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**An econometric analysis of the CDS
spreads written in different currencies
by Linear and Thresholds VECM**

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To uncle Samy...

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Chapter 1

General Introduction

Credit default swaps were engineered in 1994 by the US bank JP Morgan to transfer credit risk exposure from its balance sheet to protection sellers. At that time, hardly anyone could have imagined the extent to which CDS would occupy the daily lives of traders, regulators, and financial economists alike in the twenty-first century. As of this writing, more than one thousand working papers posted on the Social Science Research Network are directly related to the economic role of CDS or involve CDS as a research tool in one way or another. Nevertheless, some key issues on CDS are still hotly debated. [8]

1.1 General Framework

Credit Default Spreads (CDSs) are the most used derivative of the credit risk market. Credit risk is the risk of a credit event, i.e., that an obligor does not honour its obligations. A credit derivative is an instrument which pay-off is conditioned on the occurrence of a credit event.

For some large debtors to issue debts in more than one currency may be advantageous. Furthermore, thanks to the liquid market of CDS there is the possibility to trade credit protection in currencies different from the obligors local currency, even if the obligor has not issued bonds in that currency. In particular, CDS protection on many international corporations is now available in their home currency, Euro and US dollars.

CDS issued in different currencies are designed mainly to provide protection upon default of a certain entity in a given currency: for example, to avoid the Exchange Rate risk linked to the devaluation effect associated to the reference entity's default. In fact, if one of the largest US banks is near to default, an insurance payout in US dollars would likely be much less attractive than a payout denominated in euros. Alternatively, CDS can also give a trader in the credit risk market a tool to hedge risks or to enhance profits by his own vision or by exploiting a particular monetary policy. Finally, protection might be needed in a different currency from the one in which the assets of the reference entity are denominated because it serves as a hedge on a security denominated in that specific currency.

1.2 Contribute of the thesis to the literature on CDS

Although research on CDS has grown tremendously, there are gaps that offer fruitful studies, and one of these gaps is an econometric analysis of the relationship between exchange rate and corporate Credit Default Swaps.

The existence of CDSs on the same reference entity which premium and protection cash-flows are paid in different currencies can create a difference between the par-spreads of these contracts. Given this situation, it is natural to ask about the correct pricing of the credit risk in the different currencies, i.e., how credit spread is adjusted on CDS if a different currency is used? It is indeed observed that CDS written on the same underlying but with different currencies, once reported the price in the same currency, show differences for long periods of time.

This thesis analyzes empirically the persistence and significance of those differences, the equations to model them and the dynamics of the inclusion of new information in the markets with different currencies. This analysis is performed on a dataset that goes from December 2007 to December 2017. In this way it is possible to give results not only on a long interval of time, but also to observe the behaviour of these differences in price across different economic conditions and monetary policies.

The econometric approach allows to study the CDS prices time series in different currency without being constrained by specification of a particular pricing model. In particular, I performed co-integration analysis to understand if the difference is significative and if the prices in different currencies move similarly; then, I applied, if possible, the linear and/or the threshold error correction model to describe the spreads' behaviour and to study which market leads the price discovery ¹.

The innovative contribution of my work is not only to study the behaviour of corporate CDSs written in Euro and US Dollars, but also the usage of a recent and powerful model as the Threshold Vector Error Correction Model (TVECM), allowing the introduction of a more realistic non linearity. In particular, it allows the estimation of different models based on the sizes or the sign of the differences in the CDS prices issued in different currencies. This means a more realistic, but still parsimonious, model and the possibility to estimate transaction costs and to highlight arbitrage opportunities. Indeed, during financial crisis, arbitrageurs took advantage of large differences between sovereign CDSs issued in different currencies thanks to new-born instruments named Quanto CDS, i.e., a CDS in which the swap premium payments, and/or the cash-flows in the case of default are not in the same currency. It will be interesting to understand if this possibility arises also in the corporate market.

¹Price Discovery is defined by Lehmann [47] to be the efficient and timely incorporation of the information implicit in investor trading into market prices.

1.3 Research Framework

From a theoretical point of view, the issue was tackled by Philippe Ehlers and Philipp Schonbucher in [28]. They analyze the connections between local and foreign currency credit spreads on a theoretical basis in an intensity-based framework. They find that there is a persistent, significant and rather large difference between CDS rates in Yen and US dollars which cannot be explained by a purely diffusion-based dependency between default intensity and FX rate alone. Thus, they conclude that the market should price an implicit devaluation at default into these CDS spreads. The essential feature driving differences between credit spreads in different currencies is the dependency between default risk and FX risk. In order to capture empirically observed differences, they model dependency between spreads in two different ways. First, there may be correlation between the diffusion driving FX and default intensities, and second, an additional jump in the exchange rate may occur at the time of default, i.e., the default causes a devaluation of the currency.

In [18] Carr and Wu studied the role of currency volatility, developing a joint valuation framework for sovereign CDS and currency options of Mexico and Brazil. Strong positive contemporaneous correlations between CDS spreads and both the foreign options delta-neutral straddle implied volatilities and risk reversals imply that economic or political instability leads to both higher sovereign credit risk and currency return volatility. In [43] Hui and Chung document information flow from the sovereign CDS spreads of 11 EU countries to the dollars-Euro currency option prices during adverse market conditions. Furthermore, the sovereign spreads forecast well the implied volatility of dollars-Euro currency options, and leading for even better results for deep out-of-the-money options. Hui and Fong report in [44] an evidence of information flows from the sovereign CDS market to the dollars-yen currency option market during the sovereign debt crisis from September 2009 to August 2011. Finally, Pu and Zhang [57] show that the Euro-dollars exchange rate returns up to a period of ten days can be predicted by differences between US dollars and Euro denominated sovereign CDS spreads (quanto-spread) for 10 EU countries.

1.4 Structure of the thesis

This thesis is structured as follows:

The second chapter describes the main characteristics of Credit Default Swaps: the definition of the CDS contract, the regulation of its market, the main players and the market size, pricing formula and the premia that are implied in the CDS prices. The third chapter presents and motivates the choice of the dataset. The fourth chapter exposes the methodology used in this thesis. The fifth chapter shows the outcomes of my work. Finally, the sixth chapter concludes the thesis.

Chapter 2

Introduction to Credit Default Swaps

*"These increasingly complex financial instruments have contributed, especially over the recent stressful period, to the development of a far more flexible, efficient, and hence resilient financial system than existed just a quarter-century ago"*¹

*"In our view, however, derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal"*²

The first CDS contract was built in 1994, when JP Morgan off-loaded its credit risk exposure to Exxon by paying a fee to the European Bank for Reconstruction and Development, which was willing to sell protection.

CDS represent the simplest (plain vanilla) and the most widely used instrument among the broad class of credit derivatives.

2.1 Definition of CDS

A CDS [7][8] is a fixed income over the counter (OTC) derivative instrument. It is a fixed income since its payments are done at regular intervals and at predictable levels. It belongs to the OTC securities because it is not traded on a formal exchange, but through a dealer network or directly between the counterparties. The protection buyer purchases insurance against a contingent credit event on an underlying reference entity, typically a company or a state bond, by paying a quarterly annuity premium to the protection seller, generally referred to as the CDS spread, over the life of the contract. Typically the actual/360 day count convention is used.

The issuer agrees to replace the loss that the lender would incur upon a credit event of the reference entity to the buyer if the default has happened. If there is no default the issuer pays nothing.

¹ 'Economic Flexibility', Alan Greenspan, Speech given to Her Majesty's Treasury Enterprise Conference, London, January 26, 2004

²Buffett in Berkshire Hathaway's (BRK-A, BRK-B) 2002 annual letter.

If the default happens between two fee payment dates the protection seller has to pay also the accrue fraction until the time of default; this means to consider the time passed between the last payment date and the default as a fraction of the regular interval between two payments, and multiply this fraction by the spread.

In practice it is necessary to specify a set of reference assets, usually bonds of a given seniority class. They are necessary to: determine default events, specify the set of derivable assets in case of physical delivery, determine the basis of the price and recovery in case of cash settlement.

2.1.1 Difference between CDS and insurances

The concept of a CDS is similar to an insurance. The most important difference [22] is that an insurance contract provides an indemnity against the losses actually suffered by the policy holder on an asset in which it holds an insurable interest. By contrast a CDS provides an equal payout to all holders, calculated using an agreed, market-wide method. The holder does not need to own the underlying security and does not even have to suffer a loss from the default event. The CDS can therefore be used to speculate on debt objects.

Another difference is that the seller is not required to maintain reserves to cover the protection sold, they manage risk primarily by other CDS deals or the underlying bond markets.

Finally, to cancel the insurance contract the buyer can typically stop paying premiums, while for CDS the contract needs to be unwound.

2.1.2 The settlement of CDS contract

A credit event triggers a payment by the protection seller equal to the loss given default (LGD), that is the difference between the notional principal and the value of the underlying reference obligation [58].

The settlement of CDS contracts may occur in two ways: cash settlement or physical delivery. In the first case, the monetary exchange is the notional amount minus the post defaulted market value of a reference asset. In the latter case, the claimant transfers an obligation, among a set of deliverable reference obligations agreed in the contract, and receives the full notional amount of the underlying contract in return. Conceptually, this is similar to a put option seller, who receives the underlying asset upon exercise.

Cash settlement determinations is complex therefore often the physical one is preferred. The cash settlement is convenient only if there is no asset to deliver, for example when the reference entity has not issued enough bonds. A problem of the physical settlement is the artificial raise of the price due the great demand that can follow a default.

2.1.3 CDS Term structure

According to the classical Merton framework [50], the term structure of spreads should be upward sloping for high-quality credits, hump-shaped for medium-quality credits, and downward sloping for low creditworthiness. In [53] the authors noticed how the uneven liquidity across contract maturities makes more difficult the studying of the CDS term structure; CDS contracts are usually most liquid in the middle of the maturity spectrum, i.e., five-year contracts are by far

the most liquid contracts [36][13]. This is particularly true for corporate reference names, while liquidity in the sovereign market is comparatively much more balanced.

2.2 Regulation

In many cases, the liquidity of the CDS market has surpassed the liquidity of the market for the bonds of the underlying obligor [22][7]. This trading volume and liquidity has been made possible by the standardization of the documentation for CDS transactions which has been proposed by the International Swap Dealers Association (ISDA).

The contract was first standardized in 1992 by the ISDA Master Agreement, and then updated in 2002. In this way the ISDA prompted the growth of this market. Credit derivatives agreements are guided by the 2003 and 2014 ISDA Credit Derivatives Definitions. After the crisis, a regulatory overhaul has been implemented in US and EU. First the CDS 'Big Bang' and 'Small Bang', for the American and European CDS markets, came respectively on April 8, 2009 and June 20, 2009, which pushed for further standardization of the contract; then came a ban on naked³ CDS by the EU in 2011. Also Basel III and the Dodd-Frank Act acted on CDS. From 2013 in the US participants have to use the central counterparties (CCPs) and new trading platforms.

CDS Big Bang and CDS Small Bang protocols were focused on improving the efficiency and transparency of the CDS market, in particular on the standardization of the coupon payments. The fixed coupon payments were defined to be either 100 or 500 basis points, whereby any difference relative to the running par spread would be settled through an upfront payment. Another important change in the US CDS market was the exclusion of restructuring as a standard credit event in the contractual CDS clauses.

In the 2014 Definitions new credit events, were introduced as governmental intervention to bail out the financial entity and in case of bond exchanges. Moreover, senior CDS are triggered based solely on whether the senior bonds of the entity are restructured. CDS contracts become worthless following corporate reorganizations, takeovers, or IPOs, and they are named 'orphaned CDS'. To reduce the risk of orphaned CDS, ISDA has introduced the concept of universal successor to recognize the succession event when debt is transferred but identified outside the 90-day succession backstop window.

In particular when it comes to the analysis of the default risk of any given obligor in 2 or more different currencies (and thus in two different jurisdictions) this standardization is essential: bonds in domestic and foreign currency are typically issued in different jurisdictions and therefore are governed by different legal rules which has a significant impact on the resulting recovery rates of the bonds. CDS referencing the obligor on the other hand will be governed by the same standardized ISDA documentation even if they are denominated in different currencies, in particular they will have the same recovery rates. Thus, spread differences in CDS rates cannot be caused by the effects of different legal regimes and bond specification which may affect corporate bond data.

³A naked position refers to having a position in the CDS without having any exposure to the underlying reference entity. The position is said to be uncovered, or naked.

2.3 Credit Event

A credit event happens when a company failed to meet its obligations for any reference assets; default would be triggered and the payout would occur. Amid the class of qualifying credit events there are [58]:

1. bankruptcy (only for corporate);
2. failure to pay: failure of the reference entity to make, when and where due, any payments under one or more obligations. Grace periods for payment are taken into account;
3. obligation default: technical default due to provisions in the agreement, as violation of the covenants;
4. obligation acceleration: obligation that goes due and payable before its normal expiration;
5. repudiation or moratorium (for sovereign entities): compensation delayed after specified actions of a government;
6. restructuring: reduction and renegotiation of delinquent debts in order to improve or reinstate liquidity.

While the first one regards the reference obligor itself, the others are defined only with respect to the reference obligations. For restructuring thresholds and grace periods are imposed, to exclude technical glitches and minor legal disputes from the definition of default. When mergers or acquisitions occur gradually in stages, it has been introduced a 'Steps Plan' to determine successors based on a series of successions to reference entities or their obligations.

2.3.1 Restructuring Clauses

In a CDS contract there can be different types of restructuring clauses [7]. Full Restructuring (CR) was the only clause until 2001, which stipulates that any obligation with maturity of up to 30 years can be delivered. Concern raises when protection buyers obtained additional benefits by delivering the least valuable bond in the restructuring of Conseco Finance on September 22, 2000. As a consequence, in 2001 ISDA included the Modified Restructuring (MR) credit event clause for CDS contracts; the deliverable obligations are those with maturities within 30 months of the CDS remaining maturity. Finally, in March 2003 ISDA introduced also the Modified-Modified Restructuring (MMR) clause; the deliverable obligations are bonds with maturities of up to 60 months within the CDS contract's remaining maturity for restructured debt, and 30 months for other obligations. In the case the restructuring credit event is removed altogether, the clause is labeled No Restructuring (XR).

2.3.2 CDS Auctions

Big Bang and Small Bang protocols moved towards a systematic cash settlement in the contractual CDS agreements, whereby the final settlement price would be determined through an auction mechanism [8][22]. One of the key reasons was the risk of 'market squeezes', when the net notional amount outstanding of CDS would exceed the quantity of deliverable cash bonds.

ISDA, Markit, and CreditEx designed and administer a two-stage auction process. In the first round the initial market midpoint and the net open interest are determined; the participants submit bid and ask prices within 2% of par value, and the physical settlement requests to buy or sell bonds. In the second stage, the final price can not deviate more than a pre-specified quantity from the initial market midpoint and it is determined by the first-stage market orders, new orders submitted by the dealers, and the net open interest determined in the first stage. This two-stage process has been designed to control the physical settlement requests and limit the manipulation of bidding agents that may themselves have outstanding positions in the CDS market.

The existence of underpricing (overpricing) in auctions with positive (negative) net open interest is empirically observed. Chernov [21] found an underpricing of 6%, using the difference between the bond price in the auction and the one in the OTC market on the day of the auction. The authors also document a drop in bond prices of about 25% over the ten days before the auction. In [35] the authors argue that this pricing inefficiency may give rise to apparent arbitrage opportunities.

2.4 The market of Credit Default Swaps

2.4.1 CDS market size

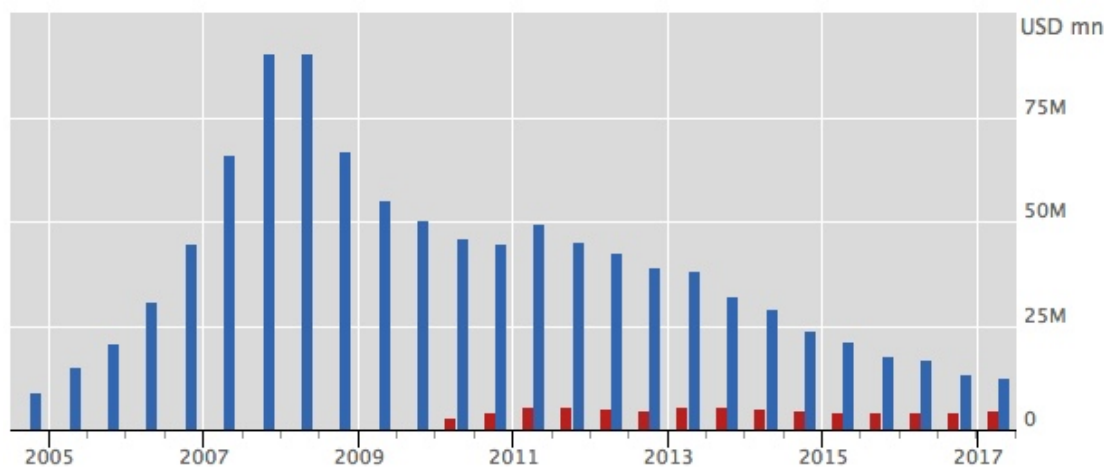


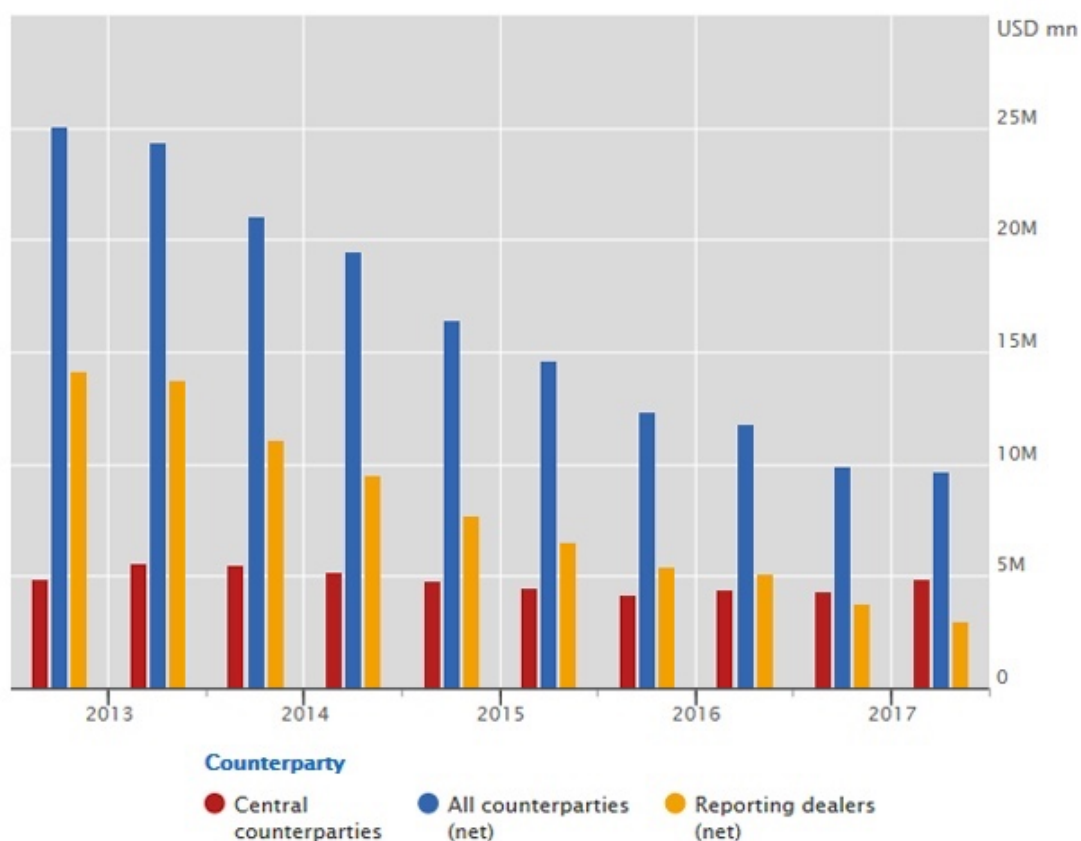
Figure 2.1: CDS history: in blue gross notional outstanding, in red the cleared segment.

In 1997 the CDS market had a small gross notional amounts outstanding, in the order of 180 billion dollars. An exponential growth started from the early 2000s up to the financial crisis, first of all because ISDA published a new set of standardized CDS contract definitions in 2003 and because in 2004 it started a huge trading of a broader class of credit derivative index products, including synthetic collateralized debt obligations (CDO), for which CDS contracts are a crucial element. At the end of 2004, the total gross notional amount of CDS outstanding was roughly

6 trillion dollars. The market witnessed three-digit growth rates and reached about 60 trillion dollars just prior to the onset of the financial crisis in 2007.

The size of gross notional amounts of CDS outstanding dropped considerably after the 2008 crisis, due to the centrality of CDS in the credit crisis and the regulators' concerns about central clearing and counterparty risk which led to significant portfolio compressions. Notional amounts for single-name CDS have also fallen to about 13 trillion dollars in 2013 and 10 trillion dollars in 2017. Overall, the CDS market remains highly sizeable and proves to be robust to the financial crisis.

Regarding to the cleared segment (red bars) rose from \$4.3 trillion to \$4.9 trillion in the first half



Outstanding notional amount of CDS, in trillions (ie in million millions, eg 10M equals 10 million millions, or 10 trillion) of US dollars | Source: BIS OTC derivatives statistics

Figure 2.2: 5 years, net CDS, CCP and reporting dealers net amounts.

of 2017, even as the total notional amount of outstanding CDS declined slightly. Consequently, the share of outstanding CDS cleared through central counterparties (CCPs) jumped from 44% at end-December 2016 to 51% at end-June 2017. Bilateral contracts between reporting dealers declined further in the first half of 2017, to \$2.9 trillion.

Most CDS contracts reference assets with S&P credit ratings ranging between A and BBB, a smallest category goes from AAA to AA.

2.4.2 Market players

Initially, insurance companies were the main CDS protection sellers while commercial banks were the main buyers. Since CDS are insurance-like contracts, it is natural for insurance companies to be market facilitators and participants in this market. They tended to sell naked credit protection, and were severely affected during the global financial crisis.

However, since the crisis, insurance companies appear to have been net buyers of protection, as of December 2013 according to a BIS survey, using CDS to hedge their bond portfolio holdings. It could be argued that banks have become even more 'too big to fail' and are net sellers, rather than buyers.

Hedge funds have increased their participation in both sides of the market. In [61] Siriwardane documents that hedge funds become net protection sellers with aggregate positions that are five times of insurance companies' positions. Besides regulatory capital relief and hedging opportunities, relaxing collateral constraints can be motivations of this exposure. Mutual funds' penetration into the CDS market has been gradual. However, they are increasingly using CDS to either hedge their credit risk or synthetically take on credit risk exposures. In particular, bond funds recently became active in the CDS market [3].

Using data from OCC's Quarterly Report on Bank Trading and Derivatives Activities, Atkeson et al. in [6] find that the CDS market is highly concentrated, with only a small number of financial institutions acting as market makers. They find that fixed entry costs, trading frictions, and the benefits of netting explain the high concentration in this market, whereby large banks act as dealers, and medium-sized banks act as customers. In [54] Peltonen et al. test the network structure of the CDS market using Depository Trust and Clearing Corporation (DTCC) data on bilateral CDS exposures on 642 sovereign and financial reference entities in 2011. They find that the CDS market is highly concentrated around 14 dealers, which suggests that the market is 'robust but fragile'. The failure of any one single dealer may impose significant contagion effects and create systemic risk.

2.5 Pricing formula

2.5.1 Set-up

Consider the filtered probability space $(\Omega, \mathbb{F}, F_t, \mathbb{P})$, with a finite time horizon $[0, T]$ and the usual assumption $\mathcal{F}_T = \mathbb{F}$.

Consider furthermore the following assumptions:

1. No default risk in the derivative's counterpart.
2. Independence between default and rates.
3. The payments can happen only on a discrete set of dates $(t_{n-1}, t_n]_{n=0, \dots, T}$.
4. If default occurs between $(t_{n-1}, t_n]$, each related payment is done in t_n .

5. The recovery is assumed constant.

Consider finally the following notation:

- R : Recovery value.
- τ : Time to default.
- $\delta(t, T)$: Period of time between t and T computed by the correct convention.
- $\mathbb{I}_{\tau > T}$: Survival indicator function.
- $P(t, T) = \mathbb{E}[\mathbb{I}_{\tau > T} | \mathcal{F}_t]$: Survival probability between t and T .
- $D(t, T) = e^{-\int_t^T r_s ds}$: Stochastic discount factor, where r is the instantaneous stochastic spot rate.
- $B(t, T) = \mathbb{E}[D(t, T) | \mathcal{F}_t]$: Default-free zero coupon bond.
- $\overline{B}_0(t, T) = \mathbb{E}[D(t, T) \mathbb{I}_{\tau > T} | \mathcal{F}_t]$: Defaultable zero coupon bond with no recovery.
- $e(t, T, T + \Delta t) = \mathbb{E}[D(t, T + \Delta t)(\mathbb{I}_{\tau > T} - \mathbb{I}_{\tau > T + \Delta t}) + \Delta t | \mathcal{F}_t]$: Value of 1 in $T + \Delta t$ if it happened a default in $(T, T + \Delta t)$.

Thanks to these hypothesis it is possible to obtain these two trivial results:

$$e(t, T, T + \Delta t) = B(t, T + \Delta) = [P(t, T) - P(t, T + \Delta t)], \quad (2.1)$$

$$\overline{B}_0(t, T) = \begin{cases} B(t, T)P(t, T), & \text{if } R = 0, \\ B(t, T)[P(t, T) + R(1 - P(t, T))], & \text{if } R \neq 0. \end{cases} \quad (2.2)$$

2.5.2 CDS Pricing

Considering the theoretical standard case that at the initiation of a CDS contract there is no exchange of cash flows between the two parties to the transaction, the CDS spread can be found imposing that the expected value of the fee leg equals the expected value of the contingent leg at the initial time t_0 . The fee leg can be priced as the discounted quarterly payments to the protection seller plus the accrued fraction payed when the default happens between two payment dates. The quarterly payments are modeled as the expected value of the sum across all the payment dates of the credit spread intensity multiplied by the interval between two payment dates (to obtain the credit spread along this period of time), discounted by the stochastic discount factor and multiplied by the survival indicator function; indeed the fee payment is due only if the underlying has not defaulted [58]:

$$\begin{aligned} \mathbb{E}\left[\sum_{n=1}^N \overline{s} D(t_0, t_n) \delta(t_{n-1} - t_n) \mathbb{I}_{\tau > t_n}\right] &= \overline{s} \sum_{n=1}^N \delta(t_{n-1} - t_n) \mathbb{E}[D(t_0, t_n) \mathbb{I}_{\tau > t_n}] = \\ &= \sum_{n=1}^N \overline{s} \overline{B}_0(t_0, t_n) \delta(t_{n-1} - t_n). \end{aligned}$$

The accrued fraction in case of default between two fee dates can be modeled in the same way, except for the substitution of the survival indicator function by the difference between the

survival indicator functions in two consecutive payment dates; indeed, it is checked if the default happens between these two dates. Anyways, the accrued time is often approximated with half of the period between two payment dates:

$$\bar{s} \sum_{n=1}^N \frac{\delta(t_{n-1} - t_n)}{2} \mathbb{E}[D(t_0, t_n)(\mathbb{I}_{\tau > t_{n-1}} - \mathbb{I}_{\tau > t_n})] = \sum_{n=1}^N \bar{s} \frac{\delta(t_{n-1} - t_n)}{2} e(t_0, t_{n-1}, t_n).$$

Following the same procedure, the contingent leg is the expected value of the sum of loss give default, i.e. the notional minus the recovery, discounted by the stochastic factor and multiplied by the difference between the survival indicator functions in two consecutive payment dates, because again it considers it only if the default happens between these two dates:

$$(1 - R) \sum_{n=1}^N \mathbb{E}[D(t_0, t_n)(\mathbb{I}_{\tau > t_{n-1}} - \mathbb{I}_{\tau > t_n})] = (1 - R) \sum_{n=1}^N e(t_0, t_{n-1}, t_n).$$

Finally the credit spread can be found as:

$$\bar{s} = \frac{(1 - R) \sum_{n=1}^N e(t_0, t_{n-1}, t_n)}{\sum_{n=1}^N \frac{\delta(t_{n-1} - t_n)}{2} e(t_0, t_{n-1}, t_n) + \sum_{n=1}^N B_0(t_0, t_n) \delta(t_{n-1} - t_n)}. \quad (2.3)$$

2.5.3 Calibration of the default probabilities or recoveries

Firm value models and intensity based models are really useful model-based pricing because they allow the measurement and use of default probabilities or recoveries.

Firm value or structural models

The structural approach to credit risk pricing is influenced by the Black and Scholes [12] and Merton [50] arbitrage pricing framework. In models of this type, the value of a firm's assets is assumed to evolve randomly over time, and is typically modeled by a stochastic process such as a geometric Brownian motion. A firm is assumed to default when its asset value falls below the default boundary. In structural models, credit spreads are determined mostly by leverage, asset volatility, and market conditions such as interest rates, which are suggested by the underlying theory.

The reason for which I do not go in deep in firm value models is that they do not fit my needs. Indeed, the main advantages of the firm value models are the possibility to price convertible bonds, link equity-debt instruments and analyze corporate finance issues; however, several papers find that structural models do a poor job in empirically explaining the magnitude of credit spreads, a result commonly referred to as the credit spread puzzle; finally, the calibration is not easy as in the intensity based models.

Reduced-form or intensity-based models

Assume that the default time for a firm is unpredictable, and that it follows a Poisson process with two characteristics:

1. $P(t, t + \Delta t) = \lambda(t)\Delta t$, where $\lambda(t)$ is the intensity; i.e. the probability of a default in a certain time interval is directly proportional to the time interval and the intensity it is their proportionality constant.
2. The probability of default is independent from the past.

Using this two assumption it can be proved that the survival probability between t_0 and T is:

$$P_{surv}(t_0, T) = e^{-\int_{t_0}^T \lambda(t) dt}. \quad (2.4)$$

The most widely calibration approach is considering the intensity a piece-wise constant function between the maturities of a bulk of CDS. This calibration consist in a bootstrap approach using (2.3):

1. Consider the CDS with the earliest expiry day T_1 : isolate the probability of default between t_0 and T_1 in (2.3), and isolate $\lambda(t)$ from (2.4) considering it constant on the interval (t_0, T_1) .
2. Consider the CDS with the second earliest expiry day T_2 : compute all the probability of default within the interval (t_0, T_1) in (2.3) using the previous λ ; then, isolate the probability of default between T_1 and T_2 from (2.4).
3. Repeat the point 2 for all the maturities available.

It is worth emphasizing that the default intensity λ is specified under the risk-neutral pricing measure, which is the relevant measure for CDS pricing. The risk-neutral default intensity differs substantially from the so-called physical or real-world default intensity. This discrepancy is also reflected in the observed CDS spreads, which represent a compensation that is higher than what is required based on the default probabilities. The difference represents a risk premium, which investors demand as a compensation for unpredictable variation in future default rates. In other words, CDS spreads represent a risk-adjusted expected loss, which is approximately equal to the sum of the expected loss given default, and a risk premium compensating for not diversifiable systematic risk and the idiosyncratic jump-at-default risk.

The main advantages of these models are the possibility to calibrate and simulate in a really easy way, and the immediate possibility to generalize them including market risks as interest rate correlations with default, jumps and FX risks.

2.6 Risk premia in CDS contracts

2.6.1 Credit risk and its usage

CDS provides a very easy way to trade credit risk essentially for three purpose, taking an outright position on spreads depending on traders' expectations over a short horizon, hedging credit risk and arbitrage trading. Corporate bonds are often hold to maturity and so secondary market liquidity is often poor; shorting credit risk is even more difficult because of the illiquidity of the repurchase agreement market for risky bonds, and also if it is possible the tenor is usually very short. CDS allows the investor to trade credit risk in a liquid market and to short easily credit risk over longer periods.

One of the reasons for which CDSs are so diffused is that they may reduce the risk-weighted assets and raise the regulatory capital ratios of financial entities. The insurance company AIG, which was a focal point of the 2008 US government bailout, disclosed that a majority of its CDS protections sold to banks were used for regulatory capital relief.[8]

There have been increasing concerns that 'naked' CDS may help speculators destabilize the debt market. For example, in a striking case, when Delphi Corporation filed for bankruptcy on October 8, 2005, the total amount of CDS contracts outstanding was roughly thirty times the face value of its bonds outstanding. Protection buyers who did not own Delphi's bonds scrambled to acquire the Delphi bonds to settle their CDS contracts through physical delivery, driving the price of these bonds up quite substantially. Furthermore, during the European debt crisis, the naked CDS positions on Greek debt also raised concerns about market manipulation by a group of hedge funds, which led to a ban on naked CDS for European sovereign debt in 2011 [7].

Acharya and Johnson [2] suggest that financial intermediaries purchase credit insurance based on superior information they obtain from their lending relationships with clients. This results in informed trading, effectively insider trading. Hakenes and Schnabel [37] showed that this incentive banks to make unprofitable loans whose risks can be transferred to other parties via CDS, leading to an increase in aggregate risk and a decrease in welfare.

Minton et al. [51] find that for US bank holding companies with assets above \$1 billion, during the period from 1999 to 2005, a substantial proportion of the CDS positions are for dealer activities and the hedging positions are rather small compared to their loan portfolios. This conjecture is supported by the well-known 'London Whale' trading fiasco in early 2012, when JP Morgan lost \$6.2 billion in CDS index trading at its chief investment office in London.

Duffee and Zhou [26] recognize this pro of CDS, but express caution on the potential downside; the banks' informational advantage may lead them to lose their incentives to monitor borrowers closely once their exposures are hedged with CDS; Beyhaghi and Massoud [48] find that banks are more likely to hedge with CDS when monitoring costs are high. Shan et al. [54] find that debt covenants are less strict if there are CDS contracts referencing the borrower's debt.

2.6.2 Non-Credit premiums

In the past, CDS were considered a direct measure of the credit risk. More recently, however, empirical evidences and research have suggested that other factors affect their price. Blanco et al. [13] concludes that CDS are upper bound on the price of credit risk.

2.6.3 Counterparty Risk

In 2007, AIG had outstanding short CDS positions valued at \$546 billion and it avoided posting collateral because of its AAA rating. However, when in 2008 it was downgraded, AIG was required to post additional collateral, which drove it into serious trouble, as described by Stulz [64]. This turmoil brings to the disposal of the central clearing of CDS contracts through clearing houses, known as central counterparties (CCPs). Central clearing operations began in March 2009. By the end of 2013, CDS contracts with central clearing accounted for 26% of all gross notional amounts of CDS outstanding; now a days, according to the BIS, it accounts for 51%.

In [1] Acharya and Bisin argue that the CCP reduces the counter-part externalizes originated by

a lack of transparency on huge short positions of insurance sellers. Duffie and Zhu [27] and Cont and Kokholm [23] observe that, on the one hand, a CCP leads to multilateral netting gains among market participants across a single class of derivatives; on the other hand, clearing through a CCP results in a loss of bilateral netting benefits across different contract types, for example, CDS and interest rate derivatives. Therefore, they confirm that the benefits from multilateral netting is sufficiently large, making CCP really valuable.

The counter-party risk reduces the value of the insurance promised by the protection seller. More specifically, if a CDS seller defaults, the CDS buyer may not receive the CDS payment. CDS sellers with higher default risk to respect their insurance commitments may be forced to sell CDS contracts at lower prices compared to similar contracts offered by financially healthier counterparties.

The economic impact of counterparty risk on CDS spreads may, however, be offset through the practice of posting collateral in the CDS market. Indeed, Arora et al. [5] find counterparty credit risk to be priced, although the magnitude is economically small. A counterparty's credit risk would have to increase by roughly six percentage points to reduce the spread by one basis point.

2.6.4 Illiquidity and Liquidity risk

Liquidity is generally defined as the degree to which assets can be traded quickly in the market without affecting the assets' current market price. Traders in the CDS market face obstacles due to information asymmetries, search costs, transaction and funding costs. Both CDS buyers and CDS sellers are affected by these frictions, although the effects may be asymmetric.

The most common measure of liquidity is the bid-ask spread. A rise in the bid-ask spread represents the evaporation of liquidity from the CDS market. An alternative measure of liquidity is the sensitivity of the price to the size of the trade, proposed by Amihud in [4]. If the market is liquid, it is expected that a large volume of an asset could be traded without very much of an effect on the asset's price. In a variation of this approach, Tang and Yan [65] capture this price impact through the ratio of spread volatility to the total number of quotes. They find that liquidity is indeed priced, and that higher illiquidity is associated with higher CDS prices. More specifically, their estimates yield a liquidity premium earned by the protection seller of approximately 11% of the mid quote. In [49] Kanga and Wilde assumed that the liquidity premium is obtained by the protection buyer. The authors of the second paper investigate the asymmetry in the liquidity premium by separating the total CDS bid-ask spread into a liquidity premium on both the bid and ask prices. The proportion of the liquidity premium attributed to the ask spread is measured as $\frac{(CDS_{ask} - CDS_{defaultpremium})}{CDS_{ask} - CDS_{bid}}$, which is the ratio of the ask liquidity premium to the bid-ask spread. The results indicate that the bid liquidity premium is, on average, larger than the ask one. Compared to the protection seller, the CDS protection buyer receives a larger liquidity premium. Moreover, the results suggest that the liquidity premium is state dependent: more liquid markets are associated with higher liquidity premia.

2.6.5 Recovery risk

While several papers have discussed methods of estimating the recovery rate from the CDS spreads, most studies assume constant recovery rates and simply neglect the recovery risk. This

is partially due to the difficulty of jointly identifying the dynamics of default and recovery risk. [29] Elkamhi et al. estimate recovery rates using CDS spreads for multiple maturities of 152 firms during 2004-2007. They find that the average recovery rate in their sample is 53.79% with substantial cross-sectional variation, which is much higher than the standard assumption in existing studies and industry practice. They further find that the estimated 5-year default probabilities are on average 67% higher than what is obtained using the standard 40% recovery assumption. Therefore, relying on long-run historical averages of recovery rates might lead to a substantial valuation bias.

2.6.6 FX risks and CDS

The influence of FX risk on CDS is really visible on sovereign CDS: the price of EUR-denominated credit default swaps referencing German sovereign bonds on April 2010 was nearly 30% cheaper than the same contract denominated in USD, according to data from Fitch Solutions. Far from reflecting a view on German default, the massive differential, up from only 7% in January of the same year, provides probably a proxy for market sentiment on the EUR as fear of contagion spreads.

The higher the differential between sovereign credit swaps in different currencies, the higher the correlation between default risk and volatility in the relevant exchange rate. Aside from expressing that somewhat obvious trend, the dual-currency CDS dynamic creates a trading opportunity using quanto swaps, FX swaps embedded in CDS contracts.

Few are the papers that try to explain the effect of this risk on the CDS prices. The main two are [28] of Philippe Ehlers and Philipp Schonbucher and [15] of Brigo, Pede and Petrelli. The first one propose a model to price credit derivative considering the possibility to have more currencies, the second a pricing formula for Quanto CDS. Conducting an empirical analysis they explain the difference of prices between CDS written on the same underlying, but with different currencies, as an effect of level of correlation between default risk and exchange rates and the risk of dramatic Foreign Exchange (FX) rate devaluation in conjunction with default events.

2.6.7 Other risks

Considering the previous discussion on the different possible restructuring clauses in CDS contracts, Berndt et al. [11] find a restructuring premium of about 6% to 8% of CDS spread without restructuring.

The protection buyer literally holds a cheapest-to-deliver (CTD) option, as he may deliver the least valuable bond among the defined set of eligible reference obligations. The effect of this option is particularly acute in the case of long-dated or convertible bonds.

Finally, if repo cost of shorting the cash bond is significant, then the credit spread on the bond underestimates the true credit risk of the bond. CDS price will tend to be greater than the measured credit spread.

Chapter 3

Dataset

*"Good is the enemy of great. And that is one of the key reasons why we have so little that becomes great. We don't have great schools, principally because we have good schools. We don't have great government, principally because we have good government. Few people attain great lives, in large part because it is just so easy to settle for a good life."*¹

3.1 Description of the dataset

The data are collected from the Thomson Reuters CDS source within Datastream. The dataset is composed by a total of 15 issuers of CDS, all investment banks based in Europe, US and Asia. Each issuer issue CDS in 10 different maturities: 6 month, 1, 2, 3, 4, 5, 7, 10, 20, 30 years; I choose to work on the 5 years, which is by far the most liquid maturity in the CDS market.

CDS can be issued in different maturities, I decided to collect the spread in euro and dollar because they are the most used currencies in the credit world.

Across the issuers, the CDS contracts do not start in the same moments, eleven start between the end of 2007 and the end of 2008, other four start between 2010 and 2011; they all end along 2017.

The CDS spreads are collected weekly, considering every Wednesday since it is the most liquid day of the week in the CDS market. The reason for which the data has been transformed is that with a so long time span weekly data are more feasible to be tested and to describe the information they contain.

An important factor to take into consideration working on OTC derivatives as CDS is the approximation done to quote their prices on Datastream; the spread is the average of the long position furnished by different dealers. If no transaction happens the price is 'fictitious', in the sense that is a quotation not determined by supply and demand. Choosing the two most liquid

¹James C. Collins, *Good to Great: Why Some Companies Make the Leap... and Others Don't*, HarperBusiness, October 16th 2001

currency in the CDS market, it should reduce drastically this possibility.

Finally, when the same bank issues more than one CDS with different restructuring clauses, the CDS is chosen by this order of priority, for liquidity reasons and in order to have coherent prices between the time series: Modified Restructuring, Modified-Modified Restructuring, Full restructuring, No restructuring.

3.2 Time periods studied

First of all, I analyze all the time-span from December 2007 to December 2017.

Then, I subdivided it in three historical periods. First, I consider the subprime financial crisis and the subsequent period of economic and monetary policies; this time interval starts from when the data are available, December 2007, to the first bailout approved by the leaders of the Eurozone to Portugal, in May 2011. The second period considered is the European sovereign debt crisis, and goes from May 2011 to March 2015, when the quantitative easing policy started. Finally, the third period starts in March 2015 with the beginning of the quantitative easing and ends when the data are available, i.e. December 2017.

The analyses of the European Sovereign crisis started with the bailout of Portugal, since it is the period in which the crisis spreaded officially from Greece to the other countries and in particular when the CDS of the bank sector rise significantly and suddenly, as it can be seen from figure C.1 and C.2 in the Appendix C.

Observing the figures C.4, C.3 and C.5 of the difference of CDS prices written in different currencies, it is possible to distinguish two different regimes. From December 2007 to January 2014 the volatility is really high, ten of the fourteen differences has mean near to zero and the values jump from negative to positive without persistence, showing a strong and fast mean reversion; while four times the differences are always positive or negative with strong persistence. The second regime coincides with the quantitative easing one. It shows less volatility and it is characterized by high persistence; thirteen on fifteen time series are persistently positive, one negative and one symmetric.

3.3 Why the banking sector

My main focus are multinational commercial and investment banks.

The bank sector is particularly interesting first of all because the majority of the studies on the relation between FX risk and CDS tended to focus on Sovereign Credit Derivative Swaps. These derivatives are quite different from the corporate ones, for example, the foreign sovereign often has negligible default risk compared to the corporate (unless it is an emerging market).

Second, Chen et al. [20] report that the largest 14 dealers account for about 90% of all CDS transactions, with more than half of these being executed within these 14 dealers. Banks are the major players in the credit derivative market. In fact, they were the main group among the proponents of CDS who lobbied for the contracts to be recognized in bank regulations as hedging instruments when calculating capital requirements, included in the Basel II capital accord.

Using data from OCC's Quarterly Report on Bank Trading and Derivatives Activities, [6] confirm these results and identify that reasons for this predominance are fixed entry costs, trading frictions

and the benefits of netting.

Chapter 4

Methodology

*"The history of Western science confirms the aphorism that the great menace to progress is not ignorance but the illusion of knowledge."*¹

4.1 Stationarity

First of all, it is important to note that stationarity is a property of a process, not of a time series.

There is one definition of strict stationary processes, and there are several definitions of weak stationary processes. Despite the fact that through this thesis I will use a weak definition, it is good to recall the strict definition to understand the general concept.

Definition 1 (Strict stationary process) *Let $\{X_t\}$ be a vector stochastic process and let $F_X(x_{t_1}, \dots, x_{t_k})$ represent its joint distribution. $\{X_t\}$ is strictly stationary if, for all k , for all τ , and for all t_1, \dots, t_k ,*

$$F_X(x_{t_1+\tau}, \dots, x_{t_k+\tau}) = F_X(x_{t_1}, \dots, x_{t_k}).$$

Since τ does not affect $F_X(\cdot)$, F_X is not a function of time; consequently, parameters such as mean and variance do not change over time.

Strict stationarity is difficult to prove, often it is sufficient and much more easier to prove a weaker form of stationarity; the definition below of weak stationarity will be used in the following.

¹Daniel J. Boorstin, *Cleopatra's Nose: Essays on the Unexpected*, Edited by Ruth F. Boorstin, 1994, Chapter 1: The Age of Negative Discovery, Start Page 3, Quote Page 7, Random House, New York.

Definition 2 (Weak stationary process) A continuous time random process $\{X_t\}$ is weak stationary if:

1. $\mathbb{E}[X_t] = m_X(t) = m_X(t + \tau) \forall \tau$
2. $\mathbb{E}[(X(t_1) - m_X(t_1))(X(t_2) - m_X(t_2))] = C_X(t_1, t_2) = C_X(t_1 - t_2, 0)$.

The first property implies that the mean function $m_X(t)$ must be constant. The second property implies that the covariance function depends only on the difference between the extremes of the time interval taken into consideration. This also implies that the variance function $var_X(t)$ must be constant and that the autocorrelation depends as well only on the difference between the extremes of the time interval.

4.2 Unit root processes and trend terms

Unit root and trend processes are two important types of not stationary processes. Now, I will briefly describe the most important examples considered in econometric analysis.

- A process y_t is said to be **trend stationary** if $y_t = f(t) + e_t$, where $f : \mathbb{R} \mapsto \mathbb{R}$ and e_t is a stationary process; f is called the trend term of the process and it can be, for example, linear, quadratic or exponential.
 y_t differs from trend by e_t at time t ; since the deviation is stationary, y_t departures only temporary.
- The most famous example of unit root process is the **random walk process** $y_t = y_{t-1} + \epsilon_t$, where its solution is $y_t = y_0 + \sum_{i=1}^t \epsilon_i$ and the stochastic trend ϵ_t is a white noise process. Its unconditional mean is y_0 , but despite this, each shock has not decaying effect; its conditional mean is y_{t-1} . The variance $t\sigma$ is time-dependent, it tends to infinity, the covariance is $\gamma_{t-s} = (t-s)\sigma$ and the correlation $\rho_s = \sqrt{\frac{t-s}{t}}$ decays slowly. From these quantities it is evident that there is no tendency to mean revert, but the first difference $\Delta y_t = \epsilon_t$ is stationary.
- The **random walk plus a drift process** $y_t = y_{t-1} + \alpha_0 + \epsilon_t$ has $y_t = y_0 + \alpha_0 t + \sum_{i=1}^t \epsilon_i$ as solution.
It has the two non-stationary components treated before: a linear deterministic and a stochastic trend. The unconditional mean is $y_0 + \alpha_0 t$. As before, the first difference is stationary.

Trend terms can be part added to stationary, unit root or other not stationary process. The main feature of a trend term is that it has a permanent effect on the series.

Before giving a more precise definition of a unit root process, it necessary to define the autoregressive model:

Definition 3 A discrete-time stochastic process y_t is an autoregressive model of order p , denoted by $AR(p)$:

$$y_t = a_0 + \sum_{i=1}^p a_i X_{t-i} + \epsilon_t,$$

where a_i are the parameters of the model, c is a constant, and ϵ_t is white noise.

Definition 4 (Unit root process) Consider a discrete-time stochastic process y_t , that can be represented by an $AR(p)$:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + \epsilon_t.$$

Here, ϵ_t is a serially uncorrelated, zero-mean stochastic process with constant variance σ^2 . If $m=1$ is a root of the characteristic equation:

$$m^p - m^{p-1}a_1 - m^{p-2}a_2 - \dots - a_p = 0$$

then the stochastic process has a unit root or, alternatively, is integrated of order one, denoted $I(1)$.

If $m=1$ is a root of multiplicity r , then the stochastic process is integrated of order r , denoted $I(r)$.

The really interesting features of a unit root process for an econometrician are the founding of Granger and Newbold in [34]. They discover that unit root process leads to spurious regression, i.e., a regression with high R^2 and t-statistics, but no economic meaning. The problem is that the residuals are not stationary, so any error does not decay, leaving deviation permanent; this means that the model has permanent errors. [55] Philips proved that this problem intensifies in larger samples.

Both unit root and trend-stationary processes have a mean that can be growing or decreasing over time; however, in the presence of a shock, trend-stationary processes are mean-reverting (i.e., transitory, the time series will converge again towards the growing mean, which was not affected by the shock) while unit-root processes have a permanent impact on the mean (due to the not decaying coefficient of the error terms).

4.2.1 Detrending and differencing

Weak stationary processes are really important in econometrics, since all the main techniques to analyze the data assume the time series are stationary.

The main way to obtain a stationary process from a unit root integrated of order one, it is to consider the time series of the difference. Considering as an example the random walk $y_t = y_{t-1} + \epsilon_t$ and taking the difference $\Delta y_t = y_t - y_{t-1}$, it follows $\Delta y_t = \epsilon_t$, that is weakly stationary. In case of a higher order of integration r , it has to be taken r times the difference of the original process. In the presence of a trend term in the process, the main practise is to estimate this term and to subtract it from the process, if the original process was a trend-stationary one, this will lead to a pure stationary process. This practise is named 'detrending'.

4.2.2 Unit Root tests

Now I will present three of the most used methods to look for unit roots in a process.

Dickey-Fuller test

Performing a test on the coefficients a_i to find the presence of a unit root is problematic, because under the hypothesis of not stationarity, the distributions of the classical statistics are not known. Moreover, the values of the coefficients are biased; for example, in an AR(1) process, where $a_1 = 1$ indicate a unit root process, a_1 is biased to be less than 1.

If the true distribution of a_1 is known under non-stationarity, the test can be performed. For this purpose, Dickey and Fuller used this Monte Carlo procedure to estimate the distribution [25]:

1. Simulate ϵ_t by generating N standard normal random numbers.
2. Compute y_t ; to purge the effect of y_0 , choose either $y_0 = \mathbb{E}[y_t]$ or $T + m$ observations and disregard the first m observations.
3. Estimate the model under the alternative hypothesis of stationarity, and then obtain the t-statistic under null hypothesis of non stationarity.
4. Repeat the first three points thousands of times.

Considering the example of an AR(1) process $y_t = \alpha_1 y_{t-1} + \epsilon_t$, subtract from the left and the right side y_{t-1} and substitute $\gamma = \alpha_x - 1$; now the test on $a_1 = 1$ is equivalent to the test $\gamma = 0$. In [25] Dickey and Fuller propose three different regression equations to test:

1. $\Delta y_t = \gamma y_{t-1} + \epsilon_t$
2. $\Delta y_t = \alpha_0 + \gamma y_{t-1} + \epsilon_t$
3. $\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \epsilon_t$

The first is a pure random walk model, the second adds an intercept, and the third includes both a drift and an intercept. The test involves estimating one of the equations above using OLS in order to estimate γ and its standard error. Comparing the resulting t-statistic with the ones previously computed by the Monte Carlo simulation described before, allows the researcher to accept or reject the null hypothesis $\gamma = 0$.

Augmented Dickey-Fuller test

If the error sequence ϵ_t is serially correlated, more lags have to be considered by an AR(p) model in the previous three regression equations. This is the reason for which it was originated the augmented Dickey-Fuller test.

To understand the concept that underlies this test, consider $y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_{p-1} y_{t-p+1} + \alpha_p y_{t-p} + \epsilon_t$, add and subtract $\alpha_p y_{t-p+1}$ to obtain $y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots - (\alpha_{p-1} + \alpha_p) y_{t-p+1} - \alpha_p \Delta y_{t-p+1} + \epsilon_t$. Now add and subtract $(\alpha_{p-1} + \alpha_p) y_{t-p+2}$ to obtain $y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots - (\alpha_{p-1} + \alpha_p) y_{t-p+2} - \alpha_p \Delta y_{t-p+1} + \epsilon_t$. Continuing in this fashion it is obtained

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \epsilon_t$$

where $\gamma = -(1 - \sum_{i=1}^p \alpha_i)$ and $\beta_i = \sum_{j=1}^p \alpha_j$. Now, you can perform the test on $\gamma = 0$ and use the statistics of the DF test modified accordingly [25].

Phillips-Perron

The Phillips-Perron (PP) test [56] involves the estimate of

$$y_t = \gamma y_{t-1} + \alpha_0 + \alpha_1 t + \epsilon_t$$

where ϵ_t may be heteroskedastic. The PP tests correct the Dickey-Fuller test for any serial correlation and heteroskedasticity in the errors.

Phillips and Perron's test statistics are the DF statistics made robust to serial correlation by using the Newey-West [52] heteroskedasticity and autocorrelation consistent estimators of the covariance matrix of ϵ_t ; under the null hypothesis $\gamma = 0$, these statistics have the same asymptotic distributions as the ADF ones.

A part being robust to general forms of heteroskedasticity and serially correlation in the error term, a great advantage of Phillips-Perron test is that it is non-parametric, i.e., it does not require to select the level of serial correlation as in ADF. It rather takes the same estimation scheme as in the DF test, but corrects the statistics. The main disadvantages of the PP test is that it is based on asymptotic theory, so it works well only in large samples, and it also shares disadvantages of ADF tests: sensitivity to structural breaks and poor power.

KPSS test

The most important limit of the ADF and PP tests is the considerably low power and the dependence of this power on α_1 . The origin of the problem is that in finite samples any trend stationary process can be approximated by a unit root process and any unit root process can be approximated by a trend stationary process [30]; consequently, it is difficult for any statistical procedure to distinguish between unit root processes and series that are highly persistent. Thus, a test that exchange the null and the alternative hypothesis is intended to complement the previous tests.

The Kwiatkowski-Phillips-Schmidt-Shin test [46] considers as null hypothesis that the series is trend-stationary and as alternative that is a unit root process. The KPSS test is based on the Lagrange Multiplier test, that is a two step procedure. In the first step you estimate the trend coefficient with a regression that do not consider the stochastic trend. In the second step, you estimate a variant of the Dickey-Fuller test using the detrended series in place of the level of y_{t-1} . Basically, it is preferable to estimate the parameters of the trend using a model without the persistent variable y_{t-1} . Once the trend is efficiently estimated, it is possible to detrend the data and perform the unit root test on the detrended data.

Choosing Lags and Deterministic Coefficients

The number of lags considered in unit root tests is really relevant. Choosing fewer lags than the true number leads to errors not distributed as a white-noise and, consequently, to wrong estimations of γ and its standard errors. Instead, considering too many lags reduces the power of the test and the degrees of freedom of the model. Practically, it is suggested to start with the greatest reasonable number of lags p , perform tests and diagnostic techniques, and if p lags don't work well, repeat everything with $p-1$ lags.

To compute the deterministic coefficients is not recommended to start with the most general

model, indeed, in [17] Cambell and Perron report that considering more deterministic coefficients than those in the true model reduces drastically the power of the test and the degrees of freedom and it enlarges the confidence intervals. But they also report that omitting the deterministic trending variable, the power of the t-statistics tends to 0 as the sample increases. If non-trending values are missed, the power of finite samples is affected. The real problem is unit roots tests are conditional on deterministic coefficients and tests on deterministic coefficients are conditional on unit roots.

It is generally recommended to base the decision on the theory and the plot of the data, to estimate the model under stationarity and then impose the restriction of unit root and, finally, to avoid compound errors testing restrictions on a model already restricted by testing other deterministic coefficients.

4.3 Cointegration

In univariate models, stochastic trends can be removed by differencing. Instead, when multivariate variables are integrated, we look for stationary linear combinations of them; such variables are said to be cointegrated. In this case, it is possible to model the long-run model and the short-run dynamics simultaneously. For this reason in [30] the authors affirm that equilibrium theories involving non-stationary variables require the existence of a combination of the variables that is stationary. Clearly, a key assumption is e_t stationary; if e_t has a stochastic trend, the errors in the model will be cumulative, so deviations from market equilibrium will not be eliminated.

Cointegration was introduced by Engle and Granger in [31]. They provide the following definition:

Definition 5 (Engle and Granger definition of Cointegration) *The components of a multivariate sequence \bar{x}_t are said to be cointegrated of order (d,b) , denoted by $CI(d, b)$ if:*

1. *All components of x_t are integrated of order d .*
2. *\exists a vector β , called the cointegrating vector, such that the linear combination $\beta \cdot \bar{x}_t = \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dots + \beta_n x_{n,t}$ is integrated of order $d-b$.*

The cointegrating vector is not unique; if β is a cointegrating vector, then for $\lambda \neq 0$, $\lambda\beta$ is also a cointegrating vector. Typically, one of the variables is used to normalize the cointegrating vector. But, there may be more than one independent cointegrating vectors. The number of cointegrating vectors is called the cointegrating rank. If x_t has n non-stationary components, the rank may be $n - 1$. Hence, if x_t contains only 2 variables, there can be at most 1 independent cointegrating vector.

Cointegration is distinguished from traditional economic equilibrium, in which a balance of forces produces stable long-term levels in the variables. Cointegrated variables are generally unstable in their levels, but exhibit mean-reverting spreads that force the variables to move around common stochastic trends. Cointegration is also distinguished from the short-term synchronize of positive covariance, which only measures the tendency to move together at each time step. A final note is that the scatter plot shows strong correlation.

Although each series is non-stationary, cointegrated series move together, in the sense that the dynamic behaviour of at least one variable must be restricted by the others, it can't simply meander away.

A very useful way to understand cointegration relationships is provided in [63], where Stock and Watson observe that cointegrated variables share common stochastic trends. Given the two variables $y_t = \mu_{yt} + e_{yt}$ and $z_t = \mu_{zt} + e_{zt}$, where μ is a RW process representing the stochastic trend in the variable i , and e is the stationary irregular component of the variable i . Considering

$$\beta_1 y_t + \beta_2 z_t = \beta_1(\mu_{yt} + e_{yt}) + \beta_2(\mu_{zt} + e_{zt}) = (\beta_1 \mu_{yt} + \beta_2 \mu_{zt}) + (\beta_1 e_{yt} + \beta_2 e_{zt})$$

For $\beta_1 y_t + \beta_2 z_t$ to be stationary, clearly, the term $\beta_1 \mu_{yt} + \beta_2 \mu_{zt}$ must vanish. It follows that for all t $\mu_{yt} = -\frac{\beta_2}{\beta_1} \mu_{zt}$. Thus, up to the scalar $-\frac{\beta_2}{\beta_1}$, the two I(1) stochastic processes y_t and z_t must have the same stochastic trend if they are cointegrated of order (1,1). The essential insight of Stock and Watson is that the parameters of the cointegrating vector must be such that they purge the trend from the linear combination.

Considering the n -dimensional case: $\bar{x}_t = \bar{\mu}_t + \bar{e}_t$. Pre-multiplying all these terms by β s.t. $\beta \bar{\mu}_t = 0$, it follows that $\beta \bar{x}_t = \beta \bar{e}_t$. Hence, the linear combination $\beta \bar{x}_t$ is stationary. The general point is that cointegration will occur whenever the trend in one variable can be expressed as a linear combination of the trends in the other variable(s).

4.4 Error-correction model

Assumed that the series are I(1)², and cointegrated, you can employ a vector error correction model (VECM) to study the joint price formation process in both markets. From the estimated error correction model you calculate measures that indicate which of the two markets is leading the price discovery process as well as examine the speed of adjustment towards the long-term equilibrium.

A principal feature of cointegrated variables is that their short-term dynamics are influenced by the extent of any deviation from long-run equilibrium; this means that the movements of at least some of the variables must respond to the magnitude of the disequilibrium. This is the core idea of the error-correction model of two cointegrated variables x and y . Following the notation of [30]:

$$\begin{aligned} \Delta x_t &= \alpha_{1,0} + \alpha_x ECT_t + \sum_{i=1}^p \alpha_{1,1}(i) \Delta x_{t-i} + \sum_{i=1}^r \alpha_{1,2}(i) \Delta y_{t-i} + c_{out,1} + \epsilon_{xt} & \alpha_x > 0 \\ \Delta y_t &= \alpha_{2,0} + \alpha_y ECT_t + \sum_{i=1}^p \alpha_{2,1}(i) \Delta x_{t-i} + \sum_{i=1}^r \alpha_{2,2}(i) \Delta y_{t-i} + c_{out,2} + \epsilon_{yt} & \alpha_y > 0 \end{aligned} \quad (4.1)$$

where the stochastic shocks ϵ_{xt} and ϵ_{yt} are white-noise disturbance terms which may be correlated; the error correction term ECT_t is equal to $(y_{t-1} - \beta_1 x_{t-1} - \beta_0)$. As specified, the variables

²I(1) i.e., an integrated process of order one, was defined in the definition 4

change in response to stochastic shocks and in response to the previous period's deviation from long-run equilibrium. α_x and α_y have the interpretation of speed of adjustment parameters; the larger they are, the greater the response of the variables to the previous period's deviation from long-run equilibrium. If $a_{1,2} = 0$, y_t changes only in response to ϵ_{yt} . The x_t sequence does all of the correction to eliminate any deviation from long-run equilibrium. Since y_t does not do any of the error-correcting, y_t is said to be weakly exogenous.

To understand the following explanation of the Johansen approach, it is important to realize that a cointegrated system can be viewed as a restricted form of a general VAR model. By collecting differences, a $VEC(q)$ model can be converted to a $VAR(p)$ model in levels with $p = q + 1$. In particular, cointegrated VAR models can be simulated and forecast using standard VAR techniques. Using directly a VAR model simply differencing is a model misspecification in the presence of cointegration, since long-term information appears in the levels. Fortunately, the cointegrated VAR model provides intermediate options between differences and levels, by mixing them together with the cointegrating relations. Since all terms of the cointegrated VAR model are stationary, problems with unit roots are eliminated.

Applying the lag operator on the VAR model, the two variables lead to the same characteristic equation; consequently, the characteristic roots determine the time paths of both variables.

- If both characteristic roots (λ_1, λ_2) lie inside the unit circle, x_t and y_t have stable solutions. If t is sufficiently large or if the initial conditions are s.t. the homogeneous solution is zero, the stability guarantees stationarity.
- If either root lies outside the unit circle, the solutions are explosive. Neither variable is difference stationary, so they cannot be $CI(1,1)$ ³.
- If both characteristic roots are 1, the second difference of each variable will be stationary. Since each is $I(2)$, the variables cannot be $CI(1,1)$.
- If $\alpha_{1,2} = \alpha_{2,1} = 0$, there will be no long-run equilibrium. For x_t and y_t to be unit root processes, it is necessary that $a_{1,1} = \alpha_{2,2} = 1$, so that $\lambda_1 = \lambda_2 = 1$. The variables cannot be cointegrated.
- For x_t and y_t to be $CI(1,1)$, it is necessary for one $\lambda = 1$ and the other $\lambda < 1$ in absolute value. They will have the same stochastic trend and the first difference of each variable will be stationary. Imposing these restrictions in the VAR coefficients leads to the VECM. This finding illustrates the Granger representation theorem: for any set of $I(1)$ variables, error correction and cointegration are equivalent representations.

4.5 Cointegration tests and VECM estimations

4.5.1 Engle-Granger Test

The main idea of the Engle-Granger cointegration test is to regress the first component $y_{1,t}$ of y_t on the other component $y_{2,t}$ and test the residuals for a unit root. The null hypothesis is that

³ $CI(1,1)$ i.e., cointegrated processes of order $(1,1)$, were defined in the definition 5

the series in y_t are not cointegrated, so if the residual test fails to find evidence against the null of a unit root, the Engle-Granger test fails to find evidence that the estimated regression relation is cointegrating.

The regression equation is

$$y_{1,t} = \beta_0 + \beta_1 y_{2,t} \quad (4.2)$$

$(1, -\beta_1, -\beta_0)$ is the cointegrating vector, where β_0 is the intercept. A complication of the Engle-Granger approach is that the residual series is estimated rather than observed, so the standard asymptotic distributions of conventional unit root statistics do not apply.

Procedure

Engle and Granger [31] propose a four-step procedure to determine if two $I(1)$ variables are cointegrated of order $CI(1, 1)$:

1. Pretest the variables for their order of integration. By definition, cointegration needs the two variables to be integrated of the same order. If both variables are stationary, it is not necessary to proceed since standard methods applies. If they are integrated of different orders, they are either not cointegrated or multi-cointegrated.
2. Estimate the long-run equilibrium. If both $y_{1,t}$ and $y_{2,t}$ are $I(1)$, the next step is to estimate the long-run equilibrium relationship in the form (4.2).
If the variables are cointegrated, Stock proves that β_0 and β_1 converge faster than they do in OLS models using stationary variables in [62].
3. If the variables are cointegrated the residual sequence, that contains the estimated values of the deviations from the long-run relationship, should be stationary. Perform a Dickey-Fuller test on these residuals: $\Delta e_t = \alpha_1 e_{t-1} + \epsilon_t$. Since e_t are residual, their mean is 0 and so there is no intercept. Notice that it is not possible to use the Dickey-Fuller tables, because the e_t sequence is generated from a regression, it is unknown the actual error e_t . If the residual are not white noise, use the augmented form.
4. Estimate the VECM (4.1). Engle and Granger (1987) circumvent the cross-equation restrictions using the residuals from the previous equilibrium regression to estimate the VECM: e_{t-1} is the deviation from long-run equilibrium in period $t-1$, and so, an estimate of $y_{1,t-1} - \beta_1 y_{2,t-1}$. In this way you obtain a VAR in first difference, that can be estimated using OLS
5. Assess Model Adequacy. The residuals of the error-correction equations should be white noise; if the residuals are serially correlated, lag lengths may be too short.

Limits

The main limits of the Engle-Granger test are the following:

1. The coefficients do not have an asymptotic t-distribution.
2. Using the residual indiscriminately $e_{1,t}$ or $e_{2,t}$ to circumvent the cross-equation restrictions as presented in step 3 works good only with large samples, because asymptotic theory indicates that the test for a unit root in the $e_{1,t}$ becomes equivalent to the test for $e_{2,t}$.

3. In practise, it is possible to find that one regression indicates that the variables are cointegrated, whereas reversing the order indicates no cointegration. This requires one of the variables to be identified as regressor. This choice, which is usually arbitrary, affects both test results and model estimation.
4. It relies on a two-step estimator. The first step is to generate the residual series, and the second step uses these generated errors. Hence, any error introduced by Step 1 is carried into Step 2. Fortunately, several methods have been developed in order to avoid these problems.
5. The test find out only one cointegrating vector. But dealing with only two variables is not a problem, since the maximum number of independent cointegrating vectors is one.

4.5.2 Johansen Test

The Johansen [45] maximum likelihood estimators circumvent the use of two-step estimators and can estimate and test the presence of multiple cointegrating vectors. Moreover, these tests allow the researcher to test restricted versions of the cointegrating vector and the speed of adjustment parameters.

Johansen procedure is conceptually a multivariate generalization of the DF test. Consider the n-dimensional equation $x_t = A_1x_{t-1} + \epsilon_t$; differentiate $\Delta x_t = A_1x_{t-1} - x_{t-1} + \epsilon_t = (A_1 - I)x_{t-1} + \epsilon_t = \pi x_{t-1} + \epsilon_t$. The rank of $\pi = (A_1 - I)$ is the number of cointegrating vectors. If $(A_1 - I) = 0$, $\text{rank}(\pi)=0$ all $x_{i,t}$ are unit root processes. Since there is no linear combination of the $x_{i,t}$ that is stationary in differences, the variables are not cointegrated. If characteristic roots are greater than unity and if $\text{rank}(\pi)=n$, all variables are stationary in levels.

In the case to include more lags is necessary, the higher order autoregressive process is:

$$x_t = A_1x_{t-1} + A_2x_{t-2} + \dots + A_px_{t-p} + \epsilon_t.$$

Adding and subtracting A_px_{t-p+1} to the right-hand side to obtain

$$x_t = A_1x_{t-1} + A_2x_{t-2} + \dots + A_{p-2}x_{t-p+2} + (A_{p-1} + A_p)x_{t-p+1} - A_p\Delta x_{t-p+1} + \epsilon_t;$$

next, add and subtract $(A_{p-1} + A_p)x_{t-p+2}$:

$$x_t = A_1x_{t-1} + A_2x_{t-2} + A_3x_{t-3} + \dots - (A_{p-1} + A_p)\Delta x_{t-p+2} - A_p\Delta x_{t-p+1} + \epsilon_t;$$

continuing in this way, finally I obtain:

$$\Delta x_t = \pi x_{t-1} + \sum_{i=1}^{p-1} (\pi_i \Delta x_{t-i} + \epsilon_t). \quad (4.3)$$

where, $\pi = -(I - \sum_{i=1}^p A_i)$ and $\pi_i = -\sum_{j=i+1}^p A_j$. Again, the key feature is $\text{rank}(\pi)$. Clearly, if $\text{rank}(\pi)=0$, the matrix is null and I obtain the usual VAR model in first differences. Instead, if the rank is n , the vector process is stationary. In intermediate cases, if $\text{rank}(\pi)=1$, there is a single cointegrating vector and the expression πx_{t-1} is the error-correction term. For $1 < \text{rank}(\pi) < n$,

there are multiple cointegrating vectors.

The main idea of the Johansen test is that the number of cointegrating vectors coincide with the rank of the matrix π . Moreover, the rank of a matrix is equal to the number of its characteristic roots that differ from zero; thanks to this observation, Johansen built the statistics of his test. Looking at $\ln(1 - \lambda_i)$, and not directly at the characteristic roots, the statistics to test if these quantities are not significantly different from one are:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^n \ln(1 - \lambda_i); \quad (4.4)$$

$$\lambda_{max}(r, r + 1) = -T(\ln(1 - \lambda_{r+1})). \quad (4.5)$$

where T is the number of observation. The first statistic tests the null hypothesis that the number of distinct cointegrating vectors is less or equal to r against the alternative hypothesis that it is greater than r . $\lambda_{trace} = 0$ when all $\lambda_i = 0$, while, the further the λ_i are from 0 the more negative is $\ln(1 - \lambda_i)$ and the larger is the trace statistic.

The second statistic tests the null hypothesis of r cointegrating vectors against the alternative of $r + 1$ cointegrating vectors. If the estimated value of the characteristic root is close to zero, λ_{max} will be small.

Critical values of the λ_{trace} and the λ_{max} statistics are obtained using the Monte Carlo approach. The distribution of these statistics depends on the number of non-stationary components under H_0 (i.e. $n - r$), and the form of A_0 .

The two tests can be contradictory; when this happens the λ_{max} has sharper H1, so it is preferred pinning down the number of cointegrated processes.

4.5.3 Johansen framework for testing restricted cointegrating vectors

Johansen allows for testing restricted forms of the cointegrating vector(s). If there are r cointegrating vectors, only these r linear combinations of the variables are stationary. All other linear combinations are non-stationary. Thus, suppose you re-estimate the model restricting the parameters of π . If the restrictions are not binding, the number of cointegrating vectors has not diminished.

Johansen framework allowed to test linear constraints on cointegrating relations, adjustment speeds and estimates VEC model parameters under the additional constraints. Those constraint testing allows you to assess the validity of relationships suggested by economic theory. An all-zero row in A indicates a variable that is weakly exogenous, it may affect other variables, but does not adjust to disequilibrium in the cointegrating relations; similarly, a standard unit vector column in A indicates a variable that is exclusively adjusting to disequilibrium in a particular cointegrating relation.

Johansen defines the two matrices α and β , s.t.

$$\pi = \alpha \cdot \beta'$$

both of dimension (n, r) , where r is the rank of π . β is the matrix of cointegrating parameters and α is the matrix of weights with which each cointegrating vector enters the n equations of

the VAR. α can be viewed as the matrix of the speed of adjustment parameters. Due to the cross-equation restrictions, it is not possible to estimate α and β using OLS; however, using MLE, the Johansen framework allowed us to:

1. estimate (4.3) as an error-correction model;
2. determine the rank of π ;
3. use the r most significant cointegrating vectors to form β ;
4. select α s.t. $\pi = \alpha\beta'$.

It is easier to understand the concept in the case of a single cointegrating vector.

If $\text{rank}(\pi)=1$, the rows of π are all linear multiples of each other. Hence, the equations in (4.3) have the form:

$$\begin{aligned}\Delta x_{1,t} &= \pi_{1,1}x_{1,t-1} + \pi_{1,2}x_{2,t-1} + \dots + \pi_{1,n}x_{n,t-1} + \epsilon_{1,t}; \\ \Delta x_{2,t} &= s_2(\pi_{1,1}x_{1,t-1} + \pi_{1,2}x_{2,t-1} + \dots + \pi_{1,n}x_{n,t-1}) + \epsilon_{2,t}; \\ &\dots; \\ \Delta x_{n,t} &= s_n(\pi_{1,1}x_{1,t-1} + \pi_{1,2}x_{2,t-1} + \dots + \pi_{1,n}x_{n,t-1}) + \epsilon_{n,t}.\end{aligned}\tag{4.6}$$

$$\tag{4.7}$$

Defining $\alpha_i = s_i\pi_{1,1}$ and $\beta_i = \frac{\pi_{1,i}}{\pi_{1,1}}$ so that each equation can be written as $\Delta x_{i,t} = \alpha_i(x_{1,t-1} + \beta_2x_{2,t-1} + \dots + \beta_nx_{n,t-1}) + \dots + \epsilon_{i,t}$ or in matrix form,

$$\Delta x_t = \sum_{i=1}^{p-1} (\pi_i \Delta x_{t-i}) + \alpha\beta'x_{t-1} + \epsilon_t$$

Now test the restrictions on β using the statistic:

$$-T \sum_{i=r+1}^n [\ln(1 - \lambda_i^r) - \ln(1 - \lambda_i^u)];\tag{4.8}$$

where λ_i^r are the characteristic roots in the case of restricted model and λ_i^u are the characteristic roots in the case of unrestricted model. This statistic has asymptotically a χ^2 distribution with number of degrees of freedom equal to the number of restrictions on β . If λ_i^r is smaller than λ_i^u , it is obtained a reduced number of cointegrated vectors, so the restriction is binding. Constraints on α work in the same way.

If there is a single cointegrating vector, the Engle-Granger and Johansen methods have the same asymptotic distribution.

4.5.4 The procedure

The procedure suggested in the case of the Johansen test is the following:

1. All variables are pretested to assess their order of integration.

2. The results of the test are sensitive to the lag length. The most common procedure is to begin with the longest lag length deemed to be reasonable and test whether it can be reduced. Even though the variables are not stationary, a likelihood ratio test statistic recommended by Sims [60] can perform lag length and the deterministic coefficient tests: $(T - c)(\ln(|L_r|) - \ln(|L_u|))$, where L_r and L_u are respectively the likelihood value of the restricted and unrestricted model; this statistic follows a χ^2 distribution with number of degrees of freedom equal to the number of coefficient restrictions. Alternatively, you can select lag length p using the multivariate generalizations of the Akaike Information Criteria, Hannan Quinn, Schwarz Information Criteria and Final Prediction Error; these criteria will be described in (5.1).
3. Estimate the model and determine the rank(π). Estimate the values of the characteristic roots of the matrix and calculate the λ_{trace} and the λ_{max} statistics and test their statistical significance.
4. Analyze the normalized cointegrating vector(s) and speed of adjustment coefficients as described in the previous section.

4.6 The price discovery

Price discovery, one of the key functions of financial markets, is defined by Lehmann [47] to be the efficient and timely incorporation of the information implicit in investor trading into market prices. When there is only one location for trading an asset, by definition all price discovery takes place in that market place. When closely related assets trade in different locations, order flow is fragmented and price discovery is split among markets. The existence of cointegration means that at least one market has to adjust by the Granger representation theorem [31]. That market is inefficient since the price reacts to publicly available information.

The appropriate method to investigate the mechanics of price discovery is not clear. The two popular common factor models due to Hasbrouck [41] and Gonzalo and Granger [33] both rely on vector error-correction models (VECM) of market prices. Hasbrouck's model of 'information shares' assumes that price volatility reflects new information, and thus the market that contributes most to the variance of the innovations to the common factor is also presumed to contribute most to price discovery. Gonzalo and Granger's approach decomposes the common factor itself, and, ignoring the correlation between the markets, attributes superior price discovery to the market that adjusts least to price movements in the other market. When price-change innovations are correlated, Hasbrouck's approach can only provide upper and lower bounds on the information shares of each market. Since neither method is considered universally superior, I will report both.

4.6.1 Information share and Hasbrouck measure

The innovation ϵ_t to the common efficient price in (4.1) impounds new information about the asset's fundamental value and has a permanent impact on the price levels. Observing this, Hasbrouck [41] proposes a measure based on the share of the variance $\phi' \Sigma \phi$ of ϵ_t that is attributable

to this market.

If Σ is diagonal, i.e., the errors are uncorrelated, then market i 's information share (IS) is defined as:

$$IS_i = \frac{\phi_i^2 \sigma_i^2}{\phi_1^2 \sigma_1^2 + \phi_2^2 \sigma_2^2} \text{ for } i = 1, 2$$

where, for construction $IS_1 + IS_2 = 1$.

If Σ is non-diagonal, to attribute the covariance terms to each market, Hasbrouck suggests to compute the Cholesky decomposition. Let F be a lower triangular matrix such that $FF' = \Sigma$, then the IS for the i_{th} market is

$$IS_i = \frac{[\phi'F]_i}{\phi'\Sigma\phi} \text{ for } i = 1, 2 \quad (4.9)$$

where $[\phi'F]$ is the i_{th} element of the row $\phi'F$. The resulting IS depends on the ordering of price variables [66]. In the bivariate case, the upper (lower) bound of the IS_i is obtained by computing the Cholesky factorization with the i_{th} price ordered first (last). With n prices, Hasbrouck [42] shows that one must examine all permutations of the variables.

4.6.2 Component Share and Gonzalo-Granger measure

The second way to measure price discovery is originated by the permanent-transitory (PT) component decomposition of p_t of Gonzalo and Granger [33]:

$$p_t = A_1 f_t + A_2 z_t \quad (4.10)$$

where f_t is the permanent component, z_t is the transitory component, and A_1 and A_2 are loading matrices; f_t is $I(1)$ and z_t is $I(0)$, they are linear combinations of p_t and z_t does not Granger cause f_t in the long run. In particular, Granger and Gonzalo define $f_t = \gamma' p_t$ and $A_1 = \beta_{\perp} (\alpha'_{\perp} \beta_{\perp})^{-1}$, where $\bar{\gamma} = (\alpha'_{\perp} \beta_{\perp})^{-1} \alpha'_{\perp}$, where $\alpha_{\perp} \alpha = 0$ and $\beta_{\perp} \beta = 0$.

Since $\beta = (1, -1)'$, one possible choice for β_{\perp} is $\mathbb{I} = (1, 1)'$ implying $\gamma = (\alpha'_{\perp} \mathbb{I})^{-1} \alpha'_{\perp}$. The permanent component becomes a weighted average of observed prices, where the component weights $\gamma_i = \frac{\alpha_{\perp, i}}{\alpha_{\perp, 1} + \alpha_{\perp, 2}}$, are suggested as a component share measure of the price discovery by Booth et. al. [14], Chu et. al. [16], and Harris et. al. [40]. It is clear that a small (large) value of CIS_i is directly related to a small (large) contribution of market i to the Granger-Gonzalo permanent component of prices f_t . In a market with uncorrelated innovations, it can be proved that IS is a variance weighted version of CS.

In [9] Baillie et. al. note that since $\alpha_{\perp} \alpha = \alpha_{\perp, 1} \alpha_x + \alpha_{\perp, 2} \alpha_y = 0$ I can express $\alpha_{\perp, 1} = -\alpha_{\perp, 2} \alpha_y / \alpha_x$, leading to:

$$CIS_1 = \frac{\alpha_y}{\alpha_y - \alpha_x} \quad CIS_2 = \frac{\alpha_x}{\alpha_x - \alpha_y} \quad (4.11)$$

Interestingly, (4.11) shows that price discovery occurs entirely in market i if $\alpha_i = 0$; that is, if the contemporary price change in market i does not respond to the lagged disequilibrium error $\beta' p_{t-1} = p_{1,t-1} - p_{2,t-1}$. Using (4.10) and $\beta' A_1 = 0$, Booth et. al. [14] note that $\beta' p_{t-1} = \alpha \beta' A_2 z_{t-1}$ and argue that the error correction coefficients measure the way in which prices adjust to lagged differences in their transitory components. The CS for market 1 reflects how sensitive market 2 is w.r.t. to market 1, to lagged transitory shocks and vice-versa.

4.6.3 The Procedure

To compute these measures, it is necessary first to estimate the VECM (4.1). If the second market is contributing significantly to the discovery, then α_x will be negative and statistically significant as the first market adjusts to incorporate this information. Similarly, if the first market is an important venue for price discovery, then α_y will be positive and statistically significant. If both coefficients are significant, then both markets contribute to price discovery.

When both coefficients are significant, I check the two measures presented previously. If the first market dominates, the Granger-Gonzalo measure will be close to 1, while if the second market dominates price discovery, the measure will be closer to zero. Considering the lower-bound furnished by Hasbrouck, the more this measure is close to 0, the more the first market is dominant.

To interpret the Hasbrouck measure in the bivariate case, the upper (lower) bound of the IS_i is obtained by computing the Cholesky factorization with the i th price ordered first (last). Then, for each market consider the average of the upper bound and the lower bound; this middle point will be considered the Information Share of the market i . The higher this middle point, the higher is the contribution of this market to price discovery.

4.7 Threshold VECM

Given that the CDS are subject to market frictions and arbitrageurs face various trading costs, it is useful to extend the linear VECM approach to a threshold vector error correction model (TVECM). Threshold cointegration was introduced by Balke and Fomby [10] as a feasible mean to combine regime switches and cointegration. The TVECM model allows for nonlinear adjustments to the long-term equilibrium in CDS and bond markets. In our case, such nonlinear adjustment dynamics should be able to capture different behaviour of the adjustment in the case of negative or positive differences; or to catch the arbitrageurs' decisions to only step into the market when the basis exceeds some critical threshold, such that the expected profit exceeds the transaction costs and the Forex risk. As a result, adjustments to the long-term equilibrium would then be regime-dependent, for example, with a relatively weak adjustment mechanism below the threshold (a 'neutral' regime) and a stronger adjustment mechanism above it.

4.7.1 The model

Consider the simple process $y_t = \alpha_1 y_{t-1} + \epsilon_{1,t}$ if $y_{t-1} \leq 0$ and $\alpha_2 y_{t-1} + \epsilon_{2,t}$ if $y_{t-1} > 0$. You can think of the equation $y_{t-1} = 0$ as being a threshold. On one side of the threshold, the y_t sequence is governed by one autoregressive process and on the other side of the threshold, there is a different autoregressive process. Shocks to $\epsilon_{1,t}$ or $\epsilon_{2,t}$ are responsible for regime switching; the larger the variance of $\epsilon_{1,t}$, the more likely is a switch from one regime to the other. If the variances of the two error terms are equal, the process can be written as $y_t = \alpha_1 I_t y_{t-1} + \alpha_y (1 - I_t) y_{t-1} + \epsilon_t$.

The generalization to a high order autoregressive process with one threshold in τ is:

$$y_{1,t} = \mathbb{I}_t[\alpha_{1,0}^L ec_{t-1} + \sum_{i=1}^p \alpha_{1,i}^L \Delta y_{t-i}] + (1 - \mathbb{I}_t)[\alpha_{1,0}^U ec_{t-1} + \sum_{i=1}^p \alpha_{1,i}^U \Delta y_{t-i}] + \epsilon_{y_1,t} \quad (4.12)$$

$$y_{2,t} = \mathbb{I}_t[\alpha_{2,0}^L ec_{t-1} + \sum_{i=1}^p \alpha_{2,i}^L \Delta y_{t-i}] + (1 - \mathbb{I}_t)[\alpha_{2,0}^U ec_{t-1} + \sum_{i=1}^p \alpha_{2,i}^U \Delta y_{t-i}] + \epsilon_{y_2,t} \quad (4.13)$$

where $\mathbb{I}_t = 1$ if $y_{t-1} \leq \tau$ and $\mathbb{I}_t = 0$ if $y_{t-1} > \tau$, $ec_t = (y_{1,t} - \beta_0 - \beta_1 y_{2,t})$ is the error correction term, $\sum_{i=1}^p \alpha_{j,i}^U \Delta y_{t-i}$ with $j=1,2$, the VAR term in difference with order p and $\epsilon_{y_j,t}$ the i.i.d. shocks. In this case the unique dummy variable determines if you are in the regime above or under the threshold.

In this thesis also the TVECM with two thresholds $\tau_1 \leq \tau_2$ will be considered:

$$y_{1,t} = \mathbb{I}_t^1[\alpha_{1,0}^L ec_{t-1} + \sum_{i=1}^p \alpha_{1,i}^L \Delta y_{t-i}] + [\alpha_{1,0}^M ec_{t-1} + \sum_{i=1}^p \alpha_{1,i}^M \Delta y_{t-i}] + \mathbb{I}_t^2[\alpha_{1,0}^U ec_{t-1} + \sum_{i=1}^p \alpha_{1,i}^U \Delta y_{t-i}] + \epsilon_{y_1,t} \quad (4.14)$$

$$y_{2,t} = \mathbb{I}_t^1[\alpha_{2,0}^L ec_{t-1} + \sum_{i=1}^p \alpha_{2,i}^L \Delta y_{t-i}] + [\alpha_{2,0}^M ec_{t-1} + \sum_{i=1}^p \alpha_{2,i}^M \Delta y_{t-i}] + \mathbb{I}_t^2[\alpha_{2,0}^U ec_{t-1} + \sum_{i=1}^p \alpha_{2,i}^U \Delta y_{t-i}] + \epsilon_{y_2,t} \quad (4.15)$$

where $\mathbb{I}_t^2 = 1$ if $y_{t-1} > \tau_2$ and $\mathbb{I}_t^2 = 0$ if $y_{t-1} \leq \tau_2$, $\mathbb{I}_t^1 = 1$ if $y_{t-1} \leq \tau_1$ and $\mathbb{I}_t^1 = 0$ if $y_{t-1} > \tau_1$. In this case the model is also called band-TVECM, and it consists of three regimes: above the higher threshold, in the band between the two threshold and under the lower threshold.

To understand the usage of this model, suppose that there is a transaction cost that prevents complete adjustment to the long-run equilibrium. If the gap between the actual value of the variable and the equilibrium is less than the cost of undertaking the transaction, it would not be profitable to switch funds between the securities. As such, there may be a neutral band within which the spread may fluctuate. Within this band, there are no economic incentives to act in a way that equates the spread with s ; outside of the band, however, there may be strong incentives to do it. So, there will be no tendency for mean reversion unless the difference with the equilibrium lies outside of the neutral band formed by adding and subtracting the transaction cost from the long-run value of the spread. Hence, inside the band, the behaviour of the spread could be a random walk.

I will check and use the TVECM constraining $\beta_1 = 1$ and $\beta_0 = 0$, to look for no arbitrage in the long term. The average transaction costs and Forex risk that arbitrageurs need to overcome, for example, in the case of one threshold and of a difference of always the same sign, is in general identified as $\tau + \beta_0$, so in our case simply as the thresholds themselves. The speed of adjustment parameters α_U , α_M and α_L characterize to what extent the price changes in each regime and in each market, allowing the studying of price discovery in each regime.

4.7.2 Estimation

Known threshold

I will tackle the estimation of the model with one threshold, knowing that the one with the two thresholds it is a simple generalization. If τ is known and the same error terms for the two

variables, the estimation of this process is simply done creating the dummy variable \mathbb{I}_t , according to whether y_{t-1} is above or below the threshold, and estimate the parameters by OLS. If you want to allow the variances of the error terms to differ across regimes, you can separate the observations according to whether y_{t-1} is above or below the threshold. Each segment of (4.12) can then be estimated using OLS.

Unknown threshold

In most instances, the value of the threshold is unknown and must be estimated along with the other parameters. Fortunately, in [19] Chan shows how to obtain a super-consistent estimate of the threshold. Clearly, the threshold has to be between the maximum and the minimum of the observations, normally excluding the highest and lowest 15%; if you have a very large dataset, you can exclude only the highest and lowest 10%. The procedure consists in trying all the possible value of $\tau = y_i$ and in estimating an equation in the form of (4.12). The regression containing the smallest residual sum of squares contains the consistent estimate of the threshold. Graphical techniques are really useful to improve this estimation. Thinking of the sum of squared residuals from any TAR model as being a function of the threshold value used in the estimation, $S(\tau)$ should be minimized at the true value of the threshold. Moreover, the sum of squared residuals will have several distinct local minima if there are several thresholds.

Unknown threshold and cointegrating vector

Hansen and Seo extend the literature by examining the case of an unknown cointegrating vector in [39]. They implement maximum likelihood estimation (MLE) of a bivariate TVECM with two regimes. Their algorithm involves a joint grid search over the threshold and the cointegrating vector while using the error-correction term as the threshold variable. All coefficients are allowed to switch between these two regimes. Only the cointegrating vector β remains fixed across all regimes, by construction. I follow this grid search estimation approach, subject to the constraint $\beta = [1, -1, 0]$, motivated by our cointegration results.

4.7.3 Testing the TVECM

The appropriate test for threshold behaviour is straightforward if the threshold value is known. Under the null hypothesis of linearity, (4.12) is an AR(n) process, as such, it is possible to use a standard F-test to determine whether $\alpha_{1,0}^L = \alpha_{1,0}^U$ and $\alpha_{2,0}^L = \alpha_{2,0}^U$.

However, if the threshold is unknown, under the null hypothesis of linearity, there is an unidentified nuisance parameter; indeed, under the null hypothesis that the model is linear (i.e., $\alpha_{1,1} = \alpha_{2,1}$), the estimate of τ can take on any value.

Following Davies [24], the test for a threshold model can be conducted using a supremum test. In fact, in [38] Hansen shows how to appropriately obtain the appropriate critical values using a bootstrapping procedure:

1. Search over all values of τ to find the best fitting TAR model.
2. Let SSR_u denote the unrestricted sum of squared residuals from the estimated threshold model. Similarly, let SSR_r denote the sum of squared residuals obtained from restricting

the model to be linear. Construct $F = \frac{(SSR_r - SSR_u)/n}{SSR_u/(T - 2n)}$, where n is the number of parameters estimated in the linear version and T the number of observation.

3. The distribution of F has to be found by one of the two alternative Hansen's bootstrapping methods [38]:
 - (a) The first technique is named 'Fixed regressor bootstrap' and sample T residuals as i.i.d. $N(0,1)$; the second bootstrap technique is named 'residual bootstrap' and uses random draws of the residuals from the linear model.
 - (b) Regress ϵ_t on a constant and y_{t-1} and call the sum of squared residuals SSR_r ;
 - (c) Regress ϵ_t on \mathbb{I}_t , $(1 - \mathbb{I}_t)$, $\mathbb{I}_t y_{t-1}$ and $(1 - \mathbb{I}_t)y_{t-1}$ for each τ and select the one that provides the best fit; call the sum of squared residuals from this supremum regression SSR_u .
 - (d) Use the two sums of squares to form F .
 - (e) Repeat this process several thousand times to obtain the distribution of F .

I will consider our model as threshold cointegrated if I can reject the null hypothesis of a linear VECM by either the 'residual bootstrap' or the 'fixed regressor bootstrap' methodology.

Chapter 5

Results

”A lot of things in life are first-order positive and second-order negative. [...] We have trouble delaying gratification, so we do a lot of things that are first-order positive, second-order negative. We buy bigger houses than we need only to find that rising interest rates make the mortgage payment untenable. We buy the sexy car only to discover later that it depreciates faster than the commuter car. A real advantage is conferred on people who can do things that are first-order negative, second-order positive. Especially if these first-order negatives are very visible costs with no immediate benefit in the short term and a non-linear benefit at some future time.”¹

This chapter is organized as follows: in the first part I will briefly run through the lag length selection and the unit root results for all the time intervals considered. Then, I will expose the core of the thesis: the results on cointegration, VECM, TVECM and price discovery. They will be presented first for all the time-span considered, then for the three historical periods selected (Sub-prime financial crisis, European Sovereign crisis and Quantitative easing period), and finally for the two time intervals selected in Chapter 3.

5.1 Lag length, Deterministic Coefficients and Stationarity

Lag length

First of all, the choice of the lag length is needed to perform unit roots and cointegration tests, and to decide how many lags include in the VECM and TVECM. I compared the results of the four most used lag selection criteria: Akaike Information Criteria (AIC), Hannan Quinn (HQC),

¹Farnam Street. (February 2018). *Your First Thought Is Rarely Your Best Thought: Lessons on Thinking* [Blog Post], retrieved from: <https://www.fs.blog/2018/02/first-thought-not-best-thought>

Shwarz Information Criteria (SIC) and Final Prediction Error (FPE). Let T be the number of observation, k be the number of estimated parameters in the model and RSS be the Residual Sum of Squares/ T .

$$AIC = 2k + T \ln(RSS) \qquad SIC = \ln(T)k - 2 \ln(RSS) \qquad (5.1)$$

$$HQC = -2RSS + 2k \ln(\ln(n)) \qquad FPE = \frac{N+q}{N-q} SSR \qquad (5.2)$$

Given a set of candidate models, for each criteria the preferred one is the model that minimize the respective quantity AIC, SIC, HQC or FPE. Between the four models selected the choice is made pondering between parsimony and statistical significance of the Fisher test of the regression model with the lag length selected.

Looking to figure C.4, C.3 and C.5 in Appendix C and considering the literature on Sovereign and Corporate CDSs, there is no theoretically reason to include the drift term; it is instead considered the intercept as in all the previous studies.

Unit root tests

All the famous econometric works on corporate and sovereign Credit Default Swaps, tested and confirmed the non stationarity of CDS spreads. Taking also in consideration the plots in Appendix C, it is straightforward to expect statistical evidences for unit root processes.

As I described in the previous chapter I performed three unit roots tests: the Augmented Dickey-Fuller, the Phillips-Perron and the KPSS test. To interpret the result is useful to remark again that the first two tests assume as null hypothesis the presence of a unit root and as alternative the stationarity, while KPSS tests the null hypothesis of trend-stationarity against the alternative of unit root process. I provided the p-values in table [reftable:urca](#) for all the time-span and in tables A.1, A.6 and A.12 of the Appendix A, for all the other time intervals on which I focused.

To give an idea of the general outcomes, consider that on 168 time series 112 times there is statistical evidence for unit roots for all the tests, 43 times only one test reject the presence of unit roots, 10 times two tests reject it and only 3 times stationarity is accepted from the three tests; how it was expected, more than 98% of the time at least one of the test shows statistical evidence for the presence of a unit root.

Performing the tests on the three historical time intervals, I find out that stationarity are concentrated during the sub-prime financial crisis, while during the quantitative easing all the tests confirm the hypothesis of unit root processes. Indeed, during the sub-prime crisis, there are some series not affected by the event; they maintain a low mean and a constant variance across all the time interval. Two of the three stationary series are an example of this observation: the dollar and euro spreads of Deutsche Bank, as it can be seen in the comparison with the CDS of CITI in figure C.13. A similar argument can be applied to the third of the stationary time series, the UBS CDS when it goes from December 2007 to January 2014.

5.2 Analysis from December 2007 to December 2017

5.2.1 Cointegration analysis

Engle-Granger approach

The first way in which I tested the CDS spreads for cointegration between the price in dollar and in euro is by the Engle-Granger approach (EG).

Taking into account the two step procedure and its limits discussed in Chapter 4, it is worth always to check the first regression, both of the dollar price on the euro one and viceversa; the result is really robust w.r.t. this choice.

The EG approach in table 5.1 testimonies strong cointegration through all the time-span, except for Nomura, where the the hypothesis of unit roots in the residual is rejected at 10%. The interpretation is that spreads in euro and dollar tends to not meander away, because it exist a linear combination of them that is stationary. Consequently, the prices could:

1. more or less coincide;
2. tend to coincide in the long-run smoothing the short-term differences;
3. behave in similar way maintaining a constant difference due to, for example, a persistent risk priced in one of the spreads;
4. behave differently without exceed to much in the distance between one another, for example because the price in the two markets is caused by different factors.

To understand in which case are the CDS spreads in euro and dollar, it is possible to take advantage of the Johansen test on restrictions on the cointegrating vector.

Johansen approach

I will start by checking again the cointegration, but in the Johansen framework and using the trace statistic. First, I test the null hypothesis of cointegration rank 0, against the alternative of cointegration rank greater than 1; all banks show statistical evidence to reject cointegration rank 0. Second, I test the null hypothesis of cointegration rank test less than one against the alternative of cointegration rank equal to two. In this case, no test rejects the null hypothesis. I can conclude that the two cointegration tests lead to coherent results.

As I described in the previous chapter, the Johansen approach allow us to test restrictions on the cointegrating vector. If Credit Default Swaps are priced equally in the long run, they should be cointegrated with cointegrating vector $[1, -1, c]$, suggesting a stationary difference. If $c = 0$, the two prices will coincide in the long run; if $c \neq 0$ one of the two markets price a constant market friction or risk factor absent in the other market. If the prices do not cointegrate with the $[1, -1, c]$ restriction imposed, then (i) the two markets price CDS differently (in excess of a constant amount), (ii) at least one market price contains one or more time varying market

friction or risk factor that reflect something other than credit risk, or (iii) at least one market price contains time-varying non-transient measurement error. From the introduction to CDS contract specifications in chapter 1, it can be suspected a priori that one of the market may price:

1. an effect of level of correlation between default risk and exchange rates, in particular in a period where timing of crisis and monetary policies were different in the two markets;
2. the risk of dramatic Foreign Exchange rate devaluation in conjunction with default events;
3. the different liquidity between the two markets.

These three factors are likely to result in a case (ii) failure of the cointegration tests. Furthermore, as I described in chapter 3, the usual way in which OTC derivatives are quoted as average of the long position furnished by different traders, also if no exchange happens, can be considered a measurement error; if one market is more liquid than the other, so that the lack of transactions to determine the price are always in the same market, can lead to a case (iii) failure; it is difficult to imagine that in two liquid markets as the euro and the dollar ones, but using other currencies this can be a factor.

The Johansen approach on constraints is based on a log-likelihood ratio statistic to test the restricted model as the null hypothesis and unrestricted one as the alternative. Looking to the table 5.1, both of the constraints are rejected six times, $[1,1,c]$ is not rejected eight times, while $[1,1,0]$ only five times. These outcomes mean that 60% of the time the euro market and the dollar market price CDS equally in the long run, while the other 40% time-varying risks or frictions induces the price to behave differently in the long run, without distancing excessively. Another potential explanation is similar to the one that Fontana and Scheicher gave to explain the lack of cointegration between CDS and bonds before the sub-prime crisis. The two prices are approximately equal in the sense that the size of the difference is similar; however, probably in part due to low trading activity in one of the two markets in certain periods of time, spreads are relatively constant in one of the two markets. Arbitrage forces do not come into play, i.e. CDS and bond spreads move in an unrelated manner because they do not move outside the arbitrage bounds determined by transaction costs.

Table 5.1: Unit roots and Cointegration analysis from December 2007 to December 2017

The first two rows present the size of the sample and the number of the observation subtracting the missing values. The third row shows the lag selection, obtained comparing the results of "VARselect" of the package "vars". In the fourth row there is the cointegration rank; if cointegration holds it will be always one, dealing with two variables. From the fifth to the seventh rows there are the p-values of the unit root tests; the function "adfTest" was used from the package "fUnitRoots", while the self-written *ppPers.test* and *kpssPers.test* were used in the other two cases; in bold: stationarity. The following three rows report the p-values of the Engle-Granger and Johansen cointegration test; the first is computed by *coint.test* of the package "aTSA"; for the Johansen approach, the trace statistic was used thanks to the *ca.jo* and *cajorls* functions of the package "urca"; in bold: rejections of cointegration. The last two rows presents the p-values of the Johansen test on restricted cointegrating vectors; they were computed by the *blrtest* and *cajorls* functions of the package "urca"; in bold: rejection of the constraints.

	Barclays		BNP Paribas		CITI		Deutsche Bank		Goldman Sachs	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	521	521	521	521	521	521	521	521	521	521
Without missing	521	521	521	521	510	510	521	521	510	510
Lags number	2	-	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,1501	0,1891	0,3884	0,3934	0,2312	0,2322	0,0446	0,0323	0,0970	0,0969
PP P-Values	0,0405	0,0767	0,5607	0,5833	0,0417	0,0417	0,1619	0,0967	0,0134	0,0126
KPSS P-Values	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	0,0518	0,0480	<0,0100	<0,0100
EG P-Values	<0,01	-	<0,01	-	<0,01	-	<0,01	-	<0,01	-
Trace P-Value r=0	0,0000	-	0,0000	-	0,0138	-	0,0000	-	0,0000	-
Trace P-Value r=1	0,1873	-	0,4190	-	0,1823	-	0,0544	-	0,0687	-
P-Values $\beta=[1, -1, c]$	0,0258	-	0,0000	-	0,4488	-	0,0946	-	0,0031	-
P-Values $\beta=[1 -1 0]$	0,0829	-	0,0000	-	0,1344	-	0,0009	-	0,0019	-

	HSBC		HSBC Holding		JP Morgan		Mediobanca		Morgan Stanley	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	521	521	521	521	521	521	521	521	521	521
Without missing	520	521	490	490	328	328	521	521	510	510
Lags number	3	-	2	-	2	-	2	-	4	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,1050	0,1069	0,0898	0,1013	0,2722	0,2773	0,3126	0,3272	0,0185	0,0167
PP P-Values	0,0534	0,0624	0,1661	0,2283	0,0876	0,0883	0,6296	0,6385	< 0,0100	< 0,0100
KPSS P-Values	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100
EG P-Values	<0,01	-	<0,01	-	0,0277	-	<0,01	-	<0,01	-
Trace P-Value r=0	0,0000	-	0,0000	-	0,1414	-	0,0000	-	0,0023	-
Trace P-Value r=1	0,1676	-	0,0500	-	0,3652	-	0,5626	-	0,2248	-
P-Values $\beta=[1, -1, c]$	0,9407	-	0,3370	-	0,7930	-	0,0003	-	0,0002	-
P-Values $\beta=[1 -1 0]$	0,0040	-	0,0000	-	0,2357	-	0,0011	-	0,0005	-

	Nomura		RBS		Standard Chartered		UBS		Wells Fargo	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	521	521	521	521	521	521	521	521	521	521
Without missing	286	286	521	521	338	338	521	521	356	356
Lags number	2	-	2	-	2	-	4	-	3	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,5415	0,5847	0,2847	0,3060	0,2861	0,2769	0,0452	0,0824	0,4867	0,4568
PP P-Values	0,7955	0,8178	0,2857	0,2869	0,0998	0,0804	< 0,0100	< 0,0100	0,4212	0,3360
KPSS P-Values	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100	<0,0100
EG P-Values	0,0749	-	<0,01	-	<0,01	-	<0,01	-	<0,01	-
Trace P-Value r=0	0,0082	-	0,0000	-	0,0002	-	0,0000	-	0,0000	-
Trace P-Value r=1	0,2705	-	0,3999	-	0,3086	-	0,0912	-	0,5969	-
P-Values $\beta=[1, -1, c]$	0,5412	-	0,0038	-	0,8971	-	0,8883	-	0,0000	-
P-Values $\beta=[1 -1 0]$	0,6707	-	0,0035	-	0,0011	-	0,4731	-	0,0003	-

5.2.2 Price Discovery

The analysis in the previous section concentrates on the long-run equilibrium behavior of the series. In the rest of the section, I analyze the dynamic behavior of CDS prices in the two markets, with a focus on lead-lag relations. The objective is to find which market provides more timely information. It is necessary first to estimate the VECM 4.1, imposing, when it is statistically significant, the restrictions on the cointegrating vector found in the previous section. The estimates of the parameters and their standard errors are presented in the first seven rows of table 5.2. Remember that it has sense to compute the measures of the contributions to price discovery only in the nine cases in which at least one of the two restrictions is valid.

Looking to table 5.3 and 5.2, in two of the nine cases, α_y is significantly positive and α_x statistically insignificant, indicating that the dollar market contributes to price discovery as the CDS euro market would adjust to correct the pricing differentials from the long-term relationship. In other words, in this case the dollar market would move ahead of the euro market as relevant information reaches investors. The euro market appears to have a significant role in the two cases in which α_x is significantly negative, while α_y is statistically insignificant; GG and the HAS measure confirm the results. In the case of Nomura, while both euro and dollar markets contribute significantly, the dollar market is slightly dominant; the HAS measure suggests more than 50% of the discovery occurs in it. In the remaining four cases where no one of the coefficients is enough significant, no lead-lag relation can be detected.

It is worth to notice that not only the price discovery happens half of the time in one market and half in the other, but also that, with the only exception of the GG measure for Deutsche Bank, less than 60% of the discovery happens in the dominant market.

Table 5.2: Linear VECM Coefficients from December 2007 to December 2017

In this table are collected the coefficients estimated for the linear VECM; the estimation was done by the function *cajorls* in the package "urca". Each coefficient is followed by its self-computed standard error. The coefficients of β are normalized by β_1 . If constraints on the cointegrating vector vectors are not rejected, the beta are fixed before estimation, and consequently the standard errors are zero. '***' = p-value < 0.01, '**' = p-value < 0.05, '*' = p-value < 0.1.

Coefficients	Barclays		BNP		CITI		Deutsche B.		Goldman S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	-0,909***	-0,3427	0,2098	1,0691***	-1,0359	-0,955	-0,1107	0,3498	3,1893	3,2835
	[0,2552]	[0,2364]	[0,4062]	[0,3981]	[2,3036]	[2,2998]	[0,2703]	[0,2848]	[2,1342]	[2,1287]
β_1 and β_2	1	-1	1	0,9825***	1	-1	1	-1	1	-1,0058***
	[0]	[0]	[0]	[0,0015]	[0]	[0]	[0]	[0]	[0]	[0,0014]
β_0	0	-	-1,3865***	-	-0,255	-	0,7661	-	0,506***	-
	[0]	-	[0,1788]	-	[0,2151]	-	[0,5671]	-	[0,2218]	-
$c_{out,1}$ and $c_{out,2}$	-	-	0,0000	0,0000	-0,0150	-0,0149	0,0000	0,0000	0,0000	0,0000

	HSBC		HSBC H.		JP Morgan		Mediob.		Morgan S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	0,4823	0,953***	0,032	0,4467**	-0,9437	-0,8583	-0,3422***	0,06758	4,4834***	4,5476***
	[0,3605]	[0,3594]	[0,1891]	[0,179]	[1,6087]	[1,5966]	[0,1314]	[0,1352]	[1,1618]	[1,1686]
β_1 and β_2	1	-1	1	-1	1	-1	1	-0,9799***	1	-1,0162***
	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0,0050]	[0]	[0,0033]
β_0	-0,4542	-	-1,6122*	-	-0,1145	-	-3,8956***	-	2,0852***	-
	[0,3537]	-	[0,8049]	-	[0,1656]	-	[0,9925]	-	[0,7002]	-
$c_{out,1}$ and $c_{out,2}$	-0,0002	-0,0002	-0,0023	-0,0021	-0,1978	-0,1968	0,0000	0,0000	0,0000	0,0000

	Nomura		RBS		Standard C.		UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	0,1431***	0,1818***	-1,0977***	-0,543*	-0,2166	0,154	-0,6943***	-0,3975***	0,2489	1,2632***
	[0,0467]	[0,0439]	[0,3050]	[0,3035]	[0,2460]	[0,2408]	[0,1533]	[0,1424]	[0,2425]	[0,2327]
β_1 and β_2	1	-1	1	-0,9919***	1	-1	1	-1	1	-1,0151***
	[0]	[0]	[0]	[0,0027]	[0]	[0]	[0]	[0]	[0]	[0,0036]
β_0	0	-	-1,5588***	-	-1,529	-	-0,5574	,,	0,9135***	,,
	[0]	-	[0,4423]	-	[1,1717]	-	[0,9300]	,,	[0,2612]	,,
$c_{out,1}$ and $c_{out,2}$	-	-	0,0000	0,0000	-0,0008	-0,0008	0,0000	0,0000	0,0000	0,0000

Table 5.3: Price Discovery from December 2007 to December 2017

The first row presents the restriction chosen on the cointegrating vector. The second and third rows contain the p-values of the speed adjustment coefficients computed by the *cajorls* function; in bold: statistically significative. In the fourth and fifth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the sixth and seventh lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share. Finally in the last row there are the markets that slightly dominates the discovery.

	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.	JP Morgan
Coint. vector	[1,-1,0]	REFUSE	[1,-1,c]	[1,-1,c]	REFUSE	[1,-1,c]	[1,-1,c]	[1,-1,c]
pval(α_1)	0,0004	-	0,6531	0,6823	-	0,1815	0,8658	0,5579
pval(α_2)	0,1479	-	0,6781	0,2199	-	0,0083	0,0129	0,5912
HANS 1	0,4909	-	0,5000	0,5304	-	0,5046	0,5390	0,5000
HANS 2	0,5091	-	0,5000	0,4696	-	0,4954	0,4610	0,5000
GG 1	-0,6051	-	-11,8020	0,7596	-	2,0248	1,0771	-10,0530
GG 2	1,6051	-	12,8020	0,2404	-	-1,0248	-0,0771	11,0530
Results	€		no	\$		\$	\$	no

	Mediob.	Morgan S.	Nomura	RBS	Standard C.	UBS	Wells F.
Coint. vector	REFUSE	REFUSE	[1,-1,0]	REFUSE	[1,-1,c]	[1,-1,c]	REFUSE
pval(α_1)	-	-	0,0024	-	0,3793	0,0000	-
pval(α_2)	-	-	0,0000	-	0,5230	0,0054	-
HANS 1	-	-	0,5153	-	0,4375	0,4945	-
HANS 2	-	-	0,4847	-	0,5625	0,5055	-
GG 1	-	-	4,6992	-	0,4156	-1,3393	-
GG 2	-	-	-3,6992	-	0,5845	2,3393	-
Results			\$		€	both	

5.2.3 TVECM and its price discovery analysis

The introduction of the TVECM model in this thesis is motivated by these three observations:

- There is a different behaviour in the adjustments when the euro spread is higher than the dollar one from when the dollar spread is higher than the euro one.
- When the difference has almost always the same sign, also when it is quite large, arbitrage forces do not come into play. This could lead to CDS written on the same underlying bond, whose prices in different currencies do not move in the same manner because they do not move outside the arbitrage bounds determined by transaction costs.
- Combining the two previous observations, it can happen that a difference, that passes from positive to negative values, is exploitable only when the value is largely negative or positive. In particular there is a band in which the difference is really persistent, and outside this band it is observed a strong mean reversion.

I will test both for one and two threshold models. In cases where there is one positive threshold, it is expected a faster adjustment process in the upper regime, in line with our reasoning on the behaviour of arbitrageurs; in the case the threshold is really small in absolute value, it indicates a separator between behaviors in case of negative or positive differences; in the case of a negative threshold, it is expected a faster adjustment process in the lower regime. Following the same argument the adjustment process of a three regimes TVECM from outside the band to inside it, should be faster than inside the band. Inside the band, the process should tend to a long run equilibrium or behave as a random walk.

In table 5.4 the results of the Hansen [38] bootstrapping test using the "Fixed regressor bootstrap" and the "Residual bootstrap" methods are presented. Using these statistical tests, it is checked the null hypothesis of a linear VECM model versus the alternative hypothesis of a TVECM. Then, it is also tested the linear VECM versus both one threshold VECM and two thresholds VECM and finally the one threshold VECM against the two thresholds VECM. All these tests are performed with 50 and 100 bootstrapped values. A Seo test to check for no cointegration versus TVECM is also performed. Each time the TVECM is tested, the p-value is given for the restricted cointegrating vector $[1,-1,0]$.

The tests lead to one threshold VECM eight times and two thresholds VECM for RBS and Deutsche Bank. It is worth to notice that four of the six times in which the Johansen approach does not accept the restrictions, the cointegrating vector $[1,-1,0]$ is not rejected the threshold vector error correction model. This means that the TVECM model extends the case in which the long term equilibrium is statistically valid and improves the modelling and price discovery analysis of the series for which the constraint was already accepted.

The estimation of the one threshold model is shown in table 5.5. It is evident the difference of the coefficients between the lower and the upper regime, they differ particularly in the cases of Nomura and Goldman Sachs.

For Nomura and Standard Chartered, the threshold is respectively negative and positive; in the case of the Japanese bank, the threshold is -17 and the coefficients of the lower regime are really high: -17 basis points can be seen as the transactions cost under which the arbitrageurs take immediately advantage of the difference in price, as it can be seen from the strong mean reversion. In all the other cases the threshold is near to 0, separating the behaviour of the adjustments when the euro spread exceeds the dollar one and when the dollar spread exceeds the euro one. From the two estimation of the band-TVECM model in table 5.7, RBS shows exactly the expected behaviour. The thresholds have opposite signs, but the negative one is really close to 0, the positive can be considered a first approximation of the transactions costs. In the lower and upper regime the adjustments are higher than inside the band, where the behaviour is almost a random walk process. Deutsche Bank has, as well, two opposite sign thresholds, but the interpretation of the coefficients is not clear, because it seems that the adjustment is done inside the band.

Finally, the price discovery is described in table 5.6 for the one threshold models and in the right part of table of 5.7 for the two thresholds models.

In the lower regime of the one thresholds model the both market lead the inclusion of new information four times, while twice the α_y is significantly positive and α_x statistically insignificant, indicating a guidance of the dollar market in the discovery, as the CDS euro market would adjust to correct the pricing differentials from the long-term relationship.

In the upper regime, three times the both coefficients are highly significatives, but in the case of Wells Fargo, the HAS measure suggest a leadership of the euro market. For HSBC Holding and Standard Chartered, all the measures and the coefficients indicate a predominance of the euro market. The price discovery of the upper regime of Nomura is strongly guided by the dollar market.

Both markets dominate the discovery in the case of the two band-VECM.

As in the linear models, except for isolate cases, none of the two CDS markets seems to lead the inclusion of new information.

Table 5.4: TVECM tests from December 2007 to December 2017

The table is subdivided in two horizontal parts: the tests ran with 50 bootstrapped values and with 100 bootstrapped values. The first and the second rows of each part present the Hansen-Seo p-values of the "Fixed regressor bootstrap" and the "Residual bootstrap" tests; the values were computed by the *TVECM.HStest* function of the "*tsDyn*" library; in bold: no evidence for TVECM. From the third to the fifth rows there are the p-values of the number of thresholds; values performed by *TVAR.LRtest* function of the "*tsDyn*" library; in bold: no rejection of H0. In the last row it is shown the choice of the model: 1 threshold, 2 thresholds, no TVECM.

50 boots	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.
p-value HS FR	0,00	0,02	0,54	0,00	0,04	0,18	0,02
p-value HS RB	0,00	0,00	0,64	0,00	0,08	0,28	0,04
lin VS 1th	0,00	0,00	0,00	0,00	0,00	0,02	0,00
lin VS 2th	0,00	0,00	0,00	0,00	0,00	0,04	0,00
1th VS 2th	0,12	0,40	0,26	0,02	0,20	0,10	0,76
100 boots							
p-value HS FR	0,00	0,00	0,40	0,00	0,04	0,17	0,05
p-value HS RB	0,01	0,00	0,67	0,00	0,17	0,15	0,03
lin VS 1th	0,00	0,00	0,01	0,01	0,00	0,01	0,00
lin VS 2th	0,02	0,00	0,00	0,00	0,00	0,01	0,00
1th VS 2th	0,10	0,26	0,32	0,01	0,19	0,07	0,69
Results	1 th	1 th	no	2 th	1 th	no	1 th

50 boots	JP Morgan	Mediob.	Morgan S.	Nomura	RBS	Standard C.	UBS	Wells F.
p-value HS FR	0,40	0,18	0,60	0,00	0,00	0,00	0,02	0,00
p-value HS RB	0,58	0,22	0,48	0,00	0,00	0,00	0,00	0,00
lin VS 1th	0,04	0,02	0,00	0,02	0,00	0,00	0,02	0,02
lin VS 2th	0,02	0,42	0,00	0,02	0,00	0,00	0,00	0,00
1th VS 2th	0,22	0,98	0,02	0,30	0,00	0,08	0,06	0,74
100 boots								
p-value HS FR	0,40	0,19	0,51	0,00	0,00	0,00	0,00	0,01
p-value HS RB	0,34	0,22	0,60	0,00	0,00	0,00	0,00	0,00
lin VS 1th	0,00	0,04	0,00	0,03	0,00	0,00	0,01	0,04
lin VS 2th	0,01	0,35	0,01	0,11	0,00	0,01	0,02	0,04
1th VS 2th	0,21	0,97	0,06	0,40	0,00	0,10	0,13	0,68
Results	no	no	no	1 th	2 th	1 th	1 th	1 th

Table 5.5: One threshold VECM coefficients from December 2007 to December 2017

In this table are collected the coefficients estimated for the 1 threshold VECM, first the ones of the lower regime, then the one of the upper regime; the estimation was done by the self-made function *persTVECM*. Each coefficient is followed by its standard error. The cointegrating vector is restricted to (1,-1,0). *** = p-value < 0.01, ** = p-value < 0.05, * = p-value < 0.1.

	Barclays		BNP Paribas		Goldman S.		HSBC H.	
α_x^L and α_y^L	0,7106***	1,0386***	0,4980	1,3115*	-25,7647	-25,3878	0,4895	1,1750***
	[0,3546]	[0,3325]	[0,5834]	[0,5730]	[64,5296]	[64,3730]	[0,5105]	[0,4846]
$c_{out,1}^L$ and $c_{out,2}^L$	0,8007	0,7466	0,0398	1,5622	-0,8357	-0,8361	-0,9574	-1,2170**
	[0,7800]	[0,7314]	[2,1122]	[2,0744]	[2,0885]	[2,0834]	[0,6299]	[0,5979]
α_x^U and α_y^U	-3,6863***	-2,6473***	-0,3639	0,3533	-1,5030	-1,4148	-0,4105	0,0152
	[0,4247]	[0,3982]	[0,7019]	[0,6893]	[1,9607]	[1,9559]	[0,2829]	[0,2686]
$c_{out,1}^U$ and $c_{out,2}^U$	5,2469***	4,3980***	0,8630	0,8101	0,5049	0,4597	1,4654	0,6635
	[1,4715]	[1,3798]	[0,6817]	[0,6695]	[1,9434]	[1,9387]	[1,0434]	[0,9904]
Threshold	1		-0,785		0,01		0,05	

	Nomura		Standard C.		UBS		Wells F.	
α_x^L and α_y^L	-6,8144***	-4,8596***	1,3656***	2,1777***	0,5156	1,2746***	1,8083***	2,7649***
	[1,7895]	[1,6952]	[0,6406]	[0,6238]	[0,7165]	[0,6563]	[0,7339]	[0,7030]
$c_{out,1}^L$ and $c_{out,2}^L$	-134,2085***	-98,3972***	-0,0911	-0,4439	1,0131	0,9766	2,6185***	3,0750***
	[33,7910]	[32,0095]	[0,7303]	[0,7111]	[0,8653]	[0,7926]	[0,8764]	[0,8395]
α_x^U and α_y^U	-0,0160	0,04614*	-0,9518*	-0,2445	-0,8923***	-0,6294***	0,5570***	1,6823***
	[0,0697]	[0,0660]	[0,5118]	[0,4983]	[0,1699]	[0,1556]	[0,2681]	[0,2568]
$c_{out,1}^U$ and $c_{out,2}^U$	-0,1610	0,0573	3,1429	0,4287	0,7919	1,2028	-0,4632	-0,6362***
	[0,4040]	[0,3827]	[2,5446]	[2,4779]	[1,0647]	[0,9752]	[0,3040]	[0,2912]
Threshold	-17		2,56		0,4		-0,375	

Table 5.6: One threshold VECM price discovery from December 2007 to December 2017

The table is subdivided in two horizontal parts: the price discovery in the lower regime and in the upper regime. The first and second rows of each part contain the p-values of the speed adjustment coefficients computed by the *persTVECM* function; in bold: statistically significant. In the third and fourth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the fifth and sixth lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share.

	Barclays	BNP	Goldman S.	HSBC H.	Nomura	Standard C.	UBS	Wells F.
$pval(\alpha_1)$	0,0456	0,3873	0,6899	0,3381	0,0031	0,0350	0,4556	0,0142
$pval((\alpha_2))$	0,0019	0,0217	0,6935	0,0157	0,0162	0,0006	0,0494	0,0001
GG 1	3,1658	1,6122	-67,3734	1,7141	-2,4860	2,6815	1,6794	2,8904
GG 2	-2,1658	-0,6122	68,3734	-0,7141	3,4860	-1,6815	-0,6794	-1,8904
HAS 1	0,5006	0,5000	0,5000	0,5004	0,5031	0,5002	0,5004	0,5014
HAS 2	0,4994	0,5000	0,5000	0,4996	0,4969	0,4998	0,4996	0,4986

	Barclays	BNP	Goldman S.	HSBC H.	Nomura	Standard C.	UBS	Wells F.
$pval(\alpha_1)$	0,0000	0,6335	0,4437	0,1475	0,3569	0,0590	0,0000	0,0385
$pval((\alpha_2))$	0,0000	0,5796	0,4698	0,9550	0,0858	0,5997	0,0001	0,0000
GG 1	-2,5481	0,4926	-16,0361	0,0356	0,7426	-0,3457	-2,3940	1,4950
GG 2	3,5481	0,5074	17,0361	0,9644	0,2574	1,3457	3,3940	-0,4950
HAS 1	0,5044	0,5436	0,5000	0,5334	0,3257	0,5133	0,5042	0,4585
HAS 2	0,4956	0,4564	0,5000	0,4666	0,6743	0,4867	0,4958	0,5415

Table 5.7: Two thresholds VECM coefficients and price discovery from December 2007 to December 2017

In left table are collected the coefficients estimated for the 2 threshold VECM, first the ones of the lower regime, second the ones of the band, third the upper regime's ones; the estimation was done by the self-made function *persTVECM*. Each coefficient is followed by its standard error. The estimation is done under the cointegrating vector restricted to (1,-1,0). '****' = p-value < 0,01, '***' = p-value < 0,05, '**' = p-value < 0,1. The right table contains the price discovery in the lower regime, inside the band and in the upper regime. the first row contains the p-values of the speed adjustment coefficients computed by the *persTVECM* function; in bold: statistically significant. In the second row there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the third line there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share.

	Deutsche B.		RBS			Deutsche B.		RBS	
α_x^L and α_y^L	0,0779	0,6461	1,4079	2,6050***	$pval(\alpha_1)$ and $pval(\alpha_2)$	0,8929	0,2884	0,0572	0,0004
	[0,5783]	[0,6079]	[0,7386]*	[0,7240]	GG1 and GG2	1,1371	-0,1371	2,1762	-1,1762
$c_{out,1}^L$ and $c_{out,2}^L$	-1,3642	1,0145	2,7814	4,0525***	HAS1 and HAS2	0,5000	0,5000	0,5001	0,4999
	[3,1059]	[3,265]	[1,8974]	[1,8601]					
α_x^M and α_y^M	-1,2735**	-1,0855**	0,3879	0,5317	$pval(\alpha_1)$ and $pval(\alpha_2)$	0,0115	0,0402	0,5614	0,4169
	[0,5021]	[0,5278]	[0,6675]	[0,6544]	GG1 and GG2	-5,7739	6,7739	3,6991	-2,6991
$c_{out,1}^U$ and $c_{out,2}^U$	0,1469	0,2280	0,0625	0,3089	HAS1 and HAS2	0,4996	0,5004	0,5001	0,4999
	[0,6405]	[0,6733]	[1,0711]	[1,0500]					
α_x^U and α_y^U	0,0518	0,9787	-5,7274***	-5,6170***	$pval(\alpha_1)$ and $pval(\alpha_2)$	0,9796	0,6467	0,0000	0,0000
	[2,0300]	[2,1340]	[0,8186]	[0,8025]	GG1 and GG2	1,0559	-0,0559	-50,8677	51,8677
$c_{out,1}^U$ and $c_{out,2}^U$	-4,6272	-4,5037	7,3804	8,5311	HAS1 and HAS2	0,4876	0,5124	0,5003	0,4997
	[6,5160]	[6,8497]	[5,4014]	[5,2952]					
Thresholds	-3,5300	2,3000	-0,3100	4,0200					

5.3 The three historical time periods

5.3.1 Cointegration tests

During the sub-prime financial crisis and the European sovereign debt crisis, all the banks show evidence of cointegration with at least one between the Johansen or EG approach. While, considering the quantitative easing period, the VAR model in difference is more feasible for RBS, Deutsche Bank and Mediobanca; indeed, the trace statistic used by the Johansen approach leads to cointegration rank equal to 0. All these results are shown in the tables A.1,A.6 and A.12 in Appendix A.

Considering the two constraints $[1,-1,0]$ and $[1,-1,c]$ on the cointegrating vector, during the sub-prime financial crisis only BNP rejects the both, while HSBC Holding rejects only the first one. When I chose this time period, I make it last till the starting of 2011 in order to consider both the crisis itself and the economic, financial and monetary policies applied to recover the economy, such as the Dodd-Frank Act. The consequences of this choice are visible in figure C.6 in Appendix C; while the difference in prices started with a really strong variance, it ended with a quite long period of lower volatility. Following this argument it seems reasonable that in the long run the prices tend to coincide.

The opposite outcome characterizes the European crisis, where nine out of fourteen CDS reject the two restrictions. Plotting one of these time series in C.9 in Appendix C, it seems that the difference between spreads is persistent, with no tendency of mean reverting, unless for large and sudden discrepancies; this behaviour is really common during this time period.

Finally, during the quantitative easing period, only three pair of CDS do not reject the second restriction, and only one does not reject the first. This period is characterized by time varying difference in prices in different currencies, strongly persistent and with low variance.

5.3.2 Estimation and Price Discovery

Starting from the estimation of the VECM in table A.2, the speed of adjustment coefficients are significant and generally high in absolute value, reflecting a strong and fast mean reversion effect. During the sub-prime financial crisis, as it can be concluded from table A.3, the dollar market owns more leadership in the discovery process. It strongly guides the inclusion of new information in the prices of four issuers, for other three issuers the discovery happens in both markets, while the euro market leads only once.

For the 5 cointegrated pairs considered in table A.7, the coefficients confirm a weaker adjustment process; this result is also due to long periods in which the prices perfectly coincide in the two currencies, so no correction happens. Table A.8 shows that in this time interval the dollar market

slightly leads the discovery: for RBS and Barclays α_y is the only significant coefficient, while for HSBC Holding and Nomura, where both speed coefficients are significant, the GG and HAS measures indicate the dominance of the greenback. The euro market slightly guides the discovery only for Deutsche Bank, while for HSBC the new information is immediately included in both markets.

Finally, for the three banks that do not reject at least one constraint through the quantitative easing regime, the results are summarized in table A.13 and A.13.

5.3.3 TVECM

The outcomes of tests on linear and threshold models in table A.4 confirm that the original VECM fits really good the period from December 2007 to May 2011; the TVECM is more feasible only for Morgan Stanley and UBS. In both cases the threshold is almost 0, indicating a strong difference in the behaviour when the euro or the dollar spread is higher. In table A.5, Morgan Stanley CDS spreads show a strong mean reversion effect in the lower regime and that price discovery happens in both markets.

Table A.9 and A.10 show that the modelling of three pairs of cointegrated CDS is improved by the TVECM during the European crisis. Moreover, Wells Fargo, Morgan Stanley and BNP Paribas, who reject the constraints in the linear model, accept it in the case of the TVECM. BNP and Wells Fargo threshold separates the negative difference regime from the positive one, while the Nomura one is strongly negative and leads to the same previous interpretation in the 2007-2017 period. Regarding the two thresholds VECM, Barclays individuates a negative band under which mean reversion is quite strong, HSBC Holdings shows a weird positive band where the speed coefficients are really high and Morgan Stanley adjusts fast its deviations from the large positive band, particularly in the upper regime. For Morgan Stanley and Nomura, the threshold can be definitely interpreted as an approximation of the transaction costs.

In table A.11, the price discovery of the lower regimes happens three times in both markets, once in the dollar one and once in the euro one. In the upper regimes, once in the euro market, once in the dollar one, once in both with a little predominance of the euro market.

The most interesting case is definitely the quantitative easing period, it clearly proves how helpful could be to complement a linear VECM analysis with the TVECM. Under linear model the restriction $[1,-1,0]$ is not rejected only three times, so someone could guess that the relation between the two variables is a different one. But, looking at the results in table A.15 and A.16, the TVECM model allow this restriction in other 9 pair of cointegrated variables. The pair of CDS written on CITI, Goldman Sachs, Morgan Stanley and UBS bonds have one threshold really close to zero; the first two show an extremely fast mean reversion in the lower regime, the other two in both regimes. Deutsche Bank selects a negative band under which the mean reversion

is strong, while Standard Chartered selects a positive one above which mean reversion is strong as well, visible also in figure C.10. HSBC Holdings, Mediobanca and Nomura hold a positive threshold close to 4 basis points, above which, the mean reversion is really strong.

The price discovery presented in table A.17 seems to happen in both markets across all the regimes and CDS, with the exemption of the lower and upper regime of Standard Chartered, where the dollar leads the discovery, and the lower regime of Nomura, where the euro market has the leadership.

5.4 The two regimes

5.4.1 Cointegration analysis

In the first period of high volatility and fast changing difference of prices between CDS in different countries, only Nomura do not reject the null hypothesis of no cointegration both in EG and Johansen tests. While in the second regime, that coincides with the quantitative easing period, when volatility is lower and the difference more persistent, the only CDSs prices that do not cointegrate are the one written on RBS bond.

During the first regime only, the restriction $[1,-1,0]$ is not rejected six times while $[1,-1,c]$ eight times. This means that, while the prices do not meander away for the fourteen pair of CDS spreads, half of the time they tend to price CDSs in the same way, the other half not. The results of the cointegration analysis are summarized in table A.18. The contribution of the TVECM is small during this period. In table A.21 BNP, HSBC Holding and Wells Fargo accept the restriction when described by a one threshold model, while UBS seems to be modelled better by a two threshold VECM than a linear VECM one.

Remember that in the quantitative easing interval the linear model does not reject a restriction only three times; it can be concluded that during the period of high volatility that coincides with the two crisis, the linear VECM describes well the behaviour of the series. This is not true for the second regime, when the economy starts to recover; from 2015 the series behaviour is described better by the TVECM.

5.4.2 Estimation and Price Discovery

The coefficients of the linear TVECM in table A.19, exhibit a stronger mean reversion in the first period compared to the second period. This is graphically confirmed also by figure C.4, C.3 and C.5. The results is exactly the reverse for the TVECM models; the regimes in which the adjustment process effect is faster, tend to be even faster in the second period.

In conclusion, the error correction is stronger during the crisis; but during the quantitative easing, when the size of the difference overtakes the threshold, it returns to equilibrium almost

immediately.

The coefficients p-values also suggest that the new information is captured more rapidly in the euro market for Barclays, Deutsche Bank and RBS, in the dollar market for HSBC and HSBC Holdings, and in both markets for Nomura and UBS. In particular, GG and HAS strengthen the interpretation in the case of Deutsche Bank.

This heterogeneity in the price discovery characterize also the outcomes in the case of the thresholds model: both markets contribute to price discovery in both regimes of Wells Fargo and UBS, with a little bit of predominance of the euro market in the upper regime of the US bank. The dollar guides the discovery of BNP and HSBC Holding.

Chapter 6

Conclusion

*”Knowing others is intelligence;
knowing yourself is true wisdom.
Mastering others is strength;
mastering yourself is true power.”*¹

In this thesis I considered the difference in price between two CDS written on the same bank’s bond in two different currencies, euro and dollar.

This difference has already been theoretically captured by an intensity based model of Philippe Ehlers and Philipp Schonbucher [28], and already exploited in the case of sovereign CDS by arbitrageurs using innovative instrument as Quanto CDS. I focused on the size behaviour of this discrepancy between CDS spreads, and to analyze its price discovery. Typical instrument to perform this analysis are cointegration and linear VECM; I chose to apply also the more sophisticated threshold model to understand if it improves the description and interpretation of this difference in prices.

First, I started studying all the time period that goes from December 2007 to December 2017. Engle-Granger [31] and Johansen [45] approach confirmed that the euro and the dollar prices are cointegrated processes. Given that the two prices do not meander away one from the other, I took advantage of the Johansen test for restrictions on coefficients of the cointegrating vector to catch the long-run behaviour of the two prices. If the restriction $[1,-1,0]$ holds, the two prices tend to coincide perfectly, instead if the constraint $[1,-1,c]$ holds, the prices evolve in the same way but with a constant difference. On fifteen cointegrated series, the first restriction was not rejected five times the second eight times, four of which did not reject the both.

Normally, at this point it would be concluded that for the other six pairs or the CDS are priced

¹Lao Tzu, *Tao Te Ching*, Ch. 33, as interpreted by Stephen Mitchell (1992)

differently in the two markets, or at least one market takes into account one or more time varying frictions or risk factors, or one spread contains time-varying non-transient measurement error. Thanks to the TVECM tests I found out that only for two cases there was statistically evidence to reject the constraints. Indeed, four of this six CDS did not reject the constraint $[1,-1,0]$ in the case they were allowed to have different regimes separated by thresholds. This non-linearity improved the modelling and interpretation of also other six CDS pairs, showing a stronger mean reversion before or after a certain value, estimating transaction costs when the difference had always the same sign, and finding a band inside which the process behaved as random walk and outside which the adjustments were fast.

Both models attested a price discovery guided half of the time by the euro market and half by the dollar; in addition, with the only exception of the GG measure for Deutsche Bank, less than 60% of the discovery happened in the dominant market.

Then, I separated the time-span in three historical periods, the sub-prime financial crisis and following period of policies, the European sovereign debt crisis and the quantitative easing regime. The linear VECM was really feasible to describe the behaviour of the difference through the first period. The third period was largely described and interpreted better by the TVECM. The second period was more heterogeneous.

Observing the path of the discrepancies, I individuated two periods: one extremely volatile and that reaches high absolute values but for short time intervals, the other lower in absolute value and volatility, but in which the difference is really more persistent. The second period coincides with the quantitative easing regime, the first one goes from December 2007 to January 2014. On this last one, again, the linear VECM was more feasible, showing robustness w.r.t the previous results.

With respect to the inclusion of new information in the markets, the dollar market has the leadership in the discovery during the subprime crisis. In all the other time intervals the results are more heterogeneous.

Thanks to the analysis on this four time periods, it can be concluded that in period of crisis, when volatility is high and the mean reversion process is really strong but not persistent, the classic linear VECM describes better the behaviour of the differences in spreads. On the opposite, when volatility is low and the difference is more persistent, as during more stable historical periods, the TVECM perfectly fits the evolution of two prices. Finally, the price discovery seems to happens in both markets.

Possible extensions

It would be interesting to repeat this analysis on less liquid maturities, widely available in the bank sector, where the more constant prices are feasible to be modeled by a threshold VECM.

Another possible extension is to perform the price discovery studying of this thesis comparing the CDS market of a liquid currency as the euro or the dollar with the CDS market of a less liquid one, as the Yen, widely studied in the literature, or the Sterling and the Australian dollar, both quite used on the credit risk market.

Regarding the weird result of some two thresholds models, in which the mean reversion is stronger in the band and in one of the two regimes outside, figure C.12 suggests that maybe it would be congenial to find new restriction to be tested using the TVECM.

Finally, enlarging the study on all the corporate sector and choosing a reasonable portfolio in which the differences are almost always with the same sign and large, it could be studied the evolution of the threshold as an estimate of the transaction costs and the possibility of arbitrages.

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Appendix A

Tables

Table A.1: Unit roots and Cointegration analysis from December 2007 to May 2011

The first two rows present the size of the sample and the number of the observation subtracting the missing values. The third row shows the lag selection, obtained comparing the results of "VARselect" of the package "vars". In the fourth row there is the cointegration rank; if cointegration holds it will be always one, dealing with two variables. From the fifth to the seventh rows there are the p-values of the unit root tests; the function "adfTest" was used from the package "fUnitRoots", while the self-written *ppPers.test* and *kpssPers.test* were used in the other two cases; in bold: stationarity. The following three rows report the p-values of the Engle-Granger and Johansen cointegration test; the first is computed by *coint.test* of the package "aTSA"; for the Johansen approach, the trace statistic was used thanks to the *ca.jo* and *cajorls* functions of the package "urca"; in bold: rejections of cointegration. The last two rows presents the p-values of the Johansen test on restricted cointegrating vectors; they were computed by the *blrtest* and *cajorls* functions of the package "urca"; in bold: rejection of the constraints.

Diagnostics	Barclays		BNP		CITI		Deutsche B.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	179	179	179	179	179	179	179	179
Without missing	179	179	179	179	168	168	179	179
Lags number	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-
ADF P-Values	0,0746	0,0893	0,0715	0,0771	0,4313	0,4316	0,0172	<0,01000
PP P-Values	0,0676	0,2185	0,0223	0,0319	0,5601	0,5603	0,0434	0,0166
KPSS P-Values	>0,1	>0,1	<0,0100	<0,0100	0,0176	0,0175	0,0986	>0,1
EG P-Values	<0,01		<0,01		<0,01		<0,01	
Trace P-Value r=0	0,0000		0,0000		0,0363		0,0000	
Trace P-Value r=1	0,0772		0,0583		0,5038		0,0077	
P-Values $\beta=[1, -1, c]$	0,5969		0,0000		0,3437		0,9293	
P-Values $\beta=[1 -1 0]$	0,7918		0,0000		0,3092		0,6970	

	Goldman S.		HSBC		HSBC H.		Mediob.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	179	179	179	179	179	179	179	179
Without missing	168	168	179	179	168	168	179	179
Lags number	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-
ADF P-Values	0,2552	0,2554	0,0909	0,0561	0,0922	0,1085	0,3412	0,3446
PP P-Values	0,0491	0,0491	0,2544	0,1804	0,2343	0,2830	0,5592	0,5762
KPSS P-Values	<0,0100	<0,0100	>0,1	>0,1	>0,1	>0,1	<0,0100	<0,0100
EG P-Values	<0,01		<0,01		<0,01		<0,01	
Trace P-Value r=0	0,0664		0,0000		0,0000		0,0000	
Trace P-Value r=1	0,1957		0,0629		0,0608		0,3027	
P-Values $\beta=[1, -1, c]$	0,2610		0,6236		0,8097		0,2814	
P-Values $\beta=[1 -1 0]$	0,2585		0,3948		0,0097		0,5339	

	Morgan S.		RBS		UBS	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	179	179	179	179	179	179
Without missing	168	168	179	179	179	179
Lags number	4	-	4	-	2	-
Coint. Rank	1	-	1	-	1	-
ADF P-Values	0,0421	0,0408	0,0372	0,0362	0,0467	0,0665
PP P-Values	<0,0100	<0,0100	<0,01	<0,01	0,0877	0,1408
KPSS P-Values	0,0328	0,0327	<0,0100	<0,0100	0,0245	0,0596
EG P-Values	0,0156		>0,1		<0,01	
Trace P-Value r=0	0,0022		0,0000		0,0195	
Trace P-Value r=1	0,6703		0,0485		0,2438	
P-Values $\beta=[1, -1, c]$	0,2145		0,0635		0,7242	
P-Values $\beta=[1 -1 0]$	0,4114		0,1052		0,6178	

Table A.2: Linear VECM Coefficients from December 2007 to May 2011

In this table are collected the coefficients estimated for the linear VECM; the estimation was done by the function *cajorls* in the package "urca". Each coefficient is followed by its standard error. The coefficients of $\bar{\beta}$ are normalized by β_1 . If constraints on the cointegrating vector coefficients are not rejected, the beta are fixed before estimation, and consequently the standard errors are zero. '***' = p-value < 0.01, '**' = p-value < 0.05, '*' = p-value < 0.1.

	Barclays		BNP		CITI		Deutsche B.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	-2,2105***	-1,6106***	-0,4693	0,9295	-3,3359	-3,0162	-0,2088	0,4067
	0,4993	[0,4496]	[0,7172]	[0,7078]	[23,358]	[23,358]	[0,4352]	[0,4665]
$\beta_1 - \beta_2$	1,0000	-1,0000	1,0000	-0,9853***	1,0000	-1,0000	1,0000	-1,0000
	[0]	[0]	[0]	[0,0032]	[0]	[0]	[0]	[0]
β_0	0,0000	-	-0,9695***	-	-0,0426	-	0,3006	-
	[0]	-	[0,2621]	-	[0,0705]	-	[1,4422]	-
$c_1 - c_2$	-	-	0,0000	0,0000	0,0647	0,0647	0,0008	0,0008

	Goldman S.		HSBC		HSBC H.		Mediob.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	-8,9682	-8,8231	-0,2913	0,5639	-0,0757	0,3616	0,1090	0,7590**
	[43,040]	[43,037]	[0,4084]	[0,3984]	[0,3229]	[0,3108]	[0,3818]	[0,3821]
$\beta_1 - \beta_2$	1,0000	-1,0000	1,0000	-1,0000	1	-1	1	-1
	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]
β_0	-0,0274	-	-0,2632	-	-1,6921	-	0,0000	-
	[0,0499]	-	[0,7011]	-	[1,7765]	-	[0]	-
$c_1 - c_2$	-0,3961	-0,3961	-0,0011	-0,0011	-0,0041	-0,0038	-	-

	Morgan S.		RBS		UBS	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	29,1510***	29,6010***	-0,8533	-0,3301	-0,7290***	-0,5232**
	[10,616]	[10,721]	[0,6369]	[0,6632]	[0,2223]	[0,2014]
$\beta_1 - \beta_2$	1	-1	1	-1	1	-1
	[0]	[0]	[0]	[0]	[0]	[0]
β_0	0,0000	-	0,0000	-	-1,6180	-
	[0]	-	[0]	-	[4,1630]	-
$c_1 - c_2$	-	-	-	-	0,0003	0,0003

Table A.3: Price Discovery from December 2007 to May 2011

The first row presents the restriction chosen on the cointegrating vector. The second and third rows contain the p-values of the speed adjustment coefficients computed by the *cajorls* function; in bold: statistically significative. In the fourth and fifth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the sixth and seventh lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share. Finally in the last row there are the markets that slightly dominates the discovery.

	Barclays	BNP	CITI	Deutsche B.	Goldman S.
Co-int. vector	[1,-1,0]	REFUSE	[1,-1,c]	[1,-1,c]	[1,-1,c]
pval(α_1)	0,0000	-	0,8866	0,6320	0,8352
pval(α_2)	0,0004	-	0,8974	0,3845	0,8378
HANS 1	0,4983	-	0,5000	0,5754	0,5000
HANS 2	0,5017	-	0,5000	0,4246	0,5000
GG 1	-2,6849	-	-9,4343	0,6608	-60,7950
GG 2	3,7067	-	10,4340	0,3392	61,7950
Results	both			no	no

	HSBC	HSBC H.	Mediob.	Morgan S.	RBS	UBS
Co-int. vector	[1,-1,c]	[1,-1,c]	[1,-1,0]	[1,-1,0]	[1,-1,0]	[1,-1,c]
pval(α_1)	0,4766	0,8149	0,7787	0,0067	0,1822	0,0013
pval(α_2)	0,1587	0,2463	0,0485	0,0064	0,6193	0,0102
HANS 1	0,5717	0,5800	0,5206	0,5000	0,4924	0,4970
HANS 2	0,4283	0,4200	0,4794	0,5000	0,5076	0,5030
GG 1	0,6594	0,8268	1,1676	65,7990	-0,6309	-2,5415
GG 2	0,3406	0,1732	-0,1676	-64,7990	1,6309	3,5415
Results	\$	\$	\$	both	€	both

Table A.4: TVECM tests from December 2007 to May 2011

The table is subdivided in 2 horizontal parts: the tests ran with 50 bootstrapped values and with 100 bootstrapped values. The first and the second rows of each part present the Hansen-Seo p-values of the "Fixed regressor bootstrap" and the "Residual bootstrap" tests; the values were computed by the *TVECM.HStest* function of the "tsDyn" library; in bold: no evidence for TVECM. From the third to the fifth rows there are the p-values of the number of Th.; values performed by *TVAR.LRtest* function of the "tsDyn" library; in bold: no rejection of H0. In the last row it is shown the choice of the model: 1 Th., 2 Th., no TVECM.

50 boots	Barclays	BNP	CITI	Deutsche B.	Goldman S.	
p-value HS FR	0,32	0,86	0,10	0,28	0,34	
p-value HS RB	0,34	0,72	0,04	0,20	0,26	
lin VS 1th	0,32	0,10	-	0,04	-	
lin VS 2th	0,32	0,40	-	0,00	-	
1th VS 2th	0,30	0,98	-	0,00	-	
100 boots						
p-value HS FR	0,39	0,89	0,23	0,35	0,28	
p-value HS RB	0,32	0,72	0,13	0,15	0,33	
lin VS 1th	0,29	0,09	-	0,05	-	
lin VS 2th	0,22	0,44	-	0,00	-	
1th VS 2th	0,34	0,97	-	0,00	-	
Results	no	no	no	no	no	
50 boots						
p-value HS FR	0,54	0,14	0,72	0,22	0,66	0,00
p-value HS RB	0,72	0,12	0,88	0,04	0,62	0,00
lin VS 1th	0,72	0,50	0,50	0,00	0,00	0,06
lin VS 2th	0,70	0,24	0,76	0,04	0,00	0,34
1th VS 2th	0,46	0,12	0,76	0,80	0,00	0,98
100 boots						
p-value HS FR	0,59	0,19	0,69	0,13	0,73	0,00
p-value HS RB	0,75	0,22	0,74	0,04	0,52	0,00
lin VS 1th	0,57	0,34	0,46	0,01	0,00	0,07
lin VS 2th	0,68	0,18	0,69	0,04	0,00	0,44
1th VS 2th	0,40	0,25	0,80	0,86	0,00	0,98
Results	no	no	no	1 th	no	1 th

Table A.5: TVECM coefficients and price discovery from December 2007 to May 2011

In left table are collected the coefficients estimated for the TVECM, first the ones of the lower regime, then the upper regime's ones; the estimation was done by the self-made function *persTVECM*. Each coefficient is followed by its standard error. The estimation is done under the cointegrating vector restricted to (1,-1,0). **** = p-value < 0,01, *** = p-value < 0,05, ** = p-value < 0,1. The right table contains the price discovery in the lower regime and in the upper regime. the first row contains the p-values of the speed adjustment coefficients computed by the *persTVECM* function; in bold: statistically significative. In the second row there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the third line there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share.

	Morgan S.		UBS		Morgan S.		UBS		
$\alpha_x^L - \alpha_y^L$	34,5189***	35,0599***	-0,9305	0,9589	pval(α_1^L) and pval(α_2^L)	0,0047	0,0045	0,5616	0,5017
	[12,0326]	[12,1514]	[1,6000]	1,4243	GG1 and GG2	64,8039	-63,8039	0,5075	0,4925
$c_1^L - c_2^L$	4,3086	4,3800	-0,3303	-0,1506	HAS1 and HAS2	0,5000	0,5000	0,6749	0,3251
	[6,3172]	[6,3796]	[2,3071]	[2,0539]		both		no	
$\alpha_x^U - \alpha_y^U$	-16,1137	-15,8222	-0,9366***	-0,7896***	pval(α_1^U) and pval(α_2^U)	0,8147	0,8198	0,0001	0,0002
	[68,6452]	[69,3231]	[0,2331]	[0,2075]	GG1 and GG2	-54,2888	55,2888	-5,3749	6,3749
$c_1^U - c_2^U$	5,5434	5,4448	3,8139	4,9475*	HAS1 and HAS2	0,5000	0,5000	0,5030	0,4970
	[31,2915]	[31,6005]	[2,8466]	[2,5341]		no		both	
Th.	0,0000	-	0,1330						

Table A.6: Unit roots and Cointegration analysis from May 2011 to March 2015

The first two rows present the size of the sample and the number of the observation subtracting the missing values. The third row shows the lag selection, obtained comparing the results of "VARselect" of the package "vars". In the fourth row there is the cointegration rank; if cointegration holds it will be always one, dealing with two variables. From the fifth to the seventh rows there are the p-values of the unit root tests; the function "adfTest" was used from the package "fUnitRoots", while the self-written *ppPers.test* and *kpssPers.test* were used in the other two cases; in bold: stationarity. The following three rows report the p-values of the Engle-Granger and Johansen cointegration test; the first is computed by *coint.test* of the package "aTSA"; for the Johansen approach, the trace statistic was used thanks to the *ca.jo* and *cajorls* functions of the package "urca"; in bold: rejections of cointegration. The last two rows presents the p-values of the Johansen test on restricted cointegrating vectors; they were computed by the *blrtest* and *cajorls* functions of the package "urca"; in bold: rejection of the constraints.

	Barclays		BNP		CITI		Deutsche B.		Goldman S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	200	200	200	200	200	200	200	200	200	200
Without missing	200	200	200	200	200	200	200	200	200	200
Lags number	2	-	2	-	3	-	2	-	3	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,6883	0,7162	0,6982	0,6982	0,6982	0,6885	0,6469	0,6268	0,6081	0,6121
PP P-Values	0,0100	0,0100	0,0222	0,0280	0,0951	0,0937	0,0433	0,0346	0,0967	0,0959
KPSS P-Values	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01
EG P-Values	<0,01		<0,01		<0,01		<0,01		<0,01	
Trace P-Value r=0	0,0000		0,0000		0,0002		0,0000		0,0000	
Trace P-Value r=1	0,8751		0,8734		0,8885		0,8818		0,8463	
P-Values $\beta=[1, -1, c]$	0,0563		0,0000		0,0000		0,2990		0,0000	
P-Values $\beta=[1 -1 0]$	0,0244		0,0000		0,0000		0,5826		0,0000	

	HSBC		HSBC H.		JP Morgan		Mediob.		Morgan S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	200	200	200	200	200	200	200	200	200	200
Without missing	200	200	200	200	185	185	200	200	200	200
Lags number	2	-	2	-	3	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,6388	0,6308	0,6471	0,6308	0,3959	0,4005	0,5765	0,5811	0,6074	0,6100
PP P-Values	0,0283	0,0276	0,0200	0,0303	0,0417	0,0418	0,0584	0,0595	0,0915	0,0899
KPSS P-Values	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01
EG P-Values	<0,01		<0,01		<0,01		<0,01		<0,01	
Trace P-Value r=0	0,0000		0,0000		0,0091		0,0000		0,2085	
Trace P-Value r=1	0,8287		0,8118		0,5725		0,9102		0,8963	
P-Values $\beta=[1, -1, c]$	0,2560		0,0106		0,0008		0,0358		0,0010	
P-Values $\beta=[1 -1 0]$	0,0358		0,0004		0,0034		0,0202		0,0030	

	Nomura		RBS		UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	200	200	200	200	200	200	200	200
Without missing	164	164	200	200	200	200	200	200
Lags number	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-
ADF P-Values	0,7204	0,7729	0,7759	0,7834	0,7104	0,7243	0,5797	0,5876
PP P-Values	0,9095	0,8641	0,0100	0,0116	0,0202	0,0226	0,0376	0,0398
KPSS P-Values	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01
EG P-Values	>0,1		<0,01		<0,01		<0,01	
Trace P-Value r=0	0,0234		0,0000		0,0000		0,0000	
Trace P-Value r=1	0,5603		0,9349		0,9502		0,8667	
P-Values $\beta=[1, -1, c]$	0,2041		0,8989		0,0011		0,0005	
P-Values $\beta=[1 -1 0]$	0,0443		0,8616		0,0009		0,0013	

Table A.7: Linear VECM Coefficients from May 2011 to March 2015

In this table are collected the coefficients estimated for the linear VECM; the estimation was done by the function *cajorls* in the package *urca*. Each coefficient is followed by its standard error. The coefficients of $\bar{\beta}$ are normalized by β_1 . If constraints on the cointegrating vector coefficients are not rejected, the beta are fixed before estimation, and consequently the standard errors are zero. '***' = p-value < 0.01, '**' = p-value < 0.05, '*' = p-value < 0.1.

	Barclays		BNP		CITI		Deutsche B.		Goldman S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	0,1617	0,9608**	0,7566	-1,5994**	-5,0758	-4,6961	-0,7534	0,0353	-6,8256**	-6,419**
	[0,4951]	[0,4589]	[0,6792]	[0,6618]	[3,8526]	[3,8126]	[0,5427]	[0,5764]	[3,2801]	[3,2407]
$\beta_1 - \beta_2$	1,0000	-1,0000	1,0000	-0,9850***	1,0000	-1,0091***	1,0000	-1,0000	1,0000	-1,0101***
	[0]	[0]	[0]	[0,0027]	[0]	[0,0007]	[0]	[0]	[0]	[0,0008]
β_0	0,4296	-	-0,7973*	-	-0,8418***	-	0	-	1,0502***	-
	[0,5192]		[0,4381]		[0,1117]		[0]		[0,1408]	
$c_1 - c_2$	-0,0083	-0,0075	0,0000	0,0000	0,0000	0,0000	-0,2590	-0,2721	0,0000	0,0000

	HSBC		HSBC H.		JP Morgan		Mediob.		Morgan S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	2,6429**	3,8795***	0,9292***	1,4562***	-10,9360***	-10,5740***	-0,3577*	0,1549	-7,1315**	-6,8612**
	[1,3167]	[1,3197]	[0,3405]	[0,3087]	[7,1524]	[7,0931]	[0,1948]	[0,2033]	[2,8868]	[2,8240]
$\beta_1 - \beta_2$	1,0000	-1,0000	1,0000	-1,0178***	1,0000	-1,0044***	1,0000	-0,9823***	1,0000	-1,0184***
	[0]	[0]	[0]	[0,0076]	[0]	[0,0005]	[0]	[0,0078]	[0]	[0,0014]
β_0	-0,0897	-	0,9319*	-	0,3567***	-	-2,1094	-	1,9057***	-
	[0,1066]		[0,6812]		[0,0428]		[2,1283]		[0,3200]	
$c_1 - c_2$	-0,0066	-0,0065	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

	Nomura		RBS		UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	0,2150***	0,2948***	0,1446	1,0336**	0,1920	0,8643*	0,9332***	1,8438***
	[0,0804]	[0,0765]	[0,6034]	[0,5674]	[0,5269]	[0,5159]	[0,2443]	[0,2314]
$\beta_1 - \beta_2$	1,0000	-1,0000	1,0000	-1,0000	1,0000	-0,9882***	1,0000	-1,0189***
	[0]	[0]	[0]	[0]	[0]	[0,0032]	[0]	[0,0051]
β_0	5,5903**	-	-0,1051	-	-0,8404***	„	1,2817***	
	[3,2295]		[0,4301]		[0,3883]		[0,4292]	
$c_1 - c_2$	-0,0188	-0,0098	-0,0118	-0,0109	0,0000	0,0000	0,0000	0,0000

Table A.8: Price Discovery from May 2011 to March 2015

The first row presents the restriction chosen on the cointegrating vector. The second and third rows contain the p-values of the speed adjustment coefficients computed by the *cajorls* function; in bold: statistically significative. In the fourth and fifth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the sixth and seventh lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share. Finally in the last row there are the markets that slightly dominates the discovery.

	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.
Coint. vector	[1,-1,c]	REFUSE	REFUSE	[1,-1,0]	REFUSE	[1,-1,c]	[1,-1,c]
pval(α_1)	0,7442	-	-	0,1667	-	0,0461	0,0030
pval(α_2)	0,0375	-	-	0,9512	-	0,0037	0,0000
HANS 1	0,5116	-	-	0,4849	-	0,5006	0,5119
HANS 2	0,4884	-	-	0,5151	-	0,4994	0,4881
GG 1	1,2024	-	-	0,0448	-	3,1372	2,4171
GG 2	-0,2024	-	-	0,9552	-	-2,1372	-1,4171
Results	\$			€		both	\$

	JP Morgan	Mediob.	Morgan S.	Nomura	RBS	Standard C.	UBS	Wells F.
Coint. vector	REFUSE	REFUSE	REFUSE	[1,-1,c]	[1,-1,c]	REFUSE	REFUSE	REFUSE
pval(α_1)	-	-	-	0,0083	0,8110	-	-	-
pval(α_2)	-	-	-	0,0000	0,0700	-	-	-
HANS 1	-	-	-	0,5134	0,5088	-	-	-
HANS 2	-	-	-	0,4866	0,4912	-	-	-
GG 1	-	-	-	3,6921	1,1627	-	-	-
GG 2	-	-	-	-2,6921	-0,1627	-	-	-
Results				\$	\$			

Table A.9: TVECM tests from May 2011 to March 2015

The table is subdivided in 2 horizontal parts: the tests ran with 50 bootstrapped values and with 100 bootstrapped values. The first and the second rows of each part present the Hansen-Seo p-values of the "Fixed regressor bootstrap" and the "Residual bootstrap" tests; the values were computed by the *TVECM.HStest* function of the *"tsDyn"* library; in bold: no evidence for TVECM. From the third to the fifth rows there are the p-values of the number of Th.; values performed by *TVAR.LRtest* function of the *"tsDyn"* library; in bold: no rejection of H0. In the last row it is shown the choice of the model: 1 Th., 2 Th., no TVECM.

50 boots		Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.	
p-value HS FR	0,06	0,00	0,86	0,64	0,80	0,56	0,00		
p-value HS RB	0,04	0,04	0,86	0,56	0,80	0,36	0,00		
lin VS 1th	0,00	0,00	0,00	0,00	0,06	0,02	0,00		
lin VS 2th	0,02	0,00	0,00	0,04	0,00	0,00	0,00		
1th VS 2th	0,02	0,66	0,00	0,70	0,04	0,00	0,00		
100 boots									
p-value HS FR	0,01	0,01	0,90	0,66	0,82	0,57	0,01		
p-value HS RB	0,06	0,05	0,84	0,61	0,79	0,42	0,00		
lin VS 1th	0,00	0,00	0,00	0,01	0,06	0,01	0,00		
lin VS 2th	0,00	0,00	0,00	0,03	0,00	0,00	0,01		
1th VS 2th	0,01	0,60	0,00	0,70	0,02	0,00	0,04		
Results	2 th	1 th	no	no	no	no	2 th		
50 boots		JP Morgan	Mediob.	Morgan S.	Nomura	RBS	Standard C.	UBS	Wells F.
p-value HS FR	0,52	0,04	0,04	0,00	0,38	1,00	0,72	0,00	
p-value HS RB	0,38	0,10	0,10	0,00	0,42	1,00	0,66	0,00	
lin VS 1th	0,02	0,26	0,00	0,02	0,00	-	0,10	0,04	
lin VS 2th	0,04	0,50	0,00	0,02	0,02	-	0,00	0,04	
1th VS 2th	0,10	0,82	0,00	0,28	0,14	-	0,00	0,14	
100 boots									
p-value HS FR	0,55	0,13	0,04	0,00	0,38	1,00	0,59	0,00	
p-value HS RB	0,44	0,07	0,09	0,00	0,46	0,99	0,52	0,00	
lin VS 1th	0,01	0,12	0,00	0,00	0,00	-	0,13	0,05	
lin VS 2th	0,03	0,51	0,00	0,04	0,00	-	0,00	0,01	
1th VS 2th	0,15	0,86	0,03	0,30	0,19	-	0,01	0,13	
Results	no	no	2 th	1 th	no	no	no	1 th	

Table A.10: TVECM coefficients from May 2011 to March 2015

In this table are collected the coefficients estimated for the TVECM, first the ones of the lower regime, then the ones of the middle regime, and finally the ones of the upper regime; the estimation was done by the self-made function *persTVECM*. Each coefficient is followed by its standard error. The cointegrating vector is restricted to (1,-1,0).
 *** = p-value < 0.01, ** = p-value < 0.05, * = p-value < 0.1.

	Barclays		BNP		HSBC H.	
$\alpha_x^L - \alpha_y^L$	2,3487***	3,3366***	0,3246	0,8247	0,9646*	1,5821
	[0,8920]	[0,8268]	[0,6495]	[0,6289]	[0,5897]	[0,5642]
$c_1^L - c_2^L$	5,6552	7,4503**	-0,9879	-0,3240	-1,2244*	-1,2941*
	[3,6796]	[3,4104]	[1,8369]	[1,7844]	[0,7196]	[0,6884]
$\alpha_x^M - \alpha_y^M$	2,0456	1,0099			22,4231**	23,5741**
	[4,6299]	[4,2913]			[9,6522]	[9,2340]
$c_1^U - c_2^U$	0,6166	0,5597			-30,1220***	-30,0497***
	[1,4593]	[1,3526]			[12,0709]	[11,5480]
$\alpha_x^U - \alpha_y^U$	-0,4012	0,6371	0,0897	1,0533	-0,9902	0,1832
	[1,0030]	[0,9296]	[1,7333]	[1,6838]	[0,8483]	[0,8115]
$c_1^U - c_2^U$	0,3974	-0,4882	7,0505***	8,2338***	6,0129	2,9646
	[3,0933]	[2,8671]	[3,0742]	[2,9865]	[3,0500]	[2,9179]
Th.	-1,2800	0,0000	-0,0100		0,7300	1,6380
	Morgan S.		Nomura		Wells F.	
$\alpha_x^L - \alpha_y^L$	-1,4407	0,1572	-6,8144**	-4,8596**	2,4715***	3,3564***
	[165,322]	[161,7334]	[3,0306]	[1,9884]	[0,9035]	[0,8571]
$c_1^L - c_2^L$	1,0023	1,1178	-134,2085***	-98,3972***	2,4791***	2,9045***
	[14,7745]	[14,4538]	[39,1304]	[37,5461]	[1,1539]	[1,0947]
$\alpha_x^M - \alpha_y^M$	1,2228	1,1764				
	[1,0053]	[0,9835]				
$c_1^U - c_2^U$	-0,2809	-0,2744				
	[2,4895]	[2,4355]				
$\alpha_x^U - \alpha_y^U$	-35,1860***	-34,1762***	0,0350	0,1306*	0,8318***	1,8600***
	[10,8642]	[10,6284]	[0,1114]	[0,1069]	[0,2661]	[0,2524]
$c_1^U - c_2^U$	251,7700***	244,7296***	0,2008	0,8848	-1,1120*	-1,4558***
	[85,6231]	[83,7645]	[0,8713]	[0,8361]	[0,5999]	[0,5691]
Th.	-0,0600	7,0200	-17,3699		-0,3750	

Table A.11: TVECM price discovery from May 2011 to March 2015

The table is subdivided in two horizontal parts: the price discovery in the lower regime and in the upper regime. The first and second rows of each part contain the p-values of the speed adjustment coefficients computed by the *persTVECM* function; in bold: statistically significant. In the third and fourth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the fifth and sixth lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share.

Lower	Barclays	BNP	HSBC H.	Morgan S.	Nomura	Wells F.
pval(α_1)	0,0091	0,6161	0,1036	0,9931	0,0102	0,0068
pval(α_2)	0,0001	0,1908	0,0056	0,9992	0,0397	0,0001
GG 1	3,3774	1,6491	2,5620	0,0984	-2,4860	3,7929
GG 2	-2,3774	-0,6491	-1,5620	0,9016	3,4860	-2,7929
HAS 1	0,5006	0,5001	0,5005	0,5000	0,5025	0,5020
HAS 2	0,4994	0,4999	0,4995	0,5000	0,4975	0,4980
Middle	both	\$	\$	no	both	both
pval(α_1)	0,6591		0,0213	0,2254		
pval(α_2)	0,8142		0,0115	0,2331		
HANS 1	-0,9750		20,4821	-25,3794		
HANS 2	1,9750		-19,4821	26,3794		
GG 1	0,5008		0,5007	0,5000		
GG 2	0,4992		0,4993	0,5000		
Upper	no		both	both		
pval(α_1)	0,6896	0,9266		0,0014	0,2700	0,0020
pval(α_2)	0,4940	0,5026	0,8216	0,0015	0,0516	0,0000
GG 1	0,6136	1,0931	0,1561	-33,8428	1,3659	1,8090
GG 2	0,3864	-0,0931		34,8428	-0,3659	-0,8090
HAS 1	0,4231	0,4896		0,5000	0,4402	0,4668
HAS 2			0,4567	0,5000	0,5598	0,5332
Results	no	no	no	€	ambiguos	€

Table A.12: Unit roots and Cointegration analysis from March 2015 to December 2017

The first two rows present the size of the sample and the number of the observation subtracting the missing values. The third row shows the lag selection, obtained comparing the results of "VARselect" of the package "vars". In the fourth row there is the cointegration rank; if cointegration holds it will be always one, dealing with two variables. From the fifth to the seventh rows there are the p-values of the unit root tests; the function "adfTest" was used from the package "fUnitRoots", while the self-written *ppPers.test* and *kpssPers.test* were used in the other two cases; in bold: stationarity. The following three rows report the p-values of the Engle-Granger and Johansen cointegration test; the first is computed by *coint.test* of the package "aTSA"; for the Johansen approach, the trace statistic was used thanks to the *ca.jo* and *cajorls* functions of the package "urca"; in bold: rejections of cointegration. The last two rows presents the p-values of the Johansen test on restricted cointegrating vectors; they were computed by the *blrtest* and *cajorls* functions of the package "urca"; in bold: rejection of the constraints.

	Barclays		BNP		CITI		Deutsche B.		Goldman S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	144	144	144	144	144	144	144	144	144	144
Without missing	144	144	144	144	144	144	144	144	144	144
Lags number	2	-	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,5278	0,4752	0,6792	0,6864	0,6243	0,6293	0,4891	0,4845	0,5097	0,5142
PP P-Values	0,5902	0,5081	0,5059	0,5028	0,2979	0,3151	0,7629	0,7603	0,3129	0,3332
KPSS P-Values	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	0,0191	0,0190	<0,01	<0,01
EG P-Values	0,0045		0,0958		0,0942		0,4602		0,0855	
Trace P-Value r=0	0,0037		0,0562		0,0166		0,3804		0,0075	
Trace P-Value r=1	0,0558		0,0739		0,1322		0,1214		0,1497	
P-Values $\beta=[1, -1, c]$	0,0597		0,0002		0,0004		0,0120		0,0002	
P-Values $\beta=[1 -1 0]$	0,1619		0,0007		0,0006		0,0319		0,0002	

	HSBC		HSBC H.		JP Morgan		Mediob.		Morgan S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	144	144	144	144	144	144	144	144	144	144
Without missing	144	144	124	124	144	144	144	144	144	144
Lags number	2	-	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,7288	0,7263	0,7765	0,7992	0,5554	0,5581	0,6155	0,6569	0,5267	0,5321
PP P-Values	0,4099	0,4015	0,8046	0,7804	0,2132	0,2264	0,6908	0,7166	0,3975	0,4153
KPSS P-Values	<0,01	<0,01	0,0307	0,0234	<0,01	<0,01	0,0192	0,0170	<0,01	<0,01
EG P-Values	0,0010		0,0018		0,0670		0,1227		0,0271	
Trace P-Value r=0	0,0010		0,0046		0,0900		0,1368		0,0068	
Trace P-Value r=1	0,0346		0,2385		0,0349		0,3204		0,1901	
P-Values $\beta=[1, -1, c]$	0,0000		0,0000		0,0177		0,0216		0,0002	
P-Values $\beta=[1 -1 0]$	0,0000		0,0000		0,0571		0,0041		0,0002	

	Nomura		RBS		Standard C.		UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	144	144	144	144	144	144	144	144	144	144
Without missing	123	123	144	144	144	144	144	144	144	144
Lags number	2	-	2	-	2	-	2	-	2	-
Coint. Rank	1	-	1	-	1	-	1	-	1	-
ADF P-Values	0,4287	0,3958	0,5307	0,5314	0,5343	0,4974	0,7899	0,8000	0,2735	0,2856
PP P-Values	0,6324	0,6049	0,6936	0,6701	0,4716	0,4529	0,4469	0,4419	0,3109	0,3279
KPSS P-Values	0,0135	0,0168	0,0218	0,0201	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01
EG P-Values	0,0170		0,6289		0,0010		0,0262		0,0290	
Trace P-Value r=0	0,0149		0,5276		0,0010		0,0076		0,0688	
Trace P-Value r=1	0,0751		0,3764		0,0328		0,0635		0,0421	
P-Values $\beta=[1, -1, c]$	0,0000		0,0261		0,0000		0,0000		0,0023	
P-Values $\beta=[1 -1 0]$	0,0000		0,0799		0,0000		0,0000		0,0082	

Table A.13: Linear VECM Coefficients from March 2015 to December 2017

In this table are collected the coefficients estimated for the linear VECM; the estimation was done by the function *cajorls* in the package *urca*. Each coefficient is followed by its standard error. The coefficients of $\bar{\beta}$ are normalized by β_1 . If constraints on the cointegrating vector are not rejected, the beta are fixed before estimation, and consequently the standard errors are zero. *** = p-value < 0.01, ** = p-value < 0.05, * = p-value < 0.1.

	Barclays		BNP		CITI		Deutsche B.		Goldman S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	-0,2715 [0,2498]	0,1463 [0,2662]	10,8875* [4,5580]	10,9981* [4,4910]	-8,4718*** [1,9480]	-8,3398*** [1,9380]	-1,9147 [3,4030]	-1,7786 [3,4750]	-9,9249*** [2,1110]	-9,8099*** [2,1000]
$\beta_1 - \beta_2$	1,0000 [0]	-1,0000 [0]	1,0000 [0]	-1,0189*** [0,0016]	1 [0]	-0,9848*** [0,0036]	1 [0]	-0,9792*** [0,0019]	1 [0]	-0,9840*** [0,0034]
β_0	0,0000 [0]	- [0]	-0,0078*** [0,0008]	- [0,0015]	0,0088*** [0,0015]	- [0,0015]	0,0062*** [0,0024]	- [0,0024]	0,0086*** [0,0012]	- [0,0012]
$c_1 - c_2$	0,0924	-0,1558	9,9210	10,0230	17,0930	16,8260	1,2290	1,1500	22,6130	22,3530

	HSBC		HSBC H.		JP Morgan		Mediob.		Morgan S.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	4,2622** [1,9260]	5,0102*** [1,8770]	-1,1668 [0,9928]	-0,3156 [0,9439]	-0,2146438 [1,4744]	-0,07240923 [1,4674]	-2,0210* [1,0770]	-1,8137* [1,0620]	-11,2360*** [2,4880]	-11,0681*** [2,4750]
$\beta_1 - \beta_2$	1 [0]	-1,0201*** [0,0020]	1 [0]	-1,0480*** [0,0054]	1,0000 [0]	-1,0000 [0]	1,0000 [0]	-0,9826*** [0,0063]	1 [0]	-0,9880*** [0,0026]
β_0	-0,0057*** [0,0012]	- [0]	-0,0227*** [0,0027]	- [0]	0,0000 [0]	- [0]	-0,0303*** [0,0059]	- [0,0059]	0,0069*** [0,0010]	- [0,0010]
$c_1 - c_2$	1,8410	2,2150	-3,1135	-1,0190	-0,0846	-0,1153	9,5970	8,5390	19,5770	19,2840

	Nomura		RBS		Standard C.		UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
$\alpha_x - \alpha_y$	-0,3612 [0,4240]	0,6033 [0,3821]	-0,8933029 [0,5009]	-0,8543234 [0,5827]	-0,0840 [0,4358]	0,9015** [0,4091]	10,8538*** [3,3820]	11,0202*** [3,3230]	-3,3304 [1,0650]	-3,2292 [1,0660]
$\beta_1 - \beta_2$	1 [0]	-1,0560*** [0,0091]	1 [0]	-1 [0]	1 [0]	-1,0439*** [0,0079]	1 [0]	-1,0227*** [0,0017]	1 [0]	-0,9667*** [0,0079]
β_0	0,01083*** [0,0038]	- [0]	0,0000 [0]	- [0]	-0,0149*** [0,0069]	- [0,0069]	-0,0087*** [0,0007]	- [0,0007]	0,0027*** [0,0014]	- [0,0014]
$c_1 - c_2$	-0,8821	0,4952	1,6875	1,5906	-0,1952	1,5654	11,4570	11,6330	6,7410	6,5350

Table A.14: Price Discovery from March 2015 to December 2017

The first row presents the restriction chosen on the cointegrating vector. The second and third rows contain the p-values of the speed adjustment coefficients computed by the *cajorls* function; in bold: statistically significant. In the fourth and fifth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the sixth and seventh lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share. Finally in the last row there are the markets that slightly dominates the discovery.

	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.
Coint. vector	[1,-1,0]	REFUSE	REFUSE	REFUSE	REFUSE	REFUSE	REFUSE
Coint. vector	0,2789	-	-	-	-	-	-
Coint. vector	0,6041	-	-	-	-	-	-
Coint. vector	0,3598	-	-	-	-	-	-
Coint. vector	0,6402	-	-	-	-	-	-
Coint. vector	0,3502	-	-	-	-	-	-
Coint. vector	0,6498	-	-	-	-	-	-
Coint. vector	€						

	JP Morgan	Mediob.	Morgan S.	Nomura	RBS	Standard C.	UBS	Wells F.
Coint. vector	[1,-1,0]	REFUSE	REFUSE	REFUSE	[1,-1,0]	REFUSE	REFUSE	REFUSE
pval(α_1)	0,8737	-	-	-	0,1430	-	-	-
pval(α_2)	0,9500	-	-	-	0,1567	-	-	-
HAS 1	0,4997	-	-	-	0,5000	-	-	-
HAS 2	0,5003	-	-	-	0,5000	-	-	-
GG 1	-0,5091	-	-	-	-21,9173	-	-	-
GG 2	1,5090	-	-	-	22,9173	-	-	-
Results					both			

Table A.15: TVECM tests from March 2015 to December 2017

The table is subdivided in 2 horizontal parts: the tests ran with 50 bootstrapped values and with 100 bootstrapped values. The first and the second rows of each part present the Hansen-Seo p-values of the "Fixed regressor bootstrap" and the "Residual bootstrap" tests; the values were computed by the *TVECM.HStest* function of the *"tsDyn"* library; in bold: no evidence for TVECM. From the third to the fifth rows there are the p-values of the number of Th.; values performed by *TVAR.LRtest* function of the *"tsDyn"* library; in bold: no rejection of H0. In the last row it is shown the choice of the model: 1 Th., 2 Th., no TVECM.

50 boots	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.
p-value HS FR	0,10	0,90	0,04	0,14	0,00	0,44	0,00
p-value HS RB	0,08	0,64	0,08	0,00	0,00	0,34	0,02
lin VS 1th	0,02	0,00	0,04	0,12	0,16	0,04	0,02
lin VS 2th	0,00	0,00	0,24	0,00	0,62	0,04	0,02
1th VS 2th	0,14	0,70	0,78	0,00	0,96	0,46	0,76
100 boots							
p-value HS FR	0,07	0,86	0,04	0,19	0,01	0,36	0,02
p-value HS RB	0,07	0,74	0,12	0,01	0,01	0,25	0,00
lin VS 1th	0,01	0,00	0,07	0,13	0,15	0,01	0,01
lin VS 2th	0,00	0,01	0,27	0,04	0,57	0,02	0,05
1th VS 2th	0,09	0,80	0,77	0,00	0,98	0,47	0,64
Results	no	no	1 th	2 th	1 th	no	1 th

50 boots	JP Morgan	Mediob.	Morgan S.	Nomura	RBS	Standard C.	UBS	Wells F.
p-value HS FR	0,44	0,06	0,00	0,06	0,76	0,00	0,10	0,56
p-value HS RB	0,28	0,02	0,08	0,02	0,72	0,00	0,06	0,48
lin VS 1th	0,02	0,00	0,22	0,54	0,00	0,22	0,00	0,08
lin VS 2th	0,02	0,02	0,14	0,06	0,02	0,74	0,00	0,10
1th VS 2th	0,12	0,38	0,28	0,02	0,24	0,94	0,06	0,52
100 boots								
p-value HS FR	0,53	0,05	0,00	0,06	0,80	0,00	0,18	0,40
p-value HS RB	0,19	0,08	0,04	0,03	0,64	0,00	0,04	0,37
lin VS 1th	0,02	0,02	0,25	0,58	0,00	0,20	0,00	0,06
lin VS 2th	0,01	0,03	0,19	0,07	0,05	0,58	0,00	0,07
1th VS 2th	0,10	0,59	0,27	0,06	0,23	0,91	0,03	0,42
Results	no	1 th	1 th	1 th	no	2 th	2 th	no

Table A.17: TVECM price discovery from March 2015 to December 2017

The table is subdivided in two horizontal parts: the price discovery in the lower regime and in the upper regime. The first and second rows of each part contain the p-values of the speed adjustment coefficients computed by the *persTVECM* function; in bold: statistically significant. In the third and fourth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the fifth and sixth lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share.

Lower	CITI	Deutsche B.	Goldman S.	HSBC H.	Mediob.	Morgan S.	Nomura	Standard C.	UBS
pval(α_1)	0,0084	0,0429	0,0018	0,0297	0,2262	0,0056	0,0348	0,794	0,1693
pval((α_2))	0,0085	0,0389	0,0018	0,1044	0,2015	0,0058	0,7625	0,1996	0,1299
GG 1	-117,2305	24,9180	-166,17	-2,5785	28,9489	-113,14	-0,1509	1,2771	13,0453
GG 2	118,2305	-23,9180	167,17	3,5785	-27,9489	114,14	1,1509	-0,2771	-12,0453
HAS 1	0,5000	0,4996	0,5000	0,5003	0,5000	0,5	0,5019	0,0572	0,5000
HAS 2	0,5000	0,5004	0,5000	0,4997	0,5000	0,5	0,4981	0,9428	0,5000
	both	both	both	€		both	€	\$	both
Middle									
pval(α_1)		0,0003						0,4817	
pval((α_2))		0,0003						0,3649	
GG 1		41,8618						5,8731	
GG 2		-40,8618						-4,8731	
HAS 1		0,5029						0,0246	
HAS 2		0,4971						0,9754	
		both							
Upper									
pval(α_1)	0,8176	0,7373	0,1603	0,3429	0,3593	0,0396	0,0198	0,086	0,0508
pval((α_2))	0,7793	0,7445	0,1635	0,2894	0,3497	0,0423	0,0197	0,0172	0,0506
GG 1	5,8134	-127,8445	-68,12	-25,4309	1055,8361	-49,413	-12,483	4,2724	-57,9388
GG 2	-4,8134	128,8445	69,12	26,4309	-1054,8361	50,413	13,483	-3,2724	58,9388
HAS 1	0,5000	0,4963	0,5000	0,5005	0,5000	0,5	0,5057	0,0265	0,5000
HAS 2	0,5000	0,5037	0,5000	0,4995	0,5000	0,5	0,4943	0,9735	0,5000
Results			both			both	both	\$	both

Table A.18: Unit roots and Cointegration analysis from December 2007 to January 2014

The first two rows present the size of the sample and the number of the observation subtracting the missing values. The third row shows the lag selection, obtained comparing the results of "VARselect" of the package "vars". In the fourth row there is the cointegration rank; if cointegration holds it will be always one, dealing with two variables. From the fifth to the seventh rows there are the p-values of the unit root tests; the function "adfTest" was used from the package "fUnitRoots", while the self-written *ppPers.test* and *kpssPers.test* were used in the other two cases; in bold: stationarity. The following three rows report the p-values of the Engle-Granger and Johansen cointegration test; the first is computed by *coint.test* of the package "aTSA"; for the Johansen approach, the trace statistic was used thanks to the *ca.jo* and *cajorls* functions of the package "urca"; in bold: rejections of cointegration. The last two rows presents the p-values of the Johansen test on restricted cointegrating vectors; they were computed by the *blrtest* and *cajorls* functions of the package "urca"; in bold: rejection of the constraints.

	Barclays		BNP		CITI		Deutsche B.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	317	317	317	317	317	317	317	317
Without missing	317	317	317	317	306	306	317	317
Lags number	2	-	2	-	2	-	2	-
CoInt. Rank	1	-	1	-	1	-	1	-
ADF P-Values	0,0505	0,0692	0,3830	0,3860	0,2805	0,2813	0,0774	0,0504
PP P-Values	0,0964	0,2677	0,7762	0,8152	0,2652	0,2648	0,2924	0,1705
KPSS P-Values	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01	<0,01
EG P-Values	<0,01		<0,01		0,0235		<0,01	
Trace P-Value r=0	0,0000		0,0000		0,1758		0,0000	
Trace P-Value r=1	0,0724		0,4264		0,2479		0,0728	
P-Values $\beta=[1, -1, c]$	0,3079		0,0000		0,6884		0,3445	
P-Values $\beta=[1 -1 0]$	0,4200		0,0000		0,3109		0,4752	

	Goldman S.		HSBC		HSBC H.		JP Morgan	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	317	317	317	317	317	317	317	317
Without missing	306	306	317	317	306	306	124	124
Lags number	2	-	3	-	2	-	3	-
CoInt. Rank	1	-	1	-	1	-	2	-
ADF P-Values	0,1442	0,1427	0,0515	0,0497	0,1155	0,1354	0,5155	0,5191
PP P-Values	0,1000	0,0978	0,1116	0,1381	0,2106	0,2864	0,0203	0,0205
KPSS P-Values	>0,1	>0,1	<0,01	<0,01	<0,01	<0,01	0,0268	0,0267
EG P-Values	<0,01		<0,01		<0,01		<0,01	
Trace P-Value r=0	0,0033		0,0000		0,0000		0,0058	
Trace P-Value r=1	0,1316		0,0941		0,0797		0,7106	
P-Values $\beta=[1, -1, c]$		0,0048	0,2920		0,6959		0,0000	
P-Values $\beta=[1 -1 0]$	0,0051		0,2556		0,0002		0,0004	

	Mediob.		Morgan S.		Nomura		RBS	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	317	317	317	317	317	317	317	317
Without missing	317	317	306	306	103	103	317	317
Lags number	2	-	2	-	2	-	2	-
CoInt. Rank	1	-	1	-	1	-	1	-
ADF P-Values	0,4218	0,4257	0,0557	0,0521	0,9317	0,9422	0,1662	0,1782
PP P-Values	0,8492	0,8504	0,0100	0,0100	0,7232	0,7202	0,5224	0,5232
KPSS P-Values	<0,01	<0,01	0,0509	0,0473	0,0247	0,0253	<0,01	<0,01
EG P-Values	<0,01		0,0253		>0,1		<0,01	
Trace P-Value r=0	0,0000		0,0068		0,1430		0,0000	
Trace P-Value r=1	0,6536		0,3412		0,7267		0,2098	
P-Values $\beta=[1, -1, c]$		0,0026	0,0002		0,4480		0,5363	
P-Values $\beta=[1 -1 0]$	0,0027		0,0004		0,2382		0,4383	

	UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)
Sample Size	317	317	317	317
Without missing	317	317	152	152
Lags number	2	-	2	-
CoInt. Rank	1	-	1	-
ADF P-Values	0,0140	0,0236	0,6305	0,6379
PP P-Values	0,0330	0,0609	0,3449	0,3737
KPSS P-Values	0,0896	>0,1	<0,01	<0,01
EG P-Values	<0,01		<0,01	
Trace P-Value r=0	0,0000		0,0000	
Trace P-Value r=1	0,1345		0,8928	
P-Values $\beta=[1, -1, c]$	0,9739		0,0056	
P-Values $\beta=[1 -1 0]$	0,8425		0,0051	

Table A.19: Linear VECM Coefficients from December 2007 to January 2014

In this table are collected the coefficients estimated for the linear VECM; the estimation was done by the function *cajorls* in the package *urca*. Each coefficient is followed by its standard error. The coefficients of $\bar{\beta}$ are normalized by β_1 . If constraints on the cointegrating vector vectors are not rejected, the beta are fixed before estimation, and consequently the standard errors are zero. **** = p-value < 0.01, *** = p-value < 0.05, ** = p-value < 0.1.

	Barclays		BNP		CITI		Deutsche B.	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	-1,2311	-0,5824	0,4337	1,4060	-0,9605	-0,8776	-0,3884	0,2783
	[0,3801]***	[0,3471]*	[0,5331]	[0,5204]***	[3,1675]	[3,1624]	[0,3589]	[0,3835]
β_1 and β_2	1,0000	-1,0000	1,0000	-0,9842	1,0000	-1,0000	1,0000	-1,0000
	[0]	[0]	[0]	[0,0019]***	[0]	[0]	[0]	[0]
β_0	0,0000	-	-0,9706	-	-0,3170	-	0,0000	-
	[0]	-	[0,2662]***	-	[0,0055]***	-	[0]	-
$c_{out,1}$ and $c_{out,2}$	-	-	0,0000	0,0000	-0,0042	-0,0042	-	-

	Goldman S.		HSBC		HSBC H.		JP Morgan	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	5,7276	5,8198	0,7415	1,3396	0,1820	0,6752	-15,1570	-14,6680
	[2,8937]**	[2,8862]**	[0,4702]	[0,4698]***	[0,2472]	[0,2325]***	[11,751]	[11,649]
β_1 and β_2	1,0000	-1,0082	1,0000	-1,0000	1,0000	-1,0000	1,0000	-1,0059
	[0]	[0,0113]	[0]	[0]	[0]	[0]	[0]	[0,0005]***
β_0	1,0437	-	-0,2007	-	-1,4426	-	0,5096	-
	[0,0011]***	-	[0,0035]***	-	[0,0077]**	-	[0,0459]	-
$c_{out,1}$ and $c_{out,2}$	0,0000	0,0000	-0,0002	-0,0002	-0,0016	-0,0014	0,0000	0,0000

	Mediob.		Morgan S.		Nomura		RBS	
	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	-0,3463	0,1813	5,0894	5,1569	0,3064	0,3646	-1,1205	-0,3912
	[0,1718]**	[0,1780]	[1,3083]***	[1,3163]***	[0,1099]***	[0,1036]***	[0,4301]***	[0,4280]
β_1 and β_2	1,0000	-0,9826	1,0000	-1,0234	1,0000	-1,0000	1,0000	-0,9919
	[0]	[0,0052]***	[0]	[0,0051]***	[0]	[0]	[0]	[0,0110]
β_0	-2,0449	-	4,3826	-	4,6179	-	0,2852	-
	[1,1866]*	-	[1,3427]***	-	[0,0459]***	-	[1,3427]	-
$c_{out,1}$ and $c_{out,2}$	0,0000	0,0000	0,0000	0,0000	-0,0296	-0,0157	-0,0001	-0,0001

	UBS		Wells F.	
	Y(\$)	Y(€)	Y(\$)	Y(€)
α_x and α_y	-0,6639	-0,4474	0,9857	1,8950
	[0,1776]***	[0,1639]***	[0,2784]***	[0,2633]***
β_1 and β_2	1,0000	-1,0000	1,0000	-1,0222
	[0]	[0]	[0]	[0,0077]***
β_0	-0,6029	„	1,6246	„
	[0,6949]	„	[0,7329]***	„
$c_{out,1}$ and $c_{out,2}$	0,0000	0,0000	0,0000	0,0000

Table A.20: Price Discovery from December 2007 to January 2014

The first row presents the restriction chosen on the cointegrating vector. The second and third rows contain the p-values of the speed adjustment coefficients computed by the *cajorls* function; in bold: statistically significant. In the fourth and fifth lines there are the Hasbrouck measures of the information share respectively of the dollar and euro market; in bold: greater share. In the sixth and seventh lines there are the Gonzalo-Granger measures of the component share respectively of the dollar and euro market; in bold: greater share. Finally in the last row there are the markets that slightly dominates the discovery.

	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.
B=[1 -1]	[1,-1,0]	REFUSE	[1,-1,c]	[1,-1,0]	REFUSE	[1,-1,c]	[1,-1,c]
pval(A1)	0,0013	-	0,7619	0,2745	-	0,1158	0,4622
pval(A2)	0,0944	-	0,7816	0,4799	-	0,0046	0,0040
HAS	0,4946	-	0,5000	0,4259	-	0,5046	0,5240
HAS	0,5054	-	0,5000	0,5741	-	0,4954	0,4760
GG1	-0,8977	-	-10,5840	0,4084	-	2,2397	1,3689
GG2	1,8977	-	11,5840	0,5916	-	-1,2397	-0,3689
Results	€			€		\$	\$

	JP Morgan	Mediob.	Morgan S.	Nomura	RBS	UBS	Wells F.
B=[1 -1]	REFUSE	REFUSE	REFUSE	[1,-1,c]	[1,-1,c]	[1,-1,c]	REFUSE
pval(A1)				0,0064	0,0096	0,0002	
pval(A2)	-	-	-	0,0007	0,3615	0,0067	-
HAS	-	-	-	0,5051	0,4934	0,4961	-
HAS	-	-	-	0,4949	0,5066	0,5039	-
GG1	-	-	-	6,2722	-0,5364	-2,0663	-
GG2	-	-	-	-5,2722	1,5364	3,0663	-
Results				both	€	both	

Table A.21: TVECM tests from December 2007 to January 2014

The table is subdivided in 2 horizontal parts: the tests ran with 50 bootstrapped values and with 100 bootstrapped values. The first and the second rows of each part present the Hansen-Seo p-values of the "Fixed regressor bootstrap" and the "Residual bootstrap" tests; the values were computed by the *TVECM.HStest* function of the "tsDyn" library; in bold: no evidence for TVECM. From the third to the fifth rows there are the p-values of the number of Th.; values performed by *TVAR.LRtest* function of the "tsDyn" library; in bold: no rejection of H0. In the last row it is shown the choice of the model: 1 Th., 2 Th., no TVECM.

50 boots	Barclays	BNP	CITI	Deutsche B.	Goldman S.	HSBC	HSBC H.
p-value HS FR	0,06	0,04	0,92	0,14	0,72	0,44	0,02
p-value HS RB	0,14	0,14	1,00	0,16	0,98	0,68	0,06
lin VS 1th	0,06	0,00	0,02	0,12	0,00	0,24	0,00
lin VS 2th	0,06	0,04	0,00	0,02	0,00	0,18	0,00
1th VS 2th	0,10	0,64	0,12	0,00	0,56	0,32	0,00
100 boots							
p-value HS FR	0,08	0,08	0,86	0,26	0,86	0,47	0,03
p-value HS RB	0,16	0,09	0,99	0,16	1,00	0,61	0,01
lin VS 1th	0,05	0,00	0,00	0,03	0,01	0,17	0,00
lin VS 2th	0,03	0,01	0,01	0,00	0,00	0,13	0,00
1th VS 2th	0,10	0,76	0,12	0,01	0,74	0,32	0,00
Results	no	1 th	no	no	no	no	no

50 boots	JP Morgan	Mediob.	Morgan S.	Nomura	RBS	UBS	Wells F.
p-value HS FR	0,34	0,64	0,52	0,34	0,78	0,00	0,00
p-value HS RB	0,40	0,54	0,76	0,08	0,62	0,00	0,00
lin VS 1th	0,08	0,74	0,04	0,10	0,00	0,14	0,04
lin VS 2th	0,06	0,98	0,08	0,04	0,00	0,06	0,02
1th VS 2th	0,18	1,00	0,90	0,04	0,06	0,24	0,08
100 boots							
p-value HS FR	0,40	0,61	0,45	0,34	0,72	0,00	0,01
p-value HS RB	0,36	0,70	0,67	0,08	0,59	0,00	0,00
lin VS 1th	0,07	0,71	0,02	0,17	0,00	0,10	0,01
lin VS 2th	0,00	0,97	0,05	0,06	0,00	0,04	0,01
1th VS 2th	0,13	0,99	0,93	0,06	0,03	0,23	0,09
Results	no	no	no	no	no	forse 2 th	1 th

Appendix B

Codes

In the first script I present an example of the code to obtain the results of this thesis. It is a pseudo-code to give an idea of how the real code is implemented.

Then I show the main function written for this work: the PersTVECM. It is a function that estimated one and two thresholds VECM by the usage of the Hansen and Seo grid procedure described in Chapter 4.

Finally I have also implemented two personal versions of the KPSS and PP unit root test, because of the lack on the libraries of R of a function that perform this test with a number of lags chosen by the user.

```
1 # Core program to obtain the results of my paper thesis
2 # It does not run, it is only a semplified guide for the implementation
3 #
4 # The hand written function indNA in the end
5
6 library("tsDyn")
7 library("vars")
8 library("fUnitRoots")
9 library("aTSA")
10
11 dataset = CDS;
12
13 # Find the indexes where are missing values
14 ind <- indNA(dataset)[[3]]
15
16 # Number of lag selected by AIC,HQ,SC,FPE. Excluding the rows with missing values
17 lagSelected = VARselect(dataset[-ind[,i],(i-1):i], lag.max = nLag, type = "const")
18   $selection
19 p = 3 #Which criteria use in the lag selection
20 nLag=lagSelected[[p]]
21
22 # Unit root test for each column
23 for(i in 1:ncol(dataset)){
24   adf <- adfTest(dataset[,i], lags=nLag[i], type="c")
25   pp <- ppPers.test(dataset[-ind[,i],i], lags=nLag[i])
26 }
```

```

25   kpss <- kpssPers.test(dataset[,i], null = "Level", lags = nLag[i])
   }
27
   # Computing the EG regressing the first variable on the second and viceversa.
29 EG1 <- coint.test(dataset[,i-1], dataset[,i], d = 0, nlag = nLag, output = FALSE)
   EG2 <- coint.test(dataset[,i], dataset[,i-1], d = 0, nlag = nLag, output = FALSE)
31
   # Computing the johansen test and estimation of VECM with the library urca
33 joh <- ca.jo(dataset[(i-1):i], type <- "trace", ecdet <- "const", K <- nLag,
   spec <- "longrun", season <- NULL, dumvar <- NULL)
   vecm <- cajorls(joh, r <- 1, reg.number <- NULL)
35
   # Computing the std dev of betas
37 res <- resid(vecm$rlm)
   N <- nrow(res)
39 sgm <- crossprod(res)/N
   beta.se <- append(beta.se, sqrt(diag(kronecker(solve(crossprod(joh@RK[, -1])), solve
   (t(alfa)%>%solve(sgm) %%% alfa))))))
41
   # Testing restriction and estimate restricted VECM
43 B1 <- matrix(c(1, -1, 0, 0, 0, 1), nrow<-3)
   B2 <- c(1, -1, 0)
45 jCon1 <- blrttest(joh, B1, 1)
   jCon2 <- blrttest(joh, B2, 1)
47 vecmCon1 <- cajorls(jCon1, r <- 1, reg.number <- NULL)
   vecmCon2 <- cajorls(jCon2, r <- 1, reg.number <- NULL)
49
   # Example of computation of price discovery measures
51 GG11 <- a2Con1 / (a2Con1-a1Con1)
   den1 <- matrix(c(a2Con1, a1Con1), nrow<-1)%%%cov(vecmCon1$rlm$residuals)%%%c(a2Con1,
   a1Con1)
53 num1 <- chol(cov(vecmCon1$rlm$residuals))%%c(a2Con1, a1Con1)
   den3 <- matrix(c(a1Con1, a2Con1), nrow<-1)%%%cbind(c(cov(vecmCon1[i-1]$rlm$residuals
   ) [2, 2], cov(vecmCon1[i-1]$rlm$residuals) [1, 2]), c(cov(vecmCon1[i-1]$rlm$
   residuals) [2, 1], cov(vecmCon1[i-1]$rlm$residuals) [1, 1]))%%c(a1Con1, a2Con1)
55 num3 <- chol(cbind(c(cov(vecmCon1[i-1]$rlm$residuals) [2, 2], cov(vecmCon1[i-1]$rlm$
   residuals) [1, 2]), c(cov(vecmCon1[i-1]$rlm$residuals) [2, 1], cov(vecmCon1[i-1]$rlm
   $residuals) [1, 1])))%%c(a1Con1, a2Con1)
   HAS <- ((num3[2] ^2) / den3 + (num1[1] ^2) / den1)/2
57
   # Testing for TVECM
59 test1<-TVECM.HStest(dataset[-ind[, i/2], ], lag=nLag, ngridTh=364, trim=0.1, nboot=
   nb, fixed.beta = 1, intercept=TRUE, boot.type="FixedReg")
   test2<-TVECM.HStest(dataset[-ind[, i/2], ], lag=nLag, ngridTh=364, trim=0.1, nboot=
   nb, fixed.beta = 1, intercept=TRUE, boot.type="ResBoot")
61 test3<-TVAR.LRtest(dataset[-ind[, i/2], ], lag = nLag, test = c("1vs"), model = c("
   TAR"), nboot = nb)
   test4<-TVAR.LRtest(dataset[-ind[, i/2], ], lag = nLag, test = c("2vs3"), model = c("
   TAR"), nboot = nb)
63
   # Estimation TVECM
65 TVECM <- PersTVECM(dataset[-ind[, i], ], lag=nLag, nthresh=1, trim=0.05, ngridTh=400,
   plot=TRUE, beta1=1, beta0=0, detVar="const")
67

```

```

69 indNA <- function(dataset){
  # Funtion that return 2 arrays: the index of the rows and the index colomns
  # where the dataset has NA.
71 # Input: dataset = dataset where you want to find the NA.
  # Outputs: rowNa = array containing the row indexes of the NA.
73 #         colNa = array containing the column indexes of the NA.
  # Function used: is.NA.
75 # Variables declaration , allocation and iniatialization.
  rowNA<-NULL # Array that will contain the row indexes of the NA.
77 colNA<-NULL # Array that will contain the colomn indexes of the NA.
  matrixRow <- matrix(0,nrow(dataset),ncol(dataset))
79 a=1
  # Computation.
81 # Visit the dataset.
  for(j in 1:ncol(dataset)){
83     b=1
      for(i in 1:nrow(dataset)){
85         # When it find a NA it saves its indexes.
          if(is.na(dataset[i,j])){
87             rowNA[a] <- i
              colNA[a] <- j
89             matrixRow[b,j] <- i
              a=a+1
91             b=b+1
          }
83     }
93 }
95 return(list(rowNA,colNA,matrixRow))
}

```

```

# Estimation of a Threshold Vector Error Correction model (TVECM)
2 #
# If thresholds and cointegrating vectors are given, the model is linear, so
4 # estimation of the regression parameters can be done directly by CLS
# (Conditional Least Squares). The search of the threshold and cointegrating
6 # parameters values which minimize the residual sum of squares (SSR) is made on
# a grid of potential values.
8 #
# If thresholds are not given, the function can estimate one as well as two
  thresholds:
10 #
# 1) the estimation of one threshold model (two regimes) is done by a grid of \var
  {ngridTh}
12 # values (default to ALL) possible thresholds. }
#
14 # 2) the estimation of two thresholds model (three regimes) is done with this
  procedure:
# The second threshold is searched conditional on the threshold found in model
  where nthresh=1.
16 # When both are found, a second grid search is made with 30 values around each
  threshold.} }
#
18 # The grid for the first threshold can be set in different ways:
#
20 # 1) given: Pre-specified value
# 2) int: Specify an interval (of length \var{ngridTh}) in which to search.
22 # 3) around: Specify to take \var{ngridTh} points around the value given.
#
24 # The default is to do an interval search. Interval bounds for the threshold
# interval are simply the \var{trim} and 1-\var{trim} percents of the sorted
26 # error correction term. For the cointegrating parameter, bounds of the
# interval are obtained from the (OLS) confidence interval of the linear
28 # cointegration case. It is often found however that this interval is too
# tight. It is hence recommended to inspect the plot of the grid search.
30 #
# The function around grid for the estimation of the thresholds
32 # and the function nameB for the layout of the output are at the bottom.
#
34 # @param data time series
# @param lag Number of lags to include in each regime
36 # @param nthresh number of threshold (see details)
# @param trim trimming parameter indicating the minimal percentage of
38 # observations in each regime
# @param ngridTh number of elements to search for the threshold value
40 # @param plot Whether the grid with the SSR of each threshold should be plotted.
# @param th1 different possibilities to pre-specify a given value, an interval
42 # or a central point for the search of the threshold (or first threshold if
# nthresh=2)
44 # @param th2 different possibilities to pre-specify an exact value or a central
# point for the search of the second threshold (used only if nthresh=2)
46 # @param beta1 so that the cointegrating vector will be c(1,\code{-beta1})
# @param restr possible restrictions on the thresholds in case three regimes
48 # @param beta0 Additional regressors to include in the cointegrating relation
# @author Stefano Moawad
50 # @references Hansen, B. and Seo, B. (2002), Testing for two-regime threshold

```

```

# cointegration in vector error-correction models, Journal of Econometrics,
52 #'110, pages 293 - 318
#
54 #'Seo, M. H. (2009) Estimation of non linear error-correction models, Working
#'paper
56 #'
PersTVECM<-function(data,lag=1,nthresh=1, trim=0.05, ngridTh=50, plot=TRUE, restr
= c("none", "signOp"),
58 , around="val"),
beta1=1, beta0=0,th1=list(given=NULL, int=c("from", "to")
, around="val"),
th2=list(given=NULL, around="val"), detVar = c("const",
60 "trend", "none", "both")) {
##### Check inputs and Create variables
62 detVar <- match.arg(detVar)
restr <- match.arg(restr)
64 ngrid <- ngridTh
if(!missing(th1) &&!is.list(th1)) th1 <- list(given=th1)
66 if(!missing(th2) &&!is.list(th2)) th2 <- list(given=th2)
68 y <- as.matrix(data)
T <- nrow(y) # Number of observations
70 nLag<- lag # Number of lags
t <- T-nLag-1 # Size of end sample
72 dim <- ncol(y) # Dimension of the time series
if(is.null(colnames(data))) colnames(data) <- paste("Var", c(1:dim), sep="")
74 ndig <- 4 #Number of digits
76 # Time series considering the lags
dy <- diff(y)[(nLag+1):(T-1),] # Consider the time series in difference
78 ymin1<-embed(y, nLag+2)[,(dim+1):(dim+dim)] # Consider the lagged y-(t-1)
lagVal<-embed(diff(y), nLag+1)[,-(1:dim)] # Lags' values
80 # Inserting the deterministic variables.
if(detVar=="const") lagVal<-cbind(rep(1,t), lagVal)
82 if(detVar=="trend") lagVal<-cbind(seq_len(t), lagVal)
if(detVar=="both") lagVal<-cbind(rep(1,t), seq_len(t), lagVal)
84
86 ##### Linear VECM estimation (Engle-Granger second step approach)
# Preparing the inputs
88 beta0 <- rep(beta0,t) # All the additional regressors of the cointegration
vector.
beta <- beta1 # Coefficients of the independent variables in the cointegration
vector.
90 if(length(beta) !=dim-1){warning("beta$given should be of same size as cols of y-
-1\n")}
ECT <- ymin1%*%c(1,-beta)-beta0 # Computation of the error correction term.
92
# Computing the regression
94 Z <- cbind(ECT,lagVal) # All the regressors: ECT and lags (t x npar)
B <- t(dy)%*%Z%*%solve(t(Z)%*%Z) # Computation of the OLS parameters, (2 x
npar)
96 npar <-ncol(B) # Number of parameter to be estimated
allpar<-ncol(B)*nrow(B) # Total nuber of parameters' componenets
98
# Diagnostic of the estimates

```

```

100 res<-dy-Z%*%t(B) # Computation of the residuals of the regression
Sigma<- matrix(crossprod(res)/t, ncol=dim, dimnames=list(colnames(data), colnames(
data))) # Var-cov matrix of the residuals
102 VarCov<-solve(crossprod(Z))%*%Sigma # Var-cov matrix of the coefficients
StDev<-matrix(diag(VarCov)^0.5, nrow=dim) # Standard deviation matrix of the
coefficients
104 Tvalue<-B/StDev # T-statistics of the coefficients
Pval<-pt(abs(Tvalue), df=(t-ncol(Z)), lower.tail=FALSE)+pt(-abs(Tvalue), df=(t-
ncol(Z)), lower.tail=TRUE) # PVal of the coefficients
106
# Names of variables and computed regressors
108 rownames(B)<-paste("Equation", colnames(data))
LagNames<-c(paste(rep(colnames(data), nLag), -rep(seq_len(nLag), each=dim)))
110 colnames(B)<-switch(detVar, "const"=c("ECT", "Intercept", LagNames), "trend"=c("
ECT", "Trend", LagNames),
"both"=c("ECT", "Intercept", "Trend", LagNames), "none"=c("ECT"
, LagNames))
112 colnames(Pval)<-colnames(B)
114
#### Set up of the grid
116 # Creation of the grid
allPoxTh<-sort(unique(ECT-beta0)) # Possible values for the treshold.
118 nTh<-length(allPoxTh)
poxTh<-allPoxTh[round(seq(from=trim, to=1-trim, length.out=ngrid)*nTh)] # grid
from lower to higher point
120
# Given threshold
122 if(is.null(th1$given)==FALSE){
if(any(allPoxTh==th1$given)==FALSE) warning("The value you gave for th does
not correspond to an existing value.")
124 poxTh<-th1$given
ngrid<-1
126 }
# Interval to search between given by user
128 if(is.numeric(th1$int)){
intDown<-which.min(abs(allPoxTh-th1$int[1]))
130 intUp<-which.min(abs(allPoxTh-th1$int[2]))
poxTh<-allPoxTh[seq(from=intDown, to=intUp, length.out=min(ngrid, intUp-intDown
))]
132 }
# Indication of value around which build a neighbourhood
134 if(is.numeric(th1$around)) poxTh<-aroundGrid(th$around, allvalues=allPoxTh, ngrid,
trim)
136
#### Search for one threshold.
138 oneSearch<-function(ECT, poxTh){
oneThresh<-function(ECT, th, dy, ymin1, lagVal){
140 zi<-cbind(ECT, lagVal) # Regressors: ECT and lag, (t x kp+1+1)
d1<-ifelse(ECT<=th, 1, 0) # Dummy vector d1=(w<=th);
142 n1<-mean(d1) # Number of elements of the ECT under the threshold
if(is.na(n1)==TRUE) n1<-0
144 if(min(n1, 1-n1)>trim) {
Z<-cbind(c(d1)*zi, zi*c(1-d1))
146 LS<-try(crossprod(c(dy-tcrossprod(Z, crossprod(dy, Z))%*%solve(crossprod(Z)))

```

```

148    )), silent=TRUE)
149     if (inherits(LS, "try-error")) {
150         warning("Error when solving for value: th=", th)
151         LS <- NA
152     }
153     else LS<-NA
154     return(LS)
155 }
156 # Grid search
157 store<-matrix(NA,nrow=length(poxTh), dimnames=list(round(poxTh,3)))
158 for (i in seq_len(length(poxTh))){
159     store[i]<-oneThresh(ECT=ECT, th=poxTh[i], lagVal=lagVal, ymin1=ymin1,dy=dy)
160 }
161 na<-sum( ifelse( is.na(store),1,0) )
162 if (na>0) cat(na,"_", na/(nrow(store)*ncol(store)), "_points_of_the_grid_lead
163 to regimes with percentage of observations < trim and were not computed\n",
164 sep="")
165 pos<-which(store==min(store, na.rm=TRUE), arr.ind=TRUE)
166 if (nrow(pos)>1) {
167     cat("There were",nrow(pos), " thresholds/cointegrating combinations(",
168     paste(poxTh[pos[,1]],"/",beta,"_","), "\nwhich minimize the SSR in the first
169 search, the first one", round(poxTh[pos[1,1]],ndig), " ",round(beta,ndig), "
170 was taken\n")
171     pos<-pos[1]
172 }
173 bestTh1<-poxTh[pos[1]]
174 # Plot results of grid search
175 if (is.null(th1$given)==FALSE){plot<-FALSE}
176 if (plot){
177     plot(poxTh,store, type="l", xlab="Threshold parameter th", ylab="Residual
178 Sum of Squares", main="Grid Search")
179     points(x=bestTh1, y=min(store, na.rm=TRUE), col=2, cex=2)
180 }
181 return(bestTh1)
182 }
183 # Run the previous funtion in case of one threshold.
184 if (nthresh==1){bestTh<-oneSearch(ECT, poxTh)}
185
186 ##### Search for two thresholds
187 if (nthresh==2){
188     two_Thresh<-function(ECT,th1,th2){
189         zi<-cbind(ECT,lagVal) # Regressors: ECT and lag.
190         d1<-ifelse(ECT<=th1, 1,0) # Dummy vector 1.
191         n1<-mean(d1) # Number of elements of the ECT under the threshold 1.
192         d2<-ifelse(ECT>th2,1,0) # Dummy vector 2.
193         n2<-mean(d2) # Number of elements of the ECT under the threshold 2.
194         if (is.na(n1)) n1<-n2<-0
195         if (min(n1,n2,1-n1-n2)>trim) {
196             Z<-cbind(c(d1)*zi, c(d2)*zi, c(1-d1-d2)*zi)
197             LS<-try( crossprod(c(dy-tcrossprod(Z, crossprod(dy,Z))%solve(crossprod(Z))
198             )), silent=TRUE)

```

```

196     if (inherits(LS, "try-error")) {
197         warning("Error when solving for value: _poxTh=", th1, th2)
198         LS <- NA
199     }
200     else LS<-NA
201     return(LS)
202 }

204 # Conditional search.
205 bestTh <- oneSearch(ECT, poxTh)
206 cat("Best threshold from first search", bestTh, "\n")
207 # Second threshold given.
208 if (!is.null(th2$given))
209     secondBestThresh<-th2$given
210 # If second threshold not given and also around.
211 if (is.null(th2$given) & !is.numeric(th2$around)) {
212     wh.thresh <- which.min(abs(allPoxTh-bestTh))
213     ninter<-round(trim*nrow(ymin1))
214     if (restr=="none") {
215         if (wh.thresh>2*ninter) {
216             thMinus<-allPoxTh[seq(from=ninter, to=wh.thresh-ninter)]
217             storeMinus<-matrix(NA,nrow=length(thMinus), dimnames=list(round(thMinus
218 ,3)))
219             for (i in seq_len(length(thMinus))) {storeMinus[i] <- two_Thresh(ECT=ECT,
220 th1=thMinus[i], th2=bestTh)}
221         }
222         else storeMinus <- NA
223         if (wh.thresh < length(allPoxTh) - 2 * ninter) {
224             thPlus <- allPoxTh[seq(from = wh.thresh + ninter, to = length(allPoxTh)
225 - ninter)]
226             storePlus<-matrix(NA,nrow=length(thPlus), dimnames=list(round(thPlus,3))
227 )
228             for (i in seq_len(length(thPlus))) {storePlus[i] <- two_Thresh(ECT=ECT,
229 th1=bestTh, th2=thPlus[i])}
230         }
231         else storePlus <- NA
232     }
233     # If threshold with opposite sign are required.
234     else if (restr == "signOp") {
235         zero <- which.min(abs(allPoxTh))
236         if (sign(bestTh) > 0) {
237             thMinus <- allPoxTh[seq(from = ninter, to = min(wh.thresh - ninter, zero
238 ))]
239             storeMinus<-matrix(NA,nrow=length(thMinus), dimnames=list(round(thMinus
240 ,3)))
241             for (i in seq_len(length(thMinus))) {storeMinus[i] <- two_Thresh(ECT=ECT,
242 th1=thMinus[i], th2=bestTh)}
243             storePlus <- NA
244         }
245         else {
246             thPlus <- allPoxTh[seq(from = max(wh.thresh + ninter, zero), to = length
247 (allPoxTh) - ninter)]
248             storePlus<-matrix(NA,nrow=length(thPlus), dimnames=list(round(thPlus,3))
249 )
250             for (i in seq_len(length(thPlus))) {storePlus[i] <- two_Thresh(ECT=ECT,

```

```

th1=bestTh , th2=thPlus [ i ] }
  storeMinus <- NA
242   }
  }
244   store2 <- c ( storeMinus , storePlus )
  positionSecond <- which ( store2 == min ( store2 , na.rm = TRUE ) )
246   if ( length ( positionSecond ) > 1 ) {
     cat ( " There_were_" , length ( positionSecond ) , "_thresholds_values_which_
minimize_the_SSR_in_the_conditional_step ,_the_first_one_was_taken\n" )
248     positionSecond <- positionSecond [ 1 ]
  }
250   if ( positionSecond <= length ( storeMinus ) ) { secondBestThresh <- thMinus [
positionSecond ] }
  else { secondBestThresh <- thPlus [ positionSecond - length ( storeMinus ) ] }
252   cat ( " Second_best_(conditionnal_on_the_first_one)" , c ( bestTh , secondBestThresh
) , "\t_SSR" , min ( store2 , na.rm = TRUE ) , "\n" )
  }
254
# If the around is given do not do all the previous conditiona search but only
start from th2.
256   if ( is.numeric ( th2$around ) )
     secondBestThresh <- th2$around
258   # Definitive search
  smallThresh <- min ( bestTh , secondBestThresh )
260   thsDown <- aroundGrid ( around = smallThresh , allPoxTh , ngrid = ngrid , trim = trim )
  bigThresh <- max ( bestTh , secondBestThresh )
262   thsUp <- aroundGrid ( around = bigThresh , allPoxTh , ngrid = ngrid , trim = trim )
  if ( ! is.null ( th2$given ) ) {
264     if ( th2$given < bestTh ) thsDown <- th2$given
     else thsUp <- th2$given
266   }
  if ( ! is.null ( th1$given ) ) {
268     if ( th1$given < secondBestThresh ) thsDown <- th1$given
     else thsUp <- th1$given
270   }
  storeIter <- matrix ( NA , ncol = length ( thsUp ) , nrow = length ( thsDown ) )
272   for ( i in seq_along ( thsDown ) ) {
     th1 <- thsDown [ i ]
274     for ( j in seq_along ( thsUp ) ) {
        th2 <- thsUp [ j ]
276         storeIter [ i , j ] <- two_Thresh ( th1 = th1 , th2 = th2 , ECT = ECT )
     }
278   }
  positionIter <- which ( storeIter == min ( storeIter , na.rm = TRUE ) , arr.ind =
TRUE )
280   if ( nrow ( positionIter ) > 1 ) {
     cat ( " There_were_" , length ( positionIter ) , "_thresholds_values_which_minimize_
the_SSR_in_the_iterative_step ,_the_first_one_was_taken\n" )
282     positionIter <- positionIter [ 1 , ]
  }
284   bestTh <- c ( thsDown [ positionIter [ 1 ] ] , thsUp [ positionIter [ 2 ] ] )
  cat ( " Second_step_best_thresholds" , bestTh , "\t\t\t_SSR" , min ( storeIter , na.rm =
TRUE ) , "\n" )
286 }
288

```

```

##### Definitive model
290 if (nthresh==1){
    Z_temp<-cbind(ECT, lagVal) # Regressors: ECT and lag.
292 d1<-ifelse (ECT<=bestTh, 1,0) # Dummy vector.
    ndown<-mean(d1) # Number of elements of the ECT under the threshold.
294 nup<-1-ndown # Number of elements of the ECT above the threshold.
    Zbest<-cbind(c(d1)*Z_temp, c(1-d1)*Z_temp)
296 }
    if (nthresh==2){
298 d1<-ifelse (ECT<=bestTh[1], 1,0) # Dummy vector #d1=(w<=th1);
    ndown<-mean(d1) # Number of elements of the ECT under the threshold 1.
300 d2<-ifelse (ECT>bestTh[2], 1,0) # Dummy vector #d1=(w>=th2);
    nup<-mean(d2) # Number of elements of the ECT under the threshold 2.
302 Z_temp<-cbind(ECT, lagVal) # Regressor: ECT and lag.
    Zbest<-cbind(c(d1)*Z_temp,(1-c(d1)-c(d2))*Z_temp, c(d2)*Z_temp)
304 }

306 # In which regime is each observation.
    reg<-if (nthresh==1) d1+2*(1-d1) else d1+2*(1-d1-d2)+3*d2
308 regime <- c(rep(NA,T-t), reg)

310 # Estimate parameters, fitted values, residuals
    Bbest<-t(dy)%*%Zbest%*%solve(t(Zbest)%*%Zbest)
312 allpar<-ncol(Bbest)*nrow(Bbest)
    fitted<-Zbest%*%t(Bbest)
314 resbest <- dy - fitted

316 # Naming the parameter matrix
    rownames(Bbest) <- paste("Equation", colnames(data))
318 lagValnames<-c(paste(rep(colnames(data), nLag), "t", -rep(1:nLag, each=dim)))
    Bcolnames <- c("ECT", switch(detVar, const="Const", trend="Trend", both=c("Const"
    , "Trend"), none=NULL), lagValnames)
320 # Partitioning the matrix following the regimes, and naming it
    Blist<-nameB(Bbest, Bnames=Bcolnames, nthresh=nthresh, npar=npar, model="TVECM",
    commonInter = FALSE)
322 BnamesVec<-if(class(Blist)=="list") c(sapply(Blist, colnames)) else colnames(
    Blist)
    colnames(Bbest)<-BnamesVec
324 naX <- rbind(matrix(NA, ncol = ncol(Zbest), nrow = nLag + 1),
    Zbest)
326 YnaX <- cbind(data, naX)
    Z<-t(as.matrix(tail.matrix(YnaX[, -c(1:dim)], t)))
328 Sigtabest<-matrix(1/t*crossprod(resbest), ncol=dim)
    SigtabestOls<-Sigtabest*(t/(t-ncol(Bbest)))
330 VarCovB<-solve(tcrossprod(Z))%*%SigtabestOls
    StDevB<-matrix(diag(VarCovB)^0.5, nrow=dim)
332 Tvalue<-beta/StDevB
    StDevB<-nameB(StDevB, commonInter=FALSE, Bnames=Bcolnames, nthresh=nthresh, npar=
    npar, model="TVECM")
334 Pval<-pt(abs(Tvalue), df=(ncol(Z)-nrow(Z)), lower.tail=FALSE)+pt(-abs(Tvalue),
    df=(ncol(Z)-nrow(Z)), lower.tail=TRUE)
    Pval<-nameB(Pval, commonInter=FALSE, Bnames=Bcolnames, nthresh=nthresh, npar=npar
    , model="TVECM")
336 #####Number of observations in each regime
    if (nthresh==1) nobs <- c(ndown=ndown, nup=nup)
338 if (nthresh==2) nobs <- c(ndown=ndown, nmiddle=1-nup-ndown, nup=nup)

```

```

340   ###elements to return
z<-list(coefficients=Blist , residuals=resbest , model=YnaX, coeffmat=Bbest , nob_
      reg=nobs , dim=dim , t=t ,T=T, nparB=allpar , fitted.values=fitted ,
342     lag=lag , thresh=bestTh , npar=npar , regime=regime , StDev=StDevB , Pval=
      Pval , varcov=VarCovB)
344   return(z)
}

346 #function to select values around a given point
aroundGrid <- function(around , allvalues , ngrid , trim){
348   nTh <- length(allvalues)
wh.around <- which.min(abs(allvalues-around))
350   if(length(which(allvalues==around))==0) cat("The value ", around , "_did not _
      match_to_existing_ones" , allvalues[wh.around] , "was taken instead\n")
352   if(length(wh.around)>1){
      cat("\n\tThere were" , length(wh.around) , "_values corresponding to the around _
      argument. The first one was taken")
354     wh.around<-wh.around[1]
}
ar <- seq(from=wh.around-round(ngrid/2) , to=(wh.around+round(ngrid/2)))
      #Values around the point
356   ar2 <- ar[ar>=round(trim*nTh)&ar<=round((1-trim)*nTh)] #
      Bounding with trim
358   values <- allvalues[ar2]
      return(values)
}

360 # Function for the layout of the output
362 nameB <- function (mat , commonInter , Bnames , nthresh , npar , model = c("TVAR" ,
      "TVECM" ) ,
      TVECMmodel = "All" , sameName = TRUE)
364 {
model <- match.arg(model)
366   addRegLetter <- if (sameName)
      NULL
368   else c("L_" , if (nthresh == 1) NULL else "M_" , "H_")
if (model == "TVAR")
370     sBnames <- Bnames[-which(Bnames == "Intercept")]
else if (model == "TVECM")
372     sBnames <- Bnames[-which(Bnames == "ECT")]
if (nthresh == 1) {
374     if (commonInter) {
if (model == "TVAR") {
376       colnames(mat) <- c("Intercept" , paste(rep(addRegLetter ,
      each = length(sBnames)) , rep(
sBnames , 2)) ,
378       sep = "")
}
380     else if (model == "TVECM") {
      colnames(mat) <- c("ECT-" , "ECT+" , sBnames)
382     }
      Blist <- mat
384   }
else {
386     colnames(mat) <- paste(rep(addRegLetter , each = length(Bnames)) ,

```

```

388         rep(Bnames, 2), sep = "")
Bdown <- mat[, c(1:npar)]
Bup <- mat[, -c(1:npar)]
390 Blist <- list(Bdown = Bdown, Bup = Bup)
}
392 }
else {
394   if (commonInter) {
     if (model == "TVAR")
396       colnames(mat) <- c("Intercept", paste(rep(addRegLetter,
                                                    each = length(sBnames)), rep(
sBnames, 3)),
398         sep = "")
     else if (model == "TVECM")
400       colnames(mat) <- c("ECT-", "ECT+", sBnames)
     Blist <- mat
402   }
   else {
404     colnames(mat) <- paste(rep(addRegLetter, each = length(Bnames)),
                            rep(Bnames, 3), sep = "")
406     Bdown <- mat[, c(1:npar)]
     Bmiddle <- mat[, c(1:npar) + npar]
408     Bup <- mat[, c(1:npar) + 2 * npar]
     colnames(Bmiddle) <- Bnames
410     Blist <- list(Bdown = Bdown, Bmiddle = Bmiddle,
                   Bup = Bup)
412   }
}
414 return(Blist)
}

```

```

1 #' Phillips–Perron unit root test
2 #'
3 #' Computes the Phillips–Perron test for the null hypothesis that data has a unit
4   root.
5 #'
6 #' The innovation w.r.t to the functions already available as the possibility to
7   choose
8   the lag number.
9 #'
10 #' It first regresses the data on the usual equation considered for unit root tests
11   ,
12   which incorporates a constant and a linear trend.
13 #' Then,  $Z(\alpha)$  or  $Z(t_\alpha)$  statistic for a first order autoregressive
14   coefficient equals one are computed.
15 #' To estimate  $\sigma^2$  the Newey–West estimator is used.
16 #' The p-values are interpolated from Table 4.1 and 4.2, p. 103 of Banerjee et al.
17   (1993).
18 #' Missing values are not handled.
19 #'
20 #' tseries_pp_sum function to compute the sum needed for the statistics is in the
21   bottom.
22 #'
23 #' @param data time series vector.
24 #' @param lag Number of lags.
25 #' @author Stefano Moawad
26 #' @references A. Banerjee, J. J. Dolado, J. W. Galbraith, and D. F. Hendry (1993):
27   Cointegration, Error Correction, and the Econometric Analysis of Non-Stationary
28   Data,
29   Oxford University Press, Oxford.
30 #'
31 #' P. Perron (1988): Trends and Random Walks in Macroeconomic Time Series.
32 #' Journal of Economic Dynamics and Control 12, 297–332.
33 #'
34 ppPers.test <- function(data, lags = nlag){
35
36   if((NCOL(data) > 1) || is.data.frame(data))
37     stop("data_is_not_a_vector_or_univariate_time_series")
38   data <- as.vector(data, mode="double")
39   z <- embed(data, 2)
40   yt <- z[,1]
41   yt1 <- z[,2]
42   n <- length(yt)
43   tt <- (1:n)-n/2
44   l <- as.integer(lags)
45
46   res <- lm(yt ~ 1 + tt + yt1)
47   if(res$rank < 3) stop("Singularities_in_regression")
48   res.sum <- summary(res)
49   u <- residuals(res)
50   ssqru <- sum(u^2)/n
51   ssqrtl <- tseries_pp_sum(as.vector(u, mode="double"), n, l, ssqru)
52   n2 <- n^2
53   trm1 <- n2*(n2-1)*sum(yt1^2)/12
54   trm2 <- n*sum(yt1*(1:n))^2
55   trm3 <- n*(n+1)*sum(yt1*(1:n))*sum(yt1)

```

```

49 trm4 <- (n*(n+1)*(2*n+1)*sum(yt1)^2)/6
Dx <- trm1-trm2+trm3-trm4
51 tstat <- (res.sum$coefficients[3,1]-1)/res.sum$coefficients[3,2]
STAT <- sqrt(ssgru)/sqrt(ssqrt1)*tstat-(n^3) / (4*sqrt(3)*sqrt(Dx)*sqrt(ssqrt1))
    *(ssqrt1-ssgru)
53
table <- cbind(c(4.38, 4.15, 4.04, 3.99, 3.98, 3.96), c(3.95, 3.80, 3.73, 3.69,
    3.68, 3.66),
55             c(3.60, 3.50, 3.45, 3.43, 3.42, 3.41), c(3.24, 3.18, 3.15, 3.13,
    3.13, 3.12),
             c(1.14, 1.19, 1.22, 1.23, 1.24, 1.25), c(0.80, 0.87, 0.90, 0.92,
57             0.93, 0.94),
             c(0.50, 0.58, 0.62, 0.64, 0.65, 0.66), c(0.15, 0.24, 0.28, 0.31,
    0.32, 0.33))
table <- -table
59 tablen <- dim(table)[2]
tableT <- c(25, 50, 100, 250, 500, 100000)
61 tablep <- c(0.01, 0.025, 0.05, 0.10, 0.90, 0.95, 0.975, 0.99)
tableipl <- numeric(tablen)
63 for(i in (1:tablen)) tableipl[i] <- approx(tableT, table[, i], n, rule=2)$y
PVAL <- approx(tableipl, tablep, STAT, rule=2)$y
65
return(list(statistic = STAT, p.value = PVAL))
67 }

69 # Funtion to compute the sum needed for the test
tseries_pp_sum <- function(u, n, l, sum){
71   tmp1 = 0
   for(i in 1:l){
73     tmp2 = 0
     for (j in (i+1):n){
75       if(!is.na(u[j]) & !is.na(u[j-i])){
         tmp2 = tmp2 + u[j]*u[j-i]
77       }
     }
79     tmp2 = tmp2 * 1-(i/(l+1))
     tmp1 = tmp1 + tmp2
81   }
   tmp1 = tmp1 / n
83   sum = sum + tmp1;
85   return(sum)
}

```

```

2 #' KPSS test for unit roots.
3 #'
4 #'Computes the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test for the null
   hypothesis
5 #'that data is level or trend stationary.
6 #'
7 #'The innovation w.r.t to the functions already available as the possibility to
   choose
8 #'the lag number.
9 #'
10 #'It first regresses the data on the null hypothesis.
11 #'Then, Z(alpha) or Z(t_alpha) statistic for a first order autoregressive
   coefficient equals one are computed.
12 #'To estimate sigma^2 the Newey–West estimator is used.
13 #'The p-values are interpolated from Table 1 of Kwiatkowski et al. (1992).
14 #'Missing values are not handled.
15 #'
16 #'@param data time series vector.
17 #'@param const Constant included in null hypothesis and must be one of "Level" (
   default) or "Trend".
18 #'@param lag Number of lags.
19 #'@author Stefano Moawad
20 #'@references D. Kwiatkowski, P. C. B. Phillips, P. Schmidt, and Y. Shin (1992):
   Testing the Null Hypothesis of Stationarity against the Alternative of a Unit
   Root.
   #'Journal of Econometrics 54, 159–178.
21
22 kpssPers.test <- function(data, const = c("Level", "Trend"), lags = nLag){
23   const <- match.arg(const)
24   data <- as.vector(data, mode="double")
25   l <- as.integer(lags)
26   n <- length(data)
27   if(const == "Trend") {
28     t <- 1:n
29     e <- residuals(lm(data ~ t))
30     table <- c(0.216, 0.176, 0.146, 0.119)
31   }
32   else if(const == "Level") {
33     e <- residuals(lm(data ~ 1))
34     table <- c(0.739, 0.574, 0.463, 0.347)
35   }
36   tablep <- c(0.01, 0.025, 0.05, 0.10)
37   s <- cumsum(e)
38   eta <- sum(s^2)/(n^2)
39   s2 <- sum(e^2)/n
40   s2 <- tseries_pp_sum(as.vector(e), n, l, s2)
41   STAT <- eta/s2
42   PVAL <- approx(table, tablep, STAT, rule=2)$y
43   if(!is.na(STAT) && is.na(approx(table, tablep, STAT, rule=1)$y))
44     if(PVAL == min(tablep))
45       warning("p-value smaller than printed p-value")
46   else
47     warning("p-value greater than printed p-value")
48   return(list(Statistic = STAT, p.value = PVAL))
49 }
50

```


Appendix C

Figures

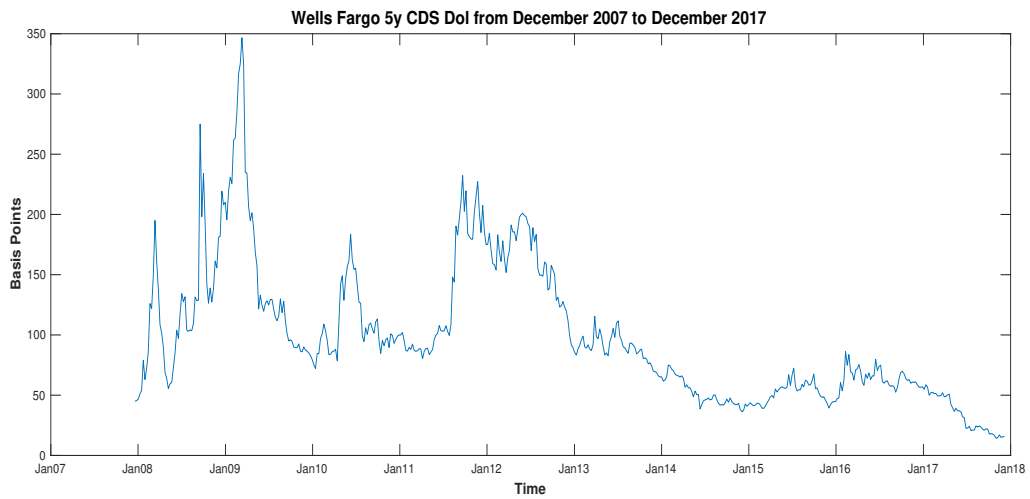


Figure C.1: Wells Fargo CDS 5y in dollars from 2008 to 2017. First of the typical shape of CDS spread through this 10 years.

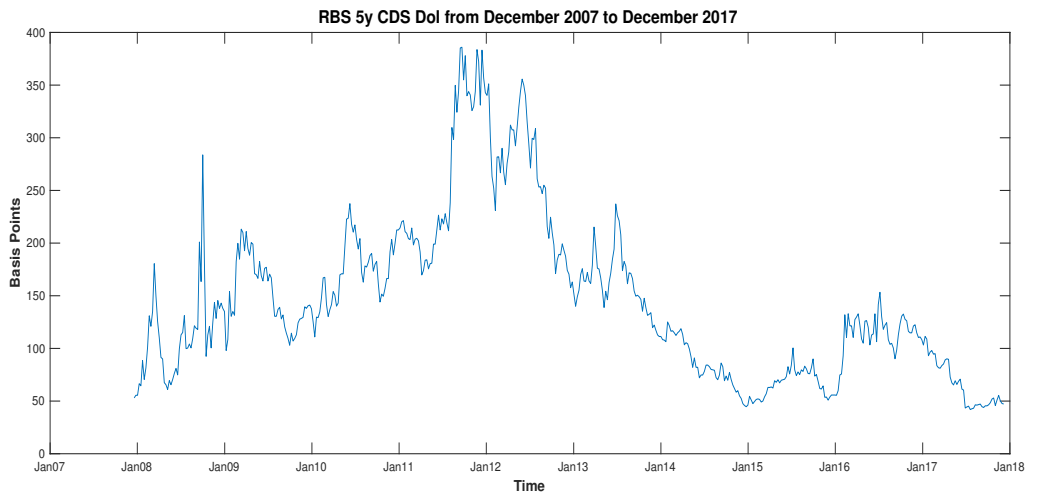


Figure C.2: RBS CDS 5y in dollars from 2008 to 2017.
 Second of the typical shape of CDS spread through this 10 years.

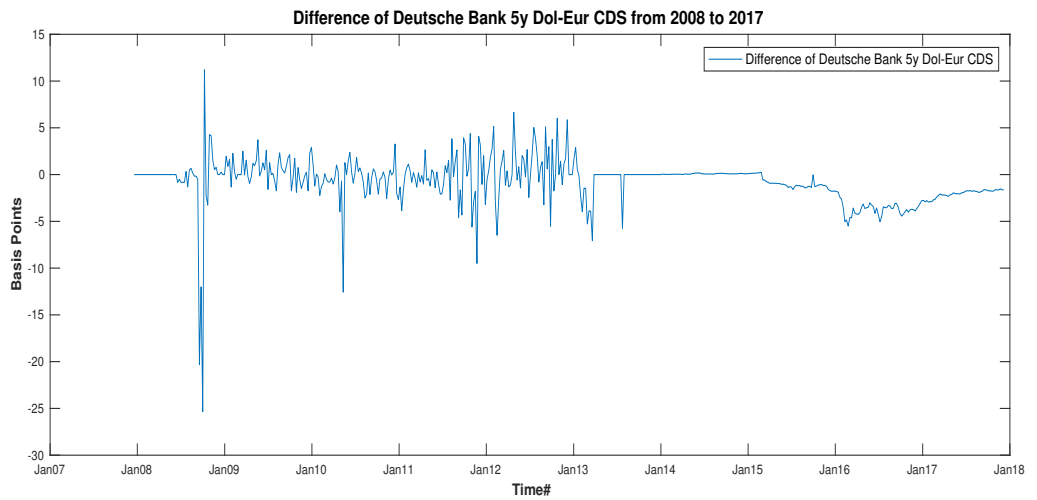


Figure C.3: Difference between Deutsche Bank CDS 5y in dollars and in euros from 2008 to 2017.

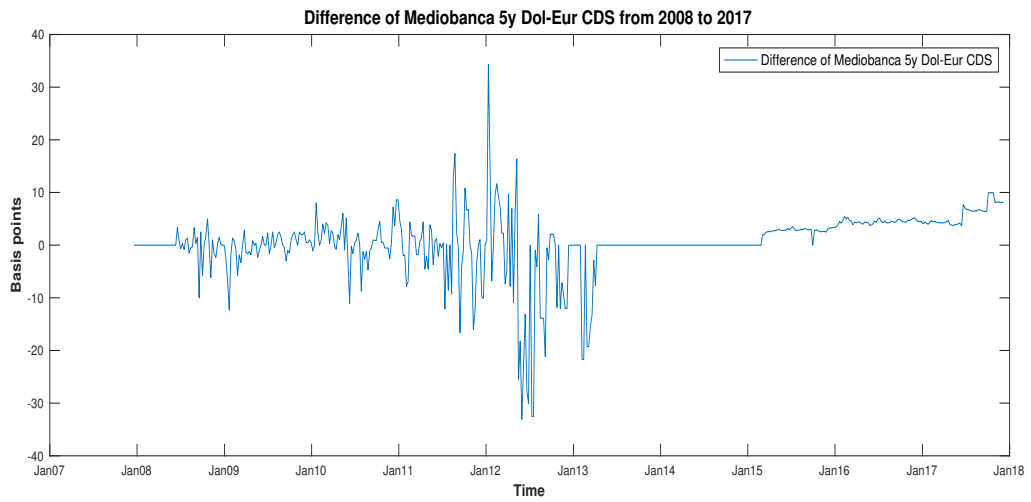


Figure C.4: Difference between Mediobanca CDS 5y in dollars and in euros from 2008 to 2017.

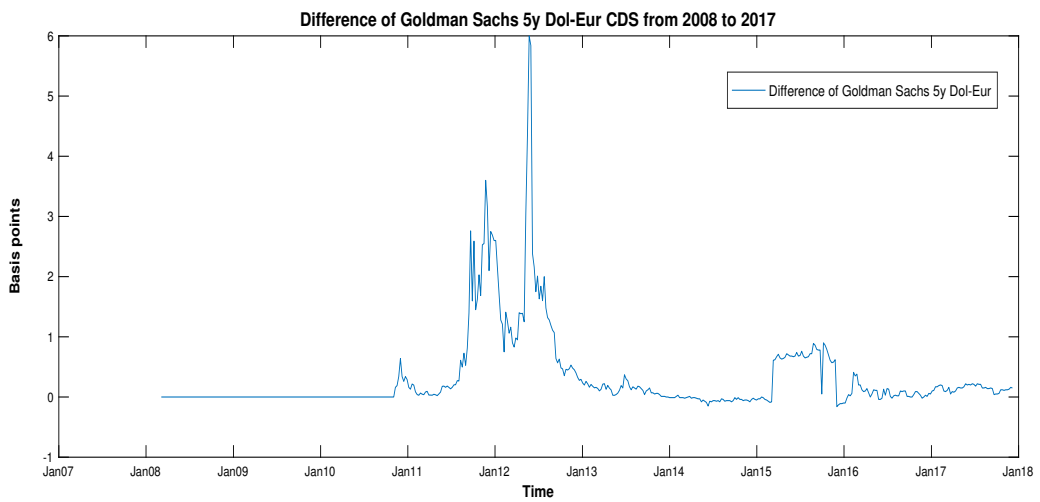


Figure C.5: Difference between Goldman Sachs CDS 5y in dollars and in euros from 2008 to 2017.

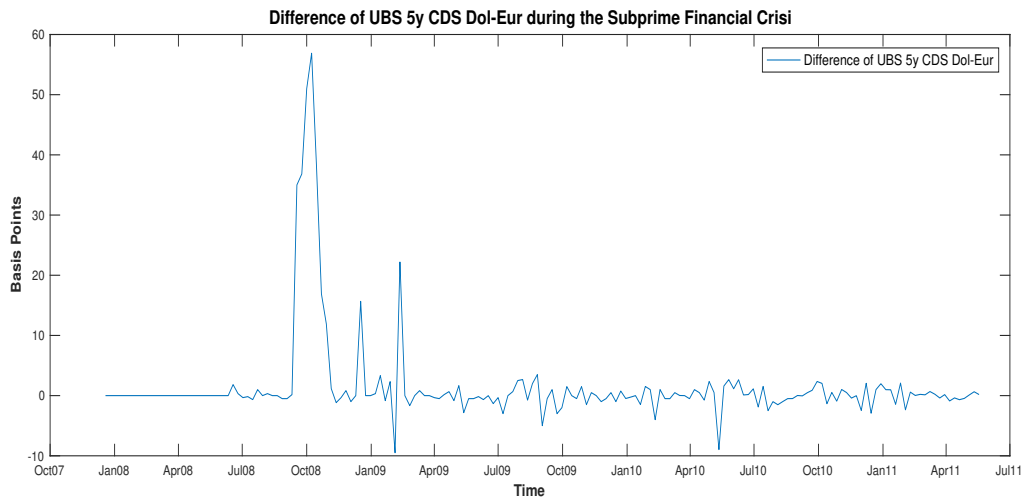


Figure C.6: Difference between UBS CDS 5y in dollars and in euros during the Sub-prime Financial Crisis.

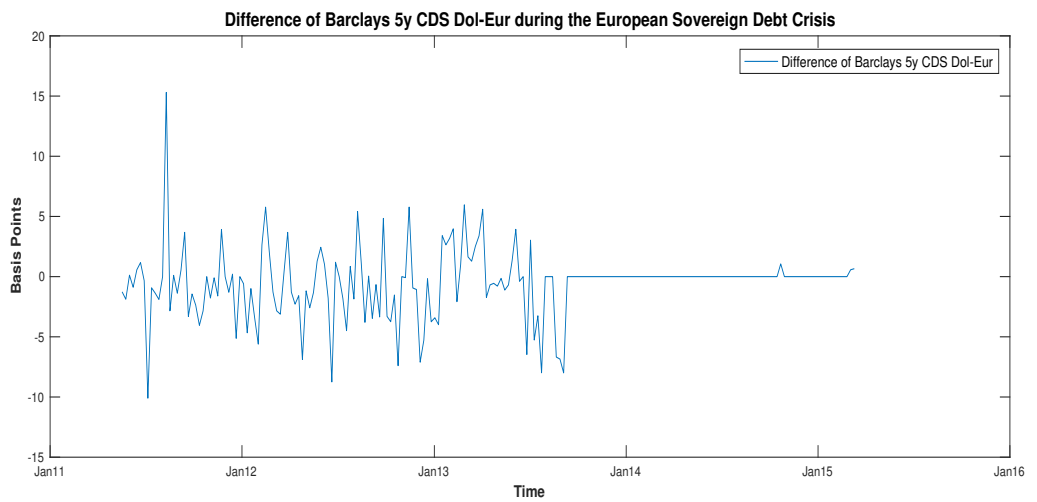


Figure C.7: Difference between Barclays CDS 5y in dollars and in euros during the European Sovereign Debt Crisis.

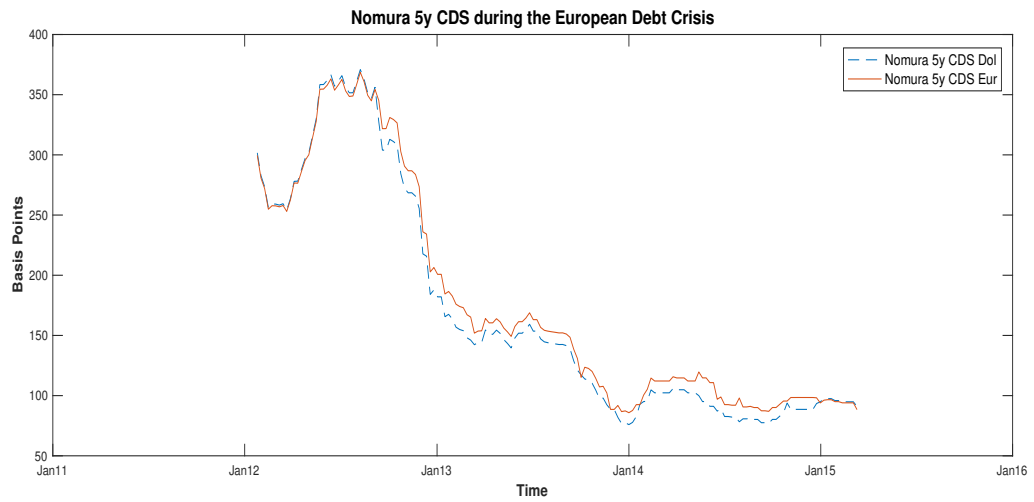


Figure C.8: Nomura CDS 5y in dollars and in euros during the European Sovereign Debt Crisis.

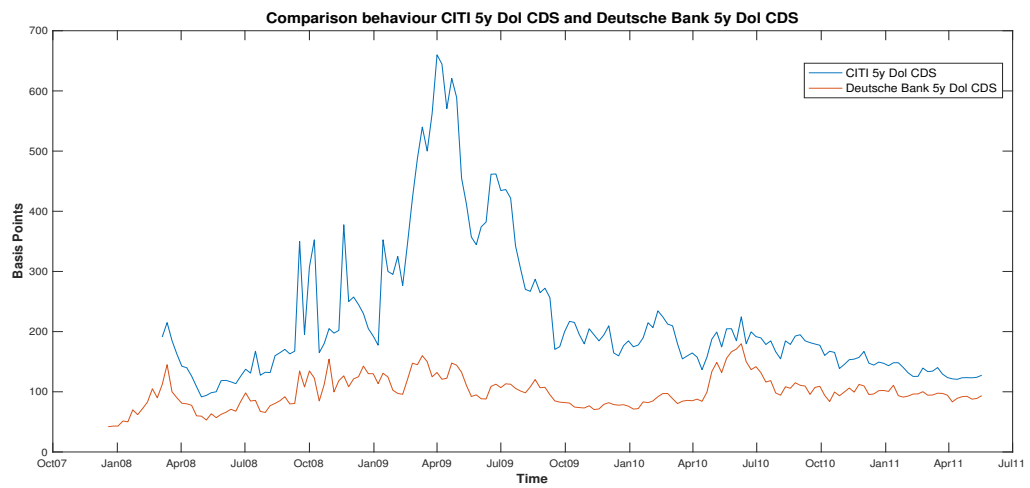


Figure C.9: Difference between CITI CDS 5y in dollars and in euros during the European Sovereign Debt Crisis.

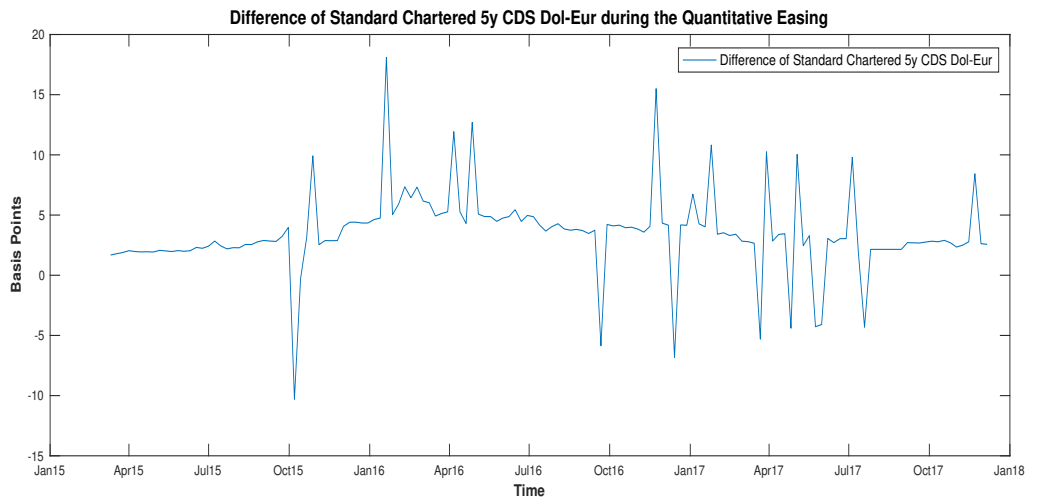


Figure C.10: Difference between Standard Chartered CDS 5y in dollars and in euros during the Quantitative Easing period.

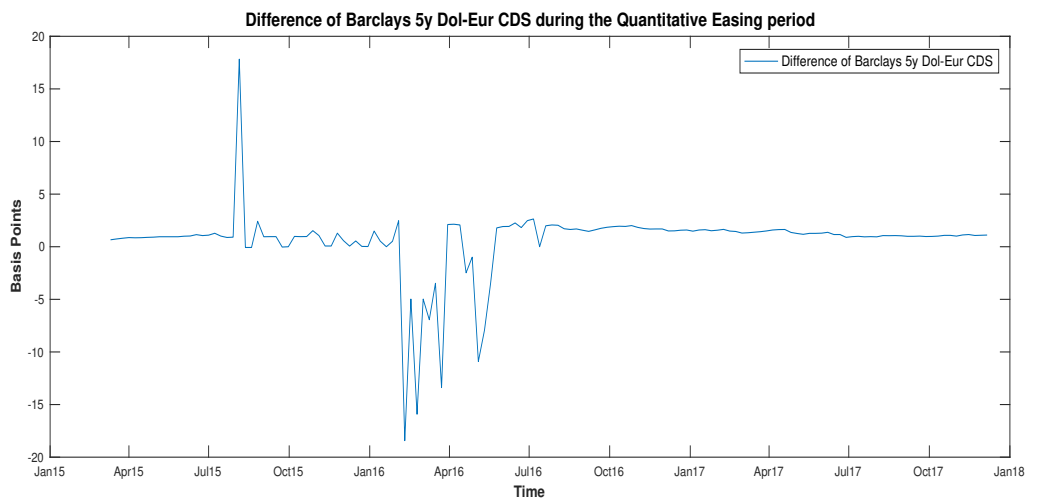


Figure C.11: Difference between Barclays CDS 5y in dollars and in euros during the Quantitative Easing period.

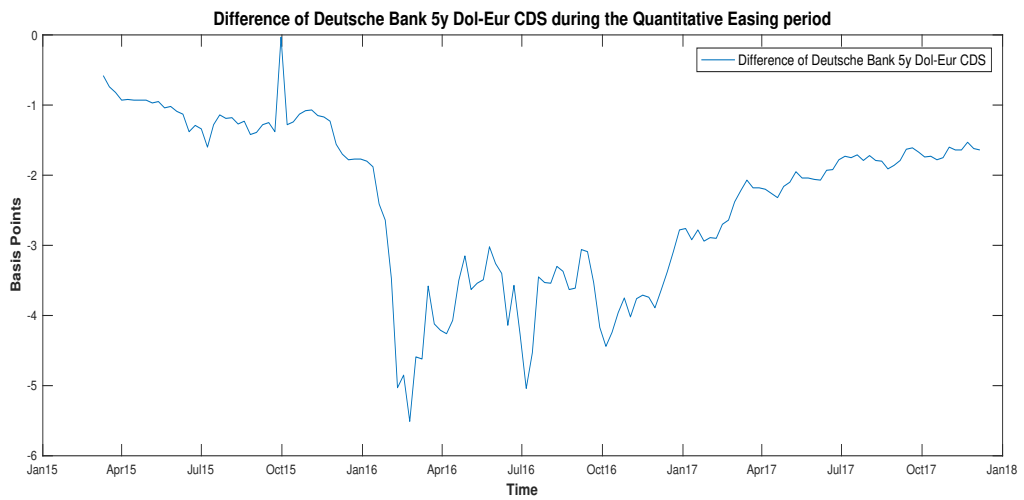


Figure C.12: Difference between Deutsche Bank CDS 5y in dollars and in euros during the Quantitative Easing period.

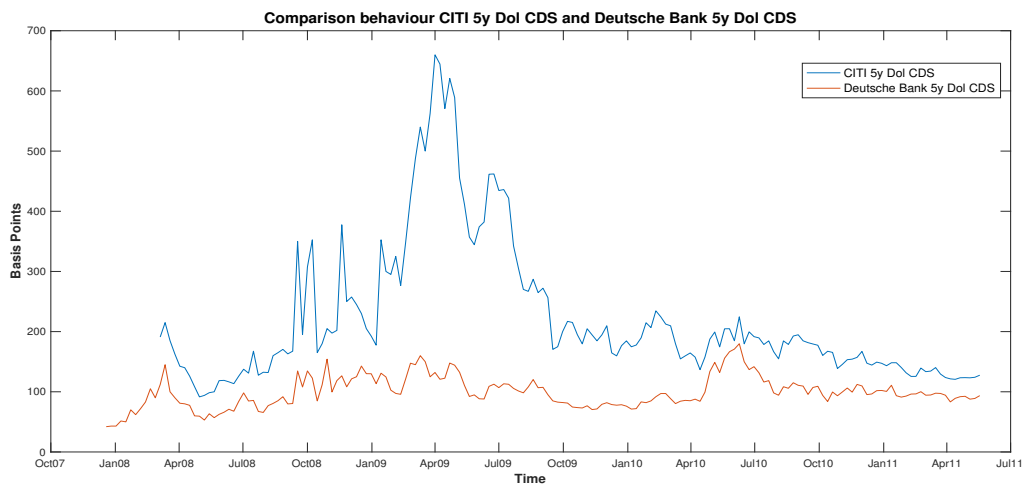


Figure C.13: Deutsche Bank CDS 5y VS CITI CDS 5y in dollars during the Sub-prime financial crisis period.