METHODS AND TECHNOLOGIES
FOR ACTIVE CONTROL OF DYNAMIC LOADS ON HIGH PERFORMANCE AIRCRAFTS

Doctoral Dissertation of:
Sheharyar Malik

Supervisor:
Prof. Sergio Ricci

Co-Supervisor:
Daniele Monti

Tutor:
Prof. Giuseppe Quaranta

The Chair of the Doctoral Program:
Prof. Luigi Vigevano

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The capability of advanced, high performance aircraft to operate at high turn rates and high angle of attack often results in emanating vortical, unsteady flow from the aircraft’s forebody and consequently the disturbed flow impinges on the wings and the vertical tail of the aircraft. The disturbed flow carries the sufficient amount of energy to excite the structural modes of the wing and the vertical tail of the aircraft. The phenomenon is named as buffeting and produces undesirable vibrations which limits the flight envelope of the high performance aircraft and reduces the fatigue life of the component under buffet loads. The dissertation presents advanced novel methodologies to actively alleviate the buffeting of the wings of the high performance aircraft. Control surfaces located on the wing are actively controlled to redistribute the aerodynamic loads. Two different aircraft are considered for application of methodologies, these include the so-called Aluminum Fighter Aircraft (AFA) and the X-DIA commercial aircraft.

The research presents two active control schemes for Aluminum Fighter Aircraft (AFA), named as Buffet Mitigation Control System (BMCS). The aeroelastic analysis of the wing is carried out by using the finite element software MSC/NASTRAN. Accurate description of aeroelastic behavior is acquired in terms of linear time invariant (LTI) state space system. Experimentally acquired pressure fluctuations on the wings of the aircraft are used to model the buffet loads as exogenous inputs to the state space system. The buffet model is defined analytically and numerically with the help of shape filter. The formulated Static Output Feedback (SoF) controller is based on simultaneous solution of Lyapunov’s equations which are numerically solved by deterministic and stochastic optimization techniques. Heuristic numerical methods employ first and second order quadratic formulation of objective function to minimize the feedback gain for the control surfaces located on the wing. The two distinct active control schemes are characterized by unconstrained and optimized actuator movement for buffet load alleviation. The process followed for the minimization of feedback gain enabled the examination of controllability and observability for the numerical conditioning of gramian with relative measure of controllability and observability.

Nowadays, high performance is also attributed to commercial aircraft. One such example is the scaled aircraft model named as X-DIA that it was built by Politecnico di Milano under European
project Active Aeroelastic Aircraft Structures (3AS) and represents the aeroelastic prototype of an advanced commercial aircraft. For the wing of X-DIA aircraft, six active control techniques are developed and named as Buffet Load Mitigation Systems (BLMS). The developed numerical schemes for X-DIA wing are followed by a comprehensive experimental validation at Politecnico di Milano-Department of Aerospace Science and Technology in the De Ponte wind tunnel. BLMS-I presents a unique modal control with an aim to of reducing structural vibrations in the first bending and first torsion mode, it ensured the performance in the flight envelope. The state space model is completed by incorporating inputs for transducers to model buffet loads. The Static Output Feedback (SoF) control law is transformed by ignoring the computationally expensive terms, that gave static output feedback controller new compressed set of equations to be efficiently computed by numerical methods. In BLMS-II, the work proposes a method to rank the efficiency of the actuators for attenuation of specific modes. A unifying theme to this phase is the application of Hankel singular values on the experimental data conducted during the buffet load mitigation research. Modal information from experimental data is used to define actuator/sensor relationships, once the actuator and sensor is decided for mode of interest, suboptimal control laws can be applied for specific performance metrics. In BLMS-III, novel technique is presented to enhance the performance of the controller in the first torsional mode, notch and peak filters are applied on the output of the system to alleviate the buffet loads in the torsion mode specifically. Robustness of the optimization algorithms is proved by analyzing the system under the effect of uncertainties in the instrumentation of the system in BLMS-IV. This approach also leads to insurance of robust controller for system under uncertainties in the instrumentation.

The work also presents the procedure for quantification of robust aeroelastic system in BLMS-V, the task is accomplished by introducing uncertainties in the aeroelastic reduced order state space system of the X-DIA aircraft. The model order is reduced by balanced truncation method. Mu synthesis approach is used to quantify the robust stability and robust performance margins of the aeroelastic system. BLMS-VI is directed towards the implementation of real-time robust controllers (H\textsubscript{\textinfty} and Mu Controller) for the Aeroelastic systems. It exemplifies the power of robust controls developed for aeroelastic system. Thanks to the decomposition approach, the order of the system is significantly reduced. Multi-input and multi-output system is reduced to single-input single-output systems based on analysis performed in BLMSII. In Mu control the perturbed model is found to be highly sensitive to small disturbances in the control systems dynamics. Such idealizations helped in the realization of robust, simple and practical based systems which corresponded to the high fidelity of the system control.
TABLE OF CONTENTS

ACKNOWLEDGEMENTS .................................................................................................................. iii
ABSTRACT ....................................................................................................................................... iv
TABLE OF CONTENTS .................................................................................................................... vii
INTRODUCTION ........................................................................................................................... 1
  1. Introduction ................................................................................................................................. 1
  2. Literature Review and Motivation.............................................................................................. 2
     I. Active Aeroelastic Control ....................................................................................................... 2
     II. Review of Static Output Feedback ...................................................................................... 3
     III. Review of Robust Control .................................................................................................... 4
     IV. Review of Multi-Surface Control .......................................................................................... 5
  3. Thesis Contributions: .............................................................................................................. 7
  4. Thesis Outline: ............................................................................................................................ 8
PART 1: ACTIVE BUFFETING MITIGATION CONTROL SYSTEM FOR ALUMINUM FIGHTER AIRCRAFT (AFA) ................................................................................................. 9
  1. Structural Model ....................................................................................................................... 10
  2. Aerodynamic Model ................................................................................................................ 11
  3. Numerical Analysis .................................................................................................................. 12
     i. Normal Mode Analysis ........................................................................................................... 12
     ii. Frequency Response Analysis .............................................................................................. 16
     iii. Dynamic Aeroelastic Response Analysis ............................................................................. 16
  4. State Space Formulation: ......................................................................................................... 16
     i. Mathematical Formulation ...................................................................................................... 16
     ii. Schematic of the Inputs and Outputs on the Wing ................................................................. 18
5. Control Law Formulation .............................................................................................................. 26
6. Optimization Algorithm ............................................................................................................... 30
7. Results: Buffet Mitigation Control System I & II ....................................................................... 37

PART 2: BUFFETING LOAD MITIGATION SYSTEM FOR ADVANCED COMMERCIAL AIRCRAFT (X-DIA) .............................................................. 41
1. Numerical Analysis ....................................................................................................................... 45
2. Buffet load Mitigation System ...................................................................................................... 55
   i. Buffet Load Mitigation System I: ............................................................................................. 57
   ii. Buffet Load Mitigation System II: ........................................................................................ 63
   iii. Buffet Load Mitigation System III: ....................................................................................... 68
   iv. Buffet Load Mitigation System IV: .................................................................................... 71
   v. Buffet Load Mitigation System V: ...................................................................................... 76
   vi. Buffet Load Mitigation System VI: .................................................................................... 82

PART 3: EXPERIMENTAL IMPLEMENTATION OF ACTIVE CONTROL SCHEMES ON (X-DIA) COMMERCIAL AIRCRAFT ........................................................................ 89
1. Experimental Set-up: .................................................................................................................. 90
   i. Wind Tunnel Specifications: .................................................................................................. 90
   ii. Airbrake: .................................................................................................................................. 93
   iii. Instrumentation of the Wing: ............................................................................................... 94
   iv. Modal Analysis ...................................................................................................................... 98
   v. Actuator System for X-DIA Wing: ...................................................................................... 102
   vi. Real Time Implementation of Control Architectures.......................................................... 106
2. Preliminary Tests .......................................................................................................................... 107
3. Validations of Control Laws ........................................................................................................ 110
   i. Buffet Load Mitigation System I: .......................................................................................... 110
   ii. Buffet Load Mitigation System II: ........................................................................................ 115
   iii. Buffet Load Mitigation System III: .................................................................................... 132
iv. Buffet Load Mitigation System IV: .................................................................134
v. Buffet Load Mitigation System V: .................................................................136
vi. Buffet Load Mitigation System VI: ...............................................................138

CONCLUSIONS .................................................................................................141

REFERENCES ..................................................................................................144
LIST OF FIGURES

Figure 1: Structural Model of Aluminum Fighter Aircraft (AFA) ........................................10
Figure 2: Structural Model (AFA) .........................................................................................11
Figure 3: Aerodynamic Model .................................................................................................12
Figure 4: AFA Aeroelastic Model .............................................................................................12
Figure 5: Rigid Body Mode - Aileron Deflection (Symmetric) ..............................................13
Figure 6: Rigid Body Mode – Aileron Deflection (Antisymmetric) ........................................13
Figure 7: First Bending (4.238 Hz) ..........................................................................................14
Figure 8: Antisymmetric First Bending Mode (5.4582 Hz) ...................................................14
Figure 9: First Torsion Mode (6.1683 Hz) .............................................................................14
Figure 10: Antisymmetric First Torsion Mode (7.6539 Hz) ..................................................15
Figure 11: Second Bending Mode (16.23 Hz) ........................................................................15
Figure 12: Antisymmetric Second Bending Mode (20.40 Hz) ...............................................15
Figure 13: Schematic: AFA Wing ..............................................................................................19
Figure 14: Sensors Positions ....................................................................................................20
Figure 15: Power Spectral Density - Sensor 7 .........................................................................20
Figure 16: Coherence Data - Pressure Sensors ......................................................................21
Figure 17: Cross Power Spectral Density - Pressure Sensors .................................................22
Figure 18: Contour Plot - Time Delay .....................................................................................23
Figure 19: Contour Plot Complete Wing – Time Delay .............................................................24
Figure 20: Spectrum Plot - Time Delay ...................................................................................24
Figure 21: Spectrum Plot – Amplitude Ratio ..........................................................................25
Figure 22: Spectrum Plot - Pressure Distribution ....................................................................25
Figure 23: Sensor configuration ...............................................................................................26
Figure 24: Buffet Mitigation Control System I .......................................................................29
Figure 25: Buffet Mitigation Control System II .....................................................................29
Figure 26: Bode plot - Second Order Actuator .....................................................................30
Figure 27: Gradient Descent Method .....................................................................................31
Figure 28: Cost Function Minimization - Heuristic Optimization ...........................................34
Figure 29: Design Variable Minimization

Figure 30: Cost Function Maximization

Figure 31: Initial Population (o) vs. Final Population (+)

Figure 32: Cost Function Maximization – actual (o), spline (–)

Figure 33: Pole Zero Map

Figure 34: Frequency Response Comparison

Figure 35: Power Spectral Density – Wing Tip Acceleration

Figure 36: Power Spectral Density – Actuator Displacement

Figure 37: Comparison of Actuator Frequency Response

Figure 38: Step Response Comparison

Figure 39: CAD Model - X-DIA Aircraft

Figure 40: X-DIA Aircraft - Wind Tunnel Testing

Figure 41: CAD Model - X-DIA wing

Figure 42: X-DIA Wing Instrumentation

Figure 43: First Bending Mode (9.39Hz)

Figure 44: First Torsion Mode (24.63Hz)

Figure 45: V-G and V-F Diagram

Figure 46: Frequency Response of X-DIA Wing

Figure 47: Phase Plot of Frequency Response

Figure 48: Magnitude Plot of Frequency Response

Figure 49: Aeroelastic Response

Figure 50: Aeroelastic Response - State Space Methods

Figure 51: Aeroelastic State Space Model

Figure 52: Aeroelastic Response - Dependency on velocities

Figure 53: Bode Diagrams - Inboard Actuators

Figure 54: Bode Diagrams - Outboard Actuators

Figure 55: Generic Control Scheme

Figure 56: BLMSI Frequency Response

Figure 57: Effectiveness of Modal Control

Figure 58: Pole zero Map of Open and Closed loop system

Figure 59: Step Response Comparison

Figure 60: SIMULINK Model Buffet Load Mitigation System

Figure 61: Time Domain Response of Trailing Edge Outboard

Figure 62: Frequency Response Comparison - Outboard Strip Vs Inboard Strip
Figure 97: Instrumentation - X-DIA Wing ................................................................. 95
Figure 98: Connectivity of the X-DIA Wing ............................................................ 96
Figure 99: Accelerometer Attachment on the Wing .................................................. 96
Figure 100: Schematic of Pressure Transducers Attachment ..................................... 97
Figure 101: Attached Pressure Transducers on the Wing ......................................... 97
Figure 102: Power Spectral Densities of Buffet Loads ............................................. 98
Figure 103: Schematic of Accelerometer Location and Hammer Roving Points .......... 99
Figure 104: Location of Reference Accelerometers on X-DIA Wing ......................... 100
Figure 105. Mode 1, 6.87 Hz .................................................................................. 101
Figure 106. Mode 2, 24.55 Hz ................................................................................ 101
Figure 107: Experimental autoMAC Matrix ............................................................. 102
Figure 108: Conceptual Scheme of PID<sub>2</sub> Controller ........................................ 103
Figure 109: Magnitude and Phase Response of PID<sub>2</sub> Controller ......................... 104
Figure 110: Magnitude and Phase Response of Bessel Filter based PID Controller .... 104
Figure 111: Actuator Dynamics for Leading Edge Inboard (LEI) .............................. 105
Figure 112: PID<sub>2</sub> controller ............................................................................. 105
Figure 113: SIMULINK Diagram of BLMS ............................................................. 106
Figure 114: Feedback Gain Matrix ......................................................................... 107
Figure 115: Effect of Airbrake .................................................................................. 108
Figure 116: Structural Accelerations from Four Accelerometers ............................ 109
Figure 117: Cross Power Spectral Density (CPSD) wrt to Trailing Edge Midspan Accelerometer ................................................................................................................. 109
Figure 118: CPSD of Phase Difference of Four Accelerometers .............................. 110
Figure 119: PSD of BLMSI .................................................................................... 111
Figure 120: Volts Consumption of each Actuator (Bottom to Top: TEO, TEI, LEO, LEI) .... 112
Figure 121: Command/Actuator Time History ....................................................... 113
Figure 122: Power Spectral Density for Bending and Torsional signals .................... 114
Figure 123: Command/Actuator Time history ......................................................... 114
Figure 124: PSD Comparison of Leading and Trailing Edge Outboard .................... 115
Figure 125: Torque Produced due to Active Control ................................................ 116
Figure 126: Command/Actuator Time History ....................................................... 116
Figure 127: PSD of Leading and Trailing Edge Control Surfaces ............................ 117
Figure 128: Time domain Response of Active Control Scheme ............................. 117
Figure 129: Time Domain Response of Active Control Scheme ............................. 118
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>PSD Comparison of Inboard and Outboard Edges</td>
<td>118</td>
</tr>
<tr>
<td>131</td>
<td>Command/Actuator Response for Inboard Edges</td>
<td>119</td>
</tr>
<tr>
<td>132</td>
<td>Command/Actuator Response for Outboard Edges</td>
<td>119</td>
</tr>
<tr>
<td>133</td>
<td>Power Spectral Density obeying ILAF concept</td>
<td>120</td>
</tr>
<tr>
<td>134</td>
<td>Volts Consumption during Actuation</td>
<td>121</td>
</tr>
<tr>
<td>135</td>
<td>Command/Actuator Time History</td>
<td>121</td>
</tr>
<tr>
<td>136</td>
<td>Torque Inputs for ILAF Concept</td>
<td>122</td>
</tr>
<tr>
<td>137</td>
<td>Frequency Response – Experimental</td>
<td>123</td>
</tr>
<tr>
<td>138</td>
<td>Frequency Response - Numerical</td>
<td>124</td>
</tr>
<tr>
<td>139</td>
<td>Control Off - Bending Mode</td>
<td>126</td>
</tr>
<tr>
<td>140</td>
<td>Control Off - Torsion Mode</td>
<td>127</td>
</tr>
<tr>
<td>141</td>
<td>Control On - Bending Mode</td>
<td>128</td>
</tr>
<tr>
<td>142</td>
<td>Control On – Actuator Movement</td>
<td>129</td>
</tr>
<tr>
<td>143</td>
<td>Control on - Torsion Mode</td>
<td>130</td>
</tr>
<tr>
<td>144</td>
<td>Control On - Actuator Movement</td>
<td>131</td>
</tr>
<tr>
<td>145</td>
<td>Application of Filters</td>
<td>133</td>
</tr>
<tr>
<td>146</td>
<td>Advantage of Application of Filters</td>
<td>133</td>
</tr>
<tr>
<td>147</td>
<td>Uncertainty in Accelerometer Sensitivity</td>
<td>135</td>
</tr>
<tr>
<td>148</td>
<td>Uncertainty in Actuator Trailing Edge Outboard Damping Ratio</td>
<td>136</td>
</tr>
<tr>
<td>149</td>
<td>Frequency Response - Mu Analysis</td>
<td>137</td>
</tr>
<tr>
<td>150</td>
<td>Command/Actuator Time History</td>
<td>137</td>
</tr>
<tr>
<td>151</td>
<td>Feedback Schematic for Robust controllers</td>
<td>138</td>
</tr>
<tr>
<td>152</td>
<td>Frequency Response H-infinity Controller</td>
<td>139</td>
</tr>
<tr>
<td>153</td>
<td>Frequency Response Mu-Controller</td>
<td>139</td>
</tr>
<tr>
<td>154</td>
<td>Command/Actuator Time History</td>
<td>140</td>
</tr>
</tbody>
</table>
INTRODUCTION

1. Introduction
The quest for improved fuel efficiency, enhanced performance in different flight conditions and green aviation has lead researchers to focus on light weight, endurable structures for aircraft. This makes the design process of aircraft a complex research and design parameters like structure and aerodynamics are often at odds with each other, hence a compromise is needed between fuel efficiency and structural strength. Wing is also targeted for weight reduction which leads to low aerodynamic drag and consequently it improves the fuel consumption in cruise condition. The light weight wings exhibit flexibility, however the benefits of lighter wings come with drawbacks like being more prone to adverse instable aerodynamics, these instabilities combined with flexible structure response and inertial loads are origin to aeroelastic related issues like divergence (static aeroelasticity), flutter and buffeting (dynamic aeroelasticity). These flow field induced phenomena limit the flight envelope of the aircraft or if these phenomena remain unaddressed they produce structural vibrations and significant deflections that are cause of structural fatigue. One such phenomenon is buffeting, which occurs because of structural modes excitation when stochastic, unsteady and vortical flow originated from nose or leading-edge extension of the aircraft at high turn rates, immerses the wings and the vertical tail in the wake of unsteady flow. The impinged flow has significant levels of energy (buffet loads) that excite the structural modes of the wings, control surfaces and vertical tail in the frequency bandwidth coinciding with lower order structural dynamics of the wing or vertical tail. If the buffet loads are unaddressed, ultimately a structure needs to be replaced due to the possible anticipated failures which in turn means increased support cost or it leads to redesigning of the aircraft structure. Premature initiation of fatigue crack is also ported in literature if buffeting is unaddressed, which leads to catastrophe. It is worth mentioning here that flow separation at the trailing edge of the wing is also responsible for structural modes excitation termed as self-induced buffeting. The buffet loads are usually experienced near the wingtip for self-induced buffeting [1, 2, 3, 4].

To address the phenomenon of buffeting, in the early years the research was focused towards the augmentation of the structure with damping materials to passively cater the aeroelastic effect. Several approaches are devised over the past decades to avoid buffeting, which are
categorized as, aerodynamic methods and structural methods. These techniques are categorized as active and passive based on structural dynamics and aerodynamic methods [5]. Notable passive aerodynamic technique includes leading edge extension fence on the F/A-18 aircraft. Tangential leading-edge blowing is an example of active aerodynamic technique [6]. Passive structural technique such as adding reinforcement to stiffen the structure are commonly used to avoid structural vibrations due to buffet loads. Fortunately, often these structural vibrations fall in the operating bandwidth of the control surfaces, which can be used to attenuate the buffeting loads by using the active structural technique.

Thanks to the recent advancements in the field of control systems and instrumentation, the undesired phenomena related to static and dynamic aeroelasticity can be reduced actively [7]. Recent years have seen a surge in development of active control techniques to alleviate the buffeting by using the control surfaces located on the wing. The objective is to enable the aircraft to operate in multiple flight conditions while insuring that structure is always directed to induced low drag during cruise flight conditions. This dissertation brings static output feedback control, heuristic optimization and robust control methods together for active aeroelastic control through multiple control surfaces located on the wing.

2. Literature Review and Motivation
   I. Active Aeroelastic Control

   Early research on active aeroelastic control relied on collocated methods where both sensors and actuators are placed at same location. The output from sensors (typically accelerometers) is used as a feedback to drive the actuators (control surfaces) while the control surfaces are deflected in a way to damp the excited mode by changing the camber of the wing. A similar method named as Identical location of acceleration and forces (ILAF) was used in the B-1 aircraft test for active structural mode damping [8]. Recently, numerous active flutter suppression control techniques have been proposed, these techniques include: dynamic inversion [9], multivariable control [10], gain scheduling [11].

   High performance aircraft F/A-18, when operated at high angles of attack (20 to 44 degrees), experienced the problem of structural fatigue due to unsteady stochastic buffet loadings. Active control system named as buffet load alleviation (BLA) was implemented under The Technical Cooperation Program (TTCP) to alleviate the buffet loads on the twin vertical tail of the aircraft. The Single Input Single Output (SISO) system used a combination of rudder and piezoelectric actuation to alleviate the excited modes ranged from 0Hz - 40Hz and 40 Hz - 100 Hz, respectively [2].
In [12], authors have developed a LQG (Linear Quadratic Gaussian) controller to attenuate the buffet loads on the vertical tail of the aircraft. They developed and compared two different approaches by using a control surface and piezoceramic actuators for tail buffeting alleviation [13]: their numerical results demonstrated a better alleviation for strain based actuation. Researchers also used a LQG controller on full-scale F/A–18 empennages with strain actuation and demonstrated 58% reduction in the torsional mode [14]. Simultaneous design approach of structure and control is presented to satisfy the aeroelastic stability objectives then optimal control is developed from steady state Linear Quadratic Regulator (LQR) [15]. Linear quadratic regulator is applied to state space realization based on Theodorsen’s unsteady aerodynamic model. The controller stabilized the system when applied during transient state but it was not efficient when applied during Limit Cycle Oscillations (LCO) [16].

Inability of piezoelectric actuators to alleviate the first bending mode is highlighted in [2], this is because the strain energy and stiffness for the first bending mode are concentrated near the root of the vertical tail, where the skin carries the bending loads and it makes piezoelectric actuation inefficient for alleviating the loads in the first bending mode. An adaptive control system on output feedback is successfully implemented in [17], it is based on pitch angle and plunge displacement to cater the effect of uncertainties for aeroelastic controls.

Modal control based on classical control technique (ILAF) is used to alleviate the aerodynamic loads on the wing of X-DIA aircraft [18]. The proposed controller damped out the first bending and first torsion mode and thus it prevents the flutter. Robust controller based on h-infinity technique is implemented for X-56 aircraft [19-22], the implemented H-infinity controller minimizes a worst-case scenario for the uncertain aeroelastic system. All the afore-mentioned researches suggested the attenuation of vibrations to some extent, measured as either in power spectral density, frequency response or in root-mean-square (rms) value.

Static output feedback and robust controllers are two of the most researched fields in the modern control theory, these techniques ensure reliability, performance and robustness for aeroelastic systems when compared to their predecessors.

II. Review of Static Output Feedback

Full state measurements are practically impossible in aeroelastic applications, in such cases the importance of static output feedback (SoF) controller is increased to provide the active control. In cases where pre-specified structure is available along with data acquisition from fixed set of sensors (accelerometers) and actuators (control surfaces), then static output feedback is the
most unique choice for active aeroelastic control. Literature review also suggest that for the
prescribed structure, static output feedback system behaves better than dynamic output
feedback systems [23, 24].

Several approaches have been proposed for static output feedback controller. A set of three
coupled equations for optimal control output feedback is presented in [25], in which one
equation is related to spectral radius coupling and the two equations are related to Riccati
equations. The problem is further simplified by reducing it to only two coupled Riccati equations
by [26]. Lyapunov equation replace the Riccati equations for suboptimal control, these
equations are simultaneously solved by heuristic optimization techniques, it solves for the
minimization problem associated to the objective function. Authors presented an approach of
optimized feedback technique based on optimal static output feedback [27]. The algorithm is
used for active control design which requires an initial stabilizing point, that is used to minimize
the associated cost function in the feasible and stable region.

Static output feedback technique with suboptimal control restricts the feedback gain subspace in
the feasible and stable region with the numerical solution provided by heuristic optimization
techniques. However, the numerical computation of suboptimal (SoF) controller is not an easy
task as it usually has convex curvature and uncertain behavior. The difficulty also arises in
initiation of the problem from stable point or asymptotically stable point for optimization
algorithms.

Recently, static output feedback controller is used to alleviate the gust loads on advance
commercial aircraft under the project Glamour [28, 29, 30]. Static output feedback controller is
also used to alleviate the buffet loads on the vertical tail of Aluminum Fighter Aircraft (AFA)
model, the buffet loads used for analytical and numerical modelling are experimentally extracted
for geometrically similar aircrafts [31].

III. Review of Robust Control
Aeroelastic systems are complicated enough that even high-fidelity models come with
approximations and uncertainties are associated with it. This underlines the importance of
robust controllers for aeroelastic systems as they take the uncertainties by default in the state
space methods. Traditional control with optimality finds a solution for nominal system while
robust controller can give a solution for worst case system with uncertainties. Recently, robust
control techniques are implemented for robust stability of active aeroelastic wing and space
launch vehicle [32, 33].
Since its formulation in 1991, H-infinity method has provided the base to robust control techniques for different application [34]. Analytically, H-infinity methods and in general robust control has now become a mature field that involves diverse techniques of analysis in presence of uncertainties to alleviate or minimize the disturbance, however, practical achievements of robust control are very rare as compared to the other classical control techniques. It is well known that performance of controller based on infinity norm is better than controller based on $H_2$ optimal controller. State space h-infinity solutions are derived for linear time invariant (LTI) models [35]. A thorough analysis of these techniques is given in [36, 37], they also suggested the output feedback using dynamic controller.

The problem associated with these controllers is usually the insufficiency of the system compatibility to meet the controller requirement, especially for the case of Mu synthesis where the order of the controller exceeds the order of plant. The difficulties are greatly reduced by MATLAB/RTAI where the dynamic control is implemented in real time environment to cater aeroelastic response of the wing under structural mode excitation, h-infinity and Mu controller are implemented in the real time for the linear time invariant state space model of 4$^{th}$ order with 2 inputs and 2 outputs. H-infinity controller ensured efficient attenuation of disturbances, while Mu controller ensured robust performance in the presence of parametric uncertainties, while operating on minimization of structured singular values. The resulting h-infinity controller order is of same order as the order of the plant [38].

Active control based on h-infinity technique takes notice of structural velocity, acceleration as the output feedback and employ actuators to achieve vibration suppression, this approach has been successfully used in defining globally robust controller for aeroelastic wing [39]. Recently, a robust controller is designed for flutter suppression. The mini MUTT (Multi Utility Technology Testbed) aircraft, has high aspect ratio wing which compromises on structural stiffness, therefore flutter speed is reduced and operational speed is limited. A schematic design procedure for h-infinity control is presented [40].

IV. Review of Multi-Surface Control

Multi-surface control is used for static and dynamic aeroelastic tailoring of the wing where, these surfaces can be used to alter the camber of the wing by redistributing the aerodynamic load.

One beneficial feature of the multi-surface control wing is that it allows the freedom of multitasking for various purposes like flutter suppression, load attenuation or flight mechanics, these possibilities make multi surface wing an attractive choice for aeroelastic wing. Multiple
control surfaces offer many ways in which these surfaces can be instructed i.e. All the surfaces can be commanded at once (one surface) or different surfaces can be made to work independently of each other. The first technique is inefficient with simple control architecture while the second technique is efficient but its control architecture is complex. Constrained optimization is helpful in maximizing the effectiveness of the of a control surface for the given mode [41].

The availability of the multi surface analysis helps to rank the suitability of the control surface for any structural mode and to reduce the multi-input multi-output system to single-input single-output system. Advanced wings that employ multiple control surfaces are used to enhance their aeroelastic response under the projects of NASA Active Flexible Wing (A.F.W.) and Active Aeroelastic Wing [42, 43, 44]. Benefits of the multi-surface controls are also highlighted by the research conducted on Sensocraft to meet mission requirements at high altitude, endurance and to reduce the noise impact for blended wing body [45, 46]. The limitless effective suppression of limit cycle oscillation motivated the authors to multi-surface control for this purpose [47]. Their methodology helped to achieve globally stabilizing control by using two control surfaces for controlling limit cycle oscillations.

Optimized placement of sensors with Identical location of acceleration and forces (ILAF) technique enables to avoid spill over into other modes by focusing on modes on interest [18]. Multi-surface control is used to improve the dynamic response of the wing by employing efficiently and swiftly by shaping normal structural modes. The combined usage of multi-surface control helped them to achieve their goal. They conducted their research on aeroelastic demonstrator named X-DIA built by Politecnico di Milano under European project Active Aeroelastic Aircraft structures (3AS). The research investigates the use of multi-surface wing for aeroelastic control by focusing on modal information for intelligent choice of load alleviation by optimal approach.

The aerodynamic force which represented the buffet loads is modelled by Gaussian white noise and shaped by a filter based on experimentation data. This buffet aerodynamic excitation is successfully implemented in the wind tunnel test. The buffet load is analytically and numerically modelled for one reference sensor to be used for numerical validation and future reference purposes [31]. The buffet forcing function can be added to the aeroservoelastic modeling by a unit magnitude [2]. Buffet loads can also be incorporated in the state space aerodynamic model by proper orthogonal based decomposition method [5].
3. Thesis Contributions:

In the presented research, each chapter of control technique is directed towards uniqueness in active control of aeroelastic systems. The main contribution of this research is to apply advance active control techniques on aeroelastic wings to damp the structural vibrations in the first bending and first torsion modes. During the first phase, active aeroelastic control based on static output feedback is applied to the wing of Aluminum Fighter Aircraft (AFA), the feedback gain is minimized by deterministic and stochastic (genetic) algorithms, it considers the optimized movement of actuators during active control for wing, which added the novelty to the research. A unique way is suggested for inclusion of buffet loads experienced on the wing in state space model.

The contribution of this research is also towards the implementation of active control techniques on the advanced unconventional unique X-DIA commercial aircraft, with forward swept slender wing. The prototype model is realized numerically by linear time invariant state space model which vary as a function of velocity and dynamic pressure. The controllers included static and dynamic types for output feedback and robust techniques, respectively. This research also contributed towards the selection of actuator/sensor for the enhanced effectiveness in the mode of interest, Hankel singular values are used on the experimental data to provide the best actuator for specific modal attenuation. Application of notch and peak filters on the output feedback provided the unique way to alleviate the buffet loading. Robustness of the optimization algorithms and aeroelastic systems is also analyzed for uncertainty in the instrumentation and through Mu synthesis, respectively.

For the applicability of robust controllers on aeroelastic systems the main contribution of this research is to adopt a new reduction procedure for reduction of state space model, the model takes advantage of the discrete states for the modes in the concerned range of frequencies. The reduced LTI system with subset of actuator and sensor (SISO) is used to design robust controllers. The followed procedure insured and verified the capability of operating system based on RTAI/Linux, to carry out the real-time implementation of dynamic robust controller.
4. Thesis Outline:

The thesis is organized as follows. The first three parts of the thesis are designated to provide the thorough background of the research performed in the three distinct phases of the PhD. Part 1 presents the application of static output feedback controller to the Aluminum Fighter Aircraft (AFA). Chapters in part 1 provide the detail of analytical model of the AFA, followed by the numerical analysis. The realization of aeroelastic model is explained with experimentally extracted buffet loads as the input to state space system. Analytical model of buffet loads is presented. Control law formulation is derived and numerical optimization techniques are thoroughly discussed. In the end, the results are presented for buffet alleviation system which works on minimization of control surface movement.

Part 2 provides the numerical implementation of advance control laws on the unconventional X-DIA wing. In the start a detailed description of the X-DIA Aeroelastic demonstrator is provided followed by analytical and numerical modeling for the aeroelastic state space system. The diverse control laws are based on static and dynamic controllers, realized from output feedback and robust controllers, respectively. Each chapter is dedicated towards each control technique. Numerical results are presented for illustration of these techniques.

Part 3 presents the experimental process performed in pursuit of validations of the numerical finding. Experimental setup details are provided with preliminary tests like model testing and buffet load data acquisition. Then each chapter is presented with results and techniques for corresponding technique conducted in the Part 2. Finally, the concluding remarks and discussions are provided.

The funding for the PhD research, themed on “Methods and technologies for active control of dynamic loads on high performance aircrafts”, is provided by TIVANO. Under the umbrella of TIVANO, the first phase of the research is conducted in collaboration of LEONARDO Company S.p.A, Aircraft division at structural dynamics group, Venegono Superior, Italy. Second and third phases are conducted at Politecnico di Milano for numerical investigation of active control schemes and De Ponte Wind tunnel laboratory for experimental validation, respectively.
PART 1: ACTIVE BUFFETING MITIGATION CONTROL SYSTEM FOR ALUMINUM FIGHTER AIRCRAFT (AFA)

This first part describes the first phase of research completed at structural dynamics group in LEONARDO Company S.P.A, Aircraft Division. This research work is based on designing of active control scheme for numerical analysis of buffet mitigation control systems. The high performance aircraft under consideration is Aluminum Fighter Aircraft (AFA). The active control deploys ailerons to cater the effect of the buffet loads on the wing of the aircraft. The buffet loads on the wings are analytically and numerically modelled.
The already available, finite element model of Aluminum Fighter Aircraft (AFA) is numerically analyzed for the preliminary investigation of the aeroelastic phenomena. Normal Modes analysis, Frequency response analysis and dynamic aeroelasticity analysis is performed on commercially available software MSC/NASTRAN. These benchmark analyses characterized the behavior of structure for the undamped system as well for the system under sinusoidal or dynamic loadings. The response of the wing establishes the platform to build the state space model. It formed the core of the active control scheme to attenuate the vibrations in aeroelastic modes by following the certain actuation based bounds. These bounds are related to efficiency and bandwidth of the actuation system. Following paragraph summarizes the detail of the analytical model of Aluminum Fighter Aircraft (AFA).

1. Structural Model

AFA model resembles light weight fighter aircraft geometrically similar to F-16. Structural model is shown in Figure 1. Analytical model shows the description of wing, horizontal tail and vertical tail. It shows only right side wing of the aircraft, the finite element model obeys symmetry around x-axis (fuselage). The wing has aspect ratio of 3.0, total planform area 330 ft$^2$, a taper ratio of 20 % and leading edge sweep of 38.7°. The length of the fuselage is 45 ft and and the wing is connected with fuselage at 19.80 ft from the aircraft nose.

![Figure 1: Structural Model of Aluminum Fighter Aircraft (AFA)](image)

The wing is given a delta shape equipped with four control surfaces, two surfaces are located at the leading edge named as leading edge inboard (LEI) and leading edge outboard (LEO), while the notations (TEI) and (TEO) are used for control surfaces located at the trailing edge Inboard and trailing edge Outboard, respectively. The skin and skeleton of the wing is formed from
aluminum properties. The number of modes for symmetric and antisymmetric boundary conditions are 36 and 35, respectively. MSC/NASTRAN elements such as CQUAD4, CSHEAR and CROD are used to model the finite element of composite skins, spars and rib caps, respectively. The complete structural model along with global and local coordinate systems is shown in commercially available software MSC/PATRAN GUI.

![Structural Model (AFA)](image)

**Figure 2: Structural Model (AFA)**

2. Aerodynamic Model

The linear aerodynamic model of AFA is shown in Figure 3, Wings, control surfaces, horizontal tail and vertical tail are modelled with CAERO1 elements. The aerodynamic model is defined by Doublet Lattice Theory for the subsonic cases. The aeroelastic model is shown in Figure 4. The interconnection between the structural and aerodynamic grids is provided with the help of SPLINE elements to provide the interface between structural and aerodynamic forces, and displacements.
3. Numerical Analysis

Numerical analyses performed on MSC/NASTRAN are briefly summarized below:

i. **Normal Mode Analysis**

MSC/NASTRAN is opted for extraction of free undamped vibration characteristics such as the natural frequencies and mode shapes of the wing’s finite element model. Although many methods are available for eigenvalue extraction, Lanczos method is used to extract the set of
eigenvalues, as it is reported in the literature to be more robust and efficient as compared to the other methods like Givens method and Inverse Power method. Dynamic reduction method is used to extract the eigenvalues for the required frequency range. Two rigid modes with aileron mechanism are shown in Figure 5 & Figure 6 for the symmetric and antisymmetric cases respectively. Figure 7 - Figure 12, shows the selected flexible modes of the aircraft. It includes first bending, first torsion and second bending modes for the symmetric and antisymmetric case.

![Figure 5: Rigid Body Mode - Aileron Deflection (Symmetric)](image)

![Figure 6: Rigid Body Mode – Aileron Deflection (Antisymmetric)](image)
Figure 7: First Bending (4.238 Hz)

Figure 8: Antisymmetric First Bending Mode (5.4582 Hz)

Figure 9: First Torsion Mode (6.1683 Hz)
Figure 10: Antisymmetric First Torsion Mode (7.6539 Hz)

Figure 11: Second Bending Mode (16.23 Hz)

Figure 12: Antisymmetric Second Bending Mode (20.40 Hz)
ii. Frequency Response Analysis

Frequency response analysis is conducted for a specified range of frequencies by using the modal method. It uncouples the equations of motion for specified mode shapes. Structural model is excited by the steady state sinusoidal and random forces in the frequency domain at the wing tips and correspondingly the response under excitation forces is measured at the wing tips. MSC/NASTRAN, is used to carry out the analysis using the following equation.

\[
\begin{bmatrix}
\omega^2 M + i \omega B + K \\
\end{bmatrix} \mathbf{u} e^{i \omega t} = \mathbf{P} e^{i \omega t}
\]

(1.1)

Where ‘\(P_d\)’ represents the input force with certain frequency ‘\(\omega\)’. ‘\(M_{dd}\)’, ‘\(B_{dd}\)’ and ‘\(K_{dd}\)’ represents mass, damping and stiffness matrices, respectively for the associated dynamic set with the dependent coordinate ‘\(u_d\)’. The response of the wing is simulated for free Ailerons and by excluding the mechanism of control surface by increasing the stiffness of the actuator.

iii. Dynamic Aeroelastic Response Analysis

Aeroelasticity is a complex phenomenon which initiates due to coupling of structural deformation, inertial and aerodynamic loads. Both static (divergence, trim conditions etc.) and dynamic aeroelastic tests (flutter, buffeting etc.) are of prime importance as they define the performance and flight envelope. Assembly of matrix equations of motion that relates aerodynamic forces, structural deformation and the corresponding inertial, elastic and viscous forces is performed on MSC/NASTRAN. The required inertial, damping and elastic forces are calculated from structural mass, damping and stiffness matrix. The aerodynamic model is casted to transform the loads by splining methods.

4. State Space Formulation:

Aeroelastic phenomena can be grasped with high fidelity by linear time invariant (LTI) state space models. The linear dynamics of numerical aeroelastic model provides the eigenvector or singular analysis for stability analysis. Low and higher order models can be built with compromise on fidelity level to predict the modes [48]. The necessary step of state space formulation is a prerequisite for the development of active aeroelastic control law.

i. Mathematical Formulation

The model of the wing has already been realized analytically [49], in the current activity, numerical analysis formed the basis of aeroelastic model. Mass, damping and stiffness matrices extracted from the dynamic aeroelastic analysis are casted to form the multi-input multi-output state space system. Both structural and aerodynamic models are coupled by the in-house
developed software MASSA, to form the aeroelastic state space model. Complete time domain aeroelastic model can be furnished by selecting appropriate structural outputs for feedback along with aerodynamic inputs. The procedure is briefly explained here, for more thorough study the reader is referred to [50]. As reported by [29], numerous strategies are available to model the unsteady aerodynamics, however rational matrix fraction approximation stands out in accuracy and thus it forms the basis of aerodynamic model in this developed code. The unsteady aerodynamic response matrix ($H_{am}$) and gust frequency response matrix ($H_{ag}$) can be derived by assuming minimal role of gust response matrix in the following equation:

$$f = H_{am}(k,M_{\infty}).q + H_{ag}(k,M_{\infty}).\frac{V_g}{V_{\infty}}$$ (1.2)

Where $'V_g'$ and $'V_{\infty}'$ refers to gust velocity and free stream velocity. Aerodynamic forces can be approximated by Rogers technique [51]:

$$H_{am} = D_0 + D_1p + D_2p^2 + \sum_{i=1}^{m} \frac{P}{p + \beta_i} E_i$$ (1.3)

$'\beta'$ is the user defined poles with real part to be negative to assure stability. In literature, aerodynamic approximations based on Pade’s principal can also be found [52]. The aeroelastic state space model of the complete system takes the form:

$$\dot{x} = Ax + Bu \quad y = Cx + Du$$ (1.4)

While solving the following aeroelastic model equations:

$$E_{ae}\dot{x}_{ae} = A_{ae}x_{ae} + B_{ae}f$$ (1.5)

$$A_{ae} = \begin{bmatrix} -C_{ae} & 1 & 0 \\ -K_{ae} & 0 & q_{\infty} C_a \\ (V_{\infty}/c)B_a & 0 & (V_{\infty}/c)C_a \end{bmatrix} \quad B_{ae} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Where

$$E_{ae} = \begin{bmatrix} M_{ae} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \{\delta_i\}$$

$$\dot{x}_{ae} = \{q\}$$

$$\{\delta_{ir}\}$$

$$\{r\}$$

$$x_{ae}$$

$$M_{ae} = M_s - q_{\infty}(c/V_{\infty})^2D_2$$ (1.7)
\[ C_{ae} = C_s - q_s (c / V_s)^2 D_1 \quad (1.8) \]
\[ K_{ae} = K_s - q_s D_o \quad (1.9) \]

where ‘q’ and ‘r’ represent the generalized degree of freedom and virtual states, respectively for the generalized structural degree of freedom and ‘\( \delta_i \)’ represents the rotations for the associated control surface. Actuator dynamics is incorporated to complete the aeroservoelastic model, the behavior of hinge moments under control surface commanded location ‘\( \delta_{ic} \)’ is given as:

\[ \{\delta_{i}\} = [H_s][\delta_{c,i}] \quad (1.10) \]

The complete aeroservoelastic model served as the basis for active aeroelastic control named as buffet load mitigation system. It consists of total 150 aeroelastic states. Summary of the extracted results is shown in Error! Reference source not found.: Table 1:

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st Bending Mode (Sym)</td>
</tr>
<tr>
<td>2</td>
<td>1st Bending Mode (AntiSym)</td>
</tr>
<tr>
<td>3</td>
<td>1st Torsion mode (Sym)</td>
</tr>
<tr>
<td>4</td>
<td>1st Bending Mode (AntiSym)</td>
</tr>
</tbody>
</table>

Table 1: First Four Flexible Body Modes

ii. Schematic of the Inputs and Outputs on the Wing

The schematic of the wing for the symbolic inputs and outputs is shown in the Figure 13. Three accelerometers are placed on the wing and two accelerometers are placed at the missile tip to measure the structural deflections, accelerations for the input forces on the wing. In total for each half of the wing, five accelerometers and one input force through control surface are selected to model the multiple-input and multiple-output system. 69 points for each of the half wing are further added to distribute the exogenous load on the structural grids. Thus, for the complete wing the total number of outputs are 10 whereas the total number of Inputs are 140. The list of input and output system is also summarized in Table 2.
The external forces due to buffet loads are developed analytically through already available 18 pressure sensor measurements, these measurements are acquired from geometrically similar aircraft, during wind-tunnel experiments. These 18 pressure sensors are positioned on the surface of the wing at different locations as shown in the Figure 14. The analytical buffet load model is used as an input to the state space model for the numerical simulations.
The wing is excited by 18 independent differential pressure signals sampled at frequency of 4096 Hz, one such measurement in terms of PSD is shown in Figure 15 for sensor 7. Power Spectral Density (PSD) calculated using Welch algorithm for each sensor measurements helped to acquire the auto PSD and cross PSD and consequently the phenomenon can be modelled analytically [31]. Figure 16 and Figure 17, shows the cross spectral density matrix for coherence and phase plots between sensor 7 and all the sensors placed on the wing.
The data matrix is beneficial to approximate the transfer function between different sensors. The gain ratio can be evaluated by the deviation ratio of the two signals. The time delay can be estimated by analyzing the low frequency content of the cross PSD.
The obtained data are scaled to the real flight conditions and wing dimensions by using the dynamic pressure ratio and the Strouhal number composed of flight velocity and the mean aerodynamic chord of the wing. The power spectra \( \Phi \) have been scaled with the following relation:

\[
\frac{\Phi^a f^a}{q^a_\infty} = \frac{\Phi^m f^m}{q^m_\infty}
\]

(1.11)

Where, superscripts ‘m’ and ‘a’ represents model and real aircraft, respectively. \( q_\infty \) represents the dynamic pressure. The scaled frequencies ‘f’ with respect to ‘f\text{m}’ are given by:

\[
f^a = f^m \frac{f^a}{f^a}
\]

(1.12)
‘\( t_a \)’ represents aerodynamic reference time, given by \( t_a = l_a/V_\infty \), where \( l_a \) represents reference length of the mean aerodynamic chord. Approximated analytical model for the single sensor is used to define the buffet load on the wing with the help of appropriate shape filter, the shape is attributed to the experimental buffet load model for the corresponding sensor. For this purpose, the number of independent signals are reduced to one by relating the sensor measurements of all the sensors with the reference sensor. This greatly simplifies the computation procedure and selection of single sensor is meant to avoid any negative time delays. Ignoring the frequency content variation in the different signals, transfer function from the reference sensor to each of the measurement point is described by the combination of amplitude variation and phase lag (time delay). The time delay between the different sensors can be observed by linear phase behavior of cross PSD, while the gain is calculated by evaluating the ratio of the standard deviation of the two signals. The reference sensor ‘7’ is selected for amplitude ratio and time delay. This sensor is selected as to avoid inclusion of negative time delay on the surface of the wing. Figure 18 - Figure 22 illustrate the time delay and amplitude ratio with contour and spectrum plots, respectively.

![Figure 18: Contour Plot - Time Delay](image-url)
Figure 19: Contour Plot Complete Wing – Time Delay

Figure 20: Spectrum Plot - Time Delay
Loads are redistributed and map on the 69 structural grids with the help of already realized time delay and amplitude ratio by using the Pade’s approximation, given as follows:

\[ Pa = KA \frac{-0.5\tau + 1}{0.5\tau + 1} \]  

(1.13)

Where, ‘K’ represents the amplitude ratio, ‘A’ represents the area for each grid point and ‘\( \tau \)’ represents the time delay. It forms the transfer function input for each point input of structural grid with respect to the reference signal.
The design point of the control is adjacent with the flight conditions at 15000 ft at 0.6 Mach. The suboptimal control is implemented by five output accelerations from wing-aileron system, with one active aileron and 69 external load points used as input to the state space system. Shape filter is designed by choosing the gain value to equate the area under curve with respect to the original PSD obtained from sensor 7. The transfer function of the shape filter comprised of two zeros and four poles, as a set on complex plane to match the spline of the original PSD from sensor 7.

5. Control Law Formulation
The aeroservoelastic model realized from dynamic aeroelastic numerical analysis in MSC/NASTRAN is formulated as linear time invariant state space system given as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bd \\
y &= Cy + Dyu + Dyd + n \\
z &= Cz + Dzu + Dzd 
\end{align*}
\]  

(1.14)

Where, \(A \in \mathbb{R}^{n \times n}\) represents the state matrix of the aircraft, \(B \in \mathbb{R}^{m}\) represents the input matrix and \(C \in \mathbb{R}^{m}\) represents the Output Matrix \(x \in \mathbb{R}^{n}\) represents state vector, \(u \in \mathbb{R}^{m}\) represents controlled input through control surfaces, \(d \in \mathbb{R}^{m_d}\) represents buffeting loads experienced by pressure transducers, \(z \in \mathbb{R}^{l_z}\) represents performance index. Feedback on the output \(u = -\hat{G}y\), forms the vectorized gain matrix \(G = (I + \hat{G}D_{yu})^{-1}\hat{G}\). The closed loop system with these assumptions is given as:
\[
\dot{x} = (A - B_u G C_y) x + [(B_d - B_u G D y_d) d - (B_u G) n] = \hat{A} x + \hat{B} d
\]

\[
z = (C_z - D_{zu} G C_y) x + [(D_{zd} - D_{zu} G D y_d) d - (D_{zu} G) n] = \hat{C} x + \hat{D} d
\]

(1.15)

User defined objective function based on performance index and inputs is specified by:

\[
F = E(z^T W_{zz} z + u^T W_{uu} u)
\]

(1.16)

Where, ‘E’ depends on the external disturbances shaped by the white noise, ‘\(W_{zz}\)’ is a positive definite (semidefinite matrix) Hermitian or real symmetric matrix. ‘\(W_{uu}\)’ is a positive definite or real symmetric matrix. The main idea is to minimize the quadratic cost function regardless of the values of weighting matrices, the cost function has a unique minimum that can be obtained by solving the Lyapunov Equation. The matrices ‘\(W_{zz}\)’ and ‘\(W_{uu}\)’ are used as a design parameters to penalize the performance index and the control signals. To satisfy the actuation boundaries set by mechanical and electrical properties, value of parameters can be increased or decreased to switch between expensive and cheap control strategy [53]. Selection of such performance index is in accordance with our research and requirements as it efficiently switch between performance extracted from accelerometers and load input on actuators of control surfaces.

With expansion of weighting functions, performance index and inputs, it is possible to express the objective function (equation 1.16) as the trace of output covariance and solution of the Lyapunov equation for controllability, above statement can be formulated as given in equation (1.17):

\[
F = \text{Tr}(W_{\Lambda} \Pi)
\]

(1.17)

It finds the solutions by simultaneously solving the two Lyapunov equations, given by the following form:

\[
A^T \Pi + \Pi A = -W_{\Pi}
\]

(1.18)

\[
\Lambda A + A^T \Lambda = -W_{\Lambda}
\]

(1.19)

where \(W_{\Lambda}\) and \(W_{\Pi}\) can be found from equation (1.20) & (1.21):

\[
W_{\Lambda} = Q_{xx} - Q_{xx} G C_y - C_y T Q_{xx} + C_y T G T Q_{uu} G C
\]

(1.20)

\[
W_{\Pi} = S_{xx} - S_{yx} T B_u T - B_u G S_{yx} + B_u G S_{yy} G T B
\]

(1.21)
Further explanation of these functions is given by these equations:

\[
Q_{xx} = C_z^T W_{zz} C_z; \quad Q_{xu} = C_{xz}^T W_{zu} D_{zu}; \quad Q_{uu} = D_{zu}^T W_{zz} D_{zu} + W_{uu} \quad (1.22)
\]

\[
S_{xx} = B_d W_{dd} B_d^T; \quad S_{yx} = D_{yd} W_{dd} B_d^T + W_{nd} B_d^T; \quad S_{yx} = D_{yd} W_{dd} D_{yd}^T + W_{nd} D_{yd}^T + D_{yd} W_{dn} + W_{nn} \quad (1.23)
\]

The quadratic formulation provides the basis of numerical optimization in this framework which can be constrained or unconstrained defined by the user. Second order quadratic formulation presented in [28], is analytically driven to use it in the optimization procedures for the static output feedback controller. Singular values of controllability and observability gramian gives the relative conditioning of controllability and observability of the system although it is not explicitly addressed to enhance the performance of this system. Gradient and hessian are formulated with respect to design variables, are presented in the impending section. Two types of active control schemes based on static output feedback are developed and applied on MATLAB, they are based on the:

- Minimization of the variance of wingtip movement (Buffet Mitigation control system I).
- Minimization of the variance of wingtip movement and actuator movement (Buffet Mitigation control system II).

Buffet mitigation control system I (BMCSI), works by solving the objective function given by equation (1.17) for coupled Lyapunov equations (1.18) & (1.19). In buffet mitigation control system II (BMCS II), Constraints on working of actuators is implemented i.e. not only the variance of accelerometers but also the variance of actuators is minimized. To achieve this task numerically, control core runs on the equations outlined by BMCS I, however the performance output matrix is incorporated with aileron variance as well. The implementation of these control schemes is executed by optimization methods explained in the upcoming section followed by the results of these two schemes.
The limitation of control system is defined by bandwidth of the actuators which work with phase delay at higher frequencies. The aileron actuator dynamics has been modelled by second order actuator dynamics, the bode plot is shown in Figure 26: Bode plot - Second Order Actuator. Pseudo integrator is used to convert accelerations to velocities which alternatively act a low pass filter to concentrate on the modes that are within the working bandwidth of the actuator.
6. Optimization Algorithm

Heuristic optimization techniques are devotedly analyzed for static output feedback control with quadratic formulation. Algorithms based on deterministic and stochastic schematic are formulated, implemented and inferences are made based on convergence rate and robustness criteria. Deterministic Algorithms include first order and second order systems.

2. Gradient Descent Method (Numerical).
4. Levenberg-Marquardt Method.
5. Genetic Algorithm.

The steepest descent method is a minimization method which moves in the “downhill” direction along ‘$P_k$’ that makes an angle less than $\pi/2$ radians with the gradient ‘$\nabla f_k$’. It consequently update the parameters in the direction opposite to the gradient of the objective function. One advantage associated with steepest descent direction is that they are computationally inexpensive as they require calculation of the gradient ‘$\nabla f_k$’ only. The gradient descent method converges well for problems with simple objective functions [54], gradient descent method established the basis for optimization. Newton’s method is a procedure for minimization of
objective function by incorporating Hessian for updating parameters. It assumes that the objective function can be approximated locally by quadratic behavior near the optimal solution [55]. Another method is reported in the literature, ‘Levenberg Marquardt’ which combines the beneficial features of gradient descent and newton’s method. It behaves like newton’s method in vicinity of the solution and subsequently as gradient descent when away from the solution and thus accelerates towards the solution. The Levenberg-Marquardt algorithm adaptively varies the parameter updates between the gradient descent and the gaussian update, i.e. it is a combination of two minimization methods. It does so by exploiting the advantages associated to variable step length.

\[ \text{Figure 27: Gradient Descent Method} \]

Given the objective function \( F(g) \in \mathbb{R}^n \), gradient \( J(g) \in \mathbb{R}^n \) can be computed by analytically formulating the method presented for second order quadratic formulation in [28]. The hessian \( H(g) \in \mathbb{R}^{n \times n} \) can be computed in the following form:

\[
H = J^T J + \lambda^2 (J^T J)
\]  

(1.24)

Numerical procedure given by \( g^{k+1} = g^k - H^{-1} \nabla J \), reduces the given objective function. Solution to gradient descent methods can found by solving the first order iterative procedure \( g^{k+1} = g^k - \alpha_k \nabla J \), \( \alpha_k \) represents the step length. Step length \( \alpha_k \) can be chosen in a variety of ways. In computing the step length \( \alpha_k \), compromise is needed between the substantial reduction for large step lengths and computation time for small step lengths. Usually, the initial step length sizes for Newton method is unit step length or represents practical equilibrium position. This choice ensures that unit step lengths are taken whenever they satisfy the termination conditions and allows the rapid rate-of-convergence properties of these methods to take effect. Initial step length is calculated as mentioned in [55]. The idea of variable step size is implemented for
gradient descent by establishing a relationship to update the gradient at the next step in the following form:

$$\alpha_k = \nabla f_k^T p_k / p_k^T Q p_k$$ \hspace{1cm} (1.25)

$\alpha_k$ represents the step size in current iteration. Efficiency of the optimization can also be increased by quasi newton method updates. The explicit computation of gradient and hessian associated with the first and second order systems is expensive and error prone, which can be avoided by either quasi newton methods or by using derivative free stochastic algorithms, such as Genetic algorithm with reportedly better global convergence rate [56, 57]. Genetic Algorithm's inspiration comes from the evolution of living things and considering the fact those fittest among the population will survive the longest, those more adaptive will adapt to the situation. Genetic algorithm only needs the objective/cost function, it does not need its gradient or hessian to complete the optimization process. This evolutionary algorithm is based on probabilistic rules rather deterministic rules. The basic steps for the algorithm are:

I. To initialize the population. (Design Variables)
II. Evaluate a population. (Binary Coding)
III. Include crossover and mutation characteristics to the population. (Offspring Features)
IV. Set the fitness criteria. (Rank etc.)
V. Set the selection criteria, usually fitness of the population.
VI. Discard those who do not meet the fitness criteria.
VII. Evaluate cost function. (Objective Function)

The execution of this algorithm proceeds by assuming a numerical value as initial guess (Design Variables) then convert this value into binary coding of 8 bits, 16 bits etc. These binary bits will mutate or cross over, depending upon the user request. Old populations with less probability of survival is replaced by new off springs. Number of elements 'k' in starting population gives clue about string length, $2^l \geq k$, Probability for crossover and mutation is between 0.6--1.0 and (1/population size – 1/string length) respectively.

Once the cost function is formulated the optimization algorithms mentioned before will optimize the cost function in the feasible region, the first order theorem will find the convergence by evaluating the gradients defined by the form [28]:

$$F_u = \text{Tr}(W_u P + P_u W_L)$$ \hspace{1cm} (1.26)
and the LM will minimize the cost functions by evaluating quadratic approximations of the form:

\[
F_{uv} = \frac{1}{2} \text{Tr}[W_{Luv}P + LW_{Puv} + 2L A_{uv} P + (W_{Lu} + L A_u + A_u' L)P_v + (W_{Lv} + L A_v + A_v' L)P_u]
\]  

(1.28)

The optimization procedure is defined for stable system, so the system should be in asymptotically stable state for commencement of optimization algorithms. Constraints in the form of maximum torque limit and power consumption of motor are dealt by the weighting functions introduced in the control system design. Levenberg-Marquardt is set as the benchmark algorithm for optimization. It has been validated as suggested by the previous findings [55, 56], that the convergence rate of Levenberg-Marquardt algorithms is faster than available optimization codes. Table 1, shows the comparison for different optimization algorithms with processor Intel(r) core(TM) i3 4150, CPU @ 3.5GHz 3.5 GHz, RAM 3.88 GB usable for all the cases. For close loop stability condition, we assume that state matrix ‘A’ is negative definite matrix. or the origin x=0 is asymptotically stable. Initiating the algorithm with G = 0, corresponding to the equilibrium position of the actuators. For all the deterministic algorithms, the equilibrium position is assumed at zero position for practical purposes. The purpose of using the available MATLAB functions (Gradient explicit) for gradient and Hessian formulation is to approximate and check the accuracy of the derived first and second order quadratic formulation.

Many numerical implementations of the above stated algorithms have been published in papers and books [56]. The convergence criteria for the numerical procedure is sought out by taking the difference of the objective function value at previous step and the next step, divided by the objective function value at previous step. It provides the slope for the objective function. The value is set at \( \epsilon < 0.001 \) where \( \epsilon = F(i)-F(i-1)/F(i-1) \).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Time(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradient – Numerical</td>
<td>3.724</td>
</tr>
<tr>
<td>Gradient - Explicit</td>
<td>7.533</td>
</tr>
<tr>
<td>Newton’s Method</td>
<td>3.639</td>
</tr>
<tr>
<td>Genetic Algorithm</td>
<td>9.097</td>
</tr>
<tr>
<td>Levenberg-Marquardt</td>
<td>3.400</td>
</tr>
</tbody>
</table>

Table 1: Comparison of Heuristic Algorithms
Figure 28, shows the optimization results for the candidate algorithms.

![Cost Function Minimization](image1)

**Figure 28: Cost Function Minimization - Heuristic Optimization**

The results for gradient-explicit and gradient-numerical showed the high-fidelity approximation with the analytical gradients provided by the method presented in [28], Gradient-numerical method is almost twice efficient computationally. Impending lines will discuss the issue of convergence while using the genetic algorithm. Penalty function guided the solution to convergence criteria, conversely it also added computational deficiency. Performance of the genetic algorithm improved as penalty is introduced to guide the algorithm towards the minimum by adjusting the step size. Figure 29 shows the effect of two approaches as reduction in design variable is smooth.

![Design Variable Minimization](image2)

**Figure 29: Design Variable Minimization**
Figure 31 shows the comparison of initial and final population for different design variables. It shows considerable minimization in the values of the design variables; these design variables are inversely proportional to the cost function. So, the minimization of these variables corresponds to the minimization of the cost function. Figure 32 shows the gradient cost function minimization as compared to Figure 30, due to the inclusion of variable step size.
Figure 32: Cost Function Maximization – actual (o), spline (–)

Figure 33, shows the pole zero map of the system to emphasize on the stability and minimum phase of the open loop system.

Figure 33: Pole Zero Map
7. Results: Buffet Mitigation Control System I & II

Numerical simulation has been run to show the effect of static output feedback controller on the open loop system. Upcoming figures will explain the effect of closed loop system along with difference of the two proposed control systems. Figure 34, presents the frequency response of the open loop and closed loop for BMCSI and BMCSII. This result is the benchmark of this phase as it is observed the amount of attenuation achieved is quite appreciable for the modes within the defined actuator bandwidth. Also, the damping is introduced by the closed loop control system. The results are also appreciable as the excited mode are not further excited by BMCS I and BMCS II for the frequencies defined within the actuator bandwidth.

![Figure 34: Frequency Response Comparison](image)

Figure 35 and Figure 36, shows the wing tip displacement for BMCSII, the performance indices defined for the optimization process are wing tip displacement and variance of the actuator movement. The percentage of attenuation achieved is listed in Table 2, and variance of actuator stroke for the case of BMCSII is 1.81 mm. Closed loop variance of wing tip displacement and acceleration are 9.67 mm and 28.94 mm/sec².

<table>
<thead>
<tr>
<th>Method</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMCS I</td>
<td>82.78 %</td>
<td>45.80 %</td>
<td>48.70 %</td>
</tr>
<tr>
<td>BMCS II</td>
<td>75.10 %</td>
<td>34.79 %</td>
<td>34.88 %</td>
</tr>
</tbody>
</table>

*Table 2: Attenuation Achieved*
The need of gain scheduling is trivial in this case as already explained that system starting from asymptotically stable condition i.e. normal operating conditions handles the changes in the state space system well and the process of finding the gain matrix is easy for optimization algorithm. With robustness of the algorithms proved it is a simple case to calculate the feedback gains for the required altitude, Mach number and consequently the dynamic pressure.

Figure 35: Power Spectral Density – Wing Tip Acceleration

Figure 36: Power Spectral Density – Actuator Displacement
The effect of incorporating minimization of aileron movement in the optimization loop is evident from Figure 37, which show the frequency response of both the systems, there is considerable decrease in the area under the curve for BMCSII, the actuator movement is almost 4-5 times more for the case of BMCSI. Optimization of actuator movement could be vital in tasks for energy efficient missions and operations.

Figure 37: Comparison of Actuator Frequency Response

Figure 38, represents time domain response of the open loop and close loop systems. BMCSI exhibits sharp response in reply to any disturbance or internal command but its settling time is more as compared to BMCSII which has more rise time and less overshoot as compared to BMCSI. So, a tradeoff is need between BMCSI and BMCSII. System with actuator bandwidth considerably high could apply BMCSI which is superior in terms of attenuating the buffet loads.
Figure 38: Step Response Comparison
PART 2: BUFFETING LOAD MITIGATION SYSTEM FOR ADVANCED COMMERCIAL AIRCRAFT (X-DIA)

This part presents the numerical analysis of X-DIA wing, which layout the basis to perform the experimental validation, the work is completed during the second phase of the PhD at Politecnico di Milano. Active control scheme is developed owing to the beneficial features offered by the multi-surface wing of the X-DIA aircraft. This part contains an introductory section of the X-DIA aircraft followed by numerical analysis. State space realization for linear time invariant dynamics is presented. The newly developed control schemes are presented for X-DIA aircraft in the following chapters. In total six controllers are implemented to mitigate buffet loads on the wing, these schemes can be categorized into static and dynamic output feedback controllers.
To test and implement the unique and novel active aeroelastic control techniques, X-DIA wing is always the first choice as the analytical model is already prepared and X-DIA aeroelastic demonstrator is already present. In addition to that the configuration of X-DIA aircraft itself makes it a unique case, as it has forward swept wing with canards and T-tail configuration. The preceding work on the X-DIA aircraft is completed under Active Aeroelastic Aircraft Structure (3AS) project funded by European Union. The model is scaled with 1/10 ratio by satisfying the Froud number, half wing is used to test and implement the diverse control laws, for this reason the wing is dismantled from the aircraft body. To provide the wing with same constrained motion as it experiences with the fuselage connection, separate mounting system is built. The Figure 39 and Figure 40, shows the X-DIA aircraft:
The wing is built by following a classical aeroelastic approach of stringers and spars with ribs placement along the span. The analytical model of the X-DIA wing is designed on the same principle. The spar is located at 30 % and 70 % of the chord length respectively. X-DIA wing is unconventional with a slender forward-swept configuration which cause it to suffer from overrelaxation of bending and torsion modes. The wing is equipped with 4 control surfaces to deal with the normal modes of the wing in addition to flight mechanics of the aircraft. Two control surfaces are situated on the leading edge, named as leading-edge outboard (LEO) and leading-edge inboard (LEI). Similarly, two control surfaces are present at the trailing edge of the wing, named as trailing edge outboard (TEO) and trailing edge inboard (TEI). Leading edge control surfaces are made from styrofoam and trailing edges are made from balsa wood. These control surfaces are rotated through motors equipped with encoder and planetary gear reduction system. The motors for driving four actuators are selected by fulfilling the physical design constraints such as weight and maximum allowable size. This arrangement allows the saturation limit of 10V for all the motors and torque saturation limit of 0.4 Nm for leading edge control surfaces and 0.1 Nm for trailing edge control surfaces. The wing is also equipped with monoaxial accelerometers, attached at predetermined locations by obeying the identically located accelerations and forces (ILAF) law. This technique suggest that deflection of a control surface should be proportional to structural velocity, as it assumes that structural accelerations and the control forces are applied at the same point. Two piezo-resistive pressure transducers are also attached on the surface of the wing to measure the dynamic loads. Figure 41 shows the CAD model and Figure 42 shows the instrumentation of the wing.
Figure 41: CAD Model - X-DIA wing

Figure 42: X-DIA Wing Instrumentation

Few of the specifications of the X-DIA wing are summarized here in Table 3:

<table>
<thead>
<tr>
<th>X-DIA right wing specification</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing span</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Wing surface</td>
<td>0.75 m²</td>
</tr>
<tr>
<td>Wing sweep</td>
<td>15 deg</td>
</tr>
<tr>
<td>Wing dihedral</td>
<td>3 deg</td>
</tr>
<tr>
<td>Total Mass</td>
<td>2.4 Kg</td>
</tr>
</tbody>
</table>

Table 3: X-DIA Wing Specifications
Grid points are precisely placed to extract structural accelerations and to apply the external forces due to buffet loads. Control surfaces are used for feedback control to attenuate the buffet loads.

The numerical analysis is carried out by using commercially available software’s MSC/NASTRAN and MSC/PATRAN. This chapter contains a description of modal analysis, frequency response analysis, static & dynamic aero elastic analysis and flutter analysis. Sol 103, Sol 111, Sol 144. Sol 145 and Sol 146, that are carried out in MSC/NASTRAN. Results are summarized here to highlight the purpose for their extraction.

1. Numerical Analysis

Two type of normal modes analysis are considered to extract the modes of the X-DIA wing i.e. 1. Modal analysis with fixed control surfaces and 2. By considering control surface mechanism. For the former case, the motion of control surface is made zero by constraining it. Stiffness of actuators is kept constant for both the cases. For the later case, the actuator is allowed to have additional rotational degree of freedom. In this way four symmetric modes are added to the rigid body modes. Results are summarized in Table 4 and Table 5:

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Eigenvalues</th>
<th>Frequency</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.819577E+00</td>
<td>2.146868E-01</td>
<td>Rigid Body Modes</td>
</tr>
<tr>
<td>2</td>
<td>2.860814E+00</td>
<td>2.691937E-01</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.041239E+00</td>
<td>2.775527E-01</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.166005E+00</td>
<td>3.248478E-01</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Normal Modes (Rigid Body)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Eigenvalues</th>
<th>Frequency</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.481183E+03</td>
<td>9.390388E+00</td>
<td>1\textsuperscript{st} Bending</td>
</tr>
<tr>
<td>2</td>
<td>4.892865E+03</td>
<td>1.113273E+01</td>
<td>1\textsuperscript{st} In-plane bending</td>
</tr>
<tr>
<td>3</td>
<td>2.395264E+04</td>
<td>2.463184E+01</td>
<td>1\textsuperscript{st} Torsion</td>
</tr>
<tr>
<td>4</td>
<td>6.268926E+04</td>
<td>3.984893E+01</td>
<td>1\textsuperscript{st} control surface bending</td>
</tr>
<tr>
<td>5</td>
<td>8.746484E+04</td>
<td>4.706920E+01</td>
<td>2\textsuperscript{nd} bending</td>
</tr>
</tbody>
</table>

Table 5: Normal Modes (Flexible Body)

With the actuator bandwidth margin, up to 30Hz, first three modes fall within the bandwidth of actuator, consequently the first two out of plane modes, i.e. first bending and first torsion are of
prime interest as their excitation not only limits the flight envelope but also introduces structural
damages and in severe cases uncontrolled first torsional mode leads up to dynamic instabilities
like flutter. These first two out-of-plane modes are shown in Figure 43 and Figure 44.

![Figure 43: First Bending Mode (9.39Hz)](image1)

![Figure 44: First Torsion Mode (24.63Hz)](image2)

Flutter analysis is also performed by using PK method. To illustrate the flutter velocity, damping
associated with the modes and frequency content of the concerned modes, V-G and V-F graphs
are presented. Figure 45 shows the behavior of first two modes for frequency and damping
content with respect to velocity, it also shows the flutter velocity.
The excitation frequency of the first torsional mode showed a gradual decrease after 15 m/s, while for the case of bending mode the excitation frequency is constant up till 50 m/s. Similarly, the damping of torsional mode gradually decreases after 40 m/s, which eventually leads it to dynamic instability at 58 m/s. In context of this research, the airspeed in the test section of the wind tunnel is kept to 30 m/s (almost half of the flutter velocity).

Static aeroelasticity analysis is performed to obtain the trim conditions and to check the divergence dynamic pressures, the propose of this analysis is to quantify the perturbed aerodynamic forces for flexible wing under static aerodynamic loads. The behavior of the wing under aerodynamic forces is assumed to be in equilibrium state. It ensured the safety margins for static aeroelasticity instability caused due to divergence. Table 6, shows the divergence summary for the respective test.

<table>
<thead>
<tr>
<th>ROOT No.</th>
<th>DIVERGENCE DYNAMIC PRESSURES</th>
<th>EIGENVALUE REAL</th>
<th>EIGENVALUE IMAGINARY</th>
<th>FREQUENCY CYCLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.22E+04</td>
<td>0.00E+00</td>
<td>1.49E+02</td>
<td>8.21E+00</td>
</tr>
<tr>
<td>2</td>
<td>2.66E+03</td>
<td>0.00E+00</td>
<td>5.16E+01</td>
<td>2.37E+01</td>
</tr>
</tbody>
</table>

*Table 6: Divergence Summary*
For trimmed flight conditions, solution includes grid point weight generator, non-dimensional stability, control derivative coefficients. It also includes aerodynamic pressure and aerodynamic pressure coefficients for each aerodynamic grid, displacements and stresses in elements as well.

Modal Frequency response analysis for the X-DIA wing is conducted by using module Sol 111 available in commercial software MSC/NASTRAN. This method is also the extension of normal modes analysis as it allows to analyze the modal shapes rather than structural grids of the X-DIA wing for the sinusoidal and random forces. It provides the efficient solution by using reduced size for couple equations of mode shapes. The extracted result for the wing tip accelerometer near to trailing edge is shown in Figure 46.

Figure 46: Frequency Response of X-DIA Wing

The first two excited modes that are excited due to sinusoidal input forces are shown above. Both these modes are of interest as control schemes are developed to alleviate the loading in these excited modes. It is clearly seen that these modes are highly undamped. Figure 47 and Figure 48, shows the magnitude and phase of the frequency response for all the modes.
Aerodynamic model based on Doublet Lattice Method (DLM) is added to the existing structural model to form the aeroelastic system. Dynamic aeroelastic response is analyzed to observe the interaction of aerodynamic instabilities with the aeroelastic structure. The dynamic aeroelastic response is shown in the Figure 49:
It can be easily inferred that damping is added with addition of aerodynamic loads to the first two modes. The magnitude of excitation remains almost constant for first bending mode, while for first torsion mode it is attenuated for aeroelastic response as compared to frequency response. With the completion of preliminary validations on already available model of the X-DIA wing, state space model is formulated based on linear time invariant systems. Linear time invariant (LTI) models captures the dynamics of the structure with high fidelity and correspondingly they can be used for stability analysis. The stability analysis is of prime importance in aeroelastic problem as well for numerical procedure adopted here for optimization techniques. The state space model is realized through in house built software MASSA, as a result of collaboration between Politecnico di Milano and LEONARDO Company S.p.A. Aeroelastic response of state space model is presented for accelerometer at wing tip trailing edge in Figure 50.
Heuristic optimization techniques, explained in “Part 1” of this thesis are used here for minimizing the gain feedback matrix that is used in static output feedback (SoF) controller. Gradient and Levenberg-Marquardt methods are used specifically due to their performance and efficiency as outlined in [28]. The optimization algorithm is quadratically or linearly minimized depending upon the kind of optimization algorithm that is under consideration.

The created state space model has 13 Inputs and 8 outputs, with 108 states. ‘5’ inputs are defined for 5 piezo resistive patches, that are attached on the upper surface of the wing to extract the external dynamic loads due to buffet phenomena. ‘4’ inputs are defined for external forces on exactly the same grid points where accelerometers are placed. Lastly, ‘4’ Inputs are provided for the four control surfaces that provide the control over the structural dynamics of the wing. ‘4’ outputs are defined for four structural accelerations corresponding to the four accelerometers attached on the wing. It is user dependent to extract structural velocities and displacements depending upon the requirement of the system. ‘4’ outputs are allocated for the rotation of control surfaces associated with active control. The generic diagram of the state space model is shown in the following Figure 51.
The aeroelastic model is prepared for three different speeds i.e. 20m/s, 30m/s, 40m/s. The 3-dimensional graph to show the variation of the open loop response with respect to the velocity is shown in Figure 52:

The buffet loads extracted from piezo-resistive pressure transducers will be used to model the external forces numerically and subsequently they define the inputs to the state space system. From the structural accelerations, pseudo integrations can be used to extract the structural
velocities or double integration can be used to extract the structural displacements. This integration technique provides the focus on concerned modes. It can be given as follows:

\[
v = \frac{s}{s^2 + 2\xi_0 \omega_o s + \omega_o^2} a_{\text{meas}}
\]

(2.1)

\[
s = \frac{s}{s^2 + 2\xi_0 \omega_o s + \omega_o^2} v
\]

(2.2)

The presented integrators act a combination of low pass and high pass filters, they will provide a high pass band at 0.3 Hz and second order low pass band at 35 Hz. The care is taken in selecting a wider band pass along with the adequate gain and phase delay.

The control surfaces are numerically modelled from already realized experimentation data of the four actuators. The response of the control surfaces is shown in Figure 53 and Figure 54. It can be observed that the actuator system (control surface and motor) has some overshoot in magnitude, that is due to the inclusion of oscillatory poles introduced by the mechanical part of the system. The leading-edge actuators provided a bandwidth of 30 Hz, while for trailing edge actuators, the bandwidth is around 80 Hz. The bandwidth of the former actuators also acts as filters on high frequency structural modes.
Figure 53: Bode Diagrams - Inboard Actuators

Figure 54: Bode Diagrams - Outboard Actuators
2. Buffet load Mitigation System

Six active aeroelastic control schemes are presented in this section for X-DIA aircraft, they are devoted to impart aerodynamic stability to the wing. The control surfaces attached to the wing are commanded by active control scheme to change the camber of the wing to redistribute aerodynamic forces for the attenuation of the excited aeroelastic modes and increasing the damping of the system. The actuator/sensor configuration for the the X-DIA is realized through Identical location of accelerations and forces (ILAF) concept. Each control surface will be controlled by motor programmed through separate PID\(_2\) controller. The PID\(_2\) controller is equipped with antiwindup scheme. The active control schemes are named as Buffet Load Mitigation System (BLMS), as variety of novel schemes are developed which belong to the family of static output feedback controllers and dynamic controllers based on advanced H-infinity and Mu synthesis methods. These controllers are characterized into six distinct phases:

1. Static output feedback control for modal control with heuristic optimization (BLMS I).
2. Comparative study of each actuator/sensor configuration by using ILAF and HSV concepts (BLMS II).
3. Incorporation of notch and peak filters in the output feedback (BLMS III).
4. Robustness of active control scheme under uncertainties in the instrumentation of the system (BLMS IV).
5. Mu synthesis of static output feedback controller (BLMS V).
6. Robust controller for attenuation of buffeting loads (BLMS VI).

To briefly summarize here, control systems based on Lyapunov stability criteria is developed to measure the performance and stability of the system. The objective is to minimize the quadratic performance index based on the variance of outputs of the system which are practically represented by the structural velocities obtained after integration of the accelerometers. The objective function is given as:

\[
F = \text{Tr}(W_A\Pi) \tag{2.3}
\]

\[
F = E(z' W_{zz} z + u' W_{uu} u) \tag{2.4}
\]

The solution to the given objective function is obtained through simultaneously solving the Lyapunov equations expressed as follows:

\[
A'\Pi + \Pi A = -W_{\Pi} \tag{2.5}
\]
\[ \Lambda A + A^T \Lambda = -W_\Lambda \]  

(2.6)

The matrices of ‘\(\Pi\)’ and ‘\(\Lambda\)’ are determined by formulating the matrices \(W_\Lambda\) and \(W_\Pi\). To improve the numerical conditioning of ‘\(\Pi\)’ and ‘\(\Lambda\)’, these matrices also provide the relative degree of controllability and observability. The terms that are computationally expensive and produce minimal effect like \(D_{yu}\), \(D_{zd}\), \(W_{uu}\), \(W_{nn}\), \(W_{ud}\) and \(W_{dn}\) are ignored and the matrices are re-arranged to be given as follows:

\[
W_\Pi = B'_d W_{dd} B_d - B'_d W_{dd} B_u G D_{yd} - D'_{yd} G B'_u W_{dd} B_d + D'_{yd} G B'_u W_{dd} B_u G D_{yd} 
\]

(2.7)

\[
W_\Lambda = C'_z W_{zz} C_z - C'_z W_{zz} D_{zu} G C_y - C'_z D_{zu} G C'_y W_{zz} + D'_{zu} G C'_y W_{zz} D_{zu} G C_y 
\]

(2.8)

Above stated equations provided the indirect access to controllability and observability to the system. The minimization of objective function takes place only in the feasible and stable region through optimization algorithms based on first order and second order techniques. This is accomplished by first and second order formulations proposed by [28], and given as follows:

\[
F_u = \text{Tr}(W_{Lu} P + P_u W_L) 
\]

(2.9)

\[
F_{uv} = \frac{1}{2} \text{Tr}[W_{Luv} P + L W_{Puv} + L A_{uv} P + (W_{Lu} + L \tilde{A}_u + \tilde{A}'_u L) P_v + (W_{Lv} + L \tilde{A}_v + \tilde{A}'_v L) P_u] 
\]

(2.10)

As, the X-DIA wing is designed for multi-surface control, so the control scheme implemented in this research takes benefit of it and a multi prong strategy is applied for the maximum advantage. Common objective to all the optimization techniques is the selection of effective objective function. BLMS is developed as such to exploit the different combinations of actuators and sensors for the beneficial attenuation of modes. The presented optimization strategy can switch between different input-output signals and it gives the advantage to readily enhance the performance and robustness of the system. The formulation of objective function also enables us to switch between control rich or performance rich criteria. This is achieved by the weighting functions that represents the trade of between control effort and performance successfully, as implemented in LQG problem for typical section airfoil [58]. With few changes in control the wing can be divided into inner and outer strip, leading edge and trailing edge strip or a single control surface working independently to attenuate the unwanted loads. The control scheme architecture is given in the Figure 55, the core of this architecture is the state space model which represents aeroelastic behavior of the wing, inputs to the state space system are classified into two types, it includes external aerodynamic forces to excite the aeroelastic modes.
and internal forces in terms of control surfaces located on the wing. The dynamics of the control surfaces is presented by nonlinear servo-actuators. The output of the actuators is guarded by saturation limit owing to the physical constraints on the working of the actuator. Outputs from the aeroelastic model are processed by anti-aliasing filter and subsequently used as performance index and feedback signal to the control surfaces.

Figure 55: Generic Control Scheme

The following chapters will provide the background, description and control implementation techniques for the above stated methods.

i. Buffet Load Mitigation System I:
The control law formulation along with optimization strategy defined in the preceding sections constitutes the unique and novel first part of the research, Buffet Load Mitigation system (BLMS I). The aim of the research is to attenuate first bending and first torsion modes as they fall well within the bandwidth of the actuator. The structural velocities can be directly used as feedback for gain matrix calculation or bending and torsional modal accelerations (velocities) can be formed individually to shift the focus of the active control on one certain mode, combination of both bending and torsional signals can also be used for attenuation of vibrations. The proposed bending and torsional signals results by arithmetic operations on structural acceleration (velocities) resulted from the combination of the signals from accelerometers [18]. The method followed in the current research is more conservative for control schematic implementation. Control signals are isolated by giving maximum weightage to accelerometer on the trailing edge at the wing tip as it is farthest from the elastic axis for bending signal and for torsional signal the
accelerometer signals are subtracted without quantifying their distance to the elastic axis. The modal accelerations (velocities) nonetheless provided the weighted mean of structural accelerations (velocities) for the first bending and first torsion modes are given as follows:

\[
\begin{bmatrix}
\text{Ben}_i \\
\text{Tor}_i \\
\text{Ben}_o \\
\text{Tor}_o
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
\text{Acc}_\text{LEI} \\
\text{Acc}_\text{TEI} \\
\text{Acc}_\text{LEO} \\
\text{Acc}_\text{TEO}
\end{bmatrix}
\]  

(2.11)

Ben\textsubscript{i}, Tor\textsubscript{i}, represents bending and torsion signals for inboard strips. While Ben\textsubscript{o} and Tor\textsubscript{o}, represents bending and torsional signals for trailing edge outboard. The proposed strategy also made the outboard and inboard control surfaces independent of each other. Structural velocities from the respective accelerometers can be derived by integrating with the pseudo integrator (that act as a combination of low pass and high pass filter). It diminishes the marginal stability introduced by the pure integrators. The balance between the control authority and the physical constraint (saturation) associated with the motor can be established by addressing the constant weighting functions present in the objective functions. As stated in [48], without compromise on generality, the weighting functions for external disturbances, feedback outputs and performance index can be set to unity (see equation 2.4). The control effort can be set arbitrarily to meet the physical constraints of the actuator, it can be done by iteratively designing the weighing functions to provide the feasibility to wide range of performance based solutions. Figure 56, shows the open loop and close loop (BLMSI active) response when all the sensors are in operation to provide the input to all the actuators. The response is shown here for sensor/actuator configuration located at trailing edge outboard.
The dominant peak in the frequency response corresponds to first bending and first torsion modes. The response of the closed loop system is highly damped and significantly attenuated due to the action of the control in the first bending and first torsion mode. Figure (57), illustrates the importance of bending and torsion signals for modal control. As apparent from the name, the output feedback due to bending mode signal and torsional mode signal is better in the bending and torsion modes, respectively. The attenuation is comparatively improved when active control does not used modal control, as shown in Figure 56. Combination of these two feedbacks will provide the closed loop response with some compromise on their respective modes, it is trivial for this case as considerable attenuation is achieved for active control scheme. The frequency response in Figure 57, is shown for trailing edge outboard with force at the wingtip.
As already described, that minimization process takes place in the stable and feasible region. To ensure the stability of the system, the check on the system is introduced on the definiteness of the state matrix, this check is numerically performed after every iteration and also when the optimization algorithms completes the minimization process. Figure 58 and Figure 59, highlight the stability of the system and reaction of the system to the input disturbance respectively. Step response is presented for time domain analysis of trailing edge outboard. Thus, it shows that active control scheme will damp out the disturbance very efficiently. Pole-zero map is presented for the open loop and closed loop systems, all the poles are well inside the stable region, infact the new dynamic systems added to the system like actuators, integrators, filters and external loads modelled with transfer functions do not destabilize the system and the stability of the system increases for further shift of the poles left plane.
Figure 58: Pole zero Map of Open and Closed loop system

Figure 59: Step Response Comparison
Once numerical simulation of the BLMSI is carried out, the complete procedure is modelled in SIMULINK, to extract the time domain results for the proposed attenuation scheme. The SIMULINK model will also provide the base for experimental testing. The feedback gain matrix is provided by the optimization scheme and input loads are assumed with 25 – 100 N variance with white noise incorporated in the signal. The sampling time is selected as 1/1000 seconds. Three distinct input blocks are represented for buffet loads, ILAF loads and actuators, respectively. Output are processed to meet the requirements of buffeting load mitigation system before being feedback to the gain matrix. Figure 60, shows the SIMULINK design for buffet load mitigation system. Whereas, Figure 61 shows the open and close loop time domain response for velocity noted at trailing edge outboard.

![Figure 60: SIMULINK Model Buffet Load Mitigation System](image-url)
ii. Buffet Load Mitigation System II:
Multi-surface and multi-sensor wing provides the opportunity to critically quantify the performance of the X-DIA wing and not to remain limited to the previous literature results related to similar multi-surface control wings. In this context, the aim of this scheme is to scratch the benefits associated with the multi-surface forward swept wing. Several combinations of control surfaces are tested to select the best control surface for the attenuation of first bending mode and first torsion mode i.e. BLMSII is developed as such to exploit the different combinations of actuators and sensors for the beneficial attenuation of modes. HSV is used to measure the effective coupling of different parameters to maximize the design goals [59], it will be used to rank the control surfaces for bending and torsion modes. The conclusion is the selection of appropriate control surfaces along with sensors that will increase the control system efficiency and performance. The dynamics of the wing can be represented symbolically by the following transfer matrix in equation (2.12):

\[
\begin{bmatrix}
  z \\
  y
\end{bmatrix} =
\begin{bmatrix}
P_{zw} & P_{zu} \\
P_{yw} & P_{yu}
\end{bmatrix}
\begin{bmatrix}
w \\
u
\end{bmatrix}
\]

(2.12)

The transfer matrix \( P_{zw} \), is the path to performance from external disturbance signals and the transfer matrix \( P_{yu} \), define the path from the input actuator or signal to the response of the sensor. The performance criteria and feedback of the system are analogous for the presented case. Identical location of accelerations and forces (ILAF) law is also validated by this control scheme. The technique is distributed in two parts, firstly frequency response is used to demonstrate the performance of different control surfaces working individually or in combination...
with other control surfaces. Secondly, analytically available model of Hankel singular value (HSV) for sensor/actuator optimized position is used to quantify the modal performance experimentally.

First comparison is shown for outboard edges and inboard edges by using the frequency response at trailing edge outboard. The results predict the superior performance of outboard control surfaces in attenuating the buffet loads as compared to inboard control surfaces specifically in the first bending mode.

![Figure 62: Frequency Response Comparison - Outboard Strip Vs Inboard Strip](image)

The result presented in Figure 63 and Figure 64, compares the response for trailing edge and leading edges. The results are noted for sensor/actuator configuration at TEO and LEO, respectively. The results for comparison of leading edges and trailing edges shows that the attenuation achieved by TEO is considerable as compared to LEO.
Figure 63: Frequency Response Trailing Edge Control Surfaces

Figure 64: Frequency Response Leading Edge Control Surfaces

Figure 65. shows the frequency response of trailing outboard sensor, calculated for all the control surfaces working individually to attenuate the loads. It also shows that TEO is the most
efficient surface to attenuate the buffet load in bending mode and it suggested that TEI is most suitable control surface to deal with the load attenuation of the torsional mode.

Figure 65: Frequency Response of All Surfaces

Figure 66: Frequency Response of All Surfaces (ILAF Concept)
Figure 66, illustrates the frequency response obtained through ILAF concept, the response is shown for each control surface input to the adjacent accelerometer output. It also predicted that TEO is most effective in the bending mode while TEI is most effective in torsion mode, like the previous results.

Frequency response of TEI and TEO is also presented to show the attenuation in deciBel scale for the buffet induced dynamic loads.

To summarize this phase, Figure 68 is presented to highlight the main features of this phase and performance of different control surface combinations i.e. leading and trailing edge outboard (LTEO), trailing edge inboard and outboard (TEIO), trailing edge outboard (TEO), all the responses are observed at trailing edge outboard.
This comprehensive and diverse phase concludes by predicting:

1. Outboard control surfaces are better for load attenuation than Inboard control surfaces.
2. Trailing edge control surfaces are better than leading edge control surfaces.
3. Performance of LEI is worst for bending mode attenuation.
4. Performance of TEI is best for attenuating the torsional mode.
5. Performance of TEO is best for attenuation in bending mode.
6. Trailing edge outer (TEO) is better than leading edge outer (LEO) in both modes.

Analysis based on Hankel singular values (HSV) is presented in ‘Part 3’ as it involves the analysis of the experimental data.

iii. **Buffet Load Mitigation System III:**

From the analysis in the BLMSII phase, preliminary numerical results showed that the efficiency of outboard trailing edge in the bending mode but its deficiency in the torsional mode. It is evident that to make trailing edge outboard (TEO) a unique surface for performance in all modes of interest, some enhancements in the open loop system are necessary to improve its performance. Several possibilities exist to achieve the target like dictating the role of trailing edge in terms of frequency dependent weighing functions. However, a new strategy is proposed here, which is novel in aeroelastic concepts. It works by application of notch/peak filters on the
output feedback of the system, the aim is to make (TEO) more performance efficient during torsional mode. It is numerically designed by selecting suitable parameters for notch/peak filter transfer function, the scheme is termed as Buffet load mitigation system III. Notch/peak filters offers variety of advantages and variety of cases can be tried with sharp notch or blunt notch. The motivation for using a notch/peak filter in the output feedback is to attenuate the participating frequencies in the torsional mode, since it is a source of dynamic instability and needs to be addressed. The proposed method forms the bending and torsional signals, which are passed out to output feedback of the system, these signals are processed by peak and notch filter at the torsional mode. Special care has been taken in the designing filters as they do not disturb the overall dynamics of the system and improvement must be noted for the proposed scheme. These filters are incorporated due to the fact of their inclusion does not affect the low frequency and transient behavior of the overall system [60, 61, 62]. The implementation of this scheme is shown as the open loop control architecture in Figure 69.

![Figure 69: Open Loop System with Notch and Peak Filters](image)

The bode magnitude and phase are shown in the Figure 70 and Figure 71.
The result of frequency response due to implementation of peak and notch filter is shown in the Figure 72. It shows that the devised strategy helped to improve the attenuation in the torsional mode.
iv. **Buffet Load Mitigation System IV:**

The performance and robustness of the control system is limited by the amount of uncertainty in the state space realization or instrumentation, governing the system [63]. Numerical optimization techniques ensure the performance and robustness of the control law even when the state space system is incorporated with uncertain and unmodeled dynamics. The suboptimal controller based on minimization of quadratic cost function behaves efficiently and robustly for the minimal changes in the system [44]. The aim of this technique is to assess the robustness of optimization algorithms and subsequently the robustness of static output feedback controller. The controller is initiated at asymptotically stable point to converge for the solution for all the same cases. Quadratic cost function is robustly minimized for any uncertainties associated with the system. This type of controller adjusts its feedback gain value for the new set of inputs and give the optimum results. The optimal solution provided by the algorithm, with same initiation point is still in feasible region.

The targets for uncertainty are related to instrumentation i.e. accelerometers and actuators, which are more prone to changes over the life span. Numerical uncertainty is introduced in the transfer functions of actuators by changing damping ratio and natural frequency. While for
accelerometers, output feedback is adjusted with the appropriate gains. The numerical results showed promise of the optimization algorithm to handle the uncertainty well. Later it will be validated by experimentation conducted under Buffet Load Mitigation system IV. Parametric uncertainty is used to model the actuator uncertain dynamics. Due to inclusion of this parametric uncertainty the system becomes uncertain and correspondingly the state space is uncertain. Several cases can be considered i.e. nominal case, worst case or from range of samples to represent the system dynamics. It is noted that outer trailing edge is more prone to uncertainty in the actuator dynamics as compared to other control surfaces. Following Figure 73, shows different cases for bode plot when TEO accelerometer is augmented with uncertainty, i.e. 20% in damping ratio and 10% natural frequency of actuator transfer function.

Figure 73: Bode Plot - Worst Case and Samples
Figure 74, shows the comparison for nominal and worst-case system. The uncertain state space (LTI) system needed to be transform as the optimization algorithms and Lyapunov stability are designed for certain state space models. The resulting controller is tested with respect to forced uncertainties and fixed initiation point for algorithm. The numerical analysis performed with test section velocity of 20 m/s, shows that the gain margin and phase margin is decreased but overall the system remains stable and as the uncertainty is increased the stability of the system is decreased. The response time, settling time, overshoot is increased as compared to the system with nominal parameters. However, the designed suboptimal controller works robustly for these uncertainties. The behavior of close loop system is shown in Figure 75, from all the uncertain scenarios only the worst-case scenario is modelled to show the behavior of TEO, it is evident that the amount of attenuation achieved by TEO decreases with increase in uncertainty levels. The attenuation levels are almost constant for uncertainty up-to 15%, followed by constant slope up till 60% uncertainty, but there is significant reduction in attenuation capabilities of trailing edge outboard for uncertainties greater than 60%, eventually for 78% the system fails to attenuate any loads in the first bending mode. It infers that the system can act robustly up to certain limit of uncertainty that can be introduced in the transfer function of the actuator.
Figure 75: Decrement of Attenuation for Uncertainty (TEO)

Figure 76: Frequency Response of Output Feedback Uncertainty
Figure 76 shows the frequency response of trailing edge wingtip accelerometer for trailing edge outboard. It can be seen in the figure that the performance of the system decreased with the increment in the uncertainty. The close loop system started behaving like open loop system for the worst amount of uncertainty.

Figure 77 shows the systems response for step input. Open loop and close responses for worst cases are compared in this figure. The closed loop response is with uncertainties, 20% in damping ratio and 10% natural frequency of actuator transfer function and 50% uncertainty in output feedback. The comparison seems to be quite satisfactory when compared with the worst case. The $2 \times 4$ matrix for step response represents two outputs form LEO and TEO accelerometers corresponding to the 4 inputs related to ILAF loads. This concludes that the step response of the system with nominal parameters is even better, which is quite appreciable.

![Step Response Comparison](image)
v. Buffet Load Mitigation System V:
The presented scheme is not only used for SoF controller with frequency dependent weighting functions but it is also used for Mu synthesis of the existing controller by minimizing the associated infinity norms. The aim of this analysis is to ensure robust performance of the existing controller under uncertainties with Mu synthesis. Under this method, the plant model (state space system of X-DIA aircraft) is made uncertain by parametric uncertainties then the output is regulated as such to modelled external disturbances and measured noises. The uncertainties in the plant model can be representative of aerodynamic, structural or stiffness uncertainties. Uncertainties related to high frequency dynamics are not considered for this case as the modes of interest are within 30Hz. In this context, the objective of this scheme is defined as disturbance rejection and noise insensitivity minimization defined by the infinity norm, the relation can be given as:

$$\min \left\| P(I + KP)^{-1} W_d, (I + PK)^{-1} W_n \right\|_\infty$$

(2.13)

where $W_d$ and $W_n$ are weighting functions reflecting the frequency content of external disturbances ‘d’ and noise ‘n’. Here ‘$W_d$’ is contributing more at low frequencies and ‘$W_n$’ is dominating at high frequencies as shown in the Figure 78. Frequency dependent weighting functions are introduced to incorporate low pass filter for external disturbances at the inputs of the aeroelastic systems and high pass filters for external noise at the output feedback of the aeroelastic system.
External disturbance is attributed to the buffet loads that are experienced by the wing, owing to this fact low pass filter is designed to represent the disturbances large up to 30 Hz. High pass filter is used to represent the noise experienced by the system. The control law BLMSV must act against these disturbances to achieve the robustness. In the context of this phase, the predefined general control schematic architecture for BLMS is changed and it is shown in the following Figure 79.
The full order of the state space model is comprised of 108 states, the system is reduced to 50 states, it is reduced with the help of multiplicative error bound. Inbuilt MATLAB function, which uses balanced stochastic model truncation via Schur method based on multiplicative (relative) error bound to achieve the target [65]. The benefit of using the method, is in the fact that for some systems with low damped poles or zeros, the balanced stochastic method produces a better reduced-order model fit in certain frequency ranges to make multiplicative error small. Whereas additive error methods only cares about minimizing the overall "absolute" peak error, they can produce a reduced-order model missing those low damped poles/zeros frequency regions. The number of states of the system is not of prime importance as the developed controller is independent of number of states in the plant. The frequency response (figure 80) and bode plots (figure 81) showed very good agreement for both original and reduced order models for frequencies of interest.

![Figure 80: Model Order Reduction](image)
In both diagrams shown above, it is evident that reduced order model captures the dynamics of the system with good approximations.

Aeroelastic system is made uncertain by introducing the parametric uncertainties in the state space matrix of the system. The uncertain system develops each performance output with respect to the amount of parametric uncertainty, the performance output deviates from the nominal (original) system output in terms of magnitude and phase. Nominal system is computed for certain system (without any uncertainty) along with samples comprised of different levels of uncertainties, worst case system is also computed for the provided uncertainty in the system. In BLMSV, the worst-case sample is considered to formulate the static output feedback controller, where robust optimization is provided by the optimization algorithms. The frequency response with nominal value, samples and worst case is shown in figure 82:
The attenuation showed by SoF is significant. The amount of attenuation achieved can be increased at the expense of more work input from the actuators until the limit that it does not exceed saturation torques.

Figure 82: Uncertain System Representation
Robust stability and Robust performance of the existing system with controller is analyzed by Mu synthesis. Inbuilt MATLAB function is used for Mu synthesis. It showed that the system achieved the robust stability for the modeled uncertainty. Following two details are directly extracted from the results of the Mu synthesis for robust stability and robust performance, respectively. It depicts the behavior of the uncertain system and presents a summary.

The system also achieved the marginal robust performance to the modelled uncertainty as shown by detail:

The robust stability and performance behavior is directly related to the modelled uncertainty, 18% in the state matrix for all the values. To achieve complete robust performance, one could
decrease the amount of uncertainty introduced in the state space matrix. The system is robustly stable for 22% of modelled uncertainty. the system performs robustly for 17% of uncertainty in the state matrix of the aeroelastic system.

The upper bounds of the mu synthesis values are shown in the Figure 84: *Structural Singular Values*, the singular values are less than one, so it specifies that that system is robust for the range of frequency under uncertainty.

![Mu plot of robust stability margins](image)

*Figure 84: Structural Singular Values*

vi. **Buffet Load Mitigation System VI:**

Robust control is a mature field analytically but rarely it has been used in experimentation or for practical purposes in dedicated aeroelastic systems. The aim of this technique is to quantify the robust performance and robust stability of the aeroelastic system by implementing dynamic robust controller based on approaches like H-Infinity and Mu analysis. The h-infinity norm of linear time invariant (LTI) aeroelastic system for specific input to output is given by:

\[
\|G(s)\| = \sup_{\omega} \sigma(G(j\omega))
\]

(2.14)

Where ‘\(\sigma\)’ represents the largest singular value throughout the frequencies ‘\(\omega\)’ of interest. The h-infinity norm measures the maximum gain or amplification of transfer function for all input and
output directions. The measured h-infinity norm can be used to assess the performance of the plant in a closed loop system. The outcome of the synthesis will be a dynamic controller as a result of solution to two Riccati equations. The purpose of the controller is to stabilize the closed loop by linear fractional transformation $F_l(P,K)$ that produces performances index ‘$\gamma$’ which provides an upper bound of the closed loop $||F_l(P,K)||_{\infty}$. Once the meaningful performance specifications are defined for desired inputs and outputs then the synthesis is efficiently performed by Robust Toolbox, MATLAB.

The objective of the robust dynamic controller is to attenuate the first bending mode. It also provides robust stability and performance to wide range of uncertainties in the aeroelastic system. The control architecture is shown below:

![Figure 85: Control Scheme - Robust Controllers](image)

Where blocks $W_{act}$ and $W_p$ represents weighting functions on the actuator bandwidth and performance of the system, respectively. The alteration is user defined, control action can be enhanced. The weighting functions are selected based on the literature review and previous experimentations. $W_d$ and $W_n$ are the exogenous inputs to the system representing aerodynamic disturbance and noise. Figure 86: Performance Parameters, shows one possible set of parameters ($W_p$ and $W_n$) that are applied on robust controllers.

In BLMSV, the order of the controller is independent from the order of the plant. However, for h-infinity controller in BLMSVI each state corresponding to the plant directly results in a state of controller or even results in more states of the controller based on Mu analysis. So the task on
hand is to decrease the number of states significantly, the reduced order modelling procedure adopted before is inefficient as even if the magnitude for the concerned frequencies is same, the phase difference is too much to be ignored, two ways are possible, one is to visit the aeroelastic model again in MASSA and change the active number of modes (states), the second method is to use the built-in MATLAB module to separate the high frequency dynamics from the low frequency dynamics. This later method is extremely time efficient as the structural dynamics nature of the system provided the breakthrough in reducing the large number of states i.e. 108 states. The number of states are decreased by keeping in view the provided actuator bandwidth. The actuator bandwidth without any significant phase delay is approximately 30 Hz. Depending upon the number of modes to be considered, high frequency and low frequency dynamics of the aeroelastic system can be separated. Each mode corresponds to 2 states, setting the limit at 30Hz, 3 modes (first bending mode, first in plane bending mode and first torsion mode) are present in the bandwidth. For a cutoff at 10 Hz, only the first bending mode is in the range of low frequency dynamics. So, the plant has correspondingly 2 states. The plant is equipped with integrators to extract the velocities and displacement signals. Each integrator added 2 more states to the plant. The resulting controller has six states, 2 states corresponding to the original plant, 2 states for actuator dynamics and 2 states are further added for the integrators associated with the accelerations to velocity conversion. The time delay associated with digitization of the signal is ignored along with time delay associated with the sensors, it helped to avoid any further states being added to the system.

The multi-input multi-output (MIMO) state space model has 13 inputs (four inputs for forces at accelerometer positions, five inputs for pressure transducers and four inputs for symmetric deflections of each actuator [LEI, LEO, TEI, and TEO]) and 8 outputs (four structural accelerometers and four rotations of control surfaces). The attenuation of first bending mode with the help of outboard trailing edge is well illustrated in previous techniques. If a dedicated trailing edge outboard is used as aileron for flight mechanics, then inboard trailing edge can also be used to attenuate the first bending and first torsion modes. The geometrical features of the wing ease this task and offers many possibilities, as one sensor and one actuator can be selected to simulate the input and output behavior of the state space model. Keeping in view the previous analysis on the same wing where it is inferred that leading edges are not beneficial to attenuate the loads so, neither LEO/LEI actuators are used nor the associated accelerometers are used by obeying the ILAF control law, the system is reduced to (SISO) single input (TEO)
single output (TE wing tip Accelerometer) system. However, the choice to switch between different outputs and inputs is in-hand and can be altered.

The dynamics of the actuator are assumed to be perfectly modeled. The second order actuator model is already identified from the frequency domain analysis performed by [18]. Uncertainty introduced in the state matrix of the system represents the uncertainty in the structural model and aerodynamic coefficients related to the system. It avoids changing the individual matrices for structural and aerodynamic uncertainties.

Robust stability and robust performance is performed with the help of H infinity and Mu controllers, to analyze its performance in the frequency domain of interest. Figure 87 and Figure 88 shown below are presenting the comparisons of original aeroservoelastic state space model and reduced order model for frequency response and bode plots as a function of frequency (radians/sec) respectively. As discussed before in this section, the target is to model the dynamics for the bending mode only, so the compromise is made on the accurate dynamics of the complete aeroelastic modes by reducing the number of states. Frequency response (figure 87) and bode plot (figure 88) showed that reduced order system exhibits the required magnitude for the bending mode.
Figure 87: Comparison of Full and Reduced Order Systems

Figure 88: Bode Plot Comparison of Full and Reduced Order State Space Systems

The figure 89 is showing the upper bounds and nominal values for the h-infinity and Mu controller. These bounds satisfy the criteria of robust stability and performance.
Remark: The results are shown for 9% uncertainty in state matrix. The closed loop gain is higher as compared to the previous analysis, due to the use of single control surface to attenuate all the loads. Both the controller shows the close loop gain less than 1. However, Mu controller seems to be more prone to uncertainty and its performance sharply decreases as the uncertainty is increased in percentage. The h-infinity controller however remains stable even with larger amount of uncertainty introduced.

The modelled system seems to handle the robust stability and robust performance of the system quite well. The margins of robust stability and performance are enhanced by the design of these controllers as compared to the previously mentioned controller.

The closed loop frequency response has shown attenuation for H-infinity and Mu controllers respectively with respect to the open loop response of the reduced state space system. The frequency response of the open and close loop response for both the controllers is shown in the following Figure 90. The result is presented for first bending mode only as this was the targeted mode owing to the reduced state space model. The attenuation predicted by robust controllers is relatively less as compared to the static output feedback controller due to the fact it is dealing with highly uncertain system, i.e. the robust controller is successful even dealing with worst
possible case. The attenuation can be increased if the controller is relaxed by considering the less amount of uncertainty and changing the performance criteria for the system.

Figure 90: Frequency Response of Open and Closed Loop Systems
PART 3: EXPERIMENTAL IMPLEMENTATION OF ACTIVE CONTROL SCHEMES ON (X-DIA) COMMERCIAL AIRCRAFT

This part starts with a detailed description of experimental setup, modal analysis and instrumentation of the wing. Later in this part, each chapter is dedicated to the experimental validation of each control scheme that is numerically developed in the ‘Part 2’. The experimental activity for the suppression of vibrations on the X-DIA wing took place at the De-Ponte Wind tunnel, Department of Aerospace Science and Technology, Politecnico di Milano, in March 2017 and September 2017. This comprehensive and diverse activity spanned over several months with twice entry to the wind-tunnel for control laws validation.
1. Experimental Set-up:
Almost 100 test runs were planned for the validation of active control schemes. In this context, comprehensive experimental setup preparation was essential, which took almost three months from installment of Linux operating system to mounting of the wing in the wind tunnel. The test section speeds for these experiments is set at 30 m/s except for the case when uncertainties in the instrumentation are considered, then the speed is set at 20 m/s. The test section speed is selected because it is approximately half of the flutter velocity as predicted by the numerical analysis of the X-DIA wing.

i. Wind Tunnel Specifications:
The wind tunnel is equipped with three motors which propels the air to the maximum speed of 55 m/s. A dedicated operating system is used to configure the wind tunnel and to acquire the experimental data with the help of commercially available software LabVIEW, specifications of the wind tunnel are summarized below:

1. Closed-circuit horizontal type.
2. Rectangular cross-section test chamber.
3. Dimensions: 1.5 m high, 1 m wide, and 3 m long.
4. Wind speed limit 55 m/s.

The wing is installed in a vertical position at the middle of the test section as shown in Figure 91 and Figure 92. Special mounting system is developed for this setup is shown in Figure 93; it provides the wing the same constrained motion as when it is attached to the fuselage of the aircraft, in addition the mounting system is also compatible with load cell platform to measure the forces at the fixed end of the cantilevered wing.
Figure 91: Test Section Wind-Tunnel

Figure 92: Downstream of Wind-Tunnel Test Section
The existing setup PC 3AS ALA provided the connections between the sensors/actuators connected with the wing, input/output boards and Simulink model of the developed control scheme, is upgraded for experimental tests along with Linux operating system and up-to date versions of commercially available MATLAB/SIMULINK and RTAI Lab software are configured. Open source drivers are developed from Linux Control & Measurement Device Interface (COMEDI), for the plugin Input/output boards provided by National Instruments. These drivers are implemented as a core Linux 2.6 kernel module (distributed with COMEDI). Data acquisition for the wing is carried out with Real Time Application Interface for Linux (RTAI) 4.0. It develops the block diagrams that can be compiled and executed on the RTAI Linux operating system, it enables host and target systems to communicate with each other. XRTAI Lab provided the real-time events by providing graphic interface for gauges, scopes, meters and oscilloscopes etc. It also provided the ability to change the parameters for real time executable files [66]. Signal conditioning for the sensors is performed by KEMO filters which also acted as anti-aliasing and noise reduction filters.
ii. **Airbrake:**

Airbrake specifically manufactured for this task is shown in Figure 94, it is placed at upstream of the wing in the test section, Figure 95 and Figure 96. The airbrake helped to produce the effect of LEX (Leading edge Extensions) and produced the vortices in the 2D directions, these vortices excited the aeroelastic modes of the X-DIA wing. Self-induced buffeting is also checked by varying the angle of attack of the wing, the maximum angle of attack is chosen to be 12° due to safety issues.

![Airbrake](image)

*Figure 94: Airbrake*

![Airbrake Installed - Upstream of the Test Section](image)

*Figure 95: Airbrake Installed - Upstream of the Test Section*
iii. **Instrumentation of the Wing:**

Wing is equipped with four PCB monoaxial accelerometers with bandwidth ranges from 0.5 Hz to 3 kHz. Two accelerometers are located near the wing tip on either side of the elastic axis and two accelerometers are located on the midspan of the wing. Two Endevco pressure transducers are also attached on the surface of the wing to measure the pressure fluctuations. Strain gauges are also attached near the root of the spar. The following Figure 97, shows the sensor/actuator schematic of the left X-DIA wing with symbolic representation of sensors, further details are furnished after the figure 97.
The electric motors for driving four control surfaces (LEI, LEO, TEI and TEO) are selected by fulfilling the criteria set by physical design constraints such as weight and maximum allowable size of the motors [18]. Comparative study of static and dynamic characteristics showed that the ideal motors for the X-DIA wing are Portescap mod 17N78-210E. Each motor is also equipped with encoder. Thanks to planetary gears in shaft/line torque transmission, chief concern of torque and gear reduction are met. Leading edge actuators are prepared with gear ratio of 88, while the trailing edge actuators are given the gear ratio of 22. The dynamic characteristics of motor are given by following electromechanical equations:

\[
L \frac{di}{dt} = -Ri - K_e w + v \tag{3.1}
\]

\[
J \frac{d\omega}{dt} = K_t i + T_f \tag{3.2}
\]

Where \(L\), \(K_e\), \(K_t\), \(T_f\) represents electrical inductance, electromotive force constant, Torque constant, frictional torque (function of angular velocity), respectively, frictional torque has no effect on frequency bandwidth, so it is assumed to be zero. The complete connectivity system for the X-DIA wing and the operating system is shown in Figure 98.
X-DIA wing model is equipped with pressure transducers to experimentally measure the pressure fluctuations on the surface of the wing and provide the numerical model for input to the state space model, two Endevco 8515-C piezo-resistive pressure transducers are provided by
LEONARDO S.p.A. company. Figure 100, depicts the patch zones on the wing, there are total of four locations where these transducers are attached. One of the transducers will be fixed on one patch (for example 3\textsuperscript{rd} patch, shown with solid circle), the other sensor will be moved to different patches to define the correlations and to extract the pressure intensity and power spectral density on the surface of the wing.

![Figure 100: Schematic of Pressure Transducers Attachment](image)

Figure 101, shows the attached pressure sensors on the surface of the X-DIA wing.

![Figure 101: Attached Pressure Transducers on the Wing](image)

The results acquired from the pressure sensors are shown as power spectral density from the four sensors, Figure 102. Four strain gauges are also attached near the root of the wing to acquire the bending stress by full bridge, half bridge or quarter bridge configuration depending upon the user demand.
Before implementation and validation of the designed control laws in the wind tunnel. Wing is put under trial to validate the numerical modal response of the wing, to specify bandwidth of the actuators. This preliminary phase is conducted outside the wind tunnel.

iv. Modal Analysis
Experimental modal analysis is performed on the wing to identify the out-of-plane bending and torsional modes. The structure is tested in hard mounted condition with the main spar fixed by an interface beam to the load cell. Wireframe technique is selected to obtain information regarding the first bending and torsional modes, reference system for the wing is defined as follows:

- x axis parallel to the chord, from rear to front edge.
- y axis parallel to the trailing edge of the main box.
- z axis normal to the previous two axes.

11 points have been measured, they are divided into 5 rows, with 2 equally spaced columns each, plus a spare point put on the constraint. Points 1 to 5 are placed at the trailing edge of the wing while the other points are approximately placed on the leading edge. The wireframe mesh is presented in Figure 103.
To obtain the out-of-plane modes, a MIMO test in roving mode is carried out. Two PCB monoaxial accelerometers are used as references while impact location has been modified step by step to obtain all the eleven FRFs needed. Sensors are glued to the structure with the sensibility axes in the Z direction. A picture of the installation is presented in Figure 104. After some preliminary verifications (on the energy introduced into the structure, on the sensor sensibility to the positions and on the peaks obtained in FRFs), the best position is identified. FRFs are acquired in the range 0-128 Hz, with a frequency resolution of 0.125 Hz. Each point is impacted five times by PCB hammer, the number is recommended by the previous experiences which yields adequate results. Also, the measured noises are reduced thanks to the averages. The PCB hammer is equipped with a load cell PCB 086B03, tip of the hammer is made from Teflon which can excite the band up to 4kHz. One front-end SCADAS 316 is used for signal conditioning and data acquisitioning.
Coherence analyses of acquired signals showed values very close to one in the whole band, even for the points on the tip. So, the method is considered suitable for test purposes. Identification and post-processing of the data is carried out with software LMS-Test.Lab Rev.13A which uses the algorithm POLIMAX. Results for first two modes are numerically reported in Table 7 and graphically showed in Figure 105 and Figure 106.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [Hz]</th>
<th>Damping (ξ%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.87</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>24.55</td>
<td>5.25</td>
</tr>
</tbody>
</table>

*Table 7: Identified Modes through Modal Testing.*
An autoMAC (Modal Assurance Criterion) matrix is also presented to illustrate the quantitative analysis of mode shapes in terms of spectrum. It is shown in Figure 107.
Modal Assurance Criterion showed the first bending and first torsion modes are perfectly uncoupled with each other. However, the excited modes at higher frequencies are coupled with each other, it includes second bending and second torsion modes but these modes are not of concern in the presented research.

v. Actuator System for X-DIA Wing:
To cater the effects of vibrations and quick response to any disturbance a derivative is added by [18], in the conventional PID controller to decrease the response time. The PID sub controller is used to maintain the control and active command authority. The conceptual scheme of PID sub controller is given in Figure 108 and represented by equation (3.3):

$$R_{PID}(s) = K_p \left(1 + \frac{k_i}{T_s} + \frac{k_d T_d s}{1 + a_d s} + k_d T_{dd} \left(\frac{s}{1 + a_{dd} s}\right)\left(\frac{s}{1 + a_{dd} s}\right)\right)$$

(3.3)
The PID\textsubscript{2} controller is also equipped with anti-windup scheme. For the present research 4\textsuperscript{th} order Bessel filter is used to model the response of PID, because it reportedly has the best response in time domain (step response), maximal flat phase delay with a compromise on sharp cutoff frequency as compared to other filters [64]. Figure 109 & Figure 110 shows comparison of two types of PID, the newly devised PID controller showed improvement in the phase delay while compromising on magnitude, but magnitude above 30Hz is of trivial case as it is not permitted by physical bandwidth of the actuator and also the first bending and first torsion modes are well inside the actuator bandwidth. Inferences can be made by comparing phase plot of Figure 109 and Figure 110 for the corresponding actuator.
Figure 109: Magnitude and Phase Response of PID$_2$ Controller

Figure 110: Magnitude and Phase Response of Bessel Filter based PID Controller
By comparison, it is clearer that much better phase response is obtained by implementation of Bessel filter for modelling of PID controller. The increment reached to almost 20 Hz for both leading and trailing edges from approximately 5 - 15 Hz in the prior case. The actual implementation scheme of the PID$_2$ controller in SIMULINK is shown in the Figure 111 and Figure 112.

**Figure 111**: Actuator Dynamics for Leading Edge Inboard (LEI)

**Figure 112**: PID$_2$ controller
vi. **Real Time Implementation of Control Architectures**

Inputs to the state space models are defined by buffet loads and actuator dynamics while outputs of the systems are structural accelerations extracted from accelerometers. Wing replaces the state space model which is configured with SIMULINK/RTAI in Linux operating system.

The generic SIMULINK diagram of buffet load mitigation system is shown in the Figure 113, actuator dynamics is already shown in the previous figure, it is worth mentioning here that all the four-actuators have four distinct actuation systems; thus, they have separate SIMULINK simulations for the experimental phase. The signals are conditioned for modal control, extraction of velocities and displacements from structural accelerations are processed in the following block to the X-DIA wing. Further blocks or subsystems are added as a prerequisite to each buffeting load mitigation system (BLMS).

Feedback gain matrix is entered as variables extracted from the optimization process obtained for the control laws, such entry helps to readily change between different optimization techniques while the test is still in progress.

![SIMULINK Diagram of BLMS](image-url)
2. Preliminary Tests

Several preliminary checks are performed before commencement of the validation phase for numerous buffet load mitigation system based controllers. These checks include data acquisition for buffet loads for pressure transducers, quantifying the performance of actuators and effect of airbrake on the flow turbulence and effect of angle of attack of the wing.

To start the experimentation, the angle of attack for the wing is adjusted to 6°. Airbrake fixed at upstream of the test section is fixed in the horizontal direction, the angle of attack of the airbrake is adjusted for adequate vortices to produce buffet loads. Following Figure 115, shows the effect of introducing airbrake with varied angle of attack. Airbrake at 40° and wing at 6° is selected as the final position for the further experimentations as the excited frequencies were adequate. Power spectral density (PSD) is shown for accelerometer located adjacent to the trailing edge outboard (TEO).
The Figure 116 shows the power spectral density (PSD) for all the four accelerometers located adjacent to the four control surfaces at the midspan and wingtip of the wing, named as trailing edge outboard (TEO), leading edge outboard (LEO), trailing edge inboard (TEI) and leading-edge inboard (LEI).

Cross power spectral density is used to highlight the excitation of the modes with respect to the buffet loads, magnitude and phase difference for the four accelerometers are presented in Figure 117 and Figure 118 respectively, with respect to accelerometer at midspan trailing edge (TEI) to show the in phase and out of phase excitation of the accelerometers.
Figure 116: Structural Accelerations from Four Accelerometers

Figure 117: Cross Power Spectral Density (CPSD) wrt to Trailing Edge Midspan Accelerometer
3. Validations of Control Laws
   
   i. Buffet Load Mitigation System I:

   The results for the first scheme of (BLMSI) are presented here, the motive of this scheme is to attenuate the first bending and first torsion modes. Two methods are followed, one method corresponds to attenuating loads by directly taking signals from accelerometers after pseudo integration. Second method is implemented by introducing modal signals through arithmetic operations on signals. As it can be clearly seen in Figure 119, the amount of attenuation for bending and torsion mode is achieved by active control scheme, also damping is added by the active control scheme. The active control scheme is employing all the control surfaces, not only adjacent accelerometers are used for feedback reading but all the other accelerometers are used to provide the feedback to the control surface. The performance of the system improves for the case of torsional signals in the torsional mode. The results agree with the numerical findings predicted by Buffet Load Mitigation system I (BLMSI).
The power consumption check is also analyzed through the SMIULINK/RTAI scheme. Figure 120, shows the power consumed by each actuator system during the active control scheme, it is evident that all the actuator systems worked well within the saturation point. Saturation of (10V) is same for all the motors, irrespective of their locations.
Encoder command and correspondingly the response of the actuator to the command are shown in Figure 121. This highlights the significance of using the efficient PID$_2$ controller. The actuator follows the command with good agreement.
Figure 121: Command/Actuator Time History

Following Figure 122, shows the effect of only bending and torsion signals alternatively on the active control scheme, PSD is used to clearly explain the difference as the bending signal provides more attenuation in bending mode and less attenuation in torsion mode. The attenuation in the torsional mode is enhanced by using the torsion signal.
The actuator response to the encoder command for this case is shown in the Figure 123. The results here showed the tremendous correlation between command and actuator. The rotations commanded by the active control system are mostly around 0.5 deg, which are handled efficiently by PID controller.
ii. **Buffet Load Mitigation System II:**

The unifying theme to this research is to quantify individual performance of actuators and sensors, characterized on the basis of power spectral densities, power consumptions, torque required and command-actuator control integration to lay out the comparative analysis for this scheme. Comparison is shown between inboard control surfaces and outboard control surfaces, trailing edge control surfaces and leading edge control surfaces, individual control surfaces with adjacent accelerometers, analytical values are presented to rank the most power efficient control surface (actuator) and to rank the most suitable control surface (actuator) to attenuate the vibrations in specific mode of concern by using Hankel singular values.

The performance of the outboard edges named as (LEO) and (TEO) is analyzed for the case of structural acceleration acquired from accelerometers located at the wingtip. Figure 124, shows the power spectral density for the roles of outboard edges in attenuating the loads, TEO outboard is more suitable for bending mode attenuation while the role of torsion mode attenuation is equal for trailing and leading edge outboard. Figure 125 and Figure 126, shows the torque required to perform the operation and command/actuator performance for this case, respectively. The saturation torque for the leading edges is 0.4 Nm and saturation torque for the trailing edges is 0.1 Nm, respectively.

![Figure 124: PSD Comparison of Leading and Trailing Edge Outboard](image)
Next, comparison is drawn for leading edge inboard - outboard (LEIO) and trailing edge inboard-outboard (TEOI), the comparison is shown in Figure 127 by the help of power spectral densities, it can be clearly seen that trailing edges are more beneficial in attenuating buffet loads while there performance is worse in the torsional mode as compared to the leading edges.
Figure 127: PSD of Leading and Trailing Edge Control Surfaces

The performance of active control scheme is shown in time domain in Figure 128 and Figure 129, where the performance is compared for variance in acceleration and variance for velocity respectively, for the cases when leading and training edge outboard are active.

Figure 128: Time domain Response of Active Control Scheme
Figure 129: Time Domain Response of Active Control Scheme

Figure 130, shows the comparison for inboard and outboard edges, again it is observed that performance of outboard edges are better in the first bending mode while the performance of inboard edges is better for torsional mode.

Figure 130: PSD Comparison of Inboard and Outboard Edges
Figure 131 & Figure 132, shows the comparison between the command/actuator authority for inboard edges and outboard edges, respectively.

Figure 131: Command/Actuator Response for Inboard Edges

Figure 132: Command/Actuator Response for Outboard Edges

The performance of individual control surface along with adjacent accelerometer (ILAF concept) is also extracted experimentally, it is shown in Figure 133, it emphasizes the role of each control
surface and accelerometer. Figure 134 and Figure 135, illustrates the command-control integration for active control scheme along with Figure 136, which highlight the torque required to carry out the command.

*Figure 133: Power Spectral Density obeying ILAF concept*
Figure 134: Volts Consumption during Actuation

Figure 135: Command/Actuator Time History
The results for the frequency response of open and closed loop systems using trailing edge outboard (TEO) with adjacent accelerometer and the trailing edge inboard (TEI) with the adjacent accelerometer are also shown in Figure 137 to quantify the performance and explicitly show the attenuation achieved by BLMSII. It can be clearly observed the amount of attenuation achieved by active control scheme along with additional damping achieved when the control system is active. For reference, numerical results are also presented for this case in Figure 138. The results show good agreement for both cases. Table 8 shows the predicted numerical absolute attenuation carried out by each actuator.
<table>
<thead>
<tr>
<th>Control surface</th>
<th>Bending mode</th>
<th>Torsion mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEI</td>
<td>15.83</td>
<td>5.60</td>
</tr>
<tr>
<td>LEO</td>
<td>4.799</td>
<td>0.18</td>
</tr>
<tr>
<td>TEI</td>
<td>38.04</td>
<td>15.92</td>
</tr>
<tr>
<td>TEO</td>
<td>82.01</td>
<td>×</td>
</tr>
</tbody>
</table>

Table 8: Percentage Attenuation Achieved

Figure 137: Frequency Response – Experimental
At this point, it is evident from power spectral densities that most suitable control surfaces for attenuation in bending mode are outboard surfaces (if comparison is between inboard and outboard edges), or trailing edges (if comparison is between leading and trailing edges). Trailing edge outboard is preferred over leading edge outboard, even if single control surface is to be used then trailing edge outboard stands out for its attenuation provided in the bending mode. However, the result is bit different for the case of torsion mode. For attenuation in the torsional mode, most suitable control surface is trailing edge inboard among all other contenders. Performance of inboard control surfaces is better for the case of torsional mode attenuation.

The second phase of BLMS II corresponds to analytical evaluation of the performances of each control surface. It is implemented by keeping in view the worst-case scenario, where one or more control surfaces are not working or getting sufficient performance from the control surface. It can be simply inferred as quantifying the suppression of vibrations with respect to the energy/cost related to the actuation of the surface.

A unifying theme to this phase is the usage of Hankel singular values (HSV) for quantitative attenuation analysis of actuator and sensor combination. This scheme is originally presented for optimized placement of actuator/sensor in the preliminary design phase. As the wing is already manufactured with predetermined location of instrumentation, HSV is used experimentally for Integrated design perspective for this research. In [67], it showed that for lightly damped
structure, its modal properties can be used to obtain HSV for modal coupling at preliminary
design phase, given by the simple expression:

\[ \sigma^2_{yui} = \frac{[b_i^T b_i][c_i^T c_i]}{(4 \zeta_i \omega_i)^2} \]  \hspace{1cm} (3.4)

With 'b_i' as the input matrix and 'c_i' as output matrix of the state space model. The numerator
provides the coupling of actuator and sensor for the respective mode. denominator defines the
time constant of the state as:

\[ \tau_i = \frac{1}{(\zeta_i \omega_i)} \]  \hspace{1cm} (3.5)

As, after experimentation the data is preferred to be analyzed in the frequency domain. Also, the
task on hand is to compare the efficient attenuation which includes the comparison of structural
vibrations for active and inactive control system along with taking into consideration the
optimized performance of the actuator. So, the task on hand is now to find the suitable
combination for modal attenuation, the equation is transformed to find the effective coupling of
actuators and sensors to damp out the first bending and first torsion modes under control law
which minimizes the variance of actuator movement:

\[ \sigma^2_{yui} = \frac{(b_{i_{on}})(c_{i_{on}} / c_{i_{on}})(c_{i_{on}})}{(4 \zeta_i \omega_i)^2} \]  \hspace{1cm} (3.6)

Where 'b_i' and 'c_i' represents the variance extracted from actuator inputs and structural outputs,
subscripts ‘on’ and ‘off’ represents the readings taken when the control system is active or
inactive. For the predetermined actuator sensor locations, the optimum coupling path of actuator
signal and sensor output between each mode \( N_m \) is given as:

\[ \sigma_{yu} = \text{diag}(\sigma_{yu1}, ..., \sigma_{yunn}) \]  \hspace{1cm} (3.7)

[50], suggested to improve the performance of control, by coupling each mode with
performance-disturbance path as:

\[ \sigma_{zw} = \text{diag}(\sigma_{zw1}, ..., \sigma_{zwnm}) \]  \hspace{1cm} (3.8)

In the design metric form, it is given as:

\[ J_{qp} = \sum_{i=1}^{N_m} \frac{\sigma_{yui}^2}{\sigma_{yui}^2} \sigma_{zw}^2 \]  \hspace{1cm} (3.9)
The essence of this task is to select the best couple of q-th sensor and p-th actuator for increase performance and efficiency. \( \sigma^2_{yu} \) defines the square of all HSV values for actuator/sensor coupling. A similar approach has been adopted to minimize the variance of the outputs: [68] showed that performance ‘z’ can be controlled by introducing the weighting function in the method with systems performance. Design metric can be utilized to switch between the robustness and performance based cost functions.

The following figures shows the process of experimentally extracting variance and Hankel singular values for already assigned actuator/sensor locations. The power spectral densities from each sensor configuration are shown in the following Figure 139, to extract the variance of sensor readings in the bending mode, the signals are filtered with equiripple filter so that the signal contains only one mode. Torsion signals are filtered in the same way, it is shown in Figure 140, the variance is calculating for the filtered signal.

![Figure 139: Control Off - Bending Mode](image)
Figure 139: Control Off - Bending Mode and Figure 140: Control Off - Torsion Mode presents the PSD of accelerometers when the control system is off and equiripple filter is used to extract the PSD of interest i.e. first bending and first torsion mode. Only the variance is extracted from the participating frequency of first bending first torsion mode.

Same procedure is followed to extract the variance for the bending signals when the control system is active. Figure 141 shows the PSD of the attenuated and filtered modes under active control. The effect of active control system is evident by observing the reduction in structural vibrations, which can be better understood by comparing with Figure 139: Control Off - Bending Mode.
Figure 141: Control On - Bending Mode

Figure 142: Control On – Actuator Movement shows the PSD of actuator movement for active control during the attenuation various modes of bending mode. PSD of actuator movement in bending mode is realized through equiripple filter. The extracted PSD is used to calculate the variance of the actuator movement in the bending mode.
To analyze the effect of active control and actuator movement during active control, the signal are filtered only for torsional modes and consequently the variance is calculated during torsional mode, effect of active control scheme on structural vibrations is shown in Figure 143 (Compare it with Figure 140). Figure 144 presents the PSD of actuator movement specifically in the torsion mode.
Figure 143: Control on - Torsion Mode
The Hankel singular Values can be calculated to show the effect of active control under optimized actuation movement for all the control surface working under ILAF concept. The values are analytically calculated by the help of the formula described in equation (3.6). It shows that most effective surface for bending mode is trailing edge outboard and most suitable surface for torsion mode is trailing edge inboard. Table 9 and Table 10, are mentioned below to signify the importance of each actuator. Significance of actuator/sensor combination for cost function is presented in Table 9, it can be readily seen that outboard trailing edge is more effective in bending mode participating frequencies while inboard trailing edge is more effective in torsional mode attenuation. Moreover, it is also noted the deficiency of leading edge strip i.e. LEO and LEI, as compared to trailing edge strip i.e. TEO and TEI, to cater the turbulent loads produced by the airbrake.
<table>
<thead>
<tr>
<th>Sensor Location / Control surface</th>
<th>Bending Mode</th>
<th>Torsion Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midspan LEI / LEI</td>
<td>×</td>
<td>0.118</td>
</tr>
<tr>
<td>Wingtip LEO / LEO</td>
<td>0.108</td>
<td>0.634</td>
</tr>
<tr>
<td>Midspan TEI / TEI</td>
<td>0.360</td>
<td>1.760</td>
</tr>
<tr>
<td>Wingtip TEO / TEO</td>
<td>2.032</td>
<td>×</td>
</tr>
</tbody>
</table>

*Table 9: Hankel Singular Values*

Effective performance factor, Table 10 can be extracted by normalizing the attenuation with the power consumption of each actuator during active control scheme.

<table>
<thead>
<tr>
<th>Sensor Location / Control surface</th>
<th>Bending Mode</th>
<th>Torsion Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midspan LEI / LEI</td>
<td>4.711</td>
<td>1.665</td>
</tr>
<tr>
<td>Wingtip LEO / LEO</td>
<td>2.661</td>
<td>0.105</td>
</tr>
<tr>
<td>Midspan TEI / TEI</td>
<td>10.91</td>
<td>4.650</td>
</tr>
<tr>
<td>Wingtip TEO / TEO</td>
<td>14.94</td>
<td>×</td>
</tr>
</tbody>
</table>

*Table 10: Performance Factor*

iii. **Buffet Load Mitigation System III:**

BLMS III is specifically designed to enhance the performance of the trailing edge outboard actuator (TEO) in the torsional mode. To improve the performance of trailing edge outboard, the idea to create an exaggerated output feedback signal at the participating frequencies of torsional mode. It is achieved by numerically introducing the notch and peak filters on the bending and torsional signals, respectively. The idea of implementing a filter for enhancement of performance in the torsional mode showed promise with the compromise in the performance in the bending mode. The performance of the peak filter helped to attenuate the vibrations in the torsional mode as compared to notch filter or when both filters are considered simultaneously.

Following Figure 145 and Figure 146, highlights the advantages associated with the use of peak filter to attenuate the vibrations, it also shows the adverse effect of using notch filter or both filters simultaneously. The result is presented for trailing edge outboard with the corresponding accelerometer, the magnitude of notch and peak filter is 0.11 dB at 27 Hz (bode plots are shown in part 2, BLMS III).
Figure 145: Application of Filters

Figure 146: Advantage of Application of Filters
iv. **Buffet Load Mitigation System IV:**

BLMS IV is dedicated to proving the robustness of the numerical optimization techniques when the instrumentation of the system is incorporated with uncertainty. The velocity in the test section is kept to 20 m/s for this phase. To quantify the performance of optimization algorithms for different scenarios, robustness of the optimization algorithm is determined for same initial point (asymptotically stable point), consequently it defined the robustness of buffet load mitigation system, instrumentation of the wing is selected as the prime target for introduction of uncertainty in the system as they are more prone to changes over the operational life. Accelerometer responsible for feedback and actuation system responsible for active control systems are incorporated with uncertainty in their respective parameters such as sensitivity of accelerometer and damping ratio of control surface. Although, numerically the simulation is performed for 50% of uncertainty in the nominal operating value for the accelerometer and the control surfaces but in the experimentation maximum value of uncertainty is limited to 20% due to safety factor and velocity of the experimentation is set at 20 m/s. Trailing edge outboard and the adjacent accelerometer at the wing tip are selected to demonstrate the results of the system under uncertainty. Experimentally dynamic gain is introduced in the SIMULINK to change the sensitivity of the accelerometer. Figure 147, highlight the fact that performance of the system deteriorates as the uncertainty in any component of the system is increased which eventually leads to instability of the system. The system however remains robustly stable. In Figure 147, it is evident that the attenuation is decreasing especially in the torsional mode for increased uncertainty level of accelerometer sensitivity. The results are shown for uncertainty ranges from 1% to 20 %. In the worst-case scenario, the system will behave as open loop system.
Figure 147: Uncertainty in Accelerometer Sensitivity

Figure 148, shows the effect of uncertainty in the parameters of trailing edge outboard, which through preliminary test in this analysis proved to be more responsive to the uncertainty in the system. The system is made uncertain by introducing dynamic gain in the SIMULINK/RTAI. The performance of the system decreases with increase in the systems uncertainty. Only two results are presented for the uncertainty case of 10% and 20%. The response time, settling time, overshoot is increased as compared to the system with nominal parameters. The designed suboptimal controller works fine for these uncertainties.
v. Buffet Load Mitigation System V:
The purpose of BLMSV is to check and quantify the robustness of the existing feedback controller. The approach is accomplished by Mu Analysis, where inputs to the state space model are derived from already existing buffet load model and noise weighting function is introduced at the output of the state space system. These frequency dependent weighting functions defined the disturbance rejection and noise insensitivity criteria. The structural singular values are less than one for the frequencies under consideration. The performance of this controller in the first bending and first torsion mode is shown in the Figure 149. The attenuation is considerable in the bending and torsional modes along with the damping added in the first bending mode. The controller is also implemented for other velocities to extract the robustness of the system.
The actuator command relation is shown in Figure 150, it can be readily seen the working of actuator to the command input.
vi. **Buffet Load Mitigation System VI:**

The objective of this active control scheme (BLMS VI) is to attenuate the bending mode only by experimentally implementing the robust controllers i.e. $H_{\infty}$ and Mu controller. The BLMS VI is implemented by only considering the single input and single output system. The model is considerably reduced as explained in part (2) of the report. Experimentally KEMO filters are used to filter the signals after 20 Hz, which leaves the problem with only one mode to attenuate i.e. the first bending mode, the performance for the controller for the bending mode is shown in the following figures for the case of $H_{\infty}$ controllers and Mu controllers. Although all the signals for all the actuator and sensors are kept active but the feedback is only provided for Trailing edge outboard from the adjacent accelerometer. The SIMULINK scheme is shown in the following Figure 151, where the switch is provided to move between control-off and control-on with choice for $H_{\infty}$ and Mu Controller.

![Figure 151: Feedback Schematic for Robust controllers](image)

The attenuation showed by these controllers are less as compared to the static feedback controllers, it is also shown by numerical analysis performed on $H_{\infty}$ and Mu controller. The advantage here lies in the fact that the controller is modelled with plant having uncertainties. Figure 152 and Figure 153 uses frequency response to demonstrate the effect of active control under $H_{\infty}$ and Mu-controllers schemes. As the objective of this scheme was to attenuate the first bending mode, it is clear that the active control scheme not only attenuated the first
bending mode but also provided additional damping. However, no attenuation is achieved for first torsional mode but this is not the demanded task from active control.

Figure 152: Frequency Response H-infinity Controller

Figure 153: Frequency Response Mu-Controller
Figure 154 shows the actuator command time history for the H-infinity (Top) and Mu controller (bottom) for the same parameters. The history is shown for trailing edge outboard (TEO).

The importance of command/actuator authority is of prime concern here, as the constant handling quality defined on the PID\textsuperscript{2} controller forces the actuator to follow the command for this case. Moreover, it is in accordance with literature that Mu-control performs robustly, as it produces the command signals larger for the same parameters for H-Infinity and Mu controller. It is observed in the experimentation phase that Mu controller is more prone to instabilities for the changes in the system dynamics. The main factor is the performance index that increase the demand to attenuate the loads efficiently, subsequently the demand on the actuators work is increased. The instability however, is produced as the demand from the actuator exceeds its bandwidth.
CONCLUSIONS

The research detailed a set of active aeroelastic control techniques for the high performance X-DIA commercial aircraft and Aluminum fighter aircraft (AFA). A unified approach is presented for the aeroelastic system, numerical analysis and control law procedures based on linear time invariant system. The three distinct phases of this research are concluded as follows: 1st phase dealt with active aeroelastic control law for Aluminum fighter aircraft (AFA). 2nd phase dealt with active aeroelastic control techniques for X-DIA commercial aircraft. In the 3rd phase, experimental validation took place for active control law techniques developed in Phase 2.

Numerical analysis is performed to comprehend the dynamics, to identify the relevant mode shapes and to extract the state space model of the aeroelastic system while the linear time invariant state space model is realized through the in house built software MASSA. The unique exogenous input to the state space system is developed analytically and numerically from the experimental data obtained for geometrically similar aircraft. The control law procedures are based on static output feedback and dynamic robust controllers with an aim of suppressing first bending and first torsion modes. Numerical procedure is followed to solve the coupled Lyapunov equations which also includes the stochastic minimization by Genetic algorithm, however, off all the optimization techniques used, Levenberg-Marquardt method is found to be the best in terms of computational time and robustness. Gradient descent method with variable step size also proved to be pivotal in the optimization process. Two active control law techniques are developed for this phase: 1. Without constraints on actuator movement, 2. With optimized actuator movement. The results demonstrated the significant attenuation, 82% reduction in the first bending and 45% reduction in the first torsion mode for the first technique. The attenuation is decreased to 75% and 35% for the first bending and torsion modes, respectively when the active controller also provided the optimized actuator movement. The variance decrease of actuator movement is reduced by 5 times (1.8179mm) as compared to unconstrained actuator movement.

In the second phase, the procedure is initiated with numerical analysis of the X-DIA commercial aircraft. Aeroelastic analysis and flutter analysis enabled to model the operating velocity and
formulate the unique state space system model based on linear time invariant (LTI) method. Keeping in view the complex nature of the X-DIA wing, multiple-input multiple-output state space systems is developed. Second order quadratic formulation is analytically derived by neglecting computationally expensive terms for numerical procedure of minimizing the feedback gain. The developed control law (BLMS I) attenuated the first bending and first torsion mode along with additional damping added to these modes. The results demonstrated the attenuation of 18dB and 10dB for the first bending and first torsion modes.

Aspect of suitable actuator/sensor configuration for modal attenuation is dealt in BLMS II. Frequency response, power consumption, effectiveness and Hankel singular values are used experimentally to rank the performance of actuators in specific mode of interest. Trailing edge outboard outperformed other control surfaces for bending mode attenuation, while contrary to this result trailing edge inboard is the most suitable surface for torsion mode attenuation.

To make trailing edge outboard as the single most efficient control surface, novel aspect of adding filters to the output feedback for participating frequency is analyzed in BLMS III. The state space system of notch and peak filters is selected as such to not disturb the overall dynamics of the system while showing enhancement in the attenuation. The results showed improvement of 1dB in the torsional mode when operated by trailing edge outboard alone.

BLMS IV is dedicated towards the robustness of optimization algorithm and subsequently for the robustness of the complete X-DIA aeroelastic system. The numerical procedure followed for parametric uncertainties in the actuators and sensors showed robustness as the same initiation point and termination criteria is held during all the cases. However, the numerical procedure predicted the decrement of 2 to 2.5dB for uncertainty of 20 % in the instrumentation.

BLMS V opted the advanced robust stability and robust performance criteria derived from Mu synthesis for the existing static output feedback controller. As a consistent procedure, the reduction of the order of aeroelastic system is achieved by balanced stochastic truncation method, model reduction exhibited significance simplification without loss of accuracy. Model reduction is followed by the 20% uncertainties in the system, to be accounted for the robust performance and robust stability of the system. The stability margin of 1.420 – 1.492 is achieved with structured values less than one for the frequencies in range of interest.

BLMSVI is dedicated towards achieving globally robust control for the aeroelastic system, in this context H-infinity and Mu Synthesis controllers are devised for the aeroelastic system. The aspect of model reduction procedure is successfully applied by decomposing the flexible body
modes of the aeroelastic system. The reduced model retained its natural linear time invariant (LTI) system behavior over the range of frequencies in interest. On the basis of BLMS II, the MIMO system is changed to SISO system. The system attenuated the bending modes with additional damping and predicted the robust performance and stability of the reduced order aeroelastic system under uncertainties. Mu-Controller proved to be more prone to sensitivity and noises.

The dedicated wind tunnel model X-DIA wing is used as a benchmark to validate all the developed controllers in the phase 2 of the research. Airbrake is specifically manufactured and placed at upstream of the test section in the wind tunnel to replicate the buffet loads experienced by the wing during flight. Pressure transducers attached on the surface of the wing provided the modeling of buffet loads. All the cases are successfully tested in comprehensive and detailed wind tunnel testing campaign. The numerically predicted results validated the performance yet limited by hardware design constraints (actuator bandwidth).

Proposed active control laws based on advanced optimization techniques produced more attenuation and damping for the X-DIA wing as compared to the classical technique i.e. identical location of accelerations and sensors (ILAF). The goal of implementing a dynamic robust controller is achieved in Linux based operating system with real time application interface (RTAI) and MATLAB, first of its kind for aeroelastic testing in Politecnico di Milano. Active control techniques summarized above has added novelty and new dimension to the active control of aeroelastic structures.

There are yet number of frontiers to be explored as this research has opened new gates for research. The performance can be further enhanced further for actuators owing to offer more bandwidth it is observed especially for the case of Mu-controller. Numerical PID design based on 4th order Bessel filter can be implemented experimentally, it enhanced the phase delay of the existing PID_2 controller. Torsional mode with SISO system can be analyzed with robust control techniques. Power consumption and efficiency of actuators can be used integrate this active control law with flight mechanics of the X-DIA aircraft with minimal energy consumption. A dedicated surface could be used for roll stabilization of the aircraft. Robustness and efficiency of the optimization techniques has enabled to implement the proposed active control for static output feedback as real-time optimization.
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