Fast and robust estimation of Atmospheric Phase Screens using C-Band spaceborne SAR and GNSS calibration

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Ph.D. Thesis of

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A mamma e papà, per aver sempre creduto in me.
A Fabio e Annalisa, per aver messo al mondo la mia stella polare.
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One of the first features mentioned when talking about Synthetic Aperture Radar (SAR) is the total independence on weather conditions. While it is true that clouds are loosely affecting SAR images, it is also true that the non-unitary refractive index in the path from the satellite to ground delays the radar signal and affects the phase of the image acquired. This delay varies spatially and temporally, and it is one of the main sources of disturbance in the interpretation of SAR interferograms (the main product of interferometric SAR, or InSAR).

In interferometry, the effect of the atmospheric delay is called Atmospheric Phase Screen (APS). For applications like ground deformation monitoring or Digital Elevation Model generation, the effect of the atmosphere must be considered as a source of noise and therefore should be at least mitigated.

The refractive index, however, changes with temperature, pressure, or humidity of the medium, therefore it carries information about the status of the atmosphere at the time of the acquisition. This peculiarity leads in recent years to the boost of a branch of meteorology called InSAR meteorology with the primary objective of producing higher quality weather forecasts by using SAR-derived water-vapor maps as an ingestion product for Numerical Weather Prediction Models (NWPM).

APS estimation from SAR images is not a novel concept: one of the by-products of Permanent Scatterers Interferometry (PSInSAR) is, indeed, the atmospheric delay. Permanent Scatterers (PS), however, are not always present in the scene, especially in rural or forested areas. PS processing also requires many images to work correctly, and the computational burden can be demanding if wide areas are employed.

To be useful for NWPM, ingestion products must be spatially and temporally dense. Moreover, the generated maps must be wide: NWPMs work in domains as big as entire countries and they need APS maps that are several hundred or thousands of km wide.

To satisfy the first requirement, not only PS but also Distributed Scatterers (DS) must be used.

The exploitation of such targets requires coherence: the spatial structure of the scene must be quite stable between two SAR acquisitions. Since the sensibility of the system to ground changes increases with the operational frequency of the SAR, the C-Band at around 5.4 GHz is particularly suited for this purpose: the operational frequency is sufficiently low, allowing moderate coherence levels while also being high enough to avoid coping with ionospheric disturbances present at very low frequencies. Other bands will suffer of severe decorrelation (X-Band) or very strong ionospheric disturbances that must be taken into account (L-Band or P-Band).

The second requirement concerning the wideness of the estimated atmospheric map is dealt by exploiting SAR images gathered with the ESA mission Sentinel-1. The Interferometric Wide (IW) acquisition mode can continuously capture a swath width of 250 km, making it the perfect instrument for this objective.
This study developed a fast and robust method to optimally estimate atmospheric phase screens from a stack of SAR images using both PS and DS. Such delay maps can be used to predict extreme weather events and to provide better accuracy in short time forecasting. To satisfy the requirement on the large size of the derived product and at the same time to keep low the computational effort, it is mandatory to degrade the resolution. At the same time, we need to exploit DS, thus it is mandatory to use all the information (looks) available in the atmospheric resolution cell. The entire procedure is based on the Phase Linking algorithm and exploits ground patches whose size compares with the desired spatial resolution. The method is suited for short revisit time, C-Band SAR mission such as Sentinel-1, where sufficient coherence is present when estimating interferometric phases using large windows.

Just a few images need to be processed with a short total temporal span: this helps reduce the effect of deformations and decorrelations, which helps the unwrapping procedure.

A cross-calibration of the data using the Global Navigation Satellite System (GNSS) is conducted in order to remove sub-centimetric orbital errors that would lead to smooth but significant errors in the final products.

InSAR by itself, included the proposed method, is unable to produce absolute Zenith Total Delay (ZTD) maps. The product is, indeed, a differential one in the sense that each image is the difference between the ZTD of a given time instant and another. In order to retrieve the absolute ZTD, a prior must be used. In this research work, we also review two procedures aimed at making the maps absolute.

The results are presented with different case studies. The first one is in the area of northern Italy where a dense GNSS network is present along with severe decorrelation and strong orography. Statistics of the estimated maps are derived and compared with the state-of-the-art PSInSAR processing. Another case study is the one in central Italy where again the maps are compared with the ones computed by the state-of-the-art processor. The third case study, instead, is a large-scale experiment involving 145,000 km² in South Africa with very few GNSS stations.
Una delle prime caratteristiche menzionate quando si parla di radar ad apertura sintetica (SAR) è la sua totale indipendenza dalle condizioni meteorologiche. Mentre è vero che le nuvole influiscono limitatamente la qualità della immagini SAR, è anche vero che un coefficiente di rifrazione non unitario nel percorso tra il satellite e un bersaglio ritarda il segnale e influenza la fase dell’immagine acquisita. Questo ritardo varia spazialmente e temporalmente ed è una delle principali sorgenti di disturbo nell’interpretazione di interferogrammi SAR (il prodotto principale dell’interferometria SAR, o InSAR).

In interferometria l’effetto atmosferico è chiamato "schermo atmosferico di fase" (atmospheric phase screen, APS). Per alcune applicazioni quali il monitoraggio delle defor- mazioni o la generazione di modelli di elevazione digitale (DEM), l’effetto dell’atmosfera viene considerato come una sorgente di rumore e quindi deve essere mitigato (o idealmente eliminato).

L’indice di rifrazione cambia con la temperatura, pressione e umidità del mezzo in cui l’onda elettromagnetica si propaga, quindi include informazioni riguardo lo stato dell’atmosfera al momento dell’acquisizione. Questa peculiarità ha recentemente portato alla nascita di una branca delle meteorologia chiamata meteorologia InSAR. Essa ha come obbiettivo principale la generazione di previsioni meteo di alta qualità usando, nel processo di assimilazione nei modelli meteo previsionali (NWPM), mappe di vapore derivate da immagini SAR. La stima di APS da immagini SAR non è un concetto nuovo: uno dei prodotti di scarto della tecnica Permanent Scatterer (PSInSAR) è, infatti, il ritardo atmosferico. I Permanent Scatterers, purtroppo, non sono sempre presenti nella scena, specialmente in aree rurali o boschive. Il processamento dati PS richiede anche diverse immagini per poter funzionare correttamente e l’onere computazionale può essere importante se lo studio richiede larghe scale spaziali.

Per essere utili ai NWPM, i prodotti assimilati devono essere spazialmente e temporalmente densi. In più, le mappe generate devono essere spazialmente estese: i modelli meteo previsionali lavorano su domini grandi come intere nazioni e quindi necessitano di mappe atmosferiche che si estendono centinaia o migliaia di km. Per soddisfare il requisito di densità spaziale, non vengono sfruttati solo PS, ma anche i DS (bersagli distribuiti o Distributed Scatterers). Sfruttare tali bersagli richiede la presenza di coerenza: la struttura spaziale della scena deve rimanere stabile tra un’acquisizione e l’altra. Dato che la sensibilità del sistema ai cambi aumenta con la frequenza operazionale del sistema SAR, la banda C, attorno ai 5.4 GHz, è particolarmente adatta a questo scopo. La frequenza è abbastanza bassa da consentire una coerenza moderata, ma è abbastanza alta da evitare effetti ionosferici importanti. L’utilizzo di altre bande comporterebbe un’effetto di decorrelazione severo (banda X) o effetti ionosferici non trascurabili (band P o band L).

Il secondo requisito riguardante la vastità delle mappe atmosferiche stimate è affrontato tramite l’utilizzo di immagini SAR della missione Sentinel-1. La modalità Interferometric
Wide (IW) può catturare un’immagine larga fino a 250 km su tutto il globo. In questo studio abbiamo sviluppato un metodo veloce e robusto per stimare in maniera ottima gli schermi atmosferici di fase partendo da uno stack di immagini SAR e sfruttando sia PS che DS. Tali mappe possono essere utilizzate per predirre eventi estrambi e per fornire migliore accuratezza nelle previsioni a corto raggio. Per soddisfare il requisito sulle dimensioni del prodotto derivato e allo stesso tempo mantenere lo sforzo computazionale moderato, è necessario ridurre la risoluzione e di conseguenza il passo di campionamento del dato SAR. Allo stesso tempo si rende necessario l’utilizzo dei DS, quindi siamo indotti ad utilizzare tutta l’informazione disponibile (looks) nella cella di risoluzione atmosferica. Il metodo è basato sull’algoritmo Phase Linking e sfrutta finestre spaziali la cui dimensione è comparabile con la risoluzione spaziale desiderata. Esso è particolarmente adatto a missioni con un tempo di rivisita breve e in Banda C come la missione Sentinel-1.

Poche immagini devono essere processate con una riduzione dell’estensione temporale complessiva dello stack: questa condizione aiuta a ridurre l’effetto delle deformazioni e della decorrelazione, che a sua volta, aiuta la procedura di srotolamento (unwrapping) della fase stessa.

Una cross-calibrazione del dato avviene tramite una rete di stazioni GNSS permettendo così la correzione di un errore orbitale sub-centimetrico che porterebbe ad un errore non trascurabile alle basse frequenze spaziali.

La tecnologia InSAR, compreso il metodo qui proposto, non è in grado di produrre mappe di ritardo assolute (Zenith Total Delay, ZTD). Il prodotto, infatti, è una mappa differenziale nel senso che ogni immagine rappresenta la differenza delle condizioni atmosferiche tra un dato istante temporale e un altro. Per stimare mappe assolute, un dato a priori deve essere utilizzato. In questo lavoro di ricerca riproponiamo due procedure utilizzate per rendere le mappe assolute.

I risultati sono presentati con differenti casi di studio. Il primo considera l’area nel nord Italia dove una rete GNSS densa è presente. In questa zona la decorrelazione è severa e ci sono forti variazioni di topografia. Le statistiche delle mappe stimate vengono mostrate e comparative con lo stato dell’arte nella generazione di mappe atmosferiche tramite PSInSAR. Un altro caso di studio è quello del centro Italia dove le mappe sono nuovamente comparative con quelle derivate dallo stato dell’arte del processamento dati PSInSAR. Il terzo caso studio, invece, è un esperimento a larga scala che coinvolge 145.000 km² in Sud Africa con poche stazioni GNSS presenti sulla scena.
Introduction

In this chapter a brief introduction of Interferometric SAR (InSAR) is given to the reader, with particular attention to InSAR meteorology. The chapter ends with a clear statement of the research objectives and the outline of the thesis.

1.1. The essence of SAR and InSAR

Synthetic Aperture Radar (SAR) is an active remote sensing system. While in classical optical systems the sensor measures the reflection of the sunlight on an object, in a SAR system an antenna brings the illumination to the scene and the echo is recorded [1]. The entire SAR system is composed of an antenna able to transmit a microwave signal and a moving platform able to synthesize a long antenna by moving in space. The platform could be a ground-based rail, a car, a drone, a plane, or a satellite. The advantages of the active nature of SAR are more than few. The illumination can be controlled electronically and therefore perfectly repeated between acquisitions. Note the this is impossible in optical systems, where the sun changes position over the horizon between acquisitions making the comparison of images a hard task [2]. Since the transmitted signal is in the microwave spectrum it can work all-weather, meaning that cloud coverage is completely absent in SAR images up to X-Band. The all-weather adjective should not be confused with weather-independent. The difference between the two will be explained in Section 1.2. The sensing can also happen during the night since the sunlight is not necessary for the measurement to happen. The most important feature of an active and coherent remote sensing system like a SAR is the capability to exploit the phase information of the echo signal [3], [4]. The mentioned phase contains information about the optical path between the sensing platform and the targets on the ground. The optical path includes the geometrical distance between sensor and target and the refractive index of the medium (i.e. atmospheric conditions).

The different scattering behavior of the targets on the ground causes the phase image to be impossible to interpret directly. For this reason, the only way to extract useful
information from such images is to use an interferometric processing (InSAR) where two images are jointly exploited. The resulting image is called interferogram and can be used to estimate ground deformation, topography, and atmospheric delay. The latter is often called Atmospheric Phase Screen and will be the main topic of this thesis.

1.2. InSAR Meteorology

The atmospheric delay (or advance) is induced by the troposphere and ionosphere during the propagation of the signal [5], [6]. Different conditions in pressure, temperature, and humidity along the signal path generate a spatio-temporal variation of the refractive index of the medium and therefore the tropospheric delay [7]. The ionospheric contribution in C-Band is spatially and temporally smooth, whereas the highest spatio-temporal variability comes from the tropospheric contribution [8]. For this reason, the main disturbance for ground deformation estimation is the tropospheric delay. This excess delay could be interpreted as ground deformation corrupting the estimate.

Several methods have been tested for the compensation of tropospheric artifacts in SAR interferograms. Some of them use the statistics of the atmospheric delay [9], [10] and some others use Numerical Weather Prediction Model (NWPM) [11], [12]. While the atmospheric delay can be a serious problem for several InSAR applications, it can also be a useful product in the field of meteorology [13], [14]. Since the APS mostly senses the variation of the water vapor, it can be convenient to include such measurements into advanced NWPM with the objective of improving weather forecasts [14]–[16].

A meteo-hydrological product must respect some requirements in terms of horizontal and temporal resolution in order to be useful in the process of ingestion into an NWPM. The Observing Systems Capability Analysis and Review (OSCAR) tool [17], developed by the World Meteorological Organization (WMO) sets the minimum horizontal resolution for Integrated Water Vapor (IWV) maps in high-resolution NWPMs at 20 km with a goal, over which further improvements are not necessary, of 0.5 km. Concerning time resolution, the minimum requirement is 6 hours with a goal at every 15 minutes. The requirements are summarized in the Table 1.1.

<table>
<thead>
<tr>
<th>Requirement for High Res NWPMs</th>
<th>Threshold</th>
<th>Breakthrough</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal resolution</td>
<td>6h</td>
<td>60 min</td>
<td>15 min</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>20 km</td>
<td>5 km</td>
<td>0.5 km</td>
</tr>
</tbody>
</table>

Table 1.1: Requirements for ingestion products into Numerical Weather Prediction Models (NWPMs).

In [18] it was proved that the ingestion of ZTD maps with high spatial and temporal resolution can significantly improve the forecast and the prediction of intense and localized rain events. In their work, the researchers found that the assimilation of SAR observations at 2.5km grid spacing every 3 or 6h provides the largest benefit to the forecasting.

Mateus et al. proved in [19] that it is sufficient to have a set of SAR images in the
NWPM domain every 12h to positively impact the performances of the model. This means that also Sentinel-1 with its 6 days repetition interval can still be useful: it is, however, mandatory to design a NWPM domain sufficiently large to contain images from different S1 tracks (different orbit cycles and/or ascending/descending). While GPS-derived Zenith Total Delay (ZTD), radio probes measurements, and ground-based radar measurements are almost point-wise but continuous in time [20], [21], a space-borne SAR system in Low Earth Orbit (LEO) is only able to fulfill the horizontal resolution requirements at the goal level, while the temporal resolution is equal to the interferometric revisit, in the order of days (6 days for the Sentinel-1 constellation). A possible innovation in this field could be the Geosynchronous SAR that provides revisit in the order of hours and not days allowing the fulfillment of both temporal and spatial resolution requirements [22].

Some care must be taken in the use of SAR water vapor maps since SAR is able to provide only differential information both in space and time are sensed (InSAR). In particular, a space-borne SAR system acquires two consecutive images with a temporal lag distance much higher than the decorrelation of the Wet delay and with a wavelength much lower than the variation of this signal in time. It is then impossible to reconstruct without ambiguity the temporal variation of this signal. The extracted APS maps are the difference of atmospheric conditions between two time instants and two points in space. The absolute maps must be estimated before the ingestion.

1.3. Research objectives and thesis outline

Provided the above discussion, the thesis objective can be stated as the following:

*Develop, test, and implement a robust and fast algorithm able to generate wide and dense atmospheric phase maps from a stack of Sentinel-1 SAR images. The final maps must also be corrected for orbital errors by using a network of GNSS stations.*

The thesis is divided as follows:

- **Chapter 2** provides an introduction to SAR and InSAR signals including the different components of the interferometric phase and the concept of coherence. It introduces also the tropospheric effects on the propagation of EM waves;

- **Chapter 3** provides a detailed overview of the implemented processing chain. The algorithm starting from the coregistration and ending with the GNSS calibration is described along with the expected performances. In this section, a simple simulation using synthetic data is also proposed;

- **Chapter 4** provides three case studies where the proposed technique is validated against APS maps derived by the state of the art PSInSAR processing;

- **Chapter 5** provides the final conclusions of this thesis.
In this chapter, a brief overview of SAR, InSAR, and the effect of the atmosphere on the SAR signal is proposed. In Section 2.1 the SAR signal model is detailed along with the concept of phase. In Section 2.1.2 the InSAR signal model is presented and each contribution to the interferometric phase is explained in detail. The concept of decorrelation is also introduced in this section of the chapter. In Section 2.2 the effects of the propagation of the Electromagnetic (EM) wave through the troposphere are explained, along with the concept of wet and dry delay and vertical stratification. In the last section (2.3) an overview of the ionospheric contribution to the interferometric phase is proposed.

2.1. SAR and InSAR signal model

This section aims at laying out a review of the SAR and InSAR signal models. This is useful to have a common theoretical basis and a set of symbols over which it is possible to develop the entire thesis.

2.1.1. SAR signal model

SAR images are acquired by an active sensor mounted on a moving platform. The radar image contains information both about the intensity of the reflection and about the sensor-target travel time [1]. Note that throughout the whole thesis, I will use the word reflection as synonym of scattering or diffuse reflection.

The intensity of the reflection is encoded in the amplitude $A$ while the travel time is encoded into the phase term $\phi$. A single pixel of the image gathered at a generic time instant can be represented as:

$$P = A \exp(j\phi)$$

(2.1)
2. SAR, InSAR and the atmospheric effects on the measurements

The complex value in Equation 2.1 is the result of the coherent sum of the contributions of the single scatterers inside the resolution cell. The amplitude of the reflection $A$ depends of several factors such as geometrical and chemical characteristics of the scatterers, wavelength, incidence angle and so on [23]. There are two main mechanisms of reflection: Point Scattering (or Point Target, from now on called PS) and Distributed Scattering (from now on called DS) [24].

In the first case, a dominant source of reflection is present in the resolution cell. Notice that the dominance of a target is not always related to its geometrical size. Even a small corner reflector could dominate the resolution cell if its Radar Cross Section (RCS) is sufficiently high. The other targets inside the resolution cell are considered as clutter.

In the second case, many scatterers with comparable RCS contribute equally to the measurement. It is a common misconception that a DS is, over time, incoherent while a Point Target is intrinsically coherent. In Figure 2.1 four categories of targets are represented. The house is dominant comparing to the surrounding trees and it is also stable in time, therefore coherent. The car dominates the resolution cell, but from one acquisition to the other, if the car moves, the signal will change (decorrelation). Some stable rocks contribute equally to the resolution cell forming in this way a distributed scatterer, but, since they do not change their physical structure they will be coherent.

The last category is represented by a forest where there are no dominant scatterers and moreover there is a significant change in the physical structure and therefore decorrelation.

It is also important to specify that the fully coherent and fully incoherent targets are just two extreme scenarios: in the middle there are the so-called partially coherent targets. Such targets show a specific decorrelation model, in other word their coherent change in time [25].

The phase of a single pixel of the SAR image can be represented as the sum of several terms:

![Figure 2.1: From left to right: a coherent point scatterer, a decorrelating point scatterer, a coherent distributed scatterer, and a decorrelating distributed scatterer. The complex pixel is given by the vector sum of the single contributions depicted in the top plots. Different colors (red and blue) represent different time instants.](image-url)
\[ \phi = \phi_{rg} + \phi_{atmo} + \phi_{scat} + \phi_n \]  

(2.2)

where

- \( \phi_{rg} \) is the phase representing the geometrical distance from the radar to the target;
- \( \phi_{atmo} \) is the phase representing the atmospheric conditions during the acquisition;
- \( \phi_{scat} \) is the scattering phase of the target: in Section 2.1.2 we will classify targets basing on the temporal variability of this phase component;
- \( \phi_n \) is the always present noise of the measure: it contains mainly clutter, but also thermal, quantization and processing noise. If the background clutter rapidly varies, this term is responsible for a rapid decorrelation of the scene [25], [26].

Notice that a phase measurement is always known modulus 2\( \pi \), therefore \( \phi \in [\pi, \pi] \). The phase measurement is not directly usable since the scattering phase \( \phi_{scat} \) is not known. Useful information can be extracted through the difference between two images. This process is called interferometric processing (InSAR).

### 2.1.2. InSAR signal model

An interferogram is computed by the complex conjugate multiplication between two coregistered SLC images

\[ I = P_m P_s^* = A_m A_s \exp(j(\phi_m - \phi_s)) \]  

(2.3)

Where the * denotes the complex conjugate and the subscripts \( m \) or \( s \) denote master and slave respectively.

The interferometric phase can be expressed as

\[
\psi = \phi_m - \phi_s \\
= (\phi_{rg,m} - \phi_{rg,s}) + (\phi_{atmo,m} - \phi_{atmo,s}) + (\phi_{scat,m} - \phi_{scat,s}) + (\phi_{n,m} - \phi_{n,s}) \\
= \psi_{rg} + \psi_{atmo} + \psi_{scat} + \psi_n
\]  

(2.4)

In this section each addend will be discussed with the exception of \( \psi_{atmo} \). This term is so central for the entire thesis that will be developed in its own section (See Section 2.2).

### Differences of geometrical distances

As previously explained, a SAR system is able to accurately measure the distance from the antenna phase center to a generic target. When the two acquisitions happen at different time instants (the so-called multipass acquisitions), the distances could change due to two main factors:

1. The platform where the antenna is mounted does not repeat the exact same trajectory in the two acquisitions;
2. The targets move (for example due to subsidence or a deformation).

In the first scenario, a so-called spatial baseline is formed. Two main effects are visible in the interferometric phase when a spatial baseline is present. Firstly a term called flat-
Earth phase arises as a strong trend in the interferogram in the range direction. In absence of topography, target motion, and by assuming flat Earth, as shown in Figure 2.2, we have that:

\[
\psi_{rg} = \phi_{rg,m} - \phi_{rg,s} = -\frac{4\pi}{\lambda} (R_m - R_s) = -\frac{4\pi}{\lambda} B_{\parallel} = -\frac{4\pi}{\lambda} B \sin(\theta)
\]

Where \(R_m\) is the geometrical distance from master \(m\) to the target \(p\), \(R_s\) is the geometrical distance from slave \(s\) to the target \(p\), \(B\) is the baseline, \(B_{\parallel}\) is the baseline projected in the direction of the line of sight, \(\theta\) is the local incidence angle. The term \(4\pi\) is typical of monostatic systems. Notice that Equation 2.5 has been derived by exploiting the far-field approximation and by assuming a baseline parallel to the ground range.

The second effect that arises when there is a spatial baseline between two acquisitions is the sensibility of the interferometric phase to topography. In Figure 2.3 a target is present at an elevation \(q\) over the reference plane.

Equation 2.5 can be expanded in Taylor’ series up to the first order with respect to
2.1. SAR and InSAR signal model

Figure 2.3: InSAR acquisition geometry in the zero Doppler plane. The master antenna is denoted by \( m \) while the slave by \( s \). The target \( p_q \) is located at height \( q \) over the reference plane (ground). A reference point on the ground is indicated by \( p_0 \).

The incidence angle \( \theta \):

\[
\psi_{rg}(\theta + \Delta \theta) = \psi_{rg}(\theta) + \frac{\partial \psi_{rg}(\theta)}{\partial \theta} \Delta \theta \\
= -\frac{4\pi}{\lambda} B_{\parallel} + \frac{4\pi}{\lambda} B \cos(\theta) \Delta \theta \\
= -\frac{4\pi}{\lambda} B_{\parallel} + \frac{4\pi}{\lambda} B \cos \theta \frac{s}{R_m} \\
= -\frac{4\pi}{\lambda} B_{\parallel} + \frac{4\pi}{\lambda} B_{\perp} \frac{q}{R_m \sin \theta} \tag{2.6}
\]

Where \( \Delta \theta \) is the variation of the look angle from the reference plane to the target \( p_q \) placed at elevation \( q \) over such reference plane, \( s \) is the cross-range (orthogonal to the line of sight) position of the target, and \( B_{\perp} \) is the component of the baseline perpendicular to the line of sight.

The second term is Equation 2.6 highlights the sensibility of InSAR to topography: the interferometric phase is proportional to the elevation of a target over a reference plane and therefore it can be used to estimate Digital Elevation Models (DEM).

When the objective is not to generate DEM over the scene, such terms must be compensated from the interferogram by using precise Orbit State Vector (OSV, to calculate \( B \)) and a known DEM of the scene (to provide \( q \) in Equation 2.6).

The sensibility of the system to elevation increases when the orthogonal baseline increases. When the objective is to generate a DEM it is preferable to have short temporal baselines to prevent temporal decorrelation and large spatial baselines to increase sensibility to elevation. The ideal condition is to have two simultaneous acquisitions: in this case, we speak about single-pass interferometry. The NASA mission SRTM and the German mission Tandem-X were designed to satisfy these requirements [27], [28].

If the objective of the studies is deformation or the atmosphere, the effect of topography
must be removed. The change of geometrical distances between the antenna phase center and the target can also be caused by the motion of the target itself. If we suppose that there is exactly zero spatial baseline between the two acquisitions \((B = 0)\) we have

\[
\psi_{rg} = \frac{4\pi}{\lambda} d_{\text{los}}
\]

(2.7)

where \(d_{\text{los}}\) is the projection of the deformation along the line of sight as shown in Figure 2.4. In this figure a target \(p\) is depicted in two time instant \(t_1\) and \(t_2\) (two consecutive SAR acquisitions) and a subsidence is present. The interferometric phase is a measure of the green segment \(d_{\text{los}}\).

Notice that the InSAR measurement is a phase, while the parameter of interest is the deformation. The length-to-phase conversion factor in the case of a monostatic SAR is the wavenumber \(4\pi/\lambda\). This conversion factor is the reason why InSAR is so popular for deformation estimation and monitoring [29], [30]. A small deformation, even of a few millimeters, is converted into a big phase difference and therefore can be detected. A deformation \(d_{\text{los}} = 1\text{cm}\) corresponds to a phase shift of 4.18 rad at X-Band, 2.26 rad at C-Band and 0.51 rad at L-Band.

**Temporal decorrelation**

From one acquisition to the successive one, the physical or geometrical characteristics of the area observed may change inside a specific resolution cell. These changes may happen in amplitude, phase, or both. A few examples are of such changes are: leaf growth in forests (See Figure 2.1), changes in the moisture of the terrain due to rain...
events, a field is plowed by a farmer, moving car in a parking lot, and so on. 
All these scenarios involve relative target displacements inside the resolution cell. 
The consequence is called *temporal* decorrelation [31], [32]. The result of such changes 
is the presence of a non-negligible and unknown term $\psi_{\text{scat}}$ in the expression of the 
interferometric phase. 
The complex *coherence* $\gamma$ is a number whose absolute value $|\gamma| \in [0, 1]$ indicates the 
degree of correlation between two images [33]. The sensibility to changes increases 
with the operating frequency: longer wavelengths are less sensible to fine changes and 
therefore temporal decorrelation is usually lower than for higher frequencies such as C-
or X-Band [34]. 
The modeling of the coherence in dependence of time and wavelength is a fundamental 
part of the proposed method and will be properly detailed in Chapter 3. Lastly, it is 
important to highlight that a uniform translation of all the scatterers inside the resolution 
cell (i.e. subsidence) does not lead to a decorrelation, but instead just to a phase shift 
as in Equation 2.7. 

**Geometric decorrelation** 
Another source of decorrelation is the so-called *geometric decorrelation* [31]. It is very 
similar to the decorrelation due to the relative target’s motion, but this time what 
moves is the satellite and not the targets. Between the two acquisitions the incidence 
angles of the EM wave on ground changes and therefore the illuminated range frequency 
spectrum. The consequence is that the spectra of the two images are not overlapped 
and therefore an additional noise term is present. The smaller $B_\perp$ the lower the noise 
will be. A critical baseline $B_{\perp,c}$ has been defined in literature: if $B_\perp$ exceeds its value 
the range spectra of the two images do not overlap and there is total decorrelation. 
For a flat topography the value of the critical baseline is computed using: 

$$B_{\perp,c} = |\lambda(B_r/c)R\tan(\theta)|$$ (2.8) 

Where $\lambda$ is the wavelength, $B_r$ is the range bandwidth, $c$ is the speed of light, $R$ is 
the slant range and $\theta$ is the local incidence angle. 
For Sentinel-1 operating in IW mode, $\lambda = 5.5$ cm, $B_r = 42 - 56$ MHz, $R = 800$ km 
and $\theta = 37$ deg. These values lead to a critical baseline of roughly 2 km, one order of 
magnitude higher than the typical interferometric baseline for this operational mode. 

**Noise in the measurement and other sources of decorrelation** 
The thermal noise due to the instrumentation, the quantization errors, and the processing 
errors are always present in any measure and a radar system makes no exception. 
The result is another source of decorrelation that contributes to the overall coherence 
value between the images [31]. 
Processing decorrelation may occur when two images are not properly coregistered, the 
Doppler centroid decorrelation is instead very similar to the geometric decorrelation but 
in the azimuth direction: the two images may be acquired within a slightly different 
angular interval. 

All these decorrelation sources contribute to the final coherence in a multiplicative 
way, resulting in a total coherence of:
\[ \gamma = \gamma_t \cdot \gamma_g \cdot \gamma_{th} \cdot \ldots \]  

(2.9)

When the coherence is high, the estimation of deformation, height, or atmospheric contribution can happen reliably, i.e., the estimates are accurate. On the other end, when the coherence is low, the measurement is too compromised by noise, therefore the resulting estimates are not reliable. The extreme case is when the coherence is close to zero, for example over water: in this case, the phase is too noisy to be used for the estimation for any parameter of interest. InSAR can’t provide any useful results over water bodies.

### 2.2. Tropospheric effects on microwave propagation

When the signal travels through the atmosphere, it is delayed by a quantity called propagation delay that depends on the refractive index of the medium itself.

If the medium is the vacuum, the refractive index along the geometrical path from the radar to ground and back is unitary \( (\eta(r) = 1) \), therefore the delay is just due to the geometrical distance between the transmitter and the target.

If the refractive index is not unitary, the optical distance experienced by the wave can be expressed as:

\[
R = \int_S^P \eta(r)dr = R_a + R_g
\]

(2.10)

Where \( S \) is the position of the sensor, \( P \) is the position of the target, \( \eta(r) \) is the range-dependent refractive index (often denoted as \( n(r) \)), \( R_g \) is the geometrical distance from the sensor to the target and \( R_a \) is the equivalent excess path generated by a non-unitary refractive index. Notice that in Equation 2.10 the excess path length provided by the ray bending has been neglected.

The excess path generated by the presence of the atmosphere can be expressed as:

\[
R_a = \int_S^P (\eta(r) - 1)dr
\]

(2.11)

Notice that, if \( \eta(r) = 1 \) (i.e. vacuum), \( R_a = 0 \) as expected from the absence of atmosphere.

A common value for \( R_a \) could be between 2.2 m and 2.7 m for zenith (vertical) propagation and increases for slanted acquisition modes. [35, p.399]

A common value for the refractive index at sea level is \( \eta = 1.0003 \), or, in other words, the velocity of propagation in air is very similar to the one in vacuum. Over long distances, however, even a slight deviation from unity can lead to significant excess path (see Equation 2.11). It is common to use the refractivity instead of the refractive index by defining it as \( N(r) = (\eta(r) - 1) \times 10^6 \) leading to a different form of equation 2.11:

\[
R_a = 10^{-6} \int_S^P N(r)dr
\]

(2.12)

The refractivity \( N \) is governed by different conditions in pressure, temperature, and humidity and therefore is space and time-dependent. \( N \) can be modeled by two terms:
2.2. Tropospheric effects on microwave propagation

a spatially and temporally smooth component called *dry* or *hydrostatic* term and a spatially turbulent one called *wet* term. While the former is independent on the water-vapor content of the atmosphere, the latter is directly proportional to the water-vapor density in the medium.

The dry delay is usually in the order of a couple of meters, while the wet delay is usually not more than 0.3 m.

Smith and Weintraub [36] defined:

\[
N = 77.6 \frac{P}{T} + 1720 \frac{\rho_v}{T} \tag{2.13}
\]

where \(P\) is the total pressure in millibar, \(T\) is the temperature in Kelvin, \(\rho_v\) is the water-vapor density in \(\text{g/m}^3\). The first term to the right of the equal sign of Equation 2.13 is often called hydrostatic refractivity and results from the induced molecular polarizations of dry air and water vapor [37].

The second term is often refereed as wet refractivity and it is induced by the effect of the permanent dipole moment of the water vapor molecule.

The water-vapor density can be calculated using:

\[
\rho_v = 217 \frac{e}{T} \tag{2.14}
\]

where \(e\) is the partial pressure of water vapor expressed in millibar. Equation 2.11 can then be expressed as the sum of two contributions, the first due to the dry delay, the second due to the wet delay:

\[
R_a = R_a^{\text{dry}} + R_a^{\text{wet}} \tag{2.15}
\]

According to [38] the dry component of \(N\) can be estimated directly from surface weather data and according to [39] the value of \(R_a^{\text{dry}}\) can be estimated with an uncertainty of 1 cm or even just a few millimetres provided that the surface pressure is measured with an accuracy of a few millibars.

The wet delay \(R_a^{\text{wet}}\), on the other end, must be investigated further.

2.2.1. Wet delay

The wet delay \(R_a^{\text{wet}}\) is the main responsible for the so-called *atmospheric artifacts* in interferograms where a deformation or elevation measure is corrupted by the effects induced on the signal by the troposphere. This contribution is also relevant for NWPM ingestion since it carries information about water-vapor content in the troposphere.

The fluctuations of this term are significant both in space and in time and we can refer at them as atmospheric turbulence. It is important to note that in this framework, the word *turbulence* is not intended as in the Kolmorogov theory as the random variation of the total refraction index. The reduced spatial scale and high temporal variability make it difficult to estimate the wet delay by using external measures in an accurate and high-resolution fashion.

It is preferable to describe the wet delay, not as a deterministic signal both in space and
time, but as a stochastic signal described by a mean and a fluctuation over the mean value [40]:

$$R_{\text{wet}}^a = \left< R_{\text{wet}}^a \right> + \xi$$  \hspace{1cm} (2.16)

where $\left< \cdot \right>$ indicates the ensemble average. The turbulent term $\xi$ is traditionally statistically modeled by a variogram [40]:

$$\nu(\xi) = \left< |\xi(p + \Delta p) - \xi(p)|^2 \right> \hspace{1cm} (2.17)$$

$$= k_1 (|\Delta p|)^{k_2} \hspace{1cm} (2.18)$$

where $p$ is the position vector in the 3D space, while $\Delta p$ is a 3D displacement vector. The two constant $k_1$ and $k_2$ indicates respectively the turbulence strength and the turbulence smoothness. Turbulence induced delay has a scale-variant power low with a transition region at approximately $\Delta p = 1.5$ km. Above this threshold, a proper value for $k_2$ is about $2/3$ suggesting a 2D turbulent process, while below the threshold is about $5/3$ indicating a 3D turbulent process [41], [42].

### 2.2.2. Vertical stratification

As already mentioned, the SAR measure is a double difference in time and space. This means that SAR data can tell us just the spatio-temporal variation of the atmosphere, not the absolute value. Let us suppose that the atmosphere at any given time $t_i$ is composed by many thin and stratified layers, each one with its own refractivity $N_{t_i}(h)$. Notice that this model does not account for the lateral variation of each layer (i.e. each layer is laterally uniform).

The double difference between time $t_1$ and $t_2$ and two targets $p$ and $g$ placed at elevation $q_p$ and $q_g$ respectively is defined as:

$$\Delta R = (R_{t_1}^t)^p - (R_{t_2}^t)^p - (R_{t_1}^t)^g + (R_{t_2}^t)^g$$

$$= \int_{q_p}^{q_g} N_{t_1}(h)dh - \int_{q_p}^{q_g} N_{t_2}(h)dh$$

$$= \int_{q_p}^{q_g} \Delta N_{t_1,t_2}^t(h)dh \hspace{1cm} (2.19)$$

where $\Delta N_{t_1,t_2}^t(h)$ is the difference of vertical stratification (layer by layer) between time $t_1$ and $t_2$. The integral in Equation 2.19 depends on the height difference between the two considered targets.

This height-dependent delay is called vertical delay stratification and it is strongly correlated with topography. In presence of strong topography this effect may dominate the lateral variation of the delay. On the other end, in absence of topography, when $q_p \approx q_g$ this effect is negligible [43].
2.3. Ionospheric effects on microwave propagation

The ionosphere is the upper part of Earth’s atmosphere and extends from 60km up to 1000km depending on the solar activity [35]. The ionosphere consists in ionized gasses containing free electrons and positively charged ions. The two main effects on the microwave propagation are the phase advance and the Faraday rotation [8], [44].

2.3.1. Phase advance

The phase velocity of microwaves is larger in the ionospheric than in free space. This condition is a direct consequence of the fact that the refractive index $\eta$ in the Earth’s ionosphere is lower than 1 [44].

The phase of the radar impulse is advanced relative to the ionosphere-free propagation. It is possible to prove that the phase advance is given by

$$\phi = \frac{4\pi K}{c} TEC$$ (2.20)

where $f$ is the carrier frequency of the radar, $c$ is the speed of light in vacuum, $K = 40.28 m^3/s^2$ and $TEC = \int_S \eta(r)dr$.

Notice that while the tropospheric phase delay is directly proportional to the carrier frequency, the ionospheric advance has an inverse relationship with it.

As an example, 1 TEC unit (1 TECU) induces in the EM wave 38.9 rad of phase advance in P-Band, 13.3 rad in L-band, 3.1 rad in C-band, and 1.8 rad in X-band.

2.3.2. Faraday rotation

A radio wave that passes through the Earth’s ionosphere changes its polarization state under the effects of Earth’s magnetic field. This effect is called Faraday effect or Faraday rotation. In this scenario, the wave’s electric field vector rotates and the amount of angular rotation depends on several factors: frequency of the wave, direction of propagation with respect to the Earth’s magnetic field, and the ionospheric TEC.

Fortunately, the effects of Faraday rotation can be completely ignored for frequencies above 3 GHz. Since this thesis will deal mostly with C-band data (5.4 GHz) from now on we will neglect this effect.
Atmospheric Phase Screen Estimation

3.1. Introduction
In this chapter, the entire processing chain for the extraction of Atmospheric Phase Screens (APS) is explained in detail. The workflow is depicted in Figure 3.1 and it is divided into pre-processing and processing of the input data. In this chapter, each step is discussed in its own section along with the useful mathematical framework and the expected performances.

3.2. Coregistration
The first step of the processing workflow is a standard coregistration [45]. Note that if the acquisition is not squinted, misregistration does not introduce a phase bias, but may increase the phase variance [46]. In [47] it is stated that an accuracy of 0.1 pixels is sufficient to obtain interferograms that do not suffer from coherence loss due to misregistration. Sentinel-1, in its standard operational mode, works in TOPS mode [48] and the requirements in terms of coregistration accuracy are much more demanding. Bara et al. showed [49] that the variation of squint angle causes large phase ramps in both azimuth and range if misregistration is present. The effect is much more pronounced in azimuth.

If we have misregistration of $\Delta t$ seconds between a master image and a given slave image we have an azimuth phase ramp over the burst of [47]:

$$\Delta \phi_{az} = 2\pi \Delta f_{DC} \Delta t$$ (3.1)

Where $\Delta f_{DC}$ is the variation in Doppler centroid over azimuth and $\Delta t$ is the misregistration. If we consider a maximum $\Delta f_{DC}$ of 5.2 kHz, in order to limit the phase ramp to 0.5 mm we need a coregistration accuracy of roughly 3.8 $\mu$s that translates in 2.6 cm or 0.0019 pixels considering the orbital velocity of Sentinel-1.
3. Atmospheric Phase Screen Estimation

Figure 3.1: Scheme of the entire workflow implemented. The input data is freely provided by the Copernicus program and the GNSS data is processed with the open source software goGPS. The pre-processing is performed using the free and open-source software SNAP, while the processing is entirely written in MATLAB®.

Since the coregistration of all the images in the stack is not the main objective of this thesis, we did not develop a custom software to achieve fine coregistration. Instead, we used the ESA Sentinel Application Platform (SNAP) tool that includes a procedure for fine coregistration able to satisfy our accuracy requirements.

3.3. Topographic phase compensation

As already discussed in Section 2.1.2 the interferometric phase contains several contributions such as the flat earth component, the topographic phase the Atmospheric Phase Screen, the deformation phase and many more. Since we are interested into atmospheric monitoring, all the other terms must be removed either a-priori, by using ancillary data, or by estimating the nuisance parameters and removing them from the data. Also, a combination of the two is possible: for example, a coarse removal of the topography component is performed using a known Digital Elevation Model (DEM), then a finer compensation can be done after the unwrapping step [50].

For what concerns the topographic phase, a Shuttle Radar Topography Mission (SRTM) 1 Sec DEM has been used in conjunction with precise orbital information to compensate for the reference (flat earth) phase and for topographic fringes. It is useful also to recognize that in our scenario, even if the used DEM is not very precise (less than 10 m RMS of vertical accuracy), the error introduced on the phase is negligible. The equation of the topographic phase (Equation 2.6) can be rewritten for assessing the sensibility of the phase to an elevation error:
3.4. Phase Linking

3.4.1. Introduction

The need for a reliable estimate of the phase over both PS and DS, requires the usage of an advanced phase estimator called Phase Linking (PL) [51], [52]. This algorithm is able to provide a high-quality estimate of the interferometric phase in presence of both PS and DS as long as the DS does not decorrelate severely at least for a few acquisitions. Different scenarios typologies may or may not satisfy the stability requirement: a desert made of rocks is stable for years, while the Amazonian forest decorrelates in less than 1 day.

In this thesis, we focus our attention on Sentinel-1: a C-Band radar with a revisit time of 6 days (considering both S1A and S1B). These features of the mission allow us to satisfy the stability requirement for most of the targets in a heterogeneous scene and therefore justify the usage of PL.

3.4.2. Theoretical background

This section provides a review of the Phase Linking (PL) algorithm for optimal interferometric phase estimation. For a comprehensive explanation of the algorithm, the reader is referred to [52].

Given a stack of \( N \) focussed and coregistered SAR SLC images, the main idea behind PL is to exploit all the possible \( N(N-1)/2 \) interferograms to find the best \( N-1 \) phases that fit the best the data covariance matrix. The resulting \( N-1 \) phases will be referred from now on as Linked Phases since they are the result of a joint processing of all the \( N(N-1)/2 \) interferogram.

We assume that each pixel in the SAR image is a Distributed Scatterer and therefore its value is determined by the presence of many independent scatterers in the resolution cell. For the central limit theorem, the time series \( x \in \mathbb{C}^N \) of such a pixel is a random process with zero mean and with circular complex normal distribution [53]:

\[
p(x|\phi) = \frac{1}{\pi^N|\mathbf{C}(\phi)|} \exp\left(-x^H \mathbf{C}(\phi)^{-1} x\right)
\]

where \( N \) is the number of images in the time series, \( \mathbf{C}(\phi) \) is the data covariance that depends on the phases \( \phi \) to be estimated.

\[
\mathbf{C}(\phi) = E[xx^H]
\]
Without loss of generality, we will suppose that all the images are normalized in power. For this reason, from now on we will speak about coherence matrix and not covariance matrix. The coherence matrix contains all the possible complex coherences between two interferometric couples in the stack. In particular, for the cell at row \( i \) and column \( j \)

\[
\gamma_{i,j} = |\gamma_{i,j}| \exp(j\phi_i - j\phi_j) = |\gamma_{i,j}| \exp(j\psi_{i,j})
\]  
(3.5)

\[
|\gamma_{i,j}| \in [0, 1] \tag{3.6}
\]

\[
\psi_{i,j} \in [-\pi, \pi) \tag{3.7}
\]

The coherence matrix can be also rewritten as

\[
C(\phi) = \phi \Gamma \phi^H \tag{3.8}
\]

Where \( \phi \) is the \( N \times N \) diagonal matrix containing all the phase terms, while \( \Gamma \) is the \( N \times N \) matrix of all the coherences (intended as absolute value).

For example, in the case of a stack of just 3 images we can write:

\[
C(\phi) = \begin{bmatrix} e^{j\phi_0} & 0 & 0 \\ 0 & e^{j\phi_1} & 0 \\ 0 & 0 & e^{j\phi_2} \end{bmatrix} \begin{bmatrix} 1 & |\gamma_{1,2}| & |\gamma_{1,3}| \\ |\gamma_{2,1}| & 1 & |\gamma_{2,3}| \\ |\gamma_{3,1}| & |\gamma_{3,2}| & 1 \end{bmatrix} \begin{bmatrix} e^{-j\phi_0} & 0 & 0 \\ 0 & e^{-j\phi_1} & 0 \\ 0 & 0 & e^{-j\phi_2} \end{bmatrix} \tag{3.9}
\]

By substituting Equation 3.8 into equation 3.3 we obtain

\[
p(\mathbf{x}|\phi) \propto \exp(-\mathbf{x}^H \phi \Gamma^{-1} \phi^H \mathbf{x}) \tag{3.10}
\]

The estimation of the parameter \( \phi \) is now performed by exploiting a suitable estimation window with \( L \) independent samples (\( L \) stands for looks). Notice that the underlying hypothesis is that the \( L \) samples must be gathered from a statistical homogeneous pool of samples and they should be independent. The first hypothesis requires that the phases \( \phi \) and the coherences \( \gamma_{i,j} \) are constant within the estimation window. Further considerations about the size of the estimation window will be detailed in the following sections. The PDF of the data within the estimation window is given by the product of the PDF of the single samples

\[
p(\mathbf{X}|\phi) \propto \prod_{i=1}^{L} \exp(-\mathbf{x}_i^H \phi \Gamma^{-1} \phi^H \mathbf{x}_i) \tag{3.11}
\]

where \( L \) is the number of independent samples within the estimation window, \( \mathbf{x}_i \) is the \( N \times 1 \) column vector representing the \( i^{th} \) sample. Equation 3.11 can be conveniently rewritten by exploiting simple matrix algebra as:
3.4. Phase Linking

\[
p(X|\phi) \propto \exp\left[-\text{Tr}\left(X^H \phi \Gamma^{-1} \phi^H X\right)\right] \\
\propto \exp\left[-\text{Tr}\left(\phi \Gamma^{-1} \phi^H XX^H\right)\right] \\
\propto \exp\left[-\text{Tr}\left(\phi \Gamma^{-1} \phi^H \hat{C}\right)\right]
\]

(3.12)

where \(\text{Tr}(\cdot)\) denotes the trace operator, \(X\) is the \(N \times L\) matrix containing all the samples in the estimation window, and \(\hat{C}\) indicates the sample estimate of \(C(\phi)\), or, more intuitively, the matrix containing all the possible interferograms averaged over the \(L\) samples. The sample estimate \(\hat{C}\) is estimated using a squared window: in the following section the size of this window will be discussed.

The estimation of the parameters of interest can be carried out in a Maximum Likelihood fashion \([54]\):

\[
\hat{\phi} = \arg\max_{\phi} p(X|\phi) \\
= \arg\min_{\phi} \text{Tr}\left(\phi \Gamma^{-1} \phi^H \hat{C}\right)
\]

(3.13)

Since the "true" coherence matrix \(\Gamma\) is not known one can use the estimate of \(\Gamma\) as the absolute value of the sample coherence matrix \(|\hat{C}|\). An iterative minimization with respect to each phase is performed to find the best \(N-1\) phases that fit the best the sample coherence matrix. More details about an efficient implementation will be given to the reader in Section 3.4.7.

Once the phases has been estimated, it is also customary to assess the "goodness of fit" of such phases to the estimated coherence matrix. One figure of merit could be, for example the following:

\[
\gamma = \frac{1}{N(N-1)} \left| \sum_{i=1}^{i=N} \sum_{k \neq i} e^{j\psi_{ik}} e^{-j(\hat{\phi}_i-\hat{\phi}_k)} \right|
\]

(3.14)

In other words, it is possible to take the estimated sample interferometric phases (the argument of each cell in the coherence matrix) and remove from those the estimated (linked) phases. If the linked phases completely explain all the sample interferometric phases we have that

\[
e^{j\psi_{ik}} e^{-j(\hat{\phi}_i-\hat{\phi}_k)} = 1 \quad \forall i \in [1, N]
\]

(3.15)

and therefore \(\gamma = 1\). Equation 3.14 is an extension to a stack of SAR images of the classical interferometric coherence.

3.4.3. Target Characterization

In Section 2.1 a qualitative explanation of possible target’s characteristics in terms of temporal stability has already be given to the reader and the concept of decorrelation
has been explained. It is useful, however, to properly characterize targets by means of suitable models. Such models are parametric and it is therefore mandatory to assess quantitatively such parameters to understand how the PL algorithm can be tuned to extract accurate and reliable APS maps.

From Equation 3.3 we can infer that the covariance/coherence matrix $\Gamma$ completely describes the target statistics. It is then mandatory to understand the "shape" of such covariance/coherence matrix.

In order to derive some statistics, we used some real images gathered by Sentinel-1 over central Italy. The area selected is quite heterogeneous in terms of scatterers nature and behavior: open field, forests, water bodies and cities are present in the scene. The Despeckled amplitude image is depicted in Figure 3.2.

![Despeckled Amplitude image](image)

Figure 3.2: Amplitude images of the considered scene. Bright spots are cities, while dimmer areas are water bodies. In between we find forested areas, agricultural fields, etc.

In this thesis we will focus on coherent Permanent Scatterers and decorrelating Distributed Scatterers. The model for the coherence matrix of the former is straightforward:

$$\Gamma = (1 - \gamma)I_N + \gamma 1_N 1_N^T$$  \hspace{1cm} (3.16)

where $I_N$ is the $N \times N$ identity matrix and $1$ is the $N \times 1$ vector of all ones. The parameter here is $\gamma$ which is the level of correlation (coherence) between all the images here considered constant between each interferometric couple. For a stable target, $\gamma$ could be ranging from 0.7 to value very close to 1. The entire PSInSAR [30] branch of SAR interferometry was born around such stable targets since they allow for a precise
and reliable estimate of ground deformation. Notice that this model is also perfectly valid for a decorrelating target: it is sufficient to set $\gamma$ to be a very low value. For water bodies, $\gamma$ is very close to 0.

In Figure 3.3 the coherence matrix of a stable PS in the scene is depicted. The coherence is very high (around 0.8 for all the image pairs) with a drop just in the last image, due to an unidentified change.

![Coherence Matrix for a stable PS](image)

Figure 3.3: Coherence matrix of a PS in the considered scene. Each image has a 6 days temporal baseline with respect to the previous one.

There are several models proposed in literature for the decorrelating and distributed target [25], [26], [32], [34], [55]. In this thesis we have chosen the most simple one proposed by Rocca in [26]. The model originates from the assumption that each scatterer inside the resolution cell moves with a Brownian motion (random walk). This motion manifests in the coherence matrix as an exponential decorrelation in time:

$$\gamma_{mn} = e^{-\frac{|m-n|\Delta T}{\tau}}$$

(3.17)

Where $m$ and $n$ are the image indexed, $\Delta T$ is the repetition interval of the SAR platform (in the case of Sentinel-1 it is 6/12 days) and $\tau$ is the decorrelation constant: smaller values represents fast decorrelating targets like water, while bigger value represents targets that decorrelate in longer times like open fields, snow packs, exposed rock formations and so on. We can also define:

$$\gamma_0 = e^{-\frac{\Delta T}{\tau}}$$

(3.18)
therefore

\[ \gamma_{mn} = \gamma_0^{|m-n|} \quad (3.19) \]

The important variable to assess here is \( \tau \): it can range from milliseconds (like for water bodies) to several months (like for the PS). In this case, however, we are interested in moderately coherent targets. Physically they are represented by agricultural fields, forests with small trees, rock formations and many more. The C-Band allows for the detection of such targets, while In X-Band the sensitivity to small changes is so high that everything decorrelates too rapidly (\( \tau \) is very small). In the area of Figure 3.2 we compute the coherence matrix for every pixel and fitted the model in 3.17 to find suitable values for \( \tau \). Such values, along with the required accuracy on the APS, will then drive the choice for the estimation window to be used in the APS retrieval procedure.

It is worth noticing that \( \tau \) is tightly related to seasonality: in winter, where there is no leaf growth or fall, forests are more stable. The same thing can be said for agricultural fields, but not for exposed rocks in mountain regions where the inverse happens: in summer they are very stable, while in winter when they are covered by snow they are very unstable [56].

Since we would like our algorithm to work even in the worst possible scenario, we have chosen the spring season to assess possible \( \tau \) values.

![Estimated decorrelation constant](image.png)

![Histogram of the decorrelation constant](image.png)

Figure 3.4: (a) Map of estimated decorrelation constants \( \tau \). Bright spots are cities and stable regions, while the vast majority (dark areas) are agricultural fields, forests, etc. (b) Histogram of figure (a).

In Figure 3.4 the results of the fitting are presented. In particular, in Figure 3.4a the map of the area is depicted while in Figure 3.4b an histogram of the values is introduced. It is clear that the bright spots are cities that are mainly populated by stable scatterers: their decorrelation constant \( \tau \) is, as expected, in the order of tens or even hundreds of days. The vast majority of the targets, instead, are much more unstable. The average decorrelation time of loosely coherent target is slightly above 6 days (we excluded from the average highly stable ones, that are not DS and a more suitable model for them is
the one in Equation 3.16). This value corresponds to an average short time coherence of 0.37. This is exactly the value found in Figure 3.5. In particular, in Figure 3.5a the map with the average short time coherence is depicted. This map has been formed by averaging the coherence of the short temporal baseline interferograms in the coherence matrix (i.e. the first diagonal outside the main one). On Figure 3.5b, instead, the histogram of such image is proposed.

![Mean short baseline coherence](image)

(a)

![Histogram of the mean short baseline coherences](image)

(b)

Figure 3.5: (a) Average coherence at 6 days of temporal baseline. Bright sports are cities and stable areas, while dark spots are agricultural fields, forests, etc. (b) Histogram of figure (a).

These values will be used later on (see Section 3.4.6) to derive a suitable window size for APS estimation.

### 3.4.4. Accuracy of the estimate

In this section the accuracy for the estimate of the interferometric phases is accesses. Three estimators that are compared against the Cramér-Rao Lower Bound (CRLB). These estimators are:

- The **Phase Linking** algorithm discussed in Section 3.4;
- The **AR(1)** estimator that consists in integrating the phases of interferograms formed using consecutive acquisitions:

\[
\psi_i = \angle \prod_{k=1}^{i} \hat{C}(k, k + 1)
\]

where \(\hat{C}(k, k+1)\) indicates the sample at row \(k\) and columns \(k+1\) of the estimated coherence matrix; We used this estimator since it is widely used in literature and it is the minimizer of equation 3.13 in the case of a target that decorrelates as an auto regressive process of order 1. From here the name AR(1).

- The classic **DInSAR** estimator that consists in evaluating interferograms at increasing temporal baselines w.r.t the first image and averaging them with \(L\) looks. In other words, just the first row (or column) of the coherence matrix is considered:
As we will demonstrate later on, the AR(1) can work even if the coherence is lost after a few time lag since it exploits only short-time interferograms. Each algorithm exploits, in fact, a different portion of the coherence matrix to obtain an estimate of the interferometric phase. A pictorial representation of the portion of the coherence matrix exploited by each algorithm is depicted in Figure 3.6.

![Figure 3.6: Each algorithm exploits a different portion of the coherence matrix. Phase Linking, for example, utilizes all the \( N(N-1)/2 \) interferograms, the AR(1) just the interferogram at the shortest temporal baseline, while the standard DInSAR method exploits all the interferogram at varying temporal baseline.](image)

The three different estimators are tested in the two different scenarios proposed in Section 3.4.3:

- A coherent Persistent Scatterer (PS) whose coherence is modeled as a constant in time
  \[ \gamma_i = \gamma \]  
  (3.22)

- A decorrelating Distributed Scatterer (DS) whose coherence is modeled as a Equation 3.17.

CRLB

First of all, in order to have a reference for the performances, we have to derive the Cramér-Rao Lower Bound of the interferometric phases. In [52] the Fisher information matrix has been already expressed as:

\[ \chi = 2L(\Gamma \circ \Gamma^{-1} - I_N) \]  
(3.23)

Where \( L \) is again the number of independent looks inside the estimation window, \( \Gamma \) is the coherence matrix, \( \circ \) is the point-wise Hadamard product and \( I_N \) is the \( N \times N \) identity matrix.

It is easy to demonstrate that the Fisher information matrix in 3.23 is always singular and therefore not invertible. The null space of \( \chi \) is the one dimensional space spanned

\[
\psi_i = \angle \hat{C}(1, i + 1)
\]  
(3.21)
by the constant vector \( \mathbf{1}_N \).

The intuition is that in each cell of the coherence matrix \( \Gamma \) there are phase differences and therefore it is impossible to estimate all the \( N \) phases: one will remain unknown (i.e. the master).

It is then simple to define a \((N \times 1) \times N\) transformation matrix that converts the absolute phases into interferometric phases. In the unknown phase is chosen to be the first one in the stack, the Fisher information matrix of the transformed phase is simply the Fisher information matrix of the original phases where the first row and the first column have been removed. From now on we will refer to \( \chi \) as the FIM of the transformed phases.

Notice that the FIM depends on the model that we choose for the considered target, in other words, it depends on its temporal statistics.

**CRLB for Permanent Scatterers**

In [52] the derivation of the CRLB for the PS has been addressed and will be reported here. Its decorrelation model can be expressed as

\[
\gamma_{nm} = \gamma_0 + (1 - \gamma_0) \delta_{n-m} \tag{3.24}
\]

Where \( \gamma_0 \) is the constant coherence and \( \delta_{n-m} \) is the discrete Dirac delta function. The inverse of the FIM can be found in closed form as

\[
\chi^{-1} = \frac{1 - \gamma_0}{2L\gamma_0^2} \left( \mathbf{I}_{N-1} + \mathbf{1}_{N-1} \mathbf{1}^T_{N-1} \right) \tag{3.25}
\]

On the diagonal of the inverse FIM we can read the lower value of the variance for the \( N - 1 \) estimated phases. It is important to notice that the elements on the diagonal of matrix 3.25 are equal, meaning that, the first estimated phase has the same quality of the last one. This is an expected behavior given the decorrelation model used. If the number of images is \( N = 2 \), then we have only one interferogram available and the CRB reduces to

\[
\chi^{-1} = \frac{1 - \gamma_0^2}{2L\gamma_0^2} \leq \sigma^2_\psi \tag{3.26}
\]

which is the well-known CRB for single baseline interferogram [57].

**CRLB for Distributed Scatterers**

As far as the author of this thesis knows, a closed form expression for the CRLB in the case of a distributed scatterer with exponential decorrelation model has not been derived.

In this derivation, the decorrelation model is the one presented in Section 3.4.3: an exponential decay that is direct consequence of the Brownian motion of the scatterers inside the resolution cell. If we use this definition for the CRLB evaluation and we eliminate the first row and first column of the FIM in 3.23 we obtain an invertible full-rank matrix \( \chi \). Its inverse is expressed as:

\[
\chi^{-1} = \frac{1 - \gamma_0^2}{2L\gamma_0^2} \sum_{i=1}^{N-1} \mathbf{v}_i \mathbf{v}_i^T \tag{3.27}
\]
where
\[ v_i = [0^T_{i-1} \ 1^T_{N-i}]^T \]  
(3.28)

Where \( 0_{i-1} \) is the vector of all zeros of length \( i - 1 \) and \( 1_{N-i} \) is the vector of all ones of length \( N - i \).

One example of equation 3.27 for \( N = 4 \) (a total of four images in the stack) is:
\[ \chi^{-1} = \frac{1 - \gamma_0^2}{2L\gamma_0^2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \]  
(3.29)

As in the PS case, the minimum variance for the estimate of the interferometric phase must be read on the diagonal. It is easy to see how the term outside the matrix in Equation 3.29 is the usual CRLB for the interferometric phase in single baseline interferometry. Looking at the diagonal of Equation 3.29, we notice that the variance of the estimate grows linearly with time.

This is an expected behaviour for the chosen coherence model. For a DS that shows exponential decay in coherence, the optimal phase estimator is the AR(1) in Equation 3.20. This is easily proven given that \( \Gamma^{-1} \) in Equation 3.13 is tri-diagonal therefore only short time interferograms are exploited for the estimate, exactly like the AR(1) estimator. The estimate, however, involves the integration of these short time interferograms which leads to error accumulation (as shown in Equation 3.29).

In Section 3.4.6 the evaluation of the minimum number of looks that guarantees an accurate estimate of the phase is proposed.

3.4.5. Performance assessment

In this section, the performances of the three estimators introduced in Section 3.4.4 are discussed.

In Figures 3.7 and 3.8 the results from a Monte-Carlo simulation are depicted. In these simulations 10 SAR images are present in each stack with a consecutive temporal baseline of 6 days (repetition interval). For the estimation of the coherence matrix 600 independent samples are taken. Since this is a simulation we can ensure the statistical independence between samples. In a real SAR image the estimation of the coherence can be done using a moving average and by considering both the resolution and the sampling spacing of the image. More on this topic will be discussed in Section 3.4.6.

This specific number of looks is the one needed for the estimation of the APS with a standard deviation of 1 mm in presence of a decorrelating DS showing temporal decorrelation (\( \tau \)) of 6 days. It will be derived in Section 3.4.6. In Figure 3.7a the coherence matrix of the simulated stable scatterer is depicted. Notice how the each image is coherent with all the others with \( \gamma = 0.5 \).

In Figure 3.7b the variance on the estimates of the interferometric phases is depicted along with the CRLB.

The variance on the estimate of the phase depends on the estimator used. Both PL and DInSAR show a constant phase noise: this behaviour is consistent with the kind of decorrelation imposed by the simulation (a constant one) and predicted by the CRLB defined in Section 3.4.4. Conversely, the AR(1) estimator shows a phase variance that
3.4. Phase Linking

Figure 3.7: (a) Absolute value of the coherence matrix in the PS scenario. (b) Variances of phase estimates for three different estimators. The CRLB is also depicted.

increase in time due to the accumulation of the error provided by the integration of several interferograms.

In Figure 3.8a the coherence matrix of a decorrelating target is depicted.

Figure 3.8: (a) Absolute value of the coherence matrix in the DS scenario. (b) Variances of phase estimates for three different estimators. The CRLB is also depicted.

The decorrelation is modeled as an exponential decorrelation and, as previously stated, the AR(1) estimator leads to a global minimizer of 3.13 and the PL does the same (Figure 3.8b). The DInSAR estimator, instead, produces an error that increases in time, as expected by this specific decorrelation model. It is interesting to notice how, in Figure 3.7b the phase variance for the AR(1) estimator increases linearly, while in Figure 3.8b the variance for the DInSAR is not increasing linearly. This behavior is expected: the AR(1) integrates (sum) phases and therefore the variances are summed and the trend is linear. On the other end the decorrelation model
in the DS is not linear, but exponential, leading to a different trend in the variance of the DInSAR estimate.

The important element to notice is that the Phase Linking always attains the lower limit for the variance on the estimate. This the most important feature for this kind of algorithm. There is no need for a detection of stable targets (PS) since the PL will always result in the best possible estimate for the interferometric phase, independently on the statistical characteristics of the area imaged.

3.4.6. Phase Linking for APS estimation

The Phase Linking algorithm is a generic estimator for interferometric phases. It can be used with an arbitrary operational frequency and with any repetition interval. Some practical precautions are needed in order to apply this algorithm for atmospheric phase screen retrieval:

- The total temporal extent of the stack must be carefully tuned;
- The size of the moving average window where the coherence matrix is estimated must be carefully selected;

The first consequence of reducing the number of images in the stack is to reduce the effect of terrain deformations. In the model of the interferometric phase, a therm due to the presence of a ground deformation is present. In particular, in presence of a constant Line Of Sight (LOS) velocity, Equation 2.7 can be rewritten as

$$\psi_{rg} = \frac{4\pi}{\lambda} v_{los} \Delta t$$  \hspace{1cm} (3.30)

where the deformation has been modeled as a linear with time (i.e. a constant velocity expressed with $v_{los}$). Here $\Delta t$ represents the temporal baseline of the interferogram. In the presence of a normal subsidence rate in the order of 10-20 mm/year, if the stack is kept short the impact of deformation on the interferometric phase in minimal. If we consider 8 images with a total temporal extent of about 50 days (in the case of Sentinel-1), the error will be less than 2 mm. This bias in the estimate is tolerable for our purposes. Notice that these considerations are valid just in the presence of a small-medium subsidence rate, not for very strong deformations or earthquake where the subsidence can be in the order of several tens of cm. In other words, a minimal ground stability is required for the algorithm to work properly.

Notice that in principle a linear deformation can also be estimated and removed from the data. In this case, however, the number of images must be large in order to correctly separate deformation, which is linear in time, from atmosphere, which is completely random in time. Following the decorrelation model explained in Section 3.4.3, we can say that the stack temporal extent needs to take into consideration the average “life” of a distributed scatterer. As already pointed out, with phase linking, we form all the possible interferograms with $N$ images and from them, we estimate $N - 1$ phases, if the coherence of the interferograms with very long temporal baseline is very low, they will bring noise into the
3.4. Phase Linking

final estimate. A solution is then to reduce the maximum temporal baseline.

The size of the estimation window must be carefully tuned. The quality of the interferometric phase is tightly related to the number of independent looks used to estimate the coherence matrix.

For a single interferometric couple, the variance on the estimate of the interferometric phase is given by [57]:

$$\sigma_{\psi}^2 = \frac{1 - \gamma^2}{2L\gamma^2}$$  \hspace{1cm} (3.31)

Given a stack of $N$ images, and in presence of a decorrelating DS, Equation 3.29 shows that the quality of the last image is the worst and it is equal to:

$$\sigma_{\psi,N-1}^2 \geq \frac{1 - \gamma_0^2}{2L\gamma_0^2} (N - 1)$$  \hspace{1cm} (3.32)

This number depends on the number of looks $L$ used (i.e. the number of independent sample in the coherence matrix estimation window) and on the true coherence of the target. A target with poor coherence can still generate a good estimate of the interferometric phase if a sufficient number of looks is used.

In order to evaluate the minimum number of looks required, we assume an exponential decay of the coherence with temporal constant $\tau$ of 6 days, a sampling interval of 6 days as for the C-Band Sentinel-1 constellation and a total of 10 images in the stack. The required standard deviation is set to be $\sigma_R = 1$ mm: this requirement is set for the last phase in the stack, namely the one that is noisier among all the estimated phases.

It is then straightforward to invert Equation 3.32 to obtain the minimum number of independent looks to be used.

$$L \geq \left( \frac{\lambda}{4\pi} \right)^2 \frac{1 - \gamma_0^2}{2\sigma_R^2\gamma_0^2} (N - 1)$$  \hspace{1cm} (3.33)

With these constraints, we obtain a number of looks that is roughly $L = 600$ (See Figure 3.9). Sentinel-1 shows a nominal resolution of $5 \times 20$ m$^2$ in IW acquisition mode in range and azimuth respectively. The images are over-sampled at about $2.5 \times 15$ m$^2$. In order to obtain a number of independent looks equal to 600, we need roughly 3000 pixels. To cope with severely decorrelating targets and to have the usual 10-15% overhead we choose a $149 \times 25$ window spanning $375 \times 375$ m$^2$.

One possible wrong interpretation of Equation 3.33 is that we need more looks with longer wavelength (lower operational frequencies). This is not a correct interpretation since the value of $\gamma_0 = e^{-\Delta T \tau}$ depends on the wavelength itself since $\tau$ depends on it. At higher frequencies the sensibility of the system to slight changes in the target is very high, therefore even the smallest change can cause decorrelation. In such systems, $\tau$ is very small. On the opposite side, at longer wavelength such as in L or P band, the sensitivity is very small, leading to images that are very coherent even after a long temporal baseline. In this scenario $\tau$ can be even in the order of months.
In Figure 3.9 the number of independent looks required at different frequencies to have a standard deviation below 1 mm. The simulated scenario is a decorrelating DS with a total of 10 images. The decorrelation constant has been modeled as in [26] as:

$$\tau = \frac{2}{\sigma_{Bd}^2} \left( \frac{\lambda}{4\pi} \right)^2$$

(3.34)

where $\sigma_{Bd}$ is the standard deviation associated to the Brownian motion of the scatterers inside the resolution cell expressed in mm/$\sqrt{\text{days}}$.

In our studies carried over several datasets with different location and scatterer characteristics and with temporal baseline of 6 days, we found that such an estimation window is big enough for allowing reliable evaluation of the APS, but small enough to comply with OSCAR requirements for spatial resolution. In this contribution, we decided to use the twin Satellites Sentinel-1 A/B: their C-Band (5.40 GHz) payload, together with the 6 days repeated geometry and closely spaced and accurate orbits make the constellation the perfect instrument for the generation of differential SAR APSs. A very interesting perspective for the future is the possibility to have a geosynchronous platform that shows a repetition interval in the order of hours and not days. The lower SNR provided by geosynchronous SAR can be compensated by the fact that the decorrelation is heavily limited by the short repetition interval. Geosynchronous SAR will open the possibility to map troposphere with an unprecedented spatial and temporal resolution.
3.4. Phase Linking

3.4.7. Efficient Phase Linking implementation for APS estimation

In this section, some suggestions for an efficient implementation of the Phase Linking algorithm are presented.

Phase Linking can be easily summarized as "Find the best rank-1 approximation of the estimated coherence matrix" and it could be, in principle, implemented in just a few lines of code. For the purpose of APS estimation, however, a few tricks must be implemented to reach higher accuracy and low computation time:

- **Weight the coherence matrix by itself.** The first step of PL is to estimate the coherence matrix using a spatial estimation window. As already mentioned, the coherence matrix contains all the possible interferograms between couples in the stack. Each cell of the coherence matrix contains a complex number (the complex coherence). While the phase is considered as the data useful for the estimation, the coherence is the quality of that data. The algorithm needs to properly weight the input data: in this way it avoids to try to justify interferometric phases with low coherence reaching high accuracy faster. One way to force the algorithm to discard interferograms with very poor coherence, is to weight the coherence matrix by itself, thus not providing just $\hat{C}$ as an input, but $\hat{C} \circ |\hat{C}|$ (where $\circ$ is the element-wise Hadamard product).

- **Spatial sub-sampling.** As already mentioned, the coherence matrix estimation window must be quite large in order to use several looks reaching in this way a good accuracy. The direct consequence is that the resolution of the final estimated APS is comparable with the size of such estimation window. Therefore, it is useless (and a waste of computational resources) to perform the estimation of every single pixel of the original product. It is instead much more efficient to under sample the data both in range and azimuth right after the estimation of the coherence matrix.

  It is suggested to choose carefully the undersampling factor. In principle, if the resolution of the APS is, for example, 500 metres, it is possible to under sample the data up to a pixel size of 500m. In the case of Sentinel-1 with 2.5m and 15m of pixel size in range and azimuth respectively, it would lead to an undersampling factors of $N_{sub}^{rg} = 200$ and $N_{sub}^{az} = 33$.

  Such strong under sampling, however, may lead to errors in the phase unwrapping procedure that follows the phase estimation. It is recommended to avoid extreme undersampling and embrace, instead, a softer one. The author found that a good balance between computational burden and unwrapping accuracy are under sampling factors of 40 e 8 in range and azimuth respectively.

- **Efficient numerical computation.** While in principle PL can be implemented in just a few lines of code, an efficient implementation is customary when exploiting stacks of images that are spatially wide and temporally long. In this case, in fact, the efficient implementation is mandatory to avoid extremely long processing time and a huge memory consumption. The implemented procedure is able to compute all the linked phases simultaneously for all the pixels and it is summarized in the following:
1. First of all we need to estimate the coherence matrices for all the pixels. The data is organized as a 3D matrix \((N_{rg} \times N_{az} \times N)\). The outer product of each pixel is formed in the third dimension leading to a 3D matrix whose third dimension is of size \(\left(\frac{N}{2}\right)^2\). A convolution in the first two dimensions (spatial dimension) is performed to estimate covariance matrices. For each pixel, the third dimension represents a vectorized covariance matrix.

The data is under sampled and the power along the diagonal is normalized to form the coherence matrix. Since all these steps can be performed by means of simple matrix multiplications, the fastest implementation relies on GPU processing. The next step is a spatial sub-sample as explained in the first point. The workflow is depicted in Figure 3.10.

![Figure 3.10: Processing workflow](image)

Figure 3.10: Processing workflow: we start from a set of coregistered and phase compensated SAR images. The outer product is computed pixelwise and a spatial average is computed to generate covariance/coherence matrices. The result is saved in a vectorized form leading to a matrix of size \(N^2\) in the third dimension. The matrix is then sub-sampled in the first two dimensions. The matrix is now provided as an input to the phase estimator (PL).

2. In order to estimate the best \(N\) phases that fit the best the sampled coherence matrix, we start from an initial guess of the \(N\) phases \(\phi_0\).

We also realize that in the first column of the coherence matrix we have all the interferograms having as a master the first image. The idea is then to remove the initial guesses of the \(N - 1\) phases (except from the first one) from the interferograms. What is left in each interferogram is the first phase, plus some noise. By coherently integrating these \(N - 1\) interferograms we obtain an updated guess of the first phase. The same process is repeated with the second column of the coherence matrix and so on up to the \(N^{th}\) column. The entire process is repeated again up to convergence or up to a maximum number of iterations \(K\).

The estimate of the \(n^{th}\) interferometric phase at the \(k^{th}\) iteration can be expressed as:

\[
\psi_n^k = \angle \left( \sum_{p \neq n} C(:, n) \exp(-j \psi^{k-1}) \right)
\]  (3.35)

In Figure 3.11 a visual interpretation on the algorithm is depicted. The advantage of this kind of method is that it can be implemented to work in
3.5. Phase Unwrapping

Once the phases are estimated, they are still only known modulo $2\pi$ (i.e., they are wrapped) [58], [59]. In order to unwrap the phases, we rely on the fact that the spatial variation of such phase field is smooth from resolution cell to resolution cell with respect to the ambiguity, which is defined as the length that makes the phase wrap. This quantity corresponds to half the operational wavelength of the system: in the case

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**Future work: split long stacks into smaller sub-stacks** As already mentioned, having a long stack of images forces the algorithm to deal with subsidence (modelling and removal) and temporal decorrelation. In some situations, however, it could be needed to have a long temporal time series of linked phases. If this is the case, it is suggested to avoid the joint processing of all the SLC simultaneously, since the number of interferograms in the coherence matrix increases quadratically with the number of images leading quickly to a very high computational burden. What is suggested is to split the data into two or sub-stacks as depicted in Figure 3.12. The phase estimation algorithm is run over each sub-stack separately and then, if the sub-stacks have at least one image in common they can be merged into one single stack.
of Sentinel-1, it is about 2.8 cm. Phase unwrapping solves the ambiguity by integrating phase differences between neighboring pixels. The phase unwrapping procedure implemented in this work is a simple inversion of a linear system of equation. It is much easier to write the equations in one dimension knowing that the extension to the 2D scenario is trivial. The model can be written as

\[
\begin{bmatrix}
\chi_0 \\
\chi_1 \\
\chi_2 \\
\vdots \\
\chi_N
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
-1 & 1 & 0 & \ldots & 0 \\
0 & -1 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & -1 & 1
\end{bmatrix}\begin{bmatrix}
\psi_0 \\
\psi_1 \\
\psi_2 \\
\vdots \\
\psi_N
\end{bmatrix}
\]  

(3.36)

Where the vector \(d\) contains the derivatives of the phase (i.e. the phase difference between a pixel and the adjacent one), \(H\) is the design matrix and \(\psi\) is the vector of the unknown phases with the ambiguity removed. Notice that the first row of the design matrix is needed to regularize the problem, fixing a one unknown, in this case \(\psi_0\) to be equal to the first data, in this case \(\chi_0\). Without this line the matrix is obviously singular, with the null-space spanned by the constant vector: a reminder that after the unwrapping the phase must again be read as a phase difference between two pixels.

It is also straightforward to see that the inverse of matrix \(H\) is the integration matrix:

\[
H^{-1} =
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
1 & 1 & 0 & \ldots & 0 \\
1 & 1 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \ldots & 1
\end{bmatrix}
\]  

(3.37)
The problem of equation 3.36 can be solved in several ways. We decided to implement a robust inversion of the problem in L1 norm using the Iterative Reweighted Least Squares (IRLS) algorithm [60]. The inversion in L1 norm requires several iterations of L2 inversion with varying weighting matrices. In the first step, however, the weighting matrix must be imposed. An obvious choice is the estimated coherence for each pixel. The algorithm is then forced to rely on phase gradients on highly coherent regions while discarding data over poorly coherent areas where the phase is noisy.

In some cases the phase unwrapping procedure may generate an error. When in the interferogram an area is surrounded by a low coherence "belt", it may happen that a phase "island" appears with a $\lambda/2$ shift with respect to the rest of the image. An example is depicted in Figure 3.13. In Figure 3.13a the phase to be unwrapped is depicted. At the center an isolated area closed by a loop of poorly coherent data is present. In a real scenario the loop can be, for example, a river or a forest. In Figure 3.13b the result of the phase after the unwrapping while in 3.13c the error with respect to the ground truth. As evident that a discontinuity is present and while outside this portion of the interferogram the phase error is zero, inside the phase error is $2\pi$, or $\lambda/2$.

In a real scenario where coherence can be low and the phase very noisy, unwrapping
errors are a serious threat. In this thesis, a combination of short temporal baselines, a C-Band radar, small spatial baselines and large phase estimation window allow to avoid, in most of the cases, these problems. In the first case study, however, a combination of strong phase gradients originated from strong topography and very low coherences originated from the presence of rivers and forests have caused the presence of phase discontinuities in the final interferogram.

3.6. Orbit correction and interferometric constant estimation: GNSS processing and integration

In this section we demonstrate how it is possible to jointly exploit SAR and GNSS atmospheric measures to estimate and remove the orbital error from interferograms. In order to properly compensate the flat-earth and topography term, precise information on the position of the platform during the acquisition (orbital information) and a precise DEM are necessary [61], [62].

The effect of a residual orbit error is usually neglected if the area of interest is very small or if the Orbit State Vectors (OSV) are known to be very precise. In the case of Sentinel-1, the precise orbital product, available after 20 days from the acquisition, allows for accuracy in the 3D orbit determination of 5 cm (RMS). This figure is referred to a single acquisition, so the accuracy in the interferometric framework is even worse. We will see in this section that the orbital error can be modelled as a plane in the interferogram. In some application is possible to exploit this information by estimating and removing a trend (plane) from the interferogram. This procedure can’t be applied when the estimate of interest is the APS, in fact, the APS itself can be modeled in the exact same way and the result would be to invalidate the atmospheric map generated: part of the signal of interest will be considered as orbital error and removed.

We rely instead on a network of GNSS stations to properly compensate for orbital errors. The meteorological applications of GNSS are well known [21], [63]. A GNSS receiver can to determine its coordinates in a global reference frame by using simultaneous observation of its distance from a number of satellites of known position [64]. The distances are derived from the time it takes to the GNSS signals to cover the satellite-receiver paths and by assuming that they travel with the constant velocity (speed of light in vacuum).

This assumption results in an observed distance different from the geometrical one by an amount called atmospheric delay. A proper combination of the dual-frequency GNSS signals can compensate the delay induced by the ionosphere leaving just a tropospheric component.

The tropospheric delay has to be accounted for in the GNSS data processing to get accurate positioning. To this aim, the tropospheric delays affecting the signals received by a GNSS station from all the satellites in view are expressed as a common delay in the zenith direction above the receiver. Each slant delay is projected onto the zenith direction with a known mapping function. The common zenith projection is then estimated, for instance by modeling it as a random walk process, resulting in a high temporal resolution time series of zenith delays for each station.

In this work, the free and open-source software goGPS [65] was used to estimate ZTD
time series from the GNSS raw observations. The procedure for orbit correction that exploits both SAR and GNSS observations is explained in the following.

The absolute interferometric phase for a target placed at off-nadir angle $\theta$ and azimuth time $t$ can be written as:

$$\psi(\theta, t) = \frac{4\pi}{\lambda} B_{\parallel}(\theta, t)$$

(3.38)

Where $B_{\parallel}(\theta, t)$ is the parallel baseline as seen by a target at position $(\theta, t)$. Notice that the compensation of the orbital error happen interferogram by interferogram. For clarity in the formulae we omit the time variable concerning the acquisition times, or, in other words:

$$\psi(\theta, t; t_1, t_2) \rightarrow \psi(\theta, t)$$

(3.39)

where $t_1$ and $t_2$ are the two acquisition time instant of the images forming the interferogram. It is easy to notice that the interferometric phase has a linear relationship with the parallel baseline, therefore:

$$\psi_e(\theta, t) = \frac{4\pi}{\lambda} B_{e\parallel}(\theta, t)$$

(3.40)

Where $B_{e\parallel}(\theta, t)$ is the error of parallel baseline as seen by a target at position $(\theta, t)$ and $\psi_e(\theta, t)$ is the phase error induced by such error in the baseline (trajectory of the radar).

By expanding Equation 3.40 in Taylor series around a reference point $(\theta_0, t_0)$ we have

$$\psi_e(\theta, t) = \frac{4\pi}{\lambda} B_{\parallel}(\theta_0, t_0) + \frac{\partial \psi_e(\theta, t)}{\partial \theta} \bigg|_{\theta=\theta_0} \Delta \theta + \frac{\partial \psi_e(\theta, t)}{\partial t} \bigg|_{t=t_0} \Delta t$$

$$= \frac{4\pi}{\lambda} B_{\parallel}(\theta_0, t_0) + \frac{4\pi}{\lambda} B_{\perp}(\theta_0, t_0) \Delta \theta + \frac{4\pi}{\lambda} \frac{\partial B_{\parallel}(\theta, t)}{\partial t} \bigg|_{t=t_0} \Delta t$$

(3.41)

Where $\Delta \theta = \theta - \theta_0$ and $\Delta t = t - t_0$.

From Equation 3.41 we can say that, with a wavelength of 5.5 cm and a variation of incidence angles from $29.1^\circ$ to $46^\circ$ in near and far range respectively, a normal baseline error of 9.4 cm can generate an interferometric fringe in range. At the same time a parallel baseline derivative of 1.1 mm/s can lead to an interferometric fringe in azimuth. These values are summarized in Table 3.1.

The model of Equation 3.41 describes only the orbital error, therefore in the data only this component should be present. This means that $\psi_e(\theta, t)$ must be an APS-free interferometric phase. When we take a few Ground Control Points (GCP) in the scene an we read the interferometric phase, it must be only due to orbital errors. In our interferogram, however, we have the sum of orbital error and the APS that are not
3. Atmospheric Phase Screen Estimation

<table>
<thead>
<tr>
<th>Sensor mode</th>
<th>λ</th>
<th>Scene size</th>
<th>∆θ</th>
<th>( b_\perp )</th>
<th>( \frac{\partial B_\perp}{\partial t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sentinel-1 IW</td>
<td>5.5 cm</td>
<td>170 × 250 km</td>
<td>16.9 °</td>
<td>9.4 cm</td>
<td>1.1 mm/s</td>
</tr>
</tbody>
</table>

Table 3.1: Baseline errors inducing one fringe of error in the interferogram.

separable since they share the same spatial and temporal statistics. We exploit a prior given by the GNSS: we take as GCP the positions of the GNSS stations in the scene. The GNSS provides the value of the APS over that area and we can subtract it from the unwrapped interferometric phase to obtain the correct value of \( \psi_e(\theta, t) \) to be used in Equation 3.41.

The result of the orbital correction estimation are heavily dependent on the quality of the GNSS-derived APS. In general, a geodetic-grade receiver allows for an accuracy in the order of 5-10 mm (\( \sigma \)) in the ZTD estimation. Now we differentiate spatially (phase locking) taking a reference point with phase \( \frac{4\pi}{\lambda} B_e^e(\theta_0, t_0) \). Equation 3.41 is then linear system of equation with \( M \) equations (being \( M \) the number of GNSS stations in the scene) and 2 unknown, namely \( \frac{\partial B_e^e(\theta, t)}{\partial t} |_{\theta=\theta_0}^{t=t_0} \) and \( B_e^e(\theta_0, t_0) \).

The system can be rewritten in matrix notation as:

\[
\begin{bmatrix}
\psi_e(\theta_1, t_1) - \psi_e(\theta_0, t_0) \\
\psi_e(\theta_2, t_2) - \psi_e(\theta_0, t_0) \\
\vdots \\
\psi_e(\theta_M, t_M) - \psi_e(\theta_0, t_0)
\end{bmatrix} =
\begin{bmatrix}
(\theta_1 - \theta_0) & (t_1 - t_0) \\
(\theta_2 - \theta_0) & (t_2 - t_0) \\
\vdots & \vdots \\
(\theta_M - \theta_0) & (t_M - t_0)
\end{bmatrix}
\begin{bmatrix}
\frac{\partial B_e^e(\theta, t)}{\partial t} |_{\theta=\theta_0}^{t=t_0} \\
B_e^e(\theta_0, t_0)
\end{bmatrix}
\]

Then, the system can easily be solved in least square sense as:

\[
\hat{b} = (A^T A)^{-1} A^T y
\]  \( (3.43) \)

Or, as proposed in the case study section of this thesis, in L1 norm, leading to a much robust estimation capable of automatically discarding outliers in the data such as malfunctioning GNSS stations or poorly coherence interferometric data.

Once the two parameters have been estimated, the forward problem is computed for the whole extent of the interferogram:

\[
y_d = A_d \hat{b}
\]  \( (3.44) \)

Where \( A_d \) is the \( (N_r \times N_{az}) \times 2 \) design matrix for the whole interferogram, and not just the pixel corresponding to the position of the GNSS stations. In this way we form \( N - 1 \) orbital phase screen that are then compensated from each of the \( N - 1 \) interferograms in the stack.

In Equation 3.42 just two unknown are present, therefore, in principle, just two active GNSS stations are needed to perform the inversion. However, as well understood in estimation theory, more data leads to an accurate estimation of the parameters. The
orbit correction procedure described here works better when it is possible to exploit 10 or more GNSS stations. It is also recommended that the stations are distributed in the scene and not concentrated around a specific area in order to obtain a good conditioning of the problem preventing noise injection into the solution.

A further consideration can be done about ionosphere compensation using GNSS. In each SAR interferogram a ionospheric component is present. Such component represents the difference in ionospheric conditions between the two acquisitions. Usually the ionosphere in the interferogram can be considered very smooth and can be approximated by a plane superimposed to the interferogram exactly as the orbital plane. If the GNSS stations work in with dual frequency, it is possible to estimate and compensate from GNSS derived ZTD products the ionosphere, leading to a ionospheric-free absolute zenith delay. When removing the GNSS-derived ZTD from the interferograms affected by ionosphere, it is no more true that the only signal present in the data is the orbital plane, in fact, also the ionospheric plane is present. The result is that ionosphere is considered as orbital plane, and therefore is estimated and corrected. Plane-like tropospheric delays are instead correctly preserved.

As a final note on orbit error correction, we can also state that it is possible to qualify the quality of the GNSS stations by looking at the residual after the inversion. Provided that we have a sufficient number of stations:

\[ e = y - A\hat{b} \tag{3.45} \]

A station that shows a very high error is already discarded thanks to the robust inversion, but it is also marked as unreliable. With a sufficient number of GNSS stations it is also possible to estimate the covariance of \( \hat{b} \). If the inversion is a estimation in L2 norm, we have:

\[ \text{cov}(\hat{b}) = (A^T A)^{-1} \sigma_w^2 \tag{3.46} \]

Where \( \sigma_w \) is the power of the noise that can be estimated using a large number of GNSS stations as:

\[ \sigma_w^2 = \frac{e^T e}{M} \tag{3.47} \]

The routine for the correction of the orbital error is able to estimate and correct an orbital plane. It is well known, however, that interferometry is differential in space: it only make sense to evaluate a phase difference between two points in space, not its absolute interferometric phase. This is the reason for the double difference in Equation 2.19. The null space is a constant: adding a constant value to an interferogram does not change its meaning.

In order to estimate the constant for each interferogram, we rely again on the network of GNSS stations. The difference between the GNSS-derived APS and the SAR-derived APS interpolated over the positions of the GNSS stations should be just a constant. It is now easy to write a linear model as:

\[ e = y - A\hat{b} \tag{3.45} \]
where the $M \times 1$ measurement vector $y$ is the vector containing all the APS residuals for each station in the network while $k$ is the interferometric shift to be estimated (one for each interferogram in the stack).

Notice that in the case of an estimation in L2 norm in presence of white Gaussian noise the result corresponds to the sample mean of the measurements. In our implementation the value of $k$ is found with a robust inversion (L1 norm) of Equation 3.48.

3.7. From differential APS to absolute ZTD

In this section, we propose a way to derive absolute Zenith Tropospheric Delay (ZTD) maps starting from differential APS. NWPM are able to ingest directly delay maps, without the necessity to convert them in Precipitable Water Vapor (PWV). The only requirement is that the maps must provide an instantaneous snapshot of the atmospheric conditions at a specific time $t_i$.

As already mentioned, once the APS have been unwrapped, corrected for orbital error and corrected for the interferometric constant, they represent the difference of atmospheric delay between an image and a reference one. In the case of a joint processing of $N$ images, $N - 1$ maps are generated and one remains unknown.

For a single pixel in the scene and supposing that the unknown master image is the first one in the stack, we have:

$$
\psi = \begin{bmatrix}
\psi_{1,2} & \psi_{1,3} & \cdots & \psi_{1,N}
\end{bmatrix}^T
$$

$$
= \begin{bmatrix}
(\phi_1 - \phi_2) & (\phi_1 - \phi_3) & \cdots & (\phi_1 - \phi_N)
\end{bmatrix}^T
$$

Where $\psi_{i,j}$ represents the differential interferometric phase between image $i$ and $j$, while $\phi_i$ is the absolute ZTD.

Given a vector of differences it is impossible to derive the original ZTD since the system would be under determined and therefore not uniquely invertible. There are, in fact, infinite solutions to the following problem:

$$
\psi = H\phi
$$

where $H$ is the $(N - 1) \times N$ matrix of differences:

$$
H = \begin{bmatrix}
1 & -1 & 0 & 0 & \cdots & 0 \\
1 & 0 & -1 & 0 & \cdots & 0 \\
& & & \ddots & \vdots & \\
1 & 0 & 0 & 0 & \cdots & -1
\end{bmatrix}
$$
and

\[ \phi = [\phi_1 \ \phi_2 \ \cdots \ \phi_N]^T \] (3.52)

In this problem the matrix \( \mathbf{H} \) is rank deficient and the null space is spanned by the constant vector.

They only way to estimate such missing master ZTD is to involve in the estimate some prior information, such as NWPM data. Once the master ZTD has been estimated, it is trivial to find the slave ZTD by simply exploiting Equation 3.49:

\[ \hat{\phi} = \hat{\phi}_1 - \psi \] (3.53)

Several algorithms have been studied in literature [66], [67], and in [68] we proposed a comparison between the time-average approach and the single master approach detailed here:

- **A time average approach.**
  The appearance of this method is in [69] and also [16] uses this method. This algorithm needs a set of \( N - 1 \) NWPM-derived absolute ZTD as an input data. For each pixel, the time average of such ZTD is taken and added to the time average of all the interferograms. In other words:

\[ \phi_1 = \frac{1}{N-1} \sum_{i=2}^{N} \psi_{1,i} + \frac{1}{N-1} \sum_{i=2}^{N} \phi_{i}^{NWPM} \] (3.54)

where \( \phi_{i}^{NWPM} \) is the absolute phase corresponding to the zenith tropospheric delay calculated at time instant \( i \) by a NWPM.

This method rely on the fact the the average of the interferogram corresponds to the master image minus the average of all the slave images. If this average is well approximated by the average of the prior ZTD (for example some ZTD given by a NWPM), then the resulting phase approximate well the missing master ZTD.

- **A single master approach** This algorithm is the most simple one and comprehends the injection of a single master image in the stack by means of a NWPM:

\[ \phi_1 = \phi_1^{NWPM} \] (3.55)

It is possible to prove that, for big \( N \), the mentioned algorithm coincides with the minimization of the following cost function:

\[ J(\phi) = \|\phi^{NWPM} - \phi\|_2^2 + \lambda^T(\psi - \mathbf{H}\phi) \] (3.56)

Where again \( \phi^{NWPM} \) is the \( N \times 1 \) vector containing all the ZTD derived by the NWPM, \( \phi \) is the unknown \( N \times 1 \) vector to be estimated, \( \psi \) is the \((N-1) \times 1\) vector of the linked phases and \( \lambda \) is the Lagrangian multiplier.

The solution to this minimization with respect to \( \phi \) is the one that is the most similar to the prior, while still respecting data reconstruction integrity.
In both methods it is customary to find a very accurate master since an error on the estimate of such map will impact directly on the estimate of all the other absolute ZTD maps. A good approach is, after having estimated the master image, to compare it with GNSS measurements. If GNSS and master image are in accordance in terms of low bias and small standard deviation, then it is possible to proceed with the estimate of the slave images. If however they are not in accordance, it is possible to change the master image that needs to be estimated. As already mentioned, in fact, in all the equations written up to now, the missing image was the first one and all the interferometric phases were referring to that unknown map. Nothing prohibits to have another image as the unknown one. Let’s suppose to have a stack of interferometric phase where the first one is unknown and therefore set to 0:

$$\psi = \begin{bmatrix} 0 & (\phi_1 - \phi_2) & (\phi_1 - \phi_3) & \cdots & (\phi_1 - \phi_N) \end{bmatrix}^T$$ (3.57)

We want to change the unknown image to, for example, the third one. It is sufficient to remove the third interferometric phase to the whole stack

$$\psi = \begin{bmatrix} 0 & (\phi_1 - \phi_2) & (\phi_1 - \phi_3) & \cdots & (\phi_1 - \phi_N) \end{bmatrix}^T - (\phi_1 - \phi_3) = \begin{bmatrix} (\phi_3 - \phi_1) & (\phi_3 - \phi_2) & 0 & \cdots & (\phi_3 - \phi_N) \end{bmatrix}^T$$ (3.58)

The new stack still contains an unknown master image to be estimated, but this time it is the first which could be easier to estimate.

The NWPM used in both techniques can be a GACOS product [12], [70], [71], ERA5 product or WRF product. In [68] we derived that the time average approach does not guarantee that the estimated master image is better: if the images included in the time average have a large bias, this error will be propagated to the master image and in turn to all the ZTD. The best method is to find the smoothest master image in the stack of NWPM-derived ZTD (the one showing lower spatial standard deviation) and use that as a master image using the single master approach.

### 3.8. Evaluation of the proposed technique using simulated InSAR data

In this section the method for the estimate of differential APS is validated with a set of simulated SAR images. While synthetic data will never be able to capture the complexity of real data, a numerical simulation can be still useful to assess the performances of the estimation method. With a simulated dataset, we are able to control every aspect and detail of the data and the scene. In the following, the simulation strategy is explained in details.

#### 3.8.1. Simulation strategy

We decided to use a real location to generated simulated data. The idea is to use Digital Elevation Model of a real scene to generate an amplitude image projected...
in radar coordinates and then add all the phase therms needed for the simulated data to be realistic: the topographic and flat earth phase, the atmospheric phase and the temporal decorrelation of the scene.

Topographic phase and DEM errors
As already mentioned one of the main component of the interferometric phase is the topographic one. This component arises only if there is a spatial (normal) baseline between two acquisitions. We started with a master (reference) flight at a particular position in space and then we selected randomly a set of $N - 1$ baselines. The baselines will stay in a orbital tube of 100m of radius in order to simulate the Sentinel-1 behavior. The DEM used is depicted in Figure 3.14a. It is extracted from a tile of SRTM 1-Sec over South Africa (the same dataset used for one of the case studies of this thesis). It shows significant topography (up to 2400 m) and topography gradients.

Once the entire stack of images has been generated, the first step will be to flatten the interferogram and remove the topographic phase. In order to properly simulate a real environment where the topography is known just up to the error on the DEM, in the topographic phase removal routine a corrupted DEM has been used. The noise over the DEM has been modeled a uniform process with zero mean and with non-zero probability density function between -20 and 20m. This is way higher than the nominal vertical accuracy of the SRTM 1-Sec used for the processing with real data. The resulting DEM error image has been then low-pass filtered to simulate a smooth error of the DEM w.r.t. the actual topography. The DEM error is shown in Figure 3.14b.

APS
There are several ways in which the APS can be simulated. One of them is to generated a spatially correlated and temporally uncorrelated signal taking into account also the
dependence of the APS on topography. A much simpler way, however, is to use directly a phase screen provided by a NWPM. One of the most used services by the InSAR community is GACOS (Generic Atmospheric Correction Online Service). It provides ZTD maps to be used for the correction of InSAR products like co-seismic interferograms.

We decided to use these maps in the opposite way: applying them to the synthetic data in order to simulate the effects of the APS. In this way we can rely on the fact that all the complexity of the atmospheric signal (like the dependency on height, the spatial and temporal correlation and so on) are considered by the simulated data stack.

Temporal decorrelation

The temporal decorrelation between images must be modelled depending on the statistics of the targets composing the scene. The two environment considered here are the one composed mainly by decorrelating distributed scatterers with short-medium decorrelation constant and a very unstable (low coherence) scenario like a forest. Their temporal coherence matrix can be modeled as the ones in Figure 3.8a and 3.7a respectively. Notice that the difference between a stable PS and a decorrelating target is the value of the coherence inside the coherence matrix of Figure 3.7a.

The generation of correlated data can then be easily performed with a simple multiplication of white noise with the Cholesky factorization of the coherence (or covariance) matrix.

Deformation

The scene could also be subjected to a subsidence. In this contribution the deformation is modeled as a linear (i.e. a constant velocity) and in this simulation we decided to follow this assumption. We selected some random circular areas in the image and we imposed a velocity field over those area. The velocity has been set to a medium/high displacement rate of about 20 mm/year.

The simulator converts the velocity in a phase therm to be added to each image. The imposed velocity field is depicted in Figure 3.15. The yellow circles try to mimic the subsidence inducted by underground oil pumping or material injection.

3.8.2. Results

In this section the results of the simulation are presented. The objective is to prove that over decorrelating distributed scatterers the proposed algorithm reaches better accuracy than standard DInSAR methods and comparable performances with respect to the standard AR(1) processor.

By generating a very unstable scene with constant and low coherence (such as a forest) we can also prove that in this case the performances of the proposed methods are very similar to the one of the standard DInSAR method while AR(1) perform worst.

The advantage of the proposed method is to automatically perform the best estimation basing on the target statistics.

Moreover, we have to prove the hypothesis that moderate/high deformation rate together with DEM errors will not hinder the correct estimation of the atmospheric phase if the total temporal extent of the stack is kept small and if the spatial baselines are
3.8. Evaluation of the proposed technique using simulated InSAR data

Figure 3.15: Simulated velocity field. All the scene is stable with the exception of the yellow circles that are areas with medium/high subsidence. The circular shapes simulates a subsidence due, for example, to an oil pumping.

small (like the ones of Sentinel-1).

First, a total of 11 images has been generated, their decorrelation is modelled as a negative exponential with decorrelation time of 10 days. The repetition interval is set to be 6 days, the baselines are defined within an orbital tube of 100m of radius and the DEM error is uniformly distributed from -30 to +30m. The slant range and azimuth resolutions are compliant with the ones of the Sentinel-1 mission. The simulated master amplitude image is depicted in Figure 3.16.

In Figure 3.17a the wrapped estimated APS are depicted along with the ground truth (See Figure 3.17d). The differential APS estimated by the proposed method show high temporal stability with a good quality throughout the whole stack. This is exactly the benefit of the Phase Linking approach: using the whole coherence matrix to obtain good phase estimate even in presence of strong temporal decorrelation. Notice that in this scenario the algorithm works exactly as the AR(1) processor (See Figure 3.17c and Figure 3.17f).

In Figure 3.17b the APS estimated with the standard DInSAR method are depicted. It is quite evident the role that the temporal decorrelation plays in the estimate. At longer temporal baseline, the images start to be uncorrelated, therefore the APS start to be corrupted by noise and after 6/7 images the information is completely lost. It is also possible to qualitatively see that the DEM error is smoothed by the estimation window used for the computation of the coherence matrix and that there are no phase "blobs" typical of the presence of deformation.

To quantitatively access the performances of both algorithms, an histogram of the error between the two method and the ground truth is presented in Figure 3.18. The proposed algorithm based on the phase linking maintains good phase quality and even in
the last image (the noisiest) where standard deviation doesn’t exceed 1 rad or about 4 mm of error in the estimate of the differential excess path. On the other end, the quality of the DInSAR APS degrade very quickly as predicted by the temporal decorrelation behavior of the simulated data. After roughly six images the interferometric phase become too noisy to be a reliable estimate of the APS. The error asymptotically approaches a uniform distribution between $-\pi$ and $\pi$ (see last sub-figure in Figure 3.18). Notice that the AR(1) perform in the same exact way of the proposed algorithm (as expected from this decorrelation model).

However, this is not the only possible decorrelation model. At C-Band there are several environments that decorrelate fast and stay incoherent. The typical scenario is a forest in the spring/summer. In this case the coherence matrix can be modelled as the one of a PS, but with very low coherence values ($\gamma \approx 0.2$) outside the main diagonal [34]. In Figure 3.19 the results of the simulation are presented. While the proposed algorithm always performs as expected, the AR(1) is unable to provide acceptable results. This result is expected: when the coherence is very low even in for small temporal baseline interferograms, the AR(1) approach simply integrates noise leading to a much less accurate estimate of the interferometric phase.

In Figure 3.20 the histograms of the errors with respect to the ground truth are presented. It is easy to see that while the classical DInSAR approach performs almost the same as the proposed method, the AR(1) quickly diverges producing very inaccurate estimates.
3.8. Evaluation of the proposed technique using simulated InSAR data

Figure 3.17: Simulation results from a short-medium time coherent scenario. (a) APS derived with the proposed algorithm. (b) APS derived by the standard DInSAR algorithm. (c) APS derived by the standard AR(1) algorithm. (d) Simulated APS stack. (e) Difference between the estimated APS and the DInSAR APS. (f) Difference between the estimated APS and the AR(1) APS.
Figure 3.18: Histograms of the errors for each image and for different estimators. Stack with short time decorrelation model.

(blue) Error of the proposed algorithm. (red) Error of the standard DInSAR processor. (yellow) Error of the AR(1) processor.
Figure 3.19: Simulation results from an highly unstable scenario. (a) APS derived with the proposed algorithm. (b) APS derived by the standard DInSAR algorithm. (c) APS derived by the standard AR(1) algorithm. (d) Simulated APS stack. (e) Difference between the estimated APS and the DInSAR APS. (f) Difference between the estimated APS and the AR(1) APS.
Figure 3.20: Histograms of the errors for each image and for different estimators. Stack with heavy decorrelation. (blue) Error of the proposed algorithm. (red) Error of the standard DInSAR processor. (yellow) Error of the AR(1) processor.
Case studies

In this chapter we assess the performances of the estimator and the processing chain by proposing different case studies carried out with real data. The algorithm and processing chain has been tested in different scenarios considering poorly and highly coherent scenes, with and without strong topography, at small and large scales and against different validation datasets.

The first one is in the area of Northern Italy, where very strong topography is present along with a dense GNSS network. In this case, the derived APS has been compared against a dataset derived by SqueeSAR®[72].

The second case study involves central Italy where we have less topography and an area that is more coherent. Even in this case, we had a SqueeSAR®dataset to compare with.

The last case study involves a large area of South Africa. This case study needs to prove the capability of the processing chain to work efficiently with areas in the order of tens of thousands of km².

4.1. Northern Italy

The first case study proposed is the one in Northern Italy. The area has been chosen due to the presence of both very strong topography and very flat areas. The former are the Alps, while the latter is the northern part of Pianura Padana.

The frame has an extent of 250 × 170 km². The scene is also very heterogeneous for what concerns the temporal stability of the scatterers: the city of Milan and a multitude of towns are present in the frame providing stable targets. The vast majority of the frame, however, is covered by vegetation of different types: from large grass fields to alpine forests. These targets, previously called decorrelating DS, provide the real challenge for the estimator.
4.1.1. Dataset description
For this experiment 11 Sentinel-1 images have been downloaded from the Alaska Satellite Facility (ASF) portal. All the images were downloaded with a 6 days temporal baseline that is the minimum interferometric revisit time allowed by the Sentinel-1 constellation. The dataset span a total extent of 60 days limiting in this way the effect of deformations and temporal decorrelation.

The time instants of the acquisitions are detailed in Table 4.1. Notice that, in order to prove the effectiveness of the method even with severe decorrelation, the dataset was taken between the end of the spring and the beginning of the summer. This period is particularly challenging in terms of temporal decorrelation due to strong vegetation growth.

<table>
<thead>
<tr>
<th>Platform</th>
<th>Date</th>
<th>Step-one temporal baseline</th>
<th>Total temporal baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1A</td>
<td>23/05/2017</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S1B</td>
<td>29/05/2017</td>
<td>6 days</td>
<td>6 days</td>
</tr>
<tr>
<td>S1A</td>
<td>04/06/2017</td>
<td>6 days</td>
<td>12 days</td>
</tr>
<tr>
<td>S1B</td>
<td>10/06/2017</td>
<td>6 days</td>
<td>18 days</td>
</tr>
<tr>
<td>S1A</td>
<td>16/06/2017</td>
<td>6 days</td>
<td>24 days</td>
</tr>
<tr>
<td>S1B</td>
<td>22/06/2017</td>
<td>6 days</td>
<td>30 days</td>
</tr>
<tr>
<td>S1A</td>
<td>28/06/2017</td>
<td>6 days</td>
<td>36 days</td>
</tr>
<tr>
<td>S1B</td>
<td>04/07/2017</td>
<td>6 days</td>
<td>42 days</td>
</tr>
<tr>
<td>S1A</td>
<td>10/07/2017</td>
<td>6 days</td>
<td>48 days</td>
</tr>
<tr>
<td>S1B</td>
<td>16/07/2017</td>
<td>6 days</td>
<td>54 days</td>
</tr>
<tr>
<td>S1A</td>
<td>22/07/2017</td>
<td>6 days</td>
<td>60 days</td>
</tr>
</tbody>
</table>

Table 4.1: Platform, date and temporal baselines for the eight acquisitions in the Northern Italy case study.

4.1.2. Orbit correction results
The first step after the estimation of the APS through PL and the unwrapping is the correction of orbital inaccuracies using a network of GNSS stations. In this specific Sentinel-1 frame, 24 stations are present. Such a large amount of station is not mandatory, however, it surely helps to improve the accuracy of the orbital plane estimation and therefore the quality of the final result.

In Figure 4.1 the results from the estimation of the orbital error are presented. As already detailed in Section 3.6, the components that are possible to estimate are the normal baseline error at the center of the trajectory and the parallel deviation along the trajectory.

In Figure 4.1a the normal baseline error for all the interferograms is depicted. First of all notice that the baseline error for the first image, the master, is forced to zero. This is due to the fact that we are not estimating an absolute trajectory error, but rather a baseline error. Unfortunately, we do not have a ground truth for the baseline error, however, the nominal accuracy of precise Orbit State Vectors (OSV) for the Sentinel-1 constellation is around 5 cm (standard deviation, 1 RMS). We expect then the baseline
error to be at least around double of this value. The estimated errors are represented in Figures 4.1a and 4.1b are within the range of acceptable values.

Figure 4.1: (a) Estimated normal baseline error (in mm). (b) Estimated time derivative of the parallel baseline.

One of the possible way to access the capability of the orbit correction algorithm to effectively improve the quality of the APS is to compare the APS itself with the network of GNSS stations. We consider the APS over the GNSS stations as ground truth and we try to understand if the baseline correction procedure makes us closer to the truth or not. The simplest figure of merit is the correlation coefficient before and after the correction. In Figure 4.2 the scatter plots for every interferogram are depicted along with the correlation coefficients in the titles of each figure. The correlation is always high even before the correction and generally increases even more after reaching values up to 94%. However this is not always the case: in Figure 4.2i, for example, the value of the correlation coefficient decreases from 0.85 to 0.78. The reason could be very simple. While the L1 inversion used for the estimate of orbital parameters is very robust to outliers, the Pearson's correlation coefficient is not so robust, in particular when the number of samples is quite low, as in this case. In Figure 4.2i the blue dots (before correction samples) that are distant from the diagonal are the points where the GNSS does not agree with the SAR. When finding the orbital errors, these points are considered as outliers, therefore they are slowly discarded by the L1 inversion from the pool of available data. In other words the algorithm does not try anymore to "pull" those points to be on the main diagonal of the scatter plot. The result is that after the correction they are even more distant from the diagonal, biasing in this way the correlation coefficient.
4. Case studies

SAR APS vs GNSS APS over stations

Correlation before = 0.766.
Correlation after = 0.806.

\[ \varphi = 13.2379 \text{ mm} \]

Pre-correction
Post-correction

Correlation before = 0.891.
Correlation after = 0.903.

\[ \varphi = 7.9949 \text{ mm} \]

Correlation before = 0.768.
Correlation after = 0.742.

\[ \varphi = 13.31 \text{ mm} \]

Correlation before = 0.907.
Correlation after = 0.909.

\[ \varphi = 10.1561 \text{ mm} \]

Correlation before = 0.790.
Correlation after = 0.679.

\[ \varphi = 16.5005 \text{ mm} \]

Correlation before = 0.830.
Correlation after = 0.848.

\[ \varphi = 10.9906 \text{ mm} \]

Correlation before = 0.722.
Correlation after = 0.718.

\[ \varphi = 11.0611 \text{ mm} \]

Correlation before = 0.770.
Correlation after = 0.819.

\[ \varphi = 18.3073 \text{ mm} \]
4.1. Estimated APS: maps, variograms and spectra

In this section the estimated maps are depicted and their spatial statistics are compared with theoretical models. 

In order to statistically characterize the maps derived, spatial variograms and spatial spectra have been computed. The wet delay has a magnitude that is much smaller than the hydrostatic one (typically less than 10% of the total slant delay), but its fluctuations are larger both in time and in space.

In Figure 4.3 the results are depicted. In particular, in Figure 4.3a we selected a random region in the Sentinel frame and the spatial variogram has been computed. For the computation we assumed the APS to be an isotropic process. In black dashed lines all the variograms for every single interferogram has been depicted, while in red and green the 2/3 and 5/3 power law are represented [7], [73]. In blue the average variogram is also depicted. A transition region is present at around 1-1.5 km of distance and is predicted by the theory.

Looking at Figure 4.3a it is evident that before the transition region the accordance between the theoretical model and the observed data is very high. After the transition region, however, the estimated semi-variance seems to have a slightly different slope, especially in some variograms. This means that the theoretical model underestimates turbulence at higher spatial scales. The effect could be due to the large presence of topography in the scene or possibly the accumulation of water vapor before an extreme rain event. Notice that it make sense only to compare the slopes of the theoretical model and the estimated variograms, not the absolute values. A missing constant (the variogram’s "nugget") that depends on the turbulence strength is missing from the theoretical model. For the plots Figure 4.3a the best fitting nugget has been estimated.

In Figure 4.3b the radially averaged power spectra are depicted in black along with the average between all the interferograms in blue. The theoretical model [73] predict a -5/3 power law and it is drawn in a red dashed line. Also in this case, the model provides just a trend, but not the absolute shift of the spectrum that instead depends on the strength of the turbulence. The observation shows very good accordance with the theory,
4.1.4. Comparison with SqueeSAR®

In the methodology section of this thesis we highlighted that, in order to prevent the effect of subsidences in the estimated phases, we reduce the total temporal length of the utilized stack. Subsidences are normally in the order of a few mm/year, with extreme cases above 150 mm/year. If the stack is very large, for example, 1 year of measurements, it is mandatory to estimate and remove the subsidences. Notice that in this case the temporal decorrelation also starts to play a significant role over DS leading to a sparse final APS map. Limiting the total number of images, instead, does not allow the estimation of target motion, however, it is not necessary in presence of a medium subsidence rate since in such a short temporal extent the phase error would be minimal. The other aspect to consider is the fact that with a short stack even severely decorrelating DS may retain some coherence, leading to wide and dense maps.

In order to prove the validity of the assumption that the target’s deformation can also be neglected and to assess the quality of the APS maps, we compared the result obtained by our processing chain with a set of SqueeSAR® derived APS [74]. SqueeSAR® is an advanced InSAR technique mainly applied in deformation estimation and displacement monitoring. The main algorithm behind SqueeSAR® is again the Phase Linking, however, the entire workflow is composed by several steps. The processing starts with the detection of statistically homogeneous pixels (SHP) by means of the Kolmogorov-Smirnov test. When the SHP has been detected it performs a space
adaptive filtering (i.e. with a non-local window of variable sizes) to generate sample coherence matrices over which the optimally linked phases are estimated. SqueeSAR® includes also a standard Permanent Scatterer (PS) processor. It allows the estimation and removal of the portion of the interferometric phase related to subsidences and residual DEM. To estimate the residual motion of the targets with high accuracy it is mandatory to use a lot of images resulting in a high computational burden. Once the subsidences are estimated and removed, the only relevant component that is left in the interferometric phase is the APS.

It is important to remember that the image will be sparse due to severe temporal decorrelation of DS when long time series are used. In those places the phase is too noisy to be meaningful, therefore it is discarded. To obtain a dense map the processing chain relies on a statistical interpolation over the area of poor coherence.

The described SqueeSAR® is surely effective and it is taken as a benchmark, as a "ground truth". On the other end, it is also true that the computational burden required to obtain SqueeSAR® maps is very high, especially when it is necessary to build a system able to provide nation-wide maps (several thousands of km², as requested by NWPM).

The method here proposed, instead, relies on the statistical properties of the atmospheric signal, performing filtering over large spatial windows to retain only the portion of the interferometric phase that plays a role in such large scales. The SHP test is thus not required. While SqueeSAR® provides long APS time series at the expense of large computational time, the method proposed provides reliable APS estimates at the expense of using a much shorter data stack leading to a much shorter APS time series. The added value provided by this work lies also in the orbit correction, interferometric constant estimation and conversion of the differential APS maps into absolute ZTD maps.

In Figure 4.4 each estimated map is shown both of the proposed method and SqueeSAR® along with the histogram of their difference. The ground truth APS is not corrected by orbital errors and since the correction would require the same dataset used for the correction of the APS under test, we decided to compare directly the uncorrected maps. The same thing is true for the missing interferometric constant. The consequence is that it is only useful to compare the lateral variation of the APS, in other words we have to look at the standard deviation of their difference, not the mean difference that is meaningless.

The degree of accordance between the two methods is generally very high and it is always below 1 cm, which is acceptable for the final purposes of the generated APS (NWPM ingestion). The accordance can be also seen qualitatively by comparing the left and middle plots in Figure 4.4.

In some images the discrepancy is higher than in others. This mismatch manifests itself as two effects easily noticeable in the right column of Figure 4.4: an increase of the standard deviation and the appearance of a bimodal structure in the histogram of the difference.

These two effects are due to different causes. First of all the proposed method result in maps containing more energy at lower spatial frequencies than SqueeSAR®. This is particularly true in regions where PS are hardly present since SqueeSAR® recurs
heavily to interpolation, possibly losing high frequency information. The consequence is that the two methods are intrinsically different, particularly in regions where there are no PS and when the APS is turbulent (high energy at high spatial frequencies).

The bimodal structure of the histograms in Figure 4.4d, 4.4f, 4.4h and 4.4j is instead due to unwrapping errors. They are easily recognizable since they have a deviation of half a wavelength or roughly 2.8 cm for Sentinel-1. These errors can hardly be detected and corrected and are due to the so-called phase islands. If can happen that in a wrapped interferogram an area is surrounded by very low coherence "strips" or "belts" like a river, a water body or a forest and there are no "bridges" of high coherence to connect this isolated area to the interferogram. The consequence is that a shift may arise in the unwrapped interferograms between the isolated area and the other portion of the interferogram. A steep gradient in the unwrapped interferometric phase arises.
4.1. Northern Italy

(a) 29-May-2017 17:13:44

(b) 04-Jun-2017 17:14:23

(c) 10-Jun-2017 17:13:45

(d) 16-Jun-2017 17:14:23

Standard deviation:

- Phase Linking: 7.22 mm
- SqueeSAR: 5.88 mm
- 7.20 mm
- 9.82 mm

Difference histogram

0 0.01 0.02 0.03 0.04 0.05 0.06 0.07

(a)

(b)

(c)

(d)
4. Case studies

Phase Linking
22-Jun-2017 17:13:45

SqueeSAR
22-Jun-2017 17:13:45

Difference histogram
Standard deviation: 11.82 mm

-100 -50 0 50 100
Difference [mm]
0
0.01
0.02
0.03
0.04
0.05
0.06

Phase Linking
28-Jun-2017 17:14:24

SqueeSAR
28-Jun-2017 17:14:24

Difference histogram
Standard deviation: 8.53 mm

-100 -50 0 50 100
Difference [mm]
0
0.02
0.04
0.06
0.08

Phase Linking
04-Jul-2017 17:13:46

SqueeSAR
04-Jul-2017 17:13:46

Difference histogram
Standard deviation: 10.57 mm

-100 -50 0 50 100
Difference [mm]
0
0.01
0.02
0.03
0.04
0.05

Phase Linking
10-Jul-2017 17:14:25

SqueeSAR
10-Jul-2017 17:14:25

Difference histogram
Standard deviation: 13.41 mm

-100 -50 0 50 100
Difference [mm]
0
0.01
0.02
0.03
0.04
0.05
Figure 4.4: Comparison between the estimated APS and SqueeSAR-derived APS. (Left column) The estimated APS. (Middle column) SqueeSAR derived APS. (Right column) histogram of their difference with mean difference and standard deviation. In some images a bimodal structure appears in the histogram. This effect is due to unwrapping errors.
4.2. Central Italy

The second case study here presented is the one carried out in central Italy. In this scenario, just one S1 frame has been processed spanning an area of roughly 250 × 170 km².

This area has been selected due to the supposed strong presence of water vapor in the atmosphere: during the week from the 10th of November to the 17th of November, strong rain with severe flooding happened in this region of Italy.

4.2.1. Dataset description

In this section the characteristics of the considered dataset are explained. Eight Sentinel-1 flights have been considered. The time instant of the acquisitions are listed in Table 4.2. Also in this case we limited the total time span of the stack by selecting a few images with a short temporal baseline between them.

<table>
<thead>
<tr>
<th>Platform</th>
<th>Date</th>
<th>Step-one temporal baseline</th>
<th>Total temporal baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1A</td>
<td>15/10/2017</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S1B</td>
<td>21/10/2017</td>
<td>6 days</td>
<td>6 days</td>
</tr>
<tr>
<td>S1A</td>
<td>27/10/2017</td>
<td>6 days</td>
<td>12 days</td>
</tr>
<tr>
<td>S1B</td>
<td>02/11/2017</td>
<td>6 days</td>
<td>18 days</td>
</tr>
<tr>
<td>S1A</td>
<td>08/11/2017</td>
<td>6 days</td>
<td>24 days</td>
</tr>
<tr>
<td>S1B</td>
<td>14/11/2017</td>
<td>6 days</td>
<td>30 days</td>
</tr>
<tr>
<td>S1A</td>
<td>20/11/2017</td>
<td>6 days</td>
<td>36 days</td>
</tr>
<tr>
<td>S1B</td>
<td>26/11/2017</td>
<td>6 days</td>
<td>42 days</td>
</tr>
</tbody>
</table>

Table 4.2: Platform, date and temporal baselines for the eight acquisitions in the central Italy case study.

The scene is quite heterogeneous for what concerns the temporal stability as shown in Figure 4.5. There are significant portions of the frame that are relatively stable in time: such areas are mainly human settlements (mainly on the coastal region) and mountains with no vegetation (at the center of the image). There are also large decorrelating spots due to forested areas and agricultural fields.

The topography is also quite heterogeneous as depicted in Figure 4.6. The Appennini mountain range is visible in the middle of the scene. The topography is smoother when approaching the sea (both easting towards the Adriatic sea and going west towards the Tirreno sea). The height gradients are for sure less steep than the first case study.
4.2. Central Italy

4.2.2. Orbit correction results

This section is dedicated to the assessment of the results and performances of the orbit correction procedure. In Figure 4.7 the absolute and differential values of the GNSS-derived atmospheric products are depicted. Notice how there is a change in pattern for all the stations at the time instants corresponding to the 14\textsuperscript{th} and 20\textsuperscript{th} of November. These are the dates where the major flooding happened in the area.

In Figure 4.7b, we have depicted the differential delays with respect to a master image in the stack (in this case the first image). In this way, we recreated the InSAR measurement. Notice how in this picture it is even more evident the change of trend in the two dates corresponding to the main rain event (third last and second last columns).

The estimated values of normal baseline error and parallel baseline derivatives are depicted in Figure 4.8. Also in this case we obtain numbers that are generally within the possible range of values of the baseline errors.

Figure 4.5: Average 6 days coherence of the scene.

Figure 4.6: DEM of the region.
In Figure 4.7 the scatter plots between the estimated SAR APS and the GNSS-derived APS are depicted. First of all, we can notice that the orbital correction helps to improve the correlation between the SAR and GNSS, in some cases (See Figure 4.9a) even with a 10% improvement. Only in one scenario (Figure 4.9c) the correlation is very poor. The orbital correction helps to improve the estimate, but the correlation still remains below 0.25.
Figure 4.8: (a) Estimated normal baseline error (in mm). (b) Estimated derivatives of the parallel baseline error (in mm/s). The values found are within a possible range of values.
Figure 4.9: Scatterplot comparing the GNSS-derived APS and the SAR-derived APS before and after the orbit correction. While the correlation coefficient is high also before the orbit correction, it generally increases after proving the effectiveness of the procedure.
4.2.3. Estimated APS: maps, variograms and spectra

In this section the final results of the estimation process are presented. In Figure 4.10 all the differential APS maps are depicted. Since we used a total of \( N = 8 \) images in the stack, the number of linked interferograms is 7. The spatial extent of each map is \( 170 \times 250 \text{ km}^2 \).

The holes that are visible in some pictures are areas where the phase is unreliable and therefore discarded. These spots are, for example, water bodies or regions with very poor interferometric coherence.

As in the previous case study, we computed the variograms and the spectra of the estimated APS and compared the results with the theoretical model.

In Figure 4.11a the variograms for each APS are depicted in dashed black lines, the average variogram is in blue while the 2/3 and 5/3 power laws are depicted in red dashed and green dashed lines. From the figure is evident that the variograms follow the theoretical model quite well both at short and long delays. There is also a transition region between 1 and 2 km, as expected from the theory. Only one variogram seems to behave in an unexpected way: that is the interferogram closer to the extreme event (20\(^{th}\) November 2017). In this image, the transition region from one power low to the other seem to be at higher distances (around 8 km) showing the presence of turbulence at bigger spatial scales.

In Figure 4.11b, instead, the radially averaged power spectra are depicted in black dashed lines, while the average spectrum is shown in blue. The theoretical model (-5/3 power law) is depicted in a red dashed line. Also for what concerns spectra, the data follows really well the theoretical model. The only discrepancy is seen at very high spatial frequency (low wavelength) where the data shows higher energy than expected by the model. The cause could be again the presence of high-frequency turbulence in the atmosphere.
Figure 4.10: Estimated APS over the area of central Italy. The spatial extent is roughly $170 \times 250 \text{ km}^2$. 
4.2. Central Italy

4.2.4. Comparison with SqueeSAR®

In this section the comparison with the SqueeSAR® APS is performed. In Figure 4.12, on the left column the Phase Linking atmospheric screens are plotted, in the middle the SqueeSAR®-derived APS and on the right the histogram of their difference. Also in this case, the SqueeSAR® stack is much longer, having 49 images. The residual topography and the subsidence has been estimated and removed. The accordance between the two methods is remarkably good with a standard deviation that remains always below 6 mm. Also in this case the discrepancy could manifest in two forms: an increase of the standard deviation and a bimodal structure of the histogram. The causes are identical to the previous case study. In presence of poor PS density SqueeSAR® could underestimate high-frequency APS turbulence leading to an increase of the standard deviation between the two methods (i.e. enlargement of the bell-shaped histogram around zero). The bimodal structure of the histogram, instead, is visible just in 4.12f. Also in this case the cause must be searched in an unwrapping error of the interferogram. Notice that in this case study the unwrapping errors are much rarer than in the previous one. This is due to the more "gentle" region, showing less topography and higher coherence. The better coherence can be due to the period of the year (November) of the physical composition of the scene made mainly by scatterers that remain coherent for longer. This proves again that highly decorrelating regions are the most difficult for the estimation of a reliable APS.

In Figure 4.13, instead, the scatter plots are depicted. On the $x$ axis the estimated APS, on the $y$ axis the SqueeSAR APS. Even with this metric, the accordance very high with correlation coefficients reaching values higher than 0.95 in 5 images out of 7. The maximum value is as high as 0.99 (See Figure 4.13b and 4.13d). Also here it is possible to see the presence of unwrapping errors that manifest themselves as biases in the scatter plot. Figure 4.13f is an example where it is evident the presence of two populations in the scatter plot, one of them with a bias of roughly half a wavelength.
Figure 4.12: Comparison between the estimated APS and SqueeSAR-derived APS. (Left column) The estimated APS. (Middle column) SqueeSAR derived APS. (Right column) histogram of their difference with a mean difference and standard deviation.

The same could be seen in the histogram of Figure 4.12f.
Figure 4.13: Scatterplot comparing the SqueeSAR-derived APS and the estimated APS.
4.3. South Africa

The third case study presented is the South African one. Differently from northern Italy and central Italy five S1 frames have been considered spanning a total area of about 800 × 250 km². The area shows also significant topography variation. Such wide area processing raises some problems in the processing chain for the extraction of APS. The merging of the single frames (250 × 170 km² each) in the pre-processing procedure leads to a computational burden in the following processing steps that is too high to be sustainable.

At the same time, the merging of the images just before the phase unwrapping it is not feasible, since, also in this case, the computational burden required in the unwrapping procedure is too high.

The approach considered here is a hybrid one: successive frames are merged in couples. If the complete image is formed by 5 frames named \( a, b, c, d, e \), then \( a \) is merged with \( b \), then \( b \) with \( c \), \( c \) with \( d \) and finally \( d \) with \( e \). A total of four, double-sized images are created with a complete frame of overlap between the two. Notice, in fact, that frame \( b \) is present in the first two images, \( c \) is present in the second and the third one and so on.

The processing follows in a slightly different way with respect to the single frame one and will be carefully explained in the following sections.

4.3.1. Dataset description

In this section the characteristics of the considered dataset are explained. Six Sentinel-1 passages have been considered for a total of five frames. The covered area is about 250 × 850 km².

The time instants of the acquisitions are listed in Table 4.3 along with the platform identifier and the temporal baselines.

<table>
<thead>
<tr>
<th>Platform</th>
<th>Date</th>
<th>Step-one temporal baseline</th>
<th>Total temporal baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1A</td>
<td>01/02/2018</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S1A</td>
<td>13/02/2018</td>
<td>12 days</td>
<td>12 days</td>
</tr>
<tr>
<td>S1A</td>
<td>25/02/2018</td>
<td>12 days</td>
<td>24 days</td>
</tr>
<tr>
<td>S1A</td>
<td>09/03/2018</td>
<td>12 days</td>
<td>36 days</td>
</tr>
<tr>
<td>S1A</td>
<td>21/03/2018</td>
<td>12 days</td>
<td>48 days</td>
</tr>
<tr>
<td>S1A</td>
<td>02/04/2018</td>
<td>12 days</td>
<td>60 days</td>
</tr>
</tbody>
</table>

Table 4.3: Platform, date and temporal baselines for the six acquisitions in the South African case study.

In the scene, 20 dual-frequency GNSS stations were present and active at the time of the Sentinel-1 acquisitions. In Figure 4.14, five Sentinel-1 frames (in blue) and all the GNSS stations (red dots) are depicted.

One of the frames considered, the one numbered as 1087 in Figure 4.14 has no active GNSS stations inside.

The area is also characterized by high temporal instability due to the presence of extended forested areas. In Figure 4.15 the temporal coherence as defined in Equation
Figure 4.14: (Blue) Sentinel-1 frames. (Red dots) Dual-frequency GNSS stations in the region.

3.14 is depicted. The majority of the scene shows a very poor coherence (lower than 0.4). Nevertheless, using wide estimation windows, it is still possible to extract useful phase signals from the images.

4.3.2. Data processing for large scale interferometry

WPMS working domain is as big as several thousands of km$^2$. The ingested products, in order to be useful, needs to cover a wide area as well. Each Sentinel-1 frame is roughly 250 $\times$ 170 km$^2$ in size and therefore several of them needs to be merged together to be useful in the ingestion.

In Figure 4.16 the entire processing chain is depicted.

While the single-frame processing is straightforward, the joint handling of several frames needs some further considerations here explained:

1. **The entire stack** composed by $M$ frames and $N$ dates is divided into substacks. The first frame is merged with the second in all the time instants. The result is a set of $N$ images double the size of the original frame. All the images are now coregistered, flattened and the topographic phase is compensated. The process is now repeated with the second and the third frame, with the third and the fourth and so on.
2. The Phase Linking algorithm process each sub-stack separately. Notice that each sub-stack has an overlap area with the following (or preceeding) sub-stack of the extension of one entire frame.

3. Each linked phase in each sub-stack is unwrapped. Since each double-size image is unwrapped separately, the computational burden remains low. The result is a set of \((N - 1) \times (M - 1)\) unwrapped APS. Immediately after the unwrapping, the overlap region between merged frames is exploited in order to find the presence of a vertical shift (i.e. a constant difference) between the phase screens. Such difference may arise if the GCP considered as a reference point in the unwrapping procedure is not in the overlapping region of the two considered sub-stacks.

4. A joint orbit correction is executed. The APS is extracted over the locations of the GNSS stations and used for orbit correction as explained in Section 3.6. Notice that, since all the sub-stack are vertically aligned (i.e. there are no phase shifts between them) we can use all the sub-stacks jointly to estimate a single baseline error.
This is an advantage for several reason:

- The geographical extent of the full image is wide and there is an high probability of finding at least a few active GNSS stations inside it. The number of required GNSS stations and their disposition in the frame is detailed in Section 3.6.

- A direct consequence of the first point is that the mentioned GNSS stations will be more sparse in the scene, leading to a better estimates of the orbital error parameters.

- Frames without any available GNSS station (see the second frame from the south in Figure 4.14) can be also calibrated. Once the orbital error parameters are estimated, the forward problem is computed for all the scene, included the frames without any active GNSS station. It is sufficient to have just a few GNSS measurements properly distributed in the scene to be able to correct orbital errors in a wide area.

5. The APS maps are merged using a simple nearest-neighbor interpolation over a common geographical grid.

4.3.3. Orbit correction results

In this section the performances and results of the orbital correction procedures are listed. As previously mentioned, adjusting orbital error is a mandatory procedure in large-scale interferometry when the objective is APS retrieval.

In Figure 4.17 absolute GNSS-derived ZTD is depicted. Notice that some black spots are present. In those time instant and in those stations the measurements where not present due to technical reasons.

In Figure 4.18 the GNSS ZTD are differentiated in time. In this scenario the measures gathered by all the stations in the first image are subtracted from all the ZTD, therefore the master image of both SAR and GNSS is the first one in the stack. It is not mandatory that the master (unknown) image is the first. In particular we always select the one that has more valid GNSS ZTD measures.

In this case study the image of the 1st of February or the one at the 25th of February.
4.3. South Africa

In Figure 4.17 the orbital error found are depicted. The accuracy of orbit state vectors are precise within a 5 cm accuracy (1 RMS) [75]. Since we are estimating a baseline error and not an absolute trajectory error, we assume that the master trajectory is perfectly known and all the error resides in the slave trajectory with doubled inaccuracy. The normal baseline error is therefore expected to be within 7 cm (1σ). The orbital error estimated are represented in Figure 4.19a.

Once the orbital parameters have been estimated, the forward problem is computed for all the pixels in the scene. The result is a set of $N - 1$ phase screens to be compensated from the interferograms.
Figure 4.18: GNSS estimates of the APS (in mm) at the time of SAR acquisitions. The master image (set to 0) is the first one. Notice that it is not mandatory that the master image is the first in the stack. The selection criteria for the reference image is the availability of the highest number of GNSS estimates possible. The black spots correspond to missing measures.

Figure 4.19: Estimated normal baseline error and estimated time derivative of the parallel baseline.
4.3.4. Estimated APS: maps, variograms and spectra
In this section the final results of the estimation process are presented. In Figure 4.20 all the differential APS maps are depicted. Since we used a total of $N = 6$ images in the stack, the number of linked interferograms is 5. The spatial extent of each map is $170 \times 850 \text{ km}^2$.

The holes that are visible in some pictures are areas where the phase is unreliable and therefore discarded. These spots are, for example, water bodies or regions with very poor interferometric coherence.

In order to statistically characterize the maps derived, spatial variograms and spatial spectra have been computed. The wet delay has a magnitude that is much smaller than the hydrostatic one (typically less than 10 % of the total slant delay), but its fluctuations are larger both in time and in space. In Figure 4.21a we selected a random region in all the estimated APS and computed the spatial variogram. For the computation, we assumed that is APS is isotropic. In black dashed line all the variograms for the five images are depicted. In blue the average variogram is represented while in red and green the $2/3$ power low and $5/3$ power low are respectively depicted. A transition region between the two models is present at around 1.5 km (intersection between red and green line).

From the figure it is evident that the data follow the theoretical model both for each image and on average.

The same goodness of fit can be found for radially average spectra. Also for this computation we assumed that the APS is isotropic and we selected a random region of the APS maps and computed the 2D Fourier transform. After that, we averaged radially with respect to the center of the transformed domain.

In Figure 4.21b the results are shown: the derived spectra fit well the theoretical model provided for the turbulent part of the APS by [73] as $1/f^\alpha$ where $\alpha = 5/3$ for radially averaged power spectra. Notice that, as in the previous case study, it is worth comparing just the trend (i.e. the slope) of the estimated spectra with the theoretical model. The amplitude, instead, depends on the strength of the turbulence. On a shorter wavelength, starting from 100 m up to 400, the model seems to underestimate the power of the turbulence. This is evident since the theoretical model follows a straight line, while the estimated power spectra follow an asymptotic behavior.

This characteristic of the power spectra has already been observed in the case study of Northern Italy (See Section 4.1.3).
4. Case studies

Estimated APS
Date: 13-Feb-2018 16:37:41

Estimated APS
Date: 25-Feb-2018 16:37:41

Estimated APS
Date: 09-Mar-2018 16:37:41

Estimated APS
Date: 21-Mar-2018 16:37:41

(a)  

(b)  

(c)  

(d)
4.3. South Africa

Figure 4.20: Estimated APS over South Africa. The spatial extent is roughly $170 \times 850 \text{ km}^2$.

Figure 4.21: (a) Variograms of each individual phase screen (in black), average of all the variograms (blue) and theoretical models for the variograms (in red and green). (b) Radially averaged power spectra of each individual phase screen (black), average of all the power spectra (blue) and theoretical model (in red).
4.3.5. Comparison with GACOS and GNSS

In this section the extracted APS are compared with the ones from the Generic Atmospheric Correction Online Service (GACOS) [12], [70], [71]. GACOS provides high resolution Zenith Total Delay (ZTD) maps to be used for correcting InSAR products. The service is globally available, it uses an high-resolution ECMWF model at 0.1-deg and 6 hours time resolution with an SRTM/ASTER DEM to retrieve height-dependent ZTD estimates. GACOS has proved several times to be a reliable tool for the mitigation of atmospheric effects in interferograms [76]–[78].

We remind to the reader that the term ZTD is referred to time absolute measurement of the delay (in mm) in the zenith direction, while APS is a relative measure and it is the output of the interferometric processing.

A set of GACOS ZTD images has been downloaded in correspondence of the SAR acquisitions. The stack is then differentiated in time according to the unknown master of the SAR stack: this step generates GACOS-based APS.

When comparing two images, it is mandatory that they have roughly the same resolution (i.e. they contain energy at the same spatial frequencies). For this reason we averaged (smoothed) both stack to have a resolution of 5 km.

The first comparison is a simple scatter plot comparing the GACOS APS with the ones estimated using the proposed method. In Figure 4.22 the scatter plots are depicted. In all the considered images the correlation coefficient is high, ranging from a moderate 0.56 in Figure 4.22c up to 0.90 of Figure 4.22e. An high correlation of the estimated APS with the APS derived by the NWPM suggests that the proposed algorithm is able to capture most of the atmospheric effects. The poor correlation coefficient of 4.22c can be due to the smaller magnitude of the phase measurement as seen from Figure 4.23c. The linear correlation coefficient is not sufficient to compare and qualify the estimated APS. A comparison in terms of mean difference and standard deviation of the difference is also computed.

In this case, contrary to the comparison with SqueeSAR®, we can assess also the mean error between the SAR-derived APS and the GACOS-derived APS. In the previous case studies, in fact, also the comparative dataset was based on SAR data suffering in this way of a lack of an absolute measure of the APS (i.e. a missing interferometric constant). In this case, instead, having an absolute APS allows to judge the quality of the interferometric shift compensation implemented with the network of GNSS.

Figure 4.23 represents the estimated APS on the left column, the GACOS one in the middle column and the histogram of the difference along with mean and standard deviation on the right column. The mean error is quite small for all the images: on average 3 mm with peaks at 11 mm in the first APS. It means that the constant shift for each interferogram and very low-frequency components like trends are correctly captured by the APS estimated. The GNSS processing aimed at estimating and compensating orbital errors and phase shift appears to be robust. If we imagine transforming the APS into absolute ZTD using a single perfect master (See Section 3.7), the error will be directly propagated to the absolute ZTD. Since the ZTD is in order of 200 cm for standard atmosphere, a bias of 11 mm accounts for roughly 0.5 %: a figure more than acceptable.

The average standard deviation of the error is acceptable, attesting itself slightly above
4.3. South Africa

To understand the origin of this deviation between the GACOS APS and the derived ones, it is customary to look at the spectra of both datasets. In Figure 4.24 the average variogram and power spectrum of the two datasets are depicted after the smoothing at 5 km. The image confirms what was already evident from Figure 4.23. The variogram in Figure 4.24a shows an higher spatial variability of the derived APS at lower spatial scales. The spectrum in Figure 4.24b also suggests that the estimated APS shows a
### Case studies

#### Estimated APS

**13-Feb-2018 16:37:41**

- **Longitude**
- **Latitude**

#### GACOS

**13-Feb-2018 16:37:41**

- **Longitude**
- **Latitude**

#### Difference histogram

- **Mean:** -11.11 mm
- **Standard deviation:** 20.70 mm

#### Estimated APS

**25-Feb-2018 16:37:41**

- **Longitude**
- **Latitude**

#### GACOS

**25-Feb-2018 16:37:41**

- **Longitude**
- **Latitude**

#### Difference histogram

- **Mean:** 2.22 mm
- **Standard deviation:** 19.83 mm

#### Estimated APS

**09-Mar-2018 16:37:41**

- **Longitude**
- **Latitude**

#### GACOS

**09-Mar-2018 16:37:41**

- **Longitude**
- **Latitude**

#### Difference histogram

- **Mean:** 2.64 mm
- **Standard deviation:** 21.07 mm

#### Estimated APS

**21-Mar-2018 16:37:41**

- **Longitude**
- **Latitude**

#### GACOS

**21-Mar-2018 16:37:41**

- **Longitude**
- **Latitude**

#### Difference histogram

- **Mean:** 3.82 mm
- **Standard deviation:** 21.41 mm
higher spatial variability with respect to GACOS, particularly at lower scales (around 10-20 km). As expected by a NWPM, the fine scale turbulence cannot be recovered correctly, while the proposed algorithm is lower bounded on the resolution just by the quality of the final estimate that requires a certain number of looks to be taken in space. For completeness, also the average variogram and average radial spectrum before the smoothing as been depicted in Figure 4.25. As expected, in these plots the differences are even more pronounced at finer spatial scales.

One last comparison is the one with the GNSS-derived APS. It is important to keep in mind that GNSS is not an independent measure for both the APSs under test. The proposed method uses the GNSS to correct for orbital error, while GACOS probably uses
GNSS to lead to the final estimate of the ZTD. Moreover, GNSS measures are point wise in space, thus a limited statistic can be derived from them.

In Figure 4.26 the scatterplots comparing GACOS, GNSS and the estimated APS are depicted. On the left column GACOS is compared with the GNSS for all the images (each time instant is a row). On the right column the estimated APS is compared with the GNSS.

It is straightforward to see how for all the images the mean error and the standard deviation is lower for the estimated APS rather then GACOS. In the comparison with the estimated APS, the mean error never exceeds 3.2mm and the standard deviation is at maximum 15mm in the last image. We argue, however, that such a large standard deviation is in reality a bias in its estimate due to the small number of samples and the presence of an outlier in the GNSS (also visible in the corresponding figure regarding the GACOS comparison).

For what concern GACOS, instead, the mean error reaches values up to 7mm, while the standard deviation values up to 22 mm.
GACOS APS vs GNSS
13-Feb-2018 16:37:41
Mean: -6.7472 mm
Std: 22.3329 mm

Estimated APS vs GNSS
13-Feb-2018 16:37:41
Mean: 2.858 mm
Std: 10.3632 mm

GACOS APS vs GNSS
25-Feb-2018 16:37:41
Mean: -3.0801 mm
Std: 15.9388 mm

Estimated APS vs GNSS
25-Feb-2018 16:37:41
Mean: -1.8762 mm
Std: 4.4304 mm

GACOS APS vs GNSS
09-Mar-2018 16:37:41
Mean: -6.4484 mm
Std: 19.839 mm

Estimated APS vs GNSS
09-Mar-2018 16:37:41
Mean: 0.045949 mm
Std: 7.9202 mm
Figure 4.26: (left column) Scatterplot of the GACOS APS and the GNSS-derived APS. (right column) Scatterplot of the estimated APS and the GNSS-derived APS. If we consider the GNSS as the ground truth, mean values and standard deviations of the error are generally much lower for the estimated APS rather than GACOS.
In this dissertation, I dealt with estimating Atmospheric Phase Screens from a stack of SAR data. The thesis relies on the peculiar characteristics of a C-Band radar with short revisit intervals. The Sentinel-1 constellation, with its six-day interferometric revisit, extensive coverage provided by the IW acquisition mode and short spatial baselines, is the perfect instrument for atmospheric monitoring. Once transformed into absolute ZTD products, the APS extracted could be ingested by NWPM to improve weather forecasts. I started with a brief overview of the characteristics of SAR images and the effect of the propagation of the radar signal through the atmosphere. After that, the complete workflow from start to the end has been detailed. A theoretical overview of the Phase Linking algorithm is proposed to the reader, along with the analytical derivation of the estimate’s accuracy and its bounds.

While the Phase Linking procedure can be implemented in just a few lines of code, an efficient implementation able to rapidly process large areas needs some more precautions. Moreover, since the objective is the estimation of the APS, some fine-tuning of the processing chain regarding the size of the estimation window and the temporal extent of the stack is needed to reach better accuracy. By exploiting a small stack of images, we obtain multiple benefits. First of all, the temporal decorrelation is limited, allowing to obtain reliable and dense maps of the atmosphere that are also easier to unwrap. Moreover, the deformation of the scene (subsidence), can be ignored in the presence of a standard subsidence rate of a few tens of mm/year. The next step is the unwrapping which is made easier thanks to the high coherence of the scene. A 2D phase unwrapper has been implemented as part of the processing chain.

When performing large-scale interferometry, the orbital error of the platform cannot be ignored. Orbital inaccuracies manifest themselves as low-frequency trends in the APS maps, leading to corruption in data in low spatial frequencies. The result can be a bias of several centimeters, leading to a complete failure in the NWPM ingestion process. This thesis proposes an orbit correction method that jointly exploits the unwrapped APS and a set of GNSS-derived APS measurements. The orbital error is modeled and then
estimated thanks to a network of GNSS stations in the scene. The APS maps are then calibrated by taking into account the estimated orbital error.

The last part of the theoretical work of this thesis regards the so-called *absolutization*. APS are differential, therefore they represent the difference of atmospheric conditions between two time instants. For NWPM ingestion, it is mandatory to transform APS into ZTD by estimating an unknown master image. In this thesis, we review two possible methods for the absolutization.

The last chapter involves three case studies where the quality of the processing chain has been assessed. Each case study is different in scene composition, temporal stability, topography, number of images, frame size, and validation dataset. The case studies proved the effectiveness of the method to capture most of the atmospheric complexity. In the first two case studies, the estimated APSs are compared with a set of SqueeSAR® APS where the elevation error and subsidence have been removed. The standard deviation of the difference never exceeds 13 mm, proving the validity of the theoretical assumptions on the size of the estimation window and the short total time span of the stack.

In the last case study, instead, the estimated APS are compared with a GACOS dataset showing good accordance in both mean (below 3 mm) and standard deviation (below 21 mm).
Bibliography


