Identification and control of an RC car for drifting purposes

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Sommario

Questa tesi si concentra sull’analisi delle manovre di derapata e sullo studio di come velocità di imbardata e angolo di deriva le influenzano. Dall’analisi del sistema dinamico di una macchina radio comandata in scala 1:10 è stato implementato un controllore e sono state effettuate diverse prove di identificazione e stima dei parametri del veicolo.

L’approccio proposto è il Pole Placement control, il quale è stato scelto per la sua efficacia e semplicità, ed è stato testato su modelli del veicolo a complessità via via crescenti. Per testare il controllore sulla macchina radio comandata è stato necessario:

- Identificare la caratteristica tra comando di sterzo della PWM, calcolato in µs, e l’angolo di sterzo δ.
- Stimare i coefficienti di attrito e le cornering stiffness confrontando i dati sperimentali delle forze laterali sulle ruote con quelli ottenuti dal modello. La stima è stata ottenuta utilizzando il metodo dei minimi quadrati non lineare.
- Realizzare un algoritmo per la stima real-time dell’angolo di deriva β.

Parole chiave: Macchina RC, identificazione dei parametri, Pole placement control, Angolo di deriva, Derapata, dinamica del veicolo, modello di Fiala
Abstract

This thesis focuses on the analysis of drifting manoeuvres and the study of how yaw rate and side-slip angle affect them. From the analysis of the dynamical system of a 1:10 scale radio controlled car, a controller has been implemented and different tests have been conducted for identification and estimation of vehicle parameters. The proposed approach is the pole placement control, which was chosen for its effectiveness and simplicity, it was examined on different models with growing complexity. To test the controller on the radio controlled vehicle, it was necessary to:

- Identify the characteristic between the PWM steering command, calculated in µs, and the steering angle δ.

- Estimate the friction coefficients and the cornering stiffness by comparing the experimental data of the lateral forces on the wheels with those obtained from the model. The estimate was computed using the non-linear least squares method.

- Realize a real-time algorithm for the estimation of the side-slip angle β

Keywords: RC car, Parameter identification, Pole placement control, Side-slip angle, Drifting, Vehicle model, Fiala model
Introduction

The world first car was patented in the 1886 by Carl Benz, his car was capable to reach a top speed of 16 km/h. Just twenty years later, the world’s fastest cars were able to run to and beyond 100 km/h and by the 1928 the speed of 192 km/h was reached. The car gained really high speed in a short time, allowing people to move fast, but also created a huge problem, the car accidents. Since the invention of the first car and with the spreading of the vehicles around the world, the number of casualties due to car accidents, grown as the number of cars circulating on the streets.

In the ’70s car producer start focusing on research how could be possible to increase the safety of their cars. So the major idea was to reduce the accelerations connected to car crash with different passive safety devices:

- Seat belts, that keep passengers in the vehicle, preventing them from continue moving at the same speed at which the car was running.
- Crumble zones, increases the distance between the passengers and the collision point and deform in order to dissipate the energy of the crash.
- Air bags, which allows the head to decelerate softly and avoiding harsh bumps.

With the development of the electronics also active systems began to play a role in car safety. The first significant introduction has been the Anti-lock braking system (ABS), which prevent the wheels from blocking and avoiding uncontrolled sliding. The ABS was followed in the ’90s by the Electronic stability control (ESC) that improve vehicle stability by detecting and reducing the loss of traction. Then in the early years, automation started to play a more and more important role in car safety with a lot of different devices, like braking assistance or autonomous cruise control.

The deaths due to a car accident drastically reduced from the introduction of active and passive equipments, but still, from the latest WHO report, in the 2013 there were estimated 1 250 000 road traffic deaths in the world, 84 589 in Europe (3721 in Italy). The majority of these accident are caused by driver errors, the only way to further reduce those numbers is to remove the human from the control cycle, thus the realization of fully autonomous vehicles.
2 Introduction

Figure 1: SAE levels of driving automation

The design of an UAV become essential in 1961 during the space race, because between the earth and the moon there is a 2.5 s delay. When a command is sent, it takes that time to get to the moon, making impossible to radio control a vehicle. The solution found was to develop the world’s first truly self-driving wheeled vehicle: The Cart, as it was called, was equipped with cameras and programmed to detect and follow a solid white line on the ground. After the Cart, rovers turn into the common way to study other planets and moons, especially Mars. All these rover are able to perform a trajectory tracking control, taking as input the path sent by earth control rooms. These extraterrestrial vehicles were the top of the state of the art of ground UAV until Darpa challenges. Darpa, the U.S. department of defence’s research arm, sponsored a series of competitions that pushed autonomous technologies forward. After those challenges, the last of which was the urban challenge in 2005, many private companies became interested in founding the research on autonomous driving.

Nowadays many companies are testing their autonomous vehicles, but with different grade of automation, as explained in figure 1. The U.S. vision is to obtain directly a full automation vehicle, while the European idea is to incrementally add automation on order to gradually reach the 5th level of SAE standard. Several producer are expected to launch their fully automated products in 2020, while lower levels features are more close to market, like automated highway driving or self-parking. While on one hand many technologies are ready to develop self-driving cars, on the other hand there are still several open challenges:

- Facing with high-complexity urban environments
- Accounting for dynamical situation, like the combustion engine effects, the kinematic vehicle constrain and the presence of other vehicles and people on the roads.
• Dealing with uncertainty due to jumping position or uncertain perception
• deal with real time operations

As well described in figure 1, a full automation car must be capable of drive in every possible condition. Not only the autonomous driver have to behave like a normal human driver, but must be able to act as the best human professional driver. This objective includes the driving on difficult weather condition, like snow, heavy rain or other low grip situation, such as on unpaved roads. At low grip the wheels tends often to slide with low lateral forces applied, to avoid problem connected to this situation there are two possible ways. The first one can be to reduce the longitudinal speed of the vehicle, but this also means to reduce the performances of the car. Then the second solution is to control this unstable situation, by allowing drifting, like a rally driver would do.

The design of an autonomous vehicle can be divided in different layers:

1. The top layer is the perception of the environment using different sensors such as radar and GPS for navigation, accelerometers and gyroscopes that allow to measure the forces acting and the orientation of the car.

2. The middle level is the trajectory planning, this means the definition of the path that the vehicle should follow. Here is where obstacle avoidance algorithms are implemented.

3. The bottom layer consists in the motion control, while in the trajectory planning is decided where the car moves, here the focus is on how the car moves.
Chapter 1

State of the art

The state of the art of the Unmanned autonomous vehicle (UAV) was demonstrated during the 2005 DARPA Grand Challenge, where twenty-three teams raced their vehicles autonomously over 131.2 miles of unpaved road in the Mojave desert and only five could complete the race within 10 hours [5]. The team from Stanford university, who won, completed the course at an average speed of approximately 19 mph (30 km/h).

Since then research focused on increase that moderate speed and to became fast as human (expert) car drivers, it was necessary to develop mathematical models and control algorithms that are able to deal with the trajectory generation and tracking problem under high-speed and/or other abnormal (e.g. off-road, rough or low grip surfaces) driving conditions.

**High side-slip manoeuvre:** The dynamic behaviour of a car negotiating a curve or running straight ahead is widely discussed in the literature. In some cases the interest is focused on the vehicle alone as Rossa, Mastinu, and Piccardi in [18], in other papers the interest is on the interaction between driver and vehicle [19] because they are two parts of a single system and their interaction can change the overall behaviour of the vehicle.

For the majority of situations, actually all those that a normal driver is likely to experience, a linear model of the vehicle and a linear model of the tyres are sufficient. There are, however, a number of important situations, mostly related to emergency manoeuvres, in which high side-slip angles are reached and the non-linear behaviour of the vehicle plays a crucial role. In such situations, a well-designed vehicle (i.e a car whose non-linear behaviour has been considered and understood) can make the difference between a tragic accident and a barely notable risky situation. Therefore knowing and studying the non-linear behaviour of the vehicle is of paramount importance in order to provide the controls and the active safety of road vehicles.
Figure 1.1: Test vehicle performing a high side-slip manoeuvre

1.1 Stability Analysis

First of all is necessary to deeply understand the powerslide, or drifting, of an automobile. Rudimentary contributions to this behaviour can be found in [16], more recent ones in [11] and in [25]. A valuable significant addition to the subject is given by Edelmann and Plöchl in [6] where they describe and analyse the high side-slip manoeuvre using a four-wheel vehicle model and validating the data with a test performed on a circular path of radius \( \rho = 50 \) m using a sports utility vehicle (Fig. 1.1), equipped with several measurement devices. Rear-wheel drive was realized by deactivating the all-wheel drive system, furthermore all electronic stability control systems have been switched off. Then Edelmann and Plöchl investigate the stability of the test-car through a root locus graph for every handling branches detected: ‘regular cornering’, ‘overdraw steering with small side slip angles of the vehicle’, ‘overdraw steering with larger side slip angles of the vehicle’ and ‘powerslide’. This study has shown that the powerslide and, more generally, high side-slip manoeuvre, are unstable motions and need to be stabilized. On the other hand, the authors do not take into consideration why should be valuable to perform high side-slip turns.

A comprehensive study of the stability is performed in [19] with the adoption of a single-track model of the vehicle, validated with a sports car, and the model of the driver described in [17], [1], [14]. The researchers considered two case studies, in the first case a vehicle is fitted with tyres whose characteristics are depicted on the left panel of Figure 1.2. Such a vehicle has an understeering behaviour at low lateral acceleration (\( \text{UN} \) case). In the second case the same vehicle is fitted with tyres whose characteristics are depicted in the right panel of Figure 1.2. The vehicle has an over-steering behaviour at any lateral acceleration (\( \text{OV} \) case).
1.2. Minimum Time Cornering

The problem of trajectory planning for high-speed ground vehicles over rough terrain presents an enormous technical challenge. A preliminary study can be found in [23], where Velenis, Tsiotras, and Lu introduce a new approach to real-time path planning of autonomous vehicles, which can be used to overcome the limitations of both numerical and analytical optimization techniques. The optimal trajectory
is computed by scheduling on-the-fly a series of pre-computed manoeuvres from a manoeuvre library using a finite state machine (manoeuvre automaton). One computes off-line a collection of aggressive manoeuvres and subsequently use them on-line for vehicle control in high-speed regimes.

In this paper Velenis, Tsiotras, and Lu conduct a mathematical analysis of rally racing techniques and they focus on high speed cornering technique, namely Trail-Braking (TB). They explored the optimality properties of TB by formulating several minimum-time cornering scenarios, which were solved using non-linear programming. They concluded that TB corresponds to the minimum-time cornering solution when the vehicle is required to return to the straight line driving condition right after it reaches the geometric end of the corner and the results are validated using the experimental data provided by expert rally drivers. Finally, and in order to reduce the numerical complexity during optimization, they proposed a parametrization of the control inputs that can be used to reproduce TB manoeuvres for a variety of corner geometries.

In a more recent paper [21], Tavernini et al. followed the idea that high-drift and even counter-steering manoeuvres may be more efficient than typical low-drift movements, under certain road–tyre characteristics and vehicle layout. The goal of this study is to find the minimum time manoeuvre (and related input history) of a given vehicle running on a given road while accounting for the environment constraints, and negotiating a $180^\circ$ turn. Non-linear optimal control techniques
will be employed, since the problem is especially non-linear when considering limit handling manoeuvres. A single-track model, with different transmission layout, including non-linear tyres and front/rear load transfer is employed. Because the same tyre behaves differently on different road surfaces, the optimization procedure is conducted on many road types. In particular, when using the same tyre on different road asphalts, the cornering stiffness keeps almost unchanged while there may be changes on the friction peak coefficient (rigid approximation). This assumption cannot be considered valid only in case of off-road surfaces, and a reduction on the cornering stiffness is therefore expected.

The minimum time manoeuvre on asphalt surfaces is characterised by low vehicle drift angles and trajectories that resemble those of racing cars on paved track circuits, where the whole road width is exploited. When moving to off-road conditions (fig. 1.3), the minimum time manoeuvre is characterised by aggressive, high-drift, even counter-steering (RWD and AWD) manoeuvres and rally like trajectories which keep the vehicle far from the outer border of the road. In other words it is proven that the powerslide is the minimum time manoeuvre under condition of low friction. For a complete understanding of the aggressive manoeuvre also the handbrake cornering technique must be taken in consideration and analysed. This is what Tavernini et al. do in [22], where they use non-linear optimal technique to model the driver behaviour. Demonstrate the capability of the model to reproduce very demanding experimental manoeuvres involving the handbrake through the comparison of simulation results with real data from a professional rally driver. It turned out that the handbrake technique allows for minimum (lateral) space and minimum time cornering on tight hairpins. The most important result of this work is given by the demonstration that the handbrake and the pendulum strategies are consistently present in numerical results and therefore are not related to a particular set of vehicle/tyre parameters, boundary conditions, and inputs limits.

In the end this work opens to the development of advanced vehicle control systems and/or autonomous vehicle able to exploit the full potential of tyre non-linearities.

### 1.3 Drifting Trajectory Tracking

In [12], they investigate optimization-based control strategies for the task of racing an autonomous vehicle around a given track. Liniger, Domahidi, and Morari focus on methods that can be implemented to run in real-time on embedded control platforms, and present experimental results using 1:43 scale Kyosho dnano RC race car that achieve top speeds of more than 3 m/s, which corresponds to an up-scaled speed of about 465 km/h. Two model-based control schemes are presented in this paper. First, they describe a hierarchical two-level control scheme consisting of a model-based path planner generating feasible trajectories for an underlying non-linear model predictive control (NMPC) trajectory tracking controller. Their second approach combines both path planning and path following into one formulation, resulting in one NMPC controller that is based on a particular formulation known from contouring control.

The objective of both approaches is to maximize the progress on the track, measured
by a projection of the vehicle’s position onto the centre line of the track. *Linear time varying* (LTV) models obtained by linearisation are employed to construct a tractable convex optimization problem to be solved at each sampling time. Efficient interior point solvers for embedded systems are employed to solve the resulting optimization problems, which makes the approaches presented in this paper suitable for use on embedded systems.

The fundamental idea of both approaches is to use a receding horizon controller, which maximizes progress on the track within the horizon as a performance measure. This is closely related to a time optimality criterion and allows for a systematic incorporation of obstacles and other constraints. It is particularly effective for overtaking opponents, as our controllers seek for a progress-optimal solution around the obstacles. In order to deal with track constraints and obstacle avoidance situations, they represent the feasible set for the position of the car at any time instance by a slab defined by two parallel linear inequalities. This ensures tractability of the sub-problems by convex programming on one hand and incorporation of track and obstacle constraints in a systematic manner on the other hand. To deal with non-convex situations such as overtaking an obstacle on the left or right, a high-level corridor planning algorithm supplies a set of appropriate convex constraints to the controllers. This constraint set is the result of a shortest path problem solved by *Dynamic programming* (DP).

### 1.4 Steering Controller Design

Race car drivers choose to operate a vehicle at its frictions limits without losing control by choosing the trade-off between path tracking and stability. The design of a steering controller must achieve the same result and the knowledge gained can be applied to the design of autonomous vehicles or future driver assistance systems, which could potentially prevent many loss-of-control accidents. Automatic steering controllers have been subject of much research, such as in [9, 10, 26], and there are multiple approaches to compute a feedforward steering input, depending on which reference point is used.

Using the *Centre of Gravity* (CG) as a reference point for the vehicle trajectory can be challenging as the feedforward steering as to predict the effect from the rear tyre force, which is highly non-linear at the limits. This is the reason why Kritayakirana and Gerdes present a new method of controlling a vehicle at the limits of handling in [10]. This study uses the *Centre of Percussion* (COP) for calculating a feedforward steering command, which reduces the complexity of the algorithm because the rear tyre force does not influence the motion of the COP. As a consequence, this approach does not require the knowledge of the rear lateral force $F_y$ and needs only path information for the calculation. The authors adopted a bicycle model to derive the equations of motions in order to compute the feedforward steering command from the COP. A coordinate transformation is then applied to convert the vehicle’s motion into path-tracking state variables, where utilising the COP eliminates all the terms associated with the rear tyre force. This creates a feedforward steering command that is robust to the disturbances from the rear tyre force and simplifies the controller structure. To provide stability to the system, a steering feedback,
based on a lane-keeping system with additional yaw damping, is then introduced. Using a state-space form and a Lyapunov function they prove that the system is asymptotically stable for a speed range of 30-50 m/s, even when the rear tyres are highly saturated. Moreover they employed a ‘g-g’ diagram, a graph in which lateral and longitudinal accelerations of the car are depicted, and slip circle for controlling throttle and brake inputs (longitudinal commands). Firstly controller’s performance are demonstrated in a vehicle simulation without disturbances, secondly with an Audi TTS is shown that the controller can operate at the friction limits while maintaining minimal tracking error.

In this paper a steering controller that can provide path-tracking ability as well as yaw stability is designed by utilising the benefits of the COP. By eliminating the effect of dynamic rear tyre force, the COP simplifies and provides robustness to the calculation of the feedforward steering controller. The COP also reduces the complexity of the state-space equation as the lateral tracking motion $e_{cop}$ is
decoupled from the vehicle’s yaw motion $\Delta \psi$. The experimental results demonstrate the controller’s ability to drive the autonomous vehicle to its friction limits with minimal tracking error. The steering controller also counter-steers to maintain yaw stability when the vehicle over-steers.

Drifting is a throttle induced over-steer, an unstable driving condition where the driven rear wheels of a vehicle break traction and slide out. While being quite dangerous for unprepared drivers, this is most enjoyable when the vehicle is handled to perfection. For this reason power-sliding, as also referred to manoeuvres with large slip angles, is part of most race driver trainings. From the industry’s point of view, it is important to understand the vehicle dynamics at the limits of handling. Many research focused on this topic has been presented from some who had the purpose of understanding the physics behind the drifting and others in which the power-slide is stabilized, at least in a particular case, like the 180° turn. However, all of the above approaches heavily rely on GPS-based slip angle estimates and fairly precise inverse models of the wheels – both not available for production vehicles on changing road conditions. To overcome these shortcomings in [26] has been proposed a steering/throttle control strategy, which approximates the travelling direction of a reference point on the vehicle by neglecting the comparably small slip angles at the front wheels of the drifting car. This simplifies the input-output dynamics and makes the derived control strategy robust to changing surface conditions, tyre parameters and vehicle load. As in [10] is used here the COP to decouple the lateral reference acceleration from the lateral rear forces. In this paper an input-output-linearising steering controller was combined with a sliding mode throttle controller in order to automatically transfer the vehicle from manual into drifting and stabilise it in this mode. The control performance was demonstrated with a production vehicle underlining the fact that suitable model simplifications can be the key to robust closed-loop control performance. As the steering and throttle controller can be independently operated, vehicle guidance may be shared with a manual driver. This is very helpful for race driving lessons, as the learner can focus on handling either the steering wheel or the gas pedal. This approach based on the knowledge of the curvature, on slip angle references and on the vehicle velocity reference at the entrance of the curve. Those values were based on manually drifted manoeuvres. The vision is to build a cascade control structure upon the presented sliding-mode strategy which is seen as a robust underlying controller.

A similar work to [26] that can be considered the continuous of Kritayakirana and Gerdes paper [10] is [9], where a feedback-feedforward steering controller capable of combined stability and accurate path tracking at the limits of handling is designed. A baseline controller with lookahead steering feedback and feedforward based on the non-linear handling diagram is presented and steady-state simulation results are used to demonstrate the relatively poor path tracking of this baseline. Then the authors considered a modified steering feedback that aims to keep the vehicle side-slip tangent to the desired path, resulting in a closed-loop steering response with zero steady-state lateral path deviation, but at the cost of poor stability margins. A better design approach is also presented that incorporate the desired side-slip behaviour as a feedforward input, which significantly improves path tracking while maintaining robust stability margins.

All the algorithms presented are tested on an Audi TTS equipped with an electronic
power steering motor for autonomous steering and active brake booster and throttle by wire for longitudinal control. An integrated differential Global Positioning System (GPS) and inertial measurement unit is used to obtain global vehicle states. A map-matching algorithm synthesises information from the GPS to determine the vehicle’s position along the desired path and obtain the lateral path deviation $e$ and vehicle heading error $\Delta \psi$.

To test the controller safety at the limits of handling, experimental data were collected on a constant radius turn in an open parking lot at two constant speeds (Fig. 1.4). Then the control algorithm has been tested on a race track, on those experimental data is conducted a histogram analysis, which quantitatively indicate that the improved feedforward command reduces lateral path deviation by more than 50%. One potential drawback is that this feedforward approach is sensitive to vehicle model uncertainty, especially at the physical limits of handling, where transient dynamics become prevalent.

### 1.5 Drifting Stabilization

Numerous studies on the dynamic behaviour and control of vehicles have appeared in the literature, and some of them were analysed here, considering the vehicle’s full handling capacity, e.g. with the tyres operating in their non-linear region. Researchers, in the area of vehicle control and automotive safety, envision that a new generation of active-safety systems, for passenger vehicles, will employ expert driving skills to actively assist the driver exploit the limits of handling of the vehicle, during emergency manoeuvring, instead of restricting the vehicle’s response within the linear region of operation of the tyres. In order to achieve those objectives it is of primary importance being able to understand and recreate a sustained drift. A preliminary study on this subject is conducted by Velenis et al. in [24], while more complete research can be considered [8, 25].

In [24], a control scheme, which stabilizes drifting equilibria of an RWD vehicle is presented, using coordinated lateral (steering) and longitudinal (drive torque) control inputs and mimicking techniques used by expert drivers. In contrast to previous works of Velenis et al., simplifications associated with the use of solely longitudinal control, or decoupled longitudinal and lateral control, are avoided. Essentially, the controller proposed herein possesses the same control authority as a human driver. The coordinated longitudinal and lateral control, which corresponds to the operator’s steering wheel and throttle commands, is motivated by the observation of data collected during the execution of drifting manoeuvres by an expert race driver. Specifically, the data revealed that experienced drivers use the throttle to induce high vehicle yaw rate and side-slip, that is, they use longitudinal inputs to control the lateral dynamics of the vehicle. The data also revealed that drifting equilibria exist at path radii considerably smaller than the kinematic turning radius of the vehicle, expanding its mobility capabilities.

The control design is based on a comprehensive four-wheel car model, instead of the simplified single-track model typically used in cornering stabilization applications, incorporating non-linear tyre friction characteristics, longitudinal and lateral weight transfer effects, and coupling of the rear wheels drive torques through the modelling
of a differential system. In order to overcome the complexity of the control design yielding from the high order vehicle model, the proposed control scheme consists of two layers:

   a. Initially, a linear controller is designed to stabilize a reduced order system, derived by neglecting the wheel rotation dynamics.

   b. The second layer of the control architecture employs a back-stepping controller to regulate the wheel speed dynamics to their desired values from a.

The controller designed in this work uses combined steering angle and drive torque inputs, correlating directly to the human driver commands. The controller was successfully validated via implementation in a high-fidelity simulation environment. Velenis et al.'s work was only theoretical, instead an example of practical realization of a controller for a sustained drift can be found in [25]. This paper presents simple analytical techniques that are used to understand and control high side-slip drift manoeuvres of road vehicles. Voser, Hindiyeh, and Gerdes utilises a two state model of the vehicle, where the front and rear tyres are lumped into a single tyre at each axle and the vehicle’s motion is planar, so pitch and roll motions are neglected. Longitudinal dynamics are also removed and the vehicle’s longitudinal velocity $v_x$ is treated as a time-varying system parameter. The vehicle states are its lateral velocity $v_y$ at the CG in body-fixed coordinates and yaw rate $r$ (rotation rate normal to the plane). The input to the model is the front tyre steer angle $\delta$. The lateral tyre forces are calculated using a variant of the Fiala model, which is simpler than the Pacejka model [16]. This vehicle model does not explicitly account for car behaviours that are significant at and beyond the limits of handling, in particular the effect of longitudinal tyre force and lateral–longitudinal tyre force coupling. Nonetheless, it is possible to account for the overall effect of these behaviours through tyre model parameters. All the experimental results for this work have been generated using an all-electric by-wire research platform (Fig. 1.5), this vehicle’s sensor suite includes an Inertial Navigation System (INS) with automotive-grade accelerometers and gyroscopes for all six degrees of freedom, a single antenna GPS receiver for position and velocity measurements, and a two
antenna GPS receiver for heading and roll angle measurements. State estimates are produced via GPS/INS sensor fusion using Kalman filters for roll, heading, and velocity states. The test were conducted on a low friction surface that facilitate experimental work at and beyond the limits of handling at lower, relatively safe speeds. In order to conduct system analysis and control design, the authors had to identify the parameters of Fiala model for the cornering stiffness and friction coefficients for each tyre. Two tests are necessary, in the first one empirical tyre curves are generated from data of a quasi-steady state ramp steer manoeuvre. Those curves are used for the identification of the cornering stiffness and the friction coefficient of the front tyre by fitting the Fiala model. A second test is necessary because the rear friction coefficient \( \mu_r \) cannot be obtained from the empirical tyre curve. Aside from the obvious lack of data in the saturation region, this is because the empirical tyre curve was obtained under conditions approximating pure cornering. Under such conditions, a relatively small amount of longitudinal force is applied at the rear tyre, whereas the high side-slip manoeuvres under consideration in this work involve significant longitudinal forces at the rear tyre. Consequently, what is desired is an ‘effective’ rear friction coefficient that reflects an average rear tyre lateral force capacity in the presence of significant applied longitudinal force and lateral-longitudinal force coupling. As a result, \( \mu_r \) is obtained by matching simulation predictions of yaw rate and side-slip with measurements of these quantities during a high side-slip manoeuvre. The time-domain fits are accomplished by taking the steering input and longitudinal velocity measurements from a high side-slip data set and using them as inputs to a bicycle model simulation with all tyre parameters, except \( \mu_r \), fixed. The yaw rate and side-slip predictions of the simulation are compared with the experimental measurements of these quantities, and the \( \mu_r \) value within the simulation is varied until close agreement is achieved. By applying techniques of non-linear system analysis to the operating regime surrounding the drift equilibria, it has been verified that those equilibria are saddle points, with unusual non-minimum phase characteristics. The success of an autonomous drifting controller designed using linearisation about one of these drift equilibria was at the time a novel experimental achievement. Moreover, it validates the utility of simple analysis techniques for predicting vehicle behaviour in this dangerous operating regime and designing control algorithms for maintaining stability within this system.

In a more recent work [8] Hindiyeh and Gerdes are focused upon refinement of control strategies for autonomous drifting that better account for the drifting techniques of skilled drivers, as well as the characteristics of the drift equilibria. This work sought to bridge the gap between model fidelity and simplicity for control design with a three-state model (instead of the two states model used in [25]) that treated rear longitudinal force as a direct input. An equilibrium analysis with this model revealed the existence of equilibria possessing all the characteristics of drifting, while also confirming the necessity of a large rear drive force at such equilibria. More importantly, this analysis facilitated a new way of looking at drifting from a control perspective, as a trade-off of vehicle stability for enhanced controllability. To be specific, the analysis demonstrated that the rear tire saturation inherent to drifting reduces vehicle stability but enhances controllability by allowing “steering” of the rear tire using the rear longitudinal force. This concept played a central
role in the design of the controller for autonomous drifting in this work, and in particular the job of rear longitudinal force as a control input. The controller in this work uses steering and rear drive force in coordination, leveraging the ability to “steer” the rear tire using the rear drive force directly in its design. Furthermore, the design has demonstrable stability characteristics, with a physically insightful invariant set calculated using simple Lyapunov techniques.

Finally, and perhaps most importantly, the controller has been experimentally validated on a by-wire test-bed (Fig. 1.5) under challenging test conditions. It achieves sustained, robust drifts (Fig. 1.6) in the presence of disturbances from significant friction variation, not only validating the control design but also bolstering the notion of using cornering with rear tire saturation to contend with uncertain conditions.
Chapter 2

Models and algorithm

2.1 Vehicle models

Before embarking into the development of mathematical models, it is perhaps advisable to discuss a little what ultimately is a drivable road vehicle. Since a road is essentially a long, fairly narrow strip, a vehicle must be an object with a clear heading direction. Moreover, roads have curves. Therefore, a vehicle must have the capability to be driven in a fairly precise way. This basically amounts to controlling simultaneously the yaw rate, the magnitude and direction of the vehicle speed. To fulfil this task a car driver can act on the brake and accelerator pedals and on the steering wheels. And here it is where vehicle dynamics comes into play, since the outcome of the driver actions strongly depends on the vehicle dynamic features and state.

A mathematical model of a vehicle should be simple, yet significant. The main point is to state clearly the assumptions behind each simplification, thus making clear which conditions the model can reliably predict the behaviour of a real vehicle. Throughout this thesis, the coordinate system used in vehicle dynamics modelling will be those used by SAE, presented in Figure 2.1. The x-axis points to the forward direction or the longitudinal direction, the y-axis, which represents the lateral direction, is positive when it points to the right of the driver, and the z-axis...
points to the ground satisfying the right hand rule. The rotation around X-Y-Z are described by the roll, pitch, yaw angles (ϕ, θ, ψ), respectively and by the angular rates (p, q, r). The following list defines relevant definitions for the variable associated with this research.

**Longitudinal direction:** forward moving direction of the vehicle. There are two different ways of looking at the forward direction, one with respect to the vehicle body itself, and another with respect to a fixed reference point. The former is often used when dealing with acceleration and velocity of the vehicle. The latter is used when the location information of the vehicle with respect to a starting or an ending point is desired.

**Lateral direction:** sideways moving direction of the vehicle. Again, there are two ways of looking at the lateral direction, with respect to the vehicle and with respect to a fixed reference point. Researchers often find this direction more interesting than the longitudinal one since extreme values of lateral acceleration or lateral velocity can decrease vehicle stability and controllability.

**Tire slip angle:** sideways motion of a tire. Occurs when the lateral forces at a tire are greater than its friction resistance, this happens during a cornering manoeuvre. The slip angle can be defined as:

\[
\alpha = \arctan \left( \frac{v_y^w}{|v_x^w|} \right)
\]  \hspace{1cm} (2.1)

where \(v_x^w\) and \(v_y^w\) are heading and lateral velocity of the wheel, respectively.

**Vehicle slip angle:** is the angle between the vehicle X-axis and the velocity vector that represents the instantaneous vehicle velocity at that point along the path. It should be emphasized that this is different from the slip angle associated with tires. Even though the concept is the same, each individual tire may have different slip angle at the same instant in time. Often the body slip angle is calculated as the ratio of lateral velocity to longitudinal velocity of the body CG.
### 2.1. Vehicle models

2.1.1 Single-track model

There are numerous vehicle models the simplest is represented by the Single-track or bicycle model, in which the left and the right wheels are joined together, both on the front and rear axles. The vehicle is supposed to be studied with the following hypothesis:

- the vehicle is a rigid body moving on a geometric plane
- the longitudinal and lateral load transfer is neglected
- the body is not subjected to rolling or pitching motions
- aerodynamic effects are neglected
- the suspension system does not affect the calculation
- the steering angle is small

The series of assumptions implies that the model is good under the condition of cornering also in case of high side-slip manoeuvre, the study of which is the main purpose of this work.

A three-degree-of-freedom (DOF) model of a RWD enables to describe the full vehicle motion in the X-Y plane, in this way only the effect of the force on the rear tire $F_y^r$ is taken in consideration. Figure 2.3 shows this model.

The three DOF represents all the possible movement of the vehicle on the plane, translations along the X and Y directions and the rotation about the Z-axis.

The three degrees of freedom model considered for this study is governed by the following equations:

$$J_z\ddot{\iota} = a F_y^f \cos \delta - b F_y^r$$  \hspace{1cm} (2.2)

$$m \left( \dot{U}_y + r \dot{U}_x \right) = F_y^f \cos \delta + F_y^r$$  \hspace{1cm} (2.3)

$$m \left( \dot{U}_x - r U_x \beta \right) = F_x^r - F_y^f \sin \delta$$  \hspace{1cm} (2.4)

Referring to Figure 2.3, the lateral and longitudinal velocities of the vehicles with respect of the fixed coordinate system XYZ can be described as in Equations 2.5
Table 2.1: Parameters and variables of the single track model

<table>
<thead>
<tr>
<th>Parameter/Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y^f, F_y^r$</td>
<td>Lateral tire force on front and rear tires</td>
</tr>
<tr>
<td>$a$</td>
<td>Distance of the CG from the front axle</td>
</tr>
<tr>
<td>$b$</td>
<td>Distance of the CG from the rear axle</td>
</tr>
<tr>
<td>$m$</td>
<td>Vehicle mass</td>
</tr>
<tr>
<td>$J_z$</td>
<td>Vehicle yaw momentum of inertia</td>
</tr>
<tr>
<td>$U_x, U_y$</td>
<td>Longitudinal and lateral velocity of vehicle in body reference frame</td>
</tr>
<tr>
<td>$x, y$</td>
<td>Longitudinal and lateral position of vehicle in inertial reference frame</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Steering angle</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Yaw angle</td>
</tr>
<tr>
<td>$r$</td>
<td>Yaw rate</td>
</tr>
</tbody>
</table>

and 2.6. Those equation represents a rotation of $\psi$ about the Z-axis.

$$
\dot{x} = -U_y \sin \psi + U_x \cos \psi \tag{2.5}
$$

$$
\dot{y} = U_y \cos \psi + U_x \sin \psi \tag{2.6}
$$

While the meaning of all the parameters and the variables of the model is summed up in table 2.1.

The three-DOF model equations 2.2-2.4 can be rearranged in order to exploit the state variable derivatives, which are: $\dot{r}$, $\dot{U}_y$ and $\dot{U}_x$. In this way the previous equations became:

$$
\dot{r} = \frac{a F_y^f \cos \delta - b F_y^r \psi}{J_z} \tag{2.7}
$$

$$
\dot{U}_y = \frac{F_y^f \cos \delta + F_y^r}{m} - r U_x \tag{2.8}
$$

$$
\dot{U}_x = \frac{F_x^r - F_y^f \sin \delta}{m} + r U_x \beta \tag{2.9}
$$

Furthermore, because the aim of this thesis is the analysis of the high side-slip manoeuvre and recalling that the relationship between $\beta$ and the velocities is:

$$
\beta = \arctan \left( \frac{U_y}{U_x} \right) \tag{2.10}
$$

The lateral velocity equation 2.8 can be rewritten using the side-slip angle as state variable, the result is the final derivation of the three-DOF single-track model, described in 2.11.

$$
\begin{align*}
\dot{r} & = \frac{a F_y^f \cos \delta - b F_y^r \psi}{J_z} \\
\dot{\beta} & = \frac{F_y^f \cos \delta + F_y^r}{m U_x} - r \\
\dot{U}_x & = \frac{F_x^r - F_y^f \sin \delta}{m} + r U_x \beta
\end{align*} \tag{2.11}
$$
2.1.2 Full-track model

The single-track model is useful, but presents also some limitations due to all the assumptions, thus it is advisable to study the vehicle dynamics using the full track model. The hypothesis described in the previous section 2.1.1 becomes:

- the vehicle is a rigid body moving on a geometric plane
- the longitudinal and lateral load transfer are considered
- the body is not subjected to rolling or pitching motions
- aerodynamic effects are neglected
- the suspension system does not affect the calculation
- the steering angle is small

The full-track model (Figure 2.4) has three DOF that describes the motion of the vehicle in the X-Y plane, which is outlined by the following Newton’s equation of forces 2.22 in the inertial reference frame:

\[
\begin{align*}
    m\ddot{X} &= F_X \\
    m\ddot{Y} &= F_Y \\
    J_z\ddot{\theta} &= M_Z
\end{align*}
\]  

(2.12)
Where

\( \ddot{X} \) = second derivative of the position along the X-axis
\( \ddot{Y} \) = second derivative of the position along the Y-axis
\( \dot{r} \) = derivative of the angular velocity about the Z-axis
\( F_X \) = total force applied on the X-axis
\( F_Y \) = total force applied on the Y-axis
\( M_Z \) = total momentum applied around the Z-axis
\( m \) = vehicle mass
\( J_Z \) = vehicle momentum of inertia on the Z-axis

The speed component can be rewritten from the inertial to the local reference frame with the same equations 2.5-2.5 used in the previous section, here has been rewritten in matrix form using the variables of the full vehicle model:

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix} =
\begin{bmatrix}
\cos\psi & -\sin\psi \\
\sin\psi & \cos\psi
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y
\end{bmatrix} = R_z(\psi)
\begin{bmatrix}
V_x \\
V_y
\end{bmatrix}
\tag{2.13}
\]

\( V_x \) and \( V_y \) are the longitudinal and lateral components of the absolute speed \( V \) respectively. \( \psi \) is the yaw angle. Deriving the matrix equation 2.13 can be obtained the description of the speed in the CG reference frame, shown in 2.14.

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix} = R_z(\psi)
\begin{bmatrix}
\dot{V}_x - rV_y \\
\dot{V}_y + rV_x
\end{bmatrix}
\tag{2.14}
\]

And given that:

\[
\begin{bmatrix}
F_X \\
F_Y
\end{bmatrix} = R_z(\psi)
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix}
\tag{2.15}
\]

The equations motions 2.12 can be fully rewritten in the local reference frame:

\[
m \left( \dot{V}_x - rV_y \right) = F_x
\tag{2.16}
\]
\[
m \left( \dot{V}_y + rV_x \right) = F_y
\tag{2.17}
\]
\[
J_z \dot{r} = M_z
\tag{2.18}
\]

By exploiting the derivatives of the state variables, the equations 2.16-2.18 turn into:

\[
\dot{V}_x = \frac{F_x}{m} + rV_y
\tag{2.19}
\]
\[
\dot{V}_y = \frac{F_y}{m} - rV_x
\tag{2.20}
\]
\[
\dot{r} = \frac{M_z}{J_z}
\tag{2.21}
\]
2.1. Vehicle models

Figure 2.5: Forces acting on the full-track vehicle model

Where:

\[ V_x, V_y = \text{longitudinal/lateral velocity in the CG reference frame} \]
\[ r = \text{yaw rate} \]
\[ F_x = \text{total longitudinal force} \]
\[ F_y = \text{total lateral force} \]
\[ M_z = \text{total momentum of the CG} \]
\[ m = \text{vehicle mass} \]
\[ J_Z = \text{vehicle yaw momentum of inertia} \]

The forces \( F_x, F_y \) and the momentum \( M_z \) are composed by sum of the forces and momenta applied at the four wheel, which are depicted in figure 2.5 together with the main vehicles angles. The total forces, along the longitudinal and lateral direction, can be computed by summing the effect of each force on the two different axes, the results are the equations 2.22-2.23. The total momentum is the sum of all the angular momenta created by the wheel forces and is the one presented in equation 2.24. The parameters shown in the figure and in the equations are described in Table 2.2.

\[
F_x = F_{x,\text{rear}} + F_{x,\text{front}}
= (F_{x,\text{L}}^R + F_{x,\text{R}}) + \left( (F_{x,\text{L}}^L + F_{x,\text{R}}) \cos \delta - (F_{y,\text{L}}^L + F_{y,\text{R}}) \sin \delta \right)
\tag{2.22}
\]

\[
F_y = F_{y,\text{front}} + F_{y,\text{rear}}
= (F_{y,\text{L}}^L + F_{y,\text{R}}^L) + \left( (F_{y,\text{L}}^L + F_{y,\text{R}}^L) \cos \delta - (F_{x,\text{L}}^R + F_{x,\text{R}}^R) \sin \delta \right)
\tag{2.23}
\]

\[
M_z = -b \left( F_{x,\text{L}}^L + F_{x,\text{R}}^R \right) + a \left( F_{x,\text{L}}^L + F_{x,\text{R}}^R \right) \cos \delta + \left[ -F_{x,\text{L}}^R \frac{C_r}{2} + F_{x,\text{R}}^R \frac{C_r}{2} \right]
+ \left( F_{x,\text{L}}^L \cos \delta - F_{y,\text{L}}^L \sin \delta \right) \left( -\frac{C_f}{2} \right)
+ \left( F_{x,\text{R}}^R \cos \delta - F_{y,\text{R}}^R \sin \delta \right) \frac{C_f}{2}
+ \left[ a \left( F_{x,\text{L}}^L \sin \delta + F_{x,\text{R}}^R \sin \delta \right) \right]
\tag{2.24}
\]
Table 2.2: Parameters and variables of the full-track model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{fL}^x$, $F_{fR}^x$, $F_{rL}^x$, $F_{rR}^x$</td>
<td>Longitudinal tire forces</td>
</tr>
<tr>
<td>$F_{fL}^y$, $F_{fR}^y$, $F_{rL}^y$, $F_{rR}^y$</td>
<td>Lateral tire forces</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Steering angle</td>
</tr>
<tr>
<td>$a$</td>
<td>Distance of the CG from the front axle</td>
</tr>
<tr>
<td>$b$</td>
<td>Distance of the CG from the rear axle</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Vehicle front track</td>
</tr>
<tr>
<td>$c_r$</td>
<td>Vehicle rear track</td>
</tr>
<tr>
<td>$\alpha_{fL}$, $\alpha_{fR}$, $\alpha_{rL}$, $\alpha_{rR}$</td>
<td>Tire side-slip angles</td>
</tr>
</tbody>
</table>

Tires vertical load:

The tire forces, that will fully characterized in the next section 2.2, are function of the tires vertical loads. In both the single-track and the full track, the quasi-static approximation in used and the dynamic of the suspensions is neglected. The latter only accounts only the static vertical load in equation 2.25, while the former takes account also the longitudinal and the lateral load transfer.

\[
F_{zf,\text{static}} = \frac{1}{2} \frac{mb}{l} \quad F_{zr,\text{static}} = \frac{1}{2} \frac{mag}{l}
\]

with: $l = a + b$ the distance between the front and the rear axles and $g$ the gravitational acceleration

\[
\Delta F_{zf} = \frac{mb}{lc_f} a_y \\
\Delta F_{zr} = \frac{mg}{lc_r} a_y \\
\Delta F_{zr} = \frac{mg}{2l} a_x
\]

The load transfer is given by contribution of the lateral acceleration $a_y = Vr$ and the one given by the longitudinal acceleration $a_x = \dot{u}$. The contribution of $a_y$ is provided by the equations 2.26-2.27, while the equation 2.28 shows the contribution of $a_x$.

\[
F_{zf} = F_{zf,\text{static}} - \Delta F_{zf} - \Delta F_{zr} \\
F_{zr} = F_{zr,\text{static}} + \Delta F_{zf} - \Delta F_{zr}
\]
2.2 Tire models

In modern highway vehicles all the primary control and disturbance forces disturbances which are applied to the vehicle, with the exception of aerodynamic forces, are generated in the tire-road contact patch. Thus a thorough understanding of the relationship between tires, their operating conditions and the resulting forces and moments developed at the contact patch is an essential aspect of the dynamic of the total vehicle.

The tire serves essentially three basic functions:

1. It supports vertical load, while softening the road shocks
2. It develops longitudinal forces for acceleration and braking
3. It develops lateral forces for cornering

Numerous simplified tire models have been developed in the past to approximate various performance properties, but for the purposes of understanding their role in vehicle dynamics it is necessary to use more complex non-linear models. In particular, a non-linear model of the tire is necessary to properly characterize the high-side slip manoeuvre.

There are different categories of models capable of representing the tire dynamics on saturation situations:

- Finite element analysis
- Through empirical data approximation, such as the 'Magic Tire Formula'
- The dynamical approximation of the tire by a 'brush' model first proposed by Fiala

In this work of thesis has been used and are here presented the Fiala model (Sec. 2.2.1) and Pacejka magic formula (Sec. 2.2.2).

2.2.1 Fiala tire model

Between the presented approaches, the third modelling technique presents a good compromise between its ability to describe tires physical properties and its complexity, since it assumes that the tire is always at steady state and tire transients are faster than chassis transients.

The Fiala tire model approximates a parabolic normal pressure distribution on the contact patch with a rectangular shape. The instantaneous value of the tire-road friction coefficient is determined by a linear interpolation in terms of resultant slip and the static friction coefficient. The influence of a camber angle on lateral force and aligning moment is not considered.

The tyre lateral forces, calculated using the Fiala lateral tire force model, are given
in 2.33.

\[
F_y(\alpha) = \begin{cases} 
-C_\alpha \tan \alpha + \frac{C_\alpha^2 (2 - \mu_s / \mu_p)}{3 \mu_p F_z} |\tan \alpha| \tan \alpha, & |\alpha| < \alpha_{sl} \\
\frac{C_\alpha^3 (1 - 2 \mu_s / 3 \mu_p)}{9 \mu_p^2 F_z^2} \tan^3 \alpha, & |\alpha| \geq \alpha_{sl}
\end{cases}
\]  
(2.33)

\[
\alpha_{sl} = \arctan \frac{3 \mu_p F_z}{C_\alpha}
\]  
(2.34)

Where:

- \(C_\alpha\) = tire cornering stiffness
- \(F_z\) = normal load applied to the tire
- \(\mu_p\) = peak friction coefficient between the tire and the ground
- \(\mu_s\) = sliding coefficient of friction between the tire and the ground
- \(\alpha\) = tire slip angle

This model does not explicitly accounts for vehicle behaviours that are significant at and beyond the limits of handling, in particular the effect of longitudinal tire force and lateral-longitudinal tire force coupling. Nonetheless, it is possible to account for the overall effect of these behaviours through different expedients:

- Introducing a derating factor \(\xi\), that accounts for the effect of the longitudinal tire force. Like in Section 3.1.
- Using a proper selection of tire model parameters, such as in Section 4.2.2.

### 2.2.2 Pacejka tire model

An excellent approximation of tire forces can be obtained through the empirical equations, first introduced by Pacejka and also known as "magic formula". This mathematical expression is named "magic formula" because there is no particular physical basis for the structure of the equations chosen, but they fit a wide variety of tire constructions and operating conditions. Each tire is characterized by 10 coefficients as a best fit between experimental data and the model. These coefficients are then used to generate equations that state the lateral and longitudinal forces, \(F_x\) and \(F_y\), as a function of the normal force \(F_z\), the camber angle \(\gamma\), the longitudinal and the lateral slip, \(\sigma\) and \(\alpha\), respectively.

#### Longitudinal tire Force

The equation that gives the longitudinal tire force is such that:

\[
F_x = F_x(F_z, \sigma, \gamma)
\]

The force is a function of \(F_z\), \(\gamma\) and the longitudinal slip \(\sigma\), which is described by the relation of 2.35.

\[
\sigma = \frac{\omega_w - \omega_w^0}{\omega_w^0} = \frac{\omega_w - \frac{V}{R_w}}{\frac{V}{R_w}}
\]  
(2.35)
2.2. Tire models

Where $V$ is the vehicle speed and $\omega_w$ the rotational speed of the wheel.
The Pacejka’s expression that computes the longitudinal force is:

$$F_x = D \sin \left( C \arctan \left\{ B (1 - E) (\sigma + S_h) + E \arctan [B (\sigma + S_h)] \right\} \right) + S_v$$

(2.36)

where $B, C, D, E, S_v$ and $S_h$ are six coefficients that depends on the load $F_z$ and the con the $\gamma$ angle. They has to be obtained experimentally and do not have any physical meaning. Especially $S_v$ and $S_h$ has been introduced to allow values of $F_x$ different from zero when $\sigma = 0$.
The $D$ coefficient just gives the maximum value of $F_x$, other than the effect of $S_v$.The product $BCD$ gives the slope of the curve for $\sigma + S_h = 0$. The values of the coefficients are expressed as a function of a certain number of $b_i$ factors, which can be considered as distinctive of the tire, but depends also on the conditions on the speed and on the road.

$$C = b_0 \quad D = \mu_p F_z$$

where for $b_0$ a value of 1.65 is suggested and

$$\mu_p = b_1 F_z + b_2 \quad BCD = \left( b_3 F_z^2 + b_4 F_z \right) e^{-b_5 F_z}$$

$$E = b_6 F_z^2 + b_7 F_z + b_8 \quad S_h = b_9 F_z + b_{10} \quad S_v = 0$$

The coefficients introduced in 2.36 and the results obtained are usually in non consistent unit: the force $F_z$ is in kN, the longitudinal slip is expressed in percentage and the force $F_x$ is in N.

**Lateral Force**

The lateral tire force is a function of:

$$F_y = F_y (F_z, \alpha, \gamma)$$

And, similarly at the expression in 2.36, in the case of the lateral tire force the magic formula is:

$$F_y = D \sin \left( C \arctan \left\{ B (1 - E) (\alpha + S_h) + E \arctan [B (\alpha + S_h)] \right\} \right) + S_v$$

(2.37)

where the products of the coefficients $B, C$ and $D$ gives directly the cornering stiffness. The values of the other coefficients, who depends on the $a_i$ factors, are:

$$C = a_0 \quad D = \mu_{yp} F_z$$

where it is suggested a value of 1.30 for $a_0$ and

$$\mu_{yp} = a_1 F_z + a_2$$

$$BCD = a_3 \sin \left[ 2 \arctan \left( \frac{F_z}{a_4} \right) \right] (1 - a_5 |\gamma|)$$

$$E = a_6 F_z + a_7 \quad S_h = a_8 \gamma + a_9 F_z + a_{10}$$

$$S_v = a_{11} \gamma F_z + a_{12} F_z + a_{13}$$
To obtain a better description of the force, the \( a_{11} \) constant is often substitute with the linear relationship

\[
a_{11} = a_{111} F_z + a_{112}
\]

(2.38)

The coefficients \( S_h \) and \( S_v \) take account for ply steer and conicity.

### Interaction between longitudinal and lateral forces

What has been said in the previous paragraphs is only valid if the tire develops only longitudinal or lateral forces. If the tire needs to develop forces at the same time in these directions, the situation may be different: the use of grip in one of the two directions decreases the grip in the other.

By applying a driving or braking force to a tire with a certain side-slip angle, the lateral force decreases and the same is true for the longitudinal force that a tire can apply if also a lateral force is present.

A model that allows to express the \( F_x, F_y \) curves at the constant \( \alpha \) is the friction ellipse approximation. Where the longitudinal and lateral force can be represented into a graphic as a function of the traction (or braking) force and as a function of the slip angle. The longitudinal and lateral force have an interaction between them. The effect of this interaction is studied into a diagram which is the friction ellipse (Fig. 2.6). The friction circle represents the force-producing limit of the tire for a set of operating conditions.

By using the friction ellipse it is possible to compute the maximum force that the tire can supply in a certain condition, such as in equation 2.39.

\[
\left( \frac{F_y}{F_{y0}} \right)^2 + \left( \frac{F_x}{F_{x0}} \right)^2 = 1
\]

(2.39)

Where the forces \( F_{y0} \) and \( F_{x0} \) are the \( F_y \) force applied, at a certain slip angle, when there is no \( F_x \) employed, and the maximum longitudinal force applied when the
side-slip angle is equal to zero.

2.2.3 Tire side-slip angle

When a tire is subjected to a lateral force it will drift to the side because of the formation of an angle between the direction of heading and the direction of travel. The angle formed is called slip angle. This is the angle on which the lateral force models relies and must be properly defined. It has the same formulation presented in 2.1, but depending on the model considered, the definition of $V_y^w$ and $V_x^w$ changes.

**Single-track model:** In case of a bicycle model there are only two tires and the contact patch is only affected by the longitudinal position of the considered wheel. The definition of the slip angles is the following:

$$
\alpha_f = \arctan \frac{V_y + ar}{V_x} - \delta \\
\alpha_r = \arctan \frac{V_y - br}{V_x}
$$

(2.40)

Where $a$ and $b$ are the distances from the CG to the front and rear axle, respectively.

**Full-track model:** In the complete model of the vehicle the slip angle are still affected by their longitudinal position inside the vehicle, but are also influenced by their lateral position. The expression of the side-slip angles is presented in 2.41 for the front tires and in 2.42 for the rear wheels.

$$
\alpha_{fL} = \arctan \frac{V_y + ar}{V_x - c_f} - \delta \\
\alpha_{fR} = \arctan \frac{V_y + ar}{V_x + c_f} - \delta \\
\alpha_{rL} = \arctan \frac{V_y - br}{V_x - c_r} \\
\alpha_{rR} = \arctan \frac{V_y - br}{V_x + c_r}
$$

(2.41)

(2.42)

Where $c_f$ and $c_r$ are the front and the rear track, respectively.

2.3 Control algorithm

In this section the control technique, implemented in this work of thesis, is developed fully in its theory for continuous time linear systems. The case of systems with measurable state is considered and an output feedback control law is developed in the case of a scalar input.

2.3.1 Pole Placement control

Consider the continuous-time system:

$$
\begin{cases}
\dot{x}(t) = Ax(t) + Bu(t), & x \in \mathbb{R}^n, & u \in \mathbb{R}^m \\
y(t) = Cx(t) + Du(t), & y \in \mathbb{R}^p,
\end{cases}
$$

(2.43)
where the state $x$ is assumed to be measurable, and the following algebraic control law:

$$u(t) = -Kx(t) + v(t), \quad K \in \mathbb{R}^{m,n}, \quad v \in \mathbb{R}^m \quad (2.44)$$

By combining the state equation and the control law, the closed loop system is:

$$\dot{x}(t) = (A - BK)x(t) + Bv(t) \quad (2.45)$$

Then, the pole placement problem consists of computing a matrix $K$ such that the eigenvalues of $A - BK$ can be arbitrarily assigned. The signal $v$ does not play any role in the problem solution, but it can be used to satisfy other requirements. For example, the previous control law can be designed to stabilize an unstable system by assigning the eigenvalues of $A - BK$, while the input $v$ can be used to close an outer control loop to meet static or dynamic specifications.

The following result holds:

**Theorem 2.3.1.** A necessary and sufficient condition for the solution of the pole placement problem (with measurable state) is that the pair $(A, B)$ is reachable.

If the system is only stabilizable, the unreachable part must have stable eigenvalues and the pole placement algorithm described in the succeeding can be applied to the reachable part only. In the following, it will always be assumed that the system is reachable.

Consider a system with one control input $m = 1$. In this case there exists a state transformation $\tilde{x} = Tx$ such that the transformation system is in the controllable (or reachable) canonical form 2.46.

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}u(t) \quad (2.46)$$

where

$$\tilde{A} = TAT^{-1} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, \quad \tilde{B} = TB = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (2.47)$$

and

$$\det(sI - A) = \det(sI - \tilde{A}) = s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0 \quad (2.48)$$

**Remark.** The transformation matrix $T$ can be determined as follows. First define the reachability matrices of the original system in the controllable canonical form.

$$M_r = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

$$\tilde{M}_r = \begin{bmatrix} \tilde{B} & \tilde{A}\tilde{B} & \cdots & \tilde{A}^{n-1}\tilde{B} \end{bmatrix}$$

$$= \begin{bmatrix} TB & TAT^{-1}TB & \cdots & (TAT^{-1}TAT^{-1} \cdots TB) \end{bmatrix}$$

$$= T \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} = TM_r$$
It follows that

\[ T = \tilde{M}_r M_r^{-1} \]  

(2.49)

Note that the matrices \( \tilde{A} \) and \( \tilde{M}_r \), can be built once the characteristic polynomial of \( A \) is known. Moreover, the inverse of \( M_r \) exists in view of the reachability assumption.

In the transformed variables, the control law (the additional input \( v \) is now omitted) can be written as

\[ u(t) = -Kx(t) = -KT^{-1}\tilde{x}(t) = -\tilde{K}\tilde{x}(t) \quad \text{with: } \tilde{K} = KT^{-1} \]  

(2.50)

which leads to the closed-loop system

\[ \dot{\tilde{x}}(t) = (\tilde{A} - \tilde{B}\tilde{K})\tilde{x}(t) \]  

(2.51)

Moreover

\[ \tilde{A} - \tilde{B}\tilde{K} = TAT^{-1} - TBKT^{-1} = T (A - BK) T^{-1} \]

then, the eigenvalues of \( \tilde{A} - \tilde{B}\tilde{K} \) and \( A - BK \) are the same, and the pole placement problem is solved provided that a matrix \( \tilde{K} \) can be found such that the eigenvalues of \( \tilde{A} - \tilde{B}\tilde{K} \) can be arbitrarily assigned.

To this end, consider the matrix

\[ \tilde{K} = [\tilde{k}_0 \ \tilde{k}_1 \ \cdots \ \tilde{k}_{n-1}] \]  

(2.52)

In view of the structure of \( \tilde{A} \) and \( \tilde{B} \) it follows that

\[
\begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_0 - \tilde{k}_0 & -a_1 - \tilde{k}_1 & -a_2 - \tilde{k}_2 & \cdots & -a_{n-1} - \tilde{k}_{n-1}
\end{bmatrix}
\]

and

\[
P(s) = \det \left( sI - (\tilde{A} - \tilde{B}\tilde{K}) \right) = s^n + (a_{n-1} + \tilde{k}_{n-1}) s^{n-1} + \cdots + (a_1 + \tilde{k}_1) s + (a_0 + \tilde{k}_0)
\]

Now, in order to impose that the eigenvalues of the closed-loop system are real or complex conjugate \( \{-\tilde{p}_1, -\tilde{p}_2, \ldots, -\tilde{p}_n\} \), its characteristic polynomial must be

\[
P(s) = (s + \tilde{p}_1)(s + \tilde{p}_2)\cdots(s + \tilde{p}_n) = s^n + p_{n-1}s^{n-1} + \cdots + p_1 + p_0
\]  

(2.53)

It is sufficient to select:

\[ a_i + \tilde{k}_i = p_i, \quad i = 0, \cdots, n - 1 \]

that is

\[ \tilde{k}_i = -a_i + p_i, \quad i = 0, \cdots, n - 1 \]

Once \( \tilde{K} \) has been computed, it is easy to obtain \( K \) as follows

\[ K = \tilde{K}T \]  

(2.54)
In summary: the problem initially stated can be solved with the following algorithm.

Given $A, B$ and the desired characteristic polynomial $P(s) = s^n + p_{n-1}s^{n-1} + \cdots + p_1 + p_0$:

1. compute the characteristic polynomial of $A$, that is its coefficients $a_i$
2. build the matrices $\tilde{A}$ and $\tilde{B}$
3. compute $M_r, \tilde{M}_r$ and $T = \tilde{M}_rM^{-1}_r$
4. set $\tilde{K} = [-a_0 + p_0, -a_1 + p_1, \cdots, -a_{n-1} + p_{n-1}]$
5. compute $K = \tilde{K}T$

However the computational of the controllable canonical form is not really necessary, as shown below.

The ackermann’s formula

Evaluate the required characteristic polynomial $P(s)$ in $\tilde{A}$

$$P(\tilde{A}) = \tilde{A}^n + p_{n-1}\tilde{A}^{n-1} + \cdots + p_1\tilde{A} + p_0I$$

In view of the Cayley-Hamilton theorem, the following expression holds

$$\tilde{A}^n = -a_{n-1}\tilde{A}^{n-1} - \cdots - a_1\tilde{A} - a_0I$$

By substituting this expression into the one of $P(\tilde{A})$ one obtains

$$P(\tilde{A}) = \left(p_{n-1} - a_{n-1}\right)\tilde{A}^{n-1} - \cdots - (p_1 - a_1)\tilde{A} + (p_0 - a_0)I$$

If the system is of order $n$, one has

$$\tilde{K} = \left[-a_0 + p_0, -a_1 + p_1, \cdots, -a_{n-2} + p_{n-2}, -a_{n-1} + p_{n-1}\right]
= \left[1, 0, \cdots, 0, 0\right]P(\tilde{A})$$

and, since $K = \tilde{K}T$,

$$K = \left[1, 0, \cdots, 0, 0\right]P(\tilde{A})T$$

Recall that $\tilde{A} = TAT^{-1}$ and $P(\tilde{A}) = P(TAT^{-1}) = TP(A)T^{-1}$, therefore, also by recalling that $T = \tilde{M}_rM^{-1}_r$,

$$K = \left[1, 0, \cdots, 0, 0\right]TP(A) = \left[1, 0, \cdots, 0, 0\right]\tilde{M}_rM^{-1}_rP(A)$$

Now consider the structure of $\tilde{M}_r$ and one has

$$\tilde{M}_r = \begin{bmatrix}
0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 0 & * \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 1 & \cdots & * & * \\
1 & * & \cdots & * & *
\end{bmatrix}$$
2.3. Control algorithm

and

\[ K = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \cdots & & 0 \end{bmatrix} \tilde{M}_r M_r^{-1} P(A) = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} M_r^{-1} P(A) \quad (2.59) \]

This is the so-called Ackermann’s formula, which allows one to compute the matrix \( K \) without the need to preliminary transform the system in controllable canonical form.

In conclusion, note that for single-input system, like the one under analysis, it is possible to solve the pole placement problem with a static (algebraic) regulator acting on the system state.
Chapter 3

Numerical simulation

In this chapter the problem of stabilizing the drift equilibria is presented and evaluated. Initially are computed the equilibria of the vehicle in order to find the drift ones characterized by high values of side-slip angles and a saturated yaw rate. Then the system is linearised and a controller is designed with the Pole Placement method seen in Section 2.3.1 considering some assumptions. In the end the hypothesis are removed and the controller is tested on a complex model, comparable to a real-life car.

3.1 Equilibria computation

The three state single-track model, presented in section 2.1.1, is considered at the equilibria. Therefore all the derivatives are equal to zero: $\dot{r} = \dot{\beta} = \dot{V_x} = 0$. Thus the differential equations 2.11 are reduced to the system of algebraic equations in:

\begin{align*}
\begin{cases}
\frac{a F_{y,l}^{eq} \cos \delta^{eq} - b F_{y,r}^{eq}}{J_z} = f_1 (r^{eq}, \beta^{eq}, F_x^{eq}, \delta^{eq}) = 0 \\
\frac{F_{y,l}^{eq} \cos \delta^{eq} + F_{y,r}^{eq}}{m V_x} - r^{eq} = f_2 (r^{eq}, \beta^{eq}, F_x^{eq}, \delta^{eq}) = 0 \\
\frac{F_{xR}^{eq} - F_{y,l}^{eq} \sin \delta^{eq}}{m} + r^{eq} V_x \beta^{eq} = f_3 (r^{eq}, \beta^{eq}, F_{xR}^{eq}, \delta^{eq}) = 0
\end{cases}
\end{align*}

(3.1)

For the tire forces the model presented in 2.2.1 is adopted considering the effect of the lateral force $F_{xR}$ and some simplifications as in [8]:

\begin{align*}
F_y &= \begin{cases}
-C_\alpha z + \frac{C_\alpha^2}{3 \mu \xi F_z} |z| z - \frac{C_\alpha^3}{27 \mu^2 \xi^2 F_z^2} z^3, & |z| < \tan \alpha_{sl} \\
-\mu F_z \text{sgn} \alpha, & |z| \geq \tan \alpha_{sl}
\end{cases} \\
z &= \tan \alpha \\
\alpha_{sl} &= \arctan \frac{3 \mu F_z}{C_\alpha}
\end{align*}

(3.2)

(3.3)

(3.4)

Where $\mu_s = \mu_p = \mu$ is the friction coefficient between the tire and the ground. $C_\alpha$ is the tire cornering stiffness, $F_z$ is the normal load applied to the tire and $\alpha$ is the tire slip angle, a function of $V_y$, $r$ and $V_x$ (Eq. 2.40).
Table 3.1: RC car parameters

<table>
<thead>
<tr>
<th>Vehicle Parameters</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>[kg]</td>
<td>1.454</td>
</tr>
<tr>
<td>CG-Front axle distance</td>
<td>[m]</td>
<td>0.1448</td>
</tr>
<tr>
<td>CG-Rear axle distance</td>
<td>[m]</td>
<td>0.1144</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>[m]</td>
<td>0.260</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>[kgm^2]</td>
<td>0.015</td>
</tr>
<tr>
<td>Wheel outer radius</td>
<td>[m]</td>
<td>0.049</td>
</tr>
<tr>
<td>Vehicle front track</td>
<td>[m]</td>
<td>0.18</td>
</tr>
<tr>
<td>Vehicle rear track</td>
<td>[m]</td>
<td>0.178</td>
</tr>
<tr>
<td>CG Height</td>
<td>[m]</td>
<td>0.1</td>
</tr>
<tr>
<td>Front cornering stiffness</td>
<td>[N/rad]</td>
<td>17</td>
</tr>
<tr>
<td>Front friction coefficient</td>
<td></td>
<td>0.26</td>
</tr>
<tr>
<td>Rear cornering stiffness</td>
<td>[N/rad]</td>
<td>17.81</td>
</tr>
<tr>
<td>Rear friction coefficient</td>
<td></td>
<td>0.22</td>
</tr>
</tbody>
</table>

The ξ in Eq. 3.2 is a derating factor (0 ≤ ξ ≤ 1) that accounts for a reduction in lateral force capacity when longitudinal force is applied. The rear tire derating factor ξ_r vary depending on the longitudinal force input \( F_{xR} \) to the rear tire and is calculated from:

\[
ξ_r = \frac{\sqrt{\left(\mu F_{zR}\right)^2 - F_{xR}^2}}{\mu F_{zR}}
\]

Equation 3.5 is a simple rearrangement of the "friction circle" equation for the rear tire, which dictates that the total force generated at the tire cannot exceed the total force available at the tire from friction. For the front tire ξ_f = 1 since there is no drive force applied at that tire. By treating lateral-longitudinal force coupling in this fashion, it is possible to treat longitudinal force as an input and eliminate the additional complexity of wheel-speed dynamics and a tire model that incorporates wheel slip.

**Computation Results:** The longitudinal velocity is assumed to be constant, such that \( V_x = 1.1 \) [m/s], while the steering angle \( δ_{eq} \) is taken as constant input. The vehicle parameters are summed up in table 3.1, a full description of the vehicle set up is made in 4.1. The Equation 3.2 becomes a system of three equations in the three unknowns \( r_{eq}, β_{eq} \) and \( F_{xR}^{eq} \) that can be solved numerically to determine the steady state cornering conditions. By repeating the process for a wide range of \( δ_{eq} \) from -25 to +25 degree The correspondent equilibria are carried out.
3.1. Equilibria computation

Figure 3.1: Vehicle equilibrium yaw rate

Figure 3.2: Vehicle equilibrium side-slip angle
The calculated equilibria are presented as a function of the steering angle in plots of equilibrium yaw rate (Fig. 3.1), side-slip (Fig. 3.2) and rear longitudinal force (Fig. 3.3). In Figs. 3.1-3.3 the three types of equilibria are depicted in different colours, in blue there are the stable cornering equilibria, while the others are the unstable cornering equilibria, also called drifting equilibria. The yellow one are the equilibria characterized by positive saturated yaw rate and negative side-slip angle. The unstable equilibria, illustrated in red, are in the opposite case with respect to the previous with negative yaw rate and positive side-slip angle. Furthermore the lateral forces has been derived considering the equilibria previously identified. In particular can be seen how the front tire lateral force (Fig. 3.4) reach the saturation limit when exits the stable equilibria at approximately ±20 degree but stays under the saturation for almost all the unstable equilibria. While the rear tire lateral force (Fig. 3.5) remain saturated from approximately 15 degree moving backwards for the yellow equilibria, and from -15 degree moving upwards for the red line.

The fact that the front tire does not saturate inside the unstable region is important because this left some openness for a drifting controller, whose aim is to stabilize the drifting cornering equilibria.
3.1. Equilibria computation

Figure 3.4: Vehicle equilibrium front lateral force

Figure 3.5: Vehicle equilibrium rear lateral force
3.2 Linearisation of drift equilibria

The equilibria computed in section 3.1 have been analysed in order to understand their dynamical behaviour. To do so the system has been linearised around each equilibria considering a dynamic bicycle model and the Fiala tire model presented respectively in eq. 2.11 and 2.33. The drifting equilibria, depicted in red and yellow in Figs. 3.1-3.5, are analysed by considering the rear tire force saturated. In other words the Fiala model is taken fixed assuming that the side-slip angle on the rear wheel is such that:

\[ |\alpha_{\text{rear}}| \geq \alpha_{\text{rear}}^{\text{sat}}. \]

This behaviour is characteristic of a drifting manoeuvre in which, due to the fast rotation and the high speed of the vehicle, the rear wheels stop rolling and start sliding. Differently for the other blue equilibria it is assumed that the wheel remains constantly unsaturated, in this case the rear side-slip is:

\[ |\alpha_{\text{rear}}| < \alpha_{\text{rear}}^{\text{sat}}. \]

The system having the steering angle \( \delta \) as input, the yaw rate \( r \) and the side-slip angle \( \beta \) as state variables is linearised around the previously computed equilibria. The longitudinal speed \( V_x \) is assumed to be constant and equal to 1.1 m/s, which is a realistic speed for the considered test vehicle. For each value of the input in the range \([-25^\circ, 25^\circ]\) the correspondent eigenvalues are carried out. In Figs. 3.6 and 3.7 are shown the eigenvalues of the drifting equilibria and of standard cornering equilibria, respectively. The drifting cornering eigenvalues in fig. 3.6 are those correspondent to a positive yaw rate and a negative side-slip angle, the eigenvalues in the dual situation has the same behaviour with all real eigenvalues, one with positive real part and one negative. It’s interesting to notice how this is an unstable system, due to the presence of the eigenvalue with positive real part, this situation is the same for each value of \( \delta \). Otherwise if the system is linearised around standard cornering equilibria it has two negative real eigenvalues, hence
3.2. Linearisation of drift equilibria

**Figure 3.7:** Eigenvalues with respect to Steering angle for Drifting Equilibria with positive yaw rate

**Figure 3.8:** Eigenvalues for Unsaturated Equilibria
the system is stable. This is verified (Fig. 3.7) for every value of the steering angle inside the admissible range. The system is linearised around the different equilibria for each value of the steering angle $\delta$, but because the eigenvalues are all real and on the complex plane is difficult to notice the effect of the steering angle on the linearised system other two figures are presented. In Figs. 3.8 and 3.9 the real part of eigenvalues are plot with respect to the steering angle $\delta$, representing the drifting and the standard. Eigenvalues carried out from the linearisation of stable equilibria (Fig. 3.9) are always negative but further values of $\delta$ go away from zero, eigenvalues became smaller till $-20^\circ$ and $+20^\circ$ where they are close to zero. This means that is easier to obtain an unstable cornering manoeuvre by turning at high speed with a large steering angle.

**Eigenvalues stability:** As described before the linearised system has a different behaviour depending on the distinct types of equilibria that are under analysis. The standard cornering equilibria once linearised have two real negative eigenvalues, while the drifting equilibria shown two real eigenvalues, one negative and the other positive. Hence the standard equilibria are stable while the drifting ones are unstable. In order to properly establish the stability of second order systems, it is possible to design the state trajectories in the so-called phase plane ($x_1, x_2$). In this way, one can study the behaviour of the system for any initial condition. The phase plane can be computed as follows

$$\frac{dx_1(t)}{dt} = \varphi_1(x_1(t), x_2(t))$$
$$\frac{dx_2(t)}{dt} = \varphi_2(x_1(t), x_2(t))$$

(3.6)
3.2. Linearisation of drift equilibria

The previous equations can be rearranged in the form

\[
\frac{dx_2(t)}{dx_1(t)} = \frac{\varphi_2(x_1, x_2)}{\varphi_1(x_1, x_2)}
\]

(3.7)

and obtain \(x_2 = \Phi(x_1)\).

This procedure has been conducted using yaw rate \(r\) and side-slip angle \(\beta\) as

![Phase plane of drifting equilibria](image1)

(a) Phase plane of drifting equilibria

![Phase plane of standard equilibria](image2)

(b) Phase plane of standard equilibria

Figure 3.10

state variables \(x_1\) and \(x_2\), respectively. The Phase plane of drifting equilibria (Fig. 3.10a) shows clearly two opposite direction of the trajectories. Arrows above the equilibrium point, depicted with a red cross, are all directed leftwards while arrows under the equilibrium are headed rightwards. This behaviour is characteristic of an unstable non diagonalisable system. The instability was suggested by the positive eigenvalue of the drifting equilibria, while the non diagonalisability of the system indicates that is not possible to control separately the two states.

In Figure 3.10b is displayed the phase plane of the standard cornering equilibrium with, as input, a steering angle of \(\delta = -13^\circ\). In this case the arrows are all oriented towards the equilibrium, for any initial state close enough to the considered equilibrium, the trajectory generated converge at the equilibrium.

By putting together the phase plane at \(\delta = -13^\circ\) degree of the three equilibria Figure 3.11 is obtained. The colours describe how high are the derivatives of \(\beta\) and \(r\), thus how fast is the movement system, from the fastest position in blue to the slowest in red. As expected the arrow are red, and really small, in correspondence of the three equilibria. The diamonds represents the drift equilibria, while the circle indicates the stable one. The arrow converge to the circle while diverge from the diamond as earlier explained in detail. In this section has been analysed the system linearised around standard and drift equilibria and has been a proof to the stability of the standard turning manoeuvre, while for the drifting equilibria has been demonstrated their instability. In the next section a pole placement controller will be implemented and tested firstly on the linearised system and secondly on models with a gradual increase of complexity.
Figure 3.11: Phase plane of vehicle equilibria at $\delta = -13^\circ$
### 3.3 Pole placement controller

The behaviour of the vehicle during a steering manoeuvre has been widely discussed in the previous section by taking into consideration the different cornering situation. In this section the focus is on the drifting manoeuvre, which has been proved to be unstable, in order to stabilize it. The stabilizing effect is given by the pole placement controller, which moves the poles of the linearized system, seen in Figure 3.6, to the negative semi-plane.

The most important part in the design of a pole placement controller is a proper choice of the new poles. In this work has been chosen to impose two constraints on the definition of poles, one on the damping and the other on the settling time.

- The damping must be greater than 0.7 in order to avoid excessive oscillations of controlled and lower than 1, which would be the ideal damping, but in real applications would stress too much the controlled variable.

- The settling time is better to have a value between 1 and 5 seconds, lower than 1 the stress on the controlled variable would be too high, while greater than 5 seconds the controller becomes too slow compared with the fast dynamics of a drifting manoeuvre.

The chosen values are presented in Table 3.2 and with this constraints has been computed the design poles for the pole placement controller, which has been implemented as described in Section 2.3.1. The result of the pole placement is a gain $K$ that has to be applied on the controlled variables.

$$
K = \begin{bmatrix} 0.02 \\ -1.47 \end{bmatrix}
$$

The first element of the vector 3.8 is the gain on the yaw rate, while the second has to be applied on the side-slip angle. Once the pole placement gain is computed, the controller can be tested. The simulations have been conducted by considering the drift equilibrium with positive yaw rate and correspondent to a steering angle of $\delta = -10^\circ$.

#### 3.3.1 Simulation on the linearised model

The first step in order to test if a controller is able to do what is designed to, is to simulate its functioning on a linearized system. For this reason has been taken in consideration the linearization of the bicycle model that considers the rear tire saturated. The results of the simulations are presented in Figures 3.12.
Chapter 3. Numerical simulation

The input, the steering angle in Figure 3.12c, goes smoothly to the reference with a negligible overshoot. The input has a direct effect on the side-slip angle, which has almost the same trajectory, while the yaw rate presents a big inverse response but the controller is able to satisfy the constraint on the settling time. The behaviour of the yaw rate is due to the pole placement gain $K$, the first element, who affects the yaw rate, is really small and that reduces the controllability. To avoid the inverse response the constraint on the settling time can be relaxed, but is better have it at the smallest possible value.

3.3.2 Simulation on single-track model

The single-track model introduces non-linearities and the complexity of a real vehicle, even if it is a simplified one. Due to those complexity the trajectories obtained is less smooth than the ones achieved with the linear model, in fact in Figures 3.13 there are 3 seconds of oscillations. Considering that the equilibria are naturally unstable this behaviour can be accepted, furthermore the friction is really small and that makes tougher to obtain a stable drift. As in the simulations on the linear model the yaw rate (Fig. 3.13a) presents an inverse response.

Another possible reason to those oscillations can be found on the assumption of constant longitudinal speed, which until now has been considered equal to $V_x = 1.1 \text{[m/s]}$. Removing this assumption is an important step in order to match the behaviour of a real vehicle.

The longitudinal dynamics is governed by the force on the rear wheels $F_x^r$, by controlling properly this variable the desired longitudinal speed $V_x$ can be obtained. A Proportional Integral controller (PI) is chosen for its simplicity and a feedforward action is added in order to get a faster system response. The value of the feedforward action is selected by the knowledge of the equilibrium value for the rear longitudinal force acquired in section 3.1. After the selection of the feedforward, the PI is tuned by selecting the proper proportional gain at first and secondly a small integral gain to guarantee the convergence of the velocity error to zero. The selected values are summed in table 3.3.

Once the longitudinal dynamics and its controller are set, the simulations can be performed, the results are presented in 3.14. The assumptions of constant longitudinal velocity $V_x$ was not a good fit for the bicycle model, in fact with the longitudinal dynamics is obtained the same performances (Fig.s 3.14a-3.14c) seen with the linearised model and the oscillations are completely removed. The only visible drawback is that the longitudinal dynamics is slower, the velocity $V_x$ (Fig. 3.14d) and force $F_x^r$ (Fig. 3.14g) take 2 seconds to go to their reference values. Despite this, the response speed of the lateral dynamics is not affected and the constrain on the settling time is satisfied.
Table 3.3: Tuning parameters of the feedforward PI controller

<table>
<thead>
<tr>
<th>Velocity Controller</th>
<th>$F_{eq_{x}}$ [N]</th>
<th>0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain $K_{pv}$ [Kg/s]</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Integral gain $K_{iv}$ [Kg/s]</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

3.3.3 Simulation on full-track model

The full-track model has been presented in section 2.1.2, while the dynamics equations has been kept the same as in 2.19-2.21, the longitudinal force has been removed because is negligible with respect to the other forces, in particular the main contribution to the longitudinal force $F_{x}$ is given by the traction force from the rear wheels. Taking in account this assumption the equations 2.22-2.24 becomes:

$$F_{x} = F_{rear}^{x} + F_{front}^{x}$$
$$= (F_{rl}^{x} + F_{rr}^{x}) - (F_{fl}^{x} + F_{fr}^{x}) \sin \delta$$

(3.9)

$$F_{y} = F_{y}^{front} + F_{y}^{rear}$$
$$= (F_{fl}^{y} + F_{fr}^{y}) + (F_{rl}^{y} + F_{rr}^{y}) \cos \delta$$

(3.10)

$$M_{z} = -b (F_{fl}^{y} + F_{fr}^{y}) + a \left( F_{fl}^{y} + F_{fr}^{y} \right) \cos \delta + \left[ -F_{rl}^{c} \frac{c_{r}}{2} + F_{rr}^{c} \frac{c_{r}}{2} \right]$$

(3.11)

The traction force $F_{x}^{r} = F_{rl}^{r} + F_{rr}^{r}$ is used as controlled variable for the control loop of the longitudinal speed as in the simulations on the single-track model. The lateral forces on the four wheels can be modelled with two different models, with distinct pros and cons:

- **Fiala tire model** presented in equations 2.33, which simply capture the behaviour of the tires, but presents a discontinuity and must be adapted for the usage on the four different wheels.

- **Pacejka tire model** explained in section 2.2.2, on the one hand has the pro of being ready to be used on the complete model of the vehicle and it is the most accurate tire model that is present in literature. On the other hand has a big drawbacks, with its accuracy introduces huge complexity and increase the computational effort during the simulations.

Evaluated all the different aspects of the two models the Fiala model has been chosen and implemented in order to compute the lateral forces $F_{fl}^{l}, F_{fr}^{l}, F_{rl}^{l}$ and $F_{rr}^{l}$. In order to do so must be computed four different side-slip angles of the tires (Eq.s 3.12-3.13) that takes in account the position of the considered tire inside the vehicle.

$$\alpha_{fl} = \arctan \frac{V_{y} + ar}{V_{x} - c_{f}r} - \delta$$

$$\alpha_{fr} = \arctan \frac{V_{y} + ar}{V_{x} + c_{f}r} - \delta$$

(3.12)

$$\alpha_{rl} = \arctan \frac{V_{y} - br}{V_{x} - c_{r}r}$$

$$\alpha_{rr} = \arctan \frac{V_{y} - br}{V_{x} + c_{r}r}$$

(3.13)
The Fiala’s force equation remain the same explained in section 2.2.1. Another modification is to be considered in the computations of the vertical load acting on each wheels, the load transfer due to the lateral and longitudinal accelerations is added as described in the following equations:

\[
F_{zl}^f = F_{zf}^{static} - dF_{zg}^{ay} - dF_{zg}^{ax} \\
F_{zr}^f = F_{zf}^{static} + dF_{zg}^{ay} - dF_{zg}^{ax} \\
F_{zl}^r = F_{zr}^{static} - dF_{zg}^{ay} + dF_{zg}^{ax} \\
F_{zr}^r = F_{zr}^{static} + dF_{zg}^{ay} + dF_{zg}^{ax}
\] (3.14) (3.15) (3.16) (3.17)

The different components are such that:

Static load:

\[
F_{zf}^{static} = \frac{mgb}{2l} \quad F_{zr}^{static} = \frac{mga}{2l}
\] (3.18)

Lateral load transfer:

\[
dF_{zg}^{ay} = \frac{mh_gb}{2lc_f} a_y \quad dF_{zg}^{ax} = \frac{mh_ga}{2lc_r} a_y
\] (3.19)

Longitudinal load transfer:

\[
dF_{zg}^{ax} = \frac{mh_ga}{2l} a_x
\] (3.20)

where the acceleration components are computed as:

\[
a_x = \frac{F_x}{m} \quad a_y = \frac{F_y}{m}
\] (3.21)

Simulations are then conducted with the described vehicle and tire models, and the results are carried out in 3.15. The red lines represents the references computed with the bicycle model, which resulted to be inaccurate but the pole placement controller still managed to stabilise the system. The full-track equilibrium is close to the bicycle equilibrium for the yaw rate and the side-slip angle (Fig.s 3.15a-3.15b), while is slightly different for the steering angle (Fig. 3.14c), which remains negative confirming that the vehicle is counter-steering in order to stabilise the drifting. Even if only little, also the longitudinal velocity and its force input (Fig.s 3.15d-3.15e) are different from the bicycle reference.

Due to the imprecision of the references has been necessary to remove the integral action from the speed controller, which kept trying to remove the error between the reference and the signal, forcing the system to leave the full vehicle equilibria. The absence of the integral action creates the oscillations, that has been removed during the simulations on the bicycle model, reappears here.

The lateral forces 3.16 presents an interesting behaviour, the forces on the left tires (Fig.s 3.16a and 3.16c) are slightly low than the forces on the right tires (Fig.s 3.16b and 3.16d). This is caused by the cornering manoeuvre, during a turning the centrifugal force makes the external tires more stressed than the internal one. In the case under analysis the vehicle is performing a left turn and, also in the case of a drifting manoeuvre, shows the described behaviour: The internal left tires are much less stressed than the right tires, which have a much greater force.
Figure 3.12: Results of simulations on linearised model
Figure 3.13: Results of simulations on bicycle model
3.3. Pole placement controller

Figure 3.14: Results of simulations on bicycle model and longitudinal velocity dynamics
Figure 3.15: Results of simulation on full-track model
Figure 3.16: Lateral forces resulting from the simulation on full-track model
Chapter 4

Experimental tests

4.1 Vehicle set-up

The core of the test vehicle in Figure 4.1 is the Radio Controlled (RC) car. The model chosen is Sakura D4, a 1:10 scale with Rear Wheel Drive (RWD) configuration, which is the optimal one for drifting. Inside the main chassis are mounted:

- **Lithium Polymer (LiPo) battery**, a 2 cell - 7.4V battery with a capacity of 4000mAh. This is the heaviest element and therefore is placed in the middle of the chassis for a perfect weight distribution.

- **Steering servo** applied on the front servo, is SC-1252MG (Fig. 4.2a) with a low profile dimension that makes it perfect for a small vehicle. It has a fast response of 0.09sec/60° at 4.8V and an applied torque of 0.628 Nm at 4.8V, that guarantee an high speed ideal for drifting operations.

- **Throttle motor** is Cheetah 1/10th60A10.5T Motor Combo (Fig. 4.2b), a synchronous three-phase brushless motor, which delivers low speed driveability
and high torque of 3250 RPM/V. The battery provides a constant current, so in order to provide the correct three-phase power an Electronic Speed Control (ESC) is used.

- **Radio command receiver**, the vehicle is remote controlled by FLYSKY RC 3 CH 2.4 GHz who sends Pulse with modulation (PWM) signals to the motor. The remote radio controller has three channels, the fist one is used to control the servo motor to provide the steer, the second commands the throttle and the third channel is used as a switch between manual and automatic mode.

### 4.1.1 Controllers

Over the chassis is mounted the control system, which reads all the data from the sensors, elaborates them and sends the commands to the motors (servo and throttle). The control system is composed by two elements, Odroid-XU4 [7] and Arduino Uno [2].

**Odroid-XU4** is the main on-board computer, it is a development platform with high computing power, open source support and the possibility to run any kind of Linux as Android Operating System (OS). The OS installed is a particular distribution of Ubuntu, called Ubuntu MATE 16.04 LTS, directly flashched on an eMMC 32GB memory that provides a faster access than standard microSD memories. Odroid connected to a potential dividers which delivers 5V by taking the power directly from the LiPo battery. The main specifications are:

- Processors: Samsung Exynos5422 ARM Cortex\textsuperscript{TM} A15 Quad 2.0GHz and Cortex\textsuperscript{TM} A7 Quad 1.4GHz
- Memory: 2Gbyte LPDDR3 RAM PoP (750 Mhz , 12GB/s memory bandwidth, 2x32bit bus)
4.1. Vehicle set-up

- 3D accelerator: Mali™-T628 MP6 OpenGL ES 3.1/3.0/2.0/1.1 and OpenCL 1.2 Full profile
- USB: 2 ports 3.0 - 1 port 2.0
- Storage: eMMC 5.0 Flash Storage (up to 64 GByte) - MicroSD Card Slot (up to 128 GByte)
- Fast Ethernet LAN: 10/100/1000Mbps Ethernet with RJ-45 Jack (Auto-MDIX support)
- WiFi: USB IEEE 802.11 ac/b/g/n 1T1R WLAN with Antenna (External USB adapter)
- Power input: 4.8V-5.2V
- Dimension: 83 × 58 × 22 mm
- Weight: 60 g including cooling fan

Odroid has not an integrated Wi-Fi antenna, thus an external one is needed. The model chosen is WT-AC9006, a compact driver with external antenna. It supports the newest Wi-Fi technology, the IEEE 802.11AC, which provides a Wi-Fi speed up to 433 Mbps (using 5GHz network) and 150 Mbps (with 2.4GHz). Thanks to this dual band capability it was possible to set a 5 GHz Wi-Fi connection to minimize any interference due to other wireless networks that usually work in 2.4GHz range.

**Arduino UNO** is the chosen micro-controlled board, it receives and elaborate signals from the radio controller in manual mode or from Odroid in automatic mode. Arduino sends the command signals to control the servos and read data from the encoders on the front wheels. Arduino is powered by Odroid and powers the radio receiver, the encoders and the steering servo. The main specifications are:

- Microcontroller: ATmega328 - 16 MHz
- Memory: Flash memory 32 KB - SRAM 2 KB - EEPROM 1KB
- Digital I/O Pins: 14 (6 PWM)
- Analog Input Pins: 6
- Power input: 7V-12V
- Dimension: 68.6 × 53.4 mm
- Weight: 25 g

4.1.2 Sensors

The vehicle is designed in order to implement a controller for drifting stabilization and a path tracking regulator, to fulfil those tasks needs different sensors to feel the physical environment. Once the data are read by the sensors, are sent to the controllers that elaborates the the right control input.
**Inertial Measurement Unit (IMU)** is an electronic device that incorporates different inertial sensors for the measures of accelerations, angular rate, orientation and magnetometers. The model used in the test vehicle is myAHRS+, which includes:

- Triple axis 16-bit gyroscope : ±2000 dps
- Triple axis 16-bit accelerometer : ±16 g
- Triple axis 13-bit magnetometer : ±1200µT

The orientation angles are computed with an embedded *Extended Kalman Filter (EKF)* and the estimated orientation is expressed using *Roll-Pitch-Yaw (RPY)* angles. The IMU has a microUSB port which allow a direct connection to Odroid.

**Rotary encoder** is an electric device which converts pulses information into angular speed of the wheel. On the car are mounted two of those devices in the two front wheels. Each rotary encoder is composed by a photo-transistor, an electronic switching that relies on exposure to light to operate, and a black and white strip which is stuck on the internal part of the wheel. The alternate colours create a difference in the luminescence that is read by the photo-transistor as a voltage step. By counting the changes in colour of the stripe over time the angular speed of the wheel can be detected.

This sensor allows to measure the heading speed of the vehicle with a small device, the chosen photo-transistor is HLC1395 design for short distance detection with high sensitivity. The resolution of the encoder depends on the number of teeth on the strip, the one chosen for the vehicle has 48 teeth, which means a resolution of 48 pulses/revolution (6.42 mm/pulse).

![Figure 4.3: Motive software for optitrack](image)
Optitrack [15] is a real time tracking system based on infrared cameras are organized around the periphery of the volume exposed. An object in order to be seen by the infrared cameras must have peculiar markers on it. Those markers are made of a particular reflective tape that mirrors infrared rays and allows the object to be tracked. Each camera captures 2D images, than the images are elaborated by by Motive, the proprietary software (Fig. 4.3), and the position data from different cameras are compared to compute the 3D position via triangulation. The software give as output position data (x-y-z) and orientation (roll-pitch-yaw) of the tracked object.

The test vehicle to be detected needs, at least, markers to create a valid rigid body, but for a precise tracking is better to have some redundant markers (4 or 5). In this way if the cameras do not detect, or lose a marker the created rigid body remains still valid. The overall system has a sub-millimetre accuracy that, along with the dedicated software, makes it the best choice in motion capture for robotic applications.

The optitrack obtain data from emitted or reflected light, to reduce disturbances, which includes sunlight, extraneous illumination or reflection surfaces, is suggested to minimize the ambient lighting and to properly calibrate the cameras. The camera calibration depends on different parameters:

- **LED**, it sets the brightness of the camera’s infrared LED ring. Higher LED values allow cameras to emit more infrared light, which can allow for better marker detection at longer ranges. However, a value that is too high may introduce reflections from non-marker objects. Generally, the value should be higher for larger volumes and lower for smaller objects.

- **EXP**, it controls how long the shutter remains open, per frame. Increasing the exposure value means to create brighter images that can increase visibility for small markers. However, setting the exposure too high can introduce merging of adjacent markers or false ones with consequent bad tracking data.

- **THR**, it determines a minimum brightness for a pixel to be seen by a camera. All pixels with a brightness below the threshold setting are ignored. On one hand increasing the limit value can help filter light interference from non-markers, while on the other hand lowering the threshold value can allow small markers to be seen by cameras.

- **FPS**, it determines the number of images a camera will capture per second. A higher FPS value will record more data within a given time. A lower FPS value will allow for higher exposure values for brighter images and will also reduce network traffic.
Table 4.1: RC car set-up parameters

<table>
<thead>
<tr>
<th>Measured Parameters</th>
<th>Value (SI Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>$m$ [Kg]</td>
</tr>
<tr>
<td>CG-Front axle distance</td>
<td>$a$ [m]</td>
</tr>
<tr>
<td>CG-Rear axle distance</td>
<td>$b$ [m]</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>$l$ [m]</td>
</tr>
<tr>
<td>Wheel outer radius</td>
<td>$R_w$ [m]</td>
</tr>
<tr>
<td>Vehicle front track</td>
<td>$c_f$ [m]</td>
</tr>
<tr>
<td>Vehicle rear track</td>
<td>$c_r$ [m]</td>
</tr>
<tr>
<td>CG height</td>
<td>$h_g$ [m]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Identified Parameters</th>
<th>Value (SI Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw moment of inertia</td>
<td>$J_z$ [Kg m$^2$]</td>
</tr>
</tbody>
</table>

4.1.3 Vehicle data

The element described in the previous sections are placed in the car as described in figure 4.1. Once the set-up has been defined the mass and the useful lengths has been measured, while the yaw moment of inertia has been identified in [4]. The data are presented in table 4.1. The wheelbase $l$ is the full length of the car, while $a$ and $b$ are the distances of the CG from the front and the rear axles, respectively. The vehicle track is the distance between the two wheels of the same axle. On the table can be noticed that there is a small difference between $c_f$ and $c_r$. The identified parameter represents the moment of inertia measured on the z-axis.

4.2 Identification procedure

Identification of different parameters or of the relations between input and output variables is a fundamental part of an automation and control project. In this work has been necessary to identify:

- the relationship between the radio command and the steering angle obtained as output (Sec. 4.2.1).

- the lateral forces and thus estimate the friction coefficients and the cornering stiffness on front and rear wheels (Sec. 4.2.2).

4.2.1 Steering angle identification

The servo, shown in 4.2a, can be controlled by variable amplitude pulses which determines the steering angle. In order to use the angle as input, it is necessary to map pulses with respect to the degree of rotation.
4.2. Identification procedure

During the test the \( \text{RC} \) car follows a circular trajectory with a constant heading speed and a variable steering angle \( \delta \). Thanks to the Optitrack \( x \) and \( y \) position are collected and used to fit a circumference and compute its radius. The circle equation is extrapolated from the position data using the Kasa fit method [20] which gives the centre and the radius of the circumference from a set of \( x \) and \( y \) data-point. Obtained the radius \( R \) and given the wheelbase of the vehicle \( l \) the steering angle can be computed as follows:

\[
\delta = \frac{l}{R} \quad (4.1)
\]

The test is repeated for all the pulse range, which is \([1240\, \mu s, 1740\, \mu s]\), at a constant straight speed of \( V_x = 1.22\, m/s \). Due to a limited space was not possible to complete a close circumference, like in 4.4a, for all the pulse range, but the circle has been computed anyway from the section realised (Fig. 4.4b).

### Experimental results

The steering angles are computed for each pulse using the wheelbase in table 4.1, and the results are presented in table 4.2. Then the data has been fitted over a linear polynomial curve using least absolute residual method (LAR) algorithm to improve the robustness of the identified relationship. Furthermore the first and the last four element has been excluded from the fitting procedure because in those regions the servo was saturated, that was no more able to increase the angle. In figure 4.5 is shown the identified linear relationship between the steer command (i.e. the pulse) and the steering angle \( \delta \). The equation of the fitted line is:

\[
y = 0.153x - 229.69 \quad (4.2)
\]

The goodness of the model has been measured using the coefficient of determination \( R^2 \). Let \( y_i \) be the experimental data and \( f_i \) the predicted ones, \( R^2 \) is defined as:

\[
R^2 = 1 - \frac{\sum_i(y_i - f_i)^2}{\sum_i(y_i - \bar{y})^2} = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}} \quad (4.3)
\]
Table 4.2: Results of circular motion test

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1240</td>
<td>-29.68</td>
<td>1500</td>
<td>0</td>
</tr>
<tr>
<td>1260</td>
<td>-28.75</td>
<td>1520</td>
<td>2.90</td>
</tr>
<tr>
<td>1280</td>
<td>-28.40</td>
<td>1540</td>
<td>5.73</td>
</tr>
<tr>
<td>1300</td>
<td>-28.45</td>
<td>1560</td>
<td>8.64</td>
</tr>
<tr>
<td>1320</td>
<td>-26.75</td>
<td>1580</td>
<td>11.72</td>
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<tr>
<td>1340</td>
<td>-24.84</td>
<td>1600</td>
<td>15.19</td>
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<td>1360</td>
<td>-22.05</td>
<td>1620</td>
<td>18.22</td>
</tr>
<tr>
<td>1380</td>
<td>-19.13</td>
<td>1640</td>
<td>21.25</td>
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<tr>
<td>1400</td>
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<td>1660</td>
<td>23.30</td>
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<tr>
<td>1420</td>
<td>-12.56</td>
<td>1680</td>
<td>24.10</td>
</tr>
<tr>
<td>1440</td>
<td>-8.95</td>
<td>1700</td>
<td>25.39</td>
</tr>
<tr>
<td>1460</td>
<td>-5.70</td>
<td>1720</td>
<td>26.75</td>
</tr>
<tr>
<td>1480</td>
<td>-2.55</td>
<td>1740</td>
<td>26.86</td>
</tr>
</tbody>
</table>

where $\bar{y}$ is the average value of the experimental data, $SS_{res}$ is the sum of squares residuals and $SS_{tot}$ is the total sum of squares. $R^2$ has values between $[0, 1]$ and express how good the identified model fits the data in comparison to the simple average, the closer is $R^2$ to 1 the better is the estimation. In this case (Eq. 4.2) the coefficient is $R^2 = 0.9987$, thus the computed model has an high accuracy.

Steering angle comparison

Consoli in his work identified the steering angle with two different tests, a circular test similar to the one presented here and a static test: where the RC motor was powered off and the steering angle was measured using millimeter paper and a goniometer. The results of those tests are presented in figure 4.6 and compared with the circular motion test conducted here. The results from the two circular tests are pretty similar and validates each other, the static test the same results for small values of $\delta$, while outside this range diverges because the friction forces acting on the wheels are lower than the ones existing in a dynamical situation like the circular test.

4.2.2 Tire model identification

In order to conduct the system analysis and the control design seen in chapter 3 with the model described in chapter 2, it is necessary to identify the parameter of the Fiala tire model (Eq. 2.33) from experimental data. The data are obtained from the circular motion test, the same used for the identification of the steering angle in section 4.2.1.

The parameter to be identified in order to use the Fiala tire model are the cornering stiffness and the friction coefficient for each tire. This is achieved through fits of empirical tire curves over the experimental data.
4.2. Identification procedure

Figure 4.5: Results of the steering identification procedure

Figure 4.6: Comparison of multiple steering identification tests
The simplest idea for conducting this identification would be to run a test with a quasi-steady state ramp manoeuvre. In this manoeuvre the front steer angle $\delta$ is slowly increased so that the vehicle can be considered to be in a steady state condition. But due to a lack of space was impossible to conduct this manoeuvre, therefore has been used the data from the circular motion test. In this way, instead of a single test with a slowly increasing steer, a sequence of different test are run with a constant $\delta$, which is different in each one of them. During the circular motion the vehicle reaches the steady state with constant inputs of steer and throttle.

In steady state condition the lateral acceleration $a_y$ can be approximated as in equation 4.4. The acceleration can be either measured with the optitrack or with the IMU, the two dataset are similar, as shown in Figure 4.7. Between the two measures the Optitrack acceleration data has been chosen to be used because it is less affected by noise, thus is more reliable.

$$a_y^{ss} = r V_x$$

(4.4)

At steady state the first two equation 2.11 of bicycle model becomes the expressions in equations 4.5 and 4.6.

$$F_y^f = \frac{mb}{(a + b) \cos \delta} V_x r = \frac{mb}{(a + b) \cos \delta} a_y^{ss}$$

(4.5)

$$F_y^r = \frac{ma}{a + b} V_x r = \frac{ma}{a + b} a_y^{ss}$$

(4.6)

The tire lateral forces (Eq 2.33) relies on the knowledge of the tire side-slip angles, which have to be computed from the inputs $\delta$ and $V_x$ and the steady state values of the state variables $r$ and $\beta$, using the equations 2.40. $\alpha_f$ and $\alpha_r$ are computed for the different values of delta correspondent to different circular test and the results...
4.2. Identification procedure

The relation between the tire side-slip and the steering angle as been approximated to a linear one and described by the equations 4.7 and 4.8, where the passage from the origin has been imposed to satisfy the physical behaviour.

\[
\alpha_f = -0.37\delta \tag{4.7}
\]

\[
\alpha_r = 0.47\delta \tag{4.8}
\]

Once the characteristics $\alpha$-$\delta$ has been derived, it is possible to generate experimental tyre curves, from the circular test data, like the one given in the figures No peak is evident in either tire curve, indicating that is possible to simplify the tire model for this test surface and validate the usage of the Fiala model instead of the more complex Pacejka tire model. The parameters to be estimated are the friction coefficients $\mu_f, \mu_r$ and the cornering stiffness $C_f, C_r$, where the subscripts indicates the front and the rear tire, respectively.

Front tire model identification: From the front empirical tire curve it is possible to identify the cornering stiffness $C_f$ and the friction coefficient $\mu_f$ by
Table 4.3: Estimated tire force parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front cornering stiffness $C_f$</td>
<td>$17.00$ [N/rad]</td>
</tr>
<tr>
<td>Rear cornering stiffness $C_r$</td>
<td>$17.81$ [N/rad]</td>
</tr>
<tr>
<td>Front friction coefficient $\mu_f$</td>
<td>$0.26$</td>
</tr>
<tr>
<td>Rear friction coefficient $\mu_r$</td>
<td>$0.22$</td>
</tr>
</tbody>
</table>

fitting the Fiala model to the front empirical data. The identification is implemented solving a non-linear least squares problem, which start from initial guesses and find the unknown parameters to best fit the non linear function, the Fiala model, to the experimental data. This identification is sufficient for the purposes of this work because the front wheel of the RC car are not driven, as a consequence the identified curve is valid up to and beyond the limit of handling. The yellow line in Figure 4.9a represents the Fiala model fit to the front experimental data.

**Rear tire model identification:** The cornering stiffness $C_r$ and the friction coefficient $\mu_r$ of the rear tire can be obtained using a fit to the rear experimental data (Fig. 4.9b). The rear friction coefficient was obtained under condition approximating pure cornering. Under those conditions a relatively small amount of longitudinal force is applied at the rear tire, while the high side-slip manoeuvre (i.e. the drifting) under consideration in this work involve significant amount of longitudinal forces at the rear tire. For this reason the effective friction coefficient, during a drifting manoeuvre, could be smaller than the one computed, but the value of $\mu_r$ can be assumed to be close to the effective friction coefficient. By seeing the value of $\mu_r$ in table 4.3, which is really small, the above assumption became acceptable.

### 4.3 Side slip angle estimation

The side-slip angle during the simulation tests has been obtained using the definition:

$$\beta = \arctan \frac{V_y}{V_x}$$  \hspace{1cm} (4.9)

Where the velocities $V_x, V_y$ has been derived from the position measured with the optitrack. This definition limits the use use of the car to the AIRLab, the laboratory where the optitrack is mounted. To extend the utilisation of the vehicle, in automatic mode, to different places $V_y$ in equation 4.9 cannot be directly measured to compute $\beta$. Thus the implementation of an estimation technique for the side-slip angle turn out to be necessary.

The test vehicle is provided with different sensor, as explained in 4.1, and these instrumentation can be used to estimate the side-slip angle. The lateral speed $V_y$ dynamical equation, recalled from 2, is:

$$\dot{V}_y = \frac{F_y}{m} - rV_x$$  \hspace{1cm} (4.10)
4.3. Side slip angle estimation

Where $F_y$ cannot be computed without the measurement of $V_y$ or $\beta$, but using the second Newton’s law, can be noted that:

$$\frac{F_y}{m} = a_y \quad (4.11)$$

The lateral acceleration $a_y$ can be measured without the optitrack and this allow to completely describe the lateral speed derivative $\dot{V}_y$ with the sensor data.

- The lateral acceleration $a_y$ measured from the IMU
- Yaw rate $r$ also obtained from the IMU
- Longitudinal velocity $V_x$ determined using the wheels encoders

Computed the derivative, $V_y$ is obtained via numerical integration and then the calculation of the side-slip angle is trivial, thanks to the 4.9. The problem connected with this algorithm is the computing of $V_y$ which, due to the noise errors, is subjected to data drifting during the integral operation. This is a common problem connected to signals affected by measurement noise, and can be solved in two ways:
• Smoothing the measured signals and, after an analysis of the spectrum of $\dot{V}_y$, filtering out the unwanted noise frequency, typically at lower frequency than the signal.

• Also with the best filters the estimate could drift after a certain time, then the only solution to the small drifting became the resetting of the integral action when the error gets too high.

In the considered case, the spectrum of the lateral speed derivative has been analysed and because it presented noise at high frequencies a low pass filter has been designed. The one sided spectrum of $\dot{V}_y$ in Figure 4.11 has the main signal under 0.05 in terms of normalised frequency. This corresponds to 2.5 Hz in absolute values of frequency.

In order to eliminate all the noises at frequencies above the one of the signal has been designed a low pass filter of the second order, with the parameters in Equation 4.12.

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^2 + b(2)s + b(3)}{a(1)s^2 + a(2)s + a(3)}$$

$$= \frac{0.01s + 5.5 \times 10^{-3}}{s^2 - 1.78s + 0.8}$$

The spectrum of the filtered signal is the red one in Figure 4.11. The main band of the signal is in the range between 0 and 0.05 ([0, 2.5] Hz). The estimate has been tested on a test in which the vehicle has been manually guided and the computed $V_y$ is presented in 4.12a and in perfectly overlap the signal measured with the optitrack. After the integration the $V_y$ is displayed in Figure 4.12b and the 4.13 image shows the estimated side-slip. $\beta$ presented some clear errors in the first 8 seconds due to
small values of $V_x$, which inside the definition 4.9 tended to create infeasible values of the side-slip angle estimate. To solve this problem has been setted a threshold between $V_x = \pm 0.01$, inside this range the estimate of $\beta$ is forced to zero, as in figure 4.13.

For the sake of completes, it must be said that this simple estimator is still affected by drift errors of the integrator in correspondence of harsh manoeuvre of the car. Thus, it is necessary to improve the robustness of this algorithm by the implementation of the high pass filter discussed above or use of the dynamic model of the car to build a state observer.

![Figure 4.12](image1.png)
(a) Estimated lateral speed derivative

![Figure 4.13](image2.png)
(b) Estimated lateral velocity

Figure 4.12

Figure 4.13: Estimated side-slip angle
Conclusions

This thesis addressed the study high side-slip manoeuvre and a drifting controller has been developed for a RC car. Moreover on the test car has been identified the characteristic of the steering angle and the lateral forces in order to estimate the friction parameters. Has been necessary to estimate the side-slip angle in order to apply the implemented controller on the car.

The dynamic model of the vehicle has been presented in chapter 2 and used in sections 3.1-3.2, where the cornering equilibria has been computed and analysed. The stability of drifting equilibria, characterized by high values of the side-slip angles, has been discussed and on the linearised system has been implemented a drifting controller.

The regulator has been designed using the Pole placement algorithm proposed in [13] and described in 2.3.1. The controller has been tested first on the linearised model, than on different models with increasing complexity. The results of the simulations on the complete vehicle model have been that the controller is able to stabilize the system, but the definition of the controller on a simplified model get errors in the complete vehicle model. Those error were small enough to be considered negligible.

In section 4.1 the set-up of the vehicle has been described and in the next sections of chapter 4 were developed some tests for identify and estimate different parameters. Using the circular motion test data the cornering stiffness and the friction coefficient of the tires has been estimated with the Maximum likelihood method [3]. Then a simple observer has been developed for the side-slip angle and has been discussed its robustness.

In conclusion the two-DOF model is suitable for control purposes, but in the drifting case shown to have a really small region of attraction due to an high coupling between the longitudinal velocity and the lateral dynamics.

For future works it is suggested to use for the controller design a more complex three-DOF vehicle model, that takes in account both the lateral and the longitudinal dynamics. This is also advisable from the experience of professional driver, who use both the steer and acceleration command to obtain and control the drifting manoeuvre.
## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ABS</td>
<td>Anti-lock braking system</td>
</tr>
<tr>
<td>ESC</td>
<td>Electronic stability control</td>
</tr>
<tr>
<td>WHO</td>
<td>World health organization</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned autonomous vehicle</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>CG</td>
<td>Centre of Gravity</td>
</tr>
<tr>
<td>COP</td>
<td>Centre of Percussion</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
</tr>
<tr>
<td>RWD</td>
<td>Rear Wheel Drive</td>
</tr>
<tr>
<td>AWD</td>
<td>All-Wheel Drive</td>
</tr>
<tr>
<td>UN</td>
<td>Under-steering</td>
</tr>
<tr>
<td>OV</td>
<td>Over-steering</td>
</tr>
<tr>
<td>TB</td>
<td>Trail-Braking</td>
</tr>
<tr>
<td>NMPC</td>
<td>non-linear model predictive control</td>
</tr>
<tr>
<td>LTV</td>
<td>Linear time varying</td>
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<tr>
<td>DP</td>
<td>Dynamic programming</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional Integral controller</td>
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<tr>
<td>PWM</td>
<td>Power with modulation</td>
</tr>
<tr>
<td>RC</td>
<td>Radio Controlled</td>
</tr>
<tr>
<td>LiPo</td>
<td>Lithium Polymer</td>
</tr>
<tr>
<td>RPM</td>
<td>Revolutions per minute</td>
</tr>
<tr>
<td>ESC</td>
<td>Electronic Speed Control</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<td>---------</td>
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</tr>
<tr>
<td>PWM</td>
<td>Pulse with modulation</td>
</tr>
<tr>
<td>OS</td>
<td>Operating System</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
</tr>
<tr>
<td>RPY</td>
<td>Roll-Pitch-Yaw</td>
</tr>
<tr>
<td>LAR</td>
<td>least absolute residual method</td>
</tr>
<tr>
<td>SAE</td>
<td>Society of Automotive Engineers</td>
</tr>
<tr>
<td>DOF</td>
<td>degree-of-freedom</td>
</tr>
<tr>
<td>AIRLab</td>
<td>Artificial Intelligence and Robotics Laboratory</td>
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Bibliography


