Performance Analysis of Intensity Modulated PAM-4 with Pre-compensation Filter and Direct Detection in Short Reach Optical Communication

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List of Acronyms

ASE Amplified Spontaneous Emission.

AWGN Additive White Gaussian Noise.

BER Bit Error Rate.

BPS Blockwise Phase Switching.

CD Chromatic Dispersion.

CSPR Carrier-to-Signal Power Ratio.

DAC Digital to Analog Converter.

DD Direct Detection.

DP-SCI Dual Polarization SCI.

DSB Double Side Band.

EVM Error Vector Magnitude.

IDFT Inverse Discrete Fourier Transform.

IFFT Inverse Fast Fourier Transform.

IFT Inverse Fourier Transform.

IM Intensity Modulation.

ISI Inter Symbol Interference.

LTI Linear Time Invariant.
NRZ  Non Return to Zero.

OFDM  Orthogonal Frequency Division Multiplexing.

OOK  On-Off Keying.

PAM  Pulse Amplitude Modulation.

PBS  Polarization Beam Splitter.

PDF  Probability Density Function.

RC  Raised Cosine.

SCI  Signal Carrier Interleaved.

SDR  Signal-to-Distortion Ratio.

SMF  Single Mode Fiber.

SNR  Signal-to-Noise Ratio.

SOP  State Of Polarization.

SRRC  Square Root Raised Cosine.

SSB  Single Side Band.

SSBI  Signal-to-Signal Beating Interference.
Sommario

La trasmissione su fibra ottica a rate 100 Gbit/s con ricezione coerente è una tecnologia commerciale ormai consolidata, che permette la trasmissione su collegamenti in fibra fino a distanze di migliaia di chilometri. Nei collegamenti a medio raggio, fino agli 80 km, il costo di questa tecnologia è troppo alto, poiché prevede l’utilizzo di un laser in ricezione, oltre alla presenza di un ibrido ottico e di quattro fotodiodi per polarizzazione. Il vantaggio dei sistemi coerenti è la possibilità di equalizzare al ricevitore tutte le distorsioni lineari subite dal segnale durante la trasmissione, in particolare la dispersione cromatica.

Recentemente, sono state state proposte diverse soluzioni a bassa complessità per ridurre i costi. In particolare, possiamo distinguere tra due principali tipi di soluzioni: i sistemi self-coherent e quelli non coerenti.

I sistemi self-coherent prevedono la trasmissione della portante insieme con il segnale di informazione. Il segnale ottico trasmesso deve essere a banda laterale singola. In questo modo, dopo il fotodiodo al ricevitore, il battimento tra il segnale e la portante non risente della dispersione cromatica.

La maggior parte dei sistemi self-coherent utilizza l’OFDM come schema di modulazione, anche se esistono esempi di sistemi self-coherent con modulazione a singola portante. Nel lavoro di tesi questo tipo di sistemi è introdotto senza specificare un particolare schema di modulazione.

I sistemi non coerenti sono ancora più semplici e prevedono la trasmissione di diversi valori di intensità della luce laser (modulazione di intensità). Dato che il ricevitore è costituito solo da un fotodiodo, il segnale elettrico generato è proporzionale al quadrato dell’ampiezza del campo ottico. La cascata tra trasmettitore, canale e ricevitore non è perciò lineare e non è più possibile equalizzare la dispersione cromatica.

Il contributo di questo lavoro di tesi è la valutazione delle prestazioni di un sistema con modulazione di intensità e ricezione coerente, che per tenere conto della dispersione
cromatica, implementa un filtro di pre-compensazione al trasmettitore. Esso utilizza uno schema di modulazione PAM-4 (Pulse Amplitude Modulation). Il filtro è generato con dei parametri nominali, tipici della banda intorno ai 1550nm (C-band) in una fibra a singolo modulo (Single Mode Fiber). Il peggioramento delle prestazioni è introdotto tramite una non perfetta corrispondenza tra questo filtro di pre-compensazione e la dispersione cromatica reale introdotta dalla trasmissione in fibra. Le prestazioni sono misurate usando come parametro principale la Bit Error Rate (BER) e cercando di trovare una sua stima tramite l'Error Vector Magnitude (EVM) misurato. Questo sistema è simulato sia con un filtro di trasmissione rettangolare sia con un filtro di trasmissione di Nyquist con banda ridotta.

La valutazione delle prestazioni serve a comprendere quanto vale il non allineamento tra il filtro di pre-compensazione e la dispersione cromatica reale. Considerando che il canale è costante nel tempo o comunque varia lentamente, l'entità del non allineamento può essere comunicata al trasmettitore tramite un canale di ritorno (non considerato in questo lavoro), permettendo perciò una pre-compensazione migliore modificando i parametri del filtro.

Dopo l'introduzione svolta nel Capitolo 1, le basi della trasmissione in fibra sono introdotte nel Capitolo 2, in particolare i tipi di ricevitore e l'origine della dispersione cromatica. Nel Capitolo 3 sono spiegati i concetti principali delle comunicazioni digitali, mentre il Capitolo 4 parla del nostro sistema PAM-4 e di alcuni esempi di trasmissione di tipo self-coherent. Nel Capitolo 5 vengono illustrati i tipi di analisi delle prestazioni e infine sono mostrati e spiegati tutti i risultati ottenuti.
Abstract

The fiber optic transmission with rate 100 Gbit/s with coherent reception is a commercially available and consolidate technology, since it allows transmission on fiber links up to distance of thousands of kilometers. In a medium reach link, up to 80 km, the cost of such a technology is too high, since it implies the use of a laser at the receiver, in addition to an optical hybrid and four photo-diodes per polarization. The advantage of coherent systems is the possibility to equalize all the linear distortions undergone by the signal during transmission at the receiver, in particular for the Chromatic Dispersion (CD).

Recently, many low complexity techniques have been proposed to reduce the costs. In particular, we can distinguish between two main types of solutions: self-coherent systems and non coherent one.

The self-coherent systems implement the transmission of the carrier along with the information signal. The transmitted optical signal must be Single Side Band (SSB). In this way, after going through the photo-detector at the receiver, the beating between signal and carrier is not affected by Chromatic Dispersion.

The biggest part of self-coherent systems uses OFDM as modulation scheme, but there are also examples of single carrier self-coherent systems. This work introduces self-coherent systems without specifying a particular modulation scheme.

Non coherent systems are even easier and exploit the transmission of different values of the laser light intensity (Intensity Modulation). Since the receiver consists only of a photo-diode, the generated electrical signal is proportional to the square of the amplitude of the optical field. The cascade between transmitter, channel and receiver is therefore non-linear and it is not possible to equalize CD anymore.

The contribute of this work is the evaluation of the performance of a system with Intensity Modulation (IM) and Direct Detection (DD) techniques, that implements a pre-compensation filter to face with Chromatic Dispersion. It uses a PAM-4 (Pulse
Amplitude Modulation) modulation scheme. This filter is built with nominal parameters, typical of the band around 1550nm (C-band) in a Single Mode Fiber (SMF). The degradation of the performance is introduced with the misalignment between this pre-compensation filter and the real CD introduced by the fiber transmission. Performance is measured using the Bit Error Rate (BER) as main parameter and trying to find an estimate of it by means of the measured Error Vector Magnitude (EVM). This system is simulated with both a rectangular transmission filter and a Nyquist filter with reduced bandwidth. The evaluation of the performance needs to understand how much the misalignment between pre-compensation filter and actual CD is. Considering that the channel is time invariant, or anyway it varies slowly, the entity of the misalignment can be communicated at the transmitter by means of a feedback channel (not considered in this work), therefore allowing a better pre-compensation with the modification of the parameters of the filter.

After introduction in Chapter 1, in Chapter 2 the fiber transmissions are introduced, in particular the types of receiver and the origin of Chromatic Dispersion. In Chapter 3 the main concepts of digital communications are explained, while Chapter 4 deals with our PAM-4 system and many self-coherent examples. In Chapter 5 the types of performance analysis are illustrated and finally all the obtained results are shown.
Chapter 1

Introduction

Fiber communications are the basis of the modern Internet. The entire transport network is composed by optical fiber links, with distances that can reach thousands of kilometers. This network allows the transferring of an enormous amount of data and the demand for bandwidth is constantly increasing, due to the success of mobile communications, HD video streaming and cloud services.

Nowadays, the technology used for the transport network is the coherent transmission at 100 Gbit/s per channel. It uses a Polarization Division Multiplexing (PDM) with Quadrature Phase Shift Keying (QPSK) modulation with a coherent receiver [1]. The coherent receiver is used to recover the entire field information and it allows, differently from the Direct Detection technique, to equalize every linear impairments. This is because every Digital Signal Processing algorithm can be applied after the coherent receiver, having access to the complete baseband signal [2]. The coherent receiver is equipped with a laser, called Local Oscillator, that should have the same wavelength of the laser used at the transmitter as optical carrier. The local oscillator beam and the information beam are the inputs of a device called Optical Hybrid that makes them interfere in order to recover the real and imaginary part of the optical field. The 100G coherent technology is mature and widely adopted for the long haul links, where high costs and energy consumption of the coherent technology are justified by the length of the fiber link.

At the opposite, when the distance decreases under 80 km, the high cost of a coherent receiver can be excessive [3]. This is the case for the short and medium reach links, present in the metro and access networks and also in short intra or inter data-center connections (also called client optics) [4]. The increasing bandwidth demand makes
these links the bottleneck of the network [5]. Therefore the switch from conventional media to fiber optics technology is necessary to satisfy this demand. Medium and short reach communication must respect some factors:

- High spectral efficiency;
- Low power consumption;
- Low costs and small components size;
- Low complexity.

Many studies have been done in the recent years and many examples can be found in literature that try to reach the 100 Gbit/s data rate while satisfying the previous requirements.

In particular, the proposed solutions can be distinguished in single-carrier Intensity Modulation (IM) with Direct Detection (DD) and self-coherent system, often implementing Orthogonal Frequency Division Multiplexing (OFDM).

The first type of solutions uses the non coherent detection, introducing high order modulation [4] to increase the total bit rate using the same bandwidth. Although a simple approach is the multiplexing of a number of wavelengths, everyone of them carrying a fraction of the total capacity [6], a better solution is to create a single wavelength 100 G channel to reduce components costs [7]. One of the main cost factor is the bandwidth of the used devices, so the reduction of bandwidth occupation is another important aspect [8], [9]. Intensity Modulation is sensitive to Chromatic Dispersion (CD), that is limiting the length of an uncompensated fiber link. So to make these systems working on longer link, the problem of CD must be addressed.

The self-coherent solution is another way to face the problem of cost reduction. They still use a non coherent receiver, but they look at the beating of the signal with the carrier, that is added at the transmitter [10].

The most studied modulation scheme for these systems is the multi-carrier OFDM [11], that allows a simpler equalization in the frequency domain and that is more flexible in power and bit allocation. Furthermore, it is easier to take care of CD with a self-coherent system, just using an optical Single Side Band (SSB) signal that does not undergo power fading in frequency domain [12].
In this work we firstly give a general view of the self-coherent transmission, presenting many examples of them.

Then we focus our attention on a simpler non coherent system. In particular, the PAM-4 single carrier IM systems with DD are used, with different filtering of the spectrum: the rectangular filter and a Nyquist FIR filter with equi-ripple characteristic.

The problem of the Chromatic Dispersion is faced with the introduction of a pre-compensation filter at the transmitter. This allows to maintain a very simple receiver. The difference between CD parameter used in this filter and the actual CD introduced by the channel creates a degradation of the performance that is studied in this work, trying to find a relation between Bit Error Rate (BER) and mismatch of pre-compensation filter.

This information can be sent back to the transmitter in order to perform a better pre-equalization (this will not be treated in this thesis).

Publication

The study and the results carried out in this work were summarized in the conference paper: A.Viganò, M.Magarini and A.Spavileri, "Performance Analysis in a PAM-4 Fiber Transmission IM-DD with Pre-compensation Filter", in Proc. European Conference on Computer Science, pp. 1-6, Rome, Italy, 21-23 October 2016.
Chapter 2

Optical Channel

In this chapter the fiber transmission channel is explained by focusing on how the light field propagates in an optical fiber. Chromatic Dispersion (CD) and its effect on the received signal are described. The theory is derived by [13].

2.1 Introduction

As a generic communication system, an optical transmission system has the goal of reliably transmitting information, i.e. a stream of bits, from the transmitter to the receiver. A generic scheme is represented in figure (2.1).

A general transmitter consists of a laser modulated with a particular modulation scheme. In particular there is an electrical stage before acting on the light emitted by the laser. The electrical part consists of taking the bit stream and mapping it in a sequence of symbols extracted from a certain constellation. The constellation has a number of symbols that represents the order of the modulation, indicated with $M$, and that

![Figure 2.1: Generic scheme of an optical transmission system](image)
depends on the number $k$ of consecutive bits mapped on a single symbol, as indicated in (2.1)

$$k = \log_2(M). \tag{2.1}$$

This sequence of symbols is then converted in an analog waveform with a Digital to Analog Converter (DAC). The waveform modulates the laser output, by means of an electro-optical modulator. The laser output is a light beam with a certain frequency $f_0$ and wavelength $\lambda_0$, that are linked by

$$c = f_0\lambda_0. \tag{2.2}$$

There’s no existing laser with a perfect monochromatic spectrum, but lasers are the best coherent light sources anyway, so the emitted light is considered to have a single frequency.

2.2 Receiver

There are two types of receiver for the optical transmission systems and they are made possible by the photo-diode, i.e. the device capable of transforming the light in an electrical current.

The two types are:

- Non Coherent receiver
- Coherent receiver

In this work, a compromise between these two conventional receivers is also studied, i.e. a solution that stays in between complexity and cost effectiveness: the self-coherent system.

2.2.1 Non Coherent Receiver

A non coherent receiver is composed only by the photo-detector (or photo-diode). The photo-diode transforms the optical signal in an electrical current, that is proportional to the intensity of the optical field impinging on the surface of the photo-diode. This technique is also called Direct Detection (DD)

$$i(t) = |e(t)|^2. \tag{2.3}$$
In order to reduce the noise produced by the optical amplifiers, called Amplified Spontaneous Emission (ASE noise), an optical filter is often put before the photo-detector. From equation (2.3) we can understand how the receiver cannot recover phase information of the optical field. With a non coherent receiver, only Intensity Modulation (IM) is possible, e.g. the Non Return to Zero - On Off Keying (NRZ OOK) [14]. This implies a reduction in spectral efficiency due to the impossibility to use complex modulation schemes. At a fixed rate, we cannot take advantage of both modulus and phase of the received symbols using DD, so the spectral efficiency is halved.

2.2.2 Coherent Receiver

A coherent receiver can recover the real and the imaginary part of the complex baseband modulation signal instead. This is done by adding a laser at the receiver, as shown in figure (2.2), that might be at the same wavelength of the transmission laser and in phase with it. The real part of the signal is recovered on one arm of the Optical Hybrid. This is done when the laser and the received signal are in phase. To cancel the additional terms, another photo-diode must be added. Its input is obtained adding to the carrier a 180° phase switch. Looking at the baseband equivalent we have that

\[ y_1(t) = |e(t) + c|^2 = |e(t)|^2 + c^2 + 2cR\{e(t)\} \]
\[ y_2(t) = |e(t) - c|^2 = |e(t)|^2 + c^2 - 2cR\{e(t)\}. \]
The output of the two photo-diodes is then subtracted in order to isolate only the real parts

\[ y_1(t) - y_2(t) = 4c\Re\{e(t)\}. \]  

(2.6)

On the other arm of the Optical Hybrid, the local laser, or the received signal as well, are delayed by \( \pi/2 \) before interfering with each other, so the imaginary part is recovered as

\[ y_3(t) = |e(t) - jc|^2 = |e(t)|^2 + c^2 + 2c\Im\{e(t)\} \]  

(2.7)

\[ y_4(t) = |e(t) +jc|^2 = |e(t)|^2 + c^2 - 2c\Im\{e(t)\}. \]  

(2.8)

As before, the two signals are subtracted after the detection

\[ y_3(t) - y_4(t) = 4c\Im\{e(t)\}. \]  

(2.9)

These operations are done by an integrated optic device called Optical 90° Hybrid.

More complexity is added if we also want to consider the possible State Of Polarization (SOP); in this case a Polarization Beam Splitter (PBS) is added before two Optical Hybrids, that operate as described above. The PBS projects the signal on two orthogonal polarizations, while the laser SOP is controlled via a Polarization Controller.

### 2.2.3 Self-Coherent

The self-coherent uses a classical non coherent receiver, but the carrier is added at the transmitter [15]. The transmitted signal is \( y(t) = e(t) + c \), so we are in the same case described by the first line of eq (2.5) at the receiver, as

\[ y(t) = |e(t) + c|^2 = |e(t)|^2 + c^2 + 2c\Re\{e(t)\} \]  

(2.10)

The term \( c^2 \) is a constant, the term \( |e(t)|^2 \) is called Signal to Signal Beating Interference (SSBI) and the remaining term is the signal we are interested in, called signal to carrier beating.

SSBI is an interfering term, since it cannot be removed optically; so it must be taken into account. The transmitted power is divided between signal and carrier: in particular, higher power in the carrier results in amplification of the signal to the carrier beating. Anyway, to avoid nonlinearities, a limited power must be concentrated in a single wavelength.
To represent the power distribution, a parameter is created: the Carrier to Signal Power Ratio (CSPR), defined as

\[ CSPR = \frac{c^2}{|e(t)|^2}. \tag{2.11} \]

Obviously, since the receiver extracts the real part of the signal, complex modulations cannot be used. The self-coherent transmissions are explained better in Chapter 4.

2.3 Fiber Propagation

The optical field \( e_{tx}(t) \) transmitted in a fiber suffers from the effects of the propagation. These act on its modulus and its phase, as we can see by looking at its Fourier Transform

\[ E_{rx}(f) = E_{tx}(f)10^{-\alpha d/20}e^{-j\beta(f)L} \tag{2.12} \]

where \( L \) is expressed in meters and represents the length of the fiber. The two parameters are:

- \( \alpha \) is an attenuation measured in [dB/m];
- \( \beta(f) \) is a phase variation evaluated in [rad/m].

2.3.1 Attenuation

The optical fiber bandwidth is commonly divided into bands defined in ITU-T series G supplement 39 [16], reported in Table (2.1). The low value of the field amplitude attenuation is one of the key elements for the success of the optical fiber technology with respect to the other transmission media. The value \( \alpha \) of the attenuation is expressed as the ratio between the input and the output power, in logarithmic units as

\[ \alpha = \frac{10}{L}\log_{10}\left(\frac{P_{in}}{P_{out}}\right) \text{ [dB/km]}. \tag{2.13} \]

The value of the attenuation varies for different wavelengths due to the different interactions of the light with the glass of the fiber. In the C-band, the attenuation reaches its lowest value, around 0.2 dB/km. The E-band and the O-band have respectively values for the attenuation around 0.4 dB/km and 3 – 4 dB/km.

A plot of the attenuation curve for the different wavelengths is reported in figure (2.3), along with three of the components (infrared absorption, ultraviolet absorption and Rayleigh scattering) that form the total attenuation.
<table>
<thead>
<tr>
<th>Band</th>
<th>Name</th>
<th>Range (nm)</th>
<th>Range (THz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-band</td>
<td>Original</td>
<td>1260-1360</td>
<td>237.9-220.4</td>
</tr>
<tr>
<td>E-band</td>
<td>Extended</td>
<td>1360-1460</td>
<td>220.4-205.3</td>
</tr>
<tr>
<td>S-band</td>
<td>Short wavelength</td>
<td>1460-1530</td>
<td>205.3-195.9</td>
</tr>
<tr>
<td>C-band</td>
<td>Conventional</td>
<td>1530-1565</td>
<td>195.9-191.6</td>
</tr>
<tr>
<td>L-band</td>
<td>Long wavelength</td>
<td>1565-1625</td>
<td>191.6-184.5</td>
</tr>
<tr>
<td>U-band</td>
<td>Ultra-long wavelength</td>
<td>1625-1675</td>
<td>184.5-179.0</td>
</tr>
</tbody>
</table>

Table 2.1: Optical fiber bands

### 2.3.2 Chromatic Dispersion

The phase variation $\beta(f)$ term indicated in (2.12) can be expanded in Taylor series as

$$\beta(f) = \beta_0 + \frac{d\beta}{df}\bigg|_{f=0} f + \frac{1}{2} \frac{d^2\beta}{df^2}\bigg|_{f=0} f^2,$$

where

- $\beta_0$ is a frequency independent phase shift;
- $\beta_1 = \frac{d\beta}{df}\bigg|_{f=0}$ is a linear phase shift term representing a delay in time, linked to the group velocity $v_g$ of signal (the speed of the information) as $v_g = \frac{2\pi}{\beta_1}$;
- $\beta_2 = \frac{1}{2} \frac{d^2\beta}{df^2}\bigg|_{f=0}$ is the change of the group velocity depending on the frequency.

So $\beta_2$ is the parameter responsible of the CD: each frequency component travels at different velocity.

The total value of CD is originated by three components of dispersion that depend on how the fiber is built:

- Waveguide Dispersion: the intrinsic dispersion induced by the propagation;
- Composite Material Dispersion: originated by the change of the refractive index of core and cladding at different wavelength;
- Composite Profile Dispersion: due to the refractive index profile of the fiber.
In the optical world the CD is commonly indicated as the function of the wavelength, so as

\[ D_\lambda = \frac{d(1/v_g)}{d\lambda}, \]  

(2.15)

and the relation between the two parameters is

\[ D_\lambda = -\frac{f_0^2 \beta_2}{2\pi c} \text{[ps/nm/km]}, \]  

(2.16)

In figure (2.4) the Waveguide dispersion and the Material dispersion of a SMF are reported, and also the total CD. The Profile dispersion has less impact with respect to the previous two. We can see that CD has the zero crossing around \( \lambda = 1310 \text{ nm} \).

The CD causes the broadening in time of a signal. The time expansion due to dispersion can be calculated as

\[ T \approx L \Delta \lambda D_\lambda, \]  

(2.17)

where \( \Delta \lambda \) is the wavelength largeness of the signal. Remembering that \( \Delta \lambda \approx \Delta f \frac{\lambda^2}{c} \) we can rewrite it as

\[ T \approx LD_\lambda \Delta f \frac{\lambda^2}{c}, \]  

(2.18)

where \( \Delta f \) is the bandwidth of the signal, \( \lambda \) is the central wavelength and \( c \) is the light speed.
A common value for the CD in a standard Single Mode Fiber (SMF) is $D_\lambda = 16.5 \text{ ps/nm/km}$ evaluated at the 1550 nm band (C-band) [17], while it is approximately zero in the O-band around 1310 nm.

It is possible now to indicate the CD transfer function of a fiber as

$$H_{cd}(f) = \exp(j \frac{1}{2} \beta_2 L f^2) = \exp(-j \frac{1}{2} \frac{\pi \lambda^2 D_\lambda L}{c} f^2)$$ (2.20)

The temporal broadening of a transmitted waveform due to CD causes a symbol to interfere with the adjacent ones, causing Inter Symbol Interference (ISI). This must be managed in order to recover the transmitted sequence correctly. There are many ways to do this. Obviously, the problem is resolved if we transmit in the O-band around the zero dispersion point. Anyway we have to use the C-band due to its lower attenuation value. Dispersion-shifted fibers can be used, that have zero CD in the C-band. However, they add several nonlinearities. Our solution is different and consists of digitally pre-filtering the baseband signal with the complex conjugate of (2.20) at the transmitter.
Chapter 3

Basics of Digital Transmission

The theory explained in this chapter is used as reference for all the work. The main source from which the arguments are derived is [15]. It is explained a general digital transmission system, especially how transmitter, channel and receiver are modeled.

3.1 Design Of The Transmitter

The purpose of a transmission system is to transfer informations from the transmitter to the receiver as more reliable as possible.

Using Fig. (3.1) as a reference, in a digital communication scheme we have to transmit a stream of bits \( b_i \) from a point to another. To do this, \( k \) bits are grouped to be mapped in one symbol extracted from a constellation of \( M \) complex symbols. This two parameters are linked as follow

\[
k = \log_2(M).
\]  

(3.1)

The constellation used from the modulator to maps bit in symbols is represented as a

![Diagram of a digital transmission system](Figure 3.1: Scheme of a digital transmission system)
symbol set
\[ A = \{a_1, a_2, a_3, \ldots a_M\}. \]  
(3.2)

These symbols have to be transmitted on the channel. Before this, they are filtered with the transmission filter to create the waveform. If we indicate with \(h_{tx}(t)\) the transmission filter, the transmitted waveform is

\[ s(t) = \sum_{n=-\infty}^{+\infty} a(n)h_{tx}(t - nT_s), \]  
(3.3)

where \(T_s\) is the symbol duration. Its inverse, \(R_s = 1/T_s\) is the symbol rate and it is measured in symbol/s or Baud (Bd). We can assume that the transmission filter has a limited time duration equal to \(T_s\), e.g. it is different from zero only in the interval \(-T_s < t < T_s\).

### 3.2 AWGN Channel Modeling

The symbols now travel along the channel, that can be represented as a Linear Time Invariant (LTI) filter with impulse response \(h_{ch}(t)\). This filter takes into account all the linear effects induced by the transmission media. In case of ideal channel, it is represented as a Dirac delta.

\[ h_{ch}(t) = \delta(t) = \begin{cases} 
1 & t = 0, \\
0 & t \neq 0.
\end{cases} \]  
(3.4)

Actually, an ideal channel cannot exist. At least, channel introduces a delay that can be represented as a delta translated by \(\tau\), where \(\tau\) indicates the time delay. A non ideal channel introduces distortion on the signal and makes symbols interfere with each others. This situation is represented by a channel filter length greater than the symbol duration. After the channel it is added the Additive White Gaussian Noise (AWGN), that is indicated with \(w(t)\).

### 3.3 Design Of The Receiver

After having gone through the channel, the signal is filtered by the receiver filter and then sampled every \(T_s\) seconds in order to recover the original symbols sequence.
If the receiver filter is indicated with $h_{rx}(t)$, the received signal by $r(t)$ and the signal at the output of the received filter by $y(t)$ we can write

$$y(t) = r(t) * h_{rx}(t), \quad (3.5)$$

$$r(t) = [(s(t) * h_{ch}(t)) + w(t)]. \quad (3.6)$$

After sampling, we obtain that

$$y(n) = a(n) * p(n) + w'(n), \quad (3.7)$$

where $w'(n)$ is the filtered and sampled noise and

$$p(n) = h_{tx}(t) * h_{ch}(t) * h_{rx}(t) \big|_{t=nT_s}. \quad (3.8)$$

In eq (3.8) $p(n)$ indicates the discrete equivalent filter obtained by the cascade of transmission filter, channel impulse response and receiver filter.

From the sequence $y(n)$, the decision device recovers the estimated sequence $\hat{a}(n)$ dividing the signal space in decision regions. Then the demodulator does the de-mapping operation, obtaining the estimated stream of bits $\hat{b}_i$.

### 3.3.1 Matched Filter

The duration $P$ of the total impulse response $p(n)$ is called channel memory and depends on the impulse responses of transmitting and receiving filter and on the channel.

In the easiest case, when the channel is ideal as in eq (3.4), $p(t)$ is given by the convolution between $h_{tx}(t)$ and $h_{rx}(t)$ and to obtain a memoryless channel, we want the total impulse response to satisfy the condition

$$p(n) = |h_{tx}(t) * h_{rx}(t)|_{t=nT_s} = \delta(n). \quad (3.9)$$

Doing the Fourier transform of this equation, we obtain the so called Nyquist’s Criterion

$$P(e^{j2\pi fT_s}) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} H_{tx}(f - \frac{k}{T_s})H_{rx}(f - \frac{k}{T_s}) = 1. \quad (3.10)$$

The ideal cardinal sine, i.e. $h_{tx}(t) * h_{rx}(t) = \text{sinc}(\frac{t}{T_s})$ would satisfy this requirement, but in practice it is not possible to implement it.
Anyway, there are many types of waveform, which have bandwidth greater than the minimum bandwidth (or Nyquist bandwidth) of the sinc $B_{\text{min}} = 1/2T_s$, that can satisfy the Nyquist Criterion. For example, a common choice is the so called Raised Cosine (RC) function that are defined as

$$RC(f) = \begin{cases} T & |f| \leq \frac{1-\alpha}{2T_s}, \\ T \cos^2\left(\frac{\pi T_s}{2\alpha} (|f| - \frac{1-\alpha}{2T_s})\right) & \frac{1-\alpha}{2T_s} < |f| \leq \frac{1+\alpha}{2T_s}, \\ 0 & |f| > \frac{1+\alpha}{2T_s}. \end{cases}$$

(3.11)

The parameter $\alpha$ is the Roll-Off factor and indicates the fraction of excess bandwidth of the waveform with respect to the Nyquist bandwidth. The total bandwidth is so $B = B_{\text{min}}(1+\alpha) = \frac{1}{2T_s}(1+\alpha)$.

To obtain $p(t)$ as a Raised Cosine function, the transmitting and receiving filter must be the so called Square Root Raised Cosine (SRRC) function. This two types of function are linked by

$$RC(f) = SRRC(f) * SRRC(f)$$

(3.12)

$$SRRC(F) = \sqrt{|RC(f)|}.$$  

(3.13)

In general, the design of the receiving filter is based on the maximization of the Signal to Noise Ratio (SNR). Assuming that we are transmitting only an isolate impulse, so we are in absence of ISI, we can define $g(t)$ as the convolution between transmitting filter and channel impulse response

$$g(t) = h_{tx}(t) * h_{ch}(t).$$

(3.14)

So, before the receiving filter, we have

$$r(t) = a(0)g(t) + w(t).$$

(3.15)

The total impulse response, evaluated in its maximum value, is

$$p(t_0) = \int_{-\infty}^{+\infty} G(f) H_{rx}(f) e^{j2\pi ft_0} df,$$ 

(3.16)

that is the inverse Fourier Transform of the cascade between the two transferring function.

The SNR after the receiver is defined as

$$\text{SNR} = \frac{|p(t_0)|^2}{\sigma_w^2} = \frac{\int_{-\infty}^{+\infty} G(f) H_{rx}(f) e^{j2\pi ft_0} df|^2}{\frac{N_0}{2} \int_{-\infty}^{+\infty} |H_{rx}(f)|^2 df},$$

(3.17)
where \( N_0/2 \) is the noise power spectral density and \( \sigma^2_w \) is the noise power spectral density after the receiving filter, while the nominator is the power of the signal. It has been proven that the maximum of this ratio can be obtained when the absolute value at the nominator is splittable (Schwarz’s inequality); it happens when

\[
H_{rx}(f) = G^*(f)e^{-j2\pi f t_0}.
\]  

(3.18)

This is the so called Matched Filter, that both minimizes ISI and maximizes the SNR. Substituting it in the SNR formula (equation (3.17)), the maximum achievable SNR is found to be

\[
SNR = \frac{2E_g}{N_0},
\]  

(3.19)

being \( E_g \) the energy of \( g(t) \).

The receiver impulse response is then obtained from the inverse Fourier Transform of eq (3.18)

\[
h_{rx}(t) = g(t_0 - t).
\]  

(3.20)

The Matched Filter is therefore the time inverted and delayed version of \( g(t) \).

### 3.3.2 Frequency Selective Channel

In the case in which the channel is not ideal, i.e. it is not a Dirac delta, two cases can be distinguished looking at the Fourier Transform of the channel impulse response \( H_{ch}(f) = \mathcal{F}\{h_{ch}(t)\} \).

- **\( H_{ch}(f) \approx 1 \) for \( |f| \leq 1/2T_s \):**
  
  In this case we can approximate \( h_{ch}(n) \approx h_{ch}(0)\delta(n) \);

- **\( H_{ch}(f) \) with rapid variation in the frequency interval \( |f| \leq 1/2T_s \):**
  
  In this case we cannot approximate the channel impulse response with only one sample. The length of the total impulse response \( p(t) \) will be greater than one.

In the case that the channel cannot be approximated with one sample, it is said that the channel has memory. Because of that, Inter Symbol Interference (ISI) arises: a received symbol is degraded by the overlapping of the previous and following symbols.

\[
y(n) = a(n) * p(n) = \sum_{i=0}^{P-1} a(n - i)p(i),
\]  

(3.21)

where \( P \) is the length of the total impulse response.
Chapter 4

Non-Coherent versus Self-Coherent

In this chapter the non coherent transmissions system are introduced. In particular, we focus on a PAM-4 as intensity modulation scheme that can be used with a non coherent receiver. We also explain the probability of error of this scheme in case of Gaussian interference.

Then the problem of the CD in a non coherent system is discussed.

In the following section, the concept of self-coherent system is presented as possibility to overcome this problem. Many examples of self-coherent are then summarized.

4.1 Non Coherent

The non coherent detection uses only a photo-detector at the receiver to generate an electrical current proportional to the intensity of the light impinging on the surface of the photo-detector, as said in chapter 2. This receiver structure is also defined as Direct Detection.

With this type of receiver, the information must be referred at the unique quantity that the receiver can distinguish: the intensity of the light.

The used modulation technique is therefore the Intensity Modulation: the electrical waveform acts on the intensity of the laser beam, making it varying according to the levels of the modulation.

There are two ways for changing the output laser power [18]. The first is the direct modulation, in which the light is modulated directly in the laser cavity, that can be a Fabry-Perot or a Distributed Feedback cavity. The other is the external modulator, a separated device that acts on the emitted light by the laser, that has a constant
power. The commercially used external modulator is the Mach-Zehnder Interferometric modulator.

In this thesis, we study a PAM-4 modulation scheme, with only positive amplitude levels, that can be distinguished by the receiver.

4.1.1 PAM

The system studied in this work uses a Pulse Amplitude Modulation (PAM). With respect to the canonical OOK, the PAM utilizes more amplitude signal level in order to increase spectral efficiency. The same bit rate can be reached, using the PAM, with a reduced Baud rate with respect to OOK.

In this particular case, since the receiver is a non coherent one, implementing the Direct Detection, it is not possible to use negative symbols. The receiver could not distinguish between optical field with same absolute value and different sign. The unipolar PAM-4 constellation is then composed by only positive symbols and it is represented in figure (4.1)

\[ A = \{1; 3; 5; 7\}. \] (4.1)

We can define the power of the constellation as the mean of the squared constellation point

\[ E_a = \frac{1}{4} \left( 1^2 + 3^2 + 5^2 + 7^2 \right) = 21. \] (4.2)

![Figure 4.1: PAM-4 IM/DD constellation](image)

Let us now assume that we are in presence of an ideal channel without memory and with additive noise. At the receiver we have a matched filter and we sample in the right time instant. Therefore, the output sequence is

\[ y(n) = a(n)E_g + w'(n). \] (4.3)

So, it is the sum between the deterministic value \( a(n)E_g \) and the random quantity \( w'(n) \), that is the noise after the receiving filter and it has variance \( \sigma_w^2 = \frac{N_0}{2}E_g \). So
the total Probability Density Function (PDF) is given by the convolution of the two PDFs: the result is the Gaussian PDF of the noise translated around the possible values of the transmitted signal, as shown in figure (4.2).

Let us assume, for simplicity, that \( E_g = 1 \)

\[
P(y) = \frac{1}{4} \frac{1}{\sqrt{2\pi\sigma_w}} \left[ \exp \left( -\frac{(y - 1)^2}{2\sigma_w^2} \right) + \exp \left( -\frac{(y - 3)^2}{2\sigma_w^2} \right) + \exp \left( -\frac{(y - 5)^2}{2\sigma_w^2} \right) + \exp \left( -\frac{(y - 7)^2}{2\sigma_w^2} \right) \right].
\]

(4.4)

The decision device has a non linear behavior. It quantizes the signal values, based on thresholds that can be chosen at the mid distance between the constellation points. Consequently, a decision is wrong if the noise has a value greater than the half distance between two point of the constellation.

To evaluate the probability of error, we must fix the transmitted symbols and then we have to integrate the area of the PDF for the value of the noise that create errors. It is known that the integral of a Gaussian function is evaluated by the so called \(Q\)-Function, defined as

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp \left( -\frac{u^2}{2} \right) du.
\]

(4.5)

For the central points, the error can happen in both directions, towards the upper or the lower point, while the for edge constellation points the error arises only in one direction. The total symbol probability is the weighted mean of the conditioned
probability of error of every point

\[ P_s(e) = \frac{1}{2} Q \left( \sqrt{\frac{2E_g}{N_0}} \right) + \frac{1}{2} 2Q \left( \sqrt{\frac{2E_g}{N_0}} \right), \quad (4.6) \]

where the first term stays for the two edge points and the second term stays for the central points.

For a general M-PAM constellation, there is a formula for evaluating the symbol error probability

\[ P_s(e) = \frac{2(M-1)}{M} Q \left( \sqrt{\frac{2E_g}{N_0}} \right). \quad (4.7) \]

From the symbol error probability we can also evaluate the bit error probability. Actually, in most cases it can be only approximated, because it depends on the mapping between symbols and bits. It can be better calculated in case of Gray mapping, when adjacent symbols differ for one bit and so, assuming that errors happen only between adjacent symbols, one symbol error corresponds to one bit error. Having \( \log_2(M) \) bits per symbol, the bit error probability is

\[ P_b(e) = \frac{1}{\log_2(M)} P_s(e) = \frac{1}{\log_2(M)} \left[ \frac{1}{2} Q \left( \sqrt{\frac{2E_g}{N_0}} \right) + \frac{1}{2} 2Q \left( \sqrt{\frac{2E_g}{N_0}} \right) \right]. \quad (4.8) \]

The formula in equation (4.6) is derived for the case of only Gaussian interference, i.e. the AWGN noise. When we are in presence of other types of interference, equation (4.6) can be used as an approximation of the symbols error probability. How good is this approximation depends on the statistical property of the interference.

### 4.1.2 Non Coherent Detection and Chromatic Dispersion

In a non coherent system the CD cannot be equalized after the photo-diode since the channel becomes quadratic [19].

Since we are using IM, the optical transmitted signal is composed by the information signal and the carrier, that is added with a bias of the modulators described in section 4.1.

The baseband equivalent of the transmitted signal can be indicated as

\[ tx(t) = s(t) + C, \quad (4.9) \]
where $s(t)$ is the real signal, so it has a complex symmetry in the frequency domain, and $C$ indicates the carrier. The total transmitted signal $tx(t)$ travels in the fiber and is convoluted with the CD impulse response $h_{cd}(t)$.

The photo-detector does the square modulus of signal $tx(t)$ and the produced current is

$$i(t) = |tx(t) * h_{cd}(t)|^2 = |(s(t) + C) * h_{cd}(t)|^2 =$$

$$= |C|^2 + |s(t)|^2 + 2C \text{Re}\{s(t) * h_{cd}(t)\}, \tag{4.10}$$

where

$$\text{Re}\{s(t) * h_{cd}(t)\} =$$

$$= s(t) * h_{cd}(t) + (s(t) * h_{cd}(t))^* \xrightarrow{\dagger}$$

$$S(f) \exp(-j\frac{1}{2}\beta_2 f^2 L) + S^*(-f) \exp(j\frac{1}{2}\beta_2 f^2 L) =$$

$$= 2S(f) \cos \left(\frac{1}{2}\beta_2 f^2 L\right). \tag{4.11}$$

We can see that the signal is degraded by the cosine term, that creates notches in the frequency domain in particular points of the spectrum. These notches correspond to the argument of the cosine equal to $\pi/2 + k\pi$.

In figures (4.3), (4.4) and (4.5) the effect of the CD on a IM-DD signal is showed, after the transmission around $\lambda = 1510\text{nm}$. We can see how the notches quantity increases with the increasing of the fiber length. Furthermore, the frequency of the first notch becomes lower with a longer transmission.

In order to avoid this degrading effect, an optical Single Side Band (SSB) signal has to be transmitted. As we will see in section 4.2, this can be easily done with a self-coherent system.
We actually implement another solution in chapter 5, that aims to move the complexity from the receiver into the transmitter.

We propose a complex digital FIR pre-compensation filter that tries to equalize at the transmitter the successive CD introduced by the fiber transmission. Its impulse response is derived as Inverse Fourier Transform (IFT) of the complex conjugate of the CD transfer function, the one indicated in equation (2.20). So we have

$$h_p(t) = \mathcal{F}^{-1} \{H_{cd}(f)^*\} = \mathcal{F}^{-1} \left\{ \exp \left( j \frac{1}{2} \frac{\pi^2 D \lambda L}{c} f^2 \right) \right\}. \quad (4.12)$$

In the simulation, discussed in chapter 5, the sampled version of filter in equation (4.12) has been implemented.
4.2 Self-Coherent

A self-coherent system consists in transmitting a carrier together with the information signal. As said in Chapter 2, this is a compromise between performance of a coherent method and easiness and low cost of a direct detection technique. This type of transmission can be generated using a scheme as in figure (4.6).

![Figure 4.6: Up-conversion scheme](image)

In this scheme we have a general modulated signal that is output by the modulator in its two baseband components: the real part $s_c(t)$ and the imaginary part $s_s(t)$. A carrier is added at frequency $f_1$ on both the In Phase and Quadrature component. Then up-conversion is done by multiplying with the laser that has an optical frequency of $f_0$.

We obtain a passband signal with only one impulse due to the carrier at frequency $f_0 + f_1$ while the impulse at frequency $f_0 - f_1$ is canceled. In equation (4.13) it is shown how one of the impulses of the carrier is canceled.

$$A \cos(2\pi f_1 t) \cos(2\pi f_0 t) - A \sin(2\pi f_0 t) \sin(2\pi f_1 t) =$$
$$= \frac{1}{2} A [\cos(2\pi (f_0 + f_1) t) + \cos(2\pi (f_0 - f_1) t)] +$$
$$- \frac{1}{2} A [\cos(2\pi (f_0 - f_1) t) - \cos(2\pi (f_0 + f_1) t)] =$$
$$= A \cos(2\pi (f_0 + f_1) t)$$

(4.13)

The resulting signal $s_{pb}(t)$ has the frequency characteristics that we see in figure (4.7).

Therefore, the passband signal is composed by the information signal whose spectrum is around frequency $f_0$ and an impulse due to the carrier at frequency $f_0 + f_1$. 

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The complete scheme of the receiver is in figure (4.8). The photo-detector is represented as a square modulus operator. After this, a low pass filter is used to cut the out of band noise, in order to reduce its power.

The resulting signal $x(t)$ is the low-pass portion of the square of the $s_{pb}(t)$ of equation (4.14)

$$x(t) = 2 \text{LP} (s_{pb}(t)^2) =$$

$$= A^2 + |\tilde{s}|^2 + 2A \cos(2\pi(f_0 + f_1)t) [s_c(t) \cos(2\pi f_0 t) - s_s(t) \sin(2\pi f_0 t)] =$$

$$= 2A s_c(t) \cos(2\pi f_1 t) + 2A s_s(t) \sin(2\pi f_1 t) + LFC,$$

where $|\tilde{s}(t)| = |s_c(t) + js_s(t)|$. In the last row, we can see the Signal to Carrier beating term, that is translated around frequency $f_1$, plus the Low Frequency Components ($LFC$), that we have defined as the square of the carrier $A^2$ and the SSBI $|\tilde{s}(t)|^2$. The spectrum of $x(t)$ is as in figure (4.9).

So it is equivalent to have a signal electronically up-converted around frequency $f_1$. To recover it, down-conversion must be done, multiplying the signal with both
the cosine and sine wave and then low pass filtering. This operation recovers In Phase and Quadrature components that are used by the I/Q demodulator to generate the estimated bit sequence.

The information signal and the added carrier must have a frequency gap in between to be better demodulated after the detection as we see in figure 4.9. The Signal to Signal Beating Interference ($|\tilde{s}(t)|^2$) has a spectrum around the zero frequency with baseband bandwidth $B$, where $B$ is the passband bandwidth of the information signal. So, to avoid the overlapping in frequency domain between $|\tilde{s}(t)|^2$ and the Signal to Carrier beating, we should choose the carrier to be

$$f_1 \geq \frac{3}{2}B.$$  \hspace{1cm} (4.16)

In this way, after the detection, the Signal to Carrier beating term and the Signal to Signal interference are not overlapped.

The advantages of self-coherent systems can be summed up in the following list:

- The receiver is non coherent and cheaper because there is not a Local Oscillator
- There is no Phase Noise because we use the same carrier in transmission and reception
- Higher power in the carrier (and so higher Carrier to Signal Power Ratio) means amplification of the Signal to Carrier beating term

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{spectrum.png}
\caption{Baseband signal spectrum after the photo-detector}
\end{figure}
There are also some issues that must be remembered:

- The SSBI is an interfering term and must be appropriately taken into account
- The transmitted power must be shared between carrier and signal

If we consider the \( s_{pb}(t) \) signal that we generate in scheme (4.6), after the channel it is multiplied by the CD transfer function, so we have

\[
s_{pb}(t) * h_{cd}(t),
\]
\[
S_{pb}(f)H(f) = S_{pb}(f) \exp(-j \frac{1}{2} \beta_2 f^2 L).
\]

Looking at the Baseband equivalent of the Signal to Carrier beating term only

\[
2 \text{Re} \{ \hat{s}(t) \exp(2 \pi f_1 t) * h_{cd}(t) \} =
\]
\[
\hat{S}(f - f_1) \exp(-j \frac{\beta_2}{2} f^2 L) + \hat{S}^*(f + f_1) \exp(j \frac{\beta_2}{2} f^2 L).
\]

The two spectra are not overlapped and the CD does not create notches, in contrast to equation (4.11). This happens because the optical passband signal is a SSB one: the information signal is only on one side of the carrier, so the double beating with opposite phase is avoided and there is no power fading due to destructive interference.

### 4.3 Other Self Coherent Methods

We have shown a way to generate a Self Coherent system, but in literature there are other examples to create this type of transmissions. All the examples reported here are described in [19].

We can identify two main types of Self Coherent methods:

**Beating methods** They aim to extract the beating term between carrier and signal and they must take care of the SSBI;

**Coherent-like methods** They use a coherent receiver equipped with an Optical Hybrid in order to recover the complex signal, but the local oscillator is substituted by the carrier added in transmission. The carrier must be separated from the signal at the receiver side.
4.3.1 Beating Methods

If we want to extract the beating term between carrier and signal, we have to create an optical Single Side Band signal, similar to what we did in scheme (4.6). This avoids the creation of notches in the spectrum as in equation (4.11).

Offset SSB

The first way to generate an SSB signal suppresses one sideband using an optical filter, after having directly modulated the laser with the signal [20]. The signal is previously complex upconverted at $f_{RF}$ as in figure (4.10). The carrier is added at the laser frequency, thanks to the bias of the intensity modulation.

![Optically filtered SSB](image)

The receiver in figure (4.11) must do IQ demodulation on the detected signal that corresponds to a baseband signal up-converted to frequency $f_{rf}$. This receiver scheme
Figure 4.11: Receiver

is the same of the one introduced before. It differs only in the way the SSB optical spectrum is generated. Here it is used a filter to eliminate one side band, while, at the opposite, the scheme proposed before implicitly eliminate one carrier without using additional filtering.

Virtual SSB

This implementation adds the carrier directly in the frequency domain, putting an higher sample close to signal spectrum [21] in the digital stage. This creates the so called Virtual SSB, because actually the signal contains both negative and positive frequencies, but stays on one side of the carrier. In this case, the laser wavelength is not transmitted.

Since there is no frequency gap, after detection the SSBI is superimposed to the Signal to Carrier beating also in the frequency domain.

An iterative algorithm as in figure (4.13) is proposed that firstly estimates the SSBI using the decision on the received signal and then subtracts it from the successive symbol.
Another approach (figure (4.14)) tries to recover a complex DSB signal with both In Phase and Quadrature parts using a time redundancy. The carrier is sent in two consecutive time instants with a 90 degrees phase difference (a cosine and then a sine wave), while the symbol is transmitted twice. So their beating term gives first the real part as

$$I = |S + C|^2 = |S|^2 + |C|^2 + 2 \Re \{SC^*\}, \quad (4.19)$$

and then the imaginary part

$$I = |S + jC|^2 = |S|^2 + |C|^2 + 2 \Im \{SC^*\}. \quad (4.20)$$
This scheme is called Block-wise Phase Switching (BPS) [22] since we can divide transmission in blocks. It is not specified how to handle the SSBI, but similarly as before a frequency separation can be applied.

4.3.2 Coherent-like Methods

This type of approaches recovers simultaneously both real and imaginary part using a coherent receiver, that naturally eliminates the SSBI. These solutions have higher costs with respect to the previous ones, since optical hybrid is used and also polarization is exploited to increase spectral efficiency. The various methods differ in the way they separate carrier and signal.

The simplest scheme acts this separation in the frequency domain. It adds the carrier introducing a bias to the laser [19]. At the receiver the total beam is split: on a branch the carrier is filtered and used as input of the optical hybrid, that combines it with the signal, isolated on the other branch, to give in output the In Phase and Quadrature parts. This means that a frequency gap must be added and that it must be used a narrow band filter at the receiver.

Signal Carrier Interleaved

It is also possible to transmit the carrier alternated with the signal, separating them in time domain. This scheme is called Signal Carrier Interleaved (SCI) [23] and is represented in figure (4.15).

At the receiver the carrier is delayed as in figure (4.16) and used as input of the Optical Hybrid. The number of consecutive signal time slots can be increased in order to have a better efficiency.
To fully utilize the time domain efficiency, another scheme has been proposed that uses the two orthogonal polarization X and Y. This scheme is called Dual Polarization - Signal Carrier Interleaved (DP-SCI) [24]. The transmitted signal has on both polarizations an alternation of information signal and carrier, as shown in figure (4.17). The receiver is an optical hybrid that has in both inputs the transmitted signal, one of them delayed by one time slot, as in figure (4.18).

The signals combine independently on the two polarizations. For this case, it is proposed a Maximum Likelihood Sequence Estimation with depth 2 in order to perfectly reconstruct the symbols sequence.
Another way [25] consists in sending a Single Polarization IQ modulated signal on the X polarization and the carrier on the Y polarization. This situation is represented in figure (4.19).

At the receiver, using a Stokes Vector (SV) receiver as in figure (4.20), it is possible to recover both real and imaginary part. So there is no loss of spectral efficiency, since in a time symbol slot a complex symbol is received and demodulated.

### 4.3.3 Overview Of Self Coherent Methods

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Speed (Gbps)</th>
<th>Distance (km)</th>
<th>SSBI Cancellation</th>
<th>Optical Spectral Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offset SSB</td>
<td>-</td>
<td>-</td>
<td>Frequency Gap</td>
<td>0.5</td>
</tr>
<tr>
<td>Virtual SSB</td>
<td>10</td>
<td>340</td>
<td>Iterative Algorithm</td>
<td>1</td>
</tr>
<tr>
<td>BPS</td>
<td>40</td>
<td>80</td>
<td>Iterative Algorithm</td>
<td>0.5</td>
</tr>
<tr>
<td>SCI</td>
<td>100</td>
<td>80</td>
<td>Balanced Receiver</td>
<td>0.5</td>
</tr>
<tr>
<td>DP-SCI</td>
<td>100</td>
<td>100</td>
<td>Balanced Receiver</td>
<td>1</td>
</tr>
<tr>
<td>SV</td>
<td>100</td>
<td>480</td>
<td>Balanced Receiver</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.1: Recap of other self-coherent methods
In the table (4.1) all the previous schemes are summed up with information on maximum throughput, reached distance, types of SSBI cancellation method and spectral efficiencies.
Chapter 5

Simulation Results

The aim of this chapter is to describe how the Montecarlo simulations have been implemented and to introduce the parameters that are used to evaluate the performance. Furthermore, the obtained results are showed and commented for two types of filtering in transmission: a rectangular filter, that has a constant value in the symbol time duration and it is zero elsewhere, and a filter similar to the RC function introduced in chapter 3, but with an equi-ripple characteristic, so with more attenuated frequency side lobes.

5.1 Introduction

The thesis activity has been carried on with the help of a simulator that was built with the MATLAB® software. Different types of Montecarlo simulations have been carried out to evaluate both numerically and graphically the effects of the Chromatic Dispersion on our system in terms of performance.

The simulations have been developed to visualize the performance changing due to variations of the mismatch percentage between the pre-compensation filter and the actual fiber CD transfer function. The main parameter that we have taken into account is the Bit Error Rate (BER). We also introduced the Signal to Distortion Ratio (SDR) and the Error Vector Magnitude (EVM) as performance indicators. The study is focused on two types of transmission filter, both respecting the Nyquist criterion. One is a rectangular filter, typically used in the NRZ transmissions, and the other is
5.2 Simulation Setup

The scheme used for the simulations is described in figure (5.1).

The first thing the simulation does is to create an array of $10^6$ random generated bits $b_i$. The bits are then grouped in groups of two and mapped to a symbols array. The symbols $a_n$ are extracted from the symbols set $A = \{1; 3; 5; 7\}$, only formed by positive symbols because of Direct Detection. The Gray mapping is used. Since MATLAB® outputs the levels $\{0; 1; 2; 3\}$, they are scaled and amplified to obtain the symbols in set $A$. The sequence of symbols is then up-sampled.

The symbols are now filtered with the transmission filter $h_{tx}(n)$. In the simulations, two different filters have been used:

1. Rectangular filter

2. Nyquist FIR filter with equi-ripple characteristic

Both of them respect the Nyquist criterion introduced in chapter 2, since in the sampling instants of previous and following symbols their impulse responses go to zero and they do not create ISI. The rectangular filter respects the Nyquist criterion limiting its time duration, while the equi-ripple filter has a longer time duration, but it is limited in bandwidth occupation.

The rectangular filter is defined as

$$h_{tx}(n) = \text{rect} \left( \frac{t - T_s/2}{T_s} \right) \big|_{t=nT_{sampling}}.$$  \hspace{1cm} (5.1)
Since we are considering to sample at the minimum possible frequency, i.e. two times in a symbol duration, the resulting filter has length 2 as in figure (5.2).

The second filter has been designed with the MATLAB® built in firnyquist function. It is a 2-band low-pass FIR filter of order 30, with a chosen roll-off of 0.3. Its impulse response is reported in figure (5.3), where it is easy to see the longer duration with respect to the rectangular filter. We can also see that the even samples have zero value, exactly when the signal will be sampled to recover the symbols sequence. This is the Nyquist criterion to avoid ISI. The shape of this filter is similar to a common Raised Cosine filter, as introduced in Chapter 3; the equi-ripple filter has the advantage of a better suppression of the side lobes in the frequency domain.
After the transmission filter, always referring as scheme in figure (5.1), the pre-compensation filter is introduced: its impulse response is derived as Inverse Discrete Fourier Transform (IDFT) of the complex conjugate of the CD transfer function, as said in chapter 4 in equation (4.12). The discrete-time impulse response \( h_p(n) \) is obtained by approximating the computation of inverse Fourier transform with Inverse Discrete Fourier Transform (IDFT), where a sampling in frequency domain is chosen to avoid aliasing in time-domain, so

\[
h_p(n) = \text{IDFT}^{-1}\{H_{cd}(f_k)^*\} = \text{IDFT}^{-1}\left\{\exp\left(j \frac{1}{2} \frac{\pi \lambda^2 D \lambda L}{c} f_k^2\right)\right\}.
\] (5.2)

The IDFT is evaluated with Inverse Fast Fourier Transform (IFFT) algorithm, with a chosen number of frequency samples equal to \( N_{FFT} = 1024 \).

The parameters that have been used in the simulations are:

- \( \lambda = 1.54294 \text{nm} \)
- \( D = 16.5 \text{ps/nm/km} \)
- \( L = 80 \text{km} \)
- \( f_{max} = 50 \text{GHz} \)

The first two parameters are typical of a C-band utilization of a Single Mode Fiber, while a length of 80 km is used as a reference value for the medium reach applications. The \( f_{max} \) is chosen equal to 50GHz because we are considering a 50Gbaud PAM-4 signal and we are sampling at the Nyquist frequency.

Since we want to represent a system with total bit rate of 100 Gbps and we are using a PAM-4 modulation, the symbol rate is found to be \( R_s = R_b / \log_2(M) = 50 \text{Gbaud} \). Up-sampling by 2 the symbols sequence, we obtain a sampling frequency of \( f_s = 100 \text{GHz} \).

After up-sampling, the CD is applied, making the convolution with the IDFT of the sampled version of equation (2.20).

The parameters used for this filter have been chosen with varying values, in order to account for misalignment in the pre-compensation. In particular, what is varying
is the percentage of the product $D\lambda * L$, so the actual CD transfer function is

$$H_{cd}(f) = \exp \left( -j \frac{1}{2} \frac{\pi \lambda^2 D\lambda L(1 + m)}{c} f^2 \right),$$

(5.3)

where $0 < m < 1$ is the percentage of mismatch.

After the CD, before the photo-detector, the array of symbols is firstly cut to take care of the delay introduced of the two convolutions and then it is down-sampled by 2.

The channel introduces the discrete time zero-mean complex Gaussian noise $w_n$ with variance $\sigma_w^2$. Then the photo-detector is represented as a square modulus operator, since it is proportional only to intensity of the optical field, as seen in equation (2.3).

Independent decisions are taken by comparison with equally spaced thresholds centered at the medium distance between constellation points. If we assume not to have noise, the output $\tilde{a}_n$ must be the square of the transmitted symbols sequence $a_n$. So a square root operator, not represented in the scheme, is used to bring back the points at they levels in set $A$. After decision, the following de-mapping extract from the estimated symbols sequence the estimated bits stream $\tilde{b}_i$. These last operations are in inverse order with respect to the sequence of operations that we have done at the beginning.

The signal before the photo-detector is the result between the two successive convolution with pre-compensation and CD filter. It can be represented as

$$r_n = a_n \otimes h_m(n) + w_n,$$

(5.4)

where $\otimes$ denotes the discrete convolution and

$$h_m(n) = h_p(n) \otimes h_{cd}(n),$$

(5.5)

is the impulse response of the cascade of the two filters. In ideal case of perfect pre-compensation, $h_m(n)$ is the Kronecker delta

$$\delta_n = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases}$$

(5.6)

In all the other cases, when the pre-compensation filter does not equalize perfectly the CD, $h_m(n)$ is longer than one and ISI arises. The received symbols are overlapped with the tails of the impulse responses of the previous symbols and this generates
performance degradations.
The results that are showed in the following are valid both for positive and negative
mismatch $m$, since the CD has a symmetry in its frequency response and also in the
time domain impulse response.

5.3 Measures of Performance

The simulations analyze this type of performance indicators:

- Signal-to-Distortion Ratio (SDR)
- Bit Error Rate (BER)
- Error Vector Magnitude (EVM)
- Scatter Plot

5.3.1 Signal-to-Distortion Ratio

The SDR measures the quality of the signal at the input of the photo-detector. It
is defined as the ratio between the power of the desired signal and the power of the
total distortion, that in our case is the sum between the power of the noise and the
power of the ISI. So we have

$$ SDR = \frac{P_a}{P_{ISI} + SNR^{-1} P_a}, \quad (5.7) $$

The power of the desired signal is calculated as average between the square of the
possible values of the constellation $a_n$, so

$$ P_a = \frac{1}{|A|} \sum_{i=0}^{|A|-1} a_i^2 = 21, \quad (5.8) $$

where $|A|$ is the cardinality of the set $A$.

The SNR is the Signal-to-Noise Ratio and it is defined as the ratio between $P_a$ and
the variance of the Gaussian noise

$$ SNR = \frac{P_a}{\sigma_w^2}. \quad (5.9) $$
In absence of ISI, when we have perfect pre-compensation, the SNR and the SDR coincide, since we have $SDR = \frac{P_a}{SNR - 1} = SNR$.

The power of the ISI term is calculated as

$$P_{ISI} = P_a \sum_k |1 - h_m(k)|^2.$$  \hfill (5.10)

Obviously, this implies to know the shape the impulse response $h_m(n)$, so to know the mismatch and the residual Chromatic Dispersion. Consequently, the SDR defined in equation (5.7) is not possible to be known at the receiver, especially for the case of Direct Detection that we are implementing. Anyway, in the simulation environment where the parameters are decided by us, it is easy to calculate the SDR. In figure (5.1) it is shown that the SDR is evaluated before the photo-detector and it uses the computing of the power of the error sequence $e_n$.

### 5.3.2 Bit Error Rate

In the simulation, the BER performance has been evaluated by comparing the transmitted bit sequence with the estimated one, produced at the output of the demodulator. The number of errors obtained by this comparing has been divided by the total number of transmitted bits.

An estimation of BER can be considered good if the total number of errors is greater or equal to 100. Therefore, having generated $10^6$ random bits, BER can be considered well estimated for values not lower than $10^{-4}$.

The evaluation of the BER has been carried out by varying the SNR or the mismatch percentage, or both.

### 5.3.3 Error Vector Magnitude (EVM)

EVM is a measure of the quality of a communication system. It describes the absolute distance between the received complex symbols and the ideal point of the constellation. It is used in the wireless communications and if it is affected by Gaussian noise, a relation with the BER has already been found. In [26] and [27] the estimation of the BER using the EVM is extended to optical communications.

EVM measurements can be data-aided or non-data-aided. The data-aided EVM can be calculated using a training sequence that is known both at the transmitter.
and at the receiver. So correct data are used to evaluate EVM. In the latter case the symbols sequence is not known and the decisions are used to be the reference points for estimating EVM.

In case of non-data-aided EVM, if the BER is too high, the measurements bring to an underestimation of the real value of the EVM. This happens because, when distortion is high, the received symbol can be very far from the transmitted one. It can bring to an error in the decision, since the decision device chooses for the nearest constellation point, and so to a smaller EVM contribute.

Consequently, non-data-aided EVM is a good performance metric only for big enough SDR.

The data-aided EVM is defined as

$$EVM = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \frac{|r_k - a_k|^2}{P_a}},$$

(5.11)

where $r_k$ is the received symbol and $a_k$ is the correspondent transmitted one. $K$ is the number of symbols used to averaging and $P_a$ is the average symbols power, evaluated in equation (5.8). As it is shown in figure (5.1), the EVM is calculated before the threshold detector, at the output of the photo-detector. From the EVM we estimate a SDR, denoted as Estimated SDR ($SDR_{est}$). They are linked by

$$EVM = \sqrt{\frac{1}{SDR_{est}}}.$$

(5.12)

This $SDR_{est}$ is different from the SDR evaluated in equation (5.7), because the last one was referred to the signal before the photo-detector and it was calculated having access at the field information. The $SDR_{est}$ is instead based only on the intensity information of the signal after the non-coherent receiver.

Similarly to equation (5.11), the non-data-aided EVM is defined as

$$EVM = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \frac{|r_k - \hat{a}_k|^2}{P_a}},$$

(5.13)

where now $\hat{a}_k$ denotes the sequence of decisions made on the received symbols.

When the SDR is high, we have that $\hat{a}_k = a_k$ and so the non-data-aided EVM corresponds to the data-aided EVM.
5.3.4 Scatter Plot

The scatter plot is a graphical tool that allows for visualizing how received symbols spread around the ideal constellation points. The larger is the distortion, the larger is the dispersion around them. It is also possible to see which are the statistical characteristics of the distortion. In fact, a Gaussian distortion creates a scatter plot where the received symbols are uniformly distributed around the reference point. The distortion introduced by the CD is inherently different, as we will see in the next section.

5.4 Numerical Results

In the following, some graphical results are plotted to understand the behavior of our PAM-4 IM-DD system with CD pre-compensation filter, using the parameters we introduced above.

5.4.1 Simulated Bit Error Rate

In figure (5.4), on each curve, the SNR in equation (5.9) is kept constant, while the mismatch between pre-compensation filter and CD is varying and so the correspondent introduced ISI changes. It is possible to appreciate how the BER performance gets worse with the increasing of the mismatch percentage. The same is done in figure (5.5) for equi-ripple Nyquist filtering. This transmission filter has a better performance at the increasing of mismatch with respect to the rectangular filter.

In figures (5.6) and (5.7), the curves have a fixed SDR, the one calculated as in equation (5.7). Since the mismatch is varying, SDR is kept constant by changing the relative powers of noise and ISI. We can observe how, at a fixed SDR, the BER decreases with the increase of the mismatch percentage. This suggests that it is in some way possible to evaluate the entity of mismatch by jointly measuring BER and SDR.

In figure (5.8) and in figure (5.9) the curves have instead constant mismatch. Again we can see how, on a same point on abscissa, so for a constant SDR, the BER is lower for higher values of the mismatch, so higher remained Chromatic Dispersion. From
these figures and the previous ones we can observe that, when the power of the total distortion is dominated by ISI that actually is not Gaussian, there are less errors than what expected from the Gaussian approximation.
5.4.2 BER Estimation From EVM

The real SDR is evaluated with equation (5.7) and it is plotted in figure (5.10) for rectangular filtering and in figure (5.11) for Nyquist equi-ripple filtering. Also the ones that are estimated from the EVM are plotted, both the data-aided (figure (5.11)) and the non data-aided (figure (5.13)) EVM, from which the SDR is estimated with equation (5.12). We can see how the estimated EVMs are greater than the true one, both for rectangular and Nyquist filtering.

This happens because the SDR estimation from EVM assumes that interference is only Gaussian, that is not true in our case. Only a Gaussian interference with less power
could generate an EVM as the one that we have measured, so the SDR estimation results in a too high value.

This situation suggests that we can infer the value of the BER, also if we cannot measure the actual SDR and we do not know a closed formula for evaluating the BER from the SDR in case of non Gaussian interference.

In figure (5.12) for rectangular filtering and in figure (5.13) for Nyquist equi-ripple filtering different BER curves are shown. One is the BER obtained by the simulation, comparing received bits and transmitted bits. The other two are estimated from the EVMs, using the analytical expression that gives the bit error probability for the
considered PAM-4 modulation scheme as

$$P_b(e) \approx \frac{2(M - 1)}{M \log_2 M} Q \left( \sqrt{\frac{\text{SDR}_{\text{est}}}{P_a}} \right),$$

(5.14)

where $Q(\cdot)$ is the $Q$-function used when the distortion is Gaussian, hence under the Gaussian approximation of ISI. The $\text{SDR}_{\text{est}}$ is evaluated with equation (5.12).

![Figure 5.10: Real and estimated SDR for rectangular filtering](image1)

![Figure 5.11: Real and estimated SDR for Nyquist equi-ripple filtering](image2)

We see that assuming that interference is all Gaussian and estimating the SDR again with Gaussian approximation, we can pretty well infer the value of the real
BER.
We have now to distinguish two cases: while the BER obtained using the data-aided estimated EVM provides a good estimate of the real BER measured by Monte Carlo simulation for all the values of mismatch, the BER obtained using estimated EVM with decisions provides a good approximation only for mismatch values below -2%, if we are in the case of rectangular transmission filter. For the Nyquist equi-ripple filter, since the BER degrades slowly with the increasing of mismatch, the estimation with non data-aided EVM can be considered good for mismatch values lower then -3%.

![Figure 5.12: Real and estimated BER for rectangular filtering](image)

![Figure 5.13: Real and estimated BER for Nyquist equi-ripple filtering](image)
Figure 5.14: BER vs mismatch for different roll-off Nyquist equi-ripple filter and for rectangular filter.

In figure (5.14) the BER vs mismatch is evaluated for different type of filtering and roll-off factor of the equi-ripple Nyquist filter, with a constant SNR. It is possible to see that the modification of the roll-off factor does not create great change of the simulated performance. As we can see by the previous figures, the equi-ripple filtering seems to have better performance against the residual Chromatic Dispersion left by the misalignment among pre-compensation and actual CD of the channel.

The filtering of the signal spectrum, and so the reducing of its bandwidth, improves the performance of the system because a larger spectrum is more affected by the effect of CD, since the difference in propagation time for each frequency components becomes bigger.

In figure (5.15) the spectra of rectangular filtered signal and the Nyquist filtered version with roll-off factor equal to 0.3 are compared. We can see in red the main lobe of the cardinal sine, obtained by the Fourier Transform of the rectangular filter, and in blue the equi-ripple Nyquist filtered spectrum that shows a reduction of bandwidth occupation.
5.4.3 Total Impulse Response With Mismatch

In figures (5.16), (5.18), (5.17) and (5.19) it is possible to appreciate the impact of the remained CD on the total impulse response of the system. In the ideal case of perfect pre-equalization of the CD, this total impulse response would be as in figures (5.2) or (5.3).

With the increasing of the mismatch, the number of non-zero values of the rectangular filter increases, so the total impulse response is larger, introducing ISI between adjacent symbols.

Also the impulse response with equi-ripple filter introduces ISI due to the fact that
the even samples are not anymore zero, because of CD.

By comparing the two types of impulse response with a constant mismatch it is possible to understand why the second filter is more performing: the even samples, that create interference with previous and following symbols, have lower values, so ISI is smaller.

---

Figure 5.17: Impulse response of the cascade among rectangular filter, pre-compensation and CD with mismatch = 4%

Figure 5.18: Impulse response of the cascade among Nyquist equi-ripple filter, pre-compensation and CD with mismatch = 2%
Figure 5.19: Impulse response of the cascade among Nyquist equi-ripple filter, pre-compensation and CD with mismatch = 4%

5.4.4 Scatter Plots After Detection

The figures (5.20), (5.21), (5.22) ans (5.23) show the scatter plot of the detected signals with the rectangular filter in transmission. The noise is not considered. We can see that the constellation points are not affected by CD in the same way. In fact, there is more dispersion around amplitude level 1 than around amplitude level 7. This happens because the lower symbols suffer the impact of higher adjacent symbols spreading in time.

Again, in figures (5.24), (5.25), (5.26) and (5.27), scatter plots of received and detected signal are shown, now for equi-ripple Nyquist filtered signal. For a certain mismatch it is clear that now the behavior is better, because there is less dispersion
around the constellation points and this creates less errors. In the figures the signal are noiseless, so all distortion is caused by CD that is remained after the cascade of pre-compensation filter and the fiber channel.
Figure 5.24: Scatter plot of the equi-ripple filtered signal after detection (mismatch = 0.5%)

Figure 5.25: Scatter plot of the equi-ripple filtered signal after detection (mismatch = 1%)

Figure 5.26: Scatter plot of the equi-ripple filtered signal after detection (mismatch = 1.5%)
5.4.5 Power of the Interference After the Photo-Diode

Now we analytically evaluate the power of the interference caused by remained CD on the detected signal.

In the following, we use the notation:

\( a_k \): transmitted symbols

\( g(t) \): impulsive response before photo-diode

\( s_k \): received sequence after the photo-diode

After the photo-diode, the received waveform is

\[
s(t) = \left| \sum_n a_n g(t - nT) \right|^2 = \sum_n a_n g(t - nT) \sum_l a_l g^*(t - lT). \tag{5.15}
\]

Sampling \( s(t) \) every \( kT \) seconds, where \( T \) is the symbol time, we obtain

\[
s_k = \sum_n a_n g(kT - nT) \sum_l a_l g^*(kT - lT). \tag{5.16}
\]

Now we introduce the variable change: \( n = k - n, l = k - l \).
We can rewrite the received sequence as

\[ s_k = \sum_n a_{k-n} g(nT) \sum_l a_{k-l} g^*(lT) = \]

\[ = \sum_n \sum_l a_{k-n} a_{k-l} g(nT) g^*(lT) = \]

\[ = \sum_n a_{k-n}^2 |g(nT)|^2 + \sum_n \sum_{l \neq n} a_{k-n} a_{k-l} g(nT) g^*(lT) = \]

\[ = a_k^2 g(0) + \sum_{n \neq 0} a_{k-n}^2 |g(nT)|^2 + \sum_n \sum_{l \neq n} a_{k-n} a_{k-l} g(nT) g^*(lT) \]

We describe the terms in the last row. The first term is the correct sample, the second one is the common ISI, introduced by the impulse response that is longer than one, and the third term is another ISI term, given by the beating between previous and successive symbols, caused by the square operation of the photo-diode.

Now we do the expectations of the square of last two terms to calculate the power of the total interference. The square operation is done by multiplying the total interference term with its complex conjugate.

\[ P_{ISI} = E \left[ \left( \sum_{n \neq 0} a_{k-n}^2 |g(nT)|^2 + \sum_n \sum_{l \neq n} a_{k-n} a_{k-l} g(nT) g^*(lT) \right) \right] \]

\[ \times \left( \sum_{q \neq 0} a_{k-q}^2 |g(qT)|^2 + \sum_q \sum_{p \neq q} a_{k-q} a_{k-p} g^*(qT) g(pT) \right) \]

\[ = \sum_{n \neq 0} \sum_{q \neq 0} a_{k-n}^2 |g(nT)|^2 a_{k-q}^2 |g(qT)|^2 \]

\[ = E[a_k^4] \sum_{n \neq 0} |g(nT)|^4 + E[a_k^2] \sum_{n \neq 0} \sum_{q \neq n} |g(nT)|^2 |g(qT)|^2 \]

\[ n = q \]

\[ + E[a_k^2]^2 \sum_{n \neq 0} \sum_{q \neq n} |g(nT)|^2 |g(qT)|^2 \]

\[ n \neq q. \]
The term $B \ast C$ is
\[
E \left[ \sum_n \sum_{l \neq n} \sum_{q \neq 0} a_k - n a_k - l a_k^2 - q g(nT) g^*(lT) |g(qT)|^2 \right] =
\]
\[
= E[a_k^2] E[a_k] \sum_{n \neq 0} \sum_{l \neq n} |g(nT)|^2 |g(nT) g^*(lT)| + \quad q = n \quad (5.20)
\]
\[
+ E[a_k^2] E[a_k] \sum_n \sum_{l \neq n} g(nT) |g(lT)|^2 g^*(lT) + \quad q = l
\]
\[
+ E[a_k^2] E[a_k] \sum_n \sum_{l \neq n} \sum_{q \neq 0, q \neq l} g(nT) g^*(lT) |g(qT)|^2 \quad q \neq l \neq n.
\]

The product $A \ast D$ is the complex conjugate of $B \ast C$.

The term $B \ast D$ is
\[
E \left[ \sum_n \sum_{l \neq n} \sum_{q} \sum_{p \neq q} a_k - n a_k - l a_k - q a_k - p g(nT) g^*(lT) g^*(qT) g(pT) \right] =
\]
\[
= E[a_k^2]^2 \sum_n \sum_{l \neq n} |g(nT)|^2 |g(lT)|^2 + \quad n = q, l = p
\]
\[
+ E[a_k^2] E[a_k] \sum_n \sum_{l \neq n} \sum_{p \neq l, p \neq q} |g(nT)|^2 g^*(lT) g(pT) + \quad n = q, l \neq p
\]
\[
+ E[a_k^2] E[a_k] \sum_n \sum_{q \neq n} \sum_{l \neq q} |g(lT)|^2 g(nT) g^*(qT) + \quad n \neq q, l = p
\]
\[
+ E[a_k^2] E[a_k] \sum_n \sum_{q \neq n} \sum_{q \neq l, q \neq n} g(nT)^2 g^*(lT) g^*(qT) + \quad (5.21)
\]
\[
+ E[a_k^2] E[a_k] \sum_n \sum_{l \neq n} \sum_{p \neq n, p \neq l} g^*(lT)^2 g(nT) g(pT) \quad q = l, n \neq p
\]
\[
+ E[a_k^2]^4 \sum_n \sum_{l \neq n} g(nT)^2 g^*(lT)^2 + \quad n = p, l = q
\]
\[
+ E[a_k^4] \sum_n \sum_{q \neq n, q \neq l} \sum_{p \neq n, p \neq l, p \neq q} g(nT) g^*(lT) g^*(qT) g(pT) \quad n \neq l \neq q \neq p.
\]

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If we put together the common expectation terms, we obtain

\[ P_{ISI} = E[a_k^4] \sum_{n \neq 0} |g(nT)|^4 + \]

\[ + E[a_k^2]^2 \left[ \sum_{n \neq 0} \sum_{q \neq n, q \neq 0} |g(nT)|^2 |g(qT)|^2 + 2 \sum_n \sum_{l \neq n} |g(nT)|^2 |g(lT)|^2 \right] + \]

\[ + E[a_k^4] \left[ \sum_n \sum_{l \neq n} \sum_{q \neq n \neq l} \sum_{p \neq n, p \neq l, p \neq q} g(nT)g^*(lT)g^*(qT)g(pT) \right] + \]

\[ + E[a_k^2]E[a_k] \left[ \sum_{n \neq 0} \sum_{l \neq n} |g(nT)|^2 g(nT)g^*(lT) + \sum_n \sum_{l \neq n, l \neq 0} g(nT)|g(lT)|^2 g^*(lT) \right] + \]

\[ + E[a_k^2]E[a_k] \left[ \sum_n \sum_{l \neq n} \sum_{q \neq l, q \neq n, q \neq 0} g(nT)g^*(lT)|g(qT)|^2 + \right. \]

\[ \left. + \sum_n \sum_{l \neq n} \sum_{q \neq l, q \neq n} (2|g(nT)|^2 g(lT)g^*(qT) + g^2(nT)g^*(lT)g(qT) + g^*(nT)^2 g(lT)g(qT)) \right] \]

(5.22)

We have here analytically derived how to calculate the power of the interference after the quadratic operation done by the photo-detector. The expression is quite long since it is composed by both the common ISI term and the beatings among the samples of the total impulse response.
5.4.6 Eye Diagrams

The better performance of the equi-ripple filter can be understood by looking at the eye diagrams of the signals after the detection, in figures (5.28), (5.29), (5.30) and (5.31). They are plotted on a two symbols time scale. With the increasing of mismatch, the eyes opening is reduced because more ISI arises. Again, the smaller signal values are more affected by ISI and so the dispersion around them is larger. Here we can also appreciate the better behavior of the second type of filtering. This is clear by looking at the transitions between a symbol and the following one, that are more centered at the correct levels.

![Figure 5.28: Eye diagram of detected signal with rectangular filter in transmission and mismatch of 2%](image1)

![Figure 5.29: Eye diagram of detected signal with rectangular filter in transmission and mismatch of 4%](image2)
Figure 5.30: Eye diagram of detected signal with equi-ripple filter in transmission and mismatch of 2%.

Figure 5.31: Eye diagram of detected signal with equi-ripple filter in transmission and mismatch of 4%.
Chapter 6

Conclusions

The problem at the basis of this work is the technology for the medium and short reach optical fiber link. This technology must respect requirements about costs and easiness, that are fundamental for the feasibility of its realization. The solution must exploit a single photo-detector. This is in contrast to coherent detection, that is used in long haul links and allows for a complete equalization of impairments at the receiver.

The target rate is fixed to 100 Gbps, in order to assure the same performance of the 100 G coherent channel. Since we are forced to use Direct Detection, the choice is among Intensity Modulation transmission and self-coherent system.

After having explained what a self-coherent system is and having introduced many examples of it, we studied the performance of an IM-DD system that has the advantage of an easy implementation.

The major impairment that afflicts the fiber transmission is the Chromatic Dispersion, so we have considered its as unique source of distortion, in addition to Gaussian noise.

To face the problem of CD, that cannot be equalized at the receiver since after the photo-detector the channel is quadratic, we introduced a digital filter placed at the transmitter that tries to pre-compensate for the estimated CD. The filter coefficients are obtained using the Inverse Discrete Fourier Transform of the conjugate of the CD transfer function with nominal parameters typical of a SMF used in the C-band.

Our study focused on the performance of a Intensity Modulation-Direct Detection PAM-4 scheme, that uses only positive levels since only intensity can be received.

We have introduced two types of transmission filter, to understand the different performance and how they change against remained CD. The first filter is a rectangular
filter while the second is a equi-ripple Nyquist filter built in MATLAB®. This second filter reduces the bandwidth occupation of the transmitted signal.

The performance has been evaluated in terms of Bit Error Rate. Then we tried to estimate the probability of error with the measured Error Vector Magnitude, obtaining some good results. The BER has been calculated for different values of the mismatch between the pre-compensation filter and the real value of the CD, that can vary due to changes of temperature or of the length of the fiber.

The results show that the equi-ripple Nyquist filter gives better BER values with respect to the rectangular one, since the lower bandwidth occupation guarantees better resistance against CD, that results in a lower Inter Symbol Interference.

The comparing of the results is also done by looking at the effect of the remained CD on the total impulse responses of the system with both transmission filters and observing the eye diagrams of the received signals.

By looking at the obtained graphical results, if it was possible to know the exact value of both Signal-to-Distortion Ratio and Bit Error Rate, we would be able to estimate the value of the mismatch of the pre-compensation filter, in order to feedback this information to the transmitter allowing for a better equalization.

The use of pre-compensation for the CD can be a solution for limiting complexity of the receiver; the results we have obtained suggest that PAM-4 with IM-DD can be a good choice for a short reach fiber link.
Bibliography


