POLITECNICO DI MILANO
Faculty of Information Engineering
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Optimizing Cell Selection in Cooperative Load-Coupled Heterogeneous Cellular Networks: Algorithm Design and Performance Study

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Graduation Thesis of:
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Bulut

Contents

List of Figures

List of Acronyms

3G Third-Generation

4G Fourth-Generation

AMC Adaptive Modulation and Coding

AWGN Additive White Gaussian Noise

BER Bit Error Rate

BS Base Station

CoMP Coordinated Multipoint

CSI Channel State Information

DCC Dynamic Cooperation Cluster

HetNet Heterogeneous Network

ICI Inter-Cell Interference

IUI Inter-User Interference

JT Joint Transmission

LTE Long Term Evolution

MC Macro Cell

MIMO Multiple Input-Multiple Output

MinMaxL Minimum of Maximum Load

MinSumL Minimum Total Load

MISO Multiple-Input Single-Output

MSE Mean Square Error

mW Milliwatt

NJT Non Joint Transmission

NP Non-Deterministic Polynomial-Time

OFDM Orthogonal Frequency Division Multiplexing

OJT Offset Joint Transmission

ONJT Offset Non Joint Transmission

QoS Quality of Service

RB Resource Block

SC Small Cell

SDMA Spatial Division Multiple Access

SIF Standard Interference Function

SINR Signal to Interference and Noise Ratio

TDMA Time Division Multiple Access

UE User Equipment

Abstract

The wireless data traffic grows with the number of user devices and as the type of traffic shifts towards multimedia applications. The amount of resources are limited because of physical and economical reasons. This growth and the limit push us to use the available resources even more efficiently each day. With the huge number of user devices, in modern multiple-input multiple-output (MIMO) systems the inter user interference is the dominant impairment over the received signal. In order to suppress the interference and the other channel impairments (multipath etc.) to improve the spectral efficiency and meet the requirements of LTE-Advanced, there are several solutions proposed through out the years.

We consider the scenario namely, Heterogeneous Networks (HetNets) where low powered nodes (referred as small cells) are deployed in the traffic hotspots to ease the load the numerous user equipments are causing to the high powered nodes (referred as macro cells). The macro cells are the base stations in the classical multi cell scenario which define the cell area. Here the load refers to the amount of resources used by the user devices while communicating with the specific node. We employ the load coupling model to define the load, where the cells interfere with each other in a recursive manner. To further improve the spectral efficiency (especially for cell edge users), we use one of the coordinated multi point (CoMP) techniques namely, joint transmission (JT), in which an user may be simultaneously served by multiple nodes.

In this thesis we study the system performance in terms of total load after optimizing the association between the user equipment (UE) and the cells and compare the results of classical single association (Non-JT) with the one from JT. Additionally, we implement cell specific offsets to the small cells (SCs) to adjust their cell ranges for a more realistic attempt in optimizing the total load.

Sommario

Il traffico di dati wireless cresce con il numero di dispositivi di utente e quando il tipo di traffico si sposta verso le applicazioni multimediali. La quantità di risorse é limitata per ragioni fisiche ed economiche. Questa crescita e tale limite ci spingono ad utilizzare le risorse disponibili in modo ancora più efficiente ogni giorno. Con l'enorme numero di dispositivi-utente, nei sistemi moderni multiple-input multiple-output (MIMO) l'interferenza tra gli utenti é il maggior disturbo sul segnale ricevuto. Per eliminare l'interferenza e gli altri disturbi di canale (multipath etc.) per migliorare l'efficienza spettrale e soddisfare i requisiti del protocollo LTE-Advanced sono state proposte diverse soluzioni nel corso degli anni.

Consideriamo lo scenario delle Heterogeneous Networks (HetNets) in cui i nodi a bassa potenza (indicati come "small cells") sono distribuiti negli hotspot di traffico per facilitare il traffico che i numerosi user device generano nei nodi ad alta potenza (chiamati "macro cells"). Le macro cells sono le stazioni base nel classico scenario a celle multiple che definisce l'area delle celle. Qui il carico si riferisce alla quantità di risorse utilizzate dai dispositivi utente durante la comunicazione con il nodo specifico. Per definire il carico impieghiamo il modello di accoppiamento carico, in cui le celle interferiscono tra loro in modo ricorsivo. Per migliorare ulteriormente l'efficienza spettrale (soprattutto per gli utenti dei bordi delle celle), usiamo una delle tecniche di tipo "coordinated multi point" (CoMP), nello specifico la trasmissione congiunta (JT), in cui un utente può essere contemporaneamente servito da più nodi.

In questa tesi si studiano le prestazioni del sistema in termini di carico totale dopo avere ottimizzato l'associazione tra il dispositivo utente (UE) e le celle e si confrontano i risultati della singola associazione classica (Non-JT) con quelli dell'approccio JT. Sono state inoltre implementate compensazioni specifiche tra le celle e le "small cell" (SC) per regolarne le distanze al fine di una più realistica ottimizzazione del carico totale.

Chapter 1

Introduction

Following the invention of smart phones and tablets, the demand for worldwide internet coverage and high speed internet has risen rapidly. Global mobile data traffic grew 74% and Fourth-Generation 4G traffic exceeded Third-Generation 3G traffic for the first time in 2015 [1]. The implemented infrastructure cannot change as fast to keep up with the fast grow rate of data demand. The high demand with a bad configuration means low free capacity and this results in poor service quality and traffic congestion.

Differently from the coverage, which is explained by received signal strengths, the capacity needs more attention. As a result of Adaptive Modulation and Coding (AMC) used in Orthogonal Frequency Division Multiplexing (OFDM) systems, the available throughput of a user is determined by the channel quality that is the Signal to Interference and Noise Ratio (SINR) and SINR is effected by the Inter-Cell Interference (ICI) while its intensity is determined by how busy the interfering cells' channels are. [35]

What we have here at hand is a system of Base Stations (BSs) mutually coupled because of ICI that they cause to each other while they all have their set of User Equipments (UEs) to serve and have to maintain an appropriate level of service experience so as to meet some Quality of Service (QoS) requirements. There has been discussed several models for this problem throughout the years and lately there has been proposed an analytical model in [51, 52] which has been shown to give a good approximation of more complicated models, especially at high user demand [14]. This model is a non-trivial system of non-linear equations and calculating the solution or determining its existence is not trivial either [52]. The optimization problem is proved to be Non-Deterministic Polynomial-Time Hard (NP-Hard) in [62].

We refer to the average level of resource usage of a cell as its load. In a Long Term Evolution (LTE) network which is governed by OFDM, these resources are quantified as units in the time-frequency domain and called as resource units. This concept has proved to be very reliable for performance characterization [22, 61].

The load in a cell is strongly related to the amount of transmission and consequently ICI. As the load of a cell grows, it causes more interference to the other cells and as a result the load of these other cells grow too. This coupling relation between the loads are called as the Load-Coupling system and this system is used to mathematically formulate the mutual influence of BSs over the interference levels throughout the network [63].

Low load indicates less chance of congestion while high load means frequent congestions and thus service outage. If a cell suffers from high load, this generally means poor planning in the deployment stage and a probable traffic hotspot in the area [52].

This Load-Coupling equation system has been shown to give a good approximation for more complicated load models that capture the dynamic nature of arrivals and service periods of data flows in the network [14], especially at high data arrival rates. Further comparison of other approximation models concluded that the Load-Coupled model is accurate yet tractable [15].

A significant step in LTE is the deployment of the Heterogeneous Networks (HetNets) integrating Macro Cells (MCs) with Small Cells (SCs) [10, 12]. SCs offload the traffic from MCs and ease the traffic overload. There are several parameters about how to utilize SCs in a network such as where to deploy them in a network or how to associate UEs with them. The deployment stage is out of the scope of this thesis work. In this work we are going to examine the performance outputs of several association scenarios in a HetNet such as: the UEs connect to the cell with the best received SINR (the usual situation), UEs freely choose between MCs/SCs to minimize the total load throughout the network and association by subjecting SCs to Cell-Specific Offsets to set their ranges.

Because of the location SC is deployed, the low output power and ICI the coverage area (range) of SC may result to be too small or far away with respect to MCs' area. When UEs select the cell with the strongest signal, SCs fail to serve UEs when there is a MC with a stronger output power nearby. This is when Cell Offsetting is proved to be useful. By implementing a cell specific offset, the UE is allowed to select a weaker cell thus easing the capacity overload in crowded areas (hotspots). We should keep in mind that connecting to SCs with very weak signal strength is not desirable either as it might happen not to meet the QoS requirements of the specific service. On the other hand SCs overload easily when serving hotspot areas due to low output power e.g., small amount of resources. This issue can be confronted to some extend by increasing the output power but if not isolated properly, it may cause ICI, increasing the loads of the other cells and resulting in a capacity reduction through out the network. [53]

The offset does not actually change the transmit power. It is used as a virtual amplification to the received signal power originated from a SC to enable it to be selected even when the best signal is coming from a MC thus providing a realistic method to adjust SC ranges.

Coordinated Multipoint (CoMP) is one of the promising concepts to improve spectral efficiency [29]. LTE uses Multiple Input-Multiple Output (MIMO) - OFDM and improves spectral efficiency in a cell for one BS. CoMP coordinates more than one BS to mitigate interference from these BSs and reduce the total interference. Single cell MIMO techniques are extended to a multi-cell setting. There are several CoMP techniques such as Joint Transmission (JT) which is done by simultaneous transmission between BSs with multi user linear pre-coding, dynamic cell selection where the UE is associated with only one BS but changes it dynamically, etc.. [50].

In this paper we are also going to re-form the mentioned above scenarios (free and offset) with JT to show the overall capacity improvement and study how the total load can be reduced by optimizing the JT pattern.

1.1 Outline

In the second chapter we provide a brief introduction to the resource allocation problem in wireless networks and introduce the reader to some important concepts and define a framework for the thesis.

In the third chapter the nature of the model used for the load coupling is explained, the problem formulation for the minimization of the total load is given and several theorems about the system at hand are demonstrated.

In the fourth chapter the different problem scenarios are illustrated and the algorithms to solve each of them are proposed.

In the last chapter the numerical results of the algorithms proposed are presented, these results are evaluated and possible future work is investigated.

Additionally, in Appendix A and B further insight to some concepts used in the thesis such as COST-231 Path Loss Model and Monte-Carlo method is given.

Chapter 2

Radio Resource Management

This chapter is both a derivation and a summary of [3]. Here we define a general framework for modelling different types of multi-cell systems and calculating their performances.

The definition for the resource allocation concept is assigning a transmit power to each user and each spatial direction while respecting some constraints which have some performance and economy wise implications. One of the main problems in resource allocation is the Inter-User Interference (IUI) when many users are served simultaneously. However, we can greatly improve the performance of multi-cell systems if we understand their nature and how to optimize the spatial domain.

Mathematical synonym for resource allocation is assigning a signal correlation matrix for each user. This way we can compute SINR of each user. This section explains the different methods to measure the performance levels and describe the arisen conflict when we try to maximize each user's performance. The performance and channel gain regions are defined. The resource allocation problem is formulated as a multi objective optimization problem.

Here the general optimization problem is formulated and different solution strategies are discussed along with the optimal solution and the performance region.

2.1 Introduction to Wireless Communications

The communication is to transmit data through a medium called the channel. This thesis focuses on wireless communications. In wireless communications the data is transmitted as electromagnetic waves through the channel.

This channel distorts the signal due to the objects present in the environment, adds interference from other signals and thermal noise. The channel is very crowded because of many devices (cellphones, computers etc..) that use its resources. Due to this, the resource licences are very expensive and scarce. Therefore, the systems should be designed to use the resources assigned to them as efficiently as possible. As the type of traffic in cellular networks is changing from low rate voice services to high rate data services, the spectral efficiency (bits/s/Hz) becomes particularly important. The overall network efficiency can be improved by dynamic resource allocation or the service providers can share the spectrum for joint spectral efficiency.

The spectral efficiency of a single link between a transmitter and a receiver is limited by the available transmit power [49] but the total spectral efficiency can be improved by simultaneous transmissions between many devices. This causes inter-user interference and if not properly controlled can degrade the performance of the specific link. The traditional way of preventing this is to simultaneously use the same resource (e.g. frequency) only when the devices are spatially separated enough as the signal strength reduces with the distance due to the path loss effect. As the radio wave from a single antenna has a fixed radiation pattern, this allows the division of space into cells and sectors. By applying fixed frequency reuse patterns appropriate with the scenario, the interference can be greatly avoided. However this approach is inefficient compared to controlling the amount of interference signals causing to each other [45].

Unlike the classical single antenna transmission, the multi antenna techniques let the allocation of resources with precise spatial separation. This is made possible by steering the signal beam with the help of an array of antennas. This is a concept introduced in signal processing and is a branch of beamforming. The steering is the transmission of the signal from different antennas in the array with different amplitudes and phases thus letting the components to add up constructively in desired directions and the contrary in the other directions. This ideally enables the global utilization of all resources removing the need for sectoring and frequency reuse patterns but leading to more complicated processing (described later in this chapter).

The spectral efficiency increases with the number of antennas (if the receiver knows the channel and has at least the same number of antennas as the transmitter) [17, 39, 55]. Strong results for the single cell downlink (broadcast) channel were derived in [9, 58]. The theoretic capacity region is characterized under general conditions in [59]. The cell scenarios are more complicated than the point-to-point ones as the transmitter needs to know the direction where the receiver is [20]. As a result of this, overhead

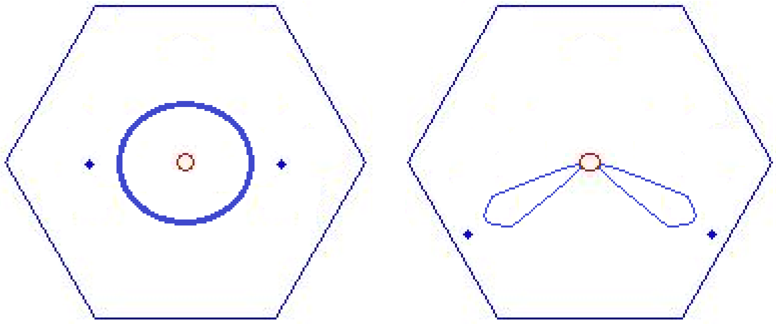


Figure 2.1: With a single antenna (left) the signal is equally strong in all directions while with multiple antennas (right) we are able steer the signal and limit the interference

signalling is required for feedback of channel information.

The multi cell scenario is more curious as with the correct coordination of the transmitters, the frequency reuse patterns can be removed and the spectral efficiency is improved even more. This causes the network to act like a big single cell [19]. This setup attempts to use the inter cell interference in its favor by allowing joint transmission from multiple cells to each user. While the optimal transmit strategy is known for the single cell scenario, it is not the case in the multi cell ones. One simple example for the multi cell scenario is the interference channel where each transmitter serves one unique user while causing coordinated interference to the others [13, 31, 33, 48].

Consider a single scenario where a base station with N antennas serve J_r users referred as UE, illustrated in Figure 2.1. kth user is denoted UE_k and has only one antenna. This scenario is very similar to the superposition of many Multiple-Input Single-Output (MISO) links, thus also referred as the MISO broadcast channel [9].

2.2 Single Cell Downlink

The channel ($\boldsymbol{h}_k \in \mathbb{C}^N$) is assumed to be flat fading which means that the frequency response of the channel is flat that the current output signal only depends on the current input. The complex valued $[\boldsymbol{h}_k]_n$ describes the channel from the nth antenna's eyes. Its magnitude is the gain and its argument is the phase shift applied to the signal by the channel. We also assume that $\boldsymbol{h}$ is quasi static which means that it is constant for the coherence time (the duration of many transmission symbols). The set $\{\boldsymbol{h}\}_{k=1}^{K_r}$ is known as the

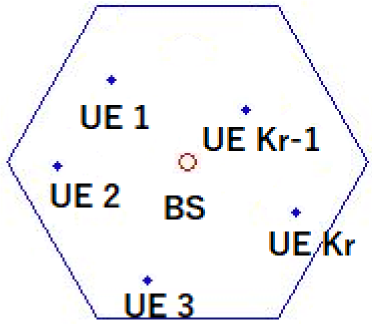


Figure 2.2: A single cell multi user system

Channel State Information (CSI) and assumed to be perfectly known at the BS. Lastly, the only degradation caused to the signal assumed to be coming from other signals (interference) and the background noise. These are not so realistic assumptions but simplify a great deal of things and help the concept to be more easily understood.

So the symbol sampled complex baseband received signal at UE_k is $y_k \in \mathbb{C}$ and defined as

$$y_k = \boldsymbol{h}_k^H \boldsymbol{x} + n_k \tag{2.1}$$

where $n_k \in \mathbb{C}$ is the combination of additive noise and interference. It is modelled as circularly symmetric complex Gaussian, $n_k \sim \mathcal{CN}(0, \sigma^2)$, where σ^2 is its power.

In a multi carrier system such as OFDM, the model (2.1) would describe one of the sub carriers.

The transmitted signal $\boldsymbol{x} \in \mathbb{C}^N$ contains datas intended for each user and given as

$$\boldsymbol{x} = \sum_{k=1}^{K_r} \boldsymbol{s}_k \tag{2.2}$$

where $\boldsymbol{s}_k \in \mathbb{C}^N$ is the signal for UE_k. These stochastic data signals are modelled as zero mean signal correlation matrices

$$\boldsymbol{S}_k = \mathbb{E}\{s_k s_k^H\} \in \mathbb{C}^{NxN} \tag{2.3}$$

This approach is known as linear multi stream beamforming (rank of $\boldsymbol{S}_k$ is the number of unique data streams) and the signal correlation matrices

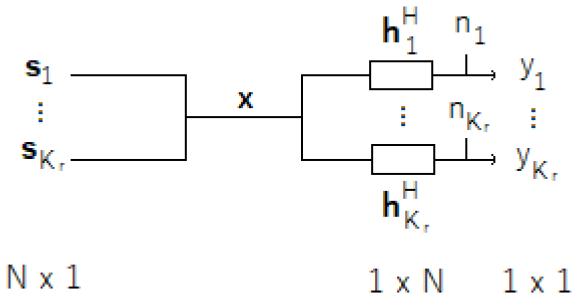


Figure 2.3: Block diagram for the downlink single cell system

are important tools to optimize the performance of the system.

Each selection of the signal correlation matrices $\boldsymbol{S}_1, ..., \boldsymbol{S}_{K_r}$ is called a transmit strategy and the average transmit power to UE$_k$ is $\mathrm{tr}(\boldsymbol{S}_k)$.

The only strategies to consider are those which satisfy the power constraints.

2.2.1 Power Constraints

The available power resources should be limited to be able model the restrictions of a practical system. The power is usually measured in Milliwatt (mW). We assume L linear power constraints

$$\sum_{k=1}^{K_r} tr(\boldsymbol{Q}_{lk}\boldsymbol{S}_k) \leq q_l \qquad l = 1, ..., L, \tag{2.4}$$

where $\boldsymbol{Q}_{lk} \in \mathbb{C}^{NxN}$ are hermitian positive semi definite weighting matrices and limits $q_l \;\; \forall l, k$. Note that $\sum_{l=1}^{L} \boldsymbol{Q}_{lk} > \mathbf{0}_N \;\; \forall k$ to ensure the power is constrained in all directions. These are decided before according to constraints such as physical limitations, economic and interference constraints.

$\boldsymbol{Q}_{lk}$ can be the same for all users but can also be used to define subspaces to limit the transmit power to some users. This would be done to limit the interference and this is called soft shaping [25, 46] because these constraints only affect the transmission when the corresponding transmit power exceed the threshold q_l.

The constraint model (2.4) can also be translated into per user power limits

$$tr(\boldsymbol{Q}_{lk}\boldsymbol{S}_k) \leq q_{lk} \qquad k = 1, ..., K_r, \quad l = 1, ..., L, \tag{2.5}$$

to be able to converted back to (2.4) it should satisfy also

$$\sum_{k=1}^{K_r} q_{lk} \leq q_l l = 1, ..., L \tag{2.6}$$

Selecting q_{lk} per user power allocation is a part of the performance optimization.

2.2.2 Resource Allocation

Selecting a transmit strategy $\boldsymbol{S}_1, ..., \boldsymbol{S}_{K_r}$ with the power constraints (2.4) is defined as resource allocation.

This selection should be based on user satisfaction. In principle, $tr(\boldsymbol{S}_k)$ is the allocated transmit power to UE_k while eigenvectors and eigenvalues of $\boldsymbol{S}_k$ define the spatial distribution of it. The rank of $\boldsymbol{S}_k$ corresponds to the number of simultaneous data streams to UE_k. The general case where multiple users are served simultaneously is called Spatial Division Multiple Access (SDMA) [44], where if only one user is allocated to a time slot for a given time with non zero power this is called Time Division Multiple Access TDMA. The N transmit antennas give N points of freedom which means to be able to transmit up to N different data simultaneously. The spectral efficiency might not be high when the maximum number of streams are transmitted due to IUI and can be sensitive to CSI. In the absence of CSI, TDMA is a good choice.

We assume to have infinite data to deliver and UEs are not limited about how high performance they can achieve. The data is delivered to the BS through a backhaul network, which is also used for BS coordination in the multi cell scenarios in the next section.

2.3 From Single Cell to Multi Cell

Traditionally in multi cell systems each user belong to one cell and the resource allocation is performed by its BS. The frequency reuse patterns make cell sectors to cause negligible interference to each other, as a result of this each sector can be seen as a single cell, if the negligible interference is seen as a part of the additive noise.

On the other hand there is the scenario where each base station uses all the frequency resources to maximize the system spectral efficiency. The branch of SDMA in multi cell scenario has several names such as network MIMO [57], CoMP [43]. It basically has the same idea of exploiting the spatial dimensions and controlling the interference. CoMP is particularly effective for users who receive the same SINR from multiple BS as usually happens on the cell edge. Initially assuming the perfect cooperative processing, we model the network as one big multi user MISO system. All the users are served by joint transmission from all BS and the multi cell constraints reduce just to per antenna power constraint. In this case ideally the optimal spectral efficiency is the same as in the single cell case [59]. Even this model mathematically seems to be acceptable, in a real scenario there are some impairments which is not possible to realize:

- Global need of CSI and data sharing [28, 36]
- Coherent joint transmission requires accurate synchronization [2, 56, 64]
- The complexity of centralized resource allocation [4]

Joint transmission also implies delay spread [66].

Several models have been proposed to characterize multi cell scenarios. The CSI requirements were reduced in [41, 26, 10] by utilizing the Wyner model (explanation below). This model has a relatively easy analysis but also leads to oversimplified results [60]. Another approach is to divide the network into static disjoint cooperation clusters (Figure 2.4). It is the same idea discussed above applied to groups of BSs instead of modelling the network as a whole.

If these clusters are small enough then the channel acquisition and synchronization within the cluster is easy however when the user distribution is not homogeneous the spectral efficiency is low [38] and there is out of cluster interference [18]. There are several methods to increase the efficiency such as creating different cluster for different frequency subcarriers [37], using a bigger cluster but not serving the users with all BS within the cluster [5] or having frequency reuse pattern at the cluster edges [32]. The main problem about this technique is however it is not user centric which means it is not differentiating the provided service according to the specific user's needs. Some work about more dynamic multi cell coordination with user centric approach is done in [2, 18, 25]. In this method each BS forms its own unique user set which implies that BSs create different clusters among themselves

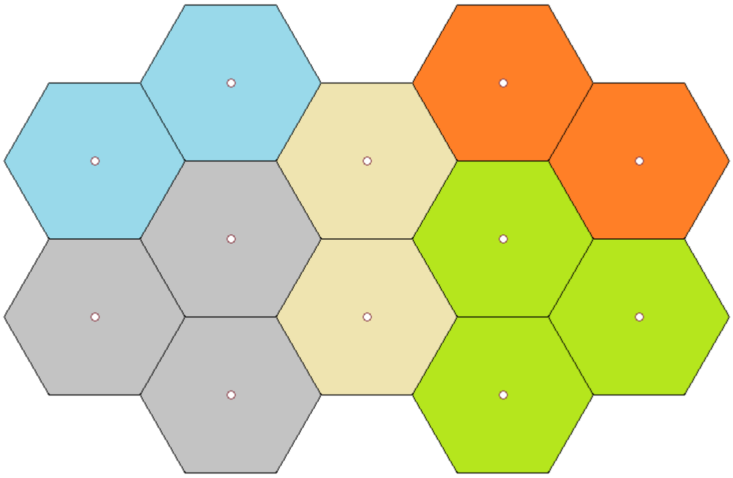


Figure 2.4: Static disjoint cooperation clusters. Each color represents a cluster

for each UE. The geographical position of UE has a large impact on this type of model [25] along with the demand levels and congestion in certain areas (hotspots). In the next chapters we will build our research on this dynamic cooperation clusters framework with an additional concept that will be introduced as heterogeneous networks.

There is another different multi cell setup comes with the name cognitive radio [21, 23]. The concept is based on having secondary systems that are using the same frequency without disturbing the primary system which owns the licence. The methods secondary devices use to achieve this are:

- Interweave: Transmit when the primary system is offline
- Underlay: Steer signals away to prevent interference
- Overlay: Compensate for the interference caused by joining the primary systems with joint transmission

This concept is also further extended to spectrum sharing between operators and will be explained in the next sections.

2.3.1 Dynamic Cooperation Clusters

We define a multi cell scenario in addition to the single cell with K_t BSs. The jth base station is denoted BS_j and has N_j antennas. The total number of antennas denoted as $N = \sum_{j=1}^{K_t} N_j$.

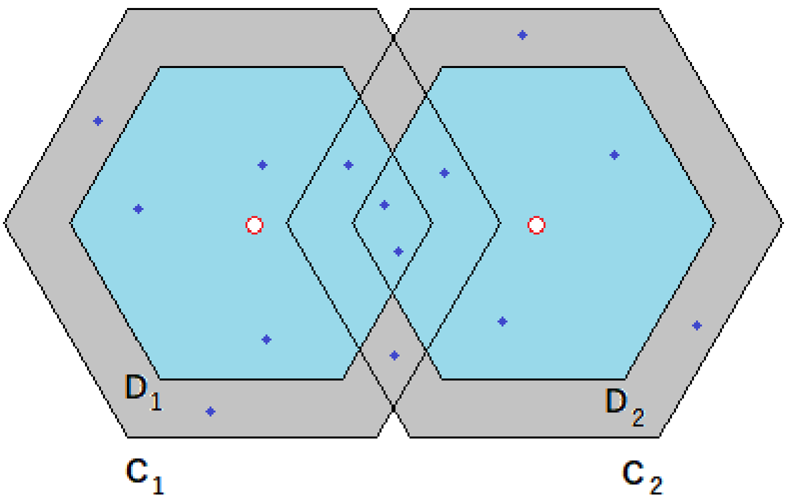


Figure 2.5: Intersection of two cells. BS_j serves users in the circle D_j, while coordinating interference to users in the outer circle C_j

We further specify two other parameters $C_j, D_j \quad \forall j$ to define the dynamic cooperation clusters. D_j is the inner set in Figure 2.5 which can contain UEs that BS_j serve with data and C_j is the outer set where BS_j has the channel estimates to UEs in this area and coordinate interference toward. While the network is working, these sets change dynamically because of user mobility and demand levels.

The selection of C_j and D_j is a complex problem [8]. In this technique the joint transmission and interference coordination provides extra degrees of freedom to separate the users spatially but this causes extra backhaul and signalling costs to obtain CSI and synchronization. As a result these techniques should be used only if there is a large improvement on the spectral efficiency. The joint transmission is more costly than the interference coordination as it needs data sharing and strict synchronization and leads to a smaller D_j set.

2.3.2 Multi Cell Downlink

Here the channel from all BSs to UE_k is denoted $\boldsymbol{h}_k = [\boldsymbol{h}_{1k}^T ... \boldsymbol{h}_{K_t k}^T]^T \in \mathbb{C}^N$, where $\boldsymbol{h}_{jk} \in \mathbb{C}^{N_j}$ is the channel from BS_j's perspective. Based on Dynamic Cooperation Cluster (DCC) only some elements of $\boldsymbol{h}_k$ will carry data or

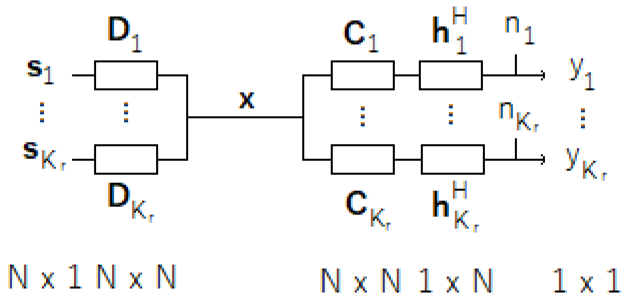


Figure 2.6: Block diagram of the multi cell system

interference. These can be selected by the diagonal matrices

$$\boldsymbol{D}_k = \begin{bmatrix} D_{1k} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & D_{K_t k} \end{bmatrix} \quad where \quad \boldsymbol{D}_{jk} = \begin{cases} \boldsymbol{I}_{N_j} & \text{if } k \in D_j, \\ \mathbf{0}_{N_j} & \text{otherwise,} \end{cases} \tag{2.7}$$

$$\boldsymbol{C}_k = \begin{bmatrix} C_{1k} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & C_{K_t k} \end{bmatrix} \quad where \quad \boldsymbol{C}_{jk} = \begin{cases} \boldsymbol{I}_{N_j} & \text{if } k \in C_j, \\ \mathbf{0}_{N_j} & \text{otherwise,} \end{cases} \tag{2.8}$$

So, $\boldsymbol{h}_k^H \boldsymbol{D}_k$ is the channel that carries data and $\boldsymbol{h}_k^H \boldsymbol{C}_k$ is the one carrying interference to UE$_k$. Extending the model from (2.1) the symbol sampled complex baseband received signal at UE$_k$ is (Figure 2.6)

$$y_k = \boldsymbol{h}_k^H \boldsymbol{C}_k \sum_{i=1}^{K_r} \boldsymbol{D}_i \boldsymbol{s}_i + n_k \tag{2.9}$$

the term $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ now contains both noise and weak uncoordinated interference from all BS$_j$ $\;k \notin C_j$. This assumption limits the amount of CSI required to realize the measurements. Another thing is that σ_k^2 is implicitly coupled with the power constraints. The uncoordinated interference increases if the network wide power usage is increased. This is important for any asymptotic analysis because multi cell systems are interference limited under high SNR [34].

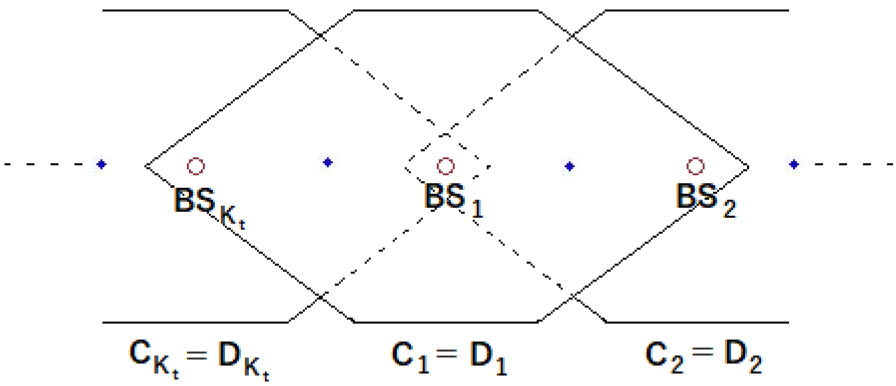


Figure 2.7: Linear Wyner Model

2.3.3 Examples of Multi Cell Scenarios

We conclude this section by giving a few examples that DCCs can include.

Wyner Model. Based on [10], it is a fairly simple model. We assume that an UE receives signals from their own BS and the other two closes neighbouring BSs to that BS.

If UE$_k$ is closest to BS$_j$ then,
$\boldsymbol{D}_k = \boldsymbol{C}_k = diag(\mathbf{0}_{N_1+\ldots+N_{j-2}}, \boldsymbol{I}_{N_{j-1}+N_j+N_{j+1}}, \mathbf{0}_{N_{j+2}+\ldots+N_{K_t}})$
which means UE$_k$ is served by BS$_{j-1}$, BS$_j$, BS$_{j+1}$ and receive interference again only from them while they serve other UEs simultaneously.

Global Joint Transmission. Here all BS serve and coordinate interference to all UEs [30, 47, 65]. If the network has multiple cells and cell sectors, this technique turns the system into one big cell with distributed antenna arrays. Only difference from a normal single cell scenario is that here the power constraints are per antenna/transmitter based. $\boldsymbol{D_k} = \boldsymbol{C_k} = \boldsymbol{I_N} \;\; \forall k$

Cognitive Radio. This scenario is where a secondary network is working underlay a primary one while causing low interference to it [21, 27].

Spectrum Sharing. Here two operators agree to share some of their frequency resources. In the figure the blue BSs serve the blue UEs while yellow BSs serve the yellow UEs.

Considering UE$_k$ suppose that apart from the signals from BS$_1$, BS$_A$,

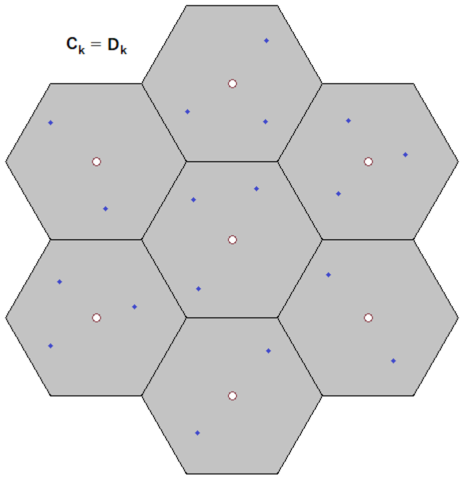


Figure 2.8: Global Joint Transmission

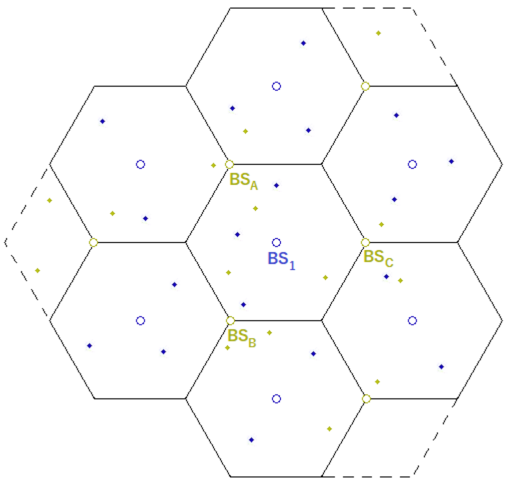


Figure 2.9: Spectrum sharing between two operators (green and blue) covering the same area

BS_B and BS_C, the signals coming from the other BSs are negligible and UE_k is served by BS_1. This leads to $\boldsymbol{D_k} = diag(\boldsymbol{I}_{N_1}, \boldsymbol{0}_{N_2}, ...)$ and $\boldsymbol{C_k} = diag(\boldsymbol{I}_{N_1}, \boldsymbol{0}, ...\boldsymbol{0}, \boldsymbol{I}_{N_A}, \boldsymbol{I}_{N_B}, \boldsymbol{I}_{N_C}, \boldsymbol{0}, ...)$.

2.4 Multi Cell Performance Measures

In this section we define a general method to measure the performance of a multi cell system. We will consider the performance from two different perspectives as the performance seen by the user side and the performance of the network as a whole. In the next chapter when we describe the load coupling formula, we will use and further detail the general concepts given in this section.

2.4.1 User Performance

We assume low complexity in the receiving device. This implies that UE is not attempting to decode and subtract interfering signals while decoding its own. This limits the spectral efficiency but demands less processing in the receiver side allowing us to manufacture simple and relatively economic UE. SINR for UE_k is

$$SINR_k(\boldsymbol{S}_1, ..., \boldsymbol{S}_{K_r}) = \frac{\boldsymbol{h}_k^H \boldsymbol{C}_k \boldsymbol{D}_k \boldsymbol{S}_k \boldsymbol{D}_k^H \boldsymbol{C}_k^H \boldsymbol{h}_k}{\boldsymbol{h}_k^H \boldsymbol{C}_k (\sum\limits_{i \in I_k} \boldsymbol{D}_i \boldsymbol{S}_i \boldsymbol{D}_i^H) \boldsymbol{C}_k^H \boldsymbol{h}_k + \sigma_k^2} \tag{2.10}$$

I_k is the set of UE being served by the BSs that coordinate interference toward UE_k.

For brevity from now on we will address SINR_k as $SINR_k(\boldsymbol{S}_1, ..., \boldsymbol{S}_{K_r})$.

Another expression is when we consider two independent data streams with correlation matrices $\boldsymbol{S}_k^{(1)}$ and $\boldsymbol{S}_k^{(2)}$ toward the same UE_k. After optimal receive processing

$$SINR_k^{2-signals}(\boldsymbol{S}_1, ..., \boldsymbol{S}_{K_r}) = \frac{\boldsymbol{h}_k^H \boldsymbol{C}_k \boldsymbol{D}_k (\boldsymbol{S}_k^{(1)} + \boldsymbol{S}_k^{(2)}) \boldsymbol{D}_k^H \boldsymbol{C}_k^H \boldsymbol{h}_k}{\boldsymbol{h}_k^H \boldsymbol{C}_k (\sum\limits_{i \in I_k} \boldsymbol{D}_i \boldsymbol{S}_i \boldsymbol{D}_i^H) \boldsymbol{C}_k^H \boldsymbol{h}_k + \sigma_k^2} \tag{2.11}$$

If all data streams are independent and not meant for multiple users then this is equal to (2.10).

Each UE has its own performance measurement function $g_k : \mathbb{R}_+ \to \mathbb{R}_+$ which is a strictly monotonically increasing function of SINR. This function is used to describe the performance experienced by UE and depends on the service in use (VoIP, Internet..).

This is a general expression and using SINR to describe a performance function is not trivial. There are different ways of defining it which have different analytical meanings. Nevertheless, it is understandable that increasing signal quality increases the performance [42] or at least wouldn't degrade it [6].

Some examples of performance functions;

Information Rate. The achievable information rate is defined as $g_k(SINR_k) = log_2(1 + SINR_k)$ and describes the number of bits that can be sent with an arbitrarily low decoding error rate [11]. We will use this performance function in the next chapter when we define the load coupling.

Mean Square Error. MSE is defined as $MSE_k = \mathbb{E}\{||\hat{\boldsymbol{s}}_k - \boldsymbol{s}_k||^2\}$, where $\hat{\boldsymbol{s}}_k$ is the estimate of $\boldsymbol{s}_k$ acquired with signal processing techniques. MSE should be minimized to maximize $g_k(SINR_k)$.

Bit Error Rate. The Bit Error Rate (BER) for Gray coded transmission of a 16-QAM constellation

$$P_{k,16-QAM} = \frac{3}{8}erfc(\sqrt{\frac{1}{10}SINR_k}) + \frac{1}{4}erfc(\sqrt{\frac{9}{10}SINR_k}) - \frac{1}{8}erfc(\sqrt{\frac{5}{2}SINR_k}) \tag{2.12}$$

where $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty$ is the complementary error function and $rank(\boldsymbol{S}_k) \leq 1$ [16, 40]. BER should be minimized to maximize $g_k(SINR_k)$.

The information rate has a simple meaning but builds on idealized coding and processing. MSE is also simple but gives only a generalized assumption on user experienced performance. BER is straight forward as its meaning but has a complicated expression. The point is there is no best way to describe the performance function but it should be selected meaningfully.

Consider the signal with the correlation matrix $\boldsymbol{S}_k$. The received signal power at UE$_i$ is $x_{ki}(\boldsymbol{S}_k) = \boldsymbol{h}_i^H \boldsymbol{C}_i \boldsymbol{D}_k \boldsymbol{S}_k \boldsymbol{D}_k^H \boldsymbol{C}_i^H \boldsymbol{h}_i$. The channel gain region of this signal

$$\Omega_k = \{(x_{k1}(\boldsymbol{S}_k), ..., x_{kK_r}(\boldsymbol{S}_k)) : \boldsymbol{S}_k \geq \boldsymbol{0}_N, \quad tr(\boldsymbol{Q}_{lk}\boldsymbol{S}_k) \leq q_{lk} \quad \forall l\} \tag{2.13}$$

It describes the impact $\boldsymbol{S}_k$ has on the other UEs.

If we re-form the SINR equation

$$SINR_k((x_{1k}(\boldsymbol{S}_1),...,x_{K_rk}(\boldsymbol{S}_{K_r})) = \frac{x_{kk}(\boldsymbol{S}_k)}{\sum\limits_{i\in I_k} x_{ik}(\boldsymbol{S}_i) + \sigma_k^2} \tag{2.14}$$

SINR is monotonically increasing in $x_{kk}(\boldsymbol{S}_k)$ and monotonically decreasing in $x_{ik}(\boldsymbol{S}_i)$ $\forall i \in I_k$. The conflict becomes visible, increasing a links channel gain x_{kk} might cause an increase in other x_{ki} and lower SINR.

2.4.2 Multi Objective Resource Allocation

The channel gain region highlights the coupling conflict the system is suffering. Each UE has its own $g_k(SINR_k)$ to be optimized and these performance function of all UE are coupled together.

We come across with multi objective optimization problems in many fields where multiple parameters need to be optimized jointly. We formulate the resource allocation problem

$$\begin{array}{cl} \underset{\boldsymbol{S}_1\geq\boldsymbol{0}_N,...,\boldsymbol{S}_{K_r}\geq\boldsymbol{0}_N}{maximize} & \{g_1(SINR_1),...,g_{K_r}(SINR_{K_r})\} \\ subject\ to & \sum\limits_{k=1}^{K_r} tr(\boldsymbol{Q}_{lk}\boldsymbol{S}_k) \leq q_l \end{array} \tag{2.15}$$

This can be seen as finding $\boldsymbol{S}_1,...,\boldsymbol{S}_{K_r}$ that optimizes the problem. However, since $g_k(SINR_k)$ is coupled with power constraints and inter user interference, generally there is no single strategy that maximizes the performance of all UEs. As we lower the transmit power of other users to lower the interference their signal cause to UE$_k$, we lower their SINR thus lowering their performance. Under this information, we define the feasible performance region

$$\mathcal{R} = \{g_1(SINR_1),...,g_{K_r}(SINR_K) : (\boldsymbol{S}_1,...,\boldsymbol{S}_{K_r}) \in \mathbb{S}\} \tag{2.16}$$

$$\mathbb{S} = \{(\boldsymbol{S}_1,...,\boldsymbol{S}_{K_r}) : \boldsymbol{S}_k \geq \boldsymbol{0}_N,\ \sum_{k=1}^{K_r} tr(\boldsymbol{Q}_{lk}\boldsymbol{S}_k) \leq q_l\} \tag{2.17}$$

This region characterizes the simultaneously achievable performance and its shape depends on channel vectors, power constraints and DCCs.

The performance region $\mathcal{R} \subseteq [\boldsymbol{0}, \boldsymbol{u}]$ where $\boldsymbol{u} = [u_1...u_{K_r}]^T$ is the utopia point. u_k is the optimum of the single user problem of UE$_k$

This is achieved by ignoring any interference caused by the other UEs.

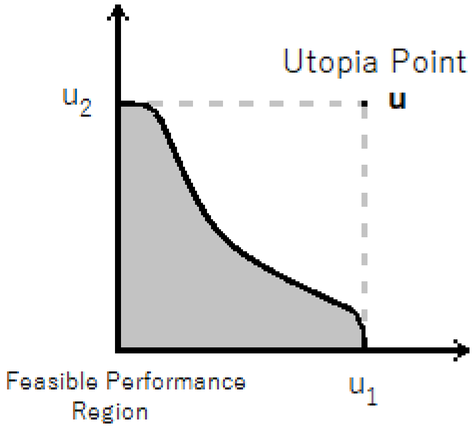


Figure 2.10: A feasible performance region. Utopia point is assembled from single user points solved from (2.18)

$$\begin{aligned} &\underset{\boldsymbol{S}_k \geq \boldsymbol{0}_N}{maximize} \quad g_k\left(\frac{\boldsymbol{h}_k^H \boldsymbol{D}_k \boldsymbol{S}_k \boldsymbol{D}_k^H \boldsymbol{h}_k}{\sigma^2}\right) \\ &subject\ to \quad tr(\boldsymbol{Q}_{lk}\boldsymbol{S}_k) \leq q_l \quad \forall l \end{aligned} \tag{2.18}$$

$\boldsymbol{u}$ is the unique optimal solution of the performance problem when all UEs can achieve maximum performance simultaneously which is not a real world scenario. The example in Figure 2.10 shows us the feasible performance region and the unattainable $\boldsymbol{u}$.

The feasible points show us that the performance cannot be improved for a specific user without degrading it for the others.

We call the set of feasible optimal points the Pareto boundary. As the Pareto boundary contains all tentative solutions, searching for Pareto optimal points is important in optimization algorithms.

Following from the monotonicity of $g_k(.)$ which depends on $x_{ki}(\boldsymbol{S}_k)$, there is a connection between The Pareto boundary of $\mathcal{R}$ and the channel gain regions Ω_k.

Suppose that the Pareto boundary of the performance region $\mathcal{R}$ is achieved by a transmit strategy $\boldsymbol{S}_1, ..., \boldsymbol{S}_{K_r}$. For each UE$_k$, $\boldsymbol{S}_k$ also achieves the upper boundary of Ω_k considering only UE$_k$.

2.5 Subjective Solutions to Resource Allocation

Whenever the utopia point is outside of the feasible performance region, there is no objectively optimal resource allocation. the points that the Pareto boundary sets contain represent several optimal solutions and none of these are distinctively better than the others. To meaningfully compare the Pareto optimal points, the system designer should create its own system utility function which follows its own optimality perspective.

A system utility function is denoted $f(g_1(SINR_1), ..., g_{K_r}(SINR_{K_r}))$ where $f : \mathcal{R} \to \mathbb{R}$ and is Lipschitz continuous and monotonically increasing in $[\mathbf{0}, \boldsymbol{u}]$.

- A function $f : [\boldsymbol{a}, \boldsymbol{b}] \to \mathbb{R}$ is defined as Lipschitz continuous if $|f(\boldsymbol{g}) - f(\boldsymbol{g}')| \leq L_f|\boldsymbol{g} - \boldsymbol{g}'|$ and L_f being the Lipschitz constant.
- A function $f : \mathbb{R}^n \to \mathbb{R}$ is defined as monotonically increasing if $f(\boldsymbol{g}) \geq f(\boldsymbol{g}')$ for any $\boldsymbol{g} \geq \boldsymbol{g}'$ where $\boldsymbol{g}, \boldsymbol{g}' \in \mathbb{R}^n$.

Chapter 3

Load Coupling Model and Definitions

3.1 Load Coupling

The level of resource usage is referred as the cell load. In OFDM using LTE systems, this can be interpreted as the fraction of the time-frequency resources (Resource Blocks) in use. A well designed network should be able to meet the target demand without overloading the cells. Therefore the cell loads can be seen as a performance metric for the network. The cell load depends on the user demands, amount of interference and the gains between the users and the cell [52]. Due to this mutual interference, the load of a cell depends on the load of other cells. This leads to a non-linear coupling relation of the cells' loads, making analytical characterization of the load challenging [24]. Load coupling refers to the dependency relation between the cell load levels as a result of the mutual interference [54].

Assuming the considered associations are feasible, we denote several parameters used in the load coupling formula:

- $\boldsymbol{x}$: The cell loads vector
- $\boldsymbol{\gamma}$: The UE SINRs vector
- x_n: The load of cell n
- γ_j: The SINR at UE_j

- h: The SINR coupling formula
- f: The load coupling formula
- I: The set of all cells
- J: The set of all UEs
- I_j: The set of cells that serve the user j ($j \in J$)
- g_{ij}: The power gain between the cell i and the user j
- r_j: The Traffic demand of the user j
- p_i: The transmit power per resource unit of cell i
- M: The total number of resource units in a cell
- B: The bandwidth per resource unit
- N: The total number of cells
- $\boldsymbol{\gamma}^*$: The SINR coupling formula's fixed point
- $\boldsymbol{x}^*$: The load coupling formula's fixed point

Additionally, The demand represents the amount of data to be delivered to the user within the time interval under consideration. If there are different QoS requirements then the demand parameter and the system model can be extended to multiple service types [51]. Here we will consider just one service type.

Here in (3.1) we model the SINR at UE_j using the same model from (2.14). This model can be accounted for both normal and JT transmission depending on the parameter I_j. If the set I_j has only one element, then UE_j is being served by only one cell, else it is served by multiple cells simultaneously.

In the nominator, the term $\sum_{i \in I_j} p_i g_{ij}$ represents the total received signal power at UE_j.

In the denominator, the term $\sum_{k \in I \setminus I_j} p_k g_{kj} x_k$ represents the total interference received by UE_j. Here the load term x_k stands for the proportion of resource units in use in cell k.

To describe the contribution of x_k we give two extreme cases

- $x_k = 0$: Means no user is associated with cell k and results in $p_k g_{kj} x_k = 0$. Therefore cell k doesn't send any interference to UE_j.

- $x_k = 1$: Means all the resource units in cell k is in use and interference results in $p_k g_{kj}$ no matter which resource unit UE$_j$ is using, it will receive interference from cell k.

x_k is seen as the likelihood that UE$_j$ receive interference from cell k $k \in (I \setminus I_j)$.

$$\gamma_j \triangleq h_j(\boldsymbol{x}) \triangleq \frac{\sum_{i \in I_j} p_i g_{ij}}{\sum_{k \in I \setminus I_j} p_k g_{kj} x_k + \sigma^2} \tag{3.1}$$

Here in (3.2) we model the load of cell $_i$ by depending it to the demands and SINR of each UE served by it.

In the denominator, the term $Blog_2(1+\gamma_j)$ is the achievable bit rate per resource unit. Then $\frac{r_j}{Blog_2(1+\gamma_j)}$ describes the required amount of resource units to satisfy the demand d_j and if we divide this number with the total amount of resource units M, this gives us the load caused to cell i by serving UE$_j$. If we sum up all the load caused to cell i by each served UE, then we end up with the load of cell i: x_i.

$$x_i \triangleq f_i(\boldsymbol{\gamma}) \triangleq \sum_{j \in J_i} \frac{r_j}{MBlog_2(1+\gamma_j)} \tag{3.2}$$

additionally, if we add the set of cells l_u to serve UE$_u$, where cell v $(v \in l_u)$ serves UE u:

$$h_u^+(\boldsymbol{x}) \triangleq \frac{\sum_{i \in I_u \cup l_u} p_i g_{iu}}{\sum_{k \in I \setminus (I_u \cup l_u)} p_k g_{ku} x_k + \sigma^2} \tag{3.3}$$

$$f_v^+(\boldsymbol{\gamma}) \triangleq \sum_{j \in J_v \cup \{u\}} \frac{r_j}{MBlog_2(1+\gamma_j)} \tag{3.4}$$

These are the extensions to (3.1) and (3.2) to describe what happens when another group of cells l_u start serving UE$_u$ simultaneously with the set I_u.

For instance, if we consider just one cell in l_u namely cell v, in (3.3) we see an additional gain term $p_v g_{vu}$ appear in the nominator while the interference term $p_v g_{vu} x_v$ is removed from the denominator, thus attempting to improve SINR at UE$_u$.

Furthermore, in (3.4) an additional term $\frac{r_u}{MBlog_2(1+\gamma_u)}$ is added to the sum as there is an additional UE that the cell v is serving, thus as a result, the term attempts to enlarge the load of cell v.

Here the verb "*attempt*" has a deeper meaning than how it may appear at the first glance. There is a complex relation about how the functions $h_j(\boldsymbol{x})$ and $f_i(\boldsymbol{\gamma})$ change quantitatively with their corresponding dependencies.

When the cell v starts to serve an additional UE_u, this UE suffers from interference from less number of cells and even receives a boost from the additional gain from simultaneous transmission (JT). However, now the cell v is serving one more UE and intuitively has more load. This increased load in cell v causes more interference to the UEs that it is not currently serving and this causes their SINRs to drop. This drop makes them cause more load to their respective cells and as a result the other cells' loads increase. While all this is happening, the other cell which was originally serving UE_u might have it's load dropped because now UE_u is being served by two cells simultaneously thus there is an attempt to improve its SINR and enabling it to cause less load to its original cell...

Anyway, this small story was also an "*attempt*" to describe the so called load coupling.

In Section 2.4.1 we did mention the user performance functions. If we recall the Information Rate example where, $g_k(SINR_k) = log_2(1+SINR_k)$

we can easily describe x_i as,

$$x_i \triangleq f_i(\boldsymbol{\gamma}) \triangleq \sum_{j \in J_i} \frac{r_j}{MBlog_2 g_j(SINR_j)} \tag{3.5}$$

so we can define x_i as a system utility function $z_i(g_j(SINR_j))\ \ j \in J_i$

Definition. The function $f(h(\mathbf{x}))$ is a Standard Interference Function (SIF).

Proof: The proof essentially relies on the framework of interference functions introduced by Yates in [61] .

- Positivity: $f(h(\mathbf{x})) > 0$
- Monotonicity: if $\mathbf{x} > \mathbf{x'}$, then $f(h(\mathbf{x})) > f(h(\mathbf{x'})) > 0$
- Scalability: $\alpha f(h(\mathbf{x})) > f(h(\alpha\mathbf{x}))$ for $\alpha > 1$

The noise contribution in $h(\mathbf{x})$ implies the positivity. The monotonicity is easily seen from the formula if we keep all the variables other than $\mathbf{x}$ as a constant. To show the scalability we have to argue that all the components of the vector $f(h(\mathbf{x}))$ is strictly concave in $\mathbf{x}$ [14]. Concavity and Positivity implies Scalability.

This definition is strongly in relation with the properties that we have defined in Section 2.5 (Lipschitz continuous and monotonically increasing) about the system utility functions.

3.2 Problem Formulation

In this section we will define two multi objective optimization problems which we will use in the upcoming sections where we investigate some theoretical aspects of the load coupling and do some simulations. They follow the same structure introduced in (2.15) where g is replaced with the system utility function z.

3.2.1 Minimum of Maximum Load (MinMaxL)

The solution to the Minimum of Maximum Load (MinMaxL) problem gives the association which minimizes the maximum load between the cells. In this thesis work we are not going to study this type of system utility function but nevertheless will include it in some of the theorems in the next section.

Formulation. MinMaxL

$$
\begin{aligned}
&\underset{\boldsymbol{K}}{min}\ \underset{i \in I}{max}\ x_i \\
&s.t. \quad \boldsymbol{x = f(h(x, K), K)} \\
&0 < x_i < 1 && i \in I \\
&K_{ij} = 0 && i \notin I_j, j \in J \\
&K_{ij} = 1 && i \in I_j, j \in J \\
&K_{ij} \in \{0, 1\} && i \in I, j \in J
\end{aligned}
$$

3.2.2 Minimum of Total Load (MinSumL)

The solution to the Minimum Total Load (MinSumL) problem gives the association which minimizes the total load throughout the network. If the purpose is to serve as much demand as possible, this problem formulation would be necessary. MinSumL is the main optimization direction that we are taking in this thesis work.

Formulation. MinSumL

$$
\underset{\boldsymbol{K}}{min}\ \sum_{i \in I} x_i
$$

$$
\begin{array}{ll}
s.t. \qquad \boldsymbol{x} = \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{K}), \boldsymbol{K}) & \\
0 < x_i < 1 & i \in I \\
K_{ij} = 0 & i \notin I_j, j \in J \\
K_{ij} = 1 & i \in I_j, j \in J \\
K_{ij} \in \{0, 1\} & i \in I, j \in J
\end{array}
$$

Here $\boldsymbol{K}$ is a $(n\ x\ m)$ binary matrix to represent the association. $K_{ij} = 1$ means that cell i is serving UE$_j$. The variable is $\boldsymbol{K}$, in the upcoming sections we will apply different methods to adjust this parameter to solve the MinSumL problem.

Please note that the association matrix constraint is given in a quite general way in the description of the formulation as it may contain classical non-JT as well as JT, because the only constraint that is shaping it, is the group of cells that is serving UE$_j$. The additional constraint which will prevent or allow the cells to simultaneously serve an UE will be given in their appropriate algorithms.

3.3 Theoretical Findings

In this section we will provide the reader with some relatively simple theoretical basis of the load coupling relation which we will relay on when we model the heuristic algorithms in the next chapter.

Theorem 1. Suppose $\boldsymbol{x}^*$ is the fixed point of the iterations $\boldsymbol{x}^{(k)} = \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}^{(k-1)}))$ $(k \geq 1)$, i.e. $\boldsymbol{x}^* = \lim_{k\to\infty} \boldsymbol{x}^{(k)}$. Then $\sum_{i=1} x_i^*$ and $max_{1\leq i\leq N} x_i^*$ are respective upper bounds of MinSumL and MinMaxL, if for any $k \geq 1$ such that:

1. $\boldsymbol{x}^k \leq \boldsymbol{x}^{k-1}$, and
2. $\sum_{i=1} x_i^{(k)}$ and $max_{1\leq i\leq N} x_i^{(k)}$ are upper bounds of MinSumL and MinMaxL, respectively.

Proof: $\boldsymbol{x}^k \leq \boldsymbol{x}^{k-1}$ being true for any $k \geq 1$ together with Monotonicity suggests that $\boldsymbol{x}^k \leq \boldsymbol{x}^{k-1}$ is true for all $k \geq 1$ constructs the sequence $\boldsymbol{x}^{(0)} \geq \boldsymbol{x}^{(1)} ... \geq \boldsymbol{x}^{(*)}$. Which leads to $\sum_{i=1} x_i^* \leq \sum_{i=1} x_i^{(k)}$ and $max_{1\leq i\leq N} x_i^* \leq max_{1\leq i\leq N} x_i^{(k)}$. Hence the conclusion.

Theorem 1 suggests that if there is a set of load functions at iteration k that surpass any other possible solution set element by element in terms of their respective problems and still improving with each iteration, it means that their fixed point will be the optimal solution. Because, the load coupling is monotonical and will follow its direction until it reaches its fixed point with each iteration.

We will use this property in the heuristic algorithm to ease the computational cost by computing only k iterations each round. This might not be the optimal strategy as we don't know if the decided load value k would be monotonically increasing or decreasing and if this behaviour lets another set take its place in the ranking but for sure will bring us to a solution which is close to the best one.

Theorem 2. Let $\boldsymbol{x}^* = \boldsymbol{f}(\boldsymbol{\gamma}^*)$ and suppose $\boldsymbol{\gamma}^*$ is the fixed point of the iterations $\boldsymbol{\gamma}^{(k)} = \boldsymbol{h}(\boldsymbol{f}(\boldsymbol{\gamma}^{(k-1)}))$ $(k \geq 1)$, i.e. $\boldsymbol{\gamma}^* = \lim_{k\to\infty} \boldsymbol{\gamma}^{(k)}$. Then $\sum_{i=1} x_i^*$ and $max_{1\leq i\leq N} x_i^*$ are respective upper bounds of MinSumL and MinMaxL, if for any $k \geq 1$ such that:

1. $\boldsymbol{\gamma}^k \geq \boldsymbol{\gamma}^{k-1}$, and
2. $\sum_{i=1} f_i(\gamma^{(k)})$ and $max_{1\leq i\leq N} f_i(\gamma^{(k)})$ are upper bounds of MinSumL and MinMaxL, respectively.

Proof: Similarly as it is in Theorem 1's proof, $\boldsymbol{\gamma}^k \geq \boldsymbol{\gamma}^{k-1}$ being true for any $k \geq 1$ together with Monotonicity suggests that $\boldsymbol{\gamma}^k \geq \boldsymbol{\gamma}^{k-1}$ is true for all $k \geq 1$ constructs the sequence $\boldsymbol{\gamma}^{(0)} \leq \boldsymbol{\gamma}^{(1)} ... \leq \boldsymbol{\gamma}^{(*)}$ where by intuition $\boldsymbol{f}(\boldsymbol{\gamma}^{(0)}) \geq \boldsymbol{f}(\boldsymbol{\gamma}^{(1)}) ... \geq \boldsymbol{f}(\boldsymbol{\gamma}^*)$ and $\boldsymbol{f}(\boldsymbol{\gamma}^{(k)}) = \boldsymbol{x}^{(k)}$ Which leads to $\sum_{i=1} x_i^* \leq \sum_{i=1} x_i^{(k)}$ and $max_{1\leq i\leq N} x_i^* \leq max_{1\leq i\leq N} x_i^{(k)}$. Hence the conclusion.

There is a strong coupling effect between the load and SINR. If the load of cell i goes up, this would mean that SINR_j $(j \in J_i)$ is reduced or in contrast, if SINR_j goes up, the load of cell i $(i \in I_j)$ reduces.

Here in Theorem 2 approaches the same idea from Theorem 1 using this property.

Theorem 3. Suppose $\boldsymbol{x}^*$ is the fixed point of the asynchronous iterations $x_{i'}^{(k)} = f_{i'}(\boldsymbol{h}(\boldsymbol{x}^{(k-1)}))$ $(\forall\ i' \in I'$ and $k \geq 1)$ where we keep $x_{i'}^{(k)} = x_{i'}^{(k-1)}$ $(\forall\ i' \in I \setminus I'$ and $k \geq 1)$. Then $\sum_{i=1} x_i^*$ and $max_{1\leq i\leq N} x_i^*$ are respective upper bounds of MinSumL and MinMaxL, if for any $k \geq 1$ such that:

1. $x_{i'}^k = x_{i'}^{k-1}\ \forall\ i' \in I \setminus I'$,
2. $x_{i'}^k \leq x_{i'}^{k-1}\ \forall\ i' \in I'$, and

3. $\sum_{i=1} x_i^{(k)}$ and $max_{1\leq i\leq N} x_i^{(k)}$ are upper bounds of MinSumL and MinMaxL, respectively.

Proof: We can divide this problem into two separate parts and treat accordingly. For the first part where $i' \in I'$ the proof is the same as in Theorem 1 and for the second part where $i' \in I \setminus I'$ the proof is trivial as $x_{i'}^{k} = x_{i'}^{k-1}$ $(k \geq 1)$. Putting together the both parts leads to

$$\boldsymbol{x}^* \leq \boldsymbol{x}^{(k)}$$

Which leads to $\sum_{i=1} x_i^* \leq \sum_{i=1} x_i^{(k)}$ and $max_{1\leq i\leq N} x_i^* \leq max_{1\leq i\leq N} x_i^{(k)}$. Hence the conclusion.

In Theorem 3 we further model the situation that we have explained above, after Theorem 2. We take into account only a group of cells i' while not including the others in the coupling and treating them as constants. We see that the nature of the coupling will stay the same and the selected group will arrive to a fixed point which is different than the global one.

We should keep in mind that the group i' can't have only one element, as the change in the load of this groups elements will result in a change in SINRs of the other elements and vice versa. Considering only one element, there can't be a coupling relation.

Theorem 4. Let $\boldsymbol{x}^* = \boldsymbol{f}(\boldsymbol{\gamma}^*)$ and suppose $\boldsymbol{\gamma}^*$ is the fixed point of the asynchronous iterations $\gamma_{i'}^{(k)} = h_{i'}(\boldsymbol{f}(\boldsymbol{\gamma}^{(k-1)}))$ $(\forall\, i' \in I'$ and $k \geq 1)$ where we keep $\gamma_{i'}^{(k)} = \gamma_{i'}^{(k-1)}$ $(\forall\, i' \in I \setminus I'$ and $k \geq 1)$. Then $\sum_{i=1} x_i^*$ and $max_{1\leq i\leq N} x_i^*$ are respective upper bounds of MinSumL and MinMaxL, if for any $k \geq 1$ such that:

1. $\gamma_{i'}^{k} = \gamma_{i'}^{k-1}$ $\forall\, i' \in I \setminus I'$,
2. $\gamma_{i'}^{k} \geq \gamma_{i'}^{k-1}$ $\forall\, i' \in I'$, and
3. $\sum_{i=1} f_i(\gamma^{(k)})$ and $max_{1\leq i\leq N} f_i(\gamma^{(k)})$ are upper bounds of MinSumL and MinMaxL, respectively.

Proof: We can divide this problem into two separate parts and treat accordingly. For the first part where $i' \in I'$ the proof is the same as in Theorem 2 and for the second part where $i' \in I \setminus I'$ the proof is trivial as $\gamma_{i'}^{k} = \gamma_{i'}^{k-1}$ $(k \geq 1)$. Putting together the both parts leads to

$$\boldsymbol{\gamma}^* \geq \boldsymbol{\gamma}^{(k)}$$

Which leads to $\sum_{i=1} x_i^* \leq \sum_{i=1} x_i^{(k)}$ and $max_{1\leq i\leq N} x_i^* \leq max_{1\leq i\leq N} x_i^{(k)}$ as in Theorem 2. Hence the conclusion.

Theorem 5. Considering the scenario where the set I_j $(j \in J)$ always have one element, suppose $\overset{m}{\boldsymbol{x}}$ is the fixed point of iterations $\overset{m(k)}{\boldsymbol{x}} = \boldsymbol{f}^m(\boldsymbol{h}^m(\overset{m(k-1)}{\boldsymbol{x}}))$ $(k \geq 1)$, i.e. $\overset{m}{\boldsymbol{x}} = \lim_{k\to\infty} \overset{m(k)}{\boldsymbol{x}}$ where m indicates the cell which the user j is associated, i.e. $I_{\{m\}} \in I_j$.

Then $\overset{m}{\boldsymbol{x}} \leq \overset{n}{\boldsymbol{x}}$, if for any iteration $k \geq 1$ we have $f_m^m(\boldsymbol{h}^m(\overset{m(k-1)}{\boldsymbol{x}})) \leq \overset{n}{x}_m$.

Proof: Let's select the initial point $\overset{m(0)}{\boldsymbol{x}} = \overset{n}{\boldsymbol{x}}$. For cell m, $\overset{m(1)}{x}_v = f_m^m(\boldsymbol{h}^m(\overset{m(0)}{\boldsymbol{x}})) = f_m^m(\boldsymbol{h}^m(\overset{n}{\boldsymbol{x}}))$ where $k = 1$ and $f_m^m(\boldsymbol{h}^m(\overset{m(k-1)}{\boldsymbol{x}})) \leq \overset{n}{x}_m = \overset{m(0)}{x}_m$ which implies

$$\overset{m(1)}{x}_m \leq \overset{m(0)}{x}_m = \overset{n}{x}_m$$

As $I_{\{n\}} \notin I_j$ while $I_{\{m\}} \in I_j$ and $\overset{m(1)}{x}_m \leq \overset{n}{x}_m$ from Monotonicity by intuition

$$\overset{m(1)}{x}_n \leq \overset{n}{x}_n$$

and as a result of the above relations again from Monotonicity

$$\overset{m(1)}{x}_i \leq \overset{n}{x}_i = \overset{m(0)}{x}_i$$

putting all these relations together for $k \to \infty$ we get

$$\overset{m}{\boldsymbol{x}} \leq \overset{n}{\boldsymbol{x}}$$

Hence the conclusion.

When we change UE$_j$'s association from cell n to m, if after k iterations cell m has less load than its former load when it wasn't serving UE$_j$ then, the new association will improve the fixed load point.

This is because when we change the association, the cell m starts to serve one more UE while n loses it. If even under this situation x_m has reduced load, it is clear that all the other cells' loads will reduce too.

If the set $v \in l_u$ from the pair (3.3)(3.4) contains only one element (we are adding a cell to jointly serve UE$_u$):

Theorem 6. Suppose $\overset{+}{\boldsymbol{x}}$ is the fixed point of the iterations $\overset{+(k)}{\boldsymbol{x}} = \boldsymbol{f}^+(\boldsymbol{h}^+(\overset{+(k-1)}{\boldsymbol{x}}))$ $(k \geq 1)$. Then $\overset{+}{\boldsymbol{x}} \leq \boldsymbol{x}^*$, if for any iteration $k \geq 1$ we have $f_v^+(\boldsymbol{h}^+(\overset{+(k-1)}{\boldsymbol{x}})) \leq x_v^*$.

Proof: Let's select the initial point $\overset{+}{\boldsymbol{x}}^{(0)} = \boldsymbol{x}^*$. For cell v, $\overset{+}{x}_v^{(1)} = f_v^+(\boldsymbol{h}^+(\overset{+}{\boldsymbol{x}}^{(0)})) = f_v^+(\boldsymbol{h}^+(\boldsymbol{x}^*))$ where $k = 1$ and $f_v^+(\boldsymbol{h}^+(\overset{+}{\boldsymbol{x}}^{(k-1)})) \leq x_v^* = \overset{+}{x}_v^{(0)}$ which implies

$$\overset{+}{x}_v^{(1)} \leq \overset{+}{x}_v^{(0)}$$

Because of monotonicity $\boldsymbol{h}^+(\boldsymbol{x}) \geq \boldsymbol{h}(\boldsymbol{x})$ and $f_i^+(\boldsymbol{h}^+(\boldsymbol{x})) = f_i(\boldsymbol{h}^+(\boldsymbol{x}))$ therefore, $f_i^+(\boldsymbol{h}^+(\boldsymbol{x})) \leq f_i(\boldsymbol{h}(\boldsymbol{x}))$ ($\forall i \neq v$ and $\forall \boldsymbol{x} \in \mathbb{R}_+^n$) which implies

$$\overset{+}{x}_i^{(1)} \leq \overset{+}{x}_i^{(0)} = x_i^* \quad (\forall i \neq v)$$

These two equations lead to $\overset{+}{\boldsymbol{x}}^{(t)} \leq \overset{+}{\boldsymbol{x}}^{(t-1)}$ ($\forall t \geq 1$) where $\overset{+}{\boldsymbol{x}} = \lim_{t\to\infty} \overset{+}{\boldsymbol{x}}^{(t)}$ and $\overset{+}{\boldsymbol{x}}^{(0)} = \boldsymbol{x}^*$ thus

$$\overset{+}{\boldsymbol{x}} \leq \boldsymbol{x}^*$$

Hence the conclusion.

Here we extend the idea from Theorem 5 to JT case. When we assign a new cell v to jointly serve an UE$_u$, if the increase in SINR$_u$ causes the original cell that was serving UE$_u$ without JT to have its load reduced enough so that it causes significantly less interference to cell v that cell v has its load reduced even while it has one more UE (specifically UE$_u$) to serve, will guarantee an improvement over the optimal solution.

Because, the only problem would be that x_v increasing because of the new connection and this could boost the interference caused to all the other cells. After removing this obstacle, the load vector will drop down globally.

Suppose we simultaneously add cells $v \in \{l_1, l_2, ..\}$ (here l stands for one cell like in Theorem 6) to serve UEs $j \in \{1, 2, ..\}$ respectively. Then the following theorem holds.

Theorem 7. Suppose $\overset{+}{\boldsymbol{x}}$ is the fixed point of the iterations $\overset{+}{\boldsymbol{x}}^{(k)} = \boldsymbol{f}^+(\boldsymbol{h}^+(\overset{+}{\boldsymbol{x}}^{(k-1)}))$ ($k \geq 1$). Then $\overset{+}{\boldsymbol{x}} \leq \boldsymbol{x}^*$, if for any iteration $k \geq 1$ and for all $v \in \{l_1, l_2, ..\}$ we have $f_v^+(\boldsymbol{h}^+(\boldsymbol{x}^{(k-1)})) \leq x_v^*$.

Proof: The proof follows the same argument flow as in Theorem 6.

Here in Theorem 7 we have extended the Theorem 6 to the case where multiple UEs are served with JT.

Chapter 4

Problems and Algorithms

The main purpose of this work is to compare the load performance levels of the different transmission scenarios such as; Non Joint Transmission (NJT), Joint Transmission (JT), Offset Non Joint Transmission (ONJT) and Offset Joint Transmission (OJT) and provide algorithms which give promising results in search of the global optimum of the MinSumL problem by changing the way how the UEs associate with the MC/SCs.

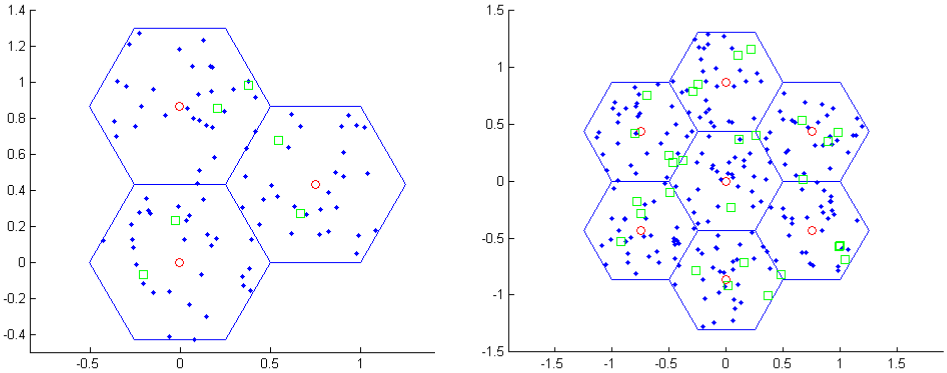
The following simulations are done for,

- a 3 cells network where each hexagonal cell area is constructed by a MC in its center, has 30 UEs and 2 SCs
- a 7 cells network where each hexagonal cell area is constructed by a MC in its center, has 30 UEs and 2 SCs
- a 7 cells network where each hexagonal cell area is constructed by a MC in its center, has 30 UEs and 4 SCs

SCs and UEs are both randomly placed inside the hexagonal areas (Figure 4.1a). We use the COST-231 path loss model while applying Additive White Gaussian Noise (AWGN) of -174 dBm/Hz spectral density and no additional shadowing.

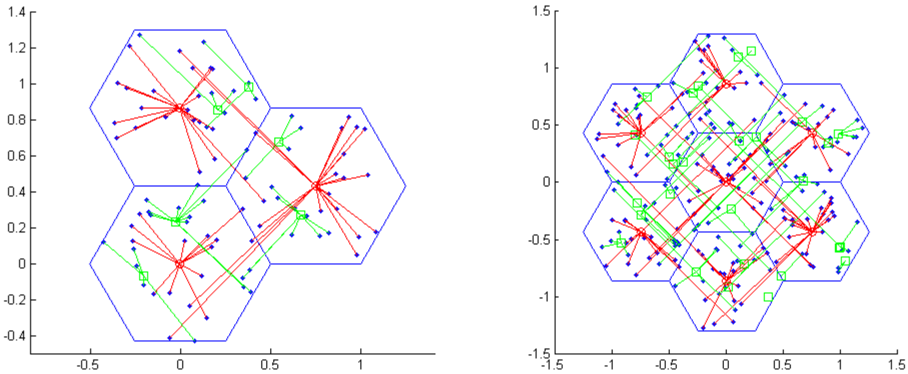
We assume that the network is operating by the LTE standard OFDM where there are 25 Resource Blocks (RBs) for each MC, SC and each RB has 180 kHz of bandwidth. The transmit power for each RB for MCs and SCs are set to 1600 mW and 400 mW for 3 cells and 4000 mW and 500 mW for 7 cells scenarios, respectively.

The following results are expected to be close to the global optimums and likely to be stuck in a local optimum as the problems at hand are NP-Hard.



(a) 3 MCs and 2 SCs each *(b) 7 MCs and 4 SCs each*

Figure 4.1: System Layout



(a) 3 MCs and 2 SCs each *(b) 7 MCs and 4 SCs each*

Figure 4.2: Maximum SINR Association

4.1 Non Joint Transmission

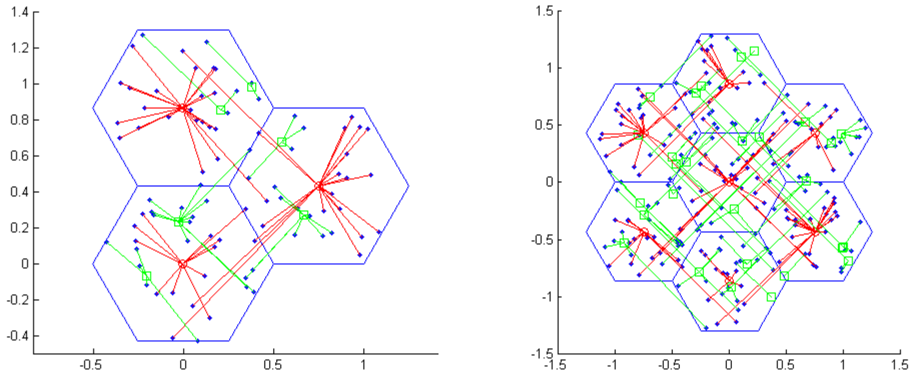
In the case of NJT, as each UE is freely associated with only one MC/SC (we assume that no UE is left free because this way those UEs wouldn't have any effect over the calculation), we propose the following greedy algorithm (Algorithm 1) for MinSumL:

For the initialization we calculate a virtual SINR (using (3.1)) of each UE-Node pair where all the nodes subjected to a fixed load value and associate each UE with the node which has the maximum SINR to build up the initial association $\boldsymbol{K}$. Starting from the association matrix $\boldsymbol{K}$, we apply α rounds of fixed point iterations to create a total load point table changing one UE-Node link pair at each cycle. After the table is full, we pick the table entry which minimizes the total load and update $\boldsymbol{K}$ accordingly. We repeat this procedure until the association and the total load stay fixed for the termination rule ($C < \beta$) is triggered.

The resulting association and the corresponding total load are the optimal association and the optimal total load resulting from Algorithm 1.

Explanation line by line:

- (Line 2-15) We build up the load matrix where we change one link every iteration and iterate the load with the load coupling formula for α times. Here we don't iterate until the fixed point to save up computational time. We will compensate for it later.
- (Line 5-7) All the possible links between UE_j and the cells is deleted to prepare the ground for the new association.
- (Line 8) We implement a new link from UE_j to the cell i thus changing the association that UE_j had.
- (Line 13) Here we define the elements of the matrix, which are the total loads for different associations.
- (Line 16) We get the indexes of the smallest element in the matrix.
- (Line 17-18) We build the association which gives the minimum total load for the current round using the previously extracted indexes.
- (Line 19-21) The control procedure to decide whether to increase the counter which triggers the end rule or not. If there is no improvement with respect to the last round, then the counter increases by one.
- (Line 22) Updating the flag to reuse in the next round when we check the end rule.



(a) 3 MCs and 2 SCs each

(b) 7 MCs and 4 SCs each

Figure 4.3: Non Joint Transmission MinSumL

- (Line 23) Updating the initial association for the next round. This will allow us to optimize the association with a greedy method, improving every round.

- (Line 1-25) Is the main body of the algorithm. One cycle of this piece of code is defined as one round and each round we change one association towards a lower total load.

- (Line 26-31) This piece of code is implemented additionally to converge the load coupling to its fixed point thus arriving to the real load value. This piece is need because in the main body of the algorithm we are iterating the load coupling formula α times and this might not be enough to get to the fixed point. Iterating the coupling formula until the convergence (fixed) point is not a good idea inside the main body of the algorithm because this causes us to lose a large computational time while iterating α rounds is enough to decide whether the result worths to consider or not. We apply α rounds of iteration before our decision about the point to let it stabilize. If the chosen association is the best association among all the other possibilities where we arrive by changing only one link, it will stay that way after the iterations in Line 26-31 as described in Theorem 1.

4.2 Joint Transmission

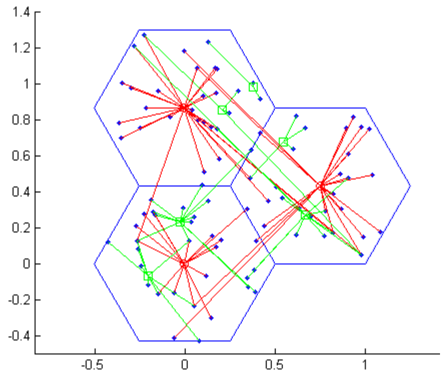
In the case of JT, where we freely associate UEs with multiple nodes, again we propose the following greedy algorithm (Algorithm 2) for MinSumL:

The algorithm differs from the former one in two main points:

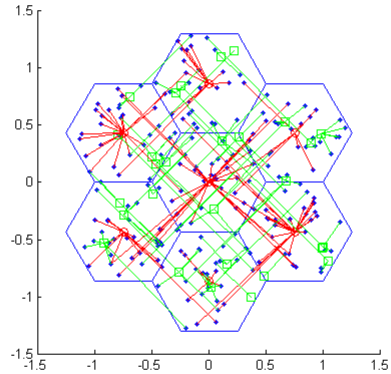
- We use the resulting association from Algorithm 1 as the initial association $\boldsymbol{K}$
- and at each cycle we add one new UE-Node link pair without cancelling any other pair from $\boldsymbol{K}$.

Explanation line by line:

- (Line 2-15) We build up the load matrix where we change one link every iteration and iterate the load with the load coupling formula for α times. Here we don't iterate until the fixed point to save up computational time. We will compensate for it later.
- (Line 5) We implement a new link from UE_j to the cell i thus allowing JT.
- (Line 10) Here we define the elements of the matrix, which are the total loads for different associations.
- (Line 13) We get the indexes of the smallest element in the matrix.
- (Line 14-15) We build the association which gives the minimum total load for the current round using the previously extracted indexes.
- (Line 16-18) The control procedure to decide whether to increase the counter which triggers the end rule or not. If there is no improvement with respect to the last round, then the counter increases by one.
- (Line 19) Updating the flag to reuse in the next round when we check the end rule.
- (Line 20) Updating the initial association for the next round. This will allow us to optimize the association with a greedy method, improving every round.
- (Line 1-22) Is the main body of the algorithm. One cycle of this piece of code is defined as one round and each round we change one association towards a lower total load.
- (Line 23) Repeats the last step from the previous algorithm to get the fixed point of the load coupling formula.



(a) 3 MCs and 2 SCs each



(b) 7 MCs and 4 SCs each

Figure 4.4: Joint Transmission MinSumL

4.3 Offset Non Joint Transmission

Here we use a different and more realistic approach to optimize the MinSumL problem by regulating the link pair choosing mechanism by virtually augmenting the power of SCs (adding offset values).

In this algorithm the idea is to calculate the virtual SINRs by using a fixed, constant load value $\overset{*}{\boldsymbol{x}}$ and the power of the considered SC plus an Offset only when it is in the nominator of (3.1). and build the association $\boldsymbol{K}$ by choosing the highest virtual SINR UE-Node pairs. In each cycle of the algorithm starting from the initial offset values of SCs ($\overset{*}{\boldsymbol{o}}$), we change one Offset value for one specific SC, compute the virtual SINRs and build $\boldsymbol{K}$. After building $\boldsymbol{K}$, we apply α rounds of fixed point iterations to create a total load point table entry by using a fixed load value $\overset{*}{\boldsymbol{x}}$ for each cell as the initial load values and $\boldsymbol{K}$. Following this we fill the table with each possible single Offset value change for each SC. After that we choose the minimum total load in the table and update $\boldsymbol{o}$. We repeat this procedure until $\boldsymbol{o}$ and the total load stay fixed for β rounds.

Explanation line by line:

- (Line 2-18) We build up the load matrix where we change one offset value each iteration to create an association matrix $\boldsymbol{K}$ and iterate the load with the load coupling formula for α times using that $\boldsymbol{K}$. Here we don't iterate until the fixed point to save up computational time. We will compensate for it later.

- (Line 4) We choose an offset value j from the predefined matrix $\boldsymbol{OS}$

for SC_i.

- (Line 5) We use this offset value to compute virtual SINRs with dummy load values to decide how to form the association $\boldsymbol{K}$ by choosing the highest SINRs.

- (Line 6-10) Building the association $\boldsymbol{K}$ using maximum selection from the virtual SINRs.

- (Line 15) Here we define the elements of the matrix, which are the total loads for different associations.

- (Line 19) We get the indexes of the smallest element in the matrix.

- (Line 20) We get the offset values which gave the best performance in terms of total load.

- (Line 21) and update the offset vector to be used as the initial offsets in the next round. This will allow us to optimize the association with a greedy method, improving every round.

- (Line 22-24) The control procedure to decide whether to increase the counter which triggers the end rule or not. If there is no improvement with respect to the last round, then the counter increases by one.

- (Line 25) Updating the flag to reuse in the next round when we check the end rule.

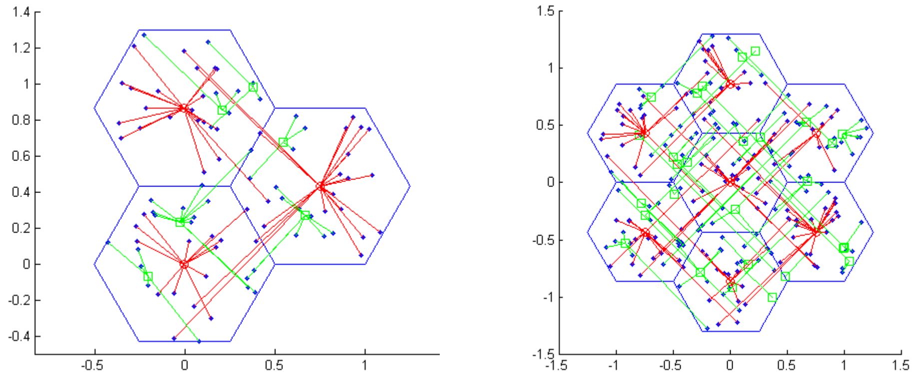
- (Line 1-26) Is the main body of the algorithm. One cycle of this piece of code is defined as one round and each round we change one association towards a lower total load.

- (Line 28) We need this repeat to build the optimal association using the optimal offsets. Because at the end of the main body of the algorithm, we just have the offsets but not the association.

- (Line 29) Repeats the last step from the first algorithm to get the fixed point of the load coupling formula.

4.4 Offset Joint Transmission

In this section to build up a joint association we divide the associations into two parts. For the first part, to keep the realistic aspect of the approach,



(a) 3 MCs and 2 SCs each

(b) 7 MCs and 4 SCs each

Figure 4.5: Non Joint Transmission MinSumL with Cell Specific Offsets

we use the resulting association $\boldsymbol{K}^{No}$ from the ONJT section. For the second part we use the same base algorithm that is used in the ONJT section to generate an association $\boldsymbol{K}^{JT}$ by the usage of the Offset values and superposition it over $\boldsymbol{K}^{No}$ to obtain a Joint association.

Algorithm 1: MinSumL - Non Joint Transmission

Input: $\overset{*}{\boldsymbol{K}}$, $\overset{*}{\boldsymbol{x}} = \mathbf{1}$, $C = 0$

Output: $\overset{*}{\boldsymbol{K}}$, $\boldsymbol{x}$

1 **while** $C < \beta$ **do**

2   **for** $\overset{*}{i} \leftarrow 1$ **to** n **do**

3   **for** $\overset{*}{j} \leftarrow 1$ **to** m **do**

4             $\boldsymbol{K} \leftarrow \overset{*}{\boldsymbol{K}}$;

5   **for** $\forall i$ **do**

6                 $K_{i\overset{*}{j}} \leftarrow 0$;

7   **end**

8             $K_{\overset{*}{i}\overset{*}{j}} \leftarrow 1$;

9             $\boldsymbol{x}^{(0)} \leftarrow \overset{*}{\boldsymbol{x}}$;

10   **for** $t \leftarrow 1$ **to** α **do**

11                 $\boldsymbol{x}^{(t)} \leftarrow \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}^{(t-1)}, \boldsymbol{K}))$;

12   **end**

13             $S_{ij} \leftarrow \sum_{ii}^{n} x_{ii}$;

14   **end**

15   **end**

16   **return** *The Row* $(\overset{*}{i})$ *and Column* $(\overset{*}{j})$ *Index of* $\min(\boldsymbol{S})$

17     $\boldsymbol{K} \leftarrow \overset{*}{\boldsymbol{K}}$;

18   **return** $\boldsymbol{K}$ *by repeating the same procedure done from Line 5 to Line 8*

19   **if** $\overset{*}{S} = S_{min}$ **then**

20         $C \leftarrow C + 1$;

21   **end**

22     $\overset{*}{S} \leftarrow S_{min}$;

23     $\overset{*}{\boldsymbol{K}} \leftarrow \boldsymbol{K}$;

24   **return** $\overset{*}{\boldsymbol{K}}$

25 **end**

26 $\boldsymbol{x}^{(0)} \leftarrow \overset{*}{\boldsymbol{x}}$;

27 $t \leftarrow 1$;

28 **while** $\boldsymbol{x}^{(t)} \neq \boldsymbol{x}^{(t-1)}$ **do**

29     $\boldsymbol{x}^{(t)} \leftarrow \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}^{(t-1)}, \overset{*}{\boldsymbol{K}}))$;

30     $t \leftarrow t + 1$;

31 **end**

32 **return** $\boldsymbol{x}$

Algorithm 2: MinSumL - Joint Transmission

Input: $\overset{*}{\boldsymbol{K}}$, $\overset{*}{\boldsymbol{x}} = \mathbf{1}$, $C = 0$

Output: $\overset{*}{\boldsymbol{K}}$, $\boldsymbol{x}$

while $C < \beta$ **do**

  for $\overset{*}{i} \leftarrow 1$ **to** n **do**

    for $\overset{*}{j} \leftarrow 1$ **to** m **do**

      $\boldsymbol{K} \leftarrow \overset{*}{\boldsymbol{K}}$;

      $K_{\overset{*}{i}\overset{*}{j}} \leftarrow 1$;

      $\boldsymbol{x}^{(0)} \leftarrow \overset{*}{\boldsymbol{x}}$;

      for $t \leftarrow 1$ **to** α **do**

        $\boldsymbol{x}^{(t)} \leftarrow \dot{\boldsymbol{f}}(\dot{\boldsymbol{h}}(\boldsymbol{x}^{(t-1)}, \boldsymbol{K}))$;

      end

      $S_{ij} \leftarrow \sum_{ii}^{n} x_{ii}$;

    end

  end

  return *The Row* $(\overset{*}{i})$ *and Column* $(\overset{*}{j})$ *Index of* $\min(\boldsymbol{S})$

  $\boldsymbol{K} \leftarrow \overset{*}{\boldsymbol{K}}$;

  $K_{\overset{*}{i}\overset{*}{j}} \leftarrow 1$;

  if $\overset{*}{S} = S_{min}$ **then**

    $C \leftarrow C + 1$;

  end

  $\overset{*}{S} \leftarrow S_{min}$;

  $\overset{*}{\boldsymbol{K}} \leftarrow \boldsymbol{K}$;

  return $\overset{*}{\boldsymbol{K}}$

end

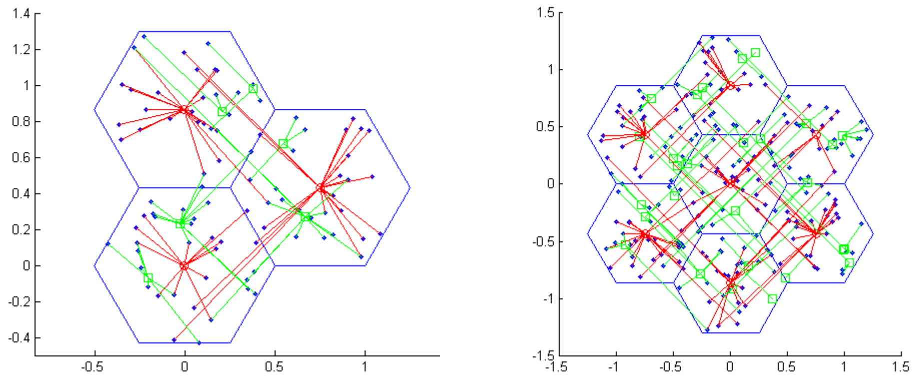
repeat the same procedure in Algorithm 1. Line 27 to Line 32;

return $\boldsymbol{x}$

Algorithm 3: MinSumL - Non Joint Transmission with Cell Specific Offsets

Input: $\overset{*}{\boldsymbol{x}} = \mathbf{1}$, $C = 0$, $\overset{*}{\boldsymbol{o}} = \mathbf{1}$, $\boldsymbol{OS}$
Output: $\boldsymbol{K}^{No}$, $\boldsymbol{x}$, $\boldsymbol{o}$

1 **while** $C < \beta$ **do**
2   **for** $i \leftarrow 1$ **to** n **do**
3   **for** $j \leftarrow 1$ **to** z **do**
4             $o_i \leftarrow os_{ij}$;
5             $\boldsymbol{\Gamma}^o \leftarrow \boldsymbol{H}^o(\overset{*}{\boldsymbol{x}}, \boldsymbol{o})$;
6             $\boldsymbol{K} \leftarrow \mathbf{0}$;
7   **for** $\overset{*}{j} \leftarrow 1$ **to** m **do**
8   **return** $\overset{*}{i}$ *of* $\max \gamma^o_{\overset{*}{i}\overset{*}{j}}$
9                 $K_{\overset{*}{i}\overset{*}{j}} \leftarrow 1$;
10   **end**
11             $\boldsymbol{x}^{(0)} \leftarrow \overset{*}{\boldsymbol{x}}$;
12   **for** $t \leftarrow 1$ **to** α **do**
13                 $\boldsymbol{x}^{(t)} \leftarrow \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}^{(t-1)}, \boldsymbol{K}))$;
14   **end**
15             $S_{ij} \leftarrow \sum_{ii}^{n} x_{ii}$;
16             $\boldsymbol{o} \leftarrow \overset{*}{\boldsymbol{o}}$;
17   **end**
18   **end**
19   **return** *The Row ($\overset{=}{i}$) and Column ($\overset{=}{j}$) Index of* $\min(\boldsymbol{S})$
20     $o_{\overset{=}{i}} \leftarrow os_{\overset{=}{i}\overset{=}{j}}$;
21     $\overset{*}{\boldsymbol{o}} \leftarrow \boldsymbol{o}$;
22   **if** $\overset{*}{S} = S_{min}$ **then**
23         $C \leftarrow C + 1$;
24   **end**
25     $\overset{*}{S} \leftarrow S_{min}$;
26 **end**
27 **return** $\boldsymbol{o}$
28 **return** $\boldsymbol{K}^{No}$ *by repeating the same procedure from Line 5 to Line 10*
29 **repeat** the same procedure in Algorithm 1. Line 27 to 32 with $\boldsymbol{K}^{No}$;
30 **return** $\boldsymbol{x}$



(a) 3 MCs and 2 SCs each

(b) 7 MCs and 4 SCs each

Figure 4.6: Joint Transmission MinSumL with Cell Specific Offsets

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Algorithm 4: MinSumL - Joint Transmission with Cell Specific Offsets

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Input: $\overset{*}{\boldsymbol{K}} = \boldsymbol{K}^{No}$, $\overset{*}{\boldsymbol{x}} = \mathbf{1}$, $C = 0$, $\overset{*}{\boldsymbol{o}} = \mathbf{1}$, $\boldsymbol{OS}$

Output: $\boldsymbol{K}^{JT}$, $\boldsymbol{x}$, $\boldsymbol{o}$

1 The algorithm follows the same procedure of Algorithm 3 with the modification of the update rule of $\boldsymbol{K}$ at each round in Line 5 and Line 28 from $\boldsymbol{K} \leftarrow \mathbf{0}$ to $\boldsymbol{K} \leftarrow \overset{*}{\boldsymbol{K}}$;

Chapter 5

Numerical Results, Conclusion

5.1 Numerical Results

In this section we will provide numerical results to the problems that are mentioned and solved by the proposed algorithms in the previous sections. We will also support the figures with some comments to let the reader grasp a better understanding of the case.

5.1.1 Three Macro Cells and Two Small Cells Each

In Figure 5.1, the results calculated by the above proposed algorithms for the problem MinSumL are given. What we see at the first glance is that the difference in total load is more evident if the cells are under heavier load. If the cells are lightly loaded, there is little to no improvement.

As expected JT is out performing all the other scenarios. However if the cell selection mechanism is subjected to Cell Specific Offsets, there is a considerable performance reduction. This reduction is based on several parameters, such as; How densely SCs are spread over the network, The power difference between SCs and MCs, etc.. To have a better performing Offset MinSumL, we would need more SCs deployed.

In Figure 5.2a and Figure 5.2b we give more detailed information about how the different scenarios perform. For NJT MinSumL, SCs are getting significant improvements over their total loads, while MCs seems to have no improvement, even tend to worsen. This is because in NJT scenario to reduce the total load MCs and SCs exchange UEs, where MCs get more UEs because of sparsely populated SCs. For JT MinSumL, there occurs a similar situation, where SCs get a much bigger improvement over MCs. This is because to minimize the total load MCs expand and serve some of the UEs using JT which were originally served by SCs. This causes these SCs to lose a high amount of load.

Another important result revealed by Figure 5.2a and Figure 5.2b is how the Offset versions of the above mentioned scenarios perform.

While the original versions of NJT MinSumL and JT MinSumL tend to reduce the loads of SCs. Offset NJT MinSumL and OJT MinSumL do the contrary and reduce the load of MCs. That happens because by implementing offsets to SCs, we expand their range and force them to serve more UEs. As a result, the algorithms minimized the total load by forcing SCs to serve more UEs. This is an important result as we get an improved total load even while there are only two randomly placed SCs in each cell.

Here in Table 5.1 we also compare the resulting total loads of the above presented algorithms with the total load of maximum SINR association. For the sake of simplicity, only the scenarios where each user has a 550 Kpbs and 200 Kpbs demand levels are printed.

Demand (Kbps)	NJT	JT	NJT Offset	JT Offset
550	14.16%	28.74%	2.04%	7.10%
200	5.40%	7.32%	0.37%	0.53%

Table 5.1: The improvement percentage of NJT, JT, NJT Offset and JToffset vs The Maximum SINR Association (3 MCs and 2 randomly placed SCs each)

We should also keep in mind that these results depend on the nature of the algorithm that is used. The initial associations used in the beginning of the algorithm has strong effects over the results, especially for JT which is built over the initial association.

5.1.2 Seven Macro Cells and Two Small Cells Each

In Figure 5.3, again the performance of the scenarios seems to follow the same route with some differences from the previous scenario. Here Offset JT MinSumL and Non-JT MinSumL have the same performance while in the previous section Offset JT case had a worse performance than Non-JT case. Also, here Offset Non-JT case has a better performance than the previous section. Additionally, the gap between the Maximum Selection and JT associations has grown bigger favoring the JT cases.

One important thing to mention is that, the dotted red line is a bound representing that the total load is above the total number of MCs deployed

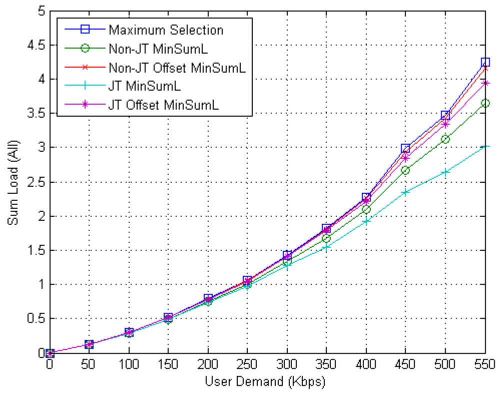
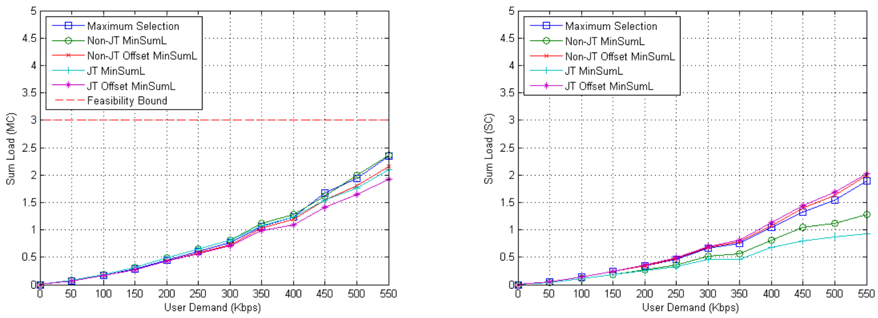


Figure 5.1: Total Loads of all the cells calculated by MinSumL for various Demand Levels (3 MCs and 2 SCs each)



(a) MCs *(b) SCs*

Figure 5.2: Total Loads of MCs and SCs calculated by MinSumL for various Demand Levels (3 MCs and 2 SCs each)

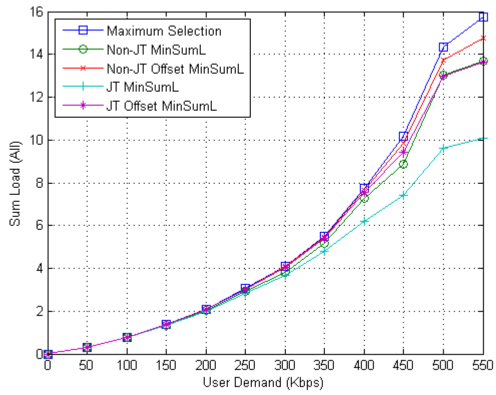


Figure 5.3: Total Loads of all the cells calculated by MinSumL for various Demand Levels (7 MCs and 2 randomly placed SCs each)

in the network and any point above that limit guarantees that some MC has its load larger than one and this implies that the current association is infeasible.

Here we see that above 450 Kbps of demand per user, only JT and JT Offset cases are feasible.

Demand (Kbps)	NJT	JT	NJT Offset	JT Offset
550	12.68%	35.12%	8.19%	11.57%
200	3.50%	5.04%	0.27%	0.21%

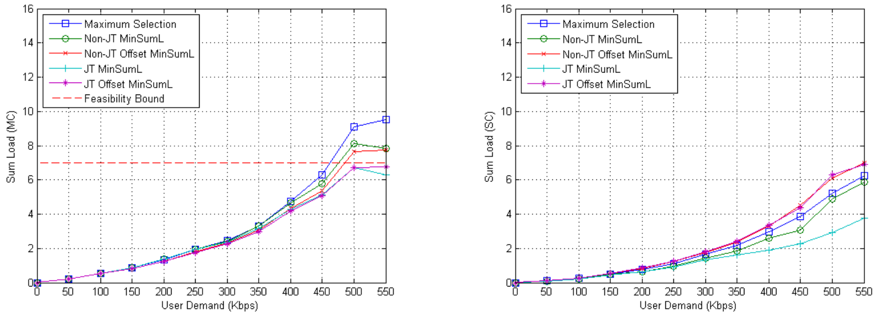
Table 5.2: The improvement percentage of NJT, JT, NJT Offset and JToffset vs The Maximum SINR Association (7 MCs and 2 randomly placed SCs each)

From Table 5.2, it is clear that there is a higher ratio of improvement when there are more cells in the area with the same amount of SCs each cell area.

5.1.3 Seven Macro Cells and Four Small Cells Each

What immediately seen at the first glance from Figure 5.5 is that the total load is significantly reduced after two additional SCs deployed in each cell with respect to the previous section.

From Figure 5.6a we see that the total load of MCs is greatly reduced and now there is no infeasible association due to MC overload, load of each MC is almost reduced to half. While the total load of SCs didn't change as much. However, we should keep in mind that in this scenario the number



(a) MCs *(b) SCs*

Figure 5.4: Total Loads of MCs and SCs calculated by MinSumL for various Demand Levels (7 MCs and 2 randomly placed SCs each)

of SCs deployed is doubled, which in turn means that the load of each SC approximately reduced to half too.

Consequently, we can make the following comment: deploying SCs greatly eases the load on MCs.

Demand (Kbps)	NJT	JT	NJT Offset	JT Offset
550	9.15%	16.03%	5.40%	8.35%
200	2.08%	2.47%	1.22%	1.22%

Table 5.3: The improvement percentage of NJT, JT, NJT Offset and JToffset vs The Maximum SINR Association (7 MCs and 4 randomly placed SCs each)

Furthermore, Figures 5.2b, 5.4b and 5.6b suggest that the offset cases always expand SCs and make them serve more users thus easing the load over MCs.

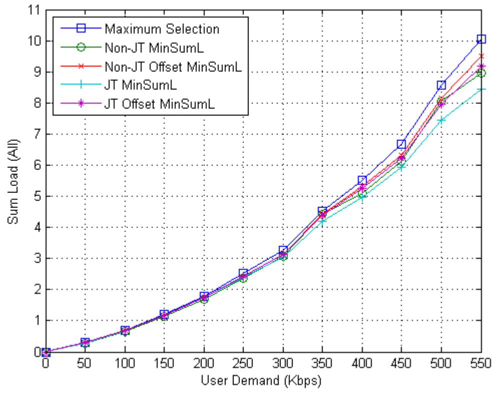
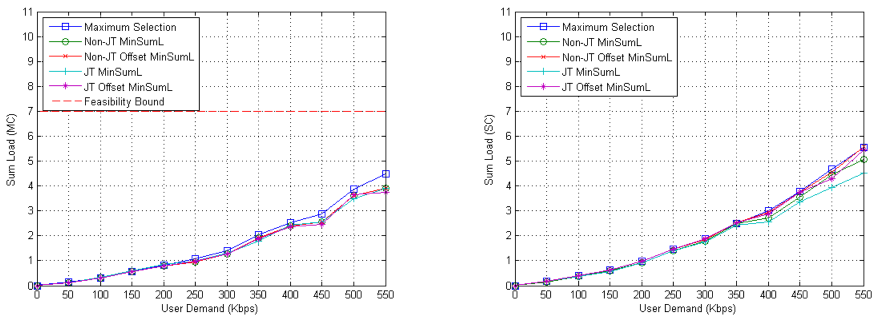


Figure 5.5: Total Loads of all the cells calculated by MinSumL for various Demand Levels (7 MCs and 4 randomly placed SCs each)



(a) MCs (b) SCs

Figure 5.6: Total Loads of MCs and SCs calculated by MinSumL for various Demand Levels (7 MCs and 4 randomly placed SCs each)

5.2 Conclusion

First of all because of having a NP-Hard problem at hand, the results we obtain suffer from being stuck in local optimas. To obtain more promising results for comparison, we run several instances with random UE, Node placement, random noise and take the average of the results (Monte-Carlo method). Further information about this technique is in Appendice B.

What we see from these results is that total load optimization is improving the total load with respect to the maximum SINR association regardless of the specific optimization scenario even if at first sight associating with the maximum SINRs seems to be the best option.

Moreover as an expected result, the Free Association is giving better results on average in terms of MinSumL over Offset Association. This is because of having a less constrained search space.

On the other hand as the main comparison objective of this project, JT versions of the above mentioned scenarios are always giving better results regardless of being stuck in local optimas, respectively over their NJT synonyms.

Offset cases are better choices when our main objective is to ease the load on MCs, additionally their performance is also considerably good and these results improve further when we deploy more SCs in the network.

Another important result is that the initial association is directly in relation with the heuristic algorithms' performance. If we start the optimization process with a worse association in terms of total load, the resulting association is generally inferior to the latter one.

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Appendix A

COST-231 Path Loss Model

COST-231 model (also known as Hata Model PCS) is a commonly used path loss model designed to be used on several Europe terrains. It is an extension of the Hata model. While Hata's model is for frequencies up to 900 MHz, COST-231 model is designed for frequencies between 1500-2000 MHz where most of the modern wireless systems are operating in.

It is given as,

$$Lp_{dB} = A + Blog_{10}(d) + C \tag{A.1}$$

Where,

- $A = 46.3 + 33.9log_{10}(fc) - 13.28log_{10}(hb) - a(hm)$
- $B = 44.9 - 6.55log_{10}(hb)$
- $C = 0$: For medium city and suburban areas or,
- $C = 3$: For metropolitan areas
- $fc = 1500 - 2000$ MHz: Carrier frequency
- $hm = 1 - 10$ m: UE antenna height
- $hb = 30 - 200$ m: BS antenna height
- $d = 1 - 20$ km: Transmission distance

Appendix B

Monte-Carlo Method

Monte Carlo is one of the strongest and most widely used numerical methods. It has a convergence rate of $O(N^{-1/2})$. This cost is independent of dimension which makes it very robust and easy to implement in a variety of scenarios but also makes it slow. The essential idea is to model the deterministic problems with a statistical approach. Without casting further confusion, for a uniformly distributed random variable x on one dimension, we give from [7],

$$I[f] = \int_0^1 f(x)dx = \bar{f} \tag{B.1}$$

and,

$$I[f] = E[f(x)] \tag{B.2}$$

If we consider that, here with the computers we generate non-continuous results. For a sampled sequence x_n from the above uniform distribution,

$$I_N[f] = \frac{1}{N}\sum_{n=1}^{N} f(x_n) \tag{B.3}$$

and from here,

$$\lim_{N\to\infty} I_N[f] \to I[f] \tag{B.4}$$

this computation is unbiased as x is uniformly distributed,

$$E[I_N[f]] = I[f] \tag{B.5}$$

but in general we define the error as,

$$e_N[f] = I[f] - I_N[f] \tag{B.6}$$

an the bias is defined as $E[e_N[f]]$.

In the thesis we implemented the Monte Carlo method in a quite simple manner by calculating the result for a specific scenario with a random noise and randomly placed UEs for several rounds while re-randomizing the noise and the UE locations at each round and taking the mean of the results in the end. This way we tried to prevent any wrong result that may occur because of the randomness of the scenario (an UE gets too favored because of the constructive contribution of the noise or a node is overloaded because of an unwanted traffic hotspot).