Control Strategies for High-Speed Tilting Body Trains

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Academic Year 2018-2019
Acknowledgements

First of all, I want to thank my relator at Politecnico di Milano, Prof. Stefano Bruni, who never failed to give me valuable advice and guided me throughout my whole thesis’ work.

I also want to express my deep gratitude to Prof. Argyrios Zolotas of the University of Lincoln, for his constant support during my work in England, and to my co-supervisor at Politecnico di Milano, Prof. Egidio di Gialleonardo, for being very supportive on a number of technical issues related to my research work.

Immense gratitude goes to my mother, my father and my sister, who supported me throughout my academic career and in everyday life, each one in a different way, despite all the “storms” we’ve been through.

I am grateful to my grandparents who always supported me.

To my close friends, to my university’s mates and to the new Erasmus’ friends: your companionship has been very important during this “journey”.

I would also like to acknowledge Ing. Paolo Colombo and all the colleagues of BLM, for allowing me to study and work at the same time, and for all they have learned to me.

Last but not least, a very big thank you to Ambra for her love, patience, support and encouragement during these years.
Abstract

Carbody tilting is nowadays a mature technology used in the field of high-speed trains allowing higher speeds in curves and thus reduced travel time. Many research works and industrial implementations on this technology are available. Among the different actuating options, in this thesis the attention is focused on a layout consisting of interconnected hydraulic actuators attached to the carbody and bogies, replacing the passive anti-roll bar used in standard railway vehicles. From the controller point of view different control strategies for tilting have been developed in the literature. The purpose of this work is to simplify the design process of the controller. A modal dominance analysis and different model reductions are performed in order to obtain a low-order system which can be used in a simpler way in the design phase of the controllers. PI controllers with and without feedforward contribution are developed analytically on the reduced model. The controllers are then tested on the higher-order linearized model in order to understand if they satisfy minimum stability requirements. Finally, the assessment of the performances is carried out on the multi-body model of the active vehicle. The analysis of the results highlight that a third-order reduction leads to a sufficiently good approximation of the behaviour of the original system, and simple but effective controllers can be designed in an easy way, reaching a good level of performances.

Keywords: railway vehicles, hydraulic suspension, carbody tilting, anti-roll bar, model reduction, modal dominance index, PI controllers.
Sommario

Il pendolamento della cassa è oggigiorno una tecnologia matura utilizzata nel campo dei treni ad alta velocità, che permette di raggiungere una velocità superiore durante le curve, riducendone il tempo di percorrenza. Molti lavori di ricerca e implementazioni industriali di questa tecnologia sono disponibili. Tra le differenti opzioni di attuazione, in questa tesi ci si è concentrati sul layout costituito da attuatori idraulici interconnessi tra di loro e attaccati alla cassa ed ai carrelli, che sostituiscono la barra anti-rollio passiva utilizzata nei veicoli ferroviari standard. Dal punto di vista del controllo, diverse strategie sono state sviluppate nella letteratura per comandare il pendolamento. L’obiettivo di questa tesi è quello di semplificare il processo di progettazione dei controllori. Una analisi degli indici di dominanza modale e differenti metodi per la riduzione del modello stati utilizzati per raggiungere un modello di ordine inferiore rispetto all’originale, per poi essere utilizzato nella seintesi dei controllori. Controllori PI con e senza contributo "feedforward" sono stati sviluppati analiticamente sul modello ridotto e successivamente testati sul modello linearizzato di ordine superiore per verificare che rispettino i requisiti minimi. Infine, la valutazione delle prestazioni è stata fatta utilizzando un modello multi-body del veicolo attivo. L’analisi dei risultati evidenzia che una riduzione del sistema originale al terzo ordine porta ad una sufficientemente buona approssimazione del comportamento del sistema originale e che controllori molto semplifici ma efficaci possono essere progettati in maniera semplificata, raggiungendo un buon livello di performance.

Parole chiave: veicolo ferroviario, sospensione idraulica, pendolamento della cassa, barra anti-rollio, riduzione del modello, indici di dominanza modale, controllori PI.
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Nomenclature

\( \theta_c \) Carbody roll

\textit{AARB} Active Anti-Roll Bar

\textit{BW} Bandwidth

\textit{DoF} Degree of Freedom

\textit{FRF} Frequency response function

\textit{GM} Gain Margin

\textit{MBS} Multi-Body System

\( P_{CT} \) Comfort Index

\textit{PID} Proportional-Derivative-Integral

\textit{PM} Phase Margin

\textit{PT} Preview tilt

\textit{SISO} Single Input - Single Output

\textit{TF} Transfer function

\( u_s \) Command to servo-valve

\textit{MDI} Modal dominance index
Chapter 1

Introduction

Nowadays the main means of transport available for medium-long distance trips remain automobiles and trains. Developing technology in automotive field allowed to reach high performance level, in terms of velocity, consumption and ride quality, maintaining an higher flexibility than trains. Moreover, good quality of highways and automobiles allows to reach velocity around 130-150 km/h (bounded by law). For all these reasons, trains lost importance because they don’t have the same flexibility of automobiles and still had high service cost, thus making rails transport an uncompetitive service in most of case. However, high-speed trains can substantially reduce journey times and the way to achieve this is either to develop new infrastructure that maximises train speeds or to use existing networks with the introduction of new technologies. Construction of new and modern infrastructures is often complicated for high cost and track problems, but also renewing already existing lines is not very convenient. The solution, where new dedicated lines are not possible, would be introducing new technologies on high-speed trains in order to let them operating at an increased speed on the existing tracks. In this way, they can regain competitiveness, increasing speed, preserving good ride-quality with a good flexibility, related to the possibility to use already existing tracks, and a better comfort than automobiles.

One of the most used solution on high-speed trains is the tilting technology. This technology follows a rather straightforward concept, i.e. by leaning the vehicle body inwards on curved section and thus reducing the lateral acceleration perceived by passengers, thereby enabling higher vehicle speeds on existing networks with the same level of comfort. A detailed description of this technology is presented in [1].
Carbody tilting is nowadays a mature technology and it is accepted by many train operators. However, this technology requires a rather complex bogie and suspension design based on a tilting bolster, which leads to increased vehicle weight and, given the constraints applied to the vehicle gauge, reduces the space available for passengers. For these reasons, simpler and lighter carbody roll actuation systems have been proposed, aiming at permitting limited tilt angles (in the range of 1-2°, compared with the 6-8° maximum tilt angle achieved by tilting trains in the strict sense) that are nevertheless sufficient to increase service speed significantly on existing lines [2].

Different solutions have been adopted to achieve this target. Noteworthy to be cited are [3], where a development of concepts for active secondary suspension is shown, and [4], where the use of air springs as active secondary suspension is proposed. Another solution proposed by Politecnico di Milano presents the use of an active hydraulic anti-roll bar [5]. This new concept, through the use of a hydraulic circuit and two vertical actuators, allows to perform the tilting manoeuvre when required and provides the same roll stiffness as a conventional anti-roll bar when operated in the passive mode.

In paper [5] different control strategies are proposed for the active hydraulic anti-roll device and for all three control gains are defined based on Genetic Algorithm (GA) optimisation, having as multiple objectives the tracking of a reference carbody tilt angle, optimising ride comfort and keeping the energy required for actuation within acceptable limits. One last work to cite is the thesis [6] where different control strategies have been developed (PID, LQR and IMC). In [6] and [5], the reference tilt angle is defined on the basis of vehicle speed and curve geometry (curvature, cant, length of transitions), under the assumption that these information are made available to the control unit through geo-localisation of the train and mapping of the line.

The aim of this thesis is to simplify the controller design for tilting technology, using model reduction. In this way, it is possible to achieve low-order models that are easier to adopt to synthesise and test new simpler controllers. The model reduction is an important part of the study because it allows to understand the different dynamics present in the plant. For this reason, this approach is useful,
both to simplify the model for a subsequent controller design and to understand how the system behaves. It is important to notice that the purpose of this work is not to synthesize controllers that achieve the best performance possible but establish stable and simple controllers that can be refined later. The advantages of this approach, adopting the reduced models instead of the original plant are: (a) definition of low-order model simpler to handle; (b) possibility to use analytical or semi-analytical methods on reduced model to synthesize robust controllers; (c) the controller’s design process becomes simpler and less time-consuming allowing the designer to evaluate quickly the impact of one controller with respect to the other; (d) possibility to use simple optimization that allows to give a physical meaning to the controller’s design process, with respect to multi-objective optimization.

Model reduction has been performed using different approaches. Starting from a 3-DoF model with AARB (active anti-roll bar) an 8-states linear model has been developed. A Modal Dominance Index (MDI) analysis has been performed on the 8-states system to understand the dominance of different modes. Using this analysis, a manual and analytical reductions are performed in order to obtain a first and third-order models. Finally, a model identification has been performed, applying external commands to a non-linear 35-states multi-body system (MBS) developed by the Department of Mechanical Engineering of Politecnico di Milano. In this way, a model reduction is performed also on a model using input-output relationship defined by a quite complex numerical model that is very close to the real behaviour. The model reductions are carried out in different ways in order to understand if consistent results are obtained. After first-order and third-order reduced model are obtained, the controllers’ synthesis has been performed. Thirteen different PI controllers (two of which with feedforward contribution) have been designed. A first assessment is performed on the 8-states linear plant in order to decide which controller leads to the best results in terms of maximum overshoot and settling time. For all the simulations performed in this thesis (both on the 8-states linear model and on the MBS) a standard case has been taken into account considering the vehicle running at 340 km/h along a single high-speed curve geometry. Finally, four out of thirteen controllers, that have led to the best results, are implemented in the MBS model of the vehicle in order to assess performances in terms of comfort, ride quality and controller-related indexes.

The results obtained in this work show that even if simple controllers like PIs
are implemented, good results both in terms of comfort indexes and controller performances can be obtained. In particular it has been shown that PI controllers, designed on the third-order reduced plant lead to better performances with respect to controllers designed on the first-order reduced model. In fact, as far as the reduction is concerned, a third-order model reduction is a good approximation for the original plant, that allows to design simple and effective controllers in an easy and fast way. It has been proven that PI’s analytical and semi-analytical design processes, used in this work, lead to comparable results in terms of comfort indexes similar to those obtained with more complex optimisation procedures, like GAs. It is important to outline that a save in terms of time and computational effort can be obtained with this methods. Finally, the robustness of two designed controllers, that lead to best results in nominal condition, has been tested. In particular, two different analysis have been performed: (a) changes in carbody inertia; (b) errors in tilt reference generation linked to GPS positioning errors. The results obtained in this part show that the controller designed are robust enough to deal with this kind of variation.

This thesis is based on the previous work [6] and paper [5], and it is organised as follow: in the first chapter an overview of tilting technology is presented, analysing different solutions nowadays adopted on railway vehicles. In the second chapter a description of 3-DoF simplified model with dynamic of the hydraulic actuator, used as starting point for simulation, and the multi-body system, used for performance assessment, are shown. In the third chapter different model reductions are performed. Manual and analytical reductions are carried out and a Modal Dominance Index analysis is performed to calculate the weight of the different dynamics. In the fifth chapter simple controllers are designed and it has been investigated which "order" of simplification is an acceptable starting point for reaching performing controllers. In the last chapter, the controllers designed are implemented in a non-linear multi-body model, previously developed by the Department of Mechanical Engineering of Politecnico di Milano, in order to assess the performances of the controllers.
Chapter 2

State of Art

High-speed trains suspensions are very complex dynamic systems. As far as typical "passive" suspension arrangement concerns, it is divided into two parts:

1. primary suspension between the wheelsets and the vehicle bogies
2. secondary suspension between the vehicle bogies and the vehicle body

The first is important as far as the running stability of the vehicle and the curving performances are concerned, while the second one is used to provide high frequency isolation in order to improve the ride-quality for the passengers. As explained in [7], increasing the speed of the trains in order to reduce travel time has some drawbacks both during travelling on straight track and curved sections. In particular two different problems are presented: irregularities of the track are perceived more and more, proportionally with the increasing of the speed, and in curved sections, that will be negotiated at higher speed, the passengers feel an unwanted centrifugal force, which is again proportional to the square of the velocity.

Passive configuration of the suspensions strongly affect ride quality [7]. In particular softer suspension increase running behavior in curved sections while worsen it in straigt paths. On the contrary, stiffer suspension improve the running behavior in straight paths decreasing it in curved sections. Hence it is important to find a good trade-off between the two extremities in order to respect the European norm [8] related to ride comfort for passengers. Thus, in general, increasing velocity in order to reduce travel time, entails a decreasing in ride quality or passengers comfort. A possible solution to overcome this problem is the introduction of an
Tilting technology was introduced in order to increase the performance of a railway vehicle traversing a curve. In fact, when a railway vehicle is running through a curve the passengers experience a centrifugal force. Similar forces act on the body and bogies, causing them to roll into or out of the curve, depending on suspension geometry and the forward speed of the vehicle. By tilting the body it is possible to reduce the lateral force perceived by the passengers, thereby either allowing the vehicle to be operated at speed higher than those that would be acceptable to passengers in a non-tilting vehicle or providing an increase in passenger comfort for the same speed. Carbody roll may be achieved by track cant or, when the track cant is insufficient, carbody tilt. Trains capable of tilting the bodies inwards in curves are often called tilting trains and can be divided into two groups: the naturally tilting trains and the actively tilting trains. A wide description of tilting technology can be found in [1].

It is important to remark that in the last years a growing development of technology and an increase of usage of electronic devices and control strategies also in the railway vehicles fields led to an improvement of the performances, both in dynamic response and in ride-quality. There is a wide bibliography about different tilting technologies and control strategies developed and adopted ( [7], [1] and [9]). In particular, in [1] are reported different tilting train technologies, and the benefits of tilting trains are shown. It is important to notice that development of tilting mechanism requires complex bogie and suspension design that increase vehicle’s weight and, consequently, reduce space available for passengers.

Some upgrade have been done in order to integrate the good results achieved by tilting technologies simplifying layout. For example in the paper [4] an active control on the pneumatic secondary suspension of an high-speed railway vehicle is introduced. Active air springs, together with lateral suspension are used to reduce the lateral acceleration perceived by passengers negotiating a curve. The combined active secondary suspensions show benefits in terms of ride-quality and speed increase. This technique has the advantage that is easy to implement. On the other hand, active air springs are quite expensive in terms of energy consumption, since this actuation system has to counteract against the anti-roll bar.
2.1 Tilting concept

The use of an active anti-roll bar in rail vehicles was first suggested by Pearson, Goodall, and Pratt in [10], with a modified layout of a mechanical bar in which either linear actuators were introduced to replace the links to the carbody, or a rotary actuator was placed in series with the torsion bar. This original concept has been explored and developed by the research group establish in Department of Mechanical Engineering of Politecnico di Milano, on which this thesis will focus on later in the text. In [2], a new concept of tilting actuation that provides a use of hydraulic system is introduced. This device is composed by hydraulic actuators interconnected with carbody and bogie and replace the more classical passive anti-roll bar. With respect to the first one suggested in [10], cross-connected actuators are adopted so that a roll torque can be generated for a theoretically null vertical force: in this way it is easier to reject track irregularity related disturbances and, at the same time, tilt actuation becomes faster, more accurate and less energy consuming compared with pneumatic actuation. In [2], a more detailed analysis of how the system is designed, developed and sized, is presented. This device permits to obtain an assimilable system to an anti-roll bar in the passive configuration and permits to roll the carbody when used in active ones. One last noteworthy work, related to this thesis, is [6] "Study of Control Strategies for Active Carbody Roll in Railway Vehicle Using Hydraulic Actuation" where three different type of controllers were designed. The results of [5] of [6] and are promising: show that it is possible to increase significantly the vehicle’s running speed (around 12% speed increase from 300 to 338 km/h) in fast curves compared with a vehicle equipped with passive suspension, maintaining the same or in the best case improve ride comfort quality.

2.1 Tilting concept

As previously said in the chapter, when a train negotiates a curve at high speed the passengers experience a centrifugal force. The suspension geometry and the forward speed of the vehicle determine the amount of roll into or out of the curve by the vehicle. The amount of lateral force experienced by the passengers can be reduced by tilting, or in simpler words leaning inwards, the vehicle body. Looking at figure 2.1 the expression of lateral acceleration perceived by passengers can be derived by assuming small angles:
where $\theta_v$ is the body roll angle, $\theta_0$ is the track cant angle, $R$ is the curve radius and $v$ is the forward speed. It is important to notice that increasing body roll angle $\theta_v$ decreases lateral acceleration perceived by passengers and provides an increase in passenger comfort [7].

In terms of mechanical configurations, the early tilt experiments on trains involved vehicle bodies with a low centre of gravity to allow for the centrifugal forces acting on the carriage to cause a passive vehicle tilt action. This approach of using inertial forces to let the train tilt was not proven to be very successful. However, passive tilting technology is undoubtedly cheaper compared to active tilting and is still used in the case of small tilt angle applications (7).

As far as mechanical arrangements, to provide an active tilting system, are concerned, there are three main possibilities:

1. Across (or through) the secondary suspension;
2. Below the secondary suspension;
3. Above the secondary suspension.

The first approach is to achieve tilt directly by applying active control to secondary suspension. It involves control of air springs or more recently AARB (active anti-roll bar). More detail on the developments of this last concept for active secondary suspension can be found in [10] and [3]. In this specific application simpler and lighter carbody roll actuation have been proposed, aiming at permitting limited tilt angles due to constraints from the suspension and the actuators used. The AARB layout uses the traditional arrangement consisting of a transversely mounted torsion bar on the bogie with vertical links to the vehicle body, except that the links are replaced by hydraulic actuators, and thereby applies tilt via the torsion tube (fig. 2.2).

![Figure 2.2: Basic scheme for active tilt applied across secondary suspension](image)

A lot of implementations use a tilting bolster to provide the tilt action. An important distinction is where this bolster is fitted compared with the secondary suspension. Both schemes (above and below the secondary suspension) use a mechanism which have inclined swing links in order to provide a rotation of the bolster. Figure 2.3 and 2.4 show respectively the typical layout of above and below the secondary suspension mechanism. The lateral track forces are inevitably increased as a consequence, but careful bogie design can mitigate this and in general railways have found that there is sufficient margin not to compromise safety.
Figure 2.3: Tilt above secondary suspension

Figure 2.4: Tilt below secondary suspension
2.2 Tilting control strategies

A number of approaches have been used for controlling the tilt system. These strategies can be divided in:

1. Nulling;
2. Command driven;
3. Command driven with precedence;
4. Tilt command from track database and geo-localisation.

Nulling control (fig. 2.5), which was the first approach proposed, attempts to drive the measured lateral body acceleration to zero on a steady curve. The negative feedback signal is provided by a body-mounted accelerometer and is used to apply tilt in a direction that will bring it towards zero. The advantage of this approach is that irregularities of the track have less influences the accelerometer measurement due to the action of the secondary suspension as a mechanical filter. The main drawback is that using a "nulling" compensation passengers experience motion sickness. For this reason partial tilt compensation control strategies have been developed. These allow to compensate only for a portion of steady-state curve transition.

Command-driven control was the next development. This approach uses both the cant deficiency, obtained from an accelerometer mounted on the bogie, and an additional feedback on tilt angle, to drive tilt actuator. In case of command-driven tilt control the sensor is not affected by the suspension response (it is now situated outside the control loop). However, due to the harsh environment of the bogie,
the accelerometer measures not only the curving acceleration but also acceleration components due to track irregularities. The tilt system responds to the effects of the track misalignments and leads to a worsening of the running behaviour. Moreover, the addition of a filter to reduce the effects of high frequency components and the required level of filtering for getting a satisfactory straight track performance introduces a significant time delay on curve transitions. This led to the development of the command-driven with precedence control approach.

*Precedence* control is a command-driven strategy which derives the tilt command signal from an accelerometer mounted on a non-tilting part of the preceding vehicle with a filter designed in such a way that the delay introduced will be compensated by the precedence effect. A scheme of the command driven with precedence control strategy is show in fig. 2.6

![Command driven with precedence control](image)

Figure 2.6: Command driven with precedence control

Finally a recent development uses a *track database* to provide the tilt command signal instead of the sensors used in the previous strategies. The accuracy of the track information provided from the database is a vital factor for this approach to produce effective results. In this thesis, the control scheme is presented in fig. 2.7. The required tilting angle is the reference which is defined on the basis of vehicle speed and curve geometry (curvature, cant, length of transitions), under the assumption that this information is made available to the control unit through geo-localisation of the train and mapping of the line ([5] and [11]). The main drawback of this strategy is that error on the position of the vehicle along the track and, therefore, an error in the generation of the reference signals may worsen the performances of the active system. In general, the range of positioning error compatible with GPS
system ([12]) is ±10 m, that is an acceptable range that the controllers need to manage. The feedback signal, in this case, is the vehicle body roll angle. Note that the way of defining reference angle is out of the scope of the thesis.

More details about tilt control strategies can be found in [7].
Chapter 3
Modelling

In this chapter two models for the study and the simulation of the Active Anti-Roll bar are illustrated. In the first section a 3-DoF linearised model is presented. In the second section, a description of a multi-body model developed by the Department of Mechanical Engineering of Politecnico di Milano is illustrated. In general a railway vehicle is a very complex mechanical system. It often consists of: a vehicle body, two bogies per vehicle and two wheel-sets per bogie. Each one of these bodies is characterised by six degree of freedom: roll, yaw, pitch, lateral, longitudinal and vertical mode. In addition to this a lot of non-linearities can affect the dynamics and the behaviour of the vehicle in unexpected ways. After have modelled the system, the results of a modal analysis are recalled from a previous thesis in order to show that there are some dynamic modes of the system that are coupled. This coupling in certain situations is very significant which unavoidably causes difficulties in mode identification (in particular vehicle lateral and roll modes).
Finally it is important to remark that obtaining a correct and detailed model is essential in order to achieve good results in simulation. This permits to design the controllers and simulate a large number of scenarios, before applying it into a real life experiment.

3.1 3-DoF vehicle model with actuator dynamic

In this section a mathematical model, based upon Newtonian approach, combined with modelling of the hydraulic circuit in place of passive anti-roll bar
mechanism, is developed. The active anti-roll device is designed to accomplish two main functions: actuate the desired carbody tilt angle when the vehicle negotiates a curve; provide the same carbody-to-bogie roll stiffness as a conventional anti-roll bar device when carbody tilt is not required. As explained in [5] the active roll function is only activated during curve negotiation and is implemented by the actuated servo-valve which controls the volume of oil in the two branches of the main hydraulic circuit, extending the linear actuator on one side while contracting the other actuator, thus providing carbody tilt. The dimensions and data of the actuator used in this thesis are taken from a previous work from Politecnico di Milano [2].

![Figure 3.1: 3-DoF model](image-url)
As can be seen from figure 3.1 the mechanical model of the vehicle is derived from a simplified end-view model version which has three degrees of freedom:

1. $x_v$: vertical displacement of the vehicle body;
2. $y_v$: lateral displacement of the vehicle body;
3. $\theta$: roll of the vehicle body.

For this model some hypothesis are done:

1. Bogie and wheel-set are assumed to be as ground and this is due to the huge stiffness of the primary suspension;
2. $k_{a1}$ and $k_{a2}$ represent the airspring’s stiffness;
3. carbody is assumed to be a rigid body.

Active Anti-Roll Bar hydraulic model is shown in fig.3.2. The details of mathematical modelling are provided in [5], [2] and [13].
The assumptions made here for modelling the hydraulic system are the following:

1. laminar and mono-dimensional flow in all pipes, losses due to fluid viscosity are neglected;
2. isothermal conditions of the fluid;
3. partially compressible fluid in the actuator chambers;
4. incompressible fluid in the pipes and reservoirs;
5. constant values for the supply and return pressures;
6. servo-valve assumed to behave as a 1st order system;
7. constant values for supply and return pressures;
8. the pressure differences between two chambers of both cylinders are the same: \( \Delta P = \Delta P_s = \Delta P_d \).

An exhaustive description of the hydraulic model can be found in the previously cited papers. Here for completeness and clearness the main equations are shown.

Considering conservation of mass for each actuator chamber it is possible to obtain the following equations:

\[
Q_I = Q_{1s} + Q_{2d} \\
Q_{II} = Q_{1d} + Q_{2s}
\]  (3.1)

Is possible to rewrite the equation considering the figure 3.2 as:

\[
\frac{V_{1s}}{\beta} \dot{P}_{1s} + \frac{V_{2d}}{\beta} \dot{P}_{2d} + C_i(\Delta P_s - \Delta P_d) + C_e(P_{1s} + P_{2d}) + A_p(\dot{y}_s - \dot{y}_d) = Q_I
\]

\[
\frac{V_{2s}}{\beta} \dot{P}_{2s} + \frac{V_{1d}}{\beta} \dot{P}_{1d} + C_i(\Delta P_s - \Delta P_d) + C_e(P_{2s} + P_{1d}) - A_p(\dot{y}_s - \dot{y}_d) = Q_{II}
\]  (3.2)

where the terms \( V_{1d}, V_{2d}, V_{1s} \) and \( V_{2s} \) are the volume of the actuator’s chambers to which the volume of the correspondent semi-reservoir is added, i.e \( \frac{V_{serb}}{2} \). \( \beta \) represent isothermal bulk modulus of the oil, \( C_i \) and \( C_e \) correspond to, respectively, internal and external leakages coefficient of the pistons.

Defining \( Q_R = \frac{Q_I - Q_{II}}{2} \), \( 2V_0 = V_{1s} + V_{2d} = V_{2s} + V_{1d} \) and considering the movement of the pistons around equilibrium position in the straight track it is possible to
simplify terms linked to compressibility and obtain only one equation $3.3$ that model the hydraulic circuit as a 1st order system with $\Delta P$ as the state variable.

$$\frac{V_0}{\beta} \Delta P_s + (2C_i + C_e)\Delta P_s + A_p(\dot{y}_s - \dot{y}_d) = Q_R \quad (3.3)$$

The model of passive anti-roll bar can be easily obtained by imposing $Q_R$ equal to zero in eq.3.3. In order to finalise the model of the active anti-roll bar it is necessary to introduce the equation of motion of the servo-valve that defines the link between flow rate and the position of servo valve. In [14] is proven that the model needed for the servo-valve can be obtained from the equation of the flow rate from an orifice:

$$Q_{or} = \frac{C_c A_0}{\sqrt{1 - (\frac{C_c A_0}{A_t})^2}} \sqrt{\frac{2\Delta P_{or}}{\rho}} \quad (3.4)$$

where $A_0$ is the area of the orifice, $C_c$ the contraction coefficient and $A_t$ is the total section of the pipe. One can clearly see that the flow rate is non-linearly dependent on the pressure drop across the servo-valve and the area. In particular, the area of the orifice is function of the position of the servo-valve spool $x_s$. Finally, after linearisation, it is possible to obtain the relationship between position of servo-valve spool, $x_s$, and flow rate, $Q_{or}$:

$$Q_{or} = Q_{or}(x_s, \Delta P_{or}) \approx K_q x_s + K_c \Delta P_{or} \quad (3.5)$$

Applying $3.5$ to the servo-valve it is possible to obtain two equations for $Q_I$ and $Q_{II}$, that are analogous to $3.2$, and in the same way as before, it is possible to calculate $Q_R$ as:

$$Q_R = K_q x_s + K_c \Delta P \quad (3.6)$$

where $x_s$ is within the range of -1 and 1 as specified in [2]. Finally it is possible to obtain the complete linear equation of motion of hydraulic anti-roll bar recalling eq.3.3:

$$\frac{V_0}{\beta} \Delta P + C_{tot} \Delta P + A_p b_{ol} \dot{\theta}_v = K_q x_s \quad (3.7)$$

where $C_{tot} = 2C_i + C_e + K_c$ is the total leakage coefficient, $\dot{y}_s - \dot{y}_d = b_{ol} \dot{\theta}_v$ with $b_{ol}$ the distance between actuators and $\dot{\theta}_v$ the carbody roll rate.
The servo-valve dynamics is described by the 1st order equation:

\[ \dot{x}_s + \frac{1}{\tau_s}x_s = \frac{k_s}{\tau_s}u_s \]  

(3.8)

with \( u_s \) the command to the servo-valve, \( \tau_s \) the time constant of the servo-valve and \( k_s \) a characteristic gain of the servo-valve.

These are the main equation that model the hydraulic system of the AARB. In particular a relationship between \( \Delta P \) and \( x_s \) is obtained.

The next purpose is to develop a mathematical model for the mechanical part of the system. The equations of motion of model in fig.3.1 are derived from Lagrange equations:

\[ \frac{d}{dt}\left( \frac{\partial E_k}{\partial \dot{x}} \right) - \left( \frac{\partial E_k}{\partial x} \right) + \left( \frac{\partial D}{\partial \dot{x}} \right) + \left( \frac{\partial V}{\partial x} \right) = \frac{\partial L}{\partial x} \]  

(3.9)

The linearised equation of motion can be formulated as:

\[ M\ddot{x} + R\dot{x} + Kx = Q \]  

(3.10)

where:

\[ x = [x_v, y_v, \theta] \]  

(3.11)

In order to obtain the linearised equations in the form (eq.3.10), \( E_k, D \) and \( K \) respectively kinetic energy, dissipation energy and potential energy are calculated:

\[ E_k = \frac{1}{2}m_v\dot{x}_v^2 + \frac{1}{2}m_v\dot{y}_v^2 + \frac{1}{2}J_v\dot{\theta}_v^2 \]  

(3.12)

\[ D = \frac{1}{2}r_v(-\dot{x}_v + \frac{b_{rv}}{2}\dot{\theta}_v)^2 + \frac{1}{2}r_v(-\dot{x}_v + \frac{b_{rv}}{2}\dot{\theta}_v)^2 + \frac{1}{2}r_v(y_v - h_v\dot{\theta}_v - \dot{y}_0)^2 + \frac{1}{2}r_{rod}\dot{\theta}_v^2 \]  

(3.13)
3.1 3-DoF vehicle model with actuator dynamic

\[ V = \frac{1}{2} k_i(y_v - h_{kl}\theta_v - y_0)^2 + \frac{1}{2} k_{tor}\theta_v^2 \]  

(3.14)

First of all the derivative of 3.12, 3.13 and 3.14 is performed:

\[ \frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{x}_v} \right) - \frac{\partial E_k}{\partial x_v} = m_v \ddot{x}_v \]

\[ \frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{y}_v} \right) - \frac{\partial E_k}{\partial y_v} = m_v \ddot{y}_v \]

\[ \frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{\theta}_v} \right) - \frac{\partial E_k}{\partial \theta_v} = J_v \ddot{\theta}_v \]

\[ \frac{\partial D}{\partial \dot{x}_v} = 2r_v \dot{x}_v \]

\[ \frac{\partial D}{\partial \dot{y}_v} = r_l \dot{y}_v - r_l h_{rl} \dot{\theta}_v - r_L \dot{y}_0 \]

\[ \frac{\partial D}{\partial \dot{\theta}_v} = -r_l h_{rl} \dot{y}_v + (r_l h_{rl}^2 + 2r_v \frac{h_{rl}^2}{4} + r_{rot}) \dot{\theta}_v - r_l h_{rl} \dot{y}_0 \]

\[ \frac{\partial V_{el}}{\partial x_v} = 0 \]

\[ \frac{\partial V_{el}}{\partial y_v} = k_{l}y_v - k_{li} h_{kl} \theta_v - k_{l}y_0 \]

\[ \frac{\partial V_{el}}{\partial \theta_v} = -k_{l} h_{kl} y_v + (k_{l} h_{kl}^2 + k_{tor}) \theta_v + k_{l} h_{kl} y_0 \]

Reformulating equations 3.10 in matrix form, it is possible define:

\[ M = \begin{bmatrix} m_v & 0 & 0 \\ 0 & m_v & 0 \\ 0 & 0 & J_v \end{bmatrix} \]  

(3.15)

\[ R = \begin{bmatrix} 2r_v & 0 & 0 \\ 0 & r_l & -r_l h_{rl} \\ 0 & -r_l h_{rl} & r_l h_{rl}^2 + 2r_v \frac{h_{rl}^2}{4} + r_{tor} \end{bmatrix} \]  

(3.16)
\[ K_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_l & -k_l h_{kl} \\ 0 & -k_l h_{kl} & k_l h_{kl}^2 + k_{tor} \end{bmatrix} \] (3.17)

Referring to \cite{2} the force generated from the airspring can be written as follows:

\[ \Delta F_{air} = k_a (x_v - \frac{b_{pm}}{2} \theta_v) \] (3.18)

where \( k_a \) is a constant. In general, the value of this stiffness coefficient is different for left and right airspring, but considering the symmetry of the problem and small oscillation of the carbody, it is possible to assume the same value for both sides. Therefore, it is possible to obtain an equivalent stiffness matrix for airspring:

\[ K_{air} = \begin{bmatrix} 2k_a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2k_a \frac{b_{pm}^2}{4} \end{bmatrix} \] (3.19)

Matrices 3.17 and 3.19 can be summed to obtain the total stiffness matrix:

\[ K_k = \begin{bmatrix} 2k_a & 0 & 0 \\ 0 & k_l & -k_l h_{kl} \\ 0 & -k_l h_{kl} & 2k_a \frac{b_{pm}^2}{4} k_l h_{kl}^2 + k_{tor} \end{bmatrix} \] (3.20)

It is important to remember that the active hydraulic anti-roll bar is used together with an active hydraulic secondary lateral suspension. The functions of this last device are to reduce the unloading of the inner wheels caused by the lateral displacement of the carbody and to prevent the lateral bump-stops to come in contact with the carbody, which would degrade the ride quality \cite{5}. The active lateral suspension envisaged here is a simple “hold-off” type described in \cite{15}, generating a lateral force in open loop which is defined to be proportional to the cant deficiency:

\[ F_{lat} = m_v \frac{v^2}{R} - m_v g \theta_0 \] (3.21)
3.1 3-DoF vehicle model with actuator dynamics

Because of the simple control strategy used for the active lateral suspension, the integration of active tilt and active lateral control at the secondary suspension as proposed in [16] is not taken into account here. Finally the virtual work can be calculated as:

$$\delta L = (-F_{lat} h_{lat}) \delta \theta_v + (A_p b_{ol} \Delta P) \delta \theta_v$$  \hspace{1cm} (3.22)

Where the second term is directly related to force generated by AARB. Thus it is possible to obtain matrix $Q$:

$$Q = \begin{bmatrix} 0 \\ 0 \\ (-m_v \frac{v^2}{R} + m_v g \theta_0) + (A_p b_{ol} \Delta P) \end{bmatrix}$$ \hspace{1cm} (3.23)

The matrix form can be formulated in another form:

Then it is possible to substitute all the previous equations in $3.9$ and obtaining:

$$\begin{align*}
    m_v \ddot{x}_v + 2 r_v \dot{x}_v + 2 k_a x_v &= 0 \\
    m_v \ddot{y}_v + r_l \dot{y}_v + r_l h_{rl} \dot{\theta}_v + k_l y_v - k_l h_{kl} \theta &= 0 \\
    J_v \ddot{\theta} - r_l h_{rl} \dot{y}_v + (r_l h_{rl}^2 + 2 r_v b_{rv}^2 + k_{rot}) \dot{\theta} - k_l h_{kl} y_v + \\
    + (2 + k_a b_{pm}^2 + k_l h_{kl}^2 \theta_v) \theta &= (-m_v \frac{v^2}{R} + m_v g \theta_0) h_{lat} + (A_p b_{ol} \Delta P) \\
\end{align*}$$ \hspace{1cm} (3.24)

It is possible to note that also disturbances ($y_0$ and $\dot{y}_0$) can be taken into account in this formulation:

$$\begin{align*}
    m_v \ddot{x}_v + 2 r_v \dot{x}_v + 2 k_a x_v &= 0 \\
    m_v \ddot{y}_v + r_l \dot{y}_v - r_l h_{rl} \dot{\theta}_v + k_l y_v - k_l h_{kl} \theta - (r_l \dot{y}_0 + k_l y_0) &= 0 \\
    J_v \ddot{\theta} - r_l h_{rl} \dot{y}_v + (r_l h_{rl}^2 + 2 r_v b_{rv}^2 + k_{rot}) \dot{\theta} + r_l h_{rl} \dot{y}_0 - k_l h_{kl} y_v + \\
    + (2 + k_a b_{pm}^2 + k_l h_{kl}^2 \theta_0) \theta + k_l h_{kl} y_0 &= (-m_v \frac{v^2}{R} + m_v g \theta_0) h_{lat} + (A_p b_{ol} \Delta P) \\
\end{align*}$$ \hspace{1cm} (3.25)

Rearranging equations $3.25$, a new formulation is obtained:
\[
\begin{align*}
\ddot{x}_v &= \frac{1}{m_v}[-2r_v \dot{x}_v - 2k_a x_v] \\
\ddot{y}_v &= \frac{1}{m_v}[-r_l \dot{y}_v + r_l h_{rl} \dot{\theta}_v - k_l y_v + k_l h_{kl} \theta + r_l y_0 - k_l y_0] \\
\ddot{\theta} &= \frac{1}{J_v} [r_l h_{rl} \dot{y}_v - (r_l h_{rl}^2 + 2r_v \frac{b_v^2}{4} + k_{rot}) \dot{\theta} - r_l h_{rl} y_0 + k_l h_{kl} y + \\
&- (2 + k_a \frac{b_{pm}^2}{4} + k_l h_{kl}^2 k_{tor}) \theta - k_l h_{kl} y_0 + (-m_v \frac{v^2}{R} + m_c g \theta_0) \dot{h}_{lat} + (A_p b_{ol} \Delta P)]
\end{align*}
\]

In (3.26) linearised equation of motion of the hydraulic anti-roll bar (3.7) is combined with the 3-DoF vehicle model (3.26).

The state vector \( \mathbf{x} \), the input vector \( \mathbf{u} \) and the disturbances vector \( \mathbf{w} \) for the model of the actuated vehicle are defined as follows:

\[
\mathbf{x} = \begin{bmatrix} \dot{x}_v & \dot{y}_v & \dot{\theta}_v & x_v & y_v & \theta_v & \Delta P & x_s \end{bmatrix}^T
\]

\[
\mathbf{u} = [u_s]^T
\]

\[
\mathbf{w} = \begin{bmatrix} R^{-1} & \theta_0 & \dot{y}_0 & y_0 \end{bmatrix}^T
\]

Finally it is possible to rearrange the equation in matrix-form in order to obtain a state-space representation:
3.1 3-DoF vehicle model with actuator dynamic

\[ \begin{align*}
\begin{bmatrix}
\dot{x}_e \\
\dot{y}_e \\
\dot{\theta}_e \\
\dot{\Delta}P
\end{bmatrix} &= \\
+ &
\begin{bmatrix}
0 \\
n_m g h_{lat} \\
-k_l h_{lat} \\
r_l h_{lat}
\end{bmatrix}
\begin{bmatrix}
\dot{\gamma}_l \\
\dot{\gamma}_r
\end{bmatrix}
\end{align*} \]

\[ \begin{bmatrix}
\dot{x}_e \\
\dot{y}_e \\
\dot{\theta}_e \\
\dot{\Delta}P
\end{bmatrix} = \\
\begin{bmatrix}
0 \\
n_m g h_{lat} \\
-k_l h_{lat} \\
r_l h_{lat}
\end{bmatrix}
\begin{bmatrix}
\dot{\gamma}_l \\
\dot{\gamma}_r
\end{bmatrix} + \\
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a
\end{bmatrix}
\]
26 Modelling

The last formulation can be arranged in short state-space form as:

\[
\dot{x} = Ax + Bu + \Gamma w
\]  \hspace{1cm} (3.28)

Where matrix \(A\), \(B\) and \(\Gamma\) can be seen from equation 3.27.

Finally, some considerations about modal analysis can be done. In particular a modal analysis based on the state space form in eq.3.28 with the parameter values listed in Appendix B is carried out in the thesis [6]. One can clearly see, from matrix \(A\) that the vertical mode is completely decoupled from the other two modes and that there is a huge coupling between lateral and roll motions.

As shown in [6], the vertical mode is completely decoupled from the other two modes. However, the upper and lower sway modes are not significantly distinguished from each other since there is a huge coupling between lateral and roll motions. For completeness, in table 3.1 are reported the system modes of the 3-DoF model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalues</th>
<th>Damping</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>(-0.64 \pm 5.39j)</td>
<td>11.8</td>
<td>0.86</td>
</tr>
<tr>
<td>Lower Sway</td>
<td>(-1.33 \pm 3.16j)</td>
<td>38.9</td>
<td>0.54</td>
</tr>
<tr>
<td>Upper Sway</td>
<td>(-1.87 \pm 8.62j)</td>
<td>21.2</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Table 3.1: System modes of the 3-DoF model without actuator dynamics

3.2 Multi-body Model

Another model was used in this thesis to assess the performance of different control logics. It consists of multi-body model of a complete rail vehicle, defined using the in-house simulation software ADTreS developed by the Department of Mechanical Engineering of Politecnico di Milano. The multi-body model was interfaced with a non-linear model of the hydraulic anti-roll devices. More detailed information about the model can be found in [5] and [17]. The rail vehicle model consists of one carbody, two bogies and four wheelsets, all considered as rigid bodies, and of primary and secondary vehicle suspensions, modelled as linear and non-linear lumped parameter elastic and viscous elements. Each rigid body has assigned
five degrees of freedom (vertical and lateral displacements, yaw, roll and pitch rotations), while the forward motion of the body’s centre of mass is assumed to have constant speed \( V \). Therefore, the model has a total of 35 degrees of freedom as shown in fig.3.3. Wheel-rail contact forces are introduced according to a non-linear multi-Hertzian contact model \[18\]. Excitation introduced by track irregularity is caused in the simulation by considering a time history of vertical and lateral displacements applied to the contact points on the rails \[5\].

The motion of each body is described with respect to a moving reference travelling with constant speed along the track centre-line. The bodies are assumed to undergo small displacements relative to the moving reference so that the equations of motion for the vehicle can be linearised with respect to kinematical non-linear effects only, and take the symbolic form:

\[
M \ddot{x} + R \dot{x} + Kx = Q(x, \dot{x}, v, t) \tag{3.29}
\]

where \( x \) is the vector of the vehicle’s 35 coordinates, \( V \) is the vehicle speed and \( Q \) contains inertial forces due to the non-inertial motion of the moving references, the forces due to non-linear elements in the suspensions and the generalised forces due to wheel-rail contact.
3.3 Simulation Parameters

Standard parameters used for all the simulations in this thesis are defined in this section. In particular track geometry refers to a set of parameters that describes both the layout and the path of a railway track. Railway vehicles are generally subject to two main categories of inputs from the track:

a. Deterministic input (or design track), i.e curves and gradient;

b. Stochastic input (or track irregularities), i.e. random changes in the track vertical, lateral, and cross-level position.

As far as deterministic inputs concern, a single high-speed curve geometry with radius 5500 m and cant 105 mm is considered, which is very common in the Italian high-speed network and represents for this scenario the most demanding case for tilting trains. Table 3.2 presents deterministic inputs used in this thesis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cant</td>
<td>105</td>
<td>mm</td>
</tr>
<tr>
<td>Maximum curve radius</td>
<td>5500</td>
<td>m</td>
</tr>
<tr>
<td>Initial rectilinear length</td>
<td>300</td>
<td>m</td>
</tr>
<tr>
<td>Transition length</td>
<td>330</td>
<td>m</td>
</tr>
<tr>
<td>Curve length</td>
<td>1000</td>
<td>m</td>
</tr>
<tr>
<td>Active train speed</td>
<td>338</td>
<td>km/h</td>
</tr>
<tr>
<td>Passive train speed</td>
<td>300</td>
<td>km/h</td>
</tr>
</tbody>
</table>

Table 3.2: Track profile

The stochastic track inputs represent the deviations of the actual track from the intended alignment, these could involve lateral alignment, vertical alignment and/or cross-level (cant deviations). These data enter into the system considered as disturbances. More details both on deterministic and stochastic input can be found in [7]. Note that stochastic inputs do not participate in simulations performed in this thesis because the main purpose was to analyse the tracking properties of the synthesised controllers with respect to deterministic input.
3.4 Tilt System Performance Assessment Parameters

Although active tilting has become a standard technology incorporated into the railway industry, a number of issues remain which need to be resolved for determining the performance of tilting trains. Qualitatively, a good tilt control system will respond principally to the deterministic track inputs, while ignoring as much as possible any random track irregularities. In order to assess different tilt control approaches in an objective manner, it is essential to define appropriate criteria and conditions. The thesis [7] proposes a rigorous overall approach for assessing the performance of tilt control systems. In particular it is possible to define different simulation outputs that are used for assessing the controller performance:

1. *PCT* factor is a measure of ride comfort on curve transition and it is defined from European Standard [8] as:

\[
P_{CT} = 100 \cdot \max[(A|\ddot{y}|_{max} + B|\dddot{y}|_{max} - C); 0] + (D|\dot{\rho}|_{max})^E \tag{3.30}
\]

where \( |\ddot{y}|_{max} \) is the maximum absolute value of lateral acceleration in the vehicle body, in the time period between the beginning of transition curve and the end plus 1.6 s, expressed in \( m/s^2 \), averaged over intervals of 1 s, \( |\dddot{y}|_{max} \) is the maximum absolute value of lateral jerk in the transition curve, in the time period between 1 s before the beginning of the transition curve, and the end of the transition, expressed in \( m/s^3 \), averaged over intervals of 1 s, \( |\dot{\rho}|_{max} \) is the maximum absolute value of roll velocity, in the time period between the beginning and the end of the transition curve, expressed in radians per second, averaged over intervals of 1 s. Two versions of this index exist: one is for evaluating the comfort of seated passengers whereas the others is used for standing passengers. The differences between this two indexes resides in the constant \( A, B, C, D \) and \( E \) that are shown in eq(3.30). An higher *PCT* value is related to a greater possibility of passengers suffering motion sickness, thus its minimization is, generally, a priority of the tilting suspensions;

2. *R.M.S. (\ddot{y})* is the root mean square of the lateral acceleration. This index is
directly related to how the system responds to track irregularity;

3. \( \text{max}(\epsilon) \) is the maximum absolute error in trajectory tracking;

4. \( R.M.S.(u) \) is the root mean square of the actuation command.

The first and second parameters are used to evaluate the ride comfort, respectively with reference to the negotiation of a curve transition and to the response to track irregularities. The third parameter defines the performance of the controllers in terms of trajectory following. The last one is adopted to evaluate energy consumption of the active anti-roll bar.

It is important to remark that ride quality is closely connected to human evaluation of comfort and its assessment is based upon given statistics and weighting factors \[19\]. More details about the parameters presented in previous numbered list, used for assessing performance and how they can be calculated, can be found in \[7\].
Chapter 4

Modelling for control (Model reduction)

4.1 Preliminary considerations

In the previous chapter a state space representation of the model is formulated. In this chapter different model-reduction techniques are proposed. Model-reduction techniques are largely adopted in control engineering (in simulation in particular) because many systems are quite complex and difficult to work with in their original form. Thus, reduced mathematical models can be used to approximate high-order systems in order to handle them easier, by means of their dominant pole-zeros in the complex plane. By reducing the complexity of the system some clear advantages can be seen: simplification of the control design process, simplifications of the simulations process, elimination of system modes that are irrelevant to control and identification of most important characteristics of the system. In addition to this, observer-based control-design methodologies are more and more used in modern control. In conclusion, different methods have been developed to estimate the so-called "dominant" part of the original system and formulate a reduced-order system representation that has a very similar dynamic behaviour to the original one.

In order to perform model-reduction, two main approaches exists: the first attempts to approximate an input-output TF of the plant by a lower order dynamic system, which automatically results in lower-order controllers; the second approach attempts to apply approximation techniques directly to the controller.

There are many papers and books in the scientific literature regarding model-reduction techniques.
reduction, such as [21], [22], [23] and [20] closely linked to tilting technology. It is important to note that even when a very detailed model is available one may choose to work with a simpler reduced order nominal model and represent the neglected dynamics as uncertainty. For this reason model-reduction should be carried out so that most dominant and critical modes of the system are not highly affected by the reduction itself. In particular, reducing a model, uncertainties are introduced due to the unmodelled dynamics, but this must have minimal effect on stability and performance; Thus, with this model are often used robust control techniques.

First of all, a quantitative measure to analyse the model dominance for continuous system, proposed in [24] is carried out. Adopting this method, information about modal dominance and most important modes of the system can be obtained. Consequently it is possible to notice which poles are dominant even when they are not the slowest. After that, a manual-reduction can be performed. In [21] another reduction method is used: based on the concepts of stability equation and important poles to find the reduced denominator and after that complex-curve fitting method to find the reduced numerator. In literature there is a specific paper on tilting problem [20], where model-reduction techniques are performed in order to design a simplified LQG controller for the specific curving performance of railway vehicles.

In the next sections a method to measure the dominance of a single mode is performed. After that a manual reduction is obtained taking into account the results obtained from modal dominance index quantitative analysis. Subsequently, analytically-based reduction and an analysis on model uncertainty is done. Finally, a reduced model is obtain identifying a set of responses obtained by the simulator and a comparison of the different reduced model is carried out.

Looking now, in particular, at our system, it is possible to extrapolate the needed transfer function from the state space formulation obtained in the previous chapter. In this chapter is taken into account the transfer function $G_{yu}(s)$ for the preview tilt (PT). This is the transfer function of the open loop between control input and vehicle body roll. It is important to remark that this TF has 11 states; This means that would be very difficult dealing with it and easily design an effective controller. For clearness it is illustrates in two different formats:
4.2 Modal Dominance Index

Zero-pole-gain format:

\[ G_{yu}^{PT}(s) = \frac{138.69(s^2 + 3.192s + 36.73)}{(s + 22)(s + 0.002693)(s^2 + 2.394s + 27.29)(s^2 + 9.321s + 119.1)} \]  \hspace{1cm} (4.1)

Time-constant format:

\[ G_{yu}^{PT}(s) = \frac{264.49(1 + 0.5267(0.1625s) + (0.165s)^2)}{(1 + 0.04547s)(1 + 3713s)(1 + 0.4585(0.1914s) + (0.1914s)^2)(1 + 0.8539(0.9161s) + (0.9161s)^2)} \]  \hspace{1cm} (4.2)

This TF will be reduced in the next sections with different method in order to reach a simpler but very effective model.

4.2 Modal Dominance Index

In this section a method to measure the dominance of a single mode is performed. In this way it is possible to understand if the model-reduction done using a manual approach is correct and coherent.

As illustrated in [24], a quantitative measure of modal dominance can be computed. In this way, it is possible to understand properly which modes dominate the behaviour of the system. One weakness of considering every slow modes always is that they may have a small contribution to the output of the system. On the contrary of a manual "qualitative" reduction, where modes are taken into account only according on experience and observation of time constants in this section new indices are used in order to indicate which poles are really dominant. With this method, in fact, it is possible to provide a quantitative measure of modal dominance, not only related on the magnitude of the time constants. The reason of performing this analysis is that the choose of a mode as dominant or not, basically not taking into account properly the steady-state behaviour of the system. For this reason may there be a fast modes, that we delete, which heavily affect the output.

As illustrates in [24], it is important to note that this method is not used in order to derive a simplified model but it can be used for checking which modes are
really dominant. This is very useful in model reduction problem.

Considering the simplified state space model\textsuperscript{4.3} obtained from transfer function\textsuperscript{4.1}:

\[
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx 
\end{aligned}
\label{4.3}
\]

where \(x \in \mathbb{R}^n\), \(u \in \mathbb{R}^v\), \(y \in \mathbb{R}^u\) and \(A\), \(B\) and \(C\) are real constant matrix of appropriate dimension, it is possible to calculate the needed matrix \(\tilde{A}\), \(\hat{B}\) and \(\hat{C}\), derived from the state matrix of the system\textsuperscript{4.3} and defined as:

\[
\tilde{A} = V^{-1}AV \\
c_j V = [\tilde{c}_1 \tilde{c}_2 \ldots \tilde{c}_n] \\
\hat{C}_j = diag[\tilde{c}_1 \tilde{c}_2 \ldots \tilde{c}_n] \quad j = 1, 2, \ldots, u \\
C = [c_1 c_2 \ldots c_u]^T \\
V^{-1}b_i = [\hat{b}_1 \hat{b}_2 \ldots \hat{b}_n]^T \\
\hat{B}_i = diag[\hat{b}_1 \hat{b}_2 \ldots \hat{b}_n] \quad i = 1, 2, \ldots, v \\
B = [b_1 b_2 \ldots b_v] \\
V = [v_1 v_2 \ldots v_n]^T
\]

where \(diag[\cdot]\) denotes diagonal matrix, \(V\) represent the modal matrix and \(v_i\) is eigenvector associated with the \(i\)-th eigenvalue of \(A\).

Finally the MDIs for the path defined from control input to the carbody roll output can be obtained by compute:

\[
\begin{bmatrix}
\sigma_{1}^{ij} \\
\sigma_{2}^{ij} \\
\vdots \\
\sigma_{n}^{ij}
\end{bmatrix} = -Re[\hat{C}_j \hat{B}_i \tilde{A}^{-1}] 
\label{4.5}
\]
where $Re$ denotes the real part.

On the paper [24] is said and proof that, for SISO model and $G(s) = C(sI - A)^{-1}B$, $\sigma_i = \gamma_i$ where $\gamma_i$ is defined as $i$-th model dominance index (the purpose of this analysis). Thus it is possible to write (4.5) as (4.6)

$$- Re[\hat{C}_j\hat{B}_i A^{-1}] = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{bmatrix}$$ (4.6)

After a brief theoretical explanation of the method, the algorithm to calculate MDI is applied to TF $G(s)$ and the results are shown in the table 4.1:

<table>
<thead>
<tr>
<th>Poles (All modes)</th>
<th>MDI</th>
<th>MDI %</th>
<th>Accumulated MDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.0003$</td>
<td>$\sim 1.0$</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$-1.1971 \pm 5.0849j$</td>
<td>$\sim 0.0$</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$-4.6600 \pm 9.8697j$</td>
<td>$\sim 0.0$</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$-21.9955$</td>
<td>$\sim 0.0$</td>
<td>0.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 4.1: Model dominance index

One can clearly see that the most important pole is the integrator (introduced by the hydraulic system). Of course, the other modes can not have a MDI of 0%, but the order of magnitude of these indexes is of $10^{-6}$, thus not appreciable. This is the confirmation that the system can described in the easiest way as an integrator. Moreover, as can be seen from the table it is not possible extrapolate information about other modes of the system. In this context, we firstly perform a slow-fast decomposition due to the fact that by default the almost integrating element of the system (actuator dynamics) is dominating the response and it is not possible to see the "dominace" of the other modes. Thus the remaining modes are splitted into $G_{yu}(j\omega) = G_{yu-s}(j\omega) + G_{yu-f}(j\omega)$ where the first term are the slow modes of the system and the latter term are the remaining faster modes (also see [20]).

It is possible to note that the poles with real part 1.19 (first row in the table representing the most dominant 'fast' mode in the system corresponding to approximately 59% MDI. From the MDI results there is an early indication that a 3rd order plant transfer function (i.e. almost integrator dynamics plus the most
dominant faster mode) approximation for control design seems an appropriate choice as a starting point. Finally it is important to be noticed that MDI can be used as a simple metric on consideration of plant order reduction for control design, however here we mainly employ it to present modal importance in the system.

### 4.3 Manual Model Reduction

In this section a manual-reduction, based on experience and observance of the transfer function form, is performed. This is done because often using analytically-based model reduction techniques we obtain a model that does not relate in an obvious way to the physical variables that typically describes the original system. By means of manual-reduction, on the contrary, it is possible to look at the original model as a sum of first and second order systems.

From theory, it is known that time-constants are related to how quick the dynamics of a particular modes behaves in time. In general time constant $\tau$ is defined as the time used for the system to reach 63% of the steady state response to a unitary step input. For first order system it is possible to say that time-constant that are very small are related with fast modes, that die out very quickly. On the contrary, large time-constant are related with very slow modes.

Looking in particular at **first order** time constant:

\[
|1 + \tau s| = |1 + \tau(j\omega)| = \sqrt{1^2 + \omega^2\tau^2}
\]  

If, in (4.7) $\tau$ is very small, $\tau^2$ is smaller and it follows that:

<table>
<thead>
<tr>
<th>Poles (fast modes)</th>
<th>MDI ($\gamma$)</th>
<th>MDI % ($\gamma%$)</th>
<th>Accumulated MDI ($\text{Accumulated}\gamma%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.1971 \pm 5.0849j$</td>
<td>0.5874</td>
<td>58.74</td>
<td>58.74</td>
</tr>
<tr>
<td>$-4.6600 \pm 9.8697j$</td>
<td>0.3328</td>
<td>33.28</td>
<td>92.02</td>
</tr>
<tr>
<td>$-21.9955$</td>
<td>0.0797</td>
<td>7.97</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Table 4.2: Model dominance index without integrator
\[ \sqrt{1^2 + \omega^2 \tau^2} \approx 1 \] (4.8)

With the same passages, it is possible understand what happen if \( \tau \) is quite large:

\[ \sqrt{1^2 + \omega^2 \tau^2} \approx \tau \omega \] (4.9)

It is important to be noticed that these real time constants are at the denominator of TF [1.2]. Thus, in the first situation [4.8] it is possible to cancel out the first time constant. On the other hand, in the second case [4.9] where \( \tau \) is very large it is possible recognize the lag dominant mode, that is clearly introduced by the linearised dynamics of the hydraulic actuator.

\[
\begin{align*}
\text{if } \tau \text{ small} & \Rightarrow \frac{1}{1 + \tau s} \approx 1 \\
\text{if } \tau \text{ large} & \Rightarrow \frac{1}{1 + \tau s} \approx \frac{1}{\tau \omega j}
\end{align*}
\]

After having analysed first order time constants, the focus is moved on the second order polynomials. In this case there are not only time constants that characterise the dynamic response of the system but also damping, that acts an important role.

Looking at time constants of **second order** polynomial:

\[
s^2 + 2\xi(\tau)s + (\tau s)^2 = \omega^{-2}s^2 + 2\xi\omega^{-2}s + 1
\] (4.10)

Setting \( \tau = \omega^{-1} \):

\[
\begin{align*}
\tau^2 s^2 + 2\xi \tau s + 1 = \\
\omega^2[\tau^2 s^2 + 2\xi \tau s + 1]
\end{align*}
\] (4.11)

The general rule that is followed with **manual reduction** is: keep the large real time constant, delete the small real time constant, keep the complex pair pole
with low damping and maintain the appropriate DC gain (generally a similar gain between reduced and original FRF). Focusing now at the denominator of eq. 4.2 it is possible to see that very different first-order time-constant are present ($\tau_1 = 0.04547$ and $\tau_2 = 3713$). In particular, $\tau_1$ is very small and $\tau_2$ is very large.

For this reason it is possible to delete $\tau_1$, that is related to a dynamic that is going to die out very quickly. On the other hand $\tau_2$, linked to the integrator (because of the denominator), is related to a very slow mode that influences in a more significant way the behaviour of the system.

Moreover, looking at $(1 + \tau_1 \omega j)$, it is possible to note that because of $\tau_1 = 0.04547$, $\tau_1 \omega j$ become relevant compared to 1 when the amplitude of $j\omega$, increase a lot. In other words when the frequency of perturbations/inputs becomes quite large. In general, usually, for mechanical problem a specific range of frequencies is taken into account. Thus in this case it is important to remark that the system works very slow according to its dynamics and in this particular case, an hydraulic actuator is present in the system and normally this kind of devices will not have a bandwidth which is more than 3 or 4 Hz ($\approx 20$ rad/s). For this reason every perturbation or input over those value will not be amplified because the hydraulic system acts like a filter and every frequencies above the threshold is going to be cut. The presence of these actuators permit to forget about this range of high frequency both as input and disturbances. These are the so called disregarded high frequencies dynamics and do not have a really big impact on our the model. For this reason it is possible to look at the disregarded poles at high frequencies as model uncertainties due to a mismodelling in the same range of frequencies.

It is also known that time constants for order over first have a similar meaning: they are related with envelope and not to actual response, but the concept is very similar. Thus, they are comparable between each other. Looking again at the denominator of 4.2 two different second order polynomials are present with two different time constant that are about 0.09 and 0.19. So basically, the one with 0.09 has a faster dynamic, that will die out twice as quick compared to the other one.

With second order poles/zeros is important to take into account also the damping factor. Looking at the same denominator, comparing 4.11 and the third bracket of 4.2 it is possible to write that $0.45 = 2\xi_1$ for the first polynomial and $0.85 = 2\xi_2$ for the second polynomial, so $\xi_1 = 0.22$ and $\xi_2 = 0.45$ can be obtained. In addition to this, the polynomial with slower time constant will have more oscillation, on the contrary of the one with faster constant that will have less oscillation due to larger
damping factor.
Finally, looking at the denominator and numerator of 4.2 together one can clearly see that complex zeros are very close to the slower dynamic of the denominator (third bracket in 4.2), both in time constant and in damping factor. Thus they are quite comparable and this can be considered an almost cancelling well-damped complex poles-zeros pair. They can be cancelled without losing so much information on the dynamic behaviour of the system. It is important to remember that poles have a lag effect; On the other hands zeros have a lead effect. If they are close, as can be seen in fig.4.1, a lag and a lead effect, that are almost cancelling each other, are present.
Another important thing to remark is that, close complex-zeros in the case of 4.2 are quite damped. This allow to cancel them. On the contrary, in the case they were not quite damped, in theory they can be cancelled (because of the previous explanation) but in practice, even if they are very close, they can cause hidden oscillation.

The same manual analysis can be performed looking at the poles-zeroz map in fig.4.1 of the transfer function eq. 4.1. The almost cancelling well-damped complex poles-zeros pair cited in the previous paragraph is clearly visible in the fig.4.1.

![Figure 4.1: Pole-zero map of $G_{yu}^{PT}$](image)

Zooming close to the origin the integrator pole can be clearly seen. Another
pole with real part -20 and a complex pair of poles with real part -5 can be seen. Is important to notice that this last pair has less damping than other complex pair of poles, thus they involve more oscillations. The time constant of this complex pair, can be seen from eq. 4.2, and is equal to 0.09.

For assessing this manual reduction the TF \( G_{00} \) is decomposed in different parts. Every parts can be considered as an independent first/second order system:

\[
G_{00} = 264.49 \quad (4.12)
\]

\[
G_{11} = \frac{1}{(1 + 0.04546s)} = \frac{21.997}{(s + 22)} \quad (4.13)
\]

\[
G_{12} = \frac{1}{(1 + 3713s)} = \frac{0.00026932}{(s + 0.0002693)} \quad (4.14)
\]

\[
G_{21} = \frac{(1 + 0.5266(0.165s) + (0.165s)^2)}{(1 + 0.4583(0.1914s) + (0.1914s)^2)} = \frac{0.74316(s^2 + 3.192s + 36.73)}{(s^2 + 2.394s + 27.3)} \quad (4.15)
\]

\[
G_{22} = \frac{1}{(1 + 0.8539(0.09162s) + (0.09162s)^2)} = \frac{119.13}{(s^2 + 9.32s + 119.1)} \quad (4.16)
\]

In particular \( G_{00} \) represent the DC gain of the original model. The TF shown in 5.3 contains the very fast real pole. On the contrary TF \( G_{11} \) is the one with slow real pole. The TF represented \( G_{12} \) is the "lag" portion of the closely placed complex pair zero and pole. Lastly, the TF \( G_{21} \) is the one with faster less damped complex pair poles.

In the figure 4.2 it is possible to see on Bode diagram different contributions of all the part in which the of original TF has been decomposed.
Looking at fig. 4.2 it is interesting to note that $G_{11}$, $G_{21}$ and $G_{22}$ seem to have small impact within the frequency range of interest. Thus, for a first rough approximation $G_{00}$ and $G_{12}$

First of all two simple approximation are taken into account: the first approximation is composed from $G_{00}$, the DC gain, and $G_{12}$, the most dominant pole, resulting in 4.17. The second one 4.18 (where i at the superscript stands for integrator) can be seen as a simplification of eq. 4.17 taking into account only a simple integrator.

$$G_{MR,I} = \frac{0.071232}{s + 0.0002693}$$  \hspace{1cm} (4.17)

$$G_{MR,I}^i = \frac{0.071232}{s}$$  \hspace{1cm} (4.18)
These are probably the simplest reduction for the TF (eq. 4.1) that is possible to perform as can be seen in figure 4.3.

![Bode Diagram](image)

**Figure 4.3**: Bode diagram of original plant and first approximations

The manual reduction permit to understand how the system is composed in terms of poles/zero. In general TF 4.18 that takes into account only the integrator, is probably a too simple and rough reduction. This first-order (I) model will be used later in the thesis to design very simple PID controllers.

Recalling MDI calculated in the previous section 4.2, one can clearly see that in order to achieve a good reduced-model formulation to assess the performance for tilting train vehicle, a more complex approximation may be required, than the simple integrator. A more complex reduced model, in order to not lose too many information, can be obtain as the composition of $G_{00}$, $G_{12}$ and $G_{22}$. This TF is composed by: DC gain, the integrator (because it is the most dominant pole) and the two most dominant complex poles, obtaining a $3^{rd}$ order system 4.19.
4.4 Analytically-based model-reduction

\[ G_{MR,III} = \frac{8.4859}{(s^3 + 9.3203s^2 + 119.1025s + 0.0321)} \]  

(4.19)

As can be seen from fig 4.4 the bode diagram of transfer function 4.19 is a better approximation of the transfer function of the original plant 4.1 than a simple integrator (TF 4.18). In addition to this, the third order approximation 4.19 seems to fit good to the original plant also at higher frequency than the less order reduced model.

Finally it is noteworthy to underline that the same approximation obtained by manual reduction can be also obtained by following MDI analysis results.

4.4 Analytically-based model-reduction

In this section we compare the analytical approach to the manual one, derived from experience, performed in the first section sec:Manual model reduction of this chapter. Another detailed and more complex analytically-based model reduction
on high-speed tilting train is presented in [20].

Here in particular a toolbox of MATLAB©, called model reducer, is adopted. This tool is very useful when, dealing with high-order models, it is necessary to obtain a reduced model in order to work with an "object" that allows a simplified analysis and controller design. As said at the beginning of this chapter, the drawbacks of using this tool or to perform an analytical reduction is that the internal description of the reduced model does not relate directly or in an obvious way to the original physical variables, which typically describe the real problem. In other words the "contact" with real variables is lost. These tools allow to follow different reduction methods, each one help to reduce model order while preserving model characteristics that are important to our application. In particular:

- **Balanced Truncation** - Remove states with relatively small energy contributions.

- **Mode Selection** - Select modes by specifying a region of interest in the complex plane.

- **Pole-Zero Simplification** - Eliminate cancelling or near-cancelling pole-zero pairs.

In this thesis only the first method is taken into account, because the focus of this chapter is to reduce the model cancelling out the states that do not give an appreciable contribution to the system response without. Finally we are going to compare model obtained from manual reduction. It is important to remark that manual reduction is helped by MDI analysis, that gives quantitative information about the dominance of a particular mode (4.2).

The other two methods presented in the previous list work in a different way, out of the interest of this work. In particular the second method (*mode selection*) works on the phase’s change rate. The problem here, is that if we include also the frequencies at which the poles less-damped are present, the system will be approximated adding two non-minimum phase poles. The third method (*pole-zero simplification*) instead, uses as maximum tolerance in order to delete pole-zero pairs that is not enough to remove any pair from the initial TF. This because the zeros-poles pairs, that can be seen in pole-zero map (fig. 4.1), are not so close to be cancelled for the algorithm implemented, but it is possible to delete them in man-
Applying balanced truncation method for first, second and third order reduction we obtain:

\[
G_{AR,I} = \frac{-0.017751(s - 8.028)}{(s + 0.0002693)}
\]
(4.20)

\[
G_{AR,II} = \frac{0.017002(s^2 - 37.83s + 371)}{(s + 0.0002693)(s + 44.26)}
\]
(4.21)

\[
G_{AR,III} = \frac{-0.0020762(s - 17.32)(s^2 + 8.131s + 152.5)}{(s + 0.0002693)(s^2 + 4.622s + 38.47)}
\]
(4.22)

Where the roman number stands for the order imposed to obtain the reduced model.

As explained at the beginning of the chapter, from TFs (eq.4.20), obtained analytically, it is difficult to understand the role of each physical variables.

Here below there are the bode diagram of the three TF obtained.

![Bode diagram](image)

Figure 4.5: Bode diagram of the original plant and $G_{AR,I}$
Figure 4.6: Bode diagram of the original plant and \( G_{AR,II} \)

![Bode Diagram](image)

Figure 4.7: Bode diagram of the original plant and \( G_{AR,III} \)

![Bode Diagram](image)

It is clearly visible that the third-order analytically reduced system differs from the low order TF only at frequency higher than \( 3Hz \) which is out from the frequency range of interest. Below this frequency the different order reduced system
are similar in terms of magnitude and phase.

4.5 Model uncertainty analysis

In this section, the uncertainty introduced by reducing the model to a lower order is analysed. The purpose is to understand the entity of uncertainty after the model reduction. It is important to remark that, in order to deal with uncertainty in model, robust control must be designed. Indeed, a control system can be considered robust if it is insensitive to differences between the original system and the system used to design the controller. More information about robust stability can be found in [25]. Note that the scope of this section is to quantify the uncertainty between the original and reduced model.

Uncertainty can be introduced by original/reduced model mismatch, i.e neglected dynamics (this is related with the thesis work), or can have other several origins: unknown parameters, non-linearities, measurements noise, unknown dynamics at high frequency. It is important to notice that the first kind of uncertainties is known by the designer that try to minimize these differences while reducing original plant. On the contrary, the latter are unwanted uncertainty that are harder to quantify. In the thesis’ case, it is supposed that the original model is not affected by uncertainty of the second type. Thus, it is possible analyse only the uncertainty derived from model reduction developed in the previous sections.

The original model can be represented by an approximated model and the remaining neglected characteristics bounded by uncertainty. Hence, uncertainty bounds can include all the uncertainties introduce by neglected frequencies. It is important to report any instabilities in the original system also in the reduced model (unstable modes must be reflected in all simplifications). Note that in this application (tilting control) there are no unstable poles, as can be seen from fig.4.1.

There are different methods to represent uncertainty regions: additive and multiplicative uncertainty. More detail on these two methods can be found in [25]. In this thesis, multiplicative uncertainty is taken into account. Thus, it is possible to represent the reduced model as the sum of the reduced plant and the multiplicative uncertainty:
\[ G_p(s) = G(s)(1 + \omega_l(s)\Delta_I(s)) \] (4.24)

Where \( G_p \) is the original plant, \( G \) is the reduced model and \( \Delta_I \) is the multiplicative weights that quantify the uncertainties between the two models at each frequency. In the thesis’ case, the original plant and reduced model are known from previous chapter and the only unknown of the eq.[4.24] is \( \Delta_I \). It is possible to calculate delta that represent the uncertainty introduced by neglected dynamics as:

\[ \Delta_I(s) \omega(s) = W(s) = \frac{G_p - G}{G} \] (4.25)

\( W \) itself showcases the uncertainties in frequencies domain magnitude wise. In particular the lesser the magnitude the lesser the uncertainty in a multiplicative sense. Normally, \(|W|\) is higher at frequencies where the reduced model mismatch more with original one. Typically multiplicative uncertainty is good in characterise unknown or unmodelled dynamics. This weight can be also used to support robust stability, i.e. if the desired close loop \(|T| < 1/|W|\) it is possible to guarantee that robust stability is achieved. This stems from small gain theorem and hence is conservative condition, that means that typically the system may still be stable for excess combination of uncertainty even if the inequality above is not strictly true. More details can be found in [25].

In the fig.[4.8] are plotted the bode diagram of the original plant (blue), of

![Multiplicative uncertainty diagram](image1)

(a) Multiplicative uncertainty between \(G_{\text{original}}\) and \(G_{MR,I}\)

![Multiplicative uncertainty diagram](image2)

(b) Multiplicative uncertainty between \(G_{\text{original}}\) and \(G_{MR,III}\)

Figure 4.8: Multiplicative uncertainty representation
the reduced model (red, in (a) of \(G_{MR,I}\) and in (b) of \(G_{MR,III}\)) and the inverse multiplicative uncertainty (dash line). As explained in [25] at frequencies where \(|W| > 1\) the uncertainties exceed 100% and the Nyquist curve may pass through the origin bringing to instability. In other words, mismatch between original/reduced plant is high where \(|W| > 1\). Looking at fig. 4.8 one can clearly see that the mismatch between TF 4.1 (original plant) and TF 4.17 (reduced one) become larger at higher frequencies. On the contrary at lower frequency the inverse multiplicative uncertainty is lower than 1, meaning that the mismatch between the two model is not appreciable and the original plant can be approximated without losing information with the reduced one.

The advantage of frequency domain uncertainty description is that one can choose to work (as in the thesis’ case) with a simple nominal model and represent neglected dynamics as uncertainties. This can be useful during robust controller’s design. In fact, the more the uncertainty, normally the more conservative (or robust) the controller should be. There are of course other ways to design controllers, i.e. adaptive, or non-linear design but in this case the controller design process is much harder to perform.

Summarizing, the purpose of this chapter is to compute reduced models that are good approximations for the original plant. These reduced models can be used in simple controller design method (e.g. chapter 5) or in model-based controllers (e.g. IMC or LQG). In this section a simple value, inverse multiplicative uncertainty, is computed in order to quantify how much the neglected dynamics affects the reduced model behaviour (fig. 4.8).

4.6 Model identification

In this section a model identification is performed in order to have another term of comparison besides the reduced model obtained manually and the analytical one.

The objective of this section is to obtain a reduced-model directly from the relationship between input and output data. These data come from simulations performed on in-house software ADTreS illustrated in chapter 3. Considering that this model is a representation of the system that better fits with the real world, the
data acquired from the results of these simulations can be seen as data very close to real measurements on the system. Thus, this process can be viewed as model identification.

The in-house software has been modified in order to have the possibility to excite the system with different external type of input applied to servo-valve. A set of responses are obtained and then used for the model identification. The typical input-output relation that is obtained is shown in the fig 4.9.

![Figure 4.9: Response from the MBS](image)

It is important to remark two things, based on results presented in previous subsections:

- the system shows an integrating behaviours;

- a third order system (that takes into account integrator and a complex pair of poles) can be considered a good approximation of the linearised higher-order system.

Two methods are used in order to identify a reduced model from the input-output relationship.

The first method proposed is based on identification performed in open loop. As a general consideration, a finite pulse, $u$, is used as input (as can be seen from
The pulse should be sufficiently short in order to avoid the system to leave the linear operating range and the output $y$ should look like a step response for a system without integrator. Thus it is possible to analyse the system like a second order one (4.26) and after the identification of oscillating part around the steady-state value the contribution of integrator is added to the identified transfer function.

$$ G_{II} = \frac{k_s}{s^2 + 2\omega_n\xi s + \omega_n^2} \quad (4.26) $$

In order to describe a second order system three parameters are needed: natural frequency $\omega_n$, damping factor $\xi$ and static gain $k_s$.

Firstly, the damped natural frequency is extracted from "numerical" experiments and then logarithmic decrement method is used to estimate the damping ratio. However, it is important to remark that logarithmic decrement method is not the best choice in this case, because as can be seen from the response in fig.4.9, the system is highly-damped. Thus it has been used for obtaining a starting value for $\delta$, that has been later refined with trial and error method, in order to obtained a similar response with respect of the original plant.

Thus, looking at the time history it is possible to compute a first value for the logarithmic decrement as the ratio of the amplitude of any two successive peaks:

$$ \delta = \ln \left( \frac{x(t_1)}{x(t_1 + T)} \right) \quad (4.27) $$

where $T$ is the period of the waveform. After that logarithmic decrement is used to find the damping ratio:

$$ \xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (4.28) $$

Secondly, a damped frequency is extracted from the time-response, and then using the relationship in eq.4.29 it is possible to find the natural frequency:

$$ \omega_n = \frac{\omega_s}{\sqrt{1 - \xi^2}} \quad (4.29) $$

All the parameters needed to identify the second order system target are derived from the previous equation and subsequently with trial and error method have been refined. The final values are shown in table 4.3. The static gain, $k_s$ is calculated simply as the ratio between steady-state value of the output and the steady-state
value of the input.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>4.53</td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>3.02</td>
<td>Hz</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>3.71</td>
<td>Hz</td>
</tr>
<tr>
<td>$k_s$</td>
<td>4.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Second order system parameters

After obtaining the second-order part of model a pure integrator is added, that is a pole in the origin ($s = 0$).

A second model identification method is performed applying as input (actuator command) a sinusoidal external command. A set of these commands at different frequency have been used as input and the output signals (carbody roll) have been obtained. In figure 4.10 the time histories of external sinusoidal command and body roll are reported. For the sake of brevity only this case, in which the frequency of the input is 0.5 Hz, is shown.

![Figure 4.10: Response to a sinusoidal excitation at 0.5 Hz](image)

This process has been repeated for frequencies between 0.05 to 3 Hz, obtaining for each sinusoidal input a related output. Using FFT algorithm (Fast Fourier
Transform) for each pair of signals (external command and carbody roll), considering the different tested frequencies, the frequency response function between output and input is calculated.

As previously said, considering a third-order model, the transfer function target has the form:

$$G_{ID,III,2} = \frac{a_0}{(-b_3 j\omega^3 - b_2 \omega^2 + b_1 j\omega + 1)}$$  \hspace{1cm} (4.30)

the objective is to find the parameters \((a_0, b_3, b_2, \text{ and } b_1)\) that minimizes the norm of the error between the evaluation of the function at the tested frequencies and the data obtained from the simulations. Figure 4.11 report the data obtained from the numerical experiments and denoted by x, as previously explained, and the identified transfer function (red line).

Figure 4.11: Comparison between identified TF and the data obtained from simulation at different frequencies

As one can clearly see from figure 4.11, the identification leads to acceptable results in fitting the data obtained for simulations. Thus, again a third-order model can be considered as a good approximation of the original plant. Finally, after having calculated the coefficient of equation 5 the third-order identified model is:

$$G_{ID,III,2} = \frac{125.8}{(-30.7s^3 - 157.36s^2 + 1689.96s + 1)}$$  \hspace{1cm} (4.31)
4.7 Comparison between reduced models

In this section the responses on the different reduced models are shown. For the sake of simplicity, are shown again the original and different reduced model, on which, the same external command to the servo-valve shown in fig.4.9 is applied on straight track:

1. original plant:

\[ G_{PT}^{yu}(s) = \frac{138.69(s^2 + 3.192s + 36.73)}{(s + 21.99)(s + 0.002693)(s^2 + 2.395s + 23.29)(s^2 + 9.321s + 119.2)} \]

2. manually reduced-plant: 4.17

\[ G_{MR,I}^{u} = \frac{0.071232}{s} \]

3. manually reduced-plant: 4.19

\[ G_{MR,III} = \frac{8.4859}{(s + 0.0002693)(s^2 + 9.32s + 119.1)} \]

4. model identified in section 4.6 with the first method (finite impulse):

\[ G_{ID,III} = \frac{15.23}{s(s^2 + 4.33s + 13.81)} \]

5. model identified in section 4.6 with the second method (sinusoidal excitations):

\[ G_{ID,III,2} = \frac{125.8}{(-30.7s^3 - 157.36s^2 + 1689.96s + 1)} \]

Here below are plotted the different responses obtained (fig.4.12), using different models shown in the previous numbered list.
Figure 4.12: Response of different models

One can clearly see that the more accurate the model is the more detailed the response to the external input would be. However, also the simplest model possible fig.4.12.a gives a quantitative representation of the system dynamics. The reduction process outlines that the system that better fits with the original plant is a third order system. In particular, it is noteworthy to remark that third order reduced-model obtained manually and obtained from model identification are very
similar, both in parameters and response in time.

In the next chapter, models reduced obtained by manual reduction are mainly adopted to design simple PI controller. In particular simple integrating plant 4.18 and third order plant 4.19 are employed in controllers synthesis.
Chapter 5

Classical control design

This chapter presents different control logic design, in particular it is focused on PID controllers. It is composed of 3 sections: in the first one the plant transfer function and stability analysis are presented, in the second section PI-type controllers are designed, in the last section a simple manual optimization of the controller gains in order to achieve defined performances is illustrates. It is noteworthy to remind that the purpose of this chapter is to adopt the reduced model, derived from the previous chapters, in order to design different controllers in a simple but effective way.

5.1 Plant transfer function and Stability analysis

Figure 5.1 represents the block diagram of the model presented in chapter 3. It is important to remark that actuator and vehicle are split in fig. 5.1 in order to outline the presence of actuator dynamics, but they are coupled with each other via pressure difference and carbody roll rate.

Figure 5.1: Block diagram of the model with actuator dynamic
The main objective of the control system is to track a reference carbody tilt angle \( \theta_{c, \text{ref}} \), that is defined on the basis of the geometry of the curve being negotiated and of the vehicle speed (fig. 5.2). Assuming the vehicle to run at constant speed, the reference tilt rate results in a trapezoidal profile, with reference tilt acceleration having impulses at the beginning and end of both entry and exit transitions. The maximum tilt angle considered for this application is 2 degrees. Note that the way of defining the reference angle is out of the scope of the thesis. More details on the derivation of references can be found in [7].

\[
G_{\text{PT plant}}(s) = \frac{277.37(s^2 + 3.192s + 36.73)}{(s + 21.99)(s + 0.002693)(s^2 + 2.395s + 23.29)} \bigg( s^2 + 9.321s + 119.2 \bigg) \quad (5.1)
\]

The Bode diagram of the plant is shown in fig. 5.3. One can clearly note the presence of the actuator dynamics (that as said in the previous chapter add a integrator to the plant) that introduce an extra-lag (-90 degrees in phase plot). In fig. 5.4 the Nichols diagram for the same plant TF 5.1 is shown.
From these two plots it is possible to extrapolate Gain Margin (GM) and Phase
Margin (PM) in order to assess the stability of the system. In Table 5.1 are shown
the stability margin for the system 5.1.
<table>
<thead>
<tr>
<th>Stability indicators</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Margin</td>
<td>70.30 dB</td>
</tr>
<tr>
<td>Phase Margin</td>
<td>89.09 deg</td>
</tr>
<tr>
<td>Gain crossover frequency</td>
<td>8.12 Hz</td>
</tr>
<tr>
<td>Phase crossover frequency</td>
<td>0.14 Hz</td>
</tr>
</tbody>
</table>

Table 5.1: Stability indicators

One can clearly see that the gain margin is largely greater than 0 and the phase margin is around 90 degrees. This means that the system is stable. Stability margins are useful to evaluate stability robustness with respect to parameters variation. The gain margin directly states how much gain variation the system can tolerate before become unstable. Phase margin is also useful to evaluate stability robustness with respect to delays in the feedback loop.

## 5.2 PID controller design

After having assessed stability, Proportional-Derivative-Integral controllers are considered. These kind of controllers are very popular and widely used in industrial and ground vehicle field. In particular, PID controllers are usually the simplest controllers that can be used for active tilt control applications \[26\]. Indeed, although more advanced controllers can provide significant improvements, a PID controller that is properly designed and tuned has proved to be satisfactory for the vast majority of industrial control loops \[27\]. There is a wide literature on PID controllers design, based on different performance criteria (\[27\], \[28\] and \[29\]). One of the first and well-known tuning method is the one proposed by Ziegler and Nicholson in \[30\].

Notewhorthy to be cited is the paper \[31\] where different methods to tune PID controllers, extending ZN method, for tilting active suspensions are presented. Furthermore, in the same paper a comparison with ZN basic tuning method is carried out. Focusing on tilting problem, different papers on control methods and approaches are available in literature (\[32\], \[9\]).
5.2 PID controller design

The usual PID controllers expression with derivative cut-off is used in this work

\[ C_{PID} = k_p \left( 1 + \frac{1}{\tau_i s} + \frac{\tau_d s}{N} + 1 \right) \]  \hspace{1cm} (5.2)

where \( k_p \) is the proportional gain, \( \tau_i \) the integral time constant, \( \tau_d \) the derivative time constant and \( N \) the derivative filter divisor.

Finally it is important to remark that the purpose of this chapter is to design simple PID controllers starting from reduced model and understand which order of reduction would be a good trade-off between simplification of control design process and good results in terms of performance. In the next section the objective is not to synthesise controllers that achieve the best performance possible (e.g. in terms of PCT, energy consumption or lateral acceleration perceived), but to establish a stable and simple controller, which can be refined later on.

5.2.1 PI controllers imposing \( PM \) (Phase Margin) and \( BW \) (Bandwidth)

In this section, a simple design method is used to obtain PI controllers. In particular PI controllers are designed for the reduced plants developed in chapter 4 and then implemented on the original plant. For this reason, tuned PI controllers may give unstable response once they are implemented for the original plant. If the results are acceptable, a simulation on the multi-body model described in 3, in order to assess performances, is carried out. Recalling model reduction illustrated in chapter 4, the simplest possible reduced-plant obtainable with manual reduction is an integrating process (5.3):

\[ G_{MR,I} = \frac{0.071232}{s} \]  \hspace{1cm} (5.3)

This model is used to design PI controllers in an analytical way, imposing a desired PM (phase margin) and BW (Bandwidth) shown in tab 5.2. This kind of design is very simple since it is easier to deal with the reduced model with respect to the original one.
Subsequently, simulations, using synthesised PI in table 5.2, are performed on original plant (3.28) using a Simulink© model. It is important to remark that for all the simulations reported in this chapter an active vehicle running at 340 km/h along a single high-speed curve geometry has been used. More information on simulation parameters can be found in 3.3. This first assessment is done in order to decide which one of the PI controllers reported in table 5.2 is worth to implement on the multi-body model.

$C_{PI}^1$, the first controller in table 5.2, does not give acceptable response when implemented on the original plant (it becomes unstable). The Bode diagrams of the closed loop with controllers designed (table 5.2) are reported in Appendix B. This can be linked to the fact that the reduced model used to design the controller has a great uncertainty with respect to the original plant (as shown in section 4.5) at high frequencies. In particular, the controller designed on the reduced model, imposing a BW of 10 rad/s, does not take into account the presence of unmodelled dynamics at high frequencies. $C_{PI}^1$ would be a good controller for the reduced plant but when implemented on the original plant it does not give acceptable results and leads to instability.

In summary, imposing greater BW, allows to design more performing, prompt and "aggressive" controller with better tracking properties but, as drawback, increasing too much the BW can lead to instability, because the reduced model on which the controller is designed, does not take into account the existence of dynamics at high frequencies, that are present in the original plant.

In the second and third controller, reported table 5.2, the bandwidth required is decreased in order to find a trade-off between performing controller and stable controlled system. In fig. 5.5 are shown the tilt reference and the time-histories.
of the body roll derived from the simulations performed on the original plant, respectively, with $C^2_{PI}$ and $C^3_{PI}$. One can clearly see that $C^2_{PI}$ allow to better tracking the tilt reference.

Figure 5.5: Body roll with $C^2_{PI}$ and $C^3_{PI}$ with respect to tilt reference

It has been decided to use $M_p$ (percentage overshoot) and $t_s$ (settling time) of the responses obtained in figure 5.5, in order to decide which controller to implement on the multi-body model. These two parameters are used to assess time domain performance, usually simulating the response to a step in the reference input. Detailed information about these parameters can be found in [25].

<table>
<thead>
<tr>
<th>PI controllers</th>
<th>$M_p$</th>
<th>$t_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C^2_{PI}$</td>
<td>10</td>
<td>3.2</td>
</tr>
<tr>
<td>$C^3_{PI}$</td>
<td>60</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.3: Performance index with $C^2_{PI}$ and $C^3_{PI}$

Finally, as can be seen from table 5.3 and figure 5.5, the best results, in simulations using original plant, are given by controller $C^2_{PI}$. This controller will be implemented on the multi-body model and results are illustrated in chapter 6.
Information in terms of Bode diagrams, for closed loop shaped with $C_2^{PI}$, both for the reduced and original plant, are shown in fig. 5.6). Whereas Bode diagrams of the closed loop shaped with controllers $C_1^{PI}$ and $C_3^{PI}$ are reported for completeness in appendix B.

![Figure 5.6: Bode diagram of Closed Loop with PI^2](image)

The same procedure is adopted with third-order reduced model obtained in chapter 4. Its transfer function is defined by

$$G_{MR,III} = \frac{8.4859}{(s^3 + 9.3203s^2 + 119.1025s + 0.0321)}$$

As before, a defined phase margin and bandwidth for the closed loop have been imposed. The results are shown in table 5.4.

<table>
<thead>
<tr>
<th>PI controllers</th>
<th>BW (rad/s)</th>
<th>PM (deg)</th>
<th>PI form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_4^{PI}$</td>
<td>10</td>
<td>45</td>
<td>$\frac{19.619(1+5.7s)}{s}$</td>
</tr>
<tr>
<td>$C_5^{PI}$</td>
<td>3</td>
<td>60</td>
<td>$\frac{32.704(1+1.2s)}{s}$</td>
</tr>
<tr>
<td>$C_6^{PI}$</td>
<td>1</td>
<td>60</td>
<td>$\frac{6.0074(1+2.1s)}{s}$</td>
</tr>
</tbody>
</table>

Table 5.4: PI controller specifications

The same consideration done for the previous case, where reduced model 5.3 is
used instead of 5.4 can be done. It is worth mentioning that a controller designed using the same target values for BW and PM but considering a different reduced model of the plant leads to different results on the original plants. In fact the third-order reduced model allow to design a more effective controller, as expected. The first controller reported in table 5.4, as in the previous case, leads to instability. Thus, only the second and third controller, reported in table 5.4 are taken into account. As for the previous case, in fig.5.5 are shown the tilt reference and the time-histories of the body roll derived from the simulations performed on the original plant, respectively, with \( C^{5}_{PI} \) and \( C^{6}_{PI} \). One can clearly see that \( C^{2}_{PI} \) allow to better tracking the tilt reference.

![Figure 5.7: Body roll with \( C^{5}_{PI} \) and \( C^{6}_{PI} \) with respect to tilt reference](image)

As before, the performance indexes used to decide which controller implement on multi-body model are \( M_p \) and \( t_s \).

<table>
<thead>
<tr>
<th>PI controllers</th>
<th>( M_p )</th>
<th>( t_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>%</td>
<td>s</td>
</tr>
<tr>
<td>( C^{5}_{PI} )</td>
<td>7.8</td>
<td>1.97</td>
</tr>
<tr>
<td>( C^{6}_{PI} )</td>
<td>18.65</td>
<td>5.29</td>
</tr>
</tbody>
</table>

Table 5.5: Performance index with \( C^{5}_{PI} \) and \( C^{6}_{PI} \)

Looking at table 5.5 and figure 5.7, \( C^{5}_{PI} \) is the controller that lead to a better
results. For this reason it will be implement on the multi-body model. Finally, information in terms of Bode diagram, are shown in fig.5.8 for closed loop shaped with $C^5_{PI}$, both for reduced and original plant. Whereas Bode diagrams for the remaining controllers in table 5.4 are reported for completeness in appendix B.

![Bode diagram of Closed Loop with $PI^5$](image)

Figure 5.8: Bode diagram of Closed Loop with $PI^5$

5.2.2 PI controllers imposing $M_p$ (maximum percentage of overshoot) and $t_s$ (settling time)

The design of the desired closed-loop is often based on designer experience. Hence, for the tilt problem, an overview of desired responses (i.e. the vehicle roll response from [6] and [5]) is taken into account in order to understand which kind of response is required to achieve for a good tilting action.

As in the previous section, the simplest model obtained in manual reduction is adopted to design controller (eq.5.5):

$$G_p = \frac{k_r}{\tau_r s}$$  \hspace{1cm} (5.5)

where $\tau_r$ and $k_r$ are respectively the gain and time constant of the reduced model (4.17).
5.2 PID controller design

Looking at the response achieved in paper [6] with respect to the ideal reference, has been decided to design simple controllers that permits to reach desired classical index of performance on the reduced-plant: settling time and maximum overshoot (more information about this parameters can be found in [25]). From experience and previous simulations, analysing typical body roll responses to actuator input signal, a starting point, in terms of maximum overshoot and maximum settling time have been chosen. In particular the maximum overshoot is taken fixed at 15%. On the contrary, the settling time varies between 1 s and 10 s. Starting from this parameter different gains for PI controllers are calculated. This design process is a kind of simple direct controller synthesis. In the literature there are a lot of paper that treat this subject. Noteworthy to be cited are [27] and [28].

The controller used is a Proportional-Integral type 5.6:

$$C_{PI} = k_p (1 + \frac{1}{\tau_Is})$$

where $\tau_I$ and $k_p$ are respectively the proportional gain and integral time constant of the PI-controller.

It is possible to easily obtain the closed loop transfer function of the system shown in fig. 5.1, where the plant is 5.5 and the controller is 5.6:

$$L = \frac{\bar{G}_PC_{PI}}{1 + \bar{G}_PC_{PI}} =$$

$$= \frac{k_r}{\tau_I k_p (1 + \frac{1}{\tau_Is})} =$$

$$= \frac{k_r k_p (s \tau_I + 1)}{\tau_I \tau_r s^2 + k_r k_p (\tau_Is + 1)} =$$

$$= \frac{s \tau_I + 1}{\tau_I \tau_r s^2 + \tau_Is + 1}$$

(5.7)

It can be clearly seen that the denominator of the loop transfer function has a typical second order form characterized by the presence of two poles. Thus, it is possible to outline time constant and damping factor that describes the behaviour of the closed loop system:
\[ \tau_L s^2 + \frac{2\xi}{\tau_L} s + 1 \quad (5.8) \]

where \( \tau_L \) is the time constant of the closed loop and \( \xi \) is the damping factor.

At this stage, the desired performance index, \( M_p \) (max overshoot) and \( t_s \) (max settling time) are prescribed (more details on these parameters can be found in [25]).

\[ M_p \approx 100 e^{-\frac{1}{\pi \xi \sqrt{1-\xi^2}}} \quad (5.9) \]

\[ t_s \approx \frac{4}{\xi \omega_n} \quad (5.10) \]

Different gains for the PI-controller are found imposing maximum overshoot to be below (15\%) and varying settling time (from 1 to 10 seconds), that is related with curve transition and length. It is important to remark that the controller synthesis is performed on the reduced model in the form 5.5, because it is more easy to manage compared to the complete model but still shows a similar response to the typical input as can be seen from chapter 4.

Using eq.5.9 and eq.5.10 it is possible to find the expression, respectively, of \( \xi \) (damping ratio) and \( \tau_L \) (time constant), passing through \( \omega_n \):

\[ \xi = \frac{\ln\left(\frac{100}{M_p}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{100}{M_p}\right)}} \quad (5.11) \]

\[ w_n = \frac{4}{\xi t_s} \quad (5.12) \]

\[ \tau_L = \frac{1}{w_n} \quad (5.13) \]

Finally, comparing the denominator of eq.5.7 with the denominator of eq.5.8 it is possible to define the controller parameters as follows:

\[ \tau_I = 2\xi \tau_L \quad (5.14) \]
5.2 PID controller design

\[ k_p = \frac{\tau_r \tau_I}{k_r \tau_L^2} \quad (5.15) \]

Adopting these last equations, imposing maximum percentage of overshoot and maximum settling time, it is easy to compute the gains for a PI controller that allow to reach a response with this performance in the reduced system.

Here below in table 5.6 are shown the target values assumed, in terms of maximum overshoot (taken fixed to 15%) and maximum settling time, and the outputs obtained, in terms of PI controllers.

<table>
<thead>
<tr>
<th>ID</th>
<th>Max. overshoot</th>
<th>Max settling time</th>
<th>PI controllers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>s</td>
<td>$s$</td>
</tr>
<tr>
<td>$C_{PI}^7$</td>
<td>15</td>
<td>10</td>
<td>45.52s+10.95s</td>
</tr>
<tr>
<td>$C_{PI}^8$</td>
<td>15</td>
<td>8</td>
<td>45.52s+13.68s</td>
</tr>
<tr>
<td>$C_{PI}^9$</td>
<td>15</td>
<td>5</td>
<td>45.52s+21.9s</td>
</tr>
<tr>
<td>$C_{PI}^{10}$</td>
<td>15</td>
<td>2</td>
<td>45.52s+54.74s</td>
</tr>
<tr>
<td>$C_{PI}^{11}$</td>
<td>15</td>
<td>1</td>
<td>45.52s+109.5</td>
</tr>
</tbody>
</table>

Table 5.6: PID controller from direct synthesis

As expected when a shorter settling time is imposed, the response is forced to be very quick as consequence of a very "aggressive" controller design. In particular, $C_{PI}^{11}$ is too "aggressive" and leads the original system to instability. Bode diagrams of the closed loop with controllers reported in table 5.6 are illustrated in appendix B. On the contrary, for long settling time, more soft controller are obtained resulting in a smoother response. As in the previous cases, the controllers of table 5.6 are implemented on the original system and simulated with Simulink©.

In fig 5.9a are shown the tilt reference and the time-histories of the body roll derived from the simulations performed on the original plant, respectively, with $C_{PI}^7$, $C_{PI}^8$, $C_{PI}^9$ and $C_{PI}^{10}$. First of all, $C_{PI}^{10}$ was disregarded because, as can be seen from fig 5.9b, when the vehicle is in steady-curve it leads to unwanted oscillations.
Finally, as in the previous cases, $M_p$ and $t_s$ are calculated in order to decide which controller to implement on the multi-body model.

<table>
<thead>
<tr>
<th>PI controllers</th>
<th>$M_p$</th>
<th>$t_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>%</td>
<td>s</td>
</tr>
<tr>
<td>$C^7_{PI}$</td>
<td>18</td>
<td>5.05</td>
</tr>
<tr>
<td>$C^8_{PI}$</td>
<td>17</td>
<td>4.75</td>
</tr>
<tr>
<td>$C^9_{PI}$</td>
<td>11.5</td>
<td>4.14</td>
</tr>
</tbody>
</table>

Table 5.7: Performance index with $C^7_{PI}$, $C^8_{PI}$ and $C^9_{PI}$

As can be seen from table 5.7, the best results, in simulations using original plant, are given by controller $C^9_{PI}$, which is a good trade-off in terms of settling time.

Finally, $C^9_{PI}$ is implemented on the multi-body model for assessing performance and obtained a closer response to the real world. The results of this analysis are reported in chapter 6. Information in terms of Bode diagram for closed loop shaped with $C^9_{PI}$, both for reduced and original plant, are shown in fig. 5.10.
5.2 PID controller design

![Bode diagram of Closed Loop with PI](image)

Figure 5.10: Bode diagram of Closed Loop with $PI$°

The design process adopted in this section can be summarized with the following steps:

1. Definition of the reduced system as simple integrator in the form of eq.5.5
2. Definition of the PI-controller in the form of eq.5.6
3. Definition of the closed loop using the plant at point (1) combined with PI-controller at point (2);
4. Definition of a desired $M_p$ (maximum overshoot) and $t_s$ for a step response;
5. Analytical computation of the PI-controller’s gains $k_p$ and $\tau_i$;
6. Firs assessment of the performance of closed loop adopting the original plant on Simulink©;

5.2.3 Effect of the derivative term in PID controllers

PI controller tends to be sufficient in many application. For stable plant usually one is not interested in letting the envelope of the response to get a very rapid transient, then a P or PI is sufficient in most of case. Another reason for not using derivative contribution is that it amplifies high frequency noise. This can be
particularly highlighted at steady-state where the controller will never “relax” and continue to work. For this reason, it is usually coupled with a low-pass filter [33]. The introduction of D-term allows to stabilize unstable plants (it is not the case of tilting control), increase stability margins and shape better transient response (i.e. rapid rise, low overshoot and fast settling). Another advantage of using D term, which provide the damping portion of the controller, can be found when one is dealing with under-damped system (again is not the case of tilting problem). On the contrary, the drawback of including D term in the controller is that another "dimension" is added to the process of control tuning that becomes more challenging. On the other hand if used properly, D term offers good potential to optimize the performance index of the controlled system.

In this thesis derivative contribution was not necessary because PI controller allow to reach good performance. In addition to this, the scope of this work is maintain a simple approach in controllers design.

5.2.4 Controllers with feed-forward action

In general feedforward aims to provide a good reference following under no uncertainty, while feedback control addresses any uncertainty or disturbances in the feedback loop. Figure 5.11 presents a 2-DoF classical feedback control set-up framework. The dash-dotted part represents the paths/blocks $K_{ff,r}(s)$ and $K_{pf,r}(s)$ that are the feed-forward and pre-filter blocks respectively. Normally the command (reference) signal is provided with a small preview time (i.e. enabling start of tilt action just before the corner section) but no preview time is utilized here following the same as in [5].

For the case of reference signal (i.e. in our case preview tilt control) the following choices facilitate independent design of $K_{ff,r}(s)$ (feedforward) and $K_f(s)$ (feedback) controllers ( [34]), respectively:

\[ K_{ff,r}(s) = k_{ff}G_{yu-}^{-1}(s)F_r(s) \]  \hspace{1cm} (5.16)

\[ K_{pf,r}(s) = G_{yu+}(s)F_r(s) \]  \hspace{1cm} (5.17)

where $G_{yu-}(s)$ is the invertible part and $G_{yu+}(s)$ the non-invertible portions of the plant. Note that $F_r(s)$ is included for properness of $K_{ff,r}(s)$. $k_{ff}$ is the
feedforward gain that is optimized by a grid search in the next section. The output signal \(Y(s)\) refers to vehicle body roll angle.

This framework illustrated in fig. 5.11 is very useful because it allow to design independently the feedforward and feedback path. In this way, it is possible to optimize the feedforward gain independently from the feedback one. It is important to remark that it would be quite difficult to define and use the invertible and non-invertible parts of the original plant 4.1. Thus, once again, these two components are derived from the reduced model.

In order to use a model that can be easily decomposed in invertible and non-invertible part, reduced model (4.18) is adopted, obtaining:

\[
G_{yu-}(s) = \frac{0.071232}{s} \tag{5.18}
\]

\[
G_{yu+}(s) = 1 \tag{5.19}
\]

\[
F_r(s) = \frac{1}{1 + \frac{s}{100}} \tag{5.20}
\]

5.3 Manual optimization by search grid

In this section, a manual optimization using grid search is performed. More details about this optimization technique can be found in [35]. It is important...
to remark that this optimization technique is very simple and allows to minimize/maximize one parameter at a time. Arbitrarily, an index of performance, $P_{CT}$, has been chosen to be optimised. The definition and more information on this index can be found in section 3.4.

The algorithm implemented is very simple:

1. An objective index performance to minimize/maximize is chosen. In this work the $P_{ct}$ factor is minimized;

2. Iteratively $k_p$ proportional gain, $\tau_i$ integral time and $k_{ff}$ feedforward gain are varied in a specific interval and with a previously chosen step;

3. A simple simulation on the original plant is performed for every cycle and the performance parameter selected at point (1) is calculated;

4. This operation is repeated for all combinations of PI-PID and FF gains;

5. Finally, all the results are sorted from the minimum value of the performance parameter found to the maximum value;

6. The combination of gains that minimizes parameter chose at point (1) is the result of the optimization.

The model used for performing simulations at point (3) has the framework shown in fig.5.11, where iteratively $k_p$ proportional gain, $\tau_i$ integral time and $k_{ff}$ feedforward gain are varied in order to optimize $P_{CT}$.

It is important to remark that the approach followed so far in this chapter is to perform a manual design for the controller, i.e. analytical design of simple controllers. However, there is a point at which, the controllers design needs to satisfy many specifications simultaneously and a higher level of optimization may be required, because the complexity of the problem makes it hard in such cases to design the controllers manually or analytically. An example of this can be found in [5]. In this paper three different control strategies are proposed for the active hydraulic anti-roll device and for all three control, gains are defined based on Genetic Algorithm (GA) optimisation, having as multiple objectives the tracking of a reference carbody tilt angle, optimising ride comfort and keeping the energy
5.3 Manual optimization by search grid

required for actuation within acceptable limits.

Using the values obtained in the previous section 5.2.1, a reasonable range has been chosen where to vary the controller’s gains. The step, with whom the gains are updated at each iteration, is decreased simulations by simulations, in order to mapping better the space where optimization is performed. At the same time, the space in which the solutions are searched is increased from first to second simulation. Obviously, increasing the space of the possible solutions and decreasing the step of update, the optimization needs more time and computational effort to be performed.

In table 5.8 the results obtained are shown.

<table>
<thead>
<tr>
<th>ID</th>
<th>$k_{ff}$</th>
<th>PI controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{PID}^{12}$</td>
<td>0.5</td>
<td>$0.48.6s+30.38$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6s</td>
</tr>
<tr>
<td>$C_{PID}^{13}$</td>
<td>0.5</td>
<td>$133.2e+37$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.6s</td>
</tr>
</tbody>
</table>

Table 5.8: PI controller from grid-search

As the in the previous cases, simulations, using synthesised PI in table 5.8 are performed on original plant using a Simulink© model. In fig.5.12a are shown the tilt reference and the time-histories of the body roll derived from the simulations performed on the original plant, respectively, with $C_{PID}^{12}$ and $C_{PID}^{13}$.

(a) Body roll with $C_{PID}^{12}$ and $C_{PID}^{13}$ with respect to tilt reference

(b) Zoom of the figure 5.12a in the steady-curve region

Figure 5.12: Body roll
One can clearly see that the two controllers give a similar response. Moreover, as reported in Table 5.9, $M_p$ and $t_s$ have similar values.

<table>
<thead>
<tr>
<th>PI controllers</th>
<th>$M_p$</th>
<th>$t_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{PI}^{12}$</td>
<td>18</td>
<td>5.05</td>
</tr>
<tr>
<td>$C_{PI}^{13}$</td>
<td>17</td>
<td>4.75</td>
</tr>
</tbody>
</table>

Table 5.9: Performance index with $C_{PI}^{13}$ and $C_{PI}^{12}$

Finally, $C_{PI}^{13}$ have been implemented on the in-house software in order to assess performance, because, even if small its response has lower $M_p$ and $t_s$ compared with the one using $C_{PI}^{12}$. The results are shown in the next chapter. Information in terms of Bode diagram, are shown in Fig. 5.13a, for closed loop shaped with $C_{PI}^{12}$, both for reduced and original plant. The same for $C_{PI}^{13}$ are shown in Fig. 5.13b.

(a) Bode diagram of Closed Loop with $PI^{12}$  (b) Bode diagram of Closed Loop with $PI^{13}$

5.4 Robustness analysis

This section analyses the robustness of the system with dynamics uncertainty (ref to sec 4.5) and assesses the conditions needed for ensuring robust stability. The base of robustness analysis for such dynamic uncertainty is the small-gain theorem. More information about robustness analysis can be found in chapter 12 of [36]. The theorem states that if:

$$ |T| < \frac{1}{|W|} $$  

(5.21)
it is then possible to guarantee that robust stability is achieved. Where $T$ is the complementary sensitivity function of the designed closed loop and $|W|$ the multiplicative uncertainty calculated in section 4.5. The physical meaning of the small-gain theorem is that when the condition is satisfied, then the external input is attenuated every time it circulates in the closed loop [36].

![Figure 5.14: Robustness analysis](image)

In fig. 5.14 are shown the different complementary sensitivity functions, $T(s)$ for the closed loops, shaped with the PIs developed in the previous section, with respect to inverse multiplicative uncertainty calculated in sec. 4.5. Recalling the robustness condition stated in the previous paragraph, one can clearly see that in all the Bode diagram in fig. 5.14 this condition is achieved. Only in fig. 5.14b around 5 rad/s, the sensitive function approaches the inverse multiplicative uncertainty. It is worth reminding that, as stated in chapter 4, the small gain theorem correspond
to a conservative condition, that means that typically the system may still be stable for excess combination of uncertainty even if the inequality above is not satisfied in strict terms. It is important to remark, that this is another assessment performed before testing this control logic on multi-body model that should give information very close to reality.
Chapter 6

Results

6.1 Results in nominal conditions

This chapter reports the results of time domain simulations on a non-linear multi-body model, for selected regulator configurations defined in chapter 5, considering the vehicle running at 340 km/h along a single high-speed curve geometry. Details on the multi-body software tool can be found in chapter 3. The vehicle considered for this assessment is a fictitious but realistic high-speed vehicle. More information both on performance assessment and simulation parameters can be found in section 3.4 and 3.3. It is important to highlight that for all the following simulations the body roll is considered as the difference between the measured carbody roll and the bogie roll. In particular, the bogie roll is definitely smaller with respect to the tilt angle on account of the high stiffness of the primary suspension.

Fig. 6.1 shows the body roll angle relative to the top-of-rail plane as a function of the vehicle position along the track, considering four different controllers implemented: \( C^2_{PI} \) (top left), \( C^5_{PI} \) (top right), \( C^9_{PI} \) (bottom left) and \( C^{14}_{PI} \) (bottom right). Refer to chapter 5 for the nomenclature. These controllers all use a preview scheme, based on the knowledge of the track, by means of geo-localisation. The first three strategies are based on simple PI controllers, whereas the fourth is a PI controller with a feedforward contribution as can be seen in section 5.3. All the strategies implement PI controllers, but they are derived in different ways, as described in chapter 5.

Time-domain simulations are performed using an ideal deterministic track
input (with no superimposed stochastic track irregularity and no sensor noise included), to analyse the quasi-static behaviour of the controlled cases. It is important to remark that in this thesis only deterministic input are taken into account.

![Figure 6.1: Carbody roll](image)

It is observed that the version of PI controller designed on the most simplified model, $C_{PI}^2$, presents an large overshoot and oscillating response around the steady-state value (fig.6.1a). On the contrary, the controller $C_{PI}^5$ designed for the third order reduced model shows a better response both in term of tracking and maximum overshoot (fig.6.1b). This is due to the fact that this last controller is developed on a plant that takes into account other dynamics as well as the integrator.

As can be seen from fig.6.1c, where the controller $C_{PI}^0$ designed imposing a Mp and a Ts is shown, this design method is too simple and does not take into account the real dynamics of the original system (note that was used a simple integrator as
the plant on which the designed is performed). In fact, the performance required in terms of settling time and maximum overshoot are obtained on the reduced plant, but is a too weak requirement to reach also good tilt control when implemented on MBS. Finally, in fig.6.1d is shown the ideal tilt profile of the body roll and the one obtained with controller $C_{PI}^{13}$ plus feedforward contribution.

![Figure 6.2: Comparison between $C_{PI}^{2}$ and $C_{PI}^{5}$](image)

The fig.6.2 presents a comparison between the ideal profile of the body roll and the ones obtained by PI controllers designed imposing the same PM and BW but on different reduced model (no sensor noise has been included in order to showcase the signal profile clearly). As said previously in the chapter the controller designed on the reduced model being closer to the ideal plant has the best response.

Finally, the dynamic behaviour of the controllers is tested in the same curve considering two different categories of parameters. The first one related to passenger ride comfort, whereas the second category is associated to the controller performance (tracking, control effort). In particular in table 6.1 PCT index as per the definition in [8] is shown. This index is a weighted summation of the maximum values of the roll rate, the lateral acceleration and the lateral jerk and provides a measure of the passenger comfort in curve transitions (essentially the percentage of passengers that may experience nausea). Note that in the other columns in table 6.1 also
the three different contribution on the PCT are reported together with the Root Mean Square (r.m.s) value of the lateral acceleration providing a global value of the lateral vibration felt by the passenger. Compared with the results illustrated in [5], the active tilt controllers proposed in this thesis produce a $P_{CT}$ value which is only slightly worse than the controllers presented in [5], despite those ones are synthesised based on a much more complex design process. It is important to remark that this controllers have been synthesised following simpler and single-objective optimization. Thus, it can be concluded that also with a controller design, that adopts reduced model, good results in terms of ride comfort levels can be achieved.

$$\begin{align*}
\text{Controller} & & P_{CT} & \text{max}(\dot{\rho}_{1s}) & \text{max}(\ddot{y}_{1s}) & \text{max}(\dot{y}_{1s}) \\
- & & \text{%} & \text{rad/s} & \text{m/s}^2 & \text{m/s}^3 \\
C_{PI}^2 & & 2.488 & 0.036 & 0.616 & 0.255 \\
C_{PI}^5 & & \textbf{2.111} & 0.033 & \textbf{0.611} & \textbf{0.226} \\
C_{PI}^9 & & 2.655 & \textbf{0.032} & 0.639 & 0.258 \\
C_{PI}^{13} & & 2.358 & \textbf{0.032} & 0.632 & 0.237 \\
\end{align*}$$

Table 6.1: Evaluation of designed system comfort indices

In terms of energy consumption, the r.m.s. of the power required for performing the manoeuvre, evaluated for a single bogie, is for all the controllers, as illustrated in table 6.2, between 1.5 and 2 kW.

As far as the comparison of alternative PI controllers is concerned, $C_{PI}^5$ regulator shows the best behaviour in terms $P_{CT}$ index. This is related to an higher level of actuation, at the expense of more power being required for the manoeuvre. On the other hand, $C_{PI}^9$ regulator shows the worse behaviour both in terms of comfort index and maximum lateral acceleration. Finally, $C_{PI}^{13}$ used simultaneously with a feedforward contribution shows the best results in terms of maximum absolute error. As expected, the feedforward contribution allows to provide a good reference tracking, maintaining the absolute error as low as possible. The r.m.s. of the power and the actuation required with this control logic are the lowest, comparable to controller $C_{PI}^9$, that however produce the worst ride comfort performance.
6.2 Robustness to changes in carbody’s inertia

| Controller | $\max(|\rho_{ref} - \rho|)$ | r.m.s.($u$) | r.m.s.($Power$) |
|------------|-----------------|------------|----------------|
| $C^2_{PI}$ | 9.080 mrad      | 0.064 V    | 1.962 kW       |
| $C^5_{PI}$ | 8.515 mrad      | 0.058 V    | 1.792 kW       |
| $C^8_{PI}$ | 8.675 mrad      | 0.056 V    | 1.724 kW       |
| $C^{13}_{PI}$ | *8.066 mrad* | *0.054 V* | *1.664 kW* |
| ref. [5] PIDSH | —            | —         | 0.940 kW       |

Table 6.2: Evaluation of controller related performance

Overall, the PI regulators proposed allow to achieve a good level of ride quality in terms of $P_{CT}$ and r.m.s. of lateral accelerations, keeping the energy required to perform the tilt action within an acceptable level.

6.2 Robustness to changes in carbody’s inertia

Analysis in terms of robustness varying crucial parameters, such as mass and roll inertia, have been performed. In particular, these two parameters, as one can easily recognise, are subjected to variations mainly related to the number of the passengers carried by the vehicle. Two limit case are analysed: laden condition and tare condition (empty vehicle). The laden case was considered the nominal one in the whole thesis (because it is the most severe for tilt actuation), while the tare condition is treated here as a variation from the nominal case. In table 6.3 are reported the mass and roll inertia of the carbody in the two cases (leaden and tare). It has been assumed that the radius of gyration is constant, thus permitting to decrease both mass and roll inertia by the same percentage of the nominal value. For this analysis only two controllers, that have proven to lead to best results in previous section, have been taken into account: $C^5_{PI}$ and $C^{14}_{PI}$. The standard parameters have been used for all the MBS simulations.
### Simulation’s conditions: change in carbody inertia

<table>
<thead>
<tr>
<th>Simulation’s condition</th>
<th>Carbody Mass</th>
<th>Carbody roll inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>laden (nominal)</td>
<td>40731 kg</td>
<td>0.880e5 kg m²</td>
</tr>
<tr>
<td>tare (perturbed)</td>
<td>35731 kg</td>
<td>0.772e5 kg m²</td>
</tr>
</tbody>
</table>

Table 6.3: Simulation’s conditions: change in carbody inertia

In figure 6.3 are shown the tilt reference and the actual body roll in the two conditions before mentioned (laden and tare case), both for system with implemented $C_{PI}^{5}$ (fig 6.3a) and $C_{PI}^{14}$ (fig 6.3b). One can clearly see that the inertia parameters changes, for both controllers, do not lead to problems of instability.

![Figure 6.3: Body roll with mass and roll inertia variations](image)

(a) Body roll in tare condition and laden condition with $C_{PI}^{5}$  
(b) Body roll in tare condition and laden condition with $C_{PI}^{14}$

Finally, table 6.4 reports the value of the performances indexes for the four cases, i.e. laden condition and tare condition with $C_{PI}^{5}$ and the same condition with $C_{PI}^{14}$. As can be seen, the laden condition (used in the whole thesis as the nominal case) is the most severe in terms of $P_{CTT}$ factor. On the contrary, the tare condition leads, as expected, to better results in terms of comfort indexes. This can be easily explained by the fact that the hydraulic system, which is controlled by the PI implemented, deals with a lower mass, allowing to achieve better results.
6.3 Results considering positioning errors

As far as energy consumption is concerned, as can be seen from table 6.5, the r.m.s. of power required for tilting the carbody is, as expected, decreased in the tare condition. As explained before, in this case the vehicle is empty thus the AARB have to deal with a lower inertia and consequently to use less energy to perform the tilt action.

<table>
<thead>
<tr>
<th>Controller</th>
<th>( P_{CT} )</th>
<th>( \max(\dot{\rho}_{1s}) )</th>
<th>( \max(\ddot{y}_{1s}) )</th>
<th>( \max(\dddot{y}_{1s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{PI}^5 ) branded cond.</td>
<td>2.111</td>
<td>0.033</td>
<td>0.611</td>
<td>0.226</td>
</tr>
<tr>
<td>( C_{PI}^5 ) tare cond.</td>
<td>1.954</td>
<td>0.033</td>
<td>0.608</td>
<td>0.213</td>
</tr>
<tr>
<td>( C_{PI}^{13} ) branded cond.</td>
<td>2.358</td>
<td>0.032</td>
<td>0.632</td>
<td>0.237</td>
</tr>
<tr>
<td>( C_{PI}^{13} ) tare cond.</td>
<td>1.754</td>
<td>0.030</td>
<td>0.610</td>
<td>0.195</td>
</tr>
</tbody>
</table>

Table 6.4: Evaluation of comfort indices with different carbody inertia parameters

| Controller | \( \max(|\rho_{ref} - \rho|) \) | r.m.s.(\( u \)) | r.m.s.\((\text{Power}) \) |
|------------|----------------|---------------|-----------------|
| \( C_{PI}^5 \) | 8.515 | 0.058 | 1.792 |
| \( C_{PI}^5 \) | 7.544 | 0.058 | 1.773 |
| \( C_{PI}^{13} \) | 8.675 | 0.056 | 1.724 |
| \( C_{PI}^{13} \) | 8.066 | 0.054 | 1.612 |

Table 6.5: Evaluation of controller related performance with different carbody inertia parameters

6.3 Results considering positioning errors

So far, the generation of references for the tilting system was assumed to be perfectly synchronised with the position of the vehicle along the curve, thanks to the use of train geo-localisation (via GPS) combined with a data-base of track geometry. In particular, the control system generates the references for the roll angle of the carbody assuming that the position of the vehicle along the curve is
known using a positioning system. The effect of an error on the position of the
vehicle along the track and, therefore, an error in the generation of the reference
signals may worsen the performances of the active system. In this respect, an
analysis related to robustness of controller designed was performed considering
the performances in presence of a positioning error of 50 m (both delayed and
anticipated), which is larger than the horizontal error obtained using GPS, usually
±10 m (12). As the previous analysis, the results are presented only for \( C_{PI}^5 \)
and \( C_{PI}^{14} \), that are the designed controllers that lead to best performances in the
nominal case.

In figure 6.4 are reported the body roll for ideal case, maximum delay and
maximum anticipation of the command with respect of ideal tilt reference
(not affected by the positioning error). The positioning error influences the actual
roll, that appear shifted with respect of the ideal desired tilt. Looking at figure
6.4, in the cases of delayed positioning signal (green line), it can be seen that the
vehicle enters the curves and leans outwards, because the tilt reference is delayed
and it behaves like it was still in straight track. In general, both controller \( C_{PI}^5 \)
and \( C_{PI}^{14} \) leads to good responses in terms of stability and robustness with respect
to positioning errors.

![Figure 6.4: Body roll in presence of GPS positioning error](image)

Table 6.6 reports the values of the performance indexes for six cases, i.e. ideal
positioning and ±50 m positioning error for the two PI controllers considered. The
±50 m positioning error is significantly larger than maximum error expected from
GPS system (12). It is observed that even in presence of a large positioning error, the controller keeps the ability to correctly follow the reference thanks to the feedback action. Slight variations are obtained also for the $P_{CT}$ index which, in any case, reaches an acceptable maximum value of 5.1%. From table 6.6 it is observed that the worst condition, in terms of level of comfort, corresponds to the case of delayed positioning error.

<table>
<thead>
<tr>
<th>Controller</th>
<th>$P_{CT}$</th>
<th>$\max(\dot{\rho}_{1s})$</th>
<th>$\max(\ddot{y}_{1s})$</th>
<th>$\max(\dddot{y}_{1s})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{PI}^5$ ideal ref</td>
<td>2.111</td>
<td>0.033</td>
<td>0.611</td>
<td>0.226</td>
</tr>
<tr>
<td>$C_{PI}^5$ delay 50 m</td>
<td>4.877</td>
<td>0.037</td>
<td>0.726</td>
<td>0.398</td>
</tr>
<tr>
<td>$C_{PI}^5$ advance 50 m</td>
<td>2.886</td>
<td>0.031</td>
<td>0.653</td>
<td>0.270</td>
</tr>
<tr>
<td>$C_{PI}^{13}$ ideal ref</td>
<td>2.358</td>
<td>0.032</td>
<td>0.632</td>
<td>0.237</td>
</tr>
<tr>
<td>$C_{PI}^{13}$ delay 50 m</td>
<td>5.134</td>
<td>0.035</td>
<td>0.752</td>
<td>0.404</td>
</tr>
<tr>
<td>$C_{PI}^{13}$ advance 50 m</td>
<td>2.934</td>
<td>0.031</td>
<td>0.653</td>
<td>0.276</td>
</tr>
</tbody>
</table>

Table 6.6: Evaluation of comfort indices in case of GPS positioning errors
Chapter 7

Conclusions and Future Developments

The present thesis proposed analytical and semi-analytical methods to design simple and robust controllers for an active hydraulic anti-roll bar used in high speed railway vehicles. The aim of this device is to apply a limited amount of carbody tilt thus reducing the exposure of passengers to non-compensated lateral acceleration in curves.

Firstly, a 3-DoF linearized model, with actuator dynamics, is illustrated. From this simplified representation, a 8-states linear model has been obtained in order to provide a very simple plant transfer function to facilitate model reduction and thus following control design process.

Subsequently, on this 8-states linear model a Modal Dominance Index (MDI) analysis have been performed in order to evaluate which are the dominant modes of the system. Two model reductions, manual and analytical, have been applied to the linearised plant (8-states linear model) and a model identification using a 35-states non-linear MBS (multi-body system) developed by Department of Mechanical Engineering of Politecnico di Milano has been performed to derive directly a low-order model. First-order and third-order reduced models have been obtained. An analysis of uncertainty introduced by these reductions, where some dynamics disregarded, was also performed to show at which frequencies the original and reduced plant mismatch. The models obtained from the model reduction process, are simpler compared to the other starting models, before mentioned, and allow to
design simple but effective controllers in analytical or semi-analytical ways.

Four PI controllers were defined for the active suspension, imposing analytically, thanks to reduced models, different requirements. Four families of controller have been obtained:

(a) PIs imposing on the first-order reduce model a required bandwidth and phase margin;

(b) PIs imposing on the third-order reduced model a required BW and PM;

(c) PIs imposing on the first-order reduced model a required maximum overshoot and settling time;

(d) PI+feedforward contribution using a search-grid optimization, minimizing $P_{CT}$ index.

For each controller a simulation on Simulink ©, using the standard case of a vehicle running at 340 km/h on a typical high-speed curve, has been performed. The responses are analysed in terms of simple performance parameters: maximum overshoot and settling time. The controller, of each family leading to best results, has been selected to be implemented on MBS. Finally, considering again the same case of a vehicle running at 340 km/h on a typical curve in a high-speed railway network, simulations on MBS have been performed in order to assess the performance of the selected synthesised controllers.

The results of these simulations show that PI controllers, that are designed in an easy and analytical way based on reduced model, achieve good performance in terms of $P_{CT}$ factor, compared to controllers derived from more complex multi-objective optimizations. In addition it has been shown that, as expected, imposing the same requirements (e.g. in terms of BW and PM), the controllers, designed on the third-order model, lead to better performance with respect to controllers designed on the first-order model.

Simulations have outlined that the mean power required to operate the active suspension with these controllers is in the range of 1.5–2.0 kW per bogie. The control strategy based on PI+feedforward provides the best trade-off between performance in terms of ride comfort and acceptable actuation power requirement.
Simple PIs, obtained imposing a required bandwidth and phase margin on the third-order reduced model, provide good results in terms of both $P_{CT}$ and power consumption, adopting an analytical and very simple approach. Finally, assessments on robustness have been performed varying some crucial parameters: carbody mass and inertia, and error in tilt reference due to errors the geo-localisation of the vehicle. The two controllers that lead to best results have been used for robustness analysis: (a) PI analytical derived from the third-order reduced model and (b) PI+feedforward. It has been proved that perturbed controllers are still able to provide acceptable results. Concluding, it has been shown that a simplified designing process adopting low-order model leads to simple and robust controllers, thus saving time and computational effort with respect to multi-objective optimization.

The possibility of future developments of this work are numerous. Further improvements can be done regarding analytical controller design, e.g. employing different target requirements or different reduced models, or investigating more in depth the contribution of the derivative term in PID controller. In terms of model identification, more specific simulations on MBS can be performed in order to obtain a simplified model that behaves as much as possible close to original plant. Finally, another possible development is the use of the reduced models for more complex model-based controllers, such as IMC (Internal Model Control) or MPC (Model Predictive Control).
## Appendix A

### Parameters of the System

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_v$</td>
<td>Carbody mass</td>
<td>20366 kg</td>
</tr>
<tr>
<td>$J_v$</td>
<td>Roll inertia</td>
<td>44000 kgm²</td>
</tr>
<tr>
<td>$r_v$</td>
<td>Vertical damping</td>
<td>13000 Ns/m</td>
</tr>
<tr>
<td>$r_l$</td>
<td>Lateral damping</td>
<td>65000 Ns/m</td>
</tr>
<tr>
<td>$r_{lor}$</td>
<td>Roll damping</td>
<td>200000 Ns/ rad</td>
</tr>
<tr>
<td>$k_l$</td>
<td>Lateral stiffness</td>
<td>248000 Ns/m</td>
</tr>
<tr>
<td>$k_{lor}$</td>
<td>Roll stiffness</td>
<td>2700000 N/ rad</td>
</tr>
<tr>
<td>$2k_a$</td>
<td>Equiv. vertical stiffness from airspring</td>
<td>600000 N/m</td>
</tr>
<tr>
<td>$2k_{a,pm}^2/4$</td>
<td>Equiv. roll stiffness from air spring</td>
<td>6000000 N/ rad</td>
</tr>
<tr>
<td>$b_{rv}$</td>
<td>Vertical suspension spacing</td>
<td>2.57 m</td>
</tr>
<tr>
<td>$b_{pm}$</td>
<td>Airsprings spacing</td>
<td>2 m</td>
</tr>
<tr>
<td>$h_{rl}$</td>
<td>Lateral damping height</td>
<td>1.23 m</td>
</tr>
<tr>
<td>$h_{kl}$</td>
<td>Lateral spring height</td>
<td>1.28 m</td>
</tr>
<tr>
<td>$h_{lat}$</td>
<td>Active lateral suspension height</td>
<td>1.35 m</td>
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Table A.1: Vehicle parameters
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{serb}$</td>
<td>Volume of each reservoir</td>
<td>$1.5 \cdot 10^{-2}$ $m^3$</td>
</tr>
<tr>
<td>$y_{lim}$</td>
<td>Maximum piston stroke</td>
<td>0.24 $m$</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Area of piston</td>
<td>$1.963 \cdot 10^{-3}$ $m^2$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Total volume of oil in the circuit</td>
<td>$8 \cdot 10^{-3}$ $m^3$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Internal leakage coefficient</td>
<td>$10^{-14}$ $\frac{m^3}{Pa}$</td>
</tr>
<tr>
<td>$C_e$</td>
<td>External leakage coefficient</td>
<td>0 $\frac{m^3}{Pa}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Oil bulk modulus</td>
<td>$1.1 \cdot 10^9$ $Pa$</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Supply pressure</td>
<td>198 $bar$</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Return pressure</td>
<td>2 $bar$</td>
</tr>
<tr>
<td>$b_{ol}$</td>
<td>Distance between actuator</td>
<td>2.57 $m$</td>
</tr>
</tbody>
</table>

Table A.2: Active anti-roll bar parameters
Appendix B

Bode diagrams

For completeness in this appendix are shown the bode diagrams of the closed loops shaped with PI controllers developed in chapter 5.

![Bode diagram of Closed Loop with $PI^1$](image)

Figure B.1: Bode diagram of Closed Loop with $PI^1$
Figure B.2: Bode diagram of Closed Loop with $PI^2$

Figure B.3: Bode diagram of Closed Loop with $PI^3$
Figure B.4: Bode diagram of Closed Loop with $PI^4$

Figure B.5: Bode diagram of Closed Loop with $PI^4$
Figure B.6: Bode diagram of Closed Loop with $PI^6$

Figure B.7: Bode diagram of Closed Loop with $PI^7$
Figure B.8: Bode diagram of Closed Loop with $Pl^8$

Figure B.9: Bode diagram of Closed Loop with $Pl^9$
Figure B.10: Bode diagram of Closed Loop with $PI^{10}$

Figure B.11: Bode diagram of Closed Loop with $PI^{11}$
Figure B.12: Bode diagram of Closed Loop with $PI^{12}$

Figure B.13: Bode diagram of Closed Loop with $PI^{13}$
Bibliography


