Vibration Control of Tall Buildings Using Eddy Current Tuned Mass Dampers

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Abstract

Advancements in materials science and construction technologies have led to an inclination towards constructing high-rise buildings. Tall buildings are slender structures that usually experience high levels of vibration, which leads to the discomfort of the occupants. Historically, stiffness and strength were intended to be increased to solve the problem of elevated accelerations, but energy dissipation systems have proven to be a sound alternative solution due to their high efficiency, reliability, feasibility, and economy. A tuned mass damper is a device that consists of mass, stiffness, and damping components, and it belongs to the passive category of energy dissipation systems. This device works in out-of-phase resonance with the structural motion when an exciting frequency matches one of the structure natural frequencies, so it is tuned to this specific frequency. Viscous fluid dampers are the most widely applied damping means for tuned mass dampers because of their good functionality, but they also have some disadvantages. Alternatively, eddy current dampers can replace the latter type to serve as the damping component of dynamic vibration absorbers. This study presents the theoretical formulation of eddy current damping and an easy-to-apply analytical model to design an eddy current tuned mass damper (ECTMD). Consequently, this model is applied in this monograph to design such a tuned mass damper to control the vibrations of the Taipei 101 super tall building. It has resulted in a remarkable reduction of the wind-induced accelerations from 31 milli-g to 2 milli-g, while the reductions in the seismic base reactions do not exceed 8%. Furthermore, feasibility and economic study of the proposed ECTMD is performed, and the advantages of using it are highlighted. It is concluded that the ECTMD studied in this manuscript significantly outperforms the existing viscous fluid tuned mass damper.
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Nomenclature

\( a_1 \)  
forced frequency ratio \( = \frac{\omega}{\omega_1} \)

\( b \)  
transversal dimension of the building

\( b_m \)  
length of the permanent magnet

\( B \)  
Length of the conductive plate

\( B_r \)  
remanent flux density of the permanent magnet

\( B_x, B_y, B_z \)  
magnetic flux density in x, y, and z directions

\( c_1 \)  
damping of the primary SDOF structure

\( c_2 \)  
damping of the TMD connected to an SDOF structure

\( c_d \)  
damping of the TMD connected to an MDOF structure

\( C \)  
damping matrix

\( C_1 \)  
modal damping of the first mode

\( f \)  
tuning frequency ratio \( = \frac{\omega_2}{\omega_1} \)

\( F \)  
damping force

\( F_{L1}(s) \)  
equivalent transversal force per unit length of height due to vortex shedding

\( g \)  
gravitational acceleration

\( h \)  
height of the structure

\( h_b \)  
thickness of the primary steel plate

\( h_c \)  
thickness of the conducting plate

\( h_f \)  
thickness of the secondary steel plate

\( h_g \)  
air-gap length between the magnets and the conductive plate

\( h_m \)  
thickness of the permanent magnet

\( J_x, J_y, J_z \)  
eddy current density in x, y, and z directions

\( k_1 \)  
stiffness of the primary SDOF structure

\( k_2 \)  
stiffness of the TMD connected to an SDOF structure

\( k_d \)  
stiffness of the TMD connected to an MDOF structure

\( k_{eq} \)  
equivalent spring stiffness of the cable of pendulum tuned mass damper

\( K \)  
stiffness matrix

\( K_1 \)  
modal stiffness of the first mode

\( L \)  
length of the cable of pendulum tuned mass damper

\( m_1 \)  
mass of the primary SDOF structure

\( m_2 \)  
mass of the TMD connected to an SDOF structure
$m_d$ mass of the TMD connected to an MDOF structure

$M$ mass matrix

$M_1$ modal mass of the first mode

$n_s$ shedding frequency

$q$ relative variable of space of the TMD with respect to the floor at which it is installed

$S_{c1}$ Scruton number of the first mode

$S_f$ uniform power spectrum density

$S_t$ Strouhal number

$t$ variable of time

$T_R$ wind return period

$v_{cr,1}$ critical wind velocity due to vortex shedding

$v_m$ mean wind speed

$x$ variable of space in the horizontal direction of the building

$X, X_1, X_2$ Amplitudes of vibration

$w_m$ width of the permanent magnet

$W$ width of the conductive plate

$z$ variable of space along the height of the building

$\sigma_c$ conductivity of the conductive plate

$\phi_1$ the first modal shape

$\mu$ mass ratio $= m_2/m_1$ for SDOF structure and $= m_d/M_1$ for MDOF structure

$\omega$ circular frequency of the external excitation

$\omega_1$ circular frequency of the primary structure $= \sqrt{k_1/m_1}$

$\omega_2$ circular frequency of the tuned mass damper connected to an SDOF structure $= \sqrt{k_2/m_2}$

$\omega_d$ circular frequency of the tuned mass damper connected to an MDOF structure $= \sqrt{k_d/m_d}$

$\rho$ density of air $= 1.25\, \text{Kg/m}^3$

$\xi_1$ primary SDOF structure damping ratio $= c_1/2m_1\omega_1$

$\xi_2$ absorber damping ratio connected to an SDOF structure $= c_2/2m_2\omega_2$

$\xi_d$ absorber damping ratio connected to an MDOF structure $= c_d/2m_d\omega_d$

$\xi_e$ equivalent damping ratio
Chapter 1: Introduction

Structures have been evolving with the advancement in construction technologies, materials and research studies being conducted on them until it has recently become a trend to build high-rise buildings. The design of such a kind of structures is governed by the lateral loads, mainly wind and seismic actions. Indeed, tall buildings are susceptible to large sway and interstorey drift, and they are subjected to high accelerations and straining actions. Thus, mitigation of these effects is required to maintain a safe design of the structure. Truly, in the past, the main concern was to ensure strength verification; nonetheless, human comfort and cost have recently become among the most important criteria of the structural design. Energy dissipation systems are the means of mitigation and can be applied by two methods. First, strengthening is the traditional method used for mitigating the effects of loads acting on a structure, and it is very effective for regular structures. This method relies entirely on the stiffness of the structure and can be realized by enlarging the structural lateral resisting elements or increasing their number, or using high-performance materials. In fact, employing this method in the design of tall buildings leads to improper use of materials and requires a high cost. In addition, another major drawback is that the stiffer a structure becomes, the higher seismic forces it drags; thus, increasing the stiffness increases the loads acting on it, so an even stiffer structure may be needed, which results in an inefficient design. Second, vibration control devices can be added to a structure, which belong to three main families that are passive, active, and semi-active control systems (Constantinou, Soong, and Dargush 1998).

The passive category of vibration control devices is the most common for structural use, such as tuned mass dampers, viscous fluid devices, friction devices, and others that are mainly of hysteretic or viscous nature. Tuned mass dampers have efficiently been used since the last century for structural vibration control, and they were first introduced by Frahm in 1909 (Frahm 1911). Indeed, the structural response depends on the mass ratio of the tuned mass damper to that of the primary structure, the frequency ratio, and the damping ratio of the tuned mass damper. Consequently, these parameters shall be carefully chosen to control the structural response. The optimal tuning and damping parameters have first been studied by Den Hartog in 1928 and introduced in his textbook (Den Hartog 1985), but this optimization theory is applicable only for tuned mass dampers attached to undamped single-degree-of-freedom structures. Then, many researchers have studied different optimization techniques to minimize
either the maximum amplitude magnification factor of the primary structure that is the $H_\infty$ norm or the area under the frequency response curve that is the $H_2$ norm. These research works studied different systems, among which are those of damped single-degree-of-freedom primary structures to find the solution of the problem in a closed-form by (Asami, Nishihara, and Baz 2002) and (Asami and Nishihara 2003). In contrast, (Lee et al. 2006) derived a numerical solution for the optimization problem of damped both single and multi-degree-of-freedom primary structures.

Stiffness and damping of the added mass can be realized in several different ways, among which is a vertical pendulum for the lateral stiffness, where the natural frequency of the tuned mass damper can be adjusted by defining the length of the pendulum, and a dashpot for the damping component. Traditionally, fluid viscous dampers are used, but they have some drawbacks, such as a continuous requirement for maintenance due to the limited service life, complexity in the connection with the structure, and sensitivity to the heat generated by the dissipated energy. Hence, a new kind of dampers has recently been introduced for structural applications to overcome such limitations of the traditional dampers, which is the eddy current damper.

When a magnet is in motion against a non-magnetic metal like copper or aluminum, the magnetic field of the metal changes. According to Faraday’s law of induction, this variation in the magnetic field induces circular electric currents called Eddy currents or an electromagnetic force. Alternatively, the eddy currents can be generated by a stationary conductor and a magnetic source that varies in intensity. Lenz’s law complements the former law by defining the direction of the electromagnetic force based on the conservation of energy. That is, the magnetic field produced by an induced current has the opposite direction to the change in the original field (Serway, R.A., Jewett 2004). Therefore, this electromagnetic force can be considered as a damping force for the driving actions in a structure, as depicted in Figure 1.1.

(Sodano and Bae 2004) reviewed the historical use of eddy current damping in several different disciplines, such as magnetic braking that is mainly applied in mechanical engineering, damping of rotating machinery, and damping for dynamic systems. Then, (Bae, Kwak, and Inman 2005) developed a design tool based on a theoretical model that is able to predict the damping characteristics and the dynamic behavior of a cantilever beam. Similarly, (Sodano et al. 2005) developed a theoretical model of an eddy current damper to suppress the
vibrations of flexible structures based on the radial magnetic flux to generate the electromagnetic damping force. They experimented a cantilever beam to verify the accuracy of the developed theoretical model, which revealed promising advantages of using this kind of damping, including a high increase in the modal damping ratio. The concept was the attachment of a conductor to a cantilever beam, which upon vibration moves perpendicularly against a permanent magnet causing the generation of eddy currents by the radial magnetic flux. After that, (Sodano et al. 2006) improved the previous theoretical model using the image method, and they also improved the concept by adding another magnet on the other side of the beam to result in a conductor moving between two magnets of opposite polarity, which significantly increased the damping ratio.

![Image of Eddy Currents Induction by a Relative Motion between a Conductor and a Magnet (Lu et al. 2017)](image)

Figure 1.1: Eddy Currents Induction by a Relative Motion between a Conductor and a Magnet (Lu et al. 2017)

Alternatively, to study the potential of this kind of damping to belong to another category of dampers than the passive one, (Sodano and Inman 2008) developed a theoretical model for an active eddy current damper by modifying the current passing through a coil, which results in a time-varying magnetic field, so the damping force is controlled by controlling the strength of the magnetic field around the conductor.

Therefore, an eddy current damper has numerous advantages that make it an important alternative for the vibration control of tall buildings. In fact, it is a non-contacting type of dampers, which leads to a very favorable situation, especially for flexible structures like high-rise buildings, by adding significant damping to the structure without changing its dynamic properties because it doesn’t add mass or stiffness (Sodano and Bae 2004). Plus, due to the
non-contacting behavior, there is no friction or wear experienced with the device, and thus the need for repair is eliminated. In addition, an eddy current damper, of a passive nature, requires no power to operate, so it is fail-safe and cost-effective. Furthermore, it is comfortable, which means it does not have a jerk or noise during the activation process. To shed the light on these advantages, (Wang, Chen, and Wang 2012) designed, analyzed, manufactured and tested a large-scale tuned mass damper using eddy current damping. They highlighted the weak points of the traditional viscous dampers usually used for tuned mass dampers, such as leakage of oil and the difficulty in adjusting the damping ratio for an operating tuned mass damper. On the other hand, their study has proven that an eddy current tuned mass damper is efficient, reliable, robust and feasible. That is, it requires no fluids for operation, and the working frequency can be easily adjusted without any additional cost, which can range from 0.1 to over 10 Hz unlike the traditional viscous dampers, which start adding stiffness to the structure beyond 3 Hz, and this dramatically reduces the control performance by detuning the tuned mass damper.

Recently, all the theoretical formulations and the studies performed on eddy current dampers were exploited by (Lu et al. 2017), who applied for the first time a large-scale eddy current tuned mass damper to one of the tallest buildings in the world, which is the Shanghai Center Tower.

Therefore, the scope of this thesis is to study the potential preeminence of eddy current tuned mass dampers over the traditional types through the design of an ECTMD for the mega tall building Taipei 101 and compare its feasibility and advantages with the existing viscous tuned mass damper. To this end, the introductory chapters discuss the serviceability issues experienced in tall buildings and the occupant comfort criteria that shall be met through their design. Then, the theoretical formulation of tuned mass dampers is generally presented to show the importance and effectiveness of their use. Furthermore, the eddy current damping theory is formulated, which serves as the basis for the design of the damping component of the tuned mass damper. Finally, the comparison between the two approaches is done and the conclusions are drawn.
Chapter 2: High-Rise Buildings

Tall buildings have historically been appealing structures, and the humankind has ever tended to build high-rise structures, such as the Tower of Babel, the Colossus of Rhodes, the Pyramids of Giza, and other structures. However, the revolution of constructing tall buildings has started since the end of the nineteenth century.

Indeed, this desire to construct tall structures comes from several factors. In the past, the main reason could be the pride of reaching high elevations. Recently, social and economic reasons have prevailed in the construction of skyscrapers. That is, a tall building increases the surrounding land value and densifies the population.

A certain criterion to define a tall structure does not exist; it is rather a relative matter. For instance, a typical city in the European continent accommodates low-rise buildings, consisting of three to four storeys, so a 20-storey building there appears as a skyscraper. In contrast, this same building if erected in Dubai or Shanghai, for example, would appear as a low-rise building. Therefore, the tallness of a structure cannot be associated with the height or the number of floors. From the structural wise, a building can be defined as tall when the analysis and design of the structure are dominated by the lateral loads rather than the gravity loads, so when the problem lies in the structural dynamics field of studies more than statics.

2.1. Historical Limitations on Tall buildings

The main limitations that historically used to limit the construction of skyscrapers are mainly the poor performance of construction materials, the lack of vertical transportation means, and the inefficient construction technologies. Before the emergence of structural steel and reinforced concrete materials, wood and masonry had been used for construction. However, wood has limited strength, and masonry has a relatively high weight. A well-known example of this limitation is the Monadnock Building in Chicago (Figure 2.1), which was built in 1891 and consists of 17 floors. This structure relies on the bearing capacity of brick to resist loads, but to achieve this target with a height of 64 m, it has required 2.13 m thick load-bearing masonry walls at the ground level, which comprise 15% of the total gross area of the ground floor (Taranath 2010). Moreover, the technologies used in the past for construction were not adequate to construct very tall buildings. However, there exist, nowadays, very advanced
technologies that facilitate construction activities in terms of safety, reducing cost, and reducing the required completion time.

Indeed, a serious limitation of constructing skyscrapers was the issue of vertical transportation before the invention of elevators in 1853 by Elisha Otis. This has lifted the constraint of erecting soaring towers.

![Figure 2.1: Monadnock Building (Taranath 2010)](image)

### 2.2. Evolution of Tall Buildings

After all the past limitations of building skyscrapers have been overcome, a significant number of super-tall buildings has arisen all over the world. Truly, when a building reaches a very high elevation, the way of approaching its analysis and design is different. Due to the enormously high stresses and strains induced in the structural elements, the conventional structural systems used in small structures are not applicable anymore. Thus, engineers and researchers have developed new structural systems to realize a tall building, such as tubular systems, mega-frame system, and others.

Another major challenge faces the designer of a super high-rise building is the large sway and vibrations at the top of the building. In fact, there are three main methods to limit
these induced effects on high-rise structures. First, an optimized aerodynamic design of the building’s shape is a desirable solution although, in many cases, it is not sufficient on its own. The aerodynamic modifications can be attained by designing a round-like shape of the tall building rather than a square or rectangular one, using slotted or chamfered corners, or using a through-building opening, which reduce mainly the vortex shedding phenomenon and the overall wind-induced effects. Second, engineers can apply structural design changes either by increasing the mass or increasing the stiffness. For instance, traditional types of solid slabs can be used to increase the mass of the structure instead of using post-tensioned or steel-concrete composite slabs, which are usually characterized by lightweight, and this ultimately enhances the structural response to wind excitation but with an inefficient use of materials and high cost. On the other hand, stiffness can be increased in different ways. For example, the traditional solution of using a higher strength material could not be the optimal solution because the modulus of elasticity is not linearly proportional to strength, so a significant increase in strength is required to limit the displacement at the upper floors. Alternatively, enlarging the structural components or introducing additional ones ends up in increasing the stiffness but again with an inefficient use of materials and a loss of the rentable space of the facility similar to the case of Monadnock Building. Third, one of the most preferable ways to limit excessive vibrations in tall buildings is the use of new innovative systems, known as energy dissipation systems. These systems have many different types, and the most typical ones used for the case of vibration control of high-rise buildings are the dynamic vibration absorbers, or the tuned mass dampers.

2.3. Vortex Shedding

Wind buffeting creates six different components of forces and moments on a bluff body in three orthogonal directions; however, the lifting force and the yawing moment corresponding to the vertical axis have minor effects on building structures. Thus, the wind analysis of a high-rise building is mainly governed by the two horizontal directions named along-wind and transverse wind (crosswind) directions, as illustrated in Figure 2.2.

Vortex shedding phenomenon occurs when the spiral vortices detach from the sides of the building into the downstream flow of wind. At relatively high wind speeds, this alternately occurs on opposite sides of the building with a frequency that is half the frequency of the along-wind impulse. The transverse impulses are equivalent to transverse loads applied alternately on the side at which the vortices are shed, as demonstrated in Figure 2.3. Since vortex shedding
is a periodic phenomenon, the mass of the building starts swinging with a small amplitude at the beginning; nevertheless, at the end of each cycle, a small impulse added to the already oscillating mass significantly increases the kinetic energy of the system and consequently the transverse deflection.

\[ n_s = \frac{S_t v_m}{b} \]  \hspace{1cm} (1)

where \( n_s \) = shedding frequency

\( S_t \) = Strouhal number
\( v_m = \text{mean wind velocity} \)

\( b = \text{transversal dimension of the building} \)

Resonance condition takes place when the shedding frequency coincides with the natural frequency of the structure. Notwithstanding, when this occurs, the Strouhal law is violated, and the structure takes control over the vortex shedding phenomenon. That is, the shedding frequency is anchored to the natural frequency of the structure in a velocity range, the so-called lock-in range, so a small change in the wind velocity will not change the shedding frequency. Therefore, the vortex shedding phenomenon is critical only in the lock-in range, and thus it is not influential for velocities above or below this velocity range.

### 2.4. Occupant Comfort Criteria in Wind-Excited Tall Buildings

As the height of a building increases, the serviceability requirements become more relevant. Evidently, the strength requirements shall assuredly be satisfied by designing each and every structural element to withstand the design loads without experiencing buckling, yielding, or fracture, according to the specifications of the design codes. Notwithstanding, the serviceability requirements are usually not codified; there are rather only recommended thresholds not to be exceeded. In fact, the serviceability requirements are usually not stringent because the failure to meet them does not lead to catastrophic results as long as they are not excessively exceeded. If the lateral displacement at the top of a skyscraper and the interstorey drifts are outrageous, a structural stability problem arises due to the second-order effects in addition to the potential of falling of non-structural elements like cladding.

Human perception and tolerance of wind-induced vibrations in tall buildings are subjective in nature and encompass several physiological and psychological factors, so there are no universally agreed occupant comfort criteria in the codes to be met as design standards. However, many researchers have conducted studies by means of field experiments and measurements, laboratory experiments by motion simulators and shake table tests, and field experiments by artificially exciting buildings, and the main target was to set recommended thresholds not to be exceeded to maintain the comfort of the occupants in tall buildings. Most of these studies were conducted at a frequency of 0.1-1.0 Hz, which is representative of the actual frequency of a typical high-rise building.

A review of different studies conducted by researches and design codes was done by (Kwok, Hitchcock, and Burton 2009), and the different recommended accelerations are plotted
Figure 2.4, where one milli-g represents an acceleration of one-thousandth of gravity. It is evident that the perception limits are frequency-dependent, and the thresholds of residential buildings are lower than those of office buildings because office buildings are usually unoccupied during the night, and they can be evacuated before the occurrence of extreme wind events. Indeed, many different factors were included in these studies, including real-time measurements in tall buildings using accelerometers during windstorms followed by interviews or survey questionnaires and experiments on test subjects. The experiments studied the perception levels of the subjects focusing on different factors, such as walking or sitting posture of subjects under unidirectional, bi-directional, and yaw vibrations, pre-conditioning the prediction of vibrations by assigning distracting tasks to the subjects, testing subjects of different genders and ages, visual cues to test the far-field and near-field observation parallax, and other physiological and psychological factors.

![Figure 2.4: Different Occupant Comfort Serviceability Criteria for a One Year Return Period Wind Storm (Kwok, Hitchcock, and Burton 2009)](image)

Therefore, different recommended levels are present referring to different guidelines and researches, and most of them set these levels based on comfort criteria rather than
perception thresholds otherwise the design cost would be significantly high. However, these recommended ranges are close to each other, and they are generally limited to 20 milli-g. If the vibration levels exceed these limits, the occupants of a tall building start to have reactions, depending on the level of acceleration in terms of, on one hand, dizziness, migraine and nausea, and on the other hand, as serviceability concerns, as tabulated in Table 2.1 by (Mendis et al. 2007).

Table 2.1: Human Perception Levels (Mendis et al. 2007)

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>ACCELERATION (m / sec^2)</th>
<th>EFFECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt; 0.05</td>
<td>Humans cannot perceive motion</td>
</tr>
<tr>
<td>2</td>
<td>0.05 - 0.1</td>
<td>a) Sensitive people can perceive motion; b) hanging objects may move slightly</td>
</tr>
<tr>
<td>3</td>
<td>0.1 - 0.25</td>
<td>a) Majority of people will perceive motion; b) level of motion may affect desk work; c) long-term exposure may produce motion sickness</td>
</tr>
<tr>
<td>4</td>
<td>0.25 - 0.4</td>
<td>a) Desk work becomes difficult or almost impossible; b) ambulation still possible</td>
</tr>
<tr>
<td>5</td>
<td>0.4 - 0.5</td>
<td>a) People strongly perceive motion; b) difficult to walk naturally; c) standing people may lose balance</td>
</tr>
<tr>
<td>6</td>
<td>0.5 - 0.6</td>
<td>Most people cannot tolerate motion and are unable to walk naturally</td>
</tr>
<tr>
<td>7</td>
<td>0.6 - 0.7</td>
<td>People cannot walk or tolerate motion</td>
</tr>
<tr>
<td>8</td>
<td>&gt; 0.85</td>
<td>Objects begin to fall and people may be injured</td>
</tr>
</tbody>
</table>
Chapter 3: Vibration Control of Tall Buildings

Structures are subjected to different loading conditions that induce accelerations and stresses in the structural elements, and they are generally designed relying on their strength and inherent ductility to resist the environmental disturbances. When the loading intensity is low, structures typically behave in the elastic range; however, when the intensity is high, different behavior is expected. For regular structures, the design engineer accepts a certain level of structural and non-structural damage, for instance, through the plasticization of specifically designed regions. Nevertheless, new and innovative energy dissipation systems have been developed to overcome this problem, especially for special structures like tall buildings.

Energy dissipation systems can be divided into seismic isolation systems, passive energy dissipation systems, and active and semi-active energy dissipation systems. In fact, passive energy dissipation systems are the most common for structural engineering applications because they do not require any externally supplied power to operate, unlike active systems, which demand a huge force to be supplied from an external power source. Passive systems are divided into different types, among which are the dynamic vibration absorbers, and these do not add a force to the structure to counteract the driving force, but they increase the damping and reduce the vibration and motion amplitudes. Tuned mass dampers are one type of dynamic vibration absorbers.

3.1. Tuned Mass Dampers

A tuned mass damper (TMD) is a device that consists of a mass, stiffness component, and damping component; it is added to a structure to control its dynamic response. The frequency of the dynamic absorber is tuned to a specific frequency, which is usually one of the natural frequencies of the primary structure, in a way which when this particular frequency is excited, the dynamic absorber resonates out of phase with respect to the primary structure.

Tuned mass dampers are typically applied to structures behaving in the elastic range subjected to wind loads; in contrast, their efficiency is significantly reduced under seismic excitations. This is true because dynamic vibration absorbers are, first, ineffective for impulsive loadings, so they cannot reach a resonance condition to dissipate energy. Second, when a structure enters into the inelastic range after a seismic event, its stiffness is decreased and thus its frequency; hence, this leads to detuning of the tuned mass damper due to the shift of the
original frequency to which it has already been tuned. Nonetheless, tall buildings generally behave in the elastic range, and their design is mainly governed by wind effects.

In this chapter, the theoretical formulation of the structure-TMD system motion is elaborated, considering different cases excluding and including damping of both the primary structure and the tuned mass damper. Herein, the primary structure is reduced into a single-degree-of-freedom (SDOF) system, which is sufficient to define the optimal parameters of the dynamic vibration absorber. So that, the SDOF-TMD system is a two-degree-of-freedom system.

3.1.1. Undamped SDOF Structure: Undamped TMD

Although a system lacking damping does not exist in real-life applications, it is worth studying its characteristics to highlight the importance of damping. Figure 3.1 illustrates a system of an undamped single-degree-of-freedom structure and an undamped tuned mass damper.

![Figure 3.1: Schematic of an Undamped TMD Attached to an Undamped SDOF Structure](image)

In Figure 3.1, $m_1$ represents the mass of the primary structure, whereas $m_2$ represents the mass of the dynamic vibration absorber. The stiffness of the structure is denoted by $k_1$, which is the spring constant or the force required to displace the spring by a unit displacement.
Similarly, $k_2$ is the stiffness of the tuned mass damper. The system is studied under the effect of a harmonic force acting directly on $m_1$. Finally, $x_1$ and $x_2$ are the displacements of the structure and the TMD, respectively, starting from the static equilibrium position.

The equations of motion of both masses at any instant of time and the equilibrium position are derived from Newton’s second law

$$\text{Force} = \text{mass} \times \text{acceleration}$$

The forces acting on $m_1$ can be written as

$$m_1 \frac{d^2 x_1}{dt^2} = m_1 \ddot{x}_1 = -k_1 x_1 - k_2 x_1 + k_2 x_2 + F \cos \omega t$$

where the two dots above the variable denotes the double differentiation with respect to time. The mass is assumed to move in the upward positive direction, the assumed sign convention, so the spring force has a negative sign in the downward direction because it pulls the mass trying to keep it in its initial position during the motion with a value of $k_1 x_1$, according to Hooke’s law. The force generated in the spring of the tuned mass damper pushes against $m_1$ due to its motion, so it also has a negative sign. Thus, since the latter spring is compressed in this case, it pushes against $m_2$ in the upward direction, which means it has a positive sign.

Similarly, the forces acting on $m_2$ are

$$m_2 \frac{d^2 x_2}{dt^2} = m_2 \ddot{x}_2 = -k_2 x_2 + k_2 x_1$$

When the motion of the tuned mass damper takes place, its spring gets extended, and it takes a negative because it pushes $m_2$ downward with a force $k_2 x_2$. The force in the same spring pushes upward mass $m_1$ by $k_2 x_1$.

By rearranging the forces acting on the two masses, the differential equations of motion can be modified to

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 x_1 - k_2 x_2 = F \cos \omega t \quad (2)$$

$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \quad (3)$$

The response is assumed to have the following form

$$x_1 = X_1 \cos \omega t \quad (4)$$

$$x_2 = X_2 \cos \omega t \quad (5)$$
where \(X_1\) and \(X_2\) are the amplitudes of the responses of the two masses \(m_1\) and \(m_2\), respectively. Thus, by differentiation of equations (4) and (5) substitution into (2) and (3), it results

\[-m_1 \omega^2 X_1 \cos \omega t + (k_1 + k_2) X_1 \cos \omega t - k_2 X_2 \cos \omega t = F \cos \omega t\]

\[-m_2 \omega^2 X_2 \cos \omega t + k_2 X_2 \cos \omega t - k_1 X_1 \cos \omega t = 0\]

The latter two equations can be simplified by dividing by \(\cos \omega t\) and rearranging to get

\[-m_1 \omega^2 X_1 \cos \omega t + (k_1 + k_2) X_1 - k_2 X_2 = F\]  \hspace{1cm} (6)

\[-k_1 X_1 + (-m_2 \omega^2 + k_2) X_2 = 0\]  \hspace{1cm} (7)

Now, equations (6) and (7) can be further simplified by dividing the first equation by \(k_1\), dividing the second equation by \(k_2\), and introducing the following symbols:

\[X_{st} = \frac{F}{k_1} = \text{static deflection of the primary structure}\]

\[\mu = \frac{m_2}{m_1} = \text{mass ratio} = \frac{\text{mass of the tuned mass damper}}{\text{mass of the primary structure}}\]

\[\omega_1 = \sqrt{\frac{k_1}{m_1}} = \text{natural circular frequency of the primary structure}\]

\[\omega_2 = \sqrt{\frac{k_2}{m_2}} = \text{natural circular frequency of the tuned mass damper}\]

\[(-m_1 \omega^2 \cos \omega t + 1 + \frac{k_2}{k_1}) X_1 - \frac{k_2}{k_1} X_2 = \frac{F}{k_1} \Rightarrow (-\frac{\omega^2}{\omega_1^2} + 1 + \frac{k_2}{k_1}) X_1 - \frac{k_2}{k_1} X_2 = X_{st}\]  \hspace{1cm} (8)

\[-X_1 + (-\frac{m_2 \omega^2}{k_2} + 1) X_2 = 0 \Rightarrow X_1 = (-\frac{\omega^2}{\omega_2^2} + 1) X_2\]  \hspace{1cm} (9)

By substituting equation (9) into equation (8), it is obtained

\[-\frac{\omega^2}{\omega_1^2} + 1 + \frac{k_2}{k_1} (-\frac{\omega^2}{\omega_2^2} + 1) X_2 - \frac{k_2}{k_1} X_2 = X_{st}\]  \hspace{1cm} (10)

Therefore, the two solutions obtained and normalized by the static deflection of the main structure are

\[\frac{X_1}{X_{st}} = \frac{1 - \frac{\omega^2}{\omega_2^2}}{(1 + \frac{k_2}{k_1} \frac{\omega^2}{\omega_1^2})(1 - \frac{\omega^2}{\omega_2^2} - \frac{k_2}{k_1})}\]  \hspace{1cm} (11)

\[\frac{X_2}{X_{st}} = \frac{1}{(1 + \frac{k_2}{k_1} \frac{\omega^2}{\omega_1^2})(1 - \frac{\omega^2}{\omega_2^2} - \frac{k_2}{k_1})}\]  \hspace{1cm} (12)
By analyzing equation (11), it can be seen that the response of the main structure is null when the numerator is zero. That is, when \((1 - \frac{\omega^2}{\omega_2^2}) = 0\), and this is attained when the frequency of the tuned mass damper coincides with the frequency of excitation.

Moreover, when \(\omega = \omega_2\), equation (12) becomes

\[
\frac{X_2}{X_{st}} = \frac{1}{\frac{k_2}{k_1}}
\]

\[
X_2 = \frac{k_1}{k_2} X_{st} = \frac{k_1}{k_2} \frac{F}{k_1} = -\frac{F}{k_2}
\]

Thus, the motion of the principal structure is null, and the motion of the tuned mass damper is expressed as

\[
x_2 = X_2 \cos \omega t = -\frac{F}{k_2} \cos \omega t
\]

and the force in its spring is

\[
k_2x_2 = -\frac{k_2}{k_2} F \cos \omega t = -F \cos \omega t
\]

which is equal and opposite the external force.

In fact, the tuned mass damper is effective only when the main structure is in resonance, so the case of \(\omega = \omega_1 = \omega_2\) is considered in the following, from which

\[
\omega_1 = \omega_2 \Rightarrow \frac{k_1}{m_1} = \frac{k_2}{m_2} \Rightarrow \frac{k_2}{k_1} = \frac{m_2}{m_1} = \mu
\]

Hence, equations (11) and (12) become

\[
\frac{X_1}{X_{st}} = \frac{1 - \frac{\omega^2}{\omega_2^2}}{(1 + \mu - \frac{\omega^2}{\omega_2^2}) (1 - \frac{\omega^2}{\omega_2^2}) - \mu}
\]

(13)

\[
\frac{X_2}{X_{st}} = \frac{1}{(1 + \mu - \frac{\omega^2}{\omega_2^2}) (1 - \frac{\omega^2}{\omega_2^2}) - \mu}
\]

(14)

The amplitudes of vibration of both the structure and the tuned mass damper become infinite when the denominator, which is equal for both of them, is set equal to zero

\[
(1 + \mu - \frac{\omega^2}{\omega_2^2}) (1 - \frac{\omega^2}{\omega_2^2}) - \mu = 0
\]
\[
\left( \frac{\omega}{\omega_2} \right)^4 - \left( \frac{\omega}{\omega_2} \right)^2 (2 + \mu) + 1 = 0 \tag{15}
\]

Equation (15) is a quadratic equation in \( \omega^2 \) and admits two solutions of the form

\[
-b \pm \sqrt{b^2 - 4ac}
\]

\[
2a
\]

where \( a, b, \) and \( c \) are coefficients representing the physical parameters of the system, such as mass and stiffness. Thus,

\[
\left( \frac{\omega}{\omega_2} \right)^2 = \left( 1 + \frac{\mu}{2} \right) \pm \sqrt{\frac{\mu}{4} + \frac{\mu^2}{4}} \tag{16}
\]

and the two solutions in equation (16) that are the resonant frequencies of the system against the mass ratio \( \mu \) are plotted in Figure 3.2.

![Figure 3.2: Resonant Frequencies as a Function of the Mass Ratio \( \mu \) for \( \omega_1 = \omega_2 \)](image)

The response of a single-degree-of-freedom system is illustrated in Figure 3.3 to compare it with that of the SDOF-TMD system. The former approaches to infinity when the exciting frequency is equal to the natural frequency of the system, which is called resonance. However, the latter that is depicted in Figure 3.4, confirms that the amplitude of vibration of the primary structure is null when the frequency of excitation coincides with the natural frequency of the system. In addition, there are two frequencies at which the response is infinite because it is a two-degree-of-freedom system, and they are located at the two solutions in equation (16), obtained in Figure 3.2.
Figure 3.3: Amplitude of Vibration of an Undamped SDOF System Versus Frequency Ratio

Figure 3.4: Amplitude of Vibration of the Main Structure of an Undamped SDOF-TMD System Versus Frequency Ratio for a Mass Ratio $\mu = 0.2$
Similarly, Figure 3.5 shows that the tuned mass damper resonates at the same frequency ratios as the principal structure; however, its motion is not null when the frequency ratio is unity.

It should be noted that both of the amplitudes are plotted for a mass ratio $\mu = 0.2$, and the values possessing negative values in Figure 3.3, Figure 3.4, and Figure 3.5 are plotted on the positive side of the axis. In fact, this adjustment is made for the ease of interpreting the plots, and it is acceptable because a negative amplitude is equivalent to a positive one with a phase shift of 180 degrees. In practical terms, when the amplitude is negative, the mass moves in the opposite direction to the exciting force, but the concern here is to control the absolute value of the amplitude of vibration.

An undamped tuned mass damper attached to an undamped structure is, therefore, applicable if the externally applied force is monoharmonic (with circular frequency $\omega_0$) or lies within a narrow bandwidth in the vicinity of $\omega_0$ without having frequency components close to the resonance zones. However, it is not usually the case in civil structures, so damping is necessary to control the structural vibration.
3.1.2. Undamped SDOF Structure: Damped TMD

A damped dynamic vibration absorber is usually required when the external excitation acts in a wide frequency band. The system considered consists of the main structure of mass $m_1$ and stiffness $k_1$, subjected to harmonic excitation, and a tuned mass damper of mass $m_2$, stiffness $k_2$, and a dashpot connected in parallel to its spring with a damping constant $c_2$, as schematized in the drawing of Figure 3.6.

![Figure 3.6: Schematic of a Damped TMD Attached to an Undamped SDOF Structure](image)

Following the same reasoning made for the system in the previous subsection, the dashpot exerts inertia forces acting in the same directions of the spring forces with absolute values equal to $c_2\dot{x}_1$ and $c_2\dot{x}_2$, acting on $m_1$ and $m_2$, respectively. Hence, by applying the second law of Newton, the governing equations of motion read

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) + i\omega c_2(x_1 - x_2) = F\cos\omega t$$  \hspace{1cm} (17)

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) = 0$$  \hspace{1cm} (18)

The responses $x_1$ and $x_2$ are harmonic motions of frequency $\omega$ and are complex numbers, whereas the other quantities are real, but all of them can be represented by vectors. Thus, the equations become

$$-m_1\omega^2x_1 + k_1x_1 + k_2(x_1 - x_2) + i\omega c_2(x_1 - x_2) = F$$
- $m_2 \omega^2 x_2 + k_2 (x_2 - x_1) + i \omega c_2 (x_2 - x_1) = 0$

and they can be written in terms of $x_1$ and $x_2$ as

$$(- m_1 \omega^2 + k_1 + k_2 + i \omega c_2) x_1 - (k_2 + i \omega c_2) x_2 = F$$  \hspace{1cm} (19)

$$- (k_2 + i \omega c_2) x_1 + (- m_2 \omega^2 + k_2 + i \omega c_2) x_2 = 0$$  \hspace{1cm} (20)

where $i = \sqrt{-1}$; equation (20) can be written in the form of $x_2$ in terms of $x_1$

$$x_2 = \frac{(k_2 + i \omega c_2)}{(- m_2 \omega^2 + k_2 + i \omega c_2)} x_1$$

and by substituting $x_2$ into equation (19), it results

$$(- m_1 \omega^2 + k_1 + k_2 + i \omega c_2) x_1 - (k_2 + i \omega c_2) \left(\frac{(k_2 + i \omega c_2)}{(- m_2 \omega^2 + k_2 + i \omega c_2)} x_1 \right) = F$$  \hspace{1cm} (21)

$$x_1 = \frac{F}{(- m_1 \omega^2 + k_1)(- m_2 \omega^2 + k_2) - m_2 \omega^2 k_2] + i \omega c_2 [- m_1 \omega^2 + k_1 - m_2 \omega^2]}$$

$$x_2 = \frac{k_2 + i \omega c_2}{(- m_1 \omega^2 + k_1)(- m_2 \omega^2 + k_2) - m_2 \omega^2 k_2] + i \omega c_2 [- m_1 \omega^2 + k_1 - m_2 \omega^2]}$$  \hspace{1cm} (22)

Equations (21) and (22) have the form of

$$x_1 = \frac{F \cdot (A + iB)}{C + iD} = \frac{F \cdot (A + iB)}{C + iD} \frac{(C + iD)}{(C + iD)} = \frac{F \cdot (AC + BD) + i(BC - AD)}{C^2 + D^2}$$

and when $x_1$ is represented as a vector, its length$^1$ is

$$\frac{X_1}{F} = \sqrt{\left(\frac{AC + BD}{C^2 + D^2}\right)^2 + \left(\frac{BC - AD}{C^2 + D^2}\right)^2} = \frac{\sqrt{A^2 C^2 + B^2 D^2 + B^2 C^2 + A^2 D^2}}{(C^2 + D^2)^2} = \sqrt{\frac{(A^2 + B^2)(C^2 + D^2)}{(C^2 + D^2)^2}} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}$$

By applying this form to equation (21), the amplitude of motion of $m_1$ becomes

$$\frac{X_1^2}{F^2} = \frac{(k_2 - m_2 \omega^2)^2 + \omega^2 c_2^2}{[(m_1 \omega^2 + k_1)(- m_2 \omega^2 + k_2) - m_2 \omega^2 k_2] + \omega^2 c_2^2 [- m_1 \omega^2 + k_1 - m_2 \omega^2]}$$  \hspace{1cm} (23)

Now, some new variables are introduced to better handle this equation.

$$f = \frac{\omega_2}{\omega_1} = \text{tuning frequency ratio}$$

$$a_1 = \frac{\omega}{\omega_1} = \text{forced frequency ratio} = \text{frequency of excitation} / \text{natural frequency of the primary structure}$$

---

$^1$ A harmonic motion of the form: $a \sin \omega t + b \cos \omega t$, can be written in the form of a vector as: $\sqrt{a^2 + b^2} \sin (\cot + \phi)$, where $\sqrt{a^2 + b^2}$ is the amplitude and $\phi$ is the phase shift as $\tan \phi = b/a$
ξ_2 = \frac{c_2}{c_c} = \text{absorber damping ratio}

where c_c = 2m_2\omega_2 = \text{critical damping}, which is the minimum amount of damping required to bring a displaced system to its original position without oscillation.

Equation (23) can be written in a dimensionless form as

\[ \left| \frac{X_1}{X_{st}} \right| = \frac{(f^2 - a_1^2)^2 + (2a_1^\xi_2f)^2}{\sqrt{[(1 - a_1^2)(f^2 - a_1^2) - \mu f^2 a_1^2]^2 + [2a_1^\xi_2f(1 - a_1^2 - \mu a_1^2)]^2}} \]  (24)

Similarly, the amplitude of vibration of the tuned mass damper is

\[ \left| \frac{X_2}{X_{st}} \right| = \frac{f^4 + (2a_1^\xi_2f)^2}{\sqrt{[(1 - a_1^2)(f^2 - a_1^2) - \mu f^2 a_1^2]^2 + [2a_1^\xi_2f(1 - a_1^2 - \mu a_1^2)]^2}} \]  (25)

Figure 3.7 is a plot of equation (24) that represents the amplitudes of vibration of the primary structure. When the absorber damping ratio ξ_2 is zero, the amplitude of vibration is infinite, which resembles the case of Figure 3.4. Nevertheless, the main structure amplitude of vibration is also infinite when the absorber damping ratio ξ_2 approaches infinity because the two masses are rigidly connected, which transforms the system into a single-degree-of-freedom one. In addition, the work done by the dissipator is given by the force multiplied by the relative displacement between the two masses, but since this relative displacement is zero when they are rigidly connected, the work is null.

Figure 3.7: Vibration Amplitudes of the Main Structure Versus the Forced Frequency Ratio \( a_1 = \omega_1/\omega \) for a Mass Ratio \( \mu = 0.05 \) and Tuning Frequency Ratio \( f = 1 \), Associated to Various Absorber Damping Ratios ξ_2
Furthermore, the amplitude of vibration decreases when the absorber damping ratio $\xi_2$ increases, but if it is relatively high, the vibration amplitude is elevated again, and the two peaks are merged since the system is approaching the case of a single-degree-of-freedom system. Therefore, there must be a certain value of the damping ratio at which the vibration amplitude of the main structure is small, and thus, the dynamic vibration absorber must not be considered as an auxiliary system providing a damping force that completely counteracts the externally applied force. It is rather a system added to a structure to dissipate a lot of energy when working in a resonant condition by increasing the damping of the overall two-degree-of-freedom system, which ultimately results in suppressing the vibration amplitudes.

It is also noteworthy that all the curves, corresponding to different absorber damping ratios $\xi_2$, intersect at the two points $P$ and $Q$. Consequently, the best curve is the one passing through the highest of the two fixed points $P$ or $Q$ with a horizontal tangent, and the ordinate of this point is the best attainable resonant amplitude, corresponding to the optimum damping.

The abscissae of the two fixed points $P$ and $Q$ can be found by looking for the solutions $a_1$ when equating equation (24) once for an absorber damping ratio $\xi_2 = 0$ and another for $\xi_2$ approaching to infinity because both curves pass through the same two points.

$$\left| \frac{X_1}{X_{st}} \right|_{\xi_2=0} = \pm \frac{(f^2 - a_1^2)}{\left[(1 - a_1^2)(f^2 - a_1^2) - \mu f^2 a_1^2\right]}$$

$$\left| \frac{X_1}{X_{st}} \right|_{\xi_2=\infty} = \pm \frac{1}{(1 - a_1^2 - \mu a_1^2)}$$

Equating these two equations, when possessing the same sign, results in

$$\left| \frac{X_1}{X_{st}} \right|_{\xi_2=0} = \left| \frac{X_1}{X_{st}} \right|_{\xi_2=\infty} \Rightarrow \frac{(f^2 - a_1^2)}{\left[(1 - a_1^2)(f^2 - a_1^2) - \mu f^2 a_1^2\right]} = \frac{1}{(1 - a_1^2 - \mu a_1^2)}$$

$$(f^2 - a_1^2)(1 - a_1^2 - \mu a_1^2) = (1 - a_1^2)(f^2 - a_1^2) - \mu f^2 a_1^2$$

$$\mu a_1^2(f^2 - a_1^2) = \mu f^2 a_1^2 \Rightarrow f^2 - a_1^2 = f \Rightarrow a_1^2 = 0$$

which is the obvious solution that corresponds to the case of $\omega = 0$, at which the response is the static one $X_{st}$.

Equating the previous two equations, when possessing the opposite signs, results in

$$\left| \frac{X_1}{X_{st}} \right|_{\xi_2=0} = \left| \frac{X_1}{X_{st}} \right|_{\xi_2=\infty} \Rightarrow \frac{(f^2 - a_1^2)}{\left[(1 - a_1^2)(f^2 - a_1^2) - \mu f^2 a_1^2\right]} = -\frac{1}{(1 - a_1^2 - \mu a_1^2)}$$
\[-(f^2 - a_1^2) (1 - a_1^2 - \mu a_1^2) = (1 - a_1^2) (f^2 - a_1^2) - \mu f^2 a_1^2\]

\[a_1^4 - a_1^2 \frac{2 + 2 f^2 (1 + \mu)}{2 + \mu} + \frac{2 f^2 (1 + \mu)}{2 + \mu} = 0\]  

(26)

Furthermore, it is of interest to obtain two equal ordinates of the two fixed points \(P\) and \(Q\), and this can be achieved by changing the tuning frequency ratio \(f\). Accordingly, an optimum tuning frequency ratio \(f_{opt}\) shall be sought for. To do this, the amplitudes of the curve corresponding to an absorber damping ratio \(\xi_2\) approaching infinity is considered, and the roots of equation (26) are named \(a_{1,P}\) and \(a_{1,Q}\).

\[\frac{1}{1 - a_{1,P}^2 (1 + \mu)} = -\frac{1}{1 - a_{1,Q}^2 (1 + \mu)}\]

\[a_{1,P}^2 + a_{1,Q}^2 = \frac{2}{1 + \mu}\]  

(27)

Since the sum of the roots of a quadratic equation is equal to the negative coefficient of the middle term\(^2\), it can be written

\[a_{1,P}^2 + a_{1,Q}^2 = \frac{2 + 2 f^2 (1 + \mu)}{2 + \mu}\]  

(28)

and by equating (27) to (28), it is obtained

\[\frac{2}{1 + \mu} = \frac{2 + 2 f_{opt}^2 (1 + \mu)}{2 + \mu} \Rightarrow f_{opt}^2 = \frac{1}{1 + \mu} \left(\frac{2 + \mu}{1 + \mu} - 1\right) = \frac{1}{(1 + \mu)^2}\]

\[f_{opt} = \frac{1}{1 + \mu}\]  

(29)

Figure 3.8 is a plot of the optimum tuning frequency ratio obtained in (29) against the mass ratio \(\mu\). It can be concluded that the higher the mass of the tuned mass damper, the lower its frequency should be to obtain the optimum vibration amplitudes.

The vibration amplitudes of the main mass are plotted in Figure 3.9 against the forced frequency ratio \(a_1\) for various absorber damping ratios \(\xi_2\) and optimum tuning frequency ratio \(f_{opt} = 0.9524\). It is shown that the two fixed points \(P\) and \(Q\) have now the same ordinate, but the

---

\(^2\) The roots of a quadratic equation in the form of \(ax^2 + bx + c = 0\) are:

\[r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}\] and \[r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}\]

so, \[r_1 + r_2 = \frac{-2b}{2a} = \frac{-b}{a}\]

---
peaks of the curves do not coincide with them, so the next target is to set these peaks on $P$ and $Q$ that can be done by optimizing the absorber damping ratio $\xi_2$.

![Figure 3.8: Optimum Tuning Frequency Ratio Versus Mass Ratio](image)

![Figure 3.9: Vibration Amplitudes of the Main Structure Versus the Forced Frequency Ratio $a_1$ for a Mass Ratio $\mu = 0.05$ and Optimum Tuning Frequency Ratio $f_{opt} = 1/(1+\mu)$, Associated to Various Absorber Damping Ratios $\xi_2$](image)

The horizontal tangent at points $P$ is set to zero by substituting the optimal tuning frequency ratio $f_{opt}$ obtained in equation (29) into equation (24), differentiating with respect to $a_1$, and then setting this new equation equal to zero. The same process is done at point $Q$, and the resulting optimum absorber damping ratios are

\[ \Delta x = \frac{x}{x_{opt}} \]
\[ \xi_{2,p} = \frac{\mu(3 - \frac{\mu}{\mu+2})}{8f_{opt}^2(1+\mu)^3} \quad \text{and} \quad \xi_{2,Q} = \frac{\mu(3 + \frac{\mu}{\mu+2})}{8f_{opt}^2(1+\mu)^3} \]

An average value can be assumed as

\[ \xi_{2,\text{opt}} = \frac{3\mu}{8f_{opt}^2(1+\mu)^3} \] (30)

The results of the optimum absorber damping ratios are used to plot Figure 3.10, and these give the lowest amplitude of vibration possible, which is gotten by substituting one root of equation (26) into (24) that results in

\[ \left| \frac{X_1}{X_{st}} \right| = \sqrt{1 + \frac{2}{\mu}} \] (31)

![Figure 3.10: Vibration Amplitudes of the Main Structure Versus the Forced Frequency Ratio \( a_1 \) for a Mass Ratio \( \mu = 0.05 \) and Optimum Tuning Frequency Ratio \( f_{opt} = 1/(1+\mu) \), Associated to Optimum Absorber Damping Ratios \( \xi_{2,\text{opt}} \)]

3.1.3. Damped SDOF Structure: Damped TMD

In practical applications, it never exists a case where the primary system completely lacks damping, so this subsection studies a system consisting of a damped single-degree-of-freedom structure, having a damping constant \( c_1 \), and a damped dynamic vibration absorber.

In the previous two cases studied for undamped structures, the optimization parameters have been obtained based on minimizing the maximum amplitude magnification factor of the
primary structure, called the $H_\infty$ norm. This is useful for suppressing the vibrations of systems subjected to harmonic excitations like mechanical systems, where the resonant frequency is the one of interest to be damped out. However, building structures are typically subjected to random excitations, such as wind and earthquake loads, which contain many frequencies, so a different optimization criterion is usually followed. Accordingly, the total vibration energy of the system over all frequencies is reduced by minimizing the area under the frequency response curve, called the $H_2$ norm. A system subjected to random excitation $f(t)$ is depicted in Figure 3.11.

![Figure 3.11: Schematic of a Damped TMD Attached to a Damped SDOF Structure](image)

A closed-form algebraic solution for the $H_2$ optimization of a system subjected to random excitation has been found by (Asami, Nishihara, and Baz 2002). The performance index intended to be reduced is defined as the ratio of the response of the main system to the excitation force with a uniform power spectrum density $S_f$ and written in the dimensionless form as

$$I_1 = \frac{E[x_1^2]}{2\pi S_f \omega_1/k_1^2} = \frac{\langle x_1^2 \rangle}{2\pi S_f \omega_1/k_1^2}$$

(32)

where $E[x_1^2]$ and $\langle x_1^2 \rangle$ are the ensemble and temporal averages of the response of the main system, respectively, so

$$\langle x_1^2 \rangle = \frac{S_f}{k_1^2} \int_{-\infty}^{\infty} \left( \frac{X_1}{X_0} \right)^2 d\omega = \frac{S_f \omega_1}{k_1^2} \int_{-\infty}^{\infty} \left( \frac{X_1}{X_0} \right)^2 d\omega$$
When the integral of equation (33) is solved, it turns out to be function of $\mu, f, \xi_1$, and $\xi_2$, so for a predefined value to each of $\mu$ and $\xi_1$, the optimum parameters $f_{opt}$ and $\xi_{2, opt}$ can be obtained by solving the equations of the derivatives of $I_1$ with respect to $f$ and $\xi_2$ when they are set equal to zero. The results of the optimum parameters are as follows

\[ f_{opt} = \sqrt{-\frac{p_2}{2} - \frac{p_2^2}{4} - q_2} \]  \tag{34} 

\[ \xi_{2, opt} = \frac{1 - (1 + \mu)^2 f_{opt}^4}{4(1 + \mu) f_{opt}^4} \left[ 1 - \frac{2 f_{opt}^2 (1 + \mu)^2 f_{opt}^4}{4(1 + \mu) f_{opt}^4} \right] \]  \tag{35} 

where

\[ p_2 = \frac{1}{2} \left[ a_1 - \sqrt{a_1^2 - 4a_2 + 4y_1} \right], \quad q_2 = \frac{1}{2} \left[ y_1 - \frac{a_1 y_1 - 2a_3}{\sqrt{a_1^2 - 4a_2 + 4y_1}} \right] \]

\[ y_1 = \frac{a_2}{3} + 2\sqrt{-Q \cos \left( \frac{\theta}{3} \right)}, \quad \theta = \cos^{-1} \left( \frac{R}{Q} \right) \]

\[ Q = -\frac{1}{9} (a_2^2 - 3a_1a_3 + 12a_4), \quad R = \frac{1}{54} (2a_3^2 - 9a_1a_2a_3 + 27a_2^2 + 27a_3^2a_4 - 72a_2a_4) \]

\[
\begin{align*}
\frac{a_1}{b_0} &= \frac{b_1}{b_0}, \quad \frac{a_2}{b_0} = \frac{b_2}{b_0}, \quad \frac{a_3}{b_0} = \frac{b_3}{b_0}, \quad \frac{a_4}{b_0} = \frac{b_4}{b_0} \\
\frac{b_0}{b_0} &= 4(1 + \mu)^5 (1 + \mu - \xi_1^2) \\
\frac{b_1}{b_0} &= 4\mu(1 + \mu)^2 (1 + \mu - \xi_1^2) (1 + \mu - 4\xi_1^2) \\
\frac{b_2}{b_0} &= -4(1 + \mu)^2 (8 + 12\mu + 3\mu^2 - 8\xi_1^2) \\
\frac{b_3}{b_0} &= -2\mu(1 + \mu) (2 + \mu - 6\xi_1^2) \\
\frac{b_4}{b_0} &= (2 + \mu)^2 - 4(1 + \mu)\xi_1^2 
\end{align*}
\]

This subsection concludes the theoretical formulation of a single-degree-of-freedom structure housing a tuned mass damper. However, in real civil engineering applications, including buildings, structures are usually composed of many degrees of freedom, so the subsequent subsection is addressed for a system a multi-degree-of-freedom (MDOF) structure and a tuned mass damper.
3.1.4. Damped MDOF Structure: Damped TMD

Building structures typically comprise many degrees of freedom with small inherent damping of just a few percentiles, so the equations of motions of a multi-degree-of-freedom structure and tuned mass damper are studied and can be written in the matrix form as

$$ M\ddot{x} + C\dot{x} + Kx = F(t) \quad (36) $$

$$ m_d\ddot{q} + c_d\dot{q} + k_dq = 0 \quad (37) $$

where \( M \) is the mass matrix, \( C \) is the damping matrix, \( K \) is the stiffness matrix, and \( \ddot{x}, \dot{x}, \) and \( x \) are the acceleration, velocity, and displacement vectors of the MDOF structure, respectively. \( F(t) \) is the vector of the external forces applied to the system. The mass, damping coefficient, and stiffness of the tuned mass damper are represented by \( m_d, c_d, \) and \( k_d \), respectively, and \( \ddot{q}, \dot{q}, \) and \( q \) are the relative acceleration, velocity, and displacement, respectively, of the tuned mass damper with respect to the floor at which it is installed.

Indeed, a TMD can be tuned only to a single structural frequency (T. T. Soong and G. F. Dargush 1997), and it is typical to be tuned to the first fundamental mode of the structure. Thus, the modal coordinates are introduced by the following relationship

$$ x = \phi_1 x_1 \quad (38) $$

where \( \phi_1 \) is the first modal shape and \( x_1 \) is the displacement of the floor at which the tuned mass damper is installed. Substituting equation (38) into (36) and pre-multiplying it by \( \phi_1^T \), the transposed vector of \( \phi_1 \), the equation of motion becomes

$$ M_1 \ddot{x}_1 + C_1 \dot{x}_1 + K_1 x_1 = F_1(t) \quad (39) $$

where \( M_1 = \phi_1^T M \phi_1, C_1 = \phi_1^T C \phi_1 = M_1 \zeta_1 \omega_1, \) and \( K_1 = \phi_1^T K \phi_1 = M_1 \omega_1^2 \), \( F_1(t) = \phi_1^T F(t), \) and \( \zeta_1 \) and \( \omega_1 \) denote the damping ratio and the natural frequency of the first mode of the structure, respectively.

Equation (39) resembles the equation of motion of a single-degree-of-freedom system, so the optimization criteria adopted in the case of a damped SDOF structure and a damped TMD in the previous subsection can be used to define the optimum parameters of the TMD for the case of an MDOF system using the modal coordinates. Ultimately, the ratio of the relative contribution of the damping parameters to the total damping is defined as the equivalent damping ratio by (J J Connor 2002) as
\[
\xi_e = \frac{\mu}{2} \sqrt{1 + \left(\frac{2\xi_d}{\mu} + \frac{1}{2\xi_d}\right)^2}
\]  

(40)

where \(\xi_d\) denotes the damping ratio of the tuned mass damper, and the optimized value \(\xi_{d,\text{opt}}\) is to be used.

### 3.2. Pendulum Tuned Mass Dampers

This monograph puts emphasis on tuned mass dampers attached to very tall buildings, which usually experience high lateral displacement at the top floors. Hence, a convenient type of tuned mass dampers used in this kind of structures is the pendulum tuned mass dampers.

The equation of motion for the horizontal direction is written by (J J Connor 2002) as

\[
T \sin \phi + m_2(\ddot{x}_1 + \ddot{x}_2) = 0
\]

(41)

where \(T\) is the tension force in the cable produced by the relative motion of the tuned mass damper with respect to the supporting structure, \(L\) is the length of the cable, and \(\phi\) is the angle of the displaced pendulum, as shown in Figure 3.12.

![Figure 3.12: Schematic of a Pendulum Tuned Mass Damper](image)

The pendulum typically extends over several floors, so its length is relatively larger than the lateral displacement of the mass. Thus, the angle of the pendulum \(\phi\) is small, and consequently, \(\sin \phi = \phi\) can be considered as an acceptable approximation to linearize the equation of motion, so
\begin{equation}
\begin{aligned}
    x_2 &= L \sin \phi = L \dot{\phi} \\
    T \sin \phi &= T \dot{\phi} = m_2 g
\end{aligned}
\end{equation}

(42)

where g is the gravitational acceleration. Equation (41) is modified by introducing (42) into it, and it becomes

\begin{equation}
    m_2 \ddot{x}_2 + \frac{m_2 g}{L} x_2 = - m_2 \ddot{x}_1
\end{equation}

(43)

Therefore, an equivalent shear spring stiffness can be defined from the equation of motion of (43) as

\begin{equation}
    k_{eq} = \frac{m_2 g}{L}
\end{equation}

(44)

which can be used to define the natural circular frequency of the pendulum that reads

\begin{equation}
    \omega_2^2 = \frac{k_{eq}}{m_2} = \frac{g}{L}
\end{equation}

(45)

The natural period of the pendulum is related to the natural circular frequency by

\begin{equation}
    T_2 = \frac{2\pi}{\omega_2} = 2\pi \sqrt{\frac{L}{g}}
\end{equation}

(46)
Chapter 4: Eddy Current Damping

Eddy current damping has been used in many different applications, especially in mechanical and aerospace engineering, whereas its application in civil engineering is still somewhat limited due to its low density of energy dissipation. Nonetheless, several improvements have been made on the configuration of the damping component and the magnetic strength of permanent magnets to increase the dissipated energy. In fact, the damping force is a function of velocity; however, the velocity of civil structures under dynamic loadings is typically small, so a proper application of the eddy current damping is to use it as a damping component of a dynamic vibration absorber that experiences resonance and thus high amplitudes of vibration.

An eddy current damper can be designed using different configurations, where the simplest form consists of a conductive plate and a set of magnets. Eddy currents are generated, according to Faraday’s law of induction, either by a stationary conductive plate with respect to the magnets that have a variable magnetic field strength, or by relative motion between the conductive plate and the magnets that have a stationary magnetic field strength, as shown in Figure 1.1. The former can be used as a damping element for an active eddy current tuned mass damper whereas the latter for a passive one, which is the subject of this manuscript. Lenz’s law defines the direction of the induced electromagnetic force as the opposite of the change in the original magnetic field. Therefore, from the structural engineering wise, this electromagnetic force functions as a damping force that dissipates energy from the structure as heat in the conducting plate.

4.1. Analytical Modeling

The magnets can have different shapes, and the analytical model used to describe the behavior of the eddy current damper depends on their shape. In this study, rectangular magnets are used, and the theoretical formulation is presented accordingly. (Wang, Chen, and Wang 2012) studied different configurations of eddy current dampers, using finite element analysis, to check which one gives the optimum results. They concluded that an alternative orientation of magnets’ poles results in a smaller magnetic circuit because when the magnetic poles are placed in the same orientation, each magnet works independently. In addition, they proved that the use of a steel plate, called secondary steel plate, of high magnetic permeability beneath the
Conductive plate increases the magnetic flux around the latter by pulling the magnetic flux away from the magnets. Consequently, (Z. W. Huang et al. 2018) developed an analytical model, based on the charge model, considering the improvements done by (Wang, Chen, and Wang 2012) but with an additional steel plate above the magnets, called primary steel plate, which results in a further shortening of the magnetic circuit. This model is adopted in this study and demonstrated in Figure 4.1, where the excitation direction is in the x-direction, and the magnetization direction is in the z-direction through the thickness of the permanent magnets. The orange color indicates the north pole (N) of a permanent magnet, while the blue color indicates the south pole (S).

Figure 4.1: Eddy Current Damper Configuration: (a) Elevation View; (b) Plan View

The velocity vector can be written as

\[ \mathbf{v} = v_x \mathbf{i} + 0 \mathbf{j} + 0 \mathbf{k} \]  

(47)
4.1.1. Magnetic Flux Density

The magnetic flux density reads

\[ \mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \]  \hspace{1cm} (48)

where the magnetic flux density at any point \( A(x, y, z) \) in space due to the positive surface charge of a single permanent magnet is given by (Z. W. Huang et al. 2018) as

\[ \overline{B}_x(x, y, z) = \frac{B_r}{4\pi} \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{m+n} \ln(-Q + T) \]  \hspace{1cm} (49)

\[ \overline{B}_y(x, y, z) = \frac{B_r}{4\pi} \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{m+n} \ln(-P + T) \]  \hspace{1cm} (50)

\[ \overline{B}_z(x, y, z) = \frac{B_r}{4\pi} \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{m+n} \arctan\left(\frac{QP}{RT}\right) \]  \hspace{1cm} (51)

where \( B_r \) is the remanent flux density and

\[ P = x - x_n \]  \hspace{1cm} (52)

\[ Q = y - y_m \]  \hspace{1cm} (53)

\[ R = z - z_k \]  \hspace{1cm} (54)

\[ T = \sqrt{P^2 + Q^2 + R^2} \]  \hspace{1cm} (55)

where \( x_n, y_m, \) and \( z_k \) are the coordinates of the corners of the permanent magnets, and the coefficients \( m \) and \( n \) are associated to the corners of the permanent magnets, according to Figure 4.2.

\[ \text{Figure 4.2: Definition of the Coefficients } m \text{ and } n \]
4.1.2. Method of Images

When a material characterized by a high permeability is placed near to a magnetic source, the method of images allows to replace it by a number of image sources that restore the boundary conditions at the interface (Furlani 2001). Thus, the method of images is used in this model to represent the effect of the primary and the secondary steel plates by replacing them by a series of images, provided that they are thick enough. As a result, the magnetic flux density of a single permanent magnet becomes the sum of the generated magnetic field by its magnetic charges and their images. Therefore, the total magnetic flux density of a number of magnets is the superposition of the magnetic fields of all the magnets, which is given by

\[
B_x(x, y, z) = \frac{B_z}{4\pi} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} (-1)^{m+n+k+j} \ln(-Q+T) 
\]

(56)

\[
B_y(x, y, z) = \frac{B_z}{4\pi} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} (-1)^{m+n+k+j} \ln(-P+T) 
\]

(57)

\[
B_z(x, y, z) = \frac{B_z}{4\pi} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} (-1)^{m+n+k+j} \arctan\left(\frac{QR}{RT}\right)
\]

(58)

where \(j\) is a counter on the number of magnets, and \(n_m\) is the total number of magnets. The number of images is \(k\), and the recommended number by (Z. W. Huang et al. 2018) is 5 to 10.

4.1.3. Eddy Current Density

The induced eddy currents in the conductive plate can be formulated as

\[
J = \sigma_c (v \times B)
\]

(59)

where \(\sigma_c\) is the electrical conductivity of the conductive plate.

Hence, the induced eddy currents in the x and y directions, neglecting the eddy currents in the z-direction due to the low frequency and speed, are written as

\[
J_x(x, y, z) = v_x \sigma_c \frac{B_z}{4\pi} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} (-1)^{m+n+k+j} \ln(R+T) 
\]

(60)

\[
J_y(x, y, z) = v_y \sigma_c \frac{B_z}{4\pi} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} (-1)^{m+n+k+j} \arctan\left(\frac{-QR}{PT}\right)
\]

(61)
4.1.4. Damping Force and Coefficient

The induced electromagnetic force or the eddy current damping force is given by (Sodano et al. 2005) as the integral over the volume \((V)\)

\[
F = \iiint_{V} J \times B \, dV \quad (62)
\]

However, since the vibration direction is in the x-direction, the x-component of the damping force is considered as

\[
F = \iiint_{V} J_y \times B_z \, dV \quad (63)
\]

and thus, the damping coefficient of the eddy current tuned mass damper reads

\[
c_d = \frac{F}{v_x} \quad (64)
\]

Therefore, the damping ratio of the eddy current tuned mass damper can be obtained by

\[
\xi_d = \frac{c_d}{2m_d\omega_d} \quad (65)
\]
Chapter 5: Case Study, Taipei 101 Tower

An eddy current tuned mass damper (ECTMD) is studied in this chapter to control the wind-induced vibrations of the Taipei 101 Tower (Figure 5.1), according to the human comfort criteria. Taipei 101 is a super-tall building reaching 509 m in height, and it actually was the tallest building in the world at the time of its completion in 2004 in Taiwan. It consists of eight canted modules bearing on a truncated pyramidal base and a pinnacle at the top.

![Figure 5.1: Taipei 101 Tower (Poon et al. 2004)](image)

The tower is well designed in terms of its aerodynamic shape to confuse the vortex shedding phenomenon induced by wind excitations, and this is attained by the double notch or saw tooth corners and the setbacks that are formed by starting the narrow base of a module from the wide head of the beneath one, as shown in Figure 5.1.

The structural system is a mega-frame that consists of a central braced core connected to outer super-columns via belt trusses, which are one to three storey deep, as shown in Figure 5.2, Figure 5.3, and Figure 5.4 from different views. The structural material used is mainly
structural steel, but concrete was also used up to a certain height to increase the overall stiffness. Since structural steel was the main material used, the inherent damping of the structure was relatively low. Thus, the structural damping ratio is assumed to be 1%, which is compliant to the field measurements performed by (Li et al. 2011), who recommended a range of 1.0-2.0% for the low vibration amplitude range.

Figure 5.2: Level 10 - Tower Framing Plan (Shieh, Chang, and Jong 2003)

Figure 5.3: Level 32 - Tower Framing Plan (Shieh, Chang, and Jong 2003)
Figure 5.4: Taipei 101 - Elevation View of a Perimeter Moment Frame Line Showing the Belt Trusses. The Shaded Elements Indicate the Concrete Filled Super-columns (Poon et al. 2004)
Although Taipei 101 has an optimized aerodynamic shape and an efficient structural system, the Taiwanese occupant comfort criteria were initially not met, which set the limit of acceleration at the top occupied floor to 5.0 cm/s² or 5.1 milli-g for a 6-month wind return period (Lago, Trabucco, and Wood 2019). Consequently, a tuned mass damper was designed to lessen the acceleration amplitudes utilizing eight traditional velocity-squared viscous damping devices.

5.1. Numerical Model

An equivalent ten-degree-of-freedom model is used in this study to design the eddy current tuned mass damper, developed by (Jerome J Connor and Kourakis 2007), as illustrated in Figure 5.5: Ten-Degree-of-Freedom Model of Taipei 101. The mass of the structure is lumped into ten point masses located at the outrigger levels of the mega-frame and rotational masses at the same levels. The stiffness is modeled by
elastic elements simulating the overall structural stiffness and forming a cantilever beam of variable stiffness with the height. The modeled structure is analyzed in SAP2000 software (Computer & Structures 2017), and the masses and stiffness values are listed in Table A.1 and Table A.2 in Appendix A.

This model accurately captures the fundamental frequency of the structure and to a lesser extent the higher modes, but the tuned mass damper is designed for the frequency of the first mode. The fundamental frequency obtained by the model is roughly 0.146 Hz that is in agreement with the full-scale measurements performed by (Tuan and Shang 2014). Figure 5.6 plots the modal shapes of the first three modes, and Table 5.1 compares the natural frequencies obtained from the model with those obtained from full-scale measurements, where (x) represents here one of the transversal directions of the building, which is considered as the vibration direction to be controlled by the TMD since the structure is not exactly symmetric in plan in terms of the natural frequencies, where there are tiny differences in their values.

![Modal Shapes](image)

Figure 5.6: The First Three Modal Shapes in the Transversal Direction

<table>
<thead>
<tr>
<th>Modal frequency [Hz]</th>
<th>1st Mode (x)</th>
<th>2nd Mode (x)</th>
<th>3rd Mode (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.14592</td>
<td>0.49218</td>
<td>0.82586</td>
</tr>
<tr>
<td>Full-scale measurement</td>
<td>0.146</td>
<td>0.435</td>
<td>0.772</td>
</tr>
</tbody>
</table>

Table 5.1: Comparison between Modal Frequencies Obtained from the Model and Full-scale Measurements
5.2. Wind Analysis

A tall building is usually characterized by a high period, so the effect of wind is expected to be more prominent than that of earthquakes. The design of Taipei 101 required the aid of a tuned mass damper to attenuate the wind-induced accelerations because of its high slenderness. Hence, the wind analysis of Taipei 101 is elaborated in this section, according to the Italian wind code (Consiglio Nazionale delle Ricerche 2008).

5.2.1. Scruton Number

Scruton number of the first mode is defined as

\[ S_{c1} = \frac{4\pi \cdot m_{e,1} \cdot \xi_1}{\rho \cdot b^2} \]  \hspace{1cm} (66)

where \( \xi_1 = \) structural damping ratio = 1%

\( \rho = \) density of air = 1.25 Kg/m³

\( m_{e,1} = \) equivalent mass per unit length corresponding to the first mode and defined by

\[ m_{e,1} = \frac{\int_0^l m(s) \cdot \varphi_1^2(s) ds}{\int_0^l \varphi_1^2(s) ds} \]  \hspace{1cm} (67)

where \( m(s) \) is the structural mass per unit length = total mass/structural height

\( l = \) height of the structure

Table 5.2: Equivalent Mass Per Unit Length Corresponding to the First Mode, where \( l \) is the Tributary Height of Each Point Mass

<table>
<thead>
<tr>
<th>Mass</th>
<th>m [t]</th>
<th>( \varphi_1 )</th>
<th>( m_s [t] )</th>
<th>( l [m] )</th>
<th>( \Phi_1^2 \cdot l )</th>
<th>( m_s \Phi_1^2 \cdot l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m₁</td>
<td>96865.38</td>
<td>0.066524</td>
<td>553.5402</td>
<td>58.8</td>
<td>0.260213</td>
<td>144.0383</td>
</tr>
<tr>
<td>m₂</td>
<td>21759.62</td>
<td>0.20118</td>
<td>553.5402</td>
<td>50.4</td>
<td>2.039864</td>
<td>1129.147</td>
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<td>m₃</td>
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<td>553.5402</td>
<td>33.6</td>
<td>2.75243</td>
<td>1523.581</td>
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<td>m₄</td>
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<td>21759.62</td>
<td>0.477736</td>
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<td>33.6</td>
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<td>m₇</td>
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<td>0.685891</td>
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<td>m₈</td>
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<td>553.5402</td>
<td>16.8</td>
<td>16.8</td>
<td>9299.475</td>
</tr>
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<td>sum</td>
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<td></td>
<td></td>
<td></td>
<td>109.7564</td>
<td>60754.58</td>
</tr>
</tbody>
</table>

\( m_{e,1} [t/m] = 553.5402 \)
The equivalent mass per unit length corresponding to the first mode is calculated in Table 5.2, and the Scruton number is then obtained in Table 5.3. The Strouhal number is approximated as 0.12 for a square shape by (Consiglio Nazionale delle Ricerche 2008).

5.2.2. Critical Wind Velocity

The critical wind velocity due to vortex shedding can be obtained from equation (1) as

\[ v_{cr,1} = \frac{S_c n_{L,1}}{b} \]  \hspace{1cm} (68)

The design wind speed with 50 years return period \( (T_R) \) is \( v_b = 43.27 \) m/s, according to (S. Huang and Li 2010). The aerodynamic parameters, including Scruton number and Strouhal number, and the critical velocity due to vortex shedding are tabulated in Table 5.3.

<table>
<thead>
<tr>
<th>( n_{L,1} ) [Hz]</th>
<th>( S_c )</th>
<th>( b ) [m]</th>
<th>( v_{cr,1} ) [m/s]</th>
<th>( S_{st,1} )</th>
<th>( v_b ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.146</td>
<td>0.12</td>
<td>45.5</td>
<td>55.35833</td>
<td>26.87981</td>
<td>43.27</td>
</tr>
</tbody>
</table>

5.2.3. Mean Wind Velocity

The 10-minute mean wind speed at 10 m height above ground is

\[ v_r = v_b \cdot c_r \]  \hspace{1cm} (69)

where \( c_r \) is the return coefficient provided by

\[ c_r = 0.75 \quad \text{for} \ T_R = 1 \ \text{year} \]

\[ c_r = 0.75 \sqrt{1 - 0.2 \cdot \ln \left[ -\ln \left( 1 - \frac{1}{T_R} \right) \right]} \quad \text{for} \ 1 \ \text{year} \leq T_R < 5 \ \text{years} \]

and two different coefficients \( c_r \) are needed, such that one corresponds to a return period of \( T_R = T_{R,0} = 6 \) months to comply with the occupant comfort criteria of the Taiwanese code, and another corresponds to \( T_R = 10T_{R,0} = 5 \) years to be used for computing the equivalent transversal loads due to vortex shedding. Thus, these values are computed with the associated mean wind speed and tabulated in Table 5.4.

<table>
<thead>
<tr>
<th>( T_R ) [years]</th>
<th>( c_r )</th>
<th>( v_r ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{R,0} )</td>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>( 10T_{R,0} )</td>
<td>5</td>
<td>0.975</td>
</tr>
</tbody>
</table>
The mean wind velocity function of height is

\[ v_m(z) = v_r \cdot c_m(z) \] (70)

where \( c_m \) is the average wind profile coefficient provided by

\[
c_m(z) = k_r \ln \left( \frac{z_{\text{min}}}{z_0} \right) \cdot c_t(\frac{z_{\text{min}}}{z_0}) \quad \text{for } z \leq z_{\text{min}}
\]

\[
c_m(z) = k_r \ln \left( \frac{z}{z_0} \right) \cdot c_t(z) \quad \text{for } z > z_{\text{min}}
\]

but \( z_{\text{min}} \), \( z_0 \) and \( k_r \) provided in the Italian wind code are related to the Italian territory, so they are obtained from the European wind standard (CEN (European Committee for Standardization) 2004), and shown in Table 5.5.

where \( k_r \) is the terrain factor calculated as

\[ k_r = 0.19 \left( \frac{z_0}{0.05} \right)^{0.07} \]

Table 5.5: Mean Wind Velocity Coefficients

<table>
<thead>
<tr>
<th>( z_0 )</th>
<th>( z_{\text{min}} )</th>
<th>( c_t )</th>
<th>( k_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.234329</td>
</tr>
</tbody>
</table>

The wind velocity profile representing \( v_m(z) \) is plotted in Figure B.1 in Appendix B.

5.2.4. Equivalent Transversal Forces

The effect of the transversal vibrations induced by the resonant detachment of the vortices on the first mode can be represented by the application of equivalent static transversal loads applied per unit length in the crosswind direction in the form of

\[
F_{L,1}(s) = m(s) \cdot (2\pi \cdot n_{L,1})^2 \cdot \phi_{L,1}(s) \cdot y_{pL,1} \cdot C_{TR,1}
\] (71)

where

\( s = \) structural coordinate with height

\( m(s) = \) mass of the structure per unit of length

---

3 Equation (70) is applicable to structures of height below 200 m. Although the height of Taipei 101 is significantly beyond this limit, this equation is used in this thesis due to the limited data available, and it is expected to be on the conservative side.
\( n_{L,1} \) = natural frequency of the first mode of vibration in the transverse direction

\( \phi_{L,1}(s) \) = modal shape of the first mode of vibration in the transverse, normalized to 1 at the level at which the TMD is attached

\( y_{pL,1} \) = peak value of the transverse displacement of the structure, evaluated at the level at which the TMD is attached

\( C_{TR,1} \) = dimensionless parameter linked to the occurrence of critical values of the mean wind velocity for high return periods \( T_R \), given by

\[
C_{TR,1} = \begin{cases} 
1 & \text{for } v_{cr,1} \leq v_{m,0} \\
\frac{v_{m,1} - v_{cr,1}}{v_{m,0} - v_{m,0}} & \text{for } v_{m,0} \leq z \leq v_{cr,1} \\
0 & \text{for } v_{m,l} \leq v_{cr,1}
\end{cases}
\]

where \( v_{m,0} \) = mean wind velocity evaluated with height and associated to the return period \( T_R = T_{R,0} \)

\( v_{m,l} \) = mean wind velocity evaluated with height and associated to the return period \( T_R = 10T_{R,0} \)

The peak value of the transverse displacement of the structure is evaluated as

\[
y_{pL,1} = g_L \cdot \sigma_L
\]

where \( g_L \) = peak factor of the transverse displacement

\( \sigma_L \) = standard deviation of the transverse displacement

- The peak factor of the transverse displacement is defined as

\[
g_L = \sqrt{2} \left\{ 1 + \left[ \arctan \left( 0.7 \left( \frac{S_e}{4\pi K_a} \right)^{2.5} \right) \right]^{1.4} \right\}
\]

\[
K_a = K_{a,max} \cdot C_I
\]

- The standard deviation of the transverse displacement is defined as

\[
\sigma_L = b \sqrt{c_1 + \sqrt{c_1^2 + c_2}}
\]

\[
c_I = \frac{a_1^2}{2} \left( 1 - \frac{S_e}{4\pi K_a} \right)
\]
\[ c_2 = \frac{a_l^2 \rho h^3}{K_a m_e h S_i^2} \]

where \( C_c \) = dimensionless parameter function of the shape of the cross section

\( h = \) height of the structure

\( a_L = \) normalized limit size = \( \frac{h}{500} \)

Accordingly, the peak value of the transverse displacement of the structure is shown in Table 5.6 with all the related coefficients.

Table 5.6: The Peak Value of the Transverse Displacement

<table>
<thead>
<tr>
<th>( c_1 )</th>
<th>( k_{a, \text{max}} )</th>
<th>( k_a )</th>
<th>( g_1 )</th>
<th>( c_c )</th>
<th>( a_1 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( \sigma_1 ) [m]</th>
<th>( y_{pL,1} ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>6</td>
<td>4.2</td>
<td>1.4945</td>
<td>0.04</td>
<td>0.022374</td>
<td>0.000123</td>
<td>3.3E-07</td>
<td>1.212</td>
<td>1.812</td>
</tr>
</tbody>
</table>

Therefore, the equivalent transversal forces can be computed along the height, and the force profile is plotted in Figure B.2, in Appendix B. It is noted that the forces are null until a certain height because the velocity is lower than the critical velocity of vortex shedding, so the vortex shedding phenomenon does not take place below that level.

5.2.5. TMD Optimized Parameters

The mass of the dynamic vibration absorber used is similar to that of the existing tuned mass damper in Taipei 101 of 660 tons (Tuan and Shang 2014) for comparative purposes. Thus, the mass ratio is defined, based on the theoretical formulation in subsection 3.1.4, as

\[ \mu = \frac{m_d}{M_1} = \frac{660}{70828.71} = 0.009318 \]

where \( M_1 \) = the modal mass of the first mode = \( \phi_1^T M \phi_1 \), and \( \phi_1 \) is indicated in Table 5.2.

The optimized parameters of the tuned mass damper are obtained by following subsection 3.1.3, where \( \zeta_{d, \text{opt}} \) is the optimal damping ratio of the TMD, obtained by equation (35). Therefore, 4.81% of damping is required for the TMD that brings equivalent damping of 5.86%, as tabulated in Table 5.7.

Table 5.7: TMD Optimum Parameters

<table>
<thead>
<tr>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( k_{eq} ) [KN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.228494</td>
<td>0.038663</td>
<td>-8.26315</td>
<td>-0.03778433</td>
<td>4.036956</td>
<td>0.009143</td>
<td>-1.95416</td>
<td>-0.00894</td>
<td>0.954703</td>
<td>547.214</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( R )</th>
<th>( y_1 )</th>
<th>( \phi )</th>
<th>( p_2 )</th>
<th>( q_2 )</th>
<th>( f_{opt} )</th>
<th>( \xi_{d, \text{opt}} )</th>
<th>( \xi_e )</th>
<th>( L ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.69727</td>
<td>2.211189</td>
<td>1.954199</td>
<td>3.332E-08</td>
<td>-1.97239</td>
<td>0.97258</td>
<td>0.992599</td>
<td>0.048098</td>
<td>0.058619</td>
<td>11.832</td>
</tr>
</tbody>
</table>
5.2.6. Wind-induced Vibrations

The equivalent transversal forces obtained are applied in the finite element model in the crosswind direction as a long time history load case, having the same period of the first mode of vibration, and the target is to find the acceleration that must be within 5 milli-g after the installation of the optimized tuned mass damper. The result of the finite element analysis of the structure with 1% inherent damping without incorporating the tuned mass damper is 0.304 m/s², which corresponds to 31 milli-g and is shown in Figure 5.7. Hence, the level of acceleration is significantly beyond the comfort limit.

![Figure 5.7: Structural Acceleration without the TMD](image)

When the tuned mass damper with the optimized parameters is installed into the structure, it brings the acceleration of the latter into 0.019 m/s² (Figure 5.8), or roughly 2 milli-g, because of the increased damping the TMD inputs to the structure that reached around 5.86% for this designed device. The obtained result complies with the occupant comfort criteria of the Taiwanese code of 5 milli-g, and this proves the effectiveness of using a tuned mass damper to control the vibrations of Taipei 101.

The velocity of the tuned mass damper is needed to obtain the eddy current density of the eddy current damper, so this velocity is depicted in Figure 5.9. The obtained steady-state velocity of the TMD is 0.66 m/s.
It shall be noted that the finite element model used in this study is a simplified equivalent model of the full 3D model, and the transversal wind forces generated by vortex shedding are obtained based on the Italian wind standards. Hence, the obtained results of accelerations and velocity are expected to be somewhat different than the ones obtained by the wind tunnel test carried out by Taipei 101 designers.
5.3. Response Spectrum Analysis

Taipei 101 tower is located in a zone moderate of moderate seismicity, and the design response spectrum is given by (Fan et al. 2009), as depicted in Figure 5.10. The structural damping is assumed to be 5% for all the considered modes.

![Image of Design Response Spectrum](image)

Figure 5.10: Design Response Spectrum of Taipei 101 (Fan et al. 2009)

The base shear and moment for each model with and without incorporating the tuned mass damper are listed in Table 5.8, in addition to the reduction gained in these values by the presence of the tuned mass damper. As it is expected, the benefits gained in reducing the seismic base reactions are not notable, where the difference in the base shear is negligible, and that of the base moments barely reaches 8%. Hence, it is confirmed that the presence of the tuned mass damper in this structure does not bring precious results in terms of the seismic behavior.

Table 5.8: Seismic Base Reactions

<table>
<thead>
<tr>
<th>Model</th>
<th>Base Shear [KN]</th>
<th>Base Moment [KNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without TMD</td>
<td>202212.33</td>
<td>12947009.68</td>
</tr>
<tr>
<td>With TMD</td>
<td>201271.748</td>
<td>11908115.36</td>
</tr>
<tr>
<td>Reduction</td>
<td>0.47</td>
<td>8.02</td>
</tr>
</tbody>
</table>

5.4. Eddy Current Tuned Mass Damper Design

Neodymium Iron Boron (NdFeB) rare earth permanent magnets are used in this study due to their very high strength. This type of permanent magnets is classified into different grades, and due to availability and economy, grade N35 is used to design the ECTMD. The
permanent magnets are assumed to be uniformly and linearly magnetized, so they can be replaced by two surface charges sheets. This aspect strongly depends on the geometric size of permanent magnets, the thick-to-length ratio of the magnets, and the air-gap length between the magnets and the conductive plate, as has been proven by (Gou, Yang, and Zheng 2004). Hence, these considerations are taken into account in this design, and the different design parameters of the ECTMD are tabulated in Table 5.9. Moreover, (Wang, Chen, and Wang 2012) studied two different materials for the conducting plate that are copper and aluminum, and they proved that copper behaves better than aluminum for such a damper, so copper is used here as the material of the conducting plate.

### Table 5.9: Design Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_m$</td>
<td>length of the permanent magnet</td>
<td>100 mm</td>
</tr>
<tr>
<td>$w_m$</td>
<td>width of the permanent magnet</td>
<td>100 mm</td>
</tr>
<tr>
<td>$h_m$</td>
<td>thickness of the permanent magnet</td>
<td>25 mm</td>
</tr>
<tr>
<td>$d$</td>
<td>end-to-end spacing of the permanent magnets</td>
<td>10 mm</td>
</tr>
<tr>
<td>$B_c$</td>
<td>length of the conductive plate</td>
<td>4000 mm</td>
</tr>
<tr>
<td>$W_c$</td>
<td>width of the conductive plate</td>
<td>4000 mm</td>
</tr>
<tr>
<td>$h_c$</td>
<td>thickness of the conductive plate</td>
<td>18.5 mm</td>
</tr>
<tr>
<td>$h_b$</td>
<td>thickness of the primary steel plate</td>
<td>40 mm</td>
</tr>
<tr>
<td>$h_f$</td>
<td>thickness of the secondary steel plate</td>
<td>40 mm</td>
</tr>
<tr>
<td>$h_g$</td>
<td>air-gap length between the magnets and the plate</td>
<td>12 mm</td>
</tr>
<tr>
<td>$B_r$</td>
<td>remanent flux density of the permanent magnets</td>
<td>1.2 T</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>conductivity of the conductive plate (copper)</td>
<td>$5.8 \times 10^7$ S/m</td>
</tr>
</tbody>
</table>

The design process is iterative in order to reach the required damping ratio of the tuned mass damper. Different design typologies were studied using different numbers, sizes, and grades of the permanent magnets, different thicknesses of the conductive plates and the steel plates, and different air-gap lengths. A proper design using 100 NdFeB grade N35 rare earth permanent magnets with geometric dimensions of 100 mm × 100 m × 25 mm is proved to be sufficient to bring the damping ratio of the tuned mass damper into 4.81%, which is in full agreement with the optimum damping ratio obtained in Table 5.7. This is the result of applying the analytical model of section 4.1. Analytical Modeling in MATLAB software.

Figure 5.11 and Figure 5.12 illustrate the elevation view and the plan view, respectively, of the eddy current tuned mass damper layout, where the dimensions are in millimeters (mm), but the copper plate and the secondary steel plate are not drawn into scale. The magnets are
spaced at 10 mm from end-to-end. The north pole of a permanent magnet is indicated by the orange color whereas the north pole by the blue color. It is noteworthy that the magnets and the primary steel plate are attached to the mass component of the TMD and constitute part of its mass.

![Diagram of ECTMD Layout Elevation View](image1)

**Figure 5.11: ECTMD Layout Elevation View**

![Diagram of ECTMD Layout Plan View](image2)

**Figure 5.12: ECTMD Layout Plan View**

The obtained magnetic field and the eddy current density of the 100 permanent magnets are shown in Figure 5.13 and Figure 5.14, respectively.
A feasibility study for the proposed design of the eddy current tuned mass damper is performed and compared to the existing viscous tuned mass damper of Taipei 101. Concerning the magnets, two different thicknesses and grades of the permanent magnets are studied, and it is proven by the available data of the prices from magnet producers that the chosen 100 magnets with a thickness of 25 mm and grade N35 are significantly more economic than 64 magnets of with a thickness of 50 mm and grade N52.
Although the mass of the magnets and the mass of the primary steel plate constitute a small portion of the massive steel sphere of Taipei 101, they are subtracted from the 660 tons total mass of the TMD. The full cost analysis is listed in Table 5.10, which includes the cost of the steel mass, the permanent magnets, the copper plate, and the steel plates. This analysis does not include the cost of the frame supporting the mass, the cables, and any other required equipment, but the included elements constitute the major cost of the whole device. It can be concluded, based on the real-time material prices, that the proposed eddy current tuned mass damper is remarkably more economic than the existing viscous fluid TMD that has a cost of US$4 million.

Table 5.10: Cost Analysis

<table>
<thead>
<tr>
<th>Neodymium Permanent Magnets NdFeB</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade b mm</td>
<td>w mm</td>
</tr>
<tr>
<td>N35</td>
<td>100</td>
</tr>
<tr>
<td>N52</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Copper Plate 4000x4000x20 mm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>4000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary Steel Plate 1100x1100x40 mm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B₀ [mm]</td>
<td>W₀ [mm]</td>
</tr>
<tr>
<td>1100</td>
<td>1100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Secondary Steel Plate 4000x4000x40 mm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁ [mm]</td>
<td>W₁ [mm]</td>
</tr>
<tr>
<td>4000</td>
<td>4000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TMD</th>
<th>Magnets</th>
<th>Steel Plate</th>
<th>Remaining Mass</th>
<th>Cost of the Remaining Mass</th>
<th>Damping component</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>660000</td>
<td>190</td>
<td>379.94</td>
<td>659430.06</td>
<td>$659,430.06</td>
<td>$26,076.34</td>
<td>$685,506.40</td>
</tr>
</tbody>
</table>

Note: the prices are in American dollars
Chapter 6: Comparison and Conclusions

High amplitudes of vibration are typically an issue that is faced in the design of high-rise building structures. Different energy dissipation systems exist for the mitigation of such vibrations, depending on the typology of the structure, the nature of excitation, and other factors. Among the passive energy dissipation systems, dynamic vibration absorbers, or tuned mass dampers, have proven a high efficiency in suppressing elevated accelerations in tall buildings due to natural excitations, such as wind.

A tuned mass damper is a device consisting of a mass element, a stiffness component, and a damping component. Viscous fluid dampers have been widely applied as the damping component of tuned mass dampers, and they serve well for this function. However, this type of dampers has some drawbacks, such as fluid degradation and leakage, sensitivity to temperature changes generated by the transformation of the dissipated energy into heat, the difficulty of attachment to the primary structure, and difficulty in adjusting the damping ratio if needed. Alternatively, a new type of damping has been introduced for the control of tall buildings, named eddy current tuned mass damper.

An eddy current tuned mass damper functions based on Faraday’s law of induction; that is, when a non-magnetic conductive metal passes through a magnetic field, circular electric currents called eddy currents are generated. The eddy currents induce a magnetic field opposite to the applied field, according to Lenz’s law. As a result, an electromagnetic force is generated, which can be seen as a damping force for structural engineering applications, such as vibration suppression via tuned mass dampers.

Indeed, this novel type of dampers has notable advantages over the traditional types, including viscous fluid dampers. First, an air gap between the magnets and the conductive plate shall always be maintained, so it is a frictionless operation. This non-contact fashion has tremendous positive attributes, such as wear-free device, complete elimination of repair necessity, and persistent dynamic behavior of the structure. Second, it is a passive device, so it does not require any external power source for operation. Third, it is very easy to be assembled and installed. Fourth, it can withstand extreme temperatures, so it does not experience any variation in its behavior because of the temperature changes. Fifth, the device can be simply retuned into different frequencies by several options, including replacing the magnets with ones.
of different geometric dimensions or grade, replacing the conductive plate by one of a different thickness or material, or adjusting the air-gap length, which is the most convenient approach and requires no cost. Sixth, the design process is quite simple, where it has been proved in this manuscript that the analytical model can be easily applied in any numerical solver software with a remarkably low computational cost. Seventh, it has also been verified by a feasibility study that an eddy current tuned mass damper is more economical than the traditional viscous type.

Therefore, it is proved that the proposed eddy current tuned mass damper is able to reduce the wind-induced accelerations from 31 milli-g to 2 milli-g; in contrast, the seismic base reactions are reduced by only 8%. In addition, the designed device in this manuscript remarkably outperforms the existing viscous fluid tuned mass damper in terms of functionality, simplicity, feasibility, and economy. Lastly, the author invites future researches to study the long-term behavior of eddy current tuned mass dampers because they are expected to have stable, consistent and non-degradable behavior with time.
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Appendix A

Table A.1: Stiffness of the Structural Elements

<table>
<thead>
<tr>
<th>Section</th>
<th>Length [m]</th>
<th>$I_{xx} = I_{yy}$ [m$^4$]</th>
<th>$I_{zz}$ [m$^4$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.4</td>
<td>8050.2586</td>
<td>20432984.96</td>
</tr>
<tr>
<td>2</td>
<td>67.2</td>
<td>5709.4043</td>
<td>3025521.822</td>
</tr>
<tr>
<td>3</td>
<td>33.6</td>
<td>5709.4043</td>
<td>3025521.822</td>
</tr>
<tr>
<td>4</td>
<td>33.6</td>
<td>5395.2246</td>
<td>3014969.188</td>
</tr>
<tr>
<td>5</td>
<td>33.6</td>
<td>4746.4438</td>
<td>2993509.281</td>
</tr>
<tr>
<td>6</td>
<td>33.6</td>
<td>4139.1317</td>
<td>2973773.915</td>
</tr>
<tr>
<td>7</td>
<td>33.6</td>
<td>2015.2462</td>
<td>2901225.842</td>
</tr>
<tr>
<td>8</td>
<td>33.6</td>
<td>1849.2625</td>
<td>2897135.181</td>
</tr>
<tr>
<td>9</td>
<td>33.6</td>
<td>1766.3441</td>
<td>2895123.472</td>
</tr>
<tr>
<td>10</td>
<td>33.6</td>
<td>1683.4794</td>
<td>2875196.6</td>
</tr>
</tbody>
</table>

Note: $I_{xx}$ = moment of inertia around x-direction

$I_{yy}$ = moment of inertia around y-direction

$I_{zz}$ = polar moment of inertia around z-direction

Table A.2: Lumped Masses

<table>
<thead>
<tr>
<th>Mass Number</th>
<th>Mass [t]</th>
<th>Rotational Mass [tm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>96865.38</td>
<td>20432984.9562</td>
</tr>
<tr>
<td>$m_2$</td>
<td>21759.62</td>
<td>3025521.8223</td>
</tr>
<tr>
<td>$m_3$</td>
<td>21759.62</td>
<td>3025521.8223</td>
</tr>
<tr>
<td>$m_4$</td>
<td>21759.62</td>
<td>3014969.1884</td>
</tr>
<tr>
<td>$m_5$</td>
<td>21759.62</td>
<td>2993509.2808</td>
</tr>
<tr>
<td>$m_6$</td>
<td>21759.62</td>
<td>2973773.9150</td>
</tr>
<tr>
<td>$m_7$</td>
<td>21759.62</td>
<td>2901225.8416</td>
</tr>
<tr>
<td>$m_8$</td>
<td>21759.62</td>
<td>2897135.1808</td>
</tr>
<tr>
<td>$m_9$</td>
<td>21759.62</td>
<td>2895123.4722</td>
</tr>
<tr>
<td>$m_{10}$</td>
<td>10809.62</td>
<td>2875196.5947</td>
</tr>
</tbody>
</table>
Appendix B

Figure B.1: Wind Velocity Profile

Figure B.2: Equivalent Transversal Forces Induced by Vortex Shedding