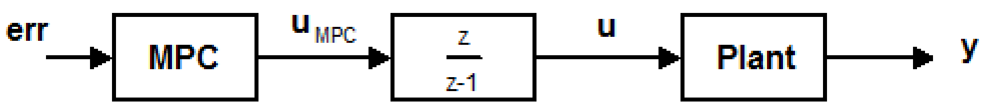

Fig.2.3 Proper integral action

On the contrary, the proper formulation of the integral action (2.10) does not produce this effect but it has the important disadvantage of not being feasible.

$$u(k) = u(k-1) + u_{MPC}(k) \tag{2.10}$$

The current value of the integral action depends on its previous value and on the current value of the input. It means that the controller should be able to evaluate the control input value in zero time not to produce a delay in the control action that can affect the stability of the system.

On other hand the integrator yields to an interesting formulation of the plant model for two reasons: it simplifies the anti-wind up technique and, more importantly, it simplifies the definition of the reference, as it will be explained in chapter 3.

In order to consider the proper integrator as part of the system for this, it is useful to rewrite the expression of the model introducing a new definition of the state variables, called delta transform [12]. It yields to rewrite the state variables as described in (2.11).

$$\begin{aligned} &\delta x_i(k+1) = x_i(k+1) - x_i(k) \\ &u_{MPC}(k) = u(k) - u(k-1) \end{aligned} \tag{2.11}$$

Starting from the state space model, after some modifications (2.12a) it is possible to rewrite the system in the new variables as in (2.12b), whose matrix form is shown in (2.13).

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + x(k) - x(k) \\ y(k) = Cx(k) + y(k+1) - y(k+1) \end{cases}$$

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + x(k) - Ax(k-1) - Bu(k-1) \\ y(k) = Cx(k) + Cx(k+1) - y(k+1) \end{cases} \tag{2.12a}$$

$$\begin{cases} x(k+1) - x(k) = A(x(k) - x(k-1)) + B(u(k) - u(k-1)) \\ y(k+1) = y(k) + C(x(k+1) - x(k)) \end{cases}$$

$$\begin{cases} \delta x(k+1) = A\delta x(k) + B\,u_{MPC}(k) \\ y(k+1) = y(k) + CA\,\delta x(k) + CB\,u_{MPC}(k) \end{cases} \quad (2.12b)$$

$$\begin{bmatrix} \delta x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix} \begin{bmatrix} \delta x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B \\ CB \end{bmatrix} u_{MPC}(k)$$
$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \delta x(k) \\ y(k) \end{bmatrix} \quad (2.13)$$

The new system presents a new state-variable that reflects the introduction of the integral action, with the difference the new variable is not the integral signal, but the output. It means the prediction of the state is now performed by the knowledge of the state vector and the output, resulting in a higher robustness against disturbance on the control variable compared to other methods.

2.5. Anti-windup techniques

As said, the constraints on the system are not taken into account explicitly in the formulation of the MPC; however, the windup problem cannot be neglected.

An integrator may suffer, in presence of saturation, from an accumulation of integral charge when the error is maintained for a long time without changing its sign. This can lead to deteriorate the performance and even to instability of the system.

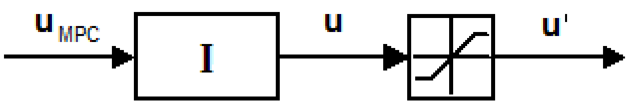

Fig.2.4Saturation without-anti windup

In order to avoid this problem there are several anti-windup techniques. A simple one, suitable only for the second type of integrator, is shown in fig.2.5.

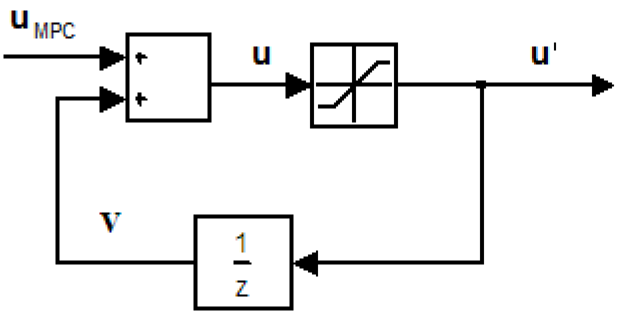

Fig.2.5Proper integrator with-anti windup

$$u(z) = \frac{1}{1 + z^{-1}} = u_{MPC}(z) \tag{2.14}$$

Without considering the saturation, it is possible to write the transfer function between the variable u and the out of the MPC (2.14). It leads to the same expression of the non-strictly proper integral action in (2.10). It means that the two systems are equivalent, but the second formulation has the advantage to avoid the windup.

Considering the system in saturation and considering that the variable v is equal to u delayed by one step it is possible to write the equations (2.15).

$$u' = u_{max} \qquad\qquad v = u_{max} \quad (2.15)$$

It means that even if the error (in this case the control action of the MPC) remains positive after saturation has been reached, there is no integral charge and both variable v and u stay limited. So when the error changes its sign the control promptly reduce the control action.

$$u(k) = v(k) + u_{MPC} = u_{max} + u_{MPC}$$

$$(2.16)$$

$$IF\ \ u_{MPC} < 0 u(k) < u_{Max}$$

For the strictly-proper regulator this method is not applicable. A simple but effective alternative technique is shown in fig.2.6.

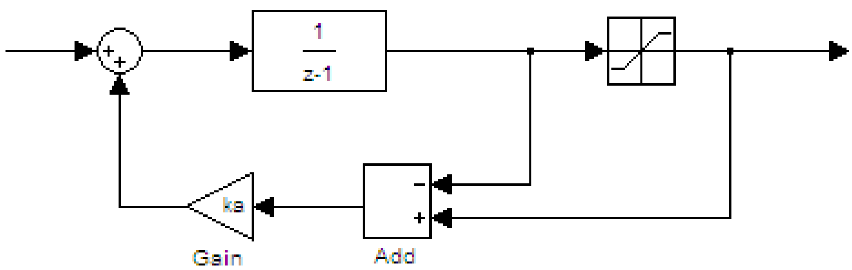

Fig.2.6 Strictly-proper integrator with-anti windup

The main disadvantage of this scheme is represented by the gain in the feed-back branch. The parameter needs to be tuned for the specific application, so it also means that the anti-windup works differently in different situation.

Chapter 3:

State reference tracking

To solve the problem of reference tracking we normally add within the function a term that weights the error between the reference and the output, as we described in chapter 2. Due to the limitations of this kind of system shown above, we should consider an alternative approach based on the state reference.

As explained above, there are different approaches in the MPC, implying different features. The most important elements of difference between all methods are the definition of the cost function and the implementation of the integral action. The proposed solution will be outlined in this chapter considering these elements.

3.1. Limits of the Output Reference tracking

In the problem of reference tracking it is natural to add a term of the error in the cost function so that the control law is able to decrease the error by minimizing it. However, the formulation in (2.1) cannot yield the error to be zero, so we need to explicitly add an integral action in the regulator.

Those advantages prompt on implementing this type of integrator, bearing in mind that we have to solve the problem of feasibility for this solution. We start implementing the proper integrator in the simulation in order to analyze the control behavior. It yields to the overall control scheme in Fig.3.1.

The regulator presents a state feed-back similar to optimum control, but it is important to underline that, for the predictive horizon of one step, the MPC with reference tracking cannot achieve the performance of the optimum control. The limit is due to the fact that the knowledge of the state variables is not exploited in an efficient way. Its measure is used in order to obtain the prediction of the output, and this is actually fed back and compared with the reference.

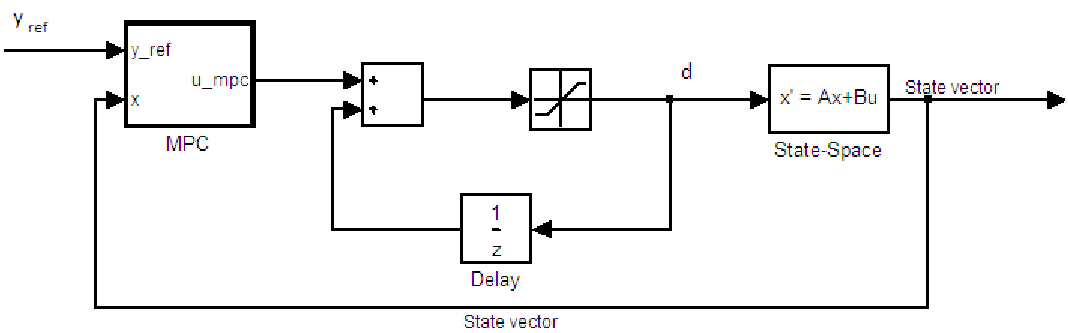

Fig.3.1Scheme of MPC with Integral Action

It is possible to see this limit by the expression of the state matrix in closed loop in (3.1). Considering a second degree system, as the Buck converter, which controls only the output voltage, it is clear that it is not possible to determine the position of the poles in closed loop with only one degree of freedom. The vector dimension of the control parameters (K_{mpc}) is, in fact, one. Therefore, combined with the integrator, the overall control system corresponds to a PI regulator. This limit can be surpassed by increasing the perdition horizon.

$$A_{cl} = A_m - B_m[1\ 0\ \dots\ 0]K_{mpc}A_c \qquad (3.1)$$

If the future values of the reference are set equal to the present voltage reference, a higher prediction horizon can improve the performance at step response. Fig.33 shows three different output voltage waveforms, for three different prediction horizons under a step in the voltage reference of 3.3V. In these conditions, we can observe that the control reaches the best performance with only two prediction steps. Even increasing the prediction horizon it is not possible to get better responses. As we will see in chapter 4, the results shown in fig.3.2 cannot be improved by changing the parameters Q and R of the regulator. We have a first clue of it by looking at the values of duty cycles in fig.3.3. Comparing the duty cycle with the voltage output waveforms we see that in almost five switching period, the output voltage reaches the steady state value. This is the best result that a digital linear control can obtain, because if we try to force the output to be faster, there will be a problem of aliasing. In other words, a digital linear regulator cannot force the output to be faster than the regulator can see.

As seen, the limit of this approach can be fixed by increasing the prediction horizon, but this leads to a higher uncertainty in the prediction. This disadvantage is even more important when the future values of the reference are known in advanced and we try to use them to improve the tracking.

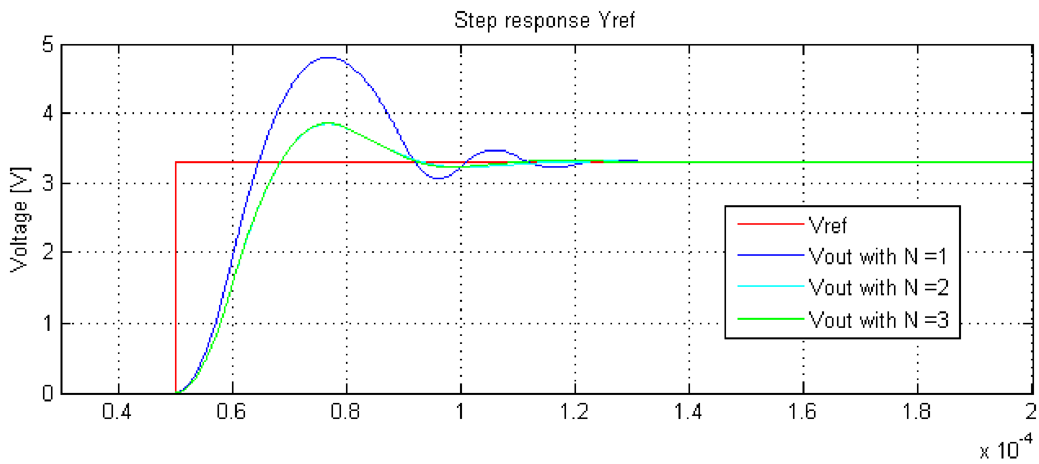

Fig.3.2 Step response for different prediction horizon

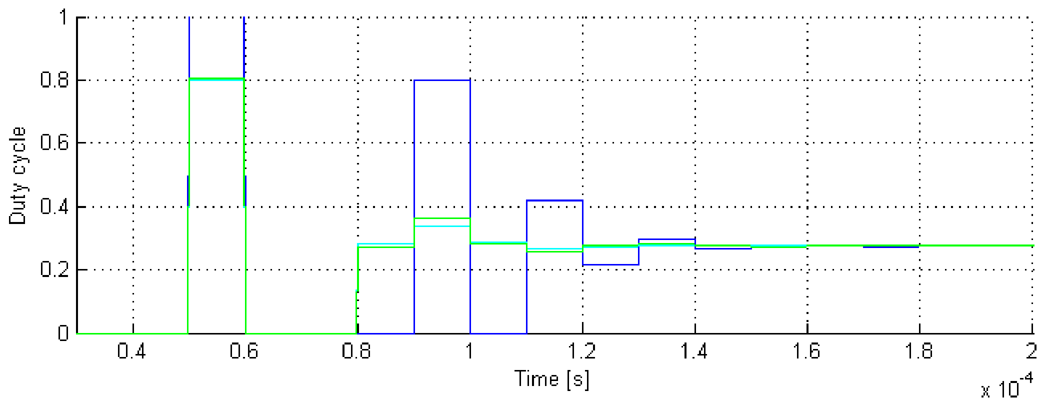

Fig.3.3 Duty cycle under step response

As we will see in the next chapter the same results can be achieved by the state reference approach with a lower prediction horizon.

3.2. The Proposed Approach

It is not convenient to measure all the state variables to control only the output one. It may be more suitable to exploit all the information in a better way, for instance using not only the knowledge of the state to predict the output, but also trying to control all the state variables as in the optimum control. In this way we should be able to control completely the behavior of the plant.

The approach consists of controlling all the state variables, assigning them a proper reference so that it is possible to reach a better performance in terms of speed and behavior of the system. The difficult part of this approach is to define a value for each variable.

The idea to give a reference to each state variable leads to a new formulation of the cost function, where the error is defined as the difference between the state reference and the state variables.

$$J(e(k),u(\cdot),k) = \sum_{i=0}^{N} [e(k+i)^T \, Q_i \, e(k+i) + u(k+i)^T R_i \, u(k+i)]$$

$$e(k+i) = x^{\circ}(k+i) - x(k+i) \tag{3.2}$$

From here on, the steps to arrive at the control law are the same in 2.2. The open loop method is used for the prediction of the state resulting in the expression in (3.3).

$$\begin{bmatrix} x(k+1) \\ \vdots \\ x(k+N) \end{bmatrix} = \begin{bmatrix} A \\ \vdots \\ A^n \end{bmatrix} x(k) + \begin{bmatrix} B & 0 & 0 \\ \vdots & \ddots & \vdots \\ A^{n-1}B & \dots & B \end{bmatrix} \begin{bmatrix} u(k) \\ \vdots \\ u(k+N-1) \end{bmatrix} \tag{3.3}$$

$$X(k+1) = A_s x(k) + B_s U(k)$$

After the minimization of the cost function, conducted as described in chapter 2.2, we finally obtain the close form of the new control law (3.4).

$$U^\circ(k) = K_{mpc}\left(X^\circ(k+1) - A_s x(k)\right) \tag{3.4}$$

The overall system scheme is the same shown in fig.3.1. The differences are in the MPC block in fig.3.4. The matrix of the open loop prediction, in this case, gives a prediction of the entire state vector. The MPC gain is composed by a number of elements, equal to the order of the system plus the number of with integral action; in case of the Buck Converter is three.

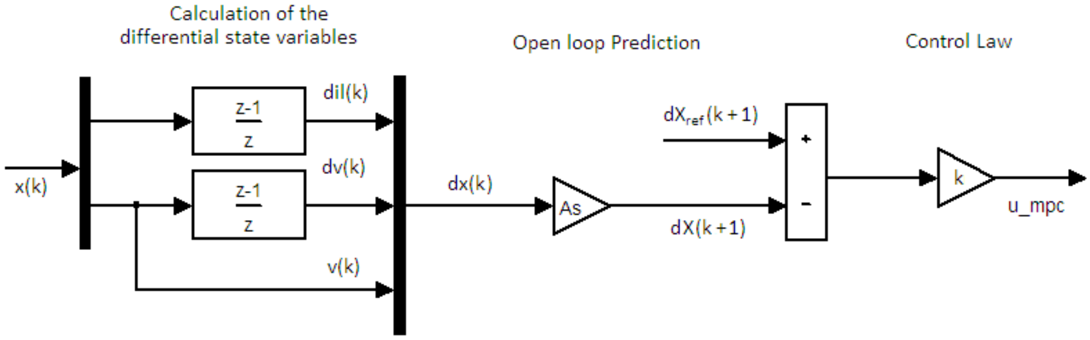

Fig.3.4 MPC block for a Buck Converter

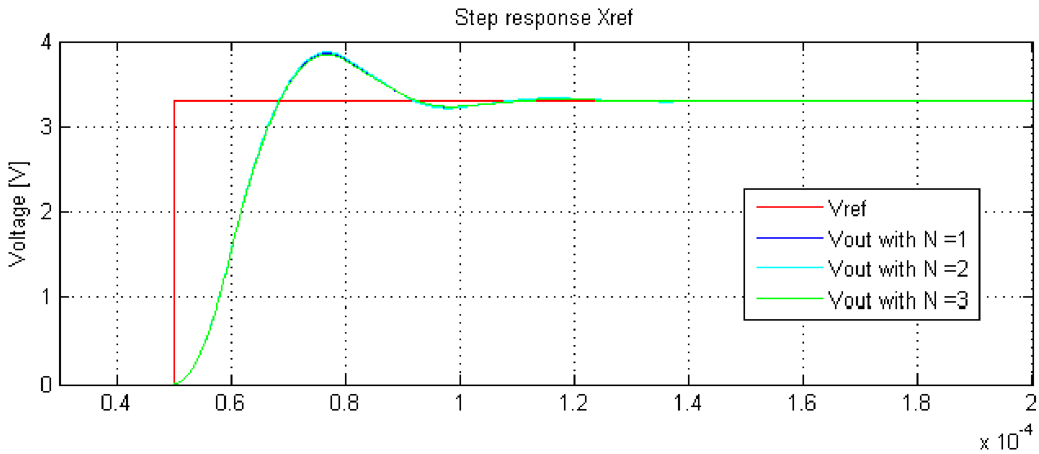

Fig.3.5 State reference tracking: output voltage

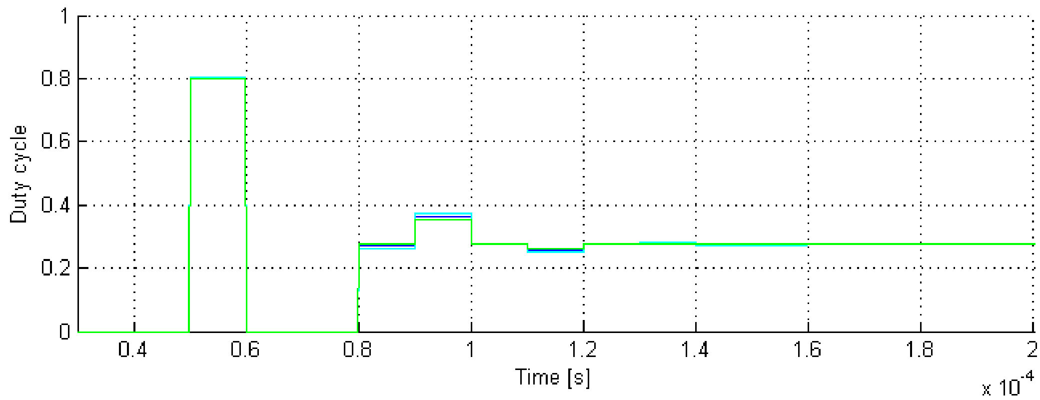

Fig.3.6 State reference tracking: duty cycle

Figure 3.5 shows how the behavior of the state reference tracking has improved compared to the previous approach. The image shows it is possible to guarantee good results with only one prediction step.

There are two problems still to be solved for a perfect functioning of the system: the definition of the state reference and the feasibility problem of the control.

3.3. Definition of the reference

When we have to define a reference for the state variable we face the problem of choosing the best solution to improve the output behavior. There are several methods to generate a state reference by the output one. Many methods imply high computation cost and, for this reason, do not represent an attractive solution to our problem. Most are based on the knowledge of the system and hence increase the sensibility toward possible changes of the plant. Here it is presented a simple but effective method to generate the state reference that should overcome these limits. It is based on the formulation of the system model in δ-transform (2.11), and in the implementation of the proper integral action (2.14) as part of the system.

Several methods have been presented in literature. One way is to generate a multi-dimensional reference trajectory from a scalar input command using the model of a switching circuit in conjunction with a feedback algorithm [13]. The algorithm generates the state references simulating in line the digital model of the system controlled by any control law. This method is not appropriate for this case, because extremely complex in terms of realizations and computing power.

A second method is based on the equivalence between a first order difference system equations and input-output mapping [14]. Because of this equivalence, it is possible, in single-input single-output (SISO) system, extracting the knowledge of the state variables by the knowledge of n consecutive values of the output and n-1 consecutive value of the input, where n defines the order of the system. In case of the Buck converter, the value of the current can be extract by the knowledge of two consecutive values of the capacitor voltage and the duty cycle.

The second method presents at least two disadvantages. Although it is simpler than the previous one, it requires an additional operation in line to obtain the current reference. Second, it is affected by disturbances on the control variable. Since the solution is based on the knowledge of load, if the load changes, the method cannot guarantee a good reference of the current inductor anymore.

We should find an alternative method to surpass the limits of the solution described above.

3.4. Deltatransform

This method is related with the implementation of the proper integrator and the reformulation of the model in delta transform.

The idea is simple as well as effective. In case of a proper integrator we are able to describe the system, enlarged with the integrator, with differential variables (2.11 and 2.13). Considering the system at steady-state condition the value of the new state variable is zero. So in case of constant reference the natural reference for the state differential state variable is zero.

There are considerable advantages in this method. First, the definition of the reference does not depend on the knowledge of the system; it means that this method is not affected by system changes. It is an important advantage that gives more generality to the solution. We shall consider, for instance, the case of the Buck converter. The load is part of the system, so we model the converter assuming that we know the value of the load. In the same way, with the previous method, we define a reference for the output current based on the nominal value of the load resistance but if it varies the reference defection based on the model would be incorrect. However, with the description model based on δ-transform we can define a reference value for the current that is correct for any change of the load, since at steady state the difference between two consecutive samples needs to be zero regardless of the load.

Another important advantage is the simplicity. In fact it does not require any additional calculation. Compared with the previous solutions described in chapter 3.3, the computational cost is zero.

If for constant references this method works perfectly, in case of sinusoidal reference tracking, the difference between two consecutive values is not zero. However, if the sampling frequency is much higher than the sinusoidal frequency, as the case of SMPS, this assumption can be still valid. Besides, setting the state reference to zero, this method makes the entire system more robust. Finally the method seems to fit better with the requirements specified for the application.

3.5. Feasibility of the solution

The solution of the feasibility problem has been examined in order not to lose the easy generation of the state reference shown in 3.4. It is represented by the implementation of an integral action that is a mix of the two shown in 2.4.

The feasibility problem is due to the computational time required by a digital regulator in order to generate the duty cycle. In a proper regulator, the current value of the control variable should depend on the actual value of the error. It requires high computational power in order to run the control law. If the digital control implemented is able to run the algorithm in a time much smaller than the sampling time, there is no problem of feasibility. In this case, we might keep a proper non-strictly regulator considering the delay as a decrease on the phase margin. However, this solution can affect considerably the closed loop performances. For this reason, in most of the cases, it is more recommendable to select a strictly proper regulator that guarantees an entire switching period to evaluate the control action.

Although we looked for a regulator light in terms of computation, the time required by the microcontroller implemented to run the control law is comparable with the switching period.

Hence, a strictly-proper integrator has been used in the final design, but it has been modified in its representation (fig. 3.7.), to allow a model description as in (2.13) and to keep the benefits related in the definition of the state reference and in the anti-windup scheme. It has been achieved taking into account an explicit delay of one step in the formulation of the law control as shown in fig.3.8.

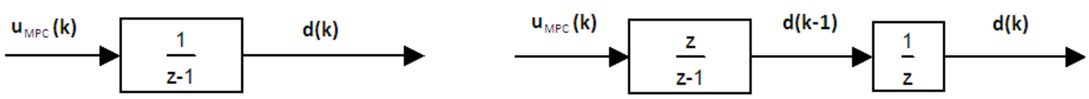

Fig.3.7 Feasible integrator

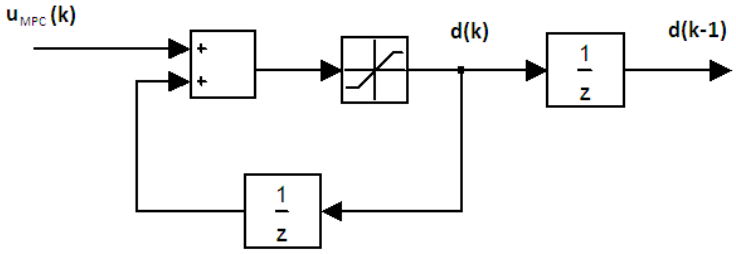

Fig.3.8 Feasible integrator with anti-windup

The scheme in fig.3.8 gives all the benefit of the representation of the model in differential variables; however we need to add a compensation of the delay to stabilize the overall system. Now the idea is to generate the control law as before, but with an important variation. In order to evaluate the error, we will not use the value of the output measured by the sensor, but we will generate a prediction of the output and pass it to the control system.

3.6. Delay compensator

The idea is that the delay compensator provides the MPC control with the prediction of one sample period of state vector. In this way the regulator evaluates the control effort based on the predicted state of the system. When the control variable acts on the converter, it will be passed one sampling time and the system will be in the predicted state. In fig.3.9 is shown the scheme of the system with the delay compensator.

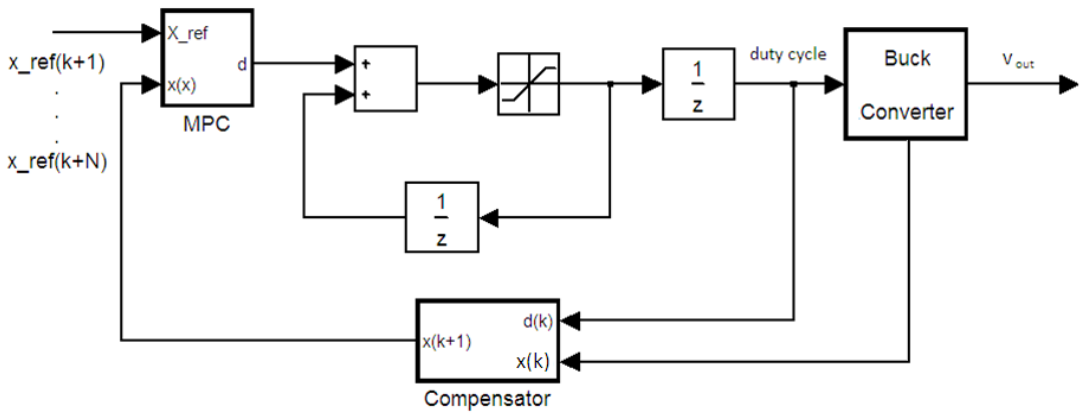

Fig.3.9 Overall system with delay compensator

The first step of prediction conducted by the compensator cannot be performed by the MPC itself. The substantial difference with the normal open loop prediction is that the current value of the input cannot be chosen, but it is already decided in the previous step. It means that in order to operate the prediction, the compensator needs to know the current value of the duty cycle. Unfortunately this method is subject to disturbance on the control variable. These considerations yield to choose a closed loop predictor, based on the Kalman filter theory [15], as shown in fig. 3.10.

This solution takes into account the error between the actual measure of the output and its previous prediction performed by the model. In this way we can add a correction in the predictor proportioned to this error, ensuring better performance in terms of estimation of the state variables.

This solution does not present a disadvantage, compared to the implement of a classical strictly-proper integrator, because in control scheme based on the feed-back of the state, as MPC is, we need a Kalman filter. Since it is not always possible to have access to all the state variables by measurement, we need to estimate their values, and it is usually carried out by mean of the Kalman filter.

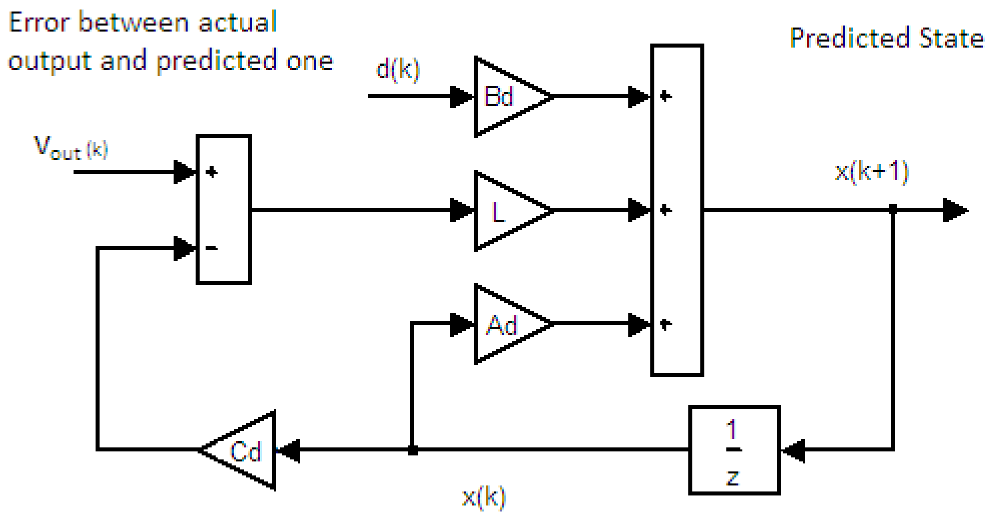

Fig.3.10 Kalman Predictor

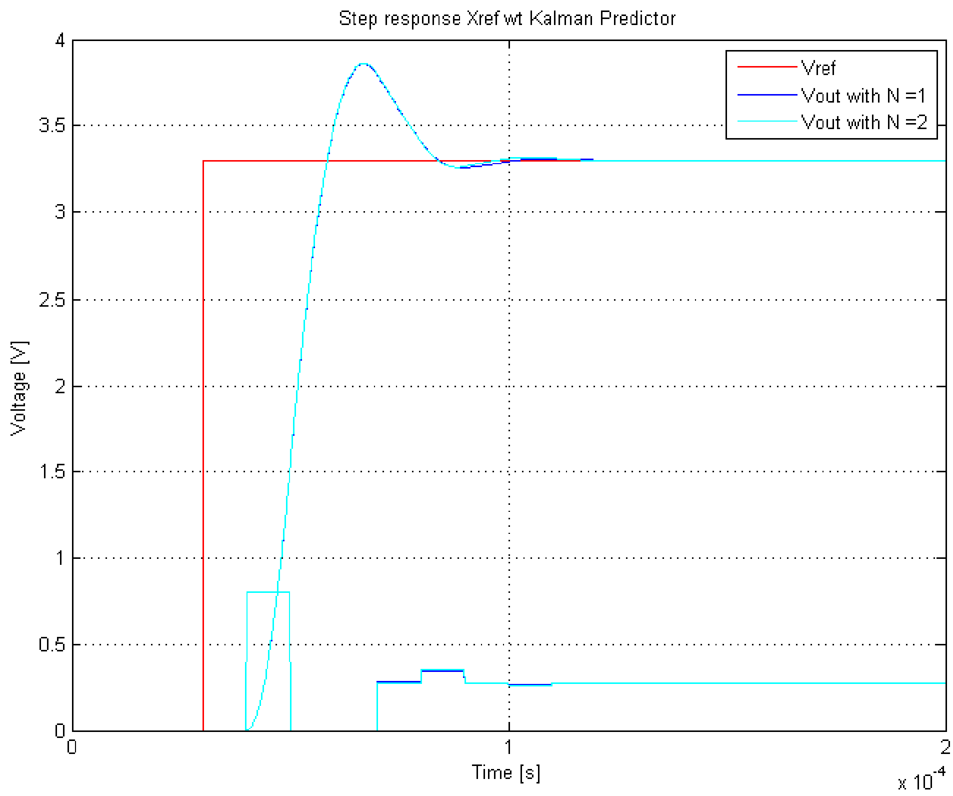

Fig.3.11 Step response of the overall regulator

Fig.3.11 shows the results of the overall regulator under the same conditions described in paragraph 3.2. We can notice that the output dynamic is really close to the previous case, in fig.3.5, reached by the ideal regulator. The main difference is that the response is delayed of one switching period. It reflects that the regulator requires an entire switching period since when a variation in the reference is noticed and an effect on the control variable is produced. The overall system with the compensator delay result to be asymptotically stable.

Chapter 4:

Method Design

The parameters Q and R of the cost function, together with the choice of the prediction horizon, can be determined during the simulation process by looking to the waveforms. If this is true in the case of MPC normal application, in our case it turns out to be really inefficient from the point of view of time cost and the results of the solutions. For this reason a Genetic Algorithm (GA) has been used to find the best set of parameters.

In this chapter some basic concepts about GA are briefly introduced. Then we will see how GA has been implemented in the design of the MPC. Finally, simulation results are presented.

4.1. The Genetic Algorithm

A GA [16] is a heuristic search that belongs to the larger class of Evolutionary Algorithm (EA). EA is a subset of optimization algorithms that have in common some mechanisms inspired by biological evolution, such as: reproduction, mutation, recombination, and selection. All the possible solutions of the optimization problem play the role of individuals in a population. At each problem is associated a cost function that assigns a value to each solution, called fitness. This value represents how well the solution for the problem is. If the fitness is higher as better is the solution, EA will look for the individual with the highest one, otherwise the algorithm will try to minimize it.

From an initial set of individuals new generations are obtained combining the best individuals according to the mechanism of evolution. After some generations the best individual will be selected by EA as best solution to the problem.

In GA the candidate solution (called phenotype or individuals) is encoded in a string (called genotype or chromosomes). The string can be codified in bits of natural or real number. The evolution usually starts from a population randomly generated in order to have a heterogeneous set of individuals, which can guarantee fast performances in the search of the solution.

At each generation, the fitness of every individual is evaluated according a proper function. After this, multiple individuals are stochastically selected from the current population and modified to form a new population. The idea is to select the best solutions of each generation and combine them to form better ones. However, the selection is performed stochastically, giving more possibility to be selected to the better solutions, but it is important that also the worse have possibility in order to avoid local optimum. The recombination is carried out with two mechanism again inspired by the nature: crossover and mutation.

In nature the crossover consists in an exchange of genetic material between homologous chromosomes. In GA, two candidate solutions are selected, coupled and part of their phenotype is mixed as shown in fig. 4.1, to create the new candidate solutions. There are different techniques to recreate this mechanism: point crossover, which twists the chromosome in one point only, multiple crossovers that cut the chromosome in multiples points and uniform crossover. The last one is most used, because it gives to each point of the chromosome the same possibility to be cut, so that inheritance is independent of the position.

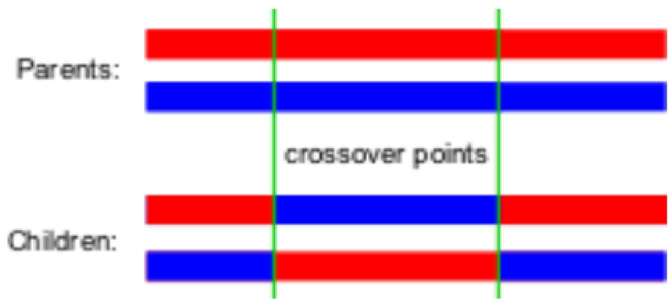

Fig.4.1 Example of multiple crossovers

The mutation consists in a punctual change of a chromosome. A simple example is shown in fig. 4.2. The string is codified in bit and the mutation can change with a certain probability the value of each bit.

parent	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
child	0	1	0	0	1	0	1	1	0	0	0	1	0	1	1	0	0	1

Fig.4.2 Example of mutation

Both mechanism of mutation and crossover are useful in different respects. The crossover helps in searching a good solution among the set of all possible solutions, while the mutation avoid a possible local optimum.

Although after some generations the GA is able to formulate a solution that is better than the initial, the fact of being a heuristic search method cannot always guarantee to find an optimum solution. Even if we consider this limit, the GA normally provides excellent results, close to the optimum.

4.2. Regulator Design

As explained in chapter 2, the MPC regulator depends on the parameters Q and R of the cost function. Without considering the future information of the reference, we can choose quite easily these parameters looking at the simulated responses. If the closed loop system is slow, it is possible to increase the bandwidth by increasing the parameter Q; if the control effort is high we can decrease it by setting a higher value for the parameter R. In this application we want something more. We would like to use the future information of the output reference to get some benefits in the reference tracking. At this stage the key point in the design is to get a good balance between the coefficients Q and R to weight the predictive error and the actual one. With the first term we try to anticipate the control action in order to reduce the phase delay between output and reference.

The manual design is appropriate to create good regulators for the classical case, but when we try to take advantage of future reference knowledge this assignation becomes more difficult and time-consuming because it is not completely clear how to modify the parameter, in order to reach the best performance. For these reasons I tried to find a solution for this problem using an advanced method of search as the genetic algorithm.

The MPC parameters are naturally encoded in a chromosome on real numbers while for the fitness there are at least two options. One possibility is given by the assignation of the dynamic. It means, for instance, trying to find the parameters that yield to specified closed loop poles. In this case, the fitness of each solution can be defined as the difference between the desired closed loop eigenvalues and the ones obtained by the solution 4.1. The GA can find a good set of parameters trying to minimize the fitness of the individuals.

$$fitness = |Re(eig_d - eig_{cl})| + |Im(eig_d - eig_{cl})| \tag{4.1}$$

The main advantage of this method consists of the possibility to specify overshoot and bandwidth of the closed loop transfer function.

A second idea consists on simulating the control law given by each solution and comparing the output waveform with the reference. In this case, we define the fitness as the quadratic sum of the difference between the reference and the output at each time (4.2).

$$fitness = \sum_t \left(y_{ref}(t) - y_{out}(t)\right)^2 \tag{4.2}$$

Therefore, by minimizing the fitness the GA is able to find the set of parameters that better tracks the reference.

The solution depends on the reference waveform. We could think that for problem of sinusoidal reference tracking, we should choose a sinusoidal waveform in order to set the MPC parameters. However, if we find a solution that is able to match the output with a sinusoidal waveform, the solution might be close to instability. A possible solution to this problem could be to choose a waveform as the one in Fig. 4.3, which has not only a sine wave, but also a step. If the system is close to instability, the step response will present oscillations that will increase the value of the fitness. Hence minimizing the fitness function, the algorithm will be able to guarantee the stability of the closed loop.

The main advantage of the second fitness definition is the possibility of choosing explicitly the reference that we want the control to follow. For this reason this method is more suitable for problems of reference tracking while the first method finds a more applicable area in problems of step response.

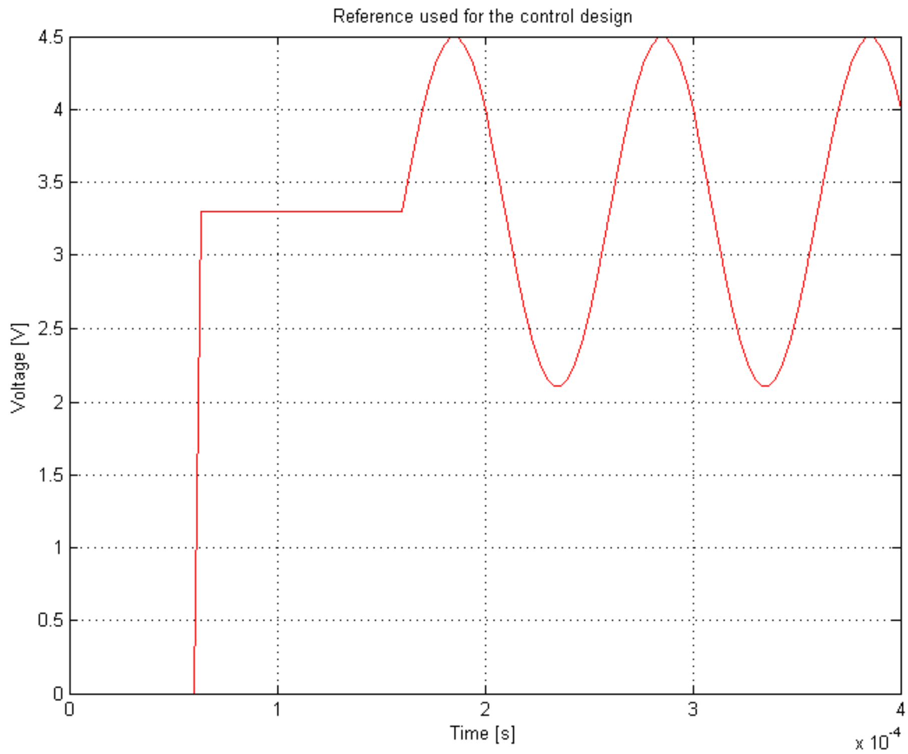

Fig.4.3 Output reference for control design

By using the future information of the reference the regulator is able to improve the tracking of a sinusoidal waveform. We compare its performance with the MPC without constraint and without using future information about the reference, discussed in 3.2. A regulator with this characteristic is equivalent to a conventional linear regulator, like a PID. The same regulator designed for the step response in 3.2, has been used to control the Buck converter with sinusoidal reference.

Fig.4.4 shows the output voltage and the sinusoidal reference at 1 kHz (1/100 of the switching frequency). We can see that the linear regulator is able to follow the reference with a small difference of phase.

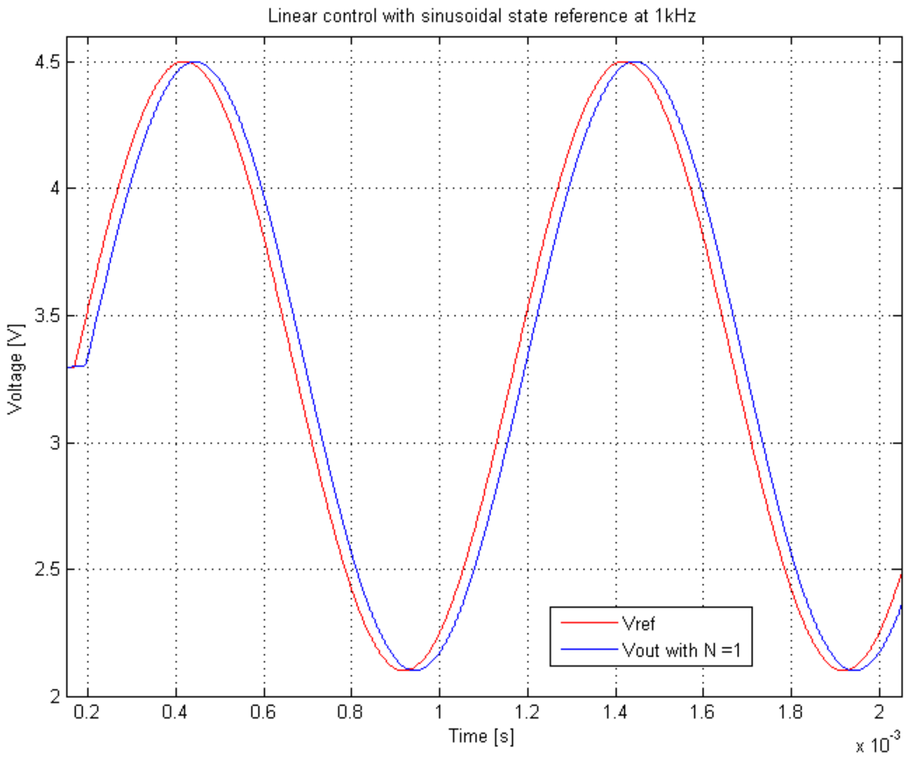

Fig.4.4 Output sinusoidal response with a conventional regulator

The behavior of the regulator in these conditions is good but when we try to increase the frequency of the sinusoidal reference the phase delay tends to increase. Fig.4.5 shows the behavior of the linear regulator when the sinusoidal reference is at 5 kHz. The output voltage has a delay of 25°.

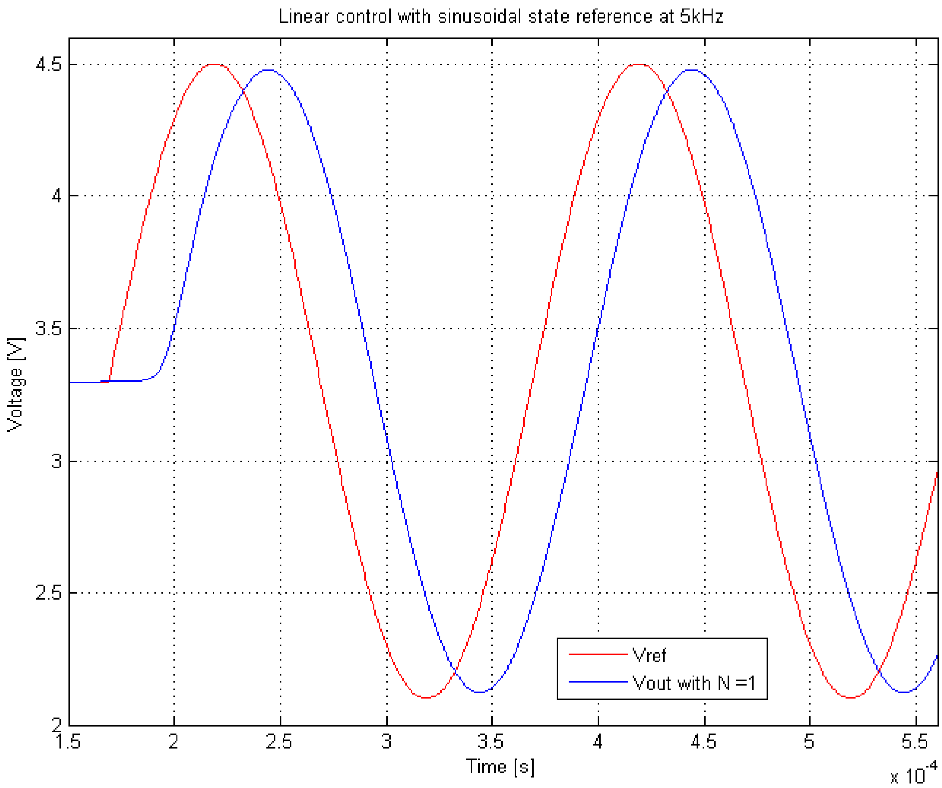

Fig.4.5 Linear control with reference at 5 kHz

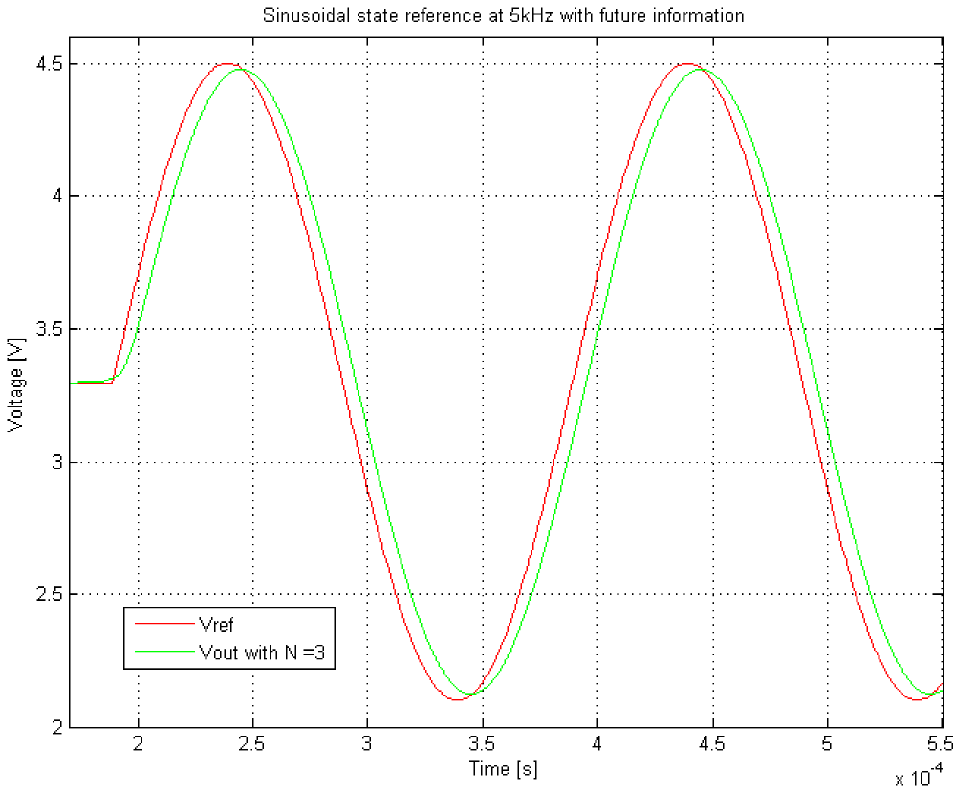

Fig.4.6Proposed approach with reference at 5 kHz

In the same conditions and taking advantage from the anticipate knowledge of the reference, the proposed control scheme, designed with GA, shows better performances in terms of tracking the sinusoidal signal, as shown in Fig. 4.6. Starting from a prediction horizon of three switching periods, the regulator is able to completely compensate the additional phase.

With a frequency of 10 kHz, one tenth of the switching frequency, the linear control follows the reference with a phase delay of 90° (fig. 4.7), which corresponds to a closed-loop bandwidth of 10 kHz. Instead, the MPC still presents a good performance with a prediction horizon of three sample periods, having a phase delay of 10°,while the phase delay is almost zero with a horizon of four (fig. 4.8).

Therefore, we can achieve better performances of reference tracking with the same switching frequency by taking advantage of reference future knowledge. From a different point of view we can say that a conventional control scheme needs a frequency ten times higher than the proposed scheme to reach the same performance. This results in a consistent improvement of the converter efficiency at high frequency, since the power consumption of a SMPS increases considerably with the increment of the switching frequency.

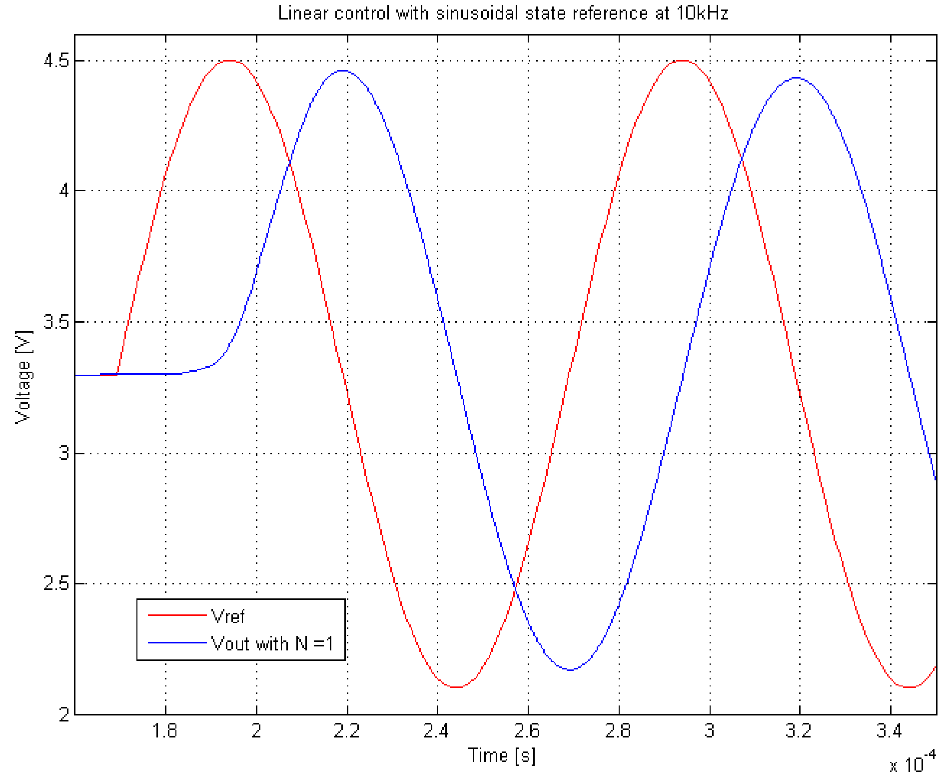

Fig.4.7 Linear control with reference at 10 kHz

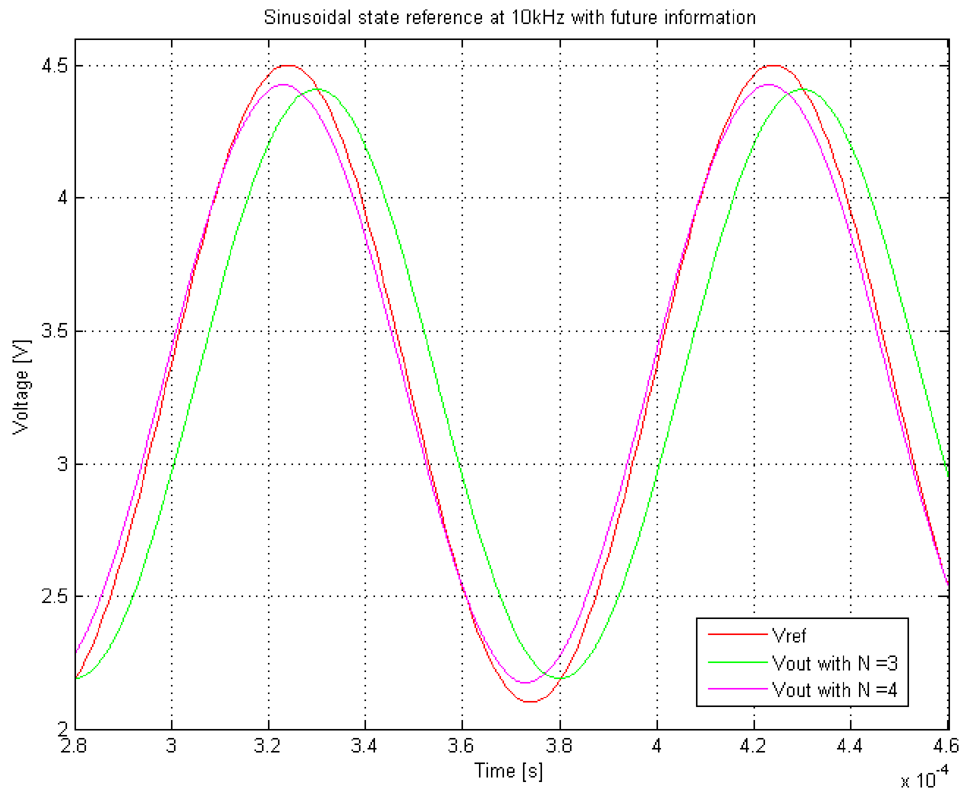

Fig.4.8Proposed approach with reference at 10 kHz

4.3. Simulation results

To validate the solution, I used PSIM as circuit simulator and MATLAB to reproduce the digital control. These software can be used in parallel trough Simulink and the SimCoupler Module of PSIM, giving us the advantage to perform any kind of control algorithm by using a powerful numerical computing environment like Matlab, and validate it on a switching model designed with PSIM.

First, the overall system with delay compensator has been validated under the step variation of the reference of 1.2V. As shown in Fig. 4.9 the system is stable and shows good performance, since the new steady state is reached in only three switching periods. We can also see the presence of one period delay due to the strictly proper property of the regulator. It affects the performance under load variation. Fig.4.10 shows the closed loop response when the load impedance changes of 20% from its nominal value. The regulator in this case needs one entire switching period before reacting, and it degrades the compensation of disturbance in the load.

After validating the overall system, we can check if the phase compensation skill of the regulator works well also with the switching model. Fig.4.11 and fig.4.12 show the response of the closed loop, implementing a MPC with a prediction horizon of three switching periods, with a sinusoidal reference respectively of 5 kHz and 10 kHz. In both cases the simulations confirm the results obtained in the design.

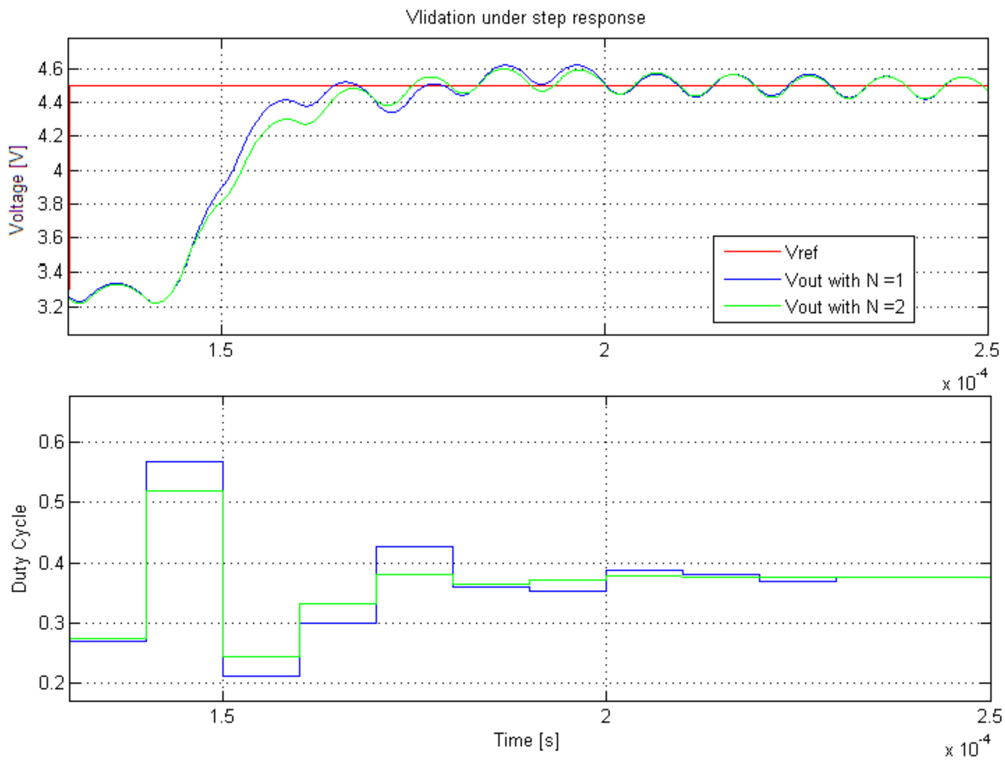

Fig.4.9Validation under reference step response

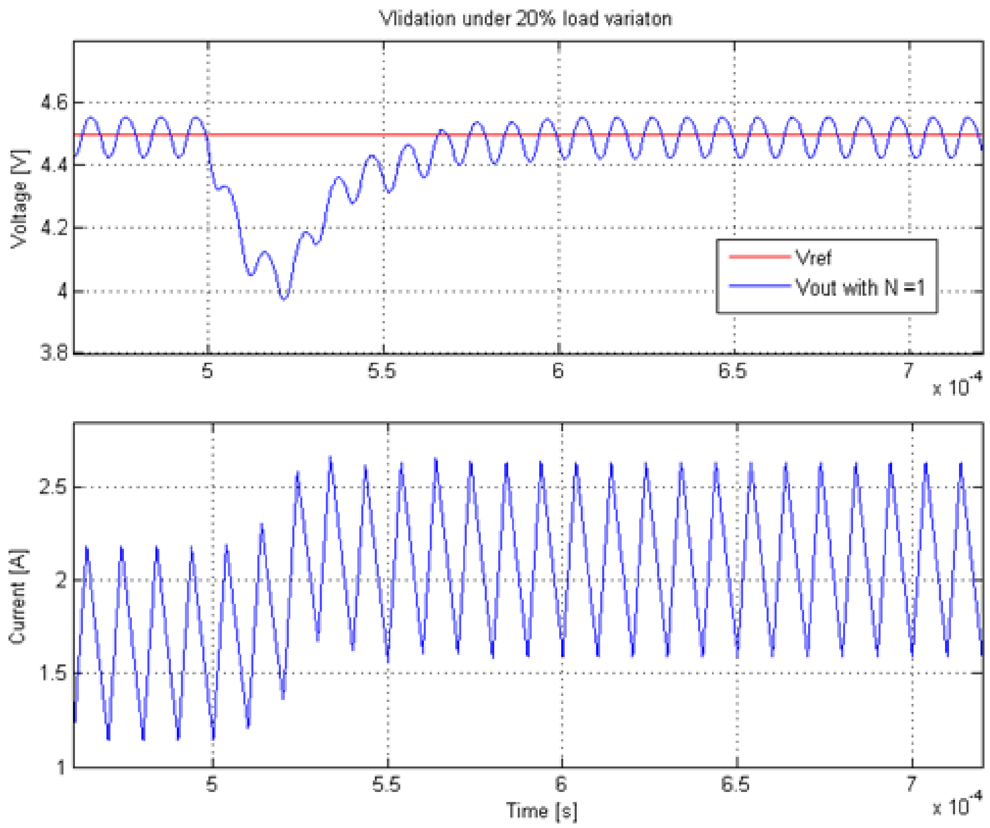

Fig.4.10Validation under load variation

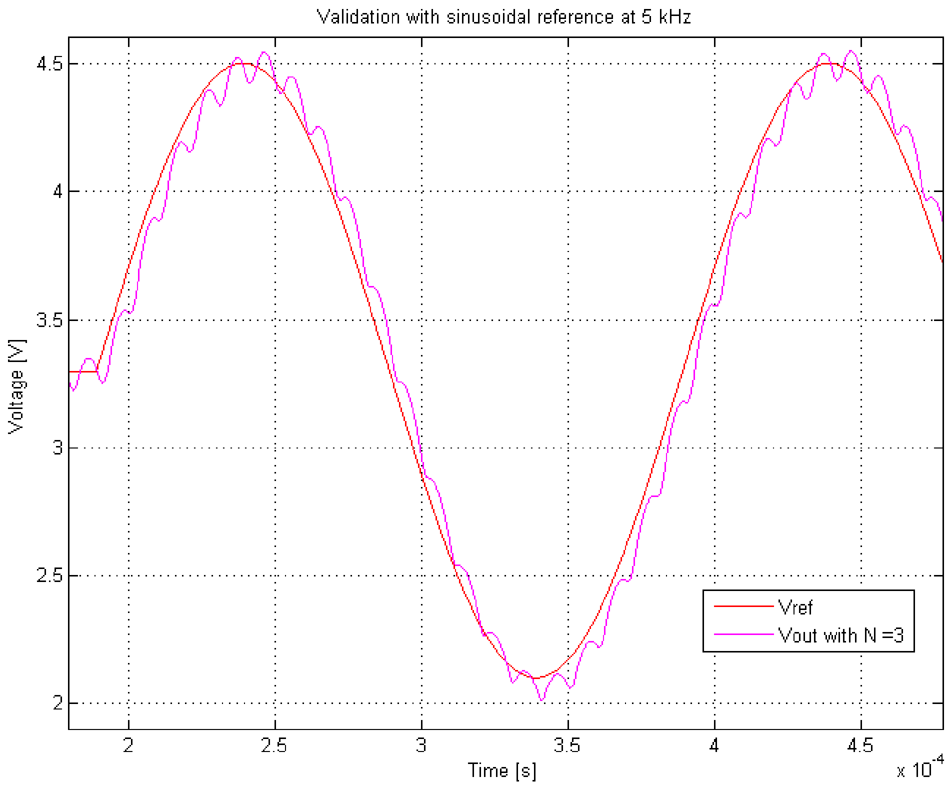

Fig.4.11Validation with sinusoidal reference at 5 kHz

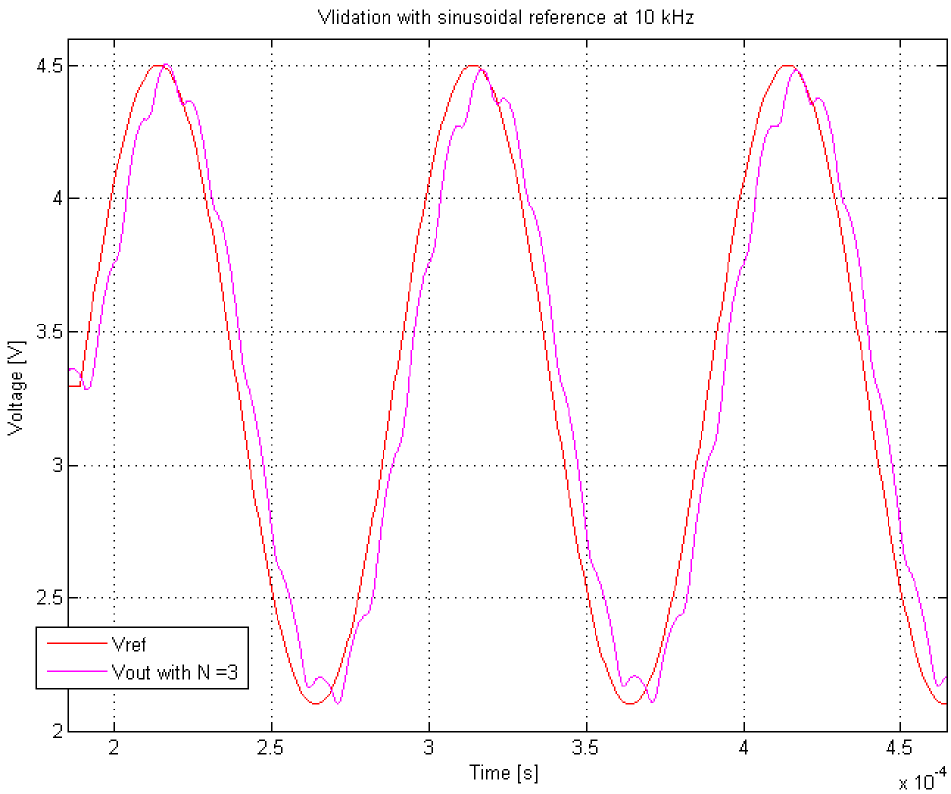

Fig.4.12Validation with sinusoidal reference at 10 kHz

As said in chapter 3, the proposed approach allows a reduction of the prediction horizon in respect to the output reference tracking approach, maintaining the same performance. Fig. 4.13 shows a comparison between the two approaches, in which the proposed one is implemented with a prediction horizon of three sampling period, while the reference tracking approach with five; both are designed with GA. In this case, we can see that the approach with state reference is able to follow the reference with 10° phase delay, while the method with output reference presets a delay of 50° and 15% attenuation of the peak value. Even increasing the prediction horizon the first method does not reach the same performance of the proposed one.

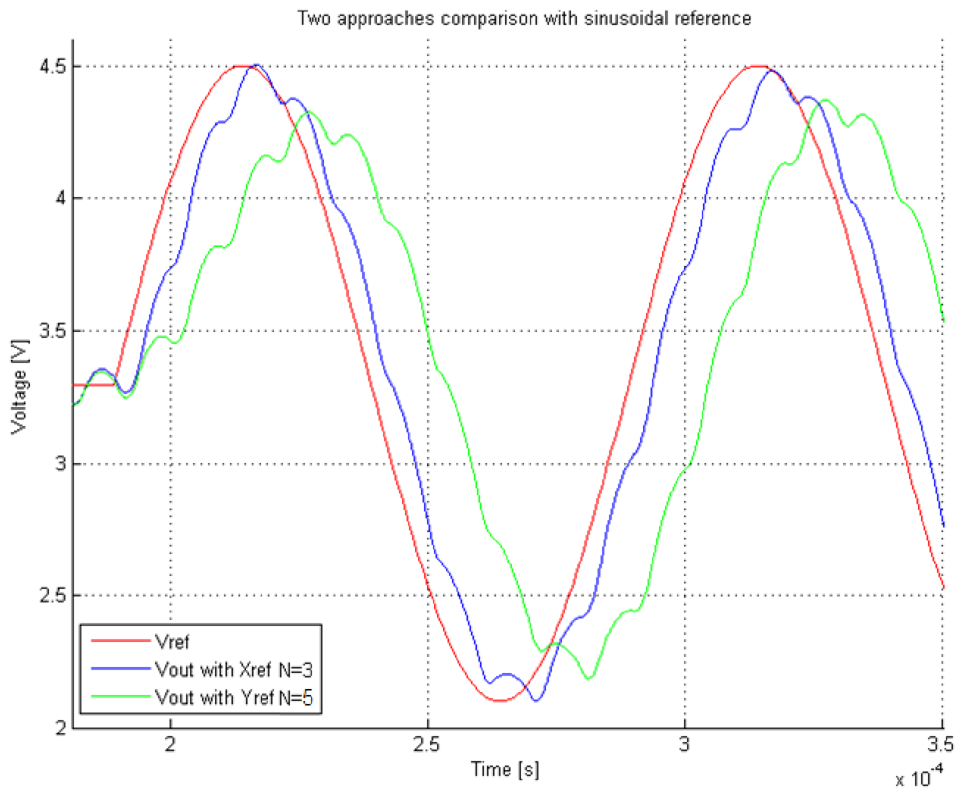

Fig.4.13Validation with sinusoidal reference at 10 kHz

For sinusoidal reference the proposed approach can bring important benefits in terms of reference tracking and reduction of switching losses compared to the conventional linear regulator. However, when the reference has a step waveform, the advantages are not so significant. Fig. 4.14 shows the comparison between step response with a predictive horizon equal to one, corresponding to a conventional regulator, and the proposed approach with predictive horizon equal to two. We can see that the main advantage is the possibility to compensate the delay of one switching period, due to the feasibility of the system. In this case the solution shows its limit for being a linear regulator.

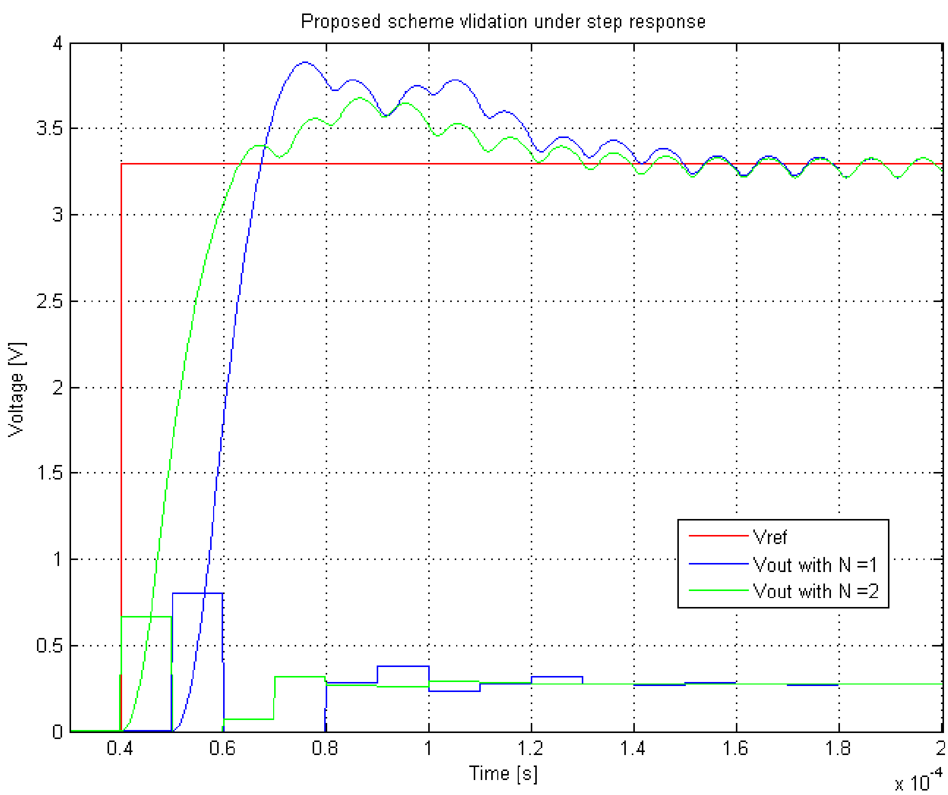

Fig.4.14Validation with step reference

Besides, under step reference the proposed approach does not improve the performance of the reference tracking approach as shown in figure 4.15.

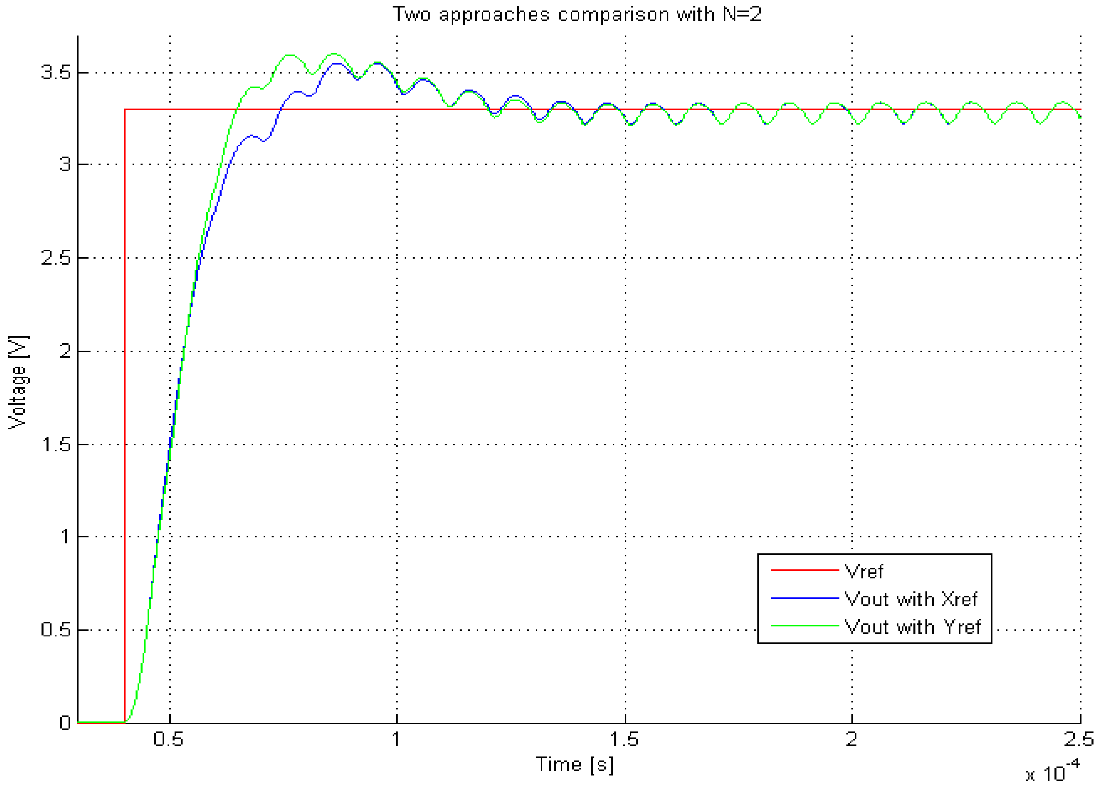

Fig.4.15 Comparison between two approaches

Chapter 5:

Experimental Results

The proposed control scheme has been implemented on a microcontroller with a prediction horizon of one step, to control a two-phase synchronous Buck converter.

5.1. System description

Once the control system has been simulated on the switching model, we can pass to the physical realization of the control. For the first implementation of the control, I used a microcontroller *Piccolo* of Texas Instrument [17], available in the laboratory. It is a low-cost device that is well suited to control power converters, due to its characteristics. Among the main features we can remark:

- 60 MHz internal clock
- 32-bit fixed point performance core
- 12-bit Analog Digital Converter (ADC) (2x7 Channels)
- 8 Pulse Width Modulation (PWM)

Fig.5.1 Microcontroller Piccolo

The presence of a specific circuit that generates the PWM signals and the ADC makes *Piccolo* particularly interesting for SMPS applications. On the other hand the working frequency is considerably lower compared to DSP or hardware-based controller as a FPGA. This limits its use to low-medium frequency applications.

The MPC control with state reference has been evaluated on a two-phase synchronous Buck converter, whose electronic circuit is shown in Fig.5.2 and characteristics are shown in Table 5.1.

As we can see from the schematic in Fig.5.3, this topology is composed by two Buck converters is parallel that share the same output capacitor and the same load. A multiphase configuration [18] is useful to reduce the output current ripple without incrementing the value of the output filter inductance. In a two-phase Buck, for instance, the MOSFETs control signals have a phase shifting of 180° (phase shedding) as well as the current ripple of the two inductors, so that the resulting output ripple is smaller due to ripple cancellation. In this way the dynamic behavior of the converter is not penalized by the presence of a higher inductance. Moreover, the topology corresponds to that of a single Buck converter with an equivalent inductance obtained by the parallel connection of the two inductors, so, the dynamics of the two-phase Buck converter are faster than those of each individual phase.

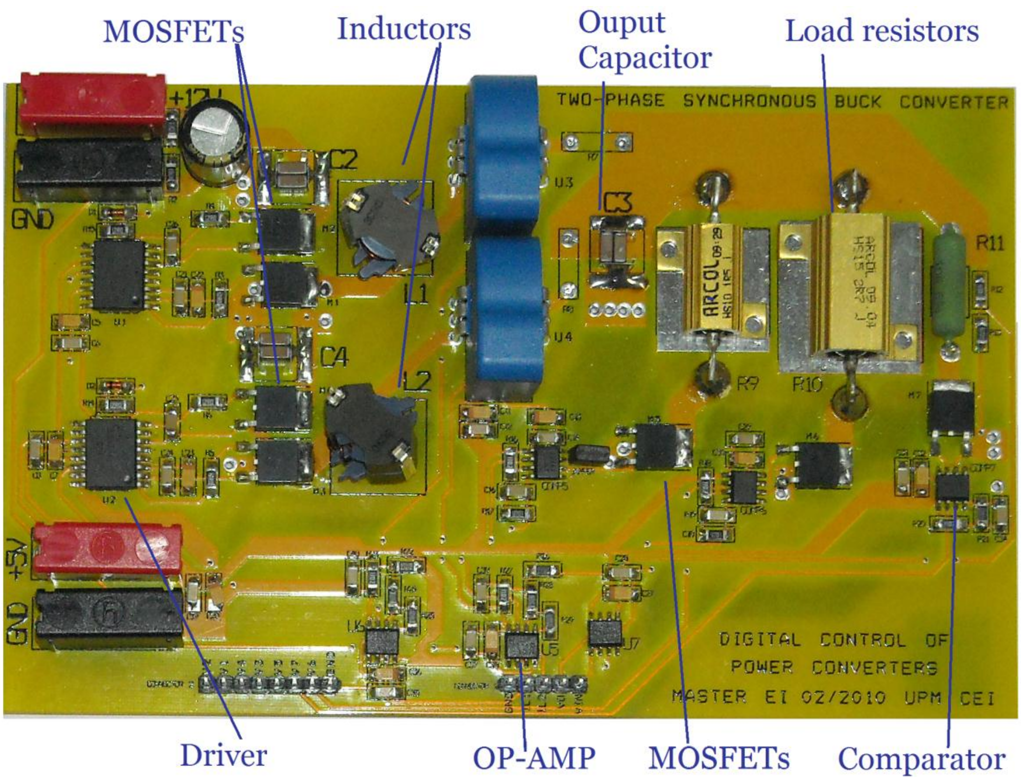

Fig.5.2 Two-phase synchronous Buck converter

C	10 uF	Drivers	IR2110
L	27 uH	Comparators	TL3016
R	2.7 Ω	OP-AMP	AD8602
$R_{load,step}$	10 Ω	MOSFETs	IRF3704Z
f_{sw}	100 kHz		

Table 5.1 Converter characteristics

A part from the power stage for the experiment have been used the board has two drivers that generate the signal for the two phase MOSFETs, three comparators that generate the control signal for the MOSFETs of the load and one OP-AMP . These MOSFETs are connected in series to three resistors in such a way that it is possible to control the value of the load resistance by connecting and disconnecting the resistor to the ground.

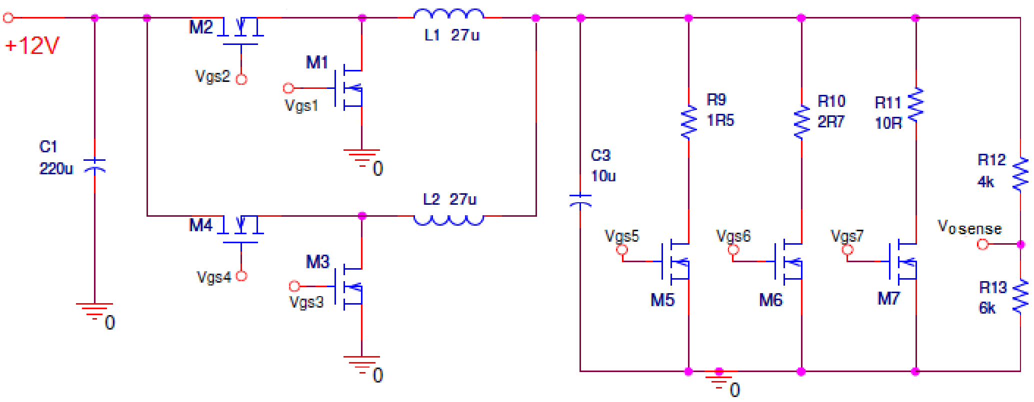

Fig.5.3 Board power stage schematic

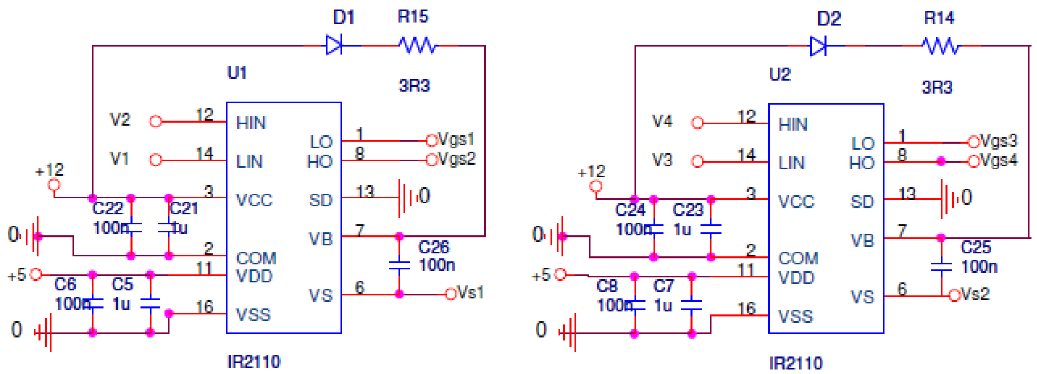

Fig.5.4 Drivers schematic

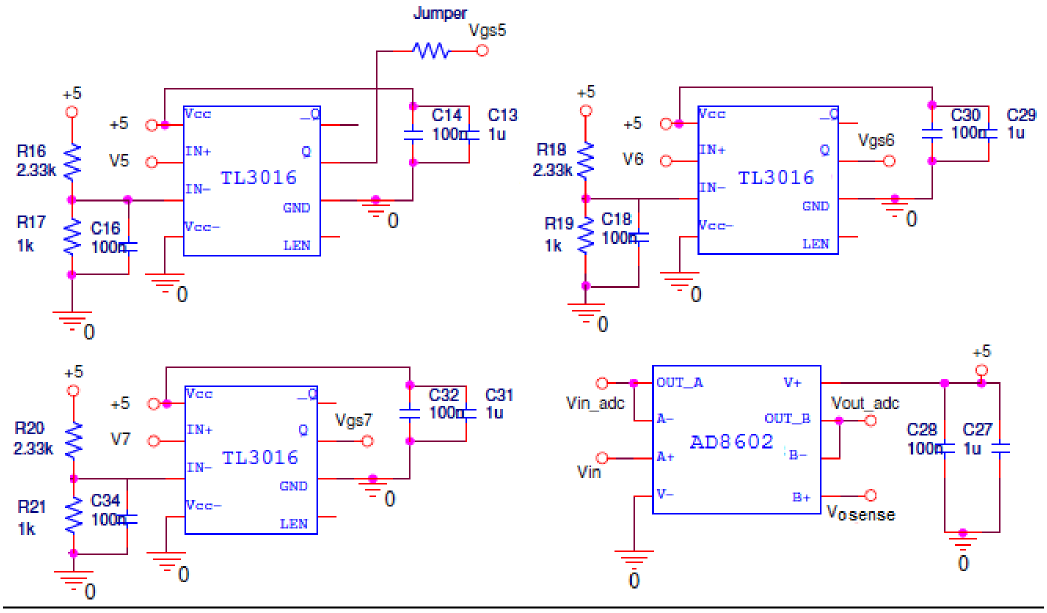

Fig.5.5 Comparators and OP-AMP

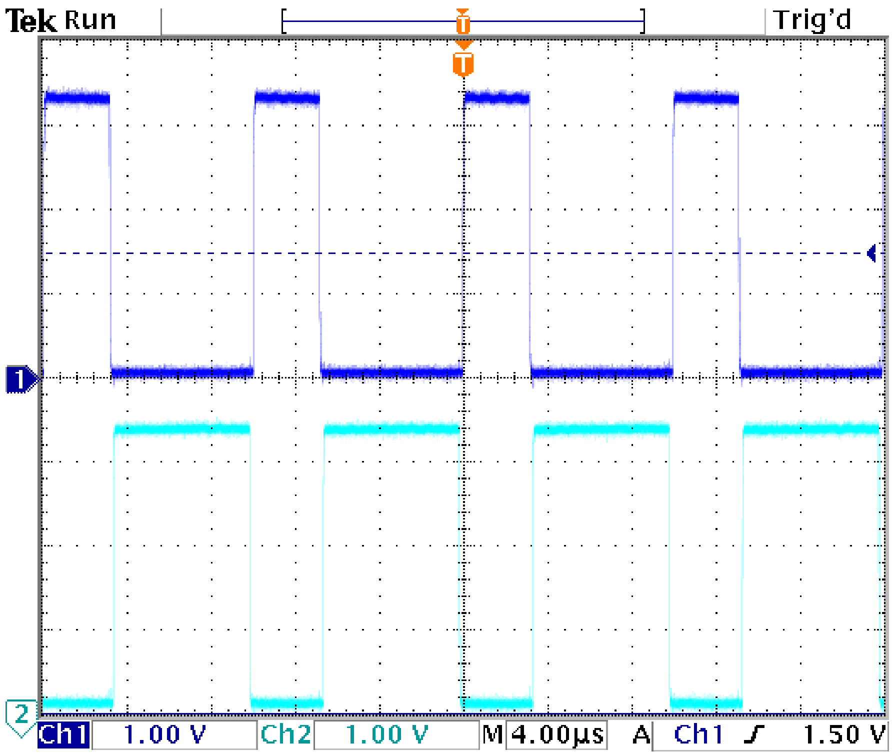

Fig.5.6 PWM with complementary control

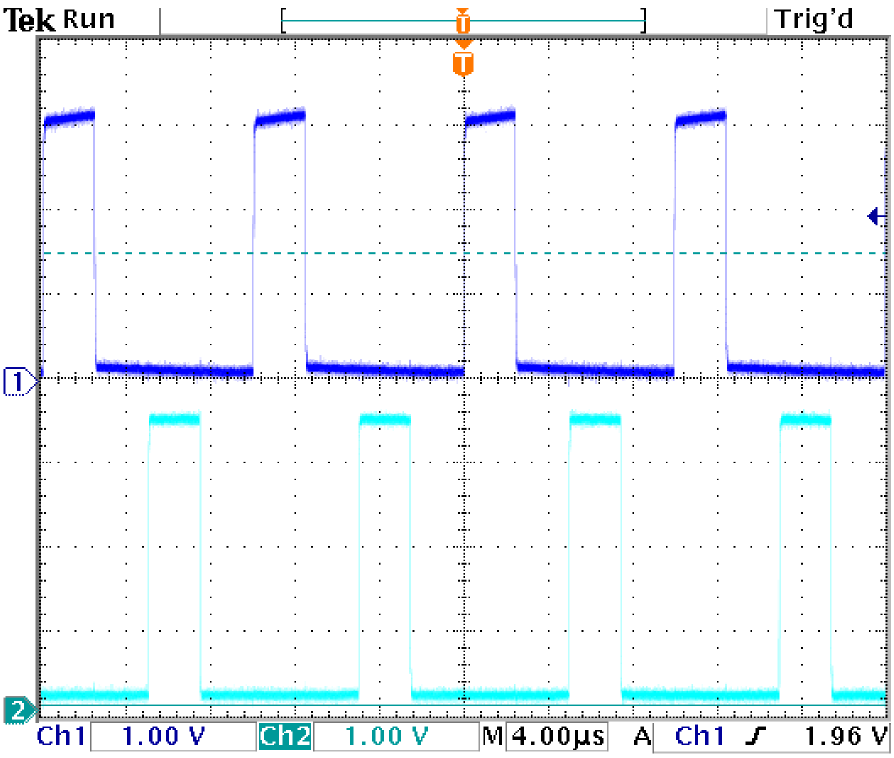

Fig.5.7 Control waveform with 180° phase delay

The two MOSFETs of one phase are complementarily driven as shown is Fig.5.6, with a proper dead time between the falling edge and the rising edge in order to avoid short circuit. The MOSFETs control signals of the second phase are the same with a phase delay of 180°, as Fig.5.7 shows, in order to obtained phase shedding.

At the beginning of each period of the first control signal, the PWM sends an interrupt to Central Processing Unit (CPU) that starts the conversion of the output voltage in a digital value by one of the 12 ADC. When the conversion is finished the ADC sends an interrupt to the CPU that starts the evaluation of the new duty cycle. At the beginning of the new period the new value of duty cycle is used.

5.2. Results

Fig.5.8 and Fig.5.9 show respectively the dynamic response under step reference variation from 1V to 3V of the open-loop system and the closed loop, in which has been implemented the control with one prediction step.

The regulator is stable and it is able to make faster the dynamic response of the converter, eliminating the oscillation caused by the second order filter. Fig. 5.10 shows a zoom in of few steps after the reference step. We can see that the response time lasts for 4-5 switching steps that mean regulator is close to its limit.

Fig.5.11 shows the simulation results obtained with the switching model of the converter and applying the same value of the regulator parameters implement in the microcontroller. Comparing both results, we can see that the simulated closed-loop behavior and the experimental one are really similar.

As last result, Fig.5.12 presents the closed-loop behavior of the system under 20% of load variation. Also in this case the control system behaves correctly rejecting the disturbance on the output current and maintaining the output voltage at the reference value.

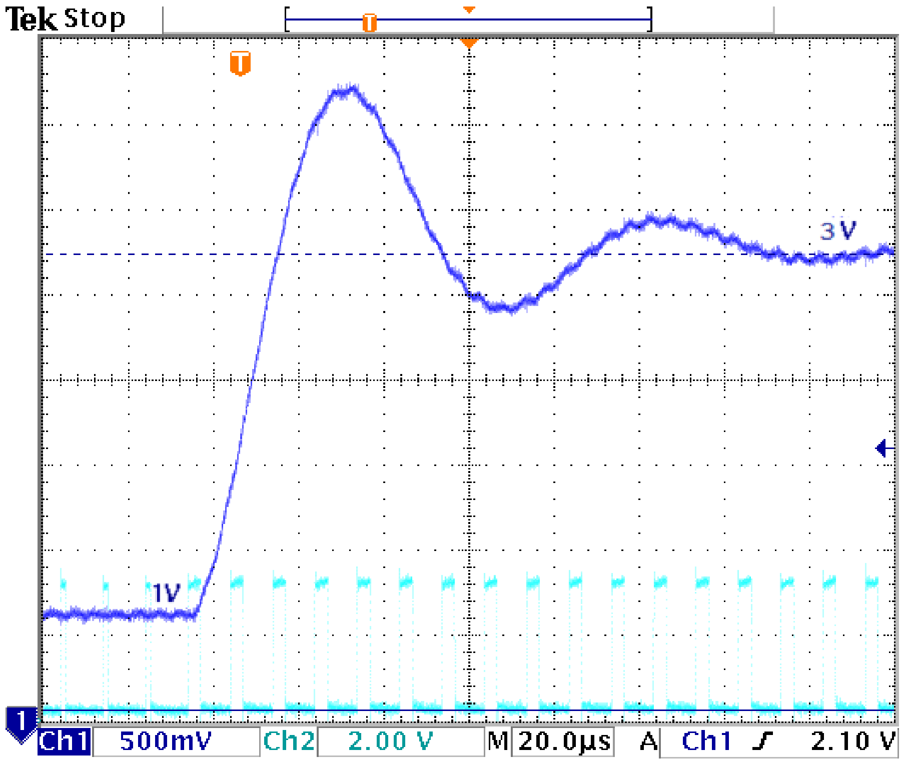

Fig.5.8 Output voltage open-loop response under reference step

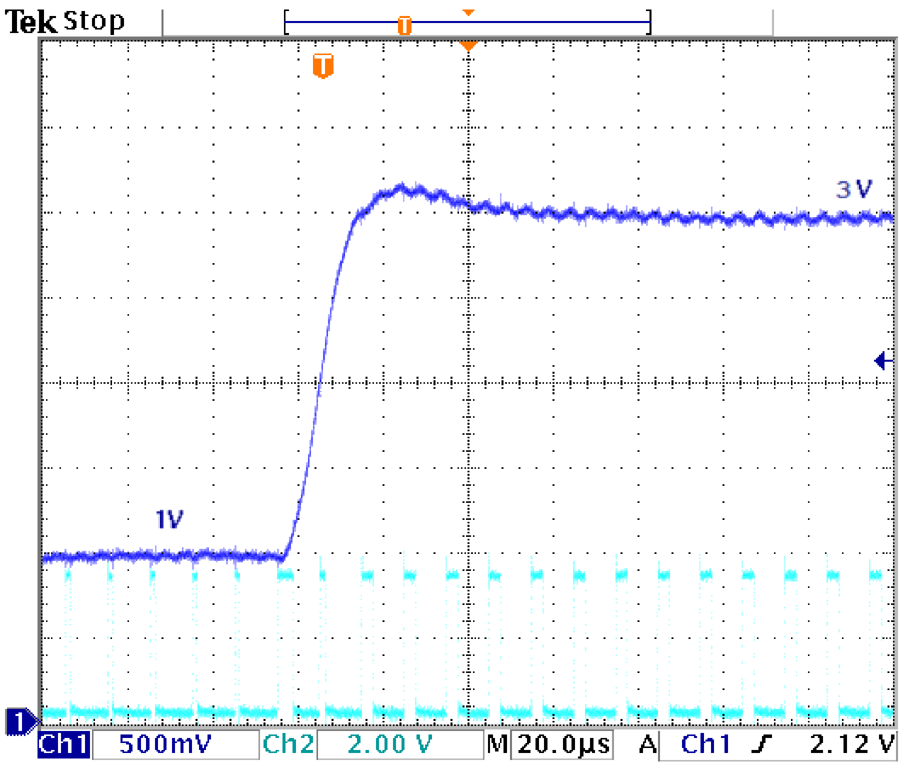

Fig.5.9 Output voltage closed-loop response under reference step

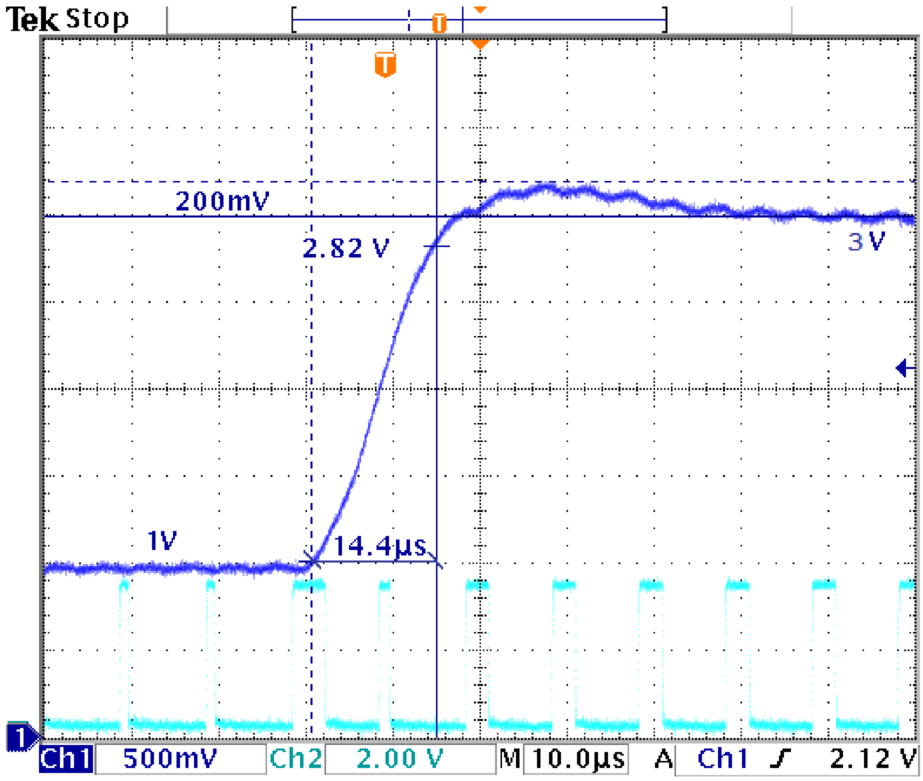

Fig.5.10 Zoom-in of the step response

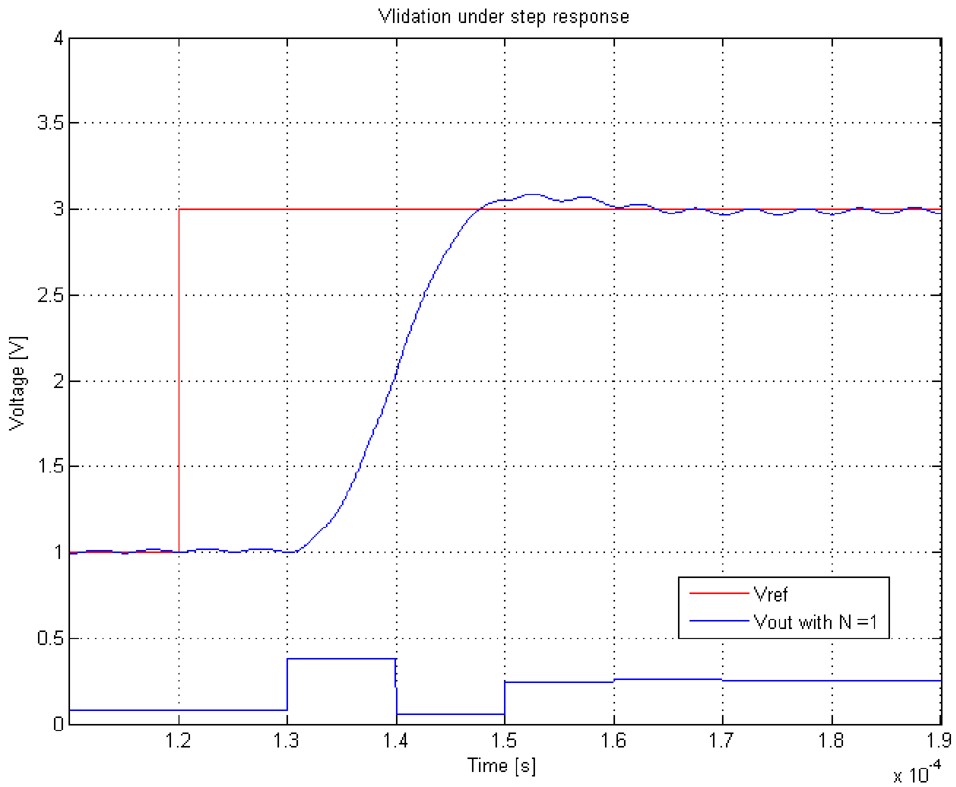

Fig.5.11 Simulation of the step response

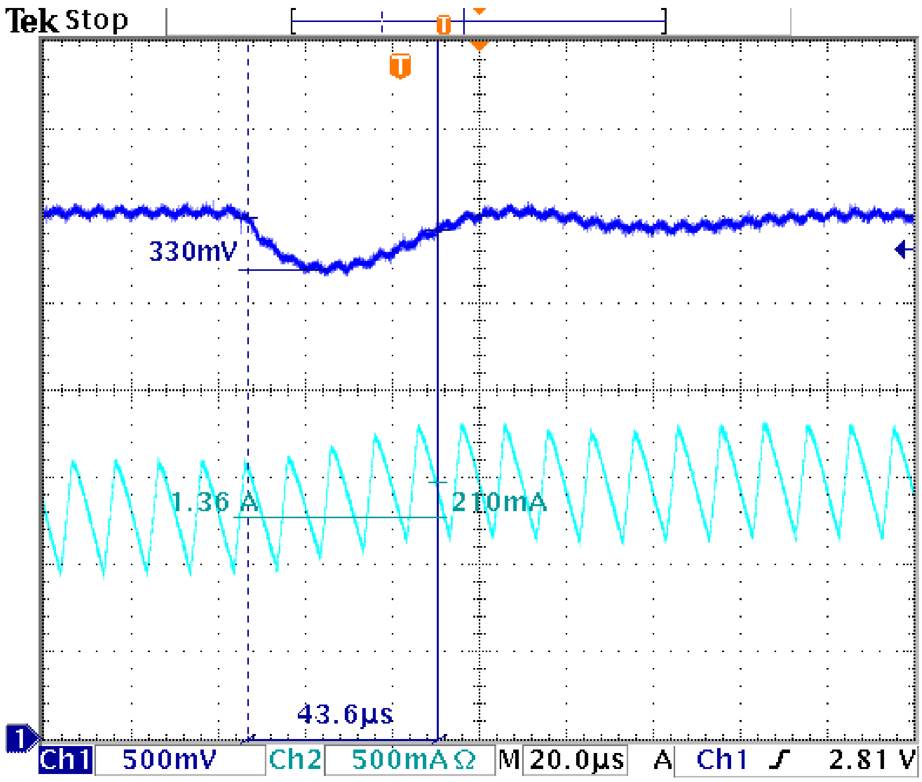

Fig.5.12 Output voltage closed-loop response under load variation

Conclusions

The target of the project was to design an efficient digital control for Buck converters, exploiting MPC theory and taking advantage of the available future information of the reference, in order to improve reference tracking. The proposed solution is able to use this information to improve the performance of the conventional linear regulator. In the simulations, a reduction of 80° in the phase delay between reference and output has been achieved compared to a conventional linear regulator.

The first benefit we can get from this approach is a consistent improvement of the converter efficiency at high frequency. Compared to a conventional regulator, we have the possibility to reach similar performance in terms of reference tracking at high frequency with a lower switching frequency and reducing in this way the power consumption caused by the switching losses.

Another minor benefit is the simplification of the state reference definition. By using the delta-transform representation of the system, we can define easily a reference for all the state variables without using additional computational power and without losing generality of the solution.

The proposed method may be easily applied to different converters as well, by simply adapting the design of the regulator to each particular case and we should expect to achieve similar results. The selection of the parameters is simplified by applying GA.

However some drawbacks can be found in this solution. The first one concerns its complexity. Although we looked for a simple solution, the number of operations increases linearly with the predictive horizon and exponentially with the order of the system. Besides, in order to stabilize the entire system we had to consider a delay compensator which complicates the implementation of the solution.

Experimental results show that it is difficult to overcome a predictive horizon of one step by implementing the algorithm in a microcontroller. However, in order to reach the expected improvement on reference tracking, we need at least a prediction horizon of three switching periods.

These considerations lead to two different possible works for the future. For low power applications, we can look for an approximate solution in order to decrease the computational power needed to run the control algorithm, for instance based on both approaches of the state and output reference, and try to implement it in a *Piccolo*-like microcontroller.

On the other hand, for high power applications, it can be interesting to implement the proposed approach on a hardware-based controller, like a FPGA. This has the benefit to be much faster than a microcontroller, so that we can easily implement more complex control algorithms and in this case we can implement a MPC with three steps predictive horizon.

As last work, there still remains to validate the robustness of the control system under possible variation of capacitor and inductors values.

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