MISSION STUDY
FOR A LOW-COST ACCESS TO MARS
USING THE VEGA SPACE SYSTEM

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Academic year 2017/2018
“A good traveler has no fixed plans, and is not intent on arriving.”

- Lao Tzu -
Acknowledgements

I would first like to thank my thesis advisor Prof. Pierluigi Di Lizia of the Department of Aerospace Science and Technology at Politecnico di Milano. Prof. Di Lizia has always been available whenever I ran into a trouble spot or had a question about my research or writing. He has also been very helpful in trying to facilitate the communications at distance and in giving me enough flexibility to develop this study in accordance with the needs imposed by the company.

I would also like to thank my company tutors of Avio S.p.A., Ing. Claudio Ferraris and Ing. Matteo Volpi. Their support has been constant and of great help to guide and direct my work. Special thanks go to my colleagues in Avio for welcoming me and providing a positive and stimulating work environment.

I must express my very profound gratitude to my parents for their support in all these years. They have always been present and at the same time have given me the freedom to make mistakes and move on by myself.

These years at Politecnico di Milano have contributed to shape my character and enrich my personality along with the experiences beyond the academic environment. A special thought goes to my university colleagues, Luciano, Jan and Pascu whom with I shared the joyful and the difficult moments of my studies.

The last thanks go to those few special people who, in these last months, left a mark in my soul by choosing to share their time and their life together with mine with the greatest intensity. Thank you.

Marco Moras
Abstract

The exploration of Mars has always played a significant role in the imaginary of the people. Nowadays the problems to be faced and the steps towards a manned mission to Mars are becoming clearer and more defined. In this context, the utilization of the electric propulsion seems to offer great possibilities to reduce the propellant and the system mass to deliver significant payload masses to the Red planet.

This project aims to assess the feasibility to employ the VEGA-C launcher to launch a mission to Mars. This analysis exploits the current design and parameters of the VEGA Orbital Transfer Vehicle (VOTV) which is a building block of the VEGA Space System (VSS). The scope of the VSS is to offer a set of services for orbital exploration and space transportation based on the VEGA family launch vehicles. In this context, the VOTV can be thought as the fifth stage of the launcher which has the purpose of carrying the payload satellite that will be injected into the orbit around Mars and at the same time, provide to the same payload mechanical and electrical interfaces.

To understand the changes to be adopted in the VOTV for an interplanetary mission, a trajectory simulation is implemented to obtain the values of the propellant required for the mission and to understand the amount of payload that can be delivered. This analysis is divided into three parts which correspond to the three phases of the ideal mission: the escape from Earth, the interplanetary transfer and the capture and closure orbit around Mars.

The analysis shows that in order to exploit the modularity of the VOTV and its sub-systems the payload that can be delivered to Mars is approximately one fifth of the payload of a GEO mission around Earth, both delivered with a VEGA family launcher and the VOTV. The other criticality that comes from this analysis is related to the transfer times which are significantly extended, from three to four years. This aspect will impact the lifetime range required for the sub-systems of the VOTV, hence it implies a difficulty to employ the same components designed for a mission in the Earth environment.
L’esplorazione di Marte ha da sempre giocato un ruolo di fondamentale importanza nell’immaginario dell’uomo. Al giorno d’oggi le problematiche e i passi da affrontare per arrivare ad una missione umana su Marte sono sempre più chiari e definiti. In questo contesto l’utilizzo della propulsione elettrica offre grandi possibilità per ridurre le masse e il propellente necessario per la missione.

Questo progetto vuole verificare la fattibilità dell’utilizzo del lanciatore VEGA-C per compiere una missione su Marte. L’analisi sfrutta il design e i parametri del VEGA Orbiter Transfer Vehicle (VOTV), una delle componenti del VEGA Space System (VSS). Lo scopo del VSS è di offrire una serie di servizi per l’esplorazione orbitale ed il trasporto spaziale ed è basato sulla famiglia dei lanciatori VEGA. Il VOTV può essere considerato come il quinto stadio del lanciatore avente il compito di immettere il payload nell’orbita attorno a Marte e allo stesso tempo il compito di fornire al payload delle interfacce meccaniche ed elettriche.

In questo studio è stata implementata una simulazione di traiettoria per comprendere le masse del propellente necessario e del payload che può essere immesso nell’orbita di arrivo. Questi valori saranno quindi utilizzati per definire i vincoli di design da applicare al VOTV per questo tipo di missione. L’analisi è divisa in tre parti che corrispondono alle tre fasi principali della missione: la fuga dalla Terra, il trasferimento interplanetario e la cattura e successiva chiusura dell’orbita attorno a Marte.

L’analisi mostra che, al fine di sfruttare la modularità del VOTV e dei suoi sottosistemi, la massa di payload che può essere immessa su Marte corrisponde circa ad un quinto di quella normalmente portata nelle missioni geostazionarie terrestri, entrambe lanciate con un lanciatore della famiglia VEGA e il modulo VOTV. Un’altra criticità che emerge da questa analisi è legata ai tempi di trasferimento che sono sensibilmente lunghi, tra i tre e i quattro anni. Questa peculiarità dell’utilizzo della propulsione elettrica, va ad impattare nel design dei singoli sottosistemi del VOTV e rende difficile sfruttare le componenti utilizzate per una missione terrestre nell’ambito di una missione interplanetaria di questo tipo.
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<th>Description</th>
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<tbody>
<tr>
<td>ALEK</td>
<td>AVUM Life Extension Kit</td>
</tr>
<tr>
<td>AU</td>
<td>Astronomical Unit</td>
</tr>
<tr>
<td>AVUM</td>
<td>Attitude and Vernier Upper Module</td>
</tr>
<tr>
<td>BOL</td>
<td>Beginning Of Life</td>
</tr>
<tr>
<td>DRA</td>
<td>Design Reference Architecture</td>
</tr>
<tr>
<td>ΔV</td>
<td>Velocity variation</td>
</tr>
<tr>
<td>ECI</td>
<td>Earth Centered Inertial</td>
</tr>
<tr>
<td>EPM</td>
<td>Electric Propulsion Module</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary Earth Orbit</td>
</tr>
<tr>
<td>GNC</td>
<td>Guidance Navigation and Control</td>
</tr>
<tr>
<td>HET</td>
<td>Hall Effect Thrusters</td>
</tr>
<tr>
<td>JD</td>
<td>Julian Date</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>LMO</td>
<td>Low Mars Orbit</td>
</tr>
<tr>
<td>MEO</td>
<td>Medium Earth Orbit</td>
</tr>
<tr>
<td>NVC</td>
<td>Normal Velocity Co-normal</td>
</tr>
<tr>
<td>P/L</td>
<td>Payload</td>
</tr>
<tr>
<td>RAAN</td>
<td>Right Ascension of the Ascending Node</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Term</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------------------------</td>
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<tr>
<td>RCT</td>
<td>Reaction Control Thrusters</td>
</tr>
<tr>
<td>S/C</td>
<td>Spacecraft</td>
</tr>
<tr>
<td>SCI</td>
<td>Sun Centered Inertial</td>
</tr>
<tr>
<td>SOI</td>
<td>Sphere Of Influence</td>
</tr>
<tr>
<td>TOF</td>
<td>Time Of Flight</td>
</tr>
<tr>
<td>VEGA</td>
<td>Vettore Europeo di Generazione Avanzata (meaning: advanced generation European carrier rocket)</td>
</tr>
<tr>
<td>VECEP</td>
<td>VEGA Consolidation and Evolution Programme</td>
</tr>
<tr>
<td>VOTV</td>
<td>VEGA Orbital Transfer Vehicle</td>
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CHAPTER 1
INTRODUCTION

This thesis work is devoted to assess the feasibility of a low-cost electrically propelled mission to Mars. The peculiarity of this mission is to exploit the VEGA-C launcher and some additional modules developed for missions in low Earth orbit (LEO’s) and geostationary Earth orbits (GEO’s).

The first part of the work is dedicated to describe the state of art of the technologies characterizing this mission. First of all, an overview on the performances of the VEGA Space System (VSS) and the configuration of the modules mounted in the payload (P/L) fairing to provide additional services and space transportation to the P/L is provided. The remaining part is devoted to understand the basic principles of the electric propulsion especially regarding the Hall-effect thrusters (HET) and the problems and peculiarities encountered while operating this technology in the space environment. Similar missions are also studied and compared to understand the limits and the payloads delivered in this scenario. Since many studies regarding the advancement of technologies are developed towards manned mission to Mars, a brief chapter is also dedicated to this intriguing field.

Then a preliminary analysis is carried out to determine some useful parameters of the propulsion system and to evaluate preliminary results regarding the time of flight (TOF) of the mission and its effect on the technologies available. This analysis is followed by the description of the code to simulate the trajectory to perform a rendezvous with Mars. The code implemented is a trajectory simulation divided into three phases which are linked at the sphere of influence (SOI) of Earth and Mars. Some realistic assumptions were implemented to obtain more trustworthy results. The first phase is the escape phase from Earth. The second phase is an optimization problem to find the appropriate thrust strategy to
reach Mars while reducing the time of flight and the propellant burned. The third phase is represented by the Mars capture and the closure of the orbit around Mars. The last part of the thesis is devoted to translate the results obtained into requirements regarding the technologies exploited especially concerning the additional propellant mass and the lifetime of some critical parts compared to the lifetime of the whole mission. The immediate result obtained from this study is the P/L mass that can be delivered to Mars, which compared to the P/L delivered in LEO or GEO orbits will decrease significantly. This is expected since the propellant mass required for this mission will definitely increase. The P/L mass is a significant information that can define the kind of mission to be carried out and determine a preliminary configuration of the system.

**Project objectives**

The objective of this thesis is to produce a preliminary mass budget and high level requirements for a mission to Mars. These requirements will come from the most significant results of the analysis which will be:

- Propellant mass required
- TOF of the whole mission
- P/L mass delivered

These results will impact several aspects of the mission and will be of great importance to understand the market that can be reached with this mission configuration. Furthermore, the target mission can in some way modify these data according to the final objective which can be either a close orbit around Mars or a fly-by to launch a satellite inside the atmosphere to land or perform an aerobraking maneuver on its own.

This work is based on a model by Richard R. Rieber “Utilization of Ion Propulsion for Mars Orbiters” [1] further expanded by X. L. Hellin in “Study of Earth-to-Mars Transfers with Low-Thrust Propulsion” [2] and is developed inside the MATLAB® environment [3]. The reference code has been drastically modified and adapted to the present analysis, only some basic assumptions were taken into considerations and were used as a starting point for the simulation.
1.1 State of the Art

1.1.1 Mission to Mars

The exploration of Mars has been of great interest since the 1960s. Several probes and satellites have been sent to the planet to increase the knowledge of the Martian environment and to fulfill specific objectives: [4]

- Understand the environment and the history of the planet
- Gather data on the presence of forms of life and water on the planet
- Determine the sustainability of a human presence on Mars
- Improve the knowledge on astrophysics and interplanetary environment through observations during the transit phase

A mission to Mars either manned or not is a highly demanding task since several important factors come into play. Time of flight, launch windows, propellant mass, payload mass and mission cost are probably the most significant aspects of a mission like this. All these aspects interact with each other and can be translated into constraints and requirements that define the mission objectives.

We can usually define two classes of missions, conjunction and opposition classes. Conjunction missions are the ones where Earth and arrival planet are in direct opposition in order to minimize the energy required for the transfer. Regarding the Earth-Mars transfer the synodic periods for these transfers occur every 780 days. Opposition transfers are those where the departure and arrival planets are closest, transfer time is very short but the energy required is much higher. [5]

Of all the studies on architectures and technologies exploitable for missions to Mars, NASA Design Reference Architecture 5.0 (DRA 5.0) has a great importance since it gives a clear footprint to follow in order to perform and design a successful mission to Mars. The DRA 5.0 is a document produced by NASA that describes the systems and operations for the first manned mission to the surface of Mars [4]. Although our interest is devoted to an unmanned mission, this document is still a cornerstone since it provides useful information on the challenges and problems that have to be dealt with for a mission to the Red planet.

Considering all the successful missions to Mars, the ones with similar launched mass and mission architecture were analyzed and grouped together. For each mission, some significant data and aspects were pointed out in order to compare these mission to our study and define
an approximate payload to dry mass ratio trend to predict roughly the result of our analysis, as it can be seen in Figure 1.1. As it has been shown in Table 1.1, another great difference from our mission is that all of these missions are inserted first into a parking orbit and then into the trans-Mars orbit by the last stage of their launcher.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Mars Global Surveyor</td>
<td>659N bipropellant</td>
<td>From elliptic orbit, third stage performed trans-Mars injection burn.</td>
<td>1030.5</td>
<td>78</td>
<td>361</td>
<td>591.5</td>
</tr>
<tr>
<td>Mars Express</td>
<td>400N bipropellant</td>
<td>200 [km] parking orbit, Injected in Mars-transfer orbit by launcher.</td>
<td>1120</td>
<td>113</td>
<td>454</td>
<td>666</td>
</tr>
<tr>
<td>2001 Mars Odyssey</td>
<td>622N Hydrazine monopropellant</td>
<td>Placed on transfer trajectory by third stage of the launcher.</td>
<td>758</td>
<td>44.5</td>
<td>381.7</td>
<td>376.3</td>
</tr>
<tr>
<td>Mars Reconnaissance Orbiter</td>
<td>1020N Hydrazine monopropellant</td>
<td>Placed on transfer trajectory by launcher.</td>
<td>2180</td>
<td>139</td>
<td>1149</td>
<td>1031</td>
</tr>
<tr>
<td>Mars Orbiter Mission</td>
<td>440N liquid fuel</td>
<td>Elliptic parking orbit 264x23903 [km], S/C engine to reach escape velocity.</td>
<td>1337.2</td>
<td>13.4</td>
<td>854.7</td>
<td>482.5</td>
</tr>
<tr>
<td>MAVEN</td>
<td>1020N Hydrazine monopropellant</td>
<td>Low Earth Parking Orbit, engine burn to insert into heliocentric transfer orbit.</td>
<td>2454</td>
<td>65</td>
<td>1645</td>
<td>809</td>
</tr>
<tr>
<td>Dawn</td>
<td>Ion propulsion Xenon + 12 0.9N hydrazine (attitude control)</td>
<td>Parking orbit 184 [km] above Earth, injected into transfer orbit by third stage of the launcher.</td>
<td>1217.7</td>
<td>45.6</td>
<td>425</td>
<td>747.1</td>
</tr>
</tbody>
</table>

Table 1.1 Comparison of missions to Mars of our interest.
These missions differ from our study since they exploit a chemical propulsion system (except from Dawn). Nevertheless, they are all unmanned and the mass data are close to the ones of our analysis.

Of the missions considered the most interesting is the Dawn probe. It is a mission launched by NASA in September 2007 to study two protoplanets of the asteroid belt Vesta and Ceres, at the moment it is in orbit around Ceres. It is the first NASA mission to use ion propulsion, it exploits three Xenon thrusters derived from NSTAR technology, using one at a time. These thrusters have a specific impulse of 3100 [s] and a thrust of 90 [mN]. The probe was injected into an escape trajectory by the third stage of the launcher Delta II, then it moved towards Mars to perform a gravity assist flyby. During this transfer, it spent 85% (270 days) of the time using its thrusters. It consumed 72 [kg] of propellant to obtain a $\Delta V$ of 1.81 [km/s].

### 1.1.2 Electric Propulsion

The basic principle of electric propulsion is to utilize electric energy to heat or to eject propellant, which is different from the electric source itself. The main subsystems of electric propulsion are: an energy source, conversion devices, a propellant system and thrusters. The energy source can be either nuclear or solar and it is followed by all the auxiliary systems to concentrate and convey the energy. The conversion devices have the purpose to transform this energy into electrical form with proper voltage, frequency and current. The propellant system has the role of storing and delivering the propellant. The thrusters convert electrical energy into kinetic energy.
Electric propulsion can be divided into three fundamental types:

1. Electrothermal: the propellant is heated electrically and expanded thermodynamically.
2. Electrostatic: acceleration is achieved by the interaction of electrostatic fields on non-neutral or charged propellant particles.
3. Electromagnetic: acceleration is achieved by interaction of electric and magnetic fields within a plasma.

In general, thrust values of electric propulsion are significantly smaller than chemical or nuclear propulsion but specific impulses are significantly higher which can be translated into a significantly longer operational life. [6]

Due to its low thrust values, electric propulsion is used in space to perform different tasks such as, stationkeeping, orbit maneuvering and nuclear or solar power orbit transfers. In Figure 1.2, the different electric thrusters are grouped according to their specific impulse and required power. Stationkeeping requires the system to overcome translational and rotational perturbations in satellite orbits, this kind of mission could require north-south station keeping in geosynchronous orbits, the alignment of telescopes or antennas or drag compensation for satellites in LEO or medium Earth orbits (MEO’s). Orbit maneuvering can be translated in the need to raise the satellite speed while overcoming the gravitational field a few distances
away from earth, it might be required to raise an orbit from LEO to GEO or the circularization of an elliptical orbit. Solar or nuclear power transfers like interplanetary or deep space missions are of current interest in lots of studies and a few missions with these characteristics have already been launched. [7]

Overall the use of electric propulsion, as an alternative of the classic chemical or nuclear propulsion, results in a decrease of mass and cost with the great benefit of the possible extension of the satellite life through the storage of additional propellant on board. [8]

1.1.3 Low-thrust propulsion

Low-thrust propulsion comes into play when highly demanding maneuvers in terms of propellant, would require too much time simply to bring all the propellant required to a LEO orbit. Hence low-thrust propulsion is a valid alternative to impulsive propulsion, it produces small and continuous accelerations with the engine operating during most of the time of flight [9]. The choice of a low-thrust propulsion system is related to the kind of mission that has to be carried out in terms of energy source and time of flight constraints. For a deep space mission the choice of nuclear propulsion is straightforward [7], for Earth missions or interplanetary missions to the inner planets solar cells can provide sufficient Mega Watts of solar energy, this limit is being increased to reach and idealistic value of 50 [MW] to be exploited for human missions to Mars [4]. The parameter that has a great importance in this choice is the mass-to-power ratio, the specific mass of the electrical source has to be limited compared to the power supplied.

1.1.4 Hall propulsion and Hall effect thrusters

The Hall thruster provides a greater thrust to power ratio than the conventional gridded ion engines, in this way it can reduce trip times and requires lower operational life when compared with other technologies in use in Earth orbit applications. The first to study and understand the importance of the Hall thrusters were the Russians in the 70’s. Both USA and Russia started studying and developing intensely the research around electric propulsion from the 60’s, but while the Americans focused on the gridded ions engine, the Russian dedicated their research around the Hall thruster. They were the first to successfully install and utilize a thruster aboard a METEOR spacecraft in 1971. Those flights and the
technology exploited went unnoticed until the Cold War was over, after that the two countries started to exchange information and technologies and to conduct joint studies to further develop this promising technology. [10]

The main difference between a gridded ion engine and a Hall thruster is the absence of the erosion phenomena. The gridded ion engine accelerates the positive ions while they pass through a positive and a negative grid, this implies significant erosion problems, the choice of the propellant can in some way mitigate this effect, but it still has a great impact on the lifetime of the system.

Regarding the Hall thruster system, it consists of three parts, the thruster, the power processor, and the propellant system. The current state of art of this technology provides power levels with a range of 1.5 [kW] to 5 [kW], future studies want to extend this power availability to 10 [kW] or even 50 [kW] for challenging missions. The physics behind this system is based on an electrostatic potential that accelerates ions up to a certain speed. Compared with the gridded ion engine, the Hall thruster doesn’t have a grid at the open end, it has instead an electron plasma that provides the negative charge to attract the ions.

![Figure 1.3 External view and quarter section of a HET. [6]](image)

To go deeper in the functioning of the Hall thruster lets analyze the quarter section in Figure 1.3. The central cylinder represents one pole of the electromagnet, it is surrounded by a hollow section where the ions flow, around this hollow section there is the second pole. From this section, we can see that the discharge chamber is not physically separated from the
accelerator region, the actual difference lies in the presence or not of ionized ions in the two regions. An electric potential is applied between the anode and the cathode, the propellant, such as Xenon, is fed through the anode which distributes the gas through several holes. The neutral atoms of the Xenon are ionized with high-energy particles and subsequently accelerated by the electric field generated by the two solenoids. This thruster can be considered as an electromagnetic device or either an electrostatic one. The two processes of ionization and acceleration can be studied separately, it can be seen that the electrons in the accelerator neutralize the ionic charge as it moves from the anode to the cathode. These thrusters take their name from the current that is generated at their open end, the orbital rotation of the electrons generate a circulating current call Hall current. The Hall effect is indeed the production of a voltage difference across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current. [6]

**SITAEL HT5K**

The Hall thruster considered in this study is produce by SITAEL and is represented in Figure 1.4. It is a 5 [kW] thruster, with high efficiency and a magnetic system based on coils, it is designed to be fed with Xe. Input power range from 2.5 to 7.5 [kW], thrust from 150 to 330 [mN] and specific impulse from 1700 to 2800 [s]. This thruster is coupled with the SITAEL HC20 hollow catode, designed to produce a maximum emission current of 20 [A]. [11], [12], [13]
1.2 VEGA and VOTV overview

Vega is a European light launcher in use by Arianespace, developed by ASI and ESA and manufactured by Avio S.p.A. It completed its inaugural flight in 2012. And recently has carried out its thirteen successful launch from Kourou in French Guiana. It can deliver payloads ranging from 1450 to 1500 [kg] in LEO orbits from equatorial to Sun synchronous orbits. The rocket is made of three solid motor stages and a fourth upper stage Attitude and Vernier Upper Module (AVUM), designed to perform the payload insertion into the required orbit. The upper stage can be ignited five times, coupled with a secondary payload adapter it can deliver several satellites into different orbits with the capability of changing orbital planes. With more than twenty successful payload insertions Vega is a reliable, versatile and efficient carrier for both public and private competitors.

The Vega Consolidation and Evolution Programme (VECEP) goal is to cover the strategic needs in terms of performance and cost for the users in the short/medium and long term, including the capability to provide cost-effective launch opportunities for small satellites. This is translated in the following objectives:

- To strengthen and enlarge the Vega market position in the short/medium term
- To decrease the dependency on non-European sources
- To be in a position to better respond to long term institutional needs
- Contribute to safeguard European industrial engineering capabilities, in particular on propulsion

The VECEP program has entered into force on 21 November 2012, following subscriptions made by participating States at the occasion of the ESA Council meeting at ministerial level in Naples. The program foresees:

- A short term consolidation of the VEGA launcher through the so called VEGA C configuration which foresees an upgrade of the first stage SRM and of the fourth stage from AVUM to AVUM + (Enlarged European Tanks)
- A longer term evolution dubbed VEGA-E constituted by the same P120, the Z40 SRM in place of the actual Z23 and a new upper stage replacing both the Z9 and the AVUM.
The new configuration of the Vega launcher under development called Vega-C, is a launcher with an increased payload mass capability of 1000 [kg]. The aim of this new configuration is to reach a higher flexibility for multiple P/Ls missions and to extend its market from 50% to 90% of accessible LEO satellites.

The scope of the VEGA Space System (VSS) is to offer a unified complete set of solutions and services for orbital exploitation and space transportation, including exploration missions, based on Vega family launch vehicles and on a set of specific modules (possibly third part), most of them existing or, to different extents, currently under development.

The distinctive characteristic of the VSS is to define standard interfaces versus payloads and a Kit approach to mission, which allows the lower cost for any mission.

VSS is composed by building blocks that can be combined together to build up the spacecraft. In principle, it is possible any choice of propulsion module (Electrical only, Extended Chemical only or Hybrid), of ALEK (1,2,3) and of Extended Services to Payloads. Of course, the correct combination depends on the mission requirements and is verified through a dedicated mission and system analysis loop.

In the following paragraphs are described the VSS building blocks. The description is limited to the building blocks that fall inside Vega Orbital Transfer Vehicle (VOTV) perimeter.

The extension of Vega capabilities is performed through the VOTV module which is installed on the launch vehicle payload interface. This additional module can enable several mission types such as GEO, HEO or interplanetary. The VOTV mission can be summarized as follows:

1. VEGA-C injects into a LEO parking orbit the upper composite formed by VOTV plus the P/L
2. VOTV is commissioned and starts providing to the P/L (kept in safe and stowed mode) survival power and a datalink for P/L housekeeping data monitoring
3. VOTV starts the orbit transfer by means of a low thrust orbit raising maneuver. GNC functions, as well as telemetry and attitude control are executed autonomously by the module avionics for the achievement of payload final orbit
4. VOTV releases the P/L into its final orbit, performs an avoidance manoeuver, a disposal manoeuver (de-orbiting or re-boost to graveyard, depending on the target orbit), then passivates.
Introduction

VOTV, presented in Figure 1.5, can be divided into two main subsystems, Avum Life Extension Kit (ALEK) and the Electrical Propulsion Module (EPM). ALEK module is available in different versions (ALEK1, ALEK2, ALEK3) according to the power needed for the mission. Overall the only difference between these versions is limited to the Electric Power subsystem, the other subsystems are the same.

Figure 1.5 VOTV System.

ALEK-3, presented in Figure 1.6, provides power generation conditioning and distribution to P/L and other services, it has been designed to produce 16 [kW] of power at beginning of life (BOL), through flexible solar panels. EPM represents the propulsion system with two gimballed HET of 4.5 [kW] class devoted to the orbital manoeuvres with Xenon as propellant gas, plus two resistojet triads for the attitude control.
The EPM module, presented in Figure 1.7, is in charge of the orbital manoeuvres and attitude control, with a reference $\Delta V$ up to 6.6 [km/s]. It is based on a cylindrical structure carrying externally four Xenon tanks and two reaction control thrusters (RCT) brackets, while all the other components are placed internally.
Introduction

The VOTV module can be considered as the fifth stage of the launcher, its main purpose is to extend the life of the spacecraft by placing it in the required orbit and to provide the survival power for P/L housekeeping and data monitoring.

VOTV has been identified as ideal for specific missions like:

- GEO mission deployment
- MEO mission, likely a Galileo constellation replenishment
- HEO mission
- Deployment of constellations in LEO
- Interplanetary mission

The reference mission designated for the VOTV performances is a GEO mission.

1.3 Manned missions to Mars

Although our study is focused on an unmanned mission, it must be said that most of the studies regarding missions to Mars are devoted to make possible a human approach to the Red planet.

According to the Design Reference Architecture (DRA) 5.0, the philosophy behind a manned mission to Mars considers a total of three missions to be launched in several decades, the first two missions would be long transfer time cargo missions while the last one would be a short transfer time mission with the crew on board [4]. The architecture for the cargo mission can therefore be taken into consideration as a reference for our study. The cargo missions analyzed in the DRA 5.0 considers a nuclear or chemical propulsion system for a transfer time ranging between 200-350 days. Overall the payload mass of each cargo mission is of the range of 100 [tons] which is delivered by a heavy lift launcher, in this case Ares V.

As highlighted by the DRA 5.0 there is the need to a near-term development in almost all areas of technology, in particular in life support systems, radiation protection and countermeasure, space transportation propulsion, utilization of locally produced consumables and power systems. Another critical factor is the total mass that must be launched and transported to Mars. At the present moment the sub-systems, especially those devoted to the life support, require a significant amount of mass, the reduction of the sub-systems mass is a long-term goal that has to be reached to significantly decrease operational costs. [15]
CHAPTER 2
MATHEMATICAL MODEL

Each phase of the simulation is carried out in a different inertial reference frame with a different gravitational model. This is possible since each phase could be simulated in a two-body gravitational model, but in order to introduce realistic constraints, different choices in the models have been made. For each phase, the forces acting on the S/C are the gravitational forces and the contribution of the thrust. A further simplification is adopted by considering Earth and Mars orbits as circular.

2.1 Earth Phase

2.1.1 Gravitational Model

Earth phase is developed in a Earth Centered Inertial (ECI) reference frame with a two-body gravitational model. This model is adopted from the departure date on the LEO until the S/C reaches the Earth SOI, at that point the integration stops and the model is substituted with a different one which considers the main celestial body as the Sun together with a different reference frame.

The gravitational force acting in this phase is the force of the Earth on the S/C. The acceleration vector acting on the S/C can be written as follows:

\[
A_{g,S/C} = -\mu_{\text{Earth}} \frac{R_{S/C}}{|R_{S/C}|^3}
\]  
Eq. (2.1)
To obtain a more realistic model the perturbation due to the non-spherical Earth is implemented in the acceleration vector. In an ideal model the Earth is considered as a symmetric sphere whereas it is bulged at the equator, flattened at the poles and is generally asymmetric.

To build the acceleration vector a different notation is used, now the two body equation is coupled with the J2 perturbation components inside a potential function \( \phi \).

\[
a = \nabla \phi = \frac{\delta \phi}{\delta x} I + \frac{\delta \phi}{\delta y} J + \frac{\delta \phi}{\delta z} K
\]

Eq. (2.2)

the acceleration vector can be obtained by computing its gradient. The potential function can be expressed in the following way:

\[
\phi = \frac{\mu}{r} \left[ 1 - \sum_{n=2}^{\infty} \frac{J_n}{n} \left( \frac{R_{\text{Earth}}}{r} \right)^n P_n \sin L \right]
\]

Eq. (2.3)

Where:

- \( \mu \) = gravitational parameter
- \( J_n \) = coefficients to be determined by experimental observation
- \( R_{\text{Earth}} \) = Earth equatorial radius
- \( P_n \) = Legendre polynomials
- \( L \) = geocentric latitude
- \( \sin L = \frac{z}{r} \)

The expression of the sine has been substituted by the ratio of the z coordinate over the norm of the radius. The first seven terms of the expression are:

\[
\phi = \frac{\mu}{r} \left[ 1 - \frac{J_2}{2} \left( \frac{R_{\text{Earth}}}{r} \right)^2 \left( 3 \sin^2 L - 1 \right) - \frac{J_2}{2} \left( \frac{R_{\text{Earth}}}{r} \right)^3 \left( 5 \sin^3 L - 3 \sin L \right) - \frac{J_4}{8} \left( \frac{R_{\text{Earth}}}{r} \right)^4 \left( 35 \sin^4 L - 30 \sin^2 L + 3 \right) - \frac{J_6}{8} \left( \frac{R_{\text{Earth}}}{r} \right)^5 \left( 63 \sin^5 L - 70 \sin^3 L + 15 \sin L \right) - \frac{J_8}{16} \left( \frac{R_{\text{Earth}}}{r} \right)^6 \left( 231 \sin^6 L - 315 \sin^4 L + 105 \sin^2 L - 5 \right) \right]
\]

Eq. (2.4)

Taking the partial derivative of \( \phi \),
\[ \ddot{x} = \frac{\delta \phi}{\delta x} = -\frac{\mu x}{r^3} \left[ 1 - J_2 \left( \frac{R_{\text{Earth}}}{r} \right)^2 \left( 3 \frac{z^2}{r^2} - 1 \right) + J_3 \frac{5}{2} \left( \frac{R_{\text{Earth}}}{r} \right)^3 \left( \frac{3 z}{r} - 7 \frac{z^3}{r^3} \right) - ight. \]
\[ J_4 \left( \frac{R_{\text{Earth}}}{r} \right)^4 \left( 3 - 42 \frac{z^2}{r^2} + 63 \frac{z^4}{r^4} \right) - J_5 \frac{3}{8} \left( \frac{R_{\text{Earth}}}{r} \right)^5 \left( 35 \frac{z}{r} - 210 \frac{z^3}{r^3} + 231 \frac{z^5}{r^5} \right) + \]
\[ J_6 \frac{1}{16} \left( \frac{R_{\text{Earth}}}{r} \right)^6 \left( 35 - 945 \frac{z^2}{r^2} + 3465 \frac{z^4}{r^4} - 3003 \frac{z^6}{r^6} \right) + \ldots \]  
[17] \[ \text{Eq. (2.5)} \]

\[ \ddot{y} = \frac{\delta \phi}{\delta y} = \frac{y}{x} \ddot{x} \]  
[17] \[ \text{Eq. (2.6)} \]

\[ \ddot{z} = \frac{\delta \phi}{\delta z} = -\frac{\mu z}{r^3} \left[ 1 + J_2 \frac{3}{2} \left( \frac{R_{\text{Earth}}}{r} \right)^2 \left( 3 - 5 \frac{z^2}{r^2} \right) + J_3 \frac{3}{2} \left( \frac{R_{\text{Earth}}}{r} \right)^3 \left( 10 \frac{z}{r} - \frac{35 z^3}{3 r^3} - \frac{r}{z} \right) - \right. \]
\[ J_4 \frac{5}{8} \left( \frac{R_{\text{Earth}}}{r} \right)^4 \left( 15 - 70 \frac{z^2}{r^2} + 63 \frac{z^4}{r^4} \right) - J_5 \frac{1}{8} \left( \frac{R_{\text{Earth}}}{r} \right)^5 \left( 315 \frac{z}{r} - 945 \frac{z^3}{r^3} + 693 \frac{z^5}{r^5} - 15 \frac{r}{z} \right) + \]
\[ J_6 \frac{1}{16} \left( \frac{R_{\text{Earth}}}{r} \right)^6 \left( 315 \frac{z}{r} - 2205 \frac{z^3}{r^3} + 4851 \frac{z^5}{r^5} - 3003 \frac{z^6}{r^6} \right) + \ldots \]  
[17] \[ \text{Eq. (2.7)} \]

The first terms in the acceleration components represent the two body acceleration and the remaining terms represent the perturbation effect. The value of the \( J_n \) coefficients are available in the literature for the Earth:

\[ J_2 = (1082.64 \pm 0.03) \cdot 10^{-6} \]  
[2.8] \[ \text{Eq. (2.8)} \]

\[ J_3 = (-2.5 \pm 0.1) \cdot 10^{-6} \]  
[2.9] \[ \text{Eq. (2.9)} \]

\[ J_4 = (-1.6 \pm 0.5) \cdot 10^{-6} \]  
[2.10] \[ \text{Eq. (2.10)} \]

\[ J_5 = (-0.15 \pm 0.1) \cdot 10^{-6} \]  
[2.11] \[ \text{Eq. (2.11)} \]

\[ J_6 = (0.57 \pm 0.1) \cdot 10^{-6} \]  
[2.12] \[ \text{Eq. (2.12)} \]

This model includes only the zonal harmonics, which are those harmonics dependent only on the mass distribution which is symmetric about the north-south axis of the Earth and so they are not longitude dependent. The actual model implemented in the code will consider only the \( J_2 \) effect since the other perturbation components are of smaller order. These equations are developed in a geocentric equatorial frame and can be directly implemented in our model. If tesseral and sectorial harmonics were to be considered, then a rotation of this frame would be required to take into account their dependency on longitude. [16]
Mathematical Model

The contribution due to the thrust is represented by the following expression:

\[ \mathbf{A}_{T,S/C} = \frac{F_{\text{thrust}}}{M_{S/C}} \mathbf{e}_T \]  

Eq. (2.13)

This acceleration comes from the mass variation due to the propellant consumption and is multiplied by a vector that represents its direction. In this phase, the thrust vector is aligned to the velocity vector in order to guarantee a fast acceleration to escape Earth SOI.

\[ \mathbf{e}_T = \frac{\mathbf{v}_{S/C}}{\| \mathbf{v}_{S/C} \|} \]  

Eq. (2.14)

The eclipse condition in the Earth phase is implemented by considering a no-thrust period when the S/C is in the eclipse zone. The model adopted to consider the shadow area is the cylindrical one. The position of the Sun is estimated at each step of the integration by computing the Julian date corresponding to the current time value of the integration. [17]

For each time instant \( t \) the following steps are performed:

1. Compute the S/C position.
2. Estimate the Sun position.

The algorithm to evaluate the position of the Sun, is based on the apparent motion of the Sun around the Earth. Two reference frames are introduced, a Geocentric-Ecliptic \( T_E \) \( (O; x_E, y_E, z_E) \) with the origin in the center of the Earth and the x axis coincident with the x axis of the Geocentric-Equatorial frame \( T_G \) \( (O; x, y, z) \). The two frames differ from each other by a rotation angle \( \varepsilon \) around the x axis, which represents the inclination angle of the ecliptic. The Sun position is computed considering as reference time the Julian date JD(2000) which corresponds to the date at 12:00 1\textsuperscript{st} Jan 2000 while the departure date considered as a reference is the 1\textsuperscript{st} Jan 2019. The steps of the algorithm are the following:

1. Compute the centuries from JD(2000) to the Julian date at the observation time \( \text{JD}_{UT} \):

\[ T_{UT} = \frac{\text{JD}_{UT} - \text{JD}(2000)}{36525} = \frac{\text{JD}_{UT} - 2451545}{36525} \]  

Eq. (2.15)
2. Compute the Sun mean longitude $\lambda_{M\odot}$ through a polynomial expression approximated to the first order:

$$\lambda_{M\odot} = 280.4606184^\circ + 36000.77005361 \, T_{UT}$$  \hspace{1cm} \text{Eq. (2.16)}

3. Compute the Sun mean anomaly $M\odot$ using another first order polynomial approximation:

$$M\odot = 357.5277233^\circ + 35999.05034 \, T_{UT}$$  \hspace{1cm} \text{Eq. (2.17)}

4. Compute the Sun longitude $\lambda_e$ along the ecliptic:

$$\lambda_e = \lambda_{M\odot} + 1.914666471^\circ \sin M\odot + 0.019994643^\circ \sin(2M\odot)$$  \hspace{1cm} \text{Eq. (2.18)}

5. Determine the Earth-Sun distance in astronomical units (AU):

$$r\odot = 1.000140612 - 0.016708617 \cos M\odot - 0.000139589 \cos(2M\odot)$$  \hspace{1cm} \text{Eq. (2.19)}

6. Compute the inclination angle of the ecliptic plane with respect to the equatorial plane:

$$\varepsilon = 23.439291^\circ - 0.0130042 \, T_{UT}$$  \hspace{1cm} \text{Eq. (2.20)}

7. Determine the Sun position vector in the $T_E'$ frame:

$$[r\odot]_{T_E'} = r\odot \begin{bmatrix} \cos \lambda_e \\ \sin \lambda_e \\ 0 \end{bmatrix}$$  \hspace{1cm} \text{Eq. (2.21)}

8. Convert the Sun position in the $T_G$ frame:

$$[r\odot]_{T_G} = r\odot \begin{bmatrix} \cos \lambda_e & \cos \varepsilon \sin \lambda_e \\ \cos \varepsilon \sin \lambda_e & \sin \varepsilon \sin \lambda_e \end{bmatrix}$$  \hspace{1cm} \text{Eq. (2.22)}
The adoption of the cylindrical model for the eclipse shadow, implies that the mutual visibility problem is simply a check to see if the vector linking the Sun and Earth position intersects the Earth surface. Considering two vectors \( r \) and \( \rho \) which represent the position of the S/C and the Sun in the ECI, the condition of mutual visibility can be reduced through the minimization of a generic vector \( c \) which represent the distance from the center of the Earth to the vector linking the two bodies considered. Vector \( c \) can be written as follows:

\[
c(\tau) = r + (\rho - r)\tau
\]

Eq. (2.23)

Where \( \tau \in [0,1] \) is a scalar parameter that represent the position of a point on the segment linking the two bodies. By minimizing vector \( c \), the mutual visibility condition is reached if the modulus of vector \( c \) is greater that the Earth radius. From Eq. (2.23) the following expression is obtained:

\[
c^2(\tau) = c(\tau) \cdot c(\tau) = r^2 + (\rho - r) \cdot (\rho - r)\tau^2 + 2r \cdot (\rho - r)\tau
\]

Eq. (2.24)

The minimization of the vector \( c \), hence \( ||c(\tau)|| \), is obtained minimizing the expression with respect to \( \tau \). By imposing the necessary condition \( dc^2/d\tau = 0 \), the following expression is obtained:

\[
\frac{dc^2}{d\tau} = 2\tau (\rho - r) \cdot (\rho - r) + 2r \cdot (\rho - r)
\]

Eq. (2.25)

The value of \( \tau \), called \( \tau^* \) which solves this equation is the following:

\[
\tau^* = \frac{r \cdot (r - \rho)}{(\rho - r) \cdot (\rho - r)}
\]

Eq. (2.26)

And the correspondent value of the minimum distance, \( c_{\text{min}} \) is given by:

\[
c_{\text{min}} = c(\tau^*) = \sqrt{|r|^2 \cdot (1 - \tau^2) + \tau \cdot \rho \times r}
\]

Eq. (2.27)

It can be observed that for \( \tau^* = 0 \), \( r \cdot (r - \rho) = 0 \) hence the vector \( (r - \rho) \) is orthogonal to \( r \). If \( \tau^* = 1 \), \( \rho \cdot (r - \rho) = 0 \) so the direction of the vector \( (r - \rho) \) is perpendicular to \( \rho \).
The remaining conditions in which $\tau^*$ is smaller than zero or greater than 1 correspond to the cases in which the Earth is not between the S/C and the Sun, hence there is mutual visibility. [18]

Considering $\tau^*$, the visibility condition lays inside these boundaries:

- If $\tau^* < 0$ or $\tau^* > 1$ and $\rho$ are surely mutually visible
- If $0 \leq \tau^* \leq 1$ the Earth can be between the two bodies. The value of $c(\tau^*)$ has to be determined, mutual visibility is hence possible if $c(\tau^*) > R_{\text{Earth}}$

### 2.1.2 System of equations

The system of equations governing this phase is characterized by a state vector of seven components, one for the mass variation and the others for the position and velocity of the S/C.

<table>
<thead>
<tr>
<th>$m$</th>
<th>Mass of the S/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{\text{S/C}}$</td>
<td>X coordinate of the S/C</td>
</tr>
<tr>
<td>$y_{\text{S/C}}$</td>
<td>Y coordinate of the S/C</td>
</tr>
<tr>
<td>$z_{\text{S/C}}$</td>
<td>Z coordinate of the S/C</td>
</tr>
<tr>
<td>$u_{\text{S/C}}$</td>
<td>X velocity coordinate of the S/C</td>
</tr>
<tr>
<td>$v_{\text{S/C}}$</td>
<td>Y velocity coordinate of the S/C</td>
</tr>
<tr>
<td>$w_{\text{S/C}}$</td>
<td>Z velocity coordinate of the S/C</td>
</tr>
</tbody>
</table>

Table 2.1 State vector components for the Earth escape model.

The seven equations to be solved are the following:

\[
m = \frac{F_{\text{thrust}}}{I_{\text{sp}} g_0} \tag{2.28}
\]

\[
\dot{x}_{\text{S/C}} = u_{\text{S/C}} \tag{2.29}
\]

\[
\dot{y}_{\text{S/C}} = v_{\text{S/C}} \tag{2.30}
\]

\[
\dot{z}_{\text{S/C}} = w_{\text{S/C}} \tag{2.31}
\]
This system of seven equations is numerically integrated to determine the S/C orbit as a function of time.

### 2.2 Interplanetary Phase

#### 2.2.1 Gravitational Model

This model is adopted from the Earth SOI up to Mars SOI, it will be explained in the later chapters that this constraint is relaxed for the rendezvous with Mars. The integration of the equations of motion isn’t stopped by an event function, as in the previous phase, the rendezvous maneuver has a margin of error hence for all the cases considered the final position of the S/C will be inside Mars SOI. The interplanetary phase analysis is developed in a Sun centered inertial (SCI) frame. The gravitational problem now considers four bodies, the Sun, Earth, Mars and the S/C.

The gravitational forces acting in this phase are:

- The force of the Sun on the S/C
- The force of the Earth on the S/C
- The force of Mars on the S/C
- The force of Mars on the Sun
- The force of the Earth on the Sun

The acceleration vector can be written as follows:

\[
A_{g,S/C} = -\frac{F_{\text{Sun}} + F_{\text{Earth}} + F_{\text{Earth/Sun}} + F_{\text{Mars}} - F_{\text{Mars/Sun}}}{M_{S/C}}
\]

Eq. (2.33)

\[
A_{g,S/C} = -\mu_{\text{Sun}} \frac{R_{S/C}}{|R_{S/C}|^3} + \mu_{\text{Earth}} \left( \frac{R_{\text{Earth}} - R_{S/C}}{|R_{\text{Earth}} - R_{S/C}|^3} - \frac{R_{\text{Earth}}}{|R_{\text{Earth}}|^3} \right) + \\
+ \mu_{\text{Mars}} \left( \frac{R_{\text{Mars}} - R_{S/C}}{|R_{\text{Mars}} - R_{S/C}|^3} - \frac{R_{\text{Mars}}}{|R_{\text{Mars}}|^3} \right)
\]

Eq. (2.34)
Where $\mu_{\text{Sun}}$, $\mu_{\text{Earth}}$ and $\mu_{\text{Mars}}$ are the standard gravitational parameters of the planets and represent the product of the gravitational constant and the mass of the celestial body.

Additional accelerations to be considered are the ones of the Earth and Mars due to the sun:

\[
A_{\text{Earth}} = \mu_{\text{Sun}} \frac{R_{\text{Earth}}}{R_{\text{Earth}}} \quad \text{Eq. (2.35)}
\]

\[
A_{\text{Mars}} = \mu_{\text{Sun}} \frac{R_{\text{Mars}}}{R_{\text{Mars}}} \quad \text{Eq. (2.36)}
\]

The contribution due to the thrust is computed through this expression:

\[
A_{T,S/C} = \frac{F_{\text{Thrust}}}{m_{S/C}} \eta_T \mathbf{e}_T \quad \text{Eq. (2.37)}
\]

Where $\mathbf{e}_T$ is the vector that represents the direction of the thrust and is computed through an optimization and $\eta_T$ is the parameter that takes into account the power degradation with respect to the distance from the Sun. The parameter $\eta_T$ is computed in the following way:

\[
\eta_T = \left( \frac{|R_{\text{Earth}}|}{|r_{S/C}|} \right)^2 \quad \text{Eq. (2.38)}
\]

### 2.2.2 System of Equations

The system of equations governing the interplanetary phase is made of nineteen equations. The components of the state vector are the following:

<p>| ( m ) | Mass of the S/C |
| ( x_{S/C} ) | X coordinate of the S/C |
| ( y_{S/C} ) | Y coordinate of the S/C |
| ( z_{S/C} ) | Z coordinate of the S/C |
| ( u_{S/C} ) | X velocity coordinate of the S/C |
| ( v_{S/C} ) | Y velocity coordinate of the S/C |
| ( w_{S/C} ) | Z velocity coordinate of the S/C |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{Earth}$</td>
<td>X coordinate of the Earth</td>
</tr>
<tr>
<td>$y_{Earth}$</td>
<td>Y coordinate of the Earth</td>
</tr>
<tr>
<td>$z_{Earth}$</td>
<td>Z coordinate of the Earth</td>
</tr>
<tr>
<td>$u_{Earth}$</td>
<td>X velocity coordinate of the Earth</td>
</tr>
<tr>
<td>$v_{Earth}$</td>
<td>Y velocity coordinate of the Earth</td>
</tr>
<tr>
<td>$w_{Earth}$</td>
<td>Z velocity coordinate of the Earth</td>
</tr>
<tr>
<td>$x_{Mars}$</td>
<td>X coordinate of Mars</td>
</tr>
<tr>
<td>$y_{Mars}$</td>
<td>Y coordinate of Mars</td>
</tr>
<tr>
<td>$z_{Mars}$</td>
<td>Z coordinate of Mars</td>
</tr>
<tr>
<td>$u_{Mars}$</td>
<td>X velocity coordinate of Mars</td>
</tr>
<tr>
<td>$v_{Mars}$</td>
<td>Y velocity coordinate of Mars</td>
</tr>
<tr>
<td>$w_{Mars}$</td>
<td>Z velocity coordinate of Mars</td>
</tr>
</tbody>
</table>

Table 2.2 State vector for the interplanetary transfer model.

The nineteen equations to be solved are the following:

\[ \dot{m} = \frac{F_{\text{thrust}}}{\text{isp} \ g_0} \]  \hspace{1cm} \text{Eq. (2.39)}

\[ \dot{x}_{S/C} = u_{S/C} \]  \hspace{1cm} \text{Eq. (2.40)}

\[ \dot{y}_{S/C} = v_{S/C} \]  \hspace{1cm} \text{Eq. (2.41)}

\[ \dot{z}_{S/C} = w_{S/C} \]  \hspace{1cm} \text{Eq. (2.42)}

\[ [\dot{u}_{S/C} \quad \dot{v}_{S/C} \quad \dot{w}_{S/C}] = A_{g,S/C} + A_{T,S/C} \]  \hspace{1cm} \text{Eq. (2.43)}

\[ \dot{x}_{Earth} = u_{Earth} \]  \hspace{1cm} \text{Eq. (2.44)}

\[ \dot{y}_{Earth} = v_{Earth} \]  \hspace{1cm} \text{Eq. (2.45)}

\[ \dot{z}_{Earth} = w_{Earth} \]  \hspace{1cm} \text{Eq. (2.46)}

\[ [\dot{u}_{Earth} \quad \dot{v}_{Earth} \quad \dot{w}_{Earth}] = A_{Earth} \]  \hspace{1cm} \text{Eq. (2.47)}

\[ \dot{x}_{Mars} = u_{Mars} \]  \hspace{1cm} \text{Eq. (2.48)}

\[ \dot{y}_{Mars} = v_{Mars} \]  \hspace{1cm} \text{Eq. (2.49)}

\[ \dot{z}_{Mars} = w_{Mars} \]  \hspace{1cm} \text{Eq. (2.50)}

\[ [\dot{u}_{Mars} \quad \dot{v}_{Mars} \quad \dot{w}_{Mars}] = A_{Mars} \]  \hspace{1cm} \text{Eq. (2.51)}
This system of nineteen equations is numerically integrated to determine the S/C orbit as a function of time.

2.3 Mars Phase

2.3.1 Gravitational Model

The model adopted for this phase starts from the final position of the S/C in the interplanetary phase and ends once the S/C reaches the final circular orbit selected. The capture and subsequent orbit closure around Mars are developed in a two-body gravitational problem. The reference frame adopted is an inertial frame centered in Mars.

The gravitational force acting in this phase is the force of Mars on the S/C. The acceleration vector acting on the S/C can be written as follows:

\[ \mathbf{A}_{g,S/C} = -\mu_{\text{Mars}} \frac{\mathbf{R}_{S/C}}{R_{S/C}^3} \]  
Eq. (2.52)

The acceleration vector provided by the thrust is the following:

\[ \mathbf{A}_{T,S/C} = \frac{F_{\text{Thrust}}}{m_{S/C}} \eta_T \mathbf{e}_T \]  
Eq. (2.53)

Where \( \eta_T \) represent the power degradation at Mars and is computed in the following way:

\[ \eta_T = \left( \frac{R_{\text{Earth}}}{R_{\text{Mars}}} \right)^2 \]  
Eq. (2.54)

This parameter can be considered as constant during the whole phase and has the following value:

\[ \eta_T = 0.43 \]  
Eq. (2.55)

The vector \( \mathbf{e}_T \) represents the thrust orientation and has different components according to the imposed thrust strategy. Two cases have been selected according to the position of the S/C.
The first case considers a vector oriented in the opposite direction with respect to the S/C velocity, and takes place when the S/C is inside the SOI of Mars and the apocenter height is greater than 500 [km]:

$$\mathbf{e}_r = -\frac{\mathbf{v}_{S/C}}{|\mathbf{v}_{S/C}|}$$  \hspace{1cm} \text{Eq. (2.56)}

The second case takes place when the S/C is outside the SOI of Mars and places the thrust vector towards Mars, in the direction of the S/C radius:

$$\mathbf{e}_r = -\frac{\mathbf{r}_{S/C}}{|\mathbf{r}_{S/C}|}$$  \hspace{1cm} \text{Eq. (2.57)}

In the first case a more elaborated thrust strategy has been selected in order to accelerate and at the same time save propellant. This strategy is explained in the next chapters.

### 2.3.2 System of equations

The system of equations governing this phase is the same as the Earth escape phase. It is characterized by seven ordinary differential equations, one for the mass variation and the others for the position and velocity of the S/C.

<table>
<thead>
<tr>
<th>(m)</th>
<th>Mass of the S/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{S/C})</td>
<td>X coordinate of the S/C</td>
</tr>
<tr>
<td>(y_{S/C})</td>
<td>Y coordinate of the S/C</td>
</tr>
<tr>
<td>(z_{S/C})</td>
<td>Z coordinate of the S/C</td>
</tr>
<tr>
<td>(u_{S/C})</td>
<td>X velocity coordinate of the S/C</td>
</tr>
<tr>
<td>(v_{S/C})</td>
<td>Y velocity coordinate of the S/C</td>
</tr>
<tr>
<td>(w_{S/C})</td>
<td>Z velocity coordinate of the S/C</td>
</tr>
</tbody>
</table>

Table 2.3 State vector for the Mars capture model.
The seven equations to be solved are the following:

\[ \dot{m} = \frac{F_{\text{thrust}}}{I_{sp} g_0} \]  
\[ \dot{x}_{S/C} = u_{S/C} \]  
\[ \dot{y}_{S/C} = v_{S/C} \]  
\[ \dot{z}_{S/C} = w_{S/C} \]  
\[ [\dot{u}_{S/C} \; \dot{v}_{S/C} \; \dot{w}_{S/C}] = A_{g,S/C} + A_{r,S/C} \]

This system of seven equations is numerically integrated to determine the S/C trajectory.
The first stage of the work is to collect the data concerning the critical systems involved in the design and to perform a preliminary analysis to assess a rough ΔV budget and hence an estimation for the required propellant.

### 3.1 ΔV estimation

To estimate the ΔV budget the analysis has to rely partly on available sources and partly on computations. Regarding the Earth escape, a rough estimation of the required ΔV can be obtained by computing the difference in terms of velocity between the departure orbit at 450 [km] and an orbit with the radius equal to the Earth SOI from where the influence of the Earth gravitational field can be neglected. The ΔV value for this phase will be the following:

\[
\Delta V = \sqrt{\frac{\mu_{\text{Earth}}}{R_{\text{LEO}}}} - \sqrt{\frac{\mu_{\text{Earth}}}{R_{\text{SOI}}}} = 6.9857 \text{ [km/s]}
\]

Eq. (3.1)

Regarding the interplanetary transfer the ΔV can be estimated from Table 3.1. These values have been presented assuming both departure and arrival C3 equal to 0 [km²/s²] and that the orbits of the departing and arrival planets are circular. These values consider planar transfers and are the same for all cases in which one or two HET or ion thrusters are used.
Table 3.1 $\Delta V$ estimation for inner planets transfers. [18]

<table>
<thead>
<tr>
<th>Planet</th>
<th>$\Delta V$ [km/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>18.02</td>
</tr>
<tr>
<td>VOTV</td>
<td>5.202</td>
</tr>
<tr>
<td>Mars</td>
<td>5.594</td>
</tr>
<tr>
<td>Mars (2 ions, Soyuz)</td>
<td>6.878</td>
</tr>
<tr>
<td>Mars (2 ion, Falcon-9)</td>
<td>6.614</td>
</tr>
</tbody>
</table>

The $\Delta V$ for the last phase can be approximated by computing the velocity variation from the Mars SOI and the low Mars orbit (LMO) selected. This value is computed assuming a desired circular arrival orbit with altitude equal to 500 [km]. The $\Delta V$ for this phase is:

$$\Delta V = 2 \frac{\mu_{\text{Mars}}}{R_{\text{LMO}}} - 2 \frac{\mu_{\text{Mars}}}{R_{\text{SOI}}} = 3.045 \text{ [km/s]}$$  \hspace{1cm} \text{Eq. (3.2)}

A preliminary value for the total $\Delta V$ required will be the following:

$$\Delta V = 6.9857 \text{ [km/s]} + 5.594 \text{ [km/s]} + 3.045 \text{ [km/s]} = 16.0307 \text{ [km/s]}$$  \hspace{1cm} \text{Eq. (3.3)}

This rough estimation doesn’t take into consideration the required $\Delta V$ for the change of plane. From this value, the propellant for the mission can be estimated through the Tsiolkovsky formula in the following way:

$$m_p = m_f \left( e^{\left( \frac{\Delta V}{g_\text{Earth}} \right)} - 1 \right) = 1567 \text{ [kg]}$$  \hspace{1cm} \text{Eq. (3.4)}

$$m_f = m_{p/L} + m_{dry} = 1240 \text{ [kg]}$$  \hspace{1cm} \text{Eq. (3.5)}

Where the preliminary P/L mass has been selected as 500 [kg] and is a value that will be refined in order to store the augmented propellant required for a mission to Mars and the consequent dry mass to take into account for the additional tanks. The value of the specific impulse selected for these calculations is of 2000 [sec] as shown in Table 3.4.
3.2 Spacecraft Significant Parameters

Part of the data for the propulsion system can be obtained directly from the Vega documentation while other parameters have to be computed. The EPM characteristics are presented in Table 3.2.

<table>
<thead>
<tr>
<th>EPM characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Propellant (Xe) mass [kg]</td>
<td>25.3% of initial mass</td>
</tr>
<tr>
<td>Inert mass [kg]</td>
<td>8% of initial mass</td>
</tr>
<tr>
<td>DeltaV provided [km/s]</td>
<td>6.6</td>
</tr>
<tr>
<td>Thruster power [kW]</td>
<td>4.5</td>
</tr>
<tr>
<td>Anodyc thrust efficiency (&gt;50%)</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 3.2 EPM data.

Some essential parameters for the propulsion system considered are computed and grouped in Table 3.3.

<table>
<thead>
<tr>
<th>EPM parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{jet}$ (W)</td>
<td>3531.6</td>
</tr>
<tr>
<td>$P/F$ (W/N)</td>
<td>9810</td>
</tr>
<tr>
<td>$P_e$ (W)</td>
<td>9800</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>0.52</td>
</tr>
<tr>
<td>$\alpha$ (W/kg)</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 3.3 EPM parameters.

The parameters provided for the single HET are presented in Table 3.4.

| Reference thruster 4.5 [kW] HET |
|-------------------------------|----------|
| Guaranteed total impulse [kNs] | 15500    |
| Minimum Isp [s]                | 2000     |
| Minimum Thrust EOL [N]         | 0.4833   |
| HET discharged power [W]       | 9000     |

Table 3.4 HET parameters.
For the power production module, the ALEK-3, the given in Table 3.5.

<table>
<thead>
<tr>
<th>ALEK-3 characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Power produced [kW]</td>
<td>16</td>
</tr>
<tr>
<td>Mass [kg]</td>
<td>12% of initial mass</td>
</tr>
</tbody>
</table>

Table 3.5 ALEK-3 data.

Hence the data considered for this analysis are summarized in Table 3.6.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry mass [kg]</td>
<td>24.2%</td>
</tr>
<tr>
<td>Reference P/L mass [kg]</td>
<td>10% of initial mass</td>
</tr>
<tr>
<td>Isp [s]</td>
<td>2000</td>
</tr>
<tr>
<td>Thrust [N]</td>
<td>0.4833</td>
</tr>
</tbody>
</table>

Table 3.6 Data considered for the analysis.

As a supplementary study an optimal flight performance analysis is performed with the aim of selecting the ideal characteristics of a thruster for this kind of mission. First of all, it is needed a relation for the reciprocal payload mass fraction:

\[
\frac{m_0}{m_{P/L}} = \frac{e^{\Delta u/v}}{1-(e^{\Delta u/v}-1)v^2/(2\pi f \eta c)}
\]

Eq. (3.6)

This relation assumes a gravity-free drag-free flight. From the derivation, the following relation is obtained:

\[
\left(\frac{v}{\Delta u}\right)\left(e^{\Delta u/v} - \frac{1}{2}\left(\frac{c}{v}\right)^2 - \frac{1}{2} = 0
\]

Eq. (3.7)

The change of the vehicle velocity \(\Delta u\) which results from the propellant exhausted at a speed \(v\) is shown in Figure 3.1. From this figure it can be seen that for a given payload fraction and characteristic speed, there is an optimum value of velocity corresponding to the peak vehicle velocity increment, this is the actual purpose of this analysis, to show the existence of a set of most desirable operating conditions. [6]
The steps of the optimum analysis are the following:

1. Given the payload mass fraction $m_{pL}/m_0 = 0.1$ from the graph the optimum value of $\Delta V/v_c$ is obtained.

$$\Delta V/v_c = 0.6$$  \hspace{1cm} Eq. (3.8)

![Figure 3.1](image1.png)

Figure 3.1 Normalized vehicle velocity increment as a function of normalized exhaust velocity for various payload fractions with zero inert mass of the propellant tank. The optima of each curve are connected by a line that represents $v \Delta u \left(e^{\Delta u/v_c} - \frac{1}{2} \left(\frac{v_c}{v}\right)^2 - \frac{1}{2}\right) = 0$  \hspace{1cm} Eq. (3.7) \hspace{0.5cm} [6]

2. Then $v_c$ is computed.

$v_c = 26.7 \,[km/s]$  \hspace{1cm} Eq. (3.9)

3. From the optimum $v/v_c = 0.7$, obtained from the graph, compute $v$ and $I_{sp}$.

<table>
<thead>
<tr>
<th>$v_{\text{optimum}} ,[km/s]$</th>
<th>18.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{sp} , [s]$</td>
<td>1906</td>
</tr>
</tbody>
</table>

Table 3.7 Optimum values obtained from the optimum curve graph.

$I_{sp} = v_{\text{optimum}} / g_0$  \hspace{1cm} Eq. (3.10)
4. Compute the thrusting time using the parameters of the EPM: $\alpha$ and $\eta_t$.

$$t_p = \frac{v_c^2}{2 \alpha \eta_t} = 661 \ [\text{days}]$$  \hspace{1cm} \text{Eq. (3.11)}

5. Compute the propellant mass

$$m_p = \frac{F}{\text{isp} \ g_0} t_p = 1476 \ [kg]$$  \hspace{1cm} \text{Eq. (3.12)}

$$m_p + 20\% = 1771 \ [kg]$$  \hspace{1cm} \text{Eq. (3.13)}

6. Compute the electric power in input. $P_e$ is computed considering the margined propellant mass.

$$P_e = \frac{v_c^2 m_p}{2 t_p \eta_t} = 10.427 \ [kW]$$  \hspace{1cm} \text{Eq. (3.14)}

From this preliminary analysis, it can be seen that the specific impulse and the power required in input are close to the actual values given by design. It is expected an increase in the propellant and also the burning time seems lower than expectations. The values coming from this preliminary analysis can be considered as a guideline to understand the direction of the final results. The development of a trajectory simulator is necessary to understand in a more precise way the consumption of propellant and the TOF of the single phases of the mission.
CHAPTER 4
LOW-THRUST MODEL

The simulation is articulated into three main sections, one for each phase of the mission. Each phase is linked to the next at the SOI of the arrival and departure planets. The final position and velocity of the S/C at each phase are the initial conditions for the integration in the next phase. The integration is assumed to start at a reference date which is the 1st Jan 2019, the planets initial positions are obtained from the ephemeris. The main functions implemented are four while others auxiliary functions complete the model.

- \textit{LT\_Model.m}: which is the structure of the low-thrust model.
- \textit{EarthPhase.m}: which models the Earth escape.
- \textit{TransferPhase.m}: which models the interplanetary transfer.
- \textit{MarsPhase.m}: which models the Mars capture and orbit closure.

4.1 Low-Thrust Analysis

The first step of the analysis is to collect the main variables and parameters that will be used in the simulation. These variables, presented in Table 4.1, are defined as global in order to be accessible by all the functions in the simulation. They can be distinguished between engine and S/C parameters, gravitational parameters and the parameters defining the departure orbit and the planets orbit.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Impulse [s]</td>
<td>2000</td>
</tr>
<tr>
<td>Thrust [N]</td>
<td>0.4833</td>
</tr>
<tr>
<td>Propulsive efficiency @ Mars</td>
<td>0.43</td>
</tr>
<tr>
<td>Initial S/C mass [kg]</td>
<td>xxxx</td>
</tr>
<tr>
<td>Earth mass [kg]</td>
<td>5.9724 e+24</td>
</tr>
<tr>
<td>Sun mass [kg]</td>
<td>1.9885 e+30</td>
</tr>
<tr>
<td>Mars mass [kg]</td>
<td>6.4171 e+23</td>
</tr>
<tr>
<td>Gravitational acceleration [m/s²]</td>
<td>9.81</td>
</tr>
<tr>
<td>Gravitational constant [km³/kg/s²]</td>
<td>6.67 e-20</td>
</tr>
<tr>
<td>Sun standard gravitational parameter [km³/s²]</td>
<td>1.3263 e+11</td>
</tr>
<tr>
<td>Earth standard gravitational parameter [km³/s²]</td>
<td>3.9836 e+05</td>
</tr>
<tr>
<td>Mars standard gravitational parameter [km³/s²]</td>
<td>4.2802 e+04</td>
</tr>
<tr>
<td>Sun radius [km]</td>
<td>695700</td>
</tr>
<tr>
<td>Earth radius [km]</td>
<td>6371</td>
</tr>
<tr>
<td>Mars radius [km]</td>
<td>3389.5</td>
</tr>
<tr>
<td>Astronomical unit [km]</td>
<td>149597870</td>
</tr>
<tr>
<td>Earth equatorial inclination [deg]</td>
<td>23.44</td>
</tr>
<tr>
<td>Departure orbit inclination [deg]</td>
<td>6.2</td>
</tr>
<tr>
<td>Mars orbit inclination [deg]</td>
<td>1.85</td>
</tr>
<tr>
<td>Earth semimajor axis [km]</td>
<td>1*AU</td>
</tr>
<tr>
<td>Mars semimajor axis [km]</td>
<td>1.523551104*AU</td>
</tr>
<tr>
<td>Earth SOI [km]</td>
<td>9.2464 e+05</td>
</tr>
<tr>
<td>Mars SOI [km]</td>
<td>5.7718 e+05</td>
</tr>
<tr>
<td>Departure orbit altitude [km]</td>
<td>450</td>
</tr>
</tbody>
</table>

Table 4.1 Parameters of the analysis.

Most of these variables are given parameters but some have been computed. Regarding the planets standard gravitational parameter, it can be obtained through this formulation:

\[ \mu_{\text{planet}} = G \times M_{\text{planet}} \]  

Eq. (4.1)
Where \( G \) is the gravitational constant and \( M \) is the mass of the planet.
The SOI of the Earth and Mars is obtained through this formula:

\[
SOI_{\text{Earth,Mars}} = a_{\text{Earth,Mars}} \left( \frac{M_{\text{Earth,Mars}}}{M_{\odot}} \right)^{\frac{2}{3}}
\]

Eq. (4.2)

Where \( a_{\text{Earth,Mars}} \) is the semimajor axis of the planet considered.
Each phase is defined in a different inertial system, always centered in the main gravitational body. The S/C is left by the launcher in a circular equatorial orbit with inclination of 6.2 [deg] and altitude of 450 [km], the other parameters are assumed and presented in Table 4.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor axis [km]</td>
<td>6821</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0</td>
</tr>
<tr>
<td>Inclination [deg]</td>
<td>6.2</td>
</tr>
<tr>
<td>Right Ascension of the Ascending Node ARAAN [deg]</td>
<td>0</td>
</tr>
<tr>
<td>Pericenter anomaly [deg]</td>
<td>0</td>
</tr>
<tr>
<td>True anomaly [deg]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2 Departure orbit parameters.

These parameters are used to compute the initial position and velocity of the S/C in the ECI frame. After the initial conditions have been defined the integration of the equations of motions starts. The solver used to integrate the equations of motion is the MATLAB® function ode113 which uses the Runge-Kutta method for a non-stiff problem. The parameters used in each integration are the following:

<table>
<thead>
<tr>
<th>ODE options</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RelTol</td>
<td>10e-06</td>
</tr>
<tr>
<td>AbsTol</td>
<td>10e-05</td>
</tr>
<tr>
<td>Refine (Earth phase)</td>
<td>2</td>
</tr>
<tr>
<td>Events</td>
<td>ESOIevent.m / MSOIevent.m</td>
</tr>
</tbody>
</table>

Table 4.3 ODE integration options.
The event functions presented in Table 4.3 are used respectively in the Earth and Mars phase to stop the integration once precise conditions occur. In the Earth phase the event function stops the integration once the S/C reaches the Earth SOI while in Mars phase the event function stops the integration once the S/C reaches the arrival orbit altitude. 
A section of the final Matlab code is devoted to plot the graphs. The graphs plotted are summarized in Table 4.4.

<table>
<thead>
<tr>
<th>Graphs Summary</th>
<th>Earth Phase</th>
<th>Transfer Phase</th>
<th>Mars Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>S/C position wrt Earth</td>
<td>Altitude variation of the S/C</td>
<td>S/C transfer orbit</td>
<td>Position wrt Mars</td>
</tr>
<tr>
<td>S/C mass vs time</td>
<td>Thrust profile / S/C mass vs time</td>
<td>S/C mass vs time</td>
<td>Altitude variation of the S/C</td>
</tr>
<tr>
<td>Orbit inclination wrt ecliptic</td>
<td>Thrust profile / S/C mass vs time</td>
<td>Altitude variation of the S/C</td>
<td>S/C mass vs time</td>
</tr>
<tr>
<td></td>
<td>Orbit inclination wrt the ecliptic / RAAN variation</td>
<td>Thrust profile / S/C mass vs time</td>
<td>Thrust profile / S/C mass vs time</td>
</tr>
<tr>
<td></td>
<td>Thrust &lt;theta&gt; angle profile</td>
<td>Thrust &lt;psi&gt; angle profile</td>
<td>Arrival orbit @ Mars</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Orbit inclination / Eccentricity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pericenter height / Eccentricity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Apocenter height / Eccentricity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Eccentricity / Thrust profile</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>True anomaly / Eccentricity</td>
</tr>
</tbody>
</table>

Table 4.4 Graphs presented in the analysis.
While the Earth and Mars phases are developed in a similar fashion, the interplanetary transfer differs significantly. The integration of the equations of motion is performed in an inertial frame centered in the Sun hence the S/C initial position and velocity that correspond to the final position and velocity of the Earth phase, have to be rotated in the new frame.

From Figure 4.1 it can be seen that the ECI frame differs from the SCI frame only by a rotation about the X-axis of the Earth equatorial angle. The initial conditions for the S/C position and velocity will be defined as follows:

\[
x_{0,S/C} = [x_{0,S/C}, y_{0,S/C}, z_{0,S/C}]_{SCI} = \mathbf{R}_1(23.44°) * [x_{f,S/C}, y_{f,S/C}, z_{f,S/C}]_{ECI} + [x_{0,Earth}, y_{0,Earth}, z_{0,Earth}]_{SCI}
\]

Eq. (4.3)  

\[
v_{0,S/C} = [u_{0,S/C}, v_{0,S/C}, w_{0,S/C}]_{SCI} = \mathbf{R}_1(23.44°) * [u_{f,S/C}, v_{f,S/C}, w_{f,S/C}]_{ECI} + [u_{0,Earth}, v_{0,Earth}, w_{0,Earth}]_{SCI}
\]

Eq. (4.4)  

Where \( \mathbf{R}_1(23.44°) \) is the rotation matrix about the X-axis:

\[
\mathbf{R}_1(23.44°) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(23.44°) & \sin(23.44°) \\
0 & -\sin(23.44°) & \cos(23.44°)
\end{bmatrix}
\]

Eq. (4.5)
The initial position and velocity of the Earth and Mars are given by the ephemeris obtained by NASA JPL’s HORIZONS system [20]. The _ephemeris.m_ function gives the position and velocity of the selected body after receiving as input the final time after the Earth phase, which is the time when the S/C reaches the Earth SOI. After the initial conditions of the S/C and planets are determined, the optimization problem can start. The optimization problem is carried out through the _fmincon.m_ function of MATLAB® which is ideal when constraints have to be imposed. The parameters that are optimized are contained in the $x_0$ vector and are the following:

$$x_0 = [tof \text{ days}_{t=0} \ \vartheta_1 \ \vartheta_2 \ \vartheta_3 \ \vartheta_4 \ \vartheta_5 \ \varphi_1 \ \varphi_2 \ \varphi_3 \ \varphi_4 \ \varphi_5]$$  \hspace{1cm} \text{Eq. (4.6)}$$

Among these parameters, the problem is devoted to the minimization of the $tof$, which is the time of flight. The second parameter is introduced in order to insert a ballistic phase in the trajectory to save propellant. This ballistic phase is assumed to happen two times during the transfer. The time interval is divided into three segments and the two no-thrust intervals are located at the conjunction of each segment as in Figure 4.2.

![Figure 4.2 Ballistic intervals structure.](image)

The other parameters represent the angles guesses to build the velocity profile. The thrust vector must include some out-of-plane components in order to perform the change of plane. To define the ideal thrust vector orientation a cubic spline interpolation is used. Four guesses are given to the spline to interpolate the angle profile, these guesses are then optimized in the problem. In the normal-velocity-co-normal (NVC) frame selected, two angles are needed to define the orientation of the thrust vector. This reference frame is oriented as in
Figure 4.3, it is centered in the S/C with the third axis oriented like the angular momentum vector, the second axis is in the velocity direction and the first axis is co-normal to the previous ones.

![Figure 4.3 Orientation of NVC and SCI reference frames.](image)

In this frame, the $\theta$ and $\varphi$ angles represent respectively the yaw and pitch angles. The initial guesses for these angles in the optimization problem have a zero value and have no constraint in their variation. The yaw angle can vary between -180 [deg] and +180 [deg] while the pitch angle range is between -90 [deg] and +90 [deg].

The ideal approach for a rendezvous with Mars is to reach the planet at its SOI with the right velocity in order to have the characteristic energy equal to zero. The characteristic energy is a measure of the excess specific energy required to escape from a planet. The actual constraint implemented in the optimization problem is the equality between the position and velocity components of Mars and the S/C. This choice has been adopted after observing that the error between the final position vectors of the two bodies at the SOI is significant. Hence by imposing the equality of the actual vectors components the S/C will unlikely collide with the planet, it is instead observed that the rendezvous actually happens inside the SOI for most of the cases considered.

Once the optimization problem converged to a feasible solution the optimized vector of the initial guesses is used as the initial conditions vector to perform an integration with the optimized values.

For completeness, the parameters that define the optimization problem are presented in Table 4.5.
4.2 Earth Escape

This function receives as input the initial conditions vector which is an array of seven components containing the initial mass of the system and the initial position and velocity.

\[
x_0 = \begin{bmatrix} m_{0,s/c} & x_{0,s/c} & \dot{x}_{0,s/c} \\ y_{0,s/c} & \dot{y}_{0,s/c} \\ z_{0,s/c} & \dot{z}_{0,s/c} \end{bmatrix}
\]

Eq. (4.7)

It is called as an input by the function ode113 to integrate the equations of motion. This function first solves the steps to approximate the Sun position at the initial date which has been selected as the 1st Jan 2019. By calling the julian.m function, this date is converted in the correspondent Julian number, the date components are defined as a global parameter in order to modify the departure date from the LT_Model.m code. The departure date can be modified as desired, but a special care has to be taken as not to exceed the range of the ephemeris files to select the arrival date at the Earth SOI, if this happens extended ephemeris have to be provided, the current range is from 1st Jan 2019 to 1st Jan 2039. During each integration step, the current date is incremented. While the
increment is given in seconds, the julian.m function considers only the year, month and day to compute the Julian date. Hence the variation of the Sun position will be updated after a day unit of time.

Then the gravitational acceleration vector is built, coupled with the J2 perturbation effect. The acceleration vector has to be summed to the thrust acceleration vector. During the Earth phase a continuous thrust is considered, the thrusters are switched off only during eclipse conditions. The eclipse conditions are computed by verifying the mutual visibility between the S/C position and the Sun position which has been previously defined.

### 4.3 Interplanetary Transfer

This function is an input to the ode113 to integrate the equations of motion. It receives as input a vector of nineteen components which represent the initial conditions for the mass and the initial position and velocity of the S/C and the planets.

\[
x_0 = \begin{bmatrix}
m_{0,S/C}
X_{0,S/C}
Y_{0,S/C}
Z_{0,S/C}
U_{0,S/C}
V_{0,S/C}
W_{0,S/C}
X_{0,Earth}
Y_{0,Earth}
Z_{0,Earth}
U_{0,Earth}
V_{0,Earth}
W_{0,Earth}
X_{0,Mars}
Y_{0,Mars}
Z_{0,Mars}
U_{0,Mars}
V_{0,Mars}
W_{0,Mars}
\end{bmatrix} \quad \text{Eq. (4.8)}
\]

The first step is to build the gravitational acceleration vector. Then a parameter is computed to take into account the reduction of power with the distance from the nominal condition at the Earth. This parameter is multiplied to the thrust vector and varies according to the position of the S/C during the transfer.

The thrust angles profiles are built through a cubic spline approximation from the guesses received as input. The angles values will be called during the integration at each integration step to build the thrust direction vector, which is the vector that defines the thrust orientation in the NVC frame.

\[
\mathbf{u}_{\text{Thrust}} = \begin{bmatrix} \cos \varphi \sin \theta & \cos \varphi \cos \theta & \sin \varphi \end{bmatrix} \quad \text{Eq. (4.9)}
\]

This vector will be rotated in the inertial frame by a rotation matrix which is built inside the matInVn.m function.
The thrust strategy implemented for the interplanetary phase aims to insert a ballistic phase during the trajectory to reduce the propellant burned. This ballistic phase is defined by assuming two no-thrust intervals located at two equally spaced intervals of the total time vector as in Figure 4.2.

The function receives as input an optimized parameter that defines the time interval around one third and two thirds of the total time span. Inside these intervals the thrust vector is multiplied by a zero-value parameter.

The last step is to build the acceleration vectors of the two planets and the total acceleration vector of the S/C which is given by the sum of the gravitational acceleration and the thrust acceleration.

### 4.4 Mars Capture and Orbit Closure

This function receives as input the initial conditions vector which is an array of seven components containing the initial mass of the system and the initial position and velocity at the beginning of Mars phase.

\[
\mathbf{x}_0 = \begin{bmatrix} x_{0,S/C} \\ y_{0,S/C} \\ z_{0,S/C} \\ u_{0,S/C} \\ v_{0,S/C} \\ w_{0,S/C} \end{bmatrix} \quad \text{Eq. (4.10)}
\]

The thrust strategy implemented for this phase deals with two alternatives which differ depending on the position of the S/C at the rendezvous with Mars. If the encounter with Mars happens outside the Mars SOI the thrust is oriented towards the opposite direction of the radius vector, whereas if the encounter happens inside the SOI a different approach is selected. Inside the SOI the thrust vector is directed in the opposite direction than the velocity vector, two conditions are exploited to define the thrust regions. If the orbit eccentricity is greater than a defined value which has been selected as 0.0001, the thrust is active $+130[\text{deg}]$ and $-130[\text{deg}]$ with respect to the pericenter, otherwise the orbit is considered as circular and the thrust is continuous. If these conditions are verified the thrust is considered as active otherwise the thrust vector will have zero components. After defining the thrust vector the total S/C acceleration vector can be built.
4.5 Auxiliary Functions of the model

The auxiliary functions and files that complete the code are:

- **constraints_fmincon.m**
  
  Function that sets the constraints for the optimization problem. The equality constraints imposed are on the S/C position and velocity components to be equal to Mars ones.

\[
\begin{align*}
x_{S/C} & = x_{Mars} & \dot{x}_{S/C} & = \dot{x}_{Mars} & x_{S/C} - x_{Mars} & = 0 & \dot{x}_{S/C} - \dot{x}_{Mars} & = 0 \\
y_{S/C} & = y_{Mars} & \dot{y}_{S/C} & = \dot{y}_{Mars} & y_{S/C} - y_{Mars} & = 0 & \dot{y}_{S/C} - \dot{y}_{Mars} & = 0 \\
z_{S/C} & = z_{Mars} & \dot{z}_{S/C} & = \dot{z}_{Mars} & z_{S/C} - z_{Mars} & = 0 & \dot{z}_{S/C} - \dot{z}_{Mars} & = 0
\end{align*}
\]

Eq. (4.11)

- **deg2rad.m**
  
  Function to convert an angle from degrees to radians.

- **ephemeris.m**
  
  Functions that reads the ephemeris files to give the planets positions at a given date.

  - **Eff_Earth_2019_01_01_2039_01_01.txt**
    
    File containing the Earth ephemeris from 1\textsuperscript{st} Jan 2019 to 1\textsuperscript{st} Jan 2039.

  - **Eff_Mars_2019_01_01_2039_01_01.txt**
    
    File containing Mars ephemeris from 1\textsuperscript{st} Jan 2019 to 1\textsuperscript{st} Jan 2039.

- **ESOIevent.m**
  
  Event function to stop the integration during the Earth phase when the Earth SOI is reached.

- **f.m**
  
  Function containing the parameters that have to be optimized for the optimization problem during the transfer phase.
• *julian.m* [21]

Function that receives in input the current date [month, day, year] and gives as output the correspondent Julian date. This function is called during the Earth phase for the estimation of the J2 perturbations.

• *MSO1event.m*

Event function to stop the integration during Mars phase when all the propellant available is burned.

• *matInVn.m*

Function that builds the rotation matrix from a body reference frame during the transfer phase to the inertial frame centered in the Sun. This function is required in order to rotate the thrust vector whose angles are defined in a body reference frame.

• *plot_orbit.m*

Function that receives in input the orbital parameters to build the position vector to plot the current orbit.

• *rad2deg.m*

Function to convert an angle from radians to degrees.

• *R1.m* [1]

Function that creates a rotation matrix about the X-axis.

• *R2.m* [1]

Function that creates a rotation matrix about the Y-axis.

• *R3.m* [1]

Function that creates a rotation matrix about the Z-axis.

• *transfer_c_k_2_sun.m*
Auxiliary Functions of the model

Function that compute the orbital parameters for the body position and velocity in an inertial frame.

- *transfer_k_c.m*

  Function that compute the body position and velocity in an inertial frame from the given orbital parameters.

- *zeroTo360.m [1]*

  Function that reduces an angle to the range of 0-360 [deg] or 0-2π [rad].
The strategy adopted to present the results of the analysis aims first to focus on an ideal Reference Mission, around which the optimization problem has been built. Then the interest shifts in making a comparison of the same mission launched at different dates. The detailed results containing the numerical values and the graphs of the trajectory and the variation with time of the main parameters, are shown only for the Reference Mission. The comparison will focus only on numerical results.

5.1 Reference Mission

The Reference Mission starts at a randomly chosen date which is the 1st Jan 2019. For each phase, the numerical results are presented followed by the graphs obtained from the analysis.

5.1.1 Earth Phase

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start date [day/month/year]</td>
<td>01/01/2019</td>
</tr>
<tr>
<td>End date [day/month/year]</td>
<td>01/05/2020</td>
</tr>
<tr>
<td>TOF [days]</td>
<td>486.2079</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>908.5</td>
</tr>
<tr>
<td>Delta V [km/s]</td>
<td>7.0775</td>
</tr>
<tr>
<td>No-thrust time [days]</td>
<td>15.5988</td>
</tr>
</tbody>
</table>
Table 5.1 Reference Mission Earth phase results.

The Earth escape phase is the most critical in terms of propellant burned and TOF. As it can be seen from Figure 5.5 nearly one year is spent to reach the GEO orbit, from that point the escape spiral increase significantly, and the escape is faster. Due to this fact the S/C is affected for most of the TOF by the J2 perturbation as it can be seen in Figure 5.5.

The eclipse events occur in the first three hundred days of the escape which correspond to the time spent by the S/C under the GEO altitude. In Figure 5.3 a close-up of the eclipse occurrences is presented.

In Figure 5.7 the thrust profile is presented. The thrust strategy adopted is a good indication of the eclipse events. The thrust is considered as continuous, and it is switched off during the eclipse phases. Also, this graph is coherent with the considerations given above, the eclipse conditions are concentrated in the first three hundred days of the simulation.

![Figure 5.1 Earth escape trajectory.](image)
Figure 5.2 Earth escape trajectory, X-Y plane.

Figure 5.3 Earth escape trajectory, X-Y plane, eclipse segments.
Results

Figure 5.4 Earth escape trajectory, X-Y plane, eclipse segments close-up.

Figure 5.5 Altitude variation during the Earth escape phase.
Figure 5.6 Mass variation during Earth escape phase.

Figure 5.7 Thrust profile compared with mass variation during Earth escape phase.
Results

Figure 5.8 Close-up of the thrust profile compared with mass variation during Earth escape phase.

Figure 5.9 S/C orbit inclination during Earth escape.
5.1.2 Interplanetary Phase

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start date [day/month/year]</td>
<td>01/05/2020</td>
</tr>
<tr>
<td>End date [day/month/year]</td>
<td>18/07/2021</td>
</tr>
<tr>
<td>TOF [days]</td>
<td>442.8428</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>555</td>
</tr>
<tr>
<td>Delta V [km/s]</td>
<td>6.055</td>
</tr>
<tr>
<td>No-thrust time [days]</td>
<td>24.714</td>
</tr>
<tr>
<td>Longest firing time [days]</td>
<td>142</td>
</tr>
<tr>
<td>Distance from Mars [km]</td>
<td>1.9552e+005</td>
</tr>
<tr>
<td>Ballistic time guess [days]</td>
<td>15</td>
</tr>
<tr>
<td>Optimized ballistic parameter [days]</td>
<td>12.3704</td>
</tr>
</tbody>
</table>

Table 5.2 Reference Mission interplanetary phase results.

In this phase, the peculiarity of the electric propulsion which is the capability of obtaining a significant $\Delta V$ thanks to an extended firing time, is exploited. To reduce the propellant burned a ballistic parameter is introduced, in Figure 5.14 the presence of the ballistic intervals can be seen in both the thrust profile and the mass variation. This parameter can vary between five and fifteen days, in the reference mission the optimized value of the parameter is around twelve days.

In Figure 5.15 the altitude variation of the S/C and the planets can be observed, the altitude of the planets is not constant since their orbits are elliptic. In Figure 5.16 the close-up of the altitude variation shows that the rendezvous with Mars happens inside Mars SOI.

In Figure 5.17 the progressive variation of the inclination is shown, this graph is related with the plots in Figure 5.18 and Figure 5.19, the thrust vector angles, defined in a local frame, have an optimized profile in order to reach progressively the inclination and the required orbit altitude.
Results

Figure 5.10 Interplanetary phase trajectory.

Figure 5.11 Interplanetary phase trajectory, X-Y plane.
Figure 5.12 Interplanetary phase trajectory, close up of the rendezvous.

Figure 5.13 Mass variation.
Figure 5.14 Thrust profile compared with the mass variation during the interplanetary phase.

Figure 5.15 Altitude variation during the interplanetary phase.
Figure 5.16 Altitude variation close up of the rendezvous.

Figure 5.17 Orbit inclination compared with the RAAN variation.
Results

Figure 5.18 $\theta$ angle variation of the thrust vector.

Figure 5.19 $\varphi$ angle variation of the thrust vector.
5.1.3 Mars Phase

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Start date [day/month/year]</td>
<td>18/07/2021</td>
</tr>
<tr>
<td>End date [day/month/year]</td>
<td>08/08/2022</td>
</tr>
<tr>
<td>TOF [days]</td>
<td>386.8401</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>190</td>
</tr>
<tr>
<td>Delta V [km/s]</td>
<td>2.5938</td>
</tr>
<tr>
<td>No-thrust time [days]</td>
<td>139.4172</td>
</tr>
<tr>
<td>Minimum time between two firings [hrs]</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 5.3 Reference Mission Mars phase results.

The last phase which is devoted to the closure and circularization of the orbit around Mars is highly dependent on the initial conditions of the S/C at the end of the transfer phase. The purpose of the thrust strategy is to minimize the propellant burned and at the same time reduce the time required to reach the arrival orbit. The choice adopted to accomplish these objectives is to activate the thrust at +130 [deg] and -130 [deg] around the pericenter. One problem encountered with this method is when the orbit eccentricity tends to zero, due to the orbit circularization, hence the definition of a pericenter becomes irrelevant. As it can be seen in Figure 5.32, the true anomaly starts to show an incoherent behavior as soon as the eccentricity tends to zero. To overcome this problem a check is performed on the eccentricity value, once these parameters is less than 0.0001 the choice of continuous thrust is adopted, as it can be seen in Figure 5.35. The value of this parameter has been tuned empirically.

In Figure 5.33 and in Figure 5.34, the variation the pericenter and apocenter are shown, the integration of this phase stops as soon as the apocenter radius reaches the required altitude, which is 500 [km].

A consideration has to be made on Figure 5.31, a variation due to a numeric error can be observe towards the end of the time interval, this variation can be neglected since is very small, of the order of $10^{-4}$. 
Results

Figure 5.20 Mars capture trajectory.

Figure 5.21 Mars capture trajectory, X-Y plane.
Figure 5.22 Mars capture trajectory, Y-Z plane.

Figure 5.23 Mars capture trajectory, X-Z plane.
Figure 5.24 Mars capture close-up.

Figure 5.25 Mars capture close-up.
Figure 5.26 Mass variation.

Figure 5.27 Orbit radius variation during Mars capture phase.
Results

Figure 5.28 Thrust profile compared with mass variation.

Figure 5.29 Thrust profile compared with mass variation close-up.
Reference Mission

Figure 5.30 Final orbit around Mars.

Figure 5.31 Orbit inclination compared with eccentricity variation.
Results

Figure 5.32 True anomaly compared with eccentricity variation.

Figure 5.33 Pericenter height compared with eccentricity variation.
Figure 5.34 Apocenter height compared with eccentricity variation.

Figure 5.35 Eccentricity variation compared with thrust profile.
5.1.4 Overall Results

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Start date [day/month/year]</td>
<td>01/01/2019</td>
</tr>
<tr>
<td>End date [day/month/year]</td>
<td>08/08/2022</td>
</tr>
<tr>
<td>TOF [years]</td>
<td>3.6052</td>
</tr>
<tr>
<td>Thrusting time [years]</td>
<td>3.1128</td>
</tr>
<tr>
<td>Longest firing time [days]</td>
<td>142</td>
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<tr>
<td>Minimum time between two firings [min]</td>
<td>2</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>1654</td>
</tr>
<tr>
<td>Delta V [km/s]</td>
<td>15.7264</td>
</tr>
<tr>
<td>Final orbit altitude [km]</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 5.4 Overall results of the Reference Mission.

5.2 Missions Comparison

After showing the results obtained for the reference mission, a series of missions are compared in order to determine the influence of the planets position on the results of the analysis. The criteria adopted to change the starting date is based on the medium duration of the Mars synodic period which is of 780 Earth days. The first Mars closest approach date is on the 6th Oct 2020 [23], the other close approach dates have been computed by adding the synodic time obtained from the reference. By considering that it takes roughly 2.5 years to reach Mars with this model, the following missions are launched approximately 2.5 years before the Mars close approach date computed, as presented in Table 5.5.

<table>
<thead>
<tr>
<th>Mars close approach [DD/MM/YY]</th>
<th>Selected departure date [DD/MM/YY]</th>
</tr>
</thead>
<tbody>
<tr>
<td>06/10/2020</td>
<td>/</td>
</tr>
<tr>
<td>25/11/2022</td>
<td>26/05/2020</td>
</tr>
<tr>
<td>13/01/2025</td>
<td>01/07/2022</td>
</tr>
<tr>
<td>04/03/2027</td>
<td>02/09/2024</td>
</tr>
<tr>
<td>22/04/2029</td>
<td>22/09/2026</td>
</tr>
<tr>
<td>11/06/2031</td>
<td>10/11/2028</td>
</tr>
</tbody>
</table>
Missions Comparison

<table>
<thead>
<tr>
<th>Date</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>30/07/2033</td>
<td>29/12/2030</td>
</tr>
<tr>
<td>18/09/2035</td>
<td>22/02/2033</td>
</tr>
<tr>
<td>06/11/2037</td>
<td>10/04/2035</td>
</tr>
<tr>
<td>26/12/2039</td>
<td>31/05/2037</td>
</tr>
</tbody>
</table>

Table 5.5 Mars close approach and departure dates selected.

The results obtained from the analysis are presented in Table 5.6, Table 5.7, Table 5.8. The results regarding the Earth phase have been neglected since the TOF and the propellant burned remain approximately the same in all of the missions considered.
<table>
<thead>
<tr>
<th>DEPARTURE DATE</th>
<th>26/05/2020</th>
<th>01/07/2022</th>
<th>02/09/2024</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANSFER PHASE</td>
<td></td>
<td></td>
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<tr>
<td>LEAVE EARTH SOI</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Ballistic time guess [days]</td>
<td>15</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>Optimized ballistic parameter [days]</td>
<td>5.3653</td>
<td>14.1788</td>
<td>14.9845</td>
</tr>
<tr>
<td>TOF [days]</td>
<td>456</td>
<td>456</td>
<td>450</td>
</tr>
<tr>
<td>No-Thrust [days]</td>
<td>0</td>
<td>27 (2 intervals)</td>
<td>30 (2 intervals)</td>
</tr>
<tr>
<td>Max firing time [days]</td>
<td>456</td>
<td>145</td>
<td>143</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>600</td>
<td>566</td>
<td>563</td>
</tr>
<tr>
<td>Mars distance [km]</td>
<td>3.1092e+05</td>
<td>9.4833e+004</td>
<td>1.2273e+05</td>
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<td>MARS PHASE</td>
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</tr>
<tr>
<td>RENDEZ-VOUS WITH MARS</td>
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<tr>
<td>TOF [days]</td>
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<td>266</td>
<td>255</td>
</tr>
<tr>
<td>No-Thrust [days]</td>
<td>59</td>
<td>40</td>
<td>26</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>215</td>
<td>204</td>
<td>207</td>
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<tr>
<td>OVERALL RESULTS</td>
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</tr>
<tr>
<td>ARRIVAL DATE</td>
<td>22/10/2023</td>
<td>21/10/2025</td>
<td>04/12/2027</td>
</tr>
<tr>
<td>TOF [years]</td>
<td>3.4</td>
<td>3.3</td>
<td>3.2</td>
</tr>
<tr>
<td>Shooting time [years]</td>
<td>3.2</td>
<td>3.1</td>
<td>2.9</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>1724</td>
<td>1677</td>
<td>1679</td>
</tr>
<tr>
<td>Minimum time between two firings [min]</td>
<td>2.6</td>
<td>2.8</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 5.6 Missions results for different launch dates.
## Missions Comparison

<table>
<thead>
<tr>
<th>DEPARTURE DATE</th>
<th>22/09/2026</th>
<th>10/11/2028</th>
<th>29/12/2030</th>
</tr>
</thead>
</table>

### TRANSFER PHASE

<table>
<thead>
<tr>
<th>LEAVE EARTH SOI</th>
<th>17/01/2028</th>
<th>11/03/2030</th>
<th>28/04/2032</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballistic time guess [days]</td>
<td>12</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Optimized ballistic parameter [days]</td>
<td>11.8433</td>
<td>6.6746</td>
<td>13.0835</td>
</tr>
<tr>
<td>TOF [days]</td>
<td>454</td>
<td>454</td>
<td>456</td>
</tr>
<tr>
<td>No-Thrust [days]</td>
<td>19 (2 intervals)</td>
<td>6.6 (1 interval)</td>
<td>26 (2 intervals)</td>
</tr>
<tr>
<td>Max firing time [days]</td>
<td>150</td>
<td>300</td>
<td>146</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>574</td>
<td>591</td>
<td>569</td>
</tr>
<tr>
<td>Mars distance [km]</td>
<td>1.9199e+005</td>
<td>2.6235e+005</td>
<td>2.9740e+005</td>
</tr>
</tbody>
</table>

### MARS PHASE

<table>
<thead>
<tr>
<th>RENDEZ-VOUS WITH MARS</th>
<th>15/04/2029</th>
<th>8/06/2031</th>
<th>28/07/2033</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOF [days]</td>
<td>348</td>
<td>413</td>
<td>306</td>
</tr>
<tr>
<td>No-Thrust [days]</td>
<td>96</td>
<td>155</td>
<td>52</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>223</td>
<td>226</td>
<td>226</td>
</tr>
</tbody>
</table>

### OVERALL RESULTS

<table>
<thead>
<tr>
<th>ARRIVAL DATE</th>
<th>29/03/2030</th>
<th>26/07/2032</th>
<th>30/05/2034</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOF [years]</td>
<td>3.5</td>
<td>3.7</td>
<td>3.4</td>
</tr>
<tr>
<td>Shooting time [years]</td>
<td>3.16</td>
<td>3.22</td>
<td>3.16</td>
</tr>
<tr>
<td>Propellant burned [kg]</td>
<td>1705</td>
<td>1725</td>
<td>1704</td>
</tr>
<tr>
<td>Minimum time between two firings [min]</td>
<td>3</td>
<td>3</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 5.7 Missions results for different launch dates.
### Results

<table>
<thead>
<tr>
<th>DEPARTURE DATE</th>
<th>22/02/2033</th>
<th>06/05/2035</th>
<th>31/05/2037</th>
</tr>
</thead>
</table>

| TRANSFER PHASE | |
|----------------|---------------|---------------|---------------|
| LEAVE EARTH SOI | Ballistic time guess [days] | 20/06/2034 | 03/09/2036 | 28/09/2038 |
| | Optimized ballistic parameter [days] | 9.6352 | 11.2390 | 8.1572 |
| | TOF [days] | 456 | 453 | 456 |
| | No-Thrust [days] | 17 (2 intervals) | 22 (2 intervals) | 16 (2 intervals) |
| | Max firing time [days] | 148 | 145 | 148 |
| | Propellant burned [kg] | 578 | 568 | 581 |
| | Mars distance [km] | 2.3059e+005 | 2.7520e+005 | 4.8866e+005 |

| MARS PHASE | RENDEZ-VOUS WITH MARS | |
|----------------|---------------------------|---------------|---------------|
| TOF [days] | 20/09/2035 | 30/11/2037 | 29/12/2039 |
| | 243 | 263 | 266 |
| No-Thrust [days] | 8.4 | 33 | 24.9 |
| Propellant burned [kg] | 209 | 207 | 214 |

| OVERALL RESULTS | |
|----------------|---------------|---------------|---------------|
| ARRIVAL DATE | 20/05/2036 | 21/08/2038 | 19/09/2040 |
| TOF [years] | 3.2 | 3.3 | 3.3 |
| Shooting time [years] | 3.1 | 3.1 | 3.1 |
| Propellant burned [kg] | 1695 | 1685 | 1702 |
| Minimum time between two firings [min] | 2 | 2.9 | 3.3 |

Table 5.8 Missions results for different launch dates.

The approach selected to obtain these results is to modify only one parameter which is the ballistic parameter of the simulation. This choice aims to verify the possibility of a rendezvous with Mars at every close approach while maintaining the TOF nearly fixed. The TOF parameter, which is optimized by the problem, has a range which spans between 1.2 and 1.25 years. This value has been empirically selected and tuned during the implementation of the simulation. Different values can be obtained by modifying the TOF initial guess and variation range but since this optimization problem is highly sensible to the initial guesses it will required a lot of tuning.
For some missions like the ones leaving on the 26th May 2020 and on the 10th Nov 2028, it can be observed that there are no ballistic phases or just one during the interplanetary transfer. This is due to the fact that the integrations step of the optimization problem is higher than the time step selected. In this case several initial guesses were considered but in none of these cases a rendezvous with Mars could be obtained hence the only case obtained with a possible rendezvous is presented anyway.
CHAPTER 6
REQUIREMENTS DEVELOPMENT

From the results obtained through the analysis a preliminary mass budget can be defined. Furthermore, some high level requirements can be defined in order to impose some boundaries to the design of the mission and understand the mission target that can be reached with the VOTV module.

6.1 Preliminary Mass Budget

To define the preliminary mass budget, three missions have been selected according to the propellant burned. The overall results of the chosen missions are grouped in Table 6.1. The best and worst cases have been identified while the mission considered to define the mass budget and the requirements is a mission with an intermediate output which is the one departing on the 31st May 2037.

<table>
<thead>
<tr>
<th>Departure date [DD/MM/YY]</th>
<th>01/07/2022</th>
<th>31/05/2037</th>
<th>22/02/2033</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall mission TOF [yrs]</td>
<td>3.3</td>
<td>3.3</td>
<td>3.2</td>
</tr>
<tr>
<td>Overall propellant burned [kg]</td>
<td>1677</td>
<td>1702</td>
<td>1712</td>
</tr>
<tr>
<td>Overall shooting time [yrs]</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Minimum time between two firings [min]</td>
<td>2.8</td>
<td>3.3</td>
<td>2.7</td>
</tr>
<tr>
<td>Maximum firing time [days]</td>
<td>145</td>
<td>148</td>
<td>148</td>
</tr>
</tbody>
</table>
Table 6.1 Reference Mission results.

To exploit the modularity of the system it is necessary to start from the reference mass budget for a GEO mission. The overall mass budget is presented in Table 6.1, whereas in Table 6.3 the mass allocated for the several subsystems is defined.

<table>
<thead>
<tr>
<th>% of initial mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry mass</td>
</tr>
<tr>
<td>Propellant</td>
</tr>
<tr>
<td>Delivered mass</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System</th>
<th>% of initial mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>3.98</td>
</tr>
<tr>
<td>Thermal</td>
<td>0.73</td>
</tr>
<tr>
<td>ACS</td>
<td>4.2</td>
</tr>
<tr>
<td>Power</td>
<td>5.19</td>
</tr>
<tr>
<td>Cabling</td>
<td>1.25</td>
</tr>
<tr>
<td>Propulsion</td>
<td>4.31</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.25</td>
</tr>
<tr>
<td>CDS</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 6.2 Reference mass budget for a GEO mission.

<table>
<thead>
<tr>
<th>System</th>
<th>% of initial mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tot dry mass</td>
<td>20.16</td>
</tr>
<tr>
<td>Tot dry mass + margin (20%)</td>
<td>24.19</td>
</tr>
</tbody>
</table>

Table 6.3 Dry mass for a GEO mission.

From this reference, a preliminary mass budget can be produced taking into account the augmented propellant mass and hence an augmented dry mass due to the tanks and structure. As a first approximation, an increment of about 100 [kg] has been applied to the dry mass.
It can be seen that a sensible reduction of the P/L deliverable is needed. The market of the mission is completely different from a GEO mission and the P/L itself could be represented by a small observational probe or some instruments mounted on VOTV module.

Regarding the propellant mass required a preliminary 10% margin has been applied. This margin considers the fact that more convenient missions in terms of propellant required can be considered. For the computation of the mass budget a mission with an intermediate propellant consumption is considered but still a 10% contingency on the propellant is considered to cover flow control tolerances, startup, and other miscellaneous affects. [23]

### 6.2 High Level Requirements

The analysis carried out in this study can be translated into high level requirements to define the shape of a hypothetical mission to Mars. This document contains VOTV system requirements and constraints concerning the system mission, performance, guidance navigation and control (GNC), design, qualification and use. Furthermore, additional requirements are added according to the results obtained from the analysis.

The requirements that can be elaborated from this analysis affect specific areas of VOTV like the propulsion system specifics in terms of firing time, and the overall operational lifetime of the system.

Another aspect that is highly influenced by the outcome of the analysis is the amount of P/L mass that can be delivered to Mars and the necessity to considered the augmented mass of several subsystems.

<table>
<thead>
<tr>
<th>REQUIREMENTS CATEGORIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
</tr>
<tr>
<td>M</td>
</tr>
</tbody>
</table>

Table 6.4 Reference Mission mass budget.
The purpose of this analysis is to produce generalized high level requirements. To define these requirements a reference P/L mission durability of 5 terrestrial years has been considered.

<table>
<thead>
<tr>
<th>REQ.ID</th>
<th>REQUIREMENT</th>
<th>JUSTIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOTV-SYS-F-R001</td>
<td>The VOTV module shall be designed for an overall reference mission duration of less than 4 years</td>
<td>The average time for a mission obtained from this simulation is smaller than 4 years, this value is defined to take into account eventual delays of the transfer mission.</td>
</tr>
<tr>
<td>VOTV-SYS-F-R002</td>
<td>The VOTV module shall be designed for an overall operational duration from launch to de-commissioning of 9 terrestrial years</td>
<td>This value considers both the transfer mission from launch to arrival at Mars and the operational mission time of the P/L.</td>
</tr>
<tr>
<td>VOTV-SYS-F-R003</td>
<td>The VOTV module shall be able to accommodate the required propellant while exploiting the modularity of its subsystems</td>
<td>Possibility of adding or removing tanks from the structure according to the mission to be performed</td>
</tr>
<tr>
<td>VOTV-SYS-F-R004</td>
<td>The VOTV module shall be able to reach the defined LMO altitude at Mars</td>
<td>Requirement to satisfy if the task is to inject a separated P/L around Mars</td>
</tr>
<tr>
<td>VOTV-SYS-F-R005</td>
<td>The VOTV module shall be able to close the defined LMO around Mars</td>
<td>Requirement to satisfy is the task is to give support to the P/L during its operative mission</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------------------------------------------------</td>
<td>-----------------------------------------------------------------</td>
</tr>
<tr>
<td>VOTV-VPS-F-R006</td>
<td>The on-board propulsion system shall satisfy the durability required by the overall mission of 9 terrestrial years</td>
<td>The current HET selected has been tested for a far smaller period than 9 years</td>
</tr>
<tr>
<td>VOTV-SYS-F-R007</td>
<td>At the beginning of the reference mission, VOTV wet mass, including its P/L cannot be greater than $m_0$, including contingencies</td>
<td>$m_0$ is the initial launchable mass of the system for a mission departing from a 450 [km] equatorial circular orbit</td>
</tr>
<tr>
<td>VOTV-SYS-F-R008</td>
<td>The solar panels shall be able to guarantee a power production of 11.3 [kW] BOL at Earth departure orbit for the EPS</td>
<td>This BOL value is defined in order to consider the power degradation due to the radiations absorbed in the continuous passages among the Van Allen belts</td>
</tr>
<tr>
<td>VOTV-SYS-F-R009</td>
<td>The solar panels shall be able to guarantee a power production of 3.87 [kW] EOL at Mars for the EPS</td>
<td>Power reduction due to distance from Sun</td>
</tr>
<tr>
<td>VOTV-SYS-F-R010</td>
<td>The VOTV shall be compatible with the whole VEGA P/L adapter family and fairing</td>
<td>The augmented dry mass shall not interfere with the adapter and fairing of the VEGA family launchers</td>
</tr>
<tr>
<td>VOTV-SYS-F-R011</td>
<td>The VOTV arrival orbit at Mars shall be greater than Mars exosphere altitude of 200 [km]</td>
<td>To ensure atmospheric drag won’t interfere with the S/C causing orbit perturbations, interferences and unwanted thermal loads</td>
</tr>
<tr>
<td>VOTV-SYS-F-R012</td>
<td>The VOTV shall be left on a departure circular orbit with altitude of 450 [km] and inclination of 6.2 [deg]</td>
<td>This departure orbit is required in order to bring the specified $m_0$ in orbit</td>
</tr>
<tr>
<td>VOTV-SYS-F-R013</td>
<td>The system adapter plus VOTV fully equipped (equipment, xenon tanks and P/L) shall be designed in order to have the 1st lateral frequency higher than 13Hz and the longitudinal frequency higher than 50Hz.</td>
<td>This requirements is defined through the loads constraints imposed by the launcher system</td>
</tr>
<tr>
<td>VOTV-SYS-F-R014</td>
<td>The first lateral frequency of the payload shall be $&gt;$ 15 Hz and the longitudinal frequency higher than 47Hz. Acceptance of lower frequencies shall be evaluated via mission coupled analysis.</td>
<td>This requirements is defined through the loads constraints imposed by the launcher system</td>
</tr>
<tr>
<td>Requirement ID</td>
<td>Description</td>
<td>Details</td>
</tr>
<tr>
<td>----------------</td>
<td>-------------</td>
<td>---------</td>
</tr>
<tr>
<td>VOTV-SYS-F-R015</td>
<td>In nominal mode, both the VOTV Module and the P/L shall be designed taking into account the following total radiation dose (TBC): 150 [krad] + (TBD) [krad] max (referring to a silicon target shielded by 4mm Al)</td>
<td>This requirement is defined by analysis performed with SPENVIS system tools for a transfer from LEO to GEO of roughly 300 days, a further addition has to be considered for the remaining time spent in the Earth SOI.</td>
</tr>
<tr>
<td>VOTV-SYS-F-R016</td>
<td>In contingency mode both the VOTV Module and the P/L shall be designed taking into account the following total radiation dose (TBC): 240 [krad] + (TBD) [krad] max (referring to a silicon target shielded by 4mm Al)</td>
<td>This requirement is defined by analysis performed with SPENVIS system tools for a transfer from LEO to GEO of roughly 300 days, a further addition has to be considered for the remaining time spent in the Earth SOI.</td>
</tr>
<tr>
<td>VOTV-SYS-F-R017</td>
<td>After the fairing jettison the VOTV shall be exposed at thermal flux less than 1135 [W]</td>
<td>This requirement is defined through the loads constraints imposed by the launcher system.</td>
</tr>
<tr>
<td>VOTV-SYS-M-R001</td>
<td>The reference mission shall be a transfer to a circular LMO of altitude 500 [km]</td>
<td>The arrival orbit has to be defined according to the P/L mission requested. This value has been assumed as a reference since it is close enough to be exploited as an operational orbit.</td>
</tr>
<tr>
<td>VOTV-SYS-M-R002</td>
<td>VOTV shall be able to perform the following tasks: • Release P/Ls in orbits around Mars • Fly-by and P/L release • Orbit insertion around Mars and assistance during P/L mission</td>
<td>The tasks are to be tuned according to the kind of mission required.</td>
</tr>
<tr>
<td>VOTV-SYS-M-R003</td>
<td>VOTV shall be designed in order to carry a P/L wet mass of maximum 300 [kg] in the reference mission</td>
<td>This comes from the mass budget, it might be relaxed for more convenient missions.</td>
</tr>
<tr>
<td>VOTV-PL-M-R004</td>
<td>The P/L mission shall last a maximum of 5 years</td>
<td>To ensure the constraint of the overall 9 years operativity of the VOTV.</td>
</tr>
<tr>
<td>VOTV-SYS-M-R005</td>
<td>The launch date shall be greater than 2020 and selected in order to reach Mars during one of its close approaches</td>
<td>The first close approach is on October 2020 but it is too close, the others close approaches could provide a shorter TOF</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>VOTV-SYS-O-R001</td>
<td>VOTV implements the following operative modes: • On-ground • Idle • Launch • Commissioning • Orbit Raising Sun mode • Orbit Raising Eclipse mode • Safe Mode • Interplanetary thrusting mode • Interplanetary no-thrusting mode • Orbit Closure • P/L release or P/L support mode • De-commissioning mode</td>
<td>This are the operative modes regarding the transfer mission, the operative modes regarding the P/L mission have to be added once the kind of mission to be carried out will be defined</td>
</tr>
<tr>
<td>VOTV-VPS-F-R001</td>
<td>The HET longest firing shall be 148 days</td>
<td>The sizing event occurs during the interplanetary phase</td>
</tr>
<tr>
<td>VOTV-VPS-F-R002</td>
<td>The firing time of the thrusters shall not exceed their maximum qualification burning time</td>
<td>The qualification burning time has to be coherent to the mission lifetime otherwise a new kind of thruster is required</td>
</tr>
<tr>
<td>VOTV-VPS-F-R003</td>
<td>The Minimum time between two consecutive HET firings shall be 3 minutes</td>
<td>This event occurs during Earth escape and is due to the eclipse conditions. Reference REQ impose a minimum time of 10 minutes</td>
</tr>
<tr>
<td>VOTV-VPS-F-R004</td>
<td>The thrusters shall guarantee the required throttability from 4.5 to 1.5 [kW]</td>
<td>Satisfy BOL and EOL power limitations</td>
</tr>
<tr>
<td>Reference</td>
<td>Text</td>
<td>Note</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>VOTV-VPS-F-R005</td>
<td>The HET shall provide a specific impulse of 2000 [sec] during the whole mission</td>
<td>The case of variable specific impulse has not been taken into consideration</td>
</tr>
<tr>
<td>VOTV-VPS-F-R006</td>
<td>The HET shall provide a thrust of 0.4833 [N] BOL at Earth departure orbit</td>
<td>This value is the design value used for the GEO mission of the VOTV</td>
</tr>
<tr>
<td>VOTV-VPS-F-R007</td>
<td>The HET shall provide a thrust of 0.2078 [N] EOL at Mars arrival orbit</td>
<td>This value is linked with the power degradation that occurs during the mission</td>
</tr>
<tr>
<td>VOTV-SYS-E-R001</td>
<td>The system shall survive to the near Earth environment: radiations, solar particles, plasma, cosmic rays, trapped particles and atomic oxygen</td>
<td>All the sub-systems have to be shielded while passing through critical environments</td>
</tr>
<tr>
<td>VOTV-SYS-E-R002</td>
<td>The system shall survive to the deep space and to the environment around Mars: radiations, solar particles, cosmic rays and plasma</td>
<td>The sub-systems have to be shielded in a deep space environment and close to Mars</td>
</tr>
<tr>
<td>VOTV-SYS-V-R001</td>
<td>The eclipse conditions of the LMO shall be verified</td>
<td>In order to determine the EOL condition for the sizing of the batteries</td>
</tr>
<tr>
<td>VOTV-VPS-V-R002</td>
<td>The HET shall undergo ground tests to assess if their operational life is compatible with the mission duration</td>
<td>A prolonged firing period shall be tested</td>
</tr>
</tbody>
</table>

Table 6.6 Reference Mission requirements.
The requirements presented in Table 6.6 are a starting point to deal with the outcomes of this analysis. From these requirements, a clear indication towards specific needs can be derived, first of all the necessity to have an overall system which can last several years in an environment far from the nominal condition at Earth. The requirements devoted to the survivability inside the Earth environment have been derived from previous analyses carried out for transfers from LEO to GEO orbits, these requirements can be considered a good approximation for the time spent to reach the GEO altitude but need to be integrated to take into account the radiation dose absorbed for the remaining time required to escape from Earth. Then clear indications are given to the EPS in order to perform the kind of maneuvers and long-time firings requested by the electric propulsion. From the results obtained regarding the firing times some requirements on the batteries and power production sub-system can be defined but the purpose of this work is to focus on high level requirements on the mission and propulsion systems. A more accurate analysis will be needed once the mission architecture will reach a more detailed level.
This study aimed to assess the feasibility of a mission to Mars using the VSS. While mostly all the mission to the Red planet have been injected into the escape orbit by the launcher itself, the use of a small launcher implies a totally different mission strategy. The S/C departs from an equatorial circular LEO and has the task to reach the escape velocity on its own. To understand the time and the propellant required for a mission of this kind, a simulation has been implemented.

The simulation is run at specific dates in order to perform the rendezvous with Mars at every close approach of the planet with Earth which occurs every Martian synodic period. Then among the best and worst cases in terms of propellant burned, a mission with an intermediate value is selected and a preliminary mass budget and high level requirements are produced.

From the results obtained, a mission to Mars launched by the VSS seems feasible although several constraints and problems emerged from the analysis.

The P/L mass that can be delivered is around the reference value of 300 [kg] which is significantly smaller than the value obtained for a GEO mission. This result is expected since it is in line with the P/L masses delivered to Mars by similar missions in terms of the dry mass of the system.

The margined propellant required is around 1874 [kg], and nearly half of it is needed to reach the escape velocity at Earth. This increment of propellant affects several aspects of the VOTV module either additional tanks or the oversizing of the currant ones shall be considered.

The other significant outcome of the analysis is the TOF which reaches the value of 4 years for the transfer from LEO to LMO. The long transfer times are a peculiarity of the electric
propulsion, in this case this condition will impact all the sub-systems of the module since they will have to be designed and qualified for an extended lifetime. This analysis is focused on the characteristics of the EPS and significant data have been obtained regarding the firing times requested by the mission. The longest firing period is similar to the one defined for a GEO mission which is around 150 days, the minimum time between two firings is of 3 minutes and is much smaller than the 10 minutes obtained for a GEO mission. It is clear that the current thrusters selected for missions inside the Earth SOI have to be tested carefully in order to be used for a mission to Mars. Firing time and lifetime are important constraints that have to be verified.

A final consideration can be given regarding the mission market reachable by a mission of this kind. It is clear that due to the long transfer times and relatively small payload deliverable a scientific mission is more likely to be carried out. The final objective of this mission can affect the final phase at Mars, several options can be considered, from a fly-by a release of a small autonomous probe inside Mars atmosphere or the injection of the same in a LMO. The more convenient solution is to redesign the VOTV from a simple bus vehicle to a support vehicle that will remain operational with a reduced power available during the payload mission around the target planet.

**Future Developments**

This study can be followed by significant improvements in different areas. First of all, the use of a more accurate trajectory simulator can be considered, exploiting a shape-based trajectory method which can optimize the number of revolutions and the sinusoid escape curve in the escape phase, in order to satisfy the required constraint on the propellant and TOF. Furthermore, a multibody approach can be considered to link the several phases of the analysis. A different optimization problem can be implemented in order to increase the integration steps around the ballistic intervals of the interplanetary transfer.

Regarding the VOTV module, a more detailed model can be produced by performing structural analysis which take into consideration the augmented dry mass of the systems and, at the same time, try to maintain the modularity constraint for the oversized or additional propellant tanks. A dedicated study on the layout of the module has to be carried out to check the compatibility with the fairing dimensions.

For what concerns the EPS a verification has to be produced to assess the capability of the current thruster to satisfy the constraints imposed and eventual alternative systems can be
selected which have better performances in terms of lifetime of the systems and propulsion parameters.

Regarding the environmental model of the Earth, a more complete study is needed to assess the expected radiation dose. In particular it is required a radiation model to cover the escape phase after the GEO altitude is reached since the actual model available has been developed for a mission from LEO to GEO.

Regarding the environmental model of Mars, the eclipse events can be simulated and taken into account for the design of the batteries and the power production sub-system during the P/L operational phase.
REFERENCES


The simulation presented in this work has been carried out in the MATLAB® environment. Here are presented all the functions implemented and utilized in the analysis.

### A.1 LT_Model.m

```matlab
%% Low-Thrust Model for a low cost access to Mars
%% using electric propulsion
%% Main Reference: "Utilization of Ion Propulsion for Mars Orbiter"
%% Richard R. Rieber

%--------------------------
% Global Variables
%--------------------------
clear all
close all

% propulsion system variables
global Isp F

% gravitational variables
global M_E M_S M_M g0 G AU J2
global SOI_E SOI_M R_earth R_mars a_earth a_mars R_sun
global mu_S mu_E mu_M

% spacecraft variable
global M0_SC eta_P

% departure and arrival orbit variables
global i_earth_eq i_orb i_mars
global h_LEO r_LEO YEAR_S MONTH_S DAY_S

Isp = 2000; % Specific Impulse [s]
F = 0.4833; % Thrusters force[N]
eta_P = 0.43; % Power degradation @ Mars
```
Appendix A
MATLAB® Code

\begin{verbatim}
M_E = 5.9724e24; \% Earth mass [kg]
M_S = 1.9885e30; \% Sun mass [kg]
M_M = 6.4171e23; \% Mars mass [kg]
g0 = 9.81; \% Gravitationnal acceleration [m/s^2]
G = 6.67e-20; \% Gravitational constant [km^3/kg/s^2]
J2 = 1082.64*10^-6; \% J2 perturbation parameter
mu_S = G * M_S; \% Sun standard gravitational parameter [km^3/s^2]
u_E = G * M_E; \% Earth standard gravitational parameter [km^3/s^2]
u_M = G * M_M; \% Mars standard gravitational parameter [km^3/s^2]
R_sun = 695700; \% Sun radius [km]
R_earth = 6371; \% Earth radius [km]
R_mars = 3389.5; \% Mars radius [km]
AU = 149597870; \% Astronomical unit [km]
i_earth_eq = deg2rad(23.44); \% Inclination of equator [rad]
i_orb = deg2rad(6.2); \% Inclination of departure orbit [rad]
i_mars = deg2rad(1.85); \% Inclination of Mars orbit [rad]
a_e = 1*AU; \% Earth semimajor axis [km]
a_m = 1.52355104*AU; \% Mars semimajor axis [km]
SOI_E = (M_E/M_S)^(2/5) * a_e; \% Earth SOI [km]
SOI_M = (M_M/M_S)^(2/5) * a_m; \% Mars SOI [km]

\% Spacecraft parameters (VOTV module)
M0_SC = xxx; \% Spacecraft mass [kg]
h_LEO = 450; \% Altitude LEO [kg]
r_LEO = R_earth + h_LEO; \% radius LEO [km]
YEAR_S = 2019; \% Departure Year
MONTH_S = 1; \% Departure Month
DAY_S = 1; \% Departure Day
ang_b1 = 15; \% Thrust window guess for ballistic phase [days]

\% Two-body problem integration with eclipses and J2 perturbations
x0 = [M0_SC R0_SC V0_SC]; \% initial condition vector
Tspan = [0,3*3600*24*365]; \% time vector
opts = odeset('RelTol',10^-6,'AbsTol',10^-6*ones(1,7),...
     'Refine',4,'Events',@ESOIevent);
[t,x] = ode113(@EarthPhase,Tspan,x0,opts);

\% Build time vector for the computations and plots

t1 = t;
tel = t(end);

\% Build integrations results vectors for the computations and plots
M_1P = x(:,1);
R1_SC = x(:,2:4);
V1_SC = x(:,5:7);

\% Build vectors to plot results
Fprof1 = []; \% Thrust profile vector
R_off = []; \% Eclipse condition vector

\% Build thrust profile by verifying mass variation at each step
for j = 2 : length(t1)
    if M_1P(j) == M_1P(j-1) == M_1P(j-2)
        Fprof1(j-l) = 0;
        R_off(j-l,1) = R1_SC(j-l,1);
        R_off(j-l,2) = R1_SC(j-l,2);
    end
\end{verbatim}
R_off(j-1,3) = R1_SC(j-1,3);
else
    Fprof1(j-1,1) = F;
    R_off(j-1,1) = NaN;
    R_off(j-1,2) = NaN;
    R_off(j-1,3) = NaN;
end
% Vectors are one value shorter
% Need to be the same size for the final plots
% This addition won’t affect results
Fprof1 = [Fprof1,Fprof1(end)];
R_off = [R_off,R_off(end,:)];

% Computation of time between firings
HET1 = [];
% Locate position when thrust goes off and turns on
for jj = 2 : length(t1)
    if Fprof1(jj-1,1)+Fprof1(jj,1) == Fprof1(jj-1,1) && Fprof1(jj-1,1) ~= 0
        HET1(jj-1,1) = t1(jj-1,1);
    elseif Fprof1(jj-1,1)+Fprof1(jj,1) == Fprof1(jj,1) && Fprof1(jj,1) ~= 0
        HET1(jj-1,1) = t1(jj,1);
    else
        HET1(jj-1,1) = 0;
    end
end
% remove zero values
HET1_si = HET1(HET1~=0);
% if number of elements is odd then the zero value need to be added
% because thrust vector starts with thrust switched off
if mod(length(HET1_si),2) ~= 0
    HET1_si = [0,HET1(HET1~=0)];
end
% compute length of each thrust-off interval
tts1 = zeros(length(HET1_si)/2,1);
for kk = 1 : length(tts1)
    tts1(kk,1) = (HET1_si(kk*2,1) - HET1_si(kk*2-1,1));
end
% compute total time when thrusters are turned on
time_shoot1 = te1 - sum(tts1);

% Compute orbit inclination
I_vect1 = [];
% inclination vector
Ang_vect1 = [];
% true anomaly vector
for k = 1:length(t1)
    [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(R1_SC(k,1),R1_SC(k,2),R1_SC(k,3),V1_SC(k,1),V1_SC(k,2),V1_SC(k,3),mu_E);
    I_vect1(k,1) = rad2deg(i);
    Ang_vect1(k,1) = rad2deg(teta);
end

disp ('Time required to reach Earth SOI [days]')
tel = te1/(3600*24);
TT1 = tel/(3600*24);
disp ('Propellant burned to reach Earth SOI [kg]')
(M_1P(1)-M_1P(end))
disp ('Delta V (Tsiolkowski) required to reach Earth SOI [km/s]')
norm(V1_SC(1,:))-norm(V1_SC(end,:));
V_Tsk = Isp*g0*log(M_1P(1)/M_1P(end))*10^-3 % Tsiolkowsky formula
DDV1 = V_Tsk;
disp('No-Thrust time [days]')
(te1-time_shoot1)/(3600*24)

disp('Minimum time between two firings [min]')
min(tts1)/60

disp('Max eclipse duration [min]')
max(tts1)/60

%--------------------------
% Plots
%--------------------------

figure()
plot3(R1_SC(:,1),R1_SC(:,2),R1_SC(:,3))
hold on
plot3(R_off(:,1),R_off(:,2),R_off(:,3),'r','linewidth',4)
hold on

%Plot Earth SOI
[x,y,z]=sphere;
x=x.*SOI_E;
y=y.*SOI_E;
z=z.*SOI_E;
h = surf(x,y,z);
set(h, 'FaceColor','k','FaceAlpha',0.1,'EdgeColor','none')
hold on

%Plot Earth
load('topo.mat','topo','topomap1');
[x,y,z]=sphere;
x=x.*R_earth;
y=y.*R_earth;
z=z.*R_earth;
surf(x,y,z);
props.AmbientStrength = 0.1;
props.DiffuseStrength = 1;
props.SpecularColorReflectance = .5;
props.SpecularExponent = 20;
props.SpecularStrength = 1;
props.FaceColor= 'texture';
props.FaceLighting = 'phong';
props.Cdata = topo;
surface(x,y,z,props);
light('position',[-1 0 1]);
light('position',[-1.5 0.5 -0.5], 'color', [.6 .2 .2]);
view(3)
title('S/C position wrt Earth')
legend('Spacecraft','Eclipse Segments','Earth SOI')
xlabel('X(ECI) [km]')
ylabel('Y(ECI) [km]')
zlabel('Z(ECI) [km]')
axis equal
grid on

figure()

h_E_vect = [];
h_SOI_E = ones(length(t1),1)*SOI_E;
h_GEO = ones(length(t1),1)*36000;
for k = 1 : length(t1)
    h_E_vect(k,1) = norm(R1_SC(k,:));
end
plot(t1,h_E_vect,t1,h_SOI_E,t1,h_GEO)
title('Altitude variation of the S/C')
legend('S/C Altitude', 'Earth SOI', 'GEO altitude', 'Location', 'Best')
xlabel('Time [days]')
ylabel('Altitude [km]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on

figure()
plot(t1, M_1P./M0_SC.*100, 'g', te1, M_1P(end)/M0_SC*100, 'og')
title('S/C mass vs time')
legend('Mass variation', 'Mass @ Earth SOI')
xlabel('Time [days]')
ylabel('Mass [%]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on

figure()
subplot(2,1,1)
plot(t1, Fprof1)
title('Thrust profile')
xlabel('Time [days]')
ylabel('Thrust [N]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on
hold on
subplot(2,1,2)
plot(t1, M_1P./M0_SC.*100, 'g', te1, M_1P(end)/M0_SC*100, 'og')
title('Spacecraft mass vs time')
legend('Mass variation', 'Mass @ Mars SOI')
xlabel('Time [days]')
ylabel('Mass [%]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on
hold off

figure()
plot(t1, I_vect1)
title('Orbit inclination wrt ecliptic')
xlabel('Time [days]')
ylabel('Inclination [deg]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on

figure()
plot(t1, Ang_vect1)
title('True anomaly [deg]')
xlabel('Time [days]')
ylabel('True Anomaly [deg]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Transfer phase %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Earth (body=1) position and velocity in SCI Ref. Frame (from ephemeris)
[x,y,z,vx,vy,vz] = ephemeris(fix(te1/(3600*24)),1);
R0_E = [x,y,z];
V0_E = [vx,vy,vz];
% S/C position and velocity from ECI to SCI Reference Frame
R0_SC = R1(i_earth_eq)*R1_SC(end,:)' + R0_E';
V0_SC = R1(i_earth_eq)*V1_SC(end,:)' + V0_E';
R0_SC = R0_SC';
V0_SC = V0_SC';
% Earth (body=2) position and velocity in SCI Ref. Frame (from ephemeris)
[x,y,z,vx,vy,vz] = ephemeris(fix(te2/(3600*24)),2);
R0_M = [x,y,z];
V0_M = [vx,vy,vz];
% Guesses for TOF, thrust angles profile and thrust window
tof = 1.2; % TOF [yrs]
tr1 = 5; % lower time range for balistic phase [days]
tr2 = 15; % upper time range for balistic phase [days]
theta1 = deg2rad(0); % Thrust angle <theta> initial guesses
theta2 = deg2rad(0);
theta3 = deg2rad(0);
theta4 = deg2rad(0);
theta5 = deg2rad(0);
psi1 = deg2rad(0); % Thrust angle <psi> initial guesses
psi2 = deg2rad(0);
psi3 = deg2rad(0);
psi4 = deg2rad(0);
psi5 = deg2rad(0);
% parameters of optimization problem
x0 = [tof, ang_bl,...
    theta1 theta2 theta3 theta4 theta5,...
    psi1 psi2 psi3 psi4 psi5];
% initial conditions vector
y0 = [M_1P(end) R0_SC V0_SC R0_E V0_E R0_M V0_M];
% lower boundaries
lb = [1.2;tr1;... % TOF, thrust window
deg2rad(0);deg2rad(0);deg2rad(0);deg2rad(0);... % theta
deg2rad(-90);deg2rad(-90);deg2rad(-90);...% psi
deg2rad(-90);deg2rad(-90)];
% upper boundaries
ub = [1.25;tr2;... % TOF, thrust window
deg2rad(180);deg2rad(180);deg2rad(180);... % theta
deg2rad(180);deg2rad(180);deg2rad(90);deg2rad(90)]; % psi
opts = optimset('Display','iter',... 'Algorithm','sqp',...
'MaxFunEvals',2000,...
'MaxSQPIter',1000,...
'ScaleProblem','none',...
'LargeScale','on');
tic
[xOTT,fval,exitflag,output] = fmincon(@(x) constraints_fmincon(x,y0),opts);
toc
[c,ceq,Tspan,Y] = constraints_fmincon(xOTT,y0);
% Read optimized parameters
theta1 = xOTT(3); theta2 = xOTT(4); theta3 = xOTT(5);
theta4 = xOTT(6); theta5 = xOTT(7);
psi1 = xOTT(8); psi2 = xOTT(9); psi3 = xOTT(10);
psi4 = xOTT(11); psi5 = xOTT(12);
% integration with optimized parameters
x0 = [M_1P(end) R0_SC V0_SC R0_E V0_E R0_M V0_M];
Tspan = [0,xOTT(1)*3600*24*365];
A_vec = xOTT(2:12);
opts = odeset('RelTol',10^-6,'AbsTol',10^-6*ones(1,19));
[tt,x] = ode113(@(t,x) TransferPhase(t,x,A_vec,Tspan(end)),Tspan,x0,opts);
t2 = tt; te2 = t2(end);

% Build results vectors for the computations and the plots
M_2P = x(:,1);
R2_SC = x(:,2:4);
V2_SC = x(:,5:7);
R_E = x(:,8:10);
V_E = x(:,11:13);
R_M = x(:,14:16);
V_M = x(:,17:19);

% Building vectors to plot results
Fprof2 = [];
eta = [];

% Build thrust profile by verifying mass variation at each step
for j = 2 : length(t2)
    eta_f = (norm(R_E(j-1,:))/norm(R2_SC(j-1,:)))^2;
    eta(j-1,1) = eta_f;
    if M_2P(j)-M_2P(j-1) == 0
        Fprof2(j-1,1) = 0;
    else
        Fprof2(j-1,1) = F*eta_f;
    end
end

% Vectors are one value shorter
% Need to be the same size for the final plots
% This addition won't affect results
Fprof2 = [Fprof2,Fprof2(end)];
eta = [eta,eta(end)];

% Computation of shooting times vector
HET2 = [];
% Locate position when thrust goes off and turns on
for jj = 2 : length(t2)
    if Fprof2(jj-1,1)+Fprof2(jj,1) == Fprof2(jj-1,1) && Fprof2(jj-1,1) ~= 0
        HET2(jj-1,1) = HET2(jj-1,1) - t2(jj-1,1);
    elseif Fprof2(jj-1,1)+Fprof2(jj,1) == Fprof2(jj,1) && Fprof2(jj,1) ~= 0
        HET2(jj-1,1) = t2(jj,1);
    else
        HET2(jj-1,1) = 0;
    end
end
% Add initial and final values to the thrust time vector
% remove zero values
HET2_si = [0;HET2(HET2~=0);t2(end)];
tts2 = zeros(length(HET2_si)/2,1);
% compute length of intervals
for kk = 1 : length(tts2)
    tts2(kk,1) = (HET2_si(kk*2,1)-HET2_si(kk*2-1,1))/(3600*24);
end

% compute total time when thrusters are turned on
HET22_si = HET2(HET2~=0);
tts22 = zeros(length(HET22_si)/2,1);
for kk = 1 : length(tts22)
    tts22(kk,1) = (HET22_si(kk*2,1)-HET22_si(kk*2-1,1))/(3600*24);
end
time_shoot2 = te2-sum(tts22)*3600*24;
% Compute orbit inclination and Apocenter radius  
I_vect2 = [];  % Inclination vector  
R_ap = [];  % Apocenter radius vector  
RAAN_vect = [];  % RAAN vector  
w_plus_theta = [];  % Pericenter anomaly vector  
for k = 1:length(t2)  
    [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(R2_SC(k,1),R2_SC(k,2),
                                             R2_SC(k,3),V2_SC(k,1),V2_SC(k,2),V2_SC(k,3),mu_S);  
    I_vect2(k,1) = rad2deg(i);  
    p = a*(1-norm_e^2);  
    R_ap(k,1) = p/(1-norm_e);  
    RAAN_vect(k,1) = rad2deg(U);  
    w_plus_theta(k,1) = rad2deg(U)+rad2deg(teta);  
end  

% Compute Mars orbital parameters  
I_vect_M = [];  % Inclination vector  
a_vect_M = [];  % Apocenter radius vector  
e_vect_M = [];  % Eccentricity vector  
for k = 1:length(t2)  
    [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(R_M(k,1),R_M(k,2),R_M(k,3),
                                             V_M(k,1),V_M(k,2),V_M(k,3),mu_S);  
    I_vect_M(k,1) = rad2deg(U);  
    a_vect_M(k,1) = a;  
    e_vect_M(k,1) = norm_e;  
end  

%------------------
% Time required to reach Mars SOI from Earth SOI [days]  
te2/(3600*24)  
TT2 = te2/(3600*24);  

% Propellant burned to reach Mars SOI [kg]  
M_1P(end) - M_2P(end)  
M_VECT = [M_1P;M_2P];

% Delta V required to reach Mars SOI [km/s]  
norm(V2_SC(:,1)) - norm(V2_SC(end,:));  
V_Tsk = Isp*g0*log(M_2P(1)/M_2P(end))*10^-3  
DDV2 = V_Tsk;  

% No-Thrust time [days]  
(te2-time_shoot2)/(3600*24)  

% Max Shooting time [days]  
max(tts2)  

% Single duration of ballistic intervals [days]  
if sum(tts22) == 0  
    0  
elseif numel(tts22) == 1  
    tts22(1)  
else  
    tts22(1)  
    tts22(2)  
end  

% Escape Velocity @ Mars SOI  
% C3 @ SOI_M  
V_SC_M = V2_SC(end,:) - V_M(end,:);
R_SC_M = R2_SC(end,:)-R_M(end,:);
C3 = norm(V_SC_M)^2-2*mu_M/norm(R_SC_M);  % C3 specific energy @ rendezvous

if abs(norm(V_SC_M) - abs(V_C30)) < 0.1
    disp('Reached Mars SOI with C3 close to 0')
    C3
else
    disp('Reached Mars SOI with hyperbolic excess of [km^2/s^2]')
    C3
end

figure()
plot3(R2_SC(:,1),R2_SC(:,2),R2_SC(:,3),R_E(:,1),R_E(:,2),R_E(:,3),...
    R_M(:,1),R_M(:,2),R_M(:,3),R2_SC(1,1),R2_SC(1,2),R2_SC(1,3),...
    R2_SC(end,1),R2_SC(end,2),R2_SC(end,3),'.b',...
    R_M(1,1),R_M(1,2),R_M(1,3),'.r',R_M(1,1),R_M(1,2),R_M(1,3),'.or')
title('Position wrt Sun')
legend('S/C Transfer Orbit','Earth','Mars','S/C t_i_n_i_t','...
    S/C t_f_i_n','Mars t_i_n_i_t','Mars t_f_i_n','Location','Best')

figure()
plot(t1,M_1P./M0_SC.*100,'g',t2+te1,M_2P./M0_SC.*100,'-m',...
    t2+te1,M_1P(end)/M0_SC*100,'og',t2+te1,M_2P(end)/M0_SC*100,'om')
title('S/C mass vs time')
legend('Earth Escape phase','Interplanetary transfer phase',...
    'Mass @ Earth SOI','Mass @ Mars SOI',...
    'Location','Best')

for k = 1 : length(t2)
    h_T_vect(k,1) = norm(R2_SC(k,:));
    h_SOI_E1(k,1) = norm(R_E(k,:));
    h_SOI_E2(k,1) = norm(R_E(k,:)+SOI_E);
    h_SOI_M1(k,1) = norm(R_M(k,:));
    h_SOI_M2(k,1) = norm(R_M(k,:)+SOI_M);
    E_pos(k,1) = norm(R_E(k,:)-SOI_E);
    M_pos(k,1) = norm(R_M(k,:)-SOI_M);
end
Appendix A
MATLAB® Code

```matlab
end
plot(t2, h_T_vect, t2, E_pos, 'c', t2, M_pos, 'k'... t2, h_SOI_E1,'g', t2, h_SOI_M1,'r', t2, h_SOI_E2,'g', t2, h_SOI_M2,'r')
title('Altitude variation')
legend('S/C Altitude', 'Earth', 'Mars', 'Earth SOI', ...
'Mars SOI', 'Location', 'Best')
xlabel('Time [days]')
ylabel('Altitude [km]')
x = get(gca, 'XTick');
set(gca, 'XTickLabel', fix(x/(3600*24)))
grid on
figure()
subplot(2,1,1)
plot(t2, Fprof2)
title('Thrust profile')
xlabel('Time [days]')
ylabel('Thrust [N]')
x = get(gca, 'XTick');
set(gca, 'XTickLabel', fix(x/(3600*24)))
grid on
hold on
subplot(2,1,2)
plot(t2, M_2P./M0_SC.*100, '-m', te2, M_2P(end)./M0_SC*100, 'om')
title('S/C mass vs time')
legend('Mass variation', 'Mass @ Mars SOI', 'Location', 'Best')
xlabel('Time [days]')
ylabel('Mass [%]')
x = get(gca, 'XTick');
set(gca, 'XTickLabel', fix(x/(3600*24)))
grid on
hold on
subplot(2,1,1)
i_M = [];
for k = 1: length(t2)
    [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(R_M(k,1),R_M(k,2),...
        R_M(k,3),V_M(k,1),V_M(k,2),V_M(k,3),mu_S);
i_M(k,1) = rad2deg(i);
end
plot(t2, I_vect2, t2, i_M,'-')
title('Orbit inclination wrt ecliptic')
legend('S/C orbit inclination', 'Mars orbit inclination', 'Location', 'Best')
xlabel('Time [days]')
x = get(gca, 'XTick');
set(gca, 'XTickLabel', fix(x/(3600*24)))
grid on
hold on
subplot(2,1,2)
RAAN_M = [];
for k = 1: length(t2)
    [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(R_M(k,1),R_M(k,2),...
        R_M(k,3),V_M(k,1),V_M(k,2),V_M(k,3),mu_S);
    RAAN_M(k,1) = rad2deg(U);
end
plot(t2, RAAN_vect, t2, RAAN_M)
title('RAAN variation')
legend('S/C RAAN', 'Mars RAAN', 'Location', 'Best')
xlabel('RAAN [deg]')
xlabel('Time [days]')
x = get(gca, 'XTick');
set(gca, 'XTickLabel', fix(x/(3600*24)))
```

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grid on

figure()
x = [0 1/4*te2 2/4*te2 3/4*te2 te2];
y = [rad2deg(theta1) rad2deg(theta2) rad2deg(theta3)...
    rad2deg(theta4) rad2deg(theta5)];
cs_theta = spline(x,y);
plot(x,y,'o',t2,ppval(cs_theta,t2),'-')
title('Thrust <theta> angle profile (z-axis)')
xlabel('Time [days]')
ylabel('Theta angle [deg]')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(x/(3600*24)))
grid on

figure()
x = [0 1/4*te2 2/4*te2 3/4*te2 te2];
y = [rad2deg(psi1) rad2deg(psi2) rad2deg(psi3)...
    rad2deg(psi4) rad2deg(psi5)];
cs_psi = spline(x,y);
plot(x,y,'o',t2,ppval(cs_psi,t2),'-')
title('Thrust <psi> angle profile (x-axis)')
xlabel('Time [days]')
ylabel('Psi angle [deg]')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(x/(3600*24)))
grid on

figure()
subplot(2,1,1)
x = [0 1/4*te2 2/4*te2 3/4*te2 te2];
y = [rad2deg(psi1) rad2deg(psi2) rad2deg(psi3)...
    rad2deg(psi4) rad2deg(psi5)];
cs_psi = spline(x,y);
plot(x,y,'o',t2,ppval(cs_psi,t2),'-')
title('Thrust <psi> angle profile (x-axis)')
xlabel('Time [days]')
ylabel('Psi angle [deg]')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(x/(3600*24)))
grid on
hold on
subplot(2,1,2)
plot(t2,w_plus_theta)
title('RAAN + True anomaly')
xlabel('Time [days]')
ylabel('deg')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(x/(3600*24)))
grid on
hold off

%--------------------------
%% Initial conditions for Mars phase
%--------------------------

disp('Initial conditions for Mars phase')
R0_SC = R2_SC(end,:)-R_M(end,:);
norm(R0_SC)
V0_SC = V2_SC(end,:)-V_M(end,:);
R_REL_M = R2_SC-R_M;

%--------------------------

disp('Optimal TOF')
xOTT(1)
xOTT(2)
%---------------------------------------------------------------
% Mars phase
%---------------------------------------------------------------

x0 = [M_2P(end) R0_SC V0_SC]; % initial conditions vector
Tspan = [0,1.5*3600*24*365]; % Time span for the integration
opts = odeset('RelTol',1e-6,'AbsTol',1e-6*ones(1,7),'Events',@MSOIevent);
t3 = t; te3 = t(end);
% Read integration results for the computations and plots
M_3P = x(:,1);
R3_SC = x(:,2:4);
V3_SC = x(:,5:7);

% Building vectors to plot results
Fprof3 = []; % Thrust profile vector
for j = 2 : length(t3)
    if M_3P(j-1)-M_3P(j) == 0
        Fprof3(j-1,1) = 0;
    else
        Fprof3(j-1,1) = F*eta_P;
    end
end
% Vector is one value shorter
% Need to be the same size for the final plots
% This addition won't affect results
Fprof3 = [Fprof3;Fprof3(end)];

% Computation of minimum time between firings
HET3 = [];
% Locate position when thrust goes off and turns on
for jj = 2 : length(t3)
    if Fprof3(jj-1,1)+Fprof3(jj,1) == Fprof3(jj-1,1) && Fprof3(jj-1,1) ~= 0
        HET3(jj-1,1) = t3(jj-1,1);
    elseif Fprof3(jj-1,1)+Fprof3(jj,1) == Fprof3(jj,1) && Fprof3(jj,1) ~= 0
        HET3(jj-1,1) = t3(jj,1);
    else
        HET3(jj-1,1) = 0;
    end
end
% remove zero values from thrust time vector
HET3_si = HET3(HET3~=0);
tts3 = zeros(length(HET3_si)/2,1);
% compute no-thrust intervals
for kk = 1 : length(tts3)
    tts3(kk,1) = (HET3_si(kk*2,1)-HET3_si(kk*2-1,1));
end
% compute total time when thrusters are turned on
time_shoot3 = te3-sum(tts3);

% Compute orbital parameters
E_m = []; % Mechanical Energy
h_a = []; % Apocenter altitude
h_p = []; % Pericenter altitude
Ang_vect3 = [];% True Anomaly
I_vect3 = []; % Inclination
RAAN_vect3 = [];% RAAN
w_vect3 = []; % True anomaly
ecc_vect3 = []; % Eccentricity
for k = 1:length(t3)
    [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(R3_SC(k,1),R3_SC(k,2),...
    R3_SC(k,3),V3_SC(k,1),V3_SC(k,2),V3_SC(k,3),mu_M);
E_mech = norm(V3_SC(k,:))^2/2 - mu_M/norm(R3_SC(k,:));
E_m(k,1) = E_mech;
h_a(k,1) = -mu_M/(2*E_mech)*(1+norm_e)-R_mars;
h_p(k,1) = -mu_M/(2*E_mech)*(1-norm_e)-R_mars;
I_vect3(k,1) = rad2deg(i);
Ang_vect3(k,1) = rad2deg(teta);
RAAN_vect3(k,1) = rad2deg(U);
w_vect3(k,1) = rad2deg(w);
ecc_vect3(k,1) = norm_e;
end

disp ('Time of flight to reach LMO [days]')
te3/(3600*24)
TT3 = te3/(3600*24);
disp ('Propellant burned to reach LMO [kg]')
M_2P(end)- M_3P(end)
M_VECT = [M_1P;M_2P;M_3P];
disp ('Delta V required to reach LMO [km/s]')
V_Tsk = Isp*g0*log(M_3P(1)/M_3P(end))*10^-3
DV3 = V_Tsk;
disp ('No-Thrust time [days]')
(te3-time_shoot3)/(3600*24)
disp ('Minimum time between two firings [min]')
[YY,II] = min(tts3);
if length(tts3) > 0
if min(tts3) < tts3(end)
tts3(II)=NaN;
end
end
min(tts3)/60
%------------------------
% Plots
%------------------------
for k = 1 : length(R_REL_M)
if norm(R_REL_M(k,:)) > SOI_M*2
R_REL_M (k,1) = NaN;
R_REL_M (k,2) = NaN;
R_REL_M (k,3) = NaN;
end
end
figure()
plot3(R_REL_M(:,1),R_REL_M(:,2),R_REL_M(:,3),'r')
hold on
plot3(R3_SC(:,1),R3_SC(:,2),R3_SC(:,3),...
R3_SC(end,1),R3_SC(end,2),R3_SC(end,3),'ob')
title('Position wrt Mars')
hold on
%x=SOI_M; y=SOI_M; z=SOI_M;
h = surf(x,y,z);
set(h, 'FaceColor','k','FaceAlpha',0.1,'EdgeColor','none')
hold on
%Plot Mars
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Appendix A

MATLAB® Code

```matlab
[x,y,z]=sphere;
x=x.*R_mars;
y=y.*R_mars;
z=z.*R_mars;
surf(x,y,z);
props.FaceColor='r';
props.EdgeColor='none';
props.FaceLighting='phong';
surface(x,y,z,props);
view(3)
legend('Interplanetary approach','Spacecraft','Final Position','Mars SOI','Location','Best')
xlabel('X(MCI) [km]')
ylabel('Y(MCI) [km]')
zlabel('Z(MCI) [km]')
axis equal
grid on
hold off
figure()
hold on
plot(t3,h_M_vect,t3,h_SOI_M,t3,h_M_surf)
hold off
plot(t1,M_1P./M0_SC.*100,'g',t2+te1,M_2P./M0_SC.*100,'o',
t3+te1+te2,M_3P./M0_SC.*100,'r',
'title('S/C mass vs time')
legend('Earth escape phase','Interplanetary transfer phase','Mars capture phase','Mass @ Earth SOI','Mass @ Mars SOI','Mass @ LMO')
xlabel('Time [days]')
ylabel('Mass [%]')
x = get(gca,'XTick');
set(gca,'XTickLabel',fix(x/(3600*24)))
grid on
figure()
subplot(2,1,1)
plot(t3,Fprof3)
title('Thrust profile')
xlabel('Time [days]')
ylabel('Thrust [N]')
x = get(gca,'XTick');
set(gca,'XTickLabel',fix(x/(3600*24)))
grid on
hold on
subplot(2,1,2)
plot(t3,M_3P./M0_SC.*100,'o',
t3+te3,M_3P(end)/M0_SC*100,'o')
title('S/C mass vs time')
legend('Mass variation','Mass @ Mars SOI')
xlabel('Time [days]')
```

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ylabel('Mass [%]')
xt = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))
grid on

[a, ecc, i, U, w, theta] = transfer_c_k_2_sun(R3_SC(end,1), R3_SC(end,2),...
R3_SC(end,3), V3_SC(end,1), V3_SC(end,2), V3_SC(end,3), mu_M);
R = plot_orbit(a, ecc, i, U, w, mu_M);

figure()
plot3(R(:,1), R(:,2), R(:,3), 'b')
hold on

%Plot Mars
[x, y, z] = sphere;
x=x.*R_mars;
y=y.*R_mars;
z=z.*R_mars;
surf(x, y, z);
props.FaceColor = 'r';
props.EdgeColor = 'r';
props.FaceLighting = 'phong';
surface(x, y, z, props);
view(3)
title('Arrival orbit @ Mars')
xlabel('X (MCI) [km]')
ylabel('Y (MCI) [km]')
zlabel('Z (MCI) [km]')
axis equal
grid on
hold off

disp('Pericenter altitude [km]')
a*(1-ecc^2)/(1+ecc)-R_mars
disp('Apocenter altitude [km]')
a*(1-ecc^2)/(1-ecc)-R_mars

figure()
subplot(2,1,1)
plot(t3, h_p)
title('Pericenter height')
xlabel('Time [days]')
ylabel('Pericenter height [km]')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))

figure()
subplot(2,1,2)
plot(t3, ecc_vect3)
title('Eccentricity variation')
xlabel('Time [days]')
ylabel('Eccentricity')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))

figure()
subplot(2,1,1)
plot(t3, I_vect3)
title('Orbit inclination')
xlabel('Time [days]')
ylabel('Inclination [deg]')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))

figure()
subplot(2,1,2)
plot(t3, ecc_vect3)
title('Eccentricity variation')
xlabel('Time [days]')
ylabel('Eccentricity')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(xt/(3600*24)))

grid on
Appendix A
MATLAB® Code

grid on
hold on
subplot(2,1,2)
plot(t3,M_3P,'r',te3,M_3P(end),'or')
plot(t3,ecc_vect3)
title('Eccentricity variation')
ylabel('Eccentricity')
x = get(gca, 'XTick');
set(gca, 'XTick', x, 'XTickLabel', fix(x/(3600*24)))
grid on
figure()
subplot(2,1,1)
plot(t3,h_a)
title('Apocenter height')
ylabel('Pericenter height [km]')
x = get(gca, 'XTick');
set(gca, 'XTick', x, 'XTickLabel', fix(x/(3600*24)))
grid on
hold on
subplot(2,1,2)
plot(t3,M_3P,'r',te3,M_3P(end),'or')
plot(t3,ecc_vect3)
title('Eccentricity variation')
ylabel('Eccentricity')
x = get(gca, 'XTick');
set(gca, 'XTick', x, 'XTickLabel', fix(x/(3600*24)))
grid on
figure()
subplot(2,1,1)
plot(t3,ecc_vect3)
title('Eccentricity variation')
ylabel('Eccentricity')
x = get(gca, 'XTick');
set(gca, 'XTick', x, 'XTickLabel', fix(x/(3600*24)))
grid on
figure()
subplot(2,1,1)
plot(t3,Fprof3)
title('Thrust profile')
ylabel('Thrust [N]')
x = get(gca, 'XTick');
set(gca, 'XTick', x, 'XTickLabel', fix(x/(3600*24)))
grid on
figure()
subplot(2,1,1)
plot(t3,Ang_vect3)
title('True anomaly')
ylabel('True anomaly [deg]')
x = get(gca, 'XTick');
set(gca, 'XTick', x, 'XTickLabel', fix(x/(3600*24)))
grid on
hold on
subplot(2,1,2)
plot(t3,ecc_vect3)
title('Eccentricity variation')
xlabel('Time [days]')
ylabel('Eccentricity')
x = get(gca, 'XTick');
set(gca, 'XTick', xt, 'XTickLabel', fix(x/(3600*24)))
grid on

%%%------------------------------------------
% Display overall results
%------------------------------------------

disp('Overall mission TOF [yrs]')
TT1/365+TT2/365+TT3/365

disp('Overall Delta V [km/s]')
DDV1+DDV2+DDV3

disp('Overall propellant burned [kg]')
M0_SC - M_3P(end)

disp('Overall shooting time [yrs]')
(time_shoot1+time_shoot2+time_shoot3)/(3600*24*365)

disp('Mission starting date [YYYY;MM;DD;HH;MM;SS]')
[YEAR_S MONTH_S DAY_S]

disp('Date when Earth SOI is reached [YYYY;MM;DD;HH;MM;SS]')
b = datestr(te1/86400+datenum(YEAR_S,MONTH_S,DAY_S));
V = datevec(b)

disp('Date when Mars SOI is reached [YYYY;MM;DD;HH;MM;SS]')
b = datestr((te1+te2)/86400+datenum(YEAR_S,MONTH_S,DAY_S));
V = datevec(b)

disp('Date when LMO is reached [YYYY;MM;DD;HH;MM;SS]')
b = datestr((te1+te2+te3)/86400+datenum(YEAR_S,MONTH_S,DAY_S));
V = datevec(b)

A.2 EarthPhase.m

function [dx] = EarthPhase(t,x0)

global Isp F g0 J2 mu_E R_earth AU YEAR_S MONTH_S DAY_S

%M Read initial conditions vector
M_SC = x0(1);
R_SC = x0(2:4);
V_SC = x0(5:7);

%%% ------------------------- %%%
%% Eclipse model from Reference
% 1) Compute r_sc(t) wrt Earth
R_SC_E = R_SC;
% 2) Compute r_sun(t) wrt Earth, using algorithm
b = datestr(t/86400+datenum(YEAR_S,MONTH_S,DAY_S));
V = datevec(b);
jdate = julian(V(2),V(3),V(1));
JD_UT = jdate;
T_UT = (JD_UT-2451545)/36525;
lambda_M_S = 280.4606184+36000.77005361*T_UT;
lambda_M_S = zeroTo360(lambda_M_S,0);
lambda_M_S = deg2rad(lambda_M_S);
M_S = 357.5277233+35999.05034*T_UT;
M_S = zeroTo360(M_S,0);
M_S = deg2rad(M_S);
lambda_e = lambda_M_S + deg2rad(1.914666471)*sin(M_S)+
           deg2rad(0.019994643)*sin(2*M_S);
M_S = 357.5277233+35999.05034*T_UT;
M_S = zeroTo360(M_S,0);
M_S = deg2rad(M_S);
r_S = 1.000140612-0.016708617*cos(M_S)-0.000139589*cos(2*M_S);
eps = 23.439291-0.0130042*T_UT;
eps = deg2rad(eps);
% r_S_vect = r_S * [cos(lambda_e) sin(lambda_e) 0];
R_S = r_S*[cos(lambda_e);cos(eps)*sin(lambda_e);sin(eps)*sin(lambda_e)];

% 3) Verify mutual visibility condition between r_sc and r sun
tau = dot(R_SC_E,(R_SC_E-R_S))/dot((R_S-R_SC_E),(R_S-R_SC_E));
c_tau = sqrt(norm(R_SC_E)^2*(1-tau)+tau*dot(R_S,R_SC_E));

% Acceleration components coupled with J2 effect
x_acc = -(mu_E*R_SC(1,1)/norm(R_SC)^3)*(1-J2*(3/2)*
           R_earth/norm(R_SC)^2*(5*R_SC(3,1)^2/norm(R_SC)^2-1));
y_acc = (R_SC(2,1)/R_SC(1,1))*x_acc;
z_acc = -(mu_E*R_SC(3,1)/norm(R_SC)^3)*(1+J2*(3/2)*
           R_earth/norm(R_SC)^2*(3-5*R_SC(3,1)^2/norm(R_SC)^2));
%A_SC_grav = [x_acc,y_acc,z_acc];

% Thrust strategy
% Continuous thrust, switched off when in eclipse condition
if tau < 0 || tau > 1 % mutual visibility condition
    A_SC_F = F/M_SC*10^-3*V_SC/norm(V_SC);
    mdot = -F/(Isp*g0);
else
    if c_tau > R_earth % mutual visibility condition
        A_SC_F = F/M_SC*10^-3*V_SC/norm(V_SC);
        mdot = -F/(Isp*g0);
    else
        A_SC_F = [0;0;0];
        mdot = 0;
    end
end
%A_SC = A_SC_grav + A_SC_F;
dx = [mdot;V_SC;A_SC];

A.3 TransferPhase.m

function [dx] = TransferPhase (t,x0,A_vec,Tf)
global Isp F g0 mu_S mu_E mu_M
% Read initial conditions vector
M_SC = x0(1);
R_SC = x0(2:4);
V_SC = x0(5:7);
R_E = x0(8:10);
V_E = x0(11:13);
R_M = x0(14:16);
V_M = x0(17:19);

% Read the parameters of the optimization problem
ang_bl = A_vec(1,1); % ballistic parameter guess
% first angle guesses
theta1 = A_vec(1,2); theta2 = A_vec(1,3); theta3 = A_vec(1,4);...
theta4 = A_vec(1,5); theta5 = A_vec(1,6);
% second angle guesses
psi1 = A_vec(1,7); psi2 = A_vec(1,8); psi3 = A_vec(1,9);...
psi4 = A_vec(1,10); psi5 = A_vec(1,11);

% Build acceleration vector in heliocentric ref frame
% acceleration terms for a 4BP
A_SC_grav = -mu_S*R_SC/norm(R_SC)^3 ... %Acceleration due to Sun
+ mu_E*{(R_E-R_SC)/norm(R_E-R_SC)^3 -... 
  R_E/norm(R_E)^3} ... %Perturbation due to Earth
+ mu_M*{(R_M-R_SC)/norm(R_M-R_SC)^3 -... 
  R_M/norm(R_M)^3}; %Perturbation due to Mars

% Power reduction factot wrt the distance
eta_f = (norm(R_E)/norm(R_SC))^2;
F_red = F*eta_f;

% Interpolation of theta angle
x = [0 1/4*Tf 2/4*Tf 3/4*Tf Tf];
y = [theta1 theta2 theta3 theta4 theta5];
theta_ang = spline(x,y,t);

% Interpolation of psi angle
x = [0 1/4*Tf 2/4*Tf 3/4*Tf Tf];
y = [psi1 psi2 psi3 psi4 psi5];
psi_ang = spline(x,y,t);

% Thrust vector in NVC frame
u_f = [cos(psi_ang)*sin(theta_ang);...
  cos(psi_ang)*cos(theta_ang);...
  sin(psi_ang)];

% Rotation matrix from NVC to SCI
InVn = matInVn(R_SC',V_SC');

% Thrust strategy with balistic phases
if t> t_span(1,2)-ang_bl*24*3600/2 && t< t_span(1,2)+ang_bl*24*3600/2
  par = 0;
else if t> t_span(1,3)-ang_bl*24*3600/2 && t< t_span(1,3)+ang_bl*24*3600/2
  par = 0;
else
  par = 1;
end

% Thrust vector in SCI frame
A_SC_F = InVn'*u_f*F_red/M_SC*10^-3.*par;
mdot = -F_red/(Isp*g0)*par;

% Build final acceleration vectors for S/C and planets
A_SC = A_SC_grav + A_SC_F;
A_E = -mu_S*R_E/norm(R_E)^3;
A_M = -mu_S*R_M/norm(R_M)^3;
dx = [mdot;V_SC;A_SC;V_E;A_E;V_M;A_M];

A.4 MarsPhase.m

function [dx] = MarsPhase (t,x0)

global Isp F g0 R_mars mu_M eta_P SOI_M

% Reduced thrust on Mars (power degradation due to distance from Earth)
F_red = F*eta_P;

% Read initial conditions vector
M_SC = x0(1);
R_SC = x0(2:4);
V_SC = x0(5:7);

% Define gravitational acceleration vector
A_SC_grav = -mu_M*R_SC/norm(R_SC)^3;

% Computation of orbital parameters
% to determine true anomaly for the thrust strategy
[a,ecc,i,U,w,teta] = transfer_c_k_2_sun(R_SC(1,1),R_SC(2,1),R_SC(3,1),
V_SC(1,1),V_SC(2,1),V_SC(3,1),mu_M);
ang = teta;

% Computation of pericenter and apocenter altitude
E_mech = norm(V_SC)^2/2 - mu_M/norm(R_SC);
ha = -mu_M/(2*E_mech)*(1+ecc) - R_mars;
hp = -mu_M/(2*E_mech)*(1-ecc) - R_mars;

% Thrust window, manual input
theta = deg2rad(130);

% Thrust strategy
% verify if S/C is inside Mars SOI and higher than 500km altitude
if norm(R_SC) < SOI_M && ha > 500
    % verify if eccentricity is higher than selected value
    % no apocenter and pericenter if orbit is circular
    % 0.0001 tuned empirically
    if ecc > 0.0001
        if ang > theta && ang < 2*pi-theta
            A_SC_F = [0;0;0];
            mdot = 0;
        else
            A_SC_F = -F_red/M_SC*10^-3*V_SC/norm(V_SC);
            mdot = -F_red/(Isp*g0);
        end
    else
        A_SC_F = -F_red/M.SC*10^-3*V.SC/norm(V.SC);
        mdot = -F_red/(Isp*g0);
    end
% if S/C is higher than Mars SOI continuous deceleration
else if norm(R.SC) >= SOI_M
    A_SC_F = -F_red/M.SC*10^-3*R.SC/norm(R.SC);
    mdot = -F_red/(Isp*g0);
else
    A_SC_F = [0;0;0];
    mdot = 0;
end
% Build final acceleration vector
A_SC = A_SC_grav + A_SC_F;

dx = [mdot;V_SC;A_SC];

A.5 Auxiliary Functions

- **constraints_fmincon.m**

    ```matlab
    function [c,ceq,Tspan,Y] = constraints_fmincon(x,y0)
    global mu_M SOI_M
    Tspan = [0,x(1)*3600*24*365];
    A_vec = x(2:12);
    opts = odeset('RelTol',10e-6,'AbsTol',10^-6*ones(1,19));
    [t,Y] = ode113(@(t,x)
    TransferPhase(t,x,A_vec,Tspan(end)),Tspan,y0,opts);
    % equality constraints on position and velocity wrt Mars
    x = Y(end,2)-Y(end,14);     vx = Y(end,5)-Y(end,17);
    y = Y(end,3)-Y(end,15);     vy = Y(end,6)-Y(end,18);
    z = Y(end,4)-Y(end,16);     vz = Y(end,7)-Y(end,19);
    % ceq = [] --> nonlinear equality constraint
    % c = [] --> nonlinear inequality constrain
    ceq = [x y z vx vy vz];
    c = [ ];
    end
    
- **deg2rad.m**

    ```matlab
    function [ans] = deg2rad(x)
    ans = x * pi/180;
    end
    
- **ephemeris.m**

    ```matlab
    function [x,y,z,vx,vy,vz] = ephemeris(d,body)
    if body == 1
        P = textread('Eff_Earth_2019_01_01_2022_01_01.txt');
        V = P(d,:);  
        x = V(1);    
        y = V(2);    
        z = V(3);    
        vx = V(4);   
        vy = V(5);   
        vz = V(6);
    ```
elseif body == 2
    D = textread('Eff_Mars_2019_01_01_2022_01_01.txt');
    V = D(:,1);
    x = V(1);
    y = V(2);
    z = V(3);
    vx = V(4);
    vy = V(5);
    vz = V(6);
else
    disp('error body')
end

• *ESOIevent.m*

    function [value,isterminal,direction] = ESOIevent(Tspan,x)
    global SOI_E

    a = norm(x(2:4))-SOI_E;

    value = a;       % The value that we want to be zero
    isterminal = 1;   % Halt integration
    direction = 0;    % The zero can be approached from either direction

end

• *f.m*

    function y = f(x)
    y = x(1);

end

• *julian.m*

    function jdate = julian(month, day, year)

    % Julian date
    % Input
    % month = calendar month [1 - 12]
    % day = calendar day [1 - 31]
    % year = calendar year [yyyy]

    % Output
    % jdate = Julian date
    % special notes
    % (1) calendar year must include all digits
    % (2) will report October 5, 1582 to October 14, 1582
% as invalid calendar dates and stop

% Orbital Mechanics with Matlab

% version 1.2.0.0 (232 KB) by David Eagle

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
y = year;
m = month;
b = 0;
c = 0;
if (m <= 2)
y = y - 1;
m = m + 12;
end
if (y < 0)
c = -.75;
end
% check for valid calendar date
if (year < 1582)
    % null
elseif (year > 1582)
a = fix(y / 100);
b = 2 - a + floor(a / 4);
elseif (month < 10)
    % null
elseif (month > 10)
a = fix(y / 100);
b = 2 - a + floor(a / 4);
elseif (day <= 4)
    % null
elseif (day > 14)
a = fix(y / 100);
b = 2 - a + floor(a / 4);
else
    clc; home;
    fprintf('\n\n this is an invalid calendar date!!\n');
    keycheck;
    return;
end
jd = fix(365.25 * y + c) + fix(30.6001 * (m + 1));
jdate = jd + day + b + 1720994.5;

• MSOIevent.m

function [value,isterminal,direction] = MSOIevent(Tspan,x)

global mu_M R_mars
M_SC = x(1);
R_SC = x(2:4);
V_SC = x(5:7);

[a,ecc,i,U,w,teta]=transfer_c_k_2_sun(R_SC(1,1),R_SC(2,1),R_SC(3,1),...
  V_SC(1,1),V_SC(2,1),V_SC(3,1),mu_M);

if eccentric == 0
  hp = a-R_mars;
  ha = a-R_mars;
elseif eccentric == 1
  hp = a*(1-ecc^2)*0.5-R_mars;
  ha = Inf;
elseif eccentric > 0 && eccentric < 1
  hp = a*(1-ecc)-R_mars;
  ha = a*(1+ecc)-R_mars;
else
  hp = a*(1-ecc)-R_mars;
  ha = Inf;
end

a = ha - 500;
value = a; % The value that we want to be zero
isterminal = 1; % Halt integration
direction = 0; % The zero can be approached from either direction

• matInVn.m

function [Innv] = matInVn(r,v)

h = cross(r,v);

xx = cross(v,h)/norm(cross(v,h));
yy = v/norm(v);
zz = cross(r,v)/norm(cross(r,v));

Innv = [xx; yy; zz];

end

• plot_orbit.m

function [A] = plot_orbit(a,e,i,U,w,mu)

angle_vect = [0:0.01:2*pi];
l = length(angle_vect);
A = zeros(l,3);

for j = 1:l
  [x,y,z,vx,vy,vz] = transfer_k_c(a,e,i,U,w,angle_vect(j),mu);
  A(j,1:3) = [x,y,z];
end

x_vect = A(1:end,1);
y_vect = A(1:end,2);
z_vect = A(1:end,3);
end

• *rad2deg.m*

```matlab
function [ans] = rad2deg(x)
ans = x * 180/pi;
end
```

• *R1.m*

```matlab
% Rotation matrix direction cosine matrix
% Richard Rieber
% September 21, 2006
% rrieber@gmail.com
% Revision 8/21/07: Deleted unneeded ; in function name.
%                  Added example.
%                  Added H1 line for lookfor functionality
% function A = R1(x)
% This function creates a rotation matrix about the 1-axis (or the X-axis)
% A = [1 0 0;
%      0 cos(x) sin(x);
%      0 -sin(x) cos(x)];
% Inputs: x - rotation angle in radians
% Outputs: A - the rotation matrix about the X-axis
% EXAMPLE:
% R1(pi/4) =
% 1.0000         0         0
% 0    0.7071    0.7071
% 0    -0.7071   0.7071

function A = R1(x)
if nargin > 1
    error('Too many inputs.  See help file')
end
A = [1 0 0;
     0 cos(x) sin(x);
     0 -sin(x) cos(x)];
```

• *R2.m*

```matlab
% Rotation matrix direction cosine matrix
% Richard Rieber
% September 21, 2006
% rrieber@gmail.com
```
% Revision 8/21/07: Deleted unneeded ; in function name.
% Added example.
% Added H1 line for lookfor functionality
% function A = R2(x)
% This function creates a rotation matrix about the 2-axis (or the Y-axis)
% Inputs:  x - rotation angle in radians
% Outputs: A - the rotation matrix about the Y-axis

function A = R2(x)

if nargin > 1
    error('Too many inputs. See help file')
end

A = [cos(x)  0      -sin(x);
     0       1      0;
     sin(x)  0      cos(x)];

•  R3.m

% Rotation matrix direction cosine matrix
% Richard Rieber
% September 21, 2006
% rrieber@gmail.com
% Revision 8/21/07: Deleted unneeded ; in function name.
% Added example.
% Added H1 line for lookfor functionality
% function A = R3(x)
% This function creates a rotation matrix about the 3-axis (or the Z-axis)
% Inputs:  x - rotation angle in radians
% Outputs: A - the rotation matrix about the Z-axis

function A = R3(x)

if nargin > 1
    error('Too many inputs. See help file')
end

A = [cos(x)  sin(x)  0;
     -sin(x) cos(x)  0;
     0       0      1];

•  transfer_c_k_2_sun.m

function [a,norm_e,i,U,w,teta] = transfer_c_k_2_sun(x,y,z,vx,vy,vz,mu)

r = [x;y;z];
v = [vx;vy;vz];
K = [0;0;1];
\[ J = [0; 1; 0]; \]
\[ I = [1; 0; 0]; \]

% Compute norms
\[ \text{norm}_r = \text{norm}(r); \]
\[ \text{norm}_v = \text{norm}(v); \]

% Compute semimajor axis
\[ a = 1/(2/\text{norm}_r - ((\text{norm}_v^2)/\mu)); \]

% Compute h vector
\[ h = \text{cross}(r, v); \]
\[ \text{norm}_h = \text{norm}(h); \]

% Compute eccentricity
\[ t = \text{cross}(v, h); \]
\[ e = (t - \mu*(r./\text{norm}_r))./\mu; \]
\[ \text{norm}_e = \text{norm}(e); \]

% Compute inclination
\[ i = \text{acos}(\text{dot}(h, K)/\text{norm}_h); \]

% Compute nodal axis
\[ n = \text{cross}(K, h)./\text{norm}(\text{cross}(K, h)); \]

% Compute RAAN
\[ \text{if dot}(n, J) > 0 \]
\[ \quad U = \text{acos}(\text{dot}(I, n)); \]
\[ \text{else} \]
\[ \quad U = -(\text{acos}(\text{dot}(I, n))) + 2*\pi; \]
\[ \text{end} \]

% Compute pericenter anomaly
\[ \text{if dot}(e, K) > 0 \]
\[ \quad w = \text{acos}(\text{dot}(n, e)/\text{norm}_e); \]
\[ \text{else} \]
\[ \quad w = -(\text{acos}(\text{dot}(n, e)/\text{norm}_e)) + 2*\pi; \]
\[ \text{end} \]

% Compute true anomaly
\[ \text{if dot}(v, r) > 0 \]
\[ \quad \text{teta} = \text{acos}((\text{dot}(r, e))/(\text{norm}_r*\text{norm}_e)); \]
\[ \text{else} \]
\[ \quad \text{teta} = -(\text{acos}((\text{dot}(r, e))/(\text{norm}_r*\text{norm}_e))) + 2*\pi; \]
\[ \text{end} \]

\[ \bullet \text{ transfer}\_k\_c.m \]
function [x,y,z,vx,vy,vz] = transfer_k_c(a,e,i,U,w,teta,mu)

%Compute p
p = a*(1-e^2);

%Compute position in perifocal system
x_pf = p*cos(teta)/(1+(e*cos(teta)));
y_pf = p*sin(teta)/(1+(e*cos(teta)));
r_pf = [x_pf;y_pf;0];

%Compute velocity in perifocal system
Vx_pf = -sqrt(mu/p)*sin(teta);
Vy_pf = sqrt(mu/p)*(e + cos(teta));
v_pf = [Vx_pf;Vy_pf;0];

%Compute rotation matrix
ROT1 = [cos(U),sin(U),0;-

- sin(U),cos(U),0;0,0,1];
ROT2 = [1,0,0;0,cos(i),sin(i);0,-

-sin(i),cos(i)];
ROT3 = [cos(w),sin(w),0;-

-sin(w),cos(w),0;0,0,1];

T_PF = ROT3*ROT2*ROT1;
T_GE = T_PF';

%Perifocal to planetocentric
r_ge = T_GE*r_pf;
v_ge = T_GE*v_pf;

x = r_ge(1);
y = r_ge(2);
z = r_ge(3);

vx = v_ge(1);
vx = v_ge(2);
vz = v_ge(3);

end

• zeroTo360.m

function y = zeroTo360(x,unit)

%Angle reduce reduction degrees radians
% Richard Rieber
% October 1, 2009
% rrieber@gmail.com
%
% function y = zeroTo360(x,unit)
%
% Purpose: This function reduces an angle to the range of 0 - 360
% degrees
% or 0 - 2*pi radians.
%
% Inputs: x - Angle to be reduced, may be an array of angles
% unit - Boolean, 1 for radians, 0 for degrees, defaults to
% degrees [OPTIONAL]
%
% Output: y - Reduced angle
%
function y = zeroTo360(x,unit)
if nargin == 1
    unit = 0;
elseif nargin > 2
    error('Too many inputs')
end

if unit
    deg = 2*pi;
else
    deg = 360;
end

y = zeros(1,length(x));

for j = 1:length(x)
    if (x(j) >= deg)
        x(j) = x(j) - fix(x(j)/deg)*deg;
    elseif (x(j) < 0)
        x(j) = x(j) - (fix(x(j)/deg) - 1)*deg;
    end
    y(j) = x(j);
end