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Development and Implementation of a GLOSA System for Urban Buses based on Finite State Machine

Tesi di Laurea Magistrale in
Mobility Engineering

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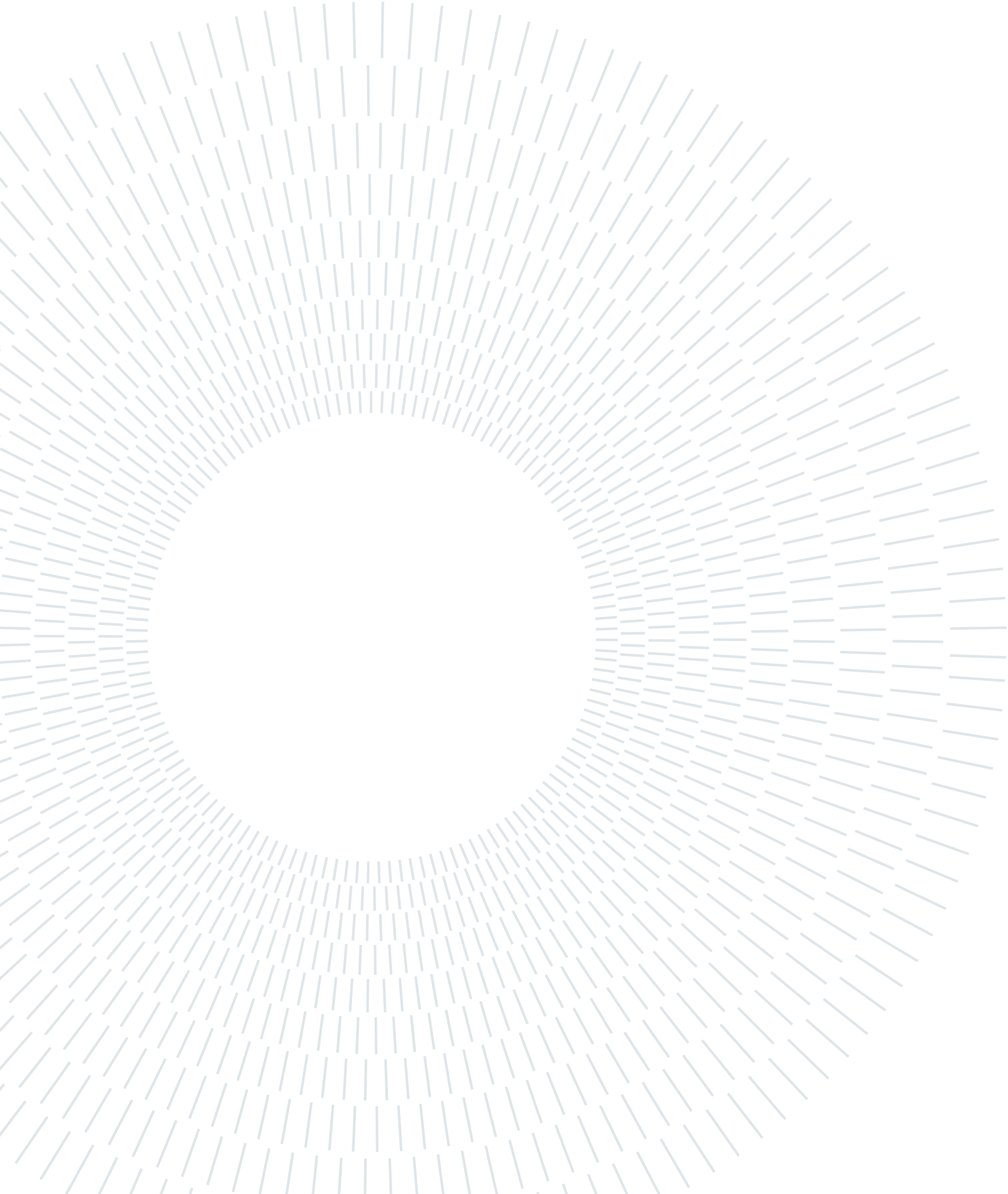
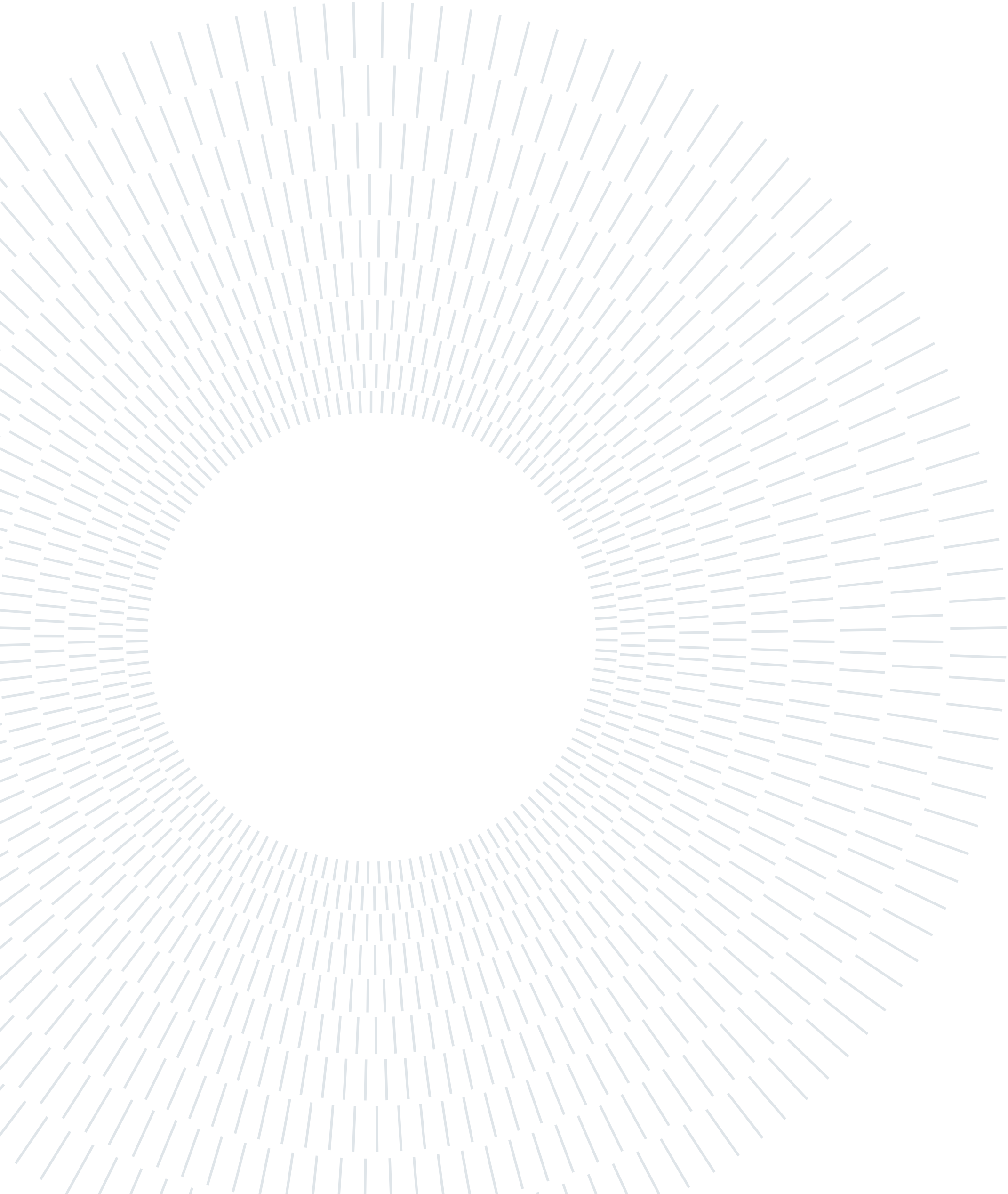


Abstract

GLOSA (Green Light Optimal Speed Advisory) is a system that suggests a speed to a driver in order to arrive at a traffic light during a green phase and avoid stopping. This thesis work proposes a GLOSA specific for buses, that takes in account the necessity of a bus of frequently stopping to let passengers boarding and alighting. The proposed algorithm is structured as a state machine to choose among possible speed profiles obtained from a kinematic set of equations, allowing the vehicle to cruise through two traffic lights with a bus stop in between and calculating the most suitable speed to cross the intersections as soon as possible during a green phase. The algorithm has been assessed with both simulations and real-world tests on the bus lines 90-91 in Milano. The results, compared to a previous algorithm, show a reduction in the number of stops at red traffic lights and in travel time, but an increase in energy consumption. Moreover, the algorithm proved to be robust and safe in case of possible traffic light plan changes.

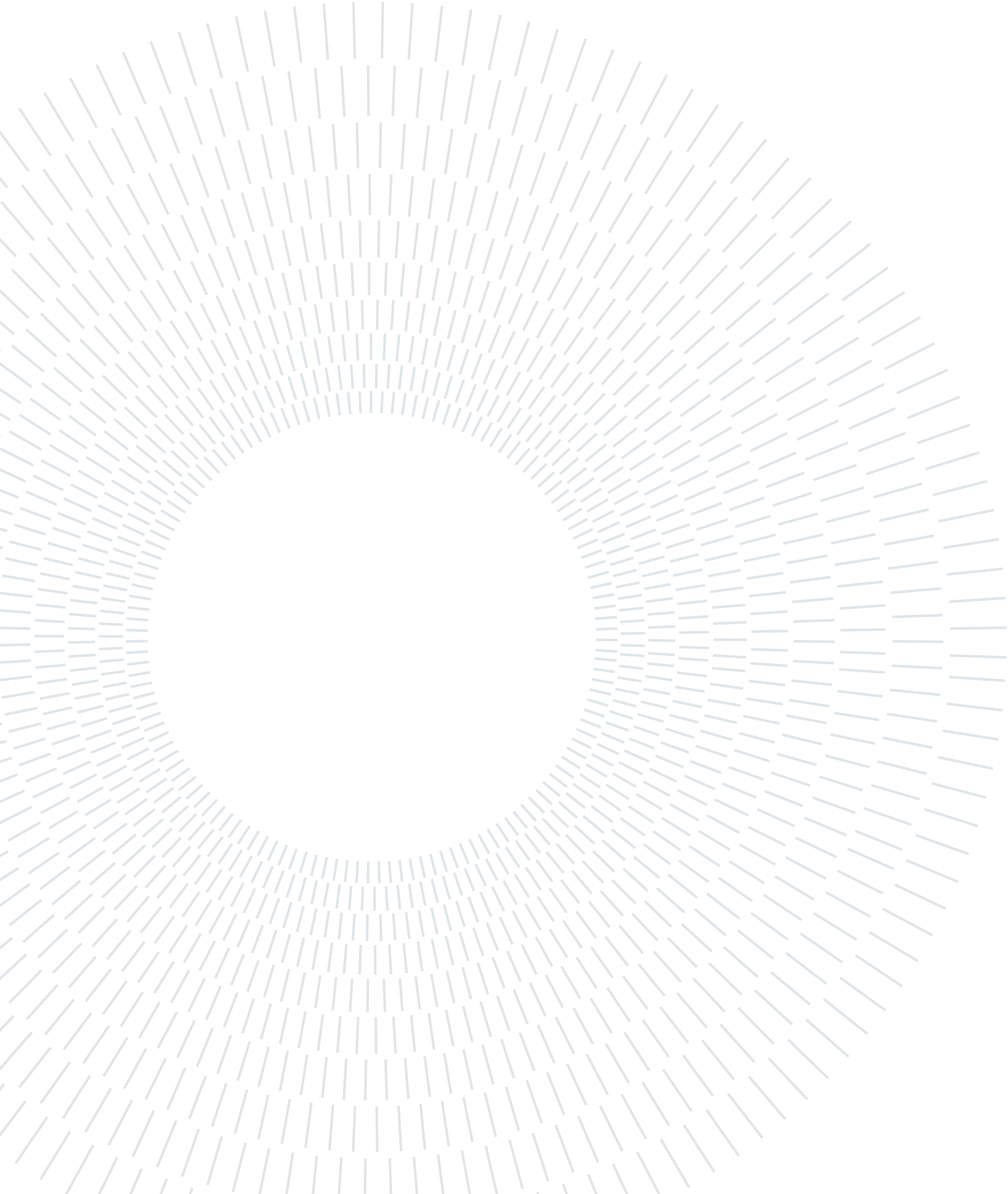
Keywords: bus, kinematic equations, traffic lights, stops, GLOSA



Sommario

GLOSA (Green Light Optimal Speed Advisory) è un sistema che suggerisce una velocità al guidatore in modo che arrivi ad un semaforo quando è verde ed eviti di fermarsi. Questo lavoro di tesi propone un GLOSA specifico per gli autobus, che tenga conto della necessità dell'autobus di fermarsi frequentemente per far salire e scendere i passeggeri. L'algoritmo proposto è strutturato come una macchina a stati per scegliere tra vari profili di velocità ottenuti da una serie di equazioni cinematiche, consentendo al veicolo di attraversare due semafori con una fermata nel mezzo e calcolando la velocità più adatta per attraversare gli incroci il prima possibile durante una fase di verde. L'algoritmo è stato testato sia con simulazioni che con test su strada sulla linea 90-91 a Milano. I risultati, confrontati con quelli di un algoritmo sviluppato in precedenza, mostrano una riduzione nel numero di soste ai semafori rossi e nel tempo di percorrenza della tratta, ma un aumento nel consumo di energia. Inoltre, l'algoritmo ha dimostrato di essere robusto e sicuro nel caso di cambiamenti del piano semaforico.

Parole chiave: autobus, equazioni cinematiche, semafori, fermate, GLOSA



Contents

Introduction

The attention to climate changes and environmental sustainability that has increased in the last decades focuses also on the reduction of vehicles emissions and fuel consumption. Together with the improvement of vehicles characteristics in term of efficiency, it is possible to further reduce energy demand by adopting a smoother driving style, keeping the speed as constant as possible and avoiding hard accelerations and brakes. A great help in achieving these results comes from Advanced Driver Assistance Systems (ADAS), among which there is also Green Light Optimal Speed Advisory (GLOSA) system. GLOSA is a system that gives speed advices to the driver with the aim of allowing the vehicle to cross a signalized intersection without stopping, in order to avoid unnecessary braking and idling at traffic lights.

From its first introduction, many different types of GLOSA have been proposed and tested, with various results. The aim of this work is to develop a GLOSA specific for buses that takes into account the presence of frequent stops on the path, often located near traffic lights. This thesis work is intended to implement an algorithm able to select the most suitable speed profile that can be followed by the bus while crossing two traffic lights, dividing the scenario into sections: crossing the upcoming intersection, slowing down to reach the bus stop and moving then towards the following traffic light.

This work is a development of previous works done in Politecnico di Milano about GLOSA [10, 27]. Moreover, thanks to the ongoing research on GLOSA systems, there was the possibility of testing the proposed algorithm on the road with a trolleybus equipped to receive traffic light information and to communicate GLOSA advices to the driver.

As mentioned, this work takes start from a GLOSA that considers multiple successive traffic lights and a uniformly accelerated motion model that allows the bus to reach the first intersection and then keep a constant speed from that point on. By inserting a stop between the two traffic lights, the GLOSA presented in this work tries to consider a speed profile that is no more composed by a single part at constant acceleration, but by many parts with different accelerations that account for the braking and restarting phases needed by the stop. This type of algorithm aims to guarantee a higher flexibility in terms of speed profiles, allowing then to better adapt to the different traffic light plan.

Since the work is intended to improve the previous algorithm, results of the proposed GLOSA system in simulations and tests are compared to those in [27].

Differently from most of the works present in literature, the algorithm developed relies just on kinematic-based set of equations to define the speed profiles. In particular the speed is the one that allows to reach the traffic lights as soon as possible during a green phase. The division of the speed profile in two parts, base on the two road sections before and after the stop, allows to easily consider all the possible scenarios in which the bus could be.

This work is developed on four main chapters:

- in the first one, an overview on the existing techniques to improve the performance of vehicles in urban scenarios is presented, with a focus on GLOSA, on some of the most recent algorithms and on GLOSA specific for buses;
- in the second chapter there is the description of the experimental setup used for tests, in particular of the instruments needed by the vehicle, the available communication with traffic lights and the path on which tests were performed;
- the third chapter contains the details of the proposed algorithm, with the description of the considered speed profiles, the decision process to choose the most suitable one and the calculations done to obtain a suggested speed as output;
- in the fourth chapter the simulation and test results are presented.

Conclusions summarise the obtained results and suggest future developments of the proposed GLOSA at the end of the work.

1 | State of the art

When a vehicle has to stop at a red traffic light, it loses time and energy, due to braking, idling at traffic light and then restarting. This is evident for cars, but most of all for buses, which often can have stops near traffic lights. So it important to avoid that a bus starts from a station and has to stop again at a red traffic light, or vice-versa, in order to save time and energy. To do this, it is possible to act on traffic lights or on the vehicle, with different methods:

- Traffic Signal Priority (TSP) → *"Traffic Signal Priority is a system that allows public transport, emergency vehicles, and other authorized vehicles to receive prioritized treatment at traffic signals."* [15].
  In this kind of systems the duration and the phase of the traffic light are modified, on request [1] or not, based on the position of the vehicle and, in case of a bus, depending on the delay accumulated over the line, in order to cross the traffic light when it is green without stopping. This method is mainly used to reduce the delay of the bus with respect to the schedule, especially during rush hours when there are many vehicles on the same line and it is important to avoid bus bunching. However, it allows also to reduce consumptions due to braking and restarting from a traffic light.
  A collateral effect of TSP is that a modification on the phases of the other traffic lights of the crossing is required, and also a change in the duration of the following phases of the same traffic light: this can create some problems of queues especially during rush hours [17].

- Eco-driving strategy based on GLOSA → these strategies act on the speed of the vehicle, in particular GLOSA (Green Light Optimal Speed Advisor) *"makes use of vehicle-to-infrastructure (V2I) communication in order to provide drivers with speed recommendations that are optimized to help them pass through a series of traffic lights without stopping."* [15]. Usually, eco-driving strategies based on GLOSA aim to reduce fuel consumption and emissions and improve the traffic flow, trying to avoid a series of acceleration and braking phases due to red traffic lights in favour of a more constant speed.

- Stop holding → it is a method that consists in keeping the bus at a stop when it is near a red traffic light until it becomes green, to avoid starting and stopping again [7]. This method is used also to regularize the headway, in case a bus is too close to the preceding one.
  A particular type of stop holding uses an algorithm called GLODTA (Green Light Optimal Dwell Time Advisory), which *"instructs the driver to prolong the dwell time at a stop so that the vehicle will arrive during green phase at the next signalized intersection"* [11]. GLODTA is typically based on information about phase and remaining time to the change of phase of the following traffic light [12], thus letting the vehicle dwell at the bus stop instead of the red traffic light.

All the techniques can be combined to achieve the best results [30], for example modifying phases duration or holding the bus at a stop when GLOSA suggestions are not enough to reach the traffic light when it is green.

1.1. GLOSA

According to [14], first studies about GLOSA were published in 2006, with a great increase in publications from 2011 on following EU directive regarding Cooperative-Intelligent Transportation Systems (C-ITS). These systems are fundamental to allow GLOSA: communication between vehicle and traffic lights is necessary to provide information about Signal Phase and Timing (SPaT), useful to adjust the speed in order to reach the traffic light when it is green and cross it without stopping.

GLOSA suggestions about the optimal speed can be given both to autonomous and manually driven vehicles, but to be able to receive those suggestions, vehicles must be connected to the infrastructure to send and receive data.

Some studies were conducted about the communication range that allows GLOSA to correctly work [19], while others explore the potentiality of 5G as a GLOSA enabling technology.

The work in [5] illustrates the requirements of 5G that make it suitable for vehicle-infrastructure communication, especially for safety applications:

- latency: low latency values, that is the time between the generation of a message and its receiving, are needed to allow real time communication. The two communication technologies proposed in the article, based on 5G, have a 10 ms end-to-end latency and 1 ms physical layer latency.

- reliability: loss rate of messages should be low to allow a highly reliable communication. 5G technologies mentioned in [5] have a reliability of 99.999%.
- message size: a bigger size of messages allows multicast and broadcast of more information.
- frequency: sending rate of messages, it is important mainly for safety applications. The technologies mentioned in the article have a band of 5.9 GHz (5.85–5.925 GHz).
- range: maximum distance for which the communication is possible, higher is better. This article does not mention any range for 5G communication, however the authors or [18] mention a GLOSA activation at around 250 m, while in [10] the activation distance is 500 m.
- speed: maximum relative and absolute speed of the vehicle under which the 5G reliability should be achieved.

GLOSA, if informative, is not a safety application of 5G communication, so many of the mentioned requirements can be relaxed.

1.1.1. Types of GLOSA

Many GLOSA were proposed with different algorithms and scenarios, they can be classified depending on some characteristics:

- multi-segment or single segment optimization: in order to reduce energy consumption, GLOSA algorithms can consider only the traffic light ahead, optimizing speed in a segment of the road. In alternative, GLOSA can optimize consumptions on the overall trip considering the entire set of traffic lights, if phases are fixed and the route is known. [11] considers only the traffic light in front of the bus, while [20] considers an optimization on the entire trajectory. GLOSA tested both on single segment and on multi-segments proves that generally best results can be obtained with multi-segment optimization [16], but the path should be known, which in real-life situations is much easier for public transport.
- deterministic or stochastic traffic light time-to-switch: remaining time to the change of phase can be exactly known or uncertain, depending if it is known in advance or it is available in real-time. [3] proposes a method to predict phase changes accurately enough to enable GLOSA, while the authors in [24] and [23] modify Dynamic Stochastic Programming in order to reduce computational costs to allow GLOSA predictions in real-time with stochastic switching times. [18] compares the same

GLOSA tested both with deterministic and stochastic traffic light switching times, showing that better results in term of fuel savings are obtained with deterministic switching times. Moreover, it is shown that the level of confidence of the switching probability highly influences the results in case of stochastic times.

- suggestion to driver based on speed or on distance: most of the proposed GLOSA suggest a target speed to the driver [2, 22], however the research work conducted in [21] proposes also a GLOSA that calculates a distance based on the current speed of the vehicle and on the remaining time to the end of the green and the red phases. In simulations, those distances are projected as red and green rectangles on the street in front of the vehicle, that can reach the traffic light during a green phase as long as it travels in the green rectangle.
- minimization of travel time, energy consumption (and CO_2 emissions) or other variables: usually GLOSA aims at minimize energy consumption, however the target variables to be minimized by the algorithms could be different and more than one. [20] takes as primary objective the reduction of travel time, but at the same time also energy consumption ends up to be reduced. [30] directly considers a multi-objective cost function that represents the operator objectives, together with a trajectory optimization model.
- traffic considered or not: GLOSA can consider the interactions with other vehicles or not, even if in case of application on real vehicles it is difficult to have no or little interactions, especially in urban areas. [4] takes into account the presence of other vehicles in front of the one equipped with the GLOSA system, showing that it is possible to achieve best results if the predicted time to reach the traffic light accounts for possible queues.

1.1.2. Examples of GLOSA algorithms

As far as algorithms are concerned, there are many different possibilities. In the following, the state of the art and the most interesting examples are reported.
The algorithm presented in [4] uses messages from traffic lights, infrastructure and collaborative vehicles to be able to estimate the level of traffic in a crossing. The model used to replicate the behaviour of the vehicle is an Intelligent Driver Model (IDM), so the vehicle behaves as the preceding one, modified to consider the stop in presence of a red traffic light. Two algorithms, rule-based and reinforcement learning based, are used to calculate a speed suitable to cross the traffic light without stopping, taking into account the presence of a preceding vehicle which reduces the maximum speed.

The work in [18] proposes a Stochastic Dynamic Programming approach to take into account the uncertainty of real-time traffic light information. The A* algorithm is used to find the optimal speed policy that minimizes the energy consumption. It considers a critical stopping distance to be sure not to cross the traffic light when it is red, decreasing the speed when this distance is overcome, and a buffer to avoid unnecessary braking due to red light. It uses an outer loop to choose the upstream policy and an inner loop to choose the downstream.

The authors in [24] and [23] propose two modified Stochastic Dynamic Programming in order to allow real-time use of GLOSA with stochastic information: Differential Dynamic Programming (DDP) proves to be the most suitable between the two proposed because it does not require discretization, bringing to a more accurate result, allowing faster calculations as well.

The work in [10], that is the starting point of this work, proposes a Multiple Traffic Light Advisory (MTLA) to find a non-optimal solution to cross multiple traffic lights and then refines it with Model Predictive Control (MPC) to find the optimal one. The non-optimal MTLA analyses four traffic lights in front of the vehicle and supposes a uniformly accelerated motion up to the first traffic light, then a constant speed motion for the next ones.

For the first traffic light, a uniformly accelerated motion is considered up to the traffic light position, so kinematic equations that link the target speed, the distance to the intersection and the time needed to reach it are written.

$$\begin{cases} d = \dfrac{1}{2}at^2 + vt \\ v_{target} = at + v \end{cases}$$

Then, depending on the phase colour, the maximum and minimum values of the target speed are calculated. If the phase is green, there are no constraints on the minimum time to reach the traffic light, so the maximum speed is set equal to road limit, and the minimum one corresponds to the velocity calculated supposing that the vehicle arrives at the traffic light in correspondence to the phase change. If the phase is red, maximum and minimum speed are calculated using the times of the ending of the red phase and the ending of the successive green phase.

When a traffic light different from the first one is analysed (called i^{th}), the kinematic equations include the ones of the accelerated motion to reach the first traffic light and the one that links the distance from the first to the i^{th} traffic light, supposing a constant speed motion.

$$\begin{cases} d_1 = \frac{1}{2}at_1^2 + vt_1 \\ v_{target} = at_1 + v \\ d_2 = v_{target}t_2 \end{cases}$$

Where d_1 is the distance of the vehicle from the first traffic light, t_1 the time needed to reach the first traffic light, v the current speed of the vehicle, d_2 the distance between the first and the i^{th} traffic light and t_2 the time needed to cover that distance at constant speed v_{target}.

Also in this case maximum and minimum target velocity values are calculated differently depending on the phase colour of the traffic light, with the same logic as before.

Previous to those calculations, some checks about localization of the vehicle and activation horizon of the algorithm are performed.

For each traffic light, the feasibility of intersection crossing is checked with respect to the first green phase, otherwise the second green phase is considered. The "Actual Velocity Check" and the "Acceleration/Deceleration Manoeuvrer Check" verify if the driver can get a green phase keeping the speed constant and, in case it is not possible, the safeness of the acceleration manoeuvrer is checked.

The non-optimal profile thus found is then used as reference for MPC, together with state constraints, to better take into account the dynamics of the vehicle and the comfort of driver and passengers.

The same logic is used for the MTLA proposed in [27], which accounts also for the impact of bus stops on the vehicle travel time.

1.1.3. GLOSA for buses

Differently from cars, buses are characterized by specific and predetermined routes and by the presence, on their path, of frequent and fixed-position stops. GLOSA applied to buses tries to take into account these specific characteristics in order to obtain a better performance and reduce energy consumption. Furthermore, this type of systems can be used to increase the adherence to the schedule [6], to avoid bus bunching [15] or to uniform the headway [30].

Often GLOSA applied to buses is evaluated with respect to or in combination with other control techniques, such as Transit Signal Priority (TSP) or holding [30], to obtain better results both in term of service and energy consumption.

The structure of a bus line makes it suitable to be considered in various way, for example it is possible to divide the line into segments between two stops and optimize the speed in each segment [8] or to consider a stop and a traffic light and optimize the speed before and after the stop [29].

In particular, in [29] a section of a path of an electric bus is divided in a first part before the stop (pre-station control section) and a second part between the stop and the traffic light (post-station control section). On the one hand, in the pre-station control section, the bus is assumed to keep a constant speed plus a variable deceleration concave curve, which allows to increase the amount of recovered energy. On the other hand, in the post-station control section the problem is formulated as a multi objective optimization problem, which considers energy consumption, travel time and passengers comfort. A convex profile with multiple accelerations is optimized by a NSGA-II algorithm, allowing a lower energy consumption at the restarting from a station with respect to constant acceleration mode. The bus is then supposed to cross the traffic light at constant speed.

Also [9] focus on electric buses: it aims at using GLOSA to extend the range of the vehicles, optimizing the energy required in the whole trip. The total energy consumption, which is the cost function to be minimized, considers also the impact of the amount of passengers on the total mass of the vehicle and so on the consumption. Constraints such as travel time and distance and maximum values for acceleration and speed are also considered. The problem formulation contains integral formulas, so it is recast as a non-linear problem and then solved as a mixed-integer linear problem through discretization into small trip segments.

[13] uses an infrastructure-enabled framework, so information are exchanged between vehicles and infrastructure and calculations for eco-driving approach are done on the infrastructure side and sent back to vehicles, thus reducing the computational effort on board the vehicles. It combines GLOSA with bus holding at stops, considering two different modules that allow to optimize the cruising speed in segments without stops or speed profile and holding time in segments with bus stops. The proposed method is first tested at segment level, considering two intersections and a bus stop, then at line level and at network level, considering two lines.

[8] estimates the energy consumption of an electric Bus Rapid Transit (BRT) accounting for the motor efficiency, then it develops a non-linear programming model to minimize the energy consumption and uses a sparrow search algorithm to find the solution. The BRT line is divided into cells, delimited by stations, and divided further by traffic lights. Considering that at each traffic light the bus can accelerate, brake, keep speed constant or stop, the force balance on the motor is formulated and used to find the optimal speed in each cell of the line.

Differently from the works above illustrated, the objective of the GLOSA algorithm proposed in this work is not directly the reduction of energy consumption or travel time, but the reduction of the number of stops at red traffic lights through a rule-based logic, which can bring a lower consumption or travel time as a consequence. The rule-based logic proposed allows to exploit the flexibility in the choice of the reference speed profile for the considered road segment.

In particular, the algorithm proposed considers a segment of a bus line composed of two traffic lights with a stop in between. Although not dealing with an actual optimization process, the focus of the thesis is to improve the rule-based control logic with a Finite State Machine while ensuring the real-time implementation based on Robotic Operating System (ROS). The suggested speed is the one that allows to reach the traffic lights as soon as possible during a green phase, following a speed profile tailored for a bus. The speed is calculated taking into account the remaining time to the change of phase and the possible speed profile that the bus could follow to cross the first traffic light, reach the stop and cross the second traffic light. The section is further divided in two parts, and so the speed profile: from the current vehicle location to the bus stop ahead, including the first upcoming traffic light, and from the stop to the second traffic light. This allows to easily consider all the possible scenarios which the bus could face, either being in front of a traffic light or of a stop.

2 | Experimental setup

This thesis work, besides conducting numerical assessments, had the opportunity to test the developed algorithm on a real bus, specifically equipped to provide an accurate localization in urban scenario and to receive traffic light messages and transmit algorithm indications to the driver.
The selected vehicle was an ATM trolleybus, shown in Figure 2.1, equipped with the following devices:

- Inertial Measurement Unit (IMU): it is placed under the chassis of the bus and it allows to measure and send to the algorithm the current values along the three inertial axes.
- Global Positioning System (GPS): it is placed in the upper part of the frontal area of the bus, in order to have an estimation of the bus position corresponding to its front end. The geographical coordinates measured by the GPS are then used to estimate the bus position thanks to a Kalman Filter developed in [25].
- Electronic Control Unit (ECU): it provides the velocity of the vehicle, obtained directly from the motor rotation speed.

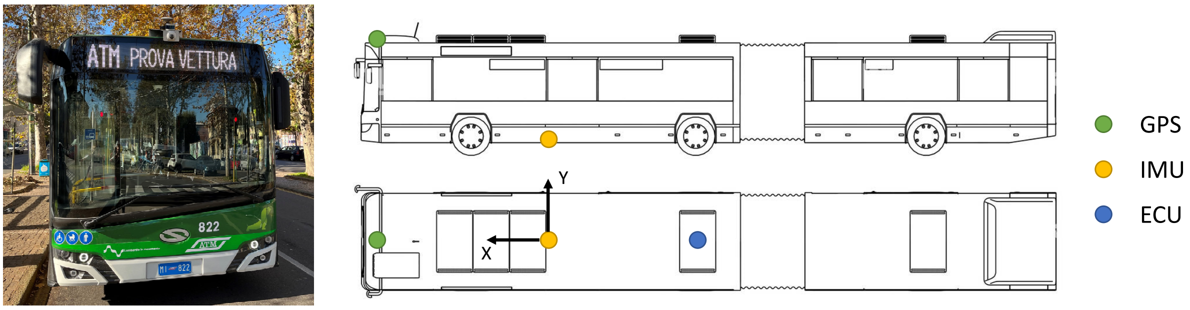


Figure 2.1: Trolleybus used for tests

- 5G modem: it was added specifically on this vehicle to perform the tests, since it allows to receive traffic light Signal Phase and Timing (SPaT) messages using 5G technology. The communication scheme is shown in Figure 2.2: all the traffic lights on the path are communicating through a Vodafone cloud platform information about their timings (SPaT messages) and the intersection geometry (MAP messages).

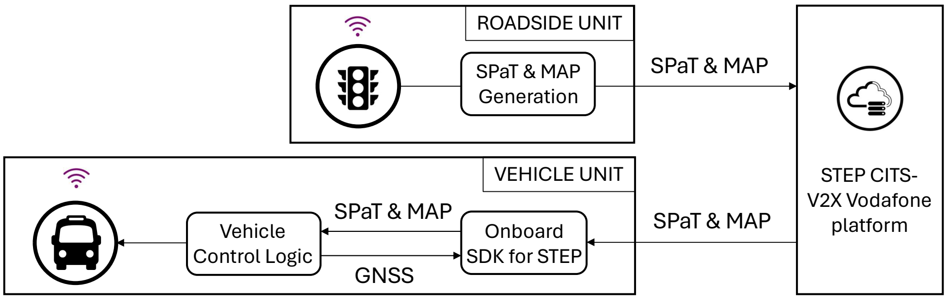


Figure 2.2: Communication scheme between trolleybus and traffic lights

Both messages are defined in a standard and each message is always associated with the traffic light ID that is sending it. In particular, SPaT messages allow traffic lights to send to the vehicle information about their current traffic light plan, as a sequence of phases and the remaining time to each phase change. An example is shown in Figure 2.3: the traffic light plant of a crossing sends information about the traffic light plan set by the Municipality of Milan, then messages are filtered in order to receive only the ones related to the traffic light on the test path (line 7 in the Figure), that are used to perform GLOSA calculations.
Further details about traffic lights communication are presented in [26].

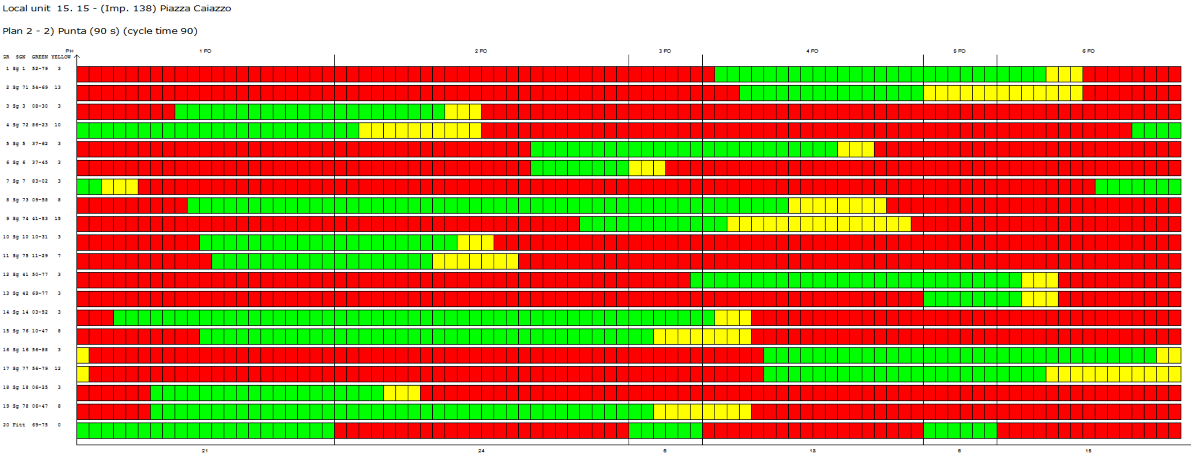


Figure 2.3: Example of traffic light plan

- on-board computer: the sensor acquisition, data processing and GLOSA algorithm all run on an onboard computational unit (Intel NUC Core i7 1165G7), operating on a soft real-time-based architecture using Robotic Operating System (ROS).

- graphical interface: the most recent buses are equipped with a little display placed on the dashboard that gives some information to the driver, as an addition to the ones given by more traditional instruments. During tests, this display was used also to give to the driver information about traffic light phase and timing and algorithm suggestions.

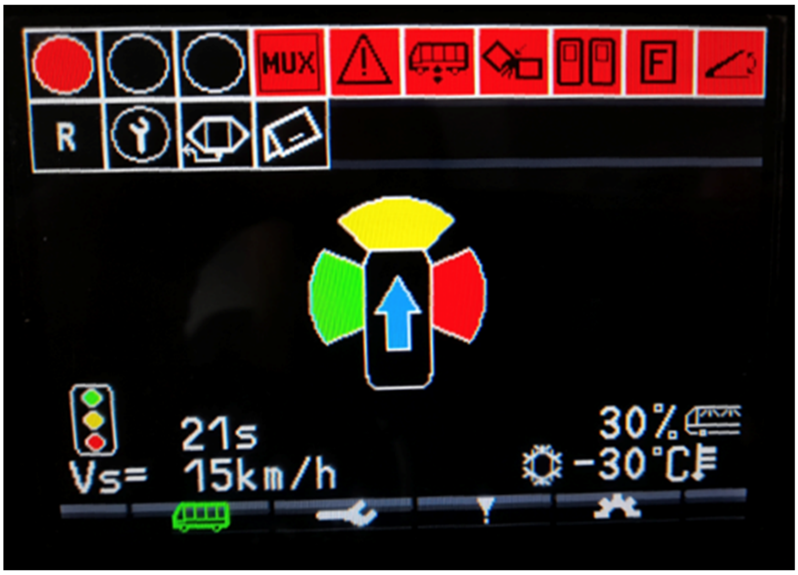


Figure 2.4: Graphical interface of the trolleybus for GLOSA suggestions

As shown in Figure 2.4, the current phase of the traffic light is displayed in the top-left angle, while in the bottom-left angle there is the remaining time to the change of the current phase. Under the remaining time indication, there is the suggested speed coming from algorithm calculations, while the arrow in the centre of the screen suggests a behaviour to the driver based on the algorithm advices: a green arrow pointing up suggests to accelerate, a blue arrow to keep the speed constant, while an orange arrow pointing down suggests to smoothly brake. Lastly, a red arrow pointing down suggests a stronger brake.

As far as the test path is concerned, it is shown in Figure 2.5. It is a part of ATM lines 90-91, from Piola to Lugano and vice-versa, for a total distance of about 5 km including 30 connected intersections and 16 stops per direction.

(a)

(b)

Figure 2.5: Test path with traffic lights (a) and stops (b)

Traffic light positions, corresponding to the stop lines, and stop positions are obtained from the map of the road and reported in Table 2.1, starting from the stop of Piola.

Traffic lights			Stops		
ID	Number	Position (m)	Name	Number	Position (m)
145	1	45.6	Piola M2	1	67.8
220	2	203.2			
219	3	384.226	Via Garofalo	2	434.8
144	4	589.410	Loreto M1 M2	3	717.9
143	5	773.596			
142	6	1004.161	Viale Andrea Doria	4	1073.2
138	7	1225.290	Caiazzo M2	5	1287.1
136	8	1419.667	Piazza Luigi di Savoia	6	1492.3
136	9	1507.799			
134	10	1739.025	Stazione Centrale	7	1803.5

134	11	1822.969			
132	12	1948.853			
132	13	2046.587	Sondrio M3	8	2130.6
148	14	2287.889	Viale Sauro	9	2402.0
148	15	2421.489			
756	16	2635.800			
756	17	2683.772	Zara M3 M5	10	2764.5
757	18	2773.864			
757	19	2800.176			
757	20	2828.378	Via Lario	11	2969.2
758	21	2981.396			
759	22	3232.562	Via Farini	12	3417.5
760	23	3420.575			
761	24	3600.386	Via Bernina	13	3750.8
762	25	3764.860			
820	26	4051.818			
774	27	4215.626	Viale Jenner	14	4280.5
888	28	4434.628	Piazzale Nigra	15	4696.2
775	29	4701.175	Piazzale Lugano	16	5100.8
777	30	5107.819			

Table 2.1: Position of traffic lights and stops on the line from Piola to Lugano

The considered bus line is mainly developed in a lane reserved to public transport (shown in Figure 2.6 from the point of view of the driver), however the access to the lane is allowed also to other vehicles with permission (for example taxis), that sometimes make it a little bit congested, especially in rush hours.

The presence of traffic and of variable phases durations made sometimes difficult the exploiting of algorithm advantages in crossing intersections without stopping, so only some parts of the entire path were then considered during result analysis.

In Figure 2.6 it is possible to see the testing lane and how the driver can look at the graphical interface shown in Figure 2.4.

Figure 2.6: Trolleybus lane during the testing campaign

3 | GLOSA algorithm

This algorithm aims at using traffic light data about phase colour and timing (SPaT messages) to calculate and suggest to the bus driver a speed to follow in order to cross the traffic light during the green phase, without stopping. The aim is to reduce the number of stops at red traffic lights, which can have as a consequence the reduction of energy consumption due to stopping and restarting or of travel time.

The algorithm is based on a series of kinematic equations that try to forecast the speed profile followed by the vehicle to reach and cross an intersection, and consequently suggest a speed that allows it to avoid stopping at the traffic light.

The algorithm considers in particular the situation of a bus, which is characterised by the presence of stops on its path. For the calculation of the suggested speed, two consecutive traffic lights with a stop in between are considered: the speed profile is thus divided in a first part, from the current position of the bus to the first stop, and a second part from the stop to the second traffic light. The two parts are calculated independently and are linked by the time required to reach the stop, where the bus has known position and speed. For each part, there are different speed profiles, chosen on the basis of some conditions.

3.1. Hypotheses

The proposed algorithm relies on the following assumptions:

- the vehicle is assumed to be a point mass, only its frontal area is considered in calculating the air resistance to evaluate the energy consumption, since the speed is low and the vehicle dynamics is not taken into account
- the path is described in a 1D curvilinear abscissa reference frame, the path is quite straight and turns have a low impact due to the low speed
- the road is assumed to be flat, being the slop of the path described in Chapter 2 not so relevant
- as far as the traffic light position used during the calculation is concerned, the stop line is considered, as it is the limit point for the vehicle when traffic light is red

- the yellow phase is included in the red phase, to keep some extra margin on the safety side
- no traffic or any interaction with other vehicles is considered, the vehicle is assumed to have no traffic ahead. It is worth highlighting that, being most of the path covered in the reserved lane, the assumption is not affecting too much the vehicle behaviour.
- in most cases, the deceleration is assumed constant and equal to $-1m/s^2$, while the maximum acceleration is assumed equal to $1m/s^2$, being values that, from experimental data, allow smooth acceleration/deceleration

3.2. Speed profile definition

3.2.1. Reaching the bus stop

As mentioned, the first part of the speed profile is the one from the current vehicle position to the upcoming bus stop eventually cruising through an intersection during a green phase. Figure 3.1 reports the set of possible speed profiles the algorithm can pick depending on the vehicle location and its distance to the upcoming traffic light: the bus can cross the intersection while it is braking to reach the stop (profiles from 1 to 3), before the braking phase (profiles from 4 to 7) or exactly during a change of acceleration (profile 8).

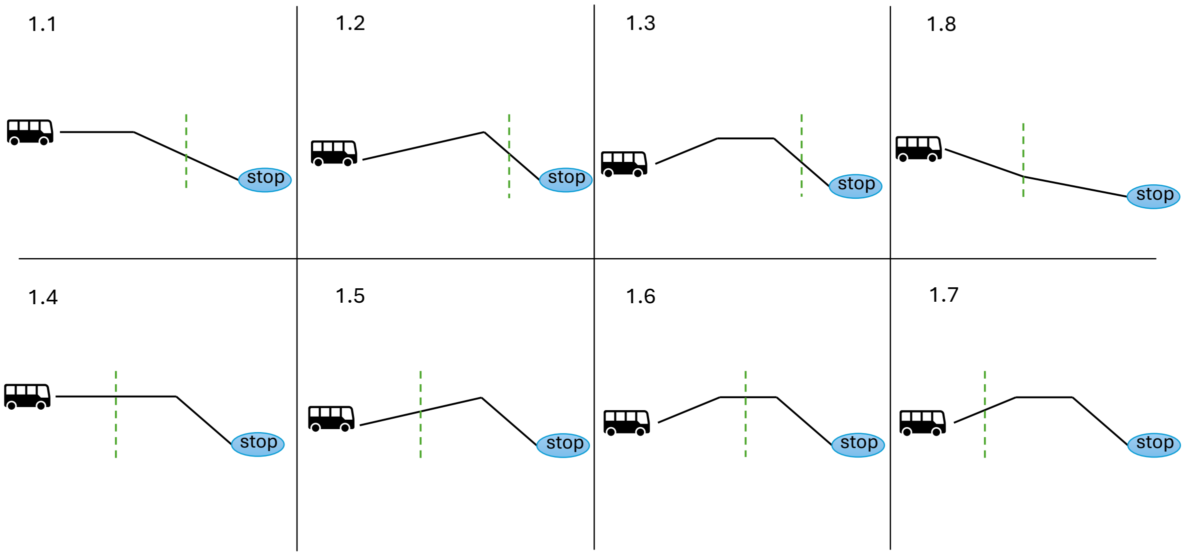


Figure 3.1: Speed profiles before the stop including the first traffic light

Since the actual braking distance is not a-priori known, as it depends on the actual speed the vehicle will keep, the expected braking distance is computed assuming to maintain the current speed of the vehicle. As a result, the braking distance turns out to be equal to:

$$dist_{br} = -\frac{v_0^2}{2a_b}$$

where v_0 is the actual speed of the vehicle and $a_b = -1m/s^2$ is the braking deceleration. So, if the distance between the first traffic light and the following stop is smaller than the braking distance, the bus is expected to cross the traffic light while braking, otherwise the vehicle will start to slow down after the intersection is passed.
After the first division, other conditions are considered to choose the speed profile.

Considering that the intersection is crossed during the braking phase, there are four options about the speed profiles:

- profile 1.1: the bus proceeds at constant speed from the current position and then start braking before crossing the intersection. Since there is no variation of speed until the bus starts braking, in this profile the braking deceleration that allows the bus to reach the traffic light during a green phase is not equal to a_b, but it is calculated.
- profile 1.2: the bus accelerates (or decelerates) until it needs to brake to reach the stop. The speed is modified before crossing the intersection, so the braking phase has fixed braking deceleration a_b.
- profile 1.3: the bus accelerates until the speed limit is reached. Since it is the maximum speed allowed in a urban road scenario, the bus keeps its speed constant and equal to the speed limit, then brakes for the stop. The speed is modified in order to cross the intersection before reaching the maximum speed, so the braking phase is characterized by fixed braking deceleration a_b.
- profile 1.8: this profile considers the case in which the bus needs to decelerate to reach the traffic light during a green phase, but the fixed braking deceleration a_b of the braking phase is too high to reach the stop. So two different negative accelerations are calculated, the first one to reach the traffic light and the second one from the traffic light to the stop.

If the braking phase begins after the bus has crossed the intersection, it is always at fixed deceleration, because the speed needs to be modified before reaching the traffic light and so before the braking phase:

- profile 1.4: the bus crosses the intersection keeping the initial speed constant, than brakes to reach the traffic light with deceleration a_b. Since there is not a speed to be modified in order to reach the traffic light during a green phase, profile 1.4 is included in profile 1.5, corresponding to the case of a null acceleration resulting from calculations.
- profile 1.5: the bus accelerates (or decelerates) while crossing the traffic light, than brakes to reach the stop with deceleration a_b.
- profile 1.6: the bus accelerates and reaches the speed limit, crossing the intersection at constant speed equal to the maximum admissible one. The speed is then kept constant until the bus brakes with deceleration a_b.
- profile 1.7: the bus accelerates and reaches the speed limit, but the intersection is crossed while it is still accelerating. After passing the traffic light, the bus reaches the maximum speed and keeps it constant until it brakes with deceleration a_b.

Among the different speed profiles, it is important to consider also the case in which the bus has to stop at a red traffic light or in which there is a stop before any traffic light. It allows to adjust the speed according to the calculations of the second part, between the stop and the following traffic light, in order to cross that intersection during a green phase.

In this kind of profile the bus is supposed to keep its speed constant, or to accelerate/decelerate according to calculations, until braking distance is reached. A modified version of profile 1.5, i.e., without the presence of traffic light, may fit the requirements. The maximum speed calculated in the profile is always checked with respect to the speed limit value, to avoid that this limit is overtaken.

In the described case, the second part of the speed profile is considered only if the bus has a stop before any traffic light, or if it has to brake at a red traffic light and there is no stop between the first and the second traffic light.

3.2.2. Restart from the bus stop

Profiles of the second part are those that a bus can undertake when leaving from the stop, to reach the following traffic light. They are similar to the ones of the first part, but less elements are considered since speed is suggested just up to the traffic light. In fact, when the bus has passed the stop, that traffic light becomes the first traffic light and a new profile is considered.

In Figure 3.2 it is possible to look at the speed profile options for this part of the trajectory prediction.

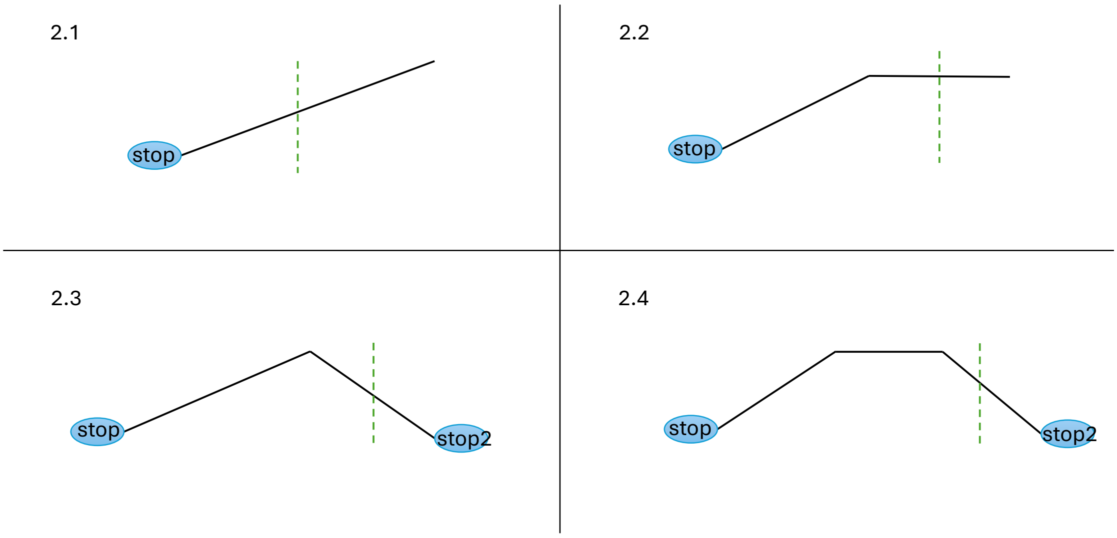


Figure 3.2: Speed profiles after the stop including the second traffic light

Also in this section, the first choice for the profile depends on the fact that the traffic light is crossed during the braking phase or not. To evaluate this, the distance between the second traffic light and the following stop is needed, and the braking distance is calculated again considering the current speed, since the actual speed at the traffic light is not yet known.
If there is no stop after the traffic light or if the braking distance is smaller than the one between the traffic light and the following stop, there is no braking phase:

- profile 2.1: the bus accelerates until the traffic light is reached.
- profile 2.2: the bus accelerates until the speed limit is reached and then cross the intersection with that constant speed.

Instead, if there is a second stop immediately after the second traffic light, the bus is supposed to be in the braking phase while crossing the intersection:

- profile 2.3: the bus accelerates and then brakes with fixed braking deceleration a_b.
- profile 2.4: the bus accelerates, reaches the maximum speed, keeps it constant for a while and then crosses the intersection while braking.

In Figure 3.3 it is shown a complete speed profile, from the current position of the vehicle to the second traffic light ahead, including the stop.

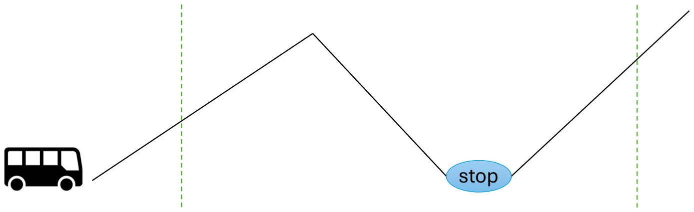


Figure 3.3: Complete speed profile

3.2.3. Additional profiles definition

The model is mainly based on the structure first traffic light-stop-second traffic light, but there could also be the situation in which there are two following traffic lights, without any stop in between. To take into account also this scenario, a speed profile with constant acceleration is considered, looking for the one that allows to cross both the traffic lights during a green phase (see Figure 3.4). A check on the speed with respect to the limit value is always performed, and acceleration is modified in order to keep the calculated speed values under the limit.

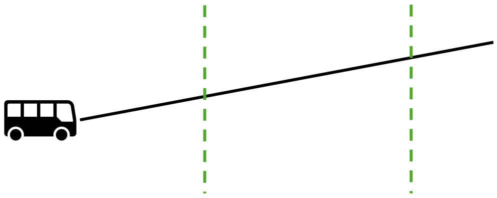


Figure 3.4: Speed profile without stops

3.3. Finite state machine

The algorithm is based on the choice of a speed profile and subsequent calculations to determine the acceleration and the speed that the driver should follow. In particular, the output to the driver is just the speed value, that is calculated integrating the acceleration previously calculated.

An easy way to select and calculate the speed profile at each time step, while the vehicle is running, is through a finite state machine implemented in Stateflow.

A finite state machine is a method of modelling that make the system stay in a state with certain characteristics until an enabling condition for the exit is verified. In Simulink it is called Stateflow and it is possible to make calculations, assign values to variables and call functions in each state. States can be executed one at a time or in parallel, and transition conditions allow the system to go from one state to another.
An example is shown in Figure 3.5.

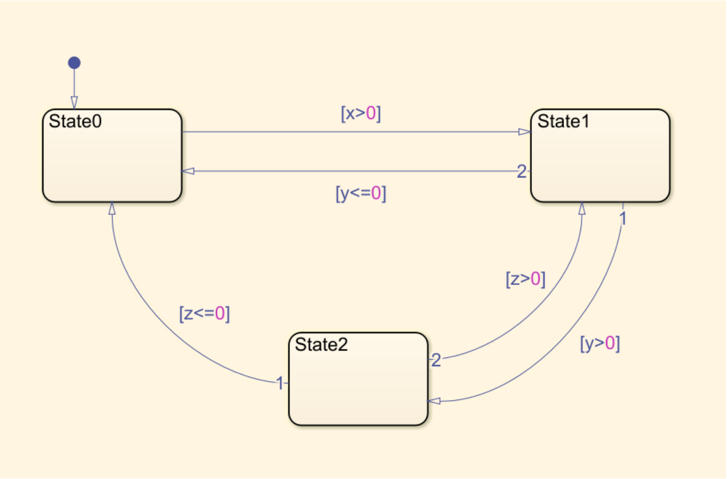


Figure 3.5: Example of a finite state machine

The algorithm is structured as a finite state machine in order to exploit the advantages of separating the speed profiles, both in term of visualization and in term of calculations. The structure made by states allow to clearly define the conditions to change speed profile and the ones for which the system should enter each state, so checking the corresponding speed profile.

Calculations of the algorithm are repeated each time step at a frequency of 10 Hz. To avoid losing time due to the choice of the profile with all the process that will be explained in subsection 3.3.2, the system stays in a state until conditions to stay in that state are verified.

3.3.1. States definition

Each speed profile of the first part has a corresponding state, in which the system remains until conditions change. The only exception is the speed profile followed when the bus has

to stop in correspondence of either a bus stop or a red traffic light: these two situations have two different states, even if the speed profile is the same, because conditions to enter and exit the states are different.
For each profile of the first part there is a function for the calculation of acceleration and speed based on kinematics, while speed profiles of the second part are all contained in an additional function, that allows to choose and calculate the most suitable speed profile. This function is called in each state of the first part. The states corresponding to speed profiles are:

- profile 1.1
- profile 1.2
- profile 1.3
- profile 1.5
- profile 1.6
- profile 1.7
- profile 1.8
- profile without stops
- profile of braking at a red traffic light
- profile of braking at a stop

In addition to the states corresponding to speed profiles, there are other states not strictly linked to the optimization of the speed profile, but that are needed to simulate correctly the behaviour of the bus, mainly during simulations.

- initial state: it is needed in the finite state machine and allows to do a first choice of the speed profile and to define the initial values of some variables. This state is used at the beginning of a simulation and after every restart from a stop or a traffic light.
- stop: it keeps the vehicle at null speed when it is at a stop or at a red traffic light, until the time set for a stop is passed (chosen as $10s$ from a previous analysis about the mean duration of stops on the line [28]) or the light switches to green. In a real application on a vehicle, this state works well only if the bus is close enough to the expected position of the stop or the traffic light.

- restart: it manages the restart suggesting an acceleration of $1m/s^2$ until the vehicle reaches a minimum speed of $1.5m/s$, before evaluating the possible speed profiles. The acceleration value of $1m/s^2$ is taken from real data gathered on the vehicle during the service, finding that it allows smooth accelerations and decelerations for passengers comfort and safety. The speed value of $1.5m/s$ is chosen in order to be small enough to be reached soon and so not to influence too much the profile evaluation, but big enough to avoid the bus to go too slowly if the selected speed profile suggests to keep the speed constant. A higher value would make the evaluation of speed profiles start later, while a lower value would make the vehicle too slow in case of suggestion of a constant speed. This state is necessary to avoid that the vehicle has a speed close to 0 if a speed profile suggests to keep speed constant.

  Stop and restart states are fundamental to correctly replicate the bus behaviour in a simulation, while could be avoided in a real application, because the driver is able to adequately keep the vehicle idling at a red traffic light or at a stop and to manage the restart until a speed is suggested.

- traffic light crossing: it guides the bus while crossing the intersection. When the vehicle is close to a traffic light and does not need to stop, the evaluation of the most suitable speed profile is difficult, because profiles are supposed from a certain distance from the traffic light. Furthermore, while crossing, there is a change in the position of the first traffic light in front of the vehicle that could bring problems with mathematical calculations. To avoid the problem, when the vehicle is close enough to the traffic light, the light is green and it is established that the vehicle is able to pass with the green, it is suggested to go at constant speed until the traffic light is crossed.

- not-connected traffic light: this state is necessary only in simulations, since it allows the bus to proceed at constant speed if the first traffic light ahead of the bus is not connected, so there is no information to evaluate the proper speed. The bus exits from this state as soon as new information arrives, and in a real case application it is useless because the driver is able to drive looking at the colour of the traffic light. However, it makes the algorithm robust with respect to the absence of traffic light data, which sometimes are not transmitted correctly.

A simplified representation of the described finite state machine is shown in Figure 3.6, where the states corresponding to speed profiles are included in the speed profile definition state.

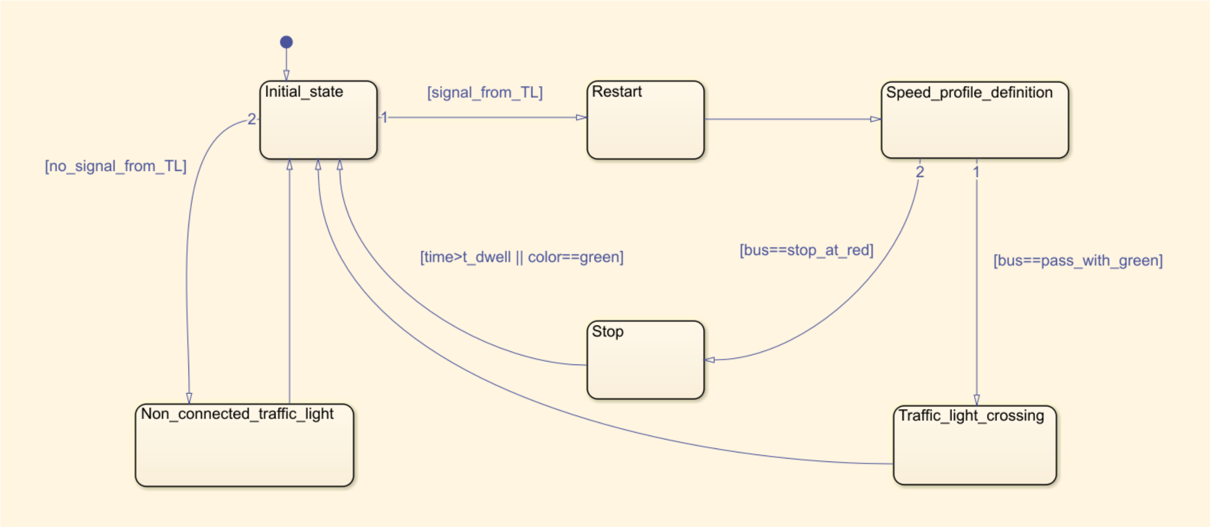


Figure 3.6: Finite state machine simplified representation

3.3.2. Choice of the profile

The choice of a speed profile consists in selecting a set of profiles based on preliminary calculations and evaluating them successively until the proper one is found, or until the only possibility is to stop at the traffic light.

In Figures 3.7 and 3.8 are reported the flowcharts explaining the process followed by the algorithm to select the proper speed profile before the stop.

All the variables mentioned in those schemes representing the speed profile choice are explained in Table 3.1.

symbol	description
a_{max}	maximum acceleration considered
v_{max}	maximum speed allowed
a_b	constant braking deceleration
S_0	initial position for the speed profile
v_0	initial speed in the speed profile
S_{sf1}	position of the first traffic light
v_{sf1}	speed at which the bus crosses the traffic light
S_{stop1}	position of the first stop
v_{stop1}	speed of the vehicle at the first stop
S_{sf2}	position of the second traffic light
t_1	time needed to cover the first trait of a speed profile

symbol	description
t_2	time needed to cover the first trait of a speed profile
v_{br}	speed at which the vehicle starts braking in a speed profile
a	acceleration in the first trait of a speed profile
a_{brake}	deceleration in the braking phase, when different from a_b
$dist_{br}$	braking distance

Table 3.1: Variables in the speed profile choice

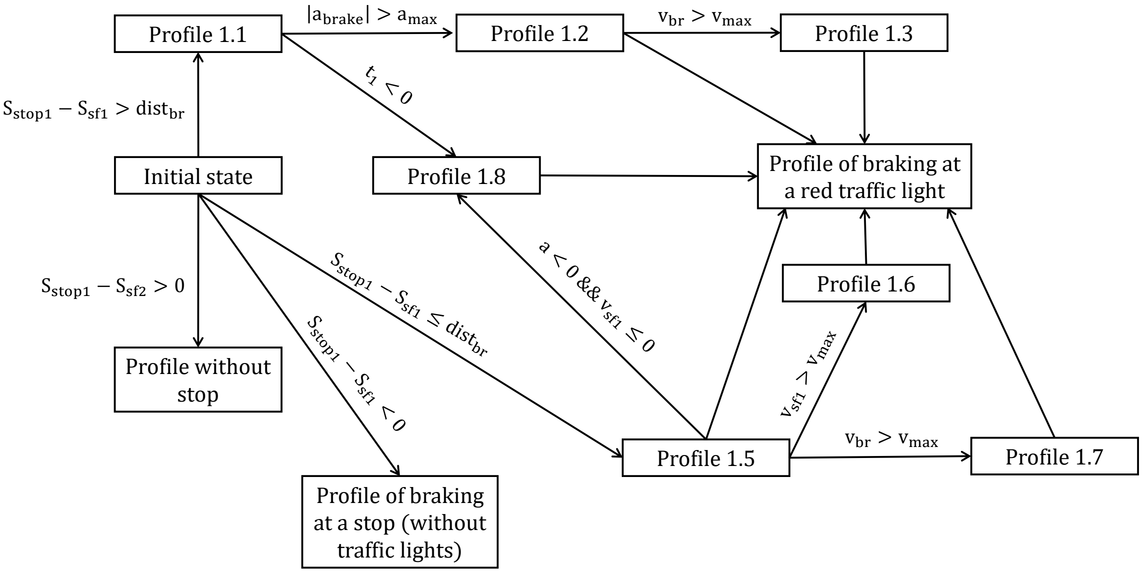


Figure 3.7: Scheme of the finite state machine for the choice of the speed profile

The first decision is about the position of the first traffic light with respect to the first stop ahead of the vehicle, so the difference $S_{stop1} - S_{sf1}$ is evaluated, as it is shown in Figure 3.8.

Then also $S_{stop1} - S_{sf2}$ is evaluated, to understand if the bus will cross two intersections in succession or a traffic light followed by a stop.

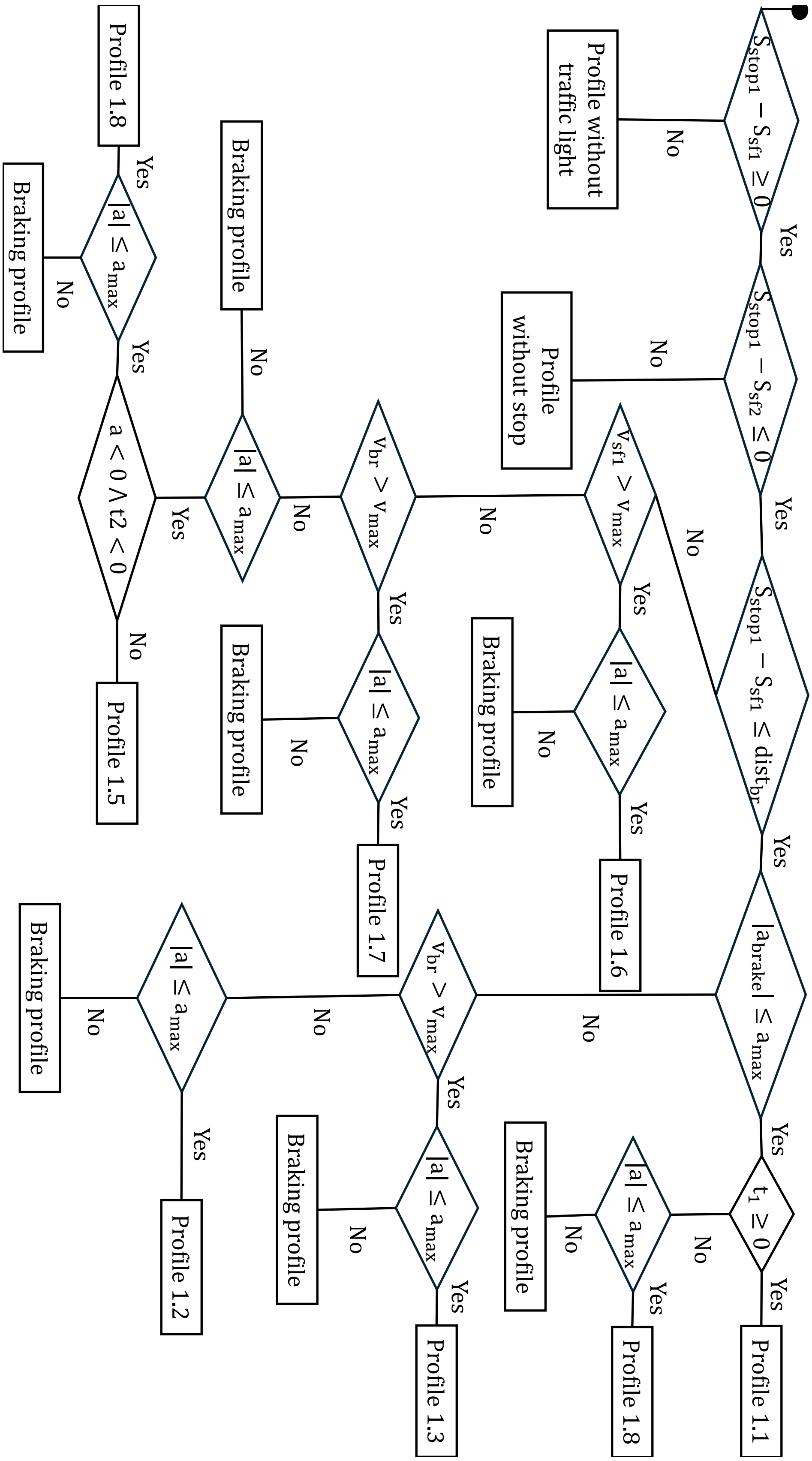


Figure 3.8: Decision process for the first part of the speed profile

As told in Section 3.2.1, the first distinction of speed profiles in two groups is about the position of the traffic light in the braking phase or not, so the braking distance is calculated as:

$$dist_{br} = -\frac{v_0^2}{2a_b}$$

where v_0 the current speed and a_b the braking deceleration, and it is compared with the distance between the stop and the first traffic light.
After these preliminary calculations, done in the initial state, the system enters a different state, which depends on the conditions shown in Figure 3.7.

In each profile some variables related to the speed profile are calculated, such as the acceleration, the maximum speed or the time needed to cover each part of the profile. All the variables linked to the profile (reported in Table 3.2) are then evaluated to understand if the profile is suitable to cross the traffic light without stopping: if every value is within the imposed limits, the bus can follow the suggestions of speed and acceleration of that profile and proceed to evaluate the second part of the profile, otherwise the system exits from the state.

If the system enters the state of profile 1.1, the braking acceleration a_{brake} and the time needed to start the braking phase t_1 are evaluated: depending on their values it is possible to stay in that state or to go to another state and consider a different profile, for example it is possible to go in the state corresponding to profile 1.2.
Here, the maximum speed (if a is positive) of the profile v_{br} is compared to the speed limit, and, if it is higher, profile 1.3 is checked.

In profile 1.5 both the speed at the traffic light v_{sf1} and the speed at the start of the braking phase v_{br}, the maximum speed reached in the profile if a is positive, are compared to the speed limit: profile 1.6 and profile 1.7 are then checked respectively.
If in profile 1.5 the value of a is negative and v_{br} is so low that the fixed braking deceleration a_b of the braking phase is too high to reach the stop without stopping before, profile 8 is evaluated.

To choose the profile of the second part, the process is similar to the one of the first part, but easier because there are less profiles, as shown in Figure 3.9.

Supposed that the second traffic light is after the first stop, so $S_{stop1} - S_{sf2}$ is negative, the distance between the second traffic light and the second stop, if there is any stop after the second traffic light, is evaluated and compared with the braking distance $dist_{br}$.

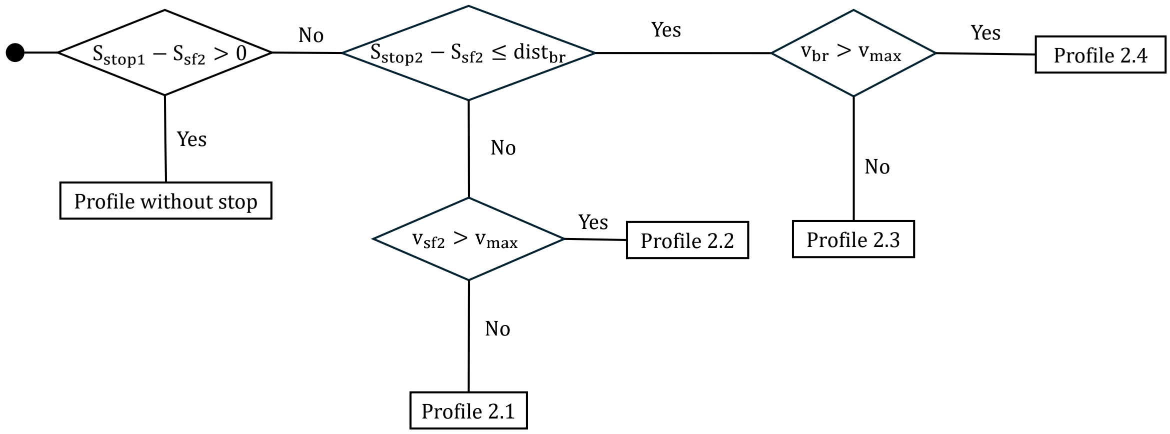


Figure 3.9: Decision process for the second part of the speed profile

Both in profile 2.1 and in profile 2.3 the maximum speed value, corresponding to v_{sf2} in profile 2.1 and 2.2 and to v_{br} in profiles 2.3 and 2.4, is calculated and compared with the speed limit.
Also the acceleration value a_2 is calculated: if it is within the limits, the profile is chosen and used to calculate again speed and acceleration of the first part, otherwise no profile for the second part is chosen.

3.4. Calculation of a speed profile

For each speed profile, the calculation of the suggested acceleration and speed to arrive at the traffic light when the light is green starts with the equations of motion of the different parts of the profile, which then are solved to find all the unknown variables.
For each part of the speed profile characterised by constant acceleration, a couple of kinematic equations is written, which express position and speed as a function of time:

$$\begin{cases} S = \frac{1}{2}at^2 + v_0 t + S_0, \\ v = at + v_0, \end{cases}$$

To better explain the process of calculation, profile 1.5 is considered as an example, since it is a general and frequent scenario. All the other equations are reported in Appendix A.

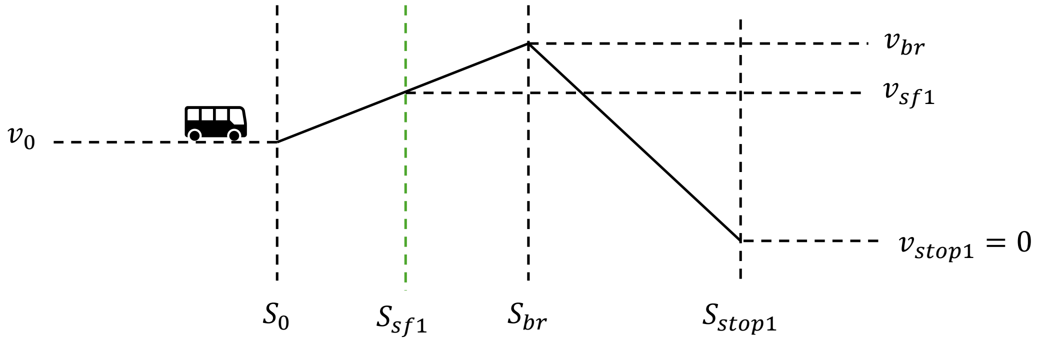


Figure 3.10: Speed profile 1.5, as a function of position

The profile, shown in Figure 3.10, is composed of three different parts, for each of them the equations of motion for position and speed are written, together with equations that link the time needed to cover each trait with the time needed to reach the traffic light, called t_{sf1}, and the time needed to reach the stop, called t_{stop1}.
So the system of equations to be solved for profile 1.5 is:

$$\begin{cases} S_{sf1} = \frac{1}{2}at_1^2 + v_0t_1 + S_0, \\ v_{sf1} = at_1 + v_0, \\ S_{br} = \frac{1}{2}at_2^2 + v_{sf1}t_2 + S_{sf1}, \\ v_{br} = at_2 + v_{sf1}, \\ S_{stop1} = \frac{1}{2}at_3^2 + v_{br}t_3 + S_{br}, \\ v_{stop1} = 0 = a_bt_3 + v_{br}, \\ t_{sf1} = t_1, \\ t_{stop1} = t_1 + t_2 + t_3. \end{cases} \tag{3.1}$$

Known: S_0,S_{sf1},S_{stop1},v_0,a_b,t_{sf1}
Unknown: a,t_1,t_2,t_3,t_{stop1},S_{br},v_{br},v_{sf1}

It is worth mentioning that, despite the speed profile reported in Figure 3.10 shows a positive initial acceleration, the profile is meant to be general for both positive and negative acceleration sign.

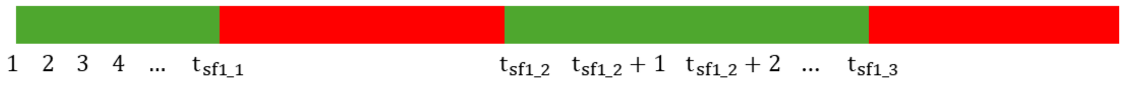
The time to reach the traffic light t_{sf1} is known because it is based on the remaining time before the phase change and it is the starting point for all the calculations. Traffic light timings are obtained directly from the SPaT messages received by the vehicle as described in Chapter 2

Since it is possible and also preferable that the bus crosses the traffic light before the end of the green phase and not exactly when it ends, a vector called $t_{sf1-vet}$ is created with one second time discretization of the green phases, depending on the current phase of the traffic light.

If the current phase is green, the first element of the vector is $t = 1s$, and the following are created with a step of 1 s until the value of the remaining green phase time, called t_{sf1_1} in Figure 3.11, is reached. Then no elements are placed between the time corresponding to the ending of the green phase (t_{sf1_1}) and the beginning of the next green phase (t_{sf1_2}). From t_{sf1_2} on, the creation of vector elements continue with a time step of 1 s up to the beginning of the next red phase (t_{sf1_3}). This process is repeated to include all the three green phases.

This vector is created in order to consider different possible times at which the bus can reach the intersection and to choose the shortest time possible according to admissible values of speed and acceleration.

Figure 3.11a shows an example of the creation of the vector starting from a green phase. If the current phase of the traffic light is red, the process to create the vector is the same, but starting from the time remaining to the end of the red phase. A margin of 2 s is added to this time, to be sure that the bus crosses the traffic light when it is green and to give more confidence to the driver while approaching a red traffic light. Figure 3.11b shows an example of creation of the vector starting from a red phase, the same process is repeated for all the three green phases.



(a)

(b)

Figure 3.11: Vector of t_{sf1} starting from a green and a red phase

After the creation of the vector, the calculation of the variables starts solving the system, in order to find all the unknowns, using as a first attempt the first element of the vector $t_{sf1-vet}$. Once variables have been computed, they are checked: if any of t_1,t_2,t_3,v_{sf1},v_{br} or of square roots contained in mathematical calculations is negative or if a is higher than a_{max}, the calculations are repeated with the next element of $t_{sf1-vet}$ until all the conditions are verified or until the end of the vector is reached.
If the end of the vector is reached and no speed is found that allows to avoid stopping, the system exits from the state, otherwise the second part of the speed profile is considered, if there is a second traffic light on the path.

About the second part of the profile, the choice process is similar to the first part. As an example, profile 2.1 is considered, meaning that there are no stops or the stop is far from the second traffic light.

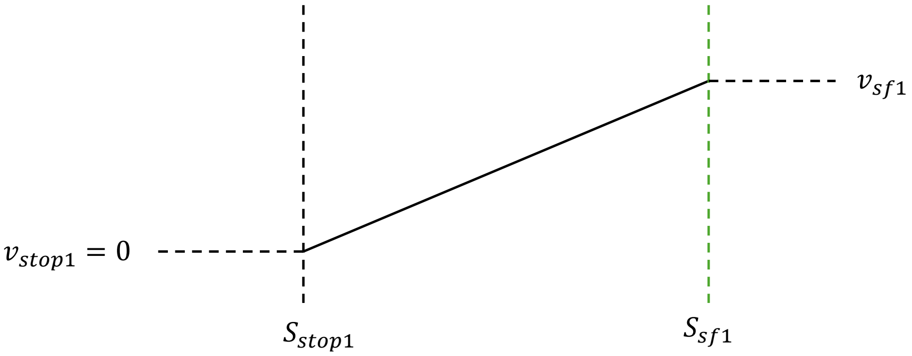


Figure 3.12: Speed profile 2.1, as a function of position

Differently from profiles of the first part, the ones of the second part are simpler, also as far as systems of equations are concerned. In particular, profile 2.1 is composed by a single trait at constant acceleration, from the stop to the second traffic light position. First and second part of a speed profile are linked by the stop, being position, speed and time to reach t_{stop1} known. The set of equation turns out to be:

$$\begin{cases} S_{sf2} = \frac{1}{2}a_2t_1^2 + S_{stop1} \\ v_{sf2} = a_2t_1, \\ t_{sf2} = t_{stop1} + t_{dwell} + t_1. \end{cases} \tag{3.2}$$

Known: S_{sf2},S_{stop1},t_{sf2},t_{stop1},t_{dwell}
Unknown: a_2,t_1,v_{sf2}

Where t_{dwell} is the time spent at the stop by the bus, for simplicity it is considered fixed and equal to $10s$, but it can also be variated based on statistical data. The variable t_{sf2} is the time to reach the second traffic light and it is taken from a vector $t_{sf2-vet}$ created as the one for the first traffic light, as illustrated in Figure 3.11.
At a first attempt, t_{stop1} is taken from calculations of the first part.
As in the first part, unknown variables are calculated taking the first element of the vector $t_{sf2-vet}$ first. If any of the unknowns is negative, calculations are repeated increasing t_{sf2} until all the variables are positive or the end on $t_{sf2-vet}$ is reached.

If a_2 is higher than a_{max}, further calculations are done: a_2 is set equal to a_{max} and t_{stop1} is calculated as unknown. Since there is a minimum time needed to reach the stop, corresponding to a profile with maximum acceleration in the first part, t_{sf2} is increased until t_{stop1} is above the limit or the end of the vector is reached.
If the calculated t_{stop1} is above the limit, it is used to calculate again the first part of the profile, to check if it is possible to cross the traffic light when it is green with the time needed to reach the stop suggested by the second part of the speed profile. Equation 3.1 is used, but with different known and unknown variables.

Known: S_0,S_{sf1},S_{stop1},v_0,a_b,t_{stop1}
Unknown: a,t_1,t_2,t_3,S_{br},v_{br},v_{sf1},t_{sf1}

So, t_{sf1} is now calculated starting from t_{stop1} and it is compared to the times that limit the duration of the green phases of the first traffic light: if the time needed by the bus to reach the traffic light is between the end of a red phase and the end of a green phase, the newly calculated speed and acceleration are suggested to the driver, otherwise the new calculations are no more considered and the speed and acceleration suggested are the ones computed before considering the second part.
The same process of calculation, shown in Figure 3.13, is followed for any profile of the first part. A similar process is used also when the bus has to stop at a stop before crossing any traffic light: in this case, the first calculations are not necessary, since there is no traffic light between the bus and the stop, and the calculations for the second part are done directly setting a_2 equal to a_{max}.

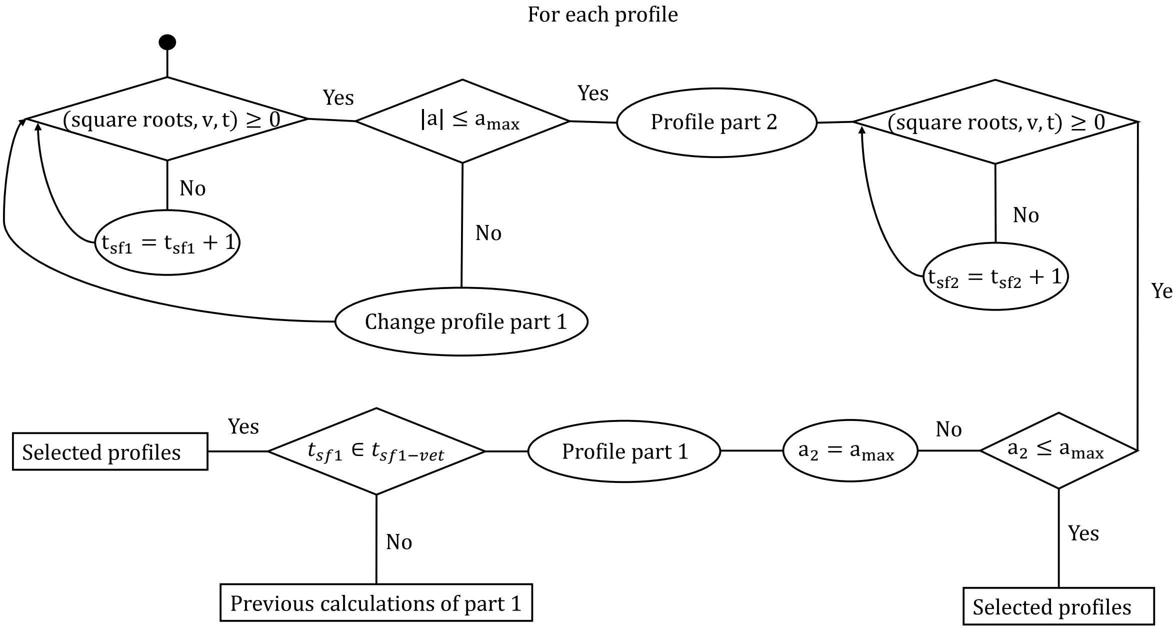


Figure 3.13: Calculation process

When the bus has to cross two traffic lights with no stop in between, calculations are similar, but with no distinction between the first and the second part.
First, acceleration necessary to cross the traffic light during the green phase is calculated. Since the speed profile from the current position to the traffic light is uniformly accelerated motion, acceleration a is simply calculated as:

$$a = 2\frac{(S_{sf1} - S_0 - v_0 t_{sf1})}{t_{sf1}^2} \tag{3.3}$$

Where t_{sf1} is taken from $t_{sf1-vet}$ until a is lower than a_{max}.
Then the time to reach the second traffic light is calculated using the same acceleration and compared to the times of beginning and ending of the green phases: if the time needed to reach the second traffic light falls in a temporal window when the traffic light is green, the acceleration is suggested to the driver, otherwise another acceleration a_2 is calculated using the equation:

$$a_2 = 2\frac{(S_{sf2} - S_0 - v_0 t_{sf2})}{t_{sf2}^2} \tag{3.4}$$

Also t_{sf2} is taken from the corresponding vector until a_2 is lower than a_{max}.
This acceleration is used to check if it allows the bus to cross the first traffic light during

a green phase: if yes, it is suggested to the bus driver instead of the one previously calculated, otherwise it is discarded.

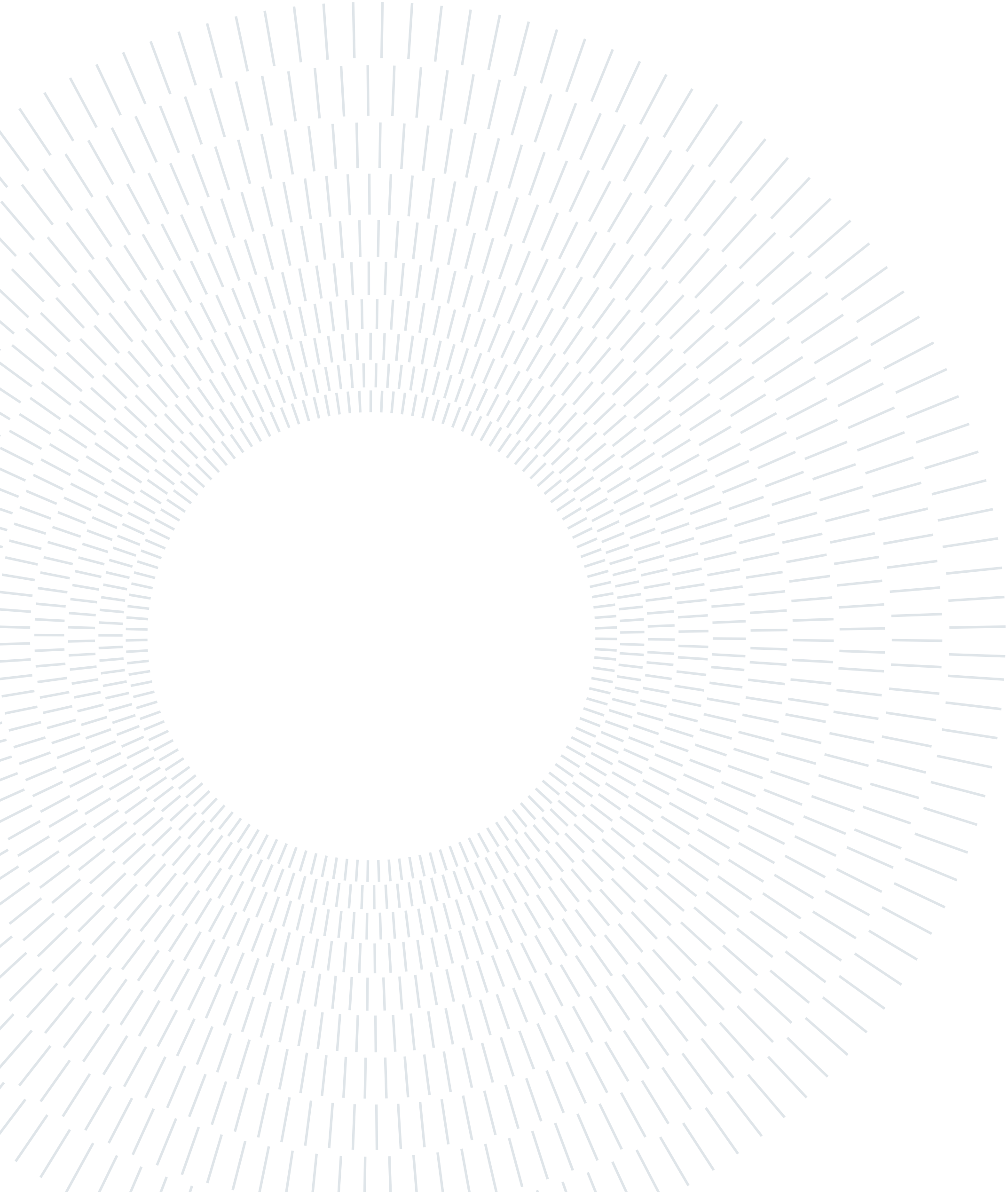
3.5. Table of variables

In this section the list of variables mentioned in this chapter is reported, together with a brief explanation and, if known, the value. Some variables have a fixed value, others are unknown in the speed profiles equations and so calculated at each time step, other ones are taken as input from the algorithm, because they are the results of previous calculations.

symbol	description	value
a_{max}	maximum acceleration considered	$1m/s^2$
v_{max}	maximum speed allowed, speed limit of the street	$50km/h$
a_b	constant acceleration chosen for the braking phase in speed profiles with fixed braking acceleration	$-1m/s^2$
t_{dwell}	time spent at each stop	$10s$
S_0	initial position for the speed profile, so current position of the bus	given as input
v_0	initial speed in the speed profile, so current speed of the vehicle	given as input
S_{sf1}	position of the first traffic light	given as input
v_{sf1}	speed at which the bus should cross the traffic light according to the chosen speed profile	computed
S_{stop1}	position of the first stop	given as input
v_{stop1}	speed of the vehicle at the first stop	$0m/s^2$
S_{sf2}	position of the second traffic light	given as input
v_{sf2}	speed at which the bus should cross the traffic light according to the chosen speed profile	computed
S_{stop2}	position of the second stop	given as input
v_{stop2}	speed of the vehicle at the second stop	$0m/s^2$
t_{sf1}	time needed to reach the first traffic light from the current position	given as input or calculated
$t_{sf1-vet}$	vector containing the possible time intervals to reach the first traffic light during a green phase	computed on an input
t_{sf2}	time needed to reach the second traffic light from the current position	given as input or calculated

symbol	description	value
$t_{sf2-vet}$	vector containing the possible time intervals to reach the second traffic light during a green phase	computed on an input
t_{stop1}	time needed to reach the first stop	computed
t_1	time needed to cover the first trait of a speed profile (up to a traffic light or a change of acceleration)	computed
t_2	time needed to cover the second trait of a speed profile (up to a traffic light or a change of acceleration or a stop)	computed
t_3	time needed to cover the third trait of a speed profile, if present (up to a traffic light or a change of acceleration or a stop)	computed
t_4	time needed to cover the fourth trait of a speed profile, if present (up to a stop)	computed
S_{br}	position at which the braking phase starts in a speed profile	computed
v_{br}	speed at which the vehicle starts braking in a speed profile (maximum speed reached)	computed
S_{cost}	position at which the speed starts to be constant in a speed profile	computed
v_{cost}	constant speed in a speed profile	v_{max}
a	acceleration in the first trait of a speed profile of the first part, it could be positive or negative	computed
a_{brake}	acceleration in the braking phase at variable acceleration, it must be negative	computed
a_2	acceleration when restarting from a stop, in the second part of the speed profile, it must be positive	computed
$dist_{br}$	braking distance	computed

Table 3.2: Variables used in the speed profiles calculations



4 | Simulations, tests and results

The algorithm presented in Chapter 3 has been assessed first in a Matlab-Simulink environment, using a pre-recorded sequence of traffic light information previously extracted, then in an actual testing campaign with the instrumented trolleybus described in Chapter 2, with real-time information coming from the traffic lights.

The testing area, described in Chapter 2, is a portion of the ATM 90-91 circular trolleybus line in Milano. The considered part of the line is mainly in a public transport reserved lane, characterized by the presence of prioritization in traffic lights for trolleybuses or trams on the line. This means that phases of traffic lights and their durations can vary a lot, in particular red and green phases can lengthen or shorten at any time, sometimes stressing the algorithm to choose a profile and to adapt to the traffic light phase plan.

Both simulation and test results deriving from following GLOSA suggestions are compared from the ones of previous simulations and tests, in which the bus follows a modified version of the MTLA algorithm presented in Chapter 1.

4.1. MTLA control logic

The Multiple Traffic Light Advisory (MTLA) algorithm is a rule-based control logic that has been previously developed in [10] to cruise through multiple intersection with the green light.

This algorithm considers multiple traffic lights to calculate a speed (and consequently an acceleration) that allows the bus to cross both of them during a green phase. The considered speed profile is a uniformly accelerated motion from the current vehicle position to reach the first traffic light ahead, followed by a constant speed to cruise trough the following intersections. By using data about the remaining time to the change of phases, the algorithm calculates an interval of speed that allows the bus to reach the first traffic light in a green phase. This interval is then restricted by the time remaining to phase changes of the second traffic light, in order to cross it also during a green phase. Additional constraints on acceleration, such as the maximum value chosen as $1.5m/s^2$, further reduce the interval of possible accelerations.

In addition, the basic MTLA presented in [10] has been then modified in [27] to take into account the presence of bus stops on the path not in the speed profile, but by considering an equivalent time needed to reach the traffic lights.

4.2. KPI definition

Both for simulations and tests, some KPIs are calculated, to extract relevant information and to compare the GLOSA algorithm proposed and the MTLA:

- travel time: one of the objectives of GLOSA is to reduce travel time and, for buses, to improve the schedule adherence. Since red traffic lights highly influence travel time, it is computed in order to compare the performance of the two algorithms.
- stop time: it is the total time "lost" at red traffic lights.
- root mean square (RMS) value of acceleration: it takes into account both positive and negative acceleration values. High positive acceleration values make energy consumption increase, while high braking deceleration values can create discomfort for passengers.
- energy consumption: it is calculated considering both the effort required to move the vehicle with a certain acceleration and the one required to win the air and rolling resistance. Calculation starts from the power:

$$P_{tot} = P_{inertia} + P_{resist} \tag{4.1}$$

Where

$$\begin{aligned} P_{inertia} &= mav \\ P_{resist} &= (0.5\rho A C_d v^2 + mgC_r)v \end{aligned} \tag{4.2}$$

Values of variables used are reported in Table 4.1.

m	mass of the bus	$19800kg$
ρ	air density	$1.225kg/m^3$
A	bus frontal area	$8.917m^2$
C_d	drag coefficient	0.8
g	gravity acceleration	$9.81m/s^2$
C_r	rolling resistance coefficient	0.015

Table 4.1: Values of variables in energy consumption calculation

In this case the power is considered only during acceleration phases, so for positive accelerations; in case it is possible to recover part of the energy during the braking phase, for electric vehicles, it is possible to consider also negative accelerations.
The required power is then integrated in time and divided by the total distance to find the energy consumption in kWh/100 km:

$$EC = P_{tot} * \frac{\Delta t}{dist_{tot}} * 100 \tag{4.3}$$

It is important to underline that the rule-based logic of this algorithm does not optimize directly the mentioned KPIs through a cost function. However, its aim is the reduction of stops at red traffic lights, which can have an indirect positive impact also on energy consumption or on other analysed KPIs.

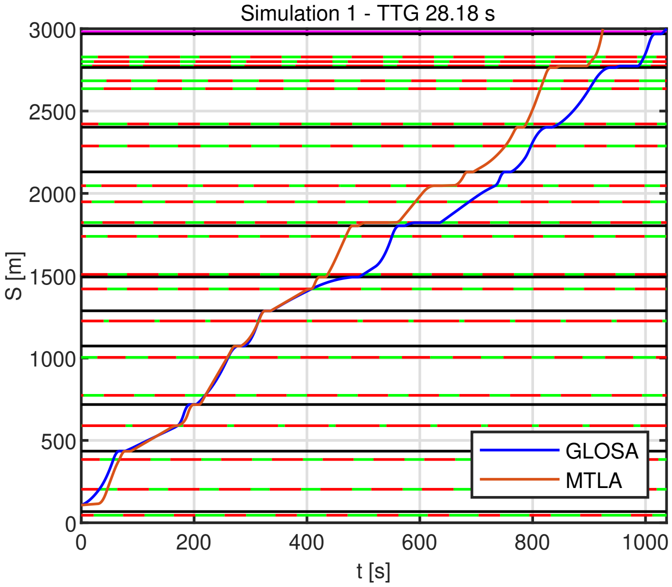
4.3. Simulation assessment

Simulations have been run on a relevant portion of the route, made of 20 traffic lights and 10 stops of line 91, between the stops of Piola M2 and Via Lario, in the direction from Piola to Lugano, for a total covered distance of about 3 km.

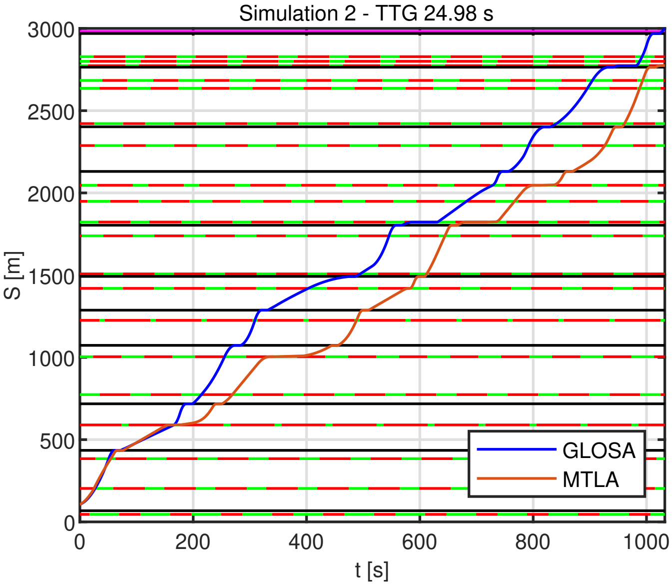
Both for GLOSA and MTLA algorithms, 16 different simulations are done with different conditions of the traffic light plan. They are obtained shifting the simulations with a time step of 5 s, in order to evaluate different initial conditions of the first traffic light, varying the phase and the remaining time to the change of phase (time-to-green TTG if the phase is red, time-to-red TTR if the phase is green), with the same traffic light plan.
For each simulation, a graph representing the position of the trolleybus as a function of time is plotted, comparing the covered distance following GLOSA and MTLA algorithms (as in Figure 4.1). On the same plot also the position of stops and of traffic lights is shown, together with traffic lights phases. Thus, it is possible to see the bus braking and leaving at stops and crossing intersections with the green light.
Figure 4.1 shows a comparison between the behaviour of the two algorithms with different traffic plans. Overall results of all the simulations are analysed below, to avoid the influence of punctual differences.

For each simulation it is possible to compare also the speed profile followed by the bus with the suggestions of GLOSA and MTLA.
In Figure 4.2, which shows the speed as a function of the position, it is possible to see an important difference between the two algorithms: using the MTLA algorithm, the

speed profile has more parts at constant speed, while with the new GLOSA algorithm the acceleration is continuously modulated, bringing to a varying speed, with more peaks. The presence of a non-constant speed has advantages and disadvantages. On the one hand, a non-null acceleration increases the energy consumption, as shown in Figure 4.5, on the other hand Figures 4.4 and 4.3 show a reduction of the stop time and consequently of travel time thanks to the GLOSA logic.

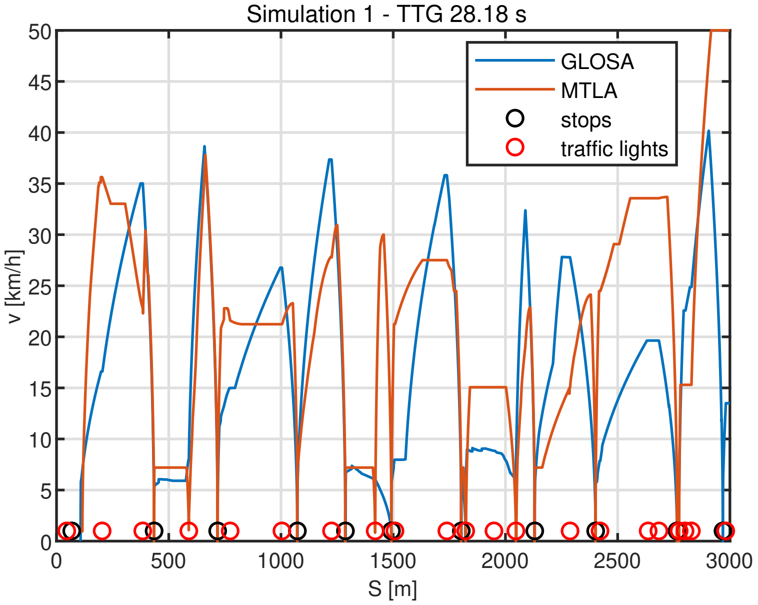


(a)

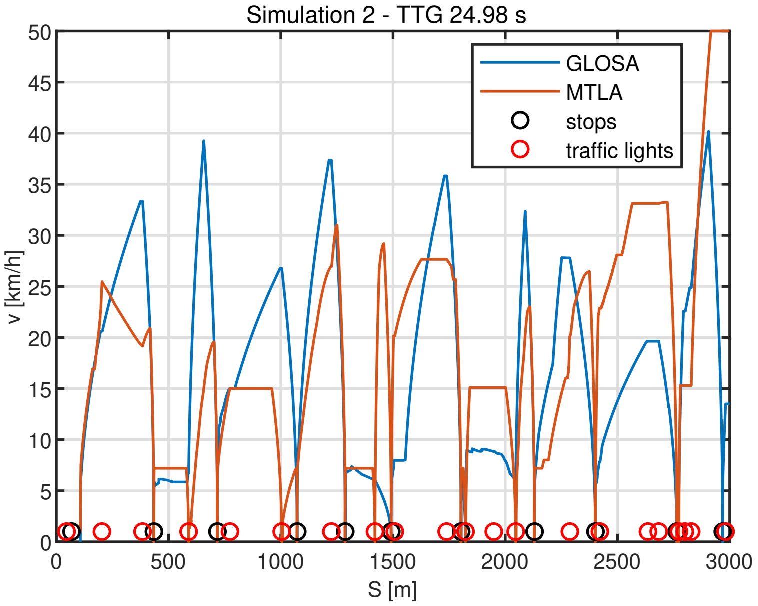


(b)

Figure 4.1: Time history of the trolleybus covered distance



(a)



(b)

Figure 4.2: Speed of the trolleybus

As far as the total travel time is concerned, Figure 4.3 shows that MTLA simulations require in general a higher time to cover the same distance, with the exception of the first simulation.

The traffic light plan can highly influence the duration of the travel: a shift of just 5 s can increase the travel time of around 3 min, as the comparison between simulation 1 (TTG 28.18 s) and simulation 2 (TTG 24.98 s) of MTLA shows.

Reduction of travel time achieved by GLOSA is a consequence also of the reduction of stop time at red traffic lights. As shown in Figure 4.4, in most of the simulations GLOSA suggestions are able to avoid stopping better than MTLA suggestions, thus reducing the loss of time.

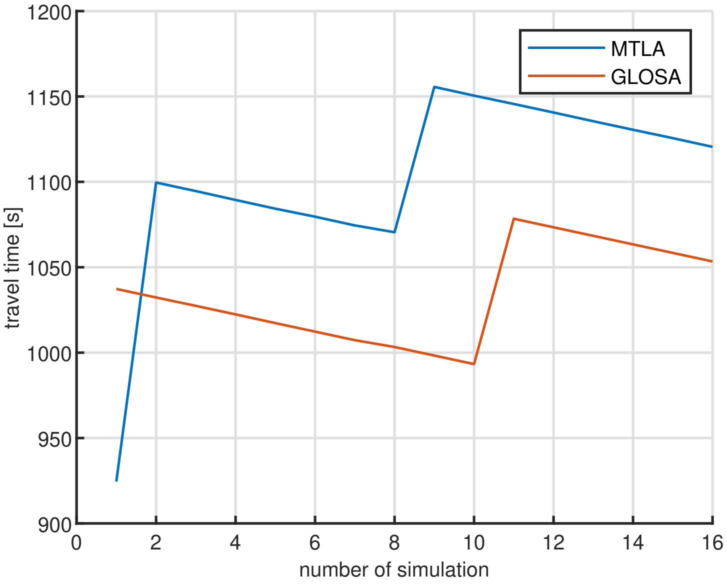


Figure 4.3: Total duration of the simulation

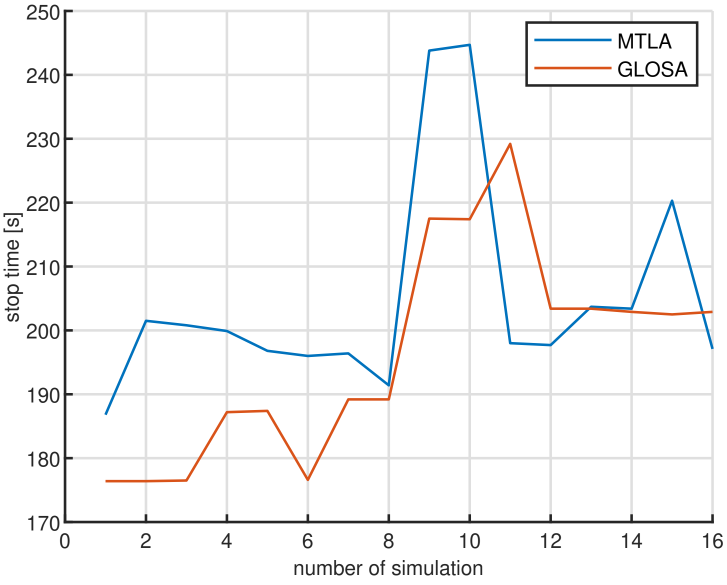


Figure 4.4: Stop time at red traffic lights

Travel time is very important in real-case applications: buses has to guarantee a service to passengers, so it is important to respect the timetable and to avoid bus bunching.

Unfortunately, a lower travel time can require a higher consumption of energy: in Figure 4.5, it can be clearly seen that in general the energy consumption is slightly higher when the new GLOSA algorithm is used, respect to MTLA. The variability in the energy values with different simulations shows the impact of traffic light plan on the vehicle behaviour and so on the total energy consumption.

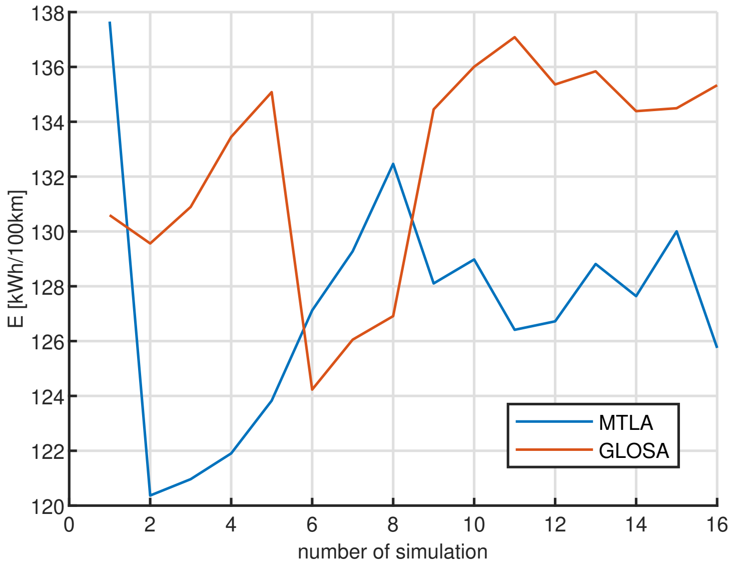


Figure 4.5: Energy consumption on 100 km

Figure 4.6 shows a comparison between the root mean square values of acceleration. It is possible to notice that the values obtained with GLOSA are slightly lower than the ones obtained with MTLA. The reason is that the acceleration proposed by GLOSA is subject to smoother variations. Moreover, the RMS value considers also the negative acceleration of the braking phase, that has different values in the algorithms: in MTLA, the braking acceleration is chosen as $-1.5m/s^2$, while in the new GLOSA algorithm it is chosen as $-1m/s^2$.

As previously told, strong variations in the acceleration values or high braking decelerations can provoke passengers discomfort, so they are taken into account in calculating the RMS and comparing the algorithms results.

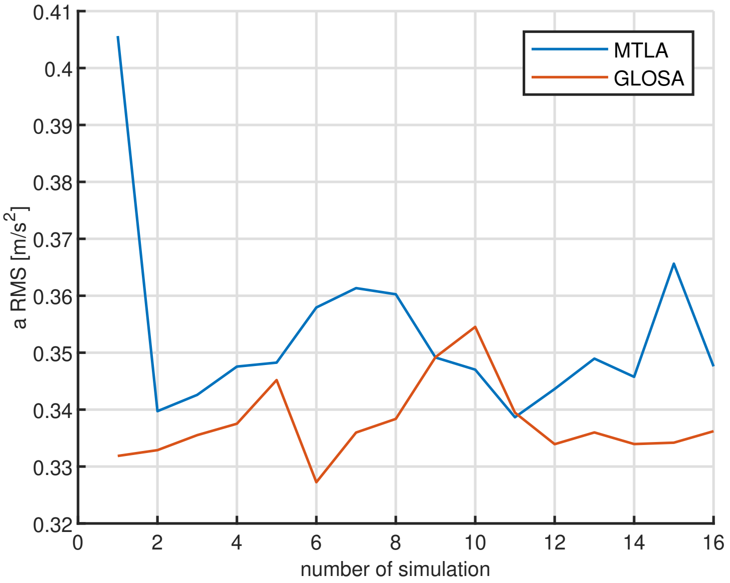


Figure 4.6: RMS of the acceleration

Results shown in graphs are resumed in Table 4.2. A comparison between the mean values of the KPIs of all the simulations highlights the reduction of travel time, stop time and RMS of acceleration thanks to the use of GLOSA instead of MTLA, but also the increase in energy consumption.

TTG/ TTR (s)	TL phase	t_{tot} (s)		t_{stop} (s)		a_{RMS} (m/s^2)		EC ($\frac{kWh}{100km}$)	
		MTLA	GLOSA	MTLA	GLOSA	MTLA	GLOSA	MTLA	GLOSA
28.18	red	924	1037	187	176	0.4056	0.3319	137.65	130.59
24.98	red	1099	1032	202	176	0.3397	0.3329	120.37	129.56
19.97	red	1094	1027	201	177	0.3426	0.3355	120.96	130.89
14.97	red	1089	1017	200	187	0.3476	0.3375	121.90	133.45
9.96	red	1084	1022	197	187	0.3483	0.3452	123.83	135.08
4.96	red	1079	1012	196	177	0.3579	0.3272	127.12	124.23
33.98	green	1074	1007	196	189	0.3613	0.3360	129.28	126.05
29.98	green	1070	1003	191	189	0.3603	0.3384	132.46	126.91
24.98	green	1155	998	244	218	0.3492	0.3492	128.10	134.45
16.98	green	1150	993	245	217	0.3470	0.3545	128.98	136.01
11.98	green	1145	1078	198	229	0.3387	0.3395	126.41	137.09
6.98	green	1140	1073	198	203	0.3436	0.3339	126.72	135.36
7.98	green	1135	1068	204	203	0.3490	0.3360	128.81	135.84
55.98	red	1130	1063	203	203	0.3458	0.3339	127.64	134.39

53.98	red	1125	1058	220	203	0.3656	0.3342	130.00	134.49
45.98	red	1120	1053	197	203	0.3476	0.3362	125.75	135.33
mean	-	1101	1034	205	196	0.3531	0.3376	127.24	132.48
σ	-	55.21	28.36	16.89	16.41	0.016	0.007	4.35	3.96
Δ%	-	-	-6.1%	-	-4.4%	-	-4.4%	-	+4.1%

Table 4.2: Simulation results

4.4. Testing campaign

As mentioned, the testing campaign has been performed on the actual path of the trolleybus, for a total of 30 traffic lights and 16 stops in both directions.

As introduced in Chapter 2, a dedicated HMI has been used to display the indications to the driver, including the colour of the next traffic light, the remaining time to the change of phase, the suggested speed and a coloured arrow that is used to suggest to accelerate, brake or keep the speed constant.

It is worth mentioning that because of external factors, such as exceptional traffic ahead or temporary lack of communication from specific traffic light plants, the analysis of the experimental campaign has been limited to three portions of the route, described in Table 4.3. As a result the experimental assessment relies on a significant amount of repetitions for each portion. Moreover, the proposed GLOSA algorithm is compared against the MTLA system tested in [27].

	Centrale-Zara	Lugano-Lancetti	Zara-Centrale
length (m)	1300	1000	1340
stops	4	2	4
traffic lights	11	4	11

Table 4.3: Portions of the testing path

For tests, the same KPIs presented in Section 4.2 are used to evaluate simulation results are calculated. They are shows in Table 4.4, both for GLOSA and MTLA algorithms.

Results of tests confirm results obtained in simulations as far as travel time in concerned. Figure 4.7 shows the mean values obtained from data gathered in simulations in the different portions of the path. In all of them, the bus following GLOSA suggestions covers the same distance in a shorter time with respect to MTLA.

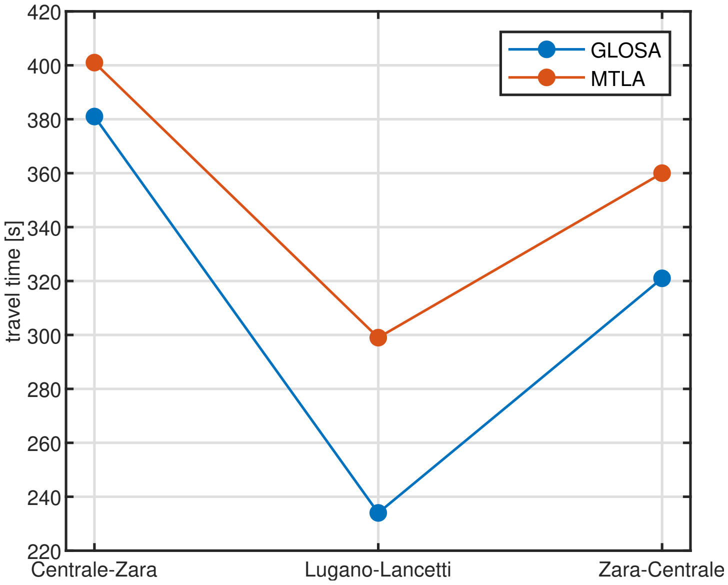


Figure 4.7: Mean travel time in tests

Also energy consumption calculations, in Figure 4.8, show results similar to simulations: mean value of energy required following GLOSA is higher than the one required by MTLA, in all the three portions of the path.

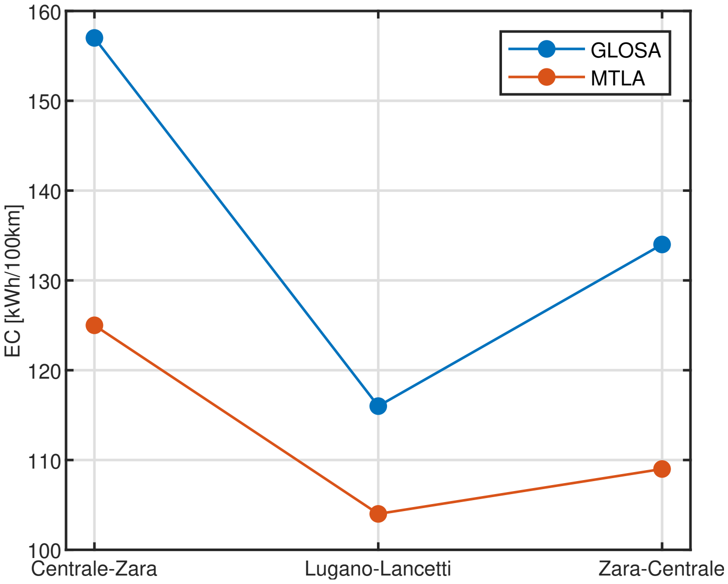


Figure 4.8: Mean energy consumption in tests

RMS of acceleration is quite different, instead: differently from simulations, the acceleration cannot be imposed to the trolleybus and so the suggested value is not perfectly followed by the driver, causing a wider variation in acceleration values (Figure 4.9)˙ Another difference with respect to simulations is the presence of decelerations lower than $-1m/s^2$ during braking phases. The reason is that the actual braking deceleration imposed by the driver could exceed the limit of $-1m/s^2$, especially if the driver does not react immediately to the suggestion given by the algorithm. In simulations, this never happened, since position of stops was fixed and the suggestions about speed and acceleration were perfectly followed. In a real case, the position of the stops is not always perfectly respected because of external conditions.

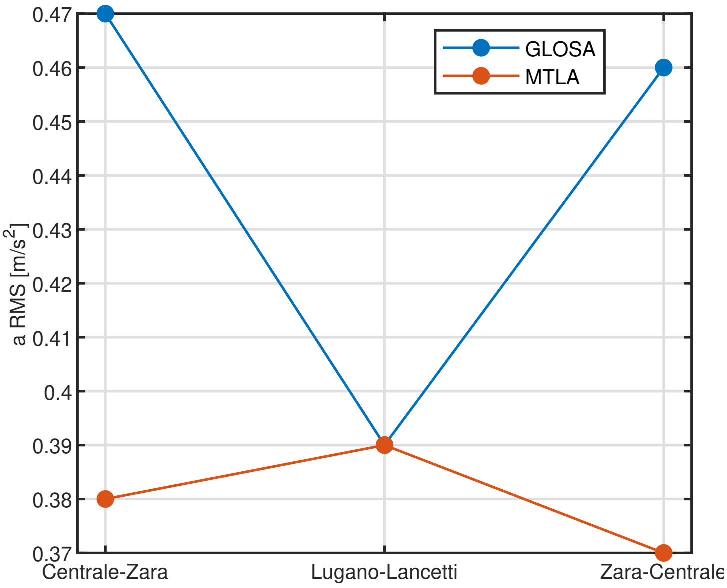


Figure 4.9: Mean values of root mean square of acceleration

Results of the testing campaign are resumed in Table 4.4 and compared with the results of MTLA tests on the same portions of the path. Percentage differences between the mean values of the results of the two algorithms are also shown.

	Centrale-Zara		Lugano-Lancetti		Zara-Centrale		Weighted average	
	MTLA	GLOSA	MTLA	GLOSA	MTLA	GLOSA	MTLA	GLOSA
runs	10	5	7	3	10	5	27	13
travel time [s]	401	381	299	234	360	321	359	324
Δ%	-	-5%	-	-22%	-	-11%	-	-10%
stop time [s]	98	128	60	42	72	69	79	85
Δ%	-	+30%	-	-30%	-	-4%	-	+8%
$a_{rms}[m/s^2]$	0.38	0.47	0.39	0.39	0.37	0.46	0.38	0.45
Δ%	-	+24%	-	0%	-	+24%	-	+18%
$EC[\frac{kWh}{100km}]$	125	157	104	116	109	134	114	139
Δ%	-	+26%	-	+12%	-	+23%	-	+22%

Table 4.4: Tests results

4.4.1. Comparison between test and simulation

To better compare the behaviour of the algorithm in a simulation and in a real test case, a part of a test was chosen and its traffic light data used in a simulation.
The chosen path was Zara-Centrale and the algorithm showed to behave similarly in the test and in the simulation, about position (Figure 4.10) and speed (Figure 4.11).
However, little differences can be expected, mainly due to the fact that in a real case there is a difference between the reference acceleration and the actual one. Indeed, speed profile is smoother in the simulation with respect to the test, but shows a similar trend, confirming the results obtained.

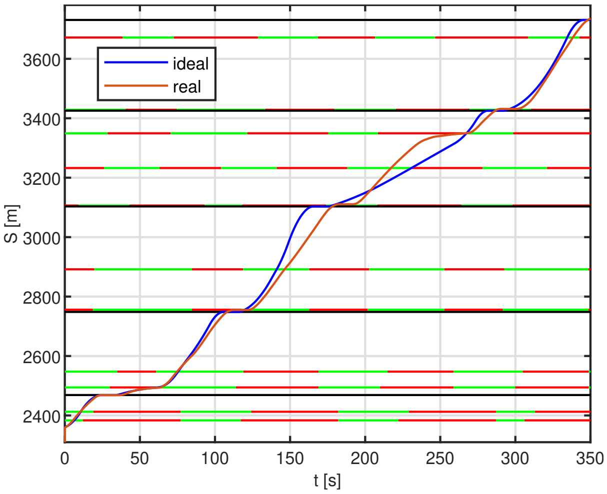


Figure 4.10: Comparison between position in the real case and in the simulation

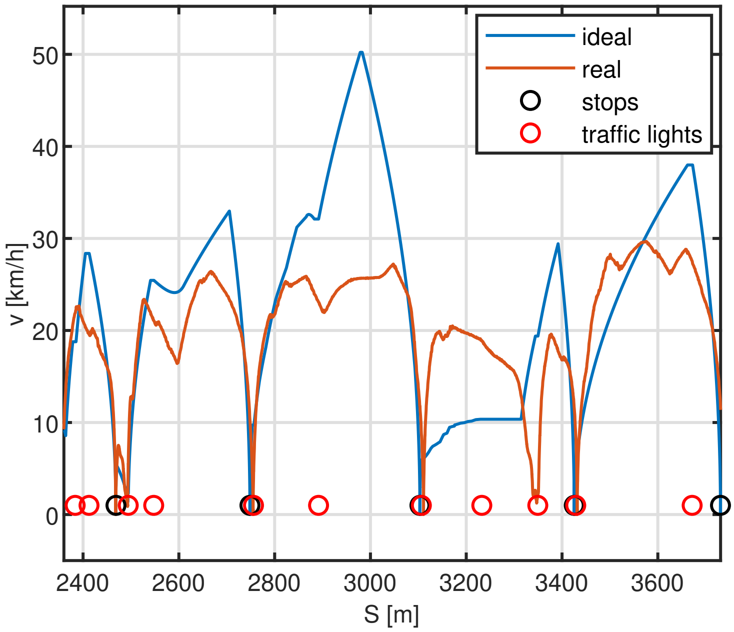


Figure 4.11: Comparison between speed in the real case and in the simulation

4.5. Algorithm robustness

As mentioned before, traffic light plans on the path used for the testing campaign could vary, sometimes very frequently. However, the results described in Section 4.4 show that the algorithm has proved to be robust to traffic light plan changes, thanks to a frequency of calculation high enough (10 Hz) and the flexibility in the choice of the speed profile.
In this section, further tests are carried out to demonstrate the robustness of the algorithm in challenging situations, in particular related to the crossing of an intersection with the red light.

The model of the algorithm does not consider the presence of the yellow light, that is included in the red light. However, yellow light lasts about 3s (from traffic light plan data given by the Municipality of Milan and shown in Chapter 2) and its presence is important in a situation of high variability of traffic light phases: it could happen that the bus receives a new message from a traffic light with a shorter time-to-red than expected when it is too close to the traffic light and it is not able to stop on the stop line, even with a higher acceleration as mentioned in Section 4.4.
Since it is not possible to stop the vehicle, the algorithm deals with this situation suggesting to the bus driver to keep the speed constant, in order to cross the traffic light as soon as possible, but without increasing the speed. When it happens, it is important to

check that the bus is able to cross the traffic light when it is yellow, so within $3s$ from the change of phase from green to red.
In simulations and tests there was not a similar dangerous situation, so an example was reproduced adjusting manually phases and times of a traffic light taken from recorded data. The resulting remaining time of the traffic light is shown in Figure 4.12.

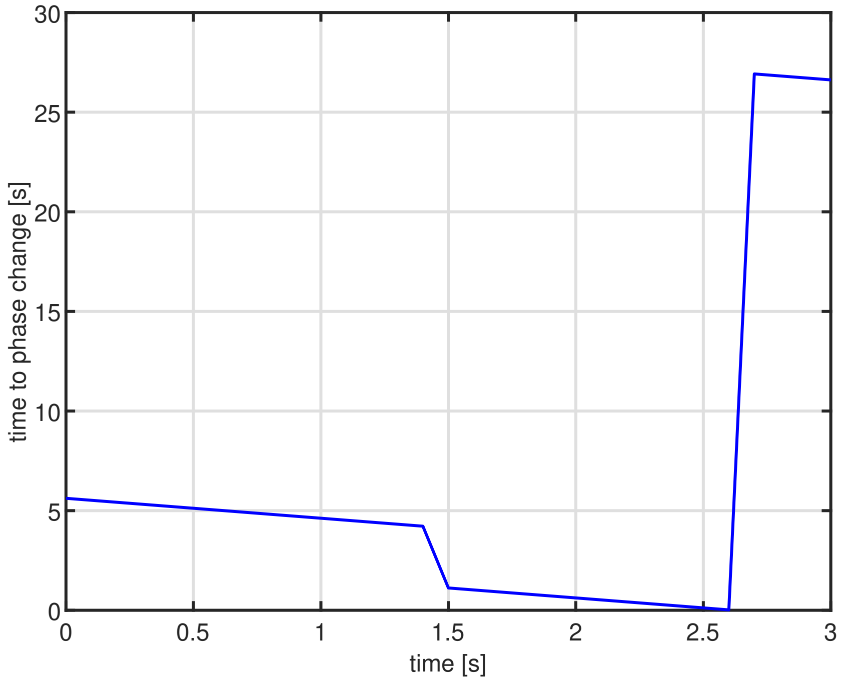


Figure 4.12: Remaining time to the change of phase of the considered traffic light

In the simulation, the trolleybus starts at a distance of $46m$ from the traffic light with a speed of $10m/s$ $(36km/h)$. The initial speed is chosen as typically average value for the trolleybus, while the initial distance is chosen high enough to avoid that the bus could cross the traffic light when it is green, but lower than the braking distance, calculated as:

$$dist_{br} = -\frac{v_0^2}{2 * a_b}$$

Figure 4.13 shows that the bus is able to cross the traffic light within 3 s from the change of phase with a constant speed.

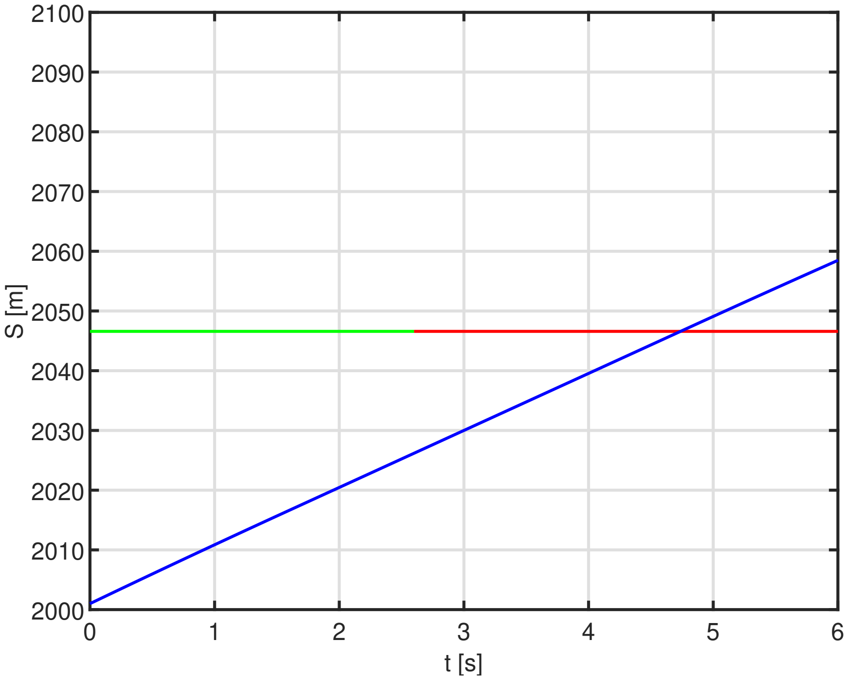


Figure 4.13: Position of the trolleybus crossing the intersection

To further assess the safety of the algorithm with respect to red traffic lights, it is supposed that the bus can receive a message that the traffic light has suddenly become red. This situation never happened in simulations or in tests, however this hypothesis allows to consider the worst case (that is also the case of drive-by-sight: the phase of the traffic light is known only when the driver sees it).
Given that the bus is travelling with speed v_0 when it receives the message, the algorithm calculates the braking distance required to completely and safely stop the vehicle:

$$dist_{br-min} = -\frac{v_0^2}{2 * a_{brake}}$$

Since in particular and possibly dangerous situations, such as a shorter time to red than expected, the algorithm can suggest to brake with accelerations up to $-1.5m/s^2$, the minimum required braking distance is calculated with $a_{brake} = -1.5m/s^2$.
Varying v_0 from 0 to the speed limit of $50km/h$, the safety green area is shown in Figure 4.14. The colour map indicates all the distances at which the bus can receive the message that the traffic light is red and consequently stop without crossing the traffic light.
If the message is received when the bus is closer to the traffic light than the minimum braking distance, it is not possible to stop before the traffic light, so the algorithm suggests to proceed at constant speed to cross the intersection. Since the first $3s$ of the red phase, in real traffic lights, corresponds to the yellow phase, the bus could be able to cross the

intersection without being dangerous for the other road users.
Also in this case it is possible to calculate a maximum distance for which the bus can receive the message and cross the traffic light safely, knowing that the bus travels with speed v_0 and the maximum available time to do the manoeuvrer is $t_{yellow} = 3s$:

$$dist_{max} = v_0 * t_{yellow}$$

Varying the speed up to the speed limit, as before, the yellow area in Figure 4.14 is obtained, showing all the distances from the traffic light at which the bus can receive the message and cross the intersection during the yellow phase in a safe manner.
Comparing the braking distance and the maximum distance to cross the traffic light, it is possible to obtain the red area in the figure: if the bus receives the message when its distance from the traffic light is in the red area, it will cross through it when it is red.
The graph indicated that the algorithm behaviour is sufficiently robust and safe, as the red area is small compared to the green and yellow ones, and that if the speed is lower than $30km/h$, which is a quite common speed in urban congested areas, the behaviour of the bus is reasonably safe.

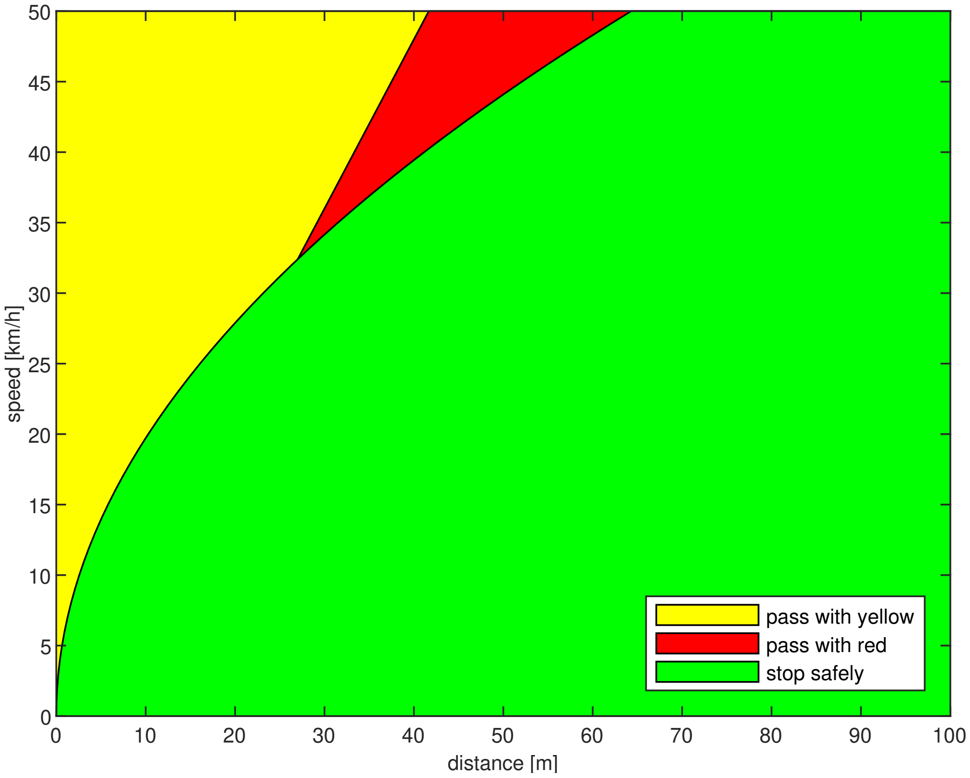
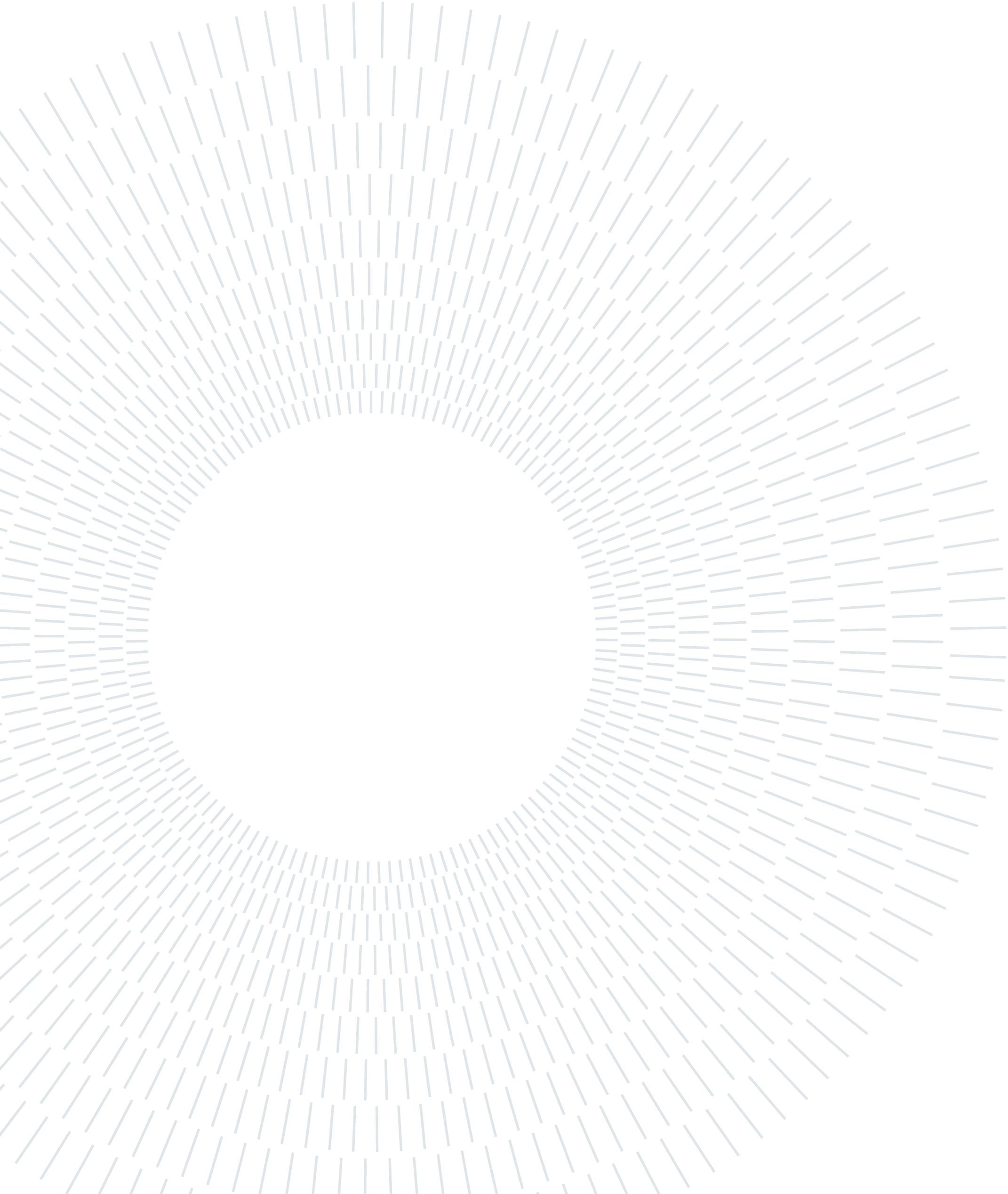


Figure 4.14: Safe and non-safe distances depending on the speed

4.6. Algorithm limitations

The proposed GLOSA algorithm is based on assumptions and hypothesis formulated to obtain a simple model, suitable for the analysed cases and scenarios. So, the work has some limits:

- Traffic and interactions with other vehicles are not considered: the trolleybus is supposed to be in a reserved lane, however the lane used for tests allowed the presence of other vehicles. In particular, the presence of queues at traffic lights influences the actual time required to reach the traffic light and so the ability to cross the intersection without stopping following the indications of the algorithm.
- The trajectory is supposed to be always rectilinear: even if on the considered line this hypothesis is largely verified, reductions of speed due to turns or to switches, in the particular case of trolleybuses, are not taken in account.
- The path is supposed to be plain: in the line used for tests, being in a flat city, the hypothesis is always verified, but for applications in different locations it could be necessary to consider also the contribution of slopes in the calculation of the required power.
- Energy recovery for electric vehicles is not considered: this could help to reduce the total amount of energy required.
- Energy for idling at red traffic lights is not considered: it is difficult to estimate and it is little with respect to the energy required to move the vehicle, so it was not considered in energy calculations, but it could change the total energy required.



Conclusions and future developments

The aim of this work was to propose a rule-based logic GLOSA, based on a flexible choice of the reference speed profile defined by kinematic equations. The objective was improving the performance of public transport vehicles in crossing traffic lights without stopping, thus increasing the chance of reducing energy consumption or travel time.

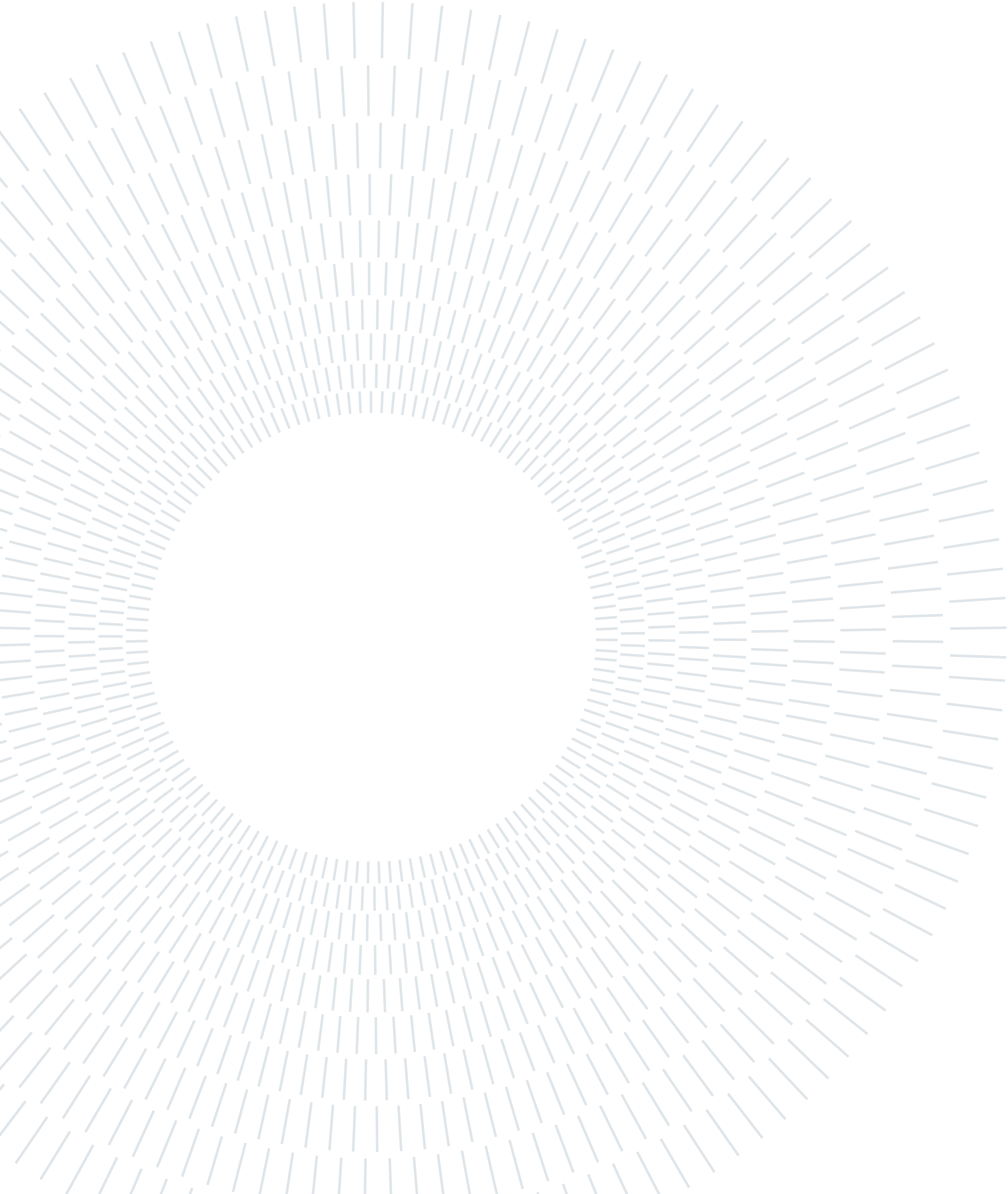
So the proposed algorithm has been developed as a Finite State Machine based on kinematic equations and it has been assessed both with simulations and tests, which allowed a comparison with the MTLA algorithm.

With respect to the MTLA, the GLOSA algorithm presented in this work achieved the objective of improving the ability of a bus to cross intersections without stopping. It increased the flexibility in the choice of the speed profile depending on the traffic light plan, with a reduction of the stop time at red traffic lights in many of the cases analysed. The energy consumption results increased, due to mean higher values of speed and acceleration, but the travel time is reduced, which is a benefit for the service.

The algorithm also proved to be robust and capable of managing critical situations related to traffic light plans.

However, the algorithm has some limits, mainly related to the fact that traffic is not taken into account and that there is not an actual cost function to be minimized, which accounts for energy consumption and travel time.

These limits can be investigated as further developments, together with an integrated use of the proposed algorithm with the prioritization for public transport vehicles at traffic lights.



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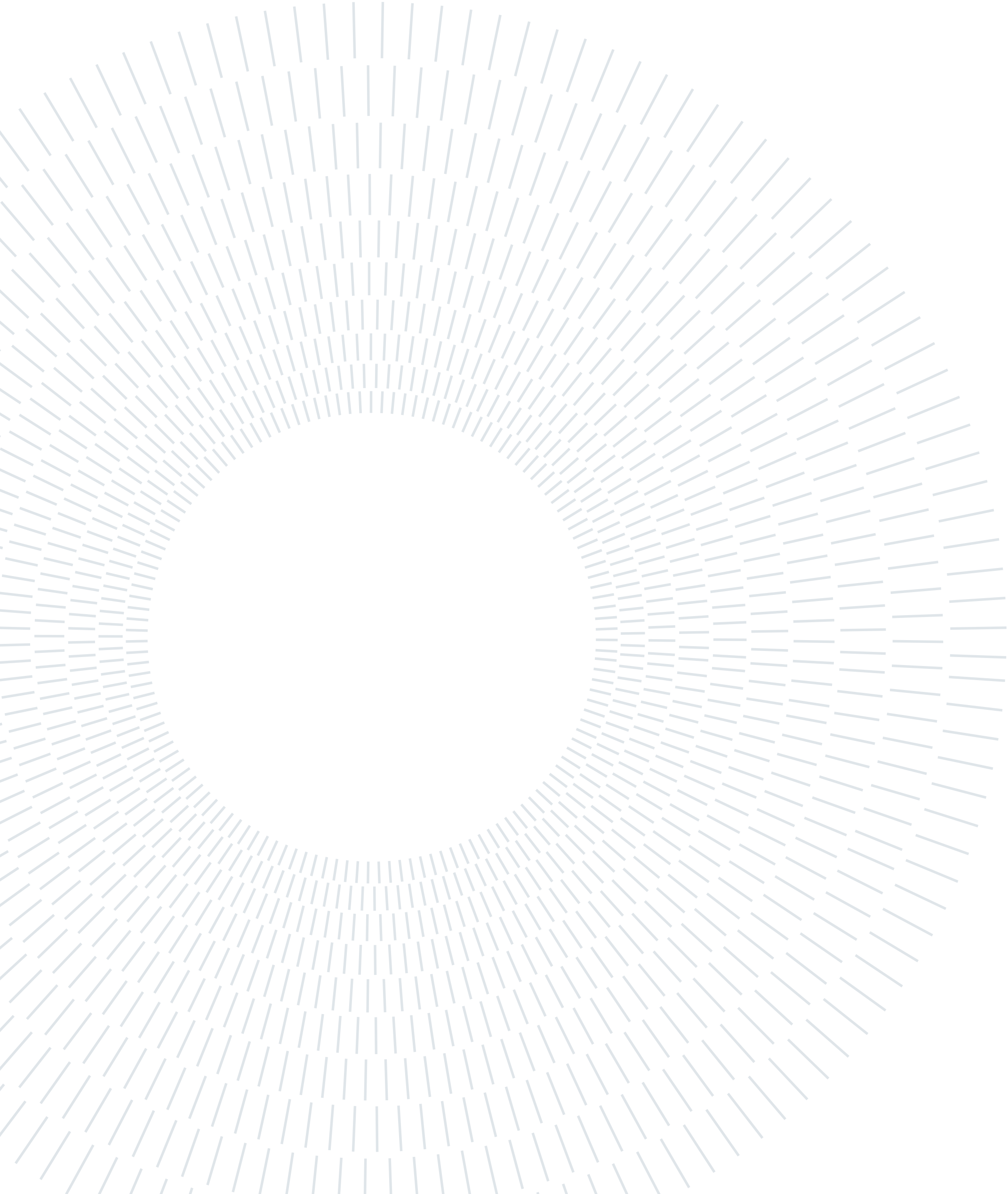

Appendix A

Table of equations of speed profiles

First part of speed profiles	
profile 1.1	$\begin{cases} S_{br} = v_0 t_1 + S_0, \\ S_{sf1} = \frac{1}{2} a_{brake} t_2^2 + v_0 t_2 + S_{br}, \\ v_{sf1} = a_{brake} t_2 + v_0, \\ S_{stop1} = \frac{1}{2} a_{brake} t_3^2 + v_{sf1} t_3 + S_{sf1}, \\ v_{stop1} = 0 = a_{brake} t_3 + v_{sf1}, \\ t_{sf1} = t_1 + t_2, \\ t_{stop1} = t_1 + t_2 + t_3. \end{cases}$
profile 1.2	$\begin{cases} S_{br} = \frac{1}{2} a t_1^2 + v_0 t_1 + S_0, \\ v_{br} = a t_1 + v_0, \\ S_{sf1} = \frac{1}{2} a_b t_2^2 + v_{br} t_2 + S_{br}, \\ v_{sf1} = a_b t_2 + v_{br}, \\ S_{stop1} = \frac{1}{2} a_b t_3^2 + v_{sf1} t_3 + S_{sf1}, \\ v_{stop1} = 0 = a_b t_3 + v_{sf1}, \\ t_{sf1} = t_1 + t_2, \\ t_{stop1} = t_1 + t_2 + t_3. \end{cases}$

profile 1.3	$\begin{cases} S_{cost} = \dfrac{1}{2}at_1^2 + v_0t_1 + S_0, \\ v_{cost} = at_1 + v_0, \\ S_{br} = v_{cost}t_2 + S_{cost}, \\ S_{sf1} = \dfrac{1}{2}a_bt_3^2 + v_{cost}t_3 + S_{br}, \\ v_{sf1} = a_bt_3 + v_{cost}, \\ S_{stop1} = \dfrac{1}{2}a_bt_4^2 + v_{sf1}t_4 + S_{sf1}, \\ v_{stop1} = 0 = a_bt_4 + v_{sf1}, \\ t_{sf1} = t_1 + t_2 + t_3, \\ t_{stop1} = t_1 + t_2 + t_3 + t_4. \end{cases}$
profile 1.5	$\begin{cases} S_{sf1} = \dfrac{1}{2}at_1^2 + v_0t_1 + S_0, \\ v_{sf1} = at_1 + v_0, \\ S_{br} = \dfrac{1}{2}at_2^2 + v_{sf1}t_2 + S_{sf1}, \\ v_{br} = at_2 + v_{sf1}, \\ S_{stop1} = \dfrac{1}{2}a_bt_3^2 + v_{br}t_3 + S_{br}, \\ v_{stop1} = 0 = a_bt_3 + v_{br}, \\ t_{sf1} = t_1, \\ t_{stop1} = t_1 + t_2 + t_3. \end{cases}$
profile 1.6	$\begin{cases} S_{cost} = \dfrac{1}{2}at_1^2 + v_0t_1 + S_0, \\ v_{cost} = at_1 + v_0, \\ S_{sf1} = v_{cost}t_2 + S_{cost}, \\ v_{sf1} = v_{cost}, \\ S_{br} = v_{cost}t_3 + S_{sf1}, \\ S_{stop1} = \dfrac{1}{2}a_bt_4^2 + v_{cost}t_4 + S_{br}, \\ v_{stop1} = 0 = a_bt_4 + v_{cost}, \\ t_{sf1} = t_1 + t_2, \\ t_{stop1} = t_1 + t_2 + t_3 + t_4. \end{cases}$

profile 1.7	$\begin{cases} S_{sf1} = \frac{1}{2}at_1^2 + v_0t_1 + S_0, \\ v_{sf1} = at_1 + v_0, \\ S_{cost} = \frac{1}{2}at_2^2 + v_{sf1}t_2 + S_{sf1}, \\ v_{cost} = at_2 + v_{sf1}, \\ S_{br} = v_{cost}t_3 + S_{cost}, \\ S_{stop1} = \frac{1}{2}a_bt_4^2 + v_{cost}t_4 + S_{br}, \\ v_{stop1} = 0 = a_bt_4 + v_{cost}, \\ t_{sf1} = t_1, \\ t_{stop1} = t_1 + t_2 + t_3 + t_4. \end{cases}$
profile 1.8	$\begin{cases} S_{sf1} = \frac{1}{2}at_1^2 + v_0t_1 + S_0, \\ v_{sf1} = at_1 + v_0, \\ S_{stop1} = \frac{1}{2}a_{brake}t_2^2 + v_{sf1}t_2 + S_{sf1}, \\ v_{stop1} = 0 = a_{brake}t_2 + v_{sf1}, \\ t_{sf1} = t_1, \\ t_{stop1} = t_1 + t_2. \end{cases}$

Table 5: Equations of the first part of a speed profile

Second part of speed profiles	
profile 2.1	$\begin{cases} S_{sf2} = \frac{1}{2}a_2t_1^2 + S_{stop1}, \\ v_{sf2} = a_2t_1, \\ t_{sf2} = t_{stop1} + t_{fix} + t_1. \end{cases}$
profile 2.2	$\begin{cases} S_{cost} = \frac{1}{2}a_2t_1^2 + S_{stop1}, \\ v_{cost} = a_2t_1, \\ S_{sf2} = v_{cost}t_2 + S_{cost}, \\ t_{sf2} = t_{stop1} + t_{fix} + t_1 + t_2. \end{cases}$
profile 2.3	$\begin{cases} S_{br} = \frac{1}{2}a_2t_1^2 + S_{stop1}, \\ v_{br} = a_2t_1, \\ S_{sf2} = \frac{1}{2}a_bt_2^2 + v_{br}t_2 + S_{br}, \\ v_{sf2} = a_bt_2 + v_{br}, \\ S_{stop2} = 1/2a_bt_3^2 + v_{sf2}t_3 + S_{sf2}, \\ v_{stop2} = 0 = a_bt_3 + v_{sf2}, \\ t_{sf2} = t_{stop1} + t_{fix} + t_1 + t_2. \end{cases}$
profile 2.4	$\begin{cases} S_{cost} = \frac{1}{2}a_2t_1^2 + S_{stop1}, \\ v_{cost} = a_2t_1, \\ S_{br} = v_{cost}t_2 + S_{cost}, \\ S_{sf2} = \frac{1}{2}a_bt_3^2 + v_{cost}t_3 + S_{br}, \\ v_{sf2} = a_bt_3 + v_{cost}, \\ S_{stop2} = \frac{1}{2}a_bt_4^2 + v_{sf2}t_4 + S_{sf2}, \\ v_{stop2} = 0 = a_bt_4 + v_{sf2}, \\ t_{sf2} = t_{stop1} + t_{fix} + t_1 + t_2 + t_3. \end{cases}$

Table 6: Equations of the second part of a speed profile

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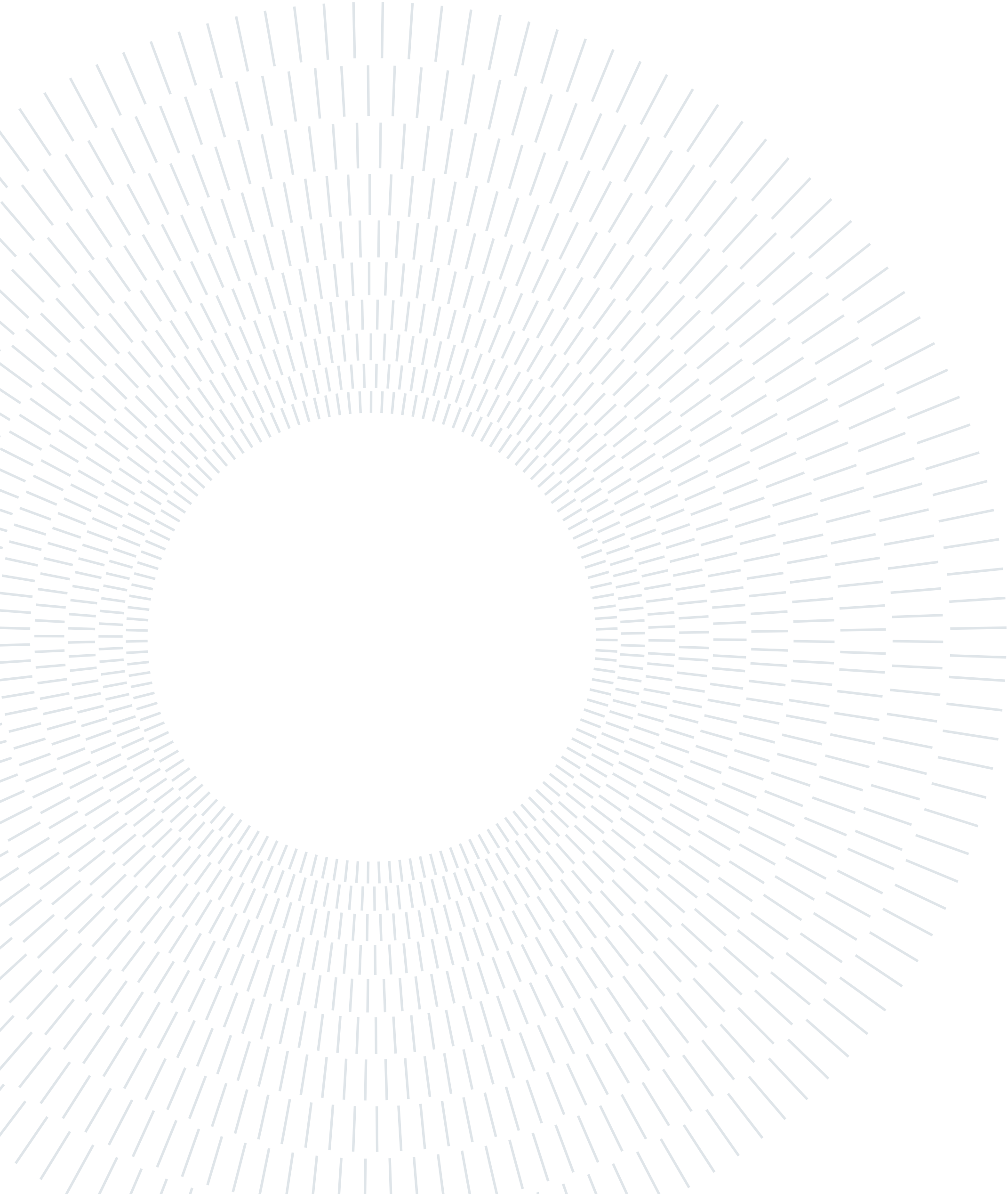

List of Symbols

Variable	**Description**	**SI unit**
a_{max}	maximum acceleration considered	m/s^2
v_{max}	maximum speed allowed, speed limit of the street	km/h
a_b	constant acceleration chosen for the braking phase in speed profiles with fixed braking acceleration	m/s^2
t_{dwell}	time spent at each stop	s
S_0	initial position for the speed profile, current position of the bus	m
v_0	initial speed in the speed profile, current speed of the vehicle	m/s
S_{sf1}	position of the first traffic light	m
v_{sf1}	speed at which the bus should cross the traffic light according to the chosen speed profile	m/s
S_{stop1}	position of the first stop	m
v_{stop1}	speed of the vehicle at the first stop	m/s^2
S_{sf2}	position of the second traffic light	m
v_{sf2}	speed at which the bus should cross the traffic light according to the chosen speed profile	m/s
S_{stop2}	position of the second stop	m
v_{stop2}	speed of the vehicle at the second stop	m/s^2
t_{sf1}	time needed to reach the first traffic light from the current position	s
$t_{sf1-vet}$	vector containing the possible time intervals to reach the first traffic light during a green phase	s
t_{sf2}	time needed to reach the second traffic light from the current position	s
$t_{sf2-vet}$	vector containing the possible time intervals to reach the second traffic light during a green phase	s
t_{stop1}	time needed to reach the first stop	s
t_1	time needed to cover the first trait of a speed profile	s

Variable	Description	SI unit
	(up to a traffic light or a change of acceleration)	
t_2	time needed to cover the second trait of a speed profile (up to a traffic light or a change of acceleration or a stop)	s
t_3	time needed to cover the third trait of a speed profile, if present (up to a traffic light or a change of acceleration or a stop)	s
t_4	time needed to cover the fourth trait of a speed profile, if present (up to a stop)	s
S_{br}	position at which the braking phase starts in a speed profile	m
v_{br}	speed at which the vehicle starts braking in a speed profile (maximum speed reached)	m/s
S_{cost}	position at which the speed starts to be constant in a speed profile	m
v_{cost}	constant speed in a speed profile	m/s
a	acceleration in the first trait of a speed profile of the first part, it could be positive or negative	m/s^2
a_{brake}	acceleration in the braking phase at variable acceleration, it must be negative	m/s^2
a_2	acceleration when restarting from a stop, in the second part of the speed profile, it must be positive	m/s^2
m	mass of the vehicle	kg
ρ	air density	$-$
A	bus frontal area	m^2
C_d	drag coefficient	$-$
g	gravity acceleration	m/s^2
C_r	rolling resistance coefficient	$-$

Table 7: Variables