Development of a Kerr Microscope for the study of magnetic micro and nanostructures

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Sommario

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Abstract

This thesis work deals with the design, implementation and characterization of a Kerr microscope for magnetic imaging of micro and nanostructures.

The Kerr microscope is an extension of a polarization contrast microscope and allows to probe the local magnetic properties of ferromagnetic specimens. Magnetic domains imaging provides the most direct access to the effective magnetic properties of materials down to the nanoscale, which is crucial to foresee the magnetic response of innovative materials and devices in the field of magnetism, magnetic memories and Spintronics. Among other techniques like Magnetic Force Microscopy and X-Ray Magnetic Circular Dichroism-Photoemission Electron Microscopy, Kerr Microscopy is of simpler implementation, gives sub-micrometric resolution, does not require a complex sample preparation and allows very fast measurements in applied magnetic field.

The activity started from a preliminary investigation of an optical Zeiss-Axiotron microscope to understand the feasibility of its use to implement a Kerr microscope. Design of a couple of water-cooled electromagnets and of a sample holder with five degrees of freedom has been done to permit the application of in-plane and out-of-plane magnetic fields to the sample in close proximity to the microscope objective.

A Matlab toolbox was developed in order to allow the full control of the experimental setup and permits the acquisition, post-processing and analysis of magnetic images. The software enables user’s scripting, allowing to perform automatic complex sequences of measurements.

An accurate characterization of the microscope performances and noise sources has been carried out to guide a microscope user to set-up custom measurements.

Finally, the capabilities of the Kerr microscope has been tested on three relevant case studies:

1. microcapacitors made by artificial multiferroics for the electric control of the magnetization;
2. permalloy zig-zag shaped micrometric conduits for lab-on-chip applications;
3. micromagnetic patterns created by thermally assisted scanning probe lithography.

These studies pointed out that the developed instrument is a state-of-art Kerr microscope.
Sommario

Il lavoro di questa tesi riguarda il design, l’implementazione e la caratterizzazione di un microscopio Kerr per l’imaging di micro e nanostrutture.

Il microscopio Kerr è un’estensione di un microscopio a contrasto di polarizzazione e permette di misurare le proprietà magnetiche di campioni ferromagnetici. La visualizzazione dei domini ferromagnetici fornisce un accesso diretto alle effettive proprietà magnetiche fino alla scala nanometrica, di cruciale importanza per prevedere la risposta magnetica di materiali innovativi e dispositivi nei campi del magnetismo, memorie magnetiche e Spintronica. Rispetto altre tecniche come MFM e XCMD-PEEM, la microscopia Kerr è di più facile implementazione, con risoluzione sotto il micrometro, non richiede una specifica preparazione del campione ed ha un elevata velocità di misura.

L’attività è iniziata da un’investigazione preliminare di un microscopio Zeiss-Axiotron al fine di implementare le modifiche richiesta da un microscopio Kerr. Il design di una coppia di elettromagneti raffreddati ad acqua e un sistema porta campioni con cinque gradi di libertà per permetter l’applicazione di un campo magnetico nel piano e fuori dal piano del campione in prossimità dell’obiettivo del microscopio.

È stato sviluppato un toolbox Matlab per il controllo del setup sperimentale e l’acquisizione, post-elaborazione e analisi magnetica delle immagini. Il software permette lo scripting permettendo complesse procedure automatiche di misura.

È stata effettuata un’accurata caratterizzazione delle performance e delle sorgenti di rumore per guidare un utente alla configurazione del microscopio.

In conclusione, sono state testate le capacità del microscopio Kerr su tre casi di studio:

1. microcapacitori contenenti multiferroici artificiali per il controllo elettrico della magnetizzazione
2. condotti micrometrici di permalloy con a forma a zig-zag per applicazioni lab-on-chip
3. pattern micromagnetici realizzati attraverso la tecnica tam-SPL

Questi studi hanno dimostrato che il microscopio Kerr sviluppatò è allo stato dell’arte.
Thesis Outline

The activity presented in this work was carried out within the NaBiS group (Nanomagnetism for Biology and Spintronics) of the Physics Department of Politecnico di Milano, headed by Prof. Riccardo Bertacco. The NaBiS group have an established experience in the development of spintronic devices for sensing, storage and signal processing, as well as in lab-on-a-chip systems for manipulation of biological entities. The group operates at Polifab, a brand new reference facility of Politecnico di Milano (400 m² of cleanroom and annexed laboratories) for micro and nanofabrication activities.

A Kerr microscope has become a fundamental instrument for groups operating in the context of magnetism and spintronics. Micromagnetic imaging permits to shed light on the complex behaviour of real micro and nano-sized devices, and goes beyond micromagnetic simulations based on finite elements methods. To this aim, my assignment was the complete development of a station for Kerr Microscopy. The results of my work is presented here, organized in three chapters:

- Chapter 1: Magneto-optic effects. The chapter presents the physics and the formalism of Magneto Optical Kerr Effect (MOKE).
- Chapter 2: Kerr Microscope. The chapter describes the hardware and software implementation of the Kerr microscope starting from the standard optical microscope Zeiss-Axiotron.
- Chapter 3: Case studies. This chapter presents the magnetic characterization of the aforementioned micrometric devices imaged by Kerr microscopy.
1. Magneto-optical effects

The interaction of light with matter involves the electronic structure of the matter. In particular, in ferromagnetic materials, the interaction depends on the magnetic state of the medium, and this gives rise to magneto-optical (MO) effects.

The Faraday effect, observed in 1843 in a piece of glass placed between the poles of a magnet, was the first of the magnetooptical effects to be discovered [1].

The Faraday effect is based on two contributions: the magnetic circular birefringence, that is a difference between the refractive indices for the left and right circular polarization; the magnetic circular dichroism, that is a difference between the absorptions of right and left circularly polarized waves propagating in a medium [2].

The magnetic circular dichroism converts a linear state of polarization into an elliptical one. The magnetic circular birefringence leads to a rotation of the major axis of polarization of the ellipse with respect to the incident linear polarization.

The rotation and ellipticity of the light due to Faraday effect can be calculated treating the incident linearly polarized light as the sum of two circular polarized components with opposite helicities. The corresponding effect in reflection was discovered by Kerr in 1876 and could be accounted for on the basis of Fresnel’s theory of reflection [3].

1.1. Polarization state of light

The direction of the electric field of an electro-magnetic wave is known as polarization of the wave. Many optical applications, such as MOKE Microscopy, depend on the nature and manipulation of the polarization.

Depending on the light source, the electromagnetic-wave can be emitted without a particular polarization, which means that the wave is the superposition of randomly distributed polarizations. This wave is said to be randomly polarized or unpolarized [4].

An electromagnetic wave whose electric field vector has a periodicity in its temporal and spatial behaviour is said to be a polarized wave. Consider the following expression of the electric field of an electromagnetic wave propagating along the z-axis:

$$ \mathbf{E} = i E_x + j E_y $$

(1.1)
\begin{equation}
E_x = E_{0x} e^{i(kz - \omega t + \phi_x)}; \quad E_y = E_{0y} e^{i(kz - \omega t + \phi_y)}
\end{equation}

\(E_{0x}\) and \(E_{0y}\) are the projection of the electric field vector along the x and y axes, while \(\phi_x\) and \(\phi_y\) represent the corresponding initial phases. Therefore, from equation (1.1) and (1.2) we can write a general polarization state as:

\begin{equation}
\mathbf{E} = \left( i E_{0x} + j E_{0y} e^{i(\phi_y - \phi_x)} \right) e^{i(kz - \omega t + \phi_y)} = \tilde{\mathbf{E}}
\end{equation}

\(\tilde{\mathbf{E}}\) is the complex vector amplitude for the polarized wave, which completely determines the polarization state of the wave. In general, eq.(1.3) represents an elliptically polarized wave: at a given position in space, the electric field vector describes an ellipse. Two parameters characterize the ellipse: the angle \(\theta\) between the major axis of the ellipse and the x-axis, and the ellipticity \(\epsilon\) defined as:

\begin{equation}
\epsilon = \left( \frac{E_{0y}}{E_{0x}} \right)
\end{equation}

Two polarizations are of particular importance. If the ellipse degenerates in a straight line the wave is said to be linearly polarized, characterized by a null phase difference between the components \(\phi_x = \phi_y\)

\begin{equation}
\mathbf{E} = \left( i E_{0x} + j E_{0y} \right) e^{i(kz - \omega t + \phi)}
\end{equation}

If the ellipse degenerates into a circle the wave is said to be circularly polarized, characterized by a phase difference of \(\pm \pi / 2\) and \(E_{0x} = E_{0y}\):

\begin{equation}
\mathbf{E} = E_0 (i \pm j) e^{i(kz - \omega t + \phi)}
\end{equation}

The plus sign is for left circularly polarized (LCP) wave and the minus sign for right circularly polarized wave (RCP). Important to note that a linearly polarized wave can be described as the sum of a RCP wave and a LCP wave:

\begin{equation}
\mathbf{E} = \frac{1}{2} E_0 (i + j) e^{i(kz - \omega t + \phi)} + \frac{1}{2} E_0 (i - j) e^{i(kz - \omega t + \phi)} = \frac{1}{2} E_0 i e^{i(kz - \omega t + \phi)}
\end{equation}
1.2. Phenomenological theory of the Faraday effect

We start our description of the magneto-optic effect with a classic treatment for dielectric non-magnetic materials [2]. The Lorentz classical theory explains the magneto optic effect in terms of dump harmonic model for the bound electrons in an oscillating electric field and in presence of a static magnetic field [5]. From the Lorentz force applied to an electron bound to an atom we will find the electron displacement, which lead to the Polarization \( P \) of the medium. From the relation between the polarization and the external electric field we can find the electric susceptibility tensor.

Once we obtain the susceptibility tensor we can obtain the difference in the indices of refraction for Left and Right Circularly Polarized Wave (LCP and RCP) which leads to the description of the Faraday and Kerr effects.

1.2.1. Susceptibility tensor

We start from the equation for the electron bound to the nucleus under the effect of an oscillating electric field and a static magnetic field.

\[
m'' \dot{r} + \left( \frac{b}{m} \dot{r} + k \right) r = -ie \omega \mu \mp E_0 + i \omega \mp E_0 \mp r \times H
\]

(1.8)

\( b \) is the damping coefficient, \( r \) is the displacement of the bound electron from its equilibrium position, \( k \) is the equivalent of an elastic constant representing the bond between the electron and the nucleous and \( \mu_0 \) is the vacuum permeability, as here we are working with non-magnetic media. We are looking for the steady state harmonic solution \( r = r_0 e^{i \omega t} \) in which the electron oscillates at the same frequency of the electric field. Substituting this solution into eq. (1.8) we obtain:

\[
-m \omega^2 r_0 + i \omega b r_0 + k r_0 = -eE_0 - i \omega \mu_0 r_0 \times H
\]

(1.9)

Assuming \( H = H_z \hat{z} \), and using the notation \( r_0 = x \hat{i} + y \hat{j} + z \hat{k} \) and \( E_0 = E_{0x} \hat{i} + E_{0y} \hat{j} + E_{0z} \hat{k} \), the equation (1.9) yields to three scalar equations, one for each coordinate

\[
\begin{cases}
(-m \omega^2 + i \omega b + k) x + i \omega \mu_0 y H = -e E_{0x} \\
(-m \omega^2 + i \omega b + k) y - i \omega \mu_0 x H = -e E_{0y} \\
(-m \omega^2 + i \omega b + k) z = -e E_{0z}
\end{cases}
\]

(1.10)
These equations can be written as

\[
\begin{align*}
\left(-\omega^2 + i\omega \frac{b}{m} + \left(\frac{k}{m}\right)^2\right)x + i\omega \frac{e\mu_0 H}{m} y &= -\frac{e}{m} E_{0x} \\
\left(-\omega^2 + i\omega \frac{b}{m} + \left(\frac{k}{m}\right)^2\right)y - i\omega \frac{e\mu_0 H}{m} y &= -\frac{e}{m} E_{0y} \\
\left(-\omega^2 + i\omega \frac{b}{m} + \left(\frac{k}{m}\right)^2\right)z &= -\frac{e}{m} E_{0z}
\end{align*}
\]

or, by using a vectorial notation

\[
\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{e}{m} \begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix}
\]

\[
\mathbf{A} = \begin{pmatrix} \omega_0^2 - \omega^2 + i\gamma\omega & +i\omega \omega_c & 0 \\ -i\omega \omega_c & \omega_0^2 - \omega^2 + i\gamma\omega & 0 \\ 0 & 0 & \omega_0^2 - \omega^2 + i\gamma\omega \end{pmatrix}
\]

\(\omega_0 = \sqrt{k/m}\) is the resonance frequency, \(\gamma = b/m\) is the damping parameter and \(\omega_c = \frac{e\mu_0 H}{m}\) is the cyclotron frequency. If \(n\) is the average number of electrons per unit volume, the electric polarization of the medium is given by \(\mathbf{P} = -n \mathbf{e} \mathbf{r}\). From the eq (1.12), it results

\[
\mathbf{P} = \frac{ne^2}{m} \mathbf{A}^{-1} \mathbf{E}_0
\]

This equation can be expressed in the following form

\[
\mathbf{P} = \varepsilon_0 \chi \mathbf{E}_0
\]
From this equation, we can define the polarizability tensor $\chi$ as the quantity that relates the polarization and the applied electric field, multiplied by the vacuum permittivity $\varepsilon_0$. We can thus write $\chi$ as:

$$\chi = \frac{ne^2}{\varepsilon_0 m} \Lambda^{-1} = \begin{pmatrix} \chi_{11} & \chi_{12} & 0 \\ -\chi_{12} & \chi_{11} & 0 \\ 0 & 0 & \chi_{33} \end{pmatrix}$$  \tag{1.16}

where

$$\chi_{11} = \frac{ne^2}{\varepsilon_0 m} \left( \frac{\omega_e^2 - \omega^2 + i\gamma \omega}{(\omega_e^2 - \omega^2 + i\gamma \omega)^2 - \omega^2 \omega_e^2} \right)$$  \tag{1.17}

$$\chi_{12} = \frac{ne^2}{\varepsilon_0 m} \left( \frac{i \omega \omega_e}{(\omega_e^2 - \omega^2 + i\gamma \omega)^2 - \omega^2 \omega_e^2} \right)$$  \tag{1.18}

$$\chi_{33} = \frac{ne^2}{\varepsilon_0 m} \left( \frac{1}{(\omega_e^2 - \omega^2 - i\gamma \omega)} \right)$$  \tag{1.19}

The effect of the applied magnetic field is to introduce non-diagonal antisymmetric terms in the polarizability tensor. For zero applied field the cyclotron frequency $\omega_e$ is zero and the polarizability reduces to a diagonal tensor with equal terms, in the form assumed by isotropic materials.

From the polarizability tensor, we can define the dielectric tensor $\varepsilon$ by the relation:

$$\varepsilon = \varepsilon_0 (I + \chi)$$  \tag{1.20}

where $I$ is the identity matrix.

1.2.2. Indices of refraction

The optical properties of materials are given by the Maxwell’s equations and the constitutive relations which express the specific material properties. The equations we need are:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$  \tag{1.21}

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}$$  \tag{1.22}
With the current density given by:

\[ \mathbf{J} = \frac{\partial \mathbf{P}}{\partial t} = \varepsilon_0 \chi \frac{\partial \mathbf{E}}{\partial t} \]  \hspace{1cm} (1.23)

\( \mathbf{B} \) and \( \mu_0 \mathbf{H} \) have been used interchangeably since at the optical frequency the relative at the optical frequencies the relative permeability \( \mu/\mu_0 \) may be considered as one. Then eq. (1.22) can be written:

\[ \nabla \times \mathbf{H} = \varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} \]  \hspace{1cm} (1.24)

where

\[ \varepsilon_r = 1 + \chi \]  \hspace{1cm} (1.25)

Note that \( \varepsilon_r \) and \( \chi \) are tensorial quantities. They will depend on the applied magnetic field or the magnetic state of the material, and it is their nonscalar character that gives rise to the magnetooptical effect.

Consider a plane wave propagating through the medium:

\[ \mathbf{E} = \mathbf{E}_0 e^{i(t-\mathbf{k} \cdot \mathbf{r})}, \quad \mathbf{H} = \mathbf{H}_0 e^{i(t-\mathbf{k} \cdot \mathbf{r})} \]  \hspace{1cm} (1.26)

we can take the curl of eq. (1.21) and use the eq. (1.24)

\[ \nabla \times \nabla \times \mathbf{E} = -\mu_0 \nabla \times \frac{\partial \mathbf{H}}{\partial t} = -\mu_0 \frac{\partial \nabla \times \mathbf{H}}{\partial t} = -\mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2 \mathbf{E}}{\partial t^2} \]  \hspace{1cm} (1.27)

through the identity

\[ \nabla \times \nabla \times \mathbf{E} = \nabla \left( \nabla \cdot \mathbf{E} \right) - \nabla^2 \mathbf{E} \]  \hspace{1cm} (1.28)

using the notation

\[ \mathbf{E}_0 = E_{0x} \hat{i} + E_{0y} \hat{j} + E_{0z} \hat{k} \]  \hspace{1cm} (1.29)

\[ \mathbf{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k} \]  \hspace{1cm} (1.30)

where \( \hat{i}, \hat{j}, \hat{k} \) are the unit vectors of the coordinate system. We can then solve eq. (1.28)

\[ \nabla \cdot \mathbf{E} = -\left( k_x E_{0x} + k_y E_{0y} + k_z E_{0z} \right) e^{i(t-\mathbf{k} \cdot \mathbf{r})} \]  \hspace{1cm} (1.31)
\[ \nabla (\nabla \cdot \mathbf{E}) = \begin{pmatrix} k_x^2 & k_x k_y & k_x k_z \\ k_y k_x & k_y^2 & k_y k_z \\ k_z k_x & k_z k_y & k_z^2 \end{pmatrix} \cdot \mathbf{E}_0 = \mathbf{k} \cdot \mathbf{E}_0 \] (1.32)

\[ \nabla^2 \mathbf{E} = \left( k_x^2 \mathbf{E}_{0x} + k_y^2 \mathbf{E}_{0y} + k_z^2 \mathbf{E}_{0z} \right) e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} = \left( k^2 \mathbf{I} \right) \cdot \mathbf{E}_0 \] (1.33)

where \( \mathbf{I} \) is the unit matrix. Thus, we can write

\[ \left\{ k^2 \mathbf{I} - k_0^2 \varepsilon_r - \mathbf{k} \mathbf{k} \right\} \cdot \mathbf{E}_0 = 0 \] (1.34)

in which we have substituted

\[ \omega / c = k_0 = 2\pi / \lambda_0 \] (1.35)

where \( \lambda_0 \) is the wavelength of the light in vacuum. The propagation vector \( \mathbf{k} \) is, in an absorbing medium, complex. Thus \( \mathbf{k} = k_1 + i k_2 \) where \( k_1 \) and \( k_2 \) are real vectors. The real part \( k_1 \) gives the normal to the wave fronts, the imaginary part \( k_2 \) gives the normal to the planes of constant intensity.

If \( k_1 \) is not parallel to \( k_2 \), the wave is said to be inhomogeneous. Inhomogeneous waves will result when light is obliquely incident on the surface of an absorbing medium. From eq. (1.35) the index of refraction is given by:

\[ n = k / k_0 = \left[ k \cdot k \right]^{1/2} / k_0 \] (1.36)

Equation (1.34) becomes

\[ \left\{ n^2 \mathbf{I} - \mathbf{k} \mathbf{k} / k_0^2 - \varepsilon_r \right\} \cdot \mathbf{E}_0 = 0 \] (1.37)

Equation (1.37) has non-trivial solution for \( \mathbf{E}_0 \) only if the determinant of the coefficient vanishes. This condition then determines \( n \) which will depend on the directions of the real and imaginary parts of \( \mathbf{k} \).

The magneto optic effect arises from the form of \( \varepsilon_r \) in the presence of a magnetic field or a spontaneous magnetization. In the following we will use the magnetization \( \mathbf{M} \) even if the same reasoning can be done for diamagnetic materials substituting \( \mathbf{H} \) to \( \mathbf{M} \). For cubic crystal \( \varepsilon_r \) has the form:
where $\varepsilon_{12}$ is an odd function of $M$, whereas $\varepsilon_{11}$ and $\varepsilon_{33}$ are even function of $M$. With this form for $\varepsilon$, the secular determinant of (1.37) is

$$
\begin{pmatrix}
\varepsilon_{11} & \varepsilon_{12} & 0 \\
-\varepsilon_{12} & \varepsilon_{11} & 0 \\
0 & 0 & \varepsilon_{33}
\end{pmatrix}
$$

(1.38)

where $\varepsilon_{12}$ is an odd function of $M$, whereas $\varepsilon_{11}$ and $\varepsilon_{33}$ are even function of $M$. With this form for $\varepsilon$, the secular determinant of (1.37) is

$$
n^4 \{ \varepsilon_{11} + (\varepsilon_{33} - \varepsilon_{11}) \cos^2 \theta' \} - n^2 \{ (\varepsilon_{11}^2 + \varepsilon_{12}^2 + \varepsilon_{11} \varepsilon_{33}) \\
- (\varepsilon_{11}^2 + \varepsilon_{12}^2 - \varepsilon_{11} \varepsilon_{33}) \cos \theta' \} + \varepsilon_{33} (\varepsilon_{11}^2 + \varepsilon_{12}^2) = 0
$$

(1.39)

where

$$
\cos \theta' = k_z / k = k_z / nk_0
$$

(1.40)

The angle $\theta'$ is, in general, complex.

1.2.3. Magneto Optic Faraday

Let us consider the simplest case, that of a homogeneous wave (real and imaginary parts of $k$ parallel to each other) propagating along the direction of $H$ (the $z$ axis). This case yields to the Faraday effect. We have $k_x = k_y = 0$ and so from eq. (1.40)

$$
\cos \theta' = 1
$$

(1.41)

Eq. (1.39) becomes

$$
n^4 \varepsilon_3 - n^2 2\varepsilon_{11} \varepsilon_{33} + \varepsilon_{33} \left( \varepsilon_{11}^2 + \varepsilon_{12}^2 \right) = n^4 - n^2 2\varepsilon_{11} + \left( \varepsilon_{11}^2 + \varepsilon_{12}^2 \right)
$$

(1.42)

And the solutions are

$$
n_z^2 = \varepsilon_{11} \pm i \varepsilon_{12}
$$

(1.43)
Substituting back to (1.37) we find the system for finding the components of $\mathbf{E}_0$

$$
\begin{pmatrix}
 n^2 - \varepsilon_{11} & -\varepsilon_{12} & 0 \\
 +\varepsilon_{12} & n^2 - \varepsilon_{11} & 0 \\
 0 & 0 & n^2 - \frac{k_z^2}{k_0^2} - \varepsilon_{33}
\end{pmatrix}
\begin{pmatrix}
 \pm i\varepsilon_{12} \\
 +\varepsilon_{12} \\
 0
\end{pmatrix}
\cdot
\mathbf{E}_0
= 0
$$

(1.44)

The solution are

$$(E_{0x}, E_{0y}) = (1, \pm i) \quad ; \quad E_{0z} = 0$$

(1.45)

The upper sign corresponds to a right circularly polarized (RCP) wave and the lower sign to an LCP wave. These are the proper modes for propagation parallel to $\mathbf{H}$. Each circular polarization has its own index of refraction, as shown by eq. 1.44.

Then, if we consider a linear polarized wave at $z = 0$ propagating along the $x$-axis, then in the medium the wave is given by

$$
\mathbf{E} = \frac{1}{2} E_0 (\hat{x} + i \hat{y}) e^{i \left( \omega t \frac{2\pi n_z}{\lambda_0} z \right)} + \frac{1}{2} E_0 (\hat{x} - i \hat{y}) e^{i \left( \omega t \frac{2\pi n_z}{\lambda_0} z \right)}
$$

(1.46)

as we expressed the linearly polarized wave as the sum of an RCP wave and a LCP wave. Eq. (1.46) can be rewritten as:

$$
\mathbf{E} = \frac{1}{2} E_0 e^{i \left( \omega t \frac{2\pi}{\lambda_0} \right)} \left( \hat{x} \cos \frac{\delta}{2} + \hat{y} \sin \frac{\delta}{2} \right)
$$

(1.47)

where

$$
\overline{n} = \frac{1}{2} (n_+ + n_-)
$$

(1.48)

and

$$
\delta = \frac{2\pi}{\lambda_0} (n_+ - n_-) z
$$

(1.49)
After propagating a distance \( l \), the wave will be elliptically polarized with a rotation angle of the major axis of the ellipse

\[
\theta_f = \text{Re} \left( \frac{\delta}{2} \right) = \text{Re} \left( \frac{\pi l}{\lambda_0} (n_+ - n_-) \right) \tag{1.50}
\]

and ellipticity [from eq. (1.4)]

\[
\epsilon = \text{Im} \left( \frac{\delta}{2} \right) = -\tanh \left( \text{Im} \left( \frac{2\pi}{\lambda_0} (n_+ - n_-) \right) \right) \tag{1.51}
\]

From equation (1.48) we can write

\[
n^2_+ - n^2_- = (n_+ + n_-)(n_+ - n_-) = i\epsilon_{12} \tag{1.52}
\]

And assuming that the difference between the \( n_+ \) and \( n_- \) is small compared to the average value \( n_0 \) and then approximate the difference as

\[
(n_+ - n_-) \approx i \frac{\epsilon_{12}}{n_0} \tag{1.53}
\]

From eq. (1.54), the definition of cyclotron frequency (eq. (1.55)) and eq. (1.25), we obtain

\[
\chi_{12} = \frac{n e^2}{\epsilon_0 m} \left( \frac{i \omega \omega_c}{(\omega_0^2 - \omega^2 + i\gamma_0 \omega)^2 - \omega^2 \omega_c^2} \right) \tag{1.54}
\]

\[
\omega_c = \frac{e\mu_c H}{m} \tag{1.55}
\]

The final result is

\[
\theta_f = VLH \tag{1.56}
\]

where \( V \) is the Verdet constant and is a characteristic of the material.

For the case of the ferromagnetic material, the same result can be achieved through a quantum theory in order to find the expression of the polarizability tensor, see Ref. [2].
1.2.4. Magneto Optic Kerr Effect

The analogous of the Faraday effect in reflection is called Kerr effect. It is possible distinguish 3 different geometries for the Kerr effect, as show in Figure 1. In the polar configuration \( M \) is normal to the surface of reflection. In longitudinal Kerr effect \( M \) is parallel to the surface and is in the plane of incidence. In the transverse effect \( M \) is parallel to the surface and is perpendicular to the plane of incidence.

![Kerr Effect configuration](image)

Figure 1: Kerr Effect configuration; (a) Polar (b) Longitudinal (c) Transversal. Figure from [2].

The calculation of the reflection coefficient involves the application of the boundary conditions at the surface to the reflecting medium. These conditions require the continuity of the normal component of \( H \) and the tangential component of \( E \). Consider a plane wave with propagation vector \( k \), as shown in Figure 2 (incoming wave). Let be \( k_r \), the wave vector of the reflected wave and \( k_m \) the propagation vector of the transmitted wave inside the medium.

The continuity of the tangential component of \( E \) on the surface leads to

\[
(E_r e^{ik_r \cdot r} + E_r e^{ik_r \cdot r} - E_m e^{ik_m \cdot r}) \times \hat{n} = 0
\]

where the normal is directed along the positive \( z \) axis. This equation must be true for all \( r \), then

\[
k_r \times \hat{n} = k_m \times \hat{n}
\]

These relations express the reflection and refractions laws. Since \( k_r = k_m \), we obtain

\[
\sin \theta_r = \sin \theta_i
\]

while with \( k_m = n_m k_i \) and assuming \( n = 1 \) for the vacuum, we get the Snell’s law in the form

\[
\sin \theta_i = n_m \sin \theta_m
\]
In an absorbing medium both $n_m$ and $\theta_m$ are complex. The complicating feature of the Kerr effect for oblique incidence is that the proper modes that are the solutions of (1.37) are not obtained (except for the transverse effect) by having an incident wave that is simply linearly or circularly polarized. With oblique incidence and with these simple polarizations for the incident light, the wave in the medium is a mixture of both proper modes. For normal incidence, however, and with M either parallel or perpendicular to the surface, the symmetry is high enough that these polarizations do give pure proper modes in the medium.

We start considering the polar Kerr effect at normal incidence. The proper modes are then RCP and LCP waves with indices of refraction given by (1.43). If $n$ is the index of refraction for a particular mode, the reflection coefficient is given by:

$$r = \frac{E_r}{E_i} = \frac{n-1}{n+1}$$

(1.61)

If we write this coefficient as

$$r = |r| e^{i\phi}$$

(1.62)

and considering RCP and LCP waves

$$\frac{r_+}{r_-} = \frac{|r_+|}{|r_-|} e^{i(\phi_+ - \phi_-)}$$

(1.63)
If the incident light is linearly polarized, it can be wrote as the sum of two circularly polarized RCP and LCP waves equal in amplitude. Then, the reflected light will be elliptically polarized because the circular components will no longer have equal amplitude after reflection and the major axis will be rotated from the direction of polarization of the incident light because of the phase difference introduced between these circular polarizations. This rotation angle is given by:

$$\theta_k = -\frac{1}{2}(\phi_+ - \phi_-)$$  \hspace{1cm} (1.64)

and the ellipticity is

$$\epsilon = \left| \frac{r_+ - |r_+|}{r_+ + |r_-|} \right|$$  \hspace{1cm} (1.65)

from the last equation, we obtain to the first order in $n_+ - n_-$

$$\theta_k = -\cos\gamma \left( \frac{n_+ - n_-}{n_+ n_- + 1} \right)$$  \hspace{1cm} (1.66)

$$\epsilon = \left( \frac{n_+ - n_-}{n_+ n_- + 1} \right)$$  \hspace{1cm} (1.67)

We present now the principal relations for the Kerr effect at oblique incidence, assuming only linear dependences on $M$. Thus, we take

$$\varepsilon_{11} = \varepsilon_{33} \approx n_0^2; \left| \varepsilon_{12} \right| \ll$$

The indices will be used to denote the polarization of the incident light, $s$ denoting polarization perpendicular to the plane of incidence, $p$ for the polarization in the plane of incidence.

**Polar Kerr Effect**

In this case, with $\mathbf{M}$ and $\hat{n}$ along the z-axis, the angle $\theta_m$ of (1.60) is the same as $\theta'$ in (1.39). We put $\gamma = \cos\theta$ and $\gamma' = \left[1 - \sin^2 \theta / n_0^2\right]^{1/2}$, the last relation being obtained from (1.60) and the approximation (1.68). Then the amplitudes of the reflected waves (the matrix of Fresnel coefficients) are given by:

$$r_{pp} = \frac{n_0^{\gamma - \gamma'}}{n_0^{\gamma + \gamma'}}$$  \hspace{1cm} (1.69)
\[ r_{ss} = \frac{\gamma - n_0 \gamma'}{\gamma + n_0 \gamma'} \quad (1.70) \]
\[ r_{sp} = r_{ps} = \frac{\varepsilon_{12}/n_0}{(\gamma + n_0 \gamma')(n_0 \gamma + \gamma')} \quad (1.71) \]

**Longitudinal Kerr Effect**

Let \( \gamma = \sin \theta \), \( \beta = \cos \theta \), and \( \beta' = \cos \theta' \), then the reflection coefficients are given by

\[ r_{pp} = \frac{n_0 \beta - \beta'}{n_0 \beta + \beta'} \quad (1.72) \]
\[ r_{ss} = \frac{\beta - n_0 \beta'}{\beta + n_0 \beta'} \quad (1.73) \]
\[ r_{sp} = r_{ps} = \frac{\gamma \beta \varepsilon_{12}/n_0^2}{\beta'(\beta' + n_0 \beta)(n_0 \beta + \beta)} \quad (1.74) \]

**Transverse Kerr Effect**

We put \( \cos \theta' = 0 \) since (1.42) since both the real and imaginary parts of \( k_m \) are perpendicular to \( M \). The proper modes from eq. (1.39) are \( s \) and \( p \) polarized wave with

\[ n_s^2 = \varepsilon_{33} = n_0^2, \quad n_p^2 = \varepsilon_{11} + \varepsilon_{12}^2 \quad (1.75) \]

Thus, for this one case of oblique incidence the incident beam can readily be chosen to yield just one of the proper modes. The reflectivity of the \( s \) wave is only slightly effected by the magnetization. The reflected \( p \) wave, on the other hand, has an amplitude which is, to first order in \( \varepsilon_{12} \)

\[ r_{pp} = \frac{n_0 \cos \theta - 1}{n_0 \cos \theta + 1} \left[ \frac{1 + \sin(2\theta)\varepsilon_{12}/n^2}{n_0^2 \cos^2 \theta - 1} \right] \quad (1.76) \]
\[ r_{ss} = \frac{\cos \theta - n_0}{\cos \theta + n_0} \quad (1.77) \]

This leads to the conclusion that if unpolarized light is incident on an absorbing magnetic medium with \( M \) perpendicular to the plane of incidence, then a reversal of \( M \) will produce a change in the reflected power.
1.3. Magneto-Optic Effects in Ferromagnet

The description of the magneto-optic effects in ferromagnets has been focused on the explanation of the unusual large effect showed by ferromagnetic materials, for which magneto-optic effects are up to five orders of magnitude more intense than in non-ferromagnetic bodies under the same external applied magnetic field [6].

These effects are definitely connected to the ferromagnetic properties of the specimen, since for temperature higher than the Curie temperature of the materials they disappear along with the ferromagnetic behaviour [7].

The first attempt to explain the much stronger magneto-optics effects in ferromagnetic materials is to replace the external field with an internal effective field, called Weiss Field. The Weiss field was postulated to account the existence of the ferromagnetic order, and the nature of this field was explained by Heisenberg that ascribed the origin of magnetism to the exchange interactions among electrons. Even if Heisenberg’s exchange interaction correctly reveals the origin of magnetism as an effective magnetic field that aligns the individual spins, this field alone cannot be used to explain the Faraday effect. This is because it is not coupled to the electron motion, which determines the dielectric properties of a material.

Hulme [8] provided the first quantum description of the magneto-optic effect in 1932. He pointed out that the magneto-optic effect in ferromagnetic materials must be a consequence of the spin-orbit interaction. This interaction couples the magnetic moment of the electron with its motion, thus, connecting the magnetic and optical properties of a ferromagnet.

Spin-orbit coupling, proportional to $\sim \mathbf{p} \cdot \mathbf{s}$, results from the interaction of the electron spin $\mathbf{s}$ with the magnetic field the electron sees as it moves through the electric field $-\nabla V$ with momentum $\mathbf{p}$ inside the medium. The Spin-Orbit interaction explain also why for nonmagnetic materials this effect is not strong, although the spin-orbit interaction is present in all matter. For non-magnetic material, the equal number of spin-up and spin-down electrons cancels the net effect. For ferromagnetic materials, however, the effect manifests itself because of the unbalanced population of electron spins. Hulme calculated the two (right- and left- circularly polarized) refractive indices by the energy splitting due to the spin-orbit interaction, neglecting, however, the change of the wave function due to the spin-orbit interaction. This theory is unsatisfying because the quenching of the orbital angular momentum in ferromagnets gives no energy splitting.
Kittel was able to show that in case of quenching of the orbital angular momentum[9], which leads to a weak spin-orbit coupling, the magneto-optic effects arise because of the change in the electronic wave function due to spin-orbit interaction, neglected from Hulme.

The full derivation of these effects in ferromagnets was given by Argyres [7] using the band theory of metals within the framework provided by the semiclassical theory of radiation, in order to evaluate the first-order effect of the spin-orbit interaction on the optical properties of ferromagnetics.

The main task of the model is to calculate the current density induced in the system by an electromagnetic wave of certain frequency and, hence to find the conductivity and polarizability tensors of the system under consideration. The assumptions taken by Argyres make his theory applicable only to visible and ultraviolet frequencies, which in general is not a limitation, because this range of the electromagnetic spectrum is characterized by energies that indeed allow to probe magneto-optic effects in ferromagnetic materials.

1.4. MOKE Experimental Setup

Sec. 20 shows that the magneto-optical Kerr Effect is proportional to the magnetization of the sample. Summarizing it is possible to study the surface magnetic properties of a bulk magnetic material, magnetic thin films or nanostructures. It is possible to probe locally the magnetic properties of the sample, while retaining its surface sensitivity, which depends on the penetration depth of the visible light (about 50nm), while it cannot be increased, MOKE measurement technique are valid also for the limit of ultra-thin magnetic layer, which can, in principle, be as thin as an atomic layer. It is possible to completely characterize the material with a quantitative evaluation of the magneto-optical effects, in terms of, arbitrary magnetization orientation and of the magneto-optical constants [10][11][12].

We start discussing, first the working principle of the experimental method, in order to introduce, later on, the instrumental setup.

Consider polar or longitudinal configuration and a linear p-polarized light reflected from a sample surface. If the sample is nonmagnetic, the reflected light is purely p polarized. If the sample is ferromagnetic then the reflection beam should consist of an s component, introduced by the magneto-optical Kerr effect, in addition to the dominant p component, with \( \frac{E_s}{E_p} \sim \) being the complex Kerr rotation. The measurement of the s component it is the goal of the experimental setup. The analyzer has to be set at a small angle from extinction. In this way, the intensity measured by the photodetector is:
\[ I \propto \left| E_p \sin \theta_a + E_s \cos \theta_a \right|^2 - |E_s|^2 \]  

(1.78)

From the definition of the Kerr angle and ellipticity of equations the equation (1.78) becomes:

\[ I \propto \left| E_p \right|^2 \left| \theta_o + \theta_k + i\epsilon \right|^2 + 2 \theta_o \theta_k \right\} = I_0 \left(1 + \frac{2 \theta_k}{\theta_o} \right) \]  

(1.79)

\[ I_0 = \left| E_p \right|^2 \cdot \theta_o \]  

is the intensity for zero Kerr effect, i.e. in the absence of magnetization, we have neglected. Since both \( \theta_k \) and \( \epsilon \) are, in first approximation, linearly proportional to the magnetization, the measured intensity as a function of \( H \) yields the magnetic hysteresis loop.

Transverse Kerr effect involves a change of the reflectivity of the light polarized parallel plane, which depends upon the component of the magnetization perpendicular to the plane of incidence [13]. First to be noted, is that, in transverse configuration, i.e. \( m_x = 1 \); \( m_y = m_z = 0 \), the only magnetization dependent quantity is the reflection coefficient \( r_{pp} \), which implies that in principles the transverse hysteresis loop can be taken without a polarizer or analyzer. Although, the difference in the transversal and longitudinal Kerr effect can be exploited by changing the analyzer angle, rotating the analyzer angle \( \theta_a \) alters the dependence of the transmitted light on the two magnetization components. Consider the case where the magnetization lie on the plane of the sample, i.e. \( m_x^2 + m_y^2 = 1 \); \( m_z = 0 \) and a \( p \)-polarized incident wave. The reflected \( p \) wave will alter in amplitude due to the component of the magnetization perpendicular to the plane of incidence, while the variation of the \( s \) component in the reflected light is a result of the magnetization component parallel to the plane of incidence. By passing the light through the analyzer and change its angle, it is possible to transmit the \( s \)- or \( p \)- component (and blocking the other) in order to selectively detect a single component of the magnetization [13]. For \( \theta_a \approx 0^\circ \) only the \( p \)-component is transmitted after reflection and the signal is proportional to transverse magnetization of the sample, while, for \( \theta_a \approx 90^\circ \) the \( s \)-component is transmitted and the signal is proportional the longitudinal magnetization.

In general, it is possible to separate the different component of the magnetization, reconstructing the magnetization vector in the 3D space, exploiting the disparity of the reflection coefficients respect to the polarizer and/or analyzer angle, this is the basic principle of magnetometry [12][14] [15][16].
Our rather simple analysis shows that a MOKE experimental setup require five basic elements: A light source, a polarizer, an analyzer, an electromagnet and a photodetector. The experimental simplicity of the setup allows great flexibility.

Figure 3: Basic MOKE setup implementation.
2. Design, Implementation and Characterization of a Kerr Microscope

2.1. Basic microscope structure

Optical Microscopes are instruments that produce a magnified image of a sample in order to render the details of the sample visible to the human eye. The magnification is achieved by the optical train, which consist of a series of lenses and diaphragm. A generic structure of a reflection Microscope can be seen in Figure 4.

![Generic scheme of a reflected light microscope](image)

The purpose of the optical train is to guide the light from the illumination source onto the sample and, after the reflection, to the imaging system. The microscope optical train typically consists of the light source and collector lens, condenser aperture diaphragm, field diaphragm, objective, sample, again the objective after the reflection onto the sample and the detector. Is it possible also to add some light condition devices (such as a polarizer), in order to change the type of contrast of the image, the contrast can be function of spatial frequency, phase, polarization, absorption, fluorescence, off-axis illumination, and/or other properties of the specimen and illumination technique.
2.2. Köhler Illumination and Illumination Source

The first stage of the optical train is the illumination source, which contains the lamp and collector lens, and is crucial in determining the final resolution and contrast on the image. An evenly dispersed illumination at the specimen is achieved using the Köhler illumination method, with this method the system is arranged so that the image of the coil filament of the lamp is brought into focus at the plane of the aperture iris diaphragm, instead the field diaphragm is imaged in the same plane as the specimen and the imaging system, so that the image of the light source is completely unfocused in the plane of the specimen, which results in an even illumination at the sample even if the light source is intrinsically not uniform, such as a filament lamp. It is also possible to introduce a sintered or frosted glass between the lamp and the collector lens to diffuse the light.

The planes that appear simultaneously in focus are called conjugate planes, this planes are imaged into each other and can collectively be observed while examining a specimen through the Imaging system. With this type of illumination, it is possible to distinguish two sets of conjugate planes, the imaging-forming set and the illumination set, Figure 6.

This distinction between the sets of conjugate plane allows the introduction in the optical train of the two diaphragms: the condenser aperture diaphragm and the field diaphragm, see Figure 5. Usually these two are an adjustable iris-type diaphragm. The first one is conjugate with the lamp, which allows
to adjusts the angles of the light rays striking the sample, crucial in Kerr microscopy. The second one, is conjugate with the sample, and controls the size of the illuminating field imaged.

Figure 6: Sets of conjugate plane. The first is the image forming conjugate plane set, the second is the illuminating conjugate plane set.

2.2.1. Objectives and Resolutions

The most important imaging component in the optical microscope is the objective, a complex multi-lens assembly that focuses and correct the aberration of light waves originating from the sample. To note, that in reflected light systems, the objective serves a dual function: on the transmission path, it serves as a matching well-corrected condenser properly aligned; on the reflection path, it is an image-forming objective.
When the light passes through the lens it interferes with itself creating a ring-shape diffraction pattern, known as the Airy pattern, shown in Figure 8, which means that the various points of the specimen appear in the image as small patterns and not as points. The Airy disk represents the ultimate resolution achievable for an optical system, free from all the other sources of non-ideality (such systems are said diffraction limited).

A quantitative expression of the resolution can be derived theoretically given the optical parameters of the system and the average wavelength of illumination, from the dimension of the Airy pattern. The distance from the central peak at which the first minimum occurs can be shown to be [4]:

\[
d = 1.22 \cdot \frac{\lambda}{n \cdot \sin\theta} = 1.22 \cdot \frac{\lambda}{NA} = \text{Resolution}
\]

where \(\lambda\) is the average wavelength of the illumination source, \(\theta\) is the half of the light cone opening angle produced by the objective and \(n\) is the refractive index of the medium between the objective and the sample (\(n=1\) in the standard case of air). The product \(n \cdot \sin\theta\) is called numerical aperture (NA) of the objective and represents the angle of acceptance of the cone light of the objective. The numeric aperture determines the ability to gather light and to resolve fine specimen details. Higher values of numerical aperture permit higher acceptance angles, meaning a higher number of oblique photons coming from one point of the surface and entering the objective front lens. Higher NA values allow smaller structures to be visualized with better clarity.

From Eq. (1.80), it is possible to introduce the Rayleigh criterion on the possibility to resolve two different points of a sample: in order to distinguish two points of the sample, the two corresponding Airy patterns in the image must be separated in such a way that the first order minimum of one pattern corresponds to the maximum of the other. According to the Rayleigh criterion, the resolution of the
optical system is given by the distance of the first minimum to the center in the Airy pattern, i.e. from $d$ of equation (1.80).

![Figure 8: Rayleigh Criterion for the resolution of a diffraction limited system](image)

Another important parameter that characterizes an objective is the image brightness that it can produce, even though the actual image brightness is determined by the whole optical train of the microscope. For an objective, the brightness is governed by its transmittance and by its light-gathering power. The transmittance of an objective, which express the transmitted intensity relatively to the incident intensity, is due to light absorption and reflection losses at optical interfaces and is wavelength-dependent. Hence, an objective must be chosen carefully in order to match its transmittance curve with the spectrum of the light source employed. Regarding the light-gathering power $F$ of an objective, it can be expressed as a function of the numerical aperture $NA$ and of the lateral magnification $M$ produced by the objective:

$$F \propto \left[ \frac{(NA)^2}{M} \right]^2$$
2.2.2. Optical Microscope for Polarization Contrast

Contrast in microscopy can be defined as the change of the intensity of a particular feature of the sample with respect to the background intensity or the surrounding of the features. The ability of a detail to stand out against the background or other adjacent details is a measure of specimen contrast, which lead to a simple definition of the contrast:

\[
\text{Contrast } C = 100 \cdot \frac{I_{\text{sample}} - I_{\text{background}}}{I_{\text{background}}} \quad (1.81)
\]

Contrast is not an intrinsic property of the specimen, but it is dependent upon interaction of the specimen with light and the ability of the optical system to distinguish the particular type of contrast. Different techniques can be employed to increase the contrast or change the type of contrast, such as, differential interference contrast, polarized light, phase contrast, Hoffman modulation contrast, and darkfield. Each technique is sensitive to a different type of contrast arising from different properties of the sample.

In particular, Kerr microscopy is a polarization-based contrast technique and relies on the rotation of the incident light polarzation plane induced upon reflection by materials with different magnetic properties or state. This type of technique requires the addition of, at least, two optical elements in the optical train, as shown in Figure 9: a polarizer and an analyzer. The polarizer set the polarization of the incidence light therefore must be placed before the sample. Instead the analyzer (a second polarizer) is placed after the sample in order to study the change of polarization state after reflection.

![Figure 9: Optical train modification for polarization contrast microscopy.](image-url)
Since magneto-optical effect arise from magnetic circular birefringence and magnetic circular dichroism it is important to ensure that lens elements, optical cements, and antireflection coatings are free of both strain and birefringent materials that might worsen the contrast arising from the specimen birefringence. In particular, as the most complex optical element in the microscope, the objective has to be strain free, since strain in the objectives lenses leads to birefringence, thus compromising the performance of the setup [23].

2.2.3. Optical Train Modification for MOKE Microscopy

The main modification regards the optical train of the microscope. Because different component of the magnetization requires different angle of incidence to be probed, it is necessary to have the possibility to select only a particular angle of incidence. The condenser aperture diaphragm is responsible for the angle of incidence of the ray bundle onto the sample. This allows the introduction of a custom slit in order to select a particular angle of incidence. The position of the slit, together with the numerical aperture of the objectives determine the final angle of incidence.

![Figure 10](image)

Figure 10: (a) change of polar Kerr intensity with numerical aperture NA for air ($n_{\text{inc}} = 1$) and oil immersion ($n_{\text{inc}} = 1.518$) objective lenses. (b) Corresponding change of longitudinal Kerr intensity. Typical values of NA for individual objective lenses of different magnification are indicated. Image adapted from [24].
The reflection coefficients strongly depend on the angle of incidence of the light. In Figure 10 are shown the dependency of the longitudinal and polar reflection coefficients.

The polar component of the magnetization, require low angle of incidence, that can be selected by taking the central portion of the ray bundle, instead, the longitudinal component requires wider angle of incidence that can be selected with an off-centered slit.

In order to correctly position the slit, it is needed an image of the condenser aperture plane, this can be obtained by the use of a Bertrand lens. The Bertrand lens allows to image the illumination set conjugate planes, which includes the condenser aperture plane. The image formed by the Bertrand lens is called Conoscopic image, characteristic of this image is that, the distance of a point from the optical axis is a (monotonous) function of the angle of beam inclination.

The conoscopic image of an optical microscope (when the polarizer and the analyzer are crossed for maximum extinction) presents a cross shaped extinction pattern (see Figure 11). Instead of a uniform dark image as in the ideal case, which means that all the beams not lying in a central incidence plane along or perpendicular to the polarization plane are reflected in an elliptical and rotated polarization state in general and cannot be extinguished by the analyzer.

Once the conoscopic image is available, it is possible properly position the slit inside the extinction pattern in order to maximize the magneto-optic contrast. Ideally the slit should be placed inside the extinction pattern, which ensure that, the ray bundle selected have a well-defined polarization, set by the polarizer. Instead, opening the slit beyond the extinction pattern, increase the illumination intensity onto the sample, due its polarization state, this component of the conoscopic image give no magneto-optic contrast, but increase the background illumination onto the imaging system.

In order to place the slit in the condenser aperture plane the condenser aperture diaphragm has been removed and replaced with a custom slit-holder. Two movement are available for the slit, it is possible to move the slit-holder along the optical axis in order to match the condenser aperture for the different objectives of the microscope, the second movement is perpendicular to the optical axis, with a micrometric stage, in order to match the extinction pattern of the microscope.

![Figure 11: Slit position for the two different components of the magnetization in plane.](image-url)
Based on the nomenclature introduced in Sec. 1.2.4, both configuration b and c of Figure 11 are longitudinal. To distinguish between the two, configuration b will be called Longitudinal (L) while configuration c will be called Longitudinal with Transverse sensitivity (LT).

2.2.4. Digital Camera

The image sensor of a digital camera (schematic in ) consists of an array of photosensitive elements (pixels). Mainly there are two different type of technology for the architecture for the image sensor, CCD and CMOS. The main difference between the two technologies is in the pixel structure. In CMOS sensors, each pixel is composed of a photodiode-amplifier pair, thus the conversion into voltage is performed inside each pixel. Instead, for CCD sensors, the photoelectrons are converted into voltage at a common port. Thus, the acquisition speed of the CCD sensor is limited respect the CMOS sensor, the latter used in our Kerr system.

The output signal of the sensor is proportional to the electrical charge accumulated, produced by photoelectric effect, during the exposure time, by each pixel in response to irradiation. Photons are reflected from the specimen and redirected to the CMOS sensor and their absorption produces electron-hole pairs with a conversion efficiency $QE$ (Quantum Efficiency). The pairs are separated by the electric field of the biased MOS structure, so that a number of electron proportional to the number of impinging photons is collected in each pixel (MOS structure).
Photoelectrons are converted into voltage by each pixel's photodiode-amplifier pair. Because the conversion from charge to voltage is done in the pixel structure, the Analog-to-Digital conversion is conducted in parallel for each row.

Several parameters describe the performance of the digital camera:

- Number of pixel of the sensor.
- Pixel size, the effective dimension of the single pixel.
- Effective sensitive area = Number of pixel · Pixel size.
- Full well capacity, the maximum amount of charge that each pixel can store before saturation.
- Dynamic range, maximum output signal (Full well capacity) over the noise of the camera, usually expressed in decibel. The dynamic range characterizes the ability of the device to capture both low and high signal levels in one image.

\[
\text{Dynamic Range} \quad DR_{\text{dB}} = 20 \cdot \log \left( \frac{S_{\text{MAX}}}{N} \right)
\]  

- Digital output bit depth, the number of bit of the Analog-to-Digital Converter, the Full well capacity divided by bit depth establishes the limit of precision for each gray level after the quantization.
- Analog gain, on-chip gain that can multiply the analog signal prior the conversion into digital signal.
- Readout noise, describe all the sources of noise associated with converting the photoelectrons in a pixel to a digital number.
- Quantum Efficiency, wavelength dependent probability that a photon is converted to a photoelectron during the charge production process in the photodiodes.
- Frame rate, number of frames per second.
- Quantum efficiency, wavelength dependent fraction of photons converted into photo electrons

In the following table, we present the camera Hamamatsu C11440-36U used in the setup:
2.2.4.1. Resolution in Digital Camera

As seen in the paragraph 2.2.1, the resolution of the system is limit by the optical system, in order not to worsen the performance of the systems, the digital camera have to match this resolution.

The spatial resolution is determined by the number of pixels of the camera, higher spatial resolution images have a greater number of pixels within the same physical dimensions. As for the optical system, the resolution of the image is regarded as the capability of the digital image to reproduce fine details that were present in the sample.

The spatial resolution of a digital image is related to the spatial density of the of pixels contained in a digital image (known as the spatial sampling interval); Features seen in the microscope that are smaller than the digital sampling interval (have a higher spatial frequency) will not be represented accurately in the digital image.

The Nyquist criterion requires a sampling interval equal to twice the highest specimen spatial frequency to accurately preserve the spatial resolution in the resulting digital image, therefore, to capture the smallest degree of detail present in a specimen, sampling frequency must be sufficient so that two samples are collected for each feature, guaranteeing that the intensity of the feature is preserved in the imaging device.

2.2.4.2. Image Brightness and Bit Depth

After sampling is completed, the resulting data is quantized by the Analog-to-Digital Converter, to assign a specific digital brightness value to each sampled data point, ranging from black, through all the intermediate gray levels, to white.
The bit depth of the ADC determines how many levels are available for the quantization. Instead, the full well capacity divided by bit depth establishes the limit of precision for each gray level, it gives the value for how many electrons per gray level. A higher number of gray levels corresponds to greater bit depth and the ability to accurately represent a greater signal dynamic range.

The dynamic range and the bit depth of the camera's analog-to-digital converter are closely interrelated in that the total available signal relative to noise governs the maximum number of gray-level steps into which the signal can be divided. In order to represent subtle intensity differences in a digital image, it is necessary to discriminate as many gray-level steps as possible, and therefore a typical approach is to match analog-to-digital conversion bit depth to dynamic range. Note that an analog-to-digital converter with a bit depth specification that exceeds the dynamic range cannot attain its full theoretical range of grayscale discrimination (bit depth) because each gray-level step must correspond to a minimum difference of one signal electron.

Brightness should not be confused with intensity, which refers to the magnitude or quantity of light energy reflected from the sample and imaged by a digital camera. Instead, in terms of digital image processing, brightness is more properly described as the measured intensity of all the pixels after the image has been captured, digitized, and displayed. Pixel brightness is the only variable that can be utilized by processing techniques to quantitatively adjust the image.

2.2.5. Sources of noise

During image acquisition with electronic sensors noise superimposed on the signal is manifested as apparently random fluctuations in signal intensity. The three primary components of noise in a CMOS imaging system are photon noise, dark noise, and read noise, all of which must be considered in a calculation of signal-to-noise ratio. These sources of noise are called Temporal Noise [25]. An additional factor to be considered is that the values of incident and background photon flux, as well as quantum efficiency, are functions of wavelength, and when broadband illumination sources are employed, the calculation of signal-to-noise ratio requires these variables to be integrated over all wavelengths utilized for imaging.

$$\text{Signal to Noise Ratio } SNR = \frac{B \cdot QE \cdot S \cdot t_{exp}}{\sqrt{B \cdot QE \cdot (S + I_b) \cdot t_{exp} + B \cdot D \cdot t_{exp} + N_r^2}}$$  \hspace{1cm} (1.83)
QE = quantum efficiency [electrons / photons]
S = Signal [photons/sec pixel]
I_b = background intensity [photons/sec pixel]
N_r = readout noise [electron rms/sec]
D = Dark Current [electrons/(pixel ∙ sec)]
\( t_{\text{exp}} \) = exposure time or integration time [sec]
B = number of pixel Binned

Regarding the particular architecture of the CMOS image sensor, other sources of noise arises from the difference in the electron to voltage conversion inside each pixel, called Fixed Pattern Noise (FPN) [25]. These sources of noise are relevant when high spatial accuracy and precision is required (such as single molecule localization). Due to the pixel-to-pixel variability, the Dark Signal, read-out noise and the Photon-response vary throughout the sensor, which means that all noise variations must be considered on an individual pixel-to-pixel basis. Correlated Double Sampling (CDS) is one of the most suitable for suppressing FPN. technique consists of taking two samples from a signal, which are closely spaced in time. Then, the first signal is subtracted from the second one, hence, removing the low-frequency noise. The two values are then used as differential signals in further stages.

The QE of a camera is the wavelength-dependent probability that a photon is converted to a photoelectron. High QE is a fundamental attribute for obtaining high SNR, since higher QEs indicate higher sensitivity, as the probability of detecting the photon increase. Conversely, lower QE means more exposure time is required to reach similar SNR levels.

A different classification distinguishes noise sources on the basis of whether they are temporal or spatial. Temporal noise, by definition, varies with time, and can be reduced by frame averaging, whereas spatial noise cannot. Spatial noise is subject to at least partial removal by various frame subtraction algorithms, or by gain and offset correction techniques. The temporal noise category includes photon noise and dark (current) noise, which are both forms of shot noise, read noise (primarily from the output amplifier), and reset noise. Among potential spatial noise sources are factors that produce non-uniformity in pixel output, including photo response non-uniformity and dark current non-uniformity.

Read-Out noise
It is a combination of system noise components inherent to the process of converting CMOS charge carriers into a voltage signal for quantification, and the subsequent processing and analog-to-digital conversion, at the output of the camera in the dark and at zero integration time. The major contribution to read noise usually originates with the on-chip preamplifier. As we have seen, the structure of the image sensor introduces pixel variation. Usually the information of the manufacturer about the Read Noise is typically expressed in term of Median or Root Mean Square (rms) value of the noise of each pixel, but due to the pixel variation the rms read noise is a more meaningful metric than the median read noise. The frequency dependency of the read-out noise depends on the output amplifier stage, which means that, the required read-out rate or frame rate partially determine the read noise.

From the read-out noise, it is possible to express the dynamic range, as the ratio of the full well capacity and the read-out noise. This express the signal to noise ratio neglecting all the others source of noise, which mean that the dynamic range represent the SNR of the camera independently of how specific operating conditions.

When read noise is the dominant noise source, the SNR becomes equal to the total signal collected during the integration time divided by the read noise value, which corresponds to the dynamic range in the limiting situation in which the full well capacity of a sensor element is reached.

Read noise is typically presented as electrons, and is not affected by the wavelength of the signal. However, because the signal to be detected are the photons reflected from the sample, and the quantum efficiency of the photons to photoelectrons conversion is wavelength dependent, the read-out noise expressed in photons depends on the wavelength as the QE. This means that higher QE camera have lower read noise in photons.

**Shot Noise**

Due to the discrete nature of photons, any measurement of them has some uncertainty. In the case of CMOS sensor, this result from the inherent statistical variation in the arrival rate of photons incident on the sensor.

In general, this source of noise is called shot noise [26], and is applied to any noise component reflecting a similar statistical variation, or uncertainty, in measurements of the number of photons collected during a given time interval.
Photons absorbed have an average photon flux per pixel $\mu_{\text{ph}}$. The random statistical fluctuations in this rate are governed by Poisson statistics, which means that, the probability for $n$ arrivals during the exposure time is given by:

$$P = \frac{\left(\mu_{\text{ph}} \cdot t_{\text{exp}}\right)^n}{n!} \cdot e^{-\mu_{\text{ph}} \cdot t_{\text{exp}}}$$

(1.84)

Therefore, they have a standard deviation $\sigma_{\text{ph}}$ that is the square root of the number of photons.

$$\sigma_{\text{ph}} = \sqrt{\mu_{\text{ph}} \cdot t_{\text{exp}}}$$

(1.85)

After the absorption of incoming photons by a pixel, the flux of photons is converted into electrons which follow the same statistics. These electrons are characterized by a noise component $\sigma_{e}$, which also has a square root relationship with $\mu_{e}$. Assuming that we are dealing with ideal noise-free imager and noise-free electronics, the performance of the image sensor based system will be limited by photon shot noise. The maximum signal-to-noise ratio can be described as follows:

$$\left(\frac{S}{N}\right)_{\text{MAX}} = \frac{\mu_{e}}{\sigma_{e}} = \frac{\mu_{e}}{\sqrt{\mu_{e}}} = \sqrt{\mu_{e}}$$

(1.86)

The shot noise is more relevant when the photon flux is limited. The amount of shot noise inherent in the signal is a function of the amount of signal, the more signal you have, the smaller the fraction of the signal comes from shot noise.

While read-out noise can be considered a noise floor with regard to electronics sources, under typical illumination conditions, the photon shot noise constitutes a natural fundamental limit on noise performance of a CMOS camera in which read noise and dark current noise are reduced to their minimum levels, that can not be reduced by camera design factors.

Consequently, it is desirable to operate an imaging system under conditions that are limited by photon noise, with other noise components being reduced to relative insignificance. Under low light-level conditions (assuming negligible dark noise), read noise is greater than photon noise and the image signal is said to be read-noise limited. The camera exposure time (integration time) can be increased to collect more photons and increase the signal-to-noise ratio, until a point is reached at which photon noise exceeds both read noise and dark noise. At longer exposure times, the image is said to be photon-noise limited.

Because the photon shot noise can be reduced increasing the number of photon absorbed, under the same illumination condition, the SNR is limited by the number of electrons that can be stored in the
Furthermore, this means that changing $t_{\text{exp}}$ keeping the number of photon per pixel constant ($\mu_{\text{ph}}$ have to change inversely respect to $t_{\text{exp}}$) will not affect the SNR in the photon-noise limited condition.

Photon noise is often modeled using a Gaussian distribution whose variance depends on the expected photon count. This approximation is typically very accurate. For small photon counts, photon noise is dominated by other signal-independent sources of noise, and for larger counts, the central limit theorem ensures that the Poisson distribution approaches a Gaussian [27].

### 2.3. Magneto-optical contrast

A useful way to represent the polarization state of a wave are the so-called Jones vector. We can express the equation (1.3) with a two element vector:

$$\begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} \sim \begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix} e^{i\phi}$$

(1.87)

If we normalize the vector dividing by the norm $\overline{E}_0^2 = E_{0x}^2 + E_{0y}^2$, we remove the information about the intensity and the vector will represent only the polarization state. For linearly polarized wave ($\phi=0$), see eq.(1.5), the normalized Jones vector representing the state has the form:

$$\overline{\mathbf{E}} = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

(1.88)

For left and right circular polarization ($\phi=\pm 90^\circ$) is:

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

(1.89)

This representation is useful because we can express the sum of two waves with different polarization as the sum of their Jones vectors. Furthermore, this representation can be used to obtain the final
polarization state modified by a series of linear optical element. Each optical element is represented by a 2x2 matrix, called Jones matrix. If $\mathbf{E}$ is the initial Jones vector of the incident light on an optical path, the emerging light is characterized by a Jones vector given by:

$$
\mathbf{E} = \mathbf{J}_1 \mathbf{E} \mathbf{J}_2 \mathbf{E} \ldots
$$

(1.90)

where $\mathbf{J}_i$, with $i=1..n$, is the Jones matrix of the i-th elements

We have seen in Sec. 1.2.4, the formalism for the Kerr effect, understanding the signal output from a conventional MOKE experimental setup. It is possible to use the same formalism for the MOKE Microscope in order to find an analytic formula the magneto-optical contrast.

In Sec. 2.2.2, we introduced the contrast as the ability of a detail to stand out against the background or other adjacent details, which lead to the simple definition of contrast, where $I_s$ is the sample point considered intensity and $I_B$ is the intensity of the background:

$$
C = \frac{I_s - I_B}{I_B} = \frac{\Delta I}{I}
$$

(1.91)

in magneto-optic application the contrast is given by two opposite magnetizations, by the use of Jones matrix, we can describe the experimental setup show in Sec.1.4. We start with a $p$-linearly polarized wave. We will first consider the $r_{pp}$ reflection coefficient independent on the transverse magnetization. The reflected wave can be expressed as:

$$
\begin{pmatrix}
E'_{p} \\
E'_{s}
\end{pmatrix} =
\begin{pmatrix}
r_{pp} & r_{px} \\
r_{sp} & r_{ss}
\end{pmatrix}
\begin{pmatrix}
E_{p}^i \\
0
\end{pmatrix} =
E_{p}^i
\begin{pmatrix}
r_{pp} \\
r_{sp}/r_{pp}
\end{pmatrix}
\times
\begin{pmatrix}
1 \\
1
\end{pmatrix}
$$

(1.92)

using the definition of the Kerr angle and ellipticity, follows that:

$$
\begin{pmatrix}
1 \\
r_{sp}/r_{pp}
\end{pmatrix} =
\begin{pmatrix}
1 \\
\theta_k^p + i\varepsilon
\end{pmatrix}
$$

(1.93)

as for the example in sec.1.4, for a classic MOKE experimental setup, the output signal is the light intensity reflected from the sample, the analysis of the polarization state of light is performed through optical devices in order to extract information about the magnetization of the sample. Without any other optical component in the optical path, the intensity of the vector (1.93) is proportional to the
square of the magnetization, losing the distinction between the two saturation state. We have already discussed how the necessity of a second linear polarizer after the sample, known as analyzer. The Jones matrix for a linear polarizer is:

\[
\begin{pmatrix}
\cos^2 \theta_a & \cos \theta_a \sin \theta_a \\
\cos \theta_a \sin \theta_a & \sin^2 \theta_a \\
\end{pmatrix}
\]

(1.94)

and the intensity will be:

\[
I \propto \left| \begin{pmatrix}
\cos^2 \theta_a & \cos \theta_a \sin \theta_a \\
\cos \theta_a \sin \theta_a & \sin^2 \theta_a \\
\end{pmatrix} \right|^2 \left( 1 + \theta_k^p + i \epsilon \right)
\]

(1.95)

where \( \theta_a \) is the angle of the polarization axis of the analyzer with respect to the p-polarization. This result in:

\[
I \propto \left( \theta_k^p \right)^2 + \left( \epsilon \right)^2 \theta_a + \cos^2 \theta_a + \theta_k^p \sin 2 \theta_a
\]

(1.96)

as results, the intensity is proportional to the Kerr rotation and so the magnetization.

From equation (1.96) it is possible to give an analytic formula the Magneto-Optic contrast. In MOKE microscopy the contrast arises from domains with opposite magnetization. We can extend the definition of contrast for MOKE microscopy as the difference intensity of two opposite magnetizations and the average intensity. Each value of the magnetization is characterized by the Kerr rotation and Ellipticity, and therefore, since, they are both proportional to the magnetization, if a magnetic domain is characterized by the couple \( \left( \theta_k^p; \epsilon \right) \), then the magnetic domain magnetized in the opposite direction is characterized by \( \left( -\theta_k^p; -\epsilon \right) \). In order to distinguish the different magnetization between these two opposite domains a contrast must arise between the intensity of the two domains, we can define the magneto-optical contrast \( C_{MO} \) (also known as fractional MOKE signal [28]) from equation (1.91):

\[
C_{MO} = \frac{\Delta I}{I_{avg}} = \frac{I(\theta_k^p; \epsilon) \epsilon}{\frac{1}{2} \left[ I(\theta_k^p; \epsilon) - \epsilon \right]}
\]

(1.97)

However, experimentally there is always a degree of imperfection to the polarized light [28][29]. The first imperfection is the cross-shaped extinction pattern, in fig. REFEREF, which results in a non-uniform dark image, and the second one is that, even in the ideal case in which all the ray are polarized
in the same way as the polarizer, the extinction efficiency of the analyzer is in general less than 100%. This result is a non-zero minimum transmitted intensity, \( I_{\text{min}} \), through the analyzer. This can be represented though a constant depolarization fraction \( \gamma_D = I_{\text{min}}/I_0 \), where \( I_0 \) is the intensity prior to the optical train, from eq. (1.93) result, \( I_0 = 1 + \left(\theta^p_K\right)^2 + \left(\epsilon\right) \).

\[
C_{\text{MO}} = \frac{\Delta I}{I_{\text{avg}}} = \frac{I(\theta^p_K;\epsilon)}{1 + \frac{1}{2} \left[ I(\theta^p_K;\epsilon) \right]}
\]

This equation can be then expressed as function of the optical parameters, in our case only the analyzer angle. We can substitute the equation (1.96), for the intensity, in equation (1.98), which result in an analytic formula for the magneto-optic contrast:

\[
C_{\text{MO}} = \frac{2\theta^p_K \sin 2\theta_a}{\left(\theta^p_K\right)^2 + \left(\epsilon\right) \theta_a + \cos^2 \theta_a + \gamma_D I_0}
\]

2.4. Kerr images acquisition, analysis and post-processing

Kerr imaging as a general challenge: to obtain great magnetic domain contrast starting from magneto-optical effect that are quite weak by nature. Therefore, it is essential to maximize the signal-to-noise ratio (SNR) by proper optimization of experimental setup, measurement parameters and post processing data-treatment techniques. For a given plane of incidence and a given magnetic material, the SNR is mainly affected by:

(i) the angle between the entrance polarizer and the analyser;
(ii) the exposure time and the noise of the camera;
(iii) the digital image processing;

This section deals with optimization of these factors. In order to model the effect of each parameter on the SNR and to be able to experimentally compare different samples and setups, we firstly introduce a quality factor and then explain how each choice can affect it.
2.4.1. The ability to distinguish between magnetic states: the quality factor $Q$

The quality factor has to take into account the possibility to distinguish different magnetic states of the sample. The signal of each pixel in a single image is composed by the sum of magnetic signal, background and noise. After proper background subtraction (that involves several delicate procedures described in section 2.4.4), the image of the sample in a saturated magnetic state has to be uniform. In reality, defects and noise affect the values read by each pixel, spreading the distribution of pixel values around the saturation level, with a standard deviation depending on the specific instrumental conditions and parameters.

In a statical framework, the ability to distinguish magnetic “levels” depends on the difference between the mean value of the pixels signal at saturation ($\mu_{\text{pixel}}$) and from the standard deviation of the distribution ($\sigma_{\text{pixel}}$). Thus, how well the two saturated states are distinguishable is quantified in terms of the quality factor $Q$ defined as

$$Q = \frac{\mu_{\text{pixel}}(H = H_{\text{max}}) - \mu_{\text{pixel}}(H = H_{\text{min}})}{\sigma_{\text{pixel}}}$$

(1.100)

It is possible to introduce an empirical criterion based on $Q$ in order to resolve at least the two saturated states. For example, $Q=1$ means that the difference between the mean value of the two saturated states distribution is equal to a single standard deviation, and situation illustrated in Figure 13 for a Ta/CoFeB/BaTiO$_3$ heterostructure (see Sec.3.4). The greater the separation of the two statistical distributions, the higher the number of grey levels distinguishable for intermediate magnetization states. $Q=1$ is in most cases not sufficient to distinguish intermediate levels and the setup conditions must be optimized to increase its value (e.g. increasing the number of pixels or the exposure time). This criterion resembles the Rayleigh criterion for the resolution of a diffraction pattern described in section 2.2.1.
Figure 13: Statistical analysis of the pixels’ values distribution at negative and positive magnetic saturation. Measurement were performed on Ta/CoFeB/BaTiO₃ capacitors in polar configuration. The magnetically active area is 70µm x 70µm, the exposure time was \( t_{\text{exp}} = 100\text{ms} \) and the analyser angle \( \vartheta_a \approx 90^\circ \).

Of course, \( Q \) depends on the number of pixels considered in the analysis. A slightly different problem is to quantify the single pixel capability to distinguish different magnetic states (giving the spatial resolution). In this case, the same analysis must be done as a function of the integration time for each pixel. The situation will be described in sec. 3.4.5, where we discuss the role of the exposure time.

2.4.2. MOKE Microscope: Signal to Noise ratio

The magneto-optical contrast (section 2.3) does not define the quality of the image, because it just considers the efficiency of the Kerr effect but it doesn’t take into account the noise introduced by the acquisition system (i.e. the camera). Thus, we need then recall the SNR for a CMOS camera (introduced in Sec.2.2.5) to evaluate the global performance obtain in a given configuration:

\[
\text{SNR} = \frac{B \cdot QE \cdot S \cdot t_{\text{exp}}}{\sqrt{B \cdot QE \cdot (S + I_b) \cdot t_{\text{exp}} + B \cdot D \cdot t_{\text{exp}} + N_f^2}}
\]  

(1.101)
where $QE$ is Quantum Efficiency, $t_{\text{exp}}$ is the exposure time, $D$ is the dark current, $N_r$ is the readout noise, $B$ is the number of pixel binned, $I_b$ is the background intensity and $S$ the signal. When the imaging system is limited by the photon shot noise, the term $B \cdot D \cdot t_{\text{exp}} + N_r^2$ becomes negligible, and we can express the SNR as [24]:

$$SNR \approx \frac{B \cdot QE \cdot S \cdot t_{\text{exp}}}{\sqrt{B \cdot QE \cdot (S + I_b) \cdot t_{\text{exp}}}}$$

(1.102)

Introducing the expression of signal and background intensity (see section 2.3):

$$S = I(\theta_K^p; \epsilon)$$

(1.103)

$$(S + I_b) = \frac{1}{2} \left[ I(\theta_K^p; \epsilon) - \epsilon \right]$$

(1.104)

and substituting these definitions into eq. (1.102), the SNR becomes:

$$SNR = \sqrt{\frac{B \cdot QE \cdot t_{\text{exp}}}{E}} \cdot \frac{\theta_K^p \sin 2\theta_a}{\sqrt{\left(\theta_K^p\right)^2 + \left(\epsilon - \theta_a + \cos^2 \theta_a + \gamma_D I_0\right)}}$$

(1.105)

where $E$ is the photon energy.

From this equation, it is possible to understand how to maximize the SNR. The first term depends from the characteristics of the light source and from the camera settings. The SNR is proportional to the squared root of the exposure time $t_{\text{exp}}$, of the quantum efficiency $QE$ and the binning of the camera, as already discussed in Sec.2.2.5. From equation (1.105) and Figure 14a, it is clear that the optimum analyzer angle is greater than the Kerr angle $\theta_K$. When the depolarization factor $\gamma_D$ increases due to the non-ideal polarizers extinction ratio, the optimum $\theta_K$ moves at larger values and the SNR is in general reduced, as shown in Figure 14a. Figure 14b shows the comparison between the fractional MOKE signal $\Delta I/I_{\text{avg}}$ (efficiency of the Kerr effect versus analyser angle) and the SNR. While the fractional MOKE signal would prefer lower angles, to increase $\theta_a$ increase the number of photons reaching the camera, and this is advantageous in terms of shot noise. It follows that including the noise in these considerations, there would exist a compromise between the Kerr sensitivity (maximum with crossed polarizers) and the shot noise. As shown quantitatively in Figure 15b, the optimum angle
to minimize the signal to noise ratio has to be larger than the optimal value for the MOKE fractional signal.

![Figure 14](image-url)

Figure 14: (a) Normalized SNR versus analyser angle calculated by using eq. (1.105) for different value of the depolarization factor $\gamma_D$, for a Kerr angle of 0.15°. (b) Comparison between the SNR and the fractional MOKE signal. While the fractional MOKE signal decrease rapidly as we move the polarizer angle away from extinction, the SNR does not change as much, since the camera is able to collect more photon which reduce the noise of the camera (even if the fraction of photons that give rise to the MOKE signal is reduced)

2.4.3. Signal model and digital image processing

Due to the weak nature of the effect, the magneto-optical contrast is mainly achieved via signal and imaging post processing. During the thesis work, a lot of attention has been dedicated to measurement of hysteresis loops and two different type of corrections were developed:

(i) the subtraction from the hysteresis loop of the Faraday effect acting on the objective due to the presence of the magnetic field nearby the sample;
(ii) removal of experimental drifts from the hysteresis loop (electronics drifts, light source intensity variations, temperature-related effects, etc.);
(iii) specific corrections to the acquired frames (illumination inhomogeneity, drifts, etc.)

The Faraday effect (correction i) caused by the objective being close to the source of the magnetic field can be corrected by removing a linear slope from the hysteresis loop. The experimental drifts (correction ii) can alter measurements in such a way a correction is not straightforward. Nevertheless, if the variation is continuous and relatively small, as a first order approximation the effect is linear in “time” and can be corrected by subtracting a straight line that makes the last and the first point of a hysteresis loop equal.
Of course, the same issues affecting the hysteresis loops are present in the acquired frames the hysteresis loop is calculated from. The first operation performed on the collected images is the subtraction of a mean image to emphasize the magnetic variations caused by the response to the magnetic field: a background image that displays no or an averaged magnetic signal or a homogeneous magnetic information, which is then subtracted from the others image of interest [24]. In order understand the effect of this subtraction, a modelling is required. We present here a simple model for the magnetic signal of each image. We define the magnetic signal $S$ extracted from the image collected at a given field $H$ as the mean value of each pixel signal $I_{ij}$ within the magnetic Region Of Interest (ROI) from which the hysteresis loop is calculated:

$$S(H) = \sum_{(i,j)\text{MAGNETIC ROI}} I_{i,j}(H) / N_{\text{pixel}}$$

where $I_{i,j}$ is the intensity of the $i$-th, $j$-th pixel and $N_{\text{pixel}}$ is the total number of pixel in the selected area.

![Figure 15: (a) Scheme of the magnetic signal as measured. (b) Magnetic signal after image subtraction.](image)

The signal $S$ can be modelled as the sum of a magnetic contribution and a background produced by the unpolarized or depolarized part of the light. As illustrated in Figure 17a, The background represents a shift of the signal $S$ from zero and has to be properly removed in order to extract only the part of the signal sensitive to magnetization. By introducing the normalized magnetization

$$\mu = m / m_{\text{sat}} = \begin{cases} +1 & \text{if } m = +m_{\text{sat}} \\ -1 & \text{if } m = -m_{\text{sat}} \end{cases}$$

the hysteresis loop of Figure 15a can be written as
\[ S(H) = I_0 \cdot \left[ 1 + \mu K m_{\text{sat}} \right] \] (1.107)

So that, at positive and negative saturation the signal \((S_+ \text{ and } S_-)\) respectively becomes

\[ S(H > H_{\text{sat}}) = S_+ = I_0 \cdot \left[ 1 + K m_{\text{sat}} \right] \] (1.108)

\[ S(H < H_{\text{sat}}) = S_- = I_0 \cdot \left[ 1 - K m_{\text{sat}} \right] \] (1.109)

The background to be subtracted to the images in order to leave only the magnetization dependent part can be obtained as the average of \(S_+\) and \(S_-\)

\[ S_0 = S_{\text{mean}} = S(H) \bigg|_{\mu=0} = \frac{S_+ + S_-}{2} = \frac{I_0 \cdot \left[ 1 + K m_{\text{sat}} \right] + I_0 \cdot \left[ 1 - K m_{\text{sat}} \right]}{2} = I_0 \] (1.110)

The new set of frames where the non-magnetic part is removed can be written as

\[ S^*(H) = S(H) - S_0 = I_0 \cdot \mu(H) K m_{\text{sat}} \propto \mu(H) \] (1.111)

and the intensity of each pixel is now directly proportional to the magnetization and the loop is centered, as represented in Figure 15b.

Finally, it happens that the intensity of the light coming from the sample is not uniform even for a non ideal uniformity of the sample. This issue can arise from incident path or sometimes because the sample is not perfectly orthogonal to the objective. Since this situation happens in real experiments, we now discuss how to compensate for non-uniform illumination. For example, we consider the case in which the intensity of the light coming from the sample has a linear increase along one direction. In order to take into account this slope, we have to add a term to the signal proportional to the “row” or “column” of the pixels reached by the same intensity:

\[ S_{ij}^*(H) = I_0 \cdot \left[ 1 + \mu K m_{\text{sat}} \right] \cdot (1 + D_{ij} j) \] (1.112)

where we introduced the coefficient \(D_{ij}\) to take into account the variation of light versus the “column” number of the pixels. It follows that the signals at positive and negative saturation become

\[ S_{ij}^*(H > H_{\text{sat}}) = S_{ij}^{*+} = I_0 \cdot \left[ 1 + K m_{\text{sat}} \right] \cdot (1 + D_{ij} j) \] (1.113)

\[ S_{ij}^*(H < H_{\text{sat}}) = S_{ij}^{*-} = I_0 \cdot \left[ 1 - K m_{\text{sat}} \right] \cdot (1 + D_{ij} j) \] (1.114)

and the mean value of them is
\[ S_{ij}^0 = \frac{S_{ij}^+ + S_{ij}^-}{2} = I_0 \cdot (1 + D_{ij}) \]  

Equation (1.115)

By subtracting \( S_{ij}^0 \) from each frame, the corrected frames will result

\[ S_{ij}(H) - S_{ij}^0 = I_0 \cdot [1 + \mu(H) K m_{sat}] \cdot (1 + D_{ij}) - I_0 \cdot (1 + D_{ij}) = 
\]

\[ = I_0 \cdot [\mu(H) K m_{sat}] \cdot (1 + D_{ij}) = S_{ij}^r \]  

Equation (1.116)

As evident from equation (1.116) this operation does not remove the background completely because the presence of \( D_{ij} \) still affects the magnetic part of the signal. Unfortunately, the operation is able to remove the component related to the non-magnetic background. Since the magneto-optical signal is a small fraction of the overall intensity, the treatment strongly increases the uniformity.

2.4.4. Post processing routines

In the previous section, we have explained the main signal correction for MOKE imaging applications. We have thus introduced a simple model that explain the effect of this correction. In this section, will be presented the post processing routine adopted for the correction of hysteresis loop measurement. In order to explain the various steps, the measurement of a system introduced later in Sec. 3.4, is used as an illustrative example.

1) Region of interest selection

In order to obtain the hysteresis loop of a micro or nanostructure, the first step is to select the magnetic region of interest (ROI) from the captured frames. The magnetic signal at each value of the external magnetic field (or time) is then obtained by averaging pixel values over the ROI. Figure 16 shows the case of a rectangular ROI (any shape is allowed by the selection of a polygonal area). Note that every frame is acquired and saved, so that the system offers the possibility to study different regions of the sample by simply choosing a different ROI during the post-processing.
2) Hysteresis loop correction

As introduced in Sec. 2.4.1, the quality factor can quantify the possibility to distinguish magnetization states and hence the effect of the corrections made to the image. For this reason, in the following we will present at each step the hysteresis loop and the pixel values distribution of images at positive and negative saturation after the application of the corrections described so far, as in where the two are presented as measured previous to any correction.

The first step consists is the removal of the linear slope caused by the Faraday effect from the raw hysteresis loop shown in Figure 17 collected on Co/Fe/BaTiO$_3$, see section 3.4. The result of the subtraction is shown in Figure 20. The histograms do not change since any correction onto the hysteresis loop do not affected the pixels distribution but only the mean value.
Figure 18: Hysteresis loop correction step. The green line has been subtracted from the blue and red hysteresis loop to obtain the black marked line.

It can be readily observed from the histogram of Figure 18, is not Gaussian and this is ascribed to a non-perfect spatial uniformity of the illumination. This sentence will become clear afterwards.

3) Magnetic contrast enhancement

This is the main step in MOKE imaging application in order to show a magnetic contrast. From eq. (1.116), it follows this increases the spatial uniformity and enhance the magnetic signal, removing the non-magnetic background $I_0$. We have to obtain an image with no magnetic contrast, or, as in our case, averaged out. As show in Figure 19a, the mean image can be obtained from as average of all the positive and negative saturation

Figure 19: (a) Selection of several images at positive and negative saturation to produce a subtraction image with no magnetic contribution. (b) Subtraction image. (c) Intensity profile along the white line on the subtraction image.
The resulting subtraction image is shown in Figure 19b, this is the image $S_0$ of eq. (1.110) presented in the model of Sec. 2.4.3. While in panel c is shown the intensity profile along the white line indicated on the subtraction image, which show that the illumination is not uniform.

After subtraction, the two distribution becomes narrower and it is then possible to distinguish between different state of the magnetization. This is confirmed by the images, as shown in Figure 21. The histogram, besides the characterization of the performances of the setup, serves also to determine the scale of the magnetic image and adjust its contrast. Our image is represented by grayscale of fixed number of level. The grey scale Upper Limit (UL) and Lower Limit (LL) can be fixed as shown in Figure 20. In this way, we ensure that our image is represented by as many levels as possible.

Figure 20: after image subtraction step.

Figure 21: Image before and after the subtraction. It is clearly visible the magnetic information
We can also verify the intensity profile shown in Figure 19c, after the subtraction for a uniform saturated image. We can conclude that, the subtraction image itself resolve most of the problem of MOKE microscopy, even if it is possible to make further adjust to keep improving the quality factor.

![Intensity profile after subtraction](image)

Figure 22: Intensity profile after subtraction.

**Optional processing options**

We present here some of the tools occasionally used for the improvement of the image, based on different average. After optimization of a MOKE microscope the better way to increase the quality of the images is to average with different measurement. This require high level of replicability of the experiment and of the magnetic behaviour of the material. Magnetic sample or microscope movements of only a few nanometers lead to a degradation of the MO image.

If averaging on different measurement is not possible, a good comprise can be achieved averaging between different images of the same measurement, or averaging over a different number of pixel. This helps to reduce the noise but lower the temporal resolution and spatial resolution respectively.
2.4.3. General considerations

In this section, we aim to implement a protocol intend to guide the setup of the Kerr microscope and to guarantee the achievement of the best configuration for each kind of magnetic measurements.

First, we start from a general discussion of the following critical parameters, and their interplay in magnetic characterization:

- analyzer-/polarizer relative angle (defined as the angle from extinction of the analyzer, $\theta_a$)
- microscope objective numerical aperture (NA)
- camera exposure time ($t_{exp}$) or frame rate ($fps$)
- probed area of the sample ($A_{rot}$)
- number of collected images/average ($N_{avg}$)

As polarization-contrast enhanced technique, the first parameter that need to be under control, in MOKE microscopy, is the illumination onto the sample. Its intensity, stability and uniformity determine the quality of the final image produced. The microscope is equipped with a standard 12V 100W halogen lamp, with the only control of the supply voltage that must be kept constant to avoid drift.

The role of the angle between the entrance polarizer and the analyzer, $\theta_a$, is the main responsible for the magneto-optical contrast and defines the quality factor $Q$ of measurements. As explained in section 2.4.2, ideally, we are interested in measuring the projection of the light polarization, which is orthogonal to the original polarization plane. Nevertheless, by setting $\theta_a = 90^\circ$, the light reaching the camera after the analyser is strongly reduced, so that the camera could suffer the predominance of shot noise over signal. As in standard MOKE, the idea is to keep $\theta_a$ small enough for the approximation $\sin(\theta_a) \approx \theta_a$ to be still valid. The signal to noise ratio will depend on the intensity of the light collected from the objective, the noise of the camera and $\theta_a$. So, once the light source and the magnification (objective) have been selected, $\theta_a$ must be optimized to achieve the maximum S/N ratio in the image.

The temporal resolution achievable is directly related to the exposure time of the camera needed to acquire images with the desired quality. Of course, there’s a lower value that depends on the minimum exposure time the camera allows for ($t_{exp} \sim s$)
The area probed from the microscope depends from the microscope objective: the greater the magnification, the smaller the area of the sample imaged on the surface of the camera sensor. This will affect the spatial resolution: since the total number of pixel of the camera is fixed, each pixel corresponds to a sample area that depends directly on the magnification. The higher the magnification, the higher the spatial resolution in terms of squared \( \mu m \) per camera’s pixel, as discussed in the Section 2.2.1. Of course, to increase the magnification costs in terms of light intensity reaching the camera because the objective acceptance cone is reduced.

The microscope objective depends on the desired spatial resolution, the magnification required and in MOKE microscopy on the particular type of measure, in order to select different incidence angle. In order to resolve small details of the sample and select wider angle of incidence, high magnification and numerical aperture are required. Also, MOKE microscopy require a custom slit in order to select a particular angle of incidence, whose introduction further reduce the illumination.

Once the objective, the slit and the analyser angle are set, the signal collected by each pixel depends on the gain constant and the exposure time of the camera.

The gain of the camera ranges from 1 to 15.8 (in 240 steps) does not contribute to SNR because it just defines the analog multiplication occurring inside the camera before analog-to-digital conversion occurs. On the other hand, it is crucial to employ the higher value of the gain that permits to avoid pixel’s signal saturation to maximize the dynamic range and increase the number of grey levels distinguishable after analog to digital conversion with the fixed number of bits used to represent the single pixel.

The exposure time of the camera is somehow related to the possibility to integrate a given signal over time. In general, if time resolution is not required (static regime), the value of the exposure time can be increased at will within the range 26.17\( \mu s \) to 10s. Anyway, while the lower bound is fixed by the camera, an upper bound can be present due to possible image saturation. Of course, the increase of the exposure time implies an increased the number of photon collected per pixel with a consequential reduction of shot noise in the image, as shown in section 2.2.5, eq.(1.85). Nevertheless, measuring hysteresis loops with very long exposure time can be detrimental due to possible drifts caused by different sources: changes in the source light intensity, mechanical instability (drift of sample position on the micrometric scale due to temperature changes, vibrations, …), etc.

After this overview, in the next section we will present how to operatively optimize camera and microscope to perform good Kerr microscopy experiments.
2.5. Magnetic imaging

The knowledge of the magnetic domain behaviour in ferro- and ferri- magnetic materials is of great importance from a fundamental as well as from an application point of view. Magnetic domains imaging provides a way to access the local magnetic properties of materials down to the nanoscale. In conjunction with macroscopic magnetometry and simulations, the knowledge of magnetic domains behaviour gives unrevealed insight into the origin of magnetization reversal mechanisms.

Different methods can be employed to observe magnetic domain [17] and each method differs from the others in terms of the physical mechanisms to interact with sample magnetization, contrast mechanism, information depth and achievable spatial or temporal resolution.

Scanning electron microscopy with polarization analysis (SEMPA) directly provides an image of the surface magnetization of a sample [18]. By using medium energy electrons (10-50 keV), small probe (<50 nm) and high currents (> 1 nA), SEMPA is able to measure the spin polarization of the secondary electrons that exit from a magnetic specimen as the finely focused (unpolarized) beam of the scanning electron microscope rasters over the sample.

SEMPA exploit the fact that the polarization of the secondary electrons reflects the net spin density of the material under investigation. Depending on the electrons emission source, a lateral resolution between 10 and 50 nm can be achieved and the image is not affected by sample topography. Anyway, the probing depth is of the order of 1 nm, corresponding to the short escape depth of secondary electron. The typical image acquisition time span from 1 to 100 min, depending on required resolution, magnetization and image size. SEMPA has several advantages with respect to standard SEM, including: a large depth of field; easily variable magnification that permits to look at regions of the surface ranging from a few millimeters to a few hundred nanometers; a large working distance. On the negative side, stray magnetic fields (> 10 Oe) must be avoided and conductive samples are required to prevent charging effects,

Both convention Transmission Electron Microscope (TEM) and Scanning Electron Microscope (STEM) can be employed for magnetic imaging [18]. As for the SEMPA sub-50 nm resolution can be achieved. As the electron beam passes through the sample, the signal is the convolution of compositional, electronic, structural and magnetic properties and to extract the magnetic contrast is challenging. This problem is more severe in the case of multilayer systems, where the properties of layers at different physical locations in the stack can differ considerably. Moreover, to study the
response of the specimen to external magnetic fields is difficult because the field cause also the
deflection of the electron beam that must be compensated.

Magnetic imaging with x-rays take advantage of the existence of powerful sources of synchrotron
radiation [19]–[21]. The technique is based on different absorption of circularly polarized x-rays
depending on the sample magnetization (from which the name x-ray magnetic circular dichroism,
XMCD). The tunability of the x-rays energy and the availability of polarized linear and circular
radiation at modern synchrotron sources is one of the main advantage of this technique. By tuning
the x-rays energy to a particular absorption edge, elemental specificity can be achieved. It is therefore
possible separate the contribution of different layers and elements to the magnetism of a system,
allowing investigation of complex, multi-element, magnetic structures. Also, because the spin and
orbital moments can be determined in XMCD, the magnitude of the magnetization can be
quantitatively determined.

One of the most widely used technique for domain imaging at the nanometer scale is Magnetic Force
Microscopy (MFM) [22]. It is a relative inexpensive add-on to commercial Atomic Force Microscope
(AFM). The standard operational mode of AFM is the contact mode, in which a sharp-tip of 10 nm-
20 nm in radius mounted on a cantilever, is scanned along the surface of the sample in a non-
destructive manner detecting the deflection of the cantilever. This deflection is used to generate an
image of the topography of the sample. While, in non-contact mode, is possible to sense the long
range van der Waals force through its effect to the resonant frequency of the cantilever. Magnetic
sensitivity is obtained by coating the tip of the cantilever with a ferromagnetic coating to add a
magnetic interaction between the tip and the surface.

An AFM microscope includes: an x-y piezoelectric scanner; a tip acting as transducer for the
particular interaction considered; a laser and a four-quadrant detector to monitor tip movements; a z
piezoelectric actuator which control the distance between the tip and the sample. The distance of the
tip from the sample is controlled through an active feedback-loop whose input is the deflection from
the photodiode. In contact mode, the height can be kept constant and the feedback signal as function
of the position can be used as output signal in order to form the image. The most common non-contact
mode, generally referred to slope detection, involves driving the cantilever at a fixed frequency, with
the height piezoelectric actuator, slightly off its resonance frequency and measure the amplitude
variation given by the interaction with the sample. The shift respect to the resonant frequency
correspond to the maximum of the slope in the frequency versus amplitude curves, i.e. maximum
sensitivity in the amplitude respect to a variation of oscillation frequency.
Magnetic force microscopy is a special mode of noncontact operation of the scanning force microscope with a ferromagnetic tip.

Interpreting an MFM image means understanding the relationship between the two distributions of magnetic multipoles, the exact nature of which may vary to minimize the total magnetostatic energy of the system, the simplified model can be found in Ref. [22]. Generally, the MFM does not respond to the magnetic field, but is usually much more sensitive to the higher spatial derivatives of the field components. This means the MFM generally produces images that denote the positions of pseudo magnetic charge, e.g., the pseudo magnetic charge present inside of domain walls. Such a charge can be thought to arise from the divergence of the magnetization, i.e., \(-\nabla \cdot \mathbf{M}\), internally and at surfaces.

In standard non-contact, the signal arises both from topography and sample magnetization. The most popular method, called interleave or liftmode, to minimize the cross-talk between topography and magnetic signal relay on two different scans of the sample surface. First, a scan of the surface is performed in contact mode to determine the topography of the signal. The second scan, in which the magnetic signal is sensed, is made in non-contact mode, in while the tip is programmed to follow a constant height contour determined by the previous topography scan. In this way, separate topographic and magnetic images can be obtained.

While, high resolution in the range of 40-90nm can be achieved, Imaging is possible over a limited area of the sample and is in general slower respect the other method (5-30min per image), which means that can be useful to have other methods of locating the area of study.

During my thesis work, I used the Keysight 5600LS AFM system available at Polifab to perform magnetic imaging of tam-SPL patterned FM/AFM exchanged-biased structures described in 3.6.1. The study of micro-patterned magnetic samples by MFM will be compared with the analysis performed by using the Kerr microscope I developed.
3. Case Studies

- **Settling Time measurements**

In general, every measurement setup needs time to reach the thermal equilibrium, when all the electronics and all the components of the setup are switched on, in order to measure in the optimum condition and minimize every type of thermal and electronic drift. In an optical apparatus, the critical component that need to reach the thermal equilibrium is the illumination source.

In order to evaluate the settling time, we can measure the intensity of the image, with both the polarizer and analyser in the optical train, as function of time. The measurements is performed on a sample with high reflectivity (Au films).

![Settling time measure](image)

**Figure 23**: Settling time measure. In figure the settling time has been indicated with a vertical dotted line.
3.4. Artificial Multiferroics

This section presents the use of Kerr microscopy to investigate the magnetic properties of Ta/CoFeB/BaTiO$_3$. These heterostructures have been produced and studied by the NaBiS group in the context of the electric control of the magnetic anisotropy in ferromagnetic thin films by exploiting. The system exploits the magneto-electric coupling at the interface with a ferroelectric material [30]. In the context of this thesis, Ta/CoFeB/BaTiO$_3$ represents a suitable test case for microscope optimization in the polar configuration, i.e. with sensitivity to the out-of-plane component of the magnetization. During my activity, several magnetic measurements were performed on Ta/CoFeB/BaTiO$_3$: acquisition of hysteresis loops; visualization of magnetic domain formation; magnetization reversal processes and characterization of the noise sources. Such studies allowed to setup a protocol to define how to characterize the microscope performances and how to set the characterization, in order to regulate microscope settings and camera parameters in order to be able to distinguish the two magnetic saturated states and the states in between. This protocol offers the possibility to compare the microscope performances with other state-of-art Kerr microscopes in terms of signal-to-noise ratio, spatial and temporal resolution, noise sources.

3.4.3. Electric control of magnetization in FE/FM systems

The electric control of magnetic properties has received great attention from technological point of view. A way to flip the magnetization by using electric fields represents a promising route toward high density and power saving memory and logic devices. The electric control of magnetization would improve the miniaturization of magnetoresistive random-access memories (MRAM) overcoming the main obstacle in this sense, i.e. the power dissipation during writing [31]–[34]. Recently, there has been a renewed interest in the exploitation of ion migration for magnetization control. Huge magneto-ionic effects have been observed in inorganic devices due to oxygen ion migration toward or away from the interface between an oxide and an ultrathin ferromagnetic layer with perpendicular to plane magnetic anisotropy [35]. In electrolyte-solid systems, sizable anisotropy variations were also found [36], [37].
A great effort is put on the investigation of multiferroics, materials that exhibit more than one of the primary ferroic order parameters: ferromagnetism, ferroelectricity, ferroelasticity in the same phase. The coexistence and coupling between ferroelectricity and ferromagnetism within the same material phase would represent the more natural way to achieve the integrated electric control of magnetization. Unfortunately, a singlephase material showing a sizable interplay between ferroelectricity and ferromagnetism at room temperature has not been found yet. Instead, artificial multiferroic heterostructures, coupling ferroelectric (FE) and ferromagnetic (FM) materials with high Curie temperatures, are promising candidates to overcome the limitations of single phase materials.

The NaBiS group has been particularly active in the context of artificial multiferroics. In 2014, the group discovered a fundamental milestone. They demonstrated the fully reversible electrically-induced change of the interfacial magnetic order exploiting the magnetization at a FE/FM interface. The system of choice was the fully epitaxial Co/Fe/BaTiO3/La0.7Sr0.3MnO3 [38], with the Fe magnetization laying in the film plane. Unfortunately, the ferroelectric polarization switching of BaTiO3 was able to affect the magnetic properties of just one/two interfacial atomic layers of Fe.

To overcome this limitation, they focused on the realization of an heterostructure such that the FM layer (still on top of BTO) presents Perpendicular Magnetic Anisotropy (PMA). Since both the magnetoelectric coupling and PMA are strongly dependent on the interface and its modifications, it was expected that such a system would have to be more sensitive to the ferroelectric switching with respect to the previously investigate Co/Fe/BaTiO3/La0.7Sr0.3MnO3 [38] with Fe magnetization in the film plane. The NaBiS group focused its attention to the Ta/CoFeB bilayer deposited on top of the ferroelectric BaTiO3. Upon proper annealing treatment, the magnetic CoFeB thin films sandwiched between a transition metal (Ta) and an oxide (BTO) acquires PMA [30].
The growth of the heterostructure, starts from a commercial SrTiO$_3$ (STO) substrate on top of which a La$_{0.7}$Sr$_{0.3}$MnO$_3$ (LSMO) serves as conductive bottom electrode and improves the epitaxy of the BTO. Then, CoFeB (~1 nm) and Ta (~1 nm) have been deposited by magnetron sputtering. The tantalum layer serves as capping layer but also plays a fundamental role to achieve the out-of-plane magnetic anisotropy of the CoFeB layer upon proper post-growth annealing of the heterostructure with an external magnetic field in vacuum [40].

In order to be able to investigate the presence of magnetoelectric effects able to change the magnetic anisotropy of the CoFeB layer, the fabrication of properly designed devices was necessary. The NaBiS group fabricated for the scope arrays of microcapacitors, whose structure is sketched in Figure 24a. The fabrication has been carried out on annealed Ta/CoFeB(1.1 nm)/BTO/LSMO/STO(001) stacks by means of optical lithography, Ion Beam Etching, deposition of a SiO$_2$ capping layer and Au/Ti contacts. Arrays of capacitors with 100 × 70 μm$^2$ area have been obtained, with a sizable portion of the top Ta/CoFeB electrode clear of the thick Au/Ti contacts, thus allowing to probe the underlying CoFeB layer by MOKE measurements. Applying an external voltage to two capacitors from the top Au contact (in a so called top-top geometry), it was possible to polarize the BTO layer and obtain its ferroelectric switching.

The group demonstrated the possibility to modulate the coercive field of the out-of-plane hysteresis loop of the CoFeB layer, i.e. its out-of-plane anisotropy. Figure 24b shows two hysteresis loop measured with +7 or -7 V applied to the capacitors: the coercivity changes between 56 Oe and 35 Oe respectively, with a relative variation of about 60%. Noteworthy, distinct magnetic anisotropy states
can be written by voltage pulses of different variable amplitude, thus allowing for the non-volatile ferroelectric control of the magnetic properties of the CoFeB films. Furthermore, the group was able to demonstrate the magnetically assisted bipolar switching of the magnetization in CoFeB electrodes, requiring a magnetic bias field as low as 10 Oe and voltage pulses compatible with complementary metal-oxide semiconductor (CMOS) electronics [30].

The mechanism is described in Figure 25, showing the general idea behind ferroelectric-driven magnetization reversal in the Ta/CoFeB/BaTiO₃ system. Suppose to start with a ferroelectric inwards polarization (corresponding in Figure 24b to the remanent state at 0 V after application of a +7V pulse), the ferromagnetic loop presents the magnetic state with higher coercivity. The system is biased with an out-of-plane magnetic field in between the high and low values of the coercive fields, corresponding to the initial state A in Figure 25a. Then, a negative voltage pulse is applied to the heterostructure, in order to switch the BaTiO3 ferroelectric remanent polarization outwards, obtaining a shrink of the hysteresis loop of the FM (Figure 24b). The system jumps into the new stable state B having opposite magnetization (Figure 25b).

![Figure 25: Sketch of the mechanism leading to the magnetic flip upon electrically induced shrinking of the magnetic hysteresis loop.](image)

To better investigate the magnetic properties of CoFeB films with PMA, X-ray photoelectron emission microscopy (X-PEEM) measurements were carried out at the CIRCE beamline of ALBA Synchrotron Light Facility in Barcelona (Spain) [41]. Figure 26b shows a few dark domains in remanence after applications of -110 Oe (inwards respect to the plane of the sample), with a labyrinth patterned structure, typical of perpendicularly magnetized samples. Upon application of +110 Oe in the opposite direction, the system showed an increased number of worm-like dark domains, covering more or less half of the sample area (the reduced remanence for positive fields is attributed to the progressive sample damage under the X-ray beam, which tends to de-stabilize the PMA).
Figure 26: a) X-ray absorption spectrum (XAS) of the CoFeB in patterned devices. X-PEEM image of CoFeB acquired in remanence after application of -110 Oe (inwards, panel b) and +110 Oe (outward, panel c)[30].

Apart from external characterization techniques only available at the synchrotron radiation facilities, the magnetic micro-characterizations carried out in the group were based on a μ-MOKE setup specifically developed to perform the magneto-optical characterization of microcapacitors as a whole, without the possibility to study the formation and propagation of magnetic domains.

The setup used was the one of a standard MOKE (see section 1.4), with the addition of an objective in front of the sample (and perpendicular to it) in order to focus the laser (λ = 635 nm) in a spot with a diameter smaller than 10 μm. The position of the beam after the objective was fixed, and the positioning of the device in front of the beam was achieved by a triaxial micrometric stage and controlled by a webcam able to show both the sample and the laser spot at the same time. The reflected beam pass through a photoelastic modulator (PEM) operating at 50 kHz and then is collected by a photodiode, which periodic signal is demodulated by a lock-in amplifier at the same or twice the frequency of the PEM. This polar μMOKE setup was able to extract Kerr ellipticity and rotation but the information does not contain any spatial information because the photodiode simply integrates the laser intensity reaching its surface.

On the contrary, the Kerr microscope developed during my thesis work collects the reflected intensity with a scientific camera and the light reaching each pixel corresponds to the magnetic properties of a given sample area. So, the instrument opens the way to in-house spatially and time-resolved magnetic measurements allowing for the study of magnetic domains, uniformity, local properties.
3.4.4. Kerr microscopy optimization with Permalloy magnetized Ta/CoFeB/BaTiO\textsubscript{3} films

3.4.5. Polar Kerr microscopy optimization on Ta/CoFeB/BaTiO\textsubscript{3}

We tested the performances of the microscope by studying the micromagnetic behaviour of Ta/CoFeB/BaTiO\textsubscript{3} capacitors. We focused on SNR and quality factor $Q$ to find the best experimental conditions as function of the various parameters (e.g. analyser angle, exposure time, etc.). As we need to distinguish different magnetization states, particular attention is given to the use of $Q$ to quantify the criterion introduced in sec. 2.4.1.

Since only the frames collected at saturation (positive and negative) are required to calculate the $Q$ factor, setup optimization measurements are performed continuously changing the field between $+H_{\text{sat}}$ and $-H_{\text{sat}}$ and acquiring one or more images at each field. It is then possible to calculate the average of an increasing number of frames at the same field in order to understand how many acquisitions are required in order to distinguish between the two saturated magnetic states (over the magnetic region of interest). For each set of experimental parameter, there are two major outputs from these measurements:

(i) the quality factor as function of the number $N_{\text{AVG}}$ of acquired frames used to compute the average (or, almost equivalent, $Q$ versus the cumulative exposure time $N_{\text{AVG}} \cdot t_{\text{exp}}$).

(ii) the time stability of the measurement over time (by looking at the evolution of the mean value of pixels’ distribution in subsequent frames).

Our characterization starts with the use of a low magnification 20x objective. Relatively low magnifications have two advantages:

(i) the wider acceptance angle permits to collect more light coming from the sample;

(ii) the longer depth of field permits to be more robust with respect to mechanical drifts causing the vertical displacement of the sample with a consequent loss of focus.

Figure 27a shows the microscope image of one of the capacitors presented in section 3.4.3 and sketched in Figure 24(a). In Figure 27c to (d) the three steps of the measurement are presented. In the first step, we measure the mean value of the pixels of inside the magnetic Region Of Interest (ROI) in the Msat and $-\text{Msat}$ states, by continuously setting the external magnetic field to a positive and negative saturating field (200 Oe). With this measurement, we obtained N couples of saturation...
images, shown in the Panel b of Figure 27. Therefore, we can make the average on a different number of images in saturation to obtain $Q$ as function of $N_{AVG}$. Averaging over an increasing number of frame reduces the noise, sharpening the pixels’ distribution until a clear separation between the two opposite saturation states is obtained (Figure 27c). Since the main source of noise in standard conditions is the shot noise, $Q$ increases approximately as $\sqrt{N_{AVG}}$, as shown in Figure 27d.

![Figure 27: (a) Microscope picture of a Ta/CoFeB/BaTiO$_3$ microcapacitor. The gold contact comes from the left and partially covers the ferromagnetic/ferroelectric pillar defined by optical lithography. The lateral dimensions of the magnetic Region Of Interest (ROI) are reported. (b) Mean value of the intensity over the pixels of the ROI measured alternating the field between $-H_{sat}$ and $+H_{sat}$. The mean value of the signals at $+M_{sat}$ and $-M_{sat}$ does not change over the measurement time, indicating the great stability of the setup and the possibility to distinguish the two magnetic levels. (c) Mean value and standard deviation of measurements versus the number of images $N_{AVG}$ over which the signal is averaged. As expected, the standard deviation reduces as $N_{AVG}$ increases. (d) Quality factor as function of $N_{AVG}$ showing a “shot-noise” limited $Q$.](image)

The effect of $Q$ on Kerr images is reported in Figure 28 for $N_{AVG} = 1$ and 100 (panels a and b, respectively). The increase of $Q$ leads to an increased contrast between positive and negative saturation. Such a contrast and distribution sharpening ensure the possibility to distinguish intermediate magnetic states. During the thesis work, we developed all the software needed to
perform this kind of analysis at the beginning of a new experiment, so that the user has the possibility to choose the best exposure time or number of averages that guarantees the feasibility of the experiment.

![Figure 28: Sharpening of the pixels’ distributions due to the averaging process](image)

(a) $Q \approx 1$ upon acquisition of just 1 frame with $t_{exp}=100\text{ms}$ ($N_{AVG}=1$): the two levels are barely distinguishable. (b) $Q \approx 6$ can be obtained by averaging over $N_{AVG}=100$ frames with the same exposure time: the two distributions become well separated and intermediate magnetic states could be resolved in an experiment.

This kind of analysis of the $Q$ factor has been also employed to characterize microscope performances and to find the optimum of the analyser angle $\theta_A$ (where $0^\circ$ corresponds to complete light extinction, if depolarization doesn’t occur). This is a great achievement because, once again, it guarantees to the user the selection of the best measurement conditions from the beginning of a new experiment.

Note that the SNR defined in Sec. 2.4.3 coincides with our definition of $Q$, so that we can recall its formula to clarify the expected dependency from the analyzer angle:

$$Q = \text{SNR} = \frac{B \cdot Q \cdot E \cdot t_{\text{exp}}}{\sqrt{B \cdot Q \cdot E \cdot t_{\text{exp}} + B \cdot D \cdot t_{\text{exp}} + N_r^2}} \approx \frac{B \cdot Q \cdot E \cdot t_{\text{exp}}}{E} \cdot \frac{I_t - I_-}{\sqrt{I_t + I_-}}$$

$$= \sqrt{\frac{B \cdot Q \cdot E \cdot t_{\text{exp}}}{E}} \cdot \frac{2 \theta_a^p \sin 2 \theta_a}{\left( \frac{\theta_a}{\epsilon} \right)^2 + \left( \epsilon \theta_a + \cos^2 \theta_a + \gamma_D \right)}$$

\[ (1.117) \]
where $I_0$ is the background intensity prior to the optical train, $\gamma_D$ the depolarization factor and $\theta_K^P$ and $\epsilon$ the Kerr angle of rotation and ellipticity.

Figure 29: (a) Quality factor as function of the analyzer angle for $t_{exp}=100$ ms and different $N_{AVG}$ (from 1 to 100, every 10). (b) Detailed $Q(\theta_A)$ for $N_{AVG}=10$. The value of the depolarization factor obtained from the fit is reported.

Figure 29 shows $Q(\theta_A)$ for $t_{exp}=100$ ms and for different $N_{AVG}$ (from 1 to 100, every 10). In panel b, we fit our experimental data of $Q$ with the analytical expression of SNR: the nice agreement confirm the role of the shot noise in the determination of the system performances. The analysis permits to define the experimental conditions (analyzer angle and number of frames) required to obtain the value of $Q$ required by a user.

3.4.6. Optimized imaging of perpendicular magnetic domains in Ta/CoFeB/BaTiO₃

Following the optimization procedure presented in the previous section, as a result we were able to demonstrate the quality of out-of-plane measurements acquired by our Kerr microscope. The measurements were performed on the CoFeB-based capacitors photographed in Figure 27a. The hysteresis loop obtained by the average signal on the magnetic region of interest is reported in Figure 30. Panel b reveals the nice contrast achievable for positive and negative magnetic saturation once the post processing routines applied the treatments described in section Kerr images acquisition, analysis and post-processing. More important, the astonishing quality factor achieved in these measurements by text= and $N_{AVG}=1$ permits to visualize the structure and evolution of worm-like magnetic domains typical of perpendicularly magnetized thin films. The Kerr microscope developed during the thesis demonstrate to produce images of the same quality of the previously introduces X-PEEM images of Figure 26 that was used to characterized the same device.
Figure 30: Magnetization reversal of the CoFeB layer after the optimization of the microscope. The worm-like magnetic domain are clearly visible. The level of quality of the images allow to compare the microscope with other magnetic imaging techniques.
3.5. Magnetic characterization of Py-based zig-zag shaped conduits

The NaBiS group has a long experience in the development of magnetic circuits for the application in the field of biology, nanomedicine and more recently mechanobiology [42]–[45]. This section presents the magnetic characterization of Permalloy based zig-zag shaped conduits for manipulation of magnetic nanoparticles for biological applications.

In particular, a 2D vectorial in-plane magnetization analysis has been performed by longitudinal Kerr microscope, thus highlighting the switching magnetization mechanism that is used with particles. Finally, it possible to compare the magnetization reversal measured by the microscope with the final-elements simulations, in order to understand the magnetization reversal process and the domain wall pinning inside the conduit.

3.5.3. Introduction to single particle manipulation via micromagnetic actuator

Part of the activity of the NaBiS group is oriented towards the realization of Lab-On-Chip platforms. The scope is the creation of magnetic manipulators able to interact with biological entities exploiting the manipulation of magnetic beads at the micro- and nanoscale, through the coupling with externally controlled magnetic domain walls.

Magnetic micro- and beads are widely employed in Lab-On-Chip devices as carriers and labels for cells, molecular manipulation [42], drug delivery, and biosensing [46]. Moreover, magnetic beads can be used to study the application of forces down to the nano-Newton range [44].

Several technologies have been developed to capture and manipulate with high resolution particles that are suspended in a biological medium. In particular, even compared to competing technologies such as electrophoresis and optical tweezers, magnetic manipulators [47]–[49] have been proven to be very effective tools because magnetic fields are not screened by biological environments and are non-invasive for cells and biomolecules.

Arrays of magnetic elements patterned on-chip have been proposed as magnetic manipulators for the transport of single magnetic particles by exploiting their capability of focusing external magnetic fields [50][51]. In addition, it has been shown that magnetic domain walls (DWs) in a ferromagnetic thin film can be used to manipulate magnetic particles at a solid–fluid interface.

In this context, the group demonstrated how micro- and nanofabricated, planar stripes made by Permalloy (Ni$_{80}$Fe$_{20}$) constitute excellent conduits where the DWs can be nucleated and moved.
between adjacent geometrical constriction, like corners or curves, under the action of external magnetic fields [42]. This technique is called magnetic Domain Wall Tweezers (DWTs) and the idea is illustrated in Figure 31a for zig-zag shaped conduits.

![Diagram of magnetic conduit with domain wall](image)

Figure 31: (a) Sketch of a magnetic conduit with a domain wall in the corner. In correspondence, there’s a formation of a potential well for magnetic particles, as depicted by the constant energy surface. (b) Optical image of micrometric zig-zag conduits. Image adapted from Ref. [45].

This platform is based on the controlled displacement of confined DWs in ferromagnetic conduits. The results of this confinement, is an inhomogeneous magnetic stray field generated by a DW, of up to several kOe, which can trap a magnetic particle. This stray field is spatially localized at the nanometer scale due to the DW’s very confined geometric structure. The gradient of the magnetic stray field generated by the DW results in an attractive force, which tends to capture and drag any magnetic particle in proximity of the DW. Therefore, a fine control of the DW position, allows for the manipulation of magnetic micro and nanoparticles with a resolution down to 100 nm [45]. By applying suitable external magnetic fields, the DW that is pinned to a corner, moves to the nearest one, thus causing the step-by-step displacement of the magnetic particle along the nanostructure.

2.5.1. MOKE measurement on zig-zag shaped Permalloy conduits

Zig-zag shaped micrometric conduits in permalloy (Ni₈₀Fe₂₀) represent a good case study to test the capabilities of the Kerr microscope in the longitudinal configuration, due to their engineered DW propagation mechanism under the application of an external magnetic field.

A vectorial characterization of the magnetization of zig-zag conduits, with different orientations with respect to the external field, has been performed by using Kerr microscope in longitudinal (L) and
longitudinal with transverse sensitivity (LT) modes: in this way it was possible to probe the in-plane magnetization components parallel and orthogonal to the applied magnetic field (see section 2.2.3).

In general, the magnetic configurations found in micrometric patterns depends on the interplay between the following contributions:

(i) the geometry of the magnetic structures (because of shape anisotropy);
(ii) magneto-crystalline anisotropy (negligible in Permalloy);
(iii) the external magnetic field orientation;
(iv) the presence of defects introduced by the microfabrication processes.

The geometry of zig-zag microstructures is shown in Figure 31b: they are 50 nm thick, 2 µm wide, with a segment length of 15 µm and an angle of 90° between two adjacent segments. Since the magneto-crystalline anisotropy is negligible in Permalloy, the micro-magnetic configuration is mostly driven by shape anisotropy, due to the strong confinement of the magnetic layer: the magnetization lies in the film plane (the film is thin) and tends to be aligned to the conduit, thus allowing to minimize the magnetostatic energy of the system.

Besides, real patterned devices can present some defects, especially at the side walls where the photoresist defines the lateral geometry. These imperfections can affect the micromagnetic configurations; for example, they are responsible for acting as pinning/depinning sites for the magnetization.

Hence, shape anisotropy and defects are expected to be crucial to determine the magnetic response of the microstructures to external magnetic fields. To disentangle these two contributions, in the following, we will describe the micromagnetic analysis performed with two different orientation of the zig-zag with respect to $H$.

In order to understand the role played by the magnetic shape anisotropy, we conduct a first experiment in which the longitudinal and transverse components of the magnetization are collected (L and LT configurations, respectively) for a couple of orthogonal arms of a zig-zag, the first parallel and the second perpendicular to the external magnetic field ($H_{ext}$), as depicted in Figure 32. This configuration highlights the role of the shape anisotropy in magnetization orientation, since it would encourage (disadvantage) the alignment of $M$ along the field in the parallel (orthogonal) segment. Furthermore, a small tilt of approximately 15 degrees with respect to $H_{ext}$ allows to better appreciate the strength of the shape anisotropy in both the segments since, also for relatively high fields, the magnetization results still partially oriented along the arms direction.
Figure 32: Orientation of the zig-zag conduit respect to the external magnetic field, in the experiment designed to understand the effect of the shape anisotropy.

Figure 33 shows the full analysis of longitudinal ($M_x$) and longitudinal with transverse sensitivity ($M_y$) Kerr microscopy with the zig-zags oriented as sketched in Figure 32 and photographed in Figure 33a.

The hysteresis loops of the magnetization for the blue and red arms of the zigzag are obtained by integrating the intensity in the two (blue and red) regions indicated in Figure 33a. The components of $M_x$ and $M_y$ are obtained from two subsequent measurements where the slit is changed to be sensitive to the one or the other projection of the magnetization, as shown in Figure 11b and c. The result is reported in Figure 33b. The measurement has been performed starting from maximum positive external applied field ($H$= 290 Oe).

The hysteresis loop of panels b and c, have been obtained through the signal post processing routine explained in Sec.2.4.4. As a first step, the linear contribution from the Faraday effect of the objectives lens has been subtracted from the hysteresis loop, in order to recover the pure magnetic signal from the sample. The slope is calculated sweeping the field above the saturation ($H_{sat}$), where the magnetic signal is expected to be constant. Afterwards, the hysteresis loops are normalized to the intensity corresponding to the saturation magnetization of the magnetic layer. Due to the different sensitivity of the L and LT configurations, two different normalization constants must be used for $M_x$ and $M_y$, respectively. For the $M_x$ (both the red and blue arms), the normalization constant was evaluated from the hysteresis loop of the blue segment (parallel to $H$), assuming full saturation at the maximum field. The same normalization applies to the $M_x$ component of the red segment. The procedure reveals that the $M_x$ signal of the red segment does not completely reach the saturation, being unfavoured by the shape anisotropy that tries to align the magnetization along the conduit, hence orthogonal to $H$. On the other hand, the normalization factor for $M_y$ is calculated from the piece of conduit perpendicular to the field (red arm), because at $H$= 0 the magnetization aligns again along the y-axis, because of the shape anisotropy.
Figure 33: Magnetization reversal process for zig-zag conduits oriented with respect to the external magnetic field in such a way to make the blue and red arms inequivalent (one arm of the zig-zag is oriented at 15 degrees with respect to the x-axis). (a) Optical image of the pattern. The blue and red rectangles indicate the regions where the hysteresis loops of panel c have been obtained from. (d)-(l) micromagnetic images of $M_x$ (left) and $M_y$ (right) at given values of the external magnetic field. The last column contains the vectorial representation of the magnetization in each arm at the corresponding values of the applied field.

Let’s now discuss the behaviour of the hysteresis loops and the micromagnetic configuration capture by Kerr microscopy. As shown in Figure 33b, the hysteresis loop for $M_x$ obtained from the blue arm is squared, with sharp transitions and the small coercivity ($H_c = 7$ Oe). Indeed, this is expected by the Zeeman interaction between the magnetization and the external field, also promoted by the favourable shape anisotropy. On the contrary, the $y$-component of the magnetization for the blue arm gives rise
to a different hysteresis loop. At zero field, the magnetization points along the arm (i.e. forming 15° with $H$) because of shape anisotropy that maximizes the $y$-component. As the external magnetic field increases along $+x$ or $-x$, the magnetization tilts toward $H$ until the magnetization is completely directed along the field orientation, and $M_y$ goes to zero: the Zeeman energy becomes the leading contribution and the magnetization is not aligned to the segments anymore, as would be favoured by the shape anisotropy. According to the signal correction, in the hysteresis loop of the blue arm (panel b of Figure 33), the maximum of the $y$ component is found at the same the field for which $M_x = 0$. Note that, it is possible to observe that the $x$-component does not exactly reach the saturation value for $H = 290$ Oe, as the $y$-component of $M$ is different from zero. This correspondence proves the validity of the normalization procedure previously explained.

The situation is different for the red arm of panel (a) because the shape anisotropy favours the orientation of $M$ along $y$. A sufficiently strong magnetic field ($H > H_c$) aligns the magnetization along $x$: the hysteresis loop for $M_x$ as a higher coercive field ($H_c = 40$ Oe) than its “blue counterpart”, with a more gradual transition compared to the blue arm. The same considerations hold for the $M_y$ component: at zero field, the magnetization is aligned along the red segment and it has a significant projection along $y$ (panel c of Figure 33). The latter is reduced almost to zero when the field is increased, without reaching a complete alignment to $H$.

From the hysteresis loops of panel b and c, and with the guide of the MOKE images of panels d-l, it is possible to reconstruct the in-plane magnetization vector as function of $H$. The result is depicted in the right column of Figure 33. Starting from maximum positive field (Figure 33d), the zig-zag in the longitudinal image appears uniformly white, indicating that the magnetization is saturated along the positive $x$ axis. On the contrary, the transverse image shows almost no contrast (except for the boundaries of the zig-zag which introduces some artefacts/shadowing effect). An external field below the negative coercive field of the blue segment ($H < -H_c = -7$ Oe) induces the magnetization reversal of both longitudinal and transverse component of the blue arms, as it is evident at $H = -25$ Oe from the hysteresis loops (panels b and c) and magnetic images (panels e-e’). In this configuration, $M_y$ is concordant in two inequivalent arms, while $M_x$ is discordant, thus the magnetization follows the zig-zag shape and all the domain walls at the corners are annihilated (panel e’’). Further reducing the field to $H = -100$ Oe (just below the negative coercive field of the red arm) also the longitudinal component of the red segment switches along the negative $x$ direction. The longitudinal magnetic image becomes uniformly dark (panel f), while the transversal image shows the difference of signs between the two arms (panel f’). At the maximum negative field ($H = -290$ Oe), the magnetic configuration (panels g-g’) is just the opposite to the one at positive saturation (panels d-d’).
Sweeping the field from negative to positive, we obtain the same configurations with the opposite sign, but something interesting happens when the positive field overcomes the coercivity of both the arms along $y$; while at $H = 25$ Oe the magnetization follows the shape of the conduits (panels h, h’, h”), at $H = 100$ Oe the magnetization vectors of the two arms point towards each other with the nucleation of a head-to-head domain wall (as it happens in panel f at -100 Oe). This arises when the $x$-components of the two arms are concordant while their $y$-components are opposite, as in the case depicted in Figure 33 (f) and (i).

In order to assess the consistency of the experimental findings, the micromagnetic configurations of zig-zag shaped conduits have been simulated with the software OOMMF (Object Oriented Micromagnetic Framework) [57] and compared with the micromagnetic images in Figure 34.

![Figure 34](image)

Figure 34: On top: sketches of zig-zag shaped conduits, where one segment is oriented at 15 degrees with respect to the x-axis. An external magnetic field (H) directed along the x-axis is swept between ±290 Oe. On bottom: micromagnetic configurations, simulated using OOMMF, of a portion of the zig-zag microstructure. The arrows represent the local magnetization direction, while the red-white-teal scale refers to the $x$-component of the magnetization.

Simulations are performed using the following parameters for Ni$_{80}$Fe$_{20}$: saturation magnetization $M_s = 680 \times 10^3$ A m$^{-1}$, exchange stiffness $A = 1.3 \times 10^{-11}$ J m$^{-1}$, damping coefficient $\tau = 0.01$ and null magneto-crystalline anisotropy. A $25 \times 25 \times 25$ nm$^3$ unit cell has been used for simulating the micromagnetic configurations of the zig-zags. Although the exchange length of Permalloy is 5.2 nm, this represents a reasonable compromise ensuring reduced computational times. The results are reported in Figure 44. As expected, when $H = +290$ Oe (see Fig.44a), the magnetization is almost saturated along the field direction (x-axis). Decreasing $H$ to -25 Oe, the magnetization in one of the two arms is reversed (see teal-pixels in Fig.44b) following the conduit direction because of the shape anisotropy, in agreement with the Kerr image of Fig.43e. At $H = -100$ Oe, the magnetization of the two segments is reversed ($M_x < 0$, see Fig.44c), the shape anisotropy still favors the alignment of M along the zig-zag segments, as previously shown in Fig.43(f). When H decreases to -290 Oe the
magnetization is aligned to the external field (almost saturated along -x see Figure 34d), as observed in the Kerr images (see Figure 34g).

The opposite scenario occurs sweeping $H$ from -290 Oe to +290 Oe (see Figure 34d-g). These simulated magnetic configurations are again in excellent agreement with those acquired by Kerr microscopy and reported in Figure 34g-l.

![Figure 35: Comparison between two different zig-zag orientation.](image)

In the following part of this section, we will compare the differences in the magnetic response of apparently “equivalent” conduits, as those depicted in Figure 35.

The hysteresis loops for $M_x$ and $M_y$ of two inequivalent arms (red and blue rectangles) of the two zig-zags of Figure 36a (dashed and solid lines) are compared in panels b-c-d-e of Figure 36. Despite the orientation with respect to the field and the saturation value are the same, “equivalent” segments display different coercivity. This confirms that each structure owns peculiar magnetic properties, thus highlighting the presence of defects.

![Figure 36: Comparison between the hysteresis loops collected in over different regions. (a) Optical image of two zig-zags. Dashed and solid lines identify equivalent regions (i.e. with the same orientation with respect to $H_{ex}$), while red and blue are used to identify the non-equivalent segments. (b)-(e) Hysteresis loops measured integrating the pixels within the rectangular red and blue area of panel a.](image)
As proved by the experiments described up to now, the first configuration (see Figure 35a) is mostly sensitive to the shape anisotropy whereas the latter (see Figure 35b) is used to highlight the presence of defects. For this reason, it is also possible to decouple the impact of defects and shape anisotropy on the magnetic behaviour of two adjacent inequivalent arms of a selected zig-zag structure, by comparing the longitudinal Kerr measurements performed adopting the two different alignments described in this section.

Figure 37: Magnetization reversal for the orthogonal orientation of the zig-zag shaped conduit inside the magnetic field. (a) Optical image with the relative orientation of the two components of the magnetization and the direction of the external magnetic field. The blue and red region of interest indicates the region where the hysteresis loop has been extracted. (b) Hysteresis loop for the two different region, with respect to both component of the magnetization. (c)-(i) MOKE images of the magnetization reversal for both components of the magnetization, on the right is depicted a sketch of the structure with the orientation of the magnetization.
Measurements in Figure 37 are recorded from the same zig-zag conduit of Figure 33 but with the perpendicular orientation (see Figure 37a). The hysteresis loop of $M_x$ in the orthogonal alignment (panel b of Figure 37) is more gradual than in the first case (panel b of Figure 33), thus indicating a stronger shape anisotropy.

In Figure 37c, the $y$-component of the magnetization for the red arm is maximum $H=0$, and goes to zero increasing the field. The complete vectorial Kerr analysis of the switching mechanism is reported in panels e-l and it is sketched in the right column of Figure 37. At maximum positive external field (panel d of Figure 37), the magnetization of the red arm is saturated while for the blue one also a positive component of the magnetization along $y$ is present. As the field is restored to zero, the magnetization aligns with the strip of the conduit as shown in panel e of Figure 37 and different configurations can occur. For example, at $H=-25$ Oe (panel f on the right, Figure 37) both the components of the magnetization vector of the red arm change sign. This means that the applied field is smaller than the negative coercive field ($H<-H_c=-5$ Oe), that is estimated from the hysteresis loop (panel b of Figure 37). On the other side, the blue segment does not switch due to the stronger coercive field ($H>-H_c=-34$ Oe) and the magnetization follows the shape of the conduit, with the annihilation of the domain wall. The magnetization along the conduits was obtained even in the first configuration, shown in panels e and h of Figure 33. Differently from the latter, in the orthogonal configuration the magnetization vector can point along opposite directions in the two adjacent zig-zags as shown in Figure 38.

Figure 38: Intermediate configuration, same of Figure 37f. The magnetization follow the shape of the conduit in the opposite direction.

This micromagnetic pattern is allowed by the different coercivity of the two inequivalent segments of the same conduit (and is valid for both the conduits shown in Figure 37), a strong indicator of the asymmetry of the structure. Since in this configuration there is no preferential direction of the $y$-component of the magnetization, defects act as pinning sites determining the final magnetic configuration. Due to the randomness of the defect distribution on the sample, each one of the four configurations listed in Figure 39 can occur.
Micromagnetic simulations have been performed to calculate the theoretical magnetic response occurring when zig-zag conduits are oriented perpendicular to $H$ (see Figure 37a). Simulated micromagnetic configurations are reported in Figure 40. As previously discussed, in this case, the real configurations (see Figure 37) are strongly affected by defects, not taken into account in simulations. Moreover, the simulated switching of $M$ happens in the two arms at the same value of $H$, due to the symmetry of the system with respect to the $x$-axis. For this reason, the magnetic configurations reported in Figure 38 cannot be carried out by simulations. The main contributions which drives the simulated micromagnetic configurations are again the shape anisotropy (Figure 40b-c-e-f) and the Zeeman energy which tends to align $M$ along the field direction (see Figure 40-d-g). In this case, the simulated coercive field is $\approx 10$ Oe, different from the values extrapolated by Kerr analysis, thus demonstrating that defects and imperfections can play a relevant role, affecting the magnetic response of micro- and nanostructures.

In conclusion, in this section we have shown how Kerr microscopy in the longitudinal (L) and longitudinal with transversal sensitivity (LT) configurations can be performed to reconstruct the
magnetization vector in response to external applied magnetic field and its time evolution. Furthermore, by modifying the relative orientation between magnetic field and patterned structure and performing a statistical analysis on several devices, it is possible to provide a comprehensive characterization of a magnetic patterned sample in terms of anisotropies, defects and propagation of domains. This example gives the fundamental reason justifying the effort for the development of a Kerr microscope: while a macroscopic MOKE investigation is absolutely not be able to identify and study the peculiar magnetic behaviour of patterns like micro-conduits, Kerr microscopy is. For a group specialized in nano-magnetism, the availability of this technique constitutes a fundamental milestone.

3.6. Thermally Assisted Scanning Probe Lithography

The NaBis group recently developed and demonstrated the feasibility of a new technique for the development of novel reprogrammable magnetic metamaterials with finely tuned magnetic properties. The technique is called thermally assisted magnetic Scanning Probe Lithography (tam-SPL) [52]. It permits to create reconfigurable magnetic nanopatterns by crafting the magnetic anisotropy landscape of a ferromagnetic layer exchange-coupled to an antiferromagnetic layer, down to the nanoscale. By performing localized field cooling with the hot tip of a scanning probe microscope, magnetic structures, with arbitrarily oriented magnetization and tunable unidirectional anisotropy, are reversibly patterned without modifying neither the film composition nor its topography. Such metamaterial would allow the creation of reconfigurable devices able to support and process spin-waves, devices for magneto plasmonics and also the creation of magnonic crystals. Before tam-SPL, the fabrication of magnetic nanostructures has been achieved mainly through structural or chemical modifications. The two conventional methods for the nano fabrication, i.e. optical or e-beam lithography and ion irradiation, were used to pattern magnetic materials by physically defining nano- and micro-sized shapes, removing or adding material [42]–[44]. Changing shapes and materials, it is possible to control magnetic anisotropy, domain nucleation, domain wall motion. An example are the zig-zag shaped conduits presented in Sec. 3.5. Another common approach consists in creating suitable masks and irradiate the magnetic material with ions to locally affect the magnetic properties [53][54]. This method induces chemical reaction within the film or simply acts on the crystalline structure modifying it or creating defects. All these techniques are in most cases irreversible, and their ability to finely control the modulation of the magnetic properties within the pattern is limited. As irreversible techniques, once the patter is
created its functionality cannot be changed. In this context, tam-SPL represent a new solution for the fabrication of reprogrammable magnetic materials. Tam-SPL combines Scanning Probe Lithography (SPL) and a suitable substrate, namely an exchange bias system. These two elements will be briefly introduced.

SPL is performed by using a custom-made cantilever for the Atomic Force Microscope. As shown in Figure 41, it is characterized by two highly-doped silicon legs with a relatively low electrical resistance connected at the apex by a segment made of low-doped silicon substrate that represents the heater region below which the real AFM tip is located. Applying a voltage across the two long legs, the flowing current heats up the tip due to Joule effect. The heater can bring the tip temperature up to 1400°C. The temperature versus power dissipated by the heater can be finely tuned, in principle allowing for a precise control of the local exchange bias produced in the bilayer.

The second element required by the technique is a suitable substrate, in this case an exchanged bias system comprising an antiferromagnet (AFM) and a ferromagnet (FM) in direct contact. An antiferromagnet has two ordered spin sublattices with the respective spins orientated antiparallel one to each other, so that the net magnetization of the material is equal to zero. For this reason, antiferromagnets are not influenced by external magnetic fields (up to several Tesla).

When a soft ferromagnetic film is strongly exchange-coupled to the antiferromagnet, it will have its interfacial spins interacting with the AFM. The exchange-coupling [56] causes a local pinning of the magnetization of the ferromagnetic layer, which is observed as shift (bias field $H_e$) and broadening ($\Delta H_C$) of the FM hysteresis loop. Note that at zero external magnetic field, if $H_e$ is greater than the coercivity, the magnetization of the FM has a well-defined orientation. This orientation can be set via a process called field cooling. During the field cooling the system is heated above its blocking temperature $T_B$, defined as the temperature at which the AFM is no more
able to pin the spins of the FM. Afterwards an external magnetic field is set and the FM is able to orient in the direction of the external field. Keeping the field applied, a cooling is performed and the antiferromagnetic order is restored and the spin of the AFM are coupled with the FM ones.

Tam-SPL is just a way to perform the field cooling at the micro- and nanoscale. The technique is based on the local heating and cooling of an exchange-biased ferromagnetic layer by bringing the hot tip of a scanning probe microscope into contact with a sample, in the presence of an external magnetic field. This local field cooling sets a unidirectional magnetic anisotropy in the ferromagnet due to exchange-coupling with the adjacent antiferromagnet. If the FM/AFM heterostructure is designed to have the blocking temperature higher than room temperature, the pattern is stable and cannot be erased by magnetic fields (up to several Tesla), thus combining endurance and reconfigurability. The detail of the magnetic patterning process is sketch in Figure 42.

First, the exchange bias system is initialized by heating the entire sample above the blocking temperature $T_B$ and subsequently cooling it in a uniform magnetic field $H_i$. This initialization sets a uniform unidirectional anisotropy axis in the ferromagnetic film, with its hysteresis loop shifted by the exchange bias field $H_e$, as shown schematically in Figure 42a.

![Figure 42: The magnetic patterning is composed by three steps. (a) During the initialization, the magnetization of the FM layer is pinned in one direction by the exchange coupling by field cooling the whole film. (b) In order to create the magnetic pattern, the heated tip is swept on the sample surface in the presence of an external magnetic field (writing field $H_w$) with a different orientation with respect to the one used for the initialization. (c) Once the external magnetic field is removed, and the field cooling finished, the magnetic configuration of the ferromagnet is stabilized by the local exchange bias. Different region of the surface present a different orientation of the remanent magnetization. The image is courtesy of [52].](image-url)
Then, the hot tip of the AFM is used to locally heat the surface of the stack above $T_B$ in a uniform static magnetic field, called writing field $H_w$, having a direction different from that of the initialization field $H_i$ (Figure 42b). The heated region of the antiferromagnetic film undergoes a ‘local field cooling’ accompanied by a local rearrangement of the spins, due to coupling with the underlying ferromagnet oriented in the direction of the external field, $H_w$, which resets the exchange bias direction. Once the temperature is reduced below the blocking temperature, the antiferromagnetic order is restored according to the actual local orientation of the FM, so that the spins of the FM layer became blocked by the AFM layer in the direction of the writing field. This mechanism requires that the blocking temperature is smaller than the Curie temperature of the ferromagnet, since during the field cooling process is the ferromagnetic layer that sets the new direction for the exchange bias, under the application of the writing field. Moving the hot tip, it is possible to create the micro-magnetic pattern.

Compared to the others techniques, the tam-SPL is fully reversible, since the magnetic pattern can be fully erased and re-written at will with a different pattern. In the seminal work about tam-SPL, the NabiS group demonstrated the reversibility of the patterning technique. They demonstrated the possibility to write, erase and rewrite in a controlled way the different magnetic patterns on the same area. The writing-erasing-rewriting process is shown in Figure 43, created by Magnetic Force Microscopy (introduced in Sec. 2.5). Two squares were initially patterned with a positive writing field. The cancellation is achieved with an opposite erasing field $H_{er}$ (of the same sign of the initializing field). The magnetic contrast disappears in the left square without changing the right one. Finally, it is possible to rewrite the erased area. Also, the magnetic pattern can be completely erased by performing uniform field cooling.

Figure 43: Writing-erasing-rewriting capability of the tam-SPL. The image is adapted from [52].
3.6.1. Polar Kerr Microscopy on magnetic pattern crafted by tam-SPL

As last case study, we had the possibility to use the microscope for the characterization of devices recently fabricated by tam-SPL. The heterostructure under investigation is grown by magnetron sputtering and constituted by Ta(5)/Ru-doped-IrMn(6.5+1.95)/Ta(0.2)/CoFeB(1)/MgO(1.6)/Ta(2) (thickness are in nanometers). The stack is conceived such that the CoFeB layer developed perpendicular magnetic anisotropy upon a proper annealing treatment (200°C in vacuum for 30 min, with an external applied field of 4 kOe in the out-of-plane direction).

Before this thesis work, the characterization of patterns created by tam-SPL was mainly done by Magnetic Force Microscopy (MFM). MFM has all the advantages and limitations of scanning probe techniques: despite the nanometric resolution, the image acquisition is infinitely slow with respect to Kerr microscopy and the imaging can be done exclusively at zero external applied field. Measurements of hysteresis loops are far from MFM possibilities. In this context, the Kerr microscope was the ideal choice to perform a fast and detailed characterization of tam-SPL patterns, enabling the optimization of heterostructure and writing conditions, without penalizing the spatial resolution (limited by diffraction to about half a micron). Moreover, Kerr microscopy allows for local hysteresis loop measurements of tam-SPL patterns.

As discussed in the previous section, tam-SPL performs a local field cooling in order to create magnetic domain patterns with arbitrary form and dimension, with no correspondence in the topography of the sample. The tricky part is the distribution of temperature created by the heated tip underneath the sample surface in the point where the field cooling is performed. The control of the temperature and its profile locally obtained in the AFM layer is strongly affected by:

(i) the stack;
(ii) the local thermal conductivity of the layers;
(iii) the accuracy of the tip calibration in terms of current-temperature characteristic;
(iv) the indentation regime, i.e. the hardness of the contact between the tip and the sample during the heating;
(v) the speed of the tip used to raster the surface and/or the time the tip spends over a single point.

Lower writing speeds, right temperatures and proper tip-sample distances can result in more effective field cooling able to shift the hysteresis loop of the pattern away from \( H=0 \), creating a fully saturated pattern at remanence different from the “blackboard” landscape of the initialization.
procedure (shown in Sec.3.6). Kerr microscope offers a way to study how well the exchange bias is modified as function of the writing parameters.

Figure 44: Exchange bias dependency on the writing parameters. (a) MOKE image of the four patterns (b) MFM image (c) Hysteresis loop as initialized from the first field cooling (c)-(g) different hysteresis loop for the four different patterns.
Figure 43a presents four different patterns written, as described in section 3.6 and produced field-cooling with a writing field $H_w$ opposite to the initialization field $H_i$. The four patterns differ in writing speed $v_w$ and the tip-sample indentation regime, as indicated in Figure 44a. The starting values $v_w$ and contact “setpoints” are the ones used for the previous system of Ref. [52]. The optimization has been performed evaluating the variation of the exchange bias of each pattern looking at its hysteresis loop, as shown in panels d-g.

The analysis of hysteresis loops shows that a lower writing speed allows a better local heating of the antiferromagnetic layer and hence a higher shift of the loop from the zero (due to exchange bias). A better field cooling permits to have a well-defined value of the magnetization within the written domain. The parameters used to obtain the “red” and “blue” squares are appropriate to obtain such a result, while the “cyan” and “green” squares are not sufficiently shifted from zero.

One has to notice that during the rastering of the sample with the tip in contact with its surface, a tip degradation occurs in time. The “wear-off” of the tip influences the effective area of the contact and hence the way the tip transfers the heat to the sample underneath. This investigation will be performed in the near future by fixing the writing parameters and looking at the magnetic behaviour of subsequently written patterns.

Here we cannot conclude about the role of the contact regime. Contrary to expectations, reducing the contact apparently increases the performances, as evident from the comparison between panels f and g. Unfortunately, this effect can also be due to the degradation of the tip during writing (the “cyan” square was written after the “green” one, the latter owning better exchange bias). This prevents a clear conclusion on the role of the contact hardness.

Finally, we tested the resolution of both tam-SPL and Kerr microscope by reducing the size of the patterns from 3 μm to 1.5 μm and producing different geometrical shapes. Figure 45 shows the patterning capability of the technique and the possibility to resolve with the Kerr microscope features below the micrometric scale, as evident for example from the well-defined corner of the triangles. The Figure 45 also states the possibility to measure the local hysteresis loop on a limited number of pixel (300 for the smallest triangle) for a very thin magnetic material (1 nm thick).
Figure 45: Pattern capability, different shape has been wrote. (a) hysteresis loop of the un-patterned area (b) MFM image, scale bar 3µm (c) MOKE image (d1-d4) hysteresis loop (e) and (f) MOKE images for different field (indicated by the black dotted line). measure performed with 100x objective (each pixel correspond to an area of 60nmx60nm)
4. Conclusions

This thesis work was devoted to the development of a Kerr microscope as high-end measurement station for magnetic domains imaging.

In the first part of the thesis work, a conventional optical microscope with polarization contrast was modified to exploit it as Kerr microscope. In particular, I designed:

(i) a couple of water-cooled electromagnets for the application of the magnetic fields in the polar and longitudinal direction;
(ii) a slit to be able to select both the plane of incident and angle of the light impinging on the sample, in order to perform longitudinal, longitudinal with transversal sensitivity and polar Kerr microscopy;
(iii) a sample holder with five degrees of freedom which allows to precisely move, rotate and tilt the specimen under investigation.

In order to control the electromagnets to generate the field and to acquire the images from the camera, I developed a full set of Matlab routines interfacing with the hardware. Matlab was the environment of choice because of its unparalleled capabilities in the field of image analysis, together with the possibility to drive hardware by scripting. Moreover, another set of functions enables Kerr images post-processing and analysis. At the end, the software constitutes a complete Matlab toolbox that allows users to perform parametric optimization of the setup, complex sequences of automatic measurements (by scripting), analysis and production of pre-print images.

The microscope was then characterized in terms of performances and noise. The definition of a quality factor for measurements permitted to establish a criterion to clearly distinguish different magnetization states and study the performances of the setup as function of various adjustable parameters. Besides, a proper modelling of signal and noise permitted to find a way to maximize the magnetic contrast for collected frames by subtracting the non-magnetic background and also correcting for non-uniform illumination, experimental drifts, spurious Faraday effect, etc..

At the end of the optimization, the spatial resolution of the microscope reaches the sub-micron range. It is possible to acquire images with a quality factor greater than unit by exposure times in the order of 100 ms, and the quality factor can easily overcome 10 for long enough exposures (scarifying the time resolution). Few seconds are generally enough to collect the hysteresis loop of magnetic materials for thin magnetic films (in the order of few nanometers).
The potential of the Kerr microscope has been exploited to study three topics of interest for the NaBiS group. First of all, we have carried out an optimization of the Kerr parameters on Ta/CoFeB/BaTiO$_3$ heterostructures, in which the ferromagnet/ferroelectric interface constitute an example of artificial multiferroic. Measurements of low noise hysteresis loops have been achieved, as well as worm-like magnetic domains have been photographed.

Then, we have studied magnetization reversal, domain wall and magnetic anisotropies in Permalloy-based conduits intended for lab-on-chip applications. It has been possible to measure the complete vectorial behaviour (orientation and magnitude) of the in-plane magnetization versus the external applied field. We have revealed that not only shape anisotropy but also randomic defects drive the micromagnetic configurations of real microstructures.

The last example of use of Kerr microscopy has been the imaging of magnetic patterns written in exchanged-biased systems by the novel technique called thermally assisted magnetic Scanning Probe Lithography, recently developed in the NaBiS group. The investigation of these micro and sub-micron structures has guided the optimization of pattern writing parameters.

The implementation of a Kerr microscopy station was the scope of this thesis. The final setup turned out to be a state of the art instrument for the magnetic characterization of micro and nanostructures.
References


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