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SCUOLA DI INGEGNERIA INDUSTRIALE
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Executive Summary of the Thesis

Design and Optimization of a Mothership-based Active Debris Removal Strategy

Laurea Magistrale in Space Engineering - Ingegneria Spaziale

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Academic year: 2024-2025

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1. Introduction

The exponential growth of the space industry and the proliferation of satellite constellations have significantly increased the amount of space debris in Low Earth Orbit (LEO). This increasing number of debris poses substantial risks to operational satellites and future space missions, both due to the limited available space and for the growing need of collision avoidance maneuvers (CAMs). These maneuvers not only represent an economic issue due to fuel consumption but also a safety concern for the mission itself and any possible human presence. For instance, the International Space Station (ISS) has performed more than one CAM per year since it began its space activity in 1999. This thesis addresses the critical issue of space debris removal, focusing on the specific scenario of a rapid-response, short-duration mission (6 hours), which could be critical for emergency. The goal is to mitigate the risk of the Kessler Syndrome [4], a scenario that hypothesizes a high density of objects in LEO could lead to a cascade of collisions, exponentially increasing debris and exacerbating the problem. According to some studies, this issue could be managed by removing five large debris pieces per year, which is considered the minimum threshold for any active debris removal mission [3].

2. Related Works

The literature presents various practical methods for Active Debris Removal (ADR), including techniques such as harpoons, magnetic fields, tethers, and deorbiting kits. However, none of these methods have been concretely implemented. One notable approach involves a mothership satellite that individually approaches debris and uses a propulsion system to perform the deorbiting duty [5]. This is the approach adopted in this thesis, as it aligns well with the goal of multiple debris removal. The mothership can target more than one debris item without directly performing a deorbit maneuver for each, thus saving both time and fuel. A critical aspect of ADR is the selection of debris for removal. Research highlights the importance of using environmental indices as the objective function to maximize risk reduction by prioritizing the removal of the most dangerous debris [2, 6]. Various optimization techniques are applied to this problem, treating it as a variant of the Traveling Salesman Problem (TSP) or its time-dependent version (TDTSP) [7]. Techniques such as genetic algorithms, Monte Carlo simulations, and reinforcement learning have been used to identify optimal paths and sequences for debris removal. Despite advancements in ADR strategies, much of the existing literature has not fully explored the problem from an Operations Research perspective. Solutions often focus on statistical methods or specific case studies without a comprehensive operational framework. This thesis introduces innovative elements, including a versatile and straightforward model that can be easily adapted to more complex scenarios. The integration of environmental indices for initial debris selection represents a significant advancement, offering a practical and sustainable approach to active debris removal and enhancing space environment management.

3. Problem Definition and Solution Methodologies

The methodology involves theoretical modeling, simulation, and optimization techniques. The space debris removal problem is formulated as a Time Dependent Orienteering Problem (TDOP). The model considers 20 major debris elements in LEO, selected based on their potential impact and position, according to their environmental indices. The mothership, equipped with deorbiting kits, follows an optimal path to rendezvous with and deorbit the selected debris within a highly constrained mission timeframe of 6 hours, simulating a rapid-response operation [1]. A mathematical programming formulation was developed in AMPL language to decide which debris to remove and determine the routing and scheduling of the mothership.

Key steps of this formulation include:

- **Instance Building:** Debris are prioritized based on size, mass, and collision risk using an index-based evaluation from existing literature. MATLAB simulations define critical parameters such as flight times and velocities, providing inputs for AMPL to ensure the model reflects real-world conditions.

- **Preprocessing:** Initial preprocessing identifies optimal paths within clusters of debris, reducing the number of debris to analyze and simplifying the optimization. In addition to that, the number of possible considered trajectories is chosen to be three: the minimum velocity path, the fastest path with a fuel consumption limit, and the path with the median value of the speed. These selections aim to minimize fuel or time consumption, also by adding a third possible alternative.

- **Solution Method Developed:** A Matheuristic approach combines mathematical optimization with heuristic methods, refining the path through multiple iterations to balance time and fuel constraints. The mission duration was set to 6 hours, divided into three smaller 2-hour optimizations. Each iteration builds on the results of the previous one, enabling optimal results within an acceptable timeframe while avoiding memory issues. The model is implemented in AMPL and solved using the CPLEX solver, known for its efficiency in complex optimization problems.

4. Mathematical Programming Formulation

The problem is mathematically formulated using integer variables (Mixed-Integer Linear Problem), an objective function to maximize the sum of environmental indices, and various constraints ensuring mission feasibility. The model has evolved to address specific requirements and improvements over the base formulation.

Variables

- Binary variables to indicate if debris is reached (y_i) and if specific trajectories are used ($x_{i,j,t,p}$).
- Continuous variables for visit times (τ_i) and waiting times (w_i).
- Integer variables to track the number of orbital periods ($\xi_{i,j}$).
- Binary variable to indicate the last debris (z_i).

Objective Function The objective is to maximize the sum of environmental indices associated with the debris removed over the set of potential debris I:

$$\text{Maximize} \sum_{i \in I} e_i y_i \tag{1}$$

Constraints

- **First debris:** Ensure the mission starts from Earth to a debris at time $t = 0$.
- **Flow coherence:** If a debris is reached, the satellite must leave it.
- **Flow conservation:** The sum of arriving paths equals the sum of departing paths plus the last debris.
- **Last debris:** Only one debris can be the last one in the mission.
- **Max fuel available:** Total fuel consumption must not exceed the initial fuel.
- **Earth leaving time:** The mission starts with the departure from Earth.
- **Waiting time:** No waiting time at Earth, and minimum operational waiting time at debris.
- **Arriving time monotony:** Arrival time at debris must account for previous arrival, waiting time, and travel time.
- **Last arriving time:** The last arrival must be within the mission duration.
- **Consistency of times:** Arrival and waiting times must align with the mission start time and orbital periods.
- **Integer number of orbital periods:** Ensure the number of completed orbital periods are integers.

5. Matheuristic

Since this optimization problem is NP-hard, with the model presente in section 4 we cannot solve, in reasonable computational amount of time, large size of real instances. For this reason, we also developed a Matheuristic, based on the previous MILP. In order to reach feasible optimal solutions while greatly reducing the number of variables to evaluate, some logical considerations and modifications have been made over the model:

- Time is discretized into $10s$ intervals, instead of the initial $1s$.
- The number of possible trajectories, p, is fixed at three, chosen based on fuel efficiency, median values of tranfer speed, and time efficiency with a maximum fuel consumption.
- The optimization is divided into three steps of 2 hours each, within the total mission time of 6 hours. Constraints on available fuel and mission duration are set to increase precision in each iteration, leading to a better overall solution.
- Debris located close to each other (according to their orbital Keplerian elements) are treated as clusters. If one debris in a cluster is reached, all in the cluster must be visited. The best internal path within a cluster is evaluated in preprocessing, depending on the debris previously removed.

6. Results

The optimization results show that the proposed model effectively removes at least 5 pieces of space debris. The model favors paths that minimize fuel consumption and time by targeting closely grouped debris. This effectiveness is evident both with and without clusters implementation, which is more beneficial for larger debris instances and longer missions.

Key findings include:

- **Efficiency:** The mothership's path is optimized to balance fuel usage and mission time, ensuring the removal of debris with maximum total environmental index.

- **Feasibility:** The mothership-based approach demonstrates feasibility and potential for real-world applications.

- **Sensitivity Analysis:** The model's robustness is tested across various scenarios. Results with and without preprocessing show that, while the mission path and removed debris' order change, the final outcomes remain consistent.

In Figure 1, the best result obtained with constrained iterations evaluating a 6-hour mission duration is presented:

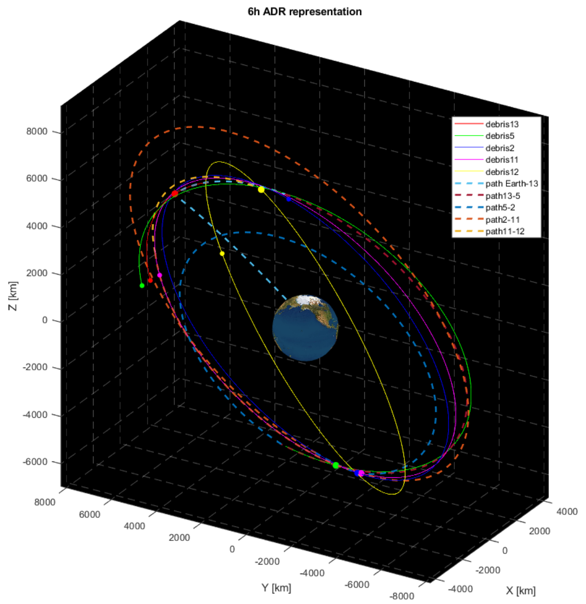

Figure 1: 6-hour constrained iterations results.

6.1. Results from CPLEX

Results from the optimization model using the CPLEX solver are summarized in the following tables, detailing performance metrics for each iteration:

- CPLEX Time Limit represents the maximum computational duration;
- $\sum e$ represents the summary of environmental indices;
- *RMG* is the Relative MIP Gap of CPLEX;
- ΔV is the velocity needed to perform the transfers;
- Δt is the time needed to perform the transfers.

<table>
<tr><th rowspan="2">CPLEX
Time Limit</th><th colspan="4">1^{ST} Iteration</th></tr>
<tr><th>$\sum e$</th><th>RMG</th><th>ΔV</th><th>Δt</th></tr>
<tr><td>100</td><td>131.5</td><td>1.0405</td><td>3.0101</td><td>7630</td></tr>
<tr><td>200</td><td rowspan="8">134.49</td><td rowspan="2">0.9943</td><td rowspan="8">3.1337</td><td rowspan="8">8900</td></tr>
<tr><td>300</td></tr>
<tr><td>500</td><td>0.9364</td></tr>
<tr><td>1000</td><td>0.6798</td></tr>
<tr><td>2000</td><td>0.6732</td></tr>
<tr><td>5000</td><td>0.6229</td></tr>
<tr><td>10000</td><td>0.3093</td></tr>
<tr><td>18000</td><td>0</td></tr>
</table>

Table 1: Results for the first iteration.

<table>
<tr><th rowspan="2">CPLEX
Time Limit</th><th colspan="4">2^{ND} Iteration</th></tr>
<tr><th>$\sum e$</th><th>RMG</th><th>ΔV</th><th>Δt</th></tr>
<tr><td>100</td><td>173.91</td><td>0</td><td>4.6911</td><td>13760</td></tr>
<tr><td>200</td><td rowspan="8">176.6</td><td rowspan="8">0</td><td rowspan="8">4.3695</td><td rowspan="8">16650</td></tr>
<tr><td>300</td></tr>
<tr><td>500</td></tr>
<tr><td>1000</td></tr>
<tr><td>2000</td></tr>
<tr><td>5000</td></tr>
<tr><td>10000</td></tr>
<tr><td>18000</td></tr>
</table>

Table 2: Results for the second iteration.

<table>
<tr><th rowspan="2">CPLEX
Time Limit</th><th colspan="4">3^{RD} Iteration</th></tr>
<tr><th>$\sum e$</th><th>RMG</th><th>ΔV</th><th>Δt</th></tr>
<tr><td>100</td><td>213.98</td><td>0.8229</td><td>6.3971</td><td>19830</td></tr>
<tr><td>200</td><td rowspan="8">218.27</td><td rowspan="8">0</td><td rowspan="8">7.9991</td><td rowspan="8">19920</td></tr>
<tr><td>300</td></tr>
<tr><td>500</td></tr>
<tr><td>1000</td></tr>
<tr><td>2000</td></tr>
<tr><td>5000</td></tr>
<tr><td>10000</td></tr>
<tr><td>18000</td></tr>
</table>

Table 3: Results for the third iteration.

6.2. Analysis of Results

The results indicate that the model is efficient and robust, as shown by the objective values and solution feasibility. The model handles varying problem sizes and complexities effectively. Although preprocessing steps offer benefits, they do not significantly alter final results compared to scenarios without it, reflecting the model's stability.

6.3. Implications and Limitations

While promising, the model has limitations. Performance may vary with different problem sizes or constraints. Future work could explore scaling the model for larger instances and incorporating additional constraints to enhance flexibility and applicability.

7. Conclusion

This thesis contributes to space debris mitigation by presenting an optimized model for active debris removal. The mothership-based strategy, supported by advanced optimization techniques, offers significant promise for addressing the growing space debris problem. The findings emphasize the importance of integrating Operations Research methodologies with computational tools for practical space sustainability solutions.

One of the key strengths of this work lies in the approach taken to manage computational limitations and ensure reasonable solution times. The model's design, which partitions the mission into smaller optimization segments, demonstrates feasibility even within tight computational constraints. This makes it especially applicable for scenarios requiring rapid decision-making, such as emergency responses to collision threats involving the ISS or other high-value satellites. Despite these computational strategies, the model and the solution remain applicable to more typical mission scenarios with longer durations and lower fuel consumption, reinforcing its versatility and practical applicability. The solution found in this specific case, characterized by a mission duration of only a few hours, is realistic given a sufficient fuel supply. This result showcases the capability to execute very short-duration ADR missions, underlining the potential utility of the approach in urgent and high-risk situations. The simplicity of the model, achieved through linearization from a more complex, time-variable problem, highlights its adaptability and accessibility. This streamlined approach ensures that mission planners can efficiently apply it without sacrificing essential mission dynamics.

Future work may refine the model to include more dynamic factors, enhance precision in transfer orbits, and explore additional optimization techniques. The potential for scaling the model to accommodate larger debris fields and more complex missions warrants further investigation, which would strengthen its utility and expand its application to a wider range of space debris management scenarios.

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