SEISMIC ASSESSMENT OF VAULTED SYSTEMS IN SAN PAOLO DI MIRABELLO CHURCH AFTER 2012 SEISMIC EVENTS

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ABSTRACT

The preservation of the cultural heritage in Italy, makes masonry structures, and specially churches, important buildings. Due to the recent seismic events in the center of Italy, it is of interest to study the seismic behavior of such structures, as the level of vulnerability will determine if additional actions are required to conserve them. The vaults in San Paolo di Mirabello Church, located in Mirabello (FE), were analyzed after the earthquake of 2012 in Emilia Romagna, which caused the partial collapse of the church and just the part of its nave and the south part of its transept remained standing. The vaults in the nave, characterized by very thin thickness, were only affected at the extremes, generating interest on their structural behavior. The seismic vulnerability for the vaulted system, as well as for the remaining part of the church was studied and out of this, the corresponding structural analysis and recommended actions to take were defined, hypothesizing different scenarios. The geometry of the vaults was generated from cloud points, from which the corresponding mesh was generated automatically. With this way it was possible to observe how the actual geometry of this kind of structures, considering imperfections, influences significantly in the results. Both linear and nonlinear analyses with three-dimensional finite elements were developed to study the seismic vulnerability, and very interesting results were obtained for both cases.

Key words: Masonry structures, Churches, Italy, Nonlinear material, Cloud of points, Vaulted systems, Pushover, Capacity curve.
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INTRODUCTION

The assessment of historical buildings in Italy has become an important issue in the last years, as they constitute an important part of the architectural heritage in numerous villages and cities. In the case of masonry churches, due to their complex and singular structural typology, the assessment of their seismic vulnerability is based on special considerations. Typical churches include wide rooms with large spans, high columns, arches, vaults, domes and bell towers, which can undergo extensible damages, and for these there are required separated analyses.

The inspection of churches is usually based on local analyses, studying the most common failure mechanisms that have occurred along the years in old churches. Different types of analyses are developed in these structures, depending on the level of exactitude that wants to be achieved, as well as the completeness of the information available and reliability of the data.

The simplest approach for the assessment of churches, in Italy, consists on the determination of a safety index (0-1), in which the seismic behavior is studied in a statistical way. The global behavior of the structure is determined with the weighted average of the behavior of the different parts composing the church. The building is assumed to be made of a good quality mortar, such that there is no disaggregation of the material, and so the resulting failure mechanisms occur due to the detachment of entire sections. In this way, the feasibility of the building, in terms of seismic bearing capacity, is determined in a qualitative way, giving a preliminary overview of the conditions of the building.

For more detailed analyses, reference is made to the local assessment of the individual parts composing the building. As (Bussi, 2011) states, the principal characteristics that influence in the resistance of an element are: its form, the geometry of its transversal section and its constituent materials. For masonry structures, especially for arches and vaulted surfaces, the geometry of these elements governs significantly their behavior, and this is directly related with their stability. For this local analysis, there are usually employed the theorems of limit analysis, and in these way, it is possible to study the state of stresses and the individual element’s vulnerability under different applied actions.

Sophisticated analyses include three-dimensional finite elements models, in which either the whole building or a specific part of it is modelled, discretizing its geometry with small simple
elements. Modeling techniques include a variety of assumptions regarding the definition the geometry and the material properties, and it is necessary to consider these to determine the level reliability of the results obtained, as well as their limitations.

Among the most susceptible parts that tend to detach from the churches, there can be identified the vaulted systems. These compression-only structures, represent a family of structures characterized by a shape-resisting behavior for which their seismic response evaluation has not been standardized yet. Thus, different methods have been developed parallelly in order to determine their safety.

Special attention will be presented in this thesis to the analysis of the masonry vaults in the central nave of San Paolo di Mirabello Church, Terre del Reno (FE). It is of interest to determine why these structures, after the earthquakes that affected the Region of Emilia Romagna in 2012, which destroyed part of the church as well as neighboring buildings, remained almost complete even though their geometry is characterized with a very thin thickness. Both limit analysis and sophisticated approaches will be implemented with the use of finite elements in order to give an overview of the vulnerability of these structures in their actual condition. Additionally, an analysis will be developed, assuming these structures will be repaired with the same original characteristics, to determine if it is necessary to implement a retrofitting intervention to guaranty their safety.

With a point cloud of the church after the 2012 earthquakes, it will be possible to determine the exact geometry of the parts in consideration. This feature will be exploited as all the imperfections present in the actual structure will be considered, giving more accurate results. At the end, there will be determined the structural reasons for the collapse mechanism of the vaulted system in the church as well as their level of vulnerability and possible interventions to improve their behavior under the action of seismic actions.
STATE OF ART

VAULTS IN ITALY

Most of the ancient constructions in Italy are made up of masonry, one of the earliest building material. Several historical and ancient masonry buildings are characterized by the presence of arches and vaults with high architectural value, which are often used as ceiling for enclosed spaces. Thin vaults are common in large part of Central Italy, like those covering the main nave in churches which are conceived as lightweight false ceilings, lacking any extrados backfilling material, thus withstanding their sole self-weight (Modena, 2016). Thin vaults are characterized by a very low thickness relative to the distance between supports, which generally are between 3 m and 10 m.

Historical buildings in general were designed for gravity loads, as in the time of their construction there were no earthquake resistant provisions. In addition, masonry structures are often associated with high seismic vulnerability due to their material. The main mechanical characteristics of masonry consist of a high specific mass, low tensile strength, moderate shear strength and low ductility (Khan M.A., 2017). The earthquake that hit the city of L'Aquila in 2009, has left in evidence that thin vaults made of a single layer of bricks, are scarcely resistant to horizontal loads and highly vulnerable to seismic actions (Kaplan, 2010). Therefore, an adequate conservation of this structures must be planned since usually vaults and arches are the most vulnerable elements in historic constructions.

The conservation of this kind of structures must be not only esthetic, maintaining their original shape, but also preserving the building techniques and the structural behavior of the original structure. Therefore, knowing the mechanical behavior of this type of constructions is of vital importance to perform an effective restoration and improvement.
GRAPHIC STATICS

During the safety assessment of masonry structures, which generally fail because of stability problems and not because of the lack of compressive strength (Block P., 2009), special attention is given to the geometry and configuration of the structural elements.

The static analysis of masonry elements, more precisely of arches, can be addressed with graphic statics in a simpler way avoiding tedious computations, other than scaling lines and angles (Fairman & Cutshall, 1932). This method is based in the implementation of reciprocal figures (form and force diagrams), which represent the actual geometry and boundary conditions of the structure in study as well as the magnitude, direction and line of action of the internal forces. Recalling Maxwell’s definition of reciprocal figures, he stated that:

“Two plane figures are reciprocal when they consist of an equal number of lines, so that corresponding lines in the two figures are parallel, and corresponding lines which converge to a point in one figure form a closed polygon in the other.”

(Konopelchenko & Schief, 2001)

In other words, referring to the problem of curved structures, the nodes and polygons representing the geometry of the structure, in the form diagram, will have reciprocal polygons and nodes representing the forces, in the force diagram. In this way, by ensuring the construction of polygons, there will be guaranteed equilibrium and just by measuring and scaling the lines in the force diagram it will be possible to determine the actual magnitude of the forces acting in the system.

This method, originated in the 19th century, was rediscovered and implemented from a mathematical point of view in the 20th century (Konstantatou & F.A., 2017). Nowadays, with the help of computational tools, its implementation has become simpler and can present some advantages compared with traditional methods. By making use of graphic statics, there is no need to compute or assemble any stiffness matrix. The number of unknowns in the equilibrium equations of the structure are reduced, and, if equilibrium is guaranteed, there will always be a solution (Beghini, Carrion, Beghini, & et al. Struct Multidisc Optim, 2014).
However, it can still become cumbersome and time-consuming if complex structures are analyzed. For analyses in 3D, the form and the force diagrams are polyhedral diagrams. The equilibrium of each node is represented by a closed force polyhedron and the area of each face represents the magnitude of the corresponding force in that node (Hablicsek, Akbarzadeh, & Guo, 2018).
LIMIT ANALYSIS

The most effective approach for analyzing vaulted masonry structures, at least for a fast and reliable evaluation, has been proven to be limit analysis (Tralli, Alessandri, & Milani, 2014). Even if there is not yet a widely accepted method to study this type of structures, with limit analysis it is possible to understand their behavior and evaluate vaulted surfaces vulnerability. For the implementation of this method it is necessary to recall the fundamental theorems of plastic analysis:

- **Static (lower bound) theorem:** Also called the safe theorem. It is based on the definition of the line of thrust of the arch (thrust surface for vaults). If there can be found a line of thrust, in equilibrium with the external loads, contained within the arch’s thickness it is sufficient to guarantee its equilibrium. “No collapse will occur if a statically admissible state of equilibrium can be found” (Tralli, Alessandri, & Milani, 2014).

\[ \lambda_s \leq \lambda_o \]

- **Kinematic (upper bound) theorem:** It studies the number of plastic hinges necessary to transform the arch into a mechanism of collapse. As normally arches have a redundancy of 3, the collapse mechanism when applied a point load will be formed with at least four hinges are formed, and if it is under symmetric loads there will be formed five hinges. Failure is assumed to occur due to the mutual rotation of blocks rather than sliding, thus only rotational hinges are formed (Taliercio, 6. Masonry arches and Arch bridges, 2017). “If a kinematically admissible mechanism can be found for which the work developed by external forces is possible or zero then the arch will collapse” (Tralli, Alessandri, & Milani, 2014).

\[ \lambda_k \geq \lambda_o \]

- **Uniqueness theorem:** Gathers the static approach and the kinematic approach together. It states that, if there can be found a line of thrust lying completely within the structure, which: represent a state of equilibrium under the external loads and allows the formation of enough hinges to transform the structure into a mechanism, then the structure is on the point of collapse and this line of thrust is unique (Clemente, 2006).

\[ \lambda_s \leq \lambda_o \leq \lambda_k \]
THRUST LINE (STATIC APPROACH)

The most simple and useful technique for studying masonry structures is the so called Thrust Line Analysis. This approach based on graphic statics, is used to determine the stability of masonry arches, as well as their load carrying capacity, and can determine collapse mechanisms without detailed information about the materials composing the structure. It is simply based in equilibrium equations, depending on the geometry and loading conditions of the structure in study, and it works under Heyman’s hypotheses (1966) which constitute the Principles of Limit Analysis of Masonry Structures:

- **Non-tensile strength.** The masonry in tension is very weak, so its strength can be assumed to be negligible. Safe assumption.

- **Infinite compressive strength.** Masonry structures are typically subjected to low stresses, an order of magnitude below the crushing values (Huerta F., 2001).

- **Sliding between masonry blocks is not allowed.** The friction between elements is high enough to avoid sliding.

According to Heyman, for an arch in equilibrium subjected to certain loading conditions, if it is possible to find a line of thrust which is completely inside the arch’s thickness, then the arch is stable and will not rotate or slide out of control. (Harvey & Maunder, 2001) (Ricci, Sacco, & Piccioni, 2016)

The previous affirmation can be described with Figure 1-1. By considering a single block (voussoir), subjected to a specific stress distribution, it is possible to determine the resulting forces at its end. Since arches are characterized as funicular systems, it is possible to state that these resultant forces, also called thrust, should always be in compression. The point of application of such forces will then be called the center of thrust and, due to equilibrium, this must be the same between adjacent blocks. (Huerta F., 2001)
If we now consider the arch in a global way, by summing up the resultants at the end of each block, we will obtain the so called thrust of the arch and this will be the compressive force at its springers. The line of action of the resulting compressive forces acting all along the arch will be the thrust line and, in order to guarantee these forces are in compression and their equilibrium, this line must fall within the arch’s thickness. The results will depend on the loading conditions and the geometry of the arch, and this is what will determine its stability.

However, it is necessary to consider that arches are statically indeterminate structures and because of this it is impossible to determine their actual thrust, just by making use of equilibrium equations. Because of this, there can be established some limits (minimum and maximum thrust), from which it is possible to evaluate the stability of arches as a function of the arch’s thickness (Huerta F., 2001).

![Figure 1-2. "Semicircular arch under its own weight. a) Minimum Thrust; b) Maximum thrust (Heyman 1995)" (Huerta F., 2001)](image)

The minimum thrust of an arch makes reference to the deepest possible solution contained within the arch’s thickness. It will have the greatest rise and the smallest clear span with three plastic hinges forming as shown in Figure 1-2. The maximum thrust instead, states for the shallowest solution with the smallest rise and largest clear span. Four plastics hinges will form in this case.

If we consider an arch subjected only to its self-weight, the resulting line of thrust will be its catenary. The limit arch, which is a semicircular arch defined with the minimum thickness in which such catenary can be contained, will be determined, and by defining the ratio between this arch and the actual thickness of the arch in study it is possible to determine its geometrical safety factor (GSF). The bigger this ratio, the safer will be the arch. For a GSF of 3, it will mean the line of thrust falls entirely within the middle third of the arch (its kern) and so there should not be tensile stresses which influence in the development of cracks. For a GSF of 2, the thrust will be contained within the middle half of the arch.
The thrust line analysis, can be applied to any kind of arch, and it is sufficient to determine just one thrust line in order to guarantee its stability (Ricci, Sacco, & Piccioni, 2016). Any line of thrust within the arch will represent an equilibrium solution, and this will be the corresponding catenary to a specific loading condition (Huerta F., 2001). This means that, as the thickness of the arch increases, the number of possible equilibrium solutions will increase too and so the stability of the arch will be easier to ensure.

The collapse of an arch will occur when no thrust line can be found within the arch’s thickness. When the line of action of the forces reaches the arch’s edge, or better when it is tangent to its intrados or extrados, it will contribute to the formation of plastic hinges. If at least four plastic hinges are formed, this will result in a mechanism of collapse. (Nobile & Bartolomeo, 2014).

Making reference now to vaulted surfaces, the same reasoning can be applied. However, as the problem turns now into a 3D problem, it gets more complex. It is still possible to implement a slicing technique, in which the vault in study is divided into a series of arches, and the vault’s behavior is now reduced into a 2D problem. For each arch, it is determined the line of thrust and this must lay within its thickness. In most of the cases it is sufficient, in order to analyze the vault’s stability, nevertheless it must be considered that it will not capture its full behavior.

A computational implementation has been developed by (Block P., 2009) as a 3D extension for the thrust-line analysis, the Thrust Network Analysis. With this method, it is possible to represent the distribution of the forces acting in the system and obtain different possible equilibrium solutions, in order to design or asses vaulted surfaces. However, it is still challenging when dealing with complex geometries and “there is a lack of a proper mathematical/algebraic formulation for the reciprocal polyhedral diagrams limiting the interactive implementation of 3DGS (3D Graphic Statics)” (Hablicsek, Akbarzadeh, & Guo, 2018).
COLLAPSE MECHANISM (KINEMATIC APPROACH)

The kinematic approach of limit analysis, seeks for the determination of the maximum load that can be applied to the structure, before it collapses. It determines which percentage of its mass can be borne, under specific loading conditions, and the most probable collapse mechanism corresponding to the limit load.

It is founded under the assumption that no collapse will occur due to sliding and so only rotational hinges will form, once the line of thrust touches the arch’s intrados or extrados as a result of the low tensile strength of masonry. The collapse mechanisms are characterized by the separation of rigid portions of the structure, and these will be formed once the fourth rotational hinge develops, making the structure unstable. Depending on the loading conditions, there will develop a different number of hinges and depending on the shape of the arch the value of the limit load will vary. Under symmetrical loads, the arch will collapse with five hinges, while for concentrated loads or nonsymmetrical loading conditions this may result in mechanisms with just four hinges.

![Figure 1-3. Different collapse mechanisms depending on the loading conditions. (Foraboschi, 2014)](image)

By simply making use of the principle of virtual works, equalizing the internal work and the external work, it is possible to determine the magnitude of the load that activates certain mechanism. Varying the position of the hinges in the structure, the collapse load multiplier, will be then determined as the one for which an admissible collapse mechanism can be determined. “If no collapse mechanism is admissible, in principle the arch can carry any load, however high it may be” (Taliercio, 6. Masonry arches and Arch bridges, 2017).
THEORY OF THIN SHELLS

The study of vaulted structures may also be approached with the theory of thin shells. However, this can only be considered assuming no restraints at the support, as this is based on the assumption that there are no moments or transverse shear forces acting on the vault.

By considering the infinitesimal portion of the vault, shown in Figure 1-4, it is possible to define the membrane stresses acting in any point. The forces $X$, $Y$ and $Z$ are the components of the tractions acting in the surface, external surface forces, corresponding to the vault’s axis, the tangent to the directrix and the normal to the mid-surface, respectively. $N_x$ and $N_\theta$ are the normal stresses, while $N_{x\theta}$ and $N_{\theta x}$ state for the in-plane shear stresses, which due to symmetry are identical. In order to evaluate these stresses, equilibrium is imposed along the three different axes, converting this problem into a statically determinate one. Then, if these stresses are known, it will be possible to determine the local ones at any point of the vault.

$$\begin{align*}
\frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_x}{\partial \theta} + X &= 0 \\
\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial N_{\theta}}{\partial \theta} + Y &= 0 \\
\frac{N_\theta}{R} &= Z
\end{align*}$$

Figure 1-5. Resulting equation from imposing equilibrium in the infinitesimal shell.
Figure 1-5 shows the equilibrium conditions for any infinitesimal shell, from which it is possible to determine the stresses as a result of specific imposed loads. As it can be seen, the circumferential stress $N_\theta$ can be simply computed if the radius of the directrix defining the curvature of the vault is known, and from this it can be also determined the thrust acting along its edges. The thrust per unit length, may be obtained by decomposing the resulting normal stresses along the tangent of the directrix. In this way, it can be expressed as $H = -N_\theta \cos(\theta)$. 

Theoretically, under gravity loads the circumferential normal stresses in the vault must be in compression. Still, if these are subjected to significant high shear stresses, tensile stresses may arise, and shear cracking may occur.

“The circumferential stresses acting in any cross-section, $N_\theta$, equilibrates the portion of the load of which the centerline is the funicular. The remaining portion is equilibrated by the other membrane stresses.” (Taliercio, 8. Masonry vaults, 2017)

If we now consider the supports at the vault’s edges, there will be identified both shear forces and bending moments acting close to these regions. For this case, as well as for the case of concentrated loads acting on the vault, the membrane analysis will not be valid, and it is necessary to study with other approach these regions.

For the cases where this analysis may be applied, the vaults will be under the action of membrane actions only, independently on the thickness. Nevertheless, the fact that vault is assumed to experience membrane stresses only may result in an underestimation of the actual stresses acting in the vault.
FINITE ELEMENTS METHOD

The analysis of structures with a complex shape, making use of analytical approaches, can become a tedious and complex process. Therefore, in most cases it is preferred to use an approximated method, such as the Finite Elements Method (FEM), which is accurate enough for structural applications.

The FEM can be applied to complex problems on fields such as heat transfer, fluid flow, mass transport, besides the application to structural problems. Usually these problems are raised in terms of differential equations. However, by using the FEM formulation these problems can be solved as a system of algebraic equations, avoiding the solution of complex equations.

The combination of structural engineering concepts with the mathematical concepts of matrix analysis, was studied in the 1930s, and further developments lead to the acceptance of computerized structural analysis in the profession. In 1959, the displacement method was proposed for the computational implementation and, 20 years after, this became the leading method in the implementation and development of most of the finite element method (FEM) programs used nowadays.

Many finite element programs are based on the approximate method of analysis called the displacement based finite element procedure (DBFE). In the first step of this method, a displacement polynomial function is assumed, based on the independent degrees of freedom in an element or domain. After that, a strain function can be obtained by using compatibility relationships and stresses which can be obtained by using the constitutive law. In the following step of the procedure, the element stiffness matrix can be found by applying the principle of virtual displacements. The final step, consists in the solution the linear system of equilibrium equations in order to determine the displacement field.

Matrix methods are based on the discretization of continuous systems. The structural system is transformed into a mathematical model with a certain number of finite sized elements, with known material and mechanical properties which can be expressed in a matrix form. The precision in the definition of the geometry of the model, the number of elements used, and the assumptions and approximations made during the definition of the model, will determine the accuracy of the results.
The steps to perform a Finite Elements Analysis on a common software are:

**Preprocessing steps**

- Definition of the element type to be used.
- Definition of the material properties of the elements.
- Definition of geometric properties of the elements.
- Definition of element connectivity (mesh the model).
- Definition of the physical constraints (boundary conditions).
- Definition of the loads and convergence criteriums.

**Solution steps**

- Computation of the unknown values of the variable. Often the variables are displacements and rotations at the nodes.
- Computation of values which are used by back substitution. Derived variables such as reaction forces, element stresses distribution.
NONLINEAR ANALYSIS

The term “stiffness” defines the fundamental difference between linear and nonlinear analysis. Stiffness is a property of the structure that characterizes its response to the applied load. Several factors affect stiffness as such as shape, material and boundary conditions. When a structure deforms under a load its stiffness can change, due to one or more of the factors listed above. If it experiments a large deformation, its shape can change. If the material reaches its failure limit, the material properties will change. On the other hand, if the displacements and stresses are small enough and produce just a small change in the stiffness, it makes sense to assume that neither the shape or the material properties change during the deformation process. This is the fundamental assumption of the Linear Analysis.

This fundamental assumption implies that during the entire deformation process, the model retains the stiffness of the undeformed configuration before loading. Then, it is the same to apply the load gradually or all at once independently of the stress state of the elements. This simplification leads to the following equation to be solved:

\[ Ku = F \]

The term \( K \) of the last equation is the rigidity of the system which, as said before, depends on the material, the shape of the sections and boundary conditions. The final solution of the linear system can be reached by solving the last equation just one time.

Nonlinear Geometry

As general rule, if the deformation of an element is lower than \( L/20 \), where \( L \) is its larger dimension, it is reasonable to assume a linear analysis on the shape. If the displacements are no longer negligible, a nonlinear analysis might be necessary to consider the changes in the shape during the deformation of the structure.

Nonlinear Material

If changes of stiffness occur only due to changes in the material properties, under the application of the load, the problem needs to consider the material non linearities. A fundamental difference between elastic and plastic material behaviors is that exist a plastic deformation on the elements and it is possible to reach a value of failure of the material itself.
CHAPTER 1. CASE OF STUDY AND PROBLEM DEFINITION

Preliminary to the seismic capacity verification of old buildings, information is collected about critical historical analysis, the geometry of the building, the type and quality of the materials, possible damages in the building and specific seismic hazard data. From this information, it is possible to define the knowledge level and limitations for the further assessment of the building.

There can be identified three different levels of knowledge. Level of knowledge 1 makes reference to limited information of the building. Information about the geometry and the material properties is obtained from visual inspections and literature data. Level of knowledge 2, refers to extended information in which some tests are developed for the characterization of the materials, in addition to the visual inspections and literature data. Finally, the level of knowledge 3, represents the exhaustive recollection of information in which in situ and lab tests are implemented (NTC 2008).

For the church in consideration, San Paolo di Mirabello Church, the level of knowledge that was possible to reach for this assessment was just Level of knowledge 1. The only information able was taken from historical data and a visual survey based on recent pictures and point clouds obtained with laser scanner. It was not possible to visit the church or develop any experimental test in order to make a material characterization, thus some assumptions were made regarding the materials. However, this limited level of knowledge and the resulting uncertainties were considered when determining the material characteristics for the assessment of the church.
1.1 HISTORICAL ANALYSIS

The historical analysis, is aimed to understand the state of damage of the building through the knowledge of the historical evolution of the church. Constructive techniques, previous interventions and natural events that have modified the structure along its life, will be a key for understanding the behavior of the building and to determine characteristics that were not possible to obtain in another way.

The first San Paolo Church was built in 1795 in the village of Sant’Agostino, by the archpriest Serra. It was built in a Tuscan style, pushed by the growth of the local economy and the increment of the population and it was completed in 1804. In the 1900s the habitants of the village of Mirabello, unable to leave their town every Sunday in order to attend Mass, demanded the raising of their local parish to local church and it was completely demolished to give place to the new building. The construction of the new, and actual, San Paolo Church, under the design of the engineer Luigi Gulli, was started in 1929 and it was completed in 1943.

The church is located in Mirabello, a village in the municipality of Terre del Reno, in the province of Ferrara in Emilia-Romagna. It has an extension of 16 km² and limits with the villages of Vigarano Mainarda, Poggio Rantico, Sant’ Agostino and Bodeno.

In 2012 Mirabello was hit by the earthquakes that occurred between May and June in the region of Emilia Romagna. There were registered two principal quakes; the first one on May 20\textsuperscript{th} at 4:03 am, with a local magnitude (M\textsubscript{L}) of 5.9 and epicenter between Finale Emilia and San Felice sul Panaro (44°51’50” N, 11°14’31” E, depth 6.3 km), and the second one on May 29 at
9:00 am, with a local magnitude (M_L) of 5.8 and epicenter between Mirandola and Medolla (44°50’00” N, 11°03’37” E, depth 13.5 km). (Massa et al., 2012)

After the quake of May 20th, there were identified important damages and collapses both in recent and historical buildings, including San Paolo di Mirabello Church. This church suffered serious damages, about € 12.062.500,00, which included the total collapse of the apse and part of the transept, and the collapse of part of the wooden cover and gable of the façade (Parisi, 2014).

Between 2012 and 2013, the church was repaired by the Regional Directorate of Cultural Heritage and Landscape of Emilia Romagna, but until now it has not been completely rebuilt. The building that can be appreciated nowadays, exhibits a still destroyed structure with some interventions in the main façade, as well as in the pillars, but it is not completely built as it was before the earthquake.
Figure 1-3. San Paolo Church, before and after the earthquake. Taken from: https://www.corriere.it/reportages/cronache/2016/prima-dopo-terremoto-emilia/

Figure 1-4. San Paolo Church, after the earthquake in Emilia Romagna 2012. (Cattaneo S., 2014)

Figure 1-1-5. San Paolo Church reparation after the earthquake of 2012. Taken from: http://www.studiocomes.it/ricostruzioni_e_interventi_post_sisma.html
1.2 GEOMETRICAL SURVEY

In order to develop a description of the geometry of the church, information was taken from pictures, point clouds after the earthquake and a survey in Google Earth. However, it was not possible to visit the place or have information of the actual conditions of the church as it was closed after the earthquake.

From what it was possible to observe, the building is composed of one nave, the main part of the church between the façade and the triumphal arch, with aisles along its sides and chapels coming out from them, two at each side. There was possible to identify the transepts, areas crossing the nave from north to south, but still the north part seem to be destroyed. At the east part of the church, there is the place where the apse used to be, which was completely destroyed after the earthquake as shown in Figure 1-6, as well as some chevets.

Figure 1-6. San Paolo di Mirabello plan, after the earthquake of 2012.

![Figure 1-6](https://upload.wikimedia.org/wikipedia/commons/thumb/a/ab/Cathedral_schematic_plan_en_vectorial.svg/500px-Cathedral_schematic_plan_en_vectorial.svg.png)

Figure 1-7. Typical parts of a church. Taken from: https://upload.wikimedia.org/wikipedia/commons/thumb/a/ab/Cathedral_schematic_plan_en_vectorial.svg/500px-Cathedral_schematic_plan_en_vectorial.svg.png
Outside of the main building, at the northwest part of the church, there is a bell tower 56 m high. The façade is characterized by a gable, triangular, roof and there can be noted three door openings and an additional opening at the top. The roof is characterized with a wooden truss structure and roof tiles, which suffered during the earthquake. Nowadays it has been repaired and it is also supported with steel ties.

Figure 1-8. West view of San Paolo di Mirabello Church. [Picture taken by Simona Belmondo] (Mirabello,2018).

Along the main nave there is a sequence of vaults, which are the main topic of this thesis, as they have remained almost complete after the past events. There are two different kinds of vaults, barrel vaults and sail vaults, and they have been erected with bricks placed in their short direction, resulting in very thin structures. These, are intercalated all along the nave, and are connected by transverse arches. Between the nave and the aisles, there are pillars supporting the vaults, exactly five at each side as it can be seen in Figure 1-10.

Figure 1-9. Vaults in the nave of the church. [Picture taken by Simona Belmondo] (Mirabello,2018).
The barrel vaults, also known as cylindrical vaults, are formed by a cylindrical surface delimited by two generatrixes. In this church, the vaults are characterized with a rise to span ratio of 0.5 with a span of 9.7 meters and a rise of 4.85 meters. The sail vaults, characterized by a double curvature, have a maximum rise of 5.5 meters and a minimum one of 2.7 meters, in their longitudinal direction supported by windows attached to a wall.

The arches have the same rise and span as the barrel vaults, but as the vaults are characterized with a thickness of 0.08 m the arches have a thickness of 0.62 meter. There is no certainty that the arches have the same thickness all along their length, but this was considered as constant. The transverse arches width is of 0.78 meters while for the barrel vaults and sail vaults it is around 3.77 meters and 4.36 meters respectively.
For the barrel vaults, their thrust must be resisted all along their perimeter while for the sail vaults it arrives only at the corners. However, due to the presence of the windows, the boundary conditions for the sail vaults turn up to be similar to the ones of the barrel vaults.

Finally, the brick courses in the vaults are arranged in a diagonal way. They have been placed at 45° with respect to the supporting pillars, but it cannot be recognized that easily as the vaults have been plastered. This arrangement of the bricks is usually used in the presence of large spans, and it might be considered as the arrangement of the courses influences both the static and dynamic behavior of the structure, in terms of stiffness and load carrying capacity.

Figure 1-12. Arrangement of the bricks in the vaults of the nave. (Picture taken by Simona Belmondo) (Mirabello, 2018).
1.3 MATERIAL CHARACTERIZATION

Regarding the material characterization is was possible to determine the material properties from what it was identified in pictures. Because of this, it was only determined the quality index of a representative portion of the masonry walls. There were followed the guidelines for the definition of the Masonry Quality Index (IQM) developed by A. Borri and A. De Maria (2014), and out of this there was identified, in an approximated and qualitative way, the quality and the typology of the masonry present in the church.

![Image](image1.png)

*Figure 1-13. Representative portion of a wall in San Paolo de Mirabello church. [Picture taken by Simona Belmondo] (Mirabello, 2018).*

The characterization of the wall is based on a visual inspection in order to determine the performance of the masonry structure, under different directions of the acting loads. Following the guidelines one has to decide if some characteristics of the wall are satisfied or not (R: Respected, PR: Partially respected, NR: Non respected) and out of this determine the quality index from the values specified in Table 1.

![Table](table1.png)

*Table 1. Punteggi da attribuire ai parametri della regola dell’arte. (Borri, A. & De Maria, A., 2016)*
The IQM makes reference to a number between 0 and 10, which synthetizes the mechanical behavior of a specific masonry typology. Depending on the resulting index, for the three different actions considered in the guidelines (vertical actions, out-of-plane horizontal actions and in-plane horizontal actions), the masonry present in the building can be classified into three different groups (A, B or C). Group A makes reference to a masonry with the best structural behavior, group B represents masonry with an intermediate quality and group C the worst situation we can have regarding the quality of the material.

\[
\text{IQM} = \text{RE.EL.} \times (\text{OR.} + \text{P.D.} + \text{F.EL.} + \text{S.G.} + \text{D.EL.} + \text{MA.})
\]

The parameters that are considered for the determination of the quality of masonry constitute what is denoted as the “rule of art”. These are a result of different construction rules in order to guarantee the good mechanical behavior, compactness and monolithicity of masonry walls, and are divided into seven categories. The quality of the mortar (MA.), the transverse gearing (P.D.), the shape of the elements constituting the wall (F.EL.), the size of the elements (D.EL.), the condition of the vertical joints (S.G.), the condition of the horizontal joints (OR.) and the quality of the resisting elements (RE.EL.).

In order to determine an appropriate characterization of the masonry, it is necessary to know in advance the type of masonry in study. Based on the guidelines, containing different charts and examples for the evaluation, and the pictures about the church, it was possible to
determine the condition for each parameter. The corresponding masonry quality indexes were computed for each kind of action, and their results are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Vertical Actions</th>
<th>Out of plane Actions</th>
<th>In-plane Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR.</td>
<td>R 2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>P.D.</td>
<td>PR 1</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>F.E.L.</td>
<td>R 3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>S.G.</td>
<td>R 1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>D.E.L.</td>
<td>PR 0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>MA.</td>
<td>PR 0.5</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>RE.EL.</td>
<td>R 1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Determination of the IQM indexes.

<table>
<thead>
<tr>
<th>Category of masonry</th>
<th>Method A</th>
<th>Method B</th>
<th>Method C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azioni verticali</td>
<td>$5 \leq IQ \leq 10$</td>
<td>$2.5 \leq IQ &lt; 5$</td>
<td>$0 \leq IQ &lt; 2.5$</td>
</tr>
<tr>
<td>Azioni ortogonali</td>
<td>$7 \leq IQ &lt; 10$</td>
<td>$4 &lt; IQ &lt; 7$</td>
<td>$0 \leq IQ &lt; 4$</td>
</tr>
<tr>
<td>Azioni orizz. complanari</td>
<td>$5 \leq IQ \leq 10$</td>
<td>$3 &lt; IQ &lt; 5$</td>
<td>$0 \leq IQ &lt; 3$</td>
</tr>
</tbody>
</table>

Table 3. Determination of the masonry category based on the IQM. (Borri, A. and De Maria, A, 2014)

From the results obtained, it can be seen that the masonry present in the church could be classified in category A for the three different types of actions. This would mean that, at least for what can be observed, the masonry composing the church is of good quality. Masonry in category A is unlikely to suffer damages under vertical and in-plane horizontal actions, and with respect to horizontal out-of-plane actions it is able to maintain a monolithic behavior. If there are guaranteed good connections between the walls composing the church, there are low probabilities of collapse in the structure.

From a first approach, the quality of the masonry in the church could be considered as a good one, at least in a general way. However, in order to determine the load carrying capacity of masonry, it would be better to develop some tests, at least non-destructing ones, in order to determine the actual condition of the church or make a visit to the place to identify it. The quality of the masonry represents one of the more relevant features when analyzing the seismic response of the building, and because of this it is important to develop a reliable characterization.

Additionally, after identifying the typology of the masonry, in order to determine its mechanical characteristics for this assessment, it is possible to make use of the values presented in Table 4, as reference values defined in the NTC 2008. As the level of knowledge in this special case is 1, the values from the table that should be used for the shear and compressive
strengths are the minimum of the two values that appear for each case. For the elastic modulus, instead, there must be computed the mean values.

From the pictures available, it could be said that the typology of the masonry present in this church is a “Brickwork of solid blocks and lime mortar”. Thus, the corresponding characteristic for such type would be the ones highlighted in Table 4, and were the ones considered for the analyses present in this thesis. Additionally, considering a mortar with good characteristics it is necessary to consider the corrective coefficients present in Table 5, and a confidence factor CF of 1.35 coming from the uncertainties of the information available.

<table>
<thead>
<tr>
<th>Masonry typology</th>
<th>$f_m$ (N/cm²)</th>
<th>$\tau_0$ (N/cm²)</th>
<th>$\varepsilon$ (N/mm²)</th>
<th>$G$ (N/mm²)</th>
<th>$w$ (kN/m³)</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disarranged masonry of cobble/boulders</td>
<td>200-280</td>
<td>3.5-5.1</td>
<td>1100-1400</td>
<td>340-480</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Masonry of rough-hewed stones</td>
<td>160-200</td>
<td>2.6-4.2</td>
<td>900-1240</td>
<td>300-420</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Masonry of cut stones</td>
<td>120-200</td>
<td>2.6-4.2</td>
<td>900-1240</td>
<td>300-420</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Masonry of squared stone blocks</td>
<td>600-800</td>
<td>9.0-12.0</td>
<td>2400-3200</td>
<td>780-940</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Brickwork of solid blocks and lime mortar</td>
<td>240-400</td>
<td>4.0-9.2</td>
<td>1200-1300</td>
<td>400-600</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Mechanical properties of masonry based on the typology (Circolare 617/09 Istruzioni per l’applicazione delle NTC08)

The final mechanical properties, used for the analysis of the church, taking into account the safety factor and corrective coefficients and are shown in Table 6.

<table>
<thead>
<tr>
<th>Masonry typology</th>
<th>$f_m$ (N/cm²)</th>
<th>$\tau_0$ (N/cm²)</th>
<th>$\varepsilon$ (N/mm²)</th>
<th>$G$ (N/mm²)</th>
<th>$w$ (kN/m³)</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brickwork of solid blocks and lime mortar</td>
<td>88.9</td>
<td>2.2</td>
<td>2250</td>
<td>750</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Vertical actions</td>
<td>113.3</td>
<td>3.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seismic actions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Corrective coefficients from Circolare 617/09 Istruzioni per l’applicazione delle NTC08.

Table 6. Mechanical characteristics masonry.
1.4 DAMAGE SURVEY AND SEISMIC VULNERABILITY

Churches along time have been affected by different by kinds of events, these causing damages such as cracking, crushing, sliding and loss of verticality of the bearing elements. After the occurrence of environmental or accidental events, change in the type of use of the building, or before developing any kind of intervention, it is necessary to develop an assessment of the actual conditions of the structure in order to determine which actions to take.

The description of the actual conditions of the building, making reference to the damage caused by the earthquake of 2012, will be presented based on the Italian guidelines specialized in the assessment of churches (Papa, S. and Di Pasquale, G., 2011). Their aim is to provide a simple approach, easily to understand and apply, where no detailed measurements, or any process that would slower the study without providing any significant information, are required. They are useful to develop a preliminary diagnosis of the building seismic response, without the need of computational models, resulting in a reliable overview of the conditions of the church.

The first guidelines were developed in 1987, after the earthquake of Parma in 1986, and they have been updated along the years after analyzing thousands of churches and their response to different seismic events. There have been identified different patterns in the failure mechanisms of churches under the action of seismic forces and nowadays the guidelines contain the 28 most common, fundamental, collapse mechanisms.

For any church, based on photographic documentation, reference can be done to macro-elements and their corresponding damage mechanisms. These parts of the church, which can coincide or not with an identifiable part under the architectural and functional aspects, experience a unitary and recognizable behavior, under the action of seismic loads, and this can be easily observed and fully described. It is always expected a local collapse of the building, local failure mechanism, and this is key for interpreting the seismic vulnerability of masonry buildings. Because of this, it is assumed that all the elements composing the building are made of a good quality mortar, such that there is no disaggregation of the material, and the resulting failure mechanisms occur due to the detachment of entire sections.

The failure mechanisms in this type of structures can be divided into two. The out-of-plane failure mechanisms, also called Mode 1 mechanisms, related to the quality of the connections
in the structure and the presence of opening. And the in-plane mechanisms, Mode 2 mechanism, which include slips and shear failure modes.

During the first part of this assessment, all the necessary information for the compliance of the form provided in the guidelines is collected, such as the location and actual condition of the building as it was described in previous sections. After that, based on graphical schemes in order to identify the 28 possible collapse mechanisms, it is developed the evaluation of each one with respect to the church in study. At the end, it is possible to determine of a safety index, ranging from 0 to 1, in which the seismic behavior is studied in a statistical way determining the global behavior of the structure with the weighted average of the behavior of the different parts composing it. With this, it is possible to evaluate if there are problems compromising the security of the structure, regarding its occupancy (utilizable, partially utilizable, non-utilizable).

The 28 failure mechanisms defined in the guidelines are subdivided in different subgroups depending on the part of the church in consideration. 4 mechanisms related to the facade, 5 mechanisms related to the nave, 3 mechanisms related to the transept, 1 mechanism related to the triumphal arch, 2 mechanisms related to the dome, 3 mechanisms related to the apse, 3 mechanisms related to the roof, 4 mechanisms related to the chapels and attached bodies, and 3 mechanisms related to the bell tower and juts. Each of these failure mechanism is shown in Table 7.
For the assessment, only the fact that the indicated elements are present in the church is enough to consider it a vulnerability, and so it is necessary to take into account the possibility of the activation of the corresponding collapse mechanisms.

In the present work, there will be only defined the possible failure mechanisms the church in its actual conditions may undergo, as the information able is not enough to make a full evaluation of the structure. As the geometry of the building and the relevant parts of it were already defined in the previous section, it is possible to identify in a faster way the possible failure mechanism relevant for the study of this structure. For the church in consideration, in its actual conditions, there were identified 16 mechanisms and they are explained below.

A definition of the level of damage, for each mechanism, will be defined depending on the condition of the church in the photographic data able. For this, there will be defined 5 different levels. Null damage (D0), representing the absence of any damage. Mild damage (D1),

### Table 7. Failure mechanisms in a church. (Papa & Di Pasquale, 2011)

<table>
<thead>
<tr>
<th>MECCANISMO DI DANNO</th>
<th>Parte della Chiesa</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1. Ribaltamento della facciata</td>
<td>FACCIATA</td>
</tr>
<tr>
<td>M2. Meccanismi nella sommità della facciata</td>
<td></td>
</tr>
<tr>
<td>M3. Meccanismi nel piano della facciata</td>
<td></td>
</tr>
<tr>
<td>M4. Profilo e Nartece</td>
<td></td>
</tr>
<tr>
<td>M5. Risposta trasversale dell’aula</td>
<td>AULA</td>
</tr>
<tr>
<td>M6. Meccanismi di taglio delle pareti laterali</td>
<td></td>
</tr>
<tr>
<td>M7. Risposta longitudinale del colonnato</td>
<td></td>
</tr>
<tr>
<td>M8. Volte dell’aula o della navata centrale</td>
<td></td>
</tr>
<tr>
<td>M9. Volte delle navate laterali</td>
<td></td>
</tr>
<tr>
<td>M10. Ribaltamento pareti del transetto</td>
<td>TRANSETTO</td>
</tr>
<tr>
<td>M11. Meccanismi di taglio del transetto</td>
<td></td>
</tr>
<tr>
<td>M12. Volte del transetto</td>
<td></td>
</tr>
<tr>
<td>M13. Arco trionfale</td>
<td>ARCO TRIONFALE</td>
</tr>
<tr>
<td>M14. Cupola e tamburo/hiburio</td>
<td>CUPOLA</td>
</tr>
<tr>
<td>M15. Lanterna</td>
<td></td>
</tr>
<tr>
<td>M16. Ribaltamento dell’abside</td>
<td>ABSIDE</td>
</tr>
<tr>
<td>M17. Meccanismi di taglio nell’abside</td>
<td></td>
</tr>
<tr>
<td>M18. Volte del presbitero o dell’abside</td>
<td></td>
</tr>
<tr>
<td>M19. Meccanismi negli elementi di copertura – Pareti laterali dell’aula</td>
<td>COPERTURA</td>
</tr>
<tr>
<td>M20. Meccanismi negli elementi di copertura – Transetto</td>
<td></td>
</tr>
<tr>
<td>M21. Meccanismi negli elementi di copertura – Abside</td>
<td></td>
</tr>
<tr>
<td>M22. Ribaltamento delle Cappelle</td>
<td>CAPPELLE</td>
</tr>
<tr>
<td>M23. Meccanismi di taglio nelle pareti delle cappelle</td>
<td>CORPI ANNESSI</td>
</tr>
<tr>
<td>M24. Volte delle cappelle</td>
<td></td>
</tr>
<tr>
<td>M25. Interazioni in prossimità di irregolarità planoo-parallelistiche (corpi adiacenti, archi rampanti)</td>
<td></td>
</tr>
<tr>
<td>M26. Aggetti (vela, guida, pinnacoli, statue)</td>
<td>AGGETTI</td>
</tr>
<tr>
<td>M27. Torre campanaria</td>
<td>CAMPANILE</td>
</tr>
<tr>
<td>M28. Cella campanaria</td>
<td></td>
</tr>
</tbody>
</table>
representing the first evidence of the activation of the mechanism in consideration, with a limited extension. Moderate damage (D2), representing a more evident activation of the collapse mechanism, but in the first phase of development. Severe damage (D3), representing a significative evidence of the activated failure mechanism, in an intermediate phase of development. Very serious damage (D4), representing mechanisms about to collapse. Collapse (D5), representing the prevalent collapse of the mechanism in consideration.

1.4.1.1 MECHANISMS IN THE MAIN FAÇADE

Figure 1-15: Facade of San Paolo church. [Picture taken by Simona Belmondo] (Mirabello, 2018).

- Façade overturning (M1): Characterized by the out of plane rotation of the plane of the façade either with the formation of a cylindrical hinge with horizontal axis or with the disconnection of the façade from the roof and side walls. The first one, may be the result of large discontinuity constituted by the presence of openings positioned in the low end of the façade. For the second one, it is the result of the breaking of the lateral walls or the formation of vertical cracks. The presence of large opening in the side walls and elements such as covering struts, vaults and arches, pushing the walls favors the activation of such mechanisms.

Figure 1-16. Façade overturning. (Papa & Di Pasquale, 2011)

There were no evidences of the activation of this type of mechanism in the church in consideration, not even after the earthquake. (D0)
• **Mechanisms in the top of the façade (M2):** This flexural mechanism occurs with the out-of-plane displacement of the top part of the façade, the tilting of the tympanum, through the formation of horizontal or inclined hinges. This may be the result of the absence of effective connections with the roof, the presence of large openings, an upper part of great size and weight, rigid curbs, or a heavy covering, among others.

![Figure 1-17. Tilting of the tympanum. (Papa & Di Pasquale, 2011)](image)

Figure 1-17. Tilting of the tympanum. (Papa & Di Pasquale, 2011)

Figure 1-18 shows clearly the activation of this failure mechanism. As the whole part at the top of the façade separated from the structure, this can be considered as a collapse situation (D5).

• **Mechanisms in the plane of the façade (M3):** This mechanism activates with in-plane deformations due to shear cracking or due to traction. The crack pattern for the shear cracking is characterized by oblique cracks along the wall and it may occur due to the presence of large or sealed openings, and significantly slender walls. For the cracking due to traction, the resulting crack pattern results in a vertical crack in the center of the facade, where a weak area is located, while the panels on the sides present diagonal cracks due to out-of-plane actions, typical in tall façades.
There were no evidences of the activation of this type of mechanism in the church in consideration. (D0)

1.4.1.2 MECHANISMS IN THE NAVE

- **Transversal response from the nave (M5):** Due to the out-of-plane displacements of either one or both of the lateral walls of the nave. It also involves the deformation and lowering in the key of the arches or vaults supported by these walls and the formation of plastic hinges. At the same time, the presence of these arches and vault favors the activation of this mechanism, as well of the presence of slender walls.

There can be identified cracks in the vaults of the nave, representing the activation of the mechanism in its first phase (D2).
• **Shear mechanisms of the side walls (M6):** Characterized by shear cracking due to in-plane actions in the walls. The crack pattern includes both single and crossed inclined cracking and may be due to the presence of large openings, thin walls, heavy roofs, among others.

![Shear cracking in the walls of the nave. (Papa & Di Pasquale, 2011)](image)

There can be identified shear cracks in the side walls of the church in consideration. From what it can be seen the level of damage related to this mechanism can be assumed as a moderate damage, representing the activation of the mechanism in its first phase (D2).

• **Longitudinal response of the colonnade (M7):** The mechanism is activated by shear cracking due to in-plane actions. The crack pattern is characterized by cracks in the longitudinal arches or lintels and possible cracking at the base of the columns, and it may be the result of the presence of heavy vaults or roofs supported by the columns.

![Shear cracking in the colonnade. (Papa & Di Pasquale, 2011)](image)
There can be identified cracking at the base of the columns, representing the activation of the mechanism in its first phase (D2).

- **Vaults of the central nave (M8):** This mechanism consists on the growth of shear cracks in the vaults of the central nave. The crack patterns are characterized by the appearance of cracks specially in the more rigid parts of the vaults and this may occur due to the presence of very slender vaults, as it is the case for the church in study, or the action of concentrated loads.
The condition of the vaults in the central nave exhibits the first evidences of the activation of this failure mechanism. Even if after the earthquake, the vaults close to the façade and to the triumphal arch collapsed, they were almost unaffected and just some cracks can be identified in the remained arches, representing the activation of the mechanism in its first phase. Moderate damage (D2).

### 1.4.1.3 MECHANISMS IN THE TRANSEPT

![East view of San Paolo de Mirabello Church.](image)

[Picture taken by Simona Belmondo](Mirabello,2018)

Regarding the transept, even if the north part collapsed completely, the south part is almost complete, and so reference will be made to the condition of the still existing part.

- **Overturning walls of the transept (M10):** This mechanism is characterized by the out-of-plane rotation of the walls of the transept resulting from the disconnection from the cover and the lateral walls of the nave. The resulting crack pattern includes the detachment of the front and side walls or the tilting or disaggregation of the top part of the walls. This mechanism may be activated due to the presence of large openings, heavy covers, among others.

![Overturning of the transept.](image)

(Papa & Di Pasquale, 2011)

There were no evidences of the activation of this type of mechanism. (D0)
• **Shear mechanisms of the transept (M11):** Characterized by shear cracking due to in-plane actions, it exhibits single and crossed inclined cracks which may cross local discontinuities. It may be activated because of the presence of large openings, walls of limited thickness and heavy roofs.

![Figure 1-30. Shear cracking in the transept. (Papa & Di Pasquale, 2011)](image)

There can be identified shear cracking in the walls of the transept, representing the activation of the mechanism in its first phase (D2).

• **Vaults of the transept (M12):** This mechanism is activated with shear cracking on the vaults of the transept. The resulting crack pattern includes cracks in the vaults or disconnection from the arches and this may develop due to the presence of concentrated loads transmitted by the cover or vaults with large spans.

![Figure 1-31. South-east view of the church. [Picture taken by Simona Belmondo] (Mirabello, 2018)](image)
There can be identified in the vault of the transept, representing the activation of the mechanism in its first phase (D2).

1.4.1.4 MECHANISMS IN THE ROOF ELEMENTS

- **Side walls of the nave** (M19): This mechanism activates with the out-of-plane rotation of the lateral walls of the nave. The resulting crack patterns are characterized by cracks and sliding near the wooden beams of the roof structure and disconnection between the curbs and masonry. Significant movements of the roof can be identified in this case.

There were no evidences of the activation of this type of mechanism in the church in consideration (D0).
• **Transept (M20):** This mechanism is activated with out-of-plane rotations of the walls of the transept. The resulting crack patterns are characterized by cracks and sliding near the wooden beams of the roof structure and disconnection between the curbs and masonry. Significant movements of the roof can be identified in this case.

![Figure 1-35. Failure mechanism of the roof in the transept. (Papa & Di Pasquale, 2011)](image)

There were no evidences of the activation of this type of mechanism in the church in consideration. (D0)

### 1.4.1.5 MECHANISMS IN THE CHAPELS AND ADJACENT BODIES

• **Overturning of the chapels (M22):** Characterized by out-of-plane rotation of the walls of the chapels, it exhibits a detachment of the front walls from the side walls. It may occur due to the presence of openings in the chapels which may weaken the walls.

![Figure 1-36. Overturning of the chapels. (Papa & Di Pasquale, 2011)](image)

There were no evidences of the activation of this type of mechanism in the church in consideration. (D0)

• **Shear mechanisms in the walls of the chapels (M23):** This mechanism activates with in-plane shear failure, with the formation of oblique cracks in the walls of the chapel. The resulting crack pattern is represented by single and crossed inclined cracks and also cracks in correspondence with wall discontinuities. The presence of heavy roofing, large openings and walls of limited thickness may influence in the activation of this mechanism.
From Figure 1-38 there can be identified shear cracks in the chapels. These may be considered to represent the first phase of development of the corresponding mechanism and be classified as a moderate damage (D2).

1.4.1.6 MECHANISMS IN THE BELL TOWER

- **Bell tower (M27):** The bell tower may undergo rotations or in-plane cracking of the perimetric walls. This may occur as a result of the presence of significant openings in several levels and a lack of symmetry at the foundation level.
- Belfry (M28): This mechanism is characterized by in-plane deformations in the arches or at the end of the piers. It exhibits the formation of cracks in the arches and rotations or sliding of the piers. It may occur due to the presence of a heavy cover or significant masses acting on it.

![Figure 1-40. Belfry failure mechanism. (Papa & Di Pasquale, 2011)](image)

There were no evidences of the activation of this type of mechanism in the church in consideration. (D0)

After a detailed assessment of all the possible failure mechanisms the building in consideration may undergo under the presence of an earthquake (or after the action of one), it is possible to determine the index of damage $i_d$ as defined in the guidelines.

$$i_d = \frac{d}{5n}$$

Where, $n$ represents the number of mechanisms analyzed ($\leq 28$) and $d$ the sum of the values assigned to each level of damage corresponding to each mechanism (between 0 and 5). If the damage index results to be equal to 1 it would mean the level of damage of the structure for each mechanism was assumed to be in the worst situation. As this value becomes smaller it would mean the structure is in a better state.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>19</td>
</tr>
<tr>
<td>$n$</td>
<td>16</td>
</tr>
<tr>
<td>$i_d$</td>
<td>0.238</td>
</tr>
</tbody>
</table>

Table 8. Determination of the damage index.

By analyzing the value obtained for the damage index it can be seen that the value is not that high. This means the part of the church that remained after the earthquake is still in acceptable conditions and just some interventions are required to guarantee its safety when reconstructing the church and avoid this type of failure mechanisms when dealing with future earthquakes.
However, it is necessary to consider that still the church was partially destroyed and so it is not completely in good conditions. Even if the church exhibits just the firsts phases of activation of certain local mechanisms, these are mainly concentrated in the nave, a so related to the vaults and pillars supporting them. The thickness of the vaults previously presented may be a disadvantage, favoring the activation of such mechanisms and it is necessary to evaluate them.

As the determination of the typology of the masonry was developed based on visual inspections, out of pictures, there could also be considered the development of laboratory or in-situ tests in order to develop a more accurate masonry characterization and determine if the quality of the actual masonry in the church is a problem or not.

Nevertheless, the type of assessment presented in the guidelines, provides a fast and global evaluation of the seismic performance of the churches, with just parameters based on observation and quantitative information easy to obtain. It is to have in mind that this type of assessment is conservative, being not always that realistic, and may end in the necessity to develop expensive strengthening interventions. Thus, this will be considered as a general overview of the actual conditions of the building.
1.5 PROBLEM STATEMENT

After developing a general evaluation of the structure to analyze, “several questions arise if trying to give a first interpretation of the observed damages” (Binda, Cardani, Gentile, Zanzi, & Massetti, 2009). It is necessary to determine what is exactly what wants to be studied and define the methodology to achieve it.

As said before, special attention will be payed to the vaults in the nave and it is of interest to determine their mechanical behavior under their dead loads as well as under a seismic action. As no laboratory test or visits to the site can be developed, in this case, the only tool that can be used is computational modelling. In this way, there will be analyzed the behavior of this structures in a global way, considering the whole building, and locally, considering just the vaults.

Three main questions will be studied. First, it is of interest to determine which would be the response of the of the actual structure to a possible earthquake, both for longitudinal and transversal motions. For this, there will be developed both linear and nonlinear analyses with models considering the geometry of vaulted surfaces in their actual conditions. With this it arises the second question, which seeks to determine why did the vault resist to the earthquakes of 2012. According to given information, these earthquakes hit the building in the longitudinal direction. For this, there will be studied a model considering the vaults in an ideal condition, and the seismic action will be taken from the real signal of the quake that occurred on May 20 of 2012. Finally, as it is expected for the church in consideration to be repaired, it will be analyzed which kind of safety would the vault provide, if repaired with the same characteristics as before. The same model used for evaluating why did the vaults resist to the earthquakes, will be used, but this time it will be evaluated with the response spectrums given by the Italian code. It will be determined if it is enough to repair the vault, leaving it with the same characteristics as before or if it is necessary to consider other kind of interventions. Knowing the case study is a historical building, which makes part of the cultural heritage, the priority is to preserve as much as possible the concept and techniques of the original church and this will be taken into account.
CHAPTER 2. MODELLING

Figure 2-1. San Paolo di Mirabello Church model in Midas Gen.

2.1 GEOMETRY

As the stability of masonry structures, specially of curved ones, is strongly dependent on they geometry it is important to develop a model which represents the structure the most precise as possible. According to (Tralli, Alessandri, & Milani, 2014), in vaulted structures the implementation of the actual imperfections reduces the collapse load up 65% if compared with a model with no imperfections. Nevertheless, due to the commonly complex geometry of this kind of structures this process becomes cumbersome and time consuming.

Because of this, for the modelling of historical buildings, which generally have a unique geometry, it useful to count on with an adequate three-dimensional reconstruction of such structures and the processing of cloud points obtained from a laser scanning is the best choice. With this, it is possible to capture the exact dimensions of an object and safe time.

Figure 2-2. Cloud points San Paolo di Mirabello Church.
The geometry of the vaulted system in study, was determined from cloud points provided by the Superintendence of Ferrara, and these were processed in the software JRC 3D Reconstructor. With the help of this software it was possible to work just with the cloud points of interest and develop the corresponding mesh in order to export it into a .dxf format which was then manipulated in the Mesh Editor of the software Rhinoceros.

Because of simplicity, in order to develop a model which was not so computationally demanding, just the geometry of the vaults was imported from the cloud points, as this was the part of the church of interest. The rest of the building was modelled with beam elements, by making use of the equivalent frame modelling method, in order to represent the walls and columns of the building.

The structural analysis was developed in the software Midas Gen, where both the frame equivalent and the vault models were assembled. Special attention was payed to the connection between the vaults and the pillars supporting them, as this is important in order to guarantee compatibility in the displacements. Because of this, there were defined rigid links at the connections. This type of constraint works by defining a master node, and the corresponding slaves that will undergo the same displacements for the constrained degrees of freedom. In this way, all the nodes corresponding to the transversal arches were constrained to the ones of their matching pillars. Additionally, in order to model the connection of the vaults with the façade, there was defined a vertical rigid diaphragm, to guarantee all the points in the vaults belonging to this end behave in the same way.

Figure 2-3. Definition of the rigid links at the connections between the vaults and the pillars in the nave.
The vaults were modelled with plate elements, thin plate elements, which are normally used in order to model both membrane and bending behavior. Thin plate, makes reference to Kirchhoff’s Thin plate theory were the following hypotheses are considered: (Biondini, 2018)

1. **Straight normal (Kirchhoff-Love).** A straight segment normal to the midplane, after deformation, remains straight, with unaltered length, and normal to the midsurface. This translates in no shear and out-of-plane strains.

2. **Small displacements.** The deflection of the element is assumed to be constant all along its thickness.

3. **Out of plane stresses are neglected.** As they are significantly smaller than the in-plane stresses.

4. **Isotropic linear elastic material.** Only to elastic constants (E,v) are necessary to characterize the linear constitutive law of the material.

5. **Homogenous cross-section.** strains vary linearly along the plate’s thickness.

6. **Parabolic shear stresses profiles.** Based on boundary conditions.

This type of elements is characterized with six degrees of freedom in each node. In Midas Gen, there is the possibility to model either with triangular elements (LST) or with quadrilateral elements (ISOP4), and for the two, their stiffness will be defined both in the in-plane and out of plane directions. For the in-plane direction there will be considered axial and shear stiffness, while for the out of plane direction there will be considered bending and shear stiffness.
In the model generated, there were used both triangular and quadrilateral elements for modelling the vaults, due to their complex geometry. However, quadrilateral elements were preferred as these guaranteed better results in places where more detailed information was required. For the vaults in the original configuration, there was not possible to modify that much the mesh generated automatically in JRC 3D Reconstructor, in order to preserve the exact geometry of the elements. Nevertheless, for the vaults in the ideal conditions it was developed a more regular mesh as it is shown in Figure 2-5.

![Figure 2-5. Destroyed and repaired vaulted system. Modelling with plate elements.](image)

Regarding the equivalent frame modelling method, the walls were represented with a frame system composed of piers (vertical elements, in charge of transferring the inertia loads), spandrels (horizontal elements, connecting the piers), and rigid nodes as a connection of the previous ones. This method is normally used in order to evaluate the global behavior of buildings, as it is assumed that no local collapse mechanisms occur. Because of this, it is necessary to guarantee a box-like behavior of the building and rigid diaphragms are implemented at each level, assuming the connections between walls are effective. As this is not the case here, and we are interested on the local mechanisms of the church, no rigid diaphragms of this type will be considered for this model.

![Figure 2-6. San Paolo di Mirabello church, modelled as an assemblage of plate and beam elements.](image)
2.2 MATERIALS

Due to the lack of information about the materials of the church, it has been decided to use the same mechanical properties for the whole structure. To each element, the material characteristics already defined in the previous chapter (Table 6), were assigned. For the rigid nodes in the frame equivalent system, there was assumed zero weight density, and an elastic modulus of an order 1000 times bigger than the one defined for masonry.

The material non-linearities where assigned directly to the material properties, by defining plastic materials both for beam elements (piers and spandrels only) and for plate elements. No plastic hinges were considered for the beam elements due to the impossibility to define a corresponding non-linearity in the plate elements, in order to develop nonlinear analyses, and so there was assumed a Mohr-Coulomb model for the plate elements and Von Mises model for the beams.

The Mohr-Coulomb model, is generally used for brittle materials which exhibit volumetric plastic deformation. Generally, it applies for materials in which the compressive strength exceeds significantly the tensile strength, such as masonry. This model may overestimate the resistance in compression, but usually the failure in masonry structures occurs due to the lack of tensile strength. The yielding function \( F \) is expressed as follow:

\[
F(\sigma, \kappa) = \tau - (c - \sigma_n \tan(\phi))
\]
Where,

- $\tau$: the magnitude of shearing stress
- $\sigma$: normal stress
- $c$: cohesion
- $\phi$: internal friction angle

![Mohr-Coulomb failure criterion](image)

**Figure 2-8. Mohr Coulomb failure criterion. (Rahman & Ueda, 2013)**

The Mohr-Coulomb criterion, defines a linear relationship between the shear and the compressive strength of the material. It states that failure is reached once the yielding function achieves a positive value, in other terms, once the failure enveloped defined by such function is reached or overpassed. The cohesion $c$, refers to the pure shear strength of the material. In masonry, both the cohesion and internal friction angle, characterize the brick-mortar interfaces.

It defines an elastic-perfectly plastic stress-strain law in which the material is considered homogenous and isotropic. For the definition of the properties to determine the plastic material, reference was made to (Almesfer, Dizhur, Lumanatana, & Ingham, 2014) and (Mosalam, Glascoe, & Bernier, 2009). The value for cohesion was determined based of proposed values resulting from the evaluation of the mechanical properties of unreinforced clay brick masonry buildings, while the angle of internal friction was defined on the basis of typical values for historical masonry buildings (high friction coefficient).
Table 9. Proposed cohesion values. (Almesfer, Dizhur, Lumantarna, & Ingham, 2014)

<table>
<thead>
<tr>
<th>Mortar hardness</th>
<th>Mortar description</th>
<th>Mortar compressive strength (MPa)</th>
<th>Cohesion (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very soft</td>
<td>Raked out by finger pressure</td>
<td>0-1</td>
<td>0.1</td>
</tr>
<tr>
<td>Soft</td>
<td>Scratches easily with finger nails</td>
<td>1-2</td>
<td>0.3</td>
</tr>
<tr>
<td>Medium</td>
<td>Scratches with finger nails</td>
<td>2-5</td>
<td>0.5</td>
</tr>
<tr>
<td>Hard</td>
<td>Scratches using aluminium pick</td>
<td>5-7</td>
<td>0.7</td>
</tr>
<tr>
<td>Very hard</td>
<td>Does not scratch with above tools</td>
<td>To be established from testing</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2-9: Mohr-Coulomb parameters set for Plate elements.

Von Mises criterium was used to model the material nonlinearity on beam elements, since the definition of plastic hinges was not compatible with the material nonlinear analysis in Midas Gen. This model is based on distortional strain energy and states that, when the shearing distortion of an element under a combined stress state is equal to, or greater than, the one of a uniaxial stress state, the material fails.

The distortion energy in a three-dimensional space can be expressed as a function of the principal stresses as follows:

$$u_d = \frac{1 + \nu}{3E} \left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]$$
Von Mises failure criteria can be then expressed as:

\[
\left[ \frac{(σ_1 - σ_2)^2 + (σ_2 - σ_3)^2 + (σ_3 - σ_1)^2}{2} \right]^{1/2} ≥ σ_y
\]

Where the left side of the inequality represents the so called Von Mises Stress \( σ_v \) and \( σ_y \) represents the uniaxial yield stress of the material. In this way, by making use of this model, just by analyzing the maximum Von Mises Stress acting in an element it is possible to determine if it reaches failure or not.

In Midas Gen, this model is represented with an elastic perfectly plastic constitutive law. It provides information about the plastic axial hinges in the beam elements as well as for the plastic bending moments about both the major and minor axes of the element. However, the couple effect between the axial forces and the moments is not considered. (Midas Gen, 2014)

Von Mises model, was used to describe the compression behavior of the beam elements, in order to control the maximum compressive stresses in the elements. Because of this, the Initial uniaxial yield stress to define in Midas Gen was the corresponding material strength in compression for seismic actions. This can be done, as we are considering that the bearing capacity under seismic actions of the walls is just defined by the in-plane behavior, and so we are considering a plane-stress problem. However, in the case of a three-dimensional state of stresses, this model is hardly accepted for brittle materials. According to (Zucchini & Lourenco, 2002) Von Mises criteria has been used for the description of the compression behavior in plane-stress problems by a number of authors.

Figure 2-10: Von Mises parameters set for beam elements
Regarding the tensile strength, it is well known that masonry elements have a very low resistance in tension. Because of this, a verification of the stress-state of the beam elements was developed directly from the results obtained in the non-lineal analysis, where no strength in tension was assumed.

It is necessary to ensure that the local axes of the elements composing both the walls and the vaults, are well defined so that a correct interpretation of the behavior in these elements can be guaranteed. All the parts conforming an element must have their local axes aligned, and this will be evidenced when analyzing the resulting stresses, as there must be seen a regular transition between the resulting values.

Figure 2-11. Von Mises Yield Criterion (Merchoir, 2006)

Figure 2-12. Check for the correct alignment of the elements local axes.
2.3 LOADS DEFINITION

When defining the acting loads in the structure, the self-weight (G1) of the elements composing the frame system, as well as the plate elements composing the vaults, were computed automatically with Midas. For this, it was necessary to define its corresponding load case and specify the direction of the load as shown in Figure 2-13.

![Figure 2-13: Self-Weight load definition](image)

The roof structures as well as the vault in the transept were not modelled, and for these there were considered their masses distributed on top of the beam elements. This additional permanent loads (G2), were applied as concentrated loads, in order to prevent an early failure of the elements composing the structure due to additional moments that could have generated when introducing the loads as distributed ones. It was used a weight of 120 kg/m$^2$ for the roof, considering a “cover with small wooden warping, sanded or brick paneling and robe of Marseilles tiles” (Spizuoco), and for the vault there was considered its weight density of 18 kN/m$^3$. In Table 10 and Figure 2-15 it is described the way in which the loads were distributed among the elements, considering a symmetrical distribution of loads along the longitudinal direction of the nave.

![Figure 2-14. Dead loads definition.](image)
Figure 2-15. Concentrated loads distribution in the model.

<table>
<thead>
<tr>
<th>REGION</th>
<th>AREA</th>
<th>LOAD (KN)</th>
<th>BEAM ELEMENTS</th>
<th>CONCENTRATED LOAD</th>
<th>LOAD 1 (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>20.70</td>
<td>24.840</td>
<td>1,2,3,4</td>
<td></td>
<td>6.210</td>
</tr>
<tr>
<td>A2</td>
<td>19.00</td>
<td>22.800</td>
<td>2,4,5,6</td>
<td></td>
<td>5.700</td>
</tr>
<tr>
<td>A3</td>
<td>14.10</td>
<td>16.920</td>
<td>5,7</td>
<td></td>
<td>4.230</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6,8,9</td>
<td></td>
<td>2.820</td>
</tr>
<tr>
<td>A4</td>
<td>15.80</td>
<td>18.960</td>
<td>10,11</td>
<td></td>
<td>4.740</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12,13,14</td>
<td></td>
<td>3.160</td>
</tr>
<tr>
<td>A5</td>
<td>3.90</td>
<td>4.680</td>
<td>15</td>
<td></td>
<td>4.680</td>
</tr>
<tr>
<td>A6</td>
<td>4.00</td>
<td>4.800</td>
<td>16</td>
<td></td>
<td>4.800</td>
</tr>
<tr>
<td>T1</td>
<td>40.00</td>
<td>48.000</td>
<td>17</td>
<td></td>
<td>24.000</td>
</tr>
<tr>
<td>T2</td>
<td>41.60</td>
<td>49.920</td>
<td>18</td>
<td></td>
<td>24.960</td>
</tr>
<tr>
<td>T3</td>
<td>32.10</td>
<td>38.520</td>
<td>19</td>
<td></td>
<td>19.260</td>
</tr>
<tr>
<td>T4</td>
<td>37.80</td>
<td>45.360</td>
<td>20</td>
<td></td>
<td>22.680</td>
</tr>
<tr>
<td>T5</td>
<td>31.25</td>
<td>37.494</td>
<td>21,23,24,26,27,29,30,32,28,31</td>
<td>4.687</td>
<td>8.174</td>
</tr>
<tr>
<td>VAULT</td>
<td>60.55</td>
<td>65.395</td>
<td></td>
<td></td>
<td>16.349</td>
</tr>
</tbody>
</table>

Table 10. Load distributions.
CHAPTER 3. LINEAR ANALYSIS

In order to guarantee the stability of the vaulted structures under their own weight, it was implemented a static analysis. There were developed two models, one with the whole structure in order to study the behavior of the vaults considering their interaction with the other parts of the church and the other one just with the vaults and the corresponding definition of their boundary conditions. For these models there was analyzed their stress state, verifying their equilibrium conditions and the strength of the materials.

It is to highlight that the same analysis developed for arches could be extended to vaults, however, turning the two-dimensional single static analysis of arches into a three-dimensional one increases its complexity significantly and this is directly related to the complexity of their geometry. From a geometrical point of view, the most single type of vaults, barrel vaults, could be seen an assemblage of single arches, and one could say their static analysis might be developed by simply analyzing isolated vaults strips of unit thickness. However, from a static point of view, even if this was the most widely used method for analyzing masonry vaults, it doesn’t capture their real behavior, as the interaction between the single strips is neglected (Taliercio, 8. Masonry vaults, 2017).

For vaults it is necessary to consider their flexural stiffness in their longitudinal direction, as the variation in their length will influence both their deformability and their non-linear behavior. As the length of the vault increases, its vertical displacements will decrease. Additionally, if a good interaction between the arches composing the vault is guaranteed, this may result in an increase of their load carry capacity.

By analyzing the Finite Elements model created for the vaults, the resulting stresses are shown below. There were considered the vaults both in their original condition, with their ends destroyed, in order to study the safety of the actual configuration, and in an ideal condition assuming they will be repaired with the same characteristics as before. As it will be seen, the definition of the geometry influences significantly in the final results.
3.1 STATIC ANALYSIS UNDER SELF-WEIGHT

3.1.1 DEFORMATED SHAPE

By analyzing first, the isolated vault models, it can be observed how the deformation of a repaired vault is reduced with respect to the one of the actual vault, from 3 mm to 1 mm when comparing the maximum displacements. This occurs as the restoration of the of the barrel vaults and transversal arches at the two extremes of the vaulted system, which were broken during the earthquake, provides additional stiffness to this arrangement reducing in a general way the displacements. The displacements in the actual vault range from 0 to 3.6 mm, and the maximum displacements can be encountered at the remaining part of the barrel vault that collapsed at the east end of the nave. Instead, in the ideal conditions the maximum displacement is around 1.6 mm and the can be appreciated at the top of the three barrel vaults in the system. From this it can be appreciated how the displacements of the vaults increase as the height increases, showing the less rigid parts in the system and the most susceptible to suffer from cracking.

![Figure 3-1: Deformed shape of actual vault. Units: meters.](image1)

![Figure 3-2: Deformed shape of repaired vault. Units: meters.](image2)
If now the whole structure is analyzed, this model shows a higher deformation on the vault, since the edges are no longer supported by fixed boundary conditions, but instead they are supported by flexible beams and columns. The highest deformation that can be observed, is located in the middle section of the vaulted system, affecting principally the central barrel vault. This is expected, since supports with higher rigidity (pillars with a larger cross-sectional area) are located at the extremes of the nave, while in the middle there are pillars with a smaller section. Additionally, this is the part of the system which is constrained by more elements.

If compared again with the model, with the vaults in ideal conditions, there can be appreciated a difference of displacements of about the double for the first one model.
From the four models, it can be said that the part of the system most susceptible to undergo large displacements corresponds to the barrel vaults. If these elements reach displacements which are considerably high, this may lead the reach an unstable equilibrium condition resulting in the collapse of the whole system or a part of it. As for the actual structure the parts that were destroyed were also the barrel vaults, the ones at the extremes, is to have in mind for further analyses that this type of vaults may correspond to the most vulnerable parts of the system.
STRESS ANALYSIS

3.1.1.1 Isolated Vault models

From the stress state in the longitudinal direction (Figure 3-7) it can be seen that the parts of the system where there is a change in the cross-section or in the slope, are the most demanded ones. This may occur due to the sudden change in the stiffness of the elements in the interface.

From the stress state in the vertical direction (Figure 3-8) it is clear how the elements of the transversal arches are the most demanded ones. It is important to highlight that these are experiencing tensile stresses, which may translate in plastic hinges at both sides. Nonetheless, the plastic hinges in the arches are not enough to activate a mechanism of collapse.

Focusing now on the local stresses, there were analyzed the membrane stresses acting in the system. From the stress state in Y (Figure 3-9), which corresponds to the circumferential stresses acting in the vaults, and the ones in X (Figure 3-10) corresponding to the normal stresses acting along the barrel vaults’ axes, it is possible to see how the vaulted systems in both the actual state and an ideal one, are only under compression stresses, which guaranties they are stable under they own weight.
By comparing the results obtained, locally, with the theoretical ones computed for a single barrel vault with a width of 3.77 meters (Figure 3-12), it can be observed that the stress states obtained with the models are in accordance with the theoretical ones. It must be underlined that the stresses computed theoretically, correspond to the ones determined with the theory of thin shells in which it considers only in plane stresses. Because of this the theoretical results underestimate the real stresses acting on the vault and this is evidenced comparing the computed theoretical values with the results obtained in the model. Additionally, it must be considered that only a barrel vault is being considered theoretically and not the whole configuration of the system.
Figure 3-12. Circumferential Stress state, computed with the theory of thin shells.

From Figure 3-12 it can be evidenced how in barrel vaults; the circumferential stresses are the ones who counteract the loads for which the midline of the cross-section is the catenary. This can be seen, as the plot of these stresses generates a surface in which it can be appreciated the vaults shape. While the normal stress Nx and the shear stresses N_αθ are in charge of supporting the remaining loads.

Figure 3-13. Normal Stress state along the longitudinal axis, computed with the theory of thin shells.

Figure 3-14. Shear Stress state, computed with the theory of thin shells.
3.1.1.2 Complete vault and church model

Studying the whole structure, there were analyzed both the vaulted system and the elements supporting it. As it can be observed in Figure 3-15, the columns that support the vaults are the more demanded ones, with the highest axial compression stresses at their base. The stress range goes from the maximum compression stress of 700 \( kN/m^2 \) to the maximum tensile stress of 96 \( kN/m^2 \) in the walls and beams in the nave. According to the compression strength considered for the model for vertical actions (890 \( kN/m^2 \)) these stresses are still under the limit; however they are really close and this may be evidenced when studying the actual conditions of the structure presented in Section 1.4. Additionally, it is necessary to pay attention to the tensile stresses that may arise in the building, which will result in the cracking of the elements involved.

![Figure 3-15: Axial force on beam elements. Units: kN/m2.](image)

For the vaults, similar values were obtained as the ones in the isolated models. As it can be seen, both in the X and Y local axes, the vaults exhibit compression stresses under their own weight, being an indicator of their stability.

![Figure 3-16: Membrane stress state in local XX direction. Units: kN/m2.](image)
If the circumferential stresses are observed from another angle, it can be identified the tendency of the thrust line of the transversal arches. As it can be seen, at the intrados in the key, the compression stresses are decreasing which means the thrust line may be close to this edge. Still, for the transversal arch at the east side of the church, it doesn’t reach positive values, and so the arch it can be said the it is still safe, with no formation of plastic hinges due to its self-weight.

There were analyzed the transverse arches with the educational tools provided in the web page “Masonry at MIT”, developed by Phillip Block, and with these it was possible to determine in an approximated way the limit thrusts. With a thickness to radius ratio of the 13% the resulting limit thrusts are shown in Figure 3-19.
Figure 3-19. Minimum and Maximum thrust for the arches in the system.

It shows the arches in consideration can bear a minimum thrust of about 18% of their total weight, and a maximum one of about 26% of their total weight. Still, it is necessary to evaluate the response of the whole system, to make inferences about it. For now, it can be said that due to the thickness of the arch its bearing capacity is really limited.

As the arches in consideration are semicircular, by following the theory, their thrust line under their self-weight should be located exactly in their midline. However, it is necessary to consider that imperfections in their geometry may change significantly the line of action of the compressive forces and so this may not be precisely true. According to (Huerta, 2006), the limit arch thickness for semicircular arches is about 1/18 the span. For the arch in consideration the limit arch would be around 0.54 meters thick and the corresponding geometrical safety factor GSF, computed as the ratio between the limit thickness and the actual one would be 1.15. As it can be seen, this ratio is really close to the unit and this means its thickness is almost close to the limit. Its line of thrust under its own weight is not even contained within the middle half of the arch (GSF=2) and any variation of loads acting in the arch may cause the thrust line to touch the boundaries of the arch generating hinges which may lead to collapse.
CHAPTER 4. KINEMATIC ANALYSIS

In masonry structures, and specially in churches, it is common to identify local collapse mechanisms when subjected to seismic actions. Because of this, it is necessary to identify the most vulnerable parts and individually study them. Making reference to the kinematic limit analysis, by transforming a part of the church into a kinematic chain, it is possible to determine its collapse multiplier for different directions of the seismic action, as well as the corresponding collapse mechanisms.

Regarding San Paolo di Mirabello Church, as the attention is mainly focused on the vaulted surfaces in the nave, only a local analysis will be performed for these region. Taking in account the low tensile strength of masonry, there are defined possible mechanism of collapse in the structure, under the action of horizontal distributed loads. Making use of the principle of virtual works, equalizing the stabilizing loads in the system with the ones activating the mechanism in consideration, it is possible to determine the corresponding load multiplier. With this, it can be determined the relationship between the applied loads and the resulting virtual displacements in the structure, as an evolution of its collapse mechanism, and determine to what extent it is able to withstand an specific action.

Local mechanisms for vaulted structures are both verified for in plane and out of plane actions. However it is not that easy to develop the computations by hand, as this becomes a complex three-dimensional problem. Because of this, there will be developed an evaluation with limit analysis, on a single transversal arch and for one of the piers supporting the vaulted system. For the whole system the resulting load at collapse and corresponding mechanism will be evaluated with other types of analyses in the next chapters.

4.1 ARCHES

There was used a Matlab script (KCLCalculator.m) developed by (Stockdale, et al., 2018) able to determine the collapse mechanism of arches both under point loads and constant horizontal acceleration conditions. For this case it was only developed a kinematic analysis for the horizontal actions. For different discretizations of the arch geometry (different number of blocks) there were obtained different results for the collapse load multiplier. However, the
collapse mechanisms in all the cases were almost the same. After a weighted average of the results obtained, it was considered a collapse multiplier of around 0.056. The corresponding collapse mechanism, is characterized with four rotational hinges, as the ones shown below.

Figure 4-1. Computation of the collapse multiplier with 11 blocks.

Figure 4-2. Computation of the collapse multiplier with 21 blocks.

Figure 4-3. Computation of the collapse multiplier with 41 blocks
The lowest is the collapse multiplier, the structure will be more susceptible to reach collapse with a lower value of the load. For this case, the values obtained for the transversal arch represent only the 5% of its total weight. If reference is made then to Figure 4-4, as the depth of the element increases, its bearing capacity will increase as well. Because of this, the values obtained from this analysis may not be considered as critical. Still, the low resistance of a single arch to transversal loads can be evidenced, and this part of the system can be considered as vulnerable, even if it is assembled with other elements. For vaults, their bearing capacity is significantly higher, and for this case it is expected a resistance at least 10 times higher than the one for the transversal arch.

![Figure 4-4. Pushover curves, for arched elements with different lengths. (Romano & Grande, 2008)](image)

4.2 PILLARS

The stability of vaulted structures is also conditioned to the stability of their supporting systems. The pillars supporting these elements must have adequate dimensions in order to resist the thrust imposed by curved structures. Because of this, it is necessary to evaluate the maximum thrust the pillars in the nave are able to resist, as the collapse of the these may cause the collapse in the vaulted structures. Either the effect of large displacements in the pillars or the cracking in a specific region, may produce the change in configuration of the system for which its stability depends mainly on its geometry.

Consider the pillar in Figure 4-5 as a rigid block, assuming friction between the pillar and the soil is large enough to prevent sliding. When this element is subjected to a horizontal load at its top part, the whole body undergoes a rotation. The maximum load this element can resist, before it fails due to overturning, can be determined by making use of the principle of virtual works, equalizing the work done by the external load ($\lambda w$) with the one done by the equilibrating load ($w$). In this way, the corresponding collapse multiplier can be expressed as:
By considering now the action of the seismic load, uniformly distributed horizontal loads, the collapse multiplier changes as the resultant of the external loads results at the middle high of the element. The boundary conditions change as well, as the beams in the transversal direction are assumed to provide an effective constraint to the displacements in the horizontal direction. In this way, the resulting collapse mechanisms for both directions are different and can be seen in Figure 4-6.

The resulting collapse multipliers for each case are shown in Table 11.

<table>
<thead>
<tr>
<th>LONGITUDINAL DIRECTION</th>
<th>TRANSVERSAL DIRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>DISTRIBUTED LOADS</td>
</tr>
<tr>
<td></td>
<td>λot</td>
</tr>
<tr>
<td>2.24</td>
<td>0.27</td>
</tr>
<tr>
<td>0.86</td>
<td>0.10</td>
</tr>
<tr>
<td>1.64</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 11. Collapse multipliers for different pillars in the nave.
CHAPTER 5. LINEAR RESPONSE SPECTRUM ANALYSIS

The linear dynamic analysis consists on the superposition of natural modes. Basically, it is about breaking down the structure into many different oscillators, each with its own frequency and with its maximum response to dynamic excitation. With this, it is possible to determine which mechanism in the structure is the governing one and its corresponding hazard. As it is a linear analysis, the structure is assumed to behave elastically just at the beginning of the excitation, then it will start to crack. The following steps were made to perform this analysis:

- Determination of the mass sources of the structure.
- Determination of the response spectrum for the structure.
- Selection of the number of modes to consider in the Eigen Value Analysis.
- Run the Linear Spectrum Analysis.
- Post-processing of the results.

5.1 MASS SOURCES

For the development of dynamic analyses such as the Response Spectrum Analysis, it is necessary to determine which will be the masses of the system to consider, as these ones will be the ones related with the inertia forces. In Midas, in order to consider both the contributions of the self-weight of the elements in the model as well as the ones of the permanent loads previously defined, it is necessary to specify this on the global control of the model and to determine the additional load case to consider.

![Figure 5-1: Used commands to convert loads to mass.](image)
5.2 RESPONSE SPECTRUM

The response spectrum, for the building in consideration, was determined based on characteristics of the place specified as follows:

<table>
<thead>
<tr>
<th>REGION</th>
<th>EMILIA-ROMAGNA</th>
<th>LONGITUDE 11.4628</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROVINCE</td>
<td>FERRARA</td>
<td>LATITUDE 44.8267</td>
</tr>
<tr>
<td>COMUNE</td>
<td>MIRABELLO</td>
<td></td>
</tr>
<tr>
<td>SEISMIC CLASSIFICATION</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Table 12. Information about the site.

![Figure 5-2. Classificazione sismica al 2015. Presidenza del Consiglio dei Ministri. Dipartimento della protezione civile. Ufficio rischio sismico e vulcanico.]

Mirabello is located in a seismic zone 3, with low probability of occurrence, according to the seismic classification of 2015 from the Department of Civil Protection of Italy. It was evaluated for a nominal life of 50 years and it was identified as class III structure.

<table>
<thead>
<tr>
<th>VITA NOMINALE (ANNI)</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLASSE D’USO</td>
<td>III</td>
</tr>
<tr>
<td>PERIODO DI RIFERIMENTO PER L’AZIONE SISMICA VR</td>
<td></td>
</tr>
<tr>
<td>COEFFICIENTE D’USO Cu</td>
<td>1.5</td>
</tr>
<tr>
<td>VR (ANNI)</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 13. Classification of the building according to the NTC.

The values shown in Table 14 were determined from the document ESPETTRI-NTC provided by the “Consiglio Superiore dei Lavori Pubblici”.

<table>
<thead>
<tr>
<th>LIMIT STATE</th>
<th>T[years]</th>
<th>ag/g</th>
<th>ag</th>
<th>Fo</th>
<th>Tc* [S]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLO</td>
<td>45</td>
<td>0.047</td>
<td>0.46107</td>
<td>2.481</td>
<td>0.268</td>
</tr>
<tr>
<td>SLD</td>
<td>75</td>
<td>0.061</td>
<td>0.59841</td>
<td>2.501</td>
<td>0.277</td>
</tr>
<tr>
<td>SLV</td>
<td>712</td>
<td>0.173</td>
<td>1.69713</td>
<td>2.561</td>
<td>0.275</td>
</tr>
<tr>
<td>SLC</td>
<td>1462</td>
<td>0.231</td>
<td>2.26611</td>
<td>2.496</td>
<td>0.283</td>
</tr>
</tbody>
</table>

Table 14. Parameter for the determination of the response spectrum depending on the limit state.

The properties for the ground were taken from (Giancalo Maselli Diagnostica & Engineering, 2016) in which there has been studied the liquefaction of the zone. Mirabello was characterized with a ground category D, with deposits of low-density coarse-grained soils or fine-grained
soils with low consistency. And additionally, it was also subclassified in the class S2 which makes reference to deposits of soil susceptible to liquefaction or sensitive clays.

<table>
<thead>
<tr>
<th>CATEGORIA SOTTOSUOLO</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>CATEGORIA SOTTOSUOLO AGGIUNTIVA</td>
<td>S2</td>
</tr>
<tr>
<td>CATEGORIA TOPOGRAFICA</td>
<td>T1</td>
</tr>
</tbody>
</table>

Table 15. Characteristics of Mirabello's soil.

For this assessment there were considered the SLD (Damage Limit State) concerning the serviceability limit state verification, with a probability of exceedance of 63% of the reference period, and the SLV (Life Safety Limit state) for the ultimate limit state with a probability of exceedance of 10%.

<table>
<thead>
<tr>
<th>ELASTIC RESPONSE SPECTRUM</th>
<th>SLS</th>
<th>ULS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Coeff ampliazione stratigrafica Ss</td>
<td>1.800</td>
<td>1.735</td>
</tr>
<tr>
<td>Cc</td>
<td>2.375</td>
<td>2.384</td>
</tr>
<tr>
<td>Coeff ampliazione topografica S</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>1.800</td>
<td>1.735</td>
</tr>
<tr>
<td>TB (s)</td>
<td>0.219</td>
<td>0.219</td>
</tr>
<tr>
<td>TC (s)</td>
<td>0.658</td>
<td>0.656</td>
</tr>
<tr>
<td>TD (s)</td>
<td>1.844</td>
<td>2.292</td>
</tr>
<tr>
<td>PGA (a)</td>
<td>1.077</td>
<td>2.945</td>
</tr>
</tbody>
</table>

Table 16. Parameters for the elastic response spectrum definition.

The q factor was defined, for a structure with ordinary masonry and one floor. The values obtained for the elastic response spectrum were reduced with this factor and this was the spectrum considered, taking into account the dissipative capacity of the structure. Still, it was considered that as (Campostrini, 2014) affirmed, this factor is still under a scientific debate for this type of structures.

<table>
<thead>
<tr>
<th>q FACTOR</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>qo</td>
<td>1.4</td>
</tr>
<tr>
<td>ao/a1</td>
<td>0.8</td>
</tr>
<tr>
<td>Kr</td>
<td>2.24</td>
</tr>
</tbody>
</table>

Table 17. Determination of the q factor for non-linear analyses.
For considering the seismic events of 2012, it was used the signal of the quake of the 20th May of 2012, at 3:02 am. This signal was taken from the closest station found to Mirabello (around 40 km), in the municipality of Mirandola (MO). The data obtained from the “Istituto Nazionale di Geofisica e Vulcanologia (Itaca)” is shown in the figures below.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Mw</th>
<th>Style of Faulting</th>
<th>Stat. Code</th>
<th>ECI</th>
<th>Rep. [km]</th>
<th>Processing</th>
<th>corr. PGA (cm/s²)</th>
<th>PSV (cm/s)</th>
<th>Location</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT-2012-0009</td>
<td>2012-05-20 03:02:47</td>
<td>5.1</td>
<td>Thrust Faulting</td>
<td>ITAPA@</td>
<td>C</td>
<td>7.400</td>
<td>manually processed</td>
<td>205.223</td>
<td>10.863</td>
<td>60</td>
<td>HN</td>
</tr>
</tbody>
</table>

Table 18. Station data, for the earthquake signal. (Itaca)

Figure 5-4. Earthquake 2012 accelerations.

Figure 5-5. Response spectrum for the earthquake of 2012.
5.3 MODAL ANALYSIS

A modal analysis was developed in order to determine the vibration modes of the structure and their corresponding periods. By analyzing the modal participation masses, it is possible to observe that the second modal shape resembles a motion in X, corresponding to the direction longitudinal to the church. While the third mode, is related to a displacement in the Y direction, transversely to the church. For these modes, the modal participation masses constitute less than the 85% of the total mass of the structure. The NTC-08 states that for dynamic linear analyses there must be considered all the modes with a significant modal participation. In this way all the modes with a participation greater than 5% must be considered and at least a total participation of 85% must be used for the analysis. Because of this, it is necessary to consider more than one mode for this analysis in both directions.

Table 19. Frequencies and periods from the eigenvalue analysis, for the model with the vault destroyed.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (rad/sec)</th>
<th>Period (sec)</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.255</td>
<td>1.075</td>
<td>0.078</td>
</tr>
<tr>
<td>2</td>
<td>17.650</td>
<td>0.570</td>
<td>0.050</td>
</tr>
<tr>
<td>3</td>
<td>18.248</td>
<td>0.547</td>
<td>0.439</td>
</tr>
<tr>
<td>4</td>
<td>21.034</td>
<td>0.491</td>
<td>0.290</td>
</tr>
<tr>
<td>5</td>
<td>27.746</td>
<td>0.360</td>
<td>0.290</td>
</tr>
<tr>
<td>6</td>
<td>24.739</td>
<td>0.411</td>
<td>0.253</td>
</tr>
<tr>
<td>7</td>
<td>25.325</td>
<td>0.389</td>
<td>0.238</td>
</tr>
<tr>
<td>8</td>
<td>27.746</td>
<td>0.360</td>
<td>0.290</td>
</tr>
<tr>
<td>9</td>
<td>33.560</td>
<td>0.305</td>
<td>0.197</td>
</tr>
<tr>
<td>10</td>
<td>37.610</td>
<td>0.260</td>
<td>0.160</td>
</tr>
<tr>
<td>11</td>
<td>38.164</td>
<td>0.255</td>
<td>0.154</td>
</tr>
<tr>
<td>12</td>
<td>39.660</td>
<td>0.250</td>
<td>0.154</td>
</tr>
</tbody>
</table>

Table 20. Frequencies and periods from the eigenvalue analysis, for the model with the vault with ideal conditions.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (rad/sec)</th>
<th>Period (sec)</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.255</td>
<td>1.075</td>
<td>0.078</td>
</tr>
<tr>
<td>2</td>
<td>17.752</td>
<td>0.570</td>
<td>0.050</td>
</tr>
<tr>
<td>3</td>
<td>18.248</td>
<td>0.547</td>
<td>0.439</td>
</tr>
<tr>
<td>4</td>
<td>21.034</td>
<td>0.491</td>
<td>0.290</td>
</tr>
<tr>
<td>5</td>
<td>27.746</td>
<td>0.360</td>
<td>0.290</td>
</tr>
<tr>
<td>6</td>
<td>24.739</td>
<td>0.411</td>
<td>0.253</td>
</tr>
<tr>
<td>7</td>
<td>25.325</td>
<td>0.389</td>
<td>0.238</td>
</tr>
<tr>
<td>8</td>
<td>27.746</td>
<td>0.360</td>
<td>0.290</td>
</tr>
<tr>
<td>9</td>
<td>33.560</td>
<td>0.305</td>
<td>0.197</td>
</tr>
<tr>
<td>10</td>
<td>37.610</td>
<td>0.260</td>
<td>0.160</td>
</tr>
<tr>
<td>11</td>
<td>38.164</td>
<td>0.255</td>
<td>0.154</td>
</tr>
<tr>
<td>12</td>
<td>39.660</td>
<td>0.250</td>
<td>0.154</td>
</tr>
</tbody>
</table>

Table 21. Modal participation masses, for the model with the vault destroyed.

<table>
<thead>
<tr>
<th>Mode</th>
<th>MASS(U)</th>
<th>SUM(U)</th>
<th>ROT(M)</th>
<th>ROT(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>2</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>3</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>4</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>5</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>6</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>7</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>8</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>9</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>10</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>11</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>12</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
<td>0.071</td>
</tr>
</tbody>
</table>
Normally, in masonry churches it is necessary to consider a higher number of modes in order to reach the minimum participation masses required by the norm. This is because of their complex geometry, which makes the dynamic response of the building to be influenced by local modes (Campostrini, 2014). If rigid diaphragms were considered, assuming good connections between the walls of the structure, the modal participation masses should result significantly high in the first modes of vibration. Nevertheless, it is of interest to determine the possible local collapse mechanisms that the building might undergo.

<table>
<thead>
<tr>
<th>Mode</th>
<th>TRAN-X</th>
<th>TRAN-Y</th>
<th>ROT-X</th>
<th>ROT-Y</th>
<th>ROT-Z</th>
<th>TRAN-Z</th>
<th>ROT-NX</th>
<th>ROT-NY</th>
<th>ROT-NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0141</td>
<td>0.0146</td>
<td>5.9150</td>
<td>5.9154</td>
<td>5.9020</td>
<td>5.9020</td>
<td>1.1907</td>
<td>1.1907</td>
<td>0.0987</td>
</tr>
<tr>
<td>2</td>
<td>70.6846</td>
<td>70.6832</td>
<td>0.6295</td>
<td>6.5953</td>
<td>0.0007</td>
<td>0.0007</td>
<td>1.0039</td>
<td>1.0039</td>
<td>0.3652</td>
</tr>
<tr>
<td>3</td>
<td>0.2780</td>
<td>71.4122</td>
<td>77.9144</td>
<td>84.4497</td>
<td>0.0376</td>
<td>0.0472</td>
<td>0.3387</td>
<td>2.1387</td>
<td>0.0191</td>
</tr>
<tr>
<td>4</td>
<td>3.3172</td>
<td>74.4657</td>
<td>4.4671</td>
<td>89.8970</td>
<td>0.0893</td>
<td>0.0955</td>
<td>1.0388</td>
<td>3.7295</td>
<td>0.0320</td>
</tr>
<tr>
<td>5</td>
<td>0.0603</td>
<td>74.4667</td>
<td>0.3360</td>
<td>89.2297</td>
<td>0.2607</td>
<td>0.1142</td>
<td>0.2430</td>
<td>3.4155</td>
<td>0.0560</td>
</tr>
<tr>
<td>6</td>
<td>0.3203</td>
<td>74.8967</td>
<td>0.0670</td>
<td>89.2948</td>
<td>0.0895</td>
<td>0.1637</td>
<td>1.7453</td>
<td>5.1608</td>
<td>0.0195</td>
</tr>
<tr>
<td>7</td>
<td>0.2344</td>
<td>76.1012</td>
<td>4.2612</td>
<td>85.5460</td>
<td>0.0009</td>
<td>0.1638</td>
<td>0.1195</td>
<td>3.1724</td>
<td>0.0345</td>
</tr>
<tr>
<td>8</td>
<td>0.6495</td>
<td>75.7516</td>
<td>2.2093</td>
<td>95.7499</td>
<td>0.0141</td>
<td>0.1979</td>
<td>3.2624</td>
<td>8.4348</td>
<td>0.0057</td>
</tr>
<tr>
<td>9</td>
<td>4.0033</td>
<td>85.2594</td>
<td>1.1851</td>
<td>86.3355</td>
<td>0.0070</td>
<td>0.2049</td>
<td>0.0731</td>
<td>8.5509</td>
<td>0.1114</td>
</tr>
<tr>
<td>10</td>
<td>3.9436</td>
<td>84.1980</td>
<td>0.0872</td>
<td>97.0166</td>
<td>0.0114</td>
<td>0.2223</td>
<td>0.0712</td>
<td>8.5911</td>
<td>0.0065</td>
</tr>
<tr>
<td>11</td>
<td>1.7198</td>
<td>85.9812</td>
<td>0.0603</td>
<td>97.0769</td>
<td>0.0172</td>
<td>0.2395</td>
<td>0.1061</td>
<td>8.6403</td>
<td>0.3297</td>
</tr>
<tr>
<td>12</td>
<td>0.3261</td>
<td>86.2442</td>
<td>0.1644</td>
<td>97.2223</td>
<td>0.0059</td>
<td>0.2453</td>
<td>0.0004</td>
<td>8.6467</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

Table 22. Modal participation masses, for the model with the vault with ideal conditions.

Figure 5-6: Second modal shape (X direction).

Figure 5-7: Third modal shape (Y direction).
5.4 LINEAR SPECTRUM ANALYSIS DATA CONTROL

The modal combination rule, specifies how to combine the results obtained for each mode in order to estimate the total response of the structure. Midas Gen allows the user to select between several combination rules such as SRSS, CQC, absolute sum, among others. Based on the NTC specifications, it was chosen to use the CQC (Complete quadratic combination). This method is usually used in cases where the natural frequencies for the different modes of vibration of the structure are close, which is the case. It can be written as:

\[ R_{\text{max}} = \left( \sum_{i=1}^{N} \sum_{j=1}^{N} R_i \rho_{ij} R_j \right)^{1/2} \]

Where \( \rho_{ij} \) represents the correlation coefficient between modes i and j, computed as follows.

\[ \rho_{ij} = \frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\xi^2r(1+r)^2} \quad ; \quad r = \frac{\omega_j}{\omega_i} \]

- \( R_{\text{max}} \): the representative maximum value for a response.
- \( R_j \): the peak value of the response for the i-th mode.
- \( r \): the ratio of the natural frequency of the i-th mode to that of the j-th mode.
- \( \xi \): damping ratio.

There were defined two different load cases corresponding to the design spectrum previously determined, one for an excitation angle of 0° (Corresponding to an excitation in the X direction) and the other for an excitation of 90° (Corresponding to an excitation in the Y direction).

As the spectrum data was generated by selecting a single response spectrum, it was necessary to develop a procedure for the spectral data correction. In this way there were calculated the damping ratios for each mode in order to generate the corresponding spectral data. For this, it was used the Direct Modal damping method, for a damping ratio of 0.05 in all modes. Additionally, the eccentricity of the structure, in terms of percentage of the plan dimensions (automatic procedure), was considered. (MIDAS, 2008)

![Figure 5-8. Spectrum Data generation for each mode. (MIDAS, 2008)]
5.5 RESPONSE OF THE ACTUAL STRUCTURE

5.5.1 DISPLACEMENTS

From the resulting displacements for excitations in both directions, it can be seen how the displacements in the vaulted system are of the order of millimeters. For an excitation in the longitudinal direction, the vaults exhibit uniform displacements and the maximum relative ones can be observed for the pillars in the main nave. For an excitation in the transversal direction, the part of the church that undergoes displacements is the upper part, evidencing how the vaulted system would be the more affected, if an earthquake hits the church in this direction.

Figure 5-9. Displacements for an excitation in X direction. Units: meters.

Figure 5-10. Displacements for an excitation in Y direction. Units: meters.

If we consider now the effect of the dead and permanent loads acting on the system, for both directions, the only part of the structure that will be affected is the part of the vaults and the pillar supporting them. As the vaulted system is destroyed at its ends, there is not evidenced a uniform behavior, when analyzing the displacements. However, the same pattern found when evaluating the structure under its self-weight only can be seen, as the higher displacements are found at the top part of the vaults.
Studying in detail the pillars, all of them undergo almost the same displacements, which vary depending on the cross-section properties. The relative displacements compared with the height of the elements constitute less than the 0.1%, for an excitation corresponding to the response spectrum at the ultimate limit state (SLV). Still it must be considered that these displacements occur both in the longitudinal and in the transversal direction. If significant displacements occur at the piers, in the transversal direction to the vaulted systems, this might be critical for its stability.
5.5.2 STRESSES

Looking at the membrane stresses in the vaults, these are under compression stresses in almost the whole surface. The maximum compressive stresses the vaults are undergoing are around 900 kN/m$^2$, except for some specific points at the destroyed ends. If compared with the compressive strength under seismic actions, 1333 $kN/m^2$, they are below the limit. Still, there are some regions subjected to tensile stresses, especially the ones at the top of the barrel vaults, which depending on their magnitude might be interpreted as cracked regions or portions which will completely collapse.
For the part of the church supporting the vaulted system, the pillars in the nave experience axial stresses in compression under acceptable ranges. However, the beams supporting the vaults and connecting the pillars are experiencing tensile axial stresses, for the two directions of excitation. As the maximum value of tensile stress is not that high, this is not considered critical but just a source of cracking. Nevertheless, when checking for bending stresses all the members in consideration are undergoing compressive stresses.

Figure 5-17. Axial and bending stresses, for an excitation in X. Units: kN/m².

Figure 5-18. Axial and bending stresses, for an excitation in Y. Units: kN/m².
5.6 RESPONSE OF THE STRUCTURE WITH THE VAULTED SYSTEM IN IDEAL CONDITIONS

5.6.1 DISPLACEMENTS

Let’s consider first the response spectrums defined in the Italian code. Figure 5-19 and Figure 5-20 show the resulting displacements, for excitations in both directions, if the sole response to the excitation is considered, without the contribution of the dead loads. It is interesting to observe, how the higher displacements in the vaults are concentrated in specific regions, showing exactly the most vulnerable parts.

As it was said before, the earthquake of 2012 hit the church in its longitudinal direction (east-west). If the resulting displacements in the X direction, corresponding to the longitudinal one, are analyzed it can be seen how the vault exhibits higher displacements exactly at the parts of the system that were destroyed after the event. For an excitation in Y, the part of the vaulted system that would undergo the higher displacements would be the east part, corresponding to the part that arrives to the transept. For this case, the displacements the vaults might undergo
in both directions are of the order of millimeters, even if the structure with an excitation in the transversal direction Y is undergoing higher displacements.

By analyzing the displacements, now considering the dead loads acting on the system. It can be identified a more uniform distribution of displacements along the vault for the two cases. A similar distribution of displacements can be seen in the two directions of excitation, as a result of the boundary conditions and it can be evidenced how the contribution of the dead loads have a significant effect on the elements. The displacements tend to vary more in the barrel vaults and are these ones the ones experiencing the higher values of displacements.

Similarly to the previous case, the relative displacements for the pillars supporting the vaults do not exceed the 0.1% of their height, for the excitations in the two directions. Because of this, it could be said, in terms of displacements, that these elements would not be critical. However, as said before, the displacements of this elements in the transversal direction of the vaults might compromise their stability, as the configuration of the vaulted structures might undergo changes.
Figure 5-23. Displacements of the pillars supporting the vaults, for an excitation in X. Units: meters.

Figure 5-24. Displacements of the pillars supporting the vaults, for an excitation in Y. Units: meters.
By analyzing now the results with the response spectrum obtained from the quake of the 20\textsuperscript{th} May of 2012, it is possible to observe how similar were the outcomes obtained. The same kind of displacements, obtained for an excitation in the X direction with the response spectrum defined by the code, can be identified in Figure 5-25 this time with the new input. Nevertheless, as the displacements for the first case are of the order of millimeters, the displacements for an excitation with the response spectrum of 2012 are of the order of centimeters.

![Figure 5-25. Displacements for the excitation of the quake of 2012, in the longitudinal direction. Units: meters.](image_url)

Though, in Figure 5-27 there can be observed a decrease in the displacements if also the contribution of the dead loads is considered. In this way, it is evidenced the influence of the dead loads and their prevalence in the total response of the system.

![Figure 5-26. Actual conditions of the vaulted system after the earthquake.](image_url)

![Figure 5-27. Displacements in the vault for the excitation of the quake of 2012, in the longitudinal direction. Considering the contribution of the dead loads. Units: meters.](image_url)
In Figure 5-28, a more detailed description of the displacements the vaulted system might undergo is shown. In its longitudinal direction, the same direction of excitation, the higher displacement values can be evidenced at the two extremes. The barrel vault at the east side will try to follow the same direction of the excitation, while the one close to the façade will try to resist against it as it is restrained. In this way, the two extremes of the system will undergo displacements with a different sign while the central part will remain almost still. If these displacements are taken into consideration, this could be one of the reasons for the kind of collapse of the barrel vaults at the extremes, after the earthquake. The displacements in the Z direction, shown in Figure 5-28, make more reference to the resulting displacements due to dead loads.

Figure 5-28. Displacements in X and Z, for the vault subjected to the excitation of the quake of 2012, in the longitudinal direction. Considering the contribution of the dead loads. Units: meters.

The displacements at the pillars will have a similar behavior as the cases studied before. Displacements of the order of millimeters, with relative displacements of less than 0.1% of the length of the elements.

Figure 5-29. Displacements in the pillars for the excitation of the quake of 2012, in the longitudinal direction. Units: meters.
5.6.2 STRESSES

Refering now to the stress distribution, the vaults remain under compression stresses almost in their whole surface. Nevethertheless, there can be observed specific points, where concentrated tensile stresses, with a significan magnitude arise. The compressive stresses experienced by the vaults are relatively low but the magnitude of the tensile ones might be a problem.

![Figure 5-30. Normal Stresses along the vault’s axis (X direction) and along the directrix of the vaults (Y direction), for an excitation in X. Units: kN/m²](image1.png)

![Figure 5-31. Normal Stresses along the vault’s axis (X direction) and along the directrix of the vaults (Y direction), for an excitation in Y. Units: kN/m²](image2.png)

Focusing on the effective stresses, it can be appreciated how their magnitude is higher for the barrel vaults, being these the more demanded portions of the system. The values obtained for an excitation in the transversal direction Y are significantly higher than the ones for an excitation in the longitudinal one X.

![Figure 5-32. Effective Stresses for an excitation in X. Units: kN/m²](image3.png)
The same situation of axial stress occurs for the two cases, were the only parts undergoing tensile stresses are the beams connecting the pillars. As the magnitude of such stresses is not that high, there is only considered a development of cracks and not the complete collapse of the elements.
In the response to the quake of 2012, the magnitude of the stresses obtained are similar with respect to the results obtained with the response spectrum of the Italian code in the X direction. However, the stresses range is significantly low. There cannot be identified the same specific points with concentrated stresses in tension as before, exhibiting a more regular distribution of stresses. The magnitude of the tensile stresses is similar as in the previous case.

Figure 5-36. Normal Stresses along the vault’s axis (X direction) and along the directrix of the vaults (Y direction), for the excitation of the quake of 2012. Units: kN/m²

Figure 5-37. Effective stresses for the excitation of the quake of 2012. Units: kN/m²

For the pillars in the nave, they are experiencing compressive stresses with values below the compressive strength. The beams are undergoing small tensile axial stresses. If bending is analyzed, all the elements are under compressive stresses and it can be seen how the pillars as well as the beams are undergoing bending.

Figure 5-38. Axial and bending stresses, for the excitation of the quake of 2012. Units: kN/m²
CHAPTER 6. NON-LINEAR STATIC ANALYSIS

In order to consider the non-linear behavior of the structure under seismic actions, there was developed a pushover analysis. The material non-linearity was considered individually for plate elements and beam elements as described in section 2.2. There were not considered geometric non-linearities, as the displacements from the previous analyses resulted to be of the order of millimeters. Geometric non linearities are used, expecting elements to undergo significant displacements when subjected to specific loading conditions. As there is still uncertainty of the magnitude of the displacements that may result in the vaults from this analysis, this will be verified at the end.

This analysis, compared with the previous ones, requires a more complex procedure for its implementation. The results can be expressed in terms of the Capacity curve of the structure, which determines the magnitude of the forces it can withstand, as well as the maximum displacements before reaching collapse. This will be compared with the response spectra already defined and, in this way, it will be possible to determine the performance of the building under specific conditions, both for the vault and the entire structure. The steps followed to perform this analysis are the following. The seismic demand will correspond to the effects of the earthquake in consideration, while the seismic capacity will be the one to resist those effects.

- Definition of nonlinear material properties.
- Definition of static load cases for the analysis.
- Nonlinear analysis control.
- Post-processing of the results.
- Bilinearization of the capacity curve and determination of the performance point.

6.1 STATIC LOAD CASES

The nonlinear static analysis consists on applying a horizontal load distribution to the structure in the direction considered for the seismic action. This load is scaled in a way that guarantees a monotonical increment, until it reaches collapse, and is defined to represent the inertia forces resulting from the seismic excitation.
For this, there must be considered at least two different load vectors which are classified in the NTC as principal distributions (Group 1) and secondary distributions (Group 2). For the first case, it was considered a distribution of forces proportional to the vibration mode governing in each direction. The second vibration mode was used for an excitation in X and the third one for an excitation in Y. For the second group, it was considered a uniform distribution of forces proportional to the masses of the structure. For these, it is necessary to consider both positive and negative directions, and the final results will be the ones which represent the most critical situation.

### 6.1.1 NON-LINEAR ANALYSIS CONTROL

In order to define the loading conditions from which the nonlinear analysis must start, it is necessary to specify a loading sequence for the analysis. In this way, it can be determined which load cases to consider as well as the sequence of application of such loads. This was done for the dead and permanent loads acting on the structure.

For each load cases, it was determined a displacement-control iteration method. In this way, by determining a specific displacement the magnitude of the applied loads can be adjusted in order to obtain that final value. For each load variation there is determined a corresponding displacement, even if the load is decreasing, and so the softening behavior can be modelled.

For determining the control point, it is necessary to use a point for which the displacements are expected to increase gradually. Because of this, it was chosen node at the top, of one of the pillars in the nave. The first definition for the maximum displacements was based on the results from the modal analysis for each mode considered. Still, this was then modified according to the resulting capacity curves in order to obtain the best representation of the behavior of the structure. Finally, it was used convergence criteria based on a force norm, as this one is usually used for rigid systems, such as masonry structures.

It is necessary to clarify, that the control point was defined at the top of one of the pillars of the nave. However, the capacity curves obtained were plotted for a point in the upper part of the vaulted system at the barrel vault in the center. This, in order to obtain information related to the elements of study.
6.1.2 BILINEARIZATION OF THE CAPACITY CURVE

The capacity curves obtained from the previous analysis were processed with the Based Capacity Spectrum method (N2 method). In this method, the demand is defined with the response spectrums from section 5.2 and the capacity will be the one of the structure resulting from this analysis. The aim, is to determine the structural response of the building to the already specified demand, and this is defined by finding the performance point. The capacity curves, obtained for a multi degree of freedom system, are transformed into one degree of freedom curves, with the bilinearization of the curves previously obtained, and it is possible to intersect both the response and the demand.

As we want to intersect the response spectrum with the capacity curve, it is required that the first one is defined considering the non-linearity of the analysis. Because of this, an appropriate reduction of the elastic response spectrum is defined. The elastic response spectrum, expressed in terms of spectral accelerations \( S_a \) and the corresponding periods \( T \), defined for a 5% damping, is expressed in terms of spectral acceleration and spectral displacements \( S_d \), with the following relation.

\[
S_d = \frac{S_a T^2}{4\pi^2}
\]

It is necessary to bi-linearize the capacity curve, in order to determine the corresponding yielding point for the one degree of freedom system. For this, both the forces and the displacements must be normalized with the participation factors of the modes corresponding to each direction of excitation. In this specific case, it will be the participation factor of the second mode for an excitation in X, and the one of the third mode for an excitation in Y. In this way, the resulting curve describes an elastic perfectly plastic behavior, by considering the following relations.

\[
d_y^* = 2\left(d_m^* - \frac{E_m^*}{F_y^*}\right)
\]

\( F_y^* \) and \( d_y^* \) represent the force and displacement at yielding, \( d_m^* \) the maximum displacement in consideration (point B in Figure 6-1) and \( E_m^* \) corresponds to the area beneath the original pushover curve up to the point defining the maximum displacement. As we have now an elastic-
perfectly plastic curve, the value for the force at yielding will be determined from the corresponding force for the maximum displacement chosen (Point B).

It is required to determine now, the effective period of the structure. And this will be defined with respect to the point at the maximum displacement. In this way, by definition, the corresponding period can be computed as a function of the force and displacement at yielding, and the equivalent mass of the structure which is defined by normalizing its total mass with the corresponding modal participation factors \( T^* = 2\pi \sqrt{\frac{m^* d^*_y}{F^*_y}} \).

![Graphical description for the bilinearization of the Capacity Curve.](Navvee, Syed, & Leslie, 2015)

Subsequently, in order to intersect the capacity of the structure with the demand, the resulting forces must be transformed into accelerations and this is achieved by diving them by the equivalent mass of the structure. By projecting the line corresponding to the elastic behavior in the resulting capacity curve, until it intersects the response spectrum curve, it is possible to determine the target displacement by taking into account the following conditions. For \( T^* < T_C \):

\[
\mu_s = (R_s - 1) \frac{T_C}{T^*} + 1
\]

\[
R_s = \frac{F_e^*}{F^*_y}
\]

\[
d^*_t = \mu_s d^*_y
\]
For $T^* \geq T_c$:

\[ d_i^* = d^* \]

In a graphical way, this can be explained as: If the intersection of the capacity curve with the response spectrum falls within the response spectrum’s plateau, it is necessary to consider an amplification of the target displacement by making use of the displacement ductility ratio $\mu_s$. If instead, the intersection point falls out of the plateau, there is no amplification required.

This becomes an iterative process, and from the definition of the target displacement for different points of the capacity curve it is possible to determine the so-called locus of the performance point, which consists of a line connecting all the possible target points. From the intersection of such curve, with the capacity curve, it will be possible to determine the performance point of the structure and this is the point that will give information about the global response of the structure for the demand in consideration. In other words, when the locus of target displacements intersects the capacity curve, this will be the performance point.
6.1.3 PUSHOVER IN THE LONGITUDINAL DIRECTION (X)

6.1.3.1 DISPLACEMENTS

For a pushover in the longitudinal direction, the displacements at the last step of the analysis, for the force distribution corresponding to the worst situation, are shown in the figures below. As it can be seen the structure experiences large displacements, up to 5 cm, but at this point the structure is already expected to fail. These figures are useful to give an idea of the mechanism of collapse the structure might experience, however it is necessary to evaluate the displacements at different steps depending on the demand of the seismic actions.

As it is possible to observe, the vault will experience more displacements at the north part, corresponding to the less rigid part of the structure. This is expected, as the walls in this side are not providing the same stiffness to the system due to the previous collapse of the transept in 2012. The longitudinal displacements of the vault seem to be a result of the displacements...
of the supporting pillars. Still, the relative displacements for the pillars are not that high compared to their height.

Figure 6-4: Plot of the undeformed and the deformed vault in the X direction. Units: meters.

6.1.3.2 STRESS ANALYSIS AND FAILURE IN ARCHES AND VAULTS

For excitations in the X direction, there were analyzed force distributions proportional to the mass of the structure and proportional to the dominant modal shape in the corresponding direction (mode 2). From the resulting capacity curves (Figure 6-5), the most critical one was taken as reference for the pushover analysis and this was the one described next. The most critical curve, in terms of acceleration, resulted to be the one in correspondence with a load distribution proportional to the dominating modal shape in the -X direction (green curve).

Figure 6-5: Capacity curve in the X direction with different force distributions.
In the capacity curve of Figure 6-6, the main points which describe the evolution of the behavior in the structure were identified and listed with the letters A, B and C. The point A, corresponds to the point in which the stiffness of the system starts to change, and the elastic behavior ends. In this, (Figure 6-7) the vaulted system is experiencing a formation of plastic hinges in the connection between arches and columns. However, failure is not considered to occur at this point. The point B, makes reference to a point already in the non-linear range, for which the system’s stiffness is almost null and the vault can be considered as already collapsed. This point will be taken as reference to compute the Risk Index of the vault. Figure 6-9 shows how the barrel vaults exhibit cracking, almost in the whole surface, and there can be identified the collapse mechanism for the transversal arch at the east end of the vaulted system, with the formation of four plastic hinges. In point C a complete collapse of the vaulted system is shown (Figure 6-11).

In Figure 6-8, Figure 6-10 and Figure 6-12, the stress state of the plate elements composing the vaults can be evaluated with respect to the failure surface of the Mohr-Coulomb criterium. As can be seen for point A, only some elements are close to the yielding surface, but none of them have reached it. This means the elements are still in the elastic range of the deformations. While for the point B a significant quantity of points is in the border of plastic deformations. Following Mohr-Coulomb criterion, the elements will fail once the failure envelop is reached.
In point C, there exist an important quantity of points in the frontier with the plastic limit, which is in agreement with the crack pattern shown in the Figure 6-11.

Figure 6-7: Effective Stress state at the point A (4/25 step). Units: kN/m².

Figure 6-8. Stresses State at point A (4/25 Step)

Figure 6-9. Effective Stress state at the point B (9/25 step). Units: kN/m².
Figure 6-10: Stresses State at point B (9/25 Step).

Figure 6-11: Effective Stress state at the point C (25/25 step). Units: kN/m².

Figure 6-12: Stresses State at point C (25/25 Step).
6.1.3.2.1 Seismic demand in X for the system under the Design Response Spectrum

Figure 6-13 shows how the vault is unable to resist the solicitations of the Design Spectrum, in the longitudinal direction X, since the maximum displacement the vault can resist \((d_{Max} = 63 \text{ mm})\) is smaller than the demanded by the seism \(d_{target} = 70 \text{ mm}\).

![Figure 6-13: Seismic demand on terms of displacement.](image)

Table 23 shows the capability to resist the Design Earthquake in terms of displacements. The worst case is for the force distribution equivalent to the dominant modal shape in -X direction, for which the corresponding risk index for the vault \(R_I_{vault}\) is around 0.36. Being this value significantly lower than 1, this means the vault would not be able to resist an excitation of this magnitude. If instead, the whole structure is evaluated, the corresponding risk index \(R_I_{structure}\) will result around 0.90. With this, it is still not possible to say that the whole structure would be able to resist the earthquake, but a value of 0.90 is still acceptable for an old masonry building.

<table>
<thead>
<tr>
<th>Number</th>
<th>Seismic Direction</th>
<th>Force Distribution</th>
<th>(d_t) [mm]</th>
<th>(d_{dc/dt})</th>
<th>(R_I_{Vault})</th>
<th>(d_{max/dt})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+X</td>
<td>Prop to Mass</td>
<td>35.0</td>
<td>0.71</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-X</td>
<td>Prop to Mass</td>
<td>48.9</td>
<td>0.51</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+X</td>
<td>Prop to Eigen Vector</td>
<td>62.5</td>
<td>0.40</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-X</td>
<td>Prop to Eigen Vector</td>
<td>70.3</td>
<td>0.36</td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

*Table 23: Summary of risk index to design spectrum in X direction for different force distributions.*
6.1.3.2.2 Seismic demand in X for the system under the Earthquake of 2012

Figure 6-14 shows the intersection of the capacity curve resulting for the worst situation, with and approximated spectrum obtained from the signal of the earthquake of 2012. From this, it can be observed how the structure was able to resist to the seismic events, since its maximum capacity of displacement $d_{Max} = 63 \text{ mm}$ is much higher than the demand $d_t = 15 \text{ mm}$. The state of the stresses corresponding to the step for the target displacement, coincide with the Figure 6-7, while the stress compared to the Mohr-Coulomb criterium are shown in the Figure 6-8.

![Figure 6-14: Seismic demand on terms of displacement.](image)

Table 24 shows the capability of the system to resist the main Earthquake in terms of displacements. For the most unfavorable case the corresponding risk index for the vaults $RI_{vault}$ resulted to be 1.24, which means that these could withstand an excitation of this magnitude with a certain slack. While the risk index for the whole structure, $RI_{structure} = 4.14$, demonstrates why it was able to resist the earthquake. In this way, the low level of damage suffered by the vault can be explained.

<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>Force Distribution</th>
<th>$d_t$ [mm]</th>
<th>$R.I._{vault} \text{ dc/dt}$</th>
<th>$R.I._{structure} \text{ dmax/dt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+X$</td>
<td>Prop to Mass</td>
<td>11.2</td>
<td>2.24</td>
<td>5.64</td>
</tr>
<tr>
<td>$-X$</td>
<td>Prop to Mass</td>
<td>12.9</td>
<td>1.94</td>
<td>4.88</td>
</tr>
<tr>
<td>$+X$</td>
<td>Prop to Eigen Vector</td>
<td>14.9</td>
<td>1.68</td>
<td>4.24</td>
</tr>
<tr>
<td>$-X$</td>
<td>Prop to Eigen Vector</td>
<td>15.2</td>
<td>1.64</td>
<td>4.14</td>
</tr>
</tbody>
</table>

Table 24: Summary of risk index to design spectrum in X direction for different force distributions.
6.1.4 PUSHOVER IN THE TRANSVERSAL DIRECTION (Y)

6.1.4.1 DISPLACEMENTS

For the transversal direction, it can be seen how the displacements, in general, are relatively higher if compared to the resulting ones in the longitudinal direction. This can be observed, as the direction of excitation for this case corresponds to the weaker direction of the structure. While for the longitudinal direction, the parts of the structure undergoing significant displacements at the top part and at the piers in the nave, in this case the whole structure is experiencing displacements. Still, as said before, these are the resultant displacements at the final step of the pushover analysis. Thus, in order to make conclusions, it is necessary to analyze first the stress states.

![Figure 6-15: Plant view, load force in the Y direction. Units: meters.](image1)

![Figure 6-16: Isoparametric view. Load force in the Y direction. Units: meters.](image2)
From the displacements of the vaults, it can be observed how the maximum displacements are obtained for the extreme that meets the façade. Additionally, it can be seen that the resulting displacement for the vaults seems to be a result of the large displacements of the supporting elements.

Figure 6-17: Plot of the undeformed and the deformed vault in the Y direction. Units: meters.
6.1.4.2  STRESS ANALYSIS AND FAILURE IN ARCHES AND VAULTS

There was analyzed the force distribution proportional to the masses of the structure, in the Y direction, both with positive and negative sign (+Y and -Y). The most critical capacity curve, resulted for -Y, and this was taken as reference for the pushover analysis as it is described on detail in Figure 6-19.

![Capacity curve in the Y direction.](image)

The point A (Figure 6-20), corresponds to the formation of plastic hinges in the connection between arches and columns. Exhibiting the change from elastic to plastic range. The point B (Figure 6-22) corresponds to the moment when collapse mechanisms are formed in the arches of the vaulted system. While Point C (Figure 6-24), describes the collapse of one half of the vaulted system, due to the previous collapse of the arches. In Point D (Figure 6-26), the final collapse of the vault can be observed.

![Points of interest in the Capacity Curve mass proportional in -Y direction.](image)
Figure 6-20: Effective Stress state at the point A (3/25 step). Units: kN/m².

Figure 6-21: Stresses State at point C (3/25 Step).

Figure 6-22: Effective Stress state at the point B (6/25 step). Units: kN/m².

Figure 6-23: Stresses State at point C (6/25 Step).
As before, the stress state of the vaulted system can be evaluated by considering the Mohr-Coulomb criterion. The progress of the failure mechanisms can be observed, as for different steps of the analysis, more elements reach the failure envelop.
6.1.4.2.1 Seismic demand in Y for the system under the Design Response Spectrum

The maximum displacement allowable for which the vaulted system remains safe, was defined as the point B \( (d_B = 24 \text{ mm}) \) and, in accordance to this point, the safety factor of the vault was computed. The maximum displacement allowable for the system was defined as the point D \( (d_D = 45 \text{ mm}) \) and, in accordance to this point, the safety factor of the structure was computed.

Figure 6-28 shows that the vault is unable to resist the solicitations of the Design Spectrum, since the maximum displacement that can resist the vault \( d_{Max} = d_D = 45 \text{ mm} \), is smaller than the demanded displacement by the seism \( d_{target} = 55 \text{ mm} \).

Table 25 shows that the vault is unable to resist the solicitations of the spectrum of design, since the maximum displacement that can be resisted is fixed at the point of performance B. However, the complete structure is less likely to collapse since the worst distribution of forces \((-Y \text{ Proportional to Masses})\) have a Risk Index of for the imminent collapse of 1.13.

<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>Force Distribution</th>
<th>( dt ) [mm]</th>
<th>R.I. ( \frac{d_c}{d_t} )</th>
<th>R.I. ( \frac{d_{Max}}{d_t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Y</td>
<td>Prop to Mass</td>
<td>34.5</td>
<td>1.02</td>
<td>1.83</td>
</tr>
<tr>
<td>-Y</td>
<td>Prop to Mass</td>
<td>55.7</td>
<td>0.63</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Table 25: Summary of risk index to design spectrum in X direction for different force distributions.
### 6.1.4.2.2 Seismic demand in Y for the system under the Earthquake Specthquake of 2012

Figure 6-29 shows that the structure was able to resist to the seismic events of 2012, since the maximum capacity of displacement $d_D = 45 \text{ mm}$, was bigger than the demand $d_t = 13 \text{ mm}$.

![Figure 6-29: Seismic demand on terms of displacement.](image)

Table 26 shows the capability of the system to resist the main Earthquake in terms of displacements. The most unfavorable case, is the one for the distributed forces equivalent to the masses of the structure in direction -Y. The corresponding risk index for the vaulted system $RI_{vault}$ is about 1.24, which means that the vault could withstand an excitation of this magnitude with a certain slack. While the risk index for the whole structure ($RI_{structure} = 4.14$), can be useful to explain the low level of damage suffered by the vault.

<table>
<thead>
<tr>
<th>Seismic Direction</th>
<th>Force Distribution</th>
<th>$dt$ [mm]</th>
<th>$R.I._{Vault}$ dc/dt</th>
<th>$R.I._{Structure}$ dmax/dt</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Y</td>
<td>Prop to Mass</td>
<td>11.4</td>
<td>2.15</td>
<td>5.53</td>
</tr>
<tr>
<td>-Y</td>
<td>Prop to Mass</td>
<td>13.2</td>
<td>1.86</td>
<td>4.78</td>
</tr>
</tbody>
</table>

Table 26: Summary of risk index to design spectrum in Y direction for different force distributions.

Table 26 shows the vault can resist the solicitations of the spectrum from the earthquake of 2012, since the maximum displacement that can be resisted is greater than the target one. Nevertheless, the vault of the model at the step corresponding with the target displacements, suffers a collapse like the real one on the west side of the vault as is shown in the following Figure:
Figure 6-30: Comparison between the cracked elements of the model with the actual vault.
CHAPTER 7. ANALYSIS OF RESULTS

After examining the results obtained from the different analyses performed on the church, it is possible to perform a general overview of the actual state of the building, the possible weaknesses and the things that must be considered when planning a reconstruction of the building, especially for the vaulted system in the nave.

A preliminary study derived from images was carried out to determine, with limited information available, the actual state of the structure and specially for the vaulted system. In this way, it was possible to identify the most critical parts, as a result of the earthquake that destroyed part of the church in 2012 and develop an accurate model to study. As it was previously mentioned, half of the transept of the church collapsed, and this is important as the part of the vaulted system which was close to this area also collapsed. At the other extreme, the roof connected to the façade, also failed, and this corresponds to the other extreme of the vaults that was destroyed.

If first, reference is made to the structure under its self-weight. As masonry has very low resistance to tension, this was analyzed by checking the entire structure was under the action of compressive stresses only. From membrane stresses, it was possible to say that the vaulted structure in study is stable and can resist its own weight. From a linear static analysis, it was confirmed that the vault can resist its own weight, since the structure is in most of the elements undergoes compressive stresses, or very low tensile stress in limited parts.

The deflections in the repaired vault are lower than in the destroyed one. One possible reason to explain this behavior is that for the repaired vault, there are two additional transverse arches at the extremes which make the system stiffer. In both models it was also observed that the highest deflections were at the central barrel vault, and in general the regions experiencing more displacements were always the barrel vaults. Since the maximum displacements on each model were of the order of millimeter, the hypothesis of small displacements is valid for this load case. Regarding the stress state, the most demanded parts of the vaulted system were the ones in which there is a change in the cross section, resulting from the different stiffness of the composing elements. From a comparation with theoretical stresses derived from the theory of thin shells, for barrel vaults, it was possible to see that the results of the software were in
accordance with the theoretical ones. Making possible the validation of the developed model. Making reference to the frame system, the columns supporting the vaults are the most demanded ones. According to the compression strength considered for the model for vertical actions these stresses are still under the limit. However, they are close, and this may be evidenced when studying the actual condition of the structure.

From limit analysis, different possible behaviors under the action of horizontal loads where studied for individual parts of the building, however it was difficult to interpret the results when thinking of the structure as an assemblage of these. Still, it was possible to observe that the thickness of the transversal arches is significantly low with respect to their span, and because of this they are not able to withstand high values of external forces, in terms of percentages of their dead load. They present low resistance under the action of transversal loads. However, the expected resistance for the whole system should be higher. The stability of vaulted structures is also conditioned to the stability of their supporting systems. The pillars supporting these elements must have adequate dimensions in order to resist the imposed thrust. The central pillars supporting the vaults, especially the ones with circular cross-section, are the most vulnerable ones as they may undergo higher displacements when subjected to horizontal loads. Thus, their resistance to this type of loads is lower. For this case, there is no problem for resisting the thrust imposed by the vaults, since this force can be transferred from the transversal beams to the perimeter walls.

A modal analysis was developed in order to determine the vibrations modes of the structure and the corresponding natural frequencies. For this type of structures, the modal participation factors for the first modal shapes not always constitute a significant percentage of the total mass of the structure. This, because due to the complex geometry of churches, local modes may influence in the dynamic response of this buildings. Therefore, it was necessary to consider 12 modal shapes in order to obtain the minimum total participation masses, both in the longitudinal and transversal directions, required by the code.

From the Response spectrum analysis, the vaults exhibit uniform displacements, for an excitation in the longitudinal direction, and the maximum ones can be observed at the pillars in the main nave. For an excitation in the transversal direction, the part of the church that undergoes displacements is mainly the upper part. If we consider now the effect of the dead
and permanent loads acting on the system, for both directions, the main part of the structure that will be affected corresponds to the vaults and the pillar supporting them. However, the relative displacements of the pillar are not significant, to consider the loss of stability of the vaulted system. Therefore, the geometric nonlinearity due to large displacements was not considered for further analyses.

From the Non-linear analysis, it was possible to evaluate the structure studying the material nonlinearities in order to consider its behavior after cracking. With this, is was possible to determine the capacity of the structure to resist seismic actions, in terms of displacements and accelerations. Comparing the structure’s capacity with the seismic demand of different excitations, it was possible to determine a risk index, both for the whole structure and the vaulted system, to determine the its level of safety.

With this it is possible to answer to the questions previously presented when defining the case study. First of all, based on the results listed on the last chapter, the remaining part of the church would not be able to resist to a seismic event as the one expected in the provisions of the NTC18. From the response spectrum analysis, even if most of the surface of the vault is subjected to compressive stresses, there are tensile stresses specially at the top of the barrel vaults. Depending on the magnitude of these stresses, this might be interpreted either as cracked regions or portions which will completely collapse.

Additionally, the resulting collapse mechanism after the earthquake of 2012 can be confirmed if the results from the Response spectrum analysis are studied. From the resulting displacements in the longitudinal direction, it is possible to see how the maximum displacements correspond to the part of the vaulted system that collapsed. A better description of this failure is provided in section 5.6.1. From the nonlinear analysis, is possible to observe how the structure was able to resist to the earthquake of 2012, since the demanded displacements by the earthquake where lower than the displacements associated to failures. Still, it is under consideration, if the parts of the vaults that collapsed might be the result of the vertical loading when the other parts of the structure failed, as these were the ones close to the main damaged parts. The most vulnerable parts, according to the results obtained in terms of displacements, are the parts of the vaulted structure that failed. Nevertheless, when checking the stress state of these elements they are still undergoing admissible stresses.
Finally, when evaluating the level of safety the church might provide, if the vaulted system is repaired with the same characteristics as before. From the Response spectrum analysis, it can be said that the same response the actual system had to the earthquake of 2012, will occur if the structure is excited in its longitudinal direction. If instead, an excitation in the transversal direction occurs, the part of the vaulted system that may suffer more damages would be the one corresponding to the barrel vault at the east end. Therefore, it is important to pay special attention to the barrel vault at the east end, when considering strengthening interventions for the vaulted system. Focusing on the effective stresses, it can be appreciated how their magnitude is higher for the barrel vaults, being these the more demanded portions of the system. The values obtained for an excitation in the transversal direction Y are significantly higher than the ones for an excitation in the longitudinal on X. From the non-linear analysis, it is possible to notice that the vaulted system, by itself, is not able to resist a seismic excitation as the one prescribed in the NTC. Instead, if the complete structure is evaluated, it can resist and excitation only in its the transversal direction. For an excitation in the longitudinal direction, the columns would form a mechanism. Therefore, there is a need to carry out important interventions in the church, to increase its ductility both in the longitudinal and transversal directions to preserve the vault. If the vaulted structures are just repaired, an earthquake as the one prescribed in the NTC would not be resisted, leading to the collapse of the complete system which will put to the limit the strength of the complete structure.

When considering interventions on masonry structures, there are some aspects that must be taken into account. First, any intervention should be as uniform as possible. Second, when executing interventions on a single part of the structure, the variation of the global stiffness and strength distribution should be avoided. Materials with physical, chemical and mechanical characteristics similar and compatible to the original masonry are preferred. Finally, for any intervention, its benefits must be proved (Crespi & Scamardo, 2018).

Additionally, for cultural heritage it is preferred to repair the existing elements instead of replacing them in order to conserve the original structure. Because of this, there are developed interventions in order to increase the element’s strength, stiffness or ductility, searching for the less invasive techniques.
Cracking and failure in arched structures may be related with the low quality of the material, inadequate supporting elements able to resist the thrust generated in these structures, or an inappropriate definition of their geometry making them unable to withstand acting loads.

For masonry vaults, a good option is to strengthen them with FRP laminates. With these, the structure’s bearing capacity will be increased by preventing a brittle failure due to the formation of plastic hinges. Laminates are placed either at the intrados or extrados, depending on the side for which the formation of plastic hinges wants to be avoided and, due to their high tensile strength, the general strength of the structure increases significantly. This laminated are characterized with a passive behavior, which only activates when traction is generated. The reinforced section will behave as a composite one, where the FRP laminates will take the tensile stresses, the epoxy used to install the laminates in the vaults will take the shears stresses and finally the vault will work under compression. Still it is necessary to consider than when applying this kind of interventions, new collapse mechanisms may form, and it is necessary to evaluate additional types of failure, due to crushing and sliding (Crespi & Scamardo, 2018). Additional intervention techniques include a lime-mortar based matrix with steel fibers or steel reinforced polymers and stiffening diaphragms reinforced with steel fibers among others. Tie-rods are also considered in order to help the supporting elements resist the thrust produced by these structures, when subjected to horizontal loads.

Before any intervention, the existing structures must be repaired in order to restore their original resistance and stiffness. Depending on the level of damage different techniques can be implemented. For this case, neglecting the extremes of the vaulted system, this could be considered as lightly damaged and so interventions regarding joints repointing and grout injections might be considered for improving the mechanical characteristics of masonry.

The implementation of FRP laminated is proposed for the three-barrel vaults in the system, which resulted to be the most vulnerable in the structure in study. With this, the tensile strength of this elements is expected to increase and the collapse mechanism for the structure will change. It is necessary to consider additional safety verifications for this type of intervention; however, the general bearing capacity of the vaulted system is expected to increase.
CONCLUSIONS

Churches have been meaningful structures in society due to the relevance of religious buildings and a reference point to provide assistance to people. Thus, the preservation of the cultural heritage, their architectural value, as well as the artistic values (painting, sculptures) present in numerous churches, make them important buildings, and so it is necessary to provide special attention to them considering their preeminent seismic vulnerability. Without mentioning the fact that usually this type of structures is made of materials with poor mechanical properties from which it is difficult to obtain the actual characteristics without affecting the building.

The procedure for the assessment of masonry churches, presented in Italian guidelines, constitute a useful tool for determining the seismic vulnerability of such structures in a fast and easy way. It is useful when defining which interventions are necessary to develop in specific parts of the structure and when planning the definitive recuperation of a building. This is an advantageous tool for starting a seismic vulnerability assessment, as it avoids the evaluation of complex quantities regarding the geometry and the characteristics of the materials present in the building, which may slow down the verification operations without substantially increasing the knowledge of the structural response. The final decision of this assessment is expressed in terms of usability and it may be used either after an earthquake, for the mapping of the damage (usability evaluation), or in view of future earthquakes, as a tool for vulnerability analyses.

For a real and preliminary diagnosis of the seismic response of the building, information about the parts composing the church is important to identify the possible collapse mechanisms the church may undergo. It is important to make a correct evaluation of each of the mechanisms potentially activatable, as it will determine the accuracy of the results obtained as well as the material properties in order to determine the resistance of the church and the different parts composing it.

Local collapse mechanisms can be analyzed by making use if the limit analysis, by means of the kinematic approach, based on the definition of the collapse mechanism and the evaluation of the horizontal action that activates such mechanisms. In this way, the maximum horizontal load the building could withstand can be determined. For systems with complex geometries, it is usually used the finite elements modelling in order to determine their seismic vulnerability both in the linear and nonlinear domains. Among these analyses there can be identified the
Response Spectrum Analysis, useful for studying the structures in their linear range, and the Nonlinear Static Analysis also known as pushover analysis. With both analyses it is possible to determine the demand of a specific seismic load and the capacity to withstand it.

For the church in consideration, due to a lack of information, there was not possible to develop a complete evaluation of the building. However, there was still possible to determine a general evaluation and deepen in the evaluation of its vaulted system.

The vaults in the nave of San Paolo de Mirabello Church, located in Mirabello (FE), were analyzed after the earthquake of 2012 in Emilia Romagna, which caused the collapse of numerous buildings in this region. This church, after the earthquake, was partially destroyed and just the part of its nave and the south part of its transept remained standing. The vaults in the nave, characterized by very thin thickness, were only affected at the extremes, generating interest on their structural behavior. Now the church is closed and has not been repaired yet.

It was studied the seismic response of the building, both for the actual configuration of the vaulted system and an ideal non-destroyed one. This, in order to determine the level of safety that the actual structure and one repaired with the same characteristics as before would provide when subjected to seismic actions. Additionally, there were determined the reasons for the partial collapse of the vault, after the 2012 seismic events.

A finite element model was developed in Midas Gen, where the vaulted system in study was modelled with plate elements and the rest of the structure with beam elements to simplify the procedure. The real geometry of the vaults was generated from cloud points, from which the corresponding mesh was generated automatically. In this way it was possible to observe how the geometry of this kind of structures influences significantly in the final results. The material non-linearities for the vaults were considered using the Mohr-Coulomb criteria, and very interesting results were obtained both for the linear and non-linear analyses made of the church.

The structure on its actual conditions showed a high vulnerability if subjected to seismic actions. Exhibiting similar behaviors in both directions of excitation. The confirmation of the resulting collapse mechanism in the vaulted structure, after the earthquake of 2012, was evidenced in the model with the complete vault for linear and non-linear analysis. The structure with the vault with ideal conditions, also showed a high seismic vulnerability when is compared
to an expected solicitation of the norm. Therefore, FRP laminates were suggested as a strengthening intervention for the barrel vaults, when repaired the whole system.

For future developments, a more refined Finite Element model, able to incorporate composite materials with more refined Non-Linear constitutive laws, could be implemented. The whole geometry of the model could be exported directly from the cloud points. This model could be used to evaluate the effect of the strengthening using FRP in the vaulted system. However, a more specialized software would be necessary. A more detailed survey of the materials constituting the church is necessary if a more complex material model wants to be used. Therefore, if possible, a set of non-destructive tests to characterize the mechanical properties of the remaining structural elements would be recommended.
REFERENCES


