Propagation Analysis in Molecular Communication Systems

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Abstract

Molecular communication (MC) is an innovative method for connecting nanoscale devices to exchange information. The incredibly reduced size of bio-nanomachines enhances their bio-compatibility, enabling a non-invasive intrabody communication and proposing MC as the best candidate for the interaction between nanonetworks and living organisms. The principles behind the idea entirely differ from the usual telecommunication paradigms: messages are carried by patterns of molecules using diffusive mechanisms rather than electrons or electromagnetic waves. However, the transmission range of molecular signals can be strongly limited by the losses and the low propagation speed of diffusion.

In this thesis we investigate quantitatively the MC and provide three different strategies to improve the communication efficiency. Specifically, we propose a Quorum Sensing-based method for communication between nanodevices distributed over a multi-hop structure, the introduction of an assisted propagation by an external energy source, or alternatively the shaping of the diffusive characteristics of the medium to favour the communication along a diffusive cable. Finally, we compare the performances of the techniques, by investigating which of the three may provide the best results, in terms of required energy and nanomachine number.
La comunicazione molecolare (MC) è un metodo innovativo per connettere dispositivi di scala nanometrica e realizzare uno scambio di informazione. La dimensione incredibilmente ridotta delle bio-nanomacchine aumenta la loro biocompatibilità, permettendo una comunicazione non invasiva all’interno del corpo e candidando la MC al primo posto per l’interazione fra le nanoreti e gli organismi viventi. I principi alla base differiscono completamente dai consueti paradigmi delle telecomunicazioni: i messaggi vengono trasportati da gruppi di molecole attraverso meccanismi diffusivi piuttosto che da elettroni o onde elettromagnetiche. Purtroppo, il range di trasmissione dei segnali molecolari può essere fortemente limitato dalle perdite e dalla ridotta velocità di propagazione della diffusione.

In questa tesi viene analizzata quantitativamente la MC e sono proposte tre differenti strategie per migliorare l’efficienza della comunicazione. Specificatamente, si presentano un metodo basato sul Quorum Sensing delle comunità batteriche per la comunicazione fra nanodispositivi in una struttura multi-hop, l’introduzione di una propagazione assistita fornita da una sorgente esterna di energia, e in alternativa la modifica delle caratteristiche diffusive del mezzo per favorire la comunicazione lungo un cavo diffusivo. Vengono poi confrontate le prestazioni delle tecniche proposte, analizzando quale delle tre può fornire i risultati migliori, in termini di energia richiesta e numero di nanodispositivi.
# Contents

List of Figures

List of Tables

1 Introduction

   1.1 Molecular communication:
       Why what and how
       Bio-nanomachines
       Communication system

   1.2 Communication channel characterization
       Particle propagation
       Nano communication parameters

   1.3 An example of MC:
       bacteria Quorum Sensing
       Quorum Sensing overview

2 Diffusion fundamentals

   2.1 Fick laws of diffusion
       Fundamental Solution
       Drift-Diffusion equation
       Diffusion in heterogeneous media

   2.2 Numerical implementation
       FTCS method for derivative implementation
       Simulations in homogeneous medium
       Heterogeneous medium: diffusive cable

3 Propagation analysis

   3.1 Diffusive channel parameters
       Pulse delay
       Pulse amplitude
       Pulse width
3.1.4 Pulse velocity ........................................... 36
3.2 Molecular vs Electromagnetic channel ......................... 38

4 Towards efficient connections ..................................... 40
  4.1 Bio-inspired multi-hop nanonetwork .......................... 41
    4.1.1 Activation model ....................................... 41
    4.1.2 Linear multi-hop ........................................ 43
    4.1.3 Lattice multi-hop ........................................ 52
    4.1.4 Hexagonal grid .......................................... 54
  4.2 Flow assisted propagation ..................................... 55
    4.2.1 Drift velocity ........................................... 56
  4.3 Diffusive cable ............................................... 59
    4.3.1 Cable velocity .......................................... 60

5 Conclusions ..................................................... 63

Bibliography ....................................................... 67
List of Figures

1.1 Scheme of a molecular communication system .......................... 3
1.2 Examples of binary source coding in molecular communication 4
1.3 Quorum Sensing unfavourable and favourable scenarios ............ 7
1.4 Quorum Sensing in Vibrio fischeri ........................................ 8
1.5 Quiescence and activation states in each bacterium ................. 9

2.1 Chemical diffusion before the barrier removal, microscopic and macroscopic view ........................................ 13
2.2 Chemical diffusion after the barrier removal, microscopic and macroscopic view ........................................ 13
2.3 Evolution of the Fundamental Solution ................................. 15
2.4 Signal diffusion [dB] in homogeneous medium, $D = 10^{-5} \text{cm}^2/\text{s}$, $\Delta t = 2.5 \cdot 10^{-2} \text{ms}$ ........................................ 20
2.5 Horizontal section of simulated and analytical solutions captured at $t = 0.5 \text{ms}$, $D = 10^{-5} \text{cm}^2/\text{s}$ .................. 21
2.6 Signal evolution in presence of drift, $D = 10^{-5} \text{cm}^2/\text{s}$, $A_x = -5.7 \text{cm}/\text{s}$ ........................................ 23
2.7 Diffusive cable, $\sigma_y = 20 \cdot \Delta_s$ ........................................ 24
2.8 Vertical section of the diffusive cable, $d = 10^{-5} \text{cm}^2/\text{s}$ .... 25
2.9 Signal diffusion in diffusive cable, $d = 10^{-5} \text{cm}^2/\text{s}$, $\sigma_y = 20 \cdot \Delta_s$, $\Delta t = 5 \cdot 10^{-2} \text{ms}$ .............. 26

3.1 Received signal as function of time, for $x = 1 \mu m$, $y = 0 \mu m$, $D = 10^{-5} \text{cm}^2/\text{s}$ ........................................ 28
3.2 Maximum amplitude-delay as function of receiver position along x axis, $D = 10^{-5} \text{cm}^2/\text{s}$ .......................... 30
3.3 Lambert W function, $z \in [-\frac{1}{e}, 6]$ .................................... 31
3.4 Threshold-delay as function of receiver position along x axis, $D = 10^{-5} \text{cm}^2/\text{s}$ .......................... 32
3.5 Received signal versus time, for multiple receiver distances, $D = 10^{-5} \text{cm}^2/\text{s}$ .......................... 34
3.6 Pulse amplitude versus receiver distance along $x$ axis, $D = 10^{-5}\, cm^2/s$ ................................. 34
3.7 Pulse width versus receiver distance along $x$ axis, $D = 10^{-5}\, cm^2/s$ 36
3.8 Maximum amplitude-velocity versus receiver distance along $x$ axis, for multiple diffusion coefficients .............................. 37
3.9 Threshold-velocity versus receiver distance along $x$ axis for multiple thresholds, $D = 10^{-5}\, cm^2/s$ ................................. 38

4.1 Quiet and activated states in nanomachines .................. 42
4.2 Multi-hop deployment and activation .................. 42
4.3 Linear multi-hop. Evolution of the molecular concentration, $\Delta = 2\mu m$, $\Delta t = 4 \cdot 10^{-2} ms$ for $N = 5$ nodes .............................. 43
4.4 Activation times versus space for $N = 100$, $\Delta = 2\mu m$ and threshold $T_{ACT}$ ................................. 45
4.5 Temporal evolution of activation, $\Delta = 2\mu m$, $T_{ACT} = 10^{-4} mol/cm^2$ 46
4.6 Diffusion velocity as function of $\Delta$ ................................. 47
4.7 Diffusion velocity as function of $T_{ACT}$ ................................. 48
4.8 Pseudo-uniform linear pattern ................................. 49
4.9 Probability of activation in pseudo-uniform distribution, elapsed time $t = 3.1 ms$ ................................. 50
4.10 Mean diffusion velocity ................................. 51
4.11 Lattice multi-hop. Evolution of the molecular concentration, $\Delta = 2\mu m$, $\Delta t = 4 \cdot 10^{-2} ms$ for $N = 16$ nodes .............................. 53
4.12 Lattice multi-hop. Avalanche effect on the diffusion velocity ................................. 53
4.13 Hexagonal grid. Evolution of the molecular concentration, $\Delta = 2\mu m$, $\Delta t = 4 \cdot 10^{-2} ms$ for $N = 37$ nodes .............................. 54
4.14 Hexagonal grid. Accumulation effect on the diffusion velocity for a dense deployment of $N = 468$ nanomachines ................................. 55
4.15 Flow assisted propagation, signal evolution for $D = 10^{-5}\, cm^2/s$, and $A_x = -5.69 cm/s$ ................................. 57
4.16 Flow assisted propagation, comparison between the different velocities, $|A_x| = V_{MH} = 5.69 cm/s$, $D = 10^{-5}\, cm^2/s$ ................................. 57
4.17 Diffusive cable velocity, $D_o = 10^{-5}\, cm^2/s$ and $d_{max} = 1.27 \cdot 10^{-4}\, cm^2/s$ ................................. 57

5.1 Channelling of diffusion along a sinusoidal cable, $\alpha = 1.2 \cdot 10^{-6}$, $d = 10^{-5}\, cm^2/s$, $\bar{g} = 20$ and $\Delta_t = 0.32 ms$ ................................. 65
5.2 $4 \times 4$ MIMO diffusive channel ................................. 66

IV
# List of Tables

1.1  Comparison between passive and active modes of molecular communication ............................................ 6

3.1  Comparison between electromagnetic and molecular propagation  39
Introduction

Molecular communication (MC) is a promising paradigm for communication in nanonetworks. Unlike conventional telecommunication techniques, MC follows a bio-inspired approach, in which molecules are used to encode, transmit and receive information at the nanoscale. This novel communication frontier is expected to be especially attractive due to its inherent biocompatibility, proposing MC as a competitive solution in problems of bio-nano medical applications, such as lab-on-a-chip devices and body area sensor networks [1, 2]. The messages consist in chemical signals of molecules and the way information travels comes in many forms. For example, molecules may propagate through an aqueous medium via Brownian motion, or may be carried by molecular motors. Moreover molecular nanoscale properties or only their macroscale properties (i.e., particle concentration) may be important for information decoding [3].

This chapter aims to give an introduction to this challenging kind of communication, justifying reasons and motivations for the choice of a molecular system instead of a common electromagnetic one.
1. Introduction

1.1 Molecular communication: Why what and how

To motivate the use of nanoscale communications, let’s suppose we are given the following design problem: performing a targeted drug delivery within the human body exactly where the medication is needed (for example, directly to malignant tumors, as chemotherapy), without affecting other healthy tissues. To accomplish this goal, thousands of tiny, blood cell-sized robots must cooperate one each other to navigate through the body, identify tumors and release their drugs to poison them. In order to cooperate, robots must be able to communicate at nanoscale distances with very small energy reserves, being careful not to disrupt healthy cells or to be destroyed by the immune system prior to complete their task. One solution to the problem can be the exchange of signals composed of molecules, rather than electrons, photons or electromagnetic waves as it commonly happens.

1.1.1 Bio-nanomachines

Since nano communication typically ranges from few nanometers to tens of micrometers, there is the need of specific devices which can interconnect one another despite the problems due to size limitations. The solution consists in using biological nanomachines, one of the primary applications of MC. Bio-nanomachines are nanoscale devices which perform simple computations, like sensing or actuation of an easy task. The main novelty is in their size: usual dimensions range from the one of a macromolecule to approximated 100\(\mu\)m (size of a biological cell [4]). Machines can be artificial, in the sense that material origin comes from a hybrid between biological and non biological tissues, or totally biological (e.g., proteins, nucleic acid, cells) [3].

An attractive aspect is that nanodevices are usually regarded as the basic functional unit for molecular communications, but they can also be used as building blocks to create more complex systems. These systems may be not strictly of nano-micro sizes, but they make use of nanomaterial properties to compute their tasks, taking advantages of these features for their purposes [5, 6].
1.1.2 Communication system

Firstly, it should be clear the concept of “molecular communication”. Even if the word “molecular” can be misleading, a MC system consists in an artificial exchange of manmade messages, conveyed from one point to another of the network. In the simplest system, there are two terminals: a transmitter which produces and sends the messages, and a receiver. To communicate, the transmitter makes a physical change in its surrounding environment that must be measurable at the receiver. The difference with respect to electromagnetic systems stays in the nature of this change. Here the change has to be molecular: released molecules propagate in a shared medium until they reach and are detected by the receiving device. So, three principal items compose the nanonetwork: the emission process, locally confined in the area of the transmitting device, the particle movement throughout the medium, and the reception process at the receiver side [3, 7].

Figure 1.1 shows an overview of a molecular system: an information source generates some messages, consequently the sending bio-nanomachines will encode information inside molecules that propagate in the system until reaching their destinations.

One possible question concerns on how information can be encoded inside the propagating molecules, so how a given message can be distinguished at
the receiver. Basically, there are three possibilities to encode and decode messages. E.g., suppose having a binary message:

- **Encoding quantities.** Transmitter can choose between two actions: generating and transmitting $n$ molecules (encoding 1), or releasing zero particles (encoding 0), so that at the receiver side the device can sense and decode a 1 or 0 respectively. The process is an ON-OFF binary modulation: ON corresponds to molecular emission, OFF to zero activity [2].

- **Encoding identities.** Transmitter uses two different kinds of molecules. Group A (circles in figure 1.2) refers to symbol 0 and group B (triangles) to 1. Receiver decodes information basing on the signal identity. This same mechanism can be used not only in binary situations but also for multilevel transmissions (if the number of different molecules is greater than two).

- **Encoding timing.** If transmitter has a single available molecule at its side, it can decide to put the information content inside the transmission times. To get a 0, it release instantaneously $n$ molecules whether to communicate 1 it waits $t > 0$ instants before releasing. Receiver decodes 0 or 1 by measuring times of arrivals.

![Figure 1.2: Examples of binary source coding in molecular communication](image-url)
1. Introduction

1.2 Communication channel characterization

1.2.1 Particle propagation

The main difference between electromagnetic propagation and molecular one stays in the way information travels throughout the medium. After the release of molecules, particles propagate towards the receiver by diffusion in an aqueous medium (intrabody communication). This diffusive process can be studied from two opposite points of view. When the number of molecules is small, propagation is basically a discrete Brownian motion, whether for a large amount of particles it is better to consider the continuous diffusion model. In practice, molecules propagate always by diffusion, but the way this phenomenon is investigated depends on where we want to focus our attention: a microscopic perspective (Brownian motion) or a macroscopic one (continuous diffusion) as a process involving concentrations of molecules rather than single particles.

Quantitative analysis of the characteristics of systems based on discrete Brownian motion requires processing that is far beyond contemporary technology because they demand sensing and manipulation of individual molecules. For this reason, it is common to use continuous diffusion laws for investigating. The method is less efficient than discrete models but more feasible for practical implementations: components really exist that can detect and respond to changes in concentration of a given molecular species [3].

1.2.2 Nano communication parameters

There exist two ways which MC inside human body can be characterized by. The first is where communication is established while the second is based on how signaling molecules propagate in the environment.

For what concerns the former case of investigation, we have three levels of communication scale: intracellular (within a cell), intercellular (between nearby cells, typically from few micrometers to $10\mu$m), and interorgan levels (between distant cells, up to few meters). When the communication range is limited to the local surroundings of the transmitter, the communication is called paracrine signaling, in the sense that the effects can be detected only
in the close vicinity of the transmitter. If we move to interorgan communication levels, it is better to talk about *endocrine signaling*, since particles propagate into the bloodstream and can circulate throughout all the body [3].

The *mode* in which molecules diffuse in the environment can be active or passive. Passive mode means that molecules randomly diffuse in all available directions. Active mode describes situations when particles directionally propagate to specific locations, typically farther than passive-mode ranges. Passive mode is particularly suited to situations in which an infrastructure for molecular communication is not available, maybe because the environments are highly dynamic and unpredictable. On the opposite side, passive mode requires a large number of molecules to reach a distant destination, and arrival times increase a lot due to the random movements of particles. If active communication provides lower delays and longer ranges, it usually requires an infrastructure establishment before the communication and a regular energy supply for the devices.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagation</td>
<td>Random</td>
<td>Directional</td>
</tr>
<tr>
<td>Range</td>
<td>Near</td>
<td>Far</td>
</tr>
<tr>
<td>Number of molecules needed</td>
<td>Large</td>
<td>Small</td>
</tr>
<tr>
<td>Communication infrastructure</td>
<td>Not required</td>
<td>Required</td>
</tr>
<tr>
<td>Energy supply</td>
<td>Not required</td>
<td>Required</td>
</tr>
</tbody>
</table>

*Table 1.1: Comparison between passive and active modes of molecular communication*

### 1.3 An example of MC: bacteria Quorum Sensing

Recent publications in microbiology show that bacteria (e.g. *V. harveyi*, *B. subtilis*, *E. coli* etc..) can communicate and synchronize each other using hormone-like signaling molecules termed *autoinducers*. Each bacterium of the colony produces and releases autoinducers, whose density in the external environment increases as the cell population grows. Bacteria have specific receptors to detect the concentration level of these molecules and alter their
1. Introduction

gene expression in response.
The whole process of producing, releasing, detecting and responding to autoinducers is termed Quorum Sensing. It has been proved that Quorum Sensing processes are unproductive when undertaken by an individual bacterium acting alone, but become beneficial when carried out simultaneously by a large number of cells. This way, bacteria coordinate their behaviour on a population-wide scale and act as a multicellular organism [8, 5].

Figure 1.3: Quorum Sensing unfavourable and favourable scenarios

1.3.1 Quorum Sensing overview

Let’s suppose a bacteria species needs to launch an attack in order to survive or spread. If a bacterium alone tries to attack, host fences will eliminate it immediately. On the contrary, if a large group of bacteria coordinate themselves and launch their attacks simultaneously, the probability of a success increases enormously. This is the power of Quorum Sensing process, which enables bacteria to “talk” one another and generate effects that would be impossible for a single bacterium. In nature, several Quorum Sensing behaviours have been detected such as motility, DNA processing, antibiotic biosynthesis and biofilm formation [5].

Maybe one of the most fascinating phenomenon is the bioluminescence activated by the marine bacterium Vibrio fischeri, which colonizes the light organ of the Bob Tail Hawaiian squid (Euprymna scolopes). This nocturnal squid uses a common method of counterillumination to produce a camouflage, not to be seen from predators or victims situated below him inside the
1. Introduction

sea. Bacteria provide the light and in return they benefit in nutrients and in quick proliferation, unachievable in seawater. The process of light production is very simple to understand from an intuitive point of view. We will not deeply investigate the biological aspect of Quorum Sensing, since it gives no contribution to our work and it is not related to the telecommunication world.

Just to give a simplified example of the agents entering the scene, figure 1.4 shows how *V. fischeri* signaling circuit achieves light production: LuxI is a protein which synthesizes the HSL autoinducers while LuxR is a receptor protein. When Quorum Sensing starts, HSL binds with LuxR, activating the transcription of the LuxICDABE operon\(^1\) which induces light and LuxI production [8].

Autoinducers have the ability of triggering the release of more of the same kind when they are sensed by bacteria. At the beginning of the process, each bacterial cell starts the autoinducer emission in its surroundings, sensing at

---

\(^1\)Operon: segment of DNA containing a cluster of genes under the control of a single operator gene. All the genes contained in the operon are either expressed together or not at all.
the same time the concentration level. There exist two meaningful thresholds for the mechanism:

- **Activation threshold.** When autoinducer density overcomes this threshold, bacteria activate the regulation of gene expression (the bioluminescence). Quorum Sensing enables the synchronization of all the colony, so that the gene expression is induced at the same time for every cell and the community can act as a multicellular organism.

- **Autocatalytic threshold.** This value is not strictly related with the gene expression but with the emission of particles. In fact, autoinducers are synthesized by default at a nominal or basal rate. When the sensed concentration reaches the autocatalytic threshold, emission rate increases dramatically and a positive feedback of particle emission and reaction to is produced.

Focusing our attention on the gene expression mechanism, the overall activity of the community can be seen as an ON-OFF switch, based on the activation value \([5, 4]\). We can identify two states: when the concentration is below the activation threshold (hereafter denoted as \(T_{ON}\)), bacterium is in \(S_{OFF}\) state, emitting autoinducers but not expressing the gene. The gene regulation happens when the density is above \(T_{ON}\) (\(S_{ON}\)).

![Figure 1.5: Quiescence and activation states in each bacterium](image)

Note that the cycle of emission and sensing of the environment is continuous: bacteria release autoinducers and sense the surroundings while being in both \(S_{ON}\) and \(S_{OFF}\) states. After the change of behaviour the sensing action continues, because activation state can also turn back into the quiescence one if concentration goes below \(T_{OFF}\). An important remark is that the two values of \(T_{ON}\) and \(T_{OFF}\) don’t have to be equal, rather \(T_{OFF}\) is usually less
than $T_{ON}$. This phenomenon is explained by the fact that an easy reversal is avoided and so a hysteresis behaviour of the cycle appears.
A deep study of the diffusion phenomenon becomes necessary to investigate the fascinating field of molecular communication.

Diffusion process has always existed in nature. Think about chemical solutions: solute particles diffuse inside the solvent until a uniform distribution is achieved in the whole space. What we can do is to give an expression of the phenomenon and be inspired by it, to build nanomachines capable to realize even intracellular communication with low energy consumption and high bio-compatibility.

In particular, the continuous diffusion model is addressed in this chapter and several are the motivations to validate the decision. As pointed out previously, the continuous process is the most direct way to understand how MC works, because it has a linear behaviour. This is an important feature giving a great help to the investigations. Just think about how superposition principle can simplify the scenario every time happens the presence of two or more simultaneous sources. Furthermore, there exist indeed bionanomachines that can implement diffusion-based operations like detection and response to changes in concentration of a given molecular species.

Since diffusion is not the conventional paradigm in the telecommunication world, a theoretical study and a discrete implementation on a mathematical simulator are both necessary steps to understand how propagation occurs. Matlab® simulator has been employed for the discrete implementation of the model.
2. Diffusion fundamentals

2.1 Fick laws of diffusion

Diffusion principle is based to the fact that a given quantity, distributed over a space, modifies its shape in time until a steady state condition is achieved, with moving flows that are balanced by opposite flows.

To give some examples, think about the change in the temperature of a room due to a sudden cold or hot source (e.g., heater starting, open window, etc...). The room temperature modifies consequently, and the provided heat distributes all over the environment by a diffusive mechanism [9]. Likewise, cancer growth, angiogenesis\(^1\) and consequent invasion of the human body are diffusion processes too: tumoral cells are hungry of nutrients and oxygen and enter the competition with healthy tissue for space and energy [10, 11].

Moving to another field of investigation, in absence of external electric field the motion of electrons and relative holes in semiconductors is a diffusive process. At macroscopic level, the chaotic movement of charges generates an electron flux which moves in a continuous diffusive action, creating the diffusion current in electronic devices [9].

All the above situations can be investigated thanks to Fick laws on diffusion, which provide a rule to explain how a certain input function diffuses over the space. Here is the principle: input will start to spread all over the surroundings, decreasing density in favour to those areas where it is much lower. Let’s write Fick’s second equation in a mono-dimensional space [12]:

\[
\frac{\partial \phi(x,t)}{\partial t} = D \frac{\partial^2 \phi(x,t)}{\partial x^2}
\]  

(2.1)

The most famous and familiar diffusion phenomenon is the chemical one: \(\phi(x,t)\) represents the concentration of some diffusive particles per unit length. Think at this situation: we have a fluid solvent in a container in which we previously introduced a barrier in the middle, and then we put some solute molecules on the left side. Solute particles immediately start to diffuse, trying to occupy all the left space in a homogeneous way. The interesting fact is that, as soon as we remove the barrier, molecules spread towards the right, modifying their density in space and time until a uniform steady state condition is reached.

\(^{1}\text{Angiogenesis: blood vessel formation}\)
Steady state condition means that, in equation (2.1), the second derivative with respect to space is null, or in general,

$$\frac{\partial \phi(x, t)}{\partial t} = 0 \quad \text{for} \quad t \to \infty \quad (2.2)$$

At an infinite time, the solution will be homogeneously distributed, that is, solute molecules will be arranged in such a way that interdistance between them will be constant all over the space.
2. Diffusion fundamentals

Coefficient $D$ in (2.1) represents the spatial diffusivity and it is connected with the velocity of particle propagation. It is dependent on the size and structure of molecules, and on the temperature and viscosity of the medium [13]. Dealing with homogeneous media, it is comfortable to choose this coefficient to be constant, both in time and space.

In the two dimension equation the second derivative is substituted with the Laplacian operator. From now on, all the investigated situations will suppose to be in a bi-dimensional space with planar diffusion [12]:

$$\frac{\partial \phi(x, y, t)}{\partial t} = D \nabla^2 \phi(x, y, t)$$  \hspace{1cm} (2.3)

Actually, to understand more deeply what happens in figures 2.1 and 2.2, first Fick law describes how the molecular flux travels in space and time.

$$\vec{J} = J_x \vec{u}_x + J_y \vec{u}_y = -D \nabla \phi(x, y, t)$$  \hspace{1cm} (2.4)

The flux of particles, defined as the amount of substances per unitary area and time (vector $\vec{J}$ in equation (2.4)), moves in the directions of negative spatial gradient until a homogeneous pattern all around the space is generated [12]. Therefore, in the previous example, once the barrier is removed, a negative concentration difference is generated which pushes the molecular flow towards the right.

### 2.1.1 Fundamental Solution

Supposing the considered medium to be spatially homogeneous, the investigation of diffusion starts with the characterization of the system impulse response. That is, finding the solution of:

$$\frac{\partial \phi(x, y, t)}{\partial t} = D \nabla^2 \phi(x, y, t) + \delta(x, y, t)$$  \hspace{1cm} (2.5)
We introduced an additional term with respect to equation (2.3) since the presence of a source signal as system input has to be considered. Function \( \phi(x, y, t) \) satisfying (2.5) was found by Fourier in his *Analytical Theory of Heat* and it is known as **Fundamental Solution** of the diffusion equation [9, 12]:

\[
\phi(x, y, t) = \frac{1}{4\pi Dt} e^{-\frac{x^2+y^2}{4Dt}}
\]  

(2.6)

It is useful to pass in Fourier domain for spatial coordinates and introduce the spatial frequencies \( f_x \) and \( f_y \):

\[
\Phi(f_x, f_y, t) = e^{-4\pi^2(f_x^2+f_y^2)Dt}
\]  

(2.7)

\( \Phi(f_x, f_y, t) \) is a Gaussian filter which smooths the initial conditions given by \( \delta(x, y, t) \) and corresponds to a low pass behaviour as time increases, so that the contribution of an instantaneous perturbation tends to vanish in time and when far from the starting point.

[Figure 2.3: Evolution of the Fundamental Solution]
2.1.2 Drift-Diffusion equation

There are two different propagation schemes in molecular communication: passive transport and active transport. In passive transport, the information carrying particles propagate from the transmitter to the receiver by diffusing in the fluid medium without using external energy. In active transport, the particles are transported by external means such as molecular motors or syringe pumps. E.g., the syringe pump is used to create a flow that assists the molecular movement [1]. Another technique for the flow assisted propagation in chemical signaling considers a tabletop fan to guide alcohol particles in air towards the receiver [2]. Other than the chemical world, the drift current in electronic devices is generated by an external electric field which pushes the motion of free electrons in the conduction band.

We can model the above instances thanks to the drift-diffusion equation, which adds to the Fundamental Solution an external contribution given by the drift.

\[
\frac{\partial \phi(x, y, t)}{\partial t} = D \nabla^2 \phi(x, y, t) + \vec{A} \cdot \nabla \phi(x, y, t) + \delta(x, y, t) \tag{2.8}
\]

\(\vec{A} (= A_x \vec{u}_x + A_y \vec{u}_y)\) is a vector indicating the direction and the amplitude of the drift. If we consider a pressure pointing towards positive horizontal coordinates, \(\vec{A} = A_x \vec{u}_x\), with \(A_x < 0\). The solution of \(2.8\) is the following:

\[
\phi(x, y, t) = \frac{1}{4\pi D t} e^{-\frac{(x + A_x t)^2 + y^2}{4Dt}} \tag{2.9}
\]

The difference to the pure homogeneous case lies in the addition of a further speed contribution which directs the flow propagation.

2.1.3 Diffusion in heterogeneous media

A more interesting and challenging situation appears when the medium is not homogeneous, that is \(D\) is not constant over space but \(D = D(x, y)\). This means that the medium has variable diffusive properties and therefore the
trend of diffusion is bound to follow its characteristics. Where the diffusion coefficient is large we have a good diffusive effect, while in the areas where it is small the signal spreads with a much lower speed. In this case Fick second equation cannot be solved just with the Laplacian, but [12]:

\[
\frac{\partial \phi(x, y, t)}{\partial t} = \nabla \cdot (D \nabla \phi(x, y, t))
\] (2.10)

Solving the equation is much more complicated than solving the former one for homogeneous-medium (subsection 2.1.1). For this reason some simplifications can be done. For example, the hypothesis of a separable diffusion coefficient \(D = D_x(x) \cdot D_y(y)\) simplifies a lot the operations:

\[
\nabla \cdot (D \nabla \phi(x, y, t)) = D_y(y) \frac{\partial}{\partial x} \left( D_x(x) \frac{\partial \phi(x, y, t)}{\partial x} \right) + D_x(x) \frac{\partial}{\partial y} \left( D_y(y) \frac{\partial \phi(x, y, t)}{\partial y} \right)
\] (2.11)

After some passages, one gets to:

\[
\frac{\partial \phi}{\partial t} = D_x(x) D_y(y) \nabla^2 \phi + D_y(y) \frac{dD_x(x)}{dx} \frac{\partial \phi}{\partial x} + D_x(x) \frac{dD_y(y)}{dy} \frac{\partial \phi}{\partial y}
\] (2.12)

The presence of additive first derivative terms suggests that, asides from diffusion, a further transport contribution appears, pushing the motion in the directions of maximum diffusivity deviation.

### 2.2 Numerical implementation

To analyze the diffusion behaviour a discretization of Fick partial differential equation (PDE) and an implementation of a mathematical simulator are needed. Both spatial and temporal samplings must be performed, being careful that stability constraints are satisfied.
2.2.1 FTCS method for derivative implementation

The conventional discretization model for a parabolic PDE like (2.3) is the FTCS (Forward-Time Central-Space) method, used to evaluate discrete versions of temporal and spatial derivatives [14, 15].

Temporal derivative is calculated by considering the difference with the next time sample (forward time):

\[
\frac{\partial \phi(x, y, t)}{\partial t} = \frac{\phi(x, y, (k + 1)T_s) - \phi(x, y, kT_s)}{T_s}
\]  \(2.13\)

Where \(T_s\) is the temporal difference between one acquisition and the following (sampling time).

Second spatial derivatives (central space) are approximated considering a uniform spatial sampling along \(x\) and \(y\) (\(\Delta_s = \Delta_x = \Delta_y\)). The derivative along the first dimension \((x)\) is here reported, being the one along \(y\) symmetrical:

\[
\frac{\partial^2 \phi(x, y, t)}{\partial x^2} = \frac{\phi((i + 1)\Delta_s, y, t) - 2\phi(i\Delta_s, y, t) + \phi((i - 1)\Delta_s, y, t)}{\Delta_s^2}
\]  \(2.14\)

Note that, in a bi-dimensional problem, stability conditions for the discretization state that \(\Delta_s\) and \(T_s\) must satisfy [14, 15]:

\[
T_s < \frac{\Delta_s^2}{4D(x, y)}, \quad \forall x, y \in \Gamma
\]  \(2.15\)

If a space of \(M\) (along \(y\)) \(\times N\) (along \(x\)) points is considered, to apply the second spatial derivative is equivalent to a matrix product between matrix \(\Phi\), which contains the concentration values (sampled every \(\Delta_s\) along \(x\) and \(y\)), and derivative matrices \(\hat{J}_x (N \times N)\), or \(\hat{J}_y (M \times M)\).

\[
\Phi_x = \Phi \cdot \hat{J}_x = \frac{\partial^2}{\partial x^2} \phi(x, y, t) = \Phi \cdot \begin{pmatrix}
-2 & 1 & 0 & 0 & \cdots & 0 \\
1 & -2 & 1 & 0 & \cdots & 0 \\
0 & 1 & -2 & 1 & \cdots & 0 \\
\vdots & 0 & 1 & -2 & \ddots & 0 \\
0 & 0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 0 & 1 & -2
\end{pmatrix}
\]  \(2.16\)
2. Diffusion fundamentals

\[ \Phi_y = \mathbf{J}_y \cdot \Phi = \frac{\partial^2}{\partial y^2} \phi(x, y, t) = \begin{pmatrix} -2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix} \cdot \Phi \quad (2.17) \]

2.2.2 Simulations in homogeneous medium

Fundamental Solution

We simulate the emission of a single molecular pulse in the center of the space. Discrete version of second Fick law for homogeneous diffusion coefficient becomes:

\[ \phi(i \Delta_s, j \Delta_s, (k + 1)T_s) = \frac{D T_s}{\Delta_s^2} \left( \phi((i + 1) \Delta_s, j \Delta_s, kT_s) - 2\phi(i \Delta_s, j \Delta_s, kT_s) + \phi((i - 1) \Delta_s, j \Delta_s, kT_s) \right) \]

By putting:

\[ \phi((i + 1) \Delta_s, j \Delta_s, kT_s) = \phi(i + 1, j, k) \quad (2.19) \]

One obtains:
2. Diffusion fundamentals

\[
\phi(i, j, k + 1) = \frac{DT_s}{\Delta s^2} \left( \phi(i + 1, j, k) - 2\phi(i, j, k) + \phi(i - 1, j, k) \right) + \\
+ \frac{DT_s}{\Delta s^2} \left( \phi(i, j + 1, k) - 2\phi(i, j, k) + \phi(i, j - 1, k) \right) + \\
+ \phi(i, j, k)
\] (2.20)

In the FTCS method, these operations can be performed by a sequence of matrix products:

\[
\Phi(k + 1) = \frac{DT_s}{\Delta s^2} \left( \Bar{\Phi_x}(k) + \Bar{\Phi_y}(k) \right) + \Phi(k)
\] (2.21)

Hereafter are reported some temporal snapshots of the simulated impulse response of the system, where \( t_0 \) is the time instant correspondent to the pulse emission.

![Temporal Snapshots](image)

*Figure 2.4: Signal diffusion [dB] in homogeneous medium, \( D = 10^{-5} \text{cm}^2/s \), \( \Delta t = 2.5 \cdot 10^{-2} \text{ms} \)*
To verify the righteousness of simulations, the comparison of numerical solution with the *Fundamental Solution* is shown in figure 2.5.

**Figure 2.5:** Horizontal section of simulated and analytical solutions captured at $t = 0.5\, ms$, $D = 10^{-5}\, cm^2/s$.

### Diffusion with drift

In addition of diffusion we introduce a negative drift ($\vec{A} = A_x \vec{u}_x$, $A_x < 0$) that directs the motion towards increasing $x$. Recalling equation (2.8), the discrete version of first spatial derivative has been implemented as follows, since it must take into account the forward propagation of the pulse.

$$
\frac{\partial \phi(x, y, t)}{\partial x} = \frac{\phi(i\Delta_s, y, t) - \phi((i - 1)\Delta_s, y, t)}{\Delta_s}
$$

(2.22)

The derivative corresponds again to a square matrix with the same dimensions of the second-derivative ones.
2. Diffusion fundamentals

\[ \dot{\Phi}_x = \Phi \cdot \dot{J}_x = \frac{\partial}{\partial x} \phi(x, y, t) = \Phi \cdot \begin{pmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 \\ \vdots & 0 & 0 & 1 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \] (2.23)

The sequence of steps is basically the same of the fundamental solution, with an additive term by the drift:

\[ \Phi(k + 1) = \frac{DT_s}{\Delta_s^2} \left( \Phi_x(k) + \Phi_y(k) \right) + \frac{A_x T_s}{\Delta_s} \Phi_x(k) + \Phi(k) \] (2.24)

We have to be careful with the discretization period because stability conditions in presence of drift are more tight. The addition of the drift brings as a consequence:

\[ T_s < \frac{\Delta_s^2}{4D - A_x \Delta_s}, \quad \forall x, y \in \Gamma \] (2.25)

Figure 2.6 shows the evolution of the concentration captured every \( \Delta_t = 5.8 \cdot 10^{-2} \text{ms} \).
2. Diffusion fundamentals

2.2.3 Heterogeneous medium: diffusive cable

If the medium is not homogeneous the Laplacian operator cannot be used, because the diffusion coefficient varies along the space and so enters into the derivatives. To provide an example of heterogeneous medium, one might think of having a space in which the diffusivity is almost constant everywhere \( D(x, y) = D_1 \quad \forall x, y \), except for a small portion where it is much higher \( D_2 \gg D_1 \). If \( D_2 \) is distributed along a line, it produces in fact a diffusive cable. The cable shown in figure 2.7 has been built according to this idea, with a separable diffusivity:

\[
D(x, y) = D_x(x) \cdot D_y(y) = \begin{cases} 
  d \cdot e^{-\frac{x^2}{2\sigma^2}} \cdot e^{-\frac{y^2}{2\sigma^2}} & x < 0 \\
  d \cdot e^{-\frac{x^2}{2\sigma^2}} & 0 \leq x \leq L_c \\
  d \cdot e^{-\frac{(x-L_c)^2}{2\sigma^2}} \cdot e^{-\frac{y^2}{2\sigma^2}} & x > L_c
\end{cases}
\]  

\( (2.26) \)
2. Diffusion fundamentals

With

$$\sigma_x = \sigma_y$$  \hspace{1cm} (2.27)

We considered a cable centred in the space, with boundary coordinates \((0, 0)\) and \((L_c, 0)\) and gaussian decay given by \(\sigma_y^2\).

![Figure 2.7: Diffusive cable, \(\sigma_y = 20 \cdot \Delta_s\)](image)

Resuming the equation for the separable heterogeneous case:

$$\frac{\partial \phi}{\partial t} = D_x(x)D_y(y)\nabla^2 \phi + D_y(y)\frac{dD_x(x)}{dx} \frac{\partial \phi}{\partial x} + D_x(x)\frac{dD_y(y)}{dy} \frac{\partial \phi}{\partial y} \hspace{1cm} (2.28)$$

First spatial derivatives are approximated with finite difference by forward space (we report only \(x\) dimension by symmetry) [14]:

$$\frac{\partial \phi(x, y, t)}{\partial x} = \frac{\phi((i + 1)\Delta_s, y, t) - \phi(i\Delta_s, y, t)}{\Delta_s} \hspace{1cm} (2.29)$$
Figure 2.8: Vertical section of the diffusive cable, $d = 10^{-5} \text{cm}^2/\text{s}$

In matrix notation:

\[
\dot{\Phi}_x = \Phi \cdot \dot{J}_x = \frac{\partial}{\partial x} \phi(x, y, t) = \Phi \cdot \begin{pmatrix}
-1 & 0 & 0 & 0 & \cdots & 0 \\
1 & -1 & 0 & 0 & \cdots & 0 \\
0 & 1 & -1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
\end{pmatrix} \tag{2.30}
\]

\[
\dot{\Phi}_y = \dot{J}_y \cdot \Phi = \frac{\partial}{\partial y} \phi(x, y, t) = \begin{pmatrix}
-1 & 1 & 0 & 0 & \cdots & 0 \\
0 & -1 & 1 & 0 & \cdots & 0 \\
0 & 0 & -1 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & -1 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 \\
\end{pmatrix} \cdot \Phi \tag{2.31}
\]

FTCS operations will be:
2. Diffusion fundamentals

\[ \Phi(k + 1) = \]

\[ \frac{T_s}{\Delta^2_s} \cdot \left( D \circ (\Phi_x(k) + \Phi_y(k)) + D_y \circ \dot{\Phi}_x(k) + D_x \circ \dot{\Phi}_y(k) \right) + \Phi(k) \]

(2.32)

Where the symbol \( \circ \) stays for the Hadamard product between 2 matrices.

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

\[ t = t_0 + \Delta t \]

\[ x [\mu m] \]

\[ y [\mu m] \]

\[ \times 10^{10} \]

\[ \times 10^5 \]

\[ \times 10^3 \]

\[ \times 10^1 \]

\[ \times 10^{-1} \]

\[ \times 10^{-3} \]

\[ \times 10^{-5} \]

\[ \times 10^{-7} \]

\[ \times 10^{-9} \]

\[ \times 10^{-11} \]

Finding an analytical solution for the problem seems to be really hard. The Gaussian diffusion problems rise because the diffusion coefficient is not constant and consequently an additive transport term appears in the equation. From the other hand, fortunately, the considered diffusivity is an infinitely derivable function, that is $D(x, y) \in C^\infty$. This is a good point, because surely the true analytical solution follows a Gaussian decay, similar to the Fundamental Solution one, with some modifications along the cable direction. There exist mathematical methods for giving an approximate solution to parabolic PDE like diffusion one (Parametrix method) but the calculation here is not faced.
Propagation analysis

The challenging aspect of molecular communication is the possibility to establish multiple links between nanomachines in environments where an electromagnetic connection is basically impossible, given the requested nano sizes. In this chapter we focus our attention on the communication channel, with the conventional tools of communication theory. To evaluate the performance of diffusion-based communications, standard parameters like channel attenuation, delay, distortion and propagation velocity are considered here. Furthermore, it is presented a comparison with the common electromagnetic channel.
3. Propagation analysis

3.1 Diffusive channel parameters

The evaluation of main channel parameters has been carried out by considering the impulse response of the system with homogeneous diffusion coefficient. Analytical solution (Fundamental Solution) and simulated one (with FTCS) have been compared, to demonstrate the reliability of simulations. If we consider the emission of a single molecule in the center of a bi-dimensional space (time $t_0 = 0$, position $(x, y) = (0, 0)$), the received signal by diffusion at a generic coordinate has the following behaviour:

![Graph showing molecular concentration over time](image)

Figure 3.1: Received signal as function of time, for $x = 1\mu m$, $y = 0\mu m$, $D = 10^{-5}cm^2/s$

The source evolution presents only one global maximum, whose position in time depends on the distance between source and receiver.

3.1.1 Pulse delay

Propagation delay is defined as the elapsed time between the transmission and reception of the signal. We provide two different definitions for the delay:
3. Propagation analysis

1 **Maximum amplitude.** Pulse delay is defined as the time instant correspondent to the maximum received amplitude.

2 **Threshold.** Pulse delay corresponds to the moment in which the received signal overcomes an assigned threshold.

**Maximum amplitude-delay**

We have to find the temporal instant in which \( \phi(x, y, t) \) gets to its global maximum. Since the received function is continuous (in fact, \( \phi(x, y, t) \in \mathbb{C}^\infty \)) the maximum is when:

\[
\frac{\partial \phi(x, y, t)}{\partial t} = \frac{\partial}{\partial t} \left( \frac{1}{4\pi D t} e^{-\frac{x^2+y^2}{4Dt}} \right) = 0 \quad (3.1)
\]

We find the propagation delay \( \tau_{\text{max}} \):

\[
\tau_{\text{max}} = \frac{(x^2 + y^2)}{4D} = \frac{\rho^2}{4D} \quad (3.2)
\]

Where \( \rho \) is the distance of a generic point in space from the source position.
3. Propagation analysis

Figure 3.2: Maximum amplitude-delay as function of receiver position along x axis, $D = 10^{-5} \text{cm}^2/s$

**Threshold-delay**

The second definition supposes that one wants to known when the received signal achieves a certain value. For example, think about the Quorum Sensing process, where the regulation of gene expression is controlled by the activation threshold (see section 1.3). Once the threshold is chosen, to find the propagation delay means:

$$\tau_{\text{thr}} : \frac{1}{4\pi Dt} e^{-\frac{x^2+y^2}{4Dt}} = \text{THR}$$  \hspace{1cm} (3.3)

The relation can be solved by means of Lambert $W$ function, which is defined as [16]:

$$W(z)e^{W(z)} = z \quad \forall z \in \mathbb{C}$$  \hspace{1cm} (3.4)
This function in general is not injective and it is complex. By restricting the study on the real values it can assume, there exist two real branches for $W$: the first one is termed $W_{-1}$ because the function is always $\leq -1$, and it is defined for $z \in [-\frac{1}{e}, 0]$; the second is the principal branch $W_0$, defined for $z \in [-\frac{1}{e}, \infty)$.

![Figure 3.3: Lambert W function, $z \in [-\frac{1}{e}, 6]$](image)

To solve equation (3.3), we can write it in a different way:

$$-\pi(x^2 + y^2) \cdot \frac{1}{4\pi Dt} e^{-\frac{x^2 + y^2}{4Dt}} = -\text{THR} \cdot \pi(x^2 + y^2) \quad (3.5)$$

In this particular case, the variable for the Lambert $W$ function becomes

$$z = -\text{THR} \cdot \pi(x^2 + y^2) \quad (3.6)$$

The function is:

$$W(z) = -\frac{x^2 + y^2}{4Dt} \quad (3.7)$$
So:

\[ W(- \text{THR} \cdot \pi(x^2 + y^2)) = -\frac{x^2 + y^2}{4Dt} \]  

Since \( z < 0 \), \( W_{-1} \) is the solution to the problem, as suggested in [17]. We derive the final expression for the propagation delay in the threshold regime:

\[ \tau_{\text{thr}} = -\frac{x^2 + y^2}{4D \cdot W_{-1}(-\text{THR} \cdot \pi(x^2 + y^2))} \]  

(3.9)

It is possible to express the threshold-delay as function of the maximum amplitude-one:

\[ \tau_{\text{thr}} = -\frac{\tau_{\text{max}}}{W_{-1}(-\text{THR} \cdot \pi(x^2 + y^2))} \]  

(3.10)

\[ x \quad \mu m \]  

\[ -10 -8 -6 -4 -2 0 2 4 6 8 10 \]  

\[ \text{Propagation delay [ms]} \]  

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5 \quad 4 \]  

\[ \tau_{\text{thr}}, \text{THR} = 1e-14 \text{ mol/cm}^2 \]  

\[ \tau_{\text{thr}}, \text{THR} = 1e-9 \text{ mol/cm}^2 \]  

\[ \tau_{\text{thr}}, \text{THR} = 1e-4 \text{ mol/cm}^2 \]  

\[ \tau_{\text{thr}}, \text{THR} = 1e1 \text{ mol/cm}^2 \]  

\[ \tau_{\text{max}} \]  

\[ x \quad \mu m \]  

\[ -10 -8 -6 -4 -2 0 2 4 6 8 10 \]  

\[ \text{Propagation delay [ms]} \]  

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5 \quad 4 \]  

Figure 3.4: Threshold-delay as function of receiver position along x axis, \( D = 10^{-5}\text{cm}^2/\text{s} \)

Since, from the definition of \( W \):
3. Propagation analysis

\[-W_{-1}(-\text{THR} \cdot \pi(x^2 + y^2)) \geq 1 \quad (3.11)\]

The behaviour of both the delays is quadratic, but the second one is always lower than the first one. The difference between the two values increases as threshold becomes smaller and smaller.

Comparison of the methods

To determine which method to use depends on the situation. The first approach (maximum amplitude) is pretty used to investigate the theoretical performances of the channel and appears more frequently when a channel characterization has to be done. Threshold-delay is the essence of all the communication techniques based on a threshold activation mechanism: e.g., the Quorum Sensing process or the multi-hop molecular communication systems (see chapter 4 for details).

Obviously, if the first delay exists by definition (the global maximum, although small, is always present in the received pulse), the last one can be calculated only if \( \text{THR} < \max \{\phi(x, y, t)\} \) for the points we want to analyze. In fact, if the threshold exceeds the pulse maximum, there will be not an existing value for the second delay. This is an issue to be aware of in practical system implementations. When the range of operation is defined, one has to be careful that every point of the space can be reached by information, and so can measure anyway a propagation delay.

3.1.2 Pulse amplitude

Pulse amplitude is defined as the maximum value of the received signal in time, at a fixed position. Once we found the maximum amplitude-delay, pulse amplitude corresponds to the Fundamental Solution evaluated in \( \tau_{\text{max}} \).

\[ \phi_{\text{max}} = \phi(x, y, t)|_{t=\tau_{\text{max}}} = \frac{1}{\pi(x^2 + y^2)e} \quad (3.12) \]
3. Propagation analysis

**Figure 3.5:** Received signal versus time, for multiple receiver distances, $D = 10^{-5} \text{cm}^2/\text{s}$

**Figure 3.6:** Pulse amplitude versus receiver distance along $x$ axis, $D = 10^{-5} \text{cm}^2/\text{s}$
3. Propagation analysis

3.1.3 Pulse width

Pulse width is computed as the width at the 50% level with respect to the maximum, i.e. the time interval when the pulse has an amplitude greater than half of its maximum:

\[
\phi(x, y, t) = \frac{1}{4\pi D t} e^{-\frac{x^2+y^2}{4Dt}} = \frac{\phi_{\text{max}}}{2} = \frac{1}{2\pi(x^2+y^2)e}
\]  

(3.13)

By rewriting the equation:

\[
-\frac{x^2 + y^2}{4\pi D t} e^{-\frac{x^2+y^2}{4Dt}} = -\frac{1}{2e}
\]  

(3.14)

Again, Lambert W function (3.4) provides the solution:

\[
W\left(-\frac{1}{2e}\right) = -\frac{x^2 + y^2}{4Dt}
\]  

(3.15)

The upper equation has two solutions, corresponding to the two time instants at which the pulse is equal to half of its maximum value. These are given by:

\[
t_1 = -\frac{x^2 + y^2}{4D \cdot W_{-1}\left(-\frac{1}{2e}\right)} , \quad t_2 = -\frac{x^2 + y^2}{4D \cdot W_0\left(-\frac{1}{2e}\right)}
\]  

(3.16)

Finally, the pulse width:

\[
t_w = t_2 - t_1
\]  

(3.17)
3. Propagation analysis

3.1.4 Pulse velocity

The velocity of propagation can be directly evaluated from the delay and is defined as the ratio between the receiver coordinate and its relative time of arrival. Therefore, there are again two possible definitions for the speed: maximum amplitude-velocity, associated to the pulse maximum and threshold-velocity, linked with the assigned threshold value.

Maximum amplitude-velocity

From $\tau_{max}$, we derive the speed:

$$V_{max}(x, y) = \frac{\sqrt{x^2 + y^2}}{\tau_{max}} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}} \cdot 4D = \frac{4D}{\sqrt{x^2 + y^2}}$$

(3.18)
Velocity is linearly dependent on the diffusivity and it is inversely proportional to the distance from source to receiver. Linearity from $D$ is shown in figure 3.8.

![Figure 3.8: Maximum amplitude-velocity versus receiver distance along x axis, for multiple diffusion coefficients](image)

**Threshold-velocity**

Velocity is calculated as follows:

$$V_{thr}(x, y) = \sqrt{\frac{x^2 + y^2}{\tau_{thr}}} = \sqrt{\frac{x^2 + y^2}{-(x^2 + y^2)}} \cdot 4D \cdot W_{-1}(-(THR \cdot \pi(x^2 + y^2))) \quad (3.19)$$

So:

$$V_{thr}(x, y) = \frac{-4D}{\sqrt{x^2 + y^2}} \cdot W_{-1}(-(THR \cdot \pi(x^2 + y^2))) \quad (3.20)$$
And

\[ V_{\text{thr}}(x, y) = V_{\text{max}}(x, y) \cdot (-W_1(-\text{THR} \cdot \pi(x^2 + y^2))) \]  (3.21)

As stated before for the delays, threshold-speed is linked with the maximum amplitude-one. In particular, \( V_{\text{thr}} > V_{\text{max}} \), with a difference that becomes greater as much as threshold gets smaller.

![Graph showing threshold-velocity versus receiver distance along x-axis for multiple thresholds, D = 10^{-5} cm^2 s^{-1}](image)

**Figure 3.9:** Threshold-velocity versus receiver distance along x-axis for multiple thresholds, \( D = 10^{-5} \text{cm}^2 \text{s}^{-1} \)

### 3.2 Molecular vs Electromagnetic channel

As a result of using biological materials and processes, MC exhibits unique features that make it distinct from the conventional telecommunication technology. For this reason, it is worth comparing the performance of diffusion to the one of the wireless electromagnetic channel. Let’s consider a 3D diffusive channel in order to make a comparison with the wave propagation in
the 3D free space. All the evaluated diffusion parameters (in section 3.1) maintain the same characteristics for the 3D case, except for the amplitude which decreases in space with the third power of the distance [17].

In the molecular channel, the pulse delay (in both of its two definitions) is proportional to the square of the transmission distance, while in the EM one the delay increases linearly as we get far from the source ($\tau_{EM} \propto r$). For what concerns the amplitude, if in diffusion $\phi_{max} \propto 1/r^3$, the path loss formula for the free space propagation claims that the signal amplitude is proportional to $1/r$. Another important difference consists in the distortion introduced by the channel: the pulse width of diffusion is $t_w \propto r^2$ whether in wave propagation the distortion is negligible.

Hence, reaching far distances using MC schemes might result infeasible. This is why the range of these systems is very small and limited to few micrometers. In table 3.1 we summarize the most significant differences between electromagnetic communication and molecular one.

<table>
<thead>
<tr>
<th>Features</th>
<th>EM communication</th>
<th>Molecular communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Devices</td>
<td>Electronic devices</td>
<td>Bio-nanomachines</td>
</tr>
<tr>
<td>Signal types</td>
<td>Optical/Electrical</td>
<td>Chemical</td>
</tr>
<tr>
<td>Propagation speed</td>
<td>$c = 3 \times 10^8 m/s$</td>
<td>Slow, $\propto 1/r$</td>
</tr>
<tr>
<td>Propagation range ($r$)</td>
<td>$10 \div 10^5 m$</td>
<td>$10^{-9} \div 10^{-6} m$</td>
</tr>
<tr>
<td>Media</td>
<td>Air</td>
<td>Aqueous</td>
</tr>
<tr>
<td>Pulse delay</td>
<td>$\propto r$</td>
<td>$\propto r^2$</td>
</tr>
<tr>
<td>Pulse amplitude</td>
<td>$\propto 1/r$</td>
<td>$\propto 1/r^2$</td>
</tr>
<tr>
<td>Pulse width</td>
<td>$\propto 1$</td>
<td>$\propto r^2$</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison between electromagnetic and molecular propagation

In order to increase the communication efficiency, in terms of range and velocity, we can think about adding to the basic diffusive mechanism further operations which enable a different evolution of the transmitted signal. Some suggestions are presented in chapter 4.
Towards efficient connections

Molecular communication is a novel and challenging idea to interconnect nano-scale devices in a bio-compatible and energy efficient way. In the simplest nanocommunication systems molecules act as information conveyors from source to destination, propagating via free diffusion throughout the medium [3, 7]. Unfortunately, this method gets to some problems due to the strong attenuation and delay on the channel, leading to great disadvantages in terms of propagation velocity with respect to the conventional electromagnetic communications. As explained in chapter 3, diffusion velocity decreases in space with the first power of the distance and therefore it is very small if compared to the light speed.

To counteract these limitations and increase the transmission efficiency, three main strategies might be adopted: building a network of bio-nanomachines where information is relayed from one machine to the others on a multi-hop fashion, introducing an external power which creates a flow assisted propagation along the desired direction, or alternatively shaping the diffusive characteristics of the medium to favour the communication, in the so called diffusive cable.

In this chapter we accurately analyze the propagation mechanisms and provide a comparison for what concerns the diffusion velocities of the three different approaches. To find efficient strategies becomes fundamental in nanocommunications, since the available waiting time of nanomachines is usually short in order to guarantee that the received signal isn’t distorted and devastated by the chemical reactions inside the human body.
4. Towards efficient connections

4.1 Bio-inspired multi-hop nanonetwork

Quorum Sensing—bacteria communicate thanks to the emission and consequent diffusion of signaling molecules termed autoinducers, capable to activate the emission of others of the same kind when sensed by bacterial cells. Quorum Sensing works with two distinct threshold values which measure the autoinducer density per unit of area. The activation threshold is connected with the regulation of gene expression (e.g., the bio-luminescence in the hawaiian squid *Euprymna Scolopes*, see 1.3.1), while the autocatalytic one is related to the autoinducer emission rate [5]. In our model we simulate this diffusion-based communication process, by activating multiple bacteria-like sensors. Right now, we focus our attention only on the activation threshold, omitting the study of autocatalysis since it introduces non linearities in the system. Multi-hop network matches the structure of the bacterial community, even if strong assumptions are present: bio-nanomachines cannot obviously self-generate (as on the contrary it happens in bacteria) and, moreover, we impose that no machine can be damaged during the communication (in bacterial world, it is like saying the death rate of the cells is null). Physical dimension of the sensors has not been considered for the results since devices have been seen as point nodes of the network topology. The whole process of sensor activation might be seen as a communication through a multi-hop channel, where information is relayed from one machine to the others with a propagation that depends on the geometry of the nanonetwork. We explore different spatial patterns of sensor distribution, varying both the inter-sensor distance and the activation threshold. The aim of the section is to investigate how diffusion velocity can be improved with respect to the homogeneous-medium scenario, as function of machine geometry, activation value and interspacing.

4.1.1 Activation model

In a bi-dimensional space with homogeneous diffusivity $D$ a finite number of nanomachines form a set of nodes arranged on a specific pattern, with all of them having the capacity to release molecules and detect their concentration level per unit of area. At time $t_0$ the first node starts the emission of
Towards efficient connections

a given amount of molecules which propagate over the space by Fick laws of diffusion. The remaining machines keep on sensing their surroundings, still silent (S\text{OFF} state in figure 4.1) until the density overcomes the activation threshold (T\text{ACT}): at that moment, devices activate and emit the same pulse of molecules (S\text{ON}). Note that the emission is instantaneous: once the machine is on, it gives no more contribution to the communication because, after that, it does not release anything.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure41.png}
\caption{Quiet and activated states in nanomachines}
\end{figure}

In all the investigated geometries, machine patterns are always considered uniform, that is, inter-sensor distance $\Delta$ keeps constant over all the involved nanostructure. Diffusion velocity $V$ is defined as the ratio between the device position and its activation time. $V$ depends on the assigned threshold but also on the geometry arrangement of the nodes.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure42.png}
\caption{Multi-hop deployment and activation}
\end{figure}
4. Towards efficient connections

4.1.2 Linear multi-hop

The first proposed solution consists in a linear multi-hop of nodes, uniformly
spaced along $x$ axis and distributed over a surface with diffusivity $D = 10^{-5}\,\text{cm}^2/\text{s}$.

At the initial instant, the far left sensor emits only one molecule. Figure 4.3 is showing some snapshots of the concentration evolution for increasing time: white dots represent the sensor positions in space while on the top we find four different diffusion evolution along $x$. Information is travelling towards the right thanks to the subsequent activation of the machines.

$$t = t_0 + \Delta t$$

$$t = t_0 + 2\Delta t$$

$$t = t_0 + 3\Delta t$$

$$t = t_0 + 4\Delta t$$

Figure 4.3: Linear multi-hop. Evolution of the molecular concentration, $\Delta = 2\mu m$, $\Delta t = 4\cdot 10^{-2}\,\text{ms}$ for $N = 5$ nodes

Activation time is defined as the temporal instant at which the received signal achieves the assigned activation threshold $T_{ACT}$. Actually, the delay for the second bio-nanomachine can be analytically evaluated by adapting the problem from chapter 3. Lambert $W$ function [16] recurs with the same principle as before:
4. Towards efficient connections

\[ t_1 = t_0 - \frac{\Delta^2}{4D \cdot W_{-1}(-\pi \Delta^2 \cdot T_{\text{ACT}})} \quad (4.1) \]

For all the other sensors the analytical calculation becomes much more complicated since multiple copies of the same pulse sum together when activation is on.

When all the machines are active, the final signal (which is the molecular concentration per unitary area) will be the superimposition of many contributions, as many as the nodes are:

\[ \phi(x, y, t) = N - \sum_{n=0}^{N-1} \frac{1}{4\pi D(t - t_n)} e^{-\frac{(x-n\Delta)^2 + y^2}{4D(t - t_n)}} \quad (4.2) \]

Where \( t_n \) is the node activation delay.

To obtain a delay for each sensor means finding the time instant in which the contribution of the previous devices, already active, reaches the threshold:

\[ \phi(n\Delta, 0, t) = \sum_{i=0}^{n-1} \frac{1}{4\pi D(t - t_i)} e^{-\frac{(x-i\Delta)^2}{4D(t - t_i)}} = T_{\text{ACT}} \quad (4.3) \]

We are not able to solve equation (4.3) analytically, but some hypothesis can be done. In fact, as activation moves far in \( x \) direction, new contributions sum to the total signal. It is reasonable to think that the difference between activation times of neighbour sensors will keep on decreasing, thanks to an avalanche effect created by the superimposition.

In spite of this, simulations (reported in figures 4.4 and 4.5) show something different: the curve of activation times linearly increases in space. This means that only the nearest active sensor influences the mechanism and no avalanche is generated. The expected phenomenon of accumulation does not appear, because very big distances and subsequently a huge number of devices would be necessary for the superimposition to be crucial.

\[ t_n - t_{n-1} = -\frac{\Delta^2}{4D \cdot W_{-1}(-\pi \Delta^2 \cdot T_{\text{ACT}})} \quad (4.4) \]
The inter-device delay remains constant for fixed $\Delta$ and $T_{\text{ACT}}$, and so the arrival times are:

$$t_n = t_0 - n \frac{\Delta^2}{4D \cdot W_1(-\pi \Delta^2 \cdot T_{\text{ACT}})} \quad (4.5)$$

*Figure 4.4: Activation times versus space for $N = 100$, $\Delta = 2\mu m$ and threshold $T_{\text{ACT}}$*
Figure 4.5: Temporal evolution of activation, $\Delta = 2 \mu m$, $T_{ACT} = 10^{-4} mol/cm^2$

Figure 4.5 is showing the temporal evolution of the signal for a structure if $N = 10$ nodes, in which every $\Delta$ each node becomes active and releases the molecular pulse. We didn’t evaluate the signal in the exact position of the machines but in their near right (with a spatial shift of $\frac{\Delta}{2}$) since otherwise we would have experimented a single pulse line for every evolution. Linear behaviour of the activation versus time is noticeable.

From the propagation time we derive the diffusion velocity:

$$V_n = \frac{n\Delta}{t_n} = -\frac{4D \cdot W_{-1}(-\pi \Delta^2 \cdot T_{ACT})}{\Delta}$$  

(4.6)

As stated before, $V$ is constant all over the network. We can change the values of $T_{ACT}$ and $\Delta$ and speed modifies consequently: in general, it decreases if threshold or interspacing increase. Figure 4.6 illustrates the results of simulations compared with the expected value of $V$ taken from equation (4.6).
4. Towards efficient connections

In a logarithmic scale, the relationship between the velocity and $\Delta$ is linear. This means that:

$$V(\Delta) = k \cdot \Delta^m \quad (4.7)$$

Actually, the parameter $k$ seems to be more strictly related with the assigned threshold, while $m$ is almost constant. In fact, we identify a slope similar to $-45^\circ$ for all the results in figure 4.6. This behaviour is justified by the presence of $\Delta$ in the denominator of (4.6).

For what concerns the relationship between velocity and $T_{ACT}$, results are shown in figure 4.7, where it is noticeable a quadratic behaviour for the speed.
4. Towards efficient connections

Activation threshold values [mol/cm$^2$]

<table>
<thead>
<tr>
<th>Value</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>$10^{-14}$</td>
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<tr>
<td>$10^{-12}$</td>
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<tr>
<td>$10^0$</td>
<td></td>
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<td>$10^2$</td>
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Diffusion velocity [cm/s]

<table>
<thead>
<tr>
<th>Value</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>$10^0$</td>
<td></td>
</tr>
<tr>
<td>$10^1$</td>
<td></td>
</tr>
<tr>
<td>$10^2$</td>
<td></td>
</tr>
</tbody>
</table>

$\Delta = 0.5 \, \mu m$

$\Delta = 1.5 \, \mu m$

$\Delta = 3 \, \mu m$

$\Delta = 5 \, \mu m$

Simulations

Figure 4.7: Diffusion velocity as function of $T_{ACT}$

Pseudo-uniform distribution

In order to investigate a more realistic configuration for the network, suppose to be in the same situation of section 4.1.2 and choose reasonable $\Delta$ and $T_{ACT}$. We introduce a small random perturbation in the position of the bio-nanomachines, so to change the original network geometry but without excessively modifying the basic structure. The investigation of this pseudo-uniform distribution is particularly tailored for highly dynamic environments, where machines might suffer of variation in their locations with respect to the nominal ones (e.g., in intrabody nanocommunications). The phenomenon can be connected to the position jitter that appears in telecommunication networks, where packets reach the destination with a delay deviating from the expected one of a random quantity (jitter). If it happens, the system must be capable to go on and operate anyway. Here we reproduce a similar scenario to investigate the multi-hop performances when unfavourable situations occur.

Specifically, as stated above, it has been supposed that every node cannot deviate too much from its nominal coordinates, that is, the spatial sorting of bio-nanomachines must remain untouched (pseudo-uniform distribution).
Towards efficient connections

For this reason, the perturbation is provided by an additive zero-mean uniform random variable, limited on its maximum range by the fact that every sensor cannot overcome the left and right neighbours. Furthermore, to get reliable results and supposing not to deal with infinitesimal devices, it has been forced a minimum value for the interspacing, given by $\Delta_{\text{min}} = \frac{\Delta}{10}$. We suppose to have at least the control on the position of the first node, so we impose the jitter for the latter machine to be zero. Taking into considerations all the assumptions, the node coordinates are:

$$x_n = n \cdot \Delta + \delta_n \quad \forall n \in [0, N - 1] \quad (4.8)$$

Where

$$\delta_n \sim \begin{cases} U[-\delta_{\text{MAX}}, \delta_{\text{MAX}}] & n > 0 \\ 0 & n = 0 \end{cases} \quad \text{with } \delta_{\text{MAX}} = \frac{\Delta - \Delta_{\text{min}}}{2} \quad (4.9)$$

Since $\delta$ is a uniform random variable, the standard deviation follows the common rules of statistics and:

$$\sigma_{\text{MAX}} = \frac{\delta_{\text{MAX}}}{3} \quad (4.10)$$

For simulations, we chose an interspacing $\Delta = 2\mu m$ and threshold $T_{\text{ACT}} = 10^{-4} \text{mol/cm}^2$. The probability of node activation is analyzed as a significant parameter:
4. Towards efficient connections

To assess the probability of activation, the propagation mechanism has been limited to a certain point in time. The reason is that actually every kind of communication has a limited life time, within which the signal must reach the destination or the link is considered inactive. So the sensors that within $t_{\text{MAX}}$ are not active yet, are never achieved by information and become useless for the communication. Figure 4.9 shows that as the jitter power increases, the active users reduce to the ones next to the origin because the activation is slower. We derive that the presence of jitter limits the communication range of the network, always worsening the performances.

In addition of the probability, it was evaluated the mean diffusion velocity. Again, as the power of perturbation grows, the curves move down, although first nodes seem to switch on faster.
4. Towards efficient connections

The behaviour of the velocity can be analytically explained by means of the Jensen inequality for convex functions. We showed before that the activation process does not present any accumulation effect, and therefore only the left neighbour influences the activation time:

\[
t_n = t_0 + \sum_{i=1}^{n} - \frac{(\Delta + \Delta_i)^2}{4D \cdot W_{-1}(-\pi(\Delta + \Delta_i)^2 \cdot T_{ACT})}
\]  

(4.11)

where

\[
\Delta_i = \delta_i - \delta_{i-1}
\]  

(4.12)

If we calculate the mean activation time over multiple realizations of \(\delta_n\), we find:

\[
E[t_n] = \sum_{i=1}^{n} E \left[ t_0 - \frac{(\Delta + \Delta_i)^2}{4D \cdot W_{-1}(-\pi(\Delta + \Delta_i)^2 \cdot T_{ACT})} \right]
\]  

(4.13)
Since the function is not linear, Jensen inequality can be applied to demonstrate that:

\[ E[t_n] \geq t_0 + \sum_{i=1}^{n} \frac{(\Delta + E[\Delta_i])^2}{4D \cdot W_{-1}(-\pi(\Delta + E[\Delta_i])^2 \cdot T_{ACT})} \] (4.14)

And:

\[ E[t_n] \geq t_0 - \frac{n\Delta^2}{4D \cdot W_{-1}(-\pi\Delta^2 \cdot T_{ACT})} \] (4.15)

The average times are always greater than the zero-jitter ones (reported in (4.5)), and this justify the decreasing in the diffusion speed and in the communication range.

### 4.1.3 Lattice multi-hop

Network nodes are now distributed over the surface on a regular square pattern \((\Delta_x = \Delta_y = \Delta)\). The activation mechanism remains the same as before, with the first nanomachine placed in the center \((0,0)\). Differently from the linear deployment, the square grid allows accumulation. This is shown in figure 4.12 where it is noticeable that, whether the radial distance \(\rho\) increases an avalanche process occurs. In the figure it is reported the diffusion propagation velocity versus angle, normalized to the horizontal propagation \((\theta = 0^o)\) for a dense deployment of \(N = 196\) nanomachines. The nodes with elevation angle \(\theta = 45^o\) measure higher delays that corner ones, but the differences move down as \(\rho\) grows.
4. Towards efficient connections

Figure 4.11: Lattice multi-hop. Evolution of the molecular concentration, \( \Delta = 2\mu\text{m} \), \( \Delta_t = 4 \cdot 10^{-2}\text{ms} \) for \( N = 16 \) nodes

Figure 4.12: Lattice multi-hop. Avalanche effect on the diffusion velocity
4. Towards efficient connections

4.1.4 Hexagonal grid

Another regular multi-hop pattern for a bi-dimensional structure is the hexagonal grid with constant inter-device distance $\Delta$. The first active machine is positioned in the center and the others are all around it. Results show that the accumulation is even greater than in the square lattice and, as $\rho$ increases, a circular propagation wavefront appears: this is reasonable given the hexagonal pattern of the structure.

Figure 4.13: Hexagonal grid. Evolution of the molecular concentration, $\Delta = 2 \mu m$, $\Delta t = 4 \cdot 10^{-2} ms$ for $N = 37$ nodes
4. Towards efficient connections

Figure 4.14: Hexagonal grid. Accumulation effect on the diffusion velocity for a dense deployment of $N = 468$ nanomachines

4.2 Flow assisted propagation

The alternative strategy to the multi-hop deployment consists in introducing a drift to guide the signaling molecules towards the receiver. As we said before, to resort the drift is not a novelty in the world of electronic devices such as in chemical signaling. Some common drift-diffusion effects in molecular communication are generated by external devices like syringe pumps or tabletop fans, which are used to push the chemicals in the direction of flow [2, 1].

In this section we provide an investigation of the velocity performances, comparing those of drift that of the linear multi-hop, since both the two systems represent a way to communicate along a line. If multi-hop produces an active transport through the network, the latter is again active but doesn’t need an infrastructure of nanomachines, even if it requires external power to assist the propagation and the boosting of the speed.
4. Towards efficient connections

4.2.1 Drift velocity

The experimental set consists in a single molecular source, placed in the center of the space. At time $t_0 = 0$, it releases a chemical pulse which diffuses by following the drift-diffusion equation (2.8). The behaviour of the concentration is:

$$\phi(x, y, t) = \frac{1}{4\pi Dt} e^{-\frac{(x + A_x t)^2 + y^2}{4Dt}}$$ (4.16)

We considered a drift contribution towards positive $x$ coordinates ($A_x < 0$) in order to experience the same propagation direction of the linear multi-hop. Once again, we can choose between two different definitions of speed, previously described in chapter 3, derived from the time of arrivals at each receiver position.

Maximum amplitude-velocity is obtained by setting to zero the first temporal derivative of $\phi(x, y, t)$ evaluated along $x$ axis. After some analytical steps, we find the expression:

$$A_x t^2 + 4Dt - x^2 = 0$$ (4.17)

The righteousness can be verified by checking the equality when the drift is null. In fact, equation (4.17) turns into the (3.2), which was the solution for the delays in case of homogeneous medium. By proceedings in the calculations, we find the channel delay that is function of drift, position and diffusivity.

$$\tau_{max} = \frac{-2D + \sqrt{4D^2 + A_x^2 x^2}}{A_x^2}$$ (4.18)

From the propagation time, it follows the diffusion velocity $V_{max}$:

$$V_{max} = \frac{A_x^2 x}{-2D + \sqrt{4D^2 + A_x^2 x^2}}$$ (4.19)

The expression we got includes both the two contributions of homogeneous diffusion and drift. Note that, whenever the ratio between the two tends to small values, $V_{max}$ approaches the drift, and the same happens as we get far
from the source. This phenomenon can be explained because homogeneous diffusion performs very well at short distances, having a great instantaneous evolution, but when the receiver is placed further away the system response is extremely slow.

The diffusion effect is noticeable in figure 4.15, where it is shown the temporal and spatial evolution of the molecular density. As we move far from the source, the pulse distortion increases as well and this deviates from the linear multi-hop, where the contribution of many pulses, summed every $\Delta$, makes the distortion to be negligible.

Threshold-velocity is harder to obtain as it requires the solution of the equality:

$$\frac{1}{4\pi Dt} e^{\frac{(x + A_x t)^2 + y^2}{4Dt}} = \text{THR}$$  \hspace{1cm} (4.20)

which leads to:

$$W_{-1}(-\pi \text{THR}(x + A_x t)^2) = -\frac{(x + A_x t)^2}{4Dt}$$  \hspace{1cm} (4.21)
Because of the complexity of the relation, we didn’t evaluate $V_{thr}$ in closed form, but by means of numerical implementations. Anyway, we expect that, exactly like the homogeneous medium at subsection 3.1.4, $V_{thr} > V_{max}$ because the received signal always reaches the threshold at first, and only after its peak.

Wanting to compare the drift performance with the linear multi-hop, we forced $|A_x|$ to equal the velocity obtained in (4.6). So,

$$A_x = -\frac{4D \cdot W_{-1}(-\pi \Delta^2 \cdot \text{THR})}{\Delta} \quad (4.22)$$

For $\Delta = 2\mu m$ and $\text{THR} = 10^{-4}\text{mol/cm}^2$.

Figure 4.16: Flow assisted propagation, comparison between the different velocities, $|A_x| = V_{MH} = 5.69\text{cm/s}$, $D = 10^{-5}\text{cm}^2/\text{s}$

Figure 4.16 shows the results and confirms what we stated before: $V_{thr}$ is always greater than $V_{max}$, while the latter basically equals the drift contribution, remaining larger especially for the distances in the neighbourhood of the source: again, this can be clarified by the rapid instantaneous behaviour
4. Towards efficient connections

of diffusion. This means that, whenever an external energy supply is available, to introduce a drift always brings superior performance than the linear deployment of network nodes.

We proved that the flow assisted propagation is an optimal strategy to guarantee at least an almost constant speed over the whole communication system. Actually the threshold-based mechanism provides always the best performances and remains the first candidate for realizing practical implementations.

4.3 Diffusive cable

Diffusion of molecules can be enabled by a non-homogeneous diffusivity \( D(x, y) \) that can favour diffusion along some predefined directions. To exemplify, the propagation along a line as for the linear multi-hop example can be reproduced by a diffusive cable with diffusivity constant only along the propagating direction, i.e. \( D(x, y) \approx D(y) \). The situation is similar to the one reported in subsection 2.2.3, with the difference that now we shape the diffusive characteristics of the medium in such a way we can make a comparison with the multi-hop line. For this reason, it has been introduced a constraint on the overall diffusivity of the medium. To clarify, note that a homogeneous space with diffusion coefficient \( D_0 \) may be analyzed from a microscopic view as a set of some “diffusive carriers” which are homogeneously distributed over the workspace. To create a cable, it is enough to rearrange the total available number of carriers by mainly concentrating them along the desired direction. So, stating that the total amount of conveyors must remain constant, we force the conservation of the surface integral for the heterogeneous diffusivity:

\[
\int_{\Gamma} D(x, y) d\Gamma = \int_{\Gamma} D_0 d\Gamma = D_0 |\Gamma| \quad (4.23)
\]

Where \( \Gamma \) is the investigated surface and \( |\Gamma| \) its area.

Recalling the analytical expression of the diffusive cable, previously reported in 2.2.3:
4. Towards efficient connections

\[ D(x, y) = D_x(x) \cdot D_y(y) = \begin{cases} 
  d_{\text{max}} \cdot e^{-\frac{x^2}{2\sigma_x^2}} \cdot e^{-\frac{x^2}{2\sigma_y^2}} & x < 0 \\
  d_{\text{max}} \cdot e^{-\frac{y^2}{2\sigma_y^2}} & 0 \leq x \leq L_c \\
  d_{\text{max}} \cdot e^{-\frac{(x-L_c)^2}{2\sigma_x^2}} \cdot e^{-\frac{y^2}{2\sigma_y^2}} & x > L_c 
\end{cases} \] (4.24)

By considering the boundaries \([-\bar{y}, \bar{y}]\) and \([-\bar{x}, L_c+\bar{x}]\), for fixed \(\sigma_y\) we obtain:

\[ d_{\text{max}} = \frac{2\bar{y}(\bar{x} + L_c/2)D_o}{\left(\sqrt{2\pi}\sigma_y - 2\sqrt{2\pi}\sigma_y Q(\frac{\bar{y}}{\sigma_y})\right) \left(\bar{x} + \frac{\sqrt{2\pi}\sigma_y}{2} - \sqrt{2\pi}\sigma_y Q(\frac{\bar{y}}{\sigma_y})\right)} \] (4.25)

Note that the above relation can be simplified by the fact that the ratio between \(\bar{y}\) and \(\sigma\) is usually big (\(\gg 1\)) in order to ensure the identification of a cable inside the environment. Even along \(x\) direction, \(\frac{\bar{x}}{\sigma_y}\) returns a value for the Q function which is very small if compared to the others. We can neglect the contribution of the Q function and affirm that:

\[ d_{\text{max}} \simeq \frac{\sqrt{2}(\bar{x} + L_c/2)D_o}{\sqrt{\pi} \left(\bar{x} + \frac{\sqrt{2\pi}\sigma_y}{2}\right)} \cdot \frac{\bar{y}}{\sigma_y} \] (4.26)

\(d_{\text{max}}\) is always greater than \(D_o\), therefore the system will surely propagate better than the simple homogeneous diffusion.

The main parameter for the maximum of diffusivity is the ratio \(\frac{\bar{y}}{\sigma_y}\), indicator of the cable width with respect to the space boundary. The more we want the width to be thin, the higher will be the carrier density in the favourite direction.

4.3.1 Cable velocity

We expect that nothing will change in the speed features with respect to a homogeneous workspace with \(D = d_{\text{max}}\). As a matter of fact, recalling the diffusive cable in figure 2.7, the diffusivity along it maintains constant \((D_x \cdot D_y = d_{\text{max}})\), given that \(D(x, y)\) is homogeneous along the cable. Then
there is no reason to think that a receiver placed at a distance $r$ from the source, with both positioned at the cable center ($y = 0$), does not receive the same signal that it would get in the homogeneous state. Therefore diffusion velocity follows the expressions evaluated in case of homogeneity. We skip the maximum amplitude-strategy because now it is clear the threshold-one is the best of the two. The velocity $V_{thr}$ is (see chapter 3 for derivations):

$$V_{thr} = \frac{-4d_{max}}{x} \cdot W_{-1}(-\text{THR} \cdot \pi x^2)$$  \hspace{1cm} (4.27)

Recalling the features for the pulse delay evaluated in chapter 3, the arrival times present a quadratic behaviour versus the receiver position whether for the multi-hop line (with diffusivity $D_o$) the relationship is linear. In spite of this, since $d_{max} > D_o$, we expect that the coordinates next to the source measure a greater velocity and this is demonstrated in figure 4.17. The border line between the methods has been evaluated numerically, by matching the two speeds.

![Figure 4.17: Diffusive cable velocity, $D_o = 10^{-5} \text{cm}^2/\text{s}$ and $d_{max} = 1.27 \cdot 10^{-4} \text{cm}^2/\text{s}$](image)

The passive transport mechanisms as homogeneous diffusion have a good
behaviour when distances remain small. Problems arise when the receiver gets far since the signal suffers of a strong propagation delay (see subsection 3.1.1 for details). In spite of this, the *diffusive cable* idea is not bad because it guarantees a limited usage of nanomachines (ideally the requested number drops to the minimal, one transmitter and one receiver), taking advantages of the possibility to shape the spatial features of diffusive carriers. Hence, it doesn’t require any external energy supply (as flow assisted propagation demands indeed) and even of an elevate number of bio-nanomachines (multi-hop). It is announced as a simple communication strategy, energy efficient and low cost, all characteristics that make it attractive in environments where it is impossible to provide energy or a network structure.
Conclusions

Molecular communication is a promising bio-inspired paradigm for communication in nanonetworks. To investigate this fascinating technology, a theoretical study of diffusion and a discrete implementation on a mathematical simulator are both mandatory steps for deeply understanding how the process occurs.

In chapter 2 we observed the diffusion phenomenon from an analytical and simulated view, by means of FTCS method for the discretization model. Moreover, we focused on the evaluation of the main channel parameters, like attenuation, distortion and propagation velocity. Since the speed is much more reduced with respect to the conventional electromagnetic propagation (it decays with the first power of the distance), some strategies to counteract the strong limitations in the communication efficiency become necessary. Therefore, we proposed three possible techniques providing an improvement on the diffusion velocity:

- Multi-hop produces a constant speed all over the linear nanostructure, whether the square and hexagonal deployments reach an accumulation effect that further enhances the propagation.
- The addition of a drift by an external device to generate a flow assisted propagation guarantees better and controlled performances than linear multi-hop.
- The diffusive cable is superior to the previous solutions only in the neighbourhood of the emission, as diffusion velocity still decades as $1/r$.

Hence, we understand there is not an overall best strategy, so we can choose one method or another according to the kind of communication we have to establish.
Future works

If we dispose of a great number of devices and this does not compromise the system bio-compatibility, building a network of nodes where information is relayed from one hop to the others is a good solution and does not require any energy supply except for feeding the bio-nanomachines. Future works will consist in a deeper analysis of the activation mechanism and molecular emission, with the target to obtain an accumulation even in the linear deployment. In fact, the released pulse can depend on all the previous emissions and not only on the neighbourhood of the node.

The diffusive cable is the less expensive option of the three, because it doesn’t require any power supply and neither a large amount of devices. One of the innovative aspects lies in the diffusive carrier rearrangement which enables the channelling of the diffusion in the desired direction. An additional benefit appears with respect to the drift: if the last one addresses the molecules through a straight connection without the possibility of changing the direction (unless another push enters the scene) the diffusive carriers can bend the communication flux as we want, depending on the possible obstacles during propagation. For example, figure 5.1 reports the evolution of the concentration for a diffusive cable bending in a sinusoidal way along $x$ dimension. The expression for the diffusivity is:

$$D(x, y) = \begin{cases} 
    d \cdot e^{-\frac{x^2}{2\sigma_x^2}} \cdot e^{-\frac{y^2}{2\sigma_y^2}} & x < 0 \\
    d \cdot e^{-\frac{(y - \alpha \cos(\frac{4\pi x}{L_c}))^2}{2\sigma_y^2}} & 0 \leq x \leq L_c \\
    d \cdot e^{-\frac{(x - L_c)^2}{2\sigma_x^2}} \cdot e^{-\frac{y^2}{2\sigma_y^2}} & x > L_c 
\end{cases} \quad (5.1)$$

Where $\alpha$ is related to the amplitude of oscillation along $y$ direction.
Furthermore, if we could manage a non-homogeneous distribution of the diffusivity along the propagation direction, we might experience an improvement on the velocity. Future investigations will focus on arranging the available carriers on increasing number as propagation occurs, in order to generate a guided diffusion enhancing the speed as much as evolution takes place. In addition, the concept of diffusive cable opens a window onto challenging researches on multiple interacting phenomena like biochemical MIMO systems. The investigation of how MIMO diffusive channels work and their possible non-linear interaction is a good starting point for many future applications. To give a first example, figure 5.2 reports a $4 \times 4$ MIMO channel, where each diffusive cable has been designed by following the analytical expression given in (4.24).
5. Conclusions

Figure 5.2: $4 \times 4$ MIMO diffusive channel
Bibliography


