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CBDC and Bank Runs in an open economy context

Tesi di Laurea Magistrale in
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Abstract

The world's central banks are studying the possibility of introducing a digital currency, a so called Central Bank Digital Currency. The aim of this work is to analyze the impact a CBDC can have on the financial stability. Firstly,all possible options are presented for the construction of a digital currency. Then, via the Diamond and Dybvig model and the Fernandez-Villaverde et all adjustment to a central bank, we get to the aim of this thesis. We study the effect on the financial stability caused by the introduction of a foreign CBDC as an alternative to the domestic CBDC, therefore we work in a competitive open economy. We suppose the foreign central bank offers a credible riskless bank deposit, therefore is not subject to bank runs. We discuss all the various possible scenarios that can occur: we find that new type of runs can happen, even in the case the domestic central bank offers a socially optimum contract. This runs could be partial if we impose a capital account constrain that limits the number of deposits in the foreign central bank, or exhaustive. Therefore we conclude that the introduction of a foreign riskfree CBDC will reduce the financial stability.

Keywords: CBDC, Central Bank Digital Currency, bank run, financial stability, open economy

Abstract in lingua italiana

Le banche centrali mondiali stanno studiando la possibilità di introdurre una moneta totalmente digitale. L'obiettivo di questo lavoro è analizzare l'impatto che tale potrà avere sulla stabilità finanziaria. Inizialmente vengono presentate tutte le possibili opzioni per la costruzione di una moneta digitale. Successivamente, passando attraverso il modello di Diamond and Dybvig e la reinterpretazione adattata alle banche centrale fornita da Fernandez-Villaverde et all, si arriva al vero obiettivo del lavoro. Studiamo l'effetto sulla stabilità finanziaria dovuto all'introduzione di una moneta digitale straniera che convive con quella domestica, pertanto andiamo a lavorare in un contesto di economia aperta in cui vi è una competizione tra le due banche centrali. Nel nostro modello supponiamo la banca centrale straniera capace di fornire un deposito privo di rischi, pertanto non soggetto a run bancari. Andiamo quindi ad analizzare tutti i vari possibili scenari che possono accadere: scopriamo che nuovi casi di run si potranno verifica. Esse saranno parziali se imponiamo un limite al numero di agenti che possono depositare nella banca straniera, oppure totale se tale restrizione non è presente. Concludiamo che, in tale economia, l'introduzione di una banca straniera che è in grado di fornire un asset privo di rischio riduce la stabilità finanziaria.

Parole chiave: CBDC, run bancarie, stabilità finanziaria, economia aperta

Contents

Introduction

"The evolution of money over the centuries and across different regions has reflected changes in economic life, in technology, and in societal beliefs and behaviours. When Marco Polo visited China in the 13th century, he was shocked to discover paper money, which we now know had already been used there for centuries. He called its creator a perfect “alchemist”. Today, digitalisation is spreading to all areas of our life, including the way we pay. So I do not expect anyone to find the idea of a digital currency as astonishing as Marco Polo found the idea of paper money." [43] This is how Fabio Panetta, executive member of the ECB, started his speech at the ECON Committee of the European Parliament and helps us understand the reason behind the necessity of a digital currency. Nowadays digitalisation is reaching all areas of our lives and the digital and online buying is becoming the most substantial form of purchase.

Even before the Covid 19, cash used in payments was declining in some advanced economies leaving the place to digital payments. Physical money remains the dominant means of payment in the euro area as a whole, but its use is declining in some countries, and preferences might change rapidly and unexpectedly.

The pandemic event accentuated this trend even more. Because of, or we can say thanks to, the lockdown periods, people made the online payments their primary source of payments, and in some situation even their only one. Therefore people has approached this technological world, even the ones that were not confident with technologies.

Given this scenario, central banks could not only have the role of spectator, they must be in the front line. They have wondered "What is the role a Central Bank can have in a contest of innovation and to what extent will its values and its mission be put into practice through these new instruments?". The answer arrives with the Central Bank Digital Currency (CBDC). This is not yet a concrete solution, but in the last few years plenty of research and tests on this topic has been made. A recent survey found that more than 80% of Central Banks are engaged in investigating CBDC and half have progressed past conceptual research to experimenting and running pilots. To coordinate and consolidate some of this work, the central banks of Canada, Japan, Sweden, Switzerland, the United Kingdom and the United States have come together, along with the European Central

Bank and the Bank for International Settlements. [42]
Therefore we can say a CBDC is a certain and immediate goal, something we will reach in the next years. And with 'next years' I do not mean ten or more years, but we can bet we will get involved with a digital money in the near future.
Consequentially, we can suppose that in the next future financial crisis, we could already have to deal with this kind of currency. Therefore, we can jump the gun and analyze how a financial crisis will behave in this situation. Starting from the already existing models of a crisis, the Diamond and Dybvig models, we will adapt it to a digital currency by exploiting the Fernandez-Villaverde et all model. Finally, we will modify this last model to introduce a foreign central bank and understand the market behaviour when a consumer can choose between a domestic central bank and a foreign risk free asset central bank. This paper contributes to the literature on international implications of CBDCs on financial stability by analyzing the behaviour of an economy composed by a domestic central bank and a foreign central bank, both providing a CBDC. We study the possible scenario in an open economy, both related to the introduction of a foreign currency or caused by the domestic bank behaviour.

Chapter 1 will analyze how a CBDC could be constructed, focusing on the risk factors. It will answer to all the questions necessarily arising when it comes to a new project: what is a CBDC? Why should it be introduced? Who will introduce it? How will it be structured? When will it be introduced indicatively? Finally, it will focus on the risks associated with this digital currency.

Chapter 2 will analyze the classical financial crisis models. It presents the Diamond and Dybvig model, an influential model of bank runs and related financial crises.

Chapter 3 will apply the Diamond and Dybvig model to central banks runs, trying to underline the feature most related to this kind of banks.

Chapter 4 will finally analyze how a central bank runs will behave in a contest where CBDCs is already a real currency. The focus will be on an economy composed by a domestic central bank issuing a CBDC, and a foreign central bank issuing a riskfree CBDC.

1 | CBDC presentation

1.1. What is a CBDC?

A Central Bank Digital Currency is a digital form of central bank money, a virtual form of a fiat currency. It is issued and regulated by a nation's monetary authority or central bank. It is a third form of base money, next to overnight deposits with a Central Bank (available only to banks, specific non-bank financial firms, and some official sector depositors) commercial bank currency and banknotes, being universally accessible but arguably of limited efficiency and relying on old technology.
Central bank interest in CBDCs comes at a critical time. Indeed in the recent years lots of innovations involving digital coins high on the agenda. First of all, since Satoshi Nakamoto drew attention on the Bitcoin factor in 2008, lots of other cryptocurrencies have been created, and this topics became one of the most discussed in the financial world. Second, the debate on a stablecoin, that is the search of stability in cryptocurrencies. According to the FSB and the BIS, stablecoins are defined as "a crypto-asset that aims to maintain a stable value relative to a specified asset, or a pool or basket of assets" and "cryptocurrencies with values tied to fiat currencies or other assets" respectively [3]. The last one is the entry of large technology firms into payment services and financial services more generally. But, as they have been created and used so far, cryptocurrencies are speculative assets rather than money, and have always been associated with various illegal activities, undermining the credibility of this new possible meter of wealth. From Dark Web transactions to price manipulations, the life of cryptocurrencies has never been relatively calm. Indeed, due to the nature of these coins, based on decentralization, lots of price manipulation has taken place and, thanks to the fact that transaction purpose is not controlled, lots of illegal activities have been paid using this technology.
Therefore, the already existing cryptocurrencies technology can only be a basis for the construction of national digital currencies, but they need improvement and changes to make it safer and more controlled.

1.2. Why do we need a CBDC?

There are countless reasons behind the digital euro project and not all of them came from central banks. Indeed, the whole world's population is at stake, and citizens are also thinking about this opportunity. Motivation of the central banks are the ones we are more interested in this thesis, since it's the objective of this work, but other reasons are mentioned anyway, as the introduction of such new currency will have an impact on everyone's life.

1.2.1. Central banks motivations

As described in the 'report no.1 about the central bank digital currencies' [42], there's a variety of reasons driving the central bank's research to CBDC . Currently, the focus is on providing a CBDC for payments, enabling broad access to central bank money and providing resilience, but there are other motivations such as financial stability risks and enhancing monetary policy tools.
Let's focus on payment motivation and the advantages CBDC could bring:

- It can be useful in a scenario where access to cash is in decline, by acting as a digital banknote, involving that there is a danger that households and business will no longer have access to risk-free central bank money.
- It could provide better means to distribute and use funds in geographically remote locations, since it does not require the physical withdrawal of banknotes from branches. Moreover, it can be useful during natural disasters, when people cannot reach the bank, or to send economic aid from outside.
- It gives the opportunity to improve financial inclusion reaching that part of the population that has not a bank deposit, especially for central banks in many emerging markets economies. To this end, significant offline capabilities would need to be developed both for CBDC system and any dependencies.
- CBDC could provide common means to transfer between fragmented closed-loop systems, reducing the costs and difficulties of paying users of other systems.
- One of the biggest improvements will be the anonymity in electronic payments. Indeed, there can be a technology thanks to which there will be no need of all the information about the figures involved in the transaction. A possible model for this purpose is one where personal and business data would not be disclosed to

third parties or government unless required by law, while maintaining capability to investigate.

What about the monetary policy motivations? The introduction of a CBDC might reinforce the transmission of monetary policy by allowing a central bank to set the remuneration rate on the digital currency, in order to directly influence the consumption and investment choices of the non-financial sector. If considered as a tool to improve the transmission of monetary policy, the digital euro should be remunerated at interest rates that the central bank can modify over time.

1.2.2. Consumers perspective

Even from the point of view of the individual citizen, the introduction of a digital currency can be seen as a necessary technology, capable of improving the economy.
CBDC should allow off-line payments, this will be easy for vulnerable groups to use, free of charge for basic use by payers and should protect privacy. Moreover, cash is the only means of payment that is provided by the public sector with no commercial perspective but we are in an economy where cash is fading, therefore we need a digital alternative that retains that feature.
In a scenario that was already opening the doors to such an alternative, the Covid event accelerated the process. Guido Sacchi, chief information officer at payment technology and software solutions firm Global Payments, said that: "Covid was a big catalyst for the adoption of digital payments and digital forms of commerce." [15] E-commerce spending in the United States grew by 93% year on year in May 2020 and e-commerce as a share of total retail sales reached a record of 33% in the United Kingdom in April and May 2020. All the data can be visualized in the figure 1.1

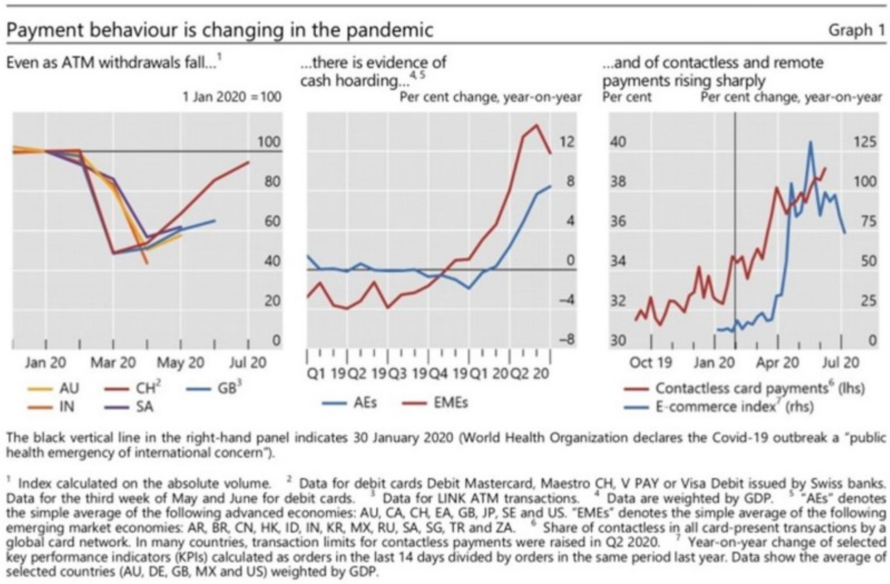


Figure 1.1: Payment behaviour during pandemic [8]

Italy has also been affected by this phenomenon, as the Bank of Italy pointed out in the report "L'impatto della pandemia sull'uso degli strumenti di pagamento in Italia". The estimates show that the pandemic has increased the use of cards compared with cash at the physical point of sale and has encouraged transactions through more innovative payment technologies that allow physical distancing, such as purchases with contactless cards, those on e-commerce sites, and those made by bank transfer. [26]

Moreover, when the pandemic shut down businesses early 2020, the need for digital, touchless payments took off. "When the pandemic came, people didn't want to touch surfaces, they didn't want to pass over their credit or debit cards or punch in their pin on the keypad," Andrew Laudato, chief operating officer of the health and wellness retailer, pointed out. Looking ahead, experts say omnichannel options that began in the pandemic, such as curbside pickup and QR code contactless ordering, are likely to remain strong along with touchless payments.[15]
These data make it clear how necessary and urgent is to introduce a currency that is not physical and is accessible to anyone, even those groups of populations more isolated and who do not have access to the most modern methods of payment, so that, even in the event of future health crises, the population is ready to face them without having to risk their health.
To conclude, it can be seen how these motivations can be translated into a digital cur-

rency. General purpose CBDCs could enhance integration in the medium term if inclusion features prominently in CBDC designs. Moreover in emerging markets general purpose CBDCs could improve financial inclusion by providing a low-cost means of payment and acting as an on ramp to broader financial services. Finally advanced economies central banks are considering these CBDCs in light of declining cash use and potential reduced access to cash.

1.3. Who will introduce a CBDC

The need of a digital currency has already been demonstrated. This demand can be met by central banks thanks to the introduction of a Central Bank Digital Currency (CBDC), a digital equivalent to banknotes. Therefore it must satisfy three major functions of money: it must provide a medium of exchange, i.e. a way to make payments, it must be a unit of account and a store of value.

Why will the central bank be the protagonist of this innovation? Because a central bank is the public institution which manages the currency of a country or group of countries and controls the supply of money, that is, the quantity of money in circulation. Therefore it is natural to think of central banks as those who will also deal with a reliable digital currency. Central banks are accountable public institution that play a pivotal role in payment systems, both wholesale and retail.

But how far along are central banks with this project? In 2017 and 2018, many central banks had a negative or dismissive stance, particularly toward retail CBDCs. Since late 2018, the number of positive mentions of retail and wholesale CBDCs in speeches has risen, and in fact there have now been more speeches with positive than a negative stance [7]. According to the Atlantic Council [18], an independent think tank headquartered in Washington, D.C., 105 countries, representing over 95% of global GDP, are exploring a CBDC. A little over two years ago, in May 2020, just 35 countries were considering a CBDC. So far, 10 countries already launched a CBDC (Nigeria, the Bahamas, Jamaica and seven countries in the Eastern Carribean), while 15 countries are on pilot.

The most relevant global cases during the past two years, are described in the maps below, to enable a comparison of this years evolution. This can help us understand how quickly the phenomenon is evolving:

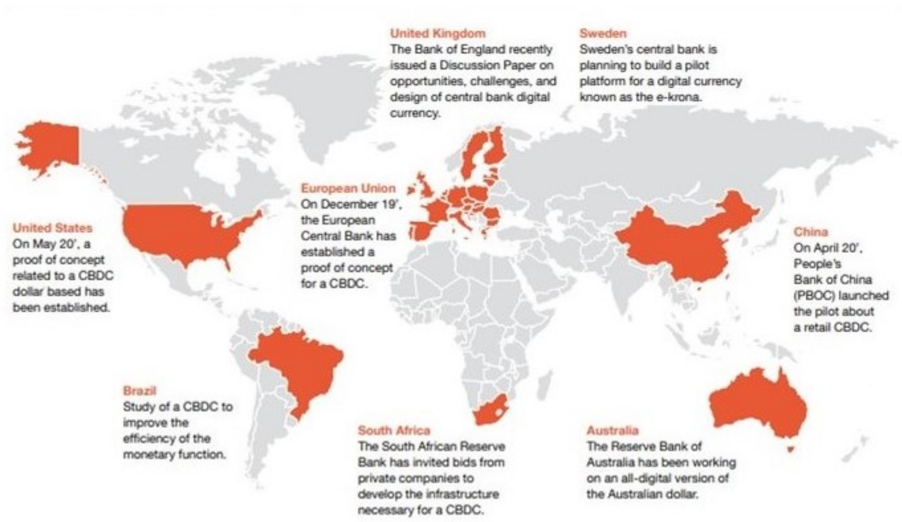


Figure 1.2: How Central Banks are moving on (from 2020)[41]

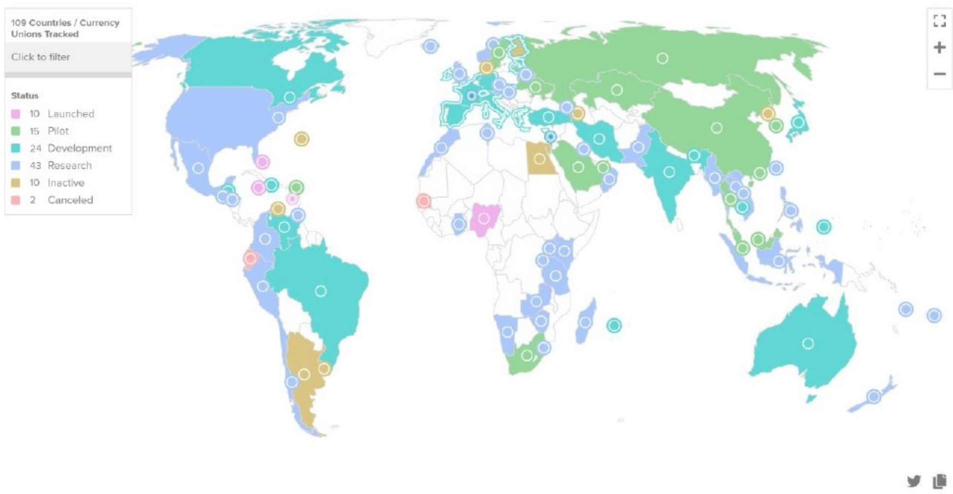


Figure 1.3: Central banks at varying stages of developing digital currencies (26 June 2022) [18]

Being this a project under development, research can lead to daily updates. To stay ahead, a site was designed to update the project progress, region by region. This site can be reached by scanning the following qrcode.

Figure 1.4: Central Bank Digital currency tracker

Now, let's analyze some particular cases.

1.3.1. The Bahamas: Sand Dollar

In December 2019, the Central Bank of the Bahamas launched the Sand Dollar project on the island of Exuma. After the successful pilot in Exuma, the digital currency fully launched across the country in October 2020, allowing the Bahamas to be the first country that experiment the digital currency. The main reason the Bahamas needed this new digital currency is its geography, as it's made of 700 islands scattered across a vast expanse of ocean. Thus, the fundamental advantage of the Sand Dollar might be that it is easier to distribute than cash, especially among the underbanked and unbanked.

As described in their relative website, "Sand Dollar allows greater flexibility and accessibility for resident that want to participate in financial services via either a mobile phone application, or using a physical payment card to access a digital wallet. It also provide and excellent record of income and spending, which can be used as supporting data for micro-loan applications."

The Sand Dollar is pegged 1-for-1 to the Bahamian dollar, the currency of The Bahamas, which is in turn pegged 1-for-1 to the U.S. dollar. Two-thirds of all jobs in The Bahamas are attributable to tourism, and since about 80% of tourists come from North America, the easy conversion rate makes many merchants accept U.S. dollar bills.

The Sand Dollar requires a technical platform to process payment transactions. NZIA Ltd. is the technical services provider to which CBOB out sources most of these technical services. Since the prepaid cards or digital wallets contain CB money and are based on DLT, the transactions can be processed directly between the eWallets of the payer and the payee.

The Sand Dollar is a wholesale CBDC for "settlements at the inter-bank level, akin to clearing house transactions". With a prepaid card recently introduced by Mastercard Inc.,

Bahamian consumers can pay with the Sand Dollar anywhere "Mastercard" is accepted around the world.
Consumers can choose between a Tier I eWallet with $500 holding limit, with a $1,500 monthly transaction limit, and a Tier II eWallet with a $8,000 holding limit, with a $10,000 monthly transaction limit. At the moment, offline transactions are not yet fully developed. [53]

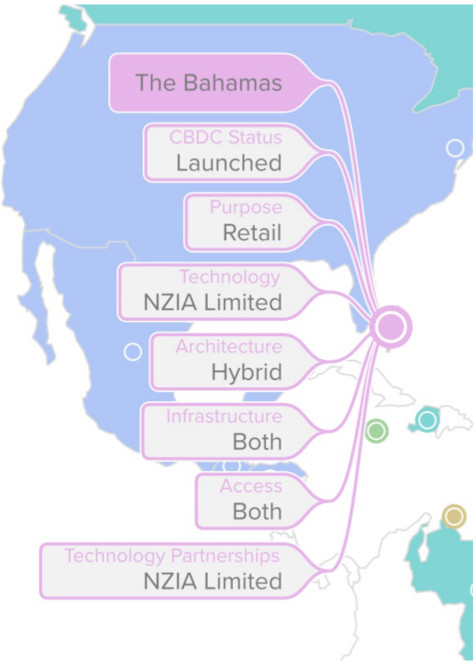


Figure 1.5: Bahamas Sand dollar characteristic[18]

1.3.2. China: e-CNY (Pilot)

China is one of the protagonists in the CBDC scenarios. It set up a Digital Currency Research Institute, that is a research team within China's central bank partly dedicated to launching the digital yuan—China's central bank digital currency (CBDC), also known as the e-CNY and formerly known as the "digital currency/electronic payments" (DCEP) project. It was established in 2016 and it is one of the most important sources of insight on the PBOC's (Peoples Bank of China) efforts to become the first major central bank to issue a digital fiat currency.
Peoples Bank of China had launched digital yuan in Luohu district in 2020 and in the city of Suzhou in 2021. Moreover, as of February 2022, the pilot has been operational in 11 regions, including the Olympic venues in Beijing and Zhangjiakou. During the Olympic

Games, both foreign and domestic visitors could use the e-CNY wallet. Use of the e-CNY during the Olympics fulfilled important policy motivations for the People's Bank of China. It was able to test the scalability and throughput of the e-CNY transactions. The current rate of transactions per second (TPS) is 10,000, but transaction capability in the future will reach 300,000 TPS.

e-CNY users can choose between individual or corporate wallets, which offer different transaction limits. Wallets can be software based, using the e-CNY mobile app, which allows users to manage their e-CNY transactions, or hardware based, with an electronic card that allows for touch-based transactions. In his speech at the Atlantic Council, Mu Changchun mentioned that foreign users primarily used hardware wallets at the Olympic Games venues.

In terms of technology choices, China is using a centralized ledger to record retail transactions and, in parallel, is implementing a tributed ledger for the reconciliation period at the end of the day. This indicates that China is exploring the use of blockchain technology in its digital currency. The PBoC, like many other Central Banks, would like to remain technology agnostic for now. However, its hybrid technology choice suggests it would be willing to move to a permissioned distributed ledger technology (DLT) in the long run. [39]

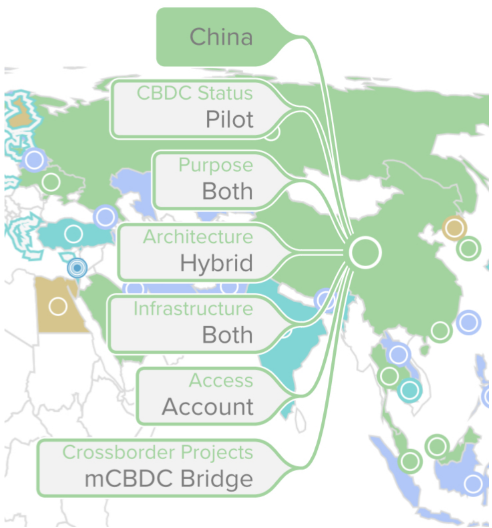


Figure 1.6: China e-CNY characteristic[18]

1.3.3. Sweden: E-krona

According to the Sweden's central bank website[50], "The Riksbank is investigating whether it is possible to issue a digital complement to cash, a so called e-krona. Just like cash, the e-krona would be issued by the Riksbank and be available to the general public. An e-krona would offer the general public continued access to state money, issued by the Riksbank, but in digital form. It would be better adapted to our digital society than cash is, and it could be used in situations where it is not possible to pay with cash now. The Riksbank started the e-krona project in 2017 to analyse the need for an e-krona. The project team has held a dialogue with several national and international agents to hear their views on an e-krona, reviewed proposals for suitable technology, and examined the legal issues that need to be dealt with to ensure the Riksbank has a clear mandate to issue an e-krona. In 2020, the Riksbank entered into a more practical phase of the e-krona project. To test how an e-krona might look and function, the Riksbank started a project, the e-krona pilot, together with the company Accenture, to construct a possible technical platform for the e-krona. The objective of the project is for the Riksbank to learn more about how a technical solution for the e-krona could work."
In conclusion of phase 2, the Riksbank underlined that it should be possible to integrate an e-krona into banks' and payment service providers' own existing systems. [51]
The Sweden example has been followed by the Bank of Japan to the computation of their digital currency. Indeed, Kazushige Kamiyama, the head of the BOJ's payment system department, said in an interview: "Sweden's staged and planned expansion of experiments is a better fit for us than China's big-scale tests from the get-go."

Figure 1.7: Sweden E-krona characteristic[18]

1.3.4. Eurosystem: Digital Euro

According to the European Central Bank, the Governing Council of the European Central Bank is working on a possible digitalization of the euro, a so called digital euro. As Fabio Panetta explained in a speech of June 15, 2022: "We are designing a digital euro that would make central bank money usable for digital payments, giving Europeans a digital means of payment that they can use throughout the euro area for their everyday payments and supporting Europe's societal objectives.[...] We are working to address at an early stage any possible undesirable consequences that may result from the issuance of a digital euro for monetary policy, financial stability and the allocation of credit to the real economy."[45]
The digital euro project may provide a suitable opportunity to establish the public-private cooperation that is needed to build the pan-European private retail payment solutions of the future. In October 2021 the ECB launched a two-year investigation phase to define the design features of a digital euro. At the end of 2023 they could decide to start a realisation phase to develop and test the appropriate technical solutions and business arrangements necessary to provide a digital euro. This phase could take three years. Therefore, the digital euro could be issued in four or five years, if the ECB's Governing Council and European authorities will decide to do so.
But how does Italy fit into this project? In late 2020, the Italian Banking Association (ABI) announced ongoing research contributions to the European Central Bank's (ECB) work on the digital euro. It was joined by the Bank of Italy in 2021, which is aiding in the ECB's efforts toward the digital Euro. In July 2021, the Bank of Italy published a report on the possible architecture of the digital euro. In July 2021, the Estonian Central Bank released a report about its experiment with the ECB and the central banks of Spain, Germany, Italy, Greece, Ireland, Latvia and the Netherlands to assess the functionality of the Digital Euro. The project was able to conduct 300,000 transactions per second, with an average rate of less than 2 seconds per transaction.

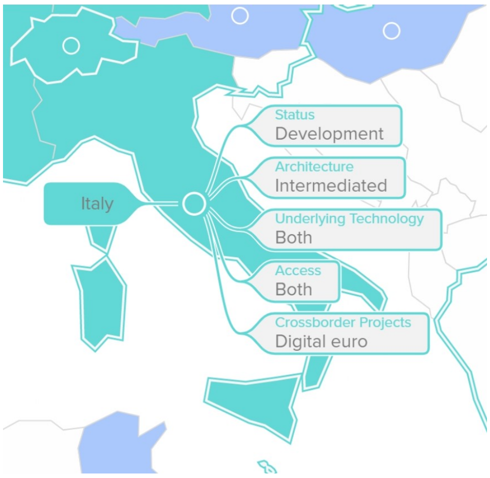


Figure 1.8: Digital Euro: Italian's project[18]

Central banks would be the only entities entailed to issue and redeem a CBDC and would bear the ultimate responsibility for the design of the CBDC system and the operational/overnight design of the core ledger.
But will central banks be the only ones who can deal with the CBDC? No, there are two possible scenarios. A one-tier CBDC system would be the one where the central bank was responsible for all aspects of the system including issuance, account-keeping, transaction verification, and so on. Alternatively, in a two-tier or 'platform' system the central bank would develop the technology to issue CBDC to private sector entities with those entities then responsible for all customer-facing activities. [49] A natural split in any tiered CBDC system would be for the central bank to be responsible for the core of the system to the extent that they could steer the system to deliver policy goals and a safe and efficient payment system. Multiple private entities would act as intermediaries, competing and offering choice within an ecosystem to drive innovation and efficiency.
Therefore, a CBDC would be introduced by the central banks, being the responsible of the currencies in the country, but they must cooperate with other entities.

1.4. How to build a CBDC

We now focus on the fundamental issue: how will a CBDC be designed?
A CBDC ecosystem would comprise multiple elements and functions.

1.4.1. Retail vs wholesale CDBD

First of all we must underline there are two kinds of CBDC that must be designed. A retail CBDC would be used like a digital extension of cash by all people and companies, whereas a wholesale CBDC could be used only by permitted institutions as a settlement asset in the interbank market. Now, let's explain these two elements a bit more.
Wholesale CBDCs are intended for settlment of interbank transfers and related wholesale transaction. They serve the same purpose as reserves held at the central bank but with additional functionality. They could make central bank money programmable, to support automation and mitigate risks. They could be implemented on new technology stacks, therefore they can be designed with international standards in mind to support interoperability. Wholesale CBDC projects have not yet reached the same maturity level as retail CBDC. However, many succesful pilots have been reported over the last 12 months. The top 10 wholesale CBDC projects so far are listed in the following figures, taken from a "PwC CBDC global index and Stablecoin overview 2022":

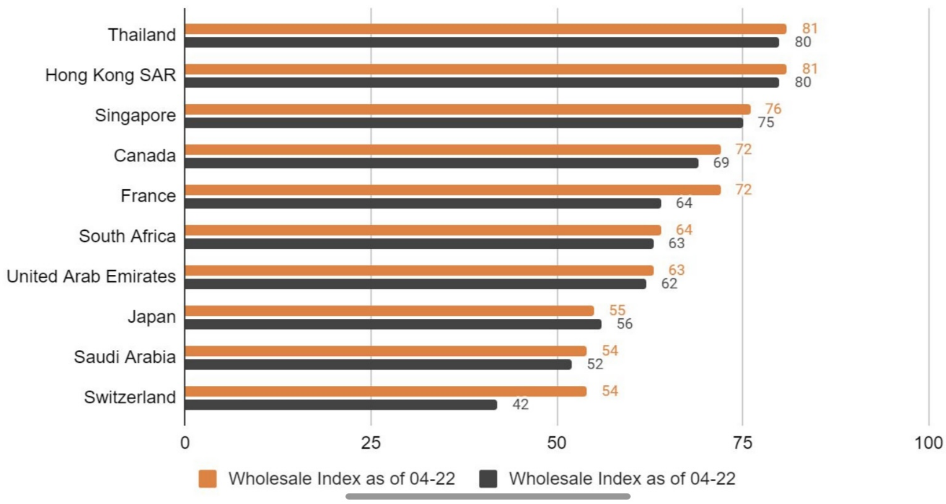


Figure 1.9: Top 10 wholesale CBDC in 04-22 vs 04-21[48]

Retail CBDCs, on the other sides, modify the two tier monetary system to make central bank digital money available to general public, just as cash is available to general public as a direct claim on central bank. An important feature is that they do not entail any credit risk for payment system participants, as they are a direct claim on the central bank. The top 10 retail CBDC projects so far are listed in the following figures:

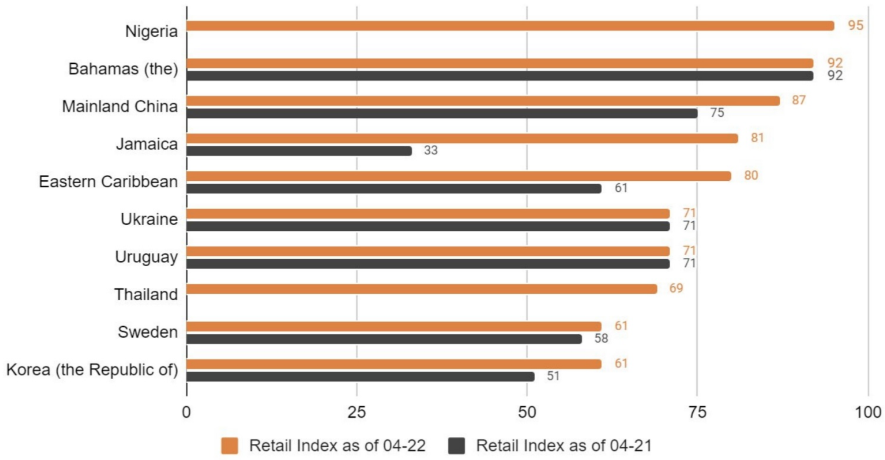


Figure 1.10: Top 10 retail CBDC in 04-22 vs 04-21[48]

1.4.2. Account-based vs token-based

Another division of a CBDC could be account based and token based. An account-based CBDC ties a CBDC to an identity system where a sender first verifies whether the receiver has an account and then verifies the identity of the receiver before making a payment or transfer. Basically, with an account-based CBDC, the person sending a CBDC amount verifies the identity of the receiver and whether the receiver holds an account (Terracciano and Somoza, 2020 [54]). An example is the Fedwire Funds Service: if a participant submits an instruction to the service to transfer funds to another participant, the instruction must be transmitted in accordance with the Reserve Banks' security procedures, which include access control features designed for the purpose of verifying that the instruction the Banks receives is an authorized one. It must establish a process for verifying the identity of the would be payer.

A token-based CBDC is not tied to an identity system; rather, a token-based CBDC ties a CBDC to an access technology based on digital tokens (Auer and Böhme, 2020 [6]). It is a digital version of the physical cash. With a token-based CBDC, a person verifies the authenticity of the token for every payment transaction (Terracciano and Somoza, 2020[54]). A CBDC token is a digital object that has a given value expressed in the national unit of account and is a claim on the central bank (Armelius et al., 2021[30]). Most times, the CBDC token is stored remotely not on local devices. An example of token based is currency. If I pay anything with the currency, the only things that must be guaranteed is the currency is not fake. In particular, no one need to know anything about me.

Moreover, both can be used for retail payment. But unlike account-based, the liabilities

are not stored at accounts at a central bank. Rather, they are stored through digital wallets.
Another difference is that token-based CBDCs can be traded offline, while account-based CBDC transactions cannot not be executed without the system remotely validating the identity of the account holder and the balance on the account.

1.4.3. Characteristics

There are three common foundational principles for a central bank's consideration of CBDC issuance that flow from their common objectives, as described in the "Central bank digital currencies: foundational principles and core features. Report no.1" [42].

- **Do no harm** : new forms of money supplied by the central bank should continue supporting the fulfilmnet of public policy objectives and should not interfere with or impede a central bank's ability to carry out its mandate for monetary and financial stability. That does not mean ”have no impact” but it means to not have a negative impact. That is, applied to a CBDC, to maintain and reinforce the singleness or uniformity of a currency, allowing the public to use different forms of money interchangeably. This principle arose from a recognition that while a CBDC has the potential to provide benefits to the operation and resilience of the financial system, particularly regarding payment services, it could also affect existing financial market structures and business models, which may pose risks to financial stability as the financial system evolves, particularly via the potential disintermediation of banks. [1]
  As an example, we can cite one of Fabio Panetta's answers in an interview about CBDC: " a CBDC that can be used outside the jurisdiction where it is issued might increase the risk of digital currency substitution, or digital “dollarisation”. If a foreign CBDC were to be widely adopted, this might lead to the domestic currency losing its function as a medium of exchange, unit of account and store of value, ultimately impairing the effectiveness of domestic monetary policy and raising financial stability risks. These risks are particularly relevant for emerging markets and less developed economies that have unstable currencies and weak fundamentals. Currency substitution could also occur in small advanced economies that are open to trade and integrated in global value chains. Since international trade and finance are complementary to each other, financial integration may matter, too. It is hard to gauge in advance how significant the risks of digital currency substitution could be, and in which currencies this substitution could occur. Trade and finance linkages with the issuers of international reserve currencies, the United States, the euro area

and China , vary considerably across countries. That, in turn, suggests that the risks of currency substitution vary significantly across countries and currencies. In any case, the introduction of a CBDC in one jurisdiction must do no harm." [44]

- **Coexistence**: Central banks have a mandate for stability and proceed cautiously in new territories. Different types of central bank money should complement one another and coexist with robust private money to support public policy objectives. It should not be imposed as the only means of payment, but it must be able to 'work together' with other currencies.
- **Innovation and efficiency**: a continuous updating of technology, innovation and competition must be guaranteed, in order to drive efficiency in a jurisdiction's payment system. Otherwise users may adopt other, less safe instruments or currencies. This could lead to economic and consumer harm if the new choice is a means of payment not controlled and not regulated. This behaviour could potentially damage monetary and financial stability.

This basis principle are not the only feature a CBDC must satisfy. Central banks have common public policy objectives that allow common principles to be agreed, therefore they must satisfy some core features, summarized in the figure 1.11:

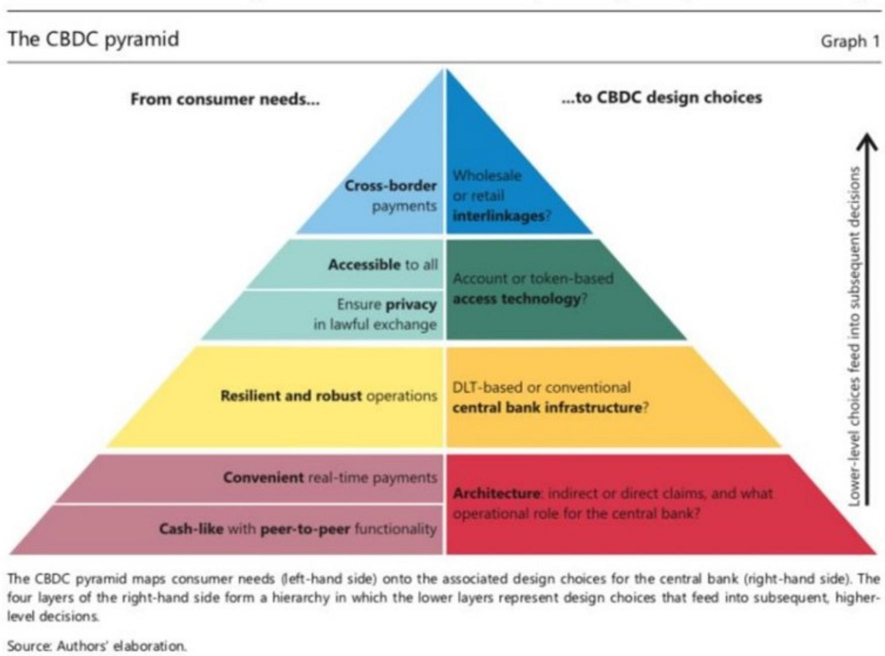


Figure 1.11: CBDC characteristic that must satisfy [8]

Let's explain some of these features:

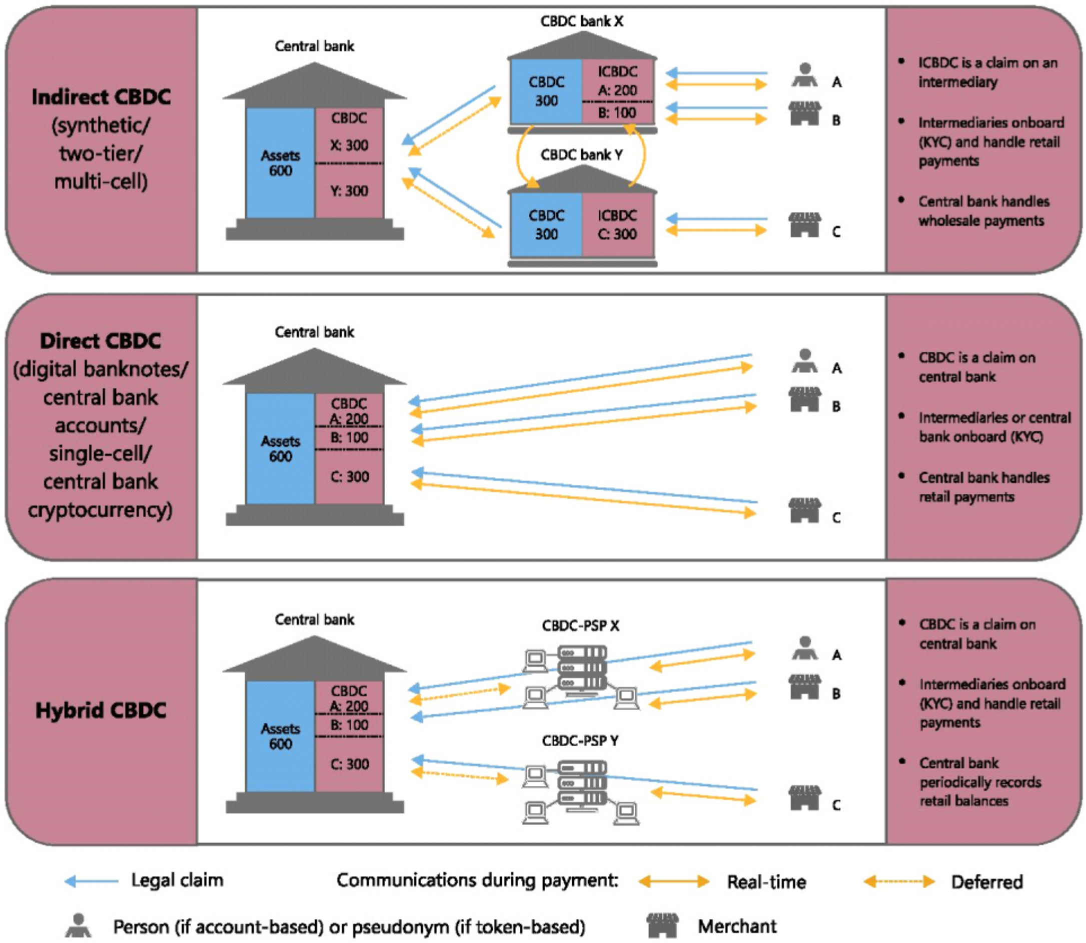
- **Cash-like payment system**: A CBDC must work as a banknote or a credit cards. Cash provides for payment features are often described as "peer-to-peer": they are instant, person-to-person and do not involve any third party. This characteristic must be behind this project since a CBDC will be used as a substitute of a currency.
- **Convenient**: CBDC payment should be as easy as using cash, tapping with a card or scanning a mobile phone to encourage adoption and accessibility. Consumers are unlikely to adopt a CBDC if it is less convenient to use than today's electronic payments/cash.
- **Resilient and robust**: A CBDC system should be resilient to operational failure and disruptions, natural disasters and, electrical outages and other issues. CBDC system should have the ability to recover from potential hardware or software failures. The resilience and robustness of network operations is a key dimension that must be taken in account for the creation of a CBDC. It must be decided whether to base the CBDC on traditional banking infrastructures or on Distributed Ledger Technology. Such choice deeply influences the structure and governance management of the infrastructure, which could either be centralized or decentralised.
- **Accessible**: A CBDC must be accessible to everyone to ensure to be used as a payment. It can't be considered if there is a part of population that is not able to be part of this project.
- **Interoperability**: such dimension must be taken into consideration in order to guarantee the possibility of interaction between different CBDC systems, therefore it impacts a higher-level layer of decision for the CBDC design.
- **Privacy-protected**:Protecting consumer privacy is critical. Any CBDC would need to strike an appropriate balance, however, between safeguarding the privacy rights of consumers and affording the transparency necessary to deter criminal activity.
- **Intermediated**: under an intermediated model, the private sector would offer accounts or digital wallets to facilitate the management of CBDC holdings and payments. Potential intermediaries could include commercial banks and regulated non-bank financial service providers, and would operate in an open market for CBDC services. Although commercial banks and non-banks would offer services to individuals to manage their CBDC holdings and payments, the CBDC itself would be a liability of the central banks.
- **Transferable**: For a CBDC to serve as a widely accessible means of payment, it

would need to be readily transferable between customers of different intermediaries. The ability to transfer value seamlessly between different intermediaries makes the payment system more efficient by allowing money to move freely throughout the economy.

1.4.4. Architecture

Besides the characteristics, the architecture a CBDC economy must have is another important features to describe.
Vital to success of a retail CBDC is an appropriate division of labour between the central bank and the private sector. Central banks and Private Services Providers (PSPs) should continue to work together in a complementary way, with each doing what they do best: the central bank providing the foundational infrastructure of the monetary system and the private sector using their creativity, infrastructure and ingenuity to serve consumer. Therefore this collaboration must be considered in the implementation of a digital currencies. Currently, three different architecture are designed, as represented in the figure 1.12 [5]:

An overview of potential retail CBDC architectures Graph 2



In all three architectures, the CBDC is issued only by the central bank. In the indirect CBDC architecture (top panel), this is done indirectly, and an ICBDC in the hands of consumers represents a claim on an intermediary. In the other two architectures, consumers have a direct claim on the central bank. In the direct CBDC model (centre panel), the central bank handles all payments in real time and thus keeps a record of all retail holdings. The hybrid CBDC model (bottom panel) is an intermediate solution providing for direct claims on the central bank while real-time payments are handled by intermediaries. In this architecture, the central bank retains a copy of all retail CBDC holdings, allowing it to transfer holdings from one payment service provider to another in the event of a technical failure. All three architectures allow for either account- or token-based access.

Source: Authors' elaboration.

Figure 1.12: Possible architectures of a CBDC [5]

- **Indirect CBDC (two-tier CBDC) model**: in this model the consumer has a claim on an intermediary, with the central bank keeping track only of wholesale accounts. For consumers, this type of CBDC is not a direct claim on the central bank. Instead, the intermediary (the "CBDC bank" in figure 1.12) is mandated to fully back each outstanding indirect CBDC-like liability to the consumer (labelled "ICBDC" in figure 1.12) to retail consumers via its holding of actual CBDCs, or other central bank money, deposited at the central bank. It is similar to commercial bank currency but it is a liability of the central bank. To get CBDC a person should exchange other money for it, cash or bank deposits. CBDC by banks at the central banks are included in reserves, i.e.monetary base. Just as in today's system, intermediaries handle all communication with retail clients, net payments and send payment messages to other intermediaries and wholesale payment instructions to the central bank. The indirect CBDC also relieves the central bank of the responsibility for dispute resolution. But one side effect could be that a central bank keeps no record of individual claims, only the intermediaries do, whereas the central bank records only wholesale holdings, nor is there any cash-like direct proof of the claim. Thus, the central bank cannot honour claims from consumers without information from the intermediary. If the intermediary is under stress, determining the legitimate owner might involve a potentially lengthy and costly legal process with an uncertain outcome.

- **Direct CBDC model**: the CBDC represents a direct claim on the central bank, which keeps a record of all balances and updates it with every transaction. In this architecture, know-your-customer and customer due diligence could be handled by the private sector or the central bank or another public sector institution. The central bank, however, would be the only institution handling payment services. The direct CBDC is attractive for its simplicity, as it eliminates dependence on intermediaries by doing away with them. However, this entails compromises in terms of the payment system's reliability, speed and efficiency.
- **Hybrid CBDC model**: is an intermediate solution providing for direct claims on the central bank while allowing intermediaries to handle payments. This solution merges elements of both the indirect and the direct CBDC. One key element of the hybrid CBDC architecture is the legal framework that underpins claims, keeps them segregated from the balance sheets of the PSPs, and allows for portability. If a PSP fails, holdings of the CBDC are not considered part of the PSP's estate available to creditors. The second key element is the technical capability to enable the portability of holdings. The hybrid CBDC would have both advantages and disadvantages vis-à-vis the indirect or direct CBDC architectures. As an intermediate solution, it might offer better resilience than the indirect CBDC, but at the cost of a more complex to operate infrastructure for the central bank. On the other hand, the hybrid CBDC is still simpler to operate than a direct CBDC. As the central bank does not directly interact with retail users, it can concentrate on a limited number of core processes, while intermediaries handle other services including instant payment confirmation.

Which architecture should be chosen? According to the Bank for International Settlement, there are good arguments against a one-tier system fully operated by the central bank. CBDCs are best designed as a part of a two-tier system, where the central bank and the private sector each play their respective role. Overall, a two-tired architecture emerges as the most promising direction for the design of the overall payment system, in which central banks provide the foundations while leaving consumer-facing tasks to the private sector. A CBDC grounded in such a two-tired system also ensures that commercial banks can maintain their vital function of inter mediating funds in the economy.

1.4.5. Technology

We have already analyzed the characteristics a CBDC must satisfy and the architecture that can be build in a contest of digital money. Let's now focus on the technology that can be exploited.

A variety of technologies could support the core feature and they depend on the architecture. Indeed, an indirect CBDC implies loads similar to those of today's system. The direct CBDC would require massive technological capabilities, as the central bank processes all transactions by itself, handling a volume of payments traffic comparable with that of today's credit or debit card operators. The hybrid CBDC architecture is more complex to operate than the indirect model, as the central bank does maintain retail balances. It could be implemented at scale using today's technology. The infrastructure could be based on a conventional centrally controlled database, or on a distributed ledger. The technology best suited to offer the required functionality is distributed ledger technology (DLT), of which the best known example is blockchain. This is a system based on a distributed ledger, where every node in the network has a copy of the database. When the database is modified, for example adding a node of transaction, every copy is modified. But this technology could not be used in any architecture. Indeed direct CBDC cannot be based on DLT, while it can be applied to the two tier ones, as the number of transactions in many wholesale payment systems is comparable with that handled by existing blockchain platforms.

There must be different technology considerations given different core features. In order to be convenient it must be available on modern smartphones and, for users without smartphones, central banks should provide devices designed for this purpose. To be resilient, it must be similar to current payment system and operate a 24/7/365 services. Moreover, in order to provide security, modern technology used in credit cards and smartphones must be a basis for the CBDC security. Another important feature it must satisfy is the ability to meet volume and velocity of modern way of payments, feature that is not already met in the cryptocurrencies world. This last feature is one of the most important one since it cannot exist a digital currency until it is not computed a technology ensuring that it is able to meet all payment requests.

1.4.6. Privacy

Privacy is an acknowledged fundamental human right in most international instruments. Therefore it must be guaranteed in every new technology where private information are exposed. What are the concerns on this topic? Identity fraud is a key factor in the digital economy, lots of them are experimented and they are always increasing. In the digital economy, the term fraud expresses several concepts: fraudulent payments, manipulation or theft of internal data, illegal payments for breach of embargoes or sanctions. Indeed, to ensure access and integrity in today's financial system, the identity must be verified. Therefore physical documents must be provided and this could lead to an identity theft. The rising incidence of major data breaches in recent years, in particular in the financial institutions, underscores the possibility that data or funds may be stolen.
Such risk would be similar for CBDC payment services. These consideration suggest that a token-based CBDC which comes with a full anonymity could facilitate illegal activity, and is therefore unlikely to serve the public interest. Therefore, central bank must demonstrate that it is able to protect the privacy of CBDC owners. Data to be protected could include personal information about the payer or payee or information about the payment itself, that can reveal personal information about the payee, their relationship or business. At this point in the study, it can be said that the most promising way of providing central bank money in the digital era is an account based CBDC built on digital ID with official sector involvement.
Moreover, this new technology will lead to a more controlled world, every single transaction will be traceable, reducing the citizens freedom. The UK House of Lords Committee, in a largely sceptical report on the desirability of a sterling CBDC, raised the spectre of a digital currency as 'an instrument of state surveillance': Governments could track all transactions between accounts held at their respective central banks. A survey of European consumers conducted by the Bank for International Settlements identified public concern about unspecified threats to privacy posed by CDBC, and a European Union consultation paper on a digital euro has explicitly cited the issue.
Althought CBDC proponents say that the ability to track transactions is generally beneficial as it allows governments to crack down on illicit activities, they also argue that governments could take steps to preserve anonymity, such as requiring warrants for searches and encrypting.
It is undoubtedly true that, in countries with a more restrictive view of citizens' rights to

privacy, similarly intrusive standards will be applied to CBDCs. But that is an intrinsic feature of the political system rather than the CBDC.

1.4.7. Remuneration

Another important feature to understand is if the CBDC will be remunerated or not. The implications on the financial stability and the banking system's ability to provide credit to the economy would differ depending on this.
It is often assumed that CBDC would be designed to have cash-like properties, including zero remuneration. Central bankers and holders of central bank money got used to banknotes representing a risk-free, short-term financial asset with a zero nominal yield, regardless of the level of nominal interest rates. While some consider this feature of banknotes to be an anomaly that could be solved with CBDC (and the discontinuation of banknotes), others argue instead that it is important to preserve this cash-like feature when issuing CBDC.
In the case of a remunerated CBDC, it could become a perfect substitute for bank deposits. The role of banks could be limited not only in the context of the payment system, but also in the context of maturity transformation and, hence, household and firm financing. The overall effects for the economic system would depend on the banks' ability to attract funds by issuing longer-term liabilities than current account deposits, or by increasing the remuneration of the latter.
In the case of a not remunerated CBDC, in phases of high financial instability it could still increase the risk of a digital run from bank deposits, which would be easily and quickly converted into central bank liabilities, making the economy more unstable. However, just as today deposit's insurance and bank resolution rules limit depositors' incentive to run to convert deposits into banknotes, and thus reduce the frequency of bank runs, these two instruments could also limit any flight of deposits to CBDC. [12]
Despite the discussion about the remuneration, we know that an account-based CBDCs can easily be designed to pay an interest rate consistent with the monetary policy objectives of the central bank, including price stability and business cycle stabilization.

In a working paper of July 2021 [4], the European Central Bank models a remunerated CBDC with different interest rates to understand the impact on the steady state, i.e. structural changes in the financial system. They construct a general equilibrium model with search and matching frictions, which require entrepreneurs to borrow inside money (bank deposits) and outside money (CBDC). They analyze these two types of money since they are needed to pay for two different kinds of inputs that are used in production. The central bank chooses the interest rate it charges on loans to entrepreneurs and the

interest rate it pays on workers' deposits of CBDC. Thanks to this simulation, they can analyse the impact of the different CBDC design parameters on credit allocation and welfare within a unified framework. They conclude that increasing the interest rate on CBDC is effective in containing bank disintermediation, in particular if inside and outside money are close substitutes.
Moreover, Christian Pfister [45] lists the main options central banks would be faced with when defining their policies regarding the remuneration of retail CBDC, as well as the main areas they would probably look at when making their choices. He assesses qualitatively the impacts of the choices made on the likely areas of interest for central banks, showing that whether the policy rate and/or the rate on CBDC is positive or null or strictly negative matters. He showed that the absence of remuneration is superior to two discretionary policies: above rule-based with a negative interest rate on retail CBDC and below rule-based with positive or null interest rate on retail CBDC. These two policies can thus be eliminated. It is also unlikely that central banks could retain either of the other discretionary policies: the above rule-based one with a positive or null interest rate on retail CBDC is inefficient in monetary policy and financial stability terms and has high administrative, management and holding costs, and the below rule-based one with a negative interest rate on retail CBDC has high political costs. Thus all discretionary policies can be eliminated, which leaves a choice between a rule-based policy and an absence of remuneration.

1.5. Risk

The advantages of introducing a CBDC are innumerable, but so the risks associated with this technology. The first risk to consider, that is also the most substantial, is the one of cyber attacks, with potential financial and business implications for several dimensions (including monetary policy, financial stability, financial risk, and the safety and efficiency of the payment system). Using DLT technology, the possibility of attacks from the outside is a very important factor at the moment. In fact, modern cryptocurrencies that follow this technology have often experienced directly the attack by external entities, only the biggest blockchains, following a large consensus, have managed to limit the damage. This hacker attacks are aimed at the theft of customer money. But a DLT can only be altered by changing the entire history of previous transactions, an enormously expensive operation. But can we say this behaviour is a new type of risk with respect to the current payments? Is the possibility of money theft not a risk even with the current forms of payment? Can we say that this risk is linked to this new currency? Of course not, the novelty resides in the technology used to steal money, but the crime is present even with normal banknote

or credit cards.
A more specific risk is that the malfunctioning of the IT infrastructure underlying the digital currency could cause loss and damage to individual users, raising questions about the responsibility of the central bank. Moreover, unauthorised payment transactions conducted in digital currency could cause inconvenience to users; in such cases, if the payer has not acted fraudulently or with gross negligence, they should be reimbursed by the payment service provider. If the payment service provider is acting on behalf of the central bank, it cannot be ruled out that the central bank could, again, be drawn in. Although the central bank would not aim at expanding its intermediation role, this possibility cannot be ruled out. In this case it could be forced to invest more in illiquid assets, ultimately taking on more credit and market risk. As profitability is not, per se, a policy objective of the Eurosystem, these considerations would have no immediate implications for the design of a digital currency. A central bank issuing a CBDC should nonetheless strengthen its risk management.[9]
The topic on risk related on a CBDC has also been discussed by Per Callesen, Governors of Denmarks Nationalbank, at CBS' 100 year celebration event. He said that Danmarks Nationalbank has initiated a forum for collaboration among the critical companies in the Danish financial sector – the Financial Sector Forum for Operational Resilience, FSOR. FSOR has as a goal to create an overview of cyberthreats to financial stability. He pointed out a way of find out this kind of attacks: "Testing is one measure that has proven to be excellent at uncovering cybersecurity weaknesses. We regularly test cross-sector crisis response plans to manage serious operational incidents, including cyberattacks. But we also seek to test the cyberresilience of individual institutions through so-called Red Team Testing. In a Red Team Test, an external team is hired to perform a series of coordinated attacks. The target institution's internal IT security then has to defend the systems to the best of their abilities. Once the test is over, the institution's vulnerabilities and its capability to detect and respond to the attacks are evaluated. A remediation plan is then put in place to eliminate weaknesses before cybercriminals are able to take advantage of them."

1.6. Issue

Besides this risk, there are other issues related to a CBDC [10].

- **Interaction with payment system**: every time a new technology is introduced in the real world, it must interact with the already existing ones and is compared to them. Therefore, when a CBDC will come out in the sunlight, it must fit in a world where there are lots of different payment technology and it must be perfectly fitting in it. It will be required to serve its role at least as well as the current currency are doing. It must be able to interact with every existing payment system used by the clients and do this with not too much additional costs.

- **Online-Offline payments**: a CBDC is born in an online world, but it must be guaranteed in the offline one, since offline functionality avoids the sharing of transaction details with parties other than the payer and payee, enabling the CBDC to become a complement to cash and providing a back-up payment solution that is available in extreme situations. Therefore the payment could be settled immediately as a transfer of pre-funded units between the devices of payer and payee. But what are the possible complications? Offline payments would require highly secure front-end standards to be drawn up to govern functionality on interoperable users' devices. Moreover they must always be connected and consuming energy as is the case with current payment terminals. Finally, an offline digital currency would need to exist online at some point, in order to load money onto the offline digital wallet through the broader payment system.
- **Legal-tender**: legal tender status would be a desirable feature of the CBDC. Without this status, the drivers of acceptance would be more similar to those of other electronic payment solutions. It would require users to be able to receive incoming payments through means that are as user-friendly as banknotes, for example by using a simple physical device that can also be used offline or, if the legal tender status were applicable also to online payments, a digital wallet service that is available to everybody.

1.7. Financial stability

After the analysis on how a CBDC would be designed and all the possible consequences related to this, its time to start the real question of this work: how a CBDC will behave in a crisis. The relationship between this currency and the financial stability is the first step of the argument. CBDCs would have implications for financial intermediation and would need careful design and implementation. As reported in the semptember 2021 report by the BIS [1], central banks are exploring safeguards that could be built into any CBDC to address financial stability risks; although such measures may need careful consideration before they are used. Central banks might consider measures to influence or control CBDC adoption or use. This could include measures such as access criteria for permitted users, limits on individuals CBDC holdings or transactions, and particular choices around CBDC remuneration. Such measures could be valuable in managing risks in any transition and could potentially have a role on a longer-term basis in some jurisdictions. However, such measures would also bring challenges. The design of any measure would likely need to balance moderating the risks from high and/or rapid take up of CBDC with other policy objectives associated with a meaningful level of usage. In some cases, there could be legal and public policy issues to consider.

In the third chapter this argument will be analyzed more in detail because the focus will be on the possibility of a crisis. This will be related to the financial stability.

There are already some literature regarding the macroeconomic and financial stability benefits of a CBDC. Kim and Kwon (2019) [36] examine the implications of CBDC for financial stability using a monetary general equilibrium model. They show that the introduction of deposits in CBDC account will decrease the supply of private credit by commercial banks, which will raise the nominal interest rate and lower the reserve-to-deposit ratio of commercial banks. This can have a negative effect for financial stability by increasing the likelihood of bank panic in which commercial banks are short of cash reserves to pay out to depositors. Regarding the risks and spillovers of CBDC, Ferrari et al. (2020) examine CBDC in an open-economy. They show that the presence of a CBDC can significantly amplify global spillovers of shocks. But the magnitude of these effects will depend on the specific design of CBDC and can be significantly dampened if the CBDC possesses risk-mitigation features.

1.8. Literature

Lot's of research are in place, and not always researchers agree on the results. This is caused by the fact is an uncharted territory, therefore there is not an unique idea to follow. The study and the prevision that can be done are all theoretical, cannot be tested in real life at the moment. Of course there are some already existing model that can be followed, but the effective behaviour of this new technology is not scientific predictable.

Let's give a general idea of the studies in progress. First of all, we will focus on the functions and objectives of a central bank that issue CBDC.

Regarding enhancing central bank objectives and functions, Cukierman (2019) [25] argue that, for central banks to preserve the effectiveness of monetary policy in a world that is increasingly flooded by private digital currencies, central banks will have to issue their own digital currencies. The paper presents two proposals for the implementation of such a currency: a moderate proposal in which only the banking sector continues to have access to deposits at the CB and a radical one in which the entire private sector is allowed to hold digital currency deposits at the CB. The paper compares and contrasts the implications of those two polar paths to a CBDC for the funding of banks, the allocation of credit to the economy and their implications for welfare as well as for political feasibility. The paper evaluates the relative merits of issuing a currency on a blockchain using a permissionless distributed ledger technology in comparison to a centralized (permissioned) blockchain ledger operated by the CB and concludes that the latter dominates the former in more than one dimension. But it does acknowledge that distributed ledger technologies have many actual and potential cost savings benefits in other segments of the financial and real sectors.

Bordo and Levin (2017) [13] investigate how a CBDC could facilitate the transparent conduct of monetary policy. They show that CBDC can become a costless medium of exchange, a secure store of value, and a stable unit of account only if central bank digital currencies are account-based and interest-bearing, and the monetary policy framework would foster true price stability.

Boar et al. (2020) [31] show that even though central banks plan to issue a CBDC, they will need to collaborate with other central banks to better understand the impact of private digital tokens for CBDC payments. In their paper they also try to answer to some questions always related to this new currency.

Fernandez-Villaverde et al. (2020) [24] show that the introduction of CBDC presents an impossible CBDC trilemma for central banks which are the goals of efficiency, financial stability and price stability. They argue that CBDC enables the attainment of only two central bank goals at the same time. In particular, they are interested in understanding how financial intermediation will be affected by the presence of CBDC. To do so, they modify the classical Diamond and Dybvig model for bank run, in order to be applicable in a central bank contest.

Also, Bjerg (2017) [11] argues that a monetary system that has two competing money creators, the central bank and the commercial banking sector, can simultaneously only pursue two out of the following three policy objectives: free convertibility between CBDC and bank money, parity between CBDC and bank money and central bank monetary sovereignty, which is the use of monetary policy for anything else than support for commercial bank credit creation. Indeed, in their paper they evaluates three different scenarios for the implementation of CBDC in terms of their monetary policy implications. In the 'money user scenario' CBDC co-exists with both cash and commercial bank deposits. In the 'money manager scenario' cash is abolished and CBDC co-exists only with commercial bank deposits. And in the 'money maker scenario' commercial bank deposits are abolished and CBDC co-exist only with cash.

Moreover, let's focus on set of studies on the argument we are most interested in this thesis: bank run.
Some observers have discussed the possibility that a CBDC will increase financial fragility because it gives banks' creditors a better option that they can "run into" at the first sign of trouble. According to this view, withdrawing funds from the banking system is currently costly. By providing a low-cost alternative to bank deposits, a CBDC would make bank creditors more likely to withdraw their funding and shift into the CBDC. As a result, this argument indicates that introducing a CBDC may increase the incidence of bank runs and financial crisis.
Kumhof and Noone (2018) [40] analyzed the possibility of a central bank digital run. The key question they try to answer is whether the presence of CBDC opens up new possibilities of system wide runs by non-banks on the banking system that are not present in a world without CBDC. During a systemic banking crisis, transfers from bank deposits into CBDC would face lower transaction costs than those associated with cash withdrawals (such as going to the ATM, waiting in line, etc.), and would provide a safe-haven destination in the form of the central bank. The lower costs of running to CBDC compared to cash imply that more depositors would quickly withdraw at a lower perceived probability of a system-wide bank solvency crisis.
Yet, the impact of a CBDC on the speed, scale and frequency of systemic bank runs depends crucially on its design, and on the credibility of deposit insurance. CBDC may facilitate a generalized run on banks by offering a readily available, safe, and liquid alternative to deposits. (Mancini-Griffoli et al (2018) [23]).
Moreover, Kim and Kwon (2019) [36] analyze the implications of central bank digital currency for financial stability using a monetary general equilibrium model in which banks provide liquidity in the form of fiat currency, and commercial bank deposits compete with the central bank deposits in CBDC account. The introduction of deposits in CBDC account essentially decreases supply of private credit by commercial banks, which raises the nominal interest rate and hence lowers a commercial bank's reserve-deposit ratio. This has negative effects on financial stability by increasing the likelihood of bank panic in which commercial banks are short of cash reserves to pay out to depositors. However, once the central bank can lend all the deposits in CBDC account to commercial banks, an increase in the quantity of CBDC which does not require reserve holdings can enhance financial stability by essentially increasing supply of private credit and hence lowering nominal interest rate.
Kumhof and Noone (2018) argue that in an imperfectly competitive deposit market, the existence of CBDC as an outside option forces banks to match the CBDC rate to retain their deposits, which would eventually have a crowd-in effect of encouraging saving.

"When banks have market power in the deposit market, issuing a deposit-like CBDC with a proper interest rate would encourage banks to pay higher interest or offer better services to keep their customers."

On the contrary, there is sizeable body of literature that argue on a positive effect of a CBDC. CBDCs can also trigger important efficiency gains for the financial sector which may improve financial stability. This may happen if the reduction in bank profits is outweighed by expansion in bank lending, itself made possible by the expansion in deposit funding through greater financial inclusion and desired saving (Andolfatto (2018)). In Chiu et al. (2019) [16] , a deposit-like CBDC with a proper interest rate would encourage banks to pay higher interest or offer better services to keep their customers, thus limiting banks' market power, and improve the efficiency of bank intermediation. Bitter (2020) also finds that while a CBDC reduces the net worth in the banking sector in normal times, it mitigates the risk of a bank run in times of crisis, by impeding the emergence of bank runs.

Garratt and Zhu (2021) [27] finds that the interest rate on CBDC puts a lower bound on banks' deposit interest rates and may level the playing field between large banks and smaller banks. They explore the implications of introducing an interest-bearing central bank digital currency through commercial banks that differ in size. Banks of heterogeneous sizes offer different convenience properties to depositors, which the CBDC adopts. The large bank gives depositors a higher convenience value and hence possesses market power. The interest rate on CBDC puts a lower bound on banks' deposit interest rates, which is particularly binding on the large bank. While a higher CBDC interest rate enhances monetary policy pass-through by raising deposit interest rates, it reduces the small bank's deposit market share and its lending volume. By contrast, a CBDC that delivers its own convenience value to users levels the playing field by shifting deposits and lending from the large bank to the small one, although it can enhance or reduce the transmission of monetary policy.

If CBCs allow for another instrument of monetary policy and can be used to implement the LOLR function, this may reduce the risk of systemic banking sector runs (Williamson (2019)) [57]. In his paper he explain that CBDC interest bearing is not an advantage, as replacement of physical currency with CBDC does not expand the attainable set of equilibrium allocations. CBDC can increase the welfare by competing with private means of payment and shifting safe assets from private banking sector to what is effectively a narrow banking facility. For example, the central bank can lend all the CBDC deposits back to commercial banks, thus a CBDC may actually enhance financial stability and

even increase supply of private credit (Brunnermeier and Niepelt (2019)) [34].
Keister and Monnet (2022) [35] present a model that captures a concern commonly raised in policy discussions: the option to hold CBDC can increase the incentive for depositors to run on weak banks. Their model highlights two countervailing effects. First, banks do less maturity transformation when depositors have access to CBDC, which leaves them less exposed to depositor runs. Second, monitoring the flow of funds into CBDC allows policymakers to more quickly identify weak banks and take appropriate action, which also decreases the incentive for depositors to run. Their results suggest that a well-designed CBDC may decrease rather than increase financial fragility.
Finally, a last study made by Ferrari at all (2022) pointed out that if a foreign CBDC were to be widely adopted internationally, it could affect also economies with stable currencies if it leads to the domestic currency losing some of its functions and traction, which could ultimately impair domestic monetary policy.

2 | Diamond and Dybvig model for a bank run

As already underlined, when a CBDC will be introduced in the economy, this will affect all the financial life's aspects. Due to this new currency all the events related to the financial world could act differently.
The event we are most interested in our project is the financial run, and we are going to analyze how this will differ once the digital money is introduced. Indeed, in case of systemic uncertainty about banks, holding risk-free CBDC could indeed become more attractive than bank deposits. Therefore, one of the concerns in the debate on central bank digital currency is whether the ability for depositors to hold an account at the central bank could trigger a run on the banking system.
A bank run occurs because the bank's assets, which are liquid but risky, no longer cover the nominally fixed liability, so depositors withdraw quickly to cut their losses. This lead to a ripple effect causing a financial crisis.
As described by the 'Borsa Italiana' [32], this phenomenon is typical of many financial crises and arises from the fear that the bank in question will become insolvent. It is a reaction, in short, that is born from the distrust in the solidity of an institute.
The real danger associated with the run is that a quick withdrawal of the funds deposited by customers leads to a liquidity deficit in the bank that can bring even the strongest institutions to their knees. Indeed, a bank does not physically hold the amount of money an investor gives, but they are invested in both short and long term policy. Therefore, the institution can't immediately have the sums a customer requests and this can lead to the failure of the single bank. But the real damage is the ripple effect. The growing interconnection of contemporary financial systems tends, in fact, to transform the crises of individual banks into crises of the entire banking sector.
Therefore a run is costly and reduce social welfare by interrupting the production and destroying optimal risk sharing among depositors. Moreover, bank runs give a better predictor of economic distress than money supply.
There are lots of historical evidence of this event. The first example is the Tulip Mania

in the 1637, then the 1929 crisis in the United States, the so called Great depression. Moreover, February 2008, the UK Finance Minister Alistair Darling had to announce the nationalisation of Northern Rock, a bank brought to its knees by the bank run unleashed by its customers. In May 2012, at the height of the European sovereign debt crisis, Spain also experienced moments of panic and in a few days, one of the most discussed and affected credit institutions, Bankia, saw customers rush to withdraw more than a billion euros, as noted by the newspaper El Mundo. At the same time, the difficult Greek crisis caused several concerns about the failure of the country and its banking system, prompting the banks' customers to queue at the counter and request their money back. An understandable reaction, but capable of leading a very solid bank to bankruptcy.
Therefore, we can conclude a bank run occurs when large groups of depositors withdraw their money from banks simultaneously based on fears that the institution will become insolvent. This means the bank's model are based on trust in the institution. If this trust starts to get lost, the system collapses.
Obviously, over the years, lots of instruments have been proposed to prevent this behaviour. Firstly, the bank deposits insurance systems insure each depositor up to a certain amount, so that depositors' savings are protected even if the bank fails. Moreover the minimal capital requirement reduces the possibility that a bank becomes insolvent. Over the years there have been updated this quantity, in the so called Basel agreements. The most recent one is the Basel III agreement that strength bank capital requirements and introduces new regulatory requirements on bank liquidity and bank leverage.

To understand how this digital currency will behave in the case of a digital run, is necessary to study how the bank system react in this kind of crisis. We use the Diamond and Dybvig model[19], an important theoretical model for bunk runs, to analyze this events. We believe the market could have a similar behaviour when a CBDC crisis would be addressed, therefore it is necessarily to understand the model in detail to adapt it to the central bank case. .

2.1. Diamond and Dybvig model

The Diamond-Dybvig model is an influential model of bank runs and related financial crises. The model, published in 1983 by Douglas W. Diamond of the University of Chicago and Philip H. Dybvig, then of Yale University and now of Washington University in St. Louis, provides a mathematical statement of the idea that an institution with long-maturity assets and short-maturity liabilities may be unstable. What drives the possibility of a run in the model is demand for liquidity, that is, a desire on the part of savers to

be able to retrieve their funds at any time[21]. Therefore it acts on the assumption that individuals have a preference for liquidity because they are uncertain about the timing of their consumption needs hence they prefer to hold liquid assets. We can define a liquid asset as an asset that can be readily converted into consumption without loss of value.
Clearly, other analysis of a bank runs have been made before this model, but they didn't explain why bank contracts are less stable than other types of financial contract, neither they investigate the strategic decision that demand depositors face. The contribute of Diamond and Dybvig is the analysis of the demand for liquidity and the transformation service provided by banks. Their model demonstrate three important points [19]:

- Bank issuing demand deposits can improve on a competitive market by providing better risk sharing among people who need to consume at different random times.
- The demand deposit contract providing this improvement has an undesirable equilibrium in which all depositors panic and withdraw immediately, including those who would prefer to leave their deposits in if they were not concerning about the bank failing. This phenomenon is the bank run, that is the focus of this work. So the equilibrium just mentioned is the main object is going to be analyzed.
- Bank runs cause real economic problems because even healthy banks can fail, causing the recall of loans and the termination of productive investment.

In their model, they also examine bank contracts that can prevent runs and their optimally. They show there is a feasible contract that allows banks both to prevent runs and to provide optimal risk sharing by converting liquidity assets. This contract is called suspension of convertibility of deposits. Moreover, they found another type of protection against runs: deposit insurance. This is able to rule out runs without reducing the ability of banks to transform asset.

2.1.1. The model

The Diamond and Dybvig model points out that there is an opposite behaviour among business investments and individual savers. The first ones often require expenditures in the present to obtain returns in the future. Therefore, they prefer loans with a long maturity, that means low liquidity. On the other hand, individual savers may have sudden, unpredictable needs for cash, due to unforeseen expenditures. So they demand liquid accounts which permit them immediate access to their deposits, therefore they prefer short maturity deposit accounts. Consequentially, their needs are not the same. But there is a third protagonist in the financial worlds, the banks, that act as intermediaries between savers who prefer to deposit in liquid accounts and borrowers who prefer to take

out long-maturity loans.
Focusing on the model, it develops on three periods time with a single homogeneous good. At the initial time there is the investment, the good is moved from the agent portfolio to the bank. Then the agent can choose whether to withdraw its good at time 1, without earning anything, or leaving it to the bank and withdraw at time 2 when an yield R>1 is added to it. This is caused by the the fact agents are rewarded if they decide to leave their money at the bank's disposal. But each agent can't know at time 0 if he will withdraw at period 1 or at period 2, it depends on random factors, on events that are not predictable since life is so unpredictable.
Therefore, the model can be schematize as follow:

- **Periods**: T = 0, 1, 2
- **Yields** R>1 : unit of output in period 2 for each unit of input in period 0.
- **Agent's types**: All consumers are identical as of period 0. In period 1, each agent learns his type, as a random variables, it has an equal and independent chance of being of type 1 or type 2:
  - **Type 1** = agents care only about consumption in period 1. They store 1 unit of good at time 0, and receive back this quantity at time 1. The decision to withdraw early even if there is no reward is made because they need liquidity. A fraction $t \in (0,1)$ of the continuum of agents are of type 1.
  - **Type 2** = agents care only about consumption in period 2. They store 1 unit of good at time 0, and receive R unit of good at time 2. They are not interested in withdrawing early until they think the bank will be insolvent.
  - We can summarize the productive technology as follow:

	T=0	**T=1**	**T=2**
Type 1	-1	1	0
Type 2	-1	0	R

Table 2.1: Productive technology in a classical Diamond and Dybvig model

- **Utility function**: Given c_T the good received by an agent at period T, the state dependent utility function of each agent is given by:

$$U(c_1, c_2, \Theta) = \begin{cases} u(c_1) & \text{if agent j is of type 1 in state } \Theta \\ \rho u(c_1 + c_2) & \text{if agent j is of type 2 in state } \Theta \end{cases} \tag{2.1}$$

Where $1 \geq \rho > R^{-1}$ and $u : R_{++} \to R$ is twice continuously differentiable, increasing, strictly concave and satisfies the condition $u'(0) = \infty$ and $u'(\infty) = 0$. Moreover, the relative risk-aversion coefficient $-\frac{cu''(c)}{u'(c)} > 1$ everywhere.
The objective is to maximize the expected utility function: $\mathbb{E}[u(c_1, c_2, \Theta)]$ conditional on their information.

- **Consumption**: Letting c_k^i be the consumption in period k of an agent of type i, the agent's choose are:

$$\begin{cases} c_1^1 = 1 & \text{and} \\ c_2^1 = 0 & \text{because agent type 1 always interrupt production at t=1} \\ c_1^2 = 0 & \text{and} \\ c_2^2 = R & \text{because agent type 2 does not interrupt production at t=1} \end{cases} \tag{2.2}$$

- **Optimal consumption**: if types were publicly observable as of period 1, it would be possible to write optimal insurance contracts. This would allow agents to insure against the unlucky outcome of being of type 1 agent. The optimal consumption c_k^{i*} satisfies:

$$\begin{cases} c_1^{1*} > 1 \\ c_2^{1*} = 0 \\ c_1^{2*} = 0 \\ c_2^{1*} < R \end{cases} \tag{2.3}$$

Moreover, we can underline that $c_1^{1*} \leq c_2^{2*}$.

- **Demand deposit contract**: given a unit of good deposited at time 0, each agent receives a fixed claim of r_1 per unit if he withdraw at time 1. Withdrawal tenders are served sequentially in random order until the bank runs out of assets. It satisfies a sequential service constrain, which specify that a bank's payoff to any agent can depend only on the agent's place in line and not on future information about agents behind him in line.

- **Payoff**: Given V_i the payoff of period i, f_j the number of withdrawers' deposits serviced before agent j as a fraction of the total demand deposits, f the total number of demand deposits withdrawn. The payoff is:

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if } \mathrm{f_j} < \mathrm{r_1}^{-1} \\ \\ 0 & \text{if } \mathrm{f_j} \geq \mathrm{r_1}^{-1} \end{cases} \tag{2.4}$$

$$V_2(f, r_1) = \max\left(\frac{R(1 - r_1 f)}{(1 - f)}, 0\right) \tag{2.5}$$

- **Consumption from deposit proceeds**: being $\mathrm{w_j}$ the fraction of agent j's deposits that he attempts to withdraw at time T=1, the total consumption is given by:

$$w_j V_1(f_j, r_1) \qquad \text{type 1 agent} \tag{2.6}$$

$$w_j V_1(f_j, r_1) + (1 - w_j) V_2(f, r_1) \qquad \text{type 2 agent} \tag{2.7}$$

2.1.2. Nash equilibrium

What is desirable is to study the equilibrium this kind of model can achieves, where as equilibrium we refer to the Nash equilibrium, that is the most common way to define the solution of a non-cooperative game involving two or more players. In a Nash equilibrium, each player is assumed to know the equilibrium strategies of the other players and no player has anything to gain by changing only their own strategy.
There are two different equilibrium this kind of model can achieves.

- **Good equilibrium: optimal risk sharing**: is the case where type 1 agents withdraw at time T=1 while type 2 agents prefer to wait. Therefore we can refer to this equilibrium as a "good" one since we achieve the objective of the model. The necessary condition to be at this balance is that the fixed payment per dollar of deposits withdrawn at T=1 is equal to the optimal consumption of a type 1 agent given full information, namely $r_1 = c_1^{1*}$.
  This condition can be achieved under ordinary circumstances, indeed savers' unpredictable needs for cash are likely to be random, as depositors' needs reflect their individual circumstances. But depositors' demand for cash are unlikely to occur at the same time, therefore a bank expect a small fraction of withdrawn in the shorter term. In this situation, the bank must own relatively small amounts of cash on hand to pay any depositors that wish to make withdrawals.
- **Bad equilibrium: Bank run**: the opposite side of the equilibrium is the "bad"

one, where all agents get scared and try to withdraw their deposits at T=1. This is because the face value of deposits is larger than the liquidation value of bank's assets.

The reason behind this event is the bank must transform illiquid assets into liquid ones to meet the demand of the agents. But banks cannot do it quickly since they lend out at long maturity. Moreover, borrowers would be unable to pay back quickly, since their loans were, by assumption, used to finance long-term investments. Therefore, if all depositors attempt to withdraw their funds simultaneously, a bank will run out of money long before it is able to pay all the depositors. The bank will be able to pay the first depositors who demand their money back, but if all others attempt to withdraw too, the bank will go bankrupt and the last depositors will be left with nothing.

This equilibrium can be achieved only if $r_1 > 1$, otherwise a bank would not be susceptible to runs since $V_1(f_j, r_1) < V_2(f, r_1)$, that is type 2 agents are not encouraged to withdrawn earlier. Moreover, if $r_1 = 1$ a run can be excluded, but this case is of no interest since the bank simply mimics direct holding of the assets and is therefore no improvement on simple competitive claims markets.

Therefore, a demand deposit contract that is not susceptible to runs provides no liquidity services. Even though an agent know there is a positive probability of a run, he will choose to invest some of his wealth in the bank since the good equilibrium will lead to a positive gain. A proof of the multiplicity of the equilibrium is given in the Appendix A.

2.1.3. How to prevent a run

Pure demand deposits contract are feasible therefore they can attract lots of investors, despite the fact the good equilibrium is very fragile and there is a positive probability of a bank run. But both banks and depositors would prefer to avoid the banking panic or at least to reduce the consequence of a run. Tho this demand Diamond and Dybvig answer with two proposals: the suspension on convertibility and the government deposit insurance.

Suspension on convertibility

Is a simple variation on the demand deposit contract which gives banks a defense against runs. Under incomplete information there is a role for suspension. Incomplete information means that depositors do not know the state of banks' investments, but use a noisy indicator to form rational expectations of deposit return rates. By suspending convertibility, banks can signal to depositors that continuation of the investments is mutually beneficial. [28]

This policy prevents runs by removing the incentive of type 2 agents to withdraw early. The contract is identical to the pure demand contract except that it states that any agents will receive nothing in t=1 if his position in the queue is over a fixed one. To impose this condition, the payoff must be changed by incorporating the condition $\hat{f} < r_1^{-1}$, that is the agents fraction beyond which any agent will receive nothing if he attempts to withdraw at T=1, $\hat{f} \in [t, \frac{R-r_1}{r_1(R-1)}]$, where t is the normal volume of withdrawals.

Given this condition, all type 1 agents will withdrawn at period 1, while type 2 agent will choose to withdrawn at period 2 only. This means there is an unique Nash equilibrium which has $f = t$, that entails the total number of demand deposits withdrawn is equal to the fraction of type 1 agents. This made the contract very stable since it is achieved the "good" demand deposit equilibrium.

The new payoff must be:

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if} f_j \leq \hat{f} \\ \\ 0 & \text{if} f_j > \hat{f} \end{cases} \tag{2.8}$$

$$V_2(f, r_1) = \max(\frac{R(1 - r_1 f)}{(1 - f)}, \frac{R(1 - r_1 \hat{f})}{(1 - \hat{f})}) \tag{2.9}$$

where there must be imposed the condition $1 - \hat{f} > 0$. The convertibility is suspended when $f_j = \hat{f}$, and no more agents are allowed to withdraw at T=1. Moreover, since $f_j \leq f$, the payoff of a type 2 agents is always higher if they wait until time 2.

But this kind of contract works perfectly only if t is known and not stochastic. Therefore, only in the case where confidence is maintained, a bank can know the exactly fraction of withdrawals and apply this policy.

On the other hand, if t is not known ex ante, there must be some different analysis. But this lead to the conclusion a bank contract cannot achieve optimal risk sharing when t is stochastic and has a non-degenerate distribution.

Government deposit Insurance

Deposit insurance is a second policy to prevent runs. It guarantees that the promised return will be paid to all who withdraw. If this is a guarantee of a real value, the amount that can be assure is constrained: the government must impose real taxes to honor a deposit guarantee.

The government is assumed to be able to levy any tax that charges every agent in the economy the same amount. In particular, it can tax those agents who withdrew early at period T=1, namely those with low values of f_j. How much tax must be raised depends on how many deposits are withdrawn at t=1 and what amount r_1 was promised to them. The tax can depend on the fraction of agents that withdrawn, therefore they are not fixed, but a function of f. It can be imposed after the agent's withdraw, and this is a marked contrast to a bank, which must provide sequential service and cannot reduce the amount of a withdrawal after it has been made.

Naturally, there can be the case where not all the collected tax are spent, so they are plowed back into the bank to minimize the fraction of assets liquidated.

It can be proven that demand deposit contracts with government deposit insurance achieve the unconstrained optimum as a unique Nash equilibrium if the government imposes an optimal tax to finance deposit insurance. This follows from the ability of tax-financed deposit insurance to duplicate the optimal consummations from the optimal risk sharing. The amount of tax that must be raised at t?1 depends on the number of withdrawals then and the asset liquidation policy. The proportionate tax $\tau : [0,1] \rightarrow [0,1]$ is given by:

$$\tau(f) = \begin{cases} 1 - \frac{c_1^{1*}(f)}{r_1} & \text{if} f \leq \hat{t} \\ \\ 1 - r_1^{-1} & \text{if} f > \hat{t} \end{cases} \tag{2.10}$$

where $\hat{t}$ is the greatest possible realization of t.

Hence, this kind of policy allows bank to follow a desirable asset liquidation policy and it prevents runs since, for all possible anticipated withdrawal policies of other agent, it never pays to participate in a bank run so type 2 agents are never induced to be part of a run.

2.2. Debate on bank runs

Since the publications on Diamond and Dybvig model, lots of studies have followed. The model has been widely used to understand bank runs and other types of financial crises, as well as ways to prevent such crises. A great deal of attention has been given to the model's multiple equilibria and interpreting them as financial fragility. As Ennis and Keister (2010a) [22] point out, it is important to find the ingredients that help understand fragility in models that follow the Diamond–Dybvig tradition as it has relevant consequences for public policy regarding how desirable government-provided safety net elements, like deposit insurance, and other interventions are.

Diamond itself wrote an article where uses narrative and numerical examples to provide a straightforward explanation of the ideas model [20].
Green and Lin (2003) [29] demonstrate that, with a finite number of individuals who receive shock realizations independently from each other, the solution of the optimal problem proposed by Diamond and Dybvig (DD) defines a deposit game that has the optimum as the unique equilibria. They use a finite-trader version of DD model both to introduce aggregate risk in a natural and explicit way, and also to provide a formulation of the sequential-service constraint. They show that a naturally defined mechanism makes it a strictly dominant strategy for each trader to communicate his type truthfully, and that this dominant strategy equilibrium implements the symmetric, ex ante efficient allocation as a unique equilibrium outcome. This is in sharp contrast to Diamond and Dybvig's deposit-with-suspension mechanism which works in the absence, but not the presence, of aggregate risk. Diamond and Dybvig assumed that the banking arrangement must give all depositors who demand early withdrawals the same amount no matter how many depositors actually claim to be impatient. On the contrary, Green and Lin allow the bank to utilize more fully the information reported by all depositors regarding their preferences. Their mechanism assigns traders the efficient allocation computed for the reported economy. Allocation rules constructed this way turn out to be capable of preventing traders from lying about their types when they decide whether or not to make early withdrawals. In light of their findings, Diamond and Dybvig's bank-run equilibrium appears to be an artifact of the simple contract modelling approach rather than a genuine feature of the economic environment that they have modelled.
Moreover, Calomiris and Kahn (1991) [14] propose a model where treats banks as individuals with commitment difficulties and who can hide resources. Their approach leads to models in which sequential service is not an ingredient for explanations of fragility as in DD but is instead an equilibrium feature that exists to discipline banks. One is led to the conclusion that there appears to exist a choice between studying the DD view of fragility when the emphasis is on the behavior of other depositors and the costly state verification model that applies not only to banking but also to optimal contracts in abstract principal agent problems.
In contrast to the Diamond and Dybvig view of runs as self-fulfilling, Chari and Jagannathan (1988) [56] argue that runs are brought on by problems of uncertainty and asymmetric information about banks' financial conditions. In their portrayal, runs reflect a signal extraction problem in which some individuals receive a noisy signal about the bank's return, which may lead them to withdraw funds early; other depositors must infer from observed withdrawals whether a negative signal was received by informed depositors, or whether liquidity needs happen to be high.

Cooper and Ross (2002) [17] used an extension of the Diamond and Dybvig model to show that full deposit insurance can lead to a moral hazard problem where intermediaries invest in speculative projects and the first-best contract cannot be achieved. It is shown that after imposing an additional capital requirement on banks the first-best contract is obtained.

Allen and Gale (2000) [2] extend the model by replacing the single intermediary environment with four identical banking regions; their goal was to investigate the possibility of financial contagion when banks are interlinked. These four regions are interlinked in the sense banks hold interregional claims on other banks to provide insurance against liquidity preference shocks, because liquidity preference shocks are imperfectly correlated across regions. It is shown that the possibility of contagion depends strongly on the completeness of the structure of interregional claims. Complete claims structures are shown to be more robust than incomplete structures, Conversely a bank run then this will become contagious and spread into the other regions if the banking system is not complete.

Investigating the possibility of systemic bank runs has been widely studied and discussed ever since the 2008 financial crisis. Uhlig (2010)[55] considers a three period model in which there are a fixed number of core banks, a continuum of local banks of mass 1, and a continuum of agents of mass 1. The interesting difference in this paper compared to others is that the local banks run on the core banks rather than having agents run on local banks. Core banks invest in asset backed securities (ABS) which are sold to outside investors. It is shown that under the case of adverse selection the probability of a systemic bank run occurring is low; under the assumption of uncertainty averse outside investors bank runs become systemic if the market share of core banks experiencing runs is too large

Moreover, Kiss et al. (2014) [33] study a small-scale environment resembling the Diamond–Dybvig setup in which bank runs are caused by coordination problems. Experimentally, they find that observability of actions affects the likelihood of bank runs, but depositors' choice is highly influenced by the particular action that is being observed. Depositors who are observed by others at the beginning of the line are more likely to keep their money deposited, leading to less bank runs. More information about previous decisions seems to reduce the likelihood of bank runs.

Markus Kinateder and Hubert János Kiss (2014) [37] modify the canonical Diamond–Dybvig model assuming that depositors have information about previous decisions. They model a sequential-move game with a finite number of depositors who contact the bank in an exogenously given fixed order to communicate whether to leave the money deposited or to withdraw it. If depositors observe the history of all previous decisions, they show that there are no bank runs in equilibrium independently of whether the realized type vector

selected by nature is of perfect or imperfect information.
Ennis and Keister (2009a, 2010b) show that suspension of convertibility is successful only if the bank can commit to use it as announced. The bank may fail to do so, since once a bank run is underway, suspension may not be ex post optimal: many depositors receive no money though they need liquidity. The bank may then attend needy depositors which are exempted from suspension as it happened during the deposit freezes in Argentina in 2001 or in the US in March 1933.
James Peck and A. Setayesh (2019) [46] answer to the questions how does the opportunity for consumers to invest in a mutual fund or otherwise invest outside the banking system affect the nature of the final allocation, affect the nature of the optimal deposit contract, and affect the fragility of the banking system? Indeed, in the many years and many published articles following the bank runs paper of Diamond and Dybvig, only a few papers have modeled the decision of whether to deposit, much less the decision of how much to deposit. They found that the withdrawal amount augments the mutual fund investment of impatient consumers by exactly the amount of consumption needed to achieve the optimal allocation. Moreover, in equilibrium achieving the efficient allocation, lower deposit levels make the banking system more fragile.
David Ristovski (2020) [52]investigate the possibility of incorporating interbank insurance among commercial banks extending the DD model. First they abandon the single intermediary environment by introducing a banking sector that is made up of a continuum of banks of mass 1; then they assume that a small proportion of banks experience runs. Next, they suppose that all banks have access to and invest in interbank insurance. Lastly, it is assumed that when banks withdraw illiquid funds prematurely they must pay a mandatory transaction cost.

2.3. Policy implications

Diamond and Dybvig's model has been widely used to understand bank runs and other types of financial crises, as well as to prevent such crises. It has been a workhorse of banking research over the last years and during the recent financial crisis it has been one that researchers and policymakers consistently turn to when interpreting financial market phenomena. Indeed, there were several runs on traditional banks in the last crisis.
During the 36th anniversary of the model, in a dedicated conference [38], it was explained that initially, the paper's main contribution was viewed as providing a rational economic justification of pre-deposit-insurance banking panics such as those in the Great Depression. It also helped to explain why deposit insurance mostly put an end to traditional bank

runs. Furthermore, although multiple equilibrium had played an important role in game theory, multiple equilibrium was largely viewed as a defect in an economic model, and Diamond and Dybvig helped to change the profession's view on this issue. More recently, the model has become an important workhorse model for studying banking and liquidity, and a framework for discussing related policy issues. The Diamond-Dybvig model is the go-to model for understanding run-like phenomena in many markets during the recent financial crisis. The model's simplicity has made it easy to extend to study a variety of issues in macroeconomics, economic theory, and financial markets.

Moreover Diamond and Dybvig argue that there are two different strategy to prevent runs: suspension on convertibility and deposit insurance. However, the first one might be the worst choice since suspension cannot be used only as a threat. Some suspension would actually occur and would be unpopular. If suspension occurred regularly, depositors would desire another way of stopping runs caused by panics. In practice, government-provided deposit insurance has been instituted following many financial crises. Such insurance pays depositors all or part of their losses in the case of a bank run. If depositors know that they will get their money back even in case of a bank run, they have no reason to participate in a bank run. Thus, sufficient deposit insurance can eliminate the possibility of bank runs. This is in contrast with the usually behaviour of a bank. Indeed, to prevent bank runs, they prefer to act by the suspension on convertibility.

3 | Central Bank run

The Diamond and Dybvig model can be applied to a central bank as well. There are only few details to be changed. The differences are in the contracts that must be nominal and in the role of the central bank as a powerful financial intermediary, that means it can manipulate the price level. As in the original model, a financial intermediary exists which offers a demand deposits contract to depositors to allow risk sharing, but the contract must be nominal rather than real, and the good can only be traded against money. Moreover, the Diamond and Dybvig model consider intermediation with private bank, while a CBDC implies Central Bank intermediation.
Why do we need to adjust the model? The focus of this thesis is strictly related to central bank, as already explained, not to private ones, the Diamond and Dybvig model cannot directly be applied to our case but needs to be adjusted.
How can we work on this adjustment? We can assume a single central bank that issues CBDC, and the agents in the economy hold accounts with CBDC balances at the central bank. The digital currency is the unit of account. As in the current economy, agents can spend their digital money by transferring to other agents or to the central bank and cannot hold negative amounts.
Another fundamental difference in the model, is the fact a central bank cannot invest in long term project, therefore it must requires a third actor, such as investment bank.
First, the difference between a central bank run and a commercial bank run is presented, then the model is schematized analyzing both the actors and the equilibrium. In this chapter we follow the Fernandez-Villaverda at all ideas [24][25], since they studied the topic in detail.

3.1. Central bank runs vs commercial bank

Let's start with the analysis of the differences between a central bank and other banks, this will underline the common point and the difference in this institutions in order to understand how to modify the model, as explained by Fernandez-Villaverda at all [25].
There are two different kind of non central bank: commercial bank and investment bank.

We focus on the first one only, since investment bank can only offer long term contracts, therefore they are not related to the model we are working with. Indeed, in the Diamond and Dybvig model both long and short term contracts are featured, and using only one of this investment cannot let the model work properly.
Contrary to the commercial bank, a central bank can invest only in a short term technology, it can never work directly with a long term investment. Therefore, it must collaborate with other investment bank to have the possibility of behave as the model wants. This will lead to a bigger problem of liquidity, since, in case of sudden requests for money from the agents, the bank can't liquidate its long term investment directly but has to contact the investment bank, which can not have the request amount available.
Moreover, it is a government-controlled institution. This means there is a seniority when liquidating a bankrupt debtor's debt, not all agents are equal. Furthermore, central bank cannot forced into liquidation, making this a positive difference in the model.
Moreover, the Diamond and Dybvig model work with real goods, not nominal contracts, but we can allow central bank to access short term asset since it has storage technology.
Finally, central bank is not fiscally backed either directly through seigniorage or indirectly through taxation.
Summing up, the contract offered by a central bank is different to a commercial bank contract. It is more rigid since it cannot call its loan to investment bank. Contrary, commercial banks have direct control on their investment, therefore they can liquidate both short and long term investments. But it has also some benefits, especially in the fact a central bank cannot forced into liquidation therefore the probability of a bankrupt is lower, moreovre it can manipulate price level.
How can all these difference impact on a run?

3.1.1. Commercial bank runs

Commercial banks are prone to self-fulfilling runs. A commercial bank run happen when the bank is forced to liquidate not only the short-term investments, but the long-term too. Indeed, if the fraction of agent that withdraws is higher than the expected one, that means also patient agents withdraw, the payoffs to rolling over and withdrawing differs from the expected one. The reason is that, to fulfill the demand of liquidity, the bank must liquidate the long-term assets, reducing the payoff of those agents that decided to wait till period 2.
Mathematically, a run on commercial bank happen if and only if:

$$(\alpha - \lambda)c_1^* > (1 - y^*)k \tag{3.1}$$

Where $\alpha \in (0,1)$ is the fraction of agents that withdrawn, c_1^* is the socially efficient contract of a type 1 agent. y^* is the portion of goods invested in the short term technology, $k \in (0,1)$ is the fraction of good payed if a long term technology is liquidated prematurely, $\lambda \in (0,1)$ is the probability an agent is an impatient consumer.
In case of a run, consumers who roll over receive zero, and consumers who withdraw receive the payoff c_1^* only with a certain likelihood due to rationing. This is because we can imagine agents that withdraws, attends on a queue at the commercial bank. If there are not enough liquidity, the bank choose randomly those agents in the queue that will receive the payment.
In conclusion, if the fraction of those agents that withdraw is less than a fixed threshold, i.e $\alpha \leq \lambda$, the payoff is higher for agent type 2. Contrary, if this threshold is exceeded, we are in a run situation and the payoff is higher for agent type 1. Therefore, a commercial bank has two equilibrium: a good one where all agents respect their type and the socially optimal contract is attained, this happens if $\alpha = \lambda$, and a bank run equilibrium where all patient agents panic and withdraw and the socially optimal contract is not attained, this happens if $\alpha = 1$.

3.1.2. Central bank runs

A central bank run acts differently from the previous one described. Indeed, conversely to the commercial bank, it cannot liquidate long-term investment directly since they are situated in an investment bank, they are not directly hold by this institution. Therefore, to fulfill the agent withdraw in a bank panic, only short term investment can be liquidated, and the fraction of agents served is lower than the commercial bank ones. Moreover, the reason behind a central bank run are different from the ones that causes a commercial bank run. Furthermore, a central bank cannot forced into liquidation.
Mathematically, a run on central bank happens if and only if:

$$\alpha > \lambda \tag{3.2}$$

where α is the fraction of agent that withdraw, and λ is the fixed threshold coming from the probability of an agent to be impatient. Consequently, only a reduced number of consumers receive their money back, the central bank can allocate only real goods of quantity y^* to a measure of α agents, consumers queue and receive c_1^* units with likelihood $\frac{\lambda}{\alpha}$.
But a central bank cannot forced into liquidation, therefore it cannot forced to fulfill all the withdraw requests. Therefore it must find solutions to deal with the agents that are

not served. They can be treated in two different ways, they do not receive zero.
The first one is the punishment behaviour. The bank decides to punish those agents that withdraw earlier, contributing to the bank run, by not paying them back. This policy enables the bank to recognise those agents that were confident in the bank and decided to maintain their investment, by dividing the returns earned in period 2 among them. In this case, there is a unique equilibrium: all patient consumers roll over and only impatient consumers withdraw, since it is worth zero of a type 2 agent to withdrawn earlier. Indeed, if $\alpha > \lambda$, we have

$$\frac{\lambda}{\alpha}u(c_1^*) < y\left(\frac{R(1-y^*)}{1-\alpha}\right) \tag{3.3}$$

that means for an agent type 2 is convenient to wait time 2. Therefore,runs on central bank cannot occur.
On the contrary, a central bank can decide to adopt an equal treatment. It can force the consumers that were not served to roll over their deposits. As in the previous case, there is a single equilibrium: only the consumers fraction that can be served withdraw, the other agents act as patient ones. Therefore a run cannot occur.
In conclusion, since the two different strategies involve a single equilibrium where no bank runs can occur and the socially optimal contract is offered, then central bank will attract all deposits in the market away from commercial banking sector. An agent will always prefer a central bank if he thinks a commercial bank has a positive probability of run. If he believe this probability is zero, then he can choose both bank indifferently.
Moreover, the central bank can offer a different contract from the socially optimal one and still obtain a monopoly of all deposits. Therefore we can summarize this behaviour in the following proposition:

Proposition 1. *A central bank will attract all deposits in the market away from the commercial banking sector, both if it offer or not the socially optimal contract.*

Proof. Given:

- U_1 = expected utility from the run-prone commercial bank deposit contract.
- U_2 = expected utility from the run-proof central bank deposit contract.

If a run occurs with a strictly positive probability, then $U_1 < U_2$, even if both contracts are socially optimal ex-ante. Thus, a central bank can find a $c_1 < c_1^*$ such that $U_1 < U(c_1) < U_2$. This means c_1 maintains run-proofness but lowers the utility of the consumer and generates positive profits for the central bank, and departs from the socially optimal allocation. □

In B, all the cases payoff' are summarized.

3.2. Diamond and Dybvig Model applied to Central Bank with a CBDC

So far we have discussed the difference between a commercial bank and a central bank, and their respective runs. We now focus on a central bank environment with the introduction of a CBDC, as explained by Fernandez-Villaverde et all[24]. The model is quite similar to the one we have already analyzed. The difference is that the demand deposit contracts offered to depositors are nominal rather than real. Moreover, the goods can only be treated against money. There is both a real investment contract and a nominal ones. In the real ones for each unit of good invested in 0, the technology yields one real unit in return when liquidated in 1, and $R > 1$ real units when held until maturity. But the contract in the case of a CBDC must be nominal.

We can assume a single central bank and no retail banks. The goods issued are CBDC that takes the form of accounts kept at the central bank. We refer to the unit of account is a digital currency. Agents can spend digital currencies on their account by transferring them to other agents or to central bank, and likewise receive digital money in the reverse transaction. As with physical money, agents cannot hold negative amounts of digital money.

Mathematically speaking, at time 0 agents have a zero balance on their CBDC account and a unit of real good that wanted to invest in $M > 0$ units of CBDC, where M is a quantity set by the central bank.

At time 1 agents learn their type and decide whether to spend all their digital currencies or to roll over. Naturally, to simplify the model, we are imposing agents cannot maintain only a part of their CBDC, and they cannot sell it to other agents. At this time period the central bank decides the fraction $y(\alpha)$ of real goods to be liquidated to fulfill the agents 1 request, and sell this quantity to the spending agents at a market clearing price P_1. Moreover it decides the nominal interest rate $i(\alpha)$ to be paid at period 2 on the CBDc balances remaining at the end of period 1.

In period 2, the type 1 agents have zero CBDC, while patient agents have $(1+i)M$ digital currency. This means that the nominal contract is given by $(M, M(1+i))$. The agents type 2 can spend their remaining balance on the remaining real goods $(1-y)R$ held to maturity by the central bank and sold to the agents at a market clearing price P_2.

The model can be schematize as follow:

- **Time**: time is discrete, there are three periods t=0,1,2.
- **Agents' type**: there is a continuum of agents [0,1], each endowed with a unit of good in period 0. Agents are symmetric in t=0, but can be of two types in t=1: patient and impatient. The second ones decide to withdraw their CBDC balance in period 1, while the patient agents wait until period 2. The agent's type is random, is drawn at the beginning of period 1 and constitute of private information. The fixed fraction of impatient agents is denoted by $\lambda \in (0, 1)$, while $\alpha \in [0, 1]$ is the equilibrium fraction of agents who decides to spend.
- **Utility function**: a strictly increasing, strictly concave and continuously differentiable function $u(\cdot) \in \mathbb{R}$, such that $\frac{-xu''(x)}{u'(x)} > 1 \forall x$, where x is the consumption levels . The agents enjoy good consumption in different periods, according to this function. Given x_i the consumption in period i, we have:

$$U(x_1, x_2) = \begin{cases} u(x_1) & \text{with probability } \lambda \\ \\ u(x_2) & \text{with probability 1 -}\lambda \end{cases} \tag{3.4}$$

- **y(α)**: $y : [0, 1] \to [0, 1]$ is the fraction of real goods to be liquidated in t=1 to fulfill the withdrawn request. The central bank must sell it to spending agents at a market clearing price P_1.
- **i(α)**: $i : [0, 1] \to [-1, \infty)$ is the nominal interest rate chose by the bank, to be paid at period 2 on the CBDC balances remaining at the end of period t=1. There is the restriction $i \geq -1$ since agents cannot hold negative amounts of CBDC.
- **Agent's nominal CBDC balance**:

	T=0	**T=1**	**T=2**	**T=2$^+$**
Type 1	M	0	0	0
Type 2	M	M	(1+i)*M	0

Table 3.1: CBDC balance of an agent

- **Agent's Real balance**

	T=0	T=1	T=2
Type 1	-1	$\frac{y}{\alpha}$	0
Type 2	-1	0	$\frac{1-y}{1-\alpha}R$

Table 3.2: Real good balance of an agent

- **Real CBDC value**

	T=0	Value	
Type 1	1	$\frac{M}{P_1}$	$\frac{M(1+i)}{P_2}$

Table 3.3: Real CBDC value

The central bank is involved at period 1. Indeed, given α the equilibrium fraction of agents who decide to spend earlier, the bank decides the fraction $y = y(\alpha)$ of real goods to be liquidated and the nominal interest rate policy. At t=2, the bank liquidates the remaining real goods $(1 - y)R$ and sells it to the agents at a market price P_2. Therefore, the central bank policy is given by $(M, y(\cdot), i(\cdot))$.
How can the two different market prices can be computed? Given α and the liquidation policy $y(\alpha)$, the bank can compute the price levels $P_1(\alpha)$ and $P_2(\alpha)$ by the following equations:

$$\begin{cases} P_1(\alpha) = \frac{\alpha M}{y(\alpha)} & \text{if t=1} \\ P_2(\alpha) = \frac{(\alpha)(1+i)M}{R(1-y(\alpha))} & \text{if t=2} \end{cases} \tag{3.5}$$

As we can see, this is a difference from the original model. It show a difference also in the behaviour of a central bank with respect to a commercial bank. Indeed, a commercial bank could not compute the price level but should take it as a given value. Therefore, a central bank has an advantages of manipulate the price level by working on the liquidation policy.
Thanks to this price equation, there can be computed the real allocation $(x_1(\alpha), x_2(\alpha))$:

$$\begin{cases} x_1(\alpha) = \frac{M}{P_1} = \frac{y(\alpha)}{\alpha} & \text{if t=1} \\ x_2(\alpha) = \frac{(1+i(\alpha))M}{P_2} = \frac{1-y(\alpha)}{1-\alpha}R & \text{if t=2} \end{cases} \tag{3.6}$$

This means the real goods $y(\alpha)$ harvested per liquidation by central bank in period 1 are equally distributed across all spending agents in period 1, and likewise in period 2.
Therefore, the model can be summerized as follow:

- **Nominal contract**: (M,M(1+i(α)))

- **Central bank policy**: $(M, y(\cdot), i(\cdot))$.
- **Price levels**: $(P_1(\alpha), P_2(\alpha)) = (\frac{\alpha M}{y(\alpha)}, \frac{(\alpha)(1+i)M}{R(1-y(\alpha))})$
- **Real contract**: $(x_1(\alpha), x_2(\alpha)) = (\frac{M}{P_1}, \frac{(1+i(\alpha))M}{P_2})$

3.2.1. Equilibrium

An equilibrium consists of a central bank policy, a liquidation policy and a nominal interest rate policy, aggregate spending behaviour and price levels $(M, y, i, \alpha, P_1, P_2)$. This quantities must satisfies some request to be optimal. The agents spending strategy α is optimal given their belief about the central bank policy (M, y, i) and the two price levels (P_1, P_2). The price levels clear the goods and money market and the central bank policy is optimal given the depositors's spending behaviour α.
As in the commercial bank case, a central bank can have two different equilibrium: the "good" one and the run. The first one lead to the social optimum that is the point on the utility possibility frontier that maximizes social welfare.
We must underline that a run in the presence of CBDC does not mean the central bank is running out of digital money, is not running out of the item it has promised to agents and that can produce freely, which is a difference with respect to the classical bank run.
As in the classical case, a run happens only if patient agents believe the expected real consumption in t=1 will exceeds the consumption in t=2, this means $x_1 > x_2$. In this case, patient agents will use the storage technology to consume x_1 in period 2. Otherwise, patient agents will wait. A nominal contract, per se, does not rule out the possibility of a run on the central bank.

Proposition 2. *Given α the fraction of agents that withdraw at period 1 and λ a fixed threshold, an equilibrium is a run on the central bank if $\alpha > \lambda$. Moreover, a run can be exhaustive if $\alpha = 1$, this means all the agents withdraw.*

Moreover, a sufficient condition for a run is:

Proposition 3. *If $x_1 > x_2$ then patient agents will choose to withdraw their CBDC balances in t=1.*

The equilibrium's cases can be schematized as follow:

- $\alpha > \lambda$: if the number of withdraw is bigger than the threshold then it can happen a run that is always an equilibrium. Let's analyze in detail the cases that can cause a run:

   - $\alpha = 1$: all the agents withdraw at t=1. This is an exhaustive run equilibrium, it can happen if and only if $x_1(1) \geq x_2(1)$.
   - $\lambda < \alpha < 1$: only some patient agents withdraw in t=1. In this case there is a partial run on the central bank, this can happen if and only if $x_1(\lambda) = x_2(\lambda)$.

- $\alpha = \lambda$: if the number of withdraw is equal to the threshold then no runs can happen. Only the good equilibrium can be reached, this is the case $x_1(\lambda) \leq x_2(\lambda)$.
- $\alpha < \lambda$: a run cannot happen, therefore there is a single equilibrium, the good one.

Social Optimum

We have concluded that there can be two different equilibrium: the bad one, also called bank run, and the good one, which can lead to the socially optimal contract. So far we focused on the bank runs, let's now briefly explain the other side of the coin. In this section, only real allocation matters, the nominal contract is of no interest to this end.

Proposition 4. *The central bank policy implements the social optimum (x_1^*, x_2^*) in equilibrium if:*

- *Given that only impatient agents spend, i.e. $\alpha = \lambda$, the central bank policy satisfies $y(\lambda) = y^*(\lambda) = x_1^*\lambda > \lambda$, implying $x_1(y, \alpha) = x_1^* = \frac{y^*}{\lambda}, x_2(y, \alpha) = x_2^* = \frac{R(1-y^*)}{1-\lambda}$ and $P_1^* < M$.*
- *Given that also patient agents spend, i.e. $\alpha > \lambda$, the central bank sets a liquidation policy that implies $x_1(\alpha) < x_2(\alpha) \quad \forall \alpha > \lambda$*

Indeed, since central bank observes aggregate spending behaviour before liquidating real assets, it is not committed to liquidate y^* if it observes that also impatient agents are spending. To deter patient agents from spending, the central bank can threaten them to implement a liquidation policy that makes spending non optimal ex post, i.e. $x_1 < x_2$ for $\alpha \in (\lambda, 1]$. If the central bank can credibly threaten the patient agents by setting such liquidation policy, it deters them from spending, by this deterring a central bank run equilibrium. In this case there is an unique equilibrium where only impatient agents spend and the socially optimum contract is always implemented. This can be possible only because the contracts between central bank and the agents are nominal, the liquidation policy is at discretion of the bank since the price levels are not given.

Proposition 5. *A liquidation policy is run-deterring if*

$$y^d(\alpha) < \frac{\alpha R}{1+\alpha(R-1)} \qquad \forall \alpha \in (\lambda, 1]. \tag{3.7}$$

Such a deterring policy choice requires the interim price level $P_1(\alpha)$ to exceed the withdrawal dependent bound

$$P_1(\alpha) > \frac{M}{R}(1+\alpha(R-1)) \qquad \forall \alpha \in (\lambda, 1] \tag{3.8}$$

Therefore, under a credible liquidation policy all agents decide to spend if and only if the agent is impatient, otherwise they wait. Thus, under rational behavior, runs do not occur. This behaviour, however, comes at a price. To offer a run-deterring policy, the central bank has to sacrifice price stability. The more agents spend, the larger the required price level threat to deter runs. The threat has to be credible to deter runs ex-ante. Agents have to believe that ex-post the central bank will give up price stability if realized spending behavior is excessive. Only then do runs and inflation not occur on the equilibrium path. This behaviour could be associated with what Diamond and Dybvig present as a possibility of deterring runs, the suspension of convertibility. But there is a important difference: suspension of convertibility requires the bank to stop paying costumers after a given fraction, on the contrary, in this case there is no suspension of accounts. Instead, it is the amount of goods traded against those digital money and the resulting change in the price level, which generates the incentives for patient agents to rather wait.

Price stability

The main objective of a central bank is to achieve price stability that means preserve the purchasing power of the domestic currency. The technique adopted is the price level targeting that is a technique in monetary policy, where the central bank increases or decreases the supply of money and credit in the economy in order to achieve a specified price level.

There can be two different price stability for a central bank policy:

- **P_1-stable at level $\overline{P}$**: if it achieves $P_1(\alpha) = \overline{P}$ for the price level target $\overline{P}$, at all spending fraction $\lambda \leq \alpha \leq 1$. This can happen if and only if

$$y(\alpha) = \frac{M}{\overline{P}}\alpha \qquad \forall n \in [0, 1] \tag{3.9}$$

$$x_1(\alpha) = \overline{x} = \frac{M}{\overline{P}} \leq 1 \tag{3.10}$$

Proposition 6. *If the central bank commits to a P_1-stable policy, then:*

- *the socially optimal allocation is not implemented.*
- *there is an unique equilibrium where only impatient agents spend, therefore no runs can happen.*

- **Price-stable at level $\overline{P}$**: if it achieves $P_1(\alpha) = P_2(\alpha) = \overline{P}$ for the price level target $\overline{P}$, for all spending fraction $\lambda \leq \alpha \leq 1$. This can happen if and only if

$$i(\alpha) = \frac{\frac{\overline{P}}{M} - \alpha}{1 - \alpha} R - 1 \qquad \text{and} \quad \overline{P} \geq M \tag{3.11}$$

Proposition 7. *If the central bank commits to a price-stable central bank policy, then the nominal interest rate is non negative for all $\alpha \in [\lambda, 1]$. The interest rate is increasing in α.*

Deposit insurance

It can be underlined that most of the explained analysis about the effect of a CBDC would remain unchanged if a deposit insurance is introduced. Indeed, it does not fully eliminate bank runs because small deposits only are covered by this insurance, but a CBDC may favor large depositors. Moreover, it does not change the deposits' expected value at a commercial bank in the absence of run.

4 | An open economy framework

Let's focus on the real goal of this thesis: how a digital run might behave in an open economy context. There is a huge quantity of the literature that has analyzed the multifaceted implications of CBDC in a closed economy context, but the literature on the international implications of CBDCs is still young. There are still lots of open questions about the open economy, as to the effects of cross-border CBDC usage on currency substitution, international capital flows, likelihood of financial crises, internationalization of currencies, degree of international risk-sharing and the global spillovers of shocks and policies. [47]
In a contest of open economy, a CBDC can be issued both by a domestic and a foreign central bank, and both be available to domestic investors. If one of the two countries has a credible and reputable central bank, this can give rise to a new type of global safe asset which, under various circumstances, can become very attractive. In our model, we suppose the foreign central bank offers a global safe asset. The presence of a foreign CBDC as an international safe asset could attract domestic deposits. In the next sections we will discuss if this may increase the risk of financial instability for the domestic banking sector.
Of course, there must be some particular design features of such a CBDC. One of the most important is that consumers can have unrestricted access to the balance sheet of both the domestic central bank and the foreign central bank, this is a fundamental difference from the case described in the chapter 3. This could occur practically in various ways. One scenario under which this could happen is if the foreign economy is a reserve-currency issuer who is also home to large international digital networks that would facilitate access to the CBDC through their platforms to agents in different countries, as explained by Popescu [47]. In his paper, he introduces a foreign central bank in the Ferdinandez-Villaverde model, but with a domestic commercial bank. We will modify it by replacing the domestic commercial bank with a domestic central bank that could be susceptible to a bank run.

4.1. The model

The actors of the model are a domestic economy populated by agents and a central bank that issues a CBDC and could be susceptible to runs, and a highly credible foreign central bank which issues a CBDC accessible to domestic residents in the form of interest-bearing deposits.
The model will generate competition for deposits between the domestic central bank and the foreign one, indeed consumers can choose to deposit in both the central banks according to their preferences and the possibility of a greater gain.

The technical model has some common point with the case of a closed economy:

- **Time**: we have a discrete time divided into three periods: t=0,1,2. At t=0 agents invest their real goods, in t=1 they understand their type and act according to this, in period t=2 agent's type 2 withdraw.
- **Agents**: there are two different type agents: a type 1 agent who decides to withdraw in t=1 since he needs immediately some liquidity, and a type 2 agent who can wait until time 2. The realization of each agent's type is private information to that agent and is i.i.d. across agents with no aggregate uncertainty.
- α: is the fraction of agents that actually withdraw earlier, meaning the type 1 agents plus a possible fraction of type 2 agents. It depends on the choice an agent made about where to invest, indeed, being the foreign central bank not susceptible to run, α corresponds to the number of type 1 agents in the foreign central bank.
- λ: probability of being of type 1. It is independent of the agent's choice about where to invest.
- **Utility function**: $u : \mathbb{R}_{++} \to \mathbb{R}$ strictly increasing, strictly concave and continuously differentiable function such that:

$$U(x_1, x_2) = \begin{cases} u(x_1) & \text{with probability } \lambda \\ \\ u(x_2) & \text{with probability } 1 \text{ -}\lambda \end{cases} \tag{4.1}$$

- **Real Investment**: both the domestic and the foreign economies allow for a short-term and a long-term investment opportunity. In the short term technology they invest 1 unit in the first period and return r units in t=1 (r can be 1 or bigger than 1, according to the investment strategy, usually is set at 1). The long term investment invests one unit in t=0 and receive $R > 1$ units in t=2.

- **y**: fraction of goods invested in the short term technology.

Then we add some features related to a foreign central bank. There must be some assumption to be done before the analysis of the model. We assume that domestic consumers can invest in CBDC deposits at the foreign central bank. Moreover, in our model we made the simplification that the foreign central bank is assumed as perfectly credible, in the sense that agents face no uncertainty or risk of loss to withdraw their deposits. This means the foreign central bank is not subject to runs contrary to the domestic central bank. A foreign central bank can create such a credible riskless deposit using one of the two solutions presented in the chapter 3: punishment behaviour or equal treatment.
The potential demand for perceived safe store-of-value foreign currencies has been exemplified by the spread of dollarization in numerous economies. Indeed, it has been conjectured that compared to hoarding cash, the potentially high accessibility and ease of availability of CBDCs could incentive further currency substitution in countries with weak credibility of the policy frameworks [47]. Finally, we assume that the foreign central bank conducts its own monetary policy regardless of the domestic central bank, and this determines the interest rate offered on CBDC deposits. Therefore, both the domestic central bank and the foreign central bank can set their interest rate independently of each other. Consumers choose the contract which delivers the highest ex-ante utility. This is the fundamental point of our analysis.
All these assumptions lead to the case where the foreign central bank can offer a safe-haven international financial asset in contrast to the run susceptible domestic financial asset.
Moreover, for simplicity's sake, we assume that domestic residents can invest, but not borrow internationally.

Therefore, there must be added some model features:

- **f**: fraction of agents who choose the foreign central bank. $f \in [0, 1]$ and $0 < f < 1$ only if consumers are indifferent between depositing at the domestic central bank or foreign central bank, therefore both central banks offer the same contract. If $f = 0$, the agents prefer the domestic asset as it gives them a higher expectation. If $f = 1$, all the agents choose the foreign asset, meaning that the domestic central bank is completely abandoned.

- $\mathbf{d_i}$: $d_i \in \{0, 1\}$ is the deposit decision of consumer i. $d_i = 0$ means the agent i prefers to invest in the domestic central bank, $d_i = 1$ the agent prefers the foreign central bank.

- **capital account constrain**: $k > 0$ limit amount that domestic residents could invest in a foreign asset. This can be useful to prevent a complete migration into the foreign central bank, by setting $k < 1$. In general, we must impose that the fraction of agents who choose the foreign central bank does not exceed the capital constrain, i.e. $k \geq f$.
- **withdrawal strategy**: $w_i \in \{1, 2\}$ denotes the time period when the consumer withdraws her deposit. An impatient agent has $w_i = 1$, while a patient one has $w_i = 2$.

The features just presented concern a general foreign bank, we must add some features related to the fact we are talking about two different CBDC. We denote with an $'$ the quantity related to a foreign currency.

- **M**: quantity of domestic CBDC an agent receives if he sells his endowment to the domestic central bank. This quantity (always positive) is set by the domestic central bank. The central bank invests the real goods received.
- **M'**: quantity of foreign CBDC an agent receives if he sells his endowment to the foreign central bank. This quantity (always positive) is set by the foreign central bank. The central bank invests the real goods received.
- **y**(α): fraction of real goods to be liquidated in period 1 by the domestic central bank in order to meet the withdrawal requests. The central bank sell this quantity to the spending agents at a market clearing price P_1.
- **y'**(α): fraction of real goods to be liquidated in period 1 by the foreign central bank in order to meet the withdrawal requests. The central bank sell this quantity to the spending agents at a market clearing price P_1'.
- **i**(α): nominal interest rate chose in period 1 by the domestic central bank, and paid in period 2 on the CBDC domestic balances remaining at the end of period 1.
- **i'**(α): nominal interest rate chose in period 1 by the foreign central bank, and paid in period 2 on the CBDC foreign balances remaining at the end of period 1.

The model can be schematized as follows.
At the beginning of t=0 an agent has a zero balance on his CBDC account and an endowment of one unit of the real good. With a probability of f, he sells his endowment to the foreign central bank, receiving a balance of $M' > 0$ of digital foreign currency, otherwise he sells his endowment to the domestic central bank receiving $M > 0$ domestic

CBDC:

$$CBDCaccount = \begin{cases} M' & \text{with probability f} \\ \\ M & \text{with probability 1 - f} \end{cases} \tag{4.2}$$

At time t=1, the agent learns his type and decides whether to spend his CBDC balance or not. The probability of being of type 1 is denoted by λ and is independent of the decision to invest in a foreign or domestic central bank. Therefore, the balance on his CBDC account is given by:

$$CBDCaccount = \begin{cases} M' & \text{with probability } f(1-\lambda) \\ \\ 0 & \text{with probability } f\lambda \\ \\ M & \text{with probability } (1-f)(1-\lambda) \\ \\ 0 & \text{with probability } (1-f)\lambda \end{cases} \tag{4.3}$$

In period t=2, the agent type 1 have a zero CBDC balance while the agent type 2 increased his balances by a factor term that depends on the corresponding nominal interest rate. Therefore, the balance on his CBDC account is given by:

$$CBDCaccount = \begin{cases} M'(1+i') & \text{with probability } f(1-\lambda) \\ \\ 0 & \text{with probability } f\lambda \\ \\ M(1+i) & \text{with probability } (1-f)(1-\lambda) \\ \\ 0 & \text{with probability } (1-f)\lambda \end{cases} \tag{4.4}$$

Therefore, we can summarize the cases of positive CBDC balances as follows, where in the right table we have given the corresponding probabilities. All other cases have a zero CBDC balance. Until now we have considered all agents to behave according to their type, now we introduce the effective fraction of agents withdrawing at equilibrium, i.e. α

instead of λ.

	t=0	**t=1**	**t=2**	**t=0**	**t=1**	**t=2**
Domestic Central Bank	M	M	M(1+i)	1-f	(1-f)(1-α)	(1-f)(1-α)
Foreign Central Bank	M'	M'	M'(1+i')	f	f(1-α)	f(1-α)

Table 4.1: CBDC balances and their probabilities

So far we have discussed the nominal contracts. Moreover, we can define the real contract (x_1, x_2). Indeed, the aggregate spending behaviour α and the liquidation policies y and y' jointly determine the real allocation of goods purchased per agents on the goods markets and thus the consumption (x_1, x_1) and (x'_1, x'_1) in periods t=1 and t=2. This can be intuitively: in t=1, a fraction y of the real goods must be liquidated to fulfill the demand of the agents' type 1 in the domestic central bank, then this quantity must be equally distributed across all spending agents in this period who have a domestic central bank deposit, i.e. $(1-f)\alpha$. The same happens in the foreign central bank, the bank liquidates a fraction y' of real the goods and equally distributes it across all spending agents in this period who have a foreign central bank deposit, i.e $f\alpha$. At t=2, the remaining fraction of real goods is liquidated and equally distributed across the agents that didn't withdraw in the previous period.
Therefore, it can be summarized as in the table below, where both the real individual supply and the real individual CBDC value are presented.

	t=1	**t=2**
Domestic Central Bank	$\frac{y}{(1-f)\alpha}$	$\frac{1-y}{(1-f)(1-\alpha)}R$
Foreign Central Bank	$\frac{y'}{f\alpha}$	$\frac{1-y'}{f(1-\alpha)}R$

Table 4.2: Real individual supply

	t=1	**t=2**
Domestic Central Bank	$\frac{M}{P_1}$	$\frac{M(1+i)}{P_2}$
Foreign Central Bank	$\frac{M'}{P'_1}$	$\frac{M'(1+i')}{P'_2}$

Table 4.3: Real individual CBDC value

Therefore, we can also compute the real allocation of goods.

$$(x_1, x_2) = \left(\frac{M}{P_1}, \frac{M(1+i)}{P_2}\right) = \left(\frac{y}{(1-f)\alpha}, \frac{1-y}{(1-f)(1-\alpha)}R\right) \tag{4.5}$$

$$(x'_1, x'_2) = \left(\frac{M'}{P'_1}, \frac{M'(1+i')}{P'_2}\right) = \left(\frac{y'}{f\alpha}, \frac{1-y'}{f(1-\alpha)}R\right) \tag{4.6}$$

Where we must impose $x'_1 < x'_2$ since we have defined the foreign central bank as perfectly safe, therefore not subject to bank runs.

Proposition 8. *The foreign central bank is run free if its liquidation policy is run-deterring, that means:*

$$x'_1 < x'_2 \qquad \Longleftrightarrow \qquad y' < \frac{\alpha R}{1-\alpha(1-R)} \quad \forall \alpha \in (\lambda, 1] \tag{4.7}$$

Therefore, it implements the socially optimum in the unique equilibrium.

Proof. Suppose we are in the general case, in which alpha can also be greater than lambda. $x'_1 < x'_2$ iff $\frac{y'}{f\alpha} < \frac{1-y'}{f(1-\alpha)}R$, where we substitute for x'_1 and x'_2 the corresponding values given by 4.8

We impose $f > 0$, that means there is at least an agent who chooses the foreign deposit, otherwise we would not be discussing this possibility. Indeed, if $f = 0$, no agent chooses the foreign central bank and this leads to a closed economy whose behaviour has already been discussed in the chapter 3.

We can then multiply both members of the equation by f:

$$\frac{y'}{\alpha} < \frac{1-y'}{(1-\alpha)}R$$

Both y' and $1-\alpha$ are positive quantity, therefore we can multiply or divide both members by them and obtain:

$$\frac{1-\alpha}{\alpha} < \frac{1-y'}{y'}R$$

Then we can isolate y' since $R > 1$ and all of the factors involved are positive($1-\alpha > 0, 1-y' > 0$, since α and y' are $\in (0,1)$), we obtain:

$$\frac{1-\alpha+\alpha R}{\alpha R} < \frac{1}{y'}$$

$y' > 0$ and the left numerator is positive, indeed $1-\alpha+\alpha R = 1-\alpha(1-R)$ where $1-R < 0$ because $R > 1$, and $\alpha > 0$ by definition, then $-\alpha(1-R) > 0$, therefore the numerator is positive, being the sum of positive quantities. We can then multiply both members by this quantities and obtain:

$$y' < \frac{\alpha R}{1 - \alpha(1 - R)}$$

□

Thus, we have shown that the foreign central bank can implement a run deterring policy, so type 2 agents have no incentive to withdraw earlier. Therefore, each agent behaves according to its own type, i.e. $\alpha = \lambda$ for the foreign central bank. This means the real allocation is given by

$$(x_1', x_2') = \left(\frac{M'}{P_1'}, \frac{M'(1+i')}{P_2'}\right) = \left(\frac{y'}{f\lambda}, \frac{1-y'}{f(1-\lambda)}R\right) \tag{4.8}$$

The consumer must decide whether to deposit with the domestic central bank, consuming either x_1 upon withdrawal in period 1 or x_2 in period 2 but with an uncertainty, or with the foreign central bank, consuming surely either x_1' upon withdrawal in period 1 or x_2' in period 2.

4.2. Equilibrium

An equilibrium consists of a demand deposit domestic contract (x_1, x_2), a demand deposit foreign contract (x_1', x_2'), deposit decision $d_i \in \{0, 1\}$ in the initial period, a strategy profile $w_i \in \{1, 2\}$ for the withdrawal game in the intermediate period, a fraction $f \in [0, 1]$ of consumers depositing in the foreign CBDC, and a fraction $\alpha \in [0, 1]$ of depositors who withdraw in period 1 such that:

- In period 0, given contract (x_1, x_2) and (x_1', x_2'), each consumer $i \in [0, 1]$ optimally choose where to deposit his unit of endowment, by selecting the contract that offers the highest expected utility. Total deposits in the foreign CBDC cannot exceed the capital account constraint, therefore $f \leq k$.
- The domestic central bank chooses the deposit contract to offer (x_1, x_2) to maximize profit in period 2, given (x_1', x_2').
- Withdrawals in period 1 satisfy the condition $\alpha = 1 - \int_{i \in [0,1]: w_i = 2} di$.
- Initial deposits in the foreign CBDC satisfy the condition $f = \int d_i di$.

4.2.1. Equilibrium with no Runs

Our analysis started from the cases where there is only a good equilibrium. This is the case where all agents behave according to their type. Of course, we must underline that, in this case, the socially optimal contract is offered either by the domestic central bank or the foreign central bank, or both. If both the central banks have customers, i.e $f \in (0, k)$, then both banks are offering the optimal contract, otherwise we would see a completely deposits migration in the bank that offers the optimal contract. This is caused by the fact agents want to maximize their expected utility, then they will choose the bank that gives them the higher gain. Therefore, if we apply this notion to our case, we can state the following proposition:

Proposition 9. *In equilibrium the domestic central bank make a zero profit and offer the optimal contract.*

Proof. If all agents behave according to their type, $\alpha = \lambda$. Let's denote by π the domestic central bank's profit, by $(y^*(\pi), x_1^*(\pi), x_2^*(\pi))$ the solution to the depositor's utility maximization problem:

$$V = \max_{(y,x_1,x_2)\in\mathbb{R}^3_+} [\lambda u(x_1) + (1-\lambda)u(x_2)] \tag{4.9}$$

subject to:

$$x_1 = \frac{y}{(1-f)\lambda} \tag{4.10}$$

$$x_2 = \frac{1-y}{(1-f)(1-\lambda)} R \tag{4.11}$$

Moreover, we must impose $0 \leq y \leq 1$, $\lambda x_1 \leq ry$, $x_1 \leq x_2$ and :

$$(1-\lambda)x_2 \leq R(1-y) + ry - \lambda x_1 - \pi \tag{4.12}$$

Given our assumption we obtain a unique interior solution characterized by:

$$u'\left(\frac{y^*}{(1-f)\lambda}\right) = Ru'\left(\frac{R(1-y^*)-\pi}{(1-\lambda)(1-f)}\right) \tag{4.13}$$

Then we have $(x_1^*(\pi), x_2^*(\pi)) = \left(\frac{y^*(\pi)}{(1-f)\lambda}, \frac{R(1-y^*(\pi))-\pi}{(1-f)(1-\lambda)}\right)$.
The envelope theorem implies $V'(\pi) \quad \forall \pi > 0$. If the domestic central bank profits is non zero, than consumers would prefers foreign central bank because the expected consump-

tion $x_2^* = \frac{R(1-y^*)-\pi}{(1-\lambda)(1-f)}$ is lower than the one they can receive if they invest in the foreign central bank, due to the factor $-\pi$,(assuming that the foreign central bank does not generate profit and is such that $X_2'^* = x_2^*$ if π is not considered). This would take away from the scenario the domestic central bank. Therefore the domestic central bank makes a zero profit.

So far we have discussed the first part of the proposition. Now, let's prove that the domestic central bank offers the optimal contract, in the case $\alpha = \lambda$. Thanks to the fact the utility function is strictly increasing, strictly concave and continuously differentiable function, the fact $R > 1$, and the equation 4.13, we can demonstrate that in equilibrium the domestic central bank offers the optimal contract, i.e $x_1^* < x_2^*$.
Indeed, starting from the 4.13, where for easy of writing we use x_1^* and x_2^* instead of their formulas, $u'(x_1^*) = Ru'(x_2^*)$, that means $\frac{u'(x_1^*)}{u'(x_2^*)} = R > 1$. This, with the fact the the first derivative of utility function is always positive being the utility function strictly increasing, leads to:

$$u'(x_1^*) > u'(x_2^*) \tag{4.14}$$

But we know the utility function is strictly concave, this means its second derivative is always negative. But the second the derivative of the utility function is the first derivative of the first derivative of the utility function. Therefore the first derivative of utility function is always decreasing i.e $u'(x_1^*) > u'(x_2^*) \iff x_1^* < x_2^*$, that means in equilibrium the domestic central bank offer the optimal contract.

Moreover, the depositors' relative risk aversion exceeding unity together with the relative contraint $R(1-\lambda x_1^*) = (1-\lambda)x_2^*$ and 4.11 implies $x_1^* > 1$ and $x_2^* < R$. The socially optimal allocation impies for central bank the socially optimal liquidation policy $y^*(\lambda) = x_1^*\lambda > \lambda$, implying $P_1 < M$. □

Therefore, in the case of only impatient agents who spend:

Proposition 10. *Central bank policy (M, y, i) implements the social optimum (x_1^*, x_2^*) in equilibrium if the central bank policy satisfies $y(\lambda) = y* = x_1^*\lambda > \lambda$, implying $x_1 = x_1^* = \frac{y*}{(1-f)\alpha}$ and $x_2 = x_2^* = \frac{1-y^*}{(1-f)(1-\alpha)}R$, and $P_1^* < M$.*

Next, we discuss the competition for deposits between domestic central bank and foreign central bank. First, it is trivial to see that there is an equilibrium in which, through its remuneration of CBDC deposits, the foreign central bank could replicate the socially optimum contract.

Proposition 11. *The foreign central bank can replicate the socially optimum domestic bank deposit contract, if the interest rate on the foreign CBDC is such that* $x'_1 = x^*_1$ *and* $x'_2 = x^*_2$.

But, since central banks can choose their interest rates, there could be also a case where the foreign central bank offers a better socially optimum bank deposit contract.

Proposition 12. *The foreign central bank can offer a better bank deposit contract if the interest rate on the foreign CBDC is such that* $x'_2 > x^*_2$.

In this case, agents prefer to move to the foreign central bank.
Therefore, in the case only impatient agents withdraw, there can be different outcomes, since there will be a competition between the domestic central bank and foreign central bank.

- If the domestic central bank offers a better socially optimum bank deposit contract, it will absorb the entire deposit market and $f = 0$.
- If the foreign central bank offers a better socially optimum bank deposit, or a better contract, then it will absorb all depositors up to the capital constraint, which means $f = k$.
- If both the domestic central bank and the foreign central bank offer the same socially optimum bank deposit, then any $f \in [0, k]$ is an equilibrium.

Therefore, in the case where all the agents act according to their type both in the domestic central bank and in the foreign central bank, these two institutions could both offer a socially optimum bank deposit contract. But, since central banks can choose their nominal interest rate, there could be three different cases. Two of them guarantee a non zero fraction of depositors who deposits in the domestic central bank. The last one leads to a partial run in the domestic central bank into the foreign central bank. Indeed, at t=1 the foreign central bank chose its interest rate and, if it leads to a higher consumption for type 2 agents with respect to the domestic consumption, agents could choose to migrate to the foreign central bank causing a domestic run.

4.2.2. Domestic Central Bank Runs

Let's now focus on the situation in which agents do not behave according to their type, but will have some doubts about the domestic central bank and start panics. Since agent's type is unobservable, it is possible for an agent to pretend to be impatient, and this could lead to a bank run. This is the case $\alpha > \lambda$. To meet the withdrawal requests, the

domestic central bank must liquidate the long term asset, but this may be a problem, indeed a central bank invest its long term asset in an investment bank that can not have immediately the necessary amount to liquidate the assets.
But, as explained in the capitol 3, a central bank can choose the nominal interest rate, therefore three different cases can happen:

Proposition 13. *Domestic central bank policy* (M, y, i) *implements:*

- *The social optimum* (x_1^*, x_2^*) *in equilibrium if the central bank sets a liquidation policy that implies* $x_1(\alpha) < x_2(\alpha)$.
- *An exhaustive run with* $\alpha = 1$ *if and only if* $x_1(1) > x_2(1)$.
- *A partial run if and only if* $x_1(\alpha) = x_2(\alpha)$ *with* $\alpha \in (\lambda, 1)$*, this means that only some patient agents withdraw.*

4.2.3. Runs in presence of a Foreign Central Bank

Relative to what we have already presented, we further assume the presence of the foreign CBDC riskless deposit which competes with domestic central bank deposits, in the case $\alpha > \lambda$.
We can state the following intuitive result:

Proposition 14. *If the foreign central bank offers a riskless deposit contract which guarantees a payoff equal to the ones offered by the domestic socially optimal contract, then it will attract all deposits up to the capital account constraint, i.e.* $f = k$.

Indeed, even if the domestic central bank offers a socially optimal contract, i.e. $x_1(\alpha) < x_2(\alpha)$, there is still a positive probability of a run that will cause a loss for the agent. Therefore, if the foreign central bank offers the same optimal contract with a zero probability of a run, it would attract all the agents. Indeed, up to the same outcome, an agent will choose the contract that has a higher probability of provide that outcome, that is the foreign one that has a probability of 1 of these results. Obviously, since we have imposed a capital account constrain, not all the agents can migrate to the foreign central bank. Therefore, the domestic economy will experience a capital outflow up to $f = k$ agents, that means a partially run.
This agents' preference to choose the foreign asset can give to the foreign central bank a market power. This means it can offer a lower payoff contract with respect to the socially optimum one. Of course there is a lower limit payoff, below which the agents are not automatically attract by the foreign asset, since their earnings would be too low therefore

they prefer to take the chance to lose their deposits with a view of a greater gain.

Proposition 15. *The foreign central bank can offer a lower payoffs contract with respect to the socially optimal one up to fixed payoff, and still attract the highest possible amount of deposits, i.e. $f = k$.*

Proof. Let's denote by U_1 the utility the agent derives from the socially optimal domestic central bank deposit contract, which can be subject to runs.

$$U_1(x_1^*, x_2^*) = \alpha u(x_1^*) + (1-\alpha)u(x_2^*) \tag{4.15}$$

and by U_2 the utility derived from the safe CBDC foreign deposit contract with the same payoff.

$$U_2(x_1'^*, x_2'^*) = \lambda u(x_1') + (1-\lambda)u(x_2'^*) = \lambda u(x_1) + (1-\lambda)u(x_2^*) \tag{4.16}$$

Where the last equality derives from the fact the contracts have the same payoff.
As runs can occur with domestic central bank, but not with the foreign central bank, it has to be that $U_1 < U_2$. Indeed $u(x_1) < u(x_2)$ being the utility function strictly increasing and the socially optimal contract entails $x_1 < x_2$. Therefore

$$U_1 - U_2 = \alpha u(x_1^*) + (1-\alpha)u(x_2^*) - \lambda u(x_1^*) - (1-\lambda)u(x_2^*) =$$

we can collect $u(x_1^*), u(x_2^*)$

$$= u(x_1^*)(\alpha - \lambda) + u(x_2^*)(1 - \alpha - 1 + \lambda) = u(x_1^*)(\alpha - \lambda) - u(x_2^*)(\alpha - \lambda)$$

we can collect $(\alpha - \lambda) > 0$ since $\alpha > \lambda$, and obtain.

$$U_1 - U_2 = (\alpha - \lambda)(u(x_1^*) - u(x_2^*))$$

Therefore $U_1 - U_2 < 0$ since $(\alpha - \lambda) > 0$ and $(u(x_1^*) - u(x_2^*)) < 0$ because $u(x_1^*) < u(x_2^*)$. Therefore $U_1 < U_2$.

In this case, the central bank can offer lower payoffs $x_1' < x_1'^*, x_2' < x_2'^*$ such that $U_1(x_1^*, x_2^*) < U_2(x_1', x_2') < U_2(x_1'^*, x_2'^*)$ and still attract all deposits up to the capital amount constraint. Indeed, following the same procedure as above, we can prove there can be the case:

$$\alpha u(x_1^*) + (1-\alpha)u(x_2^*) < \lambda u(x_1') + (1-\lambda)u(x_2') < \lambda u(x_1^*) + (1-\lambda)u(x_2^*)$$

□

This behaviour could lead to a financial instability. Indeed, in the case $\alpha > \lambda$ we can have different behaviours. First, let's focus on the case where the domestic central bank offers a socially optimal contract:

- The foreign central bank offers a riskless deposit contract which guarantees a payoff equal to the ones offered by the domestic socially optimal contract. $\rightarrow$ All the agents will be attracted by the foreign central bank, i.e. $f = k$. $\rightarrow$ There is a partial run in the domestic central bank into the foreign central bank.
- The foreign central bank offers a lower payoffs contract with respect to the socially optimal one up to fixed payoff. $\rightarrow$ All the agents will be attracted by the foreign central bank, i.e. $f = k$. $\rightarrow$ There is a partial run in the domestic central bank into the foreign central bank.
- The foreign central bank offers a payoffs contract that is lower than the fixed payoff discussed above $\rightarrow$ The agents are not automatically attracted by the foreign central bank, i.e. $f < k$ $\rightarrow$ No run.

Finally, the case where the domestic central bank does not offer a socially optimal contract has already been discussed in the chapter 3. Indeed, the agents' behaviour will be the same both if we are in a closed economy or in an open one. The presence of a foreign central bank does not impact substantially the conclusion, a run will occur anyway. The agents can choose equally to invest their goods withdrawn in the foreign central bank, or keep them.

Therefore, we can define the optimal consumption in the case of $f = k$, that is the case of a partially run into the foreign central bank. We suppose that the agents that failed to withdraw in t=1, maintain their domestic central bank deposit. Therefore the optimal consumption is given by:

$$(x_1^*, x_2^*) = \left(\frac{y^*}{(1-k)\alpha}, \frac{1-y^*}{(1-k)(1-\alpha)} R \right) \tag{4.17}$$

$$(x_1'^*, x_2'^*) = \left(\frac{y'^*}{k\lambda}, \frac{1-y'^*}{(k(1-\lambda)} R \right) \tag{4.18}$$

5 | Conclusion and future developments

Since digitalization is reaching all areas of our lives and the digital payments are prevailing on the physical ones, it seems natural to introduce a digital version of the physical currency. Central banks are working on this topic in order to create Central Bank Digital Currencies that will partially or totally substitute the physical ones. Currently, different nations are on different stages in this project: some of them have already launched it, others are on pilot or on development, others are not even taking into account the project. Since this is a new topic, there is not a unique pool of characteristics to observe, but some hypothesis have been made. As explained in the chapter 1, different architecture/technology have been adopted so far by the nations that are testing it. Of course, the related risks are not actually tested, but different theoretical research have been published.

In this scenario, we try to figure out the market behaviour in the event of a bank run. Following the classical model used to understand commercial bank runs, the Diamond and Dybvig model, we adapt it to the central bank case following the Fernandez-Villaverde et all model. We see that some features are common others are different. The main differences lie in the fact a central bank can choose its own nominal interest rate and cannot be forced into liquidation. Therefore, it can prevent a bank run under some circumstances. This can happen if the central bank adopts a punishment behaviour or an equal treatment behaviour. Through the first one, the central bank decides to punish consumers who contribute to the run by not paying any consumers who try to withdraw beyond the measure λ, therefore a consumer who roll over is rewarded when withdrawals are high. The second instrument force the agents that withdraw after a fraction λ, to roll over their deposits and the bank threats them as if they had rolled over their deposits.

Finally, we modify this last model introducing a foreign central bank, that is the real innovation of this thesis. We simplify the model by assuming the foreign central bank offers a risk-free bank deposit, therefore is not subject to runs. The foreign central bank is run free if its liquidation policy is run-deterring, under this policy, type 2 agents have no incentive to withdraw earlier.

Our model concerns an open economy composed of a domestic central bank, a foreign central bank and domestic consumers who choose where to deposit their goods. Moreover, we impose a capital account constrain that limits the number of deposits in the foreign central bank.

We find that this could lead to a new type of run, where the agents find it's convenient to move to the foreign central bank and withdraw their deposits at the domestic central bank. This can happen starting from different situation. Firstly, we analyze the case where the domestic central bank offers a socially optimal contract, regardless of fact patient agents decides to withdraw earlier or not. A foreign central bank can offer a riskless deposit contract which mimics the payoff of the domestic socially optimal contract, then it will attract all deposits up to the capital account constraint. Moreover, even if the foreign central bank offers a deposit contract that is lower with respect to the domestic one up to a limit quantity, the agents will be attracted by the foreign central bank. This is caused by the certainty the foreign central bank will not run, therefore agents feel more confident to deposit their account there. This two situation lead to a partial run in the domestic central bank toward the foreign central bank. The run can be total if there is no capital account constrain, i.e $k = 1$.

Let's now focus on the case the domestic central bank is not offering the socially optimum deposit contract and is subject to runs. This situation is not specific to the case of an open economy, indeed a run happens regardless of the fact agents move to the foreign central bank or withdraw their deposit and keep them.

Therefore, we have proven that the introduction of a foreign central bank that offers a safe deposit contract will generate new type of runs, which reduces the financial stability. Therefore, this type of economy is risky, and must impose a collaboration between the central bank to prevent the possible negative outcome of these events.

Clearly, we have analyzed a simplified case. Further analyzes must be done if we assume the foreign central bank does not offer a safe asset but could be susceptible to runs.

6 | Glossary

Account-based CBDC: a type of CBDC tied to an identification scheme, such that all users need to identify themselves to access it.

Bank Run: phenomenon that occurs when numerous customers of a bank go to a credit institution to withdraw their money in deposit.

CBDC (Central Bank Digital Currency): a digital asset issued by a central bank for the purpose of payment and settlement, in either retail or wholesale transactions.

Central Bank: financial institution given privileged control over the production and distribution of money and credit for a nation or a group of nations. In modern economies, the central bank is usually responsible for the formulation of monetary policy and the regulation of member banks.

Commercial Bank: financial institution that accepts deposits, offers checking account services, makes various loans, and offers basic financial products like certificates of deposit (CDs) and savings accounts to individuals and small businesses.

Cryptocurrency: a digital currency that uses cryptographic encryption techniques to regulate the issuance of new units, record transactions and attempt to prevent fraud.

Interoperability: is the capability to communicate execute programs, or transfer data among various functional units in a manner that requires the user to have little or no knowledge of the unique characteristics of those units and technical or legal compatibility that enables a system or mechanism to be used in conjuction with other systems or mechanisms without imposing unnecessary costs on the users.

Investment bank: financial services company that acts as an intermediary in large and complex financial transactions.

Payment Services Provider (PSP): an entity that may issue payment instruments or provide retail payment services. This can include commercial banks and non.bank financial institutions.

Payoff: Net payoff is the profit or loss from the sale of an item or service after the costs of selling it, any additional costs associated with the asset or experienced over the life of the asset, and associated accounting losses have all been subtracted. The amount that remains is considered to be the net payoff.

Retail CBDC: a CBDC for use to general public.

Token-based CBDC: a type of CBDC secured via password such as digital signatures that can be accessed anonymously.

Utility function: measure of the welfare or satisfaction of a consumer as a function of the consumption of real goods, such as food or clothing. Utility function is widely used in rational choice theory to analyze human behavior. It measures consumers' preferences for a set of goods and services.

Wholesale CBDC: a CBDC for use by financial istitutions (wholesale transactions) that is different from balances in traditional bank reserves or settlment accounts.

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A | Appendix A

Let's give a proof of the multiplicity of the equilibrium in the Diamond and Dybvig model. Since agents can behave as they want, there could be different cases: the first one is the case where all the agents believe in their bank and think everything will go according to plan, therefore everyone behave as their type. The second case is the one where it is believed that the bank will fail, therefore all the agents try to withdrawn their deposits. Therefore there are multiple equilibrium. That is, there is more than one self-fulfilling prophecy about who withdraws at date 1. There is a good equilibrium in which only the type 1 depositors withdraw and a bad equilibrium, the so called bank run, in which all withdraw at date 1 because they all expect each other to do the same.

A.1. Good Equilibrium

If all the agents believe the other will behave as their type, there will be only a good equilibrium. Indeed, this is the case where $r_1 = c_1^{1*}$, that is when the fixed payment per dollar of deposits withdrawn at T=1 is equal to the optimal consumption of a type 1 agent given full information.
Let's prove this lead to a good equilibrium. We know the agents' payoff given their type is:

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if } \mathrm{f_j} < \mathrm{r_1}^{\text{-1}} \\ \\ 0 & \text{if } \mathrm{f_j} \geq \mathrm{r_1}^{\text{-1}} \end{cases} \tag{A.1}$$

$$V_2(f, r_1) = \max(\frac{R(1 - r_1 f)}{(1 - f)}, 0) \tag{A.2}$$

Let's call $t \in (0,1)$ the fraction of the continuum of agents of type 1. We can substitute $f = t$ in the B.2, and $r_1 = c_1^{1*}$ in both the equation:

$$V_1(f_j, r_1) = \begin{cases} c_1^{1*} & \text{if } \mathrm{f_j} < (\mathrm{c_1}^{1*})^{-1} \\ \\ 0 & \text{if } \mathrm{f_j} \geq (\mathrm{c_1}^{1*})^{-1} \end{cases} \tag{A.3}$$

$$V_2(f, r_1) = \max(\frac{R(1 - c_1^{1*}t)}{(1-t)}, 0) \tag{A.4}$$

This lead to $V_1(f_j, r_1) = c_1^{1*}$ and $V_2(f, r_1) = c_2^{2*}$. Indeed the optimal consumption satisfies:

$$c_1^{2*} = c_2^{1*} = 0 \tag{A.5}$$

$$u'(c_1^{1*}) = \rho R u'(c_2^{2*}) \tag{A.6}$$

$$tc_1^{1*} + [\frac{(1-t)c_2^{2*}}{R}] = 1 \tag{A.7}$$

That are, respectively, the delay consumption for the agents who can, the marginal utility in line with the marginal productivity and the resource constraint. We can obtain c_2^{2*} from the B.7:

$$c_2^{2*} = \frac{(1 - tc_1^{1*})R}{1-t} \tag{A.8}$$

That is $v_2(f, r_1)$.

Moreover, from B.6, given that $\rho R > 1$ we can conclude that $c_1^{1*} < c_2^{2*}$. Therefore $V_1(f_j, r_1) > V_2(f, r_1)$, meaning that all the agents who does not need immediately the liquidity, prefer to wait until period 2 to withdrawn, since they will receive an higher quantity.

We have demonstrated that in the case $r_1 = c_1^{1*}$ we have a single equilibrium that is the good one.

A.2. Bad Equilibrium

As explained, a run can happen if each patient agent believes that all agents, not just the impatient, will try to withdraw their deposits in period 1. Indeed, in these case, the payoff is higher for the agent' type 1. This is the case where $r_1 \geq 1$.

Let's first focus on the case $r_1 > 1$. In this case we have

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if } \mathrm{f_j} < \mathrm{r_1}^{-1} \\ \\ 0 & \text{if } \mathrm{f_j} \geq \mathrm{r_1}^{-1} \end{cases} \tag{A.9}$$

If we are in the case $f_j \geq r_1$ the agent type 1 will not receive anything. This is the case where all the agent in the queue had been served and there are no more liquidity. Therefore the payoff at time 1 is $V_1(f_j, r_1) = 0$, and the agent could choose to wait till period 2, but we can prove that also $V_2(f, r_1) = 0$. Indeed, if $f_j \geq r_1$ then $f_j r_1 \geq 1$ and being $f \geq f_j$, also $fr_1 \geq 1$. Therefore $1 - fr_1 \leq 0$ while $1 - f > 0$ and $R > 1$. We can apply this concept in the payoff formula

$$V_2(f, r_1) = \max\left(\frac{R(1 - r_1 f)}{(1 - f)}, 0\right) \tag{A.10}$$

The quantity $\frac{R(1-r_1 f)}{(1-f)}$ is always negative therefore $V_2(f, r_1) = 0$. This is the case an agent will receive anything both if he wait until period 2 or if he try to withdrawn early.
On the contrary, if $f_j < r_1$, the agent will receive r_1 since his position in the queue grants him a remuneration. Therefore $V_1(f_j, r_1) = r_1 \geq 1$. But $f \geq f_j \forall j$ therefore $fr_1 \geq 1$ anyway and $V_2(f, r_1) = 0$. This means an agent will always choose to withdrawn at period 1 and this lead to a run.
Therefore, if $r_1 > 1$ a run is always an equilibrium.
The last case is if $r_1 = 1$, but this is the case the bank simply mimics direct holding of the assets and is therefore no improvement on simple competitive markets.

B | Appendix B

Given the two different bank analyzed, the commercial bank and the Central Bank, we can schematize the payoff related both to a run and to a not run event, as follow:

B.1. Commercial Bank

Event	**Withdraw**	**Roll over**
No Run	$u(c_1^*)$	$u(\frac{R[(1-y^*)-(\alpha-\lambda)c_1^*/k]}{1-\alpha})$
Run	$\frac{y^*+(1-y^*)k}{\alpha c_1^*} \cdot u(c_1^*)$	0

Table B.1: Payoff of a commercial bank

As the table underline, in case of a run, the payoff is higher if an agent decides to withdrawn with respect to a roll over decision. Therefore withdrawing deposits is optimal. Conversely, in $= \lambda$, that means no run can happen, the payoff of rolling over is higher, giving that for a patient agent is better to roll over.

B.2. Central Bank

In the case of a central bank, we can have two different payoff table, depending on the behaviour the institution decides to act.

Event	**Withdraw**	**Roll over**
No Run	$u(c_1^*)$	$u(\frac{R(1-y^*)}{1-\lambda})$
Run	$\frac{\lambda}{\alpha} \cdot u(c_1^*) + (1-\frac{\lambda}{\alpha}) \cdot 0$	$u(\frac{R(1-y^*)}{1-\alpha})$

Table B.2: Payoff of a Central bank, punishment case

Under this scheme, a consumer who rolls over is rewarded when withdrawals are higher. Impatient agents receives a quantity back only with a certain likelihood $\frac{\lambda}{\alpha}$, while patient consumers gain if they roll over. Therefore in equilibrium, only impatient consumers withdrawn and no run can happen.

If the bank decides to adopt a equal treatment behaviour, the payoff is the following:

Event	**Withdraw**	**Roll over**
No Run	$u(c_1^*)$	$u(\frac{R(1-y^*)}{1-\lambda})$
Run	$\frac{\lambda}{\alpha} \cdot u(c_1^*) + (1 - \frac{\lambda}{\alpha}) \cdot u(\frac{R(1-y^*)}{1-\lambda})$	$u(\frac{R(1-y^*)}{1-\lambda})$

Table B.3: Payoff of a Central bank, equal treatment case

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