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Master Thesis

Tractable Reserve Scheduling Formulations for Power Systems with Uncertain Generation

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Abstract

The increased penetration of renewable energy sources (in particular, wind generation) into the electricity grid highlights the necessity of constructing stochastic variants of the so-called unit commitment (deciding which generators to switch on at what time) and reserve scheduling problems.

In this thesis, we formulate an optimization problem for a DC power flow based model of power network and propose a technique based on randomized optimization to solve both the unit commitment and the reserve scheduling problems in a unified framework while providing probabilistic performance guarantees. This technique allows us to handle integer variables, an essential step for addressing the unit commitment problem. The resulting performance enhancement in terms of the total energy procurement cost observed in simulations was quite substantial. We then introduce affine feedback policies for reserve scheduling to assess whether more sophisticated "recourse" strategies are able to provide a further improvement in energy costs.

Finally, we extend our power network models to more accurate models based on AC power flow and propose a novel relaxation method to obtain a convex (albeit very complex) optimization problem. Empirical evidence shows that the proposed relaxation technique tends to a feasible solution for the exact problem.

The theoretical developments in all cases above were validated on realistic benchmark problems of power networks and a discussion on the tractability of the resulting optimization problems is presented.

Sommario

La presenza crescente di fonti di energia rinnovabili, ed in particolare di quella eolica, nella rete elettrica fa emergere la necessità di affrontare il problema della scelta di quali generatori utilizzare (unit commitment) e di come pianificare le riserve elettriche (reserve scheduling) secondo un approccio probabilistico, che tenga conto della natura aleatoria di tali fonti.

In questa tesi, i problemi dello unit commitment e del reserve scheduling vengono affrontati in modo congiunto, utilizzando un modello approssimato in corrente continua per il calcolo dei flussi di potenza nella rete elettrica, e formulando un problema di ottimizzazione chance-constrained in cui si richiede che i vincoli siano validi con una certa (alta) probabilità. Il problema di ottimizzazione risultante viene risolto mediante una tecnica randomizzata, che consente di ottenere garanzie sulla qualità della soluzione ottenuta anche in presenza di variabili di ottimizzazione discrete quali quelle adottate per lo unit commitment. Il miglioramento in termini di costo valutato in simulazione è notevole. Vengono anche studiate politiche di feedback per il reserve scheduling allo scopo di ottenere possibili ulteriori miglioramenti del costo.

Infine, il modello della rete elettrica viene esteso al caso di calcolo dei flussi di potenza in corrente alternata, e viene proposta una tecnica innovativa per ricondurre il problema di ottimizzazione risultante ad un problema convesso (seppure molto complesso). L'evidenza empirica mostra che la tecnica di rilassamento proposta restituisce una soluzione feasible per il problema di partenza.

Gli sviluppi teorici di questo lavoro sono validati su casi di studio realistici e vengono discusse le condizioni che rendono il problema di ottimizzazione risultante trattabile.

Preface

This report is my master thesis for completion of my master program at department of electronics, information and bioengineering (DEIB) at the Politecnico di Milano. I am indebted to many people for making my working time during the master thesis a wonderful experience.

First of all, I am deeply grateful to my advisor in Politecnico di Milano, Prof. Maria Prandini, department of electronics, information and bioengineering. She gave me a lot of trust and flexibility on the project. Without her, I would not have the chance to get involved in such a challenging project; Without her, I would not be able to get a scholarship for doing my thesis in abroad. Without her, I would not be able to reach my target.

I also want to thank Prof. John Lygeros, head of Automatic Control Laboratory, Swiss Federal Institute of Technology Zürich. I would like to thank him for being a member of his group, for his support, for insightful comments both regarding my work and the thesis, and for the many motivating discussions.

Furthermore, Dr. Kostas Margellos and Maria Vrakopoulou are really appreciated for their assistance on my thesis. Their ability to select and to approach compelling research problems, high scientific standards, and hard work set examples. All our meetings were fruitful because they are really good discussion partners. Above all, you made me feel a friend, which I appreciate from my heart.

Moreover, I would like to thank my family for their infinite support. Last but not least, I really appreciate my wife, Zahra. I finished my report and I will continue to challenge myself in the future with what I learned.

Thank you with all my heart!

Vahab Rostampour
Thursday 4th April, 2013

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Chapter 1

Introduction

The large-scale integration of wind generation in power systems represents a significant challenge for the Transmission System Operators (TSO) due to the unpredictable and highly variable pattern of wind power generation [1, 3, 21, 26]. In order to accommodate the unpredictable nature of wind power, the productions and consumptions scheduled in an electricity market need to be modified during the actual operation of the power system.

Operating reserves balance mismatches between generation and demand and their sufficient provision is crucial in terms of a secure power system operation [34]. Since the liberalization process of most parts of the European power system incorporate the distinction between the market operators and the TSO, we assume that the TSO is in charge of the power system security.

In deregulated power markets, unit commitment is one of the main tasks of the TSO. The objective is to compute the "on-off" status of the generating units, and the generation dispatch, which denotes the amount of power that each generator should produce to satisfy a given demand level. In the presence of uncertainty the TSO decides also about corrective actions to avoid any disruption of service [35].

The reserve scheduling and unit commitment problems become more challenging due to the large scale integration of wind power in the power network. This emphasizes the necessity of formulating stochastic variants of standard day-ahead planning problems, while providing probabilistic guarantees regarding the satisfaction of system constraints.

Stochastic programs are mathematical programs where some of the data incorporated into the objective or constraints are uncertain. In the optimization literature, an optimization problem having probabilistic constraints, is called a "chance-constrained" optimization problem [43, 51]. Uncertainty is usually characterized by a probability distribution on the parameters. The unpredictable fluctuations of wind power supply are naturally classified as uncertainty.

Moreover, stochastic programming provides in a probabilistic sense the optimal generation schedules and reserve schedules. In addition, a stochastic program can also be utilized for analyzing the economic impacts of wind integration and demand response in power system operations [42]. Therefore, it is an extremely useful tool both for the technical purposes of supporting the TSO tasks but also for analyzing the economic impacts of large-scale renewable power integration.

Regardless of its interesting features, two fundamental questions are evoked here: The first question is how to develop a methodology for formulating the problem of stochastic unit commitment and reserve scheduling as a chance constrained problem, and the second question refers to how to ensure that the resulting problem is tractable.

1.1 Related Work

In this section we briefly present some of the literature results related to the reserve scheduling and unit commitment problems. Starting by the work of [45, 47], different procedures are provided to determine the level of reserve required in the system in order to attack the wind generation uncertainty. In [22, 27], a similar framework is considered, with different procedures.

The work of [29] introduces a decision making for determining an optimal reserve scheduling based on decision trees that are built for a set of selected disturbances (including those pertaining to wind power uncertainty). Article [40] applies a similar model where the costs of energy and reserves are simultaneously minimized.

In the past decade, much attention has been paid to stochastic optimization approach of reserve scheduling problem incorporating the unit commitment determination, with an objective function which seeks to maximize an expected social welfare [13–15, 23, 25, 39, 42].

Articles [14,15,23] formulate a two-stage stochastic programming formulation for formulating a reserve scheduling problem by using decision variables for reserve and assuming that reserve bids as well as energy bids are available to the TSO. The authors of [13] use the model developed in [14,15] to analyze the impact of wind integration on reserve scheduling in a small-scale model with three generators and for 4-hours horizon without transmission constraints. The work of [39] employs a similar formulation to [13] in order to determine the minimum cost level of reserves with integration of wind power. By introducing unit commitment variables and using a scenario reduction technique to ensure tractability of the problem, they are able to solve the unit commitment problem for the IEEE benchmark power systems [2] with transmission constraints. Nevertheless, no guarantees regarding the robustness of the resulting solution are provided, and only an a-posteriori analysis is conducted.

In [44] the authors present a general model to estimate the impact of wind power integration in power system and formulate a two-stage stochastic program, where the first stage of the problem represents day-ahead unit commitment of slow generators, and the second stage represents hour-ahead economic dispatch of the entire system, given the fixed day-ahead schedule of slow generators. In [42] the authors use a stochastic programming unit commitment model similar to [44] and focus on a central unit commitment problem in order to minimize operating costs. Thus, they do not consider reserve bids in the model.

The work of [11,12] follows rule based approaches for reserve scheduling, whereas [46] uses a deterministic unit commitment model to perform an annual simulation of wind integration in power system.

In [34], the authors introduce a hybrid methodology, where the unit commitment and reserve scheduling problems are solved separately. They formulate the deterministic variant of the unit commitment problem, by assuming that the wind power is equal to its forecast and then fix the status of the generating units to the binary vector computed by the deterministic unit commitment program, and formulate a stochastic reserve scheduling problem.

In [53,54], the authors focused on the problem of security constrained reserve scheduling for networks with wind power generation. They formulate a chance constrained optimization program and employ the scenario approach of [17,18], in order to achieve a tractable reformulation. Following [18], and if the underlying problem is convex with respect to the decision variables, finite sample guarantees can be provided. Stochastic unit commitment models present computational challenges due to their large scale. In [9] the authors propose a robust formulation of the unit commitment problem either by assuming that the uncertainty takes values on a hyper-rectangular set, or by making certain assumptions on the underlying probability distribution.

1.2 Thesis Contribution

The contributions of this thesis are twofold:

1. We introduce a new framework for the DC power flow based reserve scheduling and stochastic unit commitment problems, that allows to inherit probabilistic guarantees even if the underlying problem is not convex (e.g. mixed integer optimization programs arising in unit commitment problems). Results rely on the methodology in [33], which is based on a mixture of randomized and robust optimization, where first probabilistic bounds for the uncertainty are computed, and then these bounds are used to solve the robust counterpart of the initial stochastic program. The latter is solved using tractable reformulations of robust optimization problems [8, 10] and [31]. This approach enables us to

incorporate the unit commitment problem and the reserve scheduling one in a unified framework, thus leading to improved solutions. Furthermore, an alternative reserve representation is discussed based on the results of disturbance feedback policies from [24] and [55].

2. We investigate some new concepts in AC optimal power flow problem formulation based on the work [30], and incorporate the reserve scheduling and unit commitment problems in the developed set-up. Optimization over AC power flow based model has not, to the best of our knowledge, been extended to the reserve scheduling and unit commitment problems of day-ahead decision making in electricity provision under uncertainty. We formulate a chance constrained optimization problem, but since the problem is infinite dimensional, we follow a heuristic approach to deal with chance constraints. Finally, we carry out a simulation study to show the efficiency of the proposed formulation versus the DC power flow based one.

1.3 Thesis Structure

This thesis is divided into four chapters. Following this introductory chapter, Chapter 2 discusses the DC power flow based reserve scheduling and unit commitment problems formulation. This chapter is divided into: (i) description of the problem set-up including the power flow equations, reserve model, and wind power model, (ii) problem formulation where we formulate a general chance constrained optimization framework for the problem of reserve scheduling and stochastic unit commitment, (iii) information on how to deal with the chance constraint which appears in our formulation, (iv) information on how to obtain a methodology that ensures tractability while providing probabilistic performance guarantees, (v) simulation study wherein we demonstrate the efficiency of the proposed approach by comparing it with an algorithm based on a deterministic unit commitment formulation, and (vi) an alternative designs for the reserve will be discussed.

Chapter 3 discusses the AC power flow based reserve scheduling and unit commitment problems formulation. This chapter investigates the following concepts: (i) a novel relaxation technique to obtain convex optimization for the AC optimal power flow problem, (ii) incorporation reserve scheduling and unit commitment problems with the AC power flow formulation, (iii) information on how to ensure that the resulting problem is tractable, and (iv) simulation study wherein we illustrate performance of the proposed approach by comparing against its DC power flow counterpart.

Finally, Chapter 4 provides a discussion on the results obtained in this thesis, and suggests some ideas for future work.

Chapter 2

DC Power Flow Based Reserve Scheduling

In this chapter we present a unified framework to deal with the problem of stochastic unit commitment and reserve scheduling using DC power flow approximation for systems with uncertain generation, while providing a-priori probabilistic certificates for the robustness properties of the resulting solution. Our methodology is based on a mixture of randomized and robust optimization and leads to a tractable problem formulation. We first compute probabilistic bounds for the uncertainty, and then employ these bounds to solve the robust counterpart of the initial stochastic program.

The rest of the chapter is organized as follows. In Section 2.1 we present the problem set-up and the mathematical model of the system. In Section 2.2 we present a formulation of the DC power flow based reserve scheduling problem as a chance constrained optimization program. In Section 2.3 we show how to deal with chance constrained problem. In Section 2.4 we describe a methodology that ensures tractability while providing probabilistic performance guarantees, whereas in Section 2.5 we demonstrate the efficiency of the proposed approach by comparing it with an algorithm based on a deterministic unit commitment formulation using Monte Carlo simulations.

2.1 Problem Set-Up and Mathematical Modeling

This section describes standard steady-state models which are common for planning problem in power network. Specifically, we consider DC models for the network elements which are obtained from AC models using linearization [4]. We use this DC power flow set-up to formulate our reserve scheduling algorithm. In this section we provide some definitions of the general structure of the power network, the power flow equations, reserve representation, and some details about the wind power model of [41].

2.1.1 Definitions and Preliminaries

We consider a power system comprising n_b buses, n_g generators, n_d loads and n_l lines. For simplicity, we consider a single wind power in-feed ($n_w = 1$) located in a certain bus. The derivations of the next sections are based on the following assumptions:

1. A linearized version of the network is considered, hence DC power flow problems are constructed based on [4].
2. Wind generation is a regulated activity and thus wind producers are not considered competitive agents in the market [39].
3. No load uncertainty is considered.
4. No $\mathcal{N} - 1$ security constraints are considered.

The first assumption is standard for reserve scheduling problems (we make this assumption only in Chapters 2, 2.6). The second one is the case in which wind power is treated in most energy markets. The third one is included so as to focus solely on the effect of wind uncertainty on the reserve level, and could be easily captured by the proposed framework, whereas the last assumption refers to the case of component outages emanating from the $\mathcal{N} - 1$ security criterion [5]; they could be modeled following the procedure outlined in [53].

2.1.2 Power Flow Equations

In this section we provide some details on the DC approximation of the power flow equations. Similar discussion can be found in [4, 54], but we describe some of the results here for sake of comparisons. We make the following assumptions:

1. The voltage at every bus of the network remains constant at 1p.u.
2. The active power losses are neglected.
3. It is assumed that $sin\ \theta_{kl} \approx \theta_{kl}$, where θ_{kl} is the angle in radians across the branch connecting the buses k and l.

Under these assumptions, the power flow equation for every line connecting bud k to bus l is given by

$$P_{f,kl} = -b_{kl}\ (\theta_k - \theta_l)\ ,$$

where θ_k,θ_l are the voltage angles at buses k,l respectively, and b_{kl} denotes the imaginary part of the admittance of the line connecting bud k to bus l. Writing $P_{f,kl}$ in a matrix form, we get

$$\mathbf{P}_f = B_f \theta\ , \tag{2.1}$$

where $\mathbf{P}_f \in \mathbb{R}^{n_l}$ is a vector containing the power flows $P_{f,kl}$ of each line, $\theta \in \mathbb{R}^{n_b}$ denotes the voltage angles at every bus, and $B_f \in \mathbb{R}^{n_l \times n_b}$. On the other hand, the active power injection at a bus k is given by

$$P_{inj,k} = \sum_{l \in N(k)} P_{f,kl} \ ,$$

where $N(k) = \{l \in \{1, 2, \cdots, n_b\} \mid k \to l \text{ is a line}\}$ and in more compact notation, we have

$$\mathbf{P}_{inj} = B_{bus}\ \theta \ , \tag{2.2}$$

where $\mathbf{P}_{inj} \in \mathbb{R}^{n_b}$ is the vector of the network power injections $P_{inj,k}$, and $B_{bus} \in \mathbb{R}^{n_b \times n_b}$ denotes the nodal admittance matrix of the power network.

To have the power flow of every line as a function of the power injection in the corresponding node, we will eliminate θ from (2.1) and (2.2). To this purpose we should solve (2.2) directly with respect to θ which is not possible since B_{bus} is singular, with rank $n_b - 1$, hence not invertible. Therefore, in order to obtain a solution, we eliminate from B_{bus} the row and column corresponding to the slack bus and reference angle, respectively. We thus get

$$\tilde{\theta} = B_{dc}^{-1}\ \tilde{\mathbf{P}}_{inj} \ , \tag{2.3}$$

where $B_{dc} \in \mathbb{R}^{(n_b-1)\times(n_b-1)}$, $\tilde{\theta} \in \mathbb{R}^{n_b-1}$ and $\tilde{\mathbf{P}}_{inj} \in \mathbb{R}^{n_b-1}$ denote the remaining parts of B_{bus},θ and $\mathbf{P}_{inj}$ respectively. Now, by substituting (2.3) into (2.1), we have

$$\left.\begin{array}{l} \theta = \begin{bmatrix} B_{dc}^{-1}\ \tilde{\mathbf{P}}_{inj} \\ 0 \end{bmatrix} \\ \mathbf{P}_f = B_f \theta \end{array}\right\} \Rightarrow \quad \mathbf{P}_f = B_f \begin{bmatrix} B_{dc}^{-1}\ \tilde{\mathbf{P}}_{inj} \\ 0 \end{bmatrix} \ . \tag{2.4}$$

The power injection vector $\mathbf{P}_{inj}$ can be written as

$$\mathbf{P}_{inj} = C_g\ \mathbf{P}_g + C_w\ \mathbf{P}_w - C_d\ \mathbf{P}_d \ , \tag{2.5}$$

where $\mathbf{P}_g \in \mathbb{R}^{n_g}$, $\mathbf{P}_w \in \mathbb{R}^{n_w}$ and $\mathbf{P}_d \in \mathbb{R}^{n_d}$ correspond to the generating power, the wind power in-feed and the demand power respectively. Matrices C_g,C_w and C_d are of appropriate dimensions, and their element (i, j) is "1" if generator j (respectively wind power/demand power) is connected to the bus i and zero otherwise.

2.1.3 Reserve Modeling

The reserves are the services traded in the market to materialize physically the required adjustments [39]. They are needed to balance generation-load mismatches, which may occur due to a difference between the actual wind power and its forecast value, or as an effect of a generator/load loss. Such power mismatch between generation-load will induce frequency deviations and activate the primary frequency controller (the active power reserves of the system).

Secondary frequency control or Automatic Frequency Control (AGC) is then activated and adjusts the production of the generators so as to compensate for the remaining frequency error and bring the tie-line power exchange back to the scheduled value. Specifically, the AGC output is distributed in a weighted way to certain generators.

Hence, in the new steady state value, the power set-point of these generators is changed by a certain percentage of the active power imbalance. In the current energy market, this percentage is the result of contracting agreements between producers and the TSO concerning the secondary frequency control reserves. The product of these weights with the worst case imbalance results in the amount of reserves that each generating unit should provide.

To take this into account in our formulation, we consider that a new steady state is reached, as an effect of the secondary frequency control action. This is a reasonable assumption, since the optimization process is carried out in hourly steps, and hence frequency deviation settles again to zero due to secondary frequency reserve deployment.

Therefore, we define $\mathbf{d}^{ds} \in \mathbb{R}^{n_g}$ and $\mathbf{d}^{us} \in \mathbb{R}^{n_g}$ be weighting vectors that the down and up-spinning respectively, distributing the excess and deficit of power among the generating units participating in the frequency control. The sum of their elements is one and if a generator is not contributing to secondary frequency control (AGC), the corresponding element in the $\mathbf{d}^{ds}$, $\mathbf{d}^{us}$ vectors is zero. The indices up and down are used to distinguish between the up and down-spinning reserves. Taking this into account, reserve representation equation (2.5) can be then written as

$$\mathbf{P}_{inj} = C_g \ (\mathbf{P}_g + \mathbf{R}) + C_w \ \mathbf{P}_w - C_d \ \mathbf{P}_d \quad . \tag{2.6}$$

Equation (2.6) implies that the power injection at every bus of the network is equal to the difference between the production and the demand, where the production is given by the sum of the outputs of the conventional units plus a power correction term $\mathbf{R}$ and the wind power output.

The power correction term $\mathbf{R} \in \mathbb{R}^{n_g}$ is related to the reserves provided by each generating unit. Following [52], we define $\mathbf{R}$ to be a linear function of the total generation-load mismatch, which is the difference between the wind power and its forecast value. Modeling the steady state behavior of this action

$$\mathbf{R} = -\mathbf{d}_t^{ds} \ |\Delta P_{w_t}|_+ \ + \mathbf{d}_t^{us} \ |\Delta P_{w_t}|_- \ , \tag{2.7}$$

where,

$$|\Delta P_{w_t}|_+ := \begin{cases} \Delta P_{w_t} & \text{if } \Delta P_{w_t} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.8a}$$

$$|\Delta P_{w_t}|_- := \begin{cases} \Delta P_{w_t} & \text{if } \Delta P_{w_t} \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.8b}$$

and

$$\Delta P_{w_t} = \mathbf{P}_{w_t} - \mathbf{P}^f_{w_t} \ . \tag{2.8c}$$

$\mathbf{P}_{w_t}$ denotes the actual wind power and $\mathbf{P}^f_{w_t}$ is the forecast value of the wind power. If $\Delta P_{w_t} \geq 0$, down-spinning reserves are provided, and the production of the generators is decreased accordingly, whereas in the opposite case the second term of (2.7) is active, providing up-spinning reserves. Introducing different distribution vectors for up-spinning and down-spinning reserves provides us with more degrees of freedom, thus avoiding feasibility issues.

Notice that $\mathbf{d}^{us}_t$ and $\mathbf{d}^{ds}_t$ are allowed have negative and positive elements respectively; the interpretation of having the elements of $\mathbf{d}^{us}_t$ being negative in case $\Delta P_{w_t} \leq 0$ is that the generators corresponding to the negative entries of the distribution vectors should provide up-spinning reserves and the rest would provide dwon-spinning. This will happen in case of network congestion. Network congestion occurs when a line or bus is flowing so much power that its quality of service takes down.

2.1.4 Wind Power Model

This section concentrates on the development of a wind power model that enables us to generate scenarios of the wind power error, while taking its temporal correlation into account. We assume that the wind power is the sum of the deterministic component, which is the available forecast, and a stochastic one, which models the error between the forecast and actual wind power.

Based on the work of [34, 41], we discretized the error between the forecast and the actual data to estimate the transition probability matrix, that enables us to generate various wind power error realizations. In this framework, we used normalized five-years hourly measured wind power data, both forecasts and actual values, for the total wind power infeed of Germany over the period 2006 - 2011. Following [41], obtaining a stochastic model directly in the wind power domain leads to a reduced number of states and to a lower order of the Markov Chain at equal power data resolution. The estimation quality of the stochastic model is positively influenced since in the power domain, a lower number of independent parameters is estimated from a given amount of recorded data. This method offers excellent fit for both the probability density function and the autocorrelation function of the generated wind power time series.

Figure 2.1 depicts the transition probability matrix using a Markov Chain with 41 states (wind power error). As noted in [41], the block triangular structure of the state transition matrix, suggests that the wind power error is strongly auto-correlated in time. Figure 2.2 shows the forecast("blue"), actual("red") wind power and 5000 wind power trajectories ("green"), based on different error realizations, generated by the Markov Chain-based model.

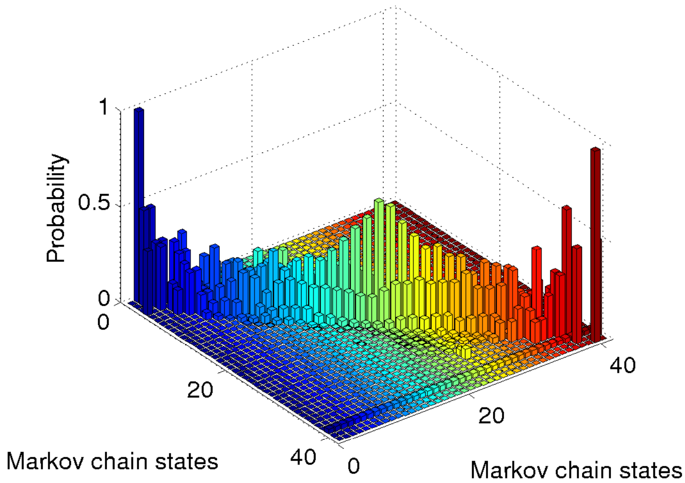

Figure 2.1. Transition probability matrix for the wind power error.

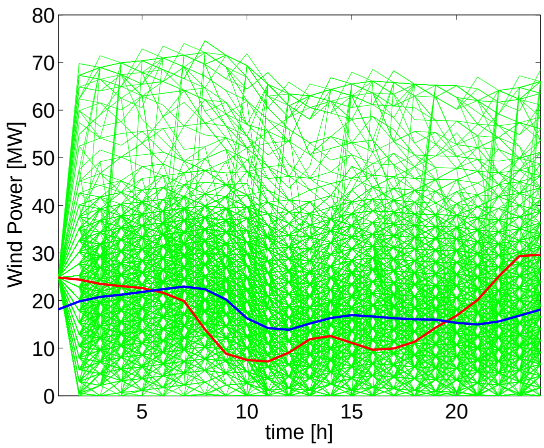

Figure 2.2. Forecast("blue"), actual("red") wind power and 5000 wind power trajectories ("green"), based on different error realizations, generated by the Markov Chain-based model.

2.2 Problem Formulation

Consider the power system comprising n_b buses, n_g generators, n_d loads and n_l lines and a single wind power in-feed $n_w = 1$ described in the previous section. $N_B := \{1, 2, 3, \cdots, n_b\}$ denote the bus indices, $N_G := \{1, 2, 3, \cdots, n_g\} \subseteq N_B$ the set of generator indices, $N_D := \{1, 2, 3, \cdots, n_d\} \subseteq N_B$ the set of load indices and $N_L \subseteq N_B \times N_B$ the set of line indices. Define the decision variables of the power system for each optimization step as follows

Decision Variables:

$$\begin{aligned} \mathbf{P}_{g,t} &\in \mathbb{R}^{n_g} \\ \mathbf{u}_t &\in \{0,1\}^{n_g} \\ \mathbf{z}_t &\in \mathbb{R}^{n_g} \\ \mathbf{d}_t^{ds} &\in \mathbb{R}^{n_g} \\ \mathbf{d}_t^{us} &\in \mathbb{R}^{n_g} \\ \mathbf{C}_t^{su} &\in \mathbb{R}^{n_g} \\ Q_t^{ds} &\in \mathbb{R} \\ Q_t^{us} &\in \mathbb{R} \end{aligned}$$

$\mathbf{P}_{g,t} = \{P_{G_{k,t}}\}_{k \in N_G}$ is a vector that contains the scheduled power output of each generator in period t. $\mathbf{u}_t = \{u_{k,t}\}_{k \in N_G}$ is a binary vector which encodes the "on-off" statues of each generating unit in period t. $\mathbf{z}_t = \{z_{k,t}\}_{k \in N_G}$ is a vector which contains auxiliary variables needed to model the minimum up and down times of each generator in period t. $\mathbf{d}_t^{ds} = \{d_{k,t}^{ds}\}_{k \in N_G}$ and $\mathbf{d}_t^{us} = \{d_{k,t}^{us}\}_{k \in N_G}$ are weighting vectors that distribute the positive and negative wind power mismatch among the generating units participating in the frequency control. If a generator is not contributing to frequency control, the corresponding element of these vectors will be zero. $\mathbf{C}_t^{su} = \{C_{k,t}^{su}\}_{k \in N_G}$ is the startup cost vector in period t and the Q_t^{ds} and Q_t^{us} serve as upper bounds of the down and up-spinning reserve cost in period t. All decision variables are defined for every time instance $t \in \{1, 2, 3, \cdots, n_t\}$ where $n_t = 24$ corresponding to a day-ahead optimization problem.

Moreover, let $\mathbf{c}_2 = \{c_{2,k}\}_{k \in N_G}$, $\mathbf{c}_1 = \{c_{1,k}\}_{k \in N_G}$, $\mathbf{c}_0 = \{c_{0,k}\}_{k \in N_G} \in \mathbb{R}^{n_g}$ are vectors contain for each generating unit cost of every Mega-Watt (MW) of production per hour and $[\mathbf{c}_2]$ denote diagonal matrix with vector $\mathbf{c}_2$ on the diagonal. For each step t of the optimization problem, define the vector of all decision variables except the binary ones to be

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{P}_{g,t}^{\mathrm{T}} & \mathbf{z}_t^{\mathrm{T}} & \mathbf{d}_t^{ds\,\mathrm{T}} & \mathbf{d}_t^{us\,\mathrm{T}} & \mathbf{C}_t^{su\,\mathrm{T}} & Q_t^{ds} & Q_t^{us} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{5n_g+2} ,$$

and binary vector $\mathbf{u}_t \in \{0,1\}_{k \in N_G}$. Following [48], a quadratic form for the production cost is considered, whereas motivated by [39] the reserve cost is considered to be linear. By considering the decision variables above, we can formulate the following optimization problem.

$$\min_{\mathbf{x}_t,\mathbf{u}_t} \sum_{t=1}^{n_t} \mathbf{P}_{g,t}^T[\mathbf{c}_2]\mathbf{P}_{g,t} + \mathbf{c}_1^T\mathbf{P}_{g,t} + \mathbf{1}_{1\times n_g}(\mathbf{c}_0 + \mathbf{C}_t^{su}) + Q_t^{ds} + Q_t^{us} \tag{2.9}$$

Subject to:

1. Up-spinning, down-spinning reserve and startup cost constraints: For all $t = 1, 2, \ldots, n_t$:

$$\mathbf{C}_t^{su} \geq 0 \tag{2.10a}$$

$$Q_t^{ds} \geq 0 \tag{2.10b}$$

$$Q_t^{us} \geq 0\ . \tag{2.10c}$$

   This constraint implies the startup cost, down and up-spinning reserve and production cost should be positive.

2. Power balance constraint in power network: For all $t = 1, 2, \ldots, n_t$:

$$\mathbf{1}_{1\times n_b}(C_g\ \mathbf{P}_g + C_w\ \mathbf{P}_w^f - C_d\ \mathbf{P}_d) = 0 \tag{2.11}$$

   where $\mathbf{P}_d = \{P_{D_k}\}_{k\in N_D}$ is a vector which contains demands real power. $\mathbf{P}_{w_t}^f \in \mathbb{R}$ denotes the wind power forecast. This constraint encodes the fact that the power balance in the network should be always satisfied when $\mathbf{P}_w = \mathbf{P}_w^f$. In other words, the total generation dispatch of the conventional units and the total wind power production, should be equal to the total load of the system.

3. Generation power units limits: For all $k \in N_G$ and $t = 1, 2, \ldots, n_t$:

$$[\mathbf{P}_g^{\min}]\ \mathbf{u}_t \leq \mathbf{P}_{g,t} \leq [\mathbf{P}_g^{\max}]\ \mathbf{u}_t \tag{2.12}$$

   where $\mathbf{P}_g^{\min} = \{P_{G_k}^{\min}\}_{k\in N_G}$, $\mathbf{P}_g^{\max} = \{P_{G_k}^{\max}\}_{k\in N_G} \in \mathbb{R}^{n_g}$ are vectors of minimum and maximum generating power of each unit respectively and $[\mathbf{P}_g^{\min}]$ and $[\mathbf{P}_g^{\max}]$ are diagonal matrices with vectors $\mathbf{P}_g^{\min}$ and $\mathbf{P}_g^{\max}$ on the diagonal respectively.

4. Transmission capacity constraints: For all $t = 1, \ldots, n_t$

$$-\mathbf{P}_{line} \leq B_f \begin{bmatrix} B_{dc}^{-1}\ \tilde{\mathbf{P}}_{inj,t} \\ 0 \end{bmatrix} \leq \mathbf{P}_{line} \tag{2.13}$$

   where $\mathbf{P}_{line} \in \mathbb{R}^{n_l}$ is the active power capacity limit of each line, $B_f \in \mathbb{R}^{n_l\times n_b}$ and

$$\tilde{\mathbf{P}}_{inj,t} = \Big[C_g\ \mathbf{P}_g + C_w\ \mathbf{P}_w^f - C_d\ \mathbf{P}_d\Big]_{n_b-1}$$

also $[.]_{n_b-1}$ denotes the first $n_b - 1$ rows of the quantity inside the brackets.

5. Probabilistic constraints:

$$\mathbb{P}\Bigg(\quad \{\Delta P_{w_t}\}_{t=1}^{n_t} \quad \Bigg| \quad -\mathbf{P}_{line} \leq \begin{bmatrix} B_{dc}^{-1} \; \tilde{\mathbf{P}}_{inj,t}^{s} \\ 0 \end{bmatrix} \leq \mathbf{P}_{line} \ , \tag{2.14a}$$

$$[\mathbf{P}_g^{\min}] \ \mathbf{u}_t \leq \mathbf{P}_{g,t} + \mathbf{R}_t \leq [\mathbf{P}_g^{\max}] \ \mathbf{u}_t \ , \tag{2.14b}$$

$$\left(\mathbf{c}_{ds}^{T} \mathbf{d}_t^{ds} \Delta P_{w_t}\right) \leq Q_t^{ds} \ , \tag{2.14c}$$

$$\left(\mathbf{c}_{us}^{T} \mathbf{d}_t^{us} \Delta P_{w_t}\right) \leq Q_t^{us} \qquad \Bigg) \geq 1 - \varepsilon \tag{2.14d}$$

where $\mathbf{c}_{ds} = \{c_{k,ds}\}_{k \in N_G}$, $\mathbf{c}_{us} = \{c_{k,us}\}_{k \in N_G} \in \mathbb{R}^{n_g}$ are vectors and contain for each generating unit cost of down and up-spinning, respectively, and

$$\tilde{\mathbf{P}}_{inj,t}^{s} = [C_g \ (\mathbf{P}_g + \mathbf{R}_t) + C_w \ \mathbf{P}_w - C_d \ \mathbf{P}_d]_{n_b-1}$$

$$\mathbf{R}_t = -\mathbf{d}_t^{ds} \left|\Delta P_{w_t}\right|_{+} + \mathbf{d}_t^{us} \left|\Delta P_{w_t}\right|_{-} \ .$$

The probabilistic constraints imply that all constraints of the power system should be satisfied for all wind power realizations except for a set of probability at most ε. The first constraint inside the probability encodes the standard transmission capacity constraints, whereas the second one provides guarantees that the scheduled generation dispatch plus the reserve contribution $\mathbf{R}_t$ will not result in a new operating point outside the generation capacity limits. The last two constraints give the upper bound for the down-spinning and up-spinning reserve costs, respectively.

6. Weighting vectors constraints: For all $k \in N_G$ and $t = 1, 2, \ldots, n_t$:

$$\mathbf{1}_{1\times n_g}(\mathbf{d}_t^{ds}) = 1, \qquad \mathbf{d}_t^{ds} \geq 0 \tag{2.15a}$$

$$\mathbf{1}_{1\times n_g}(\mathbf{d}_t^{us}) = -1, \quad \mathbf{d}_t^{us} \leq 0 \tag{2.15b}$$

These constraints dictate that the sum of the weighting vectors elements of the mismatch wind power (deviation of every wind scenario and forecast wind power) should be equal to 1 and -1, and every element of them should be positive and negative in case of the down and up-spinning, respectively.

7. Startup cost constraint for every $t = 1, 2, \ldots, n_t$:

$$[\lambda^{su}]\ (\mathbf{u}_t - \mathbf{u}_{t-1}) \leq \mathbf{C}_t^{su} \tag{2.16}$$

where $\lambda^{su} = \{\lambda_k^{su}\}_{k \in N_G}$ is a vector containing the startup costs of each generating units and $[\lambda^{su}]$ is a diagonal matrix with vector λ^{su} on the diagonal. This constraints implies that the decision vector $\mathbf{C}_t^{su}$ will be always zero unless the corresponding generator changes status from "off" to "on" within two consecutive periods.

8. Ramping constraint of each generators: For all $t = 1, 2, \ldots, n_t$:

$$-\mathbf{P}_g^{down} \leq \mathbf{P}_{g,t} - \mathbf{P}_{g,t-1} \leq \mathbf{P}_g^{up} \tag{2.17}$$

where $\mathbf{P}_g^{down}$, $\mathbf{P}_g^{up} \in \mathbb{R}^{n_g}$ are vectors that denote the ramping limits of each unit when generators decrease and increase within two consecutive periods respectively.

9. Ramping time constraint of each generators: For all $t = 1, 2, \ldots, n_t$:

$$0 \leq \mathbf{z}_t \leq 1 \tag{2.18a}$$

$$(u_{k,t} - u_{k,t-1}) \leq z_{k,t} \tag{2.18b}$$

$$\sum_{i\ =\ t-\Delta t_{up}+1}^{t} z_{k,i} \leq u_{k,i} \quad , \qquad t \geq \Delta t_{up} \tag{2.18c}$$

$$\sum_{i\ =\ t+1}^{t+\Delta t_{down}} z_{k,i} \leq 1 - u_{k,i} \quad , \quad t \leq n_t - \Delta t_{down} \tag{2.18d}$$

where Δt_{up} , $\Delta t_{down} \in \mathbb{R}^{\geq 0}$ denote the minimum time a unit needs to change status. The first constraint implies that the auxiliary variables should be positive, whereas the second and last one encode minimum up and down times for the generating units to change status [42].

The proposed optimization problem is a multi-stage, chance constrained mixed-integer quadratic program, whose stages are coupled by the binary variables and the ramping constraints (2.16), (2.17) and (2.18).

2.3 Dealing with the Chance Constraint

The chance constrained problem ($\mathrm{CCP}_1(\epsilon)$) can be rewritten in a more compact form as

$\mathbf{CCP}_1(\epsilon)$ **:** $\quad \min_{\boldsymbol{x},\boldsymbol{u}} \boldsymbol{J}(\boldsymbol{x}) \qquad$ subject to:

$$\mathbb{P}\left(\ \delta \in \Delta \mid \boldsymbol{A}(\delta)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}(\delta) \geq 0\ \right) \geq 1 - \epsilon\ ,$$

	n_D	n_s	number of constraints
Scenario Approach	912	1,8160	41,840,640

Table 2.1. Number of decision variables and number of corresponding scenarios and constraints based on [18]. ($\beta = 10^{-4}$ and $\epsilon = 0.1$)

where $\boldsymbol{x} \in \mathbb{R}^{n_x}$, with $n_x = (5n_g + 2)n_t$ is a vector including the decision variables, $\boldsymbol{u} \in \{0,1\}^{n_u}$ with $n_u = n_g n_t$ is a vector of binary variables, and $\delta \in \Delta \subset \mathbb{R}^{n_t}$ is the vector of uncertain parameters that is the wind power for every hour $t = 1, \cdots, n_t$. Variables $\boldsymbol{x}$, $\boldsymbol{u}$ and δ consist "stacked" versions of $\{\mathbf{x}_t\}_{t=1}^{nt}$, $\{\mathbf{u}_t\}_{t=1}^{nt}$ and $\{\Delta P_{wt}\}_{t=1}^{nt}$, respectively. $\boldsymbol{J}(\boldsymbol{x}) \in \mathbb{R}$ is quadratic in $\boldsymbol{x}$, and for each $\delta \in \Delta$, $\boldsymbol{A}(\delta)$, $\boldsymbol{B}$, and $\boldsymbol{C}(\delta)$ are of appropriate dimensions.

Note that all equality constraints (2.15) can be represented by double-sided inequalities (see Appendix A for more details on the compact form). It is assumed that Δ is endowed with a σ-algebra Ω, and that $\mathbb{P}$ is a probability measure defined over Ω. For all $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and $\boldsymbol{u} \in \{0,1\}^{n_u}$, the constraint functions are assumed to be measurable with respect to Ω and the Borel σ-algebra over $\mathbb{R}^{n_t}$.

In order to solve $\mathrm{CCP}_1(\epsilon)$ we have to transform the chance constrained problem to a tractable, but in some sense equivalent problem. To avoid introducing arbitrary assumptions on $\mathbb{P}$ we follow a randomized methodology inspired by the "scenario approach" and developed in [33].

The main idea of scenario approach [16, 17] is to consider only a finite number of instances (scenarios) of the uncertain parameters, and then solve the optimization problem where only the (hard) constraints corresponding to the extracted scenarios are considered. Article [18] provides a lower bound for the number of scenarios n_S that should be extracted to obtain the desired probabilistic guarantees ϵ with confidence at least $1 - \beta$ when the objective function and the constraint function are both convex in the decision variables. The number of samples that one needs to generate is

$$n_s \geq \frac{2}{\epsilon}(n_D + \ln \frac{1}{\beta}) \tag{2.19}$$

where $\epsilon \in (0,1)$ is the violation parameter, $\beta \in (0,1)$ is a confidence level, and n_D is the number of decision variables. Due to the binary vector $\boldsymbol{u}$, following such an approach would not allow us to provide any probabilistic guarantees, since convexity with respect to the decision variables is required. Moreover, even if this were possible, the number of scenarios that one needs to generate grows linearly with respect to the decision variables [18], thus hampering the applicability of the method to large scale systems. Table 2.1 shows the number of decision variables, scenarios and the resulting number of constraints for the specific problem when $\epsilon = 0.1$ and $\beta = 10^{-4}$.

To solve this problem we exploit the recent results of [33], and follow a three step procedure.

Step 1: Let $B(v) = \times_{j=1}^{n_t}[v_j^{\min}, v_j^{\max}]$ be a hyper-rectangle parametrized by

$$v = (v^{\min}, v^{\max}) \in \mathbb{R}^{2n_t}$$

where

$$v^{\min} = (v_1^{\min}, v_2^{\min}, \cdots, v_{n_t}^{\min}) \in \mathbb{R}^{n_t}$$
$$v^{\max} = (v_1^{\max}, v_2^{\max}, \cdots, v_{n_t}^{\max}) \in \mathbb{R}^{n_t} \ .$$

Consider now the following chance constrained optimization problem

$$\textbf{CCP}_2(\epsilon) \textbf{ :} \qquad \min_{v \in \mathbb{R}^{2n_t}} \sum_{i=1}^{n_t} (v_i^{\max} - v_i^{\min}) \quad \text{subject to:}$$

$$\mathbb{P}\Big(\ \delta \in \Delta \mid \delta \in [v^{\min}, v^{\max}] \ \Big) \geq 1 - \epsilon$$

Problem $\mathrm{CCP}_2(\epsilon)$ provides a parametrization v so that $B(v)$ encloses at least an $1-\epsilon$ fraction of the probability mass of the uncertainty vector δ. In general $B(v)$ could be any convex set with convex volume, and we could minimize the volume of $B(v)$ which encloses δ with probability at least $1-\epsilon$. In our case, where $B(v)$ is a hyper-rectangle, we minimize the sum of the lengths of the internals $[v_i^{\min}, v_i^{\max}]$ defining it since the volume minimization problem would not a convex problem.
Step 2: Problem $\mathrm{CCP}_2(\epsilon)$ is a convex problem by construction and we can apply the standard scenario approach to obtain a solution. Let $\beta = 10^{-4}$ and $\epsilon = 0.1$ and choose n_s that satisfies (2.19), where $n_D = 2n_t$. Consider now the scenario program that corresponds to $\mathrm{CCP}_2(\epsilon)$,

$$\textbf{RCP}_{2,n_s} \textbf{ :} \qquad \min_{v \in \mathbb{R}^{2n_t}} \sum_{i=1}^{n_t} (v_i^{\max} - v_i^{\min}) \quad \text{subject to:}$$

$$\delta^{(k)} \in [v^{\min}, v^{\max}] \ , \quad k = 1, 2, \cdots, n_s \ .$$

According [16,17], with confidence at least $1-\beta$, the optimal solution v^* of RCP_{2,n_s} is feasible for the chance constrained problem $\mathrm{CCP}_2(\epsilon)$.
Step 3: Finally, we propose the following robust counterpart of problem $\mathrm{CCP}_1(\epsilon)$

$$\textbf{RCP}_{1,n_s} \textbf{ :} \qquad \min_{\boldsymbol{x},\boldsymbol{u}} \boldsymbol{J}(\boldsymbol{x}) \qquad \text{subject to:}$$

$$\boldsymbol{A}(\delta)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}(\delta) \geq 0 \ \Big| \ \forall \delta \in \{B(v^*) \cap \Delta\} \ . \tag{2.20}$$

Note that this is not a chance constraints, and we require the constraints to be satisfied for all values of the uncertainty inside $B(v^*) \cap \Delta$. Therefore, RCP_{1,n_s} is a robust mixed-integer quadratic problem. As shown in Proposition 1 of [33], with confidence at least $1-\beta$, the solution of RCP_{1,n_s} is feasible for the initial chance constrained problem $\mathrm{CCP}_1(\epsilon)$.

2.4 Tractable Robust Reformulation

The procedure described in the previous section leads to a tractable reformulation of the chance constraints problem RCP_{1,n_s}. To ensure this, we follow the approach of [8, 10], whose authors propose a tractable reformulation of robust convex programs, that was subsequently extended in [31] to capture robust mixed-integer problems as well.

For a certain class of uncertainty sets (e.g. hyper-rectangular) it is shown in [10] that problems of the form of RCP_{1,n_s} are tractable (i.e. they do not require to enforce constraints for all 2^{n_t} vertices of the uncertainty set), and remain in the same class as the original problems, e.g. robust mixed-integer programs remain mixed-integer programs. This is achieved under the assumption that the constraint functions are concave and homogeneous with respect to the uncertainty vector, and by introducing some additional decision variables and constraints. $A(\delta)$ does not satisfy the concavity assumption due to (2.8). However, the particular structure of the constraints allows us to apply the following methodology. Specifically, since the coupling constraints are deterministic and the uncertainty and decision variables of each stage $j = 1, \ldots, n_t$ affect only that stage, it suffices to split (2.20) in two robust problems. One requiring the constraints to be satisfied for all values of the uncertainty inside the part of the hyper-rectangle $B(v^*)$ consisting of all uncertainty vectors with all their elements being positive, and one requiring constraint satisfaction for all uncertainty values inside the part of $B(v^*)$ constructed by uncertainty vectors with all their elements being negative. That way the uncertainty function $\boldsymbol{A}(\delta), \boldsymbol{C}(\delta)$ are linear with respect to δ for each sub-problem, and then we could write $\boldsymbol{A}(\alpha\delta) = \alpha\boldsymbol{A}(\delta),\ \boldsymbol{C}(\alpha\delta) = \alpha\boldsymbol{C}(\delta)$ for any $\alpha \in \mathbb{R}$.

In the sequel, to avoid introducing additional notation, we discuss the algorithm of [10] to a single robust constraint as in (2.20), but when applied to the simulation example the procedure described above (with different robust constraints according to the sign of the uncertainty) is adopted. Note that we present the algorithm of [10] as a general purpose methodology to deal with robust problems of this type. Due to the particular structure a vertex enumeration scheme can be also applied without an exponential dependence on the dimension of the uncertainty. The reason is that the uncertainty affecting the constraints of each time-step is scalar and does not influence the constraints of the other time-steps. Therefore, for each $j = 1, \ldots, n_t$ it suffices to enforce the constraints only for the extreme values of each sub-interval.

Consider $\Delta \in \mathbb{R}^{n_t}$, let $e_j \in \mathbb{R}^{n_t}$ be a unit basis vector whose j-th element is "1" for all $j = 1, \cdots, n_t$, and $v^o = 0.5(v^{\min} + v^{\max}) \in \mathbb{R}^{n_t}$ be a vector whose elements are the middle points of each interval $[v_j^{\min}\ ,\ v_j^{\max}]$, for all $j = 1, \cdots, n_t$. Let n_r denotes the number of rows of $\boldsymbol{A}(\delta)$ and $A_i(\cdot)$ corresponds to i_{th} row of $\boldsymbol{A}(\delta)$ and let $\boldsymbol{Y} = \{y_i\} \in \mathbb{R}^{n_r}$, $\boldsymbol{Q} = \{q_{ij}\} \in \mathbb{R}^{n_r \times n_t}$

be additional decision variables. We are now in a position to define an optimization problem which provides a tractable reformulation of RCP_{1,n_s}.

TRCP$_1$ **:** $\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{Y},\boldsymbol{Q}} \quad \boldsymbol{J}(\boldsymbol{x})$ subject to:

1. Upper bound for the maximum worst-case value of δ: For all $i = 1, 2, \cdots, n_r$ and $j = 1, 2, \cdots, n_t$:

$$A_i(e_j {e_j}^{\mathrm{T}} (v^{\max} - v^o))\boldsymbol{x} + C_i(e_j {e_j}^{\mathrm{T}} (v^{\max} - v^o)) \geq q_{ij}$$

2. Upper bound for the minimum worst-case value of δ: For all $i = 1, 2, \cdots, n_r$ and $j = 1, 2, \cdots, n_t$:

$$A_i(e_j {e_j}^{\mathrm{T}} (v^{\min} - v^o))\boldsymbol{x} + C_i(e_j {e_j}^{\mathrm{T}} (v^{\min} - v^o)) \geq q_{ij} \ .$$

3. Bound for sum of uncertain elements in each row: For all $i = 1, \cdots, n_r$

$$\sum_{j=1}^{n_t} q_{ij} \geq y_i \ .$$

4. Modified version of (2.20):

$$\boldsymbol{A}(v^o)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}(v^o) + \boldsymbol{Y} \geq 0$$

We discuss now the interpretation of the structure of TRCP_1. Note that we seek to transform RCP_{1,n_s} in a tractable form. Its constraints can be equivalently written as

$$\boldsymbol{A}(v^o + \delta)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}(v^o + \delta) \geq 0$$

for all $\delta \in [v^{\min} - v^o \ , \ v^{\max} - v^o]_{j=1}^{n_t}$. We have for any δ

$$\boldsymbol{A}(v^o + \delta) \geq \boldsymbol{A}(v^o) + \boldsymbol{A}(\delta) \tag{2.21}$$

$$\boldsymbol{C}(v^o + \delta) \geq \boldsymbol{C}(v^o) + \boldsymbol{C}(\delta) \ . \tag{2.22}$$

For this step to be valid we should split the hyper-rectangle $B(v^*)$ in two regions. Therefore, it suffices to require that for all admissible δ

$$\boldsymbol{A}(v^o)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}(v^o) + \boldsymbol{A}(\delta)\boldsymbol{x} + \boldsymbol{C}(\delta) \geq 0 \ . \tag{2.23}$$

To achieve this, we need to bound the term $\boldsymbol{A}(\delta)\boldsymbol{x} + \boldsymbol{C}(\delta)$. Constraints 1 and 2, impose a bound q_{ij} to the maximum and minimum worst-case values. Consider first the worst case perturbation vectors, for all $j = 1, 2, \cdots, n_t$:

$$e_j {e_j}^{\mathrm{T}} (v^{\max} - v^o) \in \mathbb{R}^{n_t}$$

$$e_j {e_j}^{\mathrm{T}} (v^{\min} - v^o) \in \mathbb{R}^{n_t} \ .$$

Note that these vectors have all their elements zero and in the j-th position include the maximum (respectively minimum) deviation of this element from the middle point v^o. Having now y_i for each $i = 1, \cdots, n_r$, as set by constraint 3, and considering the worst case superposition of the perturbation vectors, for each $i = 1, \cdots, n_r$, we have

$$\boldsymbol{A}_i(\delta)\boldsymbol{x} + \boldsymbol{C}_i(\delta) \geq \sum_{j=1}^{n_t} q_{ij} \geq y_i \ .$$

The last equation implies that $\boldsymbol{A}(\delta)\boldsymbol{x} + \boldsymbol{C}(\delta) \geq \boldsymbol{Y}$, and together with (2.23) justifies constraint 4.

Problem TRCP_1 is a mixed-integer quadratic problem, and compared to RCP_{1,n_s} has $(n_t + 1)n_r$ additional decision variables and $(2n_t + 1)n_r$ additional constraints. Due to the equivalence between TRCP_1 and RCP_{1,n_s}, the optimal solution of TRCP_1 is feasible for $\mathrm{CCP}_1(\epsilon)$ with confidence at least $1 - \beta$. The proposed procedure is summarized in Algorithm 1.

Algorithm 1 Proposed Methodology

Stochastic unit commitment and reserve scheduling.

Set $u_0 \in \{0, 1\}^{n_g}$ ▷ initial status of the generating units.

Fix $\epsilon \in (0, 1)$, $\beta \in (0, 1)$.

Define $B(v) = \times_{j=1}^{n_t}[v_j^{\min}, v_j^{\max}]$ and extract n_s scenarios $\delta^1, \cdots, \delta^{n_s}$ where $n_s \geq \frac{2}{\epsilon}(2n_t + \ln \frac{1}{\beta})$.

Solve RCP_{2,n_s} and determine v^* ▷ v^* is feasible for $\mathrm{CCP}_2(\epsilon)$ with probability at least $1 - \beta$.

Solve TRCP_1 ▷ TRCP_1 is equivalent to RCP_{1,n_s} and its solution is feasible for $\mathrm{CCP}_1(\epsilon)$ with probability $1 - \beta$.

Note that in the case where (2.14) is not a joint constraint, but we have different chance constraints with violation levels ϵ_t per stage $t = 1, \cdots, n_t$ (or also per uncertainty element), the proposed methodology is still applicable. The difference is that we should apply the procedure of the proposed method for every individual chance constraint; this will give rise to a family of hyper-rectangular sets each of them corresponding to a different ϵ_t,β_t. The resulting solution would be feasible for the problem with the separate chance constraints with confidence at least $1 - \sum_{t=1}^{n_t} \beta_t$.

2.5 Simulation Study

To illustrate the performance of our algorithm we compare it against a hybrid methodology, where the unit commitment and reserve scheduling problems are solved separately. We start by formulating the deterministic variant of the unit commitment problem, where the wind power is assumed to be equal to its forecast ($\mathbf{P}_w = \mathbf{P}_w^f$). This implies that no reserves are needed ($\mathbf{P}_w = \mathbf{P}_w^f$ leads to $\mathbf{R} = 0$). Therefore, we need to minimize

$$\sum_{t=1}^{n_t} \mathbf{P}_{g,t}^T[\mathbf{c}_2]\mathbf{P}_{g,t} + \mathbf{c}_1^T\mathbf{P}_{g,t} + \mathbf{1}_{1\times n_g}(\mathbf{c}_0 + \mathbf{C}_t^{su}) \tag{2.24}$$

in place of (2.9), with respect to the binary vector $\mathbf{u}_t = \{0,1\}^{n_g}$ and $\mathbf{x}_{1,t} = \begin{bmatrix}\mathbf{P}_{g,t}^{\mathrm{T}} & \mathbf{z}_t^{\mathrm{T}} & \mathbf{C}_t^{su\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{3n_g}$, subject to (2.10), (2.11), (2.12), (2.13), (2.16), (2.17) and (2.18) with $\mathbf{P}_w = \mathbf{P}_w^f$ for all $t = 1, 2, \cdots, n_t$. We then fix the "on-off" status of the generating units (and also $\mathbf{C}_t^{su}$, $\mathbf{z}_t$) to the binary vector computed by the deterministic unit commitment program, and formulate a stochastic reserve scheduling problem. This requires solving (2.9), (2.10), (2.11), (2.12), (2.13), (2.14), (2.15), (2.16), (2.17) and (2.18) where minimization is now carried out with respect to

$$\mathbf{x}_{2,t} = \begin{bmatrix}\mathbf{P}_{g,t}^{\mathrm{T}} & \mathbf{d}_t^{ds\mathrm{T}} & \mathbf{d}_t^{us\mathrm{T}} & Q_t^{ds\mathrm{T}} & Q_t^{us\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{3n_g+2} .$$

In order to deal with the resulting chance constrained problem we follow the standard scenario approach [18], and substitute the chance constraint with n_s hard constraints, where n_s is now chosen according to

$$n_s \geq \frac{2}{\epsilon}((3n_g + 2)n_t + ln\frac{1}{\beta}) . \tag{2.25}$$

The term $(3n_g + 2)n_t$ in (2.25) denotes the number of decision variables in the chance constrained program. The solution to this problem provides a minimum cost dispatch and reserves ($\mathbf{P}_{g,t}$, $\mathbf{d}_t^{ds}$, $\mathbf{d}_t^{ds}$) for the generators that were committed at the previous step of the algorithm, and the optimal way that these reserves should be distributed among the generating units ($\mathbf{d}_t^{ds}$, $\mathbf{d}_t^{ds}$). Given the outcome of the unit commitment problem, the resulting solution is robust, in a probabilistic sense, with respect to deviations of the wind power from its forecast. The basic steps of this procedure, which we shall refer to as "benchmark approach", are summarized in Algorithm 2.

Note that in the benchmark approach we use the standard scenario approach instead of the proposed methodology. This is motivated by the fact that our objective is to demonstrate the potential advantage of using Algorithm 1 against an algorithm that does not support chance constrained mixed-integer problems.

Algorithm 2 Benchmark approach

Deterministic unit commitment.

1: Set $u_0 \in \{0,1\}^{n_g}$ ▷ initial status of the generating units.

2: Define $\mathbf{x}_{1,t} = \begin{bmatrix}\mathbf{P}_{g,t}^{\mathrm{T}} & \mathbf{z}_t^{\mathrm{T}} & \mathbf{C}_t^{su\,\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{3n_g}$ and set $\mathbf{P}_w = \mathbf{P}_w^f$, $\mathbf{x}_t \leftarrow \mathbf{x}_{1,t}$ for all $t = 1, 2, \cdots, n_t$.

3: Minimize (2.24), subject to (2.10), (2.11), (2.12), (2.13), (2.16), (2.17) and (2.18).

Stochastic reserve scheduling.

4: Set $\{\mathbf{u}_t\}_{t=1}^{n_g n_t}$ to the value determined in Step 3. Fix $\epsilon \in (0,1)$, $\beta \in (0,1)$.

5: Define $\mathbf{x}_{2,t} = \begin{bmatrix}\mathbf{P}_{g,t}^{\mathrm{T}} & \mathbf{d}_t^{ds\,\mathrm{T}} & \mathbf{d}_t^{us\,\mathrm{T}} & Q_t^{ds\,\mathrm{T}} & Q_t^{us\,\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{3n_g+2}$ and set $\mathbf{x}_t \leftarrow \mathbf{x}_{2,t}$ for all $t = 1, 2, \cdots, n_t$.

6: Extract n_s scenarios $\delta^1, \cdots, \delta^{n_s}$ with n_s satisfying (2.25).

7: Solve the scenario program that corresponds to (2.9), (2.10), (2.11), (2.12), (2.13), (2.14), (2.15), (2.16), (2.17) and (2.18). ▷ the obtained solution is ϵ-feasible with probability $1 - \beta$.

To compare the proposed approach with the benchmark one, we carry out Monte Carlo simulations repeating the scheduling procedure for 30 different days. The forecast and actual wind power values, (required to solve RCP_{2,n_s}), for these days are retrieved from a data-set corresponding to the aggregated wind power production of Germany over the period 2006-2011. More details regarding the wind power model can be found in Section 2.1.4. All optimization problems were solved using the solver CPLEX [20] via the MATLAB interface YALMIP [32]. Algorithm 1 and 2 are applied to the IEEE 30-bus network [2] (see Figure 2.7 the circuit of this power system) with a wind power generator located to bus 22; numerical data for the reserve, start-up and production cost vectors are shown in Table 2.2, whereas $\mathbf{P}_g^{up} = \mathbf{P}_g^{down} = \mathbf{P}_g^{\max}/3$ and the elements of the minimum up and down time vectors $(\Delta t_{up}, \Delta t_{down})$ corresponding to the first two generators were chosen to be 2 hours. The remaining data are retrieved from library of MatPower which is the commercial software for solving power flow and optimal power flow problems. It is intended as a simulation tool for researchers and educators that is easy to use and modify [57]. When attempting to solve the problem using Algorithm 2 we faced memory problems due to the high number of scenarios as given by (2.25). To solve this issue, we removed the ramping constraints (2.17), (2.18) coupling the optimization stages. The linear dependence of the constraints on the uncertainty elements, together with the fact that the stages in Algorithm 2 are decoupled (since the binary variables are fixed) allows us to use for every stage only the minimum and maximum value of the scenarios extracted according to (2.25). The resulting problem is not be computationally expensive, and hence solvable by existing tools. We will reconsider constraints (2.18), (2.17) later on and repeat our

n_g	c_{ds}	c_{us}	c_{su}	c_0	c_1	c_2
1	0.5	0.5	3	0	2	0.3843
2	0.4	0.4	4	0	2	2.5
3	0.5	0.5	1	0	4	0.1
4	0.1	0.1	1.5	0	4	0.1
5	0.5	0.5	1	0	4	0.1
6	0.1	0.1	1.5	0	4	0.1

Table 2.2. Down-spinning, Up-spinning Reserve, Start-Up and Production Cost for each generating unit ($\times$ 1000$).

analysis only for the proposed method (Algorithm 1).

Figure 2.3 shows for one day of the simulated data the total scheduled cost (solid lines) as the sum of the production, reserve and start-up costs, and the production cost (dashed lines). The cost pattern follows the load profile that was employed. The "blue" lines correspond to the proposed approach (Algorithm 1), whereas the "red" lines correspond to the benchmark approach (Algorithm 2). The proposed algorithm leads to lower total cost, while the production cost is similar for both methods. The improvement in terms of cost is due to the scheduling flexibility offered by the proposed algorithm, where the unit commitment is solved together with the reserve scheduling problem, allowing us to identify better unit commitment schedules.

The unit commitment sequences generated by each method are shown in Figure 2.4, where the presence of a "blue circle" or "red square" at some time instance indicates that the corresponding generator is committed at that hour by Algorithm 1 or 2, respectively.

We repeated the procedure outlined by Algorithm 1 and 2 for 30 days of our data-set, corresponding to different forecast and actual wind power values. Figure 2.5 shows for each day the relative cost difference between the cost generated by Algorithm 2 and the cost generated by Algorithm 1, normalized with the cost of Algorithm 2. For all days this difference is always positive and a maximum improvement of 1.29% is encountered, indicating that the proposed approach leads to always lower cost compared to the benchmark approach.

Figure 2.6 shows the total cost for every day, obtained after applying Algorithm 1 while considering the ramping constraints (2.16),(2.17) and (2.18) of the generating units.

Notice that in contrast to the standard scenario approach, the proposed methodology enables us to deal with problems of higher dimension with low computational cost; due the robust problem involved at Step 3 of our method, the resulting solution is not necessarily less conservative.

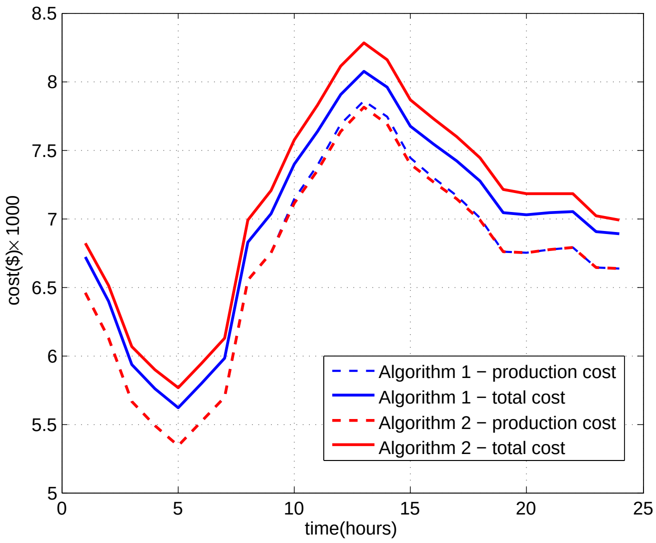

Figure 2.3. Total scheduled cost (solid line) as the sum of the production, reserve and start-up costs, and production cost (dashed line) for one day of the simulated data. The "blue" lines correspond to the proposed approach (Algorithm 1), whereas the "red" lines correspond to the benchmark approach (Algorithm 2).

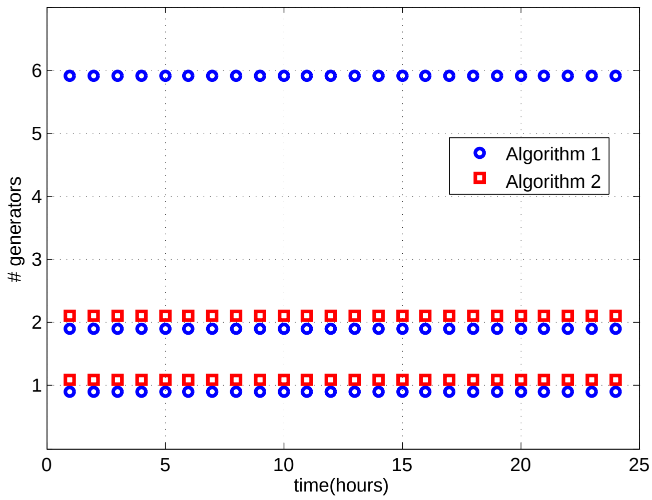

Figure 2.4. Unit commitment sequences generated by each algorithm for the set-up of Figure 2.3. The presence of a "blue circle" or "red square" indicates that a generator is committed by Algorithm 1 and 2, respectively.

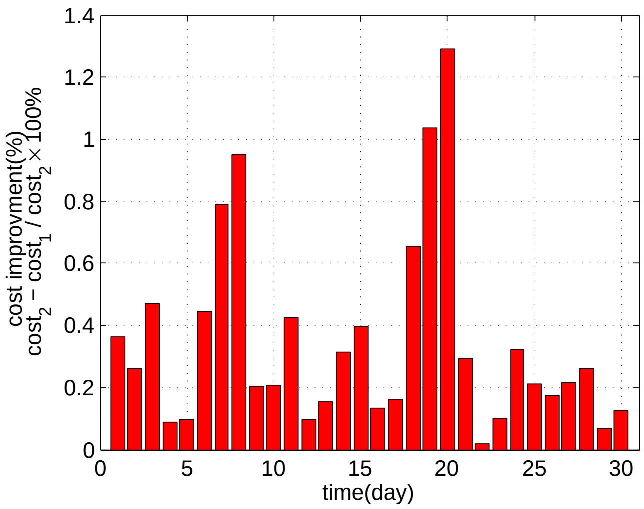

Figure 2.5. Relative cost difference between the cost generated by Algorithm 2 and the cost generated by Algorithm 1, normalized with the cost of Algorithm 2, for 30 days of the simulated data.

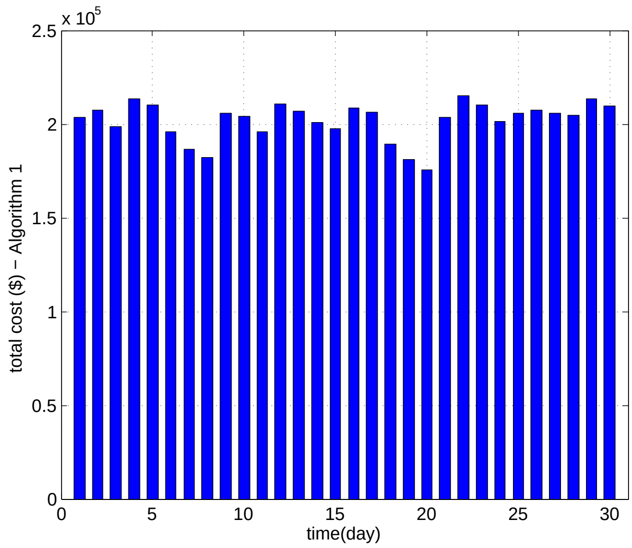

Figure 2.6. Total cost per day using Algorithm 1 and including the coupling constraints between every optimization stages (2.16),(2.17) and (2.18).

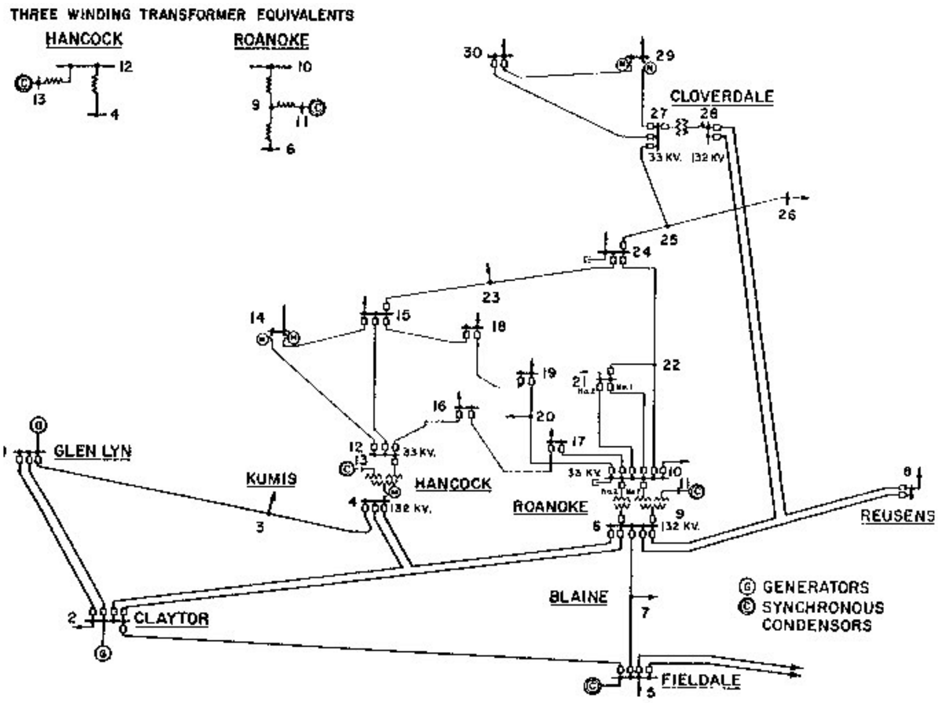

Figure 2.7. Circuit of the IEEE 30-bus system taken from [2].

2.6 Alternative Reserve Representation

We repeat here the analysis of the previous sections following, however, an alternative reserve representation based on the notion of disturbance feedback policies [24, 55]. Such a policy is designed in advance encoding the response of individual power system generators to prediction errors affecting the power system operation [55]. Specifically we consider policies for the reserves, represented as causal functions of the wind power error measurements. Since optimizing over all causal policies is intractable, we concentrate on policies that are affine functions of the past prediction errors. In the sequel we define the reserves using the concept of affine policies, and compare it against the reserve model of Section 2.1.3.

2.6.1 Affine-Policy for the Reserves

As in Section 2.1.3 we define the reserves (down and up-spinning) as

$$\mathbf{R}^{ds} = \mathbf{D}^{ds}\ \delta \quad \text{and} \quad \mathbf{R}^{us} = \mathbf{D}^{us}\ \delta \tag{2.26}$$

where $\mathbf{R}^{ds} = \begin{bmatrix} \mathbf{r}_1^{ds} & \mathbf{r}_2^{ds} & \cdots & \mathbf{r}_{n_t}^{ds} \end{bmatrix}^\top$, $\mathbf{R}^{us} = \begin{bmatrix} \mathbf{r}_1^{us} & \mathbf{r}_2^{us} & \cdots & \mathbf{r}_{n_t}^{us} \end{bmatrix}^\top$, $\delta = \begin{bmatrix} \delta_1 & \delta_2 & \cdots & \delta_{n_t} \end{bmatrix}^\top$. However, $\mathbf{D}^{ds}$ and $\mathbf{D}^{us} \in \mathbb{R}^{n_g n_t \times n_t}$ are no longer diagonal matrices, but take the lower triangular form:

$$\mathbf{D}^{us} = \begin{bmatrix} \mathbf{d}_1^{us} & 0 & \dots & 0 \\ \mathbf{d}_{2,1}^{us} & \mathbf{d}_2^{us} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{d}_{n_t,1}^{us} & \mathbf{d}_{n_t,2}^{us} & \dots & \mathbf{d}_{n_t}^{us} \end{bmatrix}, \mathbf{D}^{ds} = \begin{bmatrix} \mathbf{d}_1^{ds} & 0 & \dots & 0 \\ \mathbf{d}_{2,1}^{ds} & \mathbf{d}_2^{ds} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{d}_{n_t,1}^{ds} & \mathbf{d}_{n_t,2}^{ds} & \dots & \mathbf{d}_{n_t}^{ds} \end{bmatrix} .$$

We then formulate the same optimization problem as in Section 2.2 replacing the distribution vector constraints (2.15) with

- Down-spinning reserve balance constraints:

$$\begin{aligned} \mathbf{D}^{ds} &\geq 0 \\ \mathbf{1}_{1\times n_g}\mathbf{r}_1^{ds} &= \mathbf{1}_{1\times n_g}(\mathbf{d}_1^{ds}\delta_1) = \delta_1 \\ \mathbf{1}_{1\times n_g}\mathbf{r}_2^{ds} &= \mathbf{1}_{1\times n_g}(\mathbf{d}_{2,1}^{ds}\delta_1 + \mathbf{d}_2^{ds}\delta_2) = \delta_2 \\ \vdots \quad &\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \mathbf{1}_{1\times n_g}\mathbf{r}_{n_t}^{ds} &= \mathbf{1}_{1\times n_g}(\mathbf{d}_{n_t,1}^{ds}\delta_1 + \mathbf{d}_{n_t,2}^{ds}\delta_2 + \dots + \mathbf{d}_{n_t}^{ds}\delta_{n_t}) = \delta_{n_t} \ . \end{aligned}$$

- Up-spinning reserve balance constraints:

$$\begin{aligned} \mathbf{D}^{us} &\leq 0 \\ \mathbf{1}_{1\times n_g}\mathbf{r}_1^{us} &= \mathbf{1}_{1\times n_g}(\mathbf{d}_1^{us}\delta_1) = -\delta_1 \\ \mathbf{1}_{1\times n_g}\mathbf{r}_2^{us} &= \mathbf{1}_{1\times n_g}(\mathbf{d}_{2,1}^{us}\delta_1 + \mathbf{d}_2^{us}\delta_2) = -\delta_2 \\ \vdots \quad &\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \mathbf{1}_{1\times n_g}\mathbf{r}_{n_t}^{us} &= \mathbf{1}_{1\times n_g}(\mathbf{d}_{n_t,1}^{ds}\delta_1 + \mathbf{d}_{n_t,2}^{us}\delta_2 + \dots + \mathbf{d}_{n_t}^{us}\delta_{n_t}) = -\delta_{n_t} \ . \end{aligned}$$

These constraints imply that the sum of the reserve vectors elements for each time step should be equal to the wind power mismatch (deviation of the actual wind power with its forecast) and every element of $\mathbf{D}^{ds}$ and $\mathbf{D}^{us}$ should be positive and negative in case of the down and up-spinning, respectively.

Simulation Study To investigate whether the introduction of more sophisticated reserve policies may have an advantage in terms of cost over the reserve model of Section 2.1.3, we repeated the simulation study of Section 2.5. Motivated by [39] we assumed a linear cost for the reserves, denoted by $(\mathbf{c}_l^{us\,\mathrm{T}}\mathbf{R}^{us} + \mathbf{c}_l^{ds\,\mathrm{T}}\mathbf{R}^{ds})$ where $\mathbf{c}_l^{ds}$ and $\mathbf{c}_l^{us} \in \mathbb{R}^{n_g n_t}$ are vectors which contain down and up-spinning reserve cost for each generating unit, for each time step n_t, respectively. To achieve a more fair comparison with [55] we considered also the case of quadratic reserve costs, represented by $(\mathbf{R}^{us^{\mathrm{T}}}[\mathbf{c}_q^{us}]\mathbf{R}^{us} + \mathbf{R}^{ds^{\mathrm{T}}}[\mathbf{c}_q^{ds}]\mathbf{R}^{ds})$ where $[\mathbf{c}_q^{ds}]$ and $[\mathbf{c}_q^{us}] \in \mathbb{R}^{n_g n_t \times n_g n_t}$ are matrices with all elements zero and c_q^{us} (respectively c_q^{ds}) on the diagonal.

In [55], the introduction of such reserves policies was found to offer a significant improvement in terms of cost. However, this was not the case with our set-up. Irrespective of the choice of the objective function we obtained the same solution with those reported in Section 2.5. This is justified by the fact that we did not consider any storage dynamics or coupling costs between the time steps as in [55] ($[\mathbf{c}_q^{ds}]$, $[\mathbf{c}_q^{us}]$ are considered the full matrices when coupling costs between the time steps are assumed). This implies that the improvement reported in [55] is an effect of the particular objective function used, which is in turn dictated by the underlying market structure. Therefore, only in certain market environments the introduction of affine reserve policies would be of practical relevance.

Chapter 3

AC Power Flow Based Reserve Scheduling

In this chapter we propose a new optimization problem to jointly address the problem of reserve scheduling in conjunction with the unit commitment problem using more accurate models of the power network based on AC optimal power flow. The optimal power flow (OPF) problem is nonconvex and generally hard to solve. The propose approach for this problem involves a novel relaxation method to obtain a convex (albeit very complex) optimization problem. Theoretical and empirical evidence show that the proposed relaxation technique tends to a feasible solution for the exact problem. The theoretical developments are validated on realistic benchmark problems of power network (IEEE 30-bus) and is compared against its DC power flow counterpart by means of Monte Carlo simulations.

The rest of the chapter is organized as follows. In Section 3.1 we present a convex relaxation of the AC power flow problem. Section 3.2 introduces reserve scheduling formulation based on the AC optimal power flow model. In Section 3.3 we discuss about how to formulate the reserve scheduling and unit commitment problems as a chance constrained optimization program whereas in Section 3.4 we demonstrate a simulation study.

3.1 AC Optimal Power Flow Problem

Consider a power system with n_b buses, n_g generators, n_d loads and n_l lines. $N_B := \{1, 2, 3, \cdots, n_b\}$ denote the bus indices, $N_G := \{1, 2, 3, \cdots, n_g\} \subseteq N_B$ the set of generator indices, $N_D := \{1, 2, 3, \cdots, n_d\} \subseteq N_B$ the set of load indices and $N_L \subseteq N_B \times N_B$ the set of line indices.

AC Optimal Power Flow (AC OPF) problem deals with finding an optimal operating point of a power system that minimizes an appropriate cost function such as active power generation cost subject to certain constraints on power and voltage variables [36]. Due to the nonconvex constraints, namely magnitude constraints on complex-valued bus voltages and nonlinear equality constraints corresponding to real and reactive power injection at nodes, it can be difficult to solve for AC power networks.

The AC OPF problem has been extensively studied in the literature and numerous algorithms have been proposed for solving this highly nonconvex problem, including linear programming [37], Newton-Raphson [38], conic quadratic programming [28], nonlinear complementarity programming [50], genetic algorithm [7], interior point methods [19] and semidefinite programing [6,30]. Specifically in [30], by means of semidefinite programming, different reformulations and relaxations of the AC OPF problem were presented and their connections were discussed.

We next introduce the AC OPF set-up using SemiDefinite Programming (SDP) techniques, [6] and [30]. Define the set of variables that will be used as decision variables in this optimization context.

Decision Variables

$$\mathbf{V} \in \mathbb{R}^{n_b}$$
$$\mathbf{P}_g \in \mathbb{R}^{n_g}$$
$$\mathbf{Q}_g \in \mathbb{R}^{n_g}$$

$\mathbf{V} = \{V_k\}_{k \in N_B}$ is a vector containing complex voltage of the each bus of power system. $\mathbf{P}_g = \{P_{G_k}\}_{k \in N_G}$ and $\mathbf{Q}_g = \{Q_{G_k}\}_{k \in N_G}$ are vectors which contain the active and reactive power of each generating unit respectively. We can now formulate the optimal power flow (OPF) problem as shown in following optimization program:

OPF problem

$$\min_{\mathbf{V}, \mathbf{P}_g, \mathbf{Q}_g} \sum\nolimits_{k \in N_G} f_k(P_{G_k})$$

Subject to

$$P_k^{\min} \leq P_{G_k} \leq P_k^{\max} \qquad \forall k \in N_G \tag{3.1a}$$
$$Q_k^{\min} \leq Q_{G_k} \leq Q_k^{\max} \qquad \forall k \in N_G \tag{3.1b}$$
$$V_k^{\min} \leq |V_k| \leq V_k^{\max} \qquad \forall k \in N_B \tag{3.1c}$$
$$|V_l - V_m| \leq \Delta V_{lm}^{\max} \qquad \forall (l,m) \in N_L \tag{3.1d}$$
$$|S_{lm}| \leq S_{lm}^{\max} \qquad \forall (l,m) \in N_L \tag{3.1e}$$
$$|P_{lm}| \leq P_{lm}^{\max} \qquad \forall (l,m) \in N_L \tag{3.1f}$$

where $f_k(P_{G_k}) = c_{2,k} P_{G_k}^2 + c_{1,k} P_{G_k} + c_{0,k}$ is a quadratic cost function with given coefficients $\mathbf{c}_0 = \{c_{0,k}\}_{k \in N_G}, \mathbf{c}_1 = \{c_{1,k}\}_{k \in N_G}, \mathbf{c}_2 = \{c_{2,k}\}_{k \in N_G}$ accounting for the cost of active power generation at bus $k \in N_G$. S_{lm} and

P_{lm} are the complex apparent and active power transferred from bus $l \in N_B$ to the rest of the network through line $(l, m) \in N_L$ respectively and ΔV_{lm}^{max} is maximum voltage of every line $(l, m) \in N_L$.

Suppose the OPF problem is feasible that means $\mathbf{V} = 0$ does not satisfy its constraints (3.1c). As in [30] we can derive the circuit model of the power network by replacing every transmission line and transformer with their equivalent Π models which is an algebraic model[1]. In this circuit model, let y_{kl} denote the admittance between buses k and l and y_{kk} denote the admittance to ground at bus k. Let Y represent the admittance matrix of this equivalent circuit model, which is a $n_b \times n_b$ complex matrix whose (k, l) entry is equal to $-y_{kl}$ if $k \neq l$ and $y_{kk} + \sum_{m \in N(k)} y_{km}$ otherwise where $N(k)$ denotes the set of the all buses that are directly connected to bus k. Based on following formulations, we have that for all $k \in N_B$,

$$\begin{aligned} Y_k &= e_k e_k^T Y, \\ Y_{lm} &= (\bar{y}_{lm} + y_{lm}) e_l e_l^T - (y_{lm}) e_l e_m^T \ . \end{aligned} \tag{3.2}$$

where e_k is the standard basis vector in $\mathbb{R}^{n_b}$ and $\bar{y}_{lm}$ denotes the value of the shunt element at bus l associated with the π model of the line (l, m). Define the following system matrices:

$$\begin{aligned} \mathbf{Y}_k &= \frac{1}{2} \begin{bmatrix} Re\{Y_k + Y_k^T\} & Im\{Y_k^T - Y_k\} \\ Im\{Y_k - Y_k^T\} & Re\{Y_k + Y_k^T\} \end{bmatrix} \\ \bar{\mathbf{Y}}_k &= \frac{-1}{2} \begin{bmatrix} Im\{Y_k + Y_k^T\} & Re\{Y_k - Y_k^T\} \\ Re\{Y_k^T - Y_k\} & Im\{Y_k + Y_k^T\} \end{bmatrix} \\ \mathbf{Y}_{lm} &= \frac{1}{2} \begin{bmatrix} Re\{Y_{lm} + Y_{lm}^T\} & Im\{Y_{lm}^T - Y_{lm}\} \\ Im\{Y_{lm} - Y_{lm}^T\} & Re\{Y_{lm} + Y_{lm}^T\} \end{bmatrix} \\ \bar{\mathbf{Y}}_{lm} &= \frac{-1}{2} \begin{bmatrix} Im\{Y_{lm} + Y_{lm}^T\} & Re\{Y_{lm} - Y_{lm}^T\} \\ Re\{Y_{lm}^T - Y_{lm}\} & Im\{Y_{lm} + Y_{lm}^T\} \end{bmatrix} \\ M_k &= \begin{bmatrix} e_k e_k^T & 0 \\ 0 & e_k e_k^T \end{bmatrix} \\ M_{lm} &= \begin{bmatrix} (e_l - e_m)(e_l - e_m)^T & 0 \\ 0 & (e_l - e_m)(e_l - e_m)^T \end{bmatrix} \ . \end{aligned} \tag{3.3}$$

P_{inj_k}, Q_{inj_k} as the net active and reactive powers injected into bus $k \in N_B$

[1] The lumped-circuit line model (Π model) simplifies the description of the behavior of spatially distributed physical systems into a topology consisting of discrete entities that approximate the behavior of the distributed system under certain assumptions.

respectively, i.e. ,

$$\begin{aligned} P_{inj_k} &= P_{G_k} - P_{D_k}, \qquad \forall k \in G \\ Q_{inj_k} &= Q_{G_k} - Q_{D_k}, \qquad \forall k \in G \\ P_{inj_k} &= -P_{D_k}, \qquad \forall k \notin G \\ Q_{inj_k} &= -Q_{D_k}, \qquad \forall k \notin G \ , \end{aligned} \tag{3.4}$$

where $\mathbf{P}_d = \{P_{D_k}\}_{k \in N_D}$ and $\mathbf{Q}_d = \{Q_{D_k}\}_{k \in N_D}$ are vectors which contain the active and reactive demand power at each bus $k \in N_D$. Define now

$$\mathbf{X} = \left[Re\{\mathbf{V}\}^T \qquad Im\{\mathbf{V}\}^T \right]^T$$

Lemma 1 ([30, Lemma 1]). *For every $k \in N_B$ it holds that:*

$$P_{inj_k} = \mathrm{Tr}\{\, \boldsymbol{Y}_k \boldsymbol{X}\boldsymbol{X}^T\} \tag{3.5a}$$

$$Q_{inj_k} = \mathrm{Tr}\{\, \bar{\boldsymbol{Y}}_k \boldsymbol{X}\boldsymbol{X}^T\} \tag{3.5b}$$

$$|V_k|^2 = \mathrm{Tr}\{M_k \boldsymbol{X}\boldsymbol{X}^T\} \tag{3.5c}$$

$$|V_l - V_m|^2 = \mathrm{Tr}\{M_{lm} \boldsymbol{X}\boldsymbol{X}^T\} \tag{3.5d}$$

$$P_{lm} = \mathrm{Tr}\{\, \boldsymbol{Y}_{lm} \boldsymbol{X}\boldsymbol{X}^T\} \tag{3.5e}$$

$$|S_{lm}|^2 = (\mathrm{Tr}\{\, \boldsymbol{Y}_{lm} \boldsymbol{X}\boldsymbol{X}^T\})^2 + (\mathrm{Tr}\{\, \bar{\boldsymbol{Y}}_{lm} \boldsymbol{X}\boldsymbol{X}^T\})^2 \ . \tag{3.5f}$$

Hence, one can reformulate the OPF problem solely in terms of $\mathbf{X}$ as follows.

Optimization 1

$$\min_{\mathbf{X},W} \sum_{k \in N_G} \left\{ c_{2,k}(\mathrm{Tr}\{\mathbf{Y}_k W\} + P_{D_k})^2 + c_{1,k}(\mathrm{Tr}\{\mathbf{Y}_k W\} + P_{D_k}) + c_{0,k} \right\}$$

Subject to

$$P_{G_k}^{\min} - P_{D_k} \leq \mathrm{Tr}\{\mathbf{Y}_k W\} \leq P_{G_k}^{\max} - P_{D_k} \tag{3.6a}$$

$$Q_{G_k}^{\min} - Q_{D_k} \leq \mathrm{Tr}\{\bar{\mathbf{Y}}_k W\} \leq Q_{G_k}^{\max} - Q_{D_k} \tag{3.6b}$$

$$(V_k^{\min})^2 \leq \mathrm{Tr}\{M_k W\} \leq (V_k^{\max})^2 \tag{3.6c}$$

$$(\mathrm{Tr}\{\mathbf{Y}_{lm} W\})^2 + (\mathrm{Tr}\{\bar{\mathbf{Y}}_{lm} W\})^2 \leq (S_{lm}^{\max})^2 \tag{3.6d}$$

$$\mathrm{Tr}\{\mathbf{Y}_{lm} W\} \leq P_{lm}^{\max} \tag{3.6e}$$

$$\mathrm{Tr}\{M_{lm} W\} \leq (V_{lm})^2 \tag{3.6f}$$

$$W = \mathbf{X}\mathbf{X}^T \ , \tag{3.6g}$$

where $\mathbf{X} \in \mathbb{R}^{2n_b}$ and $W \in \mathbb{R}^{2n_b \times 2n_b}$. Optimization 1 is not quadratic in $\mathbf{X}$, since the objective function as well as constraint (3.6d) is of degree 4 with respect to $\mathbf{X}$. In order to tackle this problem, using Schur's complement formula [56], one can write constraint (3.6d) as follows,

$$\begin{bmatrix} -(S_{lm}^{\max})^2 & \mathrm{Tr}\{\mathbf{Y}_{lm}W\} & \mathrm{Tr}\{\bar{\mathbf{Y}}_{lm}W\} \\ \mathrm{Tr}\{\mathbf{Y}_{lm}W\} & -1 & 0 \\ \mathrm{Tr}\{\bar{\mathbf{Y}}_{lm}W\} & 0 & -1 \end{bmatrix} \leq 0\ . \tag{3.7}$$

Moreover, using the epigraphic formulation we can minimize $\sum_{k\in G} \alpha_k$ while imposing the constraint,

$$\begin{bmatrix} c_{1,k}\,\mathrm{Tr}\{\mathbf{Y}_k W\} - \alpha_k + a_k & \sqrt{c_{2,k}}\,\mathrm{Tr}\{\mathbf{Y}_k W\} + b_k \\ \sqrt{c_{2,k}}\,\mathrm{Tr}\{\mathbf{Y}_k W\} + b_k & -1 \end{bmatrix} \leq 0\ , \tag{3.8}$$

where $\alpha_t = \{\alpha_{k,t}\}_{k\in N_G}$ is a vector that contains an upper bound of the generation cost for every time period t and $a_k = c_{0,k} + c_{1,k}P_{D_k}$,$b_k = \sqrt{c_{2,k}}P_{D_k}$. Therefore, one can formulate the OPF problem in terms of W and α_k as follows,

Optimization 2

$$\min_{\alpha_k, W} \sum\nolimits_{k\in G} \alpha_k$$

Subject to constraints (3.6a), (3.6b), (3.6c), (3.6e), (3.6f), (3.7), (3.8) and,

$$W \geq 0 \tag{3.9a}$$

$$Rank(W) = 1\ . \tag{3.9b}$$

Optimization 2 is obtained from Optimization 1 by replacing the constraint (3.6g), i.e. , $W = \mathbf{X}\mathbf{X}^T$ with the new constraints (3.9a) and (3.9b).

Since Optimization 2 has a rank constraint ($Rank(W) = 1$), it is a nonconvex problem. It is shown in [30] that it is NP-hard and hence is not tractable. In order to ensure convexity, one can remove constraint $Rank(W) = 1$. That way, we come up with a convex AC OPF relaxation.

Optimization 3

$$\min_{\alpha_k, W} \sum\nolimits_{k\in G} \alpha_k$$

Subject to the constraints (3.6a), (3.6b), (3.6c), (3.6e), (3.6f), (3.7), (3.8) and (3.9a). Assume that Optimization 3 (convex optimization) problem has rank-one optimal solution W^{opt}, then there exists a vector $\mathbf{X}^{opt}$ such that $W^{opt} = \mathbf{X}^{opt}(\mathbf{X}^{opt})$. In that case, $\mathbf{X}^{opt}$ is a global optimum of the OPF problem. Empirical evidence shows Optimization 3 for IEEE test systems [2] always has rank two solution. The next lemma from [30] explains the reason why this occurs for IEEE systems.

Lemma 2 ([30, Lemma 2])**.** *If Optimization 3 has a rank-one solution, then it has an infinite number of rank-two solutions.*

Proof. This is taken from [30] and is included here to motivate some of the results presented in the sequel. For $\mathbf{X}^{opt}$ define $\mathbf{X}_1^{opt}$ and $\mathbf{X}_2^{opt}$ in such a way that $\mathbf{X}^{opt} = [(\mathbf{X}_1^{opt})^T \ (\mathbf{X}_2^{opt})^T]^T$. It can be verified that the matrix below $(\bar{W})$ is a solution of Optimization 3 for every real numbers w_1 and w_2 such that $w_1^2 + w_2^2 = 1$.

$$\bar{W} = \frac{1}{2}\mathbf{X}^{opt}(\mathbf{X}^{opt})^T \ + \ \frac{1}{2}\begin{bmatrix}\mathbf{X}_1^{opt}w_1 - \mathbf{X}_2^{opt}w_2 \\ \mathbf{X}_1^{opt}w_2 + \mathbf{X}_2^{opt}w_1\end{bmatrix}\begin{bmatrix}\mathbf{X}_1^{opt}w_1 - \mathbf{X}_2^{opt}w_2 \\ \mathbf{X}_1^{opt}w_2 + \mathbf{X}_2^{opt}w_1\end{bmatrix}^T \quad (3.10)$$

The proof is completed by that matrix $(\bar{W})$ has rank-two for generic values of (w_1, w_2). □

Lemma 2 states that Optimization 3 might have a rank-one solution that cannot be directly identified solving it numerically. However using Optimization 4 proposed below, one can verify that Optimization 3 always has a rank-one solution for all IEEE case study systems [2]. This implies that these power systems can be convexified using convex relaxation technique. The next theorem investigates how to recover W^{opt}, which is an optimal solution of OPF problem.

It is shown in [30], that given any arbitrary solution $\bar{W}$ of Optimization 3, the rank of $\bar{W}$ is at most 2. Moreover, if the matrix $\bar{W}$ has rank 2, then the matrix $(\rho_1 + \rho_2)EE^{\mathrm{T}}$ is a rank-one solution of Optimization 3, where ρ_1 and ρ_2 are the nonzero eigenvalues of $\bar{W}$ and E is the unit eigenvector associated with $\bar{W}$. Here, instead of post-rotating the resulting solution so that we come up with a rank-one matrix, we embed this functionality in Optimization 4.

Optimization 4

$$\min_{\alpha_k, W} \ \textstyle\sum_{k\in G} \alpha_k$$

Subject to the constraints (3.6a), (3.6b), (3.6c), (3.6e), (3.6f), (3.7) (3.8), (3.9a) and

- Voltage angle of reference bus constraint:

$$\mathrm{Tr}\{WE_k\} = 0 \ , \quad (3.11)$$

where $E_k \in \mathbb{R}^{2n_b \times 2n_b}$ is a matrix with all its entries zero except $E_{(k,k)} = 1$ that is imaginary part of the reference voltage bus.

Optimization 4 has one constraint more than Optimization 3 which corresponds to the angle of reference voltage bus and can directly identify the rank-one optimal solution W^{opt} of Optimization 3. The next theorem shows how to recover W^{opt} through Optimization 4. In order to simplify the proof, we introduce a new variable as follows,

$$\tilde{\mathbf{X}}^{opt} = \begin{bmatrix} \mathbf{X}_1^{opt} w_1 - \mathbf{X}_2^{opt} w_2 \\ \mathbf{X}_1^{opt} w_2 + \mathbf{X}_2^{opt} w_1 \end{bmatrix} , \tag{3.12}$$

then

$$\bar{W} = \frac{1}{2}\mathbf{X}^{opt}(\mathbf{X}^{opt})^T \; + \; \frac{1}{2}\tilde{\mathbf{X}}^{opt}(\tilde{\mathbf{X}}^{opt})^T \; . \tag{3.13}$$

This is simply a parameterization of the general plane spanned by $\mathbf{X}^{opt}$ and $\tilde{\mathbf{X}}^{opt}$.

Theorem 1. *If Optimization 3 has solutions with rank at most two, then forcing any arbitrary diagonal element of the matrix W to zero, Optimization 4 results in a rank-one solution W^{opt}. The fact that this is the optimal solution of Optimization 3 as well is established in Lemma 2.*

Proof. Assume that Optimization 3 has a solution whose rank is at most two. We can parametrize all rank-two solution as $\bar{W}$ in 3.13 then Theorem 1 requires setting an arbitrary diagonal element of W to 0. Therefore, to achieve this we set

$$\mathrm{Tr}\{\bar{W}E_k\} = 0 \tag{3.14}$$

where $E_k \in \mathbb{R}^{2n_b \times 2n_b}$ is a matrix with all its entries zero except $E_{(k,k)} = 1$ and we will have

$$\mathrm{Tr}\{\bar{W}E_k\} = 0 \;\Leftrightarrow\; \bar{W}_{(k,k)} = 0$$

By using (3.13), we can write as follows

$$\begin{aligned} 0 = \bar{W}_{(k,k)} &= \frac{1}{2}\mathbf{X}^{opt}(\mathbf{X}^{opt})^T_{(k,k)} \; + \; \frac{1}{2}\tilde{\mathbf{X}}^{opt}(\tilde{\mathbf{X}}^{opt})^T_{(k,k)} \\ &= \frac{1}{2}\underbrace{(X_k^{opt})^2}_{①} \; + \; \frac{1}{2}\underbrace{(\tilde{X}_k^{opt})^2}_{②} \end{aligned} \tag{3.15}$$

Equation (3.15) contains two parts ① and ②. Since ① and ② are squares, they are greater than or equal to zero. Because their sum has to be zero, both has to be zero. It follows

$$① = 0 \Rightarrow \; X_{1,k}^{opt} = 0 \tag{3.16a}$$

and

$$② = 0 \Rightarrow \underbrace{X_{1,k}^{opt}\ w_1}_{\substack{=0\\ \text{by (3.16a)}}} - X_{2,k}^{opt}\ w_2 = 0 \ \Rightarrow \ X_{2,k}^{opt}\ w_2 = 0 \tag{3.16b}$$

In this step, we check the all possible values which every one of them can get. The second part $X_{2,k}^{opt} \neq 0$ should be different from zero due to having another constraint (3.6c). In next step we show the relation of that constraint and following inequality

$$(V_k^{min})^2 \leq \mathrm{Tr}\{M_k \bar{W}\} \leq (V_k^{max})^2 \ . \tag{3.17}$$

Based on the structure of M_k given in (3.3), this operation takes two diagonal elements of $\bar{W}$, since $\mathrm{Tr}\{M_k \bar{W}\}$ is equal of with $\bar{W}_{(k,k)} + \bar{W}_{(n_b+k,n_b+k)}$. We introduce $\tilde{k} = n_b + k$ for having clear indices. Moreover by looking at these two elements of $\bar{W}$, which are given as follows

$$\begin{aligned}
\bar{W}_{(k,k)} &= \frac{1}{2}(X_k^{opt})^2 + \frac{1}{2}(\tilde{X}_k^{opt})^2 = \frac{1}{2}(X_{1,k}^{opt})^2 + \frac{1}{2}(X_{1,k}^{opt} w_1 - X_{2,k}^{opt} w_2)^2 \\
\bar{W}_{(\tilde{k},\tilde{k})} &= \frac{1}{2}(X_{\tilde{k}}^{opt})^2 + \frac{1}{2}(\tilde{X}_{\tilde{k}}^{opt})^2 = \frac{1}{2}(X_{2,k}^{opt})^2 + \frac{1}{2}(X_{1,k}^{opt} w_2 + X_{2,k}^{opt} w_1)^2 \quad ,
\end{aligned}$$

we can write

$$\begin{aligned}
\mathrm{Tr}\{M_k \bar{W}\} &= \bar{W}_{(k,k)} + \bar{W}_{(\tilde{k},\tilde{k})} \\
&= \frac{1}{2}(X_{1,k}^{opt})^2 + \frac{1}{2}(X_{2,k}^{opt})^2 + \\
&+ \frac{1}{2} \underbrace{(w_1^2 + w_2^2)}_{\substack{=1\\ \text{(by Lemma 2)}}} ((X_{1_k}^{opt})^2 + (X_{2,k}^{opt})^2) \\
&= \underbrace{(X_{1,k}^{opt})^2}_{\substack{=0\\ \text{by (3.16a)}}} + (X_{2,k}^{opt})^2 \\
&= (X_{2,k}^{opt})^2 \quad .
\end{aligned} \tag{3.18}$$

Now, based on equations (3.17) and (3.18), it follows

$$(V_k^{min})^2 \leq (X_{2_{\tilde{k}}}^{opt})^2 \leq (V_k^{max})^2 \quad . \tag{3.19}$$

Finally, as shows above, $(X_{2_{\tilde{k}}}^{opt})^2 \neq 0$ otherwise there is no feasible solution. It means, w_2 should be equal with 0. In order to fulfill $w_1^2 + w_2^2 = 1$, then w_1 should be equal with 1. In another side by having

$$\left.\begin{aligned} w_1 &= 1 \\ w_2 &= 0 \end{aligned}\right\} \Rightarrow \qquad \tilde{\mathbf{X}}^{opt} = \mathbf{X}^{opt} \tag{3.20}$$

and

$$\begin{aligned}\bar{W} &= \frac{1}{2}\mathbf{X}^{opt}(\mathbf{X}^{opt})^T + \frac{1}{2}\tilde{\mathbf{X}}^{opt}(\tilde{\mathbf{X}}^{opt})^T \\ &= \mathbf{X}^{opt}(\mathbf{X}^{opt})^T = W^{opt} \ .\end{aligned}$$

□

Now we should show the statement of Theorem 1 in OPF problem, by meaning that, the optimal objective value of OPF problem remains unaffected when including constraint (3.11). Next Lemma answers this question.

Lemma 3. *If Optimization 3 has solutions with rank at most two, then the optimal objective values of the OPF problem and Optimization 4 are equal.*

Proof. By inspection of Theorem 1 and since only the difference of the voltage angles appear in the equations of Optimization 3, fixing implicitly the voltage reference angle by adding constraint (3.11) does not affect the optimal objective value of the OPF problem. Therefore, the optimal objective values of Optimization 3 and Optimization 4 are equal. On the other hand, it is shown in [30] that the optimal objective values of the OPF problem, Optimization 1, 2 and 3 are the same if Optimization 3 has rank-two solution. Therefore, the optimal objective values of the OPF problem and the proposed optimization would be equal as well. □

Since Optimization 4 is an SDP, a global optimum can be found in polynomial time (rank-one solution). Following Theorem 1 and Lemma 3 it suffices to solve Optimization 4. We numerically solved Optimization 4 for several different arbitrary networks and all IEEE systems using "YALMIP" as Matlab interface and the solver "SeDuMi". In all cases solution W^{opt} was of rank-one. The relations among the OPF problem, Optimizations 1-4 are shown in Figure 3.1. The dashed lines correspond to the fact that this equivalence hold only if the requirements of Theorem 1 are satisfied.

3.2 Reserve Scheduling Problem Formulation

This section proposes a new methodology to determine the level of required up and down-spinning reserve of each generating unit in case of high wind power penetration. Consider a power system as introduced in Section 3.1 including one wind power generator located in a certain bus of the power network. Define the decision variables of the power system for each optimization steps as follow:

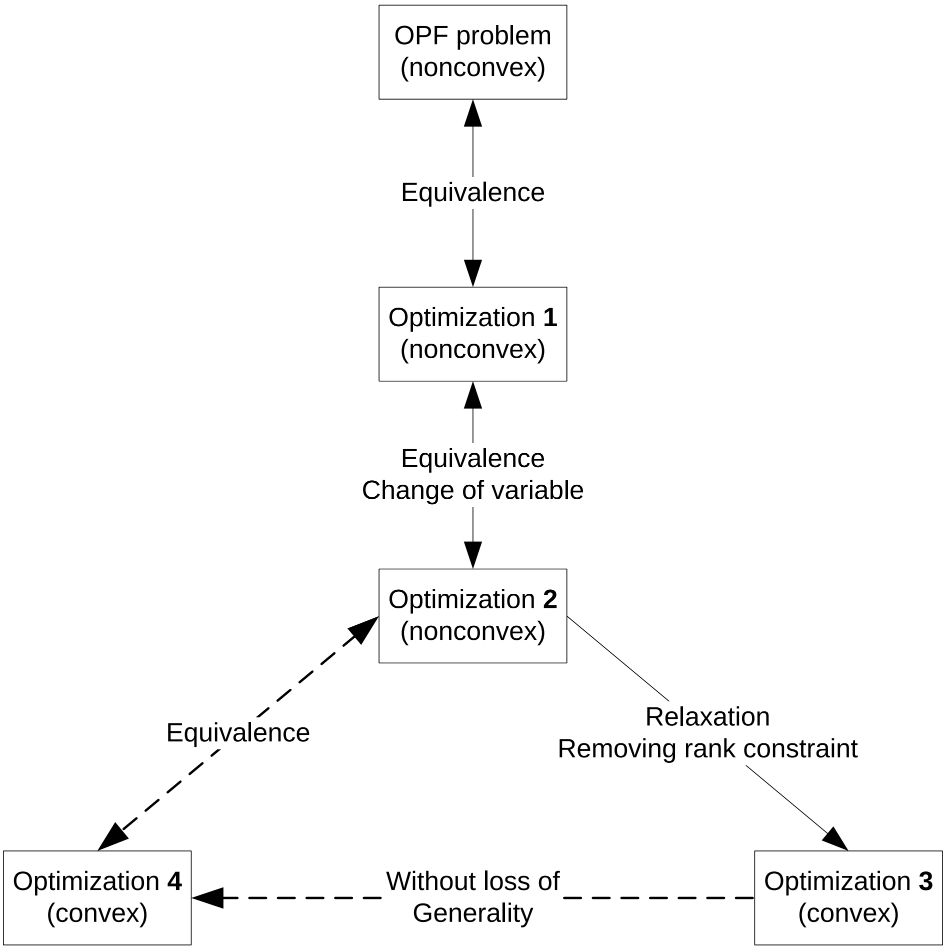

Figure 3.1. Relations among the OPF problem, Optimizations 1-4. The dashed lines correspond to the fact that this equivalence hold only if the requirements of Theorem 1 are satisfied.

Decision Variables:

$$\alpha_t \in \mathbb{R}^{n_g}$$
$$\mathbf{u}_t \in \mathbb{R}^{n_g}$$
$$\mathbf{d}_t^{ds} \in \mathbb{R}^{n_g}$$
$$\mathbf{d}_t^{us} \in \mathbb{R}^{n_g}$$
$$\mathbf{C}_t^{su} \in \mathbb{R}^{n_g}$$
$$Q_t^{ds} \in \mathbb{R}$$
$$Q_t^{us} \in \mathbb{R}$$
$$W_t^f \in \mathbb{R}^{2n_b \times 2n_b}$$
$$W_t^s(\cdot) \in \mathbb{R}^{2n_b \times 2n_b}$$

$\alpha_t = \{\alpha_{k,t}\}_{k \in N_G}$ is a vector that contains the upper bound of each power generation cost for every time period t. $\mathbf{u}_t = \{u_{k,t}\}_{k \in N_G}$ is a binary vector which encodes the "on-off" statues of each generating unit in period t. $\mathbf{d}_t^{ds} = \{d_{k,t}^{ds}\}_{k \in N_G}$ and $\mathbf{d}_t^{us} = \{d_{k,t}^{us}\}_{k \in N_G}$ are weighting vectors that are distributing coefficients of the positive and negative mismatch wind power among the generating units participating in the frequency control. If a generator is not contributing to frequency control, the corresponding element will be zero. $\mathbf{C}_t^{su} = \{C_{k,t}^{su}\}_{k \in N_G}$ is the startup cost vector in period t and the Q_t^{ds} and Q_t^{us} are upper bounds of the down and up-spinning reserves cost in period t. The last two decision variables W_t^f and $W_t^s(\cdot)$, according to the description of Section 3.1, are related to bus complex voltages for the forecast and actual wind power respectively. Notice that $W_t^s(\cdot)$ is a function of the uncertainty. For the sake of simplicity we represent it with W_t^s. All decision variables are defined for every $t = \{1, 2, 3, \cdots, n_t\}$ where $n_t = 24$ for daily-based scheduling. Now we can formulate optimization problem as follows

$$\text{minimize} \sum_{t=1}^{n_t} \Big(\sum\nolimits_{k \in N_G} \alpha_{k,t} + C_{k,t}^{su} \Big) + Q_t^{ds} + Q_t^{us} \tag{3.21}$$

over scalar variables $\alpha_{k,t}, C_{k,t}^{su}, Q_t^{ds}, Q_t^{us}$ and the matrix valued variables W_t^f, W_t^s.

Subject to:

1. Upper bound Constraint of the objective function in general OPF problem

$$\underbrace{c_{2,k} P_{G_{k,t}}^2 + c_{1,k} P_{G_{k,t}} + c_{0,k}}_{f_{k,t}(P_{G_{k,t}})} \leq \alpha_{k,t}$$

given a scalar $\alpha_{k,t}$ for all $k \in N_G$ and $t = 1, 2, \ldots, n_t$ and reformulate as follows

$$\begin{bmatrix} c_{1,k} \operatorname{Tr}\{\mathbf{Y}_k W_t^f\} - \alpha_{k,t} + a_k & \sqrt{c_{2,k}} \operatorname{Tr}\{\mathbf{Y}_k W_t^f\} + b_k \\ \sqrt{c_{2,k}} \operatorname{Tr}\{\mathbf{Y}_k W_t^f\} + b_k & -1 \end{bmatrix} \preceq 0 \quad (3.22)$$

where $\mathbf{c}_2 = \{c_{2,k}\}_{k \in N_G}$, $\mathbf{c}_1 = \{c_{1,k}\}_{k \in N_G}$, $\mathbf{c}_0 = \{c_{0,k}\}_{k \in N_G} \in \mathbb{R}^{n_g}$ are vectors contain for each generating unit cost of every MW per hour, $\mathbf{P}_{d,t} = \{P_{D_{k,t}}\}_{k \in N_D}$ is a vector which contains demands active power and

$$\begin{aligned} a_k &= c_{0,k} + c_{1,k} P_{D_{k,t}} \\ b_k &= \sqrt{c_{2,k}} P_{D_{k,t}} \quad . \end{aligned}$$

2. Productions, Reserves and Startup cost constraints $t = 1, 2, \ldots, n_t$:

$$\mathbf{C}_t^{su} \geq 0 \quad (3.23a)$$
$$Q_t^{ds} \geq 0 \quad (3.23b)$$
$$Q_t^{us} \geq 0 \quad (3.23c)$$
$$\alpha_t \geq 0 \quad . \quad (3.23d)$$

These constraints imply that the startup cost, down and up-spinning reserve and production cost should be positive.

3. Constraint of the apparent power transferred trough line (l, m) for all $(l, m) \in N_L$ and $t = 1, 2, \ldots, n_t$:

$$\begin{bmatrix} -(S_{lm}^{\max})^2 & \operatorname{Tr}\{\mathbf{Y}_{lm} W_t^f\} & \operatorname{Tr}\{\bar{\mathbf{Y}}_{lm} W_t^f\} \\ \operatorname{Tr}\{\mathbf{Y}_{lm} W_t^f\} & -1 & 0 \\ \operatorname{Tr}\{\bar{\mathbf{Y}}_{lm} W_t^f\} & 0 & -1 \end{bmatrix} \preceq 0 \quad (3.24)$$

4. Generation limits, for all $k \in N_B$ and $t = 1, 2, \ldots, n_t$:

$$P_{G_k}^{\min} u_{k,t} \leq \operatorname{Tr}\{\mathbf{Y}_k W_t^f\} + P_{D_{k,t}} - P_{w_{k,t}}^f \leq P_{G_k}^{\max} u_{k,t} \quad (3.25a)$$

$$Q_{G_k}^{\min} u_{k,t} \leq \operatorname{Tr}\{\bar{\mathbf{Y}}_k W_t^f\} + Q_{D_{k,t}} \leq Q_{G_k}^{\max} u_{k,t} \quad (3.25b)$$

where $P_{w_{k,t}}^f \in \mathbb{R}$ denotes the wind power forecast in every time period t for $k \in N_B$ and

$$\mathbf{P}_g^{\min} = \{P_{G_k}^{\min}\}_{k \in N_G} \ , \ \mathbf{P}_g^{\max} = \{P_{G_k}^{\max}\}_{k \in N_G}$$

$$\mathbf{Q}_g^{\min} = \{Q_{G_k}^{\min}\}_{k \in N_G} \ , \ \mathbf{Q}_g^{\max} = \{Q_{G_k}^{\min}\}_{k \in N_G}$$

are minimum (and maximum respectively) generating active and reactive power of each unit respectively. $\mathbf{P}_{d,t} = \{P_{D_{k,t}}\}_{k \in N_D}$ and $\mathbf{Q}_{d,t} =$

$\{Q_{D_{k,t}}\}_{k \in N_D}$ are vectors which contain active and reactive demand power respectively. $\mathbf{u}_t = \{u_{k,t}\}_{k \in N_G}$ is a binary decision vector which contains the status of each generating unit.

Note that according to the bus power injection law which is analogous to the Kirchhoff's current law [2], hence, by considering to the wind power generation unit for all $k \in N_B$ we can write:

$$P_{G_{k,t}} = P^f_{inj_{k,t}} + P_{D_{k,t}} - P^f_{w_{k,t}}$$

$$P^f_{inj_{k,t}} = \mathrm{Tr}\{\mathbf{Y}_k W^f_t\} \quad .$$

5. Bus voltage limits for all $k \in N_B$ and $t = 1, 2, \ldots, n_t$:

$$|V^{min}_k|^2 \leq \mathrm{Tr}\{M_k W^f_t\} \leq |V^{max}_k|^2 \tag{3.26}$$

where $\mathbf{V}^{\min} = \{V^{\min}_k\}_{k \in N_B}$, $\mathbf{V}^{\max} = \{V^{\max}_k\}_{k \in N_B}$ are vectors which contain minimum (and maximum respectively) of voltage of each bus.

6. Line voltage limits for all $(l, m) \in N_L$ and $t = 1, 2, \ldots, n_t$:

$$\mathrm{Tr}\{M_{lm} W^f_t\} \leq (V_{lm})^2 \tag{3.27}$$

where $\mathbf{V}_{lm} = \{V_{lm}\}_{(l,m) \in N_L}$ is a vector that contains maximum capacity of voltage of each line.

7. Probabilistic constraints for all $k \in N_B$ and $(l, m) \in N_L$:

$$\mathbb{P}\Bigg(\quad \{\Delta P_{w_t}\}^{n_t}_{t=1} \quad \Bigg| \quad \begin{bmatrix} -(S^{\max}_{lm})^2 & \mathrm{Tr}\{\mathbf{Y}_{lm} W^s_t\} & \mathrm{Tr}\{\bar{\mathbf{Y}}_{lm} W^s_t\} \\ \mathrm{Tr}\{\mathbf{Y}_{lm} W^s_t\} & -1 & 0 \\ \mathrm{Tr}\{\bar{\mathbf{Y}}_{lm} W^s_t\} & 0 & -1 \end{bmatrix} \leq 0, \tag{3.28a}$$

$$P^{min}_{G_k} u_{k,t} \leq \mathrm{Tr}\{\mathbf{Y}_k W^s_t\} + P_{D_{k,t}} - P^s_{w_{k,t}} \leq P^{max}_{G_k} u_{k,t}, \tag{3.28b}$$

$$Q^{min}_{G_k} u_{k,t} \leq \mathrm{Tr}\{\bar{\mathbf{Y}}_k W^s_t\} + Q_{D_{k,t}} \leq Q^{max}_{G_k} u_{k,t}, \tag{3.28c}$$

$$|V^{min}_k|^2 \leq \mathrm{Tr}\{M_k W^s_t\} \leq |V^{max}_k|^2, \tag{3.28d}$$

$$\mathrm{Tr}\{M_{lm} W^s_t\} \leq (V_{lm})^2, \tag{3.28e}$$

$$\left(\mathbf{c}^T_{ds} \mathbf{d}^{ds}_t \Delta P_{w_t}\right) \leq Q^{ds}_t, \tag{3.28f}$$

$$\left(\mathbf{c}^T_{us} \mathbf{d}^{us}_t \Delta P_{w_t}\right) \leq Q^{us}_t \qquad \Bigg) \geq 1 - \varepsilon \tag{3.28g}$$

[2]At any node in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node, or, the algebraic sum of currents in a network of conductors meeting at a point is zero.

where $\mathbf{S}_{lm}^{\max} = \{S_{lm}^{\max}\}_{k \in N_L}$ is a vector which contains the maximum apparent power of each line in power system and

$$\mathbf{c}_{ds} = \{c_{ds,k}\}_{k \in N_G} \ , \ \mathbf{c}_{us} = \{c_{us,k}\}_{k \in N_G}$$

are vectors that contain for each generating unit the cost of down and up-spinning reserves respectively and

$$P_{w_{k,t}} = P_{w_{k,t}}^f + \Delta P_{w_{k,t}} \quad .$$

The probabilistic constraint implies that all constraints of the power system should be satisfied with probability at least $1 - \varepsilon$. $\Delta P_{w_{k,t}}$ denotes deviation of the wind power from its forecast value. W_t^s corresponds to the complex voltage of bus for every wind power realized in each time period t by meaning that the new operating point should satisfy all network constraints.

8. Equality constraint of the generation units for all $k \in N_B$ and time period t :

$$\mathrm{Tr}\{\mathbf{Y}_k W_t^s\} - P_{w_{k,t}} = \mathrm{Tr}\{\mathbf{Y}_k W_t^f\} - P_{w_{k,t}}^f + R_{k,t} \tag{3.29a}$$

and

$$R_{k,t} = -d_{k,t}^{ds} |\Delta P_{w_{k,t}}|_+ + d_{k,t}^{us} |\Delta P_{w_{k,t}}|_- \tag{3.29b}$$

where, the power correction term $\mathbf{R} = \{R_{k,t}\} \in \mathbb{R}^{n_g}$ is related to the reserves provided by each generating unit and

$$|\Delta P_{w_{k,t}}|_+ := \begin{cases} \Delta P_{w_{k,t}} & \text{if } \Delta P_{w_{k,t}} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$|\Delta P_{w_{k,t}}|_- := \begin{cases} \Delta P_{w_{k,t}} & \text{if } \Delta P_{w_{k,t}} \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

This constraint implies when wind power is different from its forecast value, the production of the conventional generators and the wind power in-feed (if a bus k does not have wind power production the corresponding term should be zero) should be equal to the conventional and wind power production for the case where the wind power is equal to its forecast, plus a power correction term. This term was defined by weighting the mismatch to the generating units, differentiating according to its sign. This implies that we use different distribution vectors for positive and negative wind power errors. These vectors distribute the power mismatch among the generators.

9. Weighting vectors constraints for all $k \in N_G$ and $t = 1, 2, \ldots, n_t$:

$$\mathbf{1}_{1\times n_g}(\mathbf{d}_t^{ds}) = 1, \quad d_{k,t}^{ds} \geq 0 \tag{3.31a}$$
$$\mathbf{1}_{1\times n_g}(\mathbf{d}_t^{us}) = -1, \quad d_{k,t}^{us} \leq 0 \tag{3.31b}$$

   These constraints dictate that the sum of the weighting vectors elements of the mismatch wind power (deviation of every wind scenario and forecast wind power) should be equal to 1 and -1, and every element of them should be positive and negative in case of the down and up-spinning, respectively.

10. Startup cost constraint for all $k \in N_G$ and $t = 1, 2, \ldots, n_t$:

$$\lambda_k^{su}(u_{k,t} - u_{k,t-1}) \leq C_{k,t}^{su} \tag{3.32}$$

   where $\lambda^{su} = \{\lambda_k^{su}\}_{k\in N_G}$ is a vector and each element is startup cost of each generating units. This constraint implies that the decision variable $\mathbf{C}_t^{su} = \{C_{k,t}^{su}\}_{k\in N_G}$ will be always zero unless the corresponding generator changes status from "off" to "on" within two consecutive periods.

11. Reference bus angle constraint for all $t = 1, 2, \ldots, n_t$:

$$\mathrm{Tr}\{W_t^f E_k\} = 0 \tag{3.33a}$$
$$\mathrm{Tr}\{W_t^s E_k\} = 0 \tag{3.33b}$$

   where $E_k \in \mathbb{R}^{2n_b\times 2n_b}$ is a matrix with all its entries zero except $E_{(k,k)} = 1$.

The proposed optimization problem is a multi-stage chance constrained mixed-integer semidefinite program, whose stages are coupled by the binary variables constraint (3.32). Note that $W_t^s(\cdot)$ is a function of the uncertainty vector, therefore the optimization problem outlined before is an infinite dimensional problem.

3.3 Tractable Reformulation

In this section we explain the steps adopted for dealing with chance constraints in order to have a tractable formulation of the problem in Section 3.2. The problem described in the previous section is an infinite dimensional optimization problem, since it involves optimizing over the functions $W_t^s(\cdot)$. There are two main problems. The first arises due to the chance constraint, and the other refers to the fact that the problem is infinite dimensional, and hence applying the scenario based methodology of [16–18] is not straightforward.

The resulting problem is a mixed integer semidefinite program. Therefore numerical computation for systems of large scale is prohibited. To alleviate this difficulty we resorted to a suboptimal approach. We first fix the binary variables that were coupling the optimization stages by solving a deterministic unit commitment problem, and then solve the resulting reserve scheduling problem for each hour separately. We refer to the latter problem as hourly-based reserve scheduling.

To solve the chance constraint we followed a heuristic approach. We generated n_s wind power time series and since the stages are decoupled, we enforced the constraint inside the probability (see constraints (3.28)) for the minimum and maximum values at each time-step. Even though we can not provide a priori guarantees regarding the satisfaction of the system constraints the number of scenarios that we extracted was dictated by

$$n_s \geq \frac{2}{\epsilon}(2n_\delta + \ln\frac{1}{\beta}) \quad , \tag{3.34}$$

where $n_\delta = 1$ is the dimension of uncertainty since we have one wind power generator, and $\epsilon = 0.1$ and $\beta = 10^{-4}$. It was shown in [17, 34] that if the number of scenarios is chosen according to (3.34), then, with confidence at least $1 - \beta$, the uncertain constraints are satisfied with probability at least $1 - \epsilon$. Here, we loose these guarantees since we optimize over functions.

The basic steps of this procedure are summarized in Algorithm 3.

Algorithm 3 Proposed approach

Deterministic unit commitment.

1: Set $u_0 \in \{0,1\}_{k\in N_G}$ ▷ initial status of the generating units.

2: Define $\mathbf{x}_{1,t} = \begin{bmatrix}\alpha_{k,t} & C^{su}_{k,t}\end{bmatrix}, W^f_t$ and set $P_{w_{k,t}} = P^f_{w_{k,t}}$, $\mathbf{x}_t \leftarrow \mathbf{x}_{1,t}$ for each step t and $\forall k \in N_G$.

3: Solve (3.21)-(3.27), and (3.32), (3.33).

Stochastic reserve scheduling.

4: Fix $\mathbf{u}_t = \{u_{k,t}\}_{k\in N_G}$ according to Step 3, and $\epsilon \in (0,1)$, $\beta \in (0,1)$.

5: Define $\mathbf{x}_{2,t} = \begin{bmatrix}\alpha_{k,t} & d^{ds}_{k,t} & d^{us}_{k,t} & Q^{ds}_t & Q^{us}_t\end{bmatrix}, W^f_t, W_t$ and set $\mathbf{x}_t \leftarrow \mathbf{x}_{2,t}$ for each step t and $\forall k \in N_G$.

6: Generate n_s scenarios according to (3.34).

7: Solve the hourly-based reserve scheduling program that corresponds to (3.21)-(3.31), and (3.33). ▷ the resulting solution is based on an heuristic approach and with-out any guarantee.

	MatPower	Opt. **4**	Opt. **3**
Total Production Cost	9.0738	9.0738	9.0737

Table 3.1. Total Production Cost (in 1000$).

3.4 Simulation Study

This section shows the performance of the algorithms of section 3.1 and 3.2, on the IEEE 30-bus benchmark network [2]. The results of this section are attained using the following software tools, the MATLAB-based toolbox "YALMIP" together with the solver "SEDUMI" [32], [49]. For the sake of comparison, we used "MatPower" which is the commercial software for solving power flow and optimal power flow problems. It is intended as a simulation tool for researchers and educators that is easy to use and modify. The data for the 30-bus IEEE system analyzed in this example are extracted from the library of this software [57]. MatPower uses a successive quadratic programming technique with a quasi-Newton approximation for the Hessian matrix to solve optimal power flow problem.

AC Optimal Power Flow Based Results

We first compare the performance of Optimization 4 with MatPower OPF solution against the solution of Optimization 3. Here instead of post-rotating the resulting solution so that we come up with a rank-one matrix, as discussed in Section 3.1, we achieve this directly by solving Optimization 4.

Table 3.1 depicts the total production cost of the AC optimal power flow using different methods. The first column relates to MatPower, the second to Optimization 4 and the last one to Optimization 3. The total production cost is the same irrespective of the method used (i.e. Optimization 3, MatPower and Optimization 4). The advantage, however, of using Optimization 4 is that we still solve a convex program, and automatically obtain a rank-one solution. This is beneficial when applying Algorithm 3 for reserve scheduling, since the solutions that we get are of rank one; when applying Optimization 3 in conjunction with the reserve scheduling algorithm we do not necessarily get a rank-two solution which we can rotate a posteriori. This was the case for all problem instances that we examined, however, the theoretical guarantees of Optimization 4 are valid only when the requirements of Theorem 1 are satisfied.

Figure 3.2 shows active and reactive power of each generating units. The "Red" color corresponds to MatPower, the "Green" color to Optimization 4 and the "Blue" color to Optimization 3. All methods produce the same results in terms of active and reactive power productions.

The complex voltage of every node of network in terms of magnitude and angle is shown in Figure 3.3. The magnitude of voltage in every bus

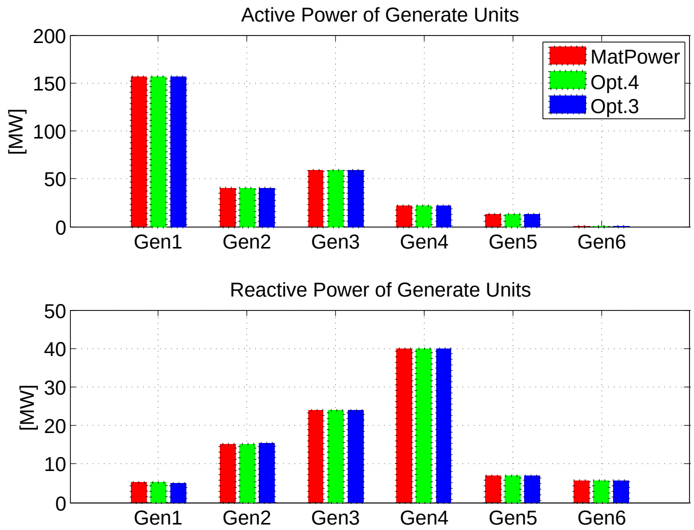

Figure 3.2. Active and Reactive Power of each Generating Units. The "Red" color corresponds to MatPower, the "Green" color to Optimization **4** and the "Blue" color to Optimization **3**.

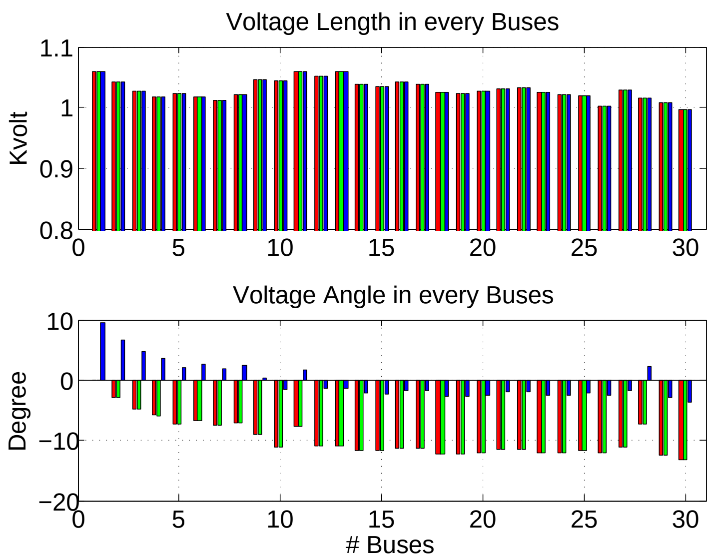

Figure 3.3. Magnitude and Angle of Bus Voltage. The "Red" color corresponds to MatPower, the "Green" color related to Optimization **4** and the "Blue" color to Optimization **3**.

of the network are exactly same. On the other hand, voltage angle of every bus of the network in MatPower program and Optimization 4 are same and voltage angle of bus with Optimization 3 is completely different with others. Since in Optimization 4 we fix the reference angle as in MatPower we get same results. Based on Optimization 3 we do not specify the reference bus voltage and the results are based on the posterior method.

AC Power Flow Based Reserve Scheduling Results

We then illustrate the efficiency of Algorithm 3 for solving reserve scheduling problem by comparing it against its DC power flow counterpart represented by Algorithm 2 in chapter 2.

We generated 10000 wind power realizations, different from the ones used in the optimization process, and evaluated the solution of each optimization problem (Algorithm 3 and its DC counterpart Algorithm 2) against those scenarios. Specifically, we seek to determine an empirical estimate of the probability of constraint violation. We first solve the corresponding optimization problems and then insert the obtained solution to the MatPower. According to the solution of the MatPower for each case, the empirical probability of constraint violation was computed as the fraction of the evaluation scenarios that resulted in violating at least one of the problem constraints.

The forecast and actual wind power values for all hours are retrieved from a data-set corresponding to the aggregated wind power production of Germany over the period 2006-2011. We assumed that the wind power is the sum of a deterministic component (forecast) and a stochastic one, which models the error between the forecast and the actual wind power. To generate scenarios for the wind power error, we employed the approach of [41], and developed a Markov chain-based model that produces wind power time series taking into account the temporal correlation of the wind power error. For more details about wind power model, the reader is referred to section 2.1.4.

The empirical probability of constraint violation for Algorithm 3 and its DC counterpart are shown in Figure 3.4. The "green" color corresponds to the results of the DC power network model, and the "red" color to the results of proposed Algorithm 3, whereas Figure 3.5 shows the amount of power with which the constraints are violated, corresponding the worst case violation encountered. The "upper" panel corresponds to the results of the DC power network model, and the "lower" panel to the results of proposed Algorithm 3. The results imply that the model mismatch (in the DC case we assumed that all voltages are 1p.u. and ignoring the active power losses) has a major effect when evaluating the solution on a more realistic simulation environment (MatPower). However, Algorithm 3, that employs an AC optimal power flow, leads to an improved performance with less frequent constraint violations.

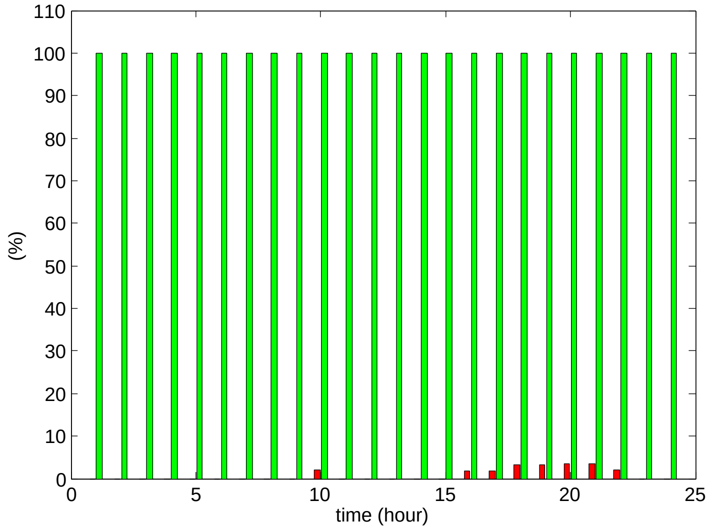

Figure 3.4. Empirical probability of constraint violation. The "green" color corresponds to the results of the DC power network model, and the "red" color to the results of proposed algorithm 3 .

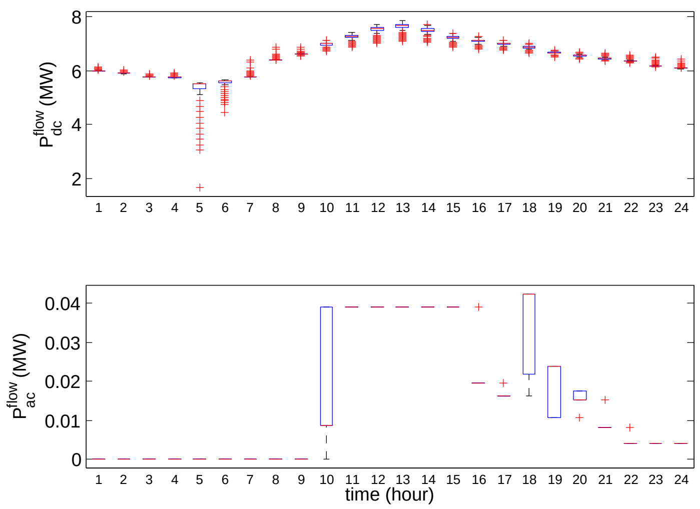

Figure 3.5. Amount of power with which the constraints are violated, corresponding the worst case violation encountered. The "upper" panel corresponds to the results of DC power network model, and the "lower" panel to the results of proposed algorithm 3 .

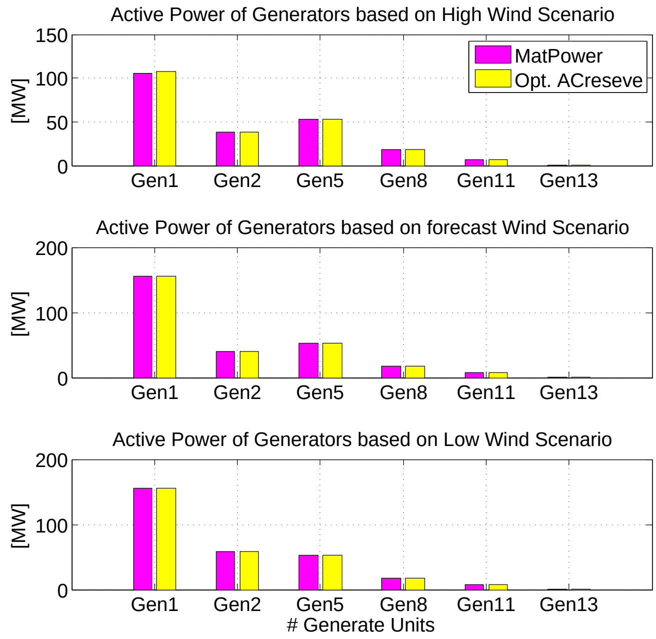

Figure 3.6. Active Power Generating Units. The "Red" color relates to the MatPower and "Yellow" color corresponds to the proposed Algorithm 3.

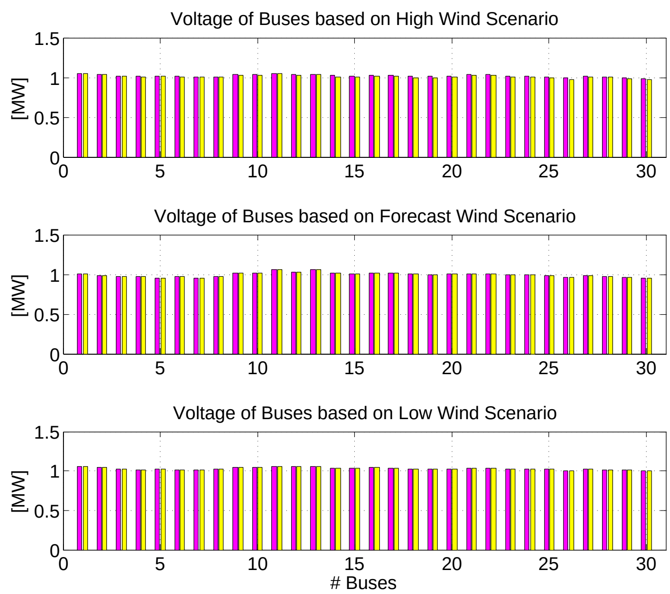

Figure 3.7. Voltage Magnitude of each Bus. The "Red" color relates to the MatPower and "Yellow" color corresponds to the proposed Algorithm 3.

To check applicability of the proposed Algorithm 3, we first get the results and then insert the obtained solution to the MatPower. We impose the min, max and forecast value of wind power in bus 22 as negative load. Figure 3.6 shows active power generating units. The "Red" color represents the solution obtained by MatPower and the "Yellow" color corresponds to the proposed Algorithm 3. Figure 3.7 shows the voltage magnitude for every bus of the network. The "Red" color represents the solution obtained by MatPower and the "Yellow" color corresponds to the proposed Algorithm 3. Each figure has three different sub-figures which show the solutions obtained by MatPower with different wind power (high, forecast, low) in-feed. The results show that the proposed algorithm has same solution as the MatPower power flow simulation.

Chapter 4

Concluding Remarks

In this thesis we introduce different reserve scheduling formulations to support the integration of renewable energy sources (in particular, wind generation) into the electricity grid.

A unified framework for solving the problem of stochastic reserve scheduling and unit commitment problem (deciding which generators to switch on at what time) is proposed in chapter 2. We provide probabilistic guarantees following a methodology based on a combination of randomized and robust optimization, and compare the proposed algorithm against a benchmark approach based on the deterministic variant of the unit commitment problem. Moreover, we incorporate affine-policy for the reserves in the developed scheme, and investigate the potential improvements.

In chapter 3 we extend the developed algorithms to AC based power flow models. The proposed approach to address this problem involves introducing a novel relaxation method to obtain a convex optimization problem and investigating its connection with the the original problem.

The theoretical developments of the aforementioned problems are validated on realistic benchmark networks [2], and we discuss under which conditions the resulting optimization problems are tractable. One future research direction would be to extend the work so as to include storage dynamics, as in [55]. Current work concentrates toward investigating the potential of substituting the heuristic relaxation-based algorithm dealing with AC based power flow models with a more sophisticated scheme, enabling one to inherit probabilistic performance guarantees regarding the satisfaction of the system constraints. Another interesting possible future work is to determine conditions under which the requirements of Theorem 1 of Chapter 3 can be relaxed.

Appendix A

Compact Format

We state here the optimization problem presented in Chapter 2 in a more compact format.

$$\boldsymbol{x} = \begin{bmatrix} x_1^{\mathrm{T}} & x_2^{\mathrm{T}} & \dots & x_{n_t}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n_t(n_x)} \tag{A.1}$$

$$\boldsymbol{u} = \begin{bmatrix} u_1^{\mathrm{T}} & u_2^{\mathrm{T}} & \dots & x_{n_t}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n_t(n_u)} \tag{A.2}$$

where for every $t = 1, \dots, n_t$

$$x_t = \begin{bmatrix} \mathbf{P}_{g,t}^{\mathrm{T}} & \mathbf{z}_t^{\mathrm{T}} & \mathbf{d}_t^{ds\,\mathrm{T}} & \mathbf{d}_t^{us\,\mathrm{T}} & \mathbf{C}_t^{su\,\mathrm{T}} & Q_t^{ds} & Q_t^{us} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{5n_g+2} , \tag{A.3}$$

$$u_t = \begin{bmatrix} \sigma_1 & \dots & \sigma_{n_g} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n_g} \tag{A.4}$$

and for every $i = 1, \dots, n_g$

$$\sigma_i \in \{0, 1\} \tag{A.5}$$

Mixed Integer Quadratic Optimization In this step consider to the following problem

$$\begin{gathered} \min_{\boldsymbol{x},\boldsymbol{u}} \boldsymbol{J}(\boldsymbol{x}) \qquad \text{subject to:} \\ \mathbb{P}\left(\ \delta \in \Delta \mid \boldsymbol{A}(\delta)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}(\delta) \geq 0\ \right) \geq 1 - \epsilon\ . \end{gathered} \tag{A.6}$$

where $\boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}} \boldsymbol{C}_2\, \boldsymbol{x} + \boldsymbol{C}_1^{\mathrm{T}} \boldsymbol{x} + \boldsymbol{C}_0$ and its components have the following structure

$$\boldsymbol{C}_2 = \begin{bmatrix} C_{2,1} & 0 & \dots & 0 \\ 0 & C_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_{2,n_t} \end{bmatrix} \in \mathbb{R}^{n_t(n_x)\times n_t(n_x)} \tag{A.7}$$

$$\boldsymbol{C}_1 = \begin{bmatrix} C_{1,1}^{\mathrm{T}} & C_{1,2}^{\mathrm{T}} & \dots & C_{1,n_t}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n_t(n_x)} \tag{A.8}$$

$$\boldsymbol{C}_0 = \mathbf{1}_{1\times n_t(n_g)}\ \boldsymbol{c}_0 \tag{A.9}$$

$$\boldsymbol{c}_0 = \begin{bmatrix} c_0^{\mathrm{T}} & c_0^{\mathrm{T}} & \dots & \dots & c_0^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n_t(n_g)} \tag{A.10}$$

and for every $t = 1, \dots, n_t$

$$C_{1,t} = \begin{bmatrix} c_1^{\mathrm{T}} & \mathbf{1}_{1\times n_g} & \mathbf{0}_{1\times n_g} & \mathbf{0}_{1\times n_g} & 1 & 1 \end{bmatrix}^{\mathrm{T}} \tag{A.11}$$

$$C_{2,t} = \begin{bmatrix} [c_2] & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \tag{A.12}$$

and $c_0, c_1, c_2 \in \mathbb{R}^{n_g}$ are vectors containing the cost of each generating unit for every MW per hour and $[c_2]$ denotes diagonal matrix with vector c_2 on the diagonal.

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