



POLITECNICO
MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Design and Optimization of a Landing Strategy on Didymos for Milani CubeSat

TESI DI LAUREA MAGISTRALE IN
SPACE ENGINEERING - INGEGNERIA SPAZIALE

Author: **Vittoria Gatti**

Student ID: 221180

Advisor: Prof. Francesco Topputo

Co-advisors: Carmine Giordano, Lucia Francesca Civati

Academic Year: 2023-2024

Abstract

The exploration of small celestial bodies, particularly asteroids, has gained increasing attention in recent years due to their significance in understanding Solar System evolution and their potential applications in planetary defense and space resource utilization. These bodies presents unique challenges for spacecraft operations, due to their weak gravitational fields, complex dynamical environments, and high levels of uncertainty.

As part of Hera mission led by ESA within the Asteroid Impact and Deflection Assessment (AIDA) collaboration, Milani CubeSat will conduct close-proximity operations around Didymos binary asteroid system. At the end of the scientific phase, it will attempt an experimental landing on one of the two asteroids. This thesis investigates the feasibility of a landing on Didymos, the primary body, assessing risks, maneuvering costs, and optimal strategies to ensure mission success.

The study begins with the identification of a suitable landing site, considering surface characteristics, dynamical constraints, and scientific objectives. A direct landing strategy is then developed, consisting of two impulsive maneuvers: the first to transition into a descent arc and the second to reduce the touchdown velocity. To evaluate the robustness of the optimized trajectory, a Monte Carlo analysis is conducted, incorporating uncertainties in navigation, maneuver execution, and environmental conditions.

The results confirm that a landing on Didymos is achievable, with the polar region emerging as the most advantageous site due to its higher return velocity, continuous illumination, and reduced risk of post-impact escape. While developed for Milani, this study provides a framework that can be applied to landing strategies on any small celestial body, offering valuable insights for future asteroid exploration missions.

Keywords: binary asteroids, Milani CubeSat, Didymos, irregular gravity, landing trajectory

Abstract in lingua italiana

L'esplorazione dei piccoli corpi celesti, in particolare degli asteroidi, ha suscitato un interesse crescente negli ultimi anni, sia per il loro ruolo nella comprensione dell'evoluzione del sistema solare, sia per le potenziali applicazioni nella difesa planetaria e nell'utilizzo delle risorse spaziali. L'esplorazione di questi corpi comporta numerose sfide a causa dei loro deboli campi gravitazionali, dell'ambiente dinamico complesso e dell'elevato grado di incertezza.

Nell'ambito della missione Hera, controparte europea della collaborazione internazionale NASA-ESA nel progetto AIDA, il CubeSat Milani condurrà operazioni ravvicinate attorno al sistema binario Didymos. Al termine della fase scientifica, tenterà un atterraggio sperimentale su uno dei due asteroidi. Questa tesi analizza la fattibilità di un atterraggio su Didymos, il corpo principale del sistema, valutando i rischi, i costi di manovra e le strategie ottimali per garantirne il successo.

Lo studio inizia con l'identificazione di un sito di atterraggio adeguato, analizzando le caratteristiche della superficie, dei vincoli dinamici e degli obiettivi scientifici. Successivamente, è sviluppata una strategia di atterraggio diretto, composta da due manovre impulsive: la prima per inserire il CubeSat in una traiettoria di discesa e la seconda per ridurre la velocità di impatto. Per valutare la robustezza della traiettoria ottimizzata, è condotta un'analisi Monte Carlo che considera incertezze legate alla navigazione, all'esecuzione delle manovre e alle condizioni dinamiche.

I risultati confermano che l'atterraggio su Didymos è realizzabile. In particolare, la regione polare emerge come il sito più vantaggioso grazie alla maggiore velocità di ritorno, all'illuminazione continua e al minor rischio di fuga dopo l'impatto. Sebbene sviluppato per Milani, questo studio fornisce un quadro metodologico applicabile a strategie di atterraggio su qualsiasi piccolo corpo celeste, offrendo spunti per future missioni di esplorazione di asteroidi.

Parole chiave: Sistema binario di asteroidi, Milani CubeSat, Didymos, gravità irregolare, traiettoria di atterraggio

Contents

Abstract	i
Abstract in lingua italiana	iii
Contents	v
Acronyms	vii
Nomenclature	ix
1 Introduction	1
1.1 Asteroids Context	1
1.2 State of the Art	4
1.3 Research Question	7
1.4 Thesis Overview	8
2 Mission Overview	9
2.1 Hera Mission	9
2.2 Milani CubeSat	11
2.3 Didymos Binary System	14
2.3.1 Why Didymos	15
2.3.2 Physical Properties	15
2.3.3 Orbital Parameters	19
2.3.4 Reference Frames	21
3 Dynamical Environment	23
3.1 Dynamical Regions	23
3.1.1 Didymos Gravitational Model	24
3.1.2 Dimorphos Gravitational Field	29
3.1.3 Solar Radiation Pressure	30

3.1.4	Third Body Effect	31
3.2	Equations of motion	31
4	Landing	33
4.1	Landing Site	33
4.1.1	Geophysical Environment	33
4.1.2	Landing sites selection	43
4.2	Landing Strategy	45
4.2.1	Problem Statement	46
4.2.2	Initial Guess	48
4.2.3	Results	51
4.2.4	Bounce Analysis	54
4.2.5	Trajectory Selection	54
5	Sensitivity Analysis	57
5.1	Monte Carlo Analysis	57
5.2	Results	58
5.2.1	Uncertainty on Initial State	58
5.2.2	Uncertainty on Maneuver	62
5.2.3	Uncertainty on Solar Radiation Pressure	64
5.2.4	Uncertainty on Gravitational Attraction	65
5.2.5	Combined Uncertainties	68
5.3	Final Considerations	70
6	Conclusions	73
6.1	Future Developments	74
	Bibliography	77
	A Appendix A	81
	List of Figures	83
	List of Tables	85

Acronyms

AHP Analytical Hierarchy Process.

AIDA Asteroid Impact and Deflection Assessment.

AU Astronomical Unit.

BM Breaking Maneuver.

CR3BP Circular Restricted Three-Body Problem.

ELM Extreme Learning Machine.

ESA European Space Agency.

GRS Guaranteed Return Speed.

HED howardite–eucrite–diogenite.

ISL Inter-Satellite Link.

LOAM Landing Orbit Acquisition Maneuver.

MBA Main Belt Asteroid.

MC Monte Carlo.

MOID Minimum Orbit Intersection Distance.

NEA Near-Earth Asteroid.

PHA Potentially Hazardous Asteroid.

SCI small carry-on impactor.

SLFN Single-Layer Feedforward Network.

SRP Solar Radiation Pressure.

SSTO Self-Stabilized Terminator Orbit.

TAG Touch-And-Go.

TNOs Trans-Neptunian Objects.

YORP Yarkovsky-O’Keefe-Radzievskii-Paddack.

Nomenclature

μ	Gravitational Parameter	$[\text{m}^3/\text{s}^2]$
Ω	Right Ascension	$[\text{deg}]$
ω	Argument of Pericenter	$[\text{deg}]$
ϕ_{slope}	Slope Angle	$[\text{deg}]$
ϕ_{tilt}	Tilt Angle	$[\text{deg}]$
ρ	Density	$[\text{kg}/\text{m}^3]$
τ	Orbital Period	$[\text{s}]$
A	Area	$[\text{m}^2]$
a	Acceleration	$[\text{km}/\text{s}^2]$
C_r	Reflectivity Coefficient	$[-]$
e	Eccentricity	$[-]$
G	Gravitational Constant	$[\text{m}^3/\text{kg s}^2]$
h	Elevation	$[\text{m}]$
i	Inclination	$[\text{deg}]$
J	Jacobi Constant	$[-]$
M	Mass	$[\text{kg}]$
n_f	Surface Normal	$[\text{km}]$
P_0	1 AU Solar Flux	$[\text{W}/\text{m}^2]$
r	Position Vector	$[\text{km}]$
r_a	Periapsis	$[\text{km}]$
r_p	Pericenter	$[\text{km}]$

U	Gravitational Potential	$[\text{m}^2/\text{s}^2]$
V	Volume	$[\text{m}^3]$
v	Velocity Vector	$[\text{km}/\text{s}]$

1 | Introduction

1.1. Asteroids Context

The Nature of Asteroids

Asteroids are small, rocky celestial objects found throughout the Solar System. They are considered minor planets because they lack atmospheres and do not display comet-like activity, such as a visible coma or tail. Generally, they are classified based on their orbits and physical characteristics [1].

The majority of asteroids reside in the Main Belt Asteroid (MBA), a region located between the orbits of Mars and Jupiter. These bodies vary greatly in size, from Ceres, which has a diameter of about 940 kilometers, to tiny fragments of rock and dust. The Main Belt contains asteroid families formed by collisions, where larger bodies break apart into smaller fragments over time. This process is ongoing, and the distribution of sizes within the belt follows a power-law relationship, with smaller bodies far outnumbering larger ones.

Some asteroids, known as Near-Earth Asteroids (NEAs), have orbits that bring them close to Earth. These bodies are particularly significant due to their potential impact risk and their accessibility for exploration. NEAs are thought to originate from the Main Belt, influenced by gravitational interactions with planets or non-gravitational forces like the Yarkovsky effect, which causes slow orbital drift due to thermal radiation forces. NEAs are considered prime targets for space missions because they offer valuable insights into planetary formation and are more accessible compared to other celestial bodies.

Beyond the Main Belt, other asteroid populations include Trojan asteroids, which share orbits with larger planets like Jupiter, and Centaurs, which orbit between Jupiter and Neptune. Additionally, Trans-Neptunian Objects (TNOs), found beyond Neptune, represent icy bodies that may transition into short-period comets when perturbed. These distant objects challenge the traditional asteroid-comet distinction, as some display characteristics of both categories.

The Importance of Asteroids in Space Exploration

Asteroids are targeted by space missions due to their significant scientific and technological potential, as they are essential for understanding the formation and evolution of the Solar System. As remnants of its early stages, they offer key insights into the processes that led to planetary development and the origins of life, revealing the building blocks of planets and the distribution of vital resources like water and organic materials through their varied composition, which range from metal-rich to carbonaceous types [2]. Additionally, the study of asteroids provides insights into primordial conditions, the delivery of essential elements to Earth, and their potential as resource hubs for future space exploration.

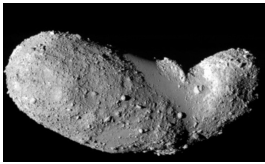
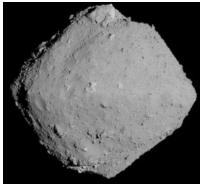
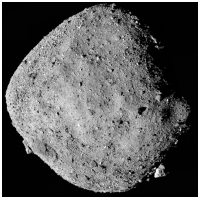
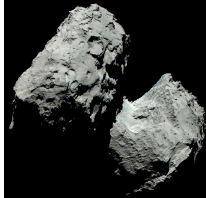
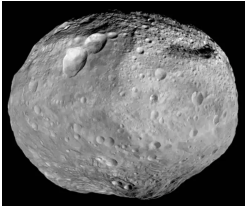
From a planetary defense perspective, asteroids are of great interest because they pose a potential impact hazard to Earth. Studying their orbits, compositions, and physical properties enables us to better assess and mitigate these risks [2]. NEAs are particularly valuable in this regard, as they are easier to reach and provide critical data for refining impact prediction models.

Key Asteroids Studied by Space Missions

Asteroids in nature exhibit a wide range of sizes, from just a few meters to hundreds of kilometers for the largest ones. So far, only three asteroids smaller than one kilometer have been investigated: Itokawa by Hayabusa (JAXA), Bennu by OSIRIS-REx (NASA), and Ryugu by Hayabusa2 (JAXA) [3]. Beyond size, asteroids also vary in composition, with the most common types being C-type, S-type, and V-type. C-type asteroids are carbon-rich, often linked to primitive materials from the early Solar System. S-type asteroids are silicate-dominated and are associated with ordinary chondrites. V-type asteroids are basaltic, suggesting volcanic activity and differentiation in their past.

The main characteristics of asteroids targeted by major missions are summarized in Table 1.1. Among these there is also 67P/Churyumov–Gerasimenko, although not an asteroid but a comet, which was also a significant target, investigated in detail by the Rosetta mission (ESA).

Table 1.1: Overview of key asteroids targeted by major space missions, highlighting their physical properties. Eros: credit NASA/JHU-APL; Itokawa: credit JAXA/ISAS; Ryugu: credit JAXA, University of Tokyo & collaborators, Bennu: credit NASA/Goddard/University of Arizona; 67P/Churyumov-Gerasimenko: credit ESA; Vesta: credit NASA.

Asteroid	Eros	Itokawa	Ryugu
Image			
Class	S-type	S-type	C-type
Dimensions	$\sim 34.4 \times 11.2 \times 11.2$ km	$\sim 535 \times 294 \times 209$ m	$\sim 1.004 \times 0.876$ km
Mission	NEAR Shoemaker	Hayabusa	Hayabusa2
Structure	Elongated shape	Elongated shape, rubble-pile ¹	Spinning-top shape, rubble-pile ¹
Bulk density	2.67 g/cm^3	1.9 g/cm^3	1.19 g/cm^3
Asteroid	Bennu	67P	Vesta
Image			
Class	C-type	Cometary	V-type
Dimensions	$\sim 505 \times 492 \times 457$ m	$\sim 4.1 \times 3.65 \times 3.5$ km	$\sim 572.6 \times 557.2 \times 446.4$ km
Mission	OSIRIS-REx	Rosetta	Dawn
Structure	Roughly spheroidal shape, rubble-pile ¹	Composed of two lobes	Oblate spheroid
Bulk density	1.194 g/cm^3	0.533 g/cm^3	3.456 g/cm^3

¹The rubble pile structure dictates a bulk density that is smaller than the material density from which it is made.

Binary Asteroid Systems

Binary asteroids, which account for approximately 15% of Near-Earth Asteroids and are similarly prevalent in the Main Belt, yield essential understanding of the dynamical and evolutionary processes of the solar system [1]. The first confirmed binary system was identified in 1993, when the Galileo mission discovered the moon Dactyl orbiting the main-belt asteroid (243) Ida [3]. Since then, the study of binary systems has become a central focus of asteroid science.

Binary NEAs are typically divided into four main classes [3]. The most common group, Group A, consists of small primary asteroids (diameter $D < 20$ km) with satellites in close orbits, characterized by diameter ratios D_s/D_p between 0.1 and 0.6. Other groups include binaries with larger satellites (Group B), wider separations (Group W), and larger primary bodies found in the Main Belt (Group L).

The YORP effect, driven by solar radiation, is a primary driver of binary formation for NEAs. It can cause small asteroids to spin up to their fission limit, leading to the ejection of material that reaggregates into a satellite [3]. Observational evidence, such as the spinning-top shapes of many binary primaries, supports this formation mechanism. Similarly, asteroid pairs, distinct from binary systems, are thought to originate from a single progenitor that underwent rotational fission, splitting due to the weak, rubble-pile structure of the body [4]. Conversely, MBA binaries often form through collisional disruption followed by reaccumulation of fragments [1].

The study of binary asteroids is essential for both scientific advancement and planetary defense. By analyzing their mass, orbital parameters, and morphology in detail, it is possible to gain deeper insights into their formation processes. This includes understanding the influence of the YORP effect on their dynamical evolution and investigating the properties of equatorial ridges, which are characteristic features of primary bodies in binary systems. Furthermore, studying material mobility provides valuable information on the mechanisms shaping fast-spinning asteroids. Beyond its scientific importance, their study also contributes to planetary defense strategies by improving the ability to assess and mitigate potential impact threats.

1.2. State of the Art

Over recent decades, space exploration has greatly advanced our understanding of asteroids and comets, overcoming unique challenges posed by their low gravity, irregular shapes, and diverse surface properties. Many missions have approached small bodies with

varying degrees of proximity. Early encounters involved far observations during flybys, while subsequent missions orbited their targets, creating detailed maps of surface and internal properties. A select few spacecraft went even further by physically touching or landing on these objects, conducting direct investigations and collecting material for analysis. These efforts have provided critical insights into the early formation and evolution of the solar system. Below is a chronological overview of some of the most significant missions in small body exploration.

NEAR Shoemaker (1996)

Launched in 1996 by NASA, NEAR Shoemaker was the first spacecraft to orbit and land on an asteroid [5]. Its target, Eros, a near-Earth asteroid, was studied extensively during a year-long orbital mission before the spacecraft executed a controlled descent onto its surface in 2001. This mission, despite not being originally designed for landing, provided revolutionary data on the shape, mass, rotational characteristics, and surface composition of Eros. Furthermore, it measured the ambient magnetic field and advanced our understanding of S-type asteroids and their relationship to meteorites, increasing our knowledge of how asteroids in this class could relate to ordinary chondrites and other small bodies in the solar system.

Hayabusa (2003)

The Japanese Hayabusa mission, launched by JAXA in 2003, was the first asteroid sample return mission [6]. Initially named MUSES-C, the spacecraft targeted the near-Earth asteroid Itokawa, an S-type asteroid about 500 meters in length. Hayabusa arrived at its target in 2005, where it conducted remote sensing observations and attempted to collect surface material. Despite significant technical challenges, Hayabusa managed to return thousands of microscopic particles from the asteroid's regolith. The capsule carrying these samples successfully re-entered Earth's atmosphere in 2010. Analyses of the returned material revealed valuable insights into the structure and composition of rubble-pile asteroids, confirming the primitive and diverse nature of these small bodies. Hayabusa also demonstrated key technologies for future planetary exploration, including ion propulsion and autonomous navigation.

Rosetta and Philae (2004)

The mission of the European Space Agency, Rosetta, was approved in 1993 as part of ESA's Horizon 2000 program and marked a pivotal advancement in comet exploration

[7]. Launched to investigate the origin of the solar system by studying comets, Rosetta targeted 67P/Churyumov-Gerasimenko. In 2014, it became the first spacecraft to rendezvous with a comet, deploying the lander Philae for a historic touchdown. While the landing faced challenges, the mission achieved its goal of analyzing the comet's nucleus and environment. Rosetta provided invaluable insights into the comet's density, dynamic properties, and chemical composition, uncovering complex interactions between volatiles and refractories. These discoveries have fundamentally reshaped our understanding of cometary nuclei, emphasizing their diverse histories and partially processed nature. The mission also facilitated in-depth investigations of the comet's activity, plasma environment, and orbital evolution, offering a comprehensive view of its behavior and transformation.

Dawn (2007)

Launched by NASA in 2007, Dawn was the first spacecraft to orbit two distinct celestial bodies, Vesta and Ceres, in the main asteroid belt [8]. From 2011 to 2012, Dawn studied Vesta, revealing its connection to HED meteorites and uncovering evidence of extensive thermal evolution driven by short-lived radionuclides. In 2015, the spacecraft reached Ceres, where it discovered bright spots and cryovolcanic features. These findings advanced our understanding of planetary differentiation and the role of water in the early history of the solar system, while demonstrating the potential habitability of icy bodies.

Hayabusa2 (2014)

Hayabusa2 was launched by JAXA in 2014 to target the carbonaceous asteroid Ryugu, a C-type near-Earth asteroid [9]. The primary objective of the mission was to investigate the origin and evolution of primitive materials in the early solar system. Hayabusa2 arrived at Ryugu in 2018 and conducted global, local, and microscopic observations through a combination of remote sensing, lander deployments, and sample collection. The spacecraft deployed several instruments, including the MASCOT lander and the small carry-on impactor (SCI), to study the asteroid's surface and subsurface. After collecting samples from Ryugu's regolith, Hayabusa2 successfully returned them to Earth in 2020. These samples have provided critical insights into the composition and physical properties of carbon-rich asteroids, including the presence of hydrated minerals and organic materials.

OSIRIS-REx (2016)

NASA's OSIRIS-REx mission, part of the New Frontiers Program, targeted Bennu, a near-Earth carbonaceous asteroid [10]. Launched in 2016, OSIRIS-REx aimed to investigate the asteroid's composition, structure, and dynamic properties to better understand the early solar system and the potential origins of life. Upon arrival at Bennu in 2018, the spacecraft conducted an extensive survey, using imaging, LIDAR, and spectroscopic instruments to create detailed maps of the asteroid's surface and gravity field. These data were used to identify and evaluate potential sampling sites. The highlight of the mission was the successful collection of regolith during a Touch-And-Go (TAG) maneuver in 2020, following meticulous preparations. The samples, returned to Earth in 2023, offer unprecedented insights into the chemical composition of carbonaceous asteroids and their role in delivering organic materials and water to Earth. Additionally, OSIRIS-REx measured the Yarkovsky effect on Bennu, improving predictions of asteroid orbits and enhancing planetary defense capabilities.

1.3. Research Question

The feasibility of landing on small celestial bodies presents numerous challenges due to their weak gravitational fields, irregular shapes, and highly uncertain surface properties. While extensive studies have focused on landing on Dimorphos, the smaller secondary body of the Didymos system, little attention has been given to the possibility of landing on Didymos itself. This thesis seeks to investigate whether a landing on Didymos is feasible and to assess the associated costs and risks.

The main research question addressed in this work is:

Is it possible to design a feasible and reliable landing strategy on Didymos for Milani CubeSat?

To comprehensively answer this question, the study also explores the following secondary objectives:

1. *What are the maneuvering costs and dynamical constraints required for a successful landing on Didymos?*
2. *Which location on the surface of Didymos provides the most favorable conditions for a safe landing?*
3. *How robust is the landing trajectory when subjected to different sources of uncertainty?*

This research leverages numerical optimization techniques and Monte Carlo simulations to evaluate the most suitable landing trajectory. While the study is specific to Milani CubeSat within the Hera mission, the methodologies developed can be extended to future small-body landings in similar environments.

1.4. Thesis Overview

The thesis is structured as follows:

- Chapter 2 provides an overview of the mission context. It introduces the AIDA collaboration, explaining the objectives of both the DART and Hera missions. Milani CubeSat is presented, highlighting its structure, payload, scientific goals, and operational phases. Finally, Didymos binary system is described, detailing its physical and dynamical properties;
- Chapter 3 focuses on modeling the dynamical environment around Didymos binary system. The chapter begins by identifying different dynamical regions based on dominant perturbations affecting the trajectory. Then, it introduces the gravitational field models for both Didymos and Dimorphos, and the equation utilized to compute the effects of solar radiation pressure and of the third-body influence of the Sun. Finally, the chapter presents the complete equations of motion governing the dynamics of Milani in DidymosJ2000 frame;
- Chapter 4 presents the design and implementation of the landing phase for Milani on Didymos. The geophysical environment of the asteroid is first analyzed, including topographic characteristics, gravitational field variations, and surface forces. The selection of a suitable landing site is then carried out by evaluating illumination conditions, slopes, local surface forces and return speed. Afterward, a direct landing strategy is developed, starting from a self-stabilized terminator orbit and minimizing the cost of the required maneuvers through optimization. The chapter concludes with a bounce analysis to ensure the spacecraft remains on the surface upon touchdown;
- Chapter 5 evaluates the robustness of the landing trajectory by performing a sensitivity analysis through Monte Carlo simulations. The chapter introduces the sources of uncertainty affecting Milani's trajectory, including maneuver execution errors, initial state variations, and dynamical model inaccuracies. A Monte Carlo approach is then applied to propagate perturbed trajectories, assessing their dispersion in terms of touchdown location, velocity at impact, and post-impact conditions.

2 | Mission Overview

2.1. Hera Mission

Overview of AIDA and DART

Hera mission is part of the Asteroid Impact and Deflection Assessment (AIDA), a collaborative effort between NASA and ESA aimed at advancing strategies for planetary defense [3]. The central goals of AIDA are the validation of the effectiveness of the kinetic impact technique for deflecting hazardous Near-Earth Asteroids and the precise quantification of the resulting deflection. This revolutionary mission represents the first deliberate attempt to measurably alter the dynamics of a celestial object.

AIDA consists of two distinct yet complementary missions:

- DART (Double Asteroid Redirection Test), led by NASA, focuses on demonstrating the feasibility of the kinetic impactor technique;
- Hera, led by ESA, is designed to conduct a detailed post-impact analysis to evaluate the results of the deflection.

On September 26, 2022, NASA's DART spacecraft successfully collided with Dimorphos, achieving a significant milestone in planetary defense. This direct impact altered the orbital period of Dimorphos around Didymos by 33 ± 1 minutes, providing definitive evidence of the viability of the kinetic impact method [11].

In addition to its primary mission, DART deployed the LICIACube, a 6U CubeSat developed by the Italian Space Agency. Released 15 days before the collision, LICIACube captured critical images of the impact event, observed the behavior of the ejecta, and documented the hemisphere of Dimorphos not visible to DART. These observations offered invaluable data, confirming the success of the mission, enhancing our understanding of impact dynamics, and improving our knowledge of the physical characteristics of the two asteroids [12].

Hera

Successfully launched on October 7, 2024, Hera is currently traveling towards Didymos binary system, with an expected arrival in early 2027. This mission represents the first comprehensive study of a binary asteroid system and aims to mark a significant milestone in small-body exploration. Its primary objectives focus on planetary defense and advancing asteroid science through detailed investigations of the DART impact and the physical and dynamic properties of Didymos system. Specifically, the scientific goals of Hera include [3]:

1. Impact analysis: characterization of the DART impact crater to refine understanding of cratering processes and the factors influencing momentum transfer;
2. Mass and structure measurements: accurate determination of the mass of Dimorphos and its internal composition to resolve uncertainties and improve impact scaling models;
3. Dynamic observations: detection of small changes in orbital and spin dynamics caused by the impact;
4. Surface and subsurface properties: investigation of the surface of Dimorphos to study regolith mobility, crater formation, and the influence of material properties on impact outcomes.

Additionally, Hera will advance asteroid science through the acquisition of unprecedented data on binary systems, performing comparative studies of Didymos and Dimorphos to investigate their structural differences, analyzing freshly exposed material from the DART impact to refine our understanding of asteroid composition and space-weathering processes, and testing theoretical models of binary asteroid formation. Furthermore, Hera's observations under low-gravity conditions will offer critical insights into the asteroid-meteorite connection and the geophysical properties of small bodies.

Upon arrival at Didymos system, Hera will deploy two CubeSats in close proximity to Didymos and Dimorphos, Juventas and Milani, which will enable a more detailed investigation of both asteroids. Juventas, is equipped with the low-frequency monostatic radar JuRa and the GRASS gravimeter, allowing it to perform the first direct measurements of the interior of Dimorphos and the gravity field. Its primary objectives include determining the asteroid's gravity field, analyzing its interior structure, and examining surface properties. Milani, designed to study the system's surface composition and properties, will be discussed in greater detail in the following section, as it is the focus of this thesis.

2.2. Milani CubeSat

Milani is a CubeSat developed by Tyvak International and launched as a piggyback payload of Hera; its mission analysis is overseen by Politecnico di Milano. By focusing on asteroid characterization, it will provide invaluable data and insights into the dynamics and composition of Didymos binary system, which will enhance the scientific return of the Hera mission, while also demonstrating CubeSat technologies.

Structure and Payload

Milani is a 6U Cubesat with a wet mass of 12 kg. It features deployable solar arrays that provide the necessary power to sustain its systems and a cold-gas thruster propulsion system, providing full six-degree-of-freedom (6-DoF) maneuvering capability for precise attitude and translational control [13]. Its structure is represented in Figure 2.1.

Milani is equipped with advanced payloads, including the ASPECT hyperspectral imager for mineralogical analysis and surface mapping, the VISTA dust analyzer for studying ejected particles, and the NavCam, originally designed for navigation but providing additional scientific imaging capabilities. Additional onboard sensors include ISL antennas, a lidar, star tracker, sun sensors, and an inertial measurement unit for navigation and orientation control [14].

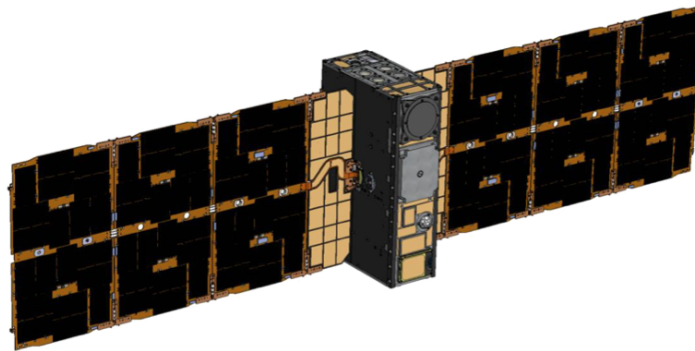


Figure 2.1: Milani CubeSat Configuration [14]

Objectives and Operational Requirements

The objectives of Milani encompass both scientific and technological goals [15]. From a scientific perspective, the CubeSat aims to support Hera in determining the global properties of Didymos binary system, mapping the surface composition of both asteroids, and evaluating space weathering phenomena. It will focus on Dimorphos and its crater, studying surface changes and properties near the impact site using high-resolution imaging

from the ASPECT payload. Additionally, Milani will detect inorganic materials, volatiles, and light organics within the asteroid environment using the VISTA dust analyzer, while contributing to gravity field estimation through range and range-rate measurements exchanged with Hera.

From a technological standpoint, Milani seeks to demonstrate the use of CubeSat technologies in deep space by testing relative positioning with respect to Hera and assessing the effects of the asteroid environment on hardware components. Crucially, it will establish an Inter-Satellite Link (ISL) with Hera to showcase the operability of an inter-satellite network under deep-space conditions.

Milani's operational constraints are shaped by its reliance on Hera and its advanced payload requirements [16]:

- It must remain within 60 km of Hera for effective ISL communications and coordinated operations;
- It must maintain a phase angle below 90° relative to the Didymos system, to optimize imaging and data acquisition;
- Its trajectory, which follows a 4–3 day operational pattern, must be synchronized with the mission team's schedule;
- Its non-rotating solar arrays require careful energy management to sustain operations.

Mission Phases

The Milani mission is divided into several sequential operational phases (illustrated in Figure 2.2), each designed to fulfill specific scientific and technological objectives [17][15]:

- **Near-Earth Commissioning Phase:** During this phase, Milani remains integrated within the Deep Space Deployer onboard Hera. Periodic health checks are conducted to ensure system integrity;
- **Mission Transfer Phase:** Following Hera's commissioning in near-Earth orbit, Milani is transported to Didymos system during a 2.2-year cruise. The CubeSat remains inactive except for periodic health assessments;
- **Ejection and Separation Phase:** Milani is deployed from Hera at approximately 30 km from Didymos, following system status checks and ISL calibration. After ejection, it operates autonomously, undergoing initial commissioning;

- **Commissioning Phase:** This 7-day phase involves system detumbling, attitude acquisition, and equipment calibration, ensuring the CubeSat is fully operational before scientific activities begin. Transfer maneuvers position Milani in its initial scientific orbit;
- **Far-Range Phase:** It is dedicated to the complete mapping of Didymos and Dimorphos at a resolution better than 2 m/pixel using the ASPECT payload. This is achieved through a trajectory comprising six hyperbolic loops, repeated three times, with a 4-3 day maneuvering pattern. Milani hovers over the asteroids at distances between 8 and 11 km, ensuring quasi-symmetric arcs relative to the Sun's direction. The 21-day phase ensures comprehensive surface mapping while balancing operational constraints like illumination and thermal stability;
- **Close-Range Phase:** It focuses on high-resolution mapping (better than 1 m/pixel) and observing the DART impact crater with sub-0.5 m/pixel resolution. To meet these goals, Milani operates at distances as close as 2.78 km from Dimorphos. The trajectory is tailored to specific configurations of the tidally locked system, ensuring crater visibility under optimal illumination near Dimorphos' dawn. A KeyPoint strategy guides Milani's positioning, balancing scientific requirements with operational constraints;
- **Experimental Phase:** Milani transitions into a Self-Stabilized Terminator Orbit (SSTO) phase for 21 days, starting at an initial distance of approximately 21 km from the system, where gravitational forces alone cannot ensure long-term stability. Gradually, through a sequence of SSTOs that reduce its distance in ballistic arcs near the terminator plane, Milani approaches closer to the asteroids, ultimately reaching a stable trajectory around 3.5 km from the barycenter. This phase demonstrates advanced navigation and imaging capabilities, providing high-resolution data while minimizing fuel consumption. The phase concludes with an attempt to land on one of the two asteroids. If landing proves unfeasible, Milani will be transferred to a stable heliocentric graveyard orbit;
- **Disposal Phase:** Upon mission completion, Milani will either remain on Dimorphos post-landing or be passivated in a heliocentric orbit. The CubeSat will then be deactivated to prevent interference with future missions.

Regarding the Experimental Phase, Milani is nominally planned to land on Dimorphos, and previous studies have focused on this scenario [18][19]. This thesis, however, aims to explore the feasibility and strategies for landing on Didymos, the larger primary body of the system.

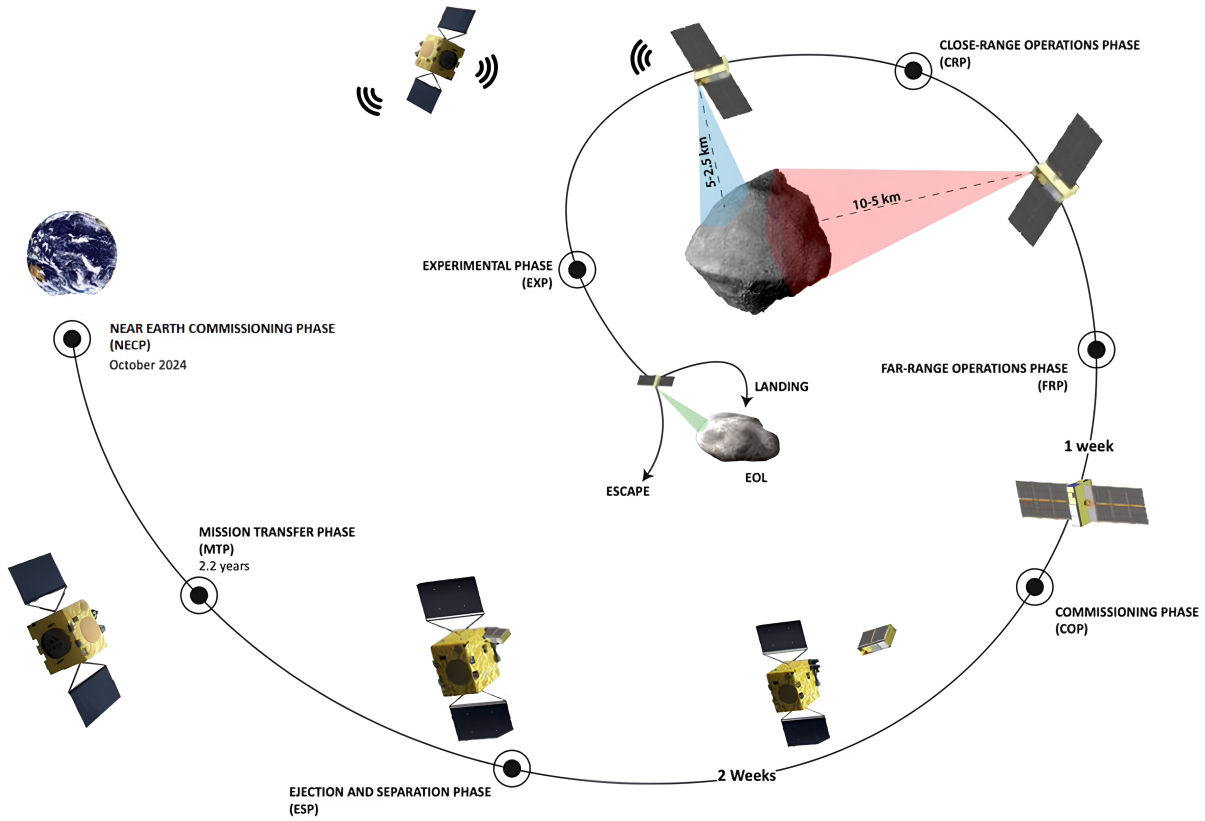


Figure 2.2: Milani CubeSat Mission Phases [15].

2.3. Didymos Binary System

The binary asteroid system (65803) Didymos is composed of the primary asteroid Didymos, also called D1, and its moon Dimorphos, also called D2. Didymos, classified as an S-type NEA [20], was discovered on April 11, 1996, by Joseph Montani of Spacewatch at Kitt Peak National Observatory, who also proposed its name, meaning “twin” in Greek.

Initial evidence of a satellite came from observations at NASA’s Goldstone Solar System Radar, which detected unusual echoes suggesting the presence of a second body. Confirmation came through analyzing optical light curves (i.e. recordings of brightness variations over time) as well as radar images captured by the Arecibo Observatory in Puerto Rico on November 23, 2003 [21].

The origins of binary asteroid systems like Didymos and Dimorphos remain uncertain. The rapid rotation of Didymos supports the hypothesis that Dimorphos may have formed through rotational fission, where material is ejected from the primary due to increasing spin. This spin-up could be caused by an uneven release of thermal radiation from the asteroid’s Sun-warmed surface, generating a gradual torque over time. Over millions of

years, this process may have released material that eventually coalesced into a satellite. However, alternative formation mechanisms remain under investigation [21].

2.3.1. Why Didymos

The mission required selecting a binary asteroid system because assessing the success of the kinetic impact experiment is easier when targeting a secondary body orbiting a primary. This configuration allows for a direct comparison of the secondary's orbit before and after impact, enabling a more precise measurement of the deflection [22].

At the time the DART mission was conceived, only a few binary systems were known, and Didymos stood out as one of the most accessible in terms of ΔV . It also offered the advantage of being one of the most well-characterized systems, both in terms of its physical and orbital properties. Additionally, Didymos' secondary moon is sufficiently small to be measurably deflected by the spacecraft, and its orbit allows for effective measurement of changes through light-curve analysis. Furthermore, the frequent close approaches of the system to Earth made it an ideal candidate for such a mission.

In addition to these considerations, two other key characteristics were necessary for the target: the system had to be a NEA, with a perihelion distance of less than 1.3 AU, and a Potentially Hazardous Asteroid (PHA), with a Minimum Orbit Intersection Distance (MOID) with Earth of less than 0.05 AU and an Absolute Magnitude H of less than 22.

Another advantage of choosing Didymos as the target lies in its composition: analysis of its visible–near-IR reflectance spectrum shows that it is consistent with L/LL chondrites [20], which are among the most common meteorite types. This characteristic is scientifically valuable, as studying such an asteroid can provide insights into the early processes that shaped the Solar System, helping to improve our understanding of planetary formation and the evolution of small bodies within it.

2.3.2. Physical Properties

Before DART mission, direct observations of Didymos and Dimorphos were limited, relying on Earth-based telescopic data and models. Didymos was known to have a "spinning top" shape with a prominent equatorial ridge, a feature that is common among binary asteroid systems. This shape is likely caused by the asteroid's rapid rotation (2.2593 hours), which drives material toward the equator, creating a characteristic bulge [21]. Dimorphos, prior to impact, was thought to have an approximately symmetrical, oblate spheroid shape, similar to a flattened sphere, wider than it is tall. However, high-resolution images

from DRACO and LICIACube captured the immediate post-impact structure, transforming Dimorphos from a symmetrical form into a more irregular, triaxial ellipsoid [23]. The pre- and post-impact shapes of D2 are shown in Figure 2.3 while Table 2.1 and Table 2.2 represent the physical parameters of D1 and D2 before DART mission. The total mass of the system was estimated to be $5.55 \times 10^{11} \text{ kg} \pm 0.42 \times 10^{11} \text{ kg}$ [22].

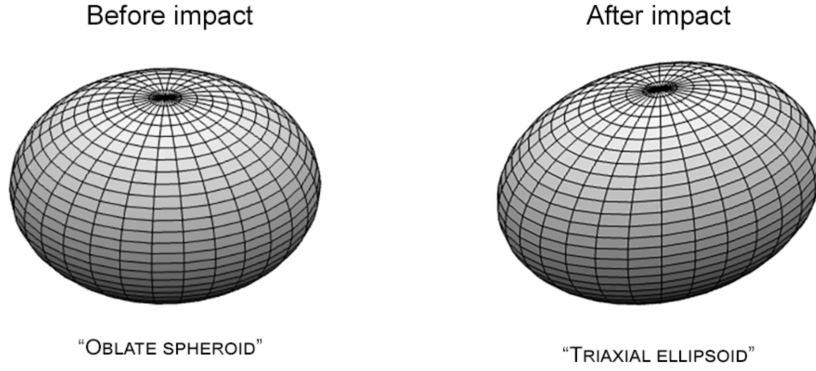


Figure 2.3: Dimorphos shape before and post DART impact [23].

Table 2.1: Pre-impact data of Didymos [22].

Parameter	Value
Extents along principal axes x,y,z	$832 \pm 25 \text{ m}; 837 \pm 25 \text{ m}; 786 \pm 25 \text{ m}$
Diameter	$780 \pm 30 \text{ m}$
Bulk density	$2170 \text{ kg/m}^3 \pm 350 \text{ kg/m}^3$

Table 2.2: Pre-impact data of Dimorphos [22].

Parameter	Value
Elongation a/b and b/c	$1.3 \pm 0.2, 1.2$
Diameter	$164 \text{ m} \pm 18 \text{ m}$
Bulk density	$2170 \text{ kg/m}^3 \pm 350 \text{ kg/m}^3$

Images captured by DRACO camera and LICIACube have provided the first detailed views of Didymos binary system, revealing significant differences between the two bodies [24] and between what was expected. D1 features a flattened shape with undulations along

its equatorial ridge, which is nearly triangular in profile. Its equatorial bulge is consistent with other fast-rotating asteroids, while its flattened poles are unusual, as similar bodies typically exhibit pointed, spinning-top-like shapes. Its surface varies with elevation: the polar regions are rough and scattered with large boulders and craters, while the equator is smoother with fewer large features. In contrast, Dimorphos exhibits a diverse mixture of boulders of all sizes, surface-wide cracks or faults, and a few craters. The surfaces of both asteroids suggest a rubble-pile structure, with boulders relatively large compared to their host bodies. Crater frequency analysis indicates that D1 is significantly older, with an estimated surface age of ~ 12.5 million years, compared to less than 0.3 million years for D2. The post-impact shapes of both asteroids are shown in Figure 2.4.

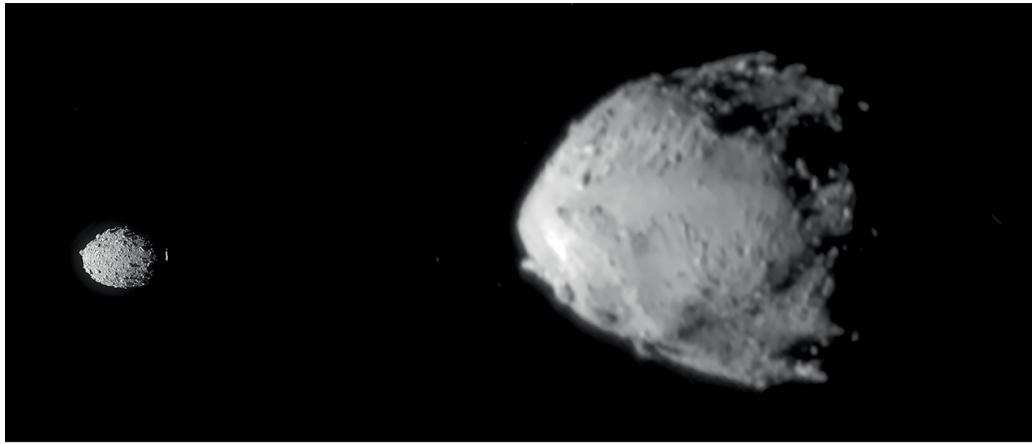


Figure 2.4: Didymos and Dimorphos as captured by DART mission [20].

Building on these detailed observations, the DART mission further refined the dimensions and shapes of both Didymos and Dimorphos. The updated measurements for both asteroids are presented in Table 2.3 and Table 2.4.

Table 2.3: Post-impact data of Didymos, calculated from the shape models, found in [25].

Parameter	Value
Extents along principal axes x,y,z	893.55 m; 811.29 m; 643.16 m
Equivalent radius	364.87 m
Volume	$2.0347 \times 10^8 \text{ m}^3$
Bulk density	2630.9 kg/m^3
Mass	$5.3531 \times 10^{11} \text{ kg}$

Table 2.4: Post-impact data of Dimorphos, calculated from the shape models, found in [25].

Parameter	Value
Extents along principal axes x,y,z	178.53 m; 174.47 m; 116.99 m
Equivalent radius	75.57 m
Volume	$1.8076 \times 10^6 \text{ m}^3$
Bulk density	2630.9 kg/m^3
Mass	$4.7556 \times 10^9 \text{ kg}$

The system mass, equal to $5.4 \times 10^{11} \text{ kg}$, is calculated using Kepler’s third law [20]:

$$M_{sys} = \frac{4\pi^2 a^3}{GP^2} \quad (2.1)$$

with a representing the post-impact semimajor axis, P as the orbital period, and G as the gravitational constant. While this approach does not account for the irregular shapes and gravitational potentials of Didymos and Dimorphos, the resulting mass estimate might only be overestimated by around 1 – 2% [20], an error considered negligible given that uncertainties in Dimorphos’s semimajor axis contribute the most to the overall uncertainty in system mass. Finally, the bulk density of the system is calculated by dividing its mass by the combined volume of both bodies:

$$\rho_{sys} = \frac{M_{sys}}{\frac{4}{3}\pi(R_{D1}^3 + R_{D2}^3)} \quad (2.2)$$

where R_{D1} and R_{D2} represent the volume-equivalent radii of Didymos and Dimorphos respectively, computed as in Equation (2.3).

$$R_{eq} = \sqrt[3]{\frac{3V}{4\pi}} \quad (2.3)$$

The density of Dimorphos is not directly known, so it is assumed to match that of Didymos. Although this assumption cannot be confirmed using DART data alone, studies of other near-Earth binary asteroid systems suggest that primary and secondary bodies often have similar bulk densities, with overlapping uncertainties. While some variations do exist, the equal-density assumption for Didymos and Dimorphos is reasonable. The Hera mission will measure their densities more accurately, allowing for verification of this

assumption [20].

2.3.3. Orbital Parameters

Didymos system belongs to the Amor group II of NEA [21], which are characterized by a semi-major axis between 1.523 and 2.12 AU and moderate orbital eccentricities, causing their orbits to intersect that of Mars. Didymos does not cross the orbit of the Earth, as seen from its heliocentric orbit illustrated in Figure 2.5, but its pericenter lies very close to it, leading to occasional close approaches to our planet. For instance, in 2003, the system approached within 0.048 AU of Earth, leading to its classification as a PHA. Conversely, at its greatest distance, when located on the far side of the Sun relative to Earth, Didymos reaches approximately 3 AU, slightly extending beyond the orbit of Mars.

The orbital parameters of Didymos system, including its perihelion, aphelion, and orbital period, are summarized in Table 2.5.

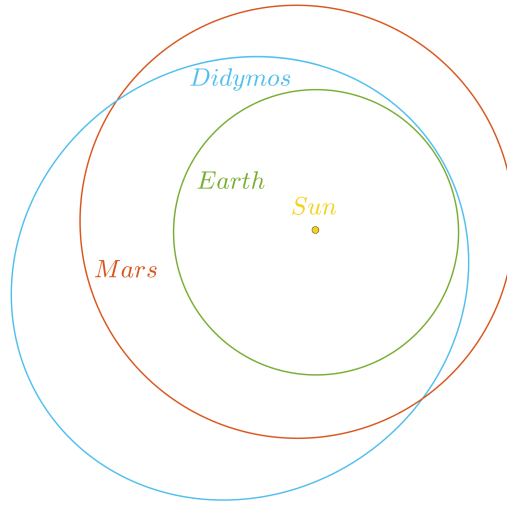


Figure 2.5: Didymos binary system heliocentric orbit.

Table 2.5: Heliocentric orbital parameters of Didymos binary system [26].

a	1.64 AU	ω	319.60 deg
e	0.38325	r_a	2.27 AU
i	3.41 deg	r_p	1.01 AU
Ω	72.98 deg	τ	768.9 day

The motion of the Didymos binary system is retrograde with respect to its orbit around the Sun, as its equatorial plane is nearly inverted relative to the ecliptic plane, with ecliptic coordinates $(\lambda, \beta) = (310^\circ, -84^\circ)$ [27]. This retrograde motion is illustrated in Figure 2.6. Before DART impact, D_2 was assumed to be tidally locked to D_1 , meaning its spin period was synchronized with its orbital period of 11.921 ± 0.0001 hours [11], and it followed a circular orbit with a distance from Didymos of 1189 ± 17 meters [28]. Following the DART impact, the orbital configuration changed significantly: Dimorphos' orbital period decreased by approximately 33.24 ± 0.07 minutes, resulting in a new period of 11.367 ± 0.001 hours [11]. The separation distance also reduced to 1152 ± 18 meters, indicating a shrinkage in the semi-major axis of approximately 37 ± 1 meters [28].

Post-impact, the orbit of Dimorphos transitioned from being circular to slightly elliptical, with an eccentricity $e \sim 0.03$ [11]. Although the impact is believed to have excited Dimorphos' rotational state, leading to a possible tumbling motion, the secondary is still generally assumed to be tidally locked with the primary. In this configuration, the binary's relative motion occurs on the equatorial plane of Didymos, which is also aligned with the orbital plane of Dimorphos [27].

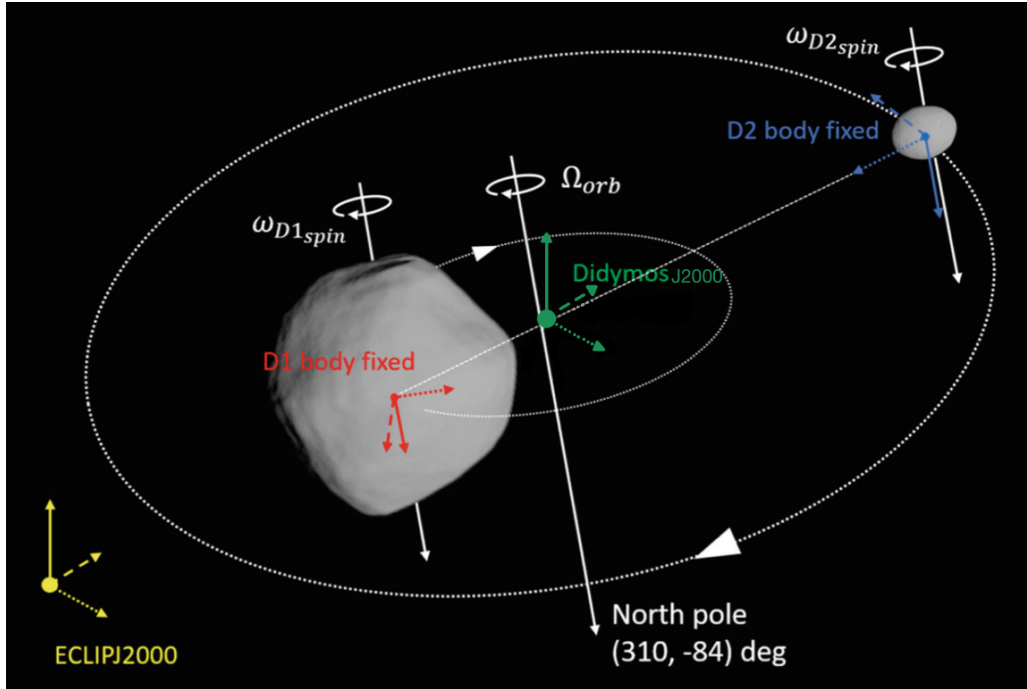


Figure 2.6: Didymos system geometry [27]

2.3.4. Reference Frames

Six main reference frames are employed to study the dynamics within Didymos binary system. These reference frames are detailed in the following list:

DidymosJ2000

The DidymosJ2000 frame is a quasi-inertial reference frame centered at the barycenter of Didymos binary system. Its orientation aligns with the ECLIPJ2000 reference frame:

- The *x-axis* points towards the Vernal line at epoch J2000, on the ecliptic plane
- The *z-axis* is perpendicular to the ecliptic plane
- The *y-axis* completes the right-hand coordinate system

Despite the non-inertial heliocentric motion of Didymos system, this frame can be approximated as inertial for the shorter timescales analyzed, enabling standard numerical integrations without adjustments to the equations of motion.

Didymos and Dimorphos Body-Fixed

The Didymos Body-Fixed and Dimorphos Body-Fixed frames are non-inertial reference frames centered at the respective bodies' centers of mass. These frames rotate with the bodies and their axes are defined as follows:

- The *x-axis* aligns with the North pole
- The *z-axis* points towards the prime meridian
- The *y-axis* completes the right-hand coordinate system

These body-fixed frames are primarily utilized for modeling the gravitational field of the respective bodies using shape models.

D1-D2 Synodic Reference Frame

The synodic system is a non-inertial rotating reference frame centered on the barycenter of Didymos binary system. The orientation of the axes is:

- The *x-axis* is aligned with the line connecting the two primary bodies
- The *z-axis* is orthogonal to the plane of the asteroids orbit

- The y -axis completes the right-hand coordinate system

This reference frame is utilized in the context of the three-body problem to compute the return velocity on the surface of Didymos.

Sun-Didymos Reference Frame

The Sun-Didymos reference frame is a Sun-fixed reference frame, centered in the barycenter of the binary system. Its orientation is:

- The x -axis aligns with the Sun-Didymos system direction
- The z -axis points toward the system's angular momentum
- The y -axis completes the right-hand coordinate system

This reference frame is used to compute the Self-Stabilized Terminator Orbit.

3 | Dynamical Environment

Asteroids, due to their small size and irregular shape, are characterized by a highly complex dynamical environment where even the smallest accelerations can significantly impact the trajectory of the CubeSat, which is particularly sensitive to such perturbations due to its limited size, autonomy, and maneuvering capabilities. To ensure safety and precision of landing operations, this chapter examines the dynamical environment surrounding Didymos binary system by accurately modeling all major accelerations.

3.1. Dynamical Regions

The dynamical environment near Didymos binary system can be divided into three main regions, each characterized by distinct dominant accelerations that influence the dynamics of Milani, depending on its distance from the barycenter of the system [27].

The first region, the *Inner region*, extends up to 1.3 km; within this area, the gravitational effects of D1 (at distances below 1.1 km) and D2 (between 1.1 and 1.3 km) dominate, and for accuracy, their gravitational fields should be computed using shape-based models. Additionally, the effect of Solar Radiation Pressure (SRP) is significant and must be taken into account.

The second region, named the *Mid-range region*, is governed primarily by the gravitational influence of D1. In this region, at distances between 1.3 and 4 km, the gravitational fields of the two asteroids can be modeled using a point-mass model for each body. However, at distances between 4 and 10 km, a single point-mass model can be considered. Similarly to the inner region, SRP must be included in the dynamics.

Finally, in the third region, called the *Outer region*, the third-body effect of the Sun can no longer be neglected. It becomes dominant at distances greater than 1000 km from the barycenter, while between 10 and 1000 km, the SRP effect prevails. In this region, the gravitational influence of D1 and D2 can be approximated using a single point-mass model.

3.1.1. Didymos Gravitational Model

To achieve precise landing on Didymos, an accurate gravitational field representation is essential. For this reason, a shape-based approach is selected as the most suitable. To identify the most appropriate method for this representation, an Analytical Hierarchy Process (AHP) is performed, evaluating four different models: the mascon model, spherical harmonics, the polyhedral model, and a machine learning approach trained on polyhedral data. Each method is assessed based on four criteria: accuracy, applicability, complexity, and computational cost. The following subsections provide an overview of these methods, highlighting their respective advantages and limitations.

Mascon Model

Mascon models approximate the gravity of irregular bodies by representing them as a finite number of point masses, each assigned a specific value to account for the total mass of the asteroid. The acceleration at a specific position $\mathbf{r}$ and time t is derived by calculating the gradient of the mascon-based potential field, which is determined by summing the contributions of all finite-volume mass concentrations [29]:

$$\mathbf{a}_{D1} = -G \sum_{i=0}^N \frac{\rho_i V_i}{|\mathbf{r} - \mathbf{r}_i|^3} (\mathbf{r} - \mathbf{r}_i) \quad (3.1)$$

where ρ_i and V_i denote the density and volume of the i -th mascon, while $\mathbf{r}_i$ represent its distance from the center of the asteroid. This model offers the advantage of being easy to implement and converges accurately to the gravitational field of the specified mass distribution, thus avoiding truncation and divergence errors. Nevertheless, it is susceptible to considerable inaccuracies, which can be partially mitigated by increasing the number of mascons, though this significantly increases computational demands. However, even with an increased number of mass elements, the model remains quite inaccurate close to the surface of the asteroid. Moreover, this method does not provide information on whether the field point is located inside or outside the body.

Spherical Harmonics

The most commonly used expression for gravitational potential is the exterior gravitational field, which is obtained from the orthogonal spherical harmonic coefficients C_{nm} and S_{nm} associated with Legendre functions P_{nm} of degree n and order m , as shown in Equation (3.2) [30]:

$$U^e(r, \phi, \lambda) = \frac{GM^*}{R_e^*} \sum_{n=0}^{\infty} \sum_{m=0}^n \left(\frac{R_e^*}{r} \right)^{n+1} P_{nm}(\sin \phi) \begin{bmatrix} \cos m\lambda \\ \sin m\lambda \end{bmatrix} \cdot \begin{bmatrix} C_{nm}^e \\ S_{nm}^e \end{bmatrix} \quad (3.2)$$

where G is the gravitational constant, M^* is the reference mass (usually the body total mass), R_e^* is the exterior reference radius, r is the spacecraft position, λ and ϕ are the longitude and latitude in the body-fixed frame.

The convergence of this series is assured only outside the Brillouin sphere, which is defined as the sphere with a radius equal to the maximum radius of the body. Consequently, these models are considered highly accurate for operations conducted at significant distances from the asteroid, but they are unsuitable for tasks near the surface, such as landing. To address these divergence issues, it is possible to calculate the potential of the interior gravity field by using a different formula:

$$U^i(r, \phi, \lambda) = \frac{GM^*}{R_i^*} \sum_{n=0}^{\infty} \sum_{m=0}^n \left(\frac{r}{R_i^*} \right)^n P_{nm}(\sin \phi) \begin{bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{bmatrix} \cdot \begin{bmatrix} C_{nm}^i \\ S_{nm}^i \end{bmatrix} \quad (3.3)$$

In this case, the series converges within the Brillouin sphere, which this time is located outside the body and tangent to a point on its surface. The size of the interior Brillouin sphere is arbitrary; however, it is essential to consider that the accuracy of the interior gravity field is influenced by the position of its center.

Figure 3.1 illustrates the regions of convergence (solid lines) for both the internal and external gravitational fields.

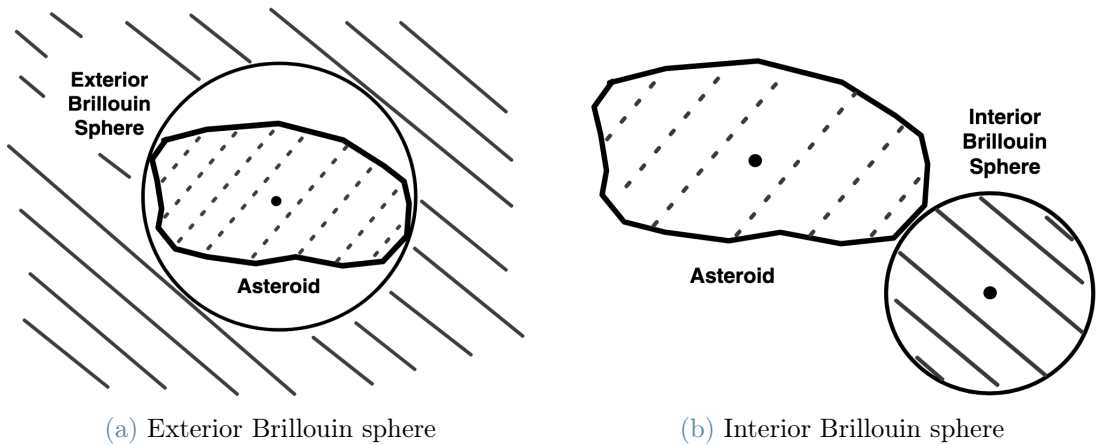


Figure 3.1: Convergence regions of the exterior and interior gravity fields [30].

A significant limitation of the interior gravity field is the need to compute the potential within the corresponding Brillouin sphere for each radius of the asteroid, as each sphere

is tangent to a single point on the surface. This requires multiple calculations to achieve a complete mapping of the surrounding gravitational field, substantially increasing computational costs.

Additionally, although the accuracy of the model can be enhanced by choosing the appropriate order and degree of the series, it remains an approximation of the true gravitational potential, subjected to truncation errors. It is important to note that for bodies with an aspherical shape, the accuracy significantly diminishes. Lastly, similar to the mascon model, the spherical harmonics approach does not offer a method to determine whether the field point lies within or outside the body.

Polyhedral Model

The polyhedral model is based on the assumption that the asteroid can be represented as a polyhedron with a constant density. A polyhedron is defined as a three-dimensional solid whose surface comprises flat faces that intersect along straight edges or meet at isolated points known as vertices [31]. The most common model consists of triangular faces, each associated with three vectors $\mathbf{r}_{ijk}^f$, which extend from the field point to the three vertices of the faces and follow a counterclockwise order around the outward-pointing face normal $\hat{\mathbf{n}}_f$. Each edge, on the other hand, has two associated position vectors, $\mathbf{r}_{ij}^e$ with respect to the field point, and the two adjacent faces, denoted with subscripts A and B . Additionally, there are the normal vectors $\hat{\mathbf{n}}_{ij}^A$ and $\hat{\mathbf{n}}_{ij}^B$, which lies in the plane of the face, are orthogonal to both the vector along the edge and the face normal vector, and point outward.

With this approach, the asteroid's gravitational field is derived analytically and expressed in closed form, providing an exact solution up to the surface of the body without divergence regions. The gravitational acceleration is then obtained as the gradient of the potential [31].

$$\mathbf{a}_{D1} = -G\rho \left[\sum_{e \in \text{edges}} (\mathbf{E}_e \cdot \mathbf{r}_e) L_e - \sum_{f \in \text{faces}} (\mathbf{F}_f \cdot \mathbf{r}_f) \omega_f \right] \quad (3.4)$$

where $\mathbf{E}_e$ and $\mathbf{F}_f$ represent the dyads associated with edge e and face f , respectively:

$$\begin{cases} \mathbf{E}_e = \hat{\mathbf{n}}_A (\hat{\mathbf{n}}_{12}^A)^T + \hat{\mathbf{n}}_B (\hat{\mathbf{n}}_{12}^B)^T \\ \mathbf{F}_f = \hat{\mathbf{n}}_f (\hat{\mathbf{n}}_f)^T \end{cases} \quad (3.5)$$

while L_e denotes the potential of a wire corresponding to edge e and ω_f is the solid angle

associated with face f . They are dimensionless scalars computed as:

$$\begin{cases} L_e = \ln \frac{r_i + r_j + e_{ij}}{r_i + r_j - e_{ij}} \\ \omega_f = 2 \arctan \frac{\mathbf{r}_i \cdot (\mathbf{r}_j \times \mathbf{r}_k)}{r_i r_j r_k + r_i (\mathbf{r}_j \cdot \mathbf{r}_k) + r_j (\mathbf{r}_k \cdot \mathbf{r}_i) + r_k (\mathbf{r}_i \cdot \mathbf{r}_j)} \end{cases} \quad (3.6)$$

Lastly, $\mathbf{r}_e$ and $\mathbf{r}_f$ are the position vectors from the field point to any point on the edge or face, expressed in the body-fixed reference frame.

Being an exact method, the errors are entirely due to the determination of the shape of the asteroid and the chosen discretization. Another advantage of the polyhedral model is the existence of a procedure to determine whether a field point is inside or outside the body. This can be assessed by calculating the Laplacian of the potential; if it is zero, then the Laplace equation is satisfied, and the point lies outside. Otherwise, the Poisson equation is satisfied, and the point lies inside:

$$\begin{cases} \nabla^2 U = 0 & \text{if } r \text{ outside} \\ \nabla^2 U = -4\pi G\sigma & \text{if } r \text{ inside} \end{cases}$$

Regarding the disadvantages, this model is computationally intensive because calculating the total potential requires summing the contributions from each tetrahedron. Moreover, an accurate model of the asteroid is essential, which may not always be available. Lastly, assuming a constant density can lead to inaccurate results, especially for bodies with significant density variations.

Machine Learning

In this approach, an Extreme Learning Machine (ELM) is employed to design, train, and validate a Single-Layer Feedforward Network (SLFN) capable of learning the non-linear relationship between the position of the spacecraft and gravitational acceleration [32]. Neural networks based on ELM are particularly efficient, as they eliminate the need for iterative tuning during training, significantly reducing computational time.

To generate the training set that captures the desired functional relationship, polyhedral models are used to accurately represent the spatial variations in the gravitational field around the small body. Sample points of the spacecraft position are used to create this training data.

Once the ELM-based SLFN is trained and validated, it can process new input positions

and quickly compute the corresponding gravitational acceleration. This method is particularly advantageous for scenarios requiring rapid and accurate calculations of the gravity field as a function of position, such as on-board trajectory propagation during spacecraft operations near the surface of highly irregular celestial bodies. However, a drawback of this approach is the high computational cost of the initial training phase, as both dataset generation and network training require significant computational resources.

Trade-off Results

The results of the AHP are shown in Table 3.1 and below is explained the methodology used to obtain them.

- **Applicability and Accuracy:** Applicability refers to how well-suited a model is for the specific mission objectives. For a landing mission, the chosen model must be as accurate as possible. Thus, the polyhedral model, which provides an exact solution, scores the highest in both applicability and accuracy. Although the machine learning approach achieves nearly the same level of accuracy as the polyhedral model, it is less suitable for this specific case. This is because its primary advantage (fast gravitational acceleration computation) makes it particularly useful for on-board applications, where real-time processing is required. However, Milani operates in an open-loop configuration, meaning all necessary computations can be performed on ground, where computational resources are not a limiting factor. Consequently, the need for rapid onboard calculations does not justify the high cost of training a neural network, which requires a significant number of polyhedral models;
- **Complexity:** Regarding the complexity of each method, the Mascon model is the simplest. Its gravitational acceleration formula is a straightforward summation of the contributions from each individual mascon. The spherical harmonics and polyhedral model are comparatively more complex. The former requires the calculation of coefficients for both the external and internal gravitational fields, along with the computation of Legendre polynomials. The polyhedral model involves geometric calculations associated with the polyhedral shape of the asteroid. In the case of machine learning, developing and training a neural network also adds to the complexity, making it a similarly challenging method;
- **Computational Cost:** The polyhedral model has the highest computational cost, as it requires a set of operations for each face and edge of the polyhedron. The Mascon model's computational cost depends on the number of mascons used, which varies according to the desired accuracy level. If both the external and internal grav-

itational fields are calculated, the computational cost for spherical harmonics also increases. Finally, when considering an already-trained network, machine learning is the least computationally demanding.

In conclusion, it can be noted that the polyhedral model achieves the highest score and is therefore the selected method for calculating the gravitational acceleration of Didymos.

Table 3.1: Analytical Hierarchy Process for Didymos gravity field model.

	Accuracy	Applicability	Complexity	Comp Cost	Results
Mascon Model	4.3%	9.1%	59.2%	25.5%	0.12
Spherical Harmonics	20.6%	24%	12.6%	25.5%	0.22
Polyhedral Model	37.8%	50.3%	21.7%	8.6%	0.40
Machine Learning	37.3%	16.6%	6.4%	40.5%	0.26
Weights	32.2%	50.1%	5.2%	12.5%	

3.1.2. Dimorphos Gravitational Field

Given that Dimorphos is smaller than Didymos and that Milani will be at a greater distance from it, a less precise model can be used to calculate its gravitational acceleration. However, since the landing area is within the inner region, a shape-based model is still required. Due to the elongated shape of D2, a triaxial ellipsoid model is applied by assuming the asteroid approximates an ellipsoid with uniform density. The gravitational acceleration can then be determined by calculating the gradient of the potential [33]:

$$\begin{aligned}
\mathbf{a}_{D2} = & \frac{2\pi G\rho abc}{\sqrt{a^2 - c^2}} \left\{ \left[1 - \frac{2x}{a^2 - b^2} + \frac{2y}{a^2 - b^2} \right] F(\omega_k, K) \right. \\
& + \left[\frac{2x}{a^2 - b^2} - \frac{2y(a^2 - c^2)}{(a^2 - b^2)(b^2 - c^2)} + \frac{2z}{b^2 - c^2} \right] E(\omega_k, K) \\
& \left. + \left[\frac{c^2 + k}{b^2 - c^2} 2y - \frac{b^2 + k}{b^2 - c^2} 2z \right] \frac{\sqrt{a^2 - c^2}}{\sqrt{(a^2 + k)(b^2 + k)(c^2 + k)}} \right\}
\end{aligned} \tag{3.7}$$

where $a \geq b \geq c$ are the semi-axes of the ellipsoid; $\mathbf{r} = [x, y, z]$ is the position vector between

the field point and the ellipsoid in the principal axis frame with the axes aligned with a , b , c ; k is the algebraically largest root of the Equation (3.8):

$$\frac{x^2}{a^2 + k} + \frac{y^2}{b^2 + k} + \frac{z^2}{c^2 + k} = 1 \quad (3.8)$$

while $F(\omega_k, K)$ and $E(\omega_k, K)$ are the Legendre's elliptic integrals of the first and second kind:

$$\begin{cases} F(\omega_k, K) &= \int_0^{\omega_k} \frac{d\varphi}{\sqrt{a - K^2 \sin^2 \varphi}} \\ E(\omega_k, K) &= \int_0^{\omega_k} \sqrt{1 - K^2 \sin^2 \varphi} d\varphi \end{cases} \quad (3.9)$$

with K and ω_k equal to:

$$\begin{cases} K &= \sqrt{\frac{a^2 - b^2}{a^2 - c^2}} \\ \omega_k &= \arcsin \left(\sqrt{\frac{a^2 - c^2}{a^2 + k}} \right) \end{cases} \quad (3.10)$$

3.1.3. Solar Radiation Pressure

The acceleration due to SRP, which pushes the CubeSat in the direction opposite to the Sun, is particularly significant around small bodies like asteroids, where the gravitational field is weaker compared to that of planets. Consequently, this effect must be considered when calculating the dynamics around Didymos binary system. It can be calculated using a cannonball model where the CubeSat is assumed to have a constant area perpendicular to the direction of the Sun [27]:

$$\mathbf{a}_{SRP} = \frac{P_0}{c} \left(\frac{AU}{r_{C-S}} \right)^2 \frac{C_r A}{M} \hat{\mathbf{r}}_{C-S} \quad (3.11)$$

where $P_0 = 1361 \text{ W/m}^2$ is the solar flux at 1 AU, $c = 2.99 \times 10^8 \text{ m/s}$ is the speed of light, $\mathbf{r}_{C-S}$ is the Cubesat-Sun position vector in DidymosJ2000, C_r is the reflectivity coefficient of Milani, A is its equivalent surface area and M is its mass. The data related to the spacecraft are reported in Table 3.2.

Table 3.2: Milani parameters for the computation of the Solar Radiation Pressure [34].

Parameter	Value
C_r	1.25
A	0.25 m ²
M	12 kg

3.1.4. Third Body Effect

As the distance from the barycenter of the binary system increases, the influence of the Sun as a third body becomes significant. The resulting gravitational acceleration accounts for the Keplerian gravity gradient between the barycenter of the Didymos system, which orbits the Sun, and the field point [27]:

$$\mathbf{a}_{3Bs} \simeq \mu_S \left(\frac{\mathbf{r}_{B-S}}{|\mathbf{r}_{B-S}|^3} - \frac{\mathbf{r}_{C-S}}{|\mathbf{r}_{C-S}|^3} \right) \quad (3.12)$$

where $\mu_S = GM_S$ is the gravitational parameter of the Sun while $\mathbf{r}_{B-S}$ is the position vector between the Sun and the barycenter of Didymos system, in DidymosJ2000. This equation represents an approximation, as the gravitational parameter of the binary system is neglected due to its significantly smaller value compared to that of the Sun.

3.2. Equations of motion

The motion of Milani near Didymos system can be described through a comprehensive set of equations. A Newtonian approach is adopted, modeling the trajectory of the CubeSat as the result of the combined effect of multiple accelerations. This equation, expressed in DidymosJ2000 reference frame and shown in Equation (3.13), provides a complete representation of the dynamics of Milani in the vicinity of Didymos system.

$$\ddot{\mathbf{r}} = \mathbf{a}_{D1} + \mathbf{a}_{D2} + \mathbf{a}_{SRP} + \mathbf{a}_{3Bs} \quad (3.13)$$

It is important to note that the previously defined equations for $\mathbf{a}_{D1}$ and $\mathbf{a}_{D2}$ are expressed in the body-fixed reference frame of the asteroids. Therefore, before these accelerations can be combined with the other two, they must first be rotated into DidymosJ2000 reference frame, as shown below:

$$\mathbf{a}_{D1,J2000} = R_{D1}\mathbf{a}_{D1,BF} \quad (3.14)$$

$$\mathbf{a}_{D2,J2000} = R_{D2}\mathbf{a}_{D2,BF} \quad (3.15)$$

where R_{D1} and R_{D2} are the Rotational matrices from Didymos and Dimorphos body-fixed reference frames to DidymosJ2000.

4 | Landing

Landing on small celestial bodies, such as asteroids and comets, presents significant challenges due to their unique physical and dynamical characteristics. Their extremely low gravity leads to escape velocities of just a few centimeters per second, increasing the risk of drifting away from the asteroid upon contact.

The complexity of landing is further amplified by the irregular shape and rapid rotation of these bodies, as well as the uncertainty in surface properties, such as strength, composition, and topography, which remain largely unknown until close-proximity observations.

This chapter analyzes the geophysical environment of D1 to assess the forces acting on its surface and to identify a suitable landing site, and implements a strategy for the landing.

4.1. Landing Site

Identifying a landing region that minimizes risks while ensuring scientific return requires analyzing geophysical environment to assess stability and safety, while considering illumination constraints to maximize scientific output, ensuring optimal lighting for image acquisition during descent and touchdown.

4.1.1. Geophysical Environment

A detailed analysis of the surface of Didymos is conducted to identify potential landing sites and understand the forces acting on the spacecraft during descent. Given the irregular shape of the asteroid, topographic features such as elevation variations and surface orientation are analyzed to ensure a stable landing. To achieve this, the shape model of the asteroid, consisting of 196 608 facets, is used to extract relevant geometric properties. However, due to the high computational cost of the polyhedral model, all calculations involving gravitational potential or acceleration are performed using a simplified 2000-face model. It is important to note that these analyses are highly dependent on the assumed density and polyhedral model, introducing a high degree of uncertainty into the results.

Radius and Elevation

Figure 4.1 shows both the radial distance and elevation of the surface, with the elevation calculated using Equation (4.1). In this equation, $\mathbf{r}$ represents the position vector of each face relative to the center of the model, while R_{eq} is the equivalent radius, computed as in Equation (2.3).

$$h = |\mathbf{r}| - R_{eq} \quad (4.1)$$

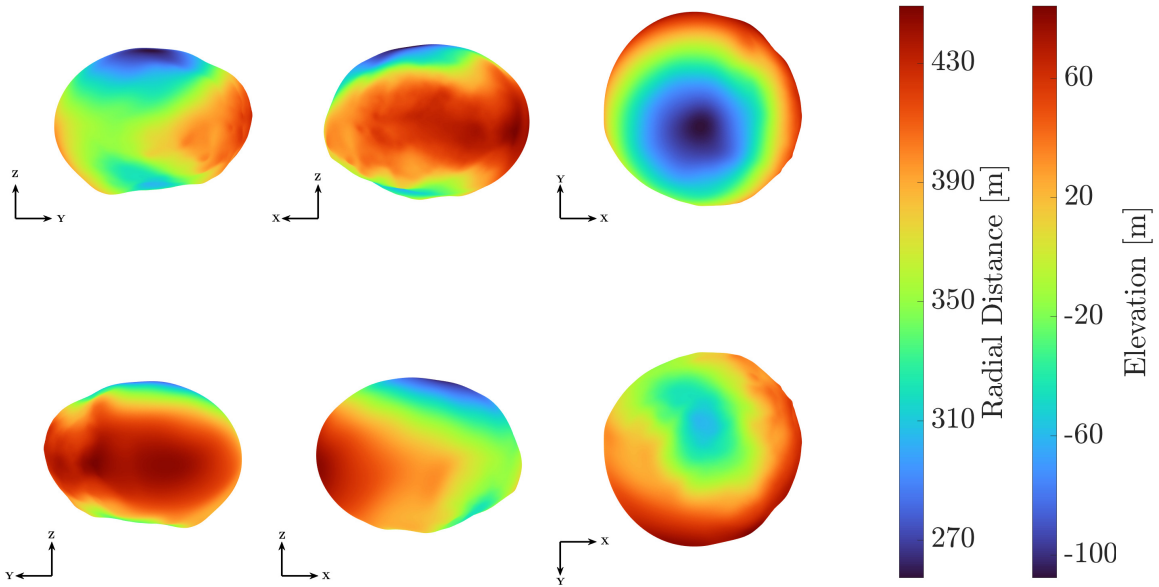


Figure 4.1: Didymos radial distance and elevation mapped over the surface of the asteroid, Didymos-fixed reference frame.

Figure 4.1 shows that the equatorial ridge stands out with elevations significantly higher than those in other areas, while the polar regions are notably flattened, with a lower radius compared to the rest of the surface. This geometric structure is important for understanding the asteroid's shape and interpreting the results of upcoming analyses.

Tilt Angle

Another important aspect is the surface orientation, characterized by the tilt angle, which represents the angle between the surface normal and the radius vector of the faces with

respect to the center of the polyhedron. It is expressed as follows [35]:

$$\phi_{\text{tilt}} = \arccos(\hat{\mathbf{r}} \cdot \hat{\mathbf{n}}_f) \quad (4.2)$$

Figure 4.2 shows the contour map of the tilt angle mapped over the surface of Didymos. This angle ranges from nearly 0° to a maximum of 40° . Selecting a region with the lowest possible tilt angle is preferable, as flatter surfaces enhance the stability of the CubeSat upon touchdown, minimizing the risk of sliding or instability during and after landing. However, the figure indicates that no areas with excessively high inclinations are present.

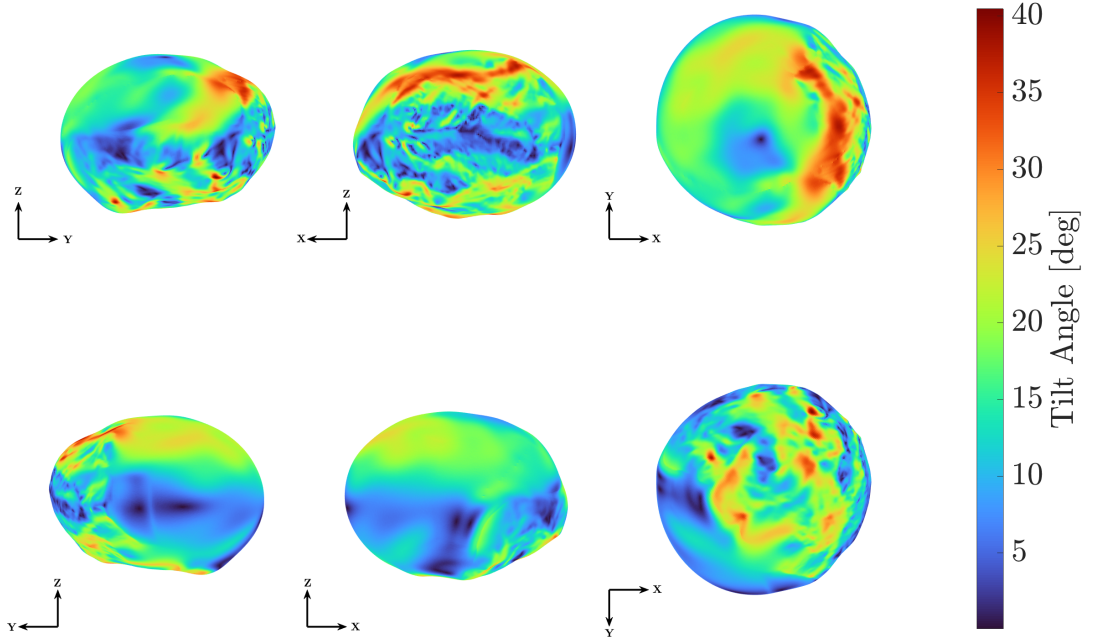


Figure 4.2: Tilt angle mapped over the surface of Didymos, Didymos-fixed.

Gravity Field

Given the need for high precision during the landing phase, the polyhedral model is selected as the most accurate approach for modeling the gravitational field, as discussed in Section 3.1.1. To further justify this choice, an analysis is conducted to quantify the deviation of the polyhedral gravity model from the point-mass gravity model, in both magnitude and direction. For magnitude, the metric parameter $\hat{a}_g$ is employed, while for direction, the separation angle θ_g between the polyhedral gravity vector and the point-mass gravity vector is calculated [36]:

$$\begin{cases} \hat{a}_g &= \frac{r^2}{\mu} a_g - 1 \\ \theta_g &= \arccos \left(-\frac{\mathbf{a}_g \cdot \hat{\mathbf{n}}_f}{a_g} \right) \end{cases} \quad (4.3)$$

where $\mathbf{a}_g$ is the gravitational acceleration, computed as in Equation (3.4), μ is the gravitational parameter of Didymos, r is the field point radius with respect to the center of the polyhedral model, $\hat{\mathbf{n}}_f$ is the face normal. A positive value of $\hat{a}_g$ indicates that the gravitational influence of the polyhedron model at the evaluation point is greater than that of the point-mass model.

The results of this analysis are shown in Figure 4.3 and Figure 4.4, where the parameters are computed for each face. It can be seen that in the equatorial regions, where the radius is larger, the point-mass model underestimates the actual gravitational acceleration computed using the shape-based model. Conversely, near the poles, where the radius is smaller, the point-mass approximation results in an overestimation of gravitational acceleration. On the other hand, the gravity direction deviation remains mostly minimal, with notable increases observed only in specific localized zones. Gravitational field discrepancies range from -49 % to 45.26 % in magnitude, with directional variations up to 29.3°. This analysis confirms that the choice of the polyhedral model is necessary for accurate surface predictions.

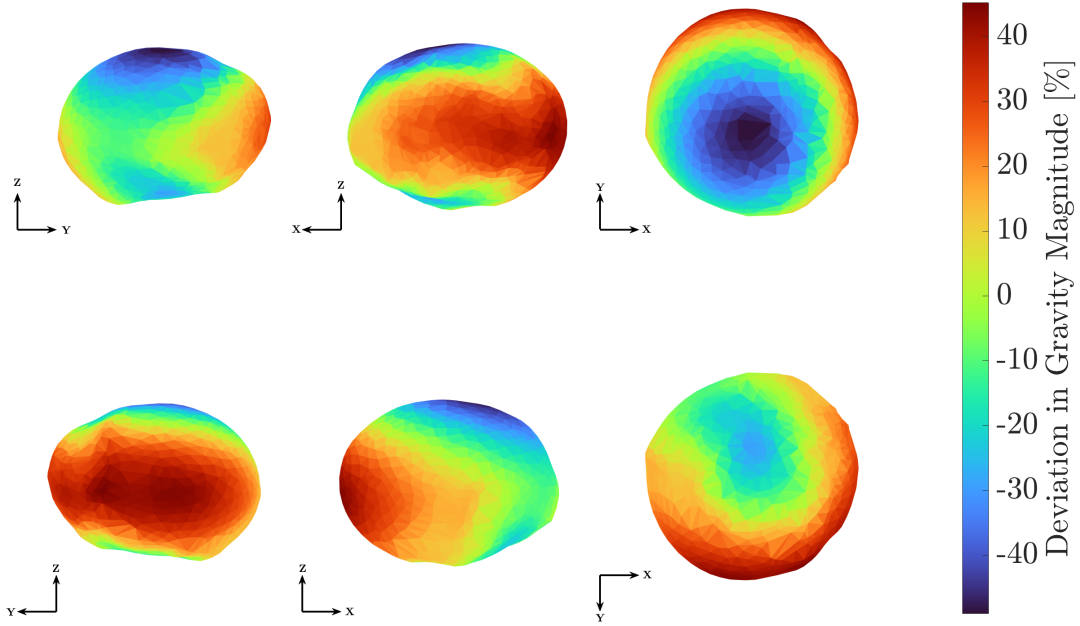


Figure 4.3: Deviation in magnitude of the gravitational acceleration, polyhedral with respect to point-mass model, Didymos-fixed.

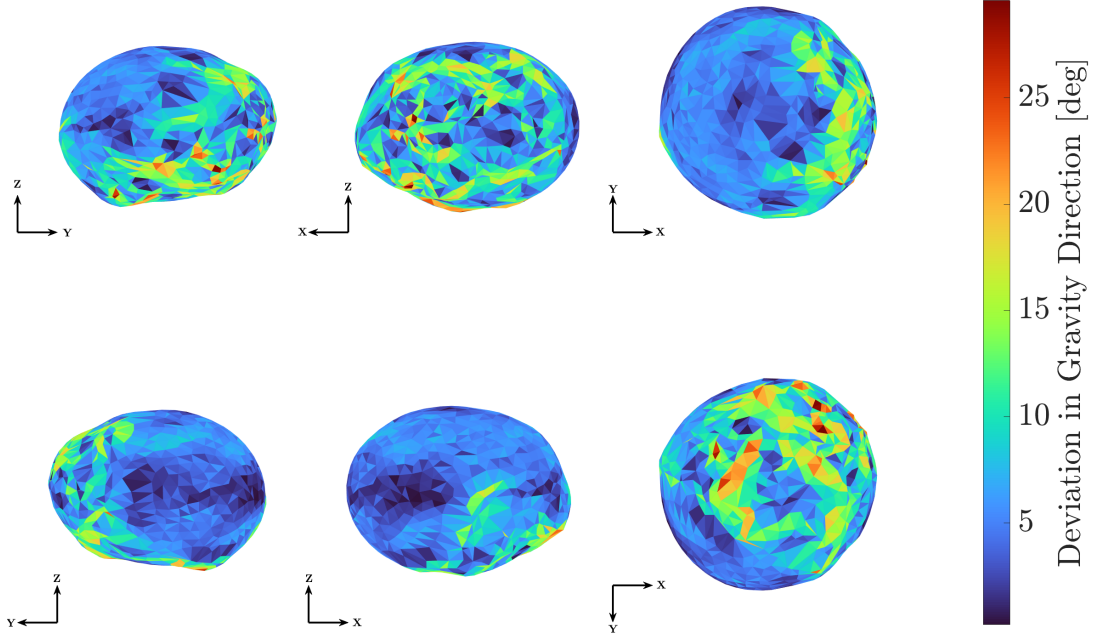


Figure 4.4: Deviation in direction of the gravitational acceleration, polyhedral with respect to point-mass model, Didymos-fixed.

Net Surface Acceleration

In the low-gravity environment of asteroids, rotational dynamics significantly influence surface conditions, affecting both the stability and motion of particles on the surface. To better understand these effects, an analysis of the forces acting on the surface is conducted. Specifically, the forces experienced by a particle at any point on the asteroid's surface result from the net surface acceleration, which combines gravitational and centrifugal effects and, for an asteroid in uniform rotation, can be calculated as [36]:

$$\mathbf{a}_s = -\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \mathbf{a}_{D1} - \frac{\mu_{D2}}{|\mathbf{r} - \mathbf{r}_{D2}|^3}(\mathbf{r} - \mathbf{r}_{D2}) \quad (4.4)$$

where $\boldsymbol{\omega}$ is the angular velocity vector of D1, $\mathbf{r}$ is the field point position vector in the body-fixed frame, μ_{D2} is the gravitational parameter of D2, $\mathbf{r}_{D2}$ is its position vector relative to D1. The net surface acceleration mapped over the surface of Didymos is shown in Figure 4.5. The highest values are found at the poles, while lower values are observed along the equator. This trend can be explained by considering the variations in gravitational and rotational contributions at different latitudes. At the poles, where the radial distance from the asteroid's center is smaller, a stronger gravitational acceleration is expected. Additionally, centrifugal forces are minimized due to the alignment of the position vector

with the rotation axis. Conversely, at the equator, where the radial distance is greater, gravitational acceleration is weaker, while the rotational contribution is more significant.

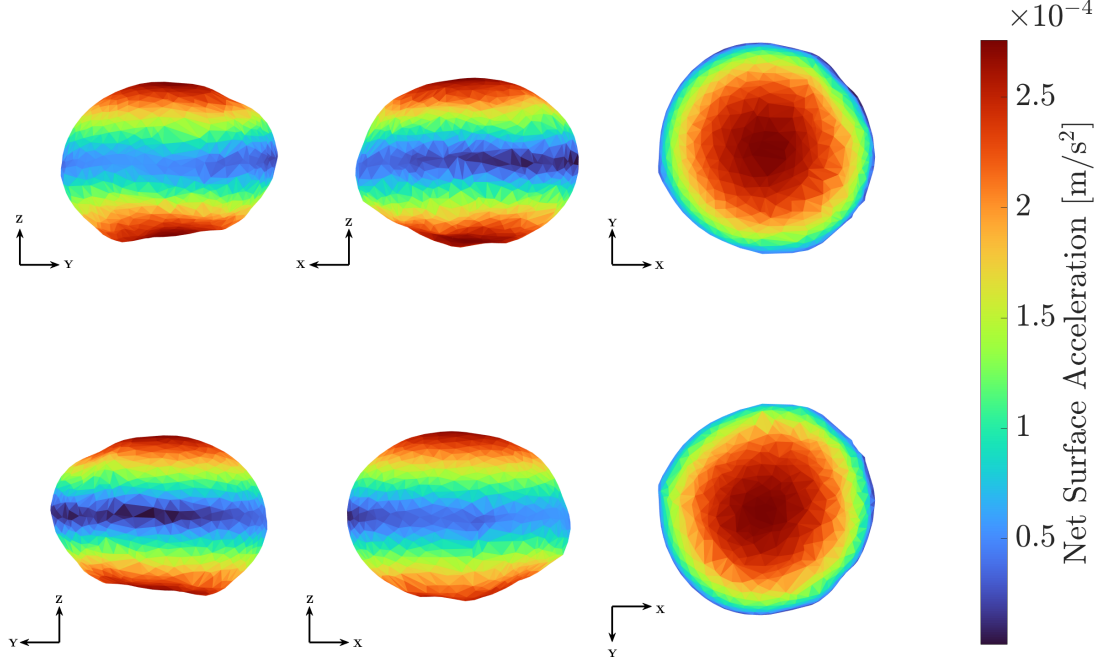


Figure 4.5: Net surface acceleration mapped over the surface of Didymos, Didymos-fixed.

To further analyze the contributions of the two effects, Figure 4.6 shows the value of the gravitational acceleration alone.

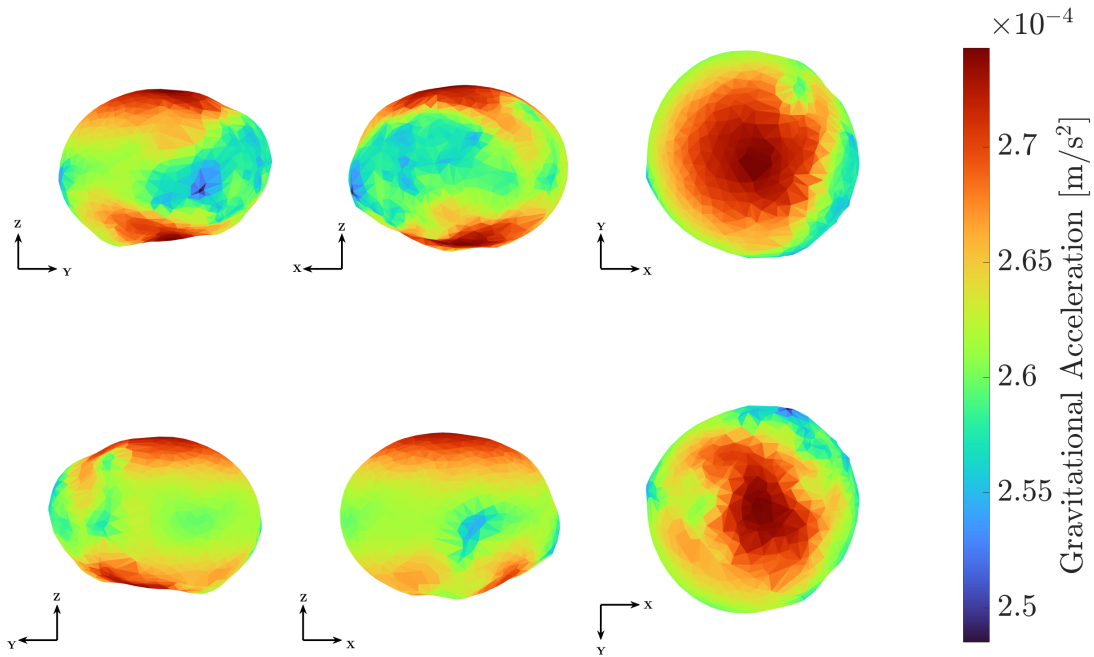


Figure 4.6: Gravitational acceleration mapped over the surface of D1, Didymos-fixed.

This comparison clearly shows that the rotational contribution acts in opposition to gravity, reducing the net acceleration. This effect is particularly evident at the equator, where the decrease in net acceleration is more pronounced than at the poles. This confirms that centrifugal forces counteract gravity more strongly at the equator than in polar regions.

As a result, the equator, where net surface acceleration is lower, is more unstable, as weak gravity or high centrifugal forces can lead to the dispersion or ejection of surface material, making it unsuitable for landing. Conversely, the poles, characterized by higher acceleration where gravity dominates, reduce the risk of slippage. To further examine the effects of these forces on surface stability, the slope angle is analyzed.

Slope Angle

The slope angle, defined as the supplement of the angle between the surface normal vector $\hat{\mathbf{n}}_f$ and the net surface acceleration vector $\mathbf{a}_s$ at a given location, is expressed as [37]:

$$\phi_{\text{slope}} = \pi - \arccos\left(\frac{\hat{\mathbf{n}}_f \cdot \mathbf{a}_s}{|\mathbf{a}_s|}\right) \quad (4.5)$$

The angle is 0° when the vectors are anti-parallel, it reaches 90° when the vectors are perpendicular, while in regions where the angle exceeds 90° , loose particles may be ejected due to the gravitational pull no longer directing them toward the surface. Figure 4.7 shows the angle mapped over the surface of Didymos.

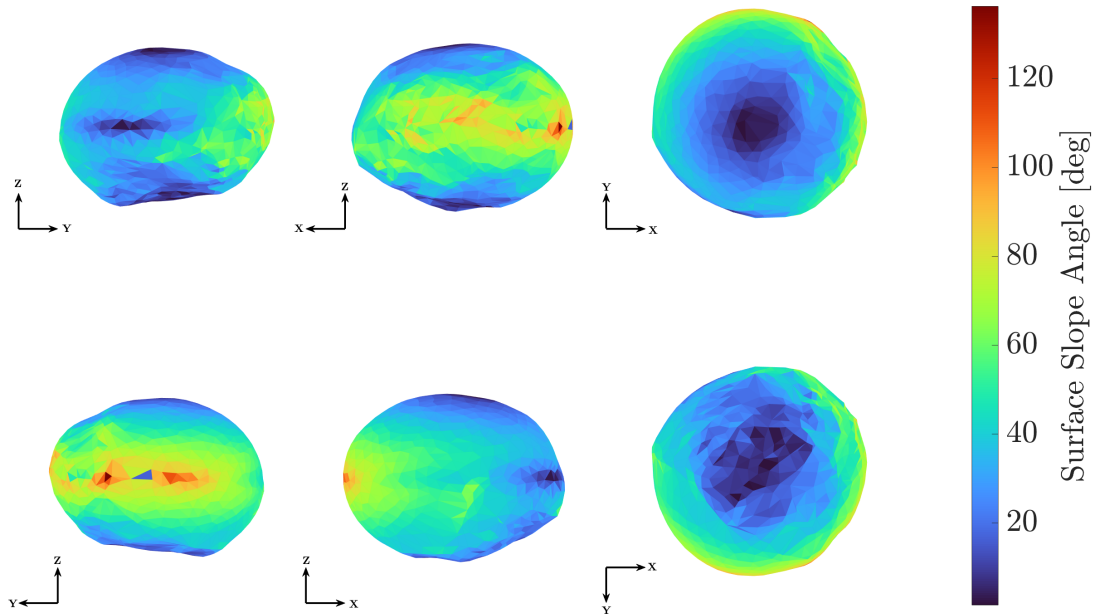


Figure 4.7: Surface slope angle mapped over the surface of Didymos, Didymos-fixed.

The acceleration slope quantifies the deviation of the net surface acceleration from the local normal, indicating how effectively gravity holds material on the surface. As shown in Figure 4.7, lower slope values near the poles suggest that the net surface acceleration is predominantly directed toward the asteroid's center. This implies that polar regions provide a more stable environment, as particles are naturally pulled toward the surface. Conversely, higher slope values are observed near the equator, where the net surface acceleration may tilt outward rather than inward. This indicates that centrifugal forces increasingly counteract gravity, creating an unstable environment.

Escape Velocity and Guaranteed Return Speed

Depending on the rotation of the asteroid and surface geometry, the velocity of a particle at rest on the surface can either direct the particle outward or into the asteroid. As a result, the velocity needed for escape varies across the surface. The escape velocity v_{esc} , defined as the speed normal to the surface that ensures a positive Keplerian energy far from the asteroid, is given by [1]:

$$v_{\text{esc}} = -\hat{\mathbf{n}} \cdot (\boldsymbol{\omega} \times \mathbf{r}) + \sqrt{[\hat{\mathbf{n}} \cdot (\boldsymbol{\omega} \times \mathbf{r})]^2 + 2U_{\text{max}}(\mathbf{r}) - (\boldsymbol{\omega} \times \mathbf{r})^2} \quad (4.6)$$

where $U_{\text{max}} = \max [U(\mathbf{r}), \frac{\mu}{r}]$ represents a conservative estimate of the maximum potential. Figure 4.8 illustrates the distribution of escape velocity across the surface of Didymos.

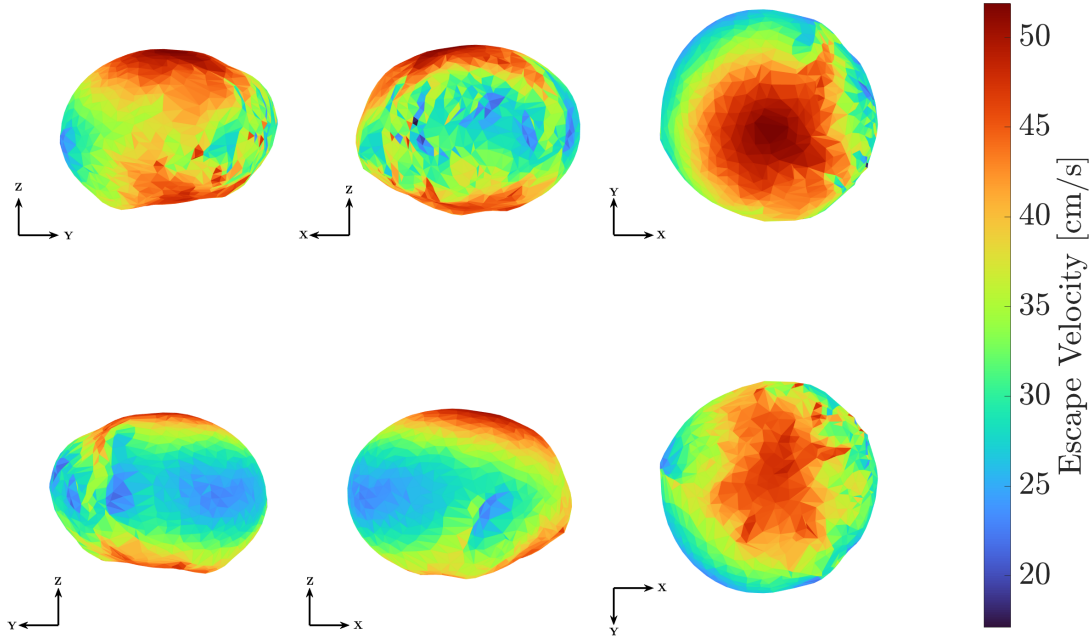


Figure 4.8: Escape velocity mapped over the surface of Didymos, Didymos-fixed.

Higher values are observed at the poles, where the rotational velocity of the asteroid is minimal, while lower values occur at the equator due to the significant rotational contribution. In these equatorial regions, particles can more easily escape the surface of the asteroid, requiring extra caution during landing operations.

It is important to note that particles with velocities below the escape velocity can still achieve orbit and eventually escape after prolonged orbital motion around the asteroid. Therefore, a lower threshold, known as the Guaranteed Return Speed (GRS), can be defined. This represents an upper conservative bounds for the velocity, to ensure that the CubeSat will stay trapped in a region around the asteroid, which is defined by the zero-velocity surface of the lowest energy Lagrangian point. A particle with a Jacobi constant greater than the value at this surface could theoretically escape, while those with lower energy remain gravitationally bound within the vicinity of the asteroid. For Didymos, L1 is the equilibrium point with the lowest energy, and its associated zero-velocity curves and surfaces are illustrated in Figure 4.9. The figure clearly highlights the region around D1 where particles become gravitationally trapped.

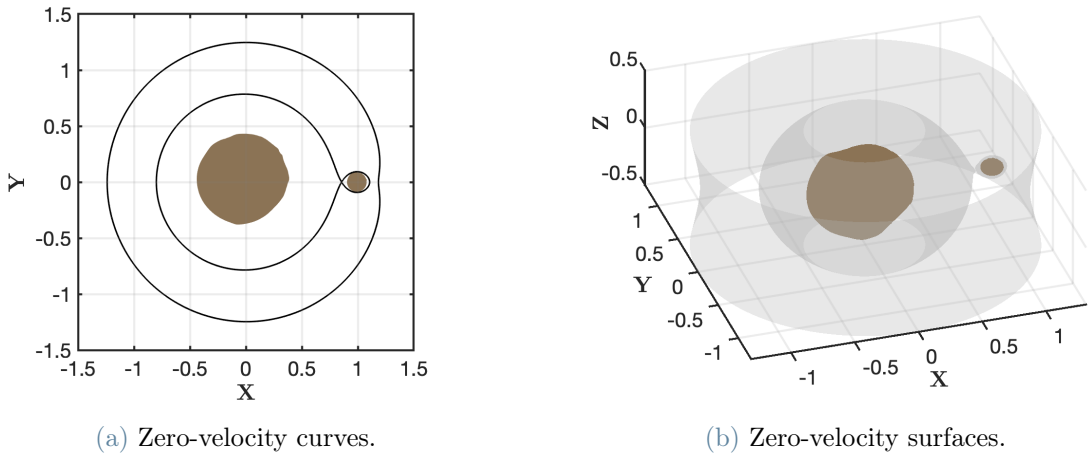


Figure 4.9: Zero-velocity curves and surfaces associated to L1.

The Guaranteed Return Speed can be determined by considering that the motion of a spacecraft within Didymos binary system can be approximated using the framework of the Circular Restricted Three-Body Problem (CR3BP), where D1 and D2 are treated as point masses orbiting their common barycenter in a circular orbit governed by Keplerian dynamics. By adopting the synodic reference frame, it is possible to calculate the Lagrange points, equilibrium positions where gravitational forces and the centrifugal acceleration balance each other and where the velocity of a particle is zero [35]. Consequently, the Jacobi constant, calculated as [18]:

$$C = \frac{1}{2}v^2 + V(\mathbf{r}) \quad (4.7)$$

equals the geopotential value at these equilibrium points. The GRS can be determined inverting Equation (4.7):

$$v_{\text{ret, synodic}} = \sqrt{2(C^* - V(\mathbf{r}))} \quad (4.8)$$

where C^* is the Jacobi constant of L1, while the geopotential is calculated as [35]:

$$V(\mathbf{r}) = -\frac{1}{2}|\boldsymbol{\omega} \times \mathbf{r}|^2 - U_{D1}(\mathbf{r}) - \frac{\mu_{D2}}{\|\mathbf{r} - \mathbf{r}_{D2}\|} \quad (4.9)$$

where $U_{D1}(\mathbf{r})$ is the gravitational potential of Didymos, modeled as¹ [31]:

$$U_{D1}(\mathbf{r}) = \frac{1}{2}G\rho \left[\sum_{e \in \text{edges}} (\mathbf{r}_e \cdot \mathbf{E}_e \cdot \mathbf{r}_e) L_e - \sum_{f \in \text{faces}} (\mathbf{r}_f \cdot \mathbf{F}_f \cdot \mathbf{r}_f) \omega_f \right] \quad (4.10)$$

The Guaranteed Return Speed calculated in the synodic reference frame is then transformed into the body-fixed frame of Didymos, with the results mapped in Figure 4.10.

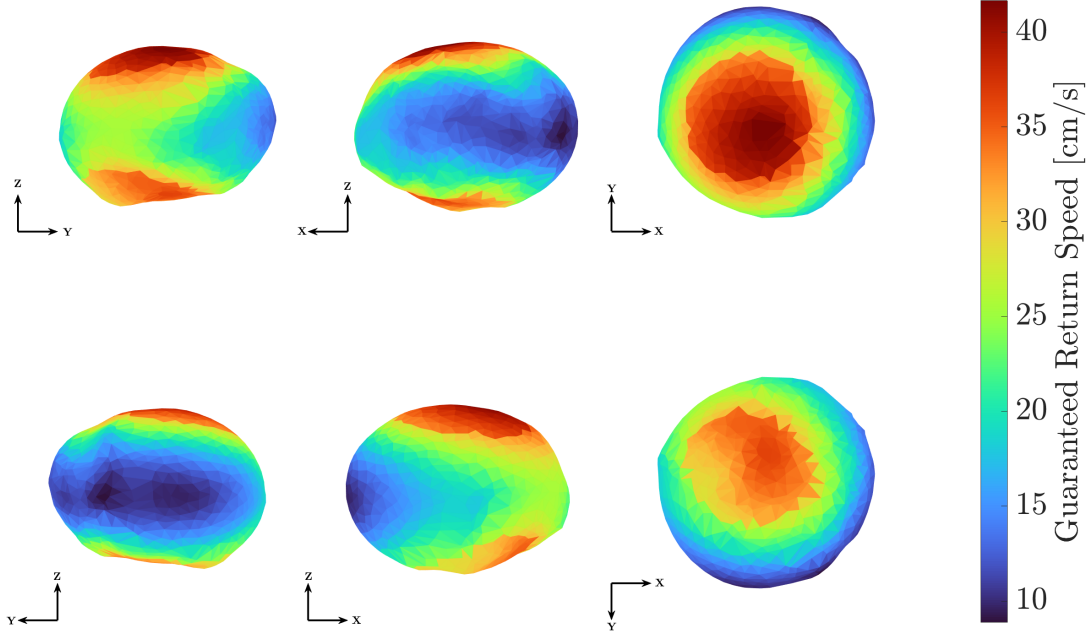


Figure 4.10: Guaranteed return speed mapped over the surface of D1, Didymos-fixed.

¹See Section 3.1.1 for the definition of the parameters.

It can be observed that at the poles, where the rotational velocity of the asteroid is lower, the guaranteed return speed is significantly high. This decreases the likelihood that an object, after the initial impact, will re-enter orbit rather than remain on the surface, as escaping from these areas requires considerably more energy.

4.1.2. Landing sites selection

After analyzing the geophysical environment, it is also necessary to consider the scientific return requirements when selecting the landing site. Since Milani is not specifically designed for landing, its survival upon impact remains highly uncertain. Therefore, to maximize scientific return, it is preferable to choose an area that remains illuminated throughout the descent, enabling the capture of images for transmission to Hera.

Two main options are analyzed for the landing site: the poles and the equatorial region, each presenting distinct advantages and disadvantages that must be carefully evaluated.

Landing at the Poles

A key advantage of selecting the poles is their continuous illumination, verified by assessing the angle between the surface normal and the vector pointing to the Sun. Feasible landing sites are identified as polyhedral facets where this angle remains below 90 degrees over a 12-hour period. Due to the rapid rotation of Didymos (2.26 hours), the only locations meeting this criterion are concentrated around the north pole along the positive Z-axis of the body-fixed frame of Didymos, as illustrated in Figure 4.11. The opposite pole is excluded, as its illumination is inconsistent due to the shape and inclination of Didymos relative to the Sun.

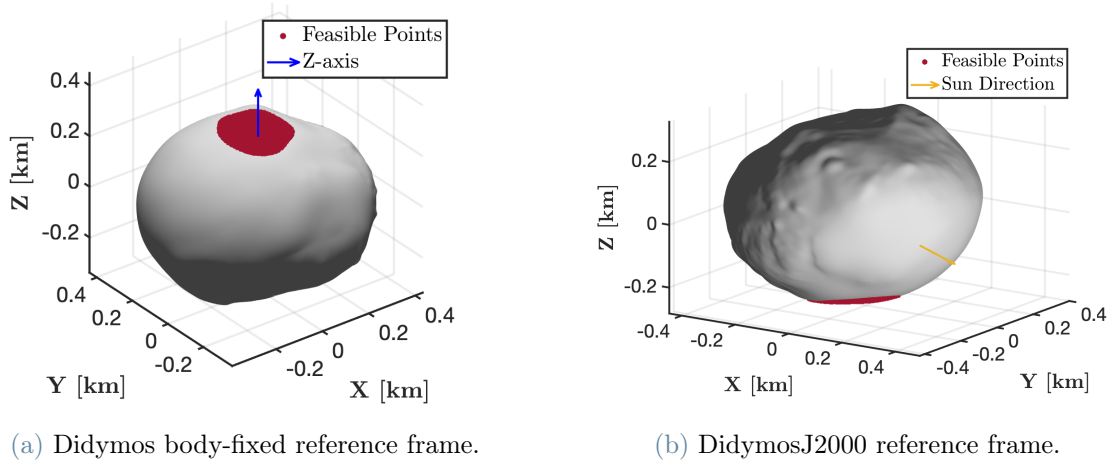


Figure 4.11: Feasible landing locations at pole.

Another important factor in site selection is the local return speed, which determines the likelihood of the spacecraft of staying trapped in a region around the system after bouncing. The poles provide a significant advantage in this regard, as the rotational contribution is minimal, leading to a more stable landing environment with a higher return speed. This allows for higher touchdown velocity while reducing the required maneuvering cost.

Moreover, selecting a landing site that minimizes risk for the CubeSat is fundamental. Among the locations that meet the illumination requirement, those with tilt and slope angles below 20 degrees can be considered feasible. However, since determining a landing trajectory for each of these points is computationally demanding, a single site is selected: the one with the lowest slope and tilt angles. The key characteristics of the selected landing site at pole are summarized in Table 4.1.

Table 4.1: Main characteristics of the selected landing site at pole.

Longitude	Latitude	Tilt Angle	Slope Angle
145.51 deg	82.62 deg	4.86 deg	4.24 deg

Landing at the Equator

The equatorial region presents a different set of challenges and opportunities. While it may offer more favorable illumination conditions at the moment of touchdown, this benefit comes with strict timing constraints.

From a dynamical perspective, the equator is more complex due to the equatorial bulge, which increases the local radius. Additionally, the rotational contribution is at its maximum, leading to lower guaranteed return speeds which, in some areas, drop to as low as 8 cm/s, significantly increasing the risk of escaping from the surface. However, a specific region at the equator, where the local radius is smaller, exhibits higher GRS of approximately 25 cm/s, as can be seen in Figure 4.10, offering a safer landing option.

Nevertheless, selecting this specific region as the landing site introduces significant risks, as the touchdown time must be carefully chosen to ensure the area is illuminated at the moment of landing. If the trajectory duration deviates from the expected value, the CubeSat could land in a different equatorial region with a lower return speed, substantially increasing the likelihood of escaping the asteroid.

Within this region, the final selection is based on identifying the point with the lowest slope and tilt angles, ensuring the safest possible landing conditions. The parameters of

the selected landing site, shown in Figure 4.12, are summarized in Table 4.2.

Table 4.2: Main characteristics of the selected landing site at equator.

Longitude	Latitude	Tilt Angle	Slope Angle
63 deg	-5.6 deg	4.33 deg	3 deg

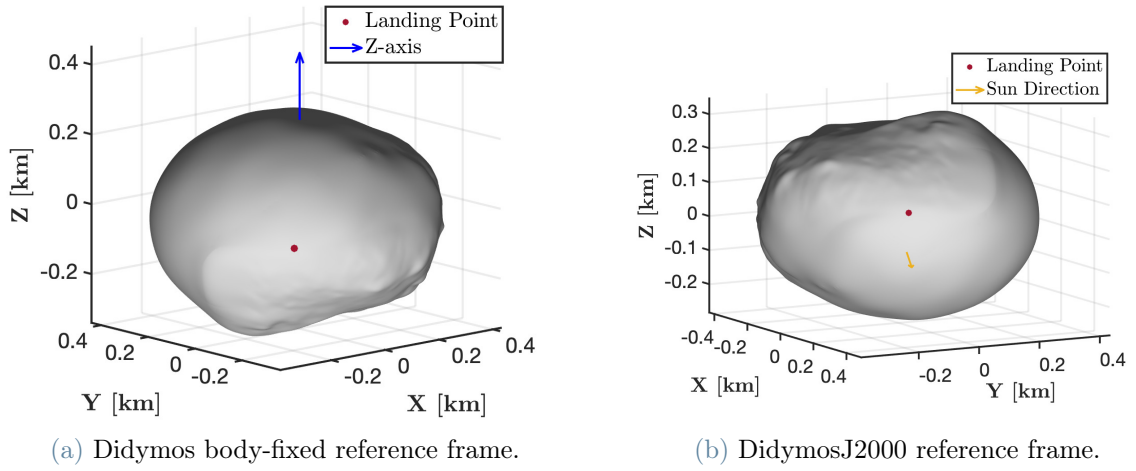


Figure 4.12: Selected landing location at equator

Finally, the choice between a polar or equatorial landing involves a trade-off between illumination constraints and landing safety.

4.2. Landing Strategy

In previous studies on landing strategies for Dimorphos, two main approaches have been investigated. The first involves an indirect method leveraging the dynamics of the three-body problem [18]. This approach exploits the invariant manifolds associated with the Lagrange point L2 to reach the asteroid. The second approach adopts a direct method, utilizing impulsive maneuvers to transition from a Self-Stabilized Terminator Orbit (SSTO), to a landing arc targeting a specific point on the surface of the asteroid [38],[19]. This work will adopt the direct method to land on Didymos.

Direct Method

From the SSTO, Milani acquires a first Landing Orbit Acquisition Maneuver (LOAM) to transition the spacecraft onto a first arc, the descent arc, directed toward Didymos.

Once the spacecraft approaches the asteroid, a Breaking Maneuver (BM) is executed to decelerate Milani, leading to the final landing arc and touch down on the surface with a reduced velocity. The scheme of the landing is illustrated in Figure 4.13.

The cost of each maneuver must comply with the capabilities of the CubeSat's cold-gas propulsion system and remain within the total ΔV budget allocated for the experimental phase. Additionally, small maneuvers are preferable, as they help minimize these inaccuracies. This is why it is important to account for uncertainties in both the magnitude of the thrust and its direction and to minimize the ΔV of both maneuvers.

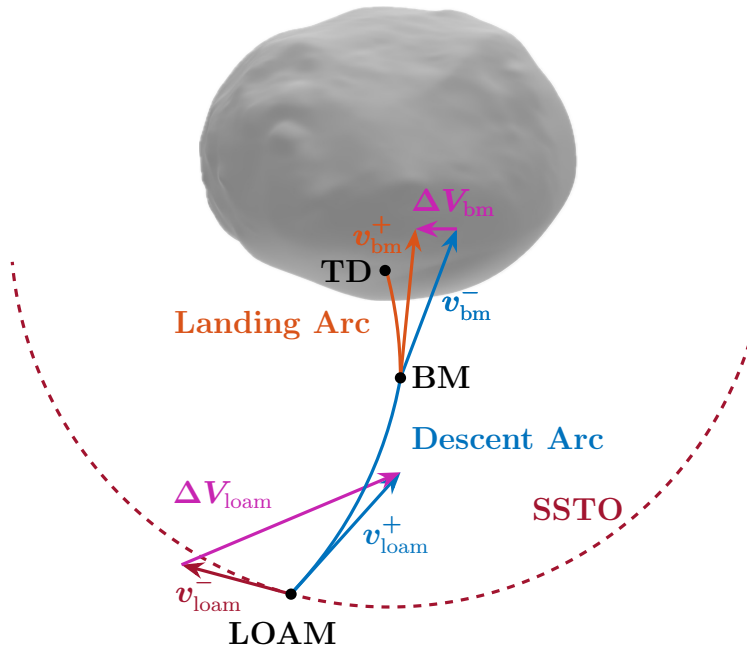


Figure 4.13: Diagrams of descent and landing arcs with associated velocity vectors and ΔV maneuvers. The figure is not to scale.

4.2.1. Problem Statement

To minimize the cost of the two maneuvers, the MATLAB optimization solver `fmincon` is utilized. The problem statement is formulated as follows:

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = \Delta V_{loam}^2 + \Delta V_{bm}^2 \quad \text{s.t.} \quad \begin{cases} c_{eq}(\mathbf{x}) = 0 \\ c(\mathbf{x}) < 0 \\ \mathbf{x}^{LB} \leq \mathbf{x} \leq \mathbf{x}^{UB} \end{cases}$$

where the squared terms in the cost function are used to mitigate numerical issues when

computing gradients. The optimization variables are defined as:

$$\mathbf{x} = (\mathbf{x}_{\text{td}}, \mathbf{x}_{\text{bm}}, t_{\text{td}}, t_{\text{bm}}, t_{\text{loam}})$$

where $\mathbf{x}_{\text{td}}$ and $\mathbf{x}_{\text{bm}}$ represent the state of the CubeSat at touchdown and before the Breaking Maneuver, respectively. The variables t_{td} , t_{bm} , and t_{loam} correspond to the times at touchdown and at the two maneuver events. The lower and upper bounds of the variables are denoted as $\mathbf{x}^{LB}$ and $\mathbf{x}^{UB}$, while $\mathbf{c}_{eq}(\mathbf{x})$ and $c(\mathbf{x})$ represent the vectors of equality and inequality nonlinear constraints. Note that the two arcs are back-propagated in time.

The equality constraints are defined as:

$$\mathbf{c}_{eq}(\mathbf{x}) = \begin{cases} \varphi(\mathbf{r}_{\text{td}}; t_{\text{td}}, t_{\text{bm}}) - \mathbf{r}_{\text{bm}} \\ \varphi(\mathbf{r}_{\text{bm}}; t_{\text{bm}}, t_{\text{loam}}) - \mathbf{r}_{\text{ssto}} \\ \mathbf{v}_{\text{td}} + |\mathbf{v}_{\text{td}}| \hat{\mathbf{n}}_f \\ \mathbf{r}_{\text{td}} - \mathbf{r}_{\text{landing}}(t_{\text{td}}) \end{cases}$$

The first constraint ensures position continuity between landing and descent arc at Breaking Maneuver by enforcing that the integration defect is zero. The second one requires the position at the end of the back-propagation of the descent arc to belong to the SSTO. The third constraint imposes that the touchdown velocity direction is opposite to the surface normal at the selected landing site, $\hat{\mathbf{n}}_f$. The fourth constraint ensures that the touchdown location matches the selected landing site.

The inequality constraint, expressed as:

$$c(\mathbf{x}) = v_{\text{td}} - v_{\text{max}}$$

ensures that the touchdown velocity does not exceed a predefined upper limit. Since a safe landing requires the post-touchdown velocity to remain below the return speed at the landing site, as will be discussed later, this constraint helps reduce the risk of the spacecraft escaping the asteroid. However, the return velocity computed in Section 4.1.1 has significant uncertainties. To account for these uncertainties and take a more conservative approach, a stricter maximum touchdown velocity is considered, lower than the theoretical limit given by the previous computation.

At the same time, a higher touchdown velocity reduces the cost of the braking maneuver, so the optimizer naturally tends to increase it as much as the constraints allow. By imposing this upper limit, the optimization process is guided toward a controlled touchdown speed that balances minimizing maneuver costs with ensuring a safe and robust landing.

4.2.2. Initial Guess

To find the optimized trajectory, a high-quality initial guess for the optimization variables is essential. The process for selecting this initial guess is detailed below.

Self-Stabilized Terminator Orbit

The Self-Stabilized Terminator Orbit is a stable, near-circular polar orbit in the terminator plane of Didymos system, with its normal pointing toward or away from the Sun. This orbit benefits from strong Solar Radiation Pressure perturbations and gravitational forces, allowing relaxed maneuvering and reducing operational demands [2].

SSTOs are computed in the Sun-Didymos reference frame, where the x-axis aligns with the Sun-Didymos system line, the z-axis with the system's angular momentum, and the y-axis completes the triad. The computation follows the algorithm described in Appendix A, ensuring a precise definition of the initial conditions. The characteristics of the SSTO make it ideal for achieving scientific objectives and preparing Milani for the landing.

The choice of the last SSTO radius is a critical factor in ensuring orbital stability. Generally, the closer the orbit is to the system, the more stable it becomes. However, there are inherent limits to this proximity. If the orbit is too close, the spacecraft risks gradually falling toward the system over time. Conversely, if it is too far, the gravitational forces become insufficient to counteract external perturbations, leading to eventual escape.

In addition to these stability considerations, the selection of the SSTO semi-major axis must also account for other two factors. A lower orbital radius increases the likelihood of intersecting the trajectory of Dimorphos, posing a significant collision risk. On the other hand, a larger radius results in higher energy requirements for the descent maneuver, making the landing more costly. A trade-off between these constraints led to the selection of a semi-major axis of 3.5 km, which provides a suitable balance between safety and maneuvering efficiency.

Landing Arc

The initial guesses for the landing arc $(\mathbf{x}_{td}, \mathbf{r}_{bm}, t_{td}, t_{bm})$ require two important considerations:

1. **Selection of the touchdown velocity magnitude.** A trade-off is considered: higher velocities reduce the ΔV required for the breaking maneuver but increase the likelihood of rebounds upon contact and the risk of escaping the asteroid;
2. **Selection of the breaking maneuver point.** To determine the breaking maneuver point, it is necessary to select an appropriate altitude relative to the surface of the asteroid where to perform the maneuver.

For what concerns the first point, to prevent the spacecraft from escaping the asteroid, the velocity after touchdown must be lower than the guaranteed return speed. Its computation starts by decomposing the pre-touchdown velocity vector into two components: the normal and the tangential ones, as shown in Equation (4.11):

$$\begin{cases} \mathbf{v}_N^- = (\hat{\mathbf{n}}_f \cdot \mathbf{v}^-) \hat{\mathbf{n}}_f \\ \mathbf{v}_T^- = \mathbf{v}^- - \mathbf{v}_N^- \end{cases} \quad (4.11)$$

where the minus indicates the pre-impact velocity vector. Thereafter, the post-touchdown normal velocity is calculated by multiplying v_N^- by the restitution coefficient ε , which ranges between zero (fully inelastic collision) and one (fully elastic condition). This coefficient quantifies the energy dissipation during an impact and is defined as follows [19]:

$$\varepsilon = \frac{v_N^+}{v_N^-} \quad (4.12)$$

where the plus indicates the post-impact velocity vector. The exact value of this parameter is typically estimated based on expected surface properties, which for Didymos are highly uncertain. Given the difficulty in accurately determining this coefficient, a conservative value of 0.8 is chosen to account for worst-case scenarios, based on the value measured for Itokawa [16].

On the other hand, simulating the behavior of the post-touchdown tangential component is particularly challenging, as it depends on surface characteristics such as rocks, boulders, and crater rims. However, since the touchdown velocity is imposed to be normal to the surface, it is assumed that its entire magnitude will be reduced by the restitution coefficient:

$$|\mathbf{v}^+| = \varepsilon |\mathbf{v}^-| \quad (4.13)$$

For this reason, the touchdown velocity magnitude is selected to be below $\frac{v_{\text{ret,td}}}{\varepsilon}$. To ensure a conservative estimate while accounting for potential uncertainties, an initial guess of 35 cm/s is selected for the pole, whereas 21 cm/s is chosen for the equatorial landing point.

Regarding the second point, backward propagation is performed starting from the candidate points and considering an initial velocity vector equal to $-|\mathbf{v}_{\text{td}}|\hat{\mathbf{n}}_f$, with an event function stopping the integration in case of re-contact with the surface of the asteroid. As illustrated in Figure 4.14, given the relatively low touchdown velocity, the resulting trajectories form small arcs that eventually return to the surface. Within these arcs, it is necessary to determine the most suitable point for executing the braking maneuver. Since performing this maneuver at the highest altitude ensures safer operation by reducing the risks associated with proximity to the asteroid and improving robustness against errors, the highest-altitude point along each trajectory is selected as the BM location.

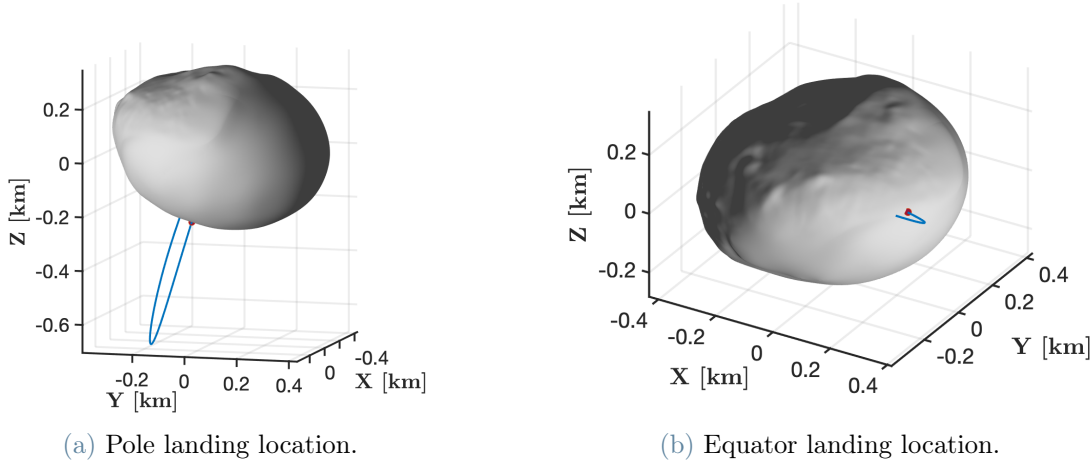


Figure 4.14: Back-propagated arcs for the selected landing points, DidymosJ2000.

Descent Arc

The initial guess for the descent arc $(\mathbf{v}_{\text{bm}}, t_{\text{loam}})$ requires the identification of the $\Delta\mathbf{V}_{\text{bm}}$ to transition toward the SSTO. Spherical coordinates are used for this purpose: the magnitude of ΔV is selected between 30 cm/s and 40 cm/s, while the azimuth and elevation angles are systematically varied, considering all possible combinations.

For each combination, the descent trajectory is propagated backward, starting from the BM point on the landing arc and applying the corresponding $\Delta\mathbf{V}_{\text{bm}}$ at this point. The resulting trajectories are filtered to keep only those that reach, or closely approach, the SSTO. Any of these trajectories can serve as a valid initial guess.

An event function is employed to terminate the integration whenever the trajectory reaches either the altitude of the final SSTO radius or the surface of the asteroid, indicating insufficient ΔV to reach the SSTO.

4.2.3. Results

The results of the landing trajectory analysis are presented for both the polar and equatorial landing sites.

Trajectory to the Pole

The trajectory used as the initial guess for the optimizer is defined by an SSTO semi-major axis of 3.5 km, a touchdown velocity magnitude of 35 cm/s, and a ΔV_{bm} of 30 cm/s. Since the selected landing sites remain continuously illuminated, there is no strict constraint on the touchdown time; however, to increase the scientific return, it would be possible to choose an epoch ensuring at least partial illumination of Dimorphos, allowing for the acquisition of images of the smaller asteroid as well.

The results of the optimization are used as initial conditions to forward-propagate the trajectory arcs. Starting from the initial time of the final SSTO, which is derived from the mission kernels [25], the trajectory is propagated forward for approximately 8 days, until the LOAM. At this point, the first $\Delta \mathbf{V}_{\text{loam}}$ maneuver is applied, defining the initial conditions for the descent arc. This arc is then propagated forward until BM time, where the second $\Delta \mathbf{V}_{\text{bm}}$ is executed. Finally, the state after the BM is propagated until the spacecraft reaches the surface of the asteroid.

The resulting trajectory is shown in Figure 4.15. The figure also illustrates the orbit and position of Dimorphos showing that there is no risk of collision during the landing.

The maneuver costs, along with the trajectory durations and touchdown velocity, are summarized in Table 4.3. It is noteworthy that the touchdown velocity magnitude reaches the maximum value allowed by the constraints. This is because reducing the touchdown velocity decreases the cost of the breaking maneuver, which is the parameter being minimized and whose value is halved after the optimization.

Overall, the total time from LOAM to touchdown is 5.8 hours, with a cumulative ΔV of 29.6 cm/s. These values offer an effective balance between time and cost, making the landing strategy both feasible and efficient.

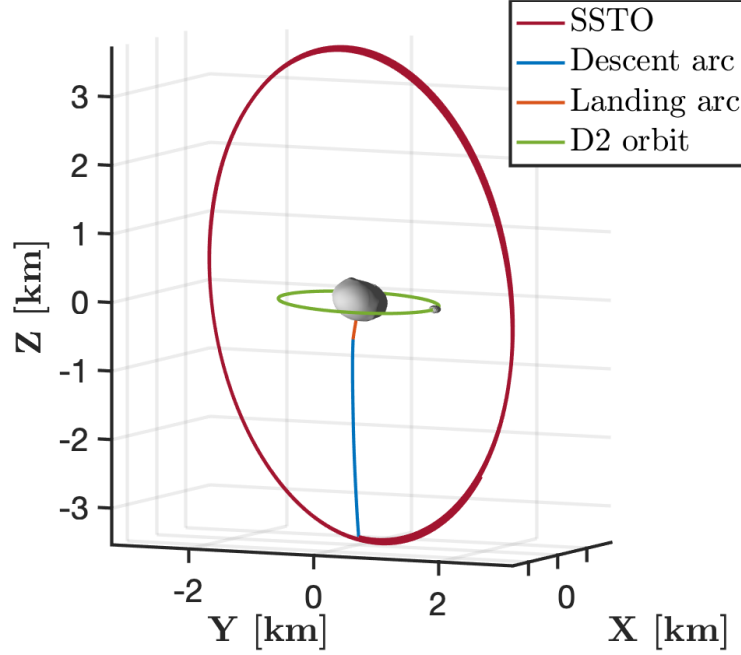


Figure 4.15: Optimized landing trajectory, DidymosJ2000.

Table 4.3: Results of the optimization analysis.

ΔV_{loam}	13.73 cm/s
ΔV_{bm}	15.87 cm/s
V_{td}	35 cm/s
Time of Descent	5.51 hours
Time of Landing	19.58 minutes

Trajectory to the equator

For the equatorial landing site, the initial trajectory guess for the optimizer is characterized by a SSTO semi-major axis of 3.5 km, a touchdown velocity magnitude of 21 cm/s, and a ΔV_{bm} of 40 cm/s. Unlike the polar site, where continuous illumination is guaranteed, selecting an appropriate touchdown time is crucial at the equator to ensure optimal illumination of the landing site. Also in this case, a timing that allows Dimorphos to be illuminated and visible from Milani can be considered.

The trajectory arcs forward-propagated using the optimized parameters are illustrated in Figure 4.16. The figure also displays the orbit and position of Dimorphos, demonstrating that no collision risk exists during the descent phase.

A summary of the maneuver costs, descent duration, and touchdown velocity is provided in Table 4.4. Notably, the touchdown velocity reaches the upper constraint limit, as minimizing it would increase the cost of the breaking maneuver, which is the primary optimization target. As a result, the required ΔV_{bm} is significantly reduced after optimization.

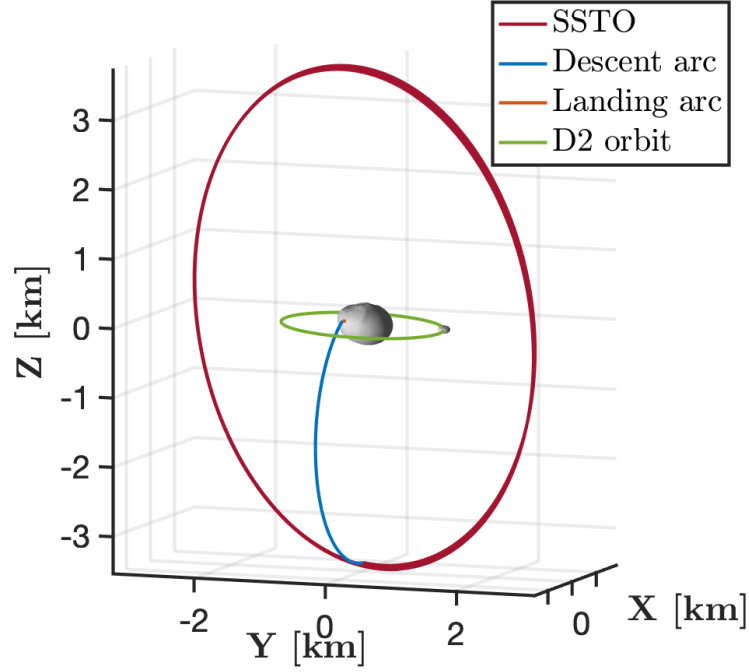


Figure 4.16: Optimized landing trajectory, DidymosJ2000.

Table 4.4: Results of the optimization analysis.

ΔV_{loam}	11.08 cm/s
ΔV_{bm}	33.16 cm/s
V_{td}	21 cm/s
Time of Descent	11.81 hours
Time of Landing	6.55 minutes

Overall, the total time from LOAM to touchdown is 11.92 hours, with a cumulative ΔV of 44.24 cm/s.

4.2.4. Bounce Analysis

To ensure that the spacecraft does not escape from the asteroid after the initial touchdown, a rebound analysis is conducted. The post-touchdown velocity is a key parameter in determining the behavior of Milani after impact. Its normal and tangential components are computed under the assumption of a locally flat surface; while the normal component can be determined using Equation (4.12), the tangential component depends on local surface features such as rocks, craters, and boulders at the landing site. Currently, these surface characteristics remain unknown.

To overcome this, the post-touchdown tangential velocity is considered unchanged, meaning that no dissipation is taken into account:

$$\mathbf{v}_T^+ = \mathbf{v}_T^- \quad (4.14)$$

Under these hypotheses, the analysis results indicate that the polar landing trajectory comes to rest approximately 3.45 hours after the initial touchdown, following 23 rebounds. While the first three to four bounces are relatively large, the subsequent ones are significantly smaller, leading to a gradual stabilization.

In contrast, the equatorial landing trajectory, influenced by the lower touchdown velocity, settles in just 28 minutes, after only three rebounds.

4.2.5. Trajectory Selection

Selecting the most suitable landing trajectory requires a careful evaluation of the advantages and disadvantages associated with each option. Both the polar and equatorial trajectories offer distinct benefits and challenges.

The polar trajectory presents several advantages that make it the most reliable and cost-effective choice. First of all, the landing site remains continuously illuminated, eliminating any strict constraint on the touchdown timing. This flexibility significantly simplifies mission planning. Additionally, the pole provides a much safer landing environment due to higher guaranteed return speeds, as shown in Section 4.1.1, which allow for a higher impact speed without increasing the risk of rebound and escape. The surface at the pole is also more stable, further enhancing landing reliability.

From an operational point of view, the polar trajectory is significantly shorter than its equatorial counterpart, reducing the propagation of uncertainties over time. The descent also results in lower maneuver costs, as a higher allowable touchdown velocity reduces the

required ΔV for the breaking maneuver.

However, since the SSTO follows a polar orbit, the trajectory leads directly to the pole, which will then be the only region observable with Milani's cameras. Additionally, the phase angle at the pole is approximately 90 degrees, meaning that while the landing site remains illuminated, the lighting conditions are relatively dim.

On the other hand, the equatorial trajectory offers the advantage of targeting a landing site that can be directly illuminated at the moment of impact. Additionally, during the descent, the spacecraft has the opportunity to observe a larger portion of the surface of the asteroid due to the rotation of Didymos.

However, the equatorial landing requires a precise timing to ensure that the selected site is fully illuminated. Any deviation in timing or trajectory behavior could result in the spacecraft landing in a region where the GRS is too low, making the risk of escape even higher. Moreover, the equatorial trajectory is twice as long as the polar one, leading to a greater accumulation of uncertainties throughout the descent. Additionally, this trajectory is more costly, as shown by the optimization results, making it less efficient from a fuel consumption perspective.

Given these considerations, the polar trajectory is chosen as the preferred landing. It provides greater safety, lower maneuver costs, and higher reliability compared to the equatorial alternative. The shorter descent minimizes the propagation of uncertainties, while the higher return velocity at the pole significantly reduces the risk of post-impact escape. Despite the dimmer lighting conditions at the pole, the continuous illumination of the landing site and the overall robustness of the trajectory make it the most viable option for ensuring mission success.

5 | Sensitivity Analysis

Milani landing mission relies on an open-loop maneuver execution, meaning all maneuvers occur at predetermined times with commands issued from the ground station, without real-time corrections. This necessitates a trajectory design that is inherently robust to uncertainties arising from maneuver execution errors, dynamical model inaccuracies, and navigation errors. To evaluate the robustness of the nominal trajectory and quantify the effects of these uncertainties, a Monte Carlo (MC) analysis is performed in this chapter.

5.1. Monte Carlo Analysis

The Monte Carlo analysis is a statistical method used to evaluate the impact of uncertainties in complex systems by performing repeated numerical simulations, where input parameters are sampled from predefined probability distributions. It is used to assess how variations in input parameters influence system behavior and to identify the most critical sources of uncertainty.

In trajectory analysis, uncertainties can be categorized into dynamic uncertainties (related to dynamical perturbations), control uncertainties (arising from maneuver execution errors in magnitude and direction), and navigation uncertainties (stemming from errors in the estimated state of the satellite). In this study, five independent sources of uncertainty are considered [16]:

- Initial state: 340 m in position and 0.0022 m/s in velocity (1σ);
- Maneuver execution: 2-degree uncertainty in direction (3σ) and 5% uncertainty in magnitude (3σ);
- SRP acceleration: 8% uncertainty in magnitude (1σ);
- Gravitational acceleration: 3.5% uncertainty, for both asteroids (1σ).

Each of these uncertainties is analyzed separately by generating an initial population of perturbed trajectories and propagating them forward to evaluate their dispersion. Since the accuracy of the results depends on the number of samples, 1000 simulations have

been selected as a balance between computational cost and statistical reliability. All uncertainties are assumed to follow a Gaussian distribution.

5.2. Results

The chosen outcome of the Monte Carlo analysis to quantify the success rate is the percentage of trajectories that reach the surface of the asteroid. This is assessed across different analyses, which account for the different uncertainties. Subsequently, for the subset of successful trajectories, the dispersion of touchdown points, touchdown velocities, and landing times is analyzed.

5.2.1. Uncertainty on Initial State

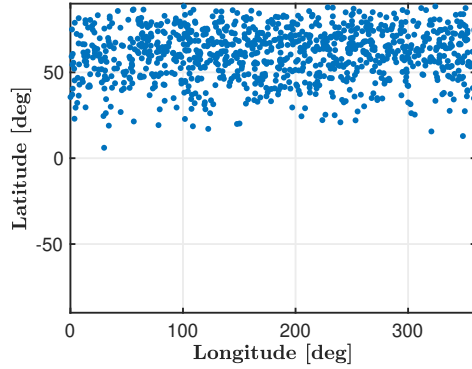
Firstly, the uncertainty on the initial state, defined as the state on the SSTO at the time of the LOAM, is analyzed. A covariance matrix is constructed based on the predefined uncertainties, and a multivariate normal distribution is generated around this nominal state.

For each sampled initial condition, the LOAM maneuver is applied, and the trajectory is propagated forward until the altitude corresponding to the nominal breaking maneuver is reached. At this final state, the breaking maneuver is executed, and the trajectory is further propagated until the spacecraft reaches the surface of the asteroid.

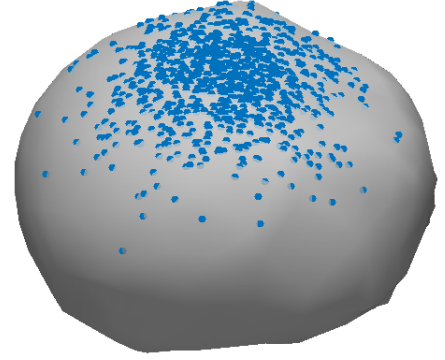
The success rate of this analysis is 100%, with all trajectories successfully reaching the surface of the asteroid. Figure 5.1 illustrates that due to the large initial uncertainties in position, the landing points are not strictly confined to the pole but extend to significantly lower latitudes, reaching even equatorial regions.

The distribution of the landing points exhibits two distinct statistical patterns:

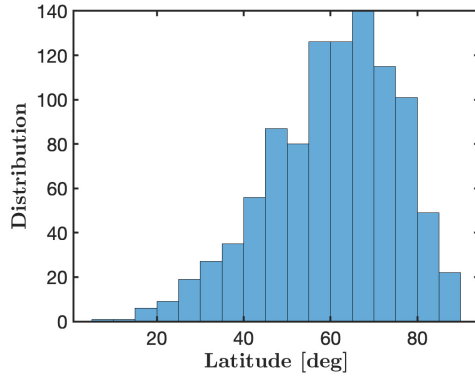
- **Longitude Dispersion:** The distribution appears nearly uniform, meaning that the landing points are evenly spread across different longitudes. This is likely due to the nearly spherical shape of the pole of Didymos, which ensures that small variations in longitude do not significantly affect the final touchdown location;
- **Latitude Dispersion:** The distribution appears asymmetrical, likely due to the high nominal latitude (83 degrees) and the 90-degree upper limit, which constrain dispersion. Additionally, the high initial uncertainty causes many points to reach lower latitudes, though the area near the pole remains the most probable landing location.



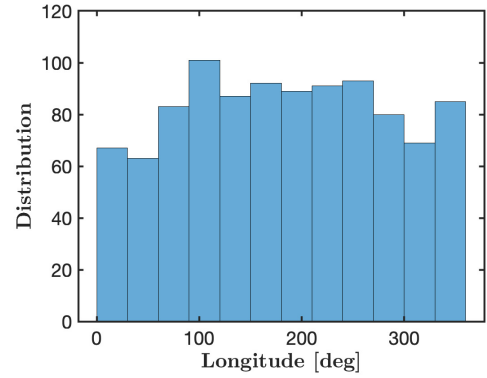
(a) 2D Dispersion.



(b) 3D Dispersion.



(c) Latitude Dispersion.



(d) Longitude Dispersion.

Figure 5.1: Landing points dispersion.

This analysis indicates that, despite the high initial uncertainty, the probability of a successful landing remains high, likely due to the gravitational attraction of Didymos, which dominates other perturbative effects as the CubeSat approaches the asteroid. Additionally, the intermediate breaking maneuver ensures that the spacecraft is directed toward the surface, further reinforcing the landing outcome. However, while the likelihood of landing is significant, the chances of touching down at a location different from the nominal site are also considerable. This could result in no scientific return during the final descent phase if the spacecraft lands in a non-illuminated area, or lead to a high risk of escaping the asteroid or experiencing multiple rebounds after touchdown.

Moreover, landing duration varies depending on the trajectory, as can be seen from the time dispersion shown in Figure 5.2, leading to different touchdown times across trajectories. This variability could pose issues if the touchdown time is fixed to achieve a specific scientific objective. However, when landing at the pole, there are no strict constraints on the touchdown timing. The only effect is on the position of Dimorphos, which, if selected

to capture images of its surface, may shift into shadow during the descent. Nevertheless, the scientific observations on Didymos remain unaffected.

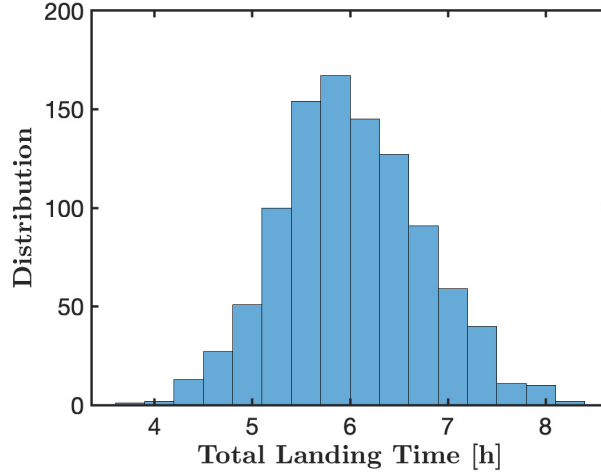


Figure 5.2: Dispersion of the total landing time.

Regarding the touchdown velocity, Figure 5.3 illustrates the dispersion of both its magnitude and the angle between the surface normal and the direction opposite to the touchdown velocity vector. In both cases, a wide range of values is observed, resulting from the high initial uncertainty, which leads to significantly different landing trajectories.

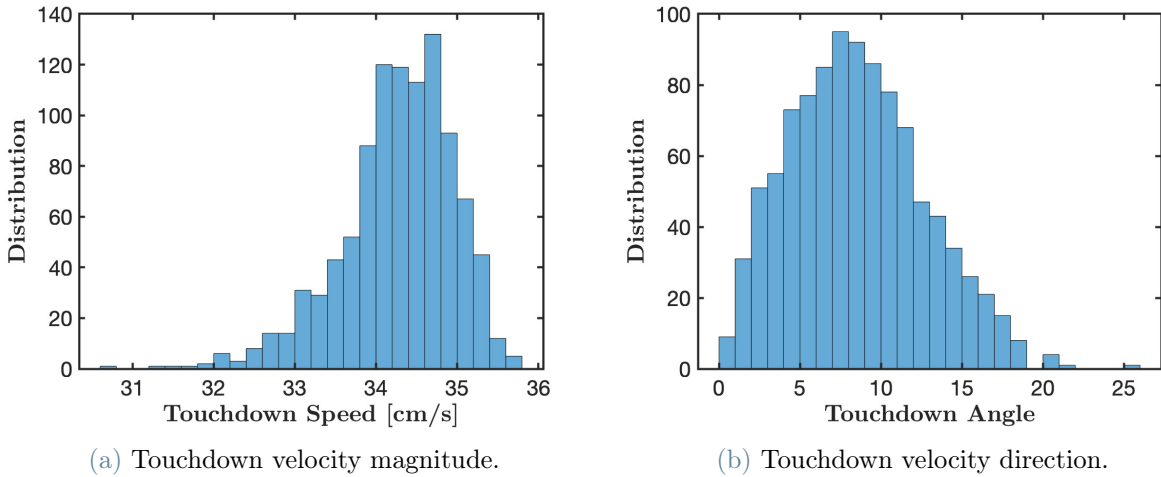


Figure 5.3: Touchdown velocity dispersion.

In particular, the magnitude of the touchdown velocity remains confined between 32 and 36 cm/s, values consistent with the nominal one. Regarding the velocity direction, the

ideal case would be an angle of 0 degrees, ensuring that the entire velocity component is reduced due to the restitution coefficient. However, the data reveals a peak between 7 and 8 degrees, with several instances of even higher values, reaching up to 26 degrees.

Due to the wide dispersion of touchdown points, both in terms of their location on the surface of the asteroid and the magnitude and direction of the velocity at touchdown, it is necessary to analyze the post-impact velocity to ensure that it remains below the guaranteed return speed. For each samples, the magnitude of the post-touchdown velocity is computed using the equations utilized in the bounce analysis (Section 4.2.4).

Figure 5.4 presents both the 3D dispersion of the touchdown points on the surface of the asteroid and a histogram depicting the difference between the guaranteed return speed at the landing site and the magnitude of the post-touchdown velocity. The red dashed line represents zero, indicating the threshold where the two velocities are equal. Notably, for some samples, represented in red in the 3D plot, this value is negative, meaning that the post-impact velocity exceeds the GRS. In these cases, specifically 31 out of 1000 trajectories, there is a risk that Milani, after the first rebound, may eventually escape the asteroid, resulting in an escape rate of approximately 3.1%. As expected, the high-risk points are located near the equator, where the rotational contribution of the asteroid is greater, resulting in a lower return speed.

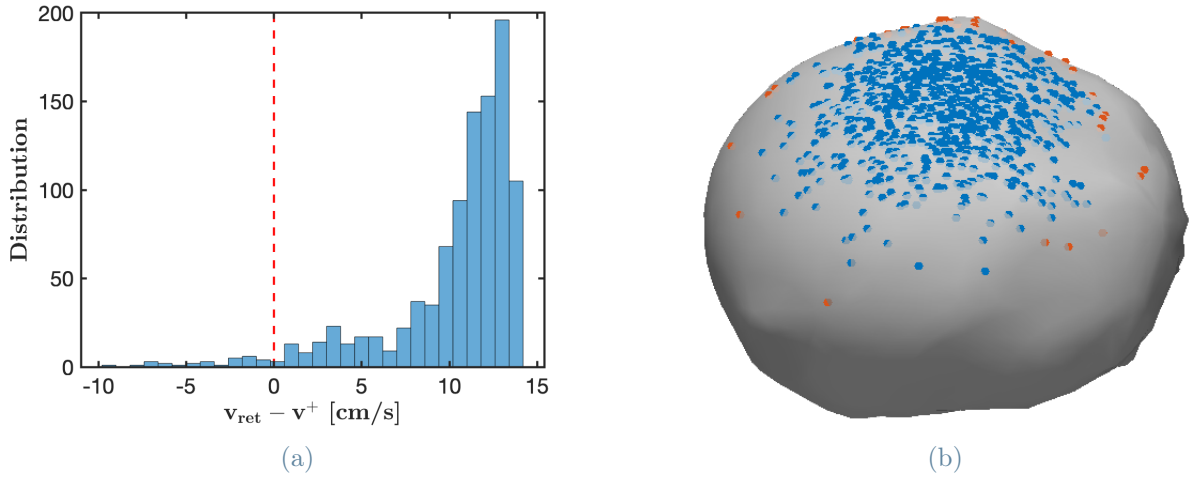
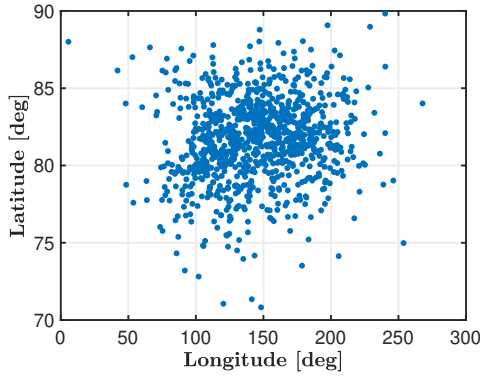


Figure 5.4: Post-touchdown velocity dispersion. (a) Dispersion of the difference between the guaranteed return speed at the landing site and the post-impact velocity magnitude. (b) Distribution of touchdown points on the surface, with red points indicating cases where the post-touchdown velocity exceeds the GRS.

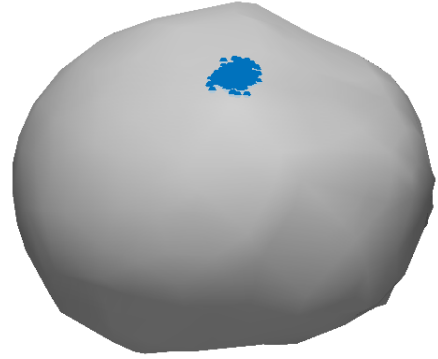
5.2.2. Uncertainty on Maneuver

To account for maneuver uncertainty, errors are considered in both magnitude and direction. The magnitude uncertainty is modeled using a normal distribution centered on the nominal value, while directional misalignment follows a normal distribution within an uncertainty cone around the nominal direction. This uncertainty is applied to both the $\Delta \mathbf{V}_{\text{loam}}$ and $\Delta \mathbf{V}_{\text{bm}}$ vectors, and the trajectory is then propagated under nominal conditions.

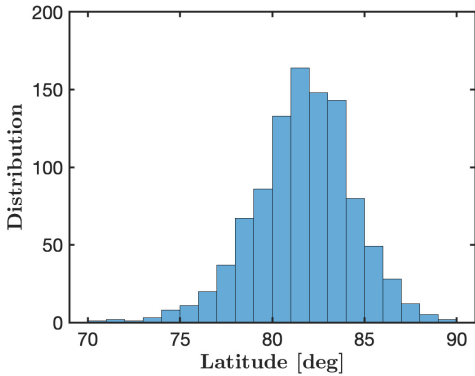
The success rate is 100%, and the dispersion of touchdown points is illustrated in Figure 5.5. Compared to the previous case, the landing dispersion area is significantly smaller. In particular, latitude remains confined between 70 and 90 degrees, indicating that even with maneuver uncertainties, the landing location does not deviate significantly from the nominal one. This is further confirmed by the fact that both the latitude and longitude distributions exhibit their highest density around the nominal values, specifically 83 degrees for latitude and 145 degrees for longitude.



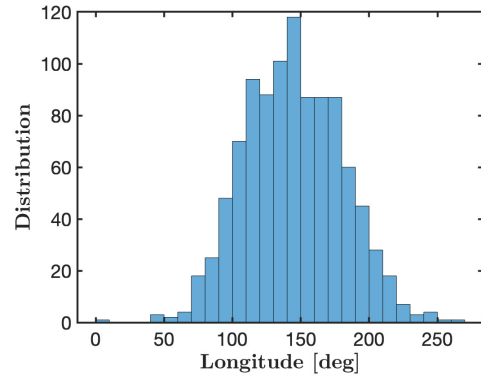
(a) 2D Dispersion.



(b) 3D Dispersion.



(c) Latitude Dispersion.



(d) Longitude Dispersion.

Figure 5.5: Landing points dispersion.

Additionally, the total landing time exhibits minimal variation across different trajectories, as shown in Figure 5.6. The observed dispersion is relatively small, with values ranging from 5.6 to 6 hours, corresponding to a maximum deviation of about 25 minutes. However, this time shift does not affect the landing conditions at the pole, where illumination remains constant, nor does it significantly alter the position of Dimorphos along its orbit, ensuring that the overall mission objectives remain unaffected.

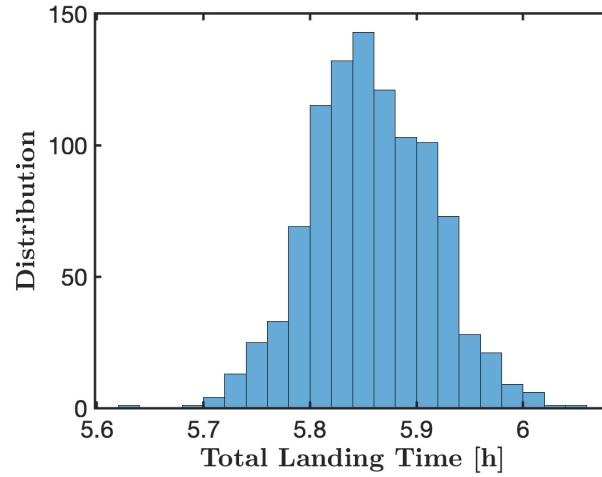


Figure 5.6: Dispersion of the total landing time.

Although the landing site dispersion is concentrated around the nominal value, it is important to analyze the touchdown velocity to assess whether it is affected by the considered uncertainty. Its dispersion is illustrated in Figure 5.7.

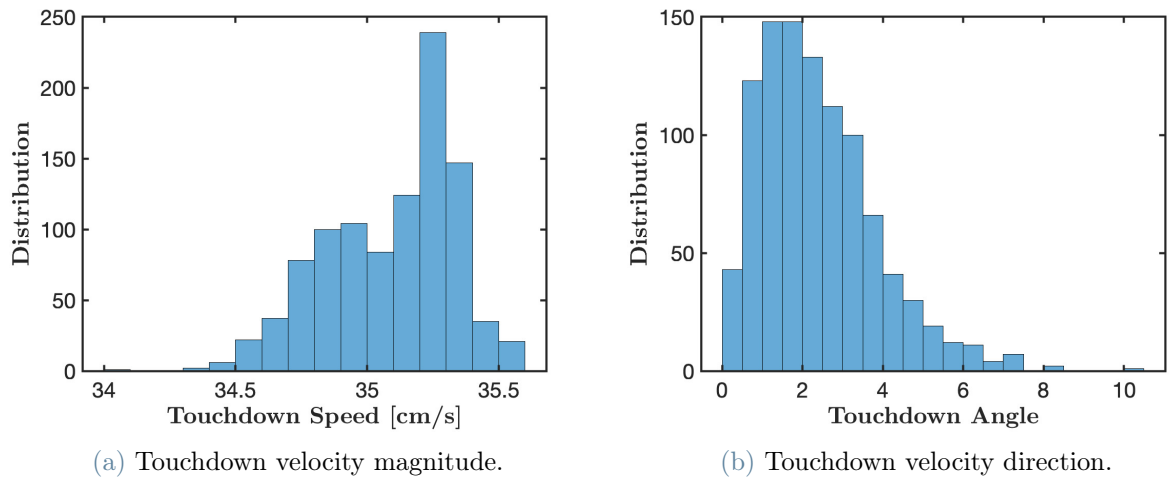


Figure 5.7: Touchdown velocity dispersion.

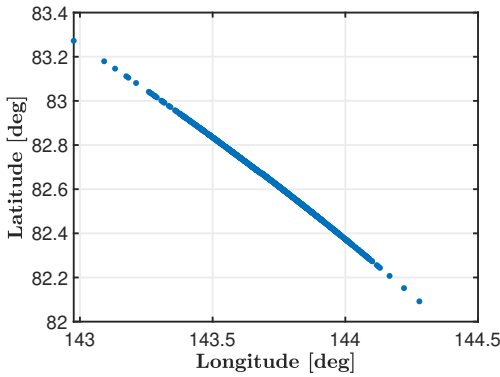
The magnitude of the touchdown velocity remains nearly the nominal value. Similarly, the angle between the surface normal and the velocity vector remains close to 0 degrees, with the exception of a few cases where it reaches up to 10 degrees.

In this case, given the limited landing region and the touchdown velocity's near-nominal magnitude and direction, an analysis of the post-impact velocity is unnecessary. There is no risk of exceeding the guaranteed return speed at the pole.

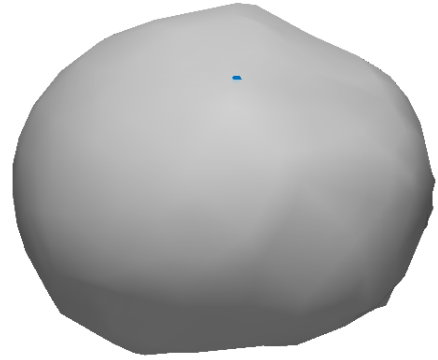
5.2.3. Uncertainty on Solar Radiation Pressure

To account for the uncertainty in SRP acceleration, only its magnitude is varied, generating a normal distribution around the nominal value, while its direction remains unchanged. This approach is justified by the short duration of the trajectory around the barycenter of the binary system, ensuring that the SRP acceleration remains constant in magnitude throughout the descent.

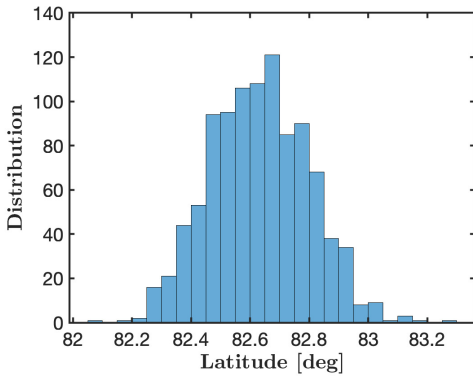
The success rate is 100% and the dispersion of the landing sites is shown in Figure 5.8.



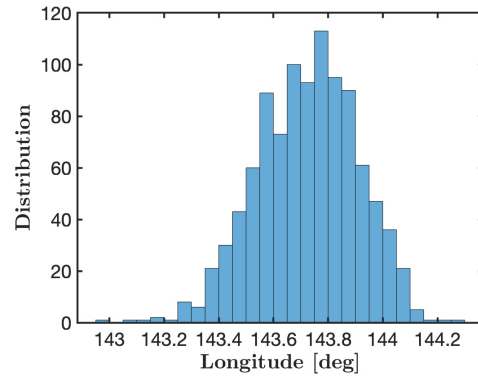
(a) 2D Dispersion.



(b) 3D Dispersion.



(c) Latitude Dispersion.



(d) Longitude Dispersion.

Figure 5.8: Landing points dispersion.

The results indicate that an 8% uncertainty in the SRP acceleration has a negligible impact on the trajectory, as the dispersion remains extremely narrow, with nearly all trajectories converging at the nominal landing site. Consequently, variations in both longitude and latitude are minimal, confirming that this level of uncertainty does not significantly alter the landing location. This is likely because the influence of Solar Radiation Pressure accumulates over time, and with the landing phase lasting only about 6 hours, the available timeframe is too short for SRP-induced deviations to become significant. Additionally, in proximity to the body, the dominant acceleration is that of D1, further limiting the effect of SRP on the trajectory.

Moreover, the total time from LOAM to touchdown remains constant, aligning with the nominal value of 5.8 hours.

Regarding the touchdown velocity, its magnitude remains unchanged, matching the nominal value. Similarly, the direction remains close to the nominal one, with the angle between the surface normal and the velocity vector ranging between 0.55 and 0.7 degrees, as can be seen in Figure 5.9.

Given the touchdown velocity values and the dispersion of landing points, an analysis of the post-impact velocity is not required.

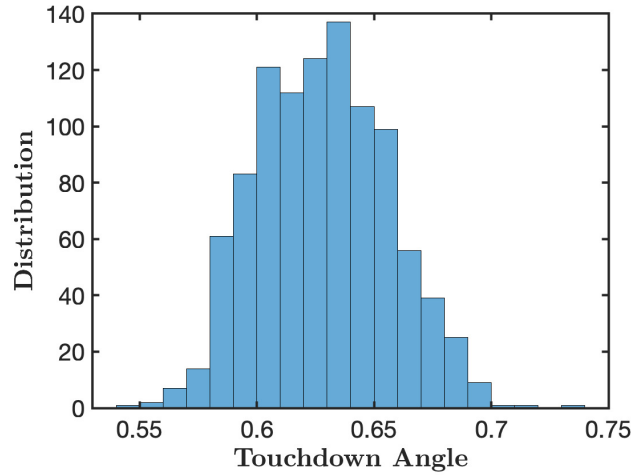


Figure 5.9: Dispersion of touchdown velocity direction.

5.2.4. Uncertainty on Gravitational Attraction

The uncertainty in gravitational attraction arises from errors in the estimated physical parameters of Didymos and Dimorphos. In this case, the uncertainty is applied to the density, around which a normal distribution is generated. Thereafter, the trajectories are

propagated using the nominal dynamics.

The success rate is 100%, and the dispersion of touchdown points is illustrated in Figure 5.10.

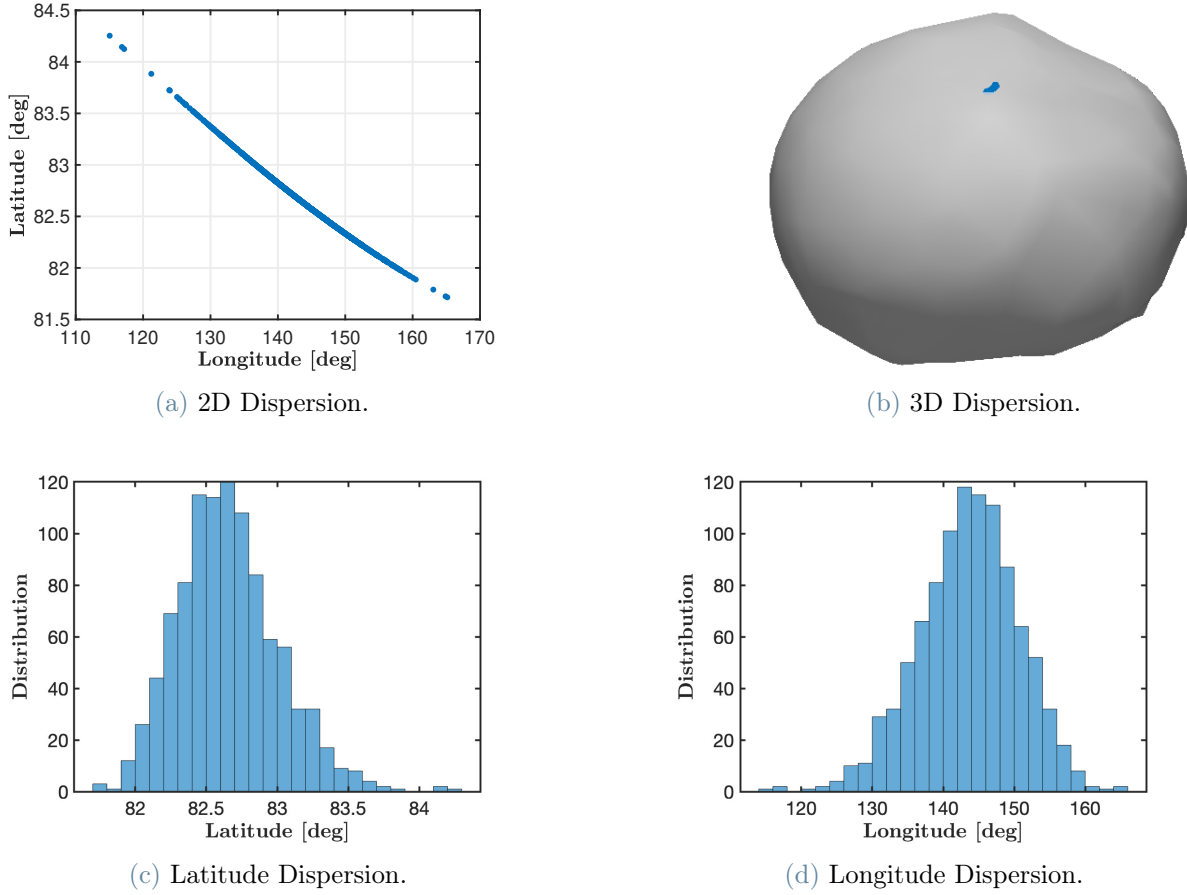


Figure 5.10: Landing points dispersion.

Although the dispersion is slightly broader than in the SRP uncertainty scenario, the variations in longitude and latitude remain minimal, with latitude deviating by less than 3 degrees.

The dispersion in total landing duration is illustrated in Figure 5.11. As in the case of maneuver uncertainties, the time variation is minimal, with a maximum difference of 24.5 minutes, meaning that it does not affect either the trajectory or the landing site.

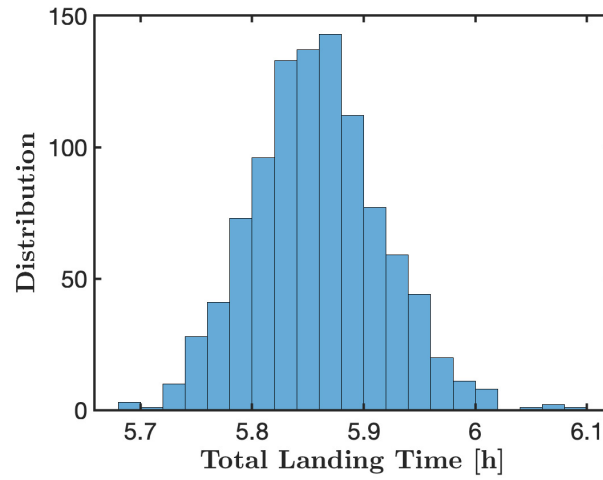


Figure 5.11: Dispersion of touchdown velocity direction.

To assess whether uncertainties in gravitational attraction affect the touchdown velocity, the dispersion of both its magnitude and direction is illustrated in Figure 5.12. The velocity magnitude remains close to the nominal value, ranging between 33 and 37 cm/s, while the angle between the surface normal and the velocity vector is predominantly concentrated between 0.6 and 0.8 degrees, with only a few values reaching up to 5 degrees. This indicates that the velocity is almost entirely normal to the surface, meaning that its full magnitude will be attenuated by the restitution coefficient. Consequently, given the narrow dispersion of landing points around the nominal value, an analysis of the post-impact velocity is not required.

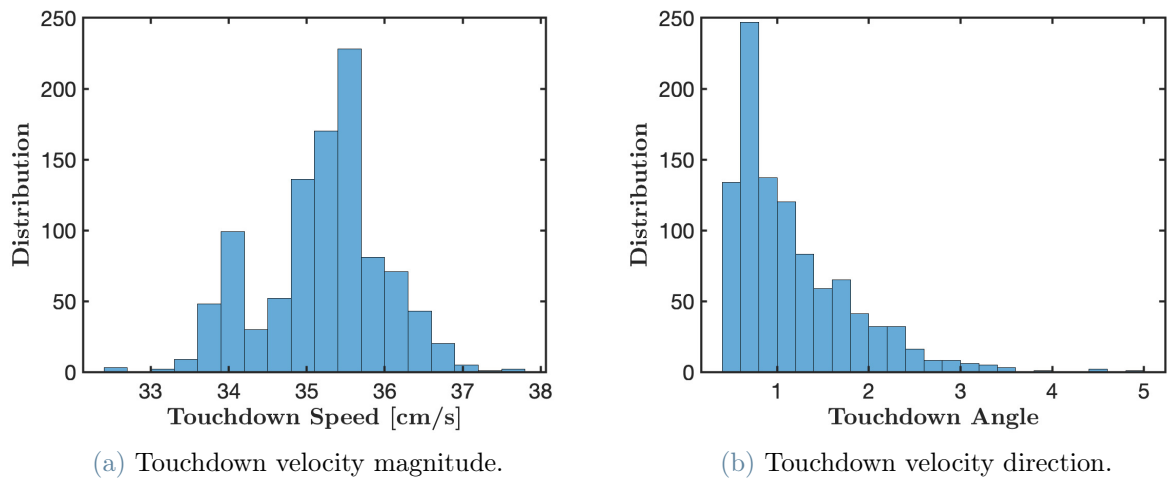


Figure 5.12: Touchdown velocity dispersion.

5.2.5. Combined Uncertainties

When considering all uncertainties together, the analysis provides a comprehensive assessment of the trajectory robustness. The success rate remains at 100%, meaning that even when accounting for all sources of uncertainty simultaneously, all 1000 simulated trajectories successfully reach the surface of the asteroid. The dispersion of touchdown points, shown in Figure 5.13, largely follows the pattern observed in the case of initial state uncertainty alone, as this is the dominant source of errors. However, the dispersion is more pronounced. While the landing points remain concentrated at higher latitudes, a greater number of trajectories extend to lower latitudes, with some reaching even below the equator. This suggests that the risk of landing in more equatorial regions increases when all uncertainties are combined.

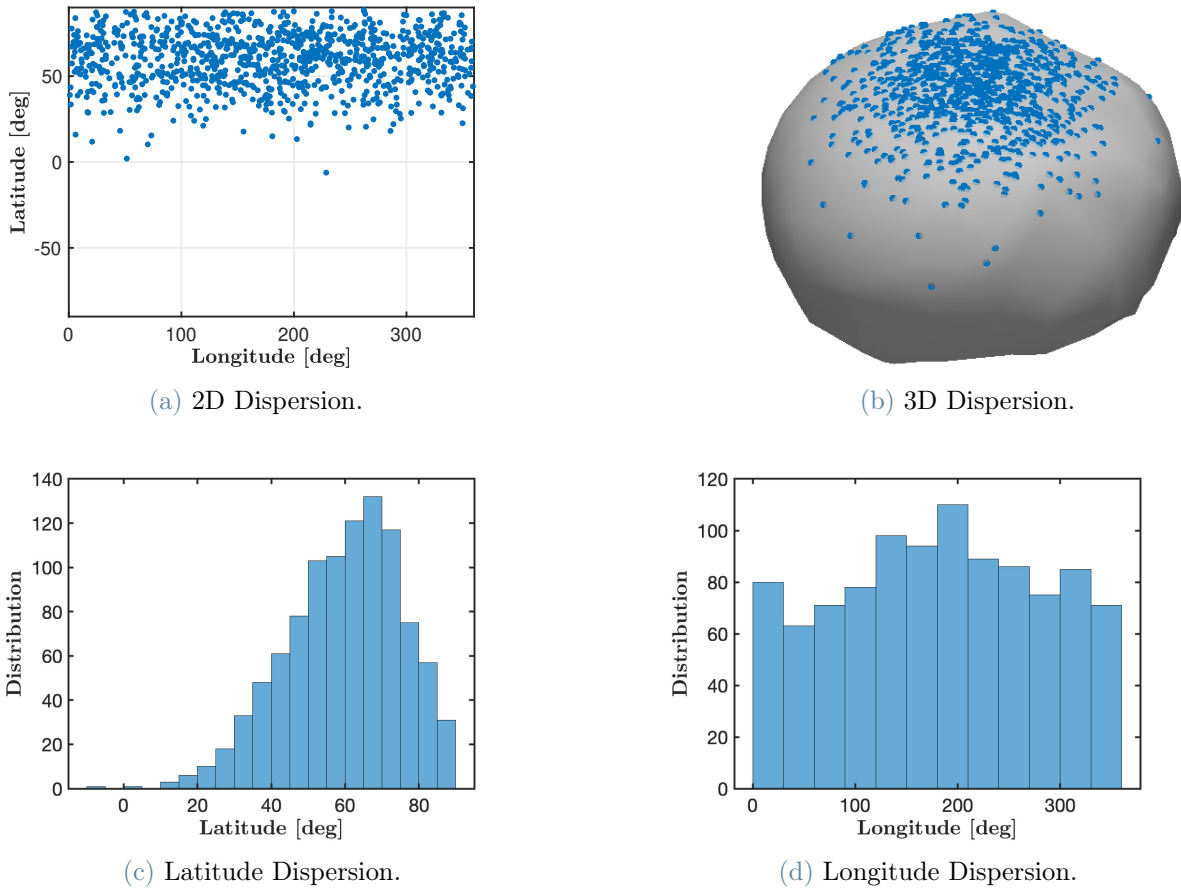


Figure 5.13: Landing points dispersion.

The distribution of total landing durations, shown in Figure 5.14, confirms that uncertainties influence the trajectory duration. However, the overall impact remains comparable to the case where only initial state uncertainty is considered, with landing times still

concentrated between 4 and 8 hours.

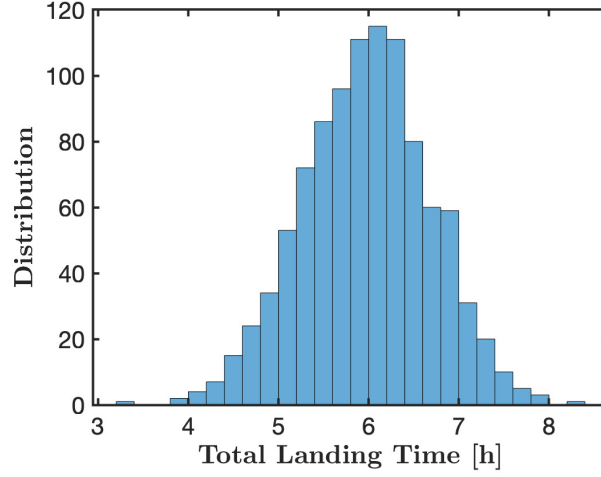


Figure 5.14: Dispersion of touchdown velocity direction.

The touchdown velocity distribution, presented in Figure 5.15, exhibits minimal variation compared to previous cases. The directional dispersion remains consistent, showing no significant deviations from the case where only initial state uncertainty was considered. However, there is a slight increase in the maximum velocity magnitude, which now reaches nearly 37 cm/s, compared to 36 cm/s in previous analysis. This 1 cm/s increase in the upper velocity bound does not pose a risk when landing at the pole, if the touchdown velocity angle remains small. However, the risk increases for trajectories deviating toward the equator, where the return velocity is lower.

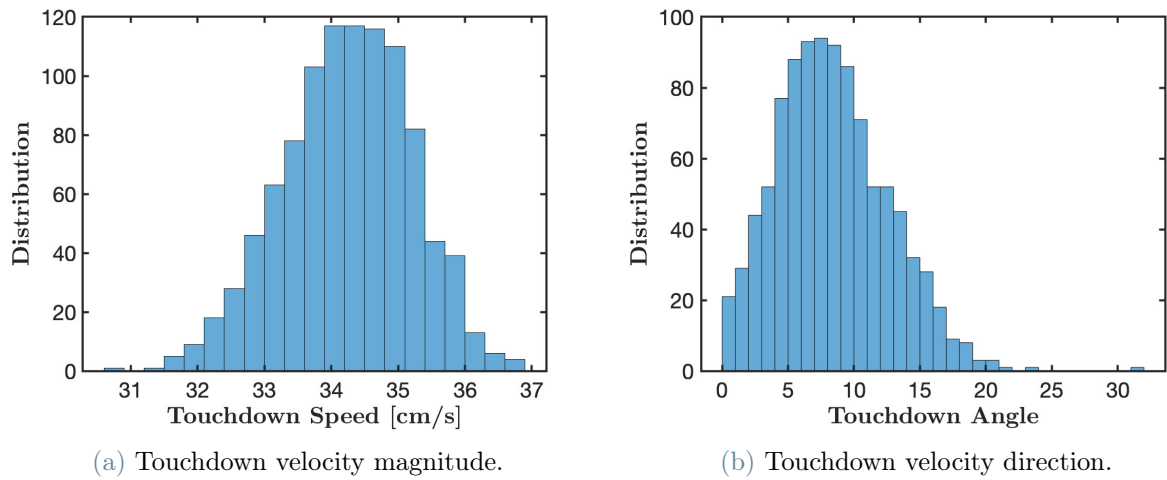


Figure 5.15: Touchdown velocity dispersion.

The analysis of post-touchdown velocities, illustrated in Figure 5.16, reveals a slight deterioration compared to the case where only initial state uncertainty is considered. Specifically, in this scenario, 37 trajectories exceed the guaranteed return speed, indicating a risk of returning to orbit after the first impact. Additionally, compared to the initial state uncertainty case, the difference between the post-touchdown velocity and the GRS is more pronounced, with some trajectories exceeding the return threshold by several cm/s. This is likely due to the fact that a larger number of trajectories reach lower latitudes, closer to the equator, where return speeds are significantly lower.

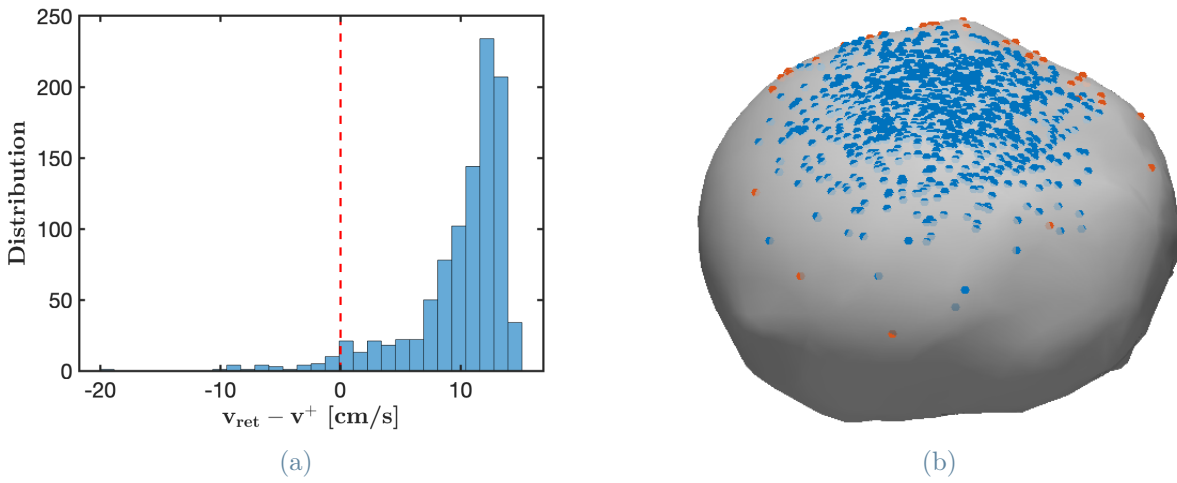


Figure 5.16: Post-touchdown velocity dispersion. (a) Dispersion of the difference between the guaranteed return speed at the landing site and the post-impact velocity magnitude. (b) Distribution of touchdown points on the surface, with red points indicating cases where the post-touchdown velocity exceeds the GRS.

5.3. Final Considerations

The sensitivity analysis reveals that small uncertainties in the dynamical model, particularly those affecting SRP and gravitational attraction, do not significantly alter the trajectory. The nominal landing site remains unchanged, and there is no substantial variation in touchdown velocity values.

Regarding maneuver execution errors, the dispersion of landing points remains confined to the polar region, ensuring that the landing objectives are not compromised. This confirms that even with minor maneuvering inaccuracies, the trajectory remains robust.

In contrast, a wider dispersion is observed when considering navigation errors, primarily due to the large initial state uncertainty. While the trajectory remains robust in terms

of reaching the asteroid, with no cases of escape, there is an increased risk of landing at a location other than the pole. This deviation increases the likelihood of exceeding the guaranteed return speed after the first bounce, potentially compromising the mission's success and safety.

When all uncertainties are considered together, the overall robustness of the trajectory is maintained, but the dispersion of touchdown points increases slightly, extending further toward lower latitudes. Consequently, the probability of exceeding the GRS and escaping after the first impact also rises, though only marginally. These findings further highlight the benefits of targeting the polar region, where higher return speeds provide a safer and more stable landing environment.

6 | Conclusions

This thesis investigated the feasibility and optimization of a landing strategy on Didymos for the Milani CubeSat. While previous studies have primarily focused on landing on Dimorphos, this work evaluated whether a landing on Didymos is feasible by analyzing its dynamical challenges, associated costs, and potential risks. Additionally, it determined the most suitable landing site.

The landing site selection process was based on two primary criteria: ensuring continuous illumination for scientific observations and maximizing landing stability and safety. Two potential landing locations were identified: one at the north pole, where continuous illumination is guaranteed, and another in a specific equatorial region, where guaranteed return speeds are higher than in surrounding areas.

The landing strategy was developed using a direct approach, consisting of two impulsive maneuvers: the Landing Orbit Acquisition Maneuver (LOAM) and the Breaking Maneuver (BM). A numerical optimization process was used to minimize the total ΔV while ensuring a safe touchdown. Results indicate that a landing on Didymos is feasible and significantly more robust when targeting the polar region; indeed, the polar trajectory is safer, requires less ΔV , and minimizes uncertainties due to a shorter descent time. Although the equatorial site provides better lighting conditions for surface imaging, it introduces strict timing constraints and a higher escape risk due to lower return velocities. For these reasons, the pole is selected as the optimal landing site for Milani CubeSat.

Thereafter, a Monte Carlo sensitivity analysis was implemented to assess the robustness of the trajectory under different sources of uncertainty, including initial state errors, maneuver execution inaccuracies, and dynamical model uncertainties. The findings that emerged from this analysis are:

1. Small dynamical uncertainties, such as variations in solar radiation pressure and gravitational attraction, have a negligible impact on the trajectory;
2. Maneuver execution errors, both in magnitude and direction, result in minor deviations in the touchdown location. However, the landing remains within the polar

region, and the touchdown velocity remains largely unaffected;

3. Navigation errors due to initial state uncertainties lead to a wider dispersion of landing points, occasionally extending to lower latitudes. This increases the risk of exceeding the guaranteed return speed after the first bounce, potentially causing the spacecraft to return to orbit.

When considering all uncertainties together, the analysis demonstrated that the trajectory remains highly robust, with a 100% success rate in reaching the surface of the asteroid. However, the dispersion of landing points increases, reinforcing the importance of targeting the polar region, where return speeds are higher, ensuring a more stable and reliable touchdown.

6.1. Future Developments

While this thesis establishes a solid foundation for a landing strategy on Didymos, several aspects should be further explored to enhance the robustness and precision of the mission:

- **Improved Landing Site Selection and Trajectory Optimization:** Once in orbit around the Didymos system, updated observations will refine key parameters, including asteroid density, surface features (e.g., craters, slopes, rocks, and boulders), and overall shape. These data will allow for a more precise selection of the landing site, improving both the touchdown accuracy and the modeling of surface interactions. Additionally, employing polyhedral models with finer discretization would improve the accuracy of gravitational field calculations while also providing a more detailed representation of the surface of Didymos, thereby reducing uncertainties in trajectory design;
- **Refined Surface Interaction Modeling:** The current bounce analysis relies on several assumptions, but a more detailed characterization of the surface properties, such as regolith composition, porosity, and local slope variations, could provide a more accurate representation of the landing dynamics. Incorporating these factors would enable a better assessment of post-touchdown behavior and the likelihood of secondary bounces;
- **Uncertainty Propagation for Trajectory:** While the robustness of the trajectory has been assessed through a Monte Carlo sensitivity analysis, future studies could benefit from a more formal uncertainty propagation framework. Implementing a covariance analysis alongside a dispersion study would offer deeper insights into the evolution of uncertainties over time and their impact on trajectory feasibility.

This approach would allow for a more precise prediction of landing dispersion and enable a structured evaluation of the flyability of the descent under real mission constraints.

Bibliography

- [1] Daniel J Scheeres. *Orbital motion in strongly perturbed environments: applications to asteroid, comet and planetary satellite orbiters*. Springer, 2016.
- [2] Shota Takahashi and Daniel J Scheeres. Higher-order corrections for frozen terminator orbit design. *Journal of Guidance, Control, and Dynamics*, 43(9):1642–1655, 2020.
- [3] Patrick Michel, Michael Küppers, Adriano Campo Bagatin, Benoit Carry, Sébastien Charnoz, Julia De Leon, Alan Fitzsimmons, Paulo Gordo, Simon F Green, Alain Hérique, et al. The esa hera mission: detailed characterization of the dart impact outcome and of the binary asteroid (65803) didymos. *The planetary science journal*, 3(7):160, 2022.
- [4] David Polishook and Oded Aharonson. Surface slopes of asteroid pairs as indicators of mechanical properties and cohesion. *Icarus*, 336:113415, 2020.
- [5] L Prockter, S Murchie, A Cheng, S Krimigis, R Farquhar, A Santo, and J Trombka. The near shoemaker mission to asteroid 433 eros. *Acta Astronautica*, 51(1-9):491–500, 2002.
- [6] Makoto Yoshikawa, Junichiro Kawaguchi, Akira Fujiwara, and Akira Tsuchiyama. Hayabusa sample return mission. *Asteroids IV*, 1(1), 2015.
- [7] Karl-Heinz Glassmeier, Hermann Boehnhardt, Detlef Koschny, Ekkehard Kührt, and Ingo Richter. The rosetta mission: flying towards the origin of the solar system. *Space Science Reviews*, 128:1–21, 2007.
- [8] Angioletta Coradini, Diego Turrini, Costanzo Federico, and Gianfranco Magni. Vesta and ceres: Crossing the history of the solar system. *Space science reviews*, 163:25–40, 2011.
- [9] Sei-ichiro Watanabe, Yuichi Tsuda, Makoto Yoshikawa, Satoshi Tanaka, Takanao Saiki, and Satoru Nakazawa. Hayabusa2 mission overview. *Space Science Reviews*, 208:3–16, 2017.

- [10] Edward Beshore, Dante Lauretta, William Boynton, Christopher Shinohara, Brian Sutter, David Everett, Jonathan Gal-Edd, Ronald Mink, Michael Moreau, and Jason Dworkin. The osiris-rex asteroid sample return mission. In *2015 ieee aerospace conference*, pages 1–14. IEEE, 2015.
- [11] Peter Scheirich, Petr Pravec, Alex J Meyer, Harrison F Agrusa, Derek C Richardson, Steven R Chesley, Shantanu P Naidu, Cristina Thomas, and Nicholas A Moskovitz. Dimorphos orbit determination from mutual events photometry. *The Planetary Science Journal*, 5(1):17, 2024.
- [12] Elisabetta Dotto, Vincenzo Della Corte, Marilena Amoroso, Ivano Bertini, John R Brucato, Andrea Capannolo, Biagio Cotugno, Gabriele Cremonese, Valerio Di Tana, Igor Gai, et al. Liciacube-the light italian cubesat for imaging of asteroids in support of the nasa dart mission towards asteroid (65803) didymos. *Planetary and Space Science*, 199:105185, 2021.
- [13] Claudio Bottiglieri, Felice Piccolo, Carmine Giordano, Fabio Ferrari, and Francesco Topputo. Applied trajectory design for cubesat close-proximity operations around asteroids: The milani case. *Aerospace*, 10(5):464, 2023.
- [14] A Felin, P Annat, J Vernière, R Pinède, A Moussi, M Rebelo, F Topputo, V Moreno-Villa, et al. Hera cubesats operational mission analysis and trajectory design around binary asteroid for small bodies characterisation. In *29th International Symposium on Space Flight Dynamics (ISSFD)*, pages 1–9, 2024.
- [15] Mattia Pugliatti, Antonio Rizza, Felice Piccolo, Vittorio Franzese, Claudio Bottiglieri, Carmine Giordano, Fabio Ferrari, and Francesco Topputo. The milani mission: overview and architecture of the optical-based gnc system. In *AIAA Scitech 2022 Forum*, page 2381, 2022.
- [16] Politecnico di Milano and Tyvak International. Milani CubeSat development, Mission Analysis Report. 2024.
- [17] Claudio Bottiglieri, Felice Piccolo, Antonio Rizza, Mattia Pugliatti, Vittorio Franzese, Carmine Giordano, Fabio Ferrari, Francesco Topputo, et al. Mission analysis and navigation assessment for hera’s milani cubesat. In *4S Symposium*, pages 1–12, 2022.
- [18] Soraya Le Roy. Mission analysis and landing strategy for hera cubesats. Master’s thesis, Politecnico di Milano, 2024.
- [19] Iosto Fodde, Jinglang Feng, Massimiliano Vasile, and Jesús Gil-Fernández. Design

- and analysis of robust ballistic landings on the secondary of a binary asteroid. *arXiv preprint arXiv:2310.19844*, 2023.
- [20] R Terik Daly, Carolyn M Ernst, Olivier S Barnouin, Nancy L Chabot, Andrew S Rivkin, Andrew F Cheng, Elena Y Adams, Harrison F Agrusa, Elisabeth D Abel, Amy L Alford, et al. Successful kinetic impact into an asteroid for planetary defence. *Nature*, 616(7957):443–447, 2023.
- [21] NASA. Didymos & dimorphos. URL <https://science.nasa.gov/solar-system/asteroids/didymos/>.
- [22] Andrew S Rivkin, Nancy L Chabot, Angela M Stickle, Cristina A Thomas, Derek C Richardson, Olivier Barnouin, Eugene G Fahnestock, Carolyn M Ernst, Andrew F Cheng, Steven Chesley, et al. The double asteroid redirection test (dart): planetary defense investigations and requirements. *The Planetary Science Journal*, 2021.
- [23] <https://www.jpl.nasa.gov>. Nasa study: Asteroid’s orbit, shape changed after dart impact. URL <https://www.jpl.nasa.gov/news/nasa-study-asteroids-orbit-shape-changed-after-dart-impact/>.
- [24] Olivier Barnouin, Ronald-Louis Ballouz, Simone Marchi, Jean-Baptiste Vincent, Harrison Agrusa, Yun Zhang, Carolyn M Ernst, Maurizio Pajola, Filippo Tusberty, Alice Lucchetti, et al. The geology and evolution of the near-earth binary asteroid system (65803) didymos. *Nature communications*, 15(1):6202, 2024.
- [25] ESA. Hera spice kernel dataset, 2024. URL https://s2e2.cosmos.esa.int/bitbucket/projects/SPICE_KERNELS/repos/hera/browse/README.md.
- [26] Nasa Horizon. Small-body database lookup. URL https://ssd.jpl.nasa.gov/tools/sbdb_lookup.html#/?sstr=didymos.
- [27] Fabio Ferrari, Vittorio Franzese, Mattia Pugliatti, Carmine Giordano, and Francesco Topputo. Trajectory options for heras milani cubesat around (65803) didymos. *The Journal of the Astronautical Sciences*, 68(4):973–994, December 2021. ISSN 2195-0571. doi: 10.1007/s40295-021-00282-z.
- [28] Nancy L Chabot, Andrew S Rivkin, Andrew F Cheng, Olivier S Barnouin, Eugene G Fahnestock, Derek C Richardson, Angela M Stickle, Cristina A Thomas, Carolyn M Ernst, R Terik Daly, et al. Achievement of the planetary defense investigations of the double asteroid redirection test (dart) mission. *The Planetary Science Journal*, 5(2):49, 2024.
- [29] Jason M Pearl and Darren L Hitt. Asteroid gravitational models using mascons

- derived from polyhedral sources. In *AIAA/AAS Astrodynamics Specialist Conference*, page 5260, 2016.
- [30] Yu Takahashi, Daniel Jay Scheeres, and Robert A Werner. Surface gravity fields for asteroids and comets. *Journal of guidance, control, and dynamics*, 36(2):362–374, 2013.
 - [31] Robert A Werner and Daniel J Scheeres. Exterior gravitation of a polyhedron derived and compared with harmonic and mascon gravitation representations of asteroid 4769 castalia. *Celestial Mechanics and Dynamical Astronomy*, 65:313–344, 1996.
 - [32] Roberto Furfaro, Riccardo Barocco, Richard Linares, Francesco Topputo, Vishnu Reddy, Jules Simo, and Lucille Le Corre. Modeling irregular small bodies gravity field via extreme learning machines and bayesian optimization. *Advances in Space Research*, 67(1):617–638, 2021.
 - [33] William Duncan MacMillan et al. *The theory of the potential*. 1958.
 - [34] Claudio Bottiglieri, Felice Piccolo, Carmine Giordano, Fabio Ferrari, and Francesco Topputo. Applied trajectory design for cubesat close-proximity operations around asteroids: The milani case. *Aerospace*, 10(5):464, 2023.
 - [35] Daniel J Scheeres, Siamak G Hesar, Simon Tardivel, M Hirabayashi, Davide Farnocchia, Jay W McMahon, Steven R Chesley, Olivier Barnouin, Richard P Binzel, William F Bottke, et al. The geophysical environment of bennu. *Icarus*, 276:116–140, 2016.
 - [36] Shota Kikuchi, Sei-ichiro Watanabe, Takanao Saiki, Hikaru Yabuta, Seiji Sugita, Tomokatsu Morota, Naru Hirata, Naoyuki Hirata, Tatsuhiro Michikami, Chikatoshi Honda, et al. Hayabusa2 landing site selection: surface topography of ryugu and touchdown safety. *Space Science Reviews*, 216:1–67, 2020.
 - [37] Daniel J Scheeres, Steven J Ostro, RS Hudson, and Robert A Werner. Orbits close to asteroid 4769 castalia. *Icarus*, 121(1):67–87, 1996.
 - [38] Maria Anna Laino. Design of landing trajectories on small bodies with application to milani cubesat in didymos binary asteroid. Master’s thesis, Politecnico di Milano, 2020.

A | Appendix A

Self-Stabilized Terminator Orbits (SSTOs) are near-circular polar orbits that maintain stability in the presence of SRP perturbation. In these orbits, the orbital plane aligns with the asteroid's terminator, ensuring that the perturbing SRP force and the orbital angular momentum remain nearly constant over time.

The method adopted in this thesis to compute the initial conditions for the last SSTO is based on the higher-order corrections proposed by Takahashi & Scheeres (2020) [2]. The algorithm considers that the SRP induces an offset between the mass center and the orbit center, leading to periodic variations in the orbital elements. To account for this offset, a correction must be applied by incorporating x_0 , the displacement along the Sun–asteroid line. This can be achieved by starting from the nominal semimajor axis a and computing an effective semimajor axis a' , which accounts for the perturbative effects of SRP, ensuring a more stable and frozen orbit over time:

$$a' = a \left(1 + \frac{a^4 a_{SRP}^2}{\mu^2} \right)^{\frac{3}{2}} \quad (\text{A.1})$$

where a_{SRP} is the magnitude of the solar radiation pressure and it is calculated using the cannonball model:

$$a_{SRP} = \frac{P_0(1 + \varepsilon)}{B} \frac{AU^2}{d^2} \quad (\text{A.2})$$

with d being the distance between the Sun and the asteroid, B the mass-to-area ratio, ε the surface reflectivity and $P_0 = 4.53981 \times 10^{-6} \text{ Nm}^{-2}$ the SRP at 1 AU.

After computing the effective semi-major axis, the other orbital parameters are recalculated accordingly. The eccentricity is calculated starting from the computation of Λ , which is an angle indicating the relative strength of SRP:

$$\begin{cases} \Lambda' = \arctan \frac{3P_0(1 + \varepsilon)AU^2}{2B} \sqrt{\frac{a'}{\mu\mu_S A(1 - E^2)}} \\ e = \cos \Lambda' \end{cases} \quad (\text{A.3})$$

where μ and μ_S are the gravitational parameters of the asteroid and of the Sun, respectively, while A and E are the semi-major axis and eccentricity of the orbit of the asteroid.

Next, the eccentricity vector, angular momentum vector, and true anomaly must be determined. In the Sun-Didymos reference frame, the angular momentum vector $\mathbf{h}$ can be defined along either $\pm x$ -axis, which aligns with the Sun-Didymos direction. Consequently, the eccentricity vector $\mathbf{e}$ will lie along the z -axis: if $\mathbf{h}$ is along $+x$, then $\mathbf{e}$ is along $-z$; conversely, if $\mathbf{h}$ is along $-x$, then $\mathbf{e}$ is along $+z$. The true anomaly is chosen equal to 0, although it could also be set to π depending on the selected convention. The final relations are given by:

$$\begin{cases} \mathbf{h} = \sqrt{1 - e^2} \hat{\mathbf{x}} \\ \mathbf{e} = -e \hat{\mathbf{z}} \end{cases} \quad (\text{A.4})$$

Subsequently, the effective orbit radius $\mathbf{R}'$ and the effective velocity vector $\mathbf{v}'$ can be computed from the orbital elements. The nominal radius $\mathbf{R}$ is then obtained from the effective radius using the following relation:

$$\mathbf{R} = \mathbf{R}' \left(1 + \frac{a^4 a_{SRP}^2}{\mu^2} \right)^{-\frac{3}{2}} \quad (\text{A.5})$$

Finally, by calculating the offset:

$$x_0 = \frac{a'^3 a_{SRP}}{\mu} (1 - e) \left(1 + \frac{e}{4} \right) \quad (\text{A.6})$$

it is possible to rescale the position vector and compute the actual state vector:

$$\begin{cases} \mathbf{r} = \mathbf{R} + x_0 \hat{\mathbf{x}} \\ \mathbf{v} = \mathbf{v}' \end{cases} \quad (\text{A.7})$$

List of Figures

2.1	Milani CubeSat Configuration	11
2.2	Milani CubeSat Mission Phases	14
2.3	Dimorphos shape before and post DART impact	16
2.4	Didymos binary system	17
2.5	Didymos binary system heliocentric orbit.	19
2.6	Didymos system geometry	20
3.1	Convergence regions of the exterior and interior gravity fields	25
4.1	Didymos radial distance and elevation	34
4.2	Tilt angle	35
4.3	Deviation in magnitude of the gravitational acceleration	36
4.4	Deviation in direction of the gravitational acceleration	37
4.5	Total surface acceleration	38
4.6	Gravitational acceleration	38
4.7	Surface slope angle	39
4.8	Escape velocity	40
4.9	Zero-velocity curves and surfaces	41
4.10	Guaranteed return speed	42
4.11	Feasible landing locations at pole	43
4.12	Selected landing location at equator	45
4.13	Diagrams of landing strategy	46
4.14	Back-propagated arcs from selected landing points	50
4.15	Optimized landing trajectory at pole	52
4.16	Optimized landing trajectory at the equator	53
5.1	Landing points dispersion, initial state uncertainty	59
5.2	Dispersion of landing duration, initial state uncertainty	60
5.3	Touchdown velocity dispersion, initial state uncertainty	60
5.4	Post-touchdown velocity dispersion, initial state uncertainty	61
5.5	Landing points dispersion, uncertainty on maneuvers	62

5.6	Dispersion of landing duration	63
5.7	Touchdown velocity dispersion, uncertainty on maneuvers	63
5.8	Landing points dispersion, uncertainty on SRP	64
5.9	Dispersion of touchdown velocity direction, uncertainty on SRP	65
5.10	Landing points dispersion, uncertainty on gravitational attraction	66
5.11	Dispersion of touchdown velocity direction, uncertainty on gravitational attraction	67
5.12	Touchdown velocity dispersion, uncertainty on gravitational attraction . .	67
5.13	Landing points dispersion, combined uncertainties	68
5.14	Dispersion of touchdown velocity direction, combined uncertainties	69
5.15	Touchdown velocity dispersion	69
5.16	Post-touchdown velocity dispersion	70

List of Tables

1.1	Main asteroids properties	3
2.1	Pre-impact data of Didymos	16
2.2	Pre-impact data of Dimorphos	16
2.3	Post-impact data of Didymos	17
2.4	Post-impact data of Dimorphos	18
2.5	Heliocentric orbital parameters of Didymos binary system	19
3.1	AHP for D1 gravity model	29
3.2	Milani SRP parameters	31
4.1	Characteristics of the selected landing point at pole	44
4.2	Characteristics of the selected landing point at equator	45
4.3	Trajectory results for the pole landing site	52
4.4	Trajectory results for the equatorial landing	53

