DYNAMIC ANALYSIS OF A PLANAR 2-DoF MANIPULATOR
DRIVEN BY PNEUMATIC ACTUATORS

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Abstract

In recent years, parallel manipulators have become increasingly popular in industries, especially, in the field of machine tools handling and, pick and place operations. In this thesis work, a planar 2-degree-of-freedom (DoF) parallel-kinematic-mechanism (PKM) manipulator, which is actuated by a pair of pneumatic drives is proposed. A full kinematic analysis of the manipulator is discussed in reference with a desired trajectory and motion law. In this analysis it is shown that the inverse and forward kinematics can be described in closed form; the velocity equation, singularity, and workspace of the manipulator are presented. Furthermore, the inverse dynamics analysis of the PKM manipulator is investigated employing the Newton-Euler approach, which targets on getting the desired driving force that should be applied at the joints by the actuators. After determining the drive forces, a model of pneumatic actuator is designed, which consists of models for a standard FESTO DGP/DGPL 32 linear actuator and for a 5/3 MYPE 1/8 proportional valve. The main work for analysis of the proposed planar manipulator is done in a Matlab/Simulink environment. In addition, a 3D physical model of the PKM manipulator which is created in a Solidworks graphic interface and then translated into a Simulink/Simmechanics environment has been used in both the kinematic and dynamic analysis stages. A numerical simulation of the manipulator is done based on the models created and the desired trajectory and law of motion chosen. Primary results obtained from this simulation are discussed, giving particular attention to the position error. And finally, an appropriate control strategy is designed meant to reduce the resulting error in position of the end-effector. At the end, a comprehensive conclusion of the entire work is presented and necessary recommendations are forwarded.
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Molalign Mhretie
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## Nomenclatures & abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$L_1$</td>
<td>length of legs 1 and 2</td>
</tr>
<tr>
<td>$L_2$</td>
<td>distance between sliders</td>
</tr>
<tr>
<td>$L$</td>
<td>actuator’s maximum stroke</td>
</tr>
<tr>
<td>$m$</td>
<td>mass of each leg</td>
</tr>
<tr>
<td>$m_s$</td>
<td>mass of each slider</td>
</tr>
<tr>
<td>$m_a$</td>
<td>mass of moving parts in each actuator</td>
</tr>
<tr>
<td>$M$</td>
<td>mass of payload</td>
</tr>
<tr>
<td>$P(x, y)$</td>
<td>vector of position of the end-effector</td>
</tr>
<tr>
<td>$Q(q_1, q_2)$</td>
<td>vector of position of the joints</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular speed</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angular acceleration</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Lagrange multiplier</td>
</tr>
<tr>
<td>$J_i$</td>
<td>moment of inertia of the $i^{th}$ leg about joint point</td>
</tr>
<tr>
<td>$P_s$</td>
<td>supply pressure</td>
</tr>
<tr>
<td>$P_e$</td>
<td>exhaust pressure</td>
</tr>
<tr>
<td>$P_{atm}$</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>$P_0$</td>
<td>initial pressure in cylinder chambers</td>
</tr>
<tr>
<td>$P_1, P_2$</td>
<td>pressure inside chamber 1 and 2, respectively</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>change in pressure of air in chamber 1 and 2</td>
</tr>
<tr>
<td>$T_s$</td>
<td>supply air temperature</td>
</tr>
<tr>
<td>$T_e$</td>
<td>exhaust air temperature</td>
</tr>
<tr>
<td>$T_c$</td>
<td>air temperature inside the chambers</td>
</tr>
<tr>
<td>$A_p$</td>
<td>piston cross-sectional area</td>
</tr>
<tr>
<td>$V_1, V_2$</td>
<td>volume of air inside chamber 1 and 2, respectively</td>
</tr>
<tr>
<td>$V_{10}, V_{20}$</td>
<td>dead-volume of air inside chamber 1 and 2, respectively</td>
</tr>
<tr>
<td>$u$</td>
<td>servo valve command signal</td>
</tr>
<tr>
<td>$y$</td>
<td>piston displacement</td>
</tr>
<tr>
<td>$q_m$</td>
<td>mass flow rate of air</td>
</tr>
<tr>
<td>$C_P$</td>
<td>constant pressure specific heat of air</td>
</tr>
<tr>
<td>$C_V$</td>
<td>constant volume specific heat of air</td>
</tr>
</tbody>
</table>
\( K=1.4 \) –heat ratio
\( R=287 \ [m.K^1] \) –gas constant
\( B=65 \ [Nsm^{-1}] \) –viscous friction coefficient of air
\( C_d=0.8 \) –valve discharge coefficient
\( K \) –total kinetic energy
\( U \) –total potential energy
\( g \) –gravitational acceleration
\( F_B \) –vector of reaction forces on the legs at the joints
\( F_P \) –vector of reaction forces on the legs at the end-effector
\( F_{ext} \) –vector of external forces acting on the end-effector
\( M_{ext} \) –vector of external moments acting on the end-effector
\( K_P \) –proportional gain of controller
\( K_I \) –integral gain of controller
\( K_D \) –derivative gain of controller

**Abbreviations**

\( PKM \) –Parallel Kinematic Mechanism
\( DoF \) –Degree of Freedom
\( PRRRP \) –Prismatic _Revolute _Revolute _Revolute _Prismatic type joint
\( IKA \) –Inverse Kinematic Analysis
\( DKA \) –Direct Kinematic Analysis
\( IDA \) –Inverse Dynamic Analysis
\( ODE \) –Ordinary Differential Equations
\( PID \) –Proportional _Integral _Derivative controller
Chapter One

1. Introduction

1.1. Background and Motivation

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of mechanical engineering, electrical engineering, systems and industrial engineering, computer science, economics, and mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics and factory automation. Robots can be classified as industrial robot manipulators, mobile robots, and other autonomous mechanical systems.

The term robot was first introduced into our vocabulary by the Czech playwright Karel Capek in his 1920 play Rossum’s Universal Robots, the word robota being the Czech word for work.[24] Since then the term has been applied to a great variety of mechanical devices, such as teleoperators, underwater vehicles, autonomous land rovers, etc. Virtually anything that operates with some degree of autonomy, usually under computer control, has at some point been called a robot. In this text the term robot will mean a computer controlled industrial manipulator.

This Thesis work is concerned with Parallel Kinematic Mechanism (PKM) type industrial robot manipulator driven by pneumatic actuators, which covers the kinematics and dynamics analysis of the PKM, motion planning and control of the manipulator under different desired trajectories.

A parallel manipulator consists of a moving platform and a fixed base, connected by several linkages (also called legs). As PKM are used for more difficult tasks, control requirements increase in complexity to meet these demands. The implementation for PKMs often differs
from their serial counterparts, and the dual relationship between serial and parallel manipulators often means one technique which is simple to implement on serial manipulators is difficult for PKMs (and vice versa). Because parallel manipulators result in a loss of full constraint at singular configurations, any control applied to a parallel manipulator must avoid such configurations. The manipulator is usually limited to a subset of the usable workspace since the required actuator torques will approach infinity as the manipulator approaches a singular configuration. Thus, some method must be in place to ensure that the manipulators avoid those configurations.

Parallel robots have many advantages comparing to the serial robots, such as high flexibility, high stiffness, and high accuracy. To achieve a higher accuracy the static and dynamic behavior must be better understood. The problems concerning kinematics and dynamics of parallel robots are as a rule more complicated than those of serial one.
Chapter Two

2. Literature Review

2.1. Parallel manipulator Design
The conceptual design of PKM manipulators can be dated back to the time when Gough established the basic principles of a manipulator with a closed-loop kinematic structure (Gough, 1956), that can generate specified position and orientation of a moving platform so as to test tire wear and tear. Stewart designed a platform manipulator for use as an aircraft simulator in 1965 (Stewart, 1965). In 1978, Hunt (1978) made a systematic study of manipulators with parallel kinematics, in which the planar 3-RPS parallel manipulator is a typical one. Since then, parallel manipulators have been studied extensively by numerous researchers.[1]

The most studied parallel manipulators are those with 6 Degree of Freedoms (DoFs). These parallel manipulators possess the advantages of high stiffness, low inertia, and large payload capacity. However, they suffer the problems of relatively small useful workspace and design difficulties (Merlet, 2000). Furthermore, their direct kinematics pose a very difficult problem; however the same problem of parallel manipulators with 2 and 3 DoFs can be described in closed form (Liu, 2001. Moreover, for a parallel manipulator with 2 and 3 DoFs, the singularities can always be identified readily. For such reasons, parallel manipulators with, especially 2 and 3 DoFs, have increasingly attracted more and more researchers’ attention with respect to industrial applications (Tonshoff et al., 1999; Siciliano, 1999; Tsai and Stamper, 1996; Ceccarelli, 1997; Liu et al., 2001). In these designs, parallel manipulators with three translational DoFs have been playing important roles in the industrial applications (Tsai and Stamper, 1996; Clavel, 1988; Hervé, 1992; Kim and Tsai, 2002; Zhao and Huang, 2000; Carricato and Parenti-Castelli, 2001; Kong and Gosselin, 2002), especially, the DELTA robot (Clavel, 1988), which is evident from the fact that the design of the DELTA robot is covered by a family of 36 patents (Bonev, 2001). Tsai’s manipulator (Tsai and Stamper, 1996), in which each of the three legs consists of a parallelogram, is the first design to solve the problem of UU chain. A 3-translational-DoF parallel manipulator, Star, was designed by Hervé based on group theory (Hervé, 1992). Such parallel manipulators have wide applications in the
industrial world, e.g., pick-and-place application, parallel kinematics machines, and medical devices.

The existing planar 2-DoF parallel manipulators (Asada and Kanade, 1983; McCloy, 1990; Gao et al., 1998) are the well-known five-bar mechanism with prismatic actuators or revolute actuators. In the case of the manipulator with revolute actuators, the mechanism consists of five revolute pairs and the two joints fixed to the base are actuated. In the case of the manipulator with prismatic actuators, the mechanism consists of three revolute pairs and two prismatic joints and the prismatic joints are actuated. The output of the manipulator is the translational motion of a point on the end-effector, i.e., the orientation of the end-effector is also changed correspondingly.

2.2. Robots mechanisms and their classification
A robot is a machine capable of physical motion for interacting with the environment. Physical interactions include manipulation, locomotion, and any other tasks changing the state of the environment or the state of the robot relative to the environment. A robot has some form of mechanisms for performing a class of tasks. A rich variety of robot mechanisms has been developed in the last few decades.

2.2.1. Joint Primitives and Serial Linkages
A robot mechanism is a multi-body system with the multiple bodies connected together. We begin by treating each body as rigid, ignoring elasticity and any deformations caused by large load conditions. Each rigid body involved in a robot mechanism is called a link, and a combination of links is referred to as a linkage. In describing a linkage it is fundamental to represent how a pair of links is connected to each other. There are two types of primitive connections between a pair of links, as shown in Figure 2.1, below. The first is a Prismatic joint where the pair of links makes a translational displacement along a fixed axis. In other words, one link slides on the other along a straight line. Therefore, it is also called a sliding joint. The second type of primitive joint is a Revolute joint where a pair of links rotates about a fixed axis. This type of joint is often referred to as a hinge, articulated, or rotational joint.

![Figure 2.1 Types of primitive connections.](image)
Combining these two types of primitive joints, one can create many useful mechanisms for robot manipulation and locomotion. These two types of primitive joints are simple to build and are well grounded in engineering design. Most of the robots that have been built so far are combinations of only these two types.

2.2.2. Robot Components
A particular kind of robot, whether Humanoid or industrial manipulator, consists of the following fundamental components.

- **Body**: the main part of the robot that helps the transformation of motion, torque or forces. In most cases, it refers to the linkages and their mechanisms.

- **Effectors**: These are the last components to receive motion and enable a robot perform the desired task by interacting with the external environments. The effectors maybe of various types, the four most common ones are:
  1. **Impactive**: – jaws or claws which physically grasp by direct impact upon the object.
  2. **Ingressive**: – pins, needles or hackles which physically penetrate the surface of the object (used in textile, carbon and glass fiber handling).
  3. **Astrictive**: – suction forces applied to the objects surface (whether by vacuum, magneto- or electro adhesion).
  4. **Contigutive**: – requiring direct contact for adhesion to take place (such as glue, surface tension or freezing).

- **Actuators**: An actuator or drive is a source of motion for a robot to perform its task. It can be of linear type (Cylinders) or rotary type (e.g. Motors). Commonly they are classified based on the kind of medium used for energy transmission, such as Electrical, Hydraulic, Pneumatic and Internal Combustion (IC) hybrids.

- **Sensors**: sensors are those components that helps to acquire a feedback action by interaction with the surrounding of the robot.

- **Controller**: The controllers are used to maintain the robot precision, stability, linearity and manageability within a desired appropriate limits.
2.2.3. Robotic Systems
A robot manipulator should be viewed as more than just a series of mechanical linkages. The mechanical arm is just one component in an overall Robotic System, illustrated in Figure 2.2, which consists of the arm, external power source, end-of-arm tooling, external and internal sensors, computer interface, and control computer. Even the programmed software should be considered as an integral part of the overall system, since the manner in which the robot is programmed and controlled can have a major impact on its performance and subsequent range of applications.

![Diagram of a typical robot/ manipulator functional system](image)

Figure 2.2 A typical robot/ manipulator functional system

2.3. Parallel Manipulator
A parallel manipulator is one in which some subset of the links form a *closed chain*. More specifically, a parallel manipulator has two or more independent kinematic chains connecting the base to the end-effector. The closed chain kinematics of parallel robots can result in *greater structural rigidity*, and hence *greater accuracy*, than open chain robots. The kinematic description of parallel robots is fundamentally different from that of serial link robots and therefore requires different methods of analysis. [22]

A parallel mechanism is one of the most active areas of current mechatronics and robotics research. Since it is promising to have those supplementary features that serial mechanisms is in lack of such as *high rigidity, high stiffness, high accuracy, high speed, high acceleration* and *high payload*. However they are much more complicated than the serial mechanisms due the presence of closed loop constraints. And our understanding of them is far behind that of serial mechanisms, which have been extensively studied. [23]
2.4. Pneumatic drives

What is an actuator? An actuator is a mechanical device used for moving or controlling something (in this case manipulators). Generally, based the medium used to transfer energy, actuators/drive are classified as:

- Electric Motors and Drives
- Hydraulic Drives
- Pneumatic Drives
- Internal Combustion hybrids

2.4.1. Pneumatic Actuators

Most of the earlier pneumatic control systems were used in the process control industries, where the low pressure air of the order 7-bar was easily obtainable and give sufficiently fast response. Pneumatic systems are extensively used in the automation of production machinery and in the field of automatic controllers. For instance, pneumatic circuits that convert the energy of compressed air into mechanical energy enjoy wide usage, and various types of pneumatic controllers are found in industry. Certain performance characteristics such as fuel consumption, dynamic response and output stiffness can be compared for general types of pneumatic actuators, such as piston-cylinder and rotary types. Figure below shows the two types of pneumatic actuators [30] The pneumatic actuator has most often been of the piston cylinder type because of its low cost and simplicity. The pneumatic power is converted to straight line reciprocating and rotary motions by pneumatic cylinders and pneumatic motors. The pneumatic position servo systems are used in numerous applications because of their ability to position loads with high dynamic response and to augment the force required moving the loads. Pneumatic systems are also very reliable. [30]

![Diagram of pneumatic actuators](image)

a) Double acting linear pneumatic actuator.  

b) Vane rotary pneumatic actuator.

**Figure 2.3** The two fundamental types of pneumatic actuators
2.4.2. Pneumatic Systems

- Pneumatic systems are designed to move loads by controlling pressurized air in distribution lines and pistons with mechanical or electronic valves.
- Air under pressure possesses energy which can be released to do useful work.
- Some examples of pneumatic systems are: dentist’s drill, pneumatic road drill, automated production systems.

![Pneumatic power supply system](image)

**Figure 2.4** Pneumatic power supply system

2.4.3. Pneumatic Drives Characteristics

- Many of the same principles as hydraulics except working fluid is compressed air
- Compressed air widely available and environmentally friendly,
- Piping installation and maintenance is easy
- Explosion proof construction
- Major disadvantage is compressibility of air, leading to low power densities and poor control properties (usually on/off)
- Pneumatic systems are suitable for light and medium loads (30N-20kN) with temperature -40 to 200 degrees Celsius

In general pneumatic systems have the following key merits

- Low cost and easy to install
- Clean and easy to maintain
- Low power densities
- Only on/off or inaccurate control necessary

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2.4.4. Pneumatic Valves
Pneumatic valve is a component placed in between the reservoir and the cylinder for the purpose of regulation the flow of air. It regulates the air flow to and from the cylinder according to an appropriate command given to it as an electrical signal. The two most common types electromechanical servo valves are the three port –two way (3-2) and the five port- three way (5-3) valves. Figure 2.5 shows the very simple type of a 5-3 servo valve connected to a double action, single rod cylinder.

![Figure 2.5 A 5-port-3-way servo valve](image)

2.5. Solid work modeling
SolidWorks is a Parasolid-based solid modeler, and utilizes a parametric feature-based approach to create models and assemblies. It is capable of carrying out numerous tasks ranging from simple components physical modeling up to simulation, motion study and sustainability analysis of a more complex assembly. However, in this thesis work the use of Solid work is limited, it will be implemented for creating a 3D physical model of the manipulator, which principally can be translated into Matlab/Simulink-SimMechanics interface. The general step used in the modeling will be discussed in the subsequent chapter that deals with the kinematic design of the PKM manipulator.
Chapter Three

3. Kinematic Analysis

This chapter discusses the formulation of equations, assumption of constraint and boundary conditions, simplified Solidworks modeling of the PKM manipulator and how these characteristic parameters can be combined in a Matlab/Simulink environment in order to perform a kinematic analysis of the manipulator. Finally, there will be a conclusion on the results obtained from the kinematic analysis, which in fact will deal with the position traction of a desired trajectory.

3.1. Kinematic mechanism description

The PRRRP 2-DoF parallel mechanism usually consists of two legs, each of which is the PRR (R-revolute joint and P-prismatic joint) chain. The two legs are connected to the end-effector point with a common R joint. The mechanism can position at a point in a plane when the P joint in each of the two legs is actuated by a linear actuator. A PRRRP mechanism that is actuated along the Y-axis direction is shown in Figure 3.1.

As illustrated in Figure 3.1. a reference frame $R : O-xy$ is fixed to the base. Vectors $q_i (i=1,2)$ are defined as the position vectors of points $B_i$ in frame $R$. The geometric parameters of the mechanism are $PB_i = L_2 (i = 1, 2)$, and the distance between two actuators is $2L_1$. The position of point $P$ in the fixed frame $R$ is given by:

$$P = [x \ y]^T \quad (3.1)$$

As shown in Figure 3.1. the position vector of the points $q_i$ in the fixed frame $R$ is denoted by:

$$Q = [q_1 \ q_2]^T \quad (3.2)$$
3.2. **Workspace of PKM Manipulator**

For determination of different characteristics of the workspace, in order to compare different existing manipulators or design a new manipulator, it is almost always necessary to determine the boundaries of the workspaces. Since workspace determination is generally an intermediate but critical step in analyzing and synthesizing manipulators, it is very important to have a theory to safeguard the estimation and conceptual design. The workspace of a manipulator is the domain of reach of its end-effector and is bounded in the 3D/2D space, depending on the DoF of the manipulator. The workspace of a manipulator has been defined in literature as the totality of positions that a particular identified point of the manipulator (end-effector) can reach. The workspace boundary is the curve (in plane) or the surface (in space) that defines the extent of reach of the end-effector.

The size of workspace has a huge impact on the possible functionality of a manipulator and it gives an insight of the performance a machine. Therefore, many scholars have proposed algebraic method and geometric method to determine the workspace of a manipulator. Wang et al. [8] presented an algorithm, called the boundary search method, to determine the workspace of a parallel machine tool. They also analyzed the boundary workspace in order to expand its operating scope.
The focuses of the many literature are mainly on finding the workspace of a manipulator, but seldom discuss the relationship between the shapes of the workspace and the geometry structure of the manipulator. There exists no theorem for spatial parallel manipulators which characterizes the geometry relationship between the reachable workspace of closed-loop manipulators and the structures of their kinematic chains. For a 2-DoF planar PKM manipulator, the workspace can be explicitly represented as a region along the horizontal (X-axis) which can be reached by the end effector without cause kinematic constraint and boundary condition violations. Hence, as shown in Figure 3.2, the workspace is a function of the arm length $L_2$ and the distance between the two actuators $2L_1$, for a given length of the actuator.

![Figure 3.2 Workspace envelop for a 2 DoF planar PKM manipulator](image)

Hence, the boundary that limits the workspace across the horizontal direction can be determined as:

\[ W = 2(L_2 - L_1) \] (3.3)

### 3.3. Inverse Kinematic Problem

The Inverse kinematic problem involves in the determination of the joint/s motion (position, velocity and acceleration) for a known value of end effector motion. Assuming that the user can generate the desired end effector motion from any trajectory of interest and an appropriate motion law, we will discuss the equation which governs the determination of joint space motion at points $B_1$ and $B_2$. 
3.3.1. Position Analysis

The equations which govern the kinematic problem of the PKM are given as:

\[(x - L_1)^2 + (y - q_1)^2 = L_2^2\] \hspace{1cm} (3.4)
\[(x + L_1)^2 + (y - q_2)^2 = L_2^2\] \hspace{1cm} (3.5)

The inverse kinematic problem can then be written as

\[q_1 = \pm \sqrt{L_2^2 - (x - L_1)^2 + y}\] \hspace{1cm} (3.6)
\[q_2 = \pm \sqrt{L_2^2 - (x + L_1)^2 + y}\] \hspace{1cm} (3.7)

3.3.2. The Jacobian Matrix

Eqs. (3.4) and (3.5) can be differentiated with respect to time to obtain the velocity equations. This leads to an equation of the form:

\[A \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = B \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}\] \hspace{1cm} (3.8)

where \(A\) and \(B\) are 2x2 matrices that can be expressed as

\[A = \begin{bmatrix} y - q_1 & 0 \\ 0 & y - q_2 \end{bmatrix}, \text{ and } B = \begin{bmatrix} x - L_1 & y - q_1 \\ x + L_1 & y - q_2 \end{bmatrix}\] \hspace{1cm} (3.9)

Matrix algebra indicates that there exists a Jacobian matrix if and only if the matrix \(A\) is not singular. Consequently, the Jacobian matrix is obtained by

\[J = A^{-1}B = \begin{bmatrix} \frac{x+L_1}{y-q_1} & 1 \\ \frac{y-q_2}{y-q_2} & 1 \end{bmatrix}\] \hspace{1cm} (3.10)

Eqs. (3.6) and (3.7) indicates that there are two different possible motions for each joint. For the working mode as shown in Figure 3.1 that the “±” in Eqs. (3.6) and (3.7) are both “+”, and the Jacobian matrix can be rewritten as:

\[J = \begin{bmatrix} \frac{x+L_1}{L_2^2-(x+L_1)^2} & 1 \\ \frac{x-L_1}{L_2^2-(x-L_1)^2} & 1 \end{bmatrix}\] \hspace{1cm} (3.11)
3.3.3. Velocity and Acceleration Analysis

Having the position vectors of the end-effector $P$ and the joints $Q$ from Eq. (3.1) and Eq. (3.2), respectively, and then, with direct kinematic analysis the end effector position $P$ can be stated as a function of the joint position $Q$ by a set of closed-form equations.

$$P = F(Q)$$ \hspace{1cm} (3.12)

Consequently, the joint space velocities at the slider-leg junctions, point $B_1$ and $B_2$ can be calculated as follows

$$\dot{Q} = J\dot{P}$$ \hspace{1cm} (3.13)

Where $J$ is the Jacobian matrix obtained in Eq. (3.11). Similarly the joint space acceleration can be obtained from the known end-effector motion and the already determined joint position $Q$ and joint velocity $\dot{Q}$.

$$\ddot{Q} = J\ddot{P} + \dot{A}(Q, \dot{Q})$$ \hspace{1cm} (3.14)

where

$$\dot{A} = \begin{bmatrix}
\frac{x^2+y^2-2yq_1+q_1^2}{y-q_1} \\
\frac{x^2+y^2-2yq_2+q_2^2}{y-q_2}
\end{bmatrix}$$ \hspace{1cm} (3.15)

3.4. Direct Kinematic Problem

3.4.1. Direct analysis for position

The direct kinematic analysis is concerned about obtaining the end-effector motion (Position, velocity and acceleration) when the joint motion is given or known from the output of actuators. In this case, for the position of the end-effector the value of $x$ and $y$ can be found as follows as a function of the joint positions $q_1$ and $q_2$.

$$y = -\frac{d-\sqrt{(d^2-4c\cdot d)}}{2c}$$ \hspace{1cm} (3.16)

$$a = \frac{(q_2 - q_1)}{2R_1}; \hspace{1cm} b = \frac{(q_1^2 - q_2^2)}{4L_1}; \hspace{1cm} c = a^2 + 1; \hspace{1cm} d = (b - L_1)^2 + q_1^2 - R_2^2;$$

$$e = 2a(b - L_1) - 2q_1$$
3.4.2. Direct analysis for velocity and acceleration

The end-effector velocity and acceleration can be determined from the joints motion. And, the equations which govern this analysis are given below in Eq. (3.18) and (3.19), respectively.

\[
\dot{p} = J^{-1} \dot{q} \tag{3.18}
\]

\[
\ddot{p} = J^{-1}[\ddot{q} - \dot{A}(q, \dot{q})] \tag{3.19}
\]

3.5. Singularity Conditions

Most of the times, kinematic singularity leads to a loss of the controllability and degradation of the natural stiffness of manipulators, as a result the analysis of parallel manipulators has drawn considerable attention. In some configurations, the robot cannot be fully controlled. Most parallel manipulator suffer from the presence of singular configurations in their workspace that limit the machine performances. In the case of parallel manipulators and closed-loop mechanisms, singularity analysis is much more difficult since such mechanisms contain unactuated joints and joints with more than one degree of freedom [ref4]. In general, closed-form solutions for singular curves/surfaces for parallel manipulators of arbitrary architecture requires elimination of unwanted variables from several nonlinear transcendental equations, and this is quite difficult.

Based on the forward and inverse Jacobian matrices, three kinds of singularities of parallel manipulators can be obtained [24]. Let \( q \) denote the actuated joint variables, and let \( p \) describe the location of the moving platform. The kinematic constraints imposed by the arms of the manipulator are expressed as \( f(x, q) = 0 \). Differentiating with respect to time, a relation between the input joint rates and the end effector output velocity is obtained as:

\[
J_p \dot{p} = J_q \dot{q} \tag{3.20}
\]

Where, \( J_p = \frac{\partial f}{\partial p} \) and \( J_q = \frac{\partial f}{\partial q} \). Which leads to the overall Jacobian matrix \( J \), can be written as:

\[
J = J_q^{-1} J_p \tag{3.21}
\]
It is said that singular configurations should be avoided at any cost. The singular configurations (also called singularities) of a parallel robot may appear inside the workspace or at its boundaries.

### 3.5.1. The first kind of singularity
The first kind of singularity occurs when the following condition is satisfied:

\[
\det(J_p) = 0, \quad \det(J_q) \neq 0
\]

This kind of singularity corresponds to the limit of the workspace. Where, the end-effector point P moves to either extremes sides of the work envelop resulting in constraint violation.

### 3.5.2. The second kind of singularity
The second kind of singularity occurs when we have following:

\[
\det(J_p) = 0, \quad \det(J_q) \neq 0
\]

The physical interpretation of this kind of singularity is that even if all of the input velocities are zero, there are still be instantaneous motion of the end-effector. In this configuration, the manipulator loses stiffness and becomes uncontrollable. This kind of singularity is located inside the workspace of the manipulator. Such a singularity is very difficult to locate only by analyzing and expanding the equation \( \det(J_q) = 0 \). A numerical method is thus a good selection for solving this problem.

### 3.5.3. The third kind of singularity
The third kind of singularity occurs when both:

\[
\det(J_p) = \det(J_q) = 0
\]

This kind of singularity corresponds to the first and second type of singularity occurring simultaneously. This singularity is both configuration and architecture dependent.

Parallel singularities are particularly undesirable because they cause the following problems:

- a high increase of forces in joints and links, that may damage the structure,
- a decrease of the mechanism stiffness that can lead to uncontrolled motions of the tool though actuated joints are locked. [44]
Figure 3.3  Singularity configuration of a PKM 2-DoF manipulator

Figure 3.4  Singularity configuration of a PKM 2-DoF manipulator

Figure 3.5  Singularity configuration of a PKM 2-DoF manipulator
3.6. Solidworks modeling

Before leading to the full inverse kinematic analysis in Matlab/Simulink it is necessary to create a physical model of the manipulator in Solidworks environment, and then, to translate that into a Matlab/Simulink-SimMechanics compatible format. All the dimensions of the robot are assumed to be as follows:

\[ 2L_1 = 500 \text{ mm} \] is the distance between sliders
\[ L = 1500 \text{ mm} \] is the length of both the cylinders (maximum stroke)
\[ L_2 = 354 \text{ mm} \] is the length of the two arms
\[ d = 30 \text{ mm} \] is the diameter of the arm rods
\[ d_c = 32 \text{ mm} \] is the bore diameter of the cylinder

The three dimensional (3D) model built in Solidworks in shown below in Figure 3.6. Each of the components are created separately in Solidworks components design module as shown in Figure 3.7, and then brought together in the assembly module to create the complete manipulator. Though it is possible to study the kinematics of the manipulator in Solidworks environment, it has not tried here because it is more direct and understandable in a Matlab/Simulink environment.

![Figure 3.6 A 3D physical model of the complete PKM manipulator created with Solidworks](image)
**Figure 3.7** Solidworks individual component model for the PKM manipulator

a) Base  
b) Slider  
c) Arm and  
d) End-effector
3.7. Numerical Simulation for the Inverse Kinematic Analysis

For the Kinematic analysis, a model is developed in Simulink including the physical model of the manipulator designed in Solidworks (shown in Figure 3.6) and exported to Simulink-SimMechanics. The necessary steps and assumptions for the development of a physical model in Solidworks are discussed in the previous chapter under the topic Solidworks modeling, hence here it is only presented the final entire model of the manipulator. Figure 3.8, below, shows the complete Matlab/Simulink + Solidworks model developed for the kinematic analysis of the parallel manipulator.

![Simulink model diagram]

**Figure 3.8** A complete model of the PKM manipulator on a Simulink environment

3.7.1. Description of sections in Simulink model

**Section 1:** This section has contained the input to the program. It is the vector of the desired trajectory \( X\text{-axis} \) and \( Y\text{-axis} \) components against the motion time \( t \). In general these inputs are functions of all known parameters, such as: the desired trajectory, the desired motion law and the motion time. More on the choice of motion law will be discussed in chapter 5. However, at this stage a circle of radius \( r=100\text{mm} \) is used as the desired trajectory. Whereas, the law of motion is assumed to be a symmetric constant acceleration law with a motion time of \( T=10 \) seconds.
Section 2: This section is the major step in the Inverse Kinematic Analysis (IKA) of the manipulator. Through the inverse Kinematic Analysis equations discussed above, the joint space motion is determined as a function of the end-effector \([ P(x, y) ]\) motion. The mathematical algorithm which cover this section in written in the Matlab code file which is named as `main_kinematic.m`. For more detail regarding the implemented Matlab codes the reader is advised to refer to the functions given in Appendix-A Part I. Similarly, the IKA can totally be done in Simulink space as shown in Figure 3.9, blow.

![Figure 3.9 Inverse Kinematic Analysis (IKA):- Simulink based model](image)

Section 3: In this section, the physical model created in Solidworks and transformed in to Matlab/Simulink environment is provided with the necessary motion input at the joints. It introduces functions \( f_1(q_1, \dot{q}_1, \ddot{q}_1) \) and \( f_2(q_2, \dot{q}_2, \ddot{q}_2) \) at points \( B_1 \) and \( B_2 \), respectively, and induces a kinematic simulation of the PKM manipulator. Figure 3.10, below, shows the translated model in a Simulink-SimMechanics environment.

Section 4: This section display the resulting trajectory simulation while the main program is executed. It is designed to display all the trajectory points in an X-Y plane plot, which in deed is lied inside the workspace of the PKM manipulator.
Figure 3.10 Model of PKM Manipulator in a SimMechanics interface

The same model with all the input motions applied at joints 1 and 2, is shown in Figure 3.11. In this model, the mechanical system is given a genaralized motion (position, velocity and acceleration) at the joints through a SimMechanics library block joint actuator attached to prismatic joints and the response motion is obtained through body sensors block attached to any of the desired linkages.

Figure 3.11 Model in Simmechanics with desired motion as an input for kinematic analysis

Section 5: This section gives the simulated plot of the desired trajectory given by the user as an input, by a similar fashion discussed in section 4, above.

3.7.2. Results and discussions

Both the Matlab based and Simulink based analysis results in same output. The desired trajectory, assumed to be a circle, is shown in Figure 3.12 below,
Figure 3.12 Desired Trajectory

Whereas, the plot of a symmetric constant acceleration law, which is chosen to be the governing motion law for the kinematic analysis is given in Figure 3.13.

Figure 3.13 Desired motion law (A symmetric Constant Acceleration Law)
**Figure 3.14** Desired motion law of the end effector

*Figure 3.14*, above depicted the desired motion of the end-effector, actually decomposed into its components along the x-axis and y-axis; and *Figure 3.15* below, shows the profile of the desired motion in the joint space.

**Figure 3.15** Desired/Reference motion law: Joint space
Chapter Four

4. Dynamic modeling and Analysis

The dynamic model of the PRRRP PKM manipulator focuses on analyzing the response motion of the manipulator for a given action of the driving actuator, in this case a pneumatic one. Therefore, this chapter deals with, first, on determining the desired action forces need to be applied by the actuators through an inverse dynamic analysis. Then, it discusses modeling of a pneumatic servo drive in the Matlab/Simulink environment. Finally, it shows how to create a complete dynamic model, analyze the model and discuss the resulting response.

4.1. Dynamic Analysis Approaches

In robotics literature, there are two basic approaches to dynamics, namely Newton-Euler and Lagrangian. For the former, one first carry out a detail force and torque analysis of each rigid link with some physical knowledge such as Newton’s third law, and then apply Newton’s law and Euler’s equation to each of the rigid links to obtain a set of 2nd order Ordinary Differential Equations (ODE) in the position and angular representation of each rigid link. Finally together with the kinematic constraints, the set of equations can be simplified or solved, till the desired form of dynamics equation is obtained. The Lagrangian approach is a more elegant and tractable one which involve the choice of a set of generalize coordinates to describe the configuration of the system, and the setting up of a scalar function called Lagrangian in the tangent bundle of the configuration space.

4.1.1. The Newton-Euler approach

In this approach, first it is necessary to isolate all the rigid links of the system. Then, attach a frame at the center of mass of each rigid link. All the forces and torques applied to each rigid link must be considered. For the forces or torques which are action and reaction pairs are considered to have the same magnitude, opposite directions and act on different bodies along the same line of action according to Newton’s third law.

The advantage of this approach is that the formulation is globally valid, i.e. independent of the choice of coordinate system. It is also more intuitive with physical meanings. In literature, there are many variations for the above method, which is basically the difference of a choice of coordinate systems (e.g. body or spatial frame) to describe the various quantities in the systems. In Newton-Euler formulation, in principle, we can account for all the forces and torques applied to each individual link in the systems and derive the equations of motion.[9]
4.1.2. **Lagrangian approach**
The Lagrangian formulation describes the behavior of a dynamic system in terms of work and energy stored in the system rather than in terms of force and moments of the individual members involved. Using this approach, the closed-form dynamical equations can be derived systematically in any coordinate system. In this paper, Lagrange’s equation using constrained coordinates (i.e. the Lagrange multiplier approach) is presented:

\[
\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{q}_j} \right) - \frac{\partial K}{\partial q_j} + \sum_{i=1}^{k} \zeta_i G_{ij} = Q_j, \quad j = 1, \ldots, n,
\]

where \( i \) is the constraint index, \( j \) is the generalized coordinate index, \( k \) is the number of constraint functions, \( K \) is the total kinetic energy of system, \( U \) is the total potential energy term, \( \zeta_i \) is the Lagrange multiplier, \( G_{ij} \) is the element of the Jacobian matrix of constraint equation, \( G_{ij} = \frac{\partial f_i}{\partial q_j} \), where \( f_i \) is a constraint equation, \( q_j \) the \( j^{th} \) generalized coordinate.

4.2. **Inverse Dynamics Analysis through the Newton-Euler approach**
The vectors closed-loop of one leg is shown in Figure 4.1. A fixed leg frame \( B-\overline{x'y'} \) is established at the point \( B \) and parallel to the global frame \( O-x y \). A local leg frame is also established at the point \( B \) while its \( x \)-axis is along the leg length direction and its \( z \)-axis is parallel to that of the global frame. The vector which represents the inverse kinematic equation of aforementioned closed-loop can be given as

\[
r = b + qe_2 + Lw
\]

![Figure 4.1. Position analysis for a single leg of the PKM manipulator](image)
where \( r = [x \ y]^T \) is the position vector of \( P \) with respect to the global frame \( O-xy \), \( b=[x_b \ 0]^T \) is the position vector of \( Q \) with respect to the global frame, and \( e_2=[0 \ 1]^T \). \( q \) is the \( y \) coordinate of point \( B \) and \( L \) and \( w \) are the length and unit vector of the leg, respectively, with respect to the global frame. As it is discussed in detail in chapter three, there are four inverse kinematic solutions for a given position of platform. In the configuration shown in Figure 3.1,

\[
w = \begin{bmatrix} w_x \\ w_y \\ \frac{R_1 \pm x}{L_2} \\ \sqrt{L_2^2-(R_1 \pm x)^2} \\ L_2 \end{bmatrix}
\]

Where \( ' \pm ' \) in Eq. 4.4, for the configuration shown in Figure 3.1, \( ' + ' \) for leg number one and \( ' - ' \) for leg number two. Then, the joint position vector \( q \) for a single leg can be obtained as:

\[
q e_2 = r - b - Lw
\]

Form the time derivative of Eq. 4.2, the velocity of the joint is obtained as:

\[
\dot{r} = \dot{q} e_2 + L \begin{bmatrix} -w_y \\ w_x \end{bmatrix}
\]

Multiplying both sides of Eq. 4.6 by \( w^T \) results in,

\[
\dot{q} w^T e_2 = w^T \dot{r}
\]

\[
\dot{q} = \frac{w^T \dot{r}}{w^T e_2} = \frac{w^T \dot{r}}{w_y}
\]

Taking the dot product of \( \begin{bmatrix} -w_y \\ w_x \end{bmatrix}^T \) with both sides of Eq. 4.6 yields in the angular velocity of the leg.

\[
\omega = \begin{bmatrix} -w_y \\ w_x \end{bmatrix} \cdot \frac{(\dot{r} - \dot{q} e_2)}{L}
\]

Further, the time derivative of Eq. 4.6 gives the following equation which, in turn can be rearranged to give a vector of accelerations of the joint, as shown:

\[
\ddot{q} = \frac{w^T \ddot{r} + L \omega^2}{w^T e_2}
\]
Similarly, by taking the dot product of $[-w_y \ w_x]^T$ with both sides of Eq. 4.10 the angular acceleration of the leg can be given as:

$$\alpha = \left[-\frac{\ddot{w}_y}{w_x}\right] \cdot \frac{(\ddot{r} - \dot{q}e_2)}{L} \tag{4.11}$$

Assuming $r_c$ is the position of the centre of gravity of the leg in the local frame $B-xy$, and $r'_c$ is the position of the centre of gravity of the leg with respect to the frame $B-x'y'$, given as:

$$r'_c = \begin{bmatrix} \dot{r}_{cx}' \\ \dot{r}_{cy}' \end{bmatrix} = T_c \dot{r}_c \tag{4.12}$$

Where $T_c$ is the rotation matrix from the frame $B-x'y'$ to the frame $B-xy$, and is given by:

$$T_c = \begin{bmatrix} w_x & -w_y \\ w_y & w_x \end{bmatrix} \tag{4.13}$$

Therefore, the position vector of the center of gravity of the leg with respect to the global frame is

$$\dot{r}_{cg} = \dot{b} + qe_2 + r'_c \tag{4.14}$$

Taking the time derivative of Eq. 4.14 yields in the velocity of the center of gravity of the leg,

$$\dot{\dot{r}}_{cg} = \dot{q}e_2 + \omega[-r''_{cy} \quad r''_{cx}]^T \tag{4.15}$$

And, from the time derivative of Eq. 4.15, the acceleration of the center of gravity of the leg can be given as:

$$\dddot{r}_{cg} = \dddot{q}e_2 + \omega[-r''_{cy} \quad r''_{cx}] \quad - \omega^2 r'_c \tag{4.16}$$

### 4.2.1. The Newton-Euler approach

The objective of this analysis is to obtain the driving forces of actuators, hence, first the force and moment equations of the legs are determined and the constraint forces at the joints are obtained. Then, from the Newton’s equation of the carriages, the driving forces of the actuator are finally obtained.

The equation for the dynamic equilibrium of forces applied on the legs can be given by a generalized formula, as shown:

$$F_B + F_P + mg + m\dddot{r}_{cg} = 0 \tag{4.17}$$
Where \( m \) is the mass of the leg, \( g \) is the gravitation acceleration, assuming a horizontal or X-Y work table, \( g=[0\ 0]^T \). \( F_p=[F_{px} \ F_{py}]^T \) and \( F_b=[F_{bx} \ F_{by}]^T \) are the forces exerted on the leg by the end effector and actuator/slider, respectively.

Applying Newton’s equation for end effector, yields in:

\[
-Mr - \sum_{i=1}^{2} F_{pi} + F_{ext} + Mg = 0
\]  \( (4.18) \)

Where \( M \) is the mass of the end-effector, and \( F_{ext}=[F_x \ F_y]^T \) is the external force exerted on the end-effector.

Besides, the moment equations of a single leg about point \( B \) are given as:

\[
F_p \cdot \begin{bmatrix} -w_y \\ w_x \end{bmatrix} - m\ddot{r}_{cg} \cdot \begin{bmatrix} -r'_{cy} \\ r'_{cx} \end{bmatrix} - mg r'_{cx} - J_B \alpha = 0
\]  \( (4.19) \)

where \( J_B = \frac{1}{3} m_l l_l^2 \) is the moment of inertia of the leg to point \( B \). Moment equation of the end-effector about point \( P \).

\[
M_{ext} - M\ddot{r} \cdot [-r'_{cy} \ R_{cx}]^T = 0
\]  \( (4.20) \)

Where \( M_{ext} \) is the external moment applied on the end-effector and \( R_c=[R_{cx} \ R_{cy}]^T \) is the position vector of the center of the end-effector with respect to the frame \( P-x'y' \), i.e. \( R_c=[0 \ 0]^T \). Finally, Eqs. 4.18, 4.19 and 4.20 are combined into a single linear system of equations in the form shown below.

\[
A \begin{bmatrix} F_{p1} \\ F_{p2} \end{bmatrix} = B
\]  \( (4.21) \)

Where,

\[
A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -y & x & -y & x \\ -Lw_{y1} & Lw_{x1} & 0 & 0 \\ 0 & 0 & -Lw_{y2} & Lw_{x2} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -M\dddot{x} + F_x \\ -M\dddot{y} + F_y + Mg \\ -Mg R_{cx} + M_{ext} + M\dddot{r}_{cy} - M\dddot{r}_{cx} \\ m_1 g r'_{cx1} + m_1 g y'_{cg1} \cdot [r'_{cy1} + J_B \alpha] \\ m_2 g r'_{cx2} + m_2 g y'_{cg2} \cdot [r'_{cy2} + J_B \alpha] \end{bmatrix}
\]

Then, the solution of Eq. 4.21 yields in the desired forces at point \( P \).
\[
[F^T_{p1} \quad F^T_{p2}]^T = A^{-1}B
\]  

(4.22)

The forces exerted on the legs by the sliders at the point B, \(F_B = [F_{B_1} \quad F_{B_2}]^T\) can be obtained as:

\[
F_B = -F_p + m\ddot{x}g
\]  

(4.23)

From Newton equation applied on the sliders, the driving forces of the actuators can be derived.

The Newton equation for the left actuator:

\[
\begin{bmatrix} N_1 \\ F_{d1} \end{bmatrix} + (m_{s1} + m_{a1})g - F_{B1} + f_{bw} - (m_{s1} + m_{a1})\ddot{q}_1e_2 = 0
\]  

(2.24)

Similarly, the Newton’s equation for the right side actuator:

\[
\begin{bmatrix} N_2 \\ F_{d2} \end{bmatrix} + (m_{s2} + m_{a2})g - F_{B2} + f_{bw} - (m_{s2} + m_{a2})\ddot{q}_2e_2 = 0
\]  

(2.25)

Where \(F_{di}\) and \(F_{dj}\) are the desired drive forces, \(N_1\) and \(N_2\) are reaction forces along the x-axis, \(m_s\) and \(m_a\) are mass of the slider and actuator, respectively. \(f_{bw}\) is the friction force due to actuator-slider interaction. Hence, the drive forces can be extracted from Eqs. 2.24 and 4.25:

\[
\begin{align*}
F_{d1} &= F_{By1} + (m_{s1} + m_{a1})\ddot{q}_1 - f_{bwy} \\
F_{d2} &= F_{By2} + (m_{s2} + m_{a2})\ddot{q}_2 - f_{bwy}
\end{align*}
\]  

(4.26)

With the desired drive forces known, the dynamic analysis of the manipulator leads to the next step of modeling a pneumatic actuator which can provide force. Hence, the purpose of the next section would be on modeling a pneumatic drive system that consists of a valve (most probably a proportional directional control valve), a cylinder-piston system and pipe lines and the various connection between them.
4.3. **Pneumatic Servo System Modeling**

4.3.1. **Pneumatic Actuators: Modeling**
Several approaches have been proposed for modeling the pneumatic actuators. One of the widely used methods for finding the mathematical model of the pneumatic actuator is a theoretical analysis. The analysis of pneumatic actuators requires a combination of thermodynamics, fluid dynamics and the dynamics of the motion. For constructing a mathematical model, three major considerations must be involved:

1. The determination of the mass flow rates through the valve.
2. The determination of the pressure, volume and temperature of the air in cylinder.
3. The determination of the dynamics of the load.

Accurate model of pneumatic actuator is an important condition both for control design and for optimizing its operation. As a result, this chapter is concerned with the mathematical modeling and numerical simulation for pneumatic actuator systems. [30]

4.3.2. **Pneumatic Actuators: The Control Strategies:**
The advantages of pneumatic systems are well known as clear, cheap, easily maintained, safe in operation, etc. But for their highly nonlinear properties such as compressibility of medium, friction effect and nonlinearity of valves, pneumatic actuators are seldom used in industrial servo applications. Moreover, some of their properties, e.g., poor damping, low stiffness, and limited bandwidth, are unfavorable in the servo control system design.

Some other difficulties in the control of pneumatic servo systems are the possible presence of unknown disturbances coming from leakage of valves, *time-varying payloads*, and external perturbations. Besides, uncertainties in system parameters make the controller design problem more challenging [30]. To cope with some of these problems, advanced control algorithms have to be proposed.

4.3.3. **The Dynamic Model**
The dynamic model used in this work is developed based on:

1. the description of the relationship between the air mass flow rate and pressure changes in the cylinder chambers, and
2. the equilibrium of the forces acting at the piston, including the friction force.
A schematic view of the system to be modeled is shown in Figure 4.2. The relationship between the air mass flow rate and the pressure changes in the chambers is obtained using energy conservation laws, and the force equilibrium is given by Newton’s second law. The friction force and the external forces are modeled in a unified way.

\[
A(p_1 - p_2)
\]

**Figure 4.2** Schematic of a Pneumatic Drive System. Source ref. [30]

### 4.3.3.1. Conservation of Energy

The internal energy of the mass flowing into chamber 1 is \( C_p q_{m1} T \), where \( C_p \) is the constant pressure specific heat of the air, \( T \) is the air supply temperature, and \( q_{m1} = \left( \frac{dm}{dt} \right) \) is the air mass flow rate into chamber 1. The rate at which work is done by the moving piston is \( p_1 \dot{V}_1 \), where \( p_1 \) is the absolute pressure in chamber 1 and \( \dot{V}_1 = \left( \frac{dV_1}{dt} \right) \) is the volumetric flow rate.

The time air internal energy change rate in the cylinder is \( \frac{d(C_V \rho_1 V_1 T)}{dt} \), where \( C_V \) is the constant volume specific heat of the air and \( \rho_1 \) is the air density. We consider the ratio between the specific heat values as \( r = C_P / C_V \) and that \( \rho_1 = C_V / (RT) \) for an ideal gas, where \( R \) is the universal gas constant. An energy balance yields:

\[
q_{m1} T - \frac{p_1}{C_P} \frac{dV_1}{dt} = \frac{1}{rR} \frac{d}{dt} (p_1 V_1) \tag{4.27}
\]
The total volume of chamber 1 is given by \( V_1 = Ay + V_{10} \), where \( A \) is the cylinder cross-sectional area, \( y \) is the piston position and \( V_{10} \) is the dead volume of air in the line and at the chamber 1 extremity. The change rate for this volume is \( \dot{V}_1 = Ay \), where \( \dot{y} = \frac{dy}{dt} \) is the piston velocity. After calculating the derivative term in the right hand side of Eq. 4.27, and assuming air is a perfect gas undergoing an isothermal process, the rate of change of the pressure inside each chamber of the cylinder can be expressed as:

\[
\dot{p}_1 = -\frac{Aky}{Ay+V_{10}} p_1 + \frac{RkT}{Ay+V_{10}} q_{m1} \\
\dot{p}_2 = \frac{Aky}{A(L-y)+V_{20}} p_2 + \frac{RkT}{A(L-y)+V_{20}} q_{m2}
\]  

(4.28)  

(4.29)

where \( C_p = \frac{(kR)}{(k - 1)} \) and \( L \) is the maximum cylinder stroke. Assuming that the mass flow rates are nonlinear functions of the servovalve control voltage \( (u) \) and of the cylinder pressures, that is, \( q_{m1} = q_{m1}(p_1, u) \) and \( q_{m2} = q_{m2}(p_2, u) \), Eq. 2.28 and Eq. 2.29 result in,

\[
\dot{p}_1 = -\frac{Aky}{Ay+V_{10}} p_1 + \frac{RkT}{Ay+V_{10}} q_{m1}(p_1, u)
\]  

(4.30)

\[
\dot{p}_2 = \frac{Aky}{A(L-y)+V_{20}} p_2 + \frac{RkT}{A(L-y)+V_{20}} q_{m2}(p_2, u)
\]  

(4.31)

Therefore, incorporation all the above nonlinear sets of equations, the resulting scheme of the pneumatic drive looks like Figure 4.3, below.

**Figure 4.3** Pneumatic drive and mechanical subsystem scheme

Where \( u \) is the command signal from the controller/regulator. The Matlab/Simulink model created based on the kinematic and dynamic equations of the PKM manipulator and equations that govern the nonlinear behavior of the pneumatic drive is shown below, in Fig. 4.4.
Figure 4.4 Complete dynamic model of the PKM manipulator built in Matlab/Simulink

The detail Matlab code for the dynamic analysis can be found in Appendix-A part I by the name of main_dynamic.m and I_dynamic.m. Also, the detail of the various subsystem blocks in Figure 4.4 are presented in Appendix-A part II.
4.4. Simulation results and Discussion

4.4.1. Thermal and physical characteristics of the pneumatic drive

In order to carry out a dynamic numerical simulation the following considerations has been taken.

<table>
<thead>
<tr>
<th>Table 4.1 Value of Physical and Thermal Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>$P_s=6\times10^5$ Pa</td>
</tr>
<tr>
<td>$P_e=1\times10^7$ Pa</td>
</tr>
<tr>
<td>$P_a=101.3\times10^3$ Pa</td>
</tr>
<tr>
<td>$P_0=3\times10^5$ Pa</td>
</tr>
<tr>
<td>$C_r=0.852$</td>
</tr>
<tr>
<td>$T_s=T_c=T_e=293 K$</td>
</tr>
<tr>
<td>$C_1=3.864$</td>
</tr>
<tr>
<td>$C_2=0.04$</td>
</tr>
<tr>
<td>$C_p$</td>
</tr>
<tr>
<td>$C_v$</td>
</tr>
<tr>
<td>$k=1.4$</td>
</tr>
<tr>
<td>$R=287 \text{ m.K}^{-1}$</td>
</tr>
<tr>
<td>$B=65 \text{ Nsm}^{-1}$</td>
</tr>
<tr>
<td>$C_d=0.8$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4.2 Pneumatic cylinder characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>Size (Bore)</td>
</tr>
<tr>
<td>Maximum stroke (Length)</td>
</tr>
<tr>
<td>Base weight</td>
</tr>
<tr>
<td>Operating Medium</td>
</tr>
<tr>
<td>Operating Pressure</td>
</tr>
<tr>
<td>Ambient Temperature</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4.3 Proportional valve characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>Valve Function</td>
</tr>
<tr>
<td>Construction design</td>
</tr>
<tr>
<td>Sealing principle</td>
</tr>
<tr>
<td>Actuation type</td>
</tr>
<tr>
<td>Type of rest</td>
</tr>
<tr>
<td>Type of pilot control</td>
</tr>
<tr>
<td>Direction of flow</td>
</tr>
<tr>
<td>Type of mounting</td>
</tr>
<tr>
<td>Mounting position</td>
</tr>
</tbody>
</table>
4.4.2. Simulink model of pneumatic system
A detailed model of the pneumatic drive system is created, in a Simulink environment, based on the equations from Eq. 4.27 to Eq. 4.31 and using the data given in Tables 4.1, 4.2 and 4.3. The resulting design is shown in Figure 4.5, below.

![Simulink model of pneumatic system](image)

**Figure 4.5** Detailed Simulink model of a pneumatic actuator

This subsystems receives a command $u$, position and speed feedback $q$ and $\dot{q}$, and it gives the change in pressure between chamber $A$ and $B$. Which then multiplied by the cross-sectional area of the piston $A_p$ and results in the desired force $F$ need to be applied at the manipulators joint.
Chapter Five

5. Control Strategies

The advantages of pneumatic systems are well known such as clear, cheap, easily maintained, safe in operation, etc. But for their highly nonlinear properties such as compressibility of medium, friction effect and nonlinearity of valves, pneumatic actuators are seldom used in industrial servo applications. Moreover, some of their properties, e.g., poor damping, low stiffness, and limited bandwidth, are unfavorable in the servo control system design. Some other difficulties in the control of pneumatic servo systems are the possible presence of unknown disturbances coming from leakage of valves, time-varying payloads, and external perturbations. Besides, uncertainties in system parameters make the controller design problem more challenging.

5.1. A Review of Control strategies

The problem of controlling robots has been extensively important in most of industrial applications. A great variety of control approaches have been proposed throughout the years. The most common is a decentralized "proportional, integral, derivative" (PID) control for each degree of freedom. More sophisticated nonlinear control schemes have been developed, such as so-called computed torque control, termed inverse dynamic control, which linearizes and decouples the equation of motion of the robot. In this chapter, first it is discussed the pros and cons PID with respect to other control schemes, i.e. a classical PID control, Sliding mode control, and then the nonlinear linearizing and decoupling control schemes. Then, it focuses on PID controller modeling and tuning based on the need to obtain a good trajectory tracking.

In case of pneumatic drive PID controller is still the most widely used approach due to its ease of implementation, the need for overcoming highly nonlinear phenomena turns away the use of classical PID controllers nowadays, see [4]. Therefore modern control techniques were designed and tested in pneumatic actuators in order to improve the performance of such systems considering position accuracy and repeatability as the two main performance characteristics. Fuzzy logic control, neural networks method, adaptive control, self-tuning or
gain scheduling, the so-called “Soft Computing” control techniques, are approaches that have attracted many researchers.

5.1.1. PID Controller
The dynamic model is described by a system of $n$ coupled nonlinear second order differential equations, $n$ being the number of joints. However, for most of today's industrial robots, a local decentralized "proportional, integral, derivative" (PID) control with constant gains is implemented for each joint. The advantages of such a technique are the simplicity of implementation and the low computational cost. The drawbacks are that the dynamic performance of the robot varies according to its configuration, and poor dynamic accuracy when tracking a high velocity trajectory. In many applications, these drawbacks are not of much significance.

The control law is given by:

$$u = K_p(q^d - q) + K_i(q^d - \dot{q}) + K_i \int_0^t (q^d - q) \, dt \quad (5.1)$$

Where $e = q^d - q$ is the position error at the joint. The objective function is, then, to get a quality trajectory tracking by minimizing the error in joint position through a compensation of the nonlinearities in the pneumatic drive system.

Practically, the block diagrams of such a control scheme is shown in Figure 5.1.

![Figure 5.1 PID controller block diagram](image)

5.2. PID Controller Design and tuning
The design is done using a Simulink PID Controller Block, from library: Simulink/Continuous in Matlab. This block takes an error signal and tends to give a tuned value of the gain constants. The PID block is shown in Figure 5.2.
Figure 5.2 PID controller Block in Simulink library

5.2.1. Position Control
As it can be seen from Figure 4.4, the control for trajectory tracking is performed on the joint position. Indirectly, the position of the actuated joints which is also known as the cylinder piston displacement is compared with the desired joint position.

To be Continued …
Conclusions

The objective of this thesis work was to carry out a full kinematic and dynamic analysis of a 2 DoF planar PKM manipulator driven by a pneumatic actuator. Thereby, to devise an appropriate control strategy in order to eliminate the position error that might be induced due to the nonlinearity in the drive mechanism.

Accordingly, in third chapter, it is presented detailed kinematic analysis. The model used in the kinematic simulation is designed in a Matlab/Simulink environment and it includes a physical representation of the manipulator created in Solidworks interface and then translated into a Simmechanics equivalent. The response of the manipulator for a desired input trajectory, in this case a circle under the effective work-envelop and desired motion law, i.e. a symmetric constant acceleration law, was as expected. Means, there isn’t position error due to only the kinematic of the mechanism because, it has been assumed that all the linkages in the PKM manipulator are rigid.

Whereas, as it is presented in the subsequent chapters, four and five, it has been tried to make a full dynamic analysis, which includes modeling of a pneumatic drive in a Matlab/Simulink environment. A model of the pneumatic actuator is created, as it is stated in chapter four, based on a set of nonlinear state flow and thermal equations. However, at the end of the analysis it appears to have a problem on the control of the resulting response. i.e. the manipulator very hardly follows the desired circular trajectory. As a result, I have faced difficulties on designing a control scheme that can compensate this huge gap in position.

Having spent too much effort and time on finding/creating a pneumatic actuator model that may result in a reasonable response, I still couldn’t overcome this difficulty. Therefore, due to lack of additional time, my limited scope of knowledge on pneumatics and also the economic situation I am in now, I am forced to compile the unfinished work that I have done so far. Which I hope leads me to my graduation.
Appendix A

1. Part I - Program Details

```
\begin{verbatim}
% Master's Thesis
% Title: Dynamic Analysis of a Planar Manipulator Driven by Pneumatic Actuators
% By: Mhreite Moalign Mulu
% Supervisors: Eng Hermes Giberti and Ing Simone Cinquemani
% NOTE: This Matlab program is written for the kinematic analysis of the manipulator. To be fully functional, this program requires the following
% Matlab and Simulink files: 1) Constant_Acceleration_ND.m
% 2) sim_kinematics.mdl
% All the above files should be placed in same folder(directory) that contains this main program.
% clear all
% close all
clc

% Robot geometric parameters
l1=0.500; % Horizontal distance between the two sliders/actuators [m]
l2=0.354; % Length of a single arm [m]
l3=1.500; % Maximum length of actuator [m]
R1=l1/2;
R2=l2;

T=10; % Simulation time [s]
t=0:0.1:T;

% Desired trajectory
Rc=0.1; % Radius of the circle [m]
angle=2*pi; % Total angle covered by a circle [rad]
angle_0=0; % Angle corresponding to initial position [rad]

% Initial conditions of the manipulator
xc=0; % x-position of the center of the circle [m]
yc=-0.0790919; % y-position of the center of the circle [m]
Dangle=angle-angle_0;
Xo=xc+Rc;
Yo=yc;
vXo=0;
vYo=0;

%max_kinematic.m
\end{verbatim}
```
% Motion law of the End-Effector (Constant Acceleration Law)
ndt=t./T;
par=[1/3,1/3];

[D,V,A] = Constant_Acceleration_ND(ndt,par);

theta=Dangle*D;
thetaD=Dangle/T*V;
thetaDD=Dangle/T^2*A;

xx=Rc*sin(theta);
yy=Rc*(cos(theta));
xd=Rc*thetaD.*cos(theta);
xd=thetaD.*cos(theta)-thetaD.*(thetaD.^2).*sin(theta);
yd=Rc*thetaD.*sin(theta)+(thetaD.^2).*cos(theta);

%% PLOT OF THE DESIRED TRAJECTORY
figure('Number','Off','Name','DESIRED TRAJECTORY')
plot(xx,yy);grid on;axis equal

%% END EFFECTOR DESIRED MOTION LAW
figure('Number','Off','Name','END-EFFECTOR DESIRED MOTION LAW [SYMMETRIC CONSTANT ACCELERATION LAW]

subplot(311);
plot(t,theta,'LineWidth',2);grid on;xlabel('t [s]');ylabel('theta [rad]')

subplot(312);
plot(t,thetaD,'g','LineWidth',2); grid on;xlabel('t [s]');ylabel('d(theta) [rad/s]')

subplot(313);
plot(t,thetaDD,'r','LineWidth',2); grid on;xlabel('t [s]');ylabel('d^2(theta) [rad/s^2]')

%% END EFFECTOR DESIRED MOTION LAW DECOMPOSED FOR THE X-AXIS AND Y-AXIS
figure('Number','Off','Name','END-EFFECTOR DESIRED MOTION LAW [DECOMPOSED TO X & Y AXES]

subplot(321);
plot(t,xx,'LineWidth',2); title('Axis-X');grid on;xlabel('t [s]');ylabel('X [m]')

subplot(323);
plot(t,xd,'g','LineWidth',2); grid on;xlabel('t [s]');ylabel('dX [m/s]')

subplot(325);
plot(t,xdd,'r','LineWidth',2); grid on;xlabel('t [s]');ylabel('d^2X [m/s^2]')

subplot(322);
plot(t,yy,'LineWidth',2); title('Axis-Y');grid on; xxlabel('t [s]');ylabel('X [m]')

subplot(324);
plot(t,yd,'g','LineWidth',2); grid on;xlabel('t [s]');ylabel('dX [m/s]')

subplot(326);
plot(t,ydd,'r','LineWidth',2); grid on;xlabel('t [s]');ylabel('d^2X [m/s^2]')

%---------------------------------------------------------------
%%% INVERSE KINEMATIC ANALYSIS

% joint space position
q1 = yy - sqrt(R2^2 - (xx + R1).^2);
q2 = yy - sqrt(R2^2 - (xx - R1).^2);
Q = [q1; q2];

for i=1:length(t)
% Jacobian matrix
J = [(xx(i)-R1)/sqrt(R2^2-(xx(i)-R1).^2) 1;
     (xx(i)+R1)/sqrt(R2^2-(xx(i)+R1).^2) 1];

% Joint space velocity
Qd(:,i)=J*[xd(i);yd(i)];
q1d = Qd(1,:);
q2d = Qd(2,:);

% Matrix A(p,pd)
A_A = [(xx(i)^2 + yy(i)^2 - 2*yd(i)*Qd(1) + Qd(1).^2)/(yy(i) - q1(i));
       (xx(i)^2 + yy(i)^2 - 2*yd(i)*Qd(2) + Qd(2).^2)/(yy(i) - q2(i))];

% Joint space acceleration
Qdd(:,i)=J*[xdd(i);ydd(i)] + A_A;
end
q1dd = Qdd(1,:);
q2dd = Qdd(2,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% DESIRED MOTION LOW IN THE JOINT SPACE. MATLAB BASED ANALYSIS
figure('Number','Off','Name','REFERENCE MOTION LAW [ JOINT SPACE ]_MATLAB')
subplot(321);
plot(t,q1); title('q_1_R'); grid on; xlabel('t [s]'); ylabel('q_1_R [m]')
subplot(323)
plot(t,Qd(1,:), 'g'); grid on; xlabel('t [s]'); ylabel('dq_1_R [m/s]')
subplot(325)
plot(t,Qdd(1,:), 'r'); grid on; xlabel('t [s]'); ylabel('d^2q_1_R [m/s^2]')
subplot(322);
plot(t,q2); title('q_2_R'); grid on; xlabel('t [s]'); ylabel('q_2_R [m]')
subplot(324)
plot(t,Qd(2,:), 'g'); grid on; xlabel('t [s]'); ylabel('dq_2_R [m/s]')
subplot(326)
plot(t,Qdd(2,:), 'r'); grid on; xlabel('t [s]'); ylabel('d^2q_2_R [m/s^2]')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% VARIABLES THAT CAN BE USED IN SIMULINK AS AN INPUT
xx_r=[t;xx]';
yy_r=[t;yy]';
q1_r=[t;q1]';
q2_r=[t;q2]';
q1d_r=[t;Qd(1,:)]';
q2d_r=[t;Qd(2,:)]';
q1dd_r=[t;Qdd(1,:)]';
q2dd_r=[t;Qdd(2,:)]';
q2dd_r=[t;Qdd(2,:)];

%% SIMULINK BASED KINEMATIC ANALYSIS
open('sim_kinematic.mdl')
sim('sim_kinematic')
xp=x.signals.values;
yp=y.signals.values;
xr=x_r.signals.values;
yr=y_r.signals.values;
tt=x.time;
q1_=q1_r1.signals.values;
q2_=q2_r1.signals.values;
q1d=q1dot_r.signals.values;
q2d=q2dot_r.signals.values;
q1dd=q12dot_r.signals.values;
q2dd=q22dot_r.signals.values;

%% DESIRED MOTION LAW IN THE JOINT SPACE. SIMULINK BASED ANALYSIS
figure('Number','Off','Name','REFERENCE MOTION LAW [ JOINT SPACE ]_SIMULINK')
subplot(321);
plot(tt,q1_,'LineWidth',2); title('q_1_R');grid on;xlabel('t [s]');ylabel('q_1_R [m]')
subplot(323)
plot(tt,q1d,'g','LineWidth',2); grid on;xlabel('t [s]');ylabel('dq_1_R [m/s]')
subplot(325)
plot(tt,q1dd,'r','LineWidth',2); grid on;xlabel('t [s]');ylabel('d^2q_1_R [m/s^2]')
subplot(322);
plot(tt,q2_,'LineWidth',2); title('q_2_R');grid on;xlabel('t [s]');ylabel('q_2_R [m]')
subplot(324)
plot(tt,q2d,'g','LineWidth',2); grid on;xlabel('t [s]');ylabel('dq_2_R [m/s]')
subplot(326)
plot(tt,q2dd,'r','LineWidth',2); grid on;xlabel('t [s]');ylabel('d^2q_2_R [m/s^2]')

%% TRAJECTORY COMPARISON AND POSITION ERROR ANALYSIS
figure('Number','Off','Name','TRAJECTORY COMPARISON...KINEMATIC ANALYSIS')
hold on
plot(xr,yr);grid on
plot(xp,yp,'r');grid on
axis equal

ex=xr-xp;
ey=yr-yp;

figure('Number','Off','Name','End-effector Position Error in the X-axis & Y-axis')
subplot(2,1,1);
plot(tt,ex),ylim([-2 2]);title('Error along x-axis');grid on;
subplot(2,1,2);
plot(tt,ey,'r'), ylim([-2 2]); title('Error along y-axis'); grid on;

--------------------------------------------------------

Detail of the Matlab Code Written for a Symmetric Constant Acceleration Law
Constant_Acceleration_ND.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Master's Thesis
% Title: Dynamic Analysis of a Planar Manipulator Driven by Pneumatic
% Actuators
% By: Mhretie Molalign Mulusew
% Supervisors: Eng Hermes Giberti and Ing Simone Cinquemanii
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% NOTE: This Matlab Function is written for generating a Symmetric Constant
% Acceleration Law.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [D,V,A] = Constant_Acceleration_ND(x,par)

% Non-dimensional General Constant Acceleration Motion Law
% % x = t/tm non-dimensional time
% % D = non-dimensional displacement
% % V = non-dimensional velocity
% % A = non-dimensional acceleration
% % par(1) = xa+
% % par(2) = xa-
% par=[1/3,1/3];

xap = par(1); % Positive acceleration part
xam = par(2); % Negative acceleration part

cap = 1/(xap*(1-(xap+xam)/2));
cam = 1/(xam*(1-(xap+xam)/2));

for i=1:length(x)

    if x(i)>=0 && x(i)<=xap
        A(i)=cap;
        V(i)=cap*x(i);
        D(i)=0.5*cap*x(i)^2;
    end

    if x(i)>xap && x(i)<(1-xam)
        A(i)=0;
        V(i)=cap*xap;
        D(i)=cap*xap*(x(i)-xap/2);
    end

    if x(i)>(1-xam) && x(i)<=1
        A(i)=-cam;
        V(i)=cap*xap-cam*(x(i)-1+xam);
        D(i)=cap*xap*(x(i)-xap/2)-cam/2*(x(i)-1+xam)^2;
    end

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Master's Thesis
% Title: Dynamic Analysis of a Planar Manipulator Driven by Pneumatic
% Actuators
% By: Mhretie Molalign Mulusew
% Supervisors: Eng Hermes Giberti and Ing Simone Cinquemanii
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% NOTES: This Matlab Function is written for generating a Symmetric Constant
% Acceleration Law.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
### Detail of the Matlab Code Written for Direct Kinematic Analysis

**D_kinematic.m**

```matlab
function [PP]=D_kinematic(Q1,Q2)

global R1 R2 t

for i=1:(length(t))
    q1=Q1(i);
    q2=Q2(i);

    a=(q2 - q1)/(2*R1);
    b=(q1^2 - q2^2)/(4*R1);
    e=a^2 + 1;
    g=(b-R1)^2 + q1^2 - R2^2;
    f=2*a*(b - R1) - q1*2;

    y(i)=-((f - sqrt(f^2 - 4*e*g))/(2*e));
    x(i)=a*y(i)+b;
end

PP=[x;y];
```

### Detail of the Matlab Code Written for Workspace Generation

**Workspace.m**

```matlab
close all
clear all
clc

%Input parameters
l1=0.500; %mm
l2=0.354; %mm
l3=1.500; %mm
R1=l1/2;
```
R2=l2;

%%workspace
q=0:0.1:1.25;
for i=1:length(q)
    q1=q(i);
    for j=1:length(q)
        q2=q(j);
        Q=[q1;q2];
        [x,y]=D_kinematic(Q);
        plot(x,y,'*')
    end
end

%%----------------------

Detail of the Matlab Code Written for Dynamic Analysis

main_dynamic.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Master's Thesis
%Title: Dynamic Analysis of a Planar Manipulator Driven by Pneumatic
%Actuators
%By: Mhretie Molalign Mulusew
%Supervisors: Eng Hermes Giberti and Ing Simone Cinquemanii
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%NOTE: This Matlab program is written for the dynamic analysis of the
%manipulator. To be fully functional, this program requires the following
%Matalb and Simulink files: 1) Constant_Acceleration_ND.m
%                           2) Inverse_Dynamics.m , and
%                           3) sim_dynamics.mdl
%All the above files should be placed in same folder(directory) that
%contains this main program.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all
close all
clc

global l1 l2 R1 J1 J2 M m1 m2 ms mcy g

%Robot geometric parameters
l1=0.500;  %Horizontal distance between the two sliders/actuators [m]
l2=0.354;  %Length of a single arm [m]
l3=1.500;  %Maximum length of actuator [m]

%Physical and Thermal Constants
Ps=6e5;    %Cylinder Supply pressure [Pa]
Pe=1e5;    %Cylinder Exhaust pressure [Pa]
P0=3e5;    %Initial pressure in the cylinder [Pa]
Patm=101.3e3;  %average atmospheric pressure [Pa]
Cr=0.852;  %Critical pressure ratio[]
Ts=293;    %Supply temperature [K]
\[ \text{Tc} = \text{Ts}; \quad \% \text{Cylinder temperature [K]} \ldots \text{Isothermal process} \\
\text{Tp} = \text{Ts}; \quad \% \text{Pipe temperature [K]} \ldots \text{Isothermal process} \\
Ck = 3.864; \quad \% \text{Valve constant1[]} \\
Co = 0.04; \quad \% \text{Valve constant2[]} \\
Cd = 0.8; \quad \% \text{valve coefficient of discharger[]} \\
Cs = Cd; \\
Cc = 0.7; \quad \% \text{Connection port discharge coefficient []} \\
Cv = 3.15745e-4; \quad \% \text{Valve constant[m/V]} \\
B = 65; \quad \% \text{Viscous friction coefficient of air[Ns/m]} \\
k = 1.4; \quad \% \text{Heat ratio} \\
\gamma = k; \\
R = 287; \quad \% \text{Gas constant [m/K]} \\
g = 0; \quad \% \text{Gravitational Acceleration [m/s^2]} \\
ms = 1; \quad \% \text{Mass of slider [kg]} \\
mcy = 1; \quad \% \text{Mass of Cylinder piston [kg]} \\
ml = 2; \quad \% \text{Mass of leg one [kg]} \\
m2 = 2; \quad \% \text{Mass of leg two [kg]} \\
M = 5; \quad \% \text{Mass of end-effector [kg]} \\
Wa = 5e-3; \quad \% \text{valve opening width for intake valve [m]} \\
Wb = 5e-3; \quad \% \text{valve opening width for exhaust valve}[m] \\
dp = 0.016; \quad \% \text{Cylinder bore or Piston diameter [m]} \\
L = 1.5; \quad \% \text{Cylinder length [m]} \\
\% \text{Calculated geometric parameters related to actuator} \\
Ap = \pi \ast dp^2; \quad \% \text{Cylinder area or piston cross-sectional area [m^2]} \\
Acy = Ap; \\
Vc = L \ast Ap; \quad \% \text{cylinder volume [m^3]} \\
Vp = 0.5 \ast \pi \ast 0.02^2; \quad \% \text{Pipe effective volume [m^3]} \\
\% \text{Calculated robot geometric parameters} \\
R1 = 11/2; \\
R2 = 12; \\
J1 = (1/3) \ast m1 \ast l2^3; \quad \% \text{Moment of inertia of leg #one about point B1 [kg m^3]} \\
J2 = (1/3) \ast m2 \ast l2^3; \quad \% \text{Moment of inertia of leg #two about point B1 [kg m^3]} \\
\% \text{Control Parameters} \\
Kp = 1; \quad \% \text{Proportional gain []} \\
Ki = 0; \quad \% \text{Integral gain []} \\
Kd = 0; \quad \% \text{derivative gain []} \\
T = 10; \quad \% \text{Simulation time[s]} \\
t = 0:0.1:T; \\
\% \text{Desired trajectory} \\
\% \text{Circle} \\
Rc = 0.1; \quad \% \text{Radius of the circle[m]} \\
angle = 2 \ast \pi; \quad \% \text{Total angle covered by a circle angle[rad]} \\
angle_0 = 0; \quad \% \text{Angle corresponding to initial position [rad]} \\
\% \text{Initial conditions of the manipulator} \\
\text{xc} = 0; \quad \% \text{x position of the center of the circle [m]}
\[ y_c = 0.0790919; \] % y_position of the center of the circle [m]
\[ \text{Dangle=}\text{angle-0}; \] % Initial x-position of the end effector [m]
\[ x_0 = x_0 + R_c; \] % Initial y-position of the end effector [m]
\[ v_{x_0} = 0; \] % Initial velocity of the manipulator along x-axis [m/s]... considering that the robot start from rest
\[ v_{y_0} = 0; \] % Initial velocity of the manipulator along y-axis [m/s]... considering that the robot start from rest

\[
\begin{align*}
q_{10} &= -0.0840919; & \% \text{Yo} - \sqrt{R_2^2 - (X_0 - R_1)^2}; \\
q_{20} &= -0.0840919; & \% \text{Yo} - \sqrt{R_2^2 - (X_0 + R_1)^2};
\end{align*}
\]

% Motion law of the End-Effector (Constant Acceleratin Law)

\[
\text{ndt} = \frac{t_0}{T};
\]
\[
\text{par} = [1/3, 1/3];
\]
\[
[D,V,A] = \text{Constant\_Acceleration\_ND}(\text{ndt}, \text{par});
\]
\[
\begin{align*}
\theta &= \text{Dangle} \times D; \\
\theta_D &= \text{Dangle} / T \times V; \\
\theta_{DD} &= \text{Dangle} / T^2 \times A;
\end{align*}
\]
\[
\begin{align*}
xx &= R_c \times \sin(\theta); \\
yy &= R_c \times \cos(\theta); \\
x_d &= R_c \times \theta_D \times \cos(\theta); \\
y_d &= R_c \times \theta_D \times \sin(\theta);
\end{align*}
\]
\[
\begin{align*}
xx &= R_c \times (\theta_{DD} \times \cos(\theta) - (\theta_D)^2 \times \sin(\theta)); \\
y_d &= R_c \times (\theta_{DD} \times \sin(\theta) + (\theta_D)^2 \times \cos(\theta));
\end{align*}
\]

% PLOT OF THE DESIRED TRAJECTORY
figure('Number', 'Off', 'Name', 'DESIRED TRAJECTORY')
plot(xx, yy); grid on; axis equal

% PLOT OF THE DESIRED MOTION LAW
figure('Number', 'Off', 'Name', 'END-EFFECTOR DESIRED MOTION LAW [SYMMETRIC CONSTANT ACCELERATION LAW]')
subplot(311);
plot(t, theta); grid on; xlabel('t [s]'); ylabel('theta [rad]')
subplot(312)
plot(t, thetaD, 'g'); grid on; xlabel('t [s]'); ylabel('d(\theta) [rad/s]')
subplot(313)
plot(t, thetaDD, 'r'); grid on; xlabel('t [s]'); ylabel('d^2(\theta) [rad/s^2]')

% PLOT OF THE DESIRED MOTION OF THE END EFFECTOR
figure('Number', 'Off', 'Name', 'END-EFFECTOR DESIRED MOTION LAW [DECOMPOSED TO X & Y AXES]')
subplot(321);
%%-----------------------------------------------------
%% % INVERSE KINEMATIC ANALYSIS
%% Joint space position
q1 = yy - sqrt(R2^2 - (xx + R1).^2);  % considering only one (+) solution out of the four possible results
q2 = yy - sqrt(R2^2 - (xx - R1).^2);  % based on the configuration discussed in the third chapter of the report
Q = [q1; q2];

for i = 1:length(t)
    % Jacobian matrix
    J = [-(xx(i) - R1)/sqrt(R2^2 - (xx(i) - R1).^2) 1;
         -(xx(i) + R1)/sqrt(R2^2 - (xx(i) + R1).^2) 1];

    % Joint space velocity
    Qd(:,i) = J*[xd(i) yd(i)]';

    % Matrix A(p,pd)
    A_A = [(xx(i)^2 + yy(i)^2 - 2*yd(i)*Qd(1) + Qd(1).^2)/(yy(i) - q1(i));
           (xx(i)^2 + yy(i)^2 - 2*yd(i)*Qd(2) + Qd(2).^2)/(yy(i) - q2(i))];

    % Joint space acceleration
    Qdd(:,i) = J*[xdd(i);ydd(i)] + A_A;
end
q1d = Qd(1,:);
q2d = Qd(2,:);
q1dd = Qdd(1,:);
q2dd = Qdd(2,:);

%%-----------------------------------------------------
%% % PLOTS OF JOINT SPACE MOTION (POSITION, VELOCITY & ACCELERATION)
figure('Number','Off','Name','REFERENCE MOTION LAW [ JOINT SPACE ]')
subplot(321);
    plot(t,q1); title('q_1_R'); grid on; xlabel('t [s]'); ylabel('q_1_R [m]')
subplot(323)
    plot(t,q1d,'g'); grid on; xlabel('t [s]'); ylabel('dq_1_R [m/s]')
subplot(325)
    plot(t,q1dd,'r'); grid on; xlabel('t [s]'); ylabel('d^2q_1_R [m/s^2]')
subplot(322);
    plot(t,yy); title('Axis-Y'); grid on; xlabel('t [s]'); ylabel('X [m]')
subplot(324)
    plot(t,yd,'g'); grid on; xlabel('t [s]'); ylabel('dX [m/s]')
subplot(326)
    plot(t,ydd,'r'); grid on; xlabel('t [s]'); ylabel('d^2X [m/s^2]')
subplot(322); title('q_2_R'); grid on; xlabel('t [s]'); ylabel('q_2_R [m]') subplot(324) plot(t,q2d,'g'); grid on; xlabel('t [s]'); ylabel('dq_2_R [m/s]') subplot(326) plot(t,q2dd,'r'); grid on; xlabel('t [s]'); ylabel('d^2q_2_R [m/s^2]')

%% INVERSE DYNAMIC ANALYSIS
[FB1 FB2 Fd1 Fd2]=Inverse_Dynamics(t,xx,yy,xd,yd,xdd,ydd);

%%plots of desired drive forces
figure('Number','Off','Name','Desired Force at Slider Joint 1') subplot(211) plot(t,FB1(1,:)); title('X-axis component'); xlabel('t [s]'); ylabel('F [N]'); grid on subplot(212) plot(t,FB1(2,:)); title('Y-axis component'); xlabel('t [s]'); ylabel('F [N]'); grid on

figure('Number','Off','Name','Desired Force at Slider Joint 2') subplot(211) plot(t,FB2(1,:)); title('X-axis component'); xlabel('t [s]'); ylabel('F [N]'); grid on subplot(212) plot(t,FB2(2,:)); title('Y-axis component'); xlabel('t [s]'); ylabel('F [N]'); grid on

figure('Number','Off','Name','Desired Drive Force') subplot(211) plot(t,Fd1); title('Drive Force at Slider Joint#1'); xlabel('t [s]'); ylabel('F_d_1 [N]'); grid on subplot(212) plot(t,Fd2); title('Drive Force at Slider Joint#2'); xlabel('t [s]'); ylabel('F_d_2 [N]'); grid on

%%VARIABLE USED AS AN INPUT TO THE SIMULINK MODELS
xx_r=[t;xx]';
yy_r=[t;yy]';
q1_r=[t;q1]';
q2_r=[t;q2]';
q1d_r=[t;Qd(1,:)]';
q2d_r=[t;Qd(2,:)]';
q1dd_r=[t;Qdd(1,:)]';
q2dd_r=[t;Qdd(2,:)]';
F1_r=[t;Fd1]';
F2_r=[t;Fd2]';

%%-------------------E--N--d-------------------%
This Matlab Code is Written for Inverse Dynamic Analysis using Newton-Euler Approach

\( I_{\text{Dynamic.m}} \)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\%Master’s Thesis
\%Title: Dynamic Analysis of a Planar Manipulator Driven by Pneumatic
\%Actuators
\%By: Mhretie M. Molalign Mulusew
\%Supervisors: Eng Hermes Giberti and Ing Simone Cinquemanii
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\%NOTE: This Matlab Function is written for Inverse Dynamic Analysis using
\%the Newton–Euler Approach
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

\function [FB1 FB2 Fd1 Fd2] = Inverse_Dynamics(t, xx_, yy_, xd_, yd_, xdd_, ydd_)

\global l1 l2 R1 J1 J2 M m1 m2 ms mcy g
l=12;
Rcg=12/2;

for \( i = 1: \text{length}(t) \)
    \( xx=xx_(:,i); \)
    \( yy=yy_(:,i); \)
    \( xd=xd_(:,i); \)
    \( yd=yd_(:,i); \)
    \( xdd=xdd_(:,i); \)
    \( ydd=ydd_(:,i); \)
    \( r=[xx \ yy]'; \)
    \( rd=[xd \ yd]'; \)
    \( rdd=[xd \ yd]'; \)
    \( Rc=[Rcg 0]'; \)
    \( a=[0 0]'; \)
    \( e2=[0 1]'; \)
    \( b=[11/2 0]'; \)

    \( lx1=R1+r(1); \)
    \( lx2=R1-r(1); \)
    \( w1=(1/l2)*[lx1 \ \text{sqrt}(l2^2-lx1^2)]'; \)
    \( w2=(1/l2)*[lx2 \ \text{sqrt}(l2^2-lx2^2)]'; \)

    % velocity Analysis
    \( q1d=(w1'*rd)/w1(2); \)
    \( q2d=(w2'*rd)/w2(2); \)

    \( rqd1=rd-qld*e2; \)
    \( omega1=(1/l2)*(-w1(2)*\text{rd}(1)+w1(1)*\text{rd}(2)); \)
    \( rqd2=rd-\text{rd}(2)*e2; \)
    \( omega2=(1/l2)*(-w2(2)*\text{rd}(1)+w2(1)*\text{rd}(2)); \)

    % Acceleration Analysis
    \( q1dd=(1/\text{w1}(2))*(\text{w1}^2*\text{rd} + 1*(omega1^2)); \)
    \( q2dd=(1/\text{w2}(2))*(\text{w2}^2*\text{rd} + 1*(omega2^2)); \)

    \( rqdd1=\text{rd}(1)*\text{rd}(2)*e2; \)
    \( alfa1=(1/\text{l2})*(-w1(2)*\text{rd}(1)+w1(1)*\text{rd}(2)); \)
    \( rqdd2=\text{rd}(2)*\text{rd}(2)*e2; \)
alfa2=(1/12)*(-w2(2)*rqdd2(1)+w2(1)*rqdd2(2));

%%Position, Velocity and Acceleration analysis at the center of gravity of legs

%%Position
Tc1=[w1(1) -w1(2);
    w1(2)  w1(1)];

Rc1=Tc1*Rc;

Tc2=[w2(1) -w2(2);
    w2(2)  w2(1)];

Rc2=Tc2*Rc;
Rcg1=b+q;

%%Velocity
Rcg1d=q1d*e2 + omega1*[-Rc1_(2);Rc1_(1)];
Rcg2d=q2d*e2 + omega2*[-Rc2_(2);Rc2_(1)];

%%Acceleration
Rcg1dd=q1dd*e2 + alfa1*[-Rc1_(2);Rc1_(1)]-Rc1_*omega1^2;
Rcg2dd=q2dd*e2 + alfa2*[-Rc2_(2);Rc2_(1)]-Rc2_*omega2^2;

%%Evaluation of the forces acting on each linkage

Fext=[0 0]';
Mext=0;
Rcp=[0 0]';
AID=[ 1 0 1 0 ;
    0 1 0 1 ;
    -yy xx -yy xx ;
    -12*w1(2) 12*w1(1) 0 0 ;
    0 0 -12*w2(2) 12*w2(1)];

BID=[-M*rdd(1)+Fext(1);
    -M*rdd(2)+Fext(2)-M*g;
    -M*g*Rcp(1) + Mext + M*rdd(1)*Rcp(2)-M*rdd(2)*Rcp(1);
    m1*g*Rc1_(1)+m1*(Rcg1dd(1)*(-Rc1_(2))+Rcg1dd(2)*Rc1_(1))+J1*alfa1;
    m2*g*Rc2_(1)+m2*(Rcg2dd(1)*(-Rc2_(2))+Rcg2dd(2)*Rc2_(1))+J2*alfa2];

FA=AID\BID;
FA1=[FA(1) FA(2)]';
FA2=[FA(3) FA(4)]';

FB1(:,i)=-m1*g-FA1+m1*Rcg1dd;
FB2(:,i)=-m2*g-FA2+m2*Rcg2dd;

Fd1(1,i)=FB1(2,i)+ (ms+mcy)*q1dd;
Fd2(1,i)=FB2(2,i)+ (ms+mcy)*q2dd;
end

%%-----------------------------------E-N-D----------------------------------
1. Part II – Simulink Blocks Detail

2.1 Main Simulink model of the manipulator for kinematic analysis

2.1.1 Inverse Kinematic Block
2.1.2 Manipulator Block

![Diagram of Manipulator Block]

2.2 Main Simulink of the Manipulator for Dynamic Analysis

![Diagram of Main Simulink]

2.2.1 Controller Block

![Diagram of Controller Block]
2.2.1.1 PID Block

2.2.2 Cylinder Block

2.2.1.1 Flow rate Block
2.2.1.2 Block Diagram for Chambers Under the Flow rate Block

2.2.2 Chamber Block Under Cylinder Block Diagram
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