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Performance evaluation of production lines with unreliable batch machines and random processing time

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*A*bstract

The thesis considered a production flow line system consists of machines that produce in batches and is characterized with random processing time and multiple failure modes. An exact analytical method for the performance evaluation of two-machine building blocks is presented as well as a decomposition method for the performance evaluation of long production lines. The key feature of the proposed method is the capability to deal with machines that produces multiple pieces in a time and also considering the impact blocking and starvation caused by breakdown and long processing time. The proposed method also present a different modeling of remote failures considering the mechanism of the phenomenon. Simulation and numerical experiments are carried out to show the performance of the proposed method in comparison with simulation and existing ones. Characteristics and behavior of batch production lines are also analysis and discussed. A case study is also carried out to illustrate the application of proposed method into real case.

Abstract

La tesi considerato un sistema di linea di flusso di produzione consiste di macchine che producono in batch ed è caratterizzato da tempi di elaborazione casuale e molteplici modalità di guasto . Un metodo esatto analitico per la valutazione delle prestazioni di blocchi di due macchine si presenta così come un metodo di decomposizione per la valutazione delle prestazioni delle linee di produzione lunghe . La caratteristica fondamentale del metodo proposto è la capacità di trattare con le macchine che produce più pezzi in un tempo e considerando anche l'impatto di blocco e la fame causata dalla composizione e lungo tempo di elaborazione . Il metodo proposto presenta anche una diversa modellazione di guasti remoti considerando il meccanismo del fenomeno . Simulazione e esperimenti numerici sono effettuati per mostrare le prestazioni del metodo proposto in confronto con la simulazione e quelli esistenti . Caratteristiche e comportamento delle linee di produzione dei lotti sono anche l'analisi e discussi . Un caso di studio viene effettuata anche per illustrare l'applicazione del metodo proposto nel caso reale.

Acknowledgements

I would be thankful to my supervisor professor Marcello Colledani for his help (and patience) during the period of my study. It is him who helped me concrete my rough idea and provided me numerous valuable advices. Also I should not forget my parents who let me stay two year here in Italy and support me always whatever I did. For us it is a pain to stay away from each other. Moreover, I would like to say thank you to professor Chanaka and doctor Andrea for their helps on the modeling, writing and many other issues, without whom I cannot complete the thesis.

Yanxin Tu

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Chapter 1

Introduction

A production lines, or more formally called *manufacturing flow line* system, is a tandem system that consists of a serial of work areas and storage areas among which materials are transferred. In such a system material flows from work area to storage area to work area and it visits each work and storage area exactly once sequentially. There is a first work area through which material enters the system and a last work area through which it leaves. Figure 1.1 depicts a flow line system consists of 4 machines and 3 buffers.

If machine behavior were perfectly predictable and regular, there would be no need for buffers. However, the work area can be subject to some random failures which results in the unavailability of this work area. Some work areas require an unpredictable, or predictable but not constant, amount of time to complete their

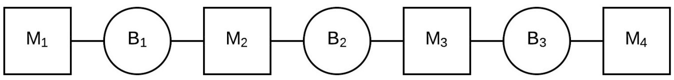

FIGURE 1.1: An example of four-machine flow line system

operations. This unpredictability or irregularity has the potential for disrupting the operations of adjacent machines, or even machines further away, and buffers are used to reduce the potential impact of this unpredictability or irregularity.

Since this is a tandem system, materials are not allow to skip the unavailable work area and reach other work areas which remain available meanwhile. And those work areas which remain available will not be affected (at least in a short period) by the unavailable work area, which means the system is unsynchronized and the production resources are not constrained to start or stop their operations at the same instant. Due to the previous reasons, when a failure occurs, or when a machine takes an exceptionally long time to complete an operation, the level in the adjacent upstream storage area may rise and the level in the adjacent downstream storage area may decrease. If the disruption persists long enough, the upstream storage area fills up and force the machine upstream of it to stop processing. Such a forced down machine is called *blocked*. And correspondingly, if the disruption persists long enough, the downstream storage area gets empty and force the machine downstream of it to stop processing. Such a forced down machine is called *starved*. These effects propagate up and down the line if repair is not done promptly.

By supplying both workpieces and space for workpieces, interstage buffer storages partially decouple adjacent machines. Therefore the effect of a failure or a exceptionally long processing time of one of the machines on the operation of others is mitigated by the buffer storages. When storages are empty or full, however, the decoupling effect cannot take place. Thus, as storage sizes increase, the probability of storages being empty or full decreases and the effects of failures on the production rate of the system are reduced. However, buffer size has its own cost: it brings together the increase *work-in-process* (WIP). As buffer sizes increase, more partially completed material is present between processing stages.

Line inventory or WIP is undesirable because:

1. it costs money to create, but as long as it sits in buffers, it generates no revenue (the opportunity cost).

2. the average lead time or the average amount of time between when an item is ordered and when it is delivered is increased. This is because of the *Little's law* which states that the average lead time of a flow system is proportional to the average amount of inventory in the system. The increase in average lead time is undesirable because

   (a) customers do not want to wait; and

   (b) if there is a problem in production, this much time elapses before the problem is found. During that time, many faulty pieces are made; and

   (c) if the line is supposed to be reconfigured to meet the new market needs, it takes more time to clean up the line and launch the new production.

3. inventory in a factory or a warehouse are vulnerable to damage or shrinkage (theft). The more items, the more time they spend, the more vulnerable they are.

4. the space and the infrastructure and labor force for inventory costs money.

In addition, there are some general assumptions that we took into account in the modeling of the system:

1. *Conservative system* In the system there is no creation of workpieces except the entrance of the line and there is no destroy or removal of workpieces in any point of the line and workpieces are not allow to exit the system except they finish processing in the last work area of the line.

2. *Reusable workpieces* If one of the work area becomes not available (thus subject to a failure) the workpieces in the machine is returned to its previous buffer and is regarded as new workpieces without significant inspection time.

3. *Unlimited repair personnel* In general, we assign a repair rate or probability to each machine, which is independent of the state of the rest of the system. But in real case repair staff is assigned to a group of machine and the repair could be constrained by the availability of repair personnel. Essentially, this means that the repair process at teach machine depends only on the characteristics of the machine, and not on any system-wide properties.

4. *Uncorrelated failures* This assumption is analogous to the assumption of unlimited repair personnel. Each machine's failure process is assumed to be independent of the state of the rest of the system. This excludes such events as a power failure or loss of coolant that affects the whole line, a shipment of poor quality raw parts, which causes failures of machines, or a tool that is so worn that it degrades workpiece quality badly enough to cause excess wer on the next tool. In that case, both tools are likely to wear out together, and thus precipitate machine failures.

5. *Operational dependent failures* It means that during a time unit when machine is working, it is subject to failures. When a machine is not working, due to starvation or blockage, it cannot failure. The mean time to fail (MTTF) is on working time (not clock time) while mean time to repair is on clock time.

6. *Perfect yield* Quality is not reated in the models presented here. All parts are assumed perfect. There is no inspection procedure, no rework, and no reject. There phenomena are certainly important, but it is not discussed here as quality issue is not the major focus of our study.

7. *The first machine is never starved and the last is never blocked* This is a widespread assumption in the transfer line literature. In reality, vendors sometimes failure to deliver, and sales are sometimes less than expected. This could be remedied by describing a model with a random arrival process to the first machine and a random departure process from the last machine. An easier approach would be to use the first machine in the model to represent the arrivals of materials. In that case, the first buffer in the model represents the raw material inventory of the factory, and the second machine of the model represnets that first machine of the line. Similarly, the last machine of the model could represent the demand or sales process, and the last buffer is the finished goods inventory. The next-to-last machine of the model is the last machine of the line. Either way, in a modeling point of view, is equivalent to the model that takes the assumption.

8. *Blocking before service, BBS* There are two check points during the production process of a machine, one before production start and one after production ends. Starvation is always checked in the first check point since if there is no enough pieces in the upstream buffer it is not possible to begin processing. But blocking can be checked in both. When performed in the first check point, machine do not start processing unless it makes sure that there is enough space in downstream after it finish this batch. When performed in the second check point, machine start processing as soon as there is enough pieces in upstream

buffer and if it identifies that there is no enough spaces in downstream buffer, the processed pieces remains in the machine until enough spaces is available in downstream buffer.

In real cases, the work areas (production resources) are usually called machines. Storage areas are often called buffers. The material in most cases consists of discrete parts. It is usually assumed that there is only a single kind of material in the system so that each piece of material travels the same sequence of machines and buffers, but each may experience different delays at each point in the system. A major example of the use of transfer lines is in the high volume production of metal parts of automobiles, but flow lines can be found throughout manufacturing industry.

the time that parts spend in the production system could be random due to the random availability of the machine or the random processing time or the random time spend in each storage. And this is the only randomness in the system. Performance analysis of this kinds of production line is important for the design, operation and management of manufacturing systems. Previous studies in industrial engineering and operations research indicate that planning appropriate decision tools based on production line models (or service line models or queueing network models) with finite buffers is of great economic importance.

Effective performance evaluation is of even greater importance in the contemporary context of *reconfigurable manufacturing system*. *Reconfigurable manufacturing system* is a machining system which can be created by incorporating basic process modules - both hardware and software - that can be rearranged or replaced quickly and reliably. Reconfiguration will allow adding, removing, or modifying specific process capabilities, controls, software, or machine structure to adjust production capacity in response to changing market demands or technologies. This type of system will provide customized flexibility for a particular part family, and will be open-ended, so that it can be improved, upgraded, and reconfigured, rather than replaced. But in the mean time, frequent reconfiguration of the production system need to be carefully planned and optimized to reach multiple economic and production objectives, which out-stands the importance of appropriate decision tools based on production line models.

Effective performance evaluation is also of importance in terms of studying teh interactions of manufacturing stages and their decoupling by means of buffers. By studying these phenomenon we may learn something useful that we can apply to more complex system. Due to its importance, a wide variance of configuration model of manufacturing flow line productions systems ha been studied. These analysis of production systems can be primary categorised into the study of systems with reliable machines and with unreliable machines, both with finite and infinite buffer capacities. Constant cycle times are more suitable for automated production lines, whereas random processing times are more applicable to manual operations like inspection. For manual inspection systems, the inspection time varies as a function of the special requirements of each item or batch, operator's skill, and other factors. Moreover, the model can be further categorized into single-item machines which process only one item a time and batch machines which process many items simultaneously. (See 1.2)

1.1 Problem description

Our model focuses on performance evaluation of production lines with unreliable batch machine which can be subjective to multiple failure modes and characterized

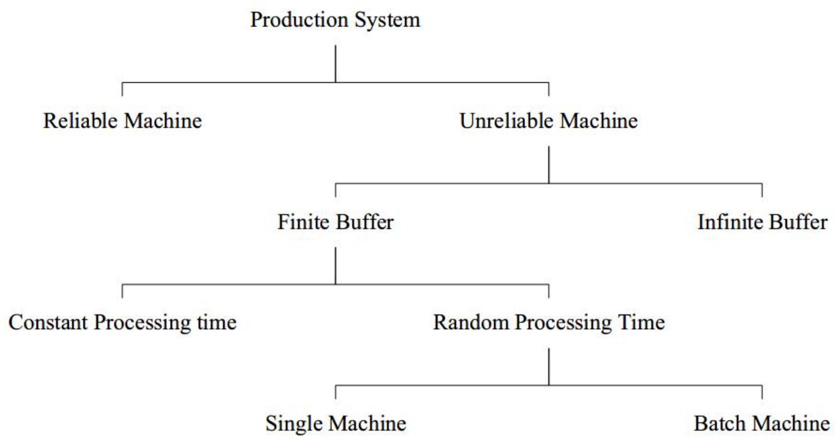

FIGURE 1.2: Categorisation of manufacturing flow line systems in performance evaluation

by random processing time. Our models is motivated by potential applications in the areas of inspection and packaging processes [1], semiconductor manufacturing processes [2], computer aided manufacturing [3], transportation systems [4], and any other systems or processes in which customers (or items in a factory) are shipped to a service area (or machine) in batches, obtain the service they require, and then move on to other service areas in accordance with a certain pre-determined policy or leave the system in batches.

In semiconductor wafer fabrication processes, furnaces or deposition processes accumulate jobs in a buffer, and then process them as a batch of predetermined size. In the burn-in operation in manufacturing processes of very large-scale integrated (VLSI) circuits, VLSI chips are usually processed in batches. In computer aided manufacturing, items to be processed are coded and collected into groups prior to processing . In transportation problems such as dispatching, products arriving at a station are dispatched in batches by a vehicle. About 78 percent of all manufacturing activities in the United States fall into the classification of batch production, with batch sizes ranging from less than 10 to many thousands [1]. Other examples include inspection and packaging equipment/operators, automated guided vehicles in material handling systems, vehicles in dispatching stations, etc. However, stochastic production lines involving batch processing and a finite buffer are, in general, difficult to analyze.

To model these production processes, we adopt exponential distributions for modeling the service times of each machine since an exponential distribution would seem to be a plausible representation of these types of service situations. Exponential assumption is also adopted to model the behavior of machine failure and repair (failure rate and repair rate) since it is a common practice.

Few studies has been carried out in the area of production lines of batch machines. Change and Gershwin addressed this issue for performance evaluation of two-machine lines (Change and Gershwin, 2010) and our model extends the application to multiple machine lines. The purpose of this paper is therefore (1) to present a effective model of these systems and its exact analysis; (2) to present qualitative and quantitative insights and interpretations of system behavior.

Chapter 2

Literature Review

Since performance evaluation is important for the design, operation and management of production system, a large number of works have been dedicated to different kinds of model and various configuration of the manufacturing flow line system. The great variety of models in part reflects the variety of different kinds of systems; in part it reflects the fact that different models lend themselves to analysis more or less easily for different purposes. Take the automotive paint shops as an example [5], the paint shop is a system bottleneck in many automotive assembly plants due to complexity inherent in the process, production control policies and rigorous quality requirements. In automotive paint shops, rework loops are often required when a job needs multiple passes or is defective. Jobs can enter the painting booths multiple times, either for repaint or for "tutone" operation (i.e., to have different colors painted). Such production line can be modeled as an variance of manufacturing flow line system and the rework of painting process due to defectiveness can be modeled as a failure mode of the machine.

But meanwhile, there is not so much attention on batch machine model. Few models are presented for production line consists of batch machines and the discuss is limited to batch two-machine line. Therefore we limit the focus of this chapter to present an overview of important precedent models that is related to the batch production model.

Firstly, from the model point of view, we present a categorization of these production system models with 5 dimensions.

1. material flow: depends on the application, three different type of material flow modeling exists: single-item which means machines produce a single piece each time; batch which means machines produce multiple pieces each time and continuous which means the production is not discrete.

2. machine parameter: different assumptions are taken to model various productions systems especially in term of modeling machine production rate. Some typical assumptions adopted are deterministic, exponential and Bernoulli.

3. number of failure modes, due to the complexity of the application, some models are presented with only one failure mode each machine while some other models allow multiple failure modes.

4. number of machines in the line: since the exact modeling of long line is hardly achievable, almost all the models consider alternative approach of decomposition. Therefore it split the model of the line into two steps: the exact modeling of basic two-machine model and the decomposition model for a long line.

5. additional features of the line: some additional features that is specific to certain production system are presented, such as paralleled machine, multiple types of products, non-exponential repair rate and open/close loop system.

Note that some assumptions, like unreliable machines and finite buffer size are not mentioned in the categorization since they are so common that appears in every model. Some other assumptions, such as time dependent or operational dependent failure and blocking before service or after service is not included because they usually do not have significant impact on modeling.

Table 2.1 show an categorization of recent works on modeling and performance evaluation of manufacturing flow line system on the basis of aforementioned frameworks.

Gershwin [6] and Dallery [7], [8] presented an method for evaluating performance measures for a class of tandem queueing systems with finite buffers which took into count blocking and starvation. These models deal with machines with deterministic processing time and single failure mode on each machine. S. Yeralan and B. Tan [9] provided a computational algorithm as an realization of this method. Yang, Sheng et al. [10] extends the Gershwin's model to machine with exponential processing time but still limited by single failure mode.

In 2002 Tolio [11] presented a model of two-machine lines that is capable to deal with multiple failure modes and soon R. Levantesi et al. [12] extended the basic two-machine line into multi-stage system. This model also considered machines that produces continuous products like liquid and gas. Levantesi, R. et al. [13], [14] then further explored the model of exponential machine lines with multiple failure modes.

Colledani, M. [15] considered the model of machine lines with deterministic processing time but produces two different types of products. Li and Meekov [16] presented the same model of deterministic processing time but taking into account that the repair rate is not necessarily exponential. This topic is further developed by Colledani

Paper	Materials	num. machines	machine parameter
Tolio, 2002	discrete	two	deterministic
Levantesi, 2003	discrete	multiple	deterministic
Levantesi, 2003	discrete	multiple	exponential
Colledani, 2005	discrete	multiple	deterministic
Li, 2005	discrete	multiple	homogeneous
Vuuren, 2006	discrete	multiple	exponential
Gershwin, 2007	discrete	multiple	deterministic
Barış, 2009	continuous	two	exponential
Alexandros, 2009	continuous	two	exponential
Chang, 2010	discrete	two	exponential
Colledani, 2011	discrete	multiple	homogeneous
Colledani, 2013	continuous	multiple	exponential
John Benedict, 2013	discrete	two	exponential

Paper	num. failure	other features
Tolio, 2002	multiple	
Levantesi, 2003	multiple	
Levantesi, 2003	multiple	
Colledani, 2005	multiple	two products
Li, 2005	multiple	non-exponential failure rate
Vuuren, 2006	none	parallel machines
Gershwin, 2007	multiple	close loop
Barış, 2009	multiple	
Alexandros, 2009	multiple	parallel machines
Chang, 2010	multiple	batch machine
Colledani, 2011	multiple	non-exponential failure rate
Colledani, 2013	multiple	
John Benedict, 2013	multiple	batch machine

Table 2.1: Categorization of recent works

and Tolio [17] to more general cases. Vuuren, Marcel et al. [18] treated with the machine lines with exponential processing time and multiple machines operates in parallel in one station. Gershwin and Werner [19] deals with a close loop system that consists of unreliable machines with deterministic processing time.

Barış Tan and Gershwin [20] presented a more general two-machine model of machine line with continuous products. Colledani and Gershwin [21] extended this solution to the decomposition of long line. Alexandros and Chrissoleon [22] considered a two-machine model of continuous production line with paralleled machines.

In terms of batch machine, Chang and Gershwin [23] presented an modelling of two-machine line with exponential processing time and multiple failure modes. John Benedict et al. [24] also discussed on this two-machine model with incompatible job families. But till now there is no model developed on the performance evaluation of long production lines in which machines product in batches.

Chapter 3

Model Description and Assumptions

This method considers tandem transfer lines consisting of multiple unreliable machines that produce in discrete batches and are separated by buffers of finite capacity. A discrete flow of material from outside is supposed to enter the system to process starting from the first machine, then visits first buffer, then visits the other machines and buffers sequentially before leaving the system. No workpieces could be destroyed or removed at any stage of the transfer line and there is not partly processed workpieces. In case a machines fails during processing, all workpieces in this machine are returned to previous buffer and are treated as new workpieces.

Each machine is capable to process a certain amount of workpieces simultaneously and the capacity is called batch size. A machine is only allowed to process when it loads an amount of pieces exactly equals its batch size but not lower, or produces in full batch size (defined by Gershwin [23]). It takes each machine a random amount of time to process a batch and this process time is shared among all pieces in the batch. Each machine can also be affected by a set of different and random failures. It is assumed that the machine is subject to failures only when it is operational, or operation dependent failures (ODFs). So a machine idle or failed cannot experience a failure before it returns to work. If a machine fails in a certain failure mode, a repair process starts immediately which also takes a random amount of time to complete. This means a machine cannot fail in more than one mode at the same time. Therefore, each machine is characterized by a set of random variables, namely the processing time and for each failure mode, the mean time between failures(MTTF) and the mean time to repair(MTTR) and these random variables are all assumed to be exponentially distributed.

When a machine is available to process a batch but there is not enough pieces in the upstream buffer, the machines is said to be *starved*. While if there is not enough space in the downstream buffer, it is said to be *blocked*. Blocking and starvation mainly occur because of failures in the portion of the line upstream or downstream the machine under examination. However, it may also happen when none of the upstream and downstream machine fails, which is due to the difference and randomness in processing times of various machines. For example, when a certain machines inside the line takes a long time to process a piece, the upstream buffer tends to become full while the downstream buffer tends to become empty. If the phenomenon persists, it will finally lead to block in upstream and starve in downstream. In addition, it is assumed that there are always available material at the input of the system and always available storage space at the output , therefore the first machine never gets starved and the last machine never blocked.

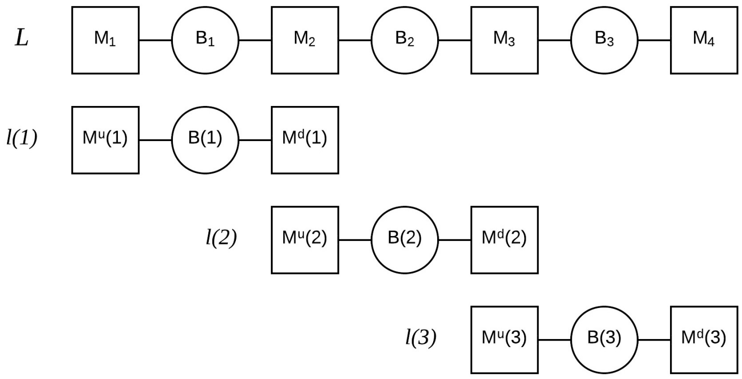

FIGURE 3.1: Decomposition of a four-machine transfer line system

3.1 Outline of the approach

The approach is actually an extension of Gershwin decomposition method. Detailed descriptions can be found in the papers of Gershwin [6], Choong and Gershwin [25], Dallery et al. [8] and Tolio et al.[11] The rationale of the decomposition approach is to decompose the original transfer line L made of K machines into a set of two-machine lines $l(i)$, for $i = 1, \ldots, K-1$, as shown in fig. 3.1. Each two-machine line, called building block, is characterized by a buffer $B(i)$ of the same size of the corresponding buffer of the original line, an upstream pseudo machine $M^u(i)$ and a downstream pseudo machine $M^d(i)$. With the upstream and downstream pseudo machines modeling the behavior of the portion of the original line upstream and downstream of buffer B_i, or in other words, modeling how workpieces enter and leave buffer B_i, the decomposed building blocks are supposed to reproduce the dynamic behavior of corresponding machines observed in original line. Therefore the performance and characteristics of the original line can be derived by solving the building blocks.

Therefore the main difficulty here is to find out the unknown parameters of building blocks. Computation algorithms, referred to as Dallery–David–Xie (DDX) algorithms (Dallery et [8]) and the accelerated DDX (ADDX, Burman [26]) algorithm, have been introduced to solve these equations. The main idea of the algorithm is to repetitively use the result of previous building blocks to generate the parameters of current building blocks until all parameters converge. For example, in the four-machine transfer line system shown above in fig. 3.1, the algorithm takes the following steps:

1. set initial values for all parameters in the system;
2. calculate the performance of building block $l(1)$ and update the parameters of building block $l(2)$;
3. calculate the performance of building block $l(2)$ and update the parameters of building block $l(3)$;
4. calculate the performance of building block $l(3)$ and update the parameters of building block $l(2)$;
5. calculate the performance of building block $l(2)$ and update the parameters of building block $l(1)$;
6. repeat the previous steps until the performance measures converge.

Since Gershwin decomposition and DDX algorithm is existing approach that have been adopted in many analysis, the main challenge for our model is to represent the characteristics of random processing time and production batch in such a long production line. To be more specific, the model is dedicated to solve the modeling of the behavior of buffer considering different batch size of upstream and downstream machines and the modeling of exponential processing time in a long production line.

As a conclusion, the key of the approach is to find out proper modeling of the behavior of building blocks, extend it efficiently to production line and adopt DDX-like algorithm to find out the unknown parameters of the system. Therefore we can split the approach into two steps:

1. modeling of building blocks as a two-stage production lines system;

2. modeling of long lines using the result of building block model.

The detailed modeling will be discussed in the following sections.

Chapter 4

Performance evaluation of building blocks

4.1 Model description and assumptions

In this section the proposed method focuses on two-stage production lines with unreliable batch pseudo machines decoupled by a buffer of finite capacity that represents a building block. The modeling of building blocks is a variance of the modeling of basic two-stage production line which is the basic element of a long production line. Therefore it inherits a lot of characters, behaviors and assumptions from the two-stage production line model. To be specific, the system is modeled as a continuous time, discrete state Markov process and is based of the following assumptions and behaviors.

Each part enters the system through the first machine then goes to the buffer waiting for the availability of the second machine and then leave the system.

A machine whose upstream buffer cannot satisfy its production batch is said to be *starved* while a machines whose downstream buffer does not have enough capacity to contain its production batch is said to be *blocked*. It is not allowed that a machine takes piece less than its production batch from its upstream buffer to process or a machine processes piece less than its production batch to be stored in its downstream buffer. It means that full batch policy is adopted. It is also assumed that there is always enough pieces before the upstream machine and there are always enough space after the downstream machine, therefore the upstream machine of the line is never starved while the downstream machine is never blocked.

Processing, failure and repair times for the machines are assumed to be exponential random variables with parameters representing, respectively, processing rate, failure rates and repair rates.

A machine can be in one of the following conditions: *operational*, *idle* or *failed in a certain mode*. An operational machines can be subject to all failure modes but an idle machine cannot fail in any mode. This means that operation dependent failures (*ODFs*) is assumed.

When a machines is operational, it works on a batch and it continues working until either it completes the batch or one of the different failures occurs. If a failure happens during processing, the pieces in the machine is returned to the upstream buffer. The machine checks the states of the downstream buffer before it loads pieces. That means the machines take blocking before service (*BBS*) control policy.

A machine can be subjected to two kinds of failure modes: local failure, representing the failure of the physical machine and remote failure, representing the down time caused by other components of the pseudo machine. A machine cannot fail in more than one mode at the same time. Also, a machine failed in a certain mode cannot experience a different mode before being repaired. It is also assumed that all failure modes are independent to each other.

When a machine is under repair, it remains in that state for a period of time which is exponentially distributed with parameters depending on the type of failure that occurred.

Workpieces cannot be destroyed or rejected or removed from the system at any stage in the line. Partly processed workpieces are not added into the line.

4.2 Notations

Before we start discussing the modeling, it is necessary to clarify the notations we take. The state of the system is given by (n, α_u, α_d) where n is an integer that indicates the number of pieces in the buffer plus the piece on the working area of the downstream machine while α_u and α_d indicate the state of upstream and downstream machine, respectively. The convention for blocking is $n = N - c_d + 1 \ldots N$, where c_d is the batch size of downstream machine and N is the buffer capacity. When the upstream machine is operational or idle $\alpha_u = 1$ and when $\alpha_u = u_i$, for $i = 1, \ldots, s$, it means that the upstream machine is down due to failure mode i. Similarly, α_d can assume that values $1, d_1, d_2, \ldots, d_t$. The steady state probability of the system being in state (n, α_u, α_d) is indicated by $p(n, \alpha_u, \alpha_d)$.

If the upstream machine is working on a batch at time t, during the time interval $(t, t+\delta t)$ it can complete the batch of c_i pieces (batch size) with probability approximately $\mu_u \delta t$ or it can fail in mode u_i with probability approximately $p_{u_i} \delta t$. If the upstream machine is failed in mode u_i at time t, it can be repaired during the time interval $(t, t + \delta t)$ with an approximate probability of $r_{u_i}\delta t$, for small δt. Similarly, the downstream machine is working on a batch at time t, during the time interval $(t, t + \delta t)$ it can complete the batch of c_{i+1} pieces with probability approximately $\mu_d \delta t$ or it can fail in mode d_i with probability approximately $p_{d_i}\delta t$. If the downstream machine is failed in mode d_i at time t, it can be repaired during the time interval $(t, t + \delta t)$ with an approximate probability of $r_{d_i}\delta t$, for small δt.

Since the failure modes are independent to each other, the total failure rate P_U of the upstream machine, i.e. the approximate probability of a failure during the time interval $(t, t+\delta t)$ regardless of the mode in which the machine fails, can be given by $P_U = \sum_{i=1}^{s} p_{u_i}$. Similarly, the total failure rate P_D can be given by $P_D = \sum_{i=1}^{s} p_{d_i}$.

4.3 Performance Measures

The efficiency is defined as the steady state probability that a machine is working on a piece and it can be seen as the fraction of time in which the machine processes parts. Therefore, for the upstream machine the operational probability can be given by:

$$E_u = prob\,[a_u = 1, n \leq N - c_u] = \sum_{n=0}^{N-c_u} \left[p(n,1,1) + \sum_{j=1}^{t} p(n,1,d_j) \right] \tag{4.1}$$

since the upstream machine, being the first machine of the line, cannot be starved.

Similarly, the efficiency of the downstream machine can be given by:

$$E_d = prob\,[a_d = 1, n \geq c_d] = \sum_{n=c_d}^{N} \left[p(n,1,1) + \sum_{u=1}^{s} p(n,u_i,1) \right] \tag{4.2}$$

since the downstream machine, being the last machine of the line, cannot be blocked.

On the basis of the efficiency, it is possible to evaluate the production (throughput rate of the upstream and downstream machines) as shown below:

$$P_u = \mu_u c_u E_u \tag{4.3}$$

$$P_d = \mu_d c_d E_d \tag{4.4}$$

In additional, since there is no workpieces created or destroyed or removed, as a result, it can be shown, extending to the case of multiple failures the proof given by Gershwin, that the part flow in the two machine line is conserved:

$$P_u = \mu_u c_u E_u = P_d = \mu_d c_d E_d \tag{4.5}$$

Finally, the average buffer level can be written as:

$$\bar{n} = \sum_{n=0}^{N} n \left[p(n,1,1) + \sum_{u=1}^{s} p(n,u_i,1) + \sum_{j=1}^{t} p(n,1,d_j) + \sum_{u=1}^{s} \sum_{j=1}^{t} p(n,u_i,d_j) \right] \tag{4.6}$$

4.4 Balance Equations

Since we have to deal with a continuous time, discrete space Markov process, to obtain the steady state distribution we make use of balance equations, which equate the rate of leaving a state with the rate of entering the state.

The complete state space can be defined by

$$\pi(n,1,1),\ \ for\ n=0,\ldots,N$$

$$\pi(n,1,d_j),\ \ for\ n=0,\ldots,N,j=1,\ldots,t$$

$$\pi(n,u_i,1),\ \ for\ n=0,\ldots,N,i=1,\ldots,s$$

$$\pi(n,u_i,d_j),\ \ for\ n=0,\ldots,N,i=1,\ldots,s,j=1,\ldots,t$$

and the corresponding steady probability of the state is defined by

$$\pi(n,1,1),\ \pi(n,1,d_j),\ \pi(n,u_i,1),\ \pi(n,u_i,d_j)$$

We distinguish four sets of equations on the basis of the different values of the state variables α_u and α_d. In addition, we define the states $\pi(n,\alpha_u,\alpha_d)$ where $c_d \le n \le N - c_u$ as *internal state*, which mean there is no blocking or starvation involved. Correspondingly, we we define the states $\pi(n,\alpha_u,\alpha_d)$ that is not internal state as *boundary state*. Obviously, they can be classified into two types: the upper boundary where $N-c_u+1 \le n \le N$ and the lower boundary where $0 \le n \le c_d - 1$. Moreover, we can further classify balance equations that do not involve boundary states as *internal equations*, while all the others are *boundary equations*.

The complete set of balance equations of internal states for the Markov chain that models the two-machine line can be given by:

$$p(n,u_i,d_j)(r_{u_i}+r_{d_j}) = p(n,u_i,1)p_{d_j} + p(n,1,d_j)p_{u_i} \tag{4.7}$$

$$\begin{aligned} p(n,u_i,1)(\mu_d + r_{u_i} + \sum_{j=1}^{t} p_{d_j}) = \sum_{j=1}^{t} p(n,u_i,d_j)r_{d_j} \\ +p(n,1,1)p_{u_i} + p(n+c_d,u_i,1)\mu_d \end{aligned} \tag{4.8}$$

$$p(n,1,d_j)(\mu_u + \sum_{i=1}^{s} p_{u_i} + r_{d_j}) = \sum_{i=1}^{s} p(n,u_i,d_j) r_{u_i} \\ + p(n,1,1)p_{d_j} + p(n-c_u,u_i,1)\mu_u \tag{4.9}$$

$$p(n,1,1)(\mu_u + \mu_d + \sum_{i=1}^{s} p_{u_i} + \sum_{j=1}^{t} p_{d_j}) = \sum_{i=1}^{s} p(n,u_i,d_j) r_{u_i} \\ + \sum_{j=1}^{t} p(n,u_i,d_j) r_{d_j} + p(n-c_u,u_i,1)\mu_u + p(n+c_d,u_i,1)\mu_d \tag{4.10}$$

where $c_d \leq n \leq N - c_u$

On the left side of these equations we have the rate at which the system leaves state $\pi(n,u_i,d_j)$, $\pi(n,u_i,1)$, $\pi(n,1,d_j)$ and $\pi(n,1,1)$. Take state $\pi(n,u_i,d_j)$ as an example, this happens whenever a machine, upstream or downstream, is recovered from a certain failure mode. Conversely, the right side stands for the rate at which the system enter state $\pi(n,u_j,d_j)$, $\pi(n,u_i,1)$, $\pi(n,1,d_j)$ and $\pi(n,1,1)$. Take state $\pi(n,u_i,d_j)$ as an example, this state can be reached either from state $\pi(n,1,d_j)$ if the upstream machine fails in mode i or from state $\pi(n,u_i,1)$ if the downstream machine fails in mode j. The behaviour is similar for state $\pi(n,u_i,1)$, $\pi(n,1,d_j)$ and $\pi(n,1,1)$, except that they have a machine operational, therefore the system can leave the state by producing a batch with processing rate μ_u or μ_d, it can also be reached by state with lower buffer level $n - c_u$ (representing that the upstream machine produces a batch) or by state with higher buffer level $n + c_d$ (representing that the downstream machine produces a batch).

In order to build the balance equations of boundary states for the Markov chain that models the two-machine line, we need to consider the following constraints:

1. If $n \leq c_d - 1$ the downstream machine is starved and given the assumption of operational dependent failures, cannot fail.

2. Similarly, when $n \geq N - c_u + 1$, the upstream machine is blocked and it cannot be affected by any failure mode.

Therefore the balance equations of upper boundary states for the Markov chain can be given by:

$$p(n, u_i, d_j)(r_{u_i} + r_{d_j}) = p(n, u_i, 1)p_{d_j} + p(n, 1, d_j)p_{u_i} \tag{4.11}$$

$$p(n, u_i, 1)(\mu_d + r_{u_i} + \sum_{j=1}^{t} p_{d_j}) = \sum_{j=1}^{t} p(n, u_i, d_j)r_{d_j} \tag{4.12}$$

$$p(n, 1, d_j)r_{d_j} = \sum_{i=1}^{s} p(n, u_i, d_j)r_{u_i} + p(n, 1, 1)p_{d_j} + p(n - c_u, u_i, 1)\mu_u \tag{4.13}$$

$$\begin{aligned} p(n, 1, 1)(\mu_d + \sum_{j=1}^{t} p_{d_j}) &= \sum_{i=1}^{s} p(n, u_i, d_j)r_{u_i} \\ &+ \sum_{j=1}^{t} p(n, u_i, d_j)r_{d_j} + p(n - c_u, u_i, 1)\mu_u \end{aligned} \tag{4.14}$$

where $N - c_u + 1 \leq n \leq N$

Similarly the balance equations of lower boundary states for the Markov chain can be given by:

$$p(n, u_i, d_j)(r_{u_i} + r_{d_j}) = p(n, u_i, 1)p_{d_j} + p(n, 1, d_j)p_{u_i} \tag{4.15}$$

$$p(n, u_i, 1)r_{u_i} = \sum_{j=1}^{t} p(n, u_i, d_j)r_{d_j} + p(n, 1, 1)p_{u_i} + p(n + c_d, u_i, 1)\mu_d \tag{4.16}$$

$$p(n, 1, d_j)(\mu_u + \sum_{i=1}^{s} p_{u_i} + r_{d_j}) = \sum_{i=1}^{s} p(n, u_i, d_j)r_{u_i} \tag{4.17}$$

$$
\begin{aligned}
p(n,1,1)(\mu_u + \sum_{i=1}^{s} p_{u_i}) = &\sum_{i=1}^{s} p(n,u_i,d_j) r_{u_i} \\
&+ \sum_{j=1}^{t} p(n,u_i,d_j) r_{d_j} + p(n+c_d,u_i,1)\mu_d
\end{aligned} \tag{4.18}
$$

where $0 \leq n \leq c_d - 1$

4.4.1 Modeling of remote failures

While the above equations are valid also for local failures of two-machine production line, in building block we need to consider also the remote failures. In building block, remote failures are a set of failure modes created to approximate the behaviour of the portion of the original line that is not directly connect with the buffer investigated. Remote failures takes the form of simple failure with a failure rate and a repair rate (Note that the real behaviour is far more complex which will be discussed in the following section.) and each remote failure rate represents the rate of which a failure of upstream/downstream machine of the original line or a starvation/blocking in upstream/downstream buffer of the original line could result in the stoppage of the physical machine that is directly connected to the buffer investigated. For example, in a four-machine line shown in fig. 4.1, the remote failures of building block $l(2)$ represent: (1) the failure of physical machine M_1 that results in the stoppage of physical machine M_2 (therefore the stoppage of pseudo machine $M^u(2)$); (2) the failure of physical machine M_4 that results in the stoppage of physical machine M_3(therefore the stoppage of pseudo machine $M^d(2)$).

The failure rate of remote failure are not equal to the real failure rate of the physical machine since to cause the stoppage of a remote machine, the stoppage time should be long enough to cause a starvation in downstream buffer or a blocking in

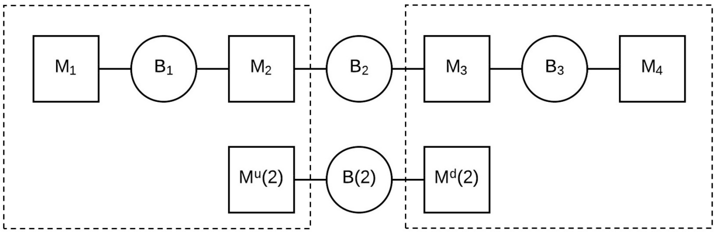

FIGURE 4.1: An example of remote failures in a four-machine line

upstream buffer. In other word, the remote failure modes are the spread to starvation/blocking status, caused by a breakdown or long processing time of physical machine. Therefore it is easy to understand that the event that the system enter a remote failure state always happens together with the event that the system changes the buffer level (produce a part in upstream/downstream machine, respectively) because by definition the machine checks the status of block or starvation only after they finish processing a batch. Therefore the complete set of balance equation with remote failure modes can be given by:
assume that $\alpha_u = u_i (i = 1, \ldots, s)$ and $\alpha_d = d_j (j = 1, \ldots, t)$ are all remote failures

$$p(n, u_i, d_j)(r_{u_i} + r_{d_j}) = p(n + c_d, u_i, 1)p_{d_j} + p(n - c_u, 1, d_j)p_{u_i} \tag{4.19}$$

$$\begin{aligned} p(n, u_i, 1)(\mu_d + r_{u_i}) &= \sum_{j=1}^{t} p(n, u_i, d_j) r_{d_j} \\ + p(n - c_u, 1, 1)p_{u_i} + p(n + c_d, u_i, 1)(\mu_d &- \sum_{j=1}^{t} p_{d_j}) \end{aligned} \tag{4.20}$$

$$\begin{aligned} p(n, 1, d_j)(\mu_u + r_{d_j}) &= \sum_{i=1}^{s} p(n, u_i, d_j) r_{u_i} \\ + p(n + c_d, 1, 1)p_{d_j} + p(n - c_u, u_i, 1)(\mu_u &- \sum_{i=1}^{s} p_{u_i}) \end{aligned} \tag{4.21}$$

$$
\begin{aligned}
p(n,1,1)(\mu_u+\mu_d) &= \sum_{i=1}^{s} p(n,u_i,d_j)r_{u_i} + \sum_{j=1}^{t} p(n,u_i,d_j)r_{d_j} \\
&+p(n-c_u,u_i,1)(\mu_u - \sum_{i=1}^{s} p_{u_i}) + p(n+c_d,u_i,1)(\mu_d - \sum_{j=1}^{t} p_{d_j})
\end{aligned}
\tag{4.22}
$$

where $c_d \leq n \leq N - c_u$

$$p(n,u_i,d_j)(r_{u_i}+r_{d_j}) = p(n+c_d,u_i,1)p_{d_j} + p(n-c_u,1,d_j)p_{u_i} \tag{4.23}$$

$$p(n,u_i,1)(\mu_d + r_{u_i}) = \sum_{j=1}^{t} p(n,u_i,d_j)r_{d_j} \tag{4.24}$$

$$
\begin{aligned}
p(n,1,d_j)r_{d_j} &= \sum_{i=1}^{s} p(n,u_i,d_j)r_{u_i} + p(n+c_d,1,1)p_{d_j} \\
&+p(n-c_u,u_i,1)(\mu_u - \sum_{i=1}^{s} p_{u_i})
\end{aligned}
\tag{4.25}
$$

$$
\begin{aligned}
p(n,1,1)\mu_d &= \sum_{i=1}^{s} p(n,u_i,d_j)r_{u_i} + \sum_{j=1}^{t} p(n,u_i,d_j)r_{d_j} \\
&+p(n-c_u,u_i,1)(\mu_u - \sum_{i=1}^{s} p_{u_i})
\end{aligned}
\tag{4.26}
$$

where $N - c_u + 1 \leq n \leq N$

$$p(n,u_i,d_j)(r_{u_i}+r_{d_j}) = p(n+c_d,u_i,1)p_{d_j} + p(n-c_u,1,d_j)p_{u_i} \tag{4.27}$$

$$
\begin{aligned}
p(n,u_i,1)r_{u_i} &= \sum_{j=1}^{t} p(n,u_i,d_j)r_{d_j} + p(n-c_u,1,1)p_{u_i} \\
&+p(n+c_d,u_i,1)(\mu_d - \sum_{j=1}^{t} p_{d_j})
\end{aligned}
\tag{4.28}
$$

$$p(n,1,d_j)(\mu_u + r_{d_j}) = \sum_{i=1}^{s} p(n,u_i,d_j)r_{u_i} \tag{4.29}$$

$$\begin{aligned} p(n,1,1)\mu_u = \sum_{i=1}^{s} p(n,u_i,d_j)r_{u_i} + \sum_{j=1}^{t} p(n,u_i,d_j)r_{d_j} \\ +p(n+c_d,u_i,1)(\mu_d - \sum_{j=1}^{t} p_{d_j}) \end{aligned} \tag{4.30}$$

where $0 \leq n \leq c_d - 1$

Finally, if we merge the balance equations for local failures and remote failures, we can derive the complete set of balance equations for the building blocks.

4.5 Solving algorithm

The solving methodology is to build the transition map Q of Markov chain using the previous equations. Since the exponential distribution is adopted in assumption, the Markov chain is a continuous time Markov chain and can be solved by the following system of equations:

$$\begin{cases} \pi\mathbf{Q} = 0 \\ \sum_{x_j \in \Omega} \pi_j = 1 \end{cases} \tag{4.31}$$

where Ω is the state space.

It is also possible that an extension to the case of multiple failure modes of the one originally proposed by Gershwin and Berman (REF) can be adopted. The first step consists of the analysis of the Markov chain and the formulation of a guess on the form of the steady state probabilities of the internal states. Then substituting this guess into the internal balance equations, a set of s+t+2 solutions (vectors that satisfy the internal equations) can be found. At this point, if the guess is correct it must be possible to find a linear combination of these internal solutions that also satisfied the boundary conditions. The advantage of this approach is that the computational effor depends only on the total number of failure modes taken in account and not on the buffer capacity N.

Chapter 5

Performance evaluation of production lines

5.1 Model description and assumptions

This method considers tandem transfer lines consisting of multiple unreliable machines that produce in batches and are separated by buffers of finite capacity. A discrete flow of material from outside is supposed to enter the system to process starting from the first machine, then visits first buffer, then visits the other machines and buffers sequentially before leaving the system. No workpieces could be destroyed or removed at any stage of the transfer line and there is not partly processed workpieces. In case a machines fails during processing, all workpieces in this machine are returned to previous buffer and are treated as new workpieces.

Each machine is capable to process a certain amount of workpieces simultaneously and the capacity is called batch size. A machine is only allowed to process when it loads an amount of pieces exactly equals its batch size but not lower, or produces in full batch size (defined by Gershwin [23]). It takes each machine a random amount of time to process a batch and this process time is shared among all pieces in the batch. Each machine can also be affected by a set of different and random failures. It is assumed that the machine is subject to failures only when it is operational, or operation dependent failures (ODFs). So a machine idle or failed cannot experience a failure before it returns to work. If a machine fails in a certain failure mode, a repair process starts immediately which also takes a random amount of time to complete. This means a machine cannot fails in more than one mode at the same time. Therefore, each machine is characterized by a set of random variables, namely the processing time and for each failure mode, the mean time between failures(MTTF) and the mean time to repair(MTTR) and these random variables are all assumed to be exponentially distributed.

When a machine is available to process a batch but there is not enough pieces in the upstream buffer, the machines is said to be *(*starved). While if there is not enough space in the downstream buffer, it is said to be *(*blocked). Blocking and starvation maily occur because of failures in the portion of the line upstream or downstream the machine under examination. However, it may also happen when none of the upsteam and downstream machine fails, which is due to the difference and randomness in processing times of various machines. For example, when a certain machines inside the line takes a long time to process a piece, the upstream buffer tends to become full while the downstream buffer tends to become empty. If the phenomenon persists, it will finally lead to block in upstream and starve in downstream. In addition, it is assumed that there are always available material at the input of the system and always available storage space at the output , therefore the first machine never gets starved and the last machine never blocked.

5.2 Machine parameters and performance measures

A machine M_i can be either operational, idle or failed in certain mode. When a machine is operational, it processes a batch of c_i pieces and it goes on working until either it completes the piece or a failure mode f occurs, whichever happen first. Either events can take place during the time interval $(t, t + \delta t)$ with probability approximately $\mu_i \delta t$ or $p_{i,f} \delta t$, for a small δt. If a machine is failed in mode f at time t, it can be repaired during the time interval $(t, t+\delta t)$ with probability approximately $r_{i,f} \delta t$.

Given the failure and the repair rates for each failure mode of machine M_i it is possible to obtain the *efficiency in isolation*, that is the fraction of time the machine would be productive if it was not subject to failure or blocking/starvation:

$$e_i = \frac{1}{1 + \sum_{f=1}^{F_i} \frac{p_{i,f}}{r_{i,f}}} \tag{5.1}$$

The *efficiency of the machine inside the line*, that is the fraction of time in which the machine effectively processes workpieces is given by:

$$E_i = e_i(1 - Ps_i - Pb_i) \tag{5.2}$$

taking into account the losses cased by blocking (Ps_i) and starvation(Pb_i).Note that this equation known in literature as $flowrate - idletime$ represents an approximation of operational time because it is based on the assumption that a machine cannot be starved and blocked at the same time.

On the basis of the efficiency it is possible to give the isolated production rate p_i and production rate in line P_i of machine M_i:

$$p_i = \mu_i c_i e_i \tag{5.3}$$

$$P_i = \mu_i c_i E_i \tag{5.4}$$

5.3 Outline of the method

The rationale of the decomposition approach is to decompose the original transfer line L made of K machines into a set of two-machine lines $l(i)$, for $i = 1, \ldots, K-1$, as shown in fig. 5.1. Each two-machine line, or building block, is characterized by a buffer $B(i)$ of the same size of the corresponding buffer B_i of the original line, an upstream pseudo machine $M^u(i)$ and a downstream pseudo machine $M^d(i)$. With the upstream and downstream pseudo machines modeling the behavior of the portion of the original line upstream and downstream of buffer B_i, or in other words, modeling how workpieces enter and leave buffer B_i, the decomposed building blocks are supposed to reproduce the dynamic behavior of the material flow observed in original line. Therefore the performance and characteristics of the original line can be derived by solving the building blocks.

In order to reproduce the dynamic behavior of machines and materials, the flow of material in the original line is studied. The flow of material into a buffer B_i of the original line can be interrupted because 1) the upstream machine M_i fails in certain failure mode; 2) it takes a long time for the upstream machine M_i to process a batch; 3) the upstream machine is starved, which may be due to a failure or long

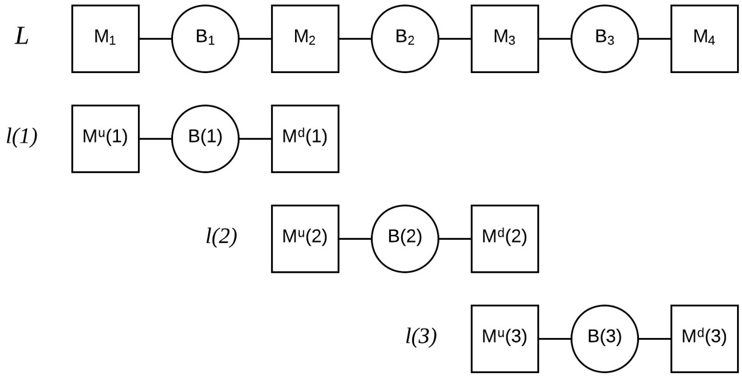

Figure 5.1: Decomposition of a four-machine transfer line system

processing time of the upstream line. Similarly, the flow of material out of a buffer B_i of the original line can be interrupted because 1) the downstream machine M_i fails in certain failure mode; 2) it takes a long time for the downstream machine M_i to process a batch; 3) the downstream machine is starved, which may due to a failure or long processing time of the downstream line. However, in decomposed building blocks, since the upstream machine is the first machine of the line and the downstream machine is the last, both machines cannot be starved/blocked by assumption. Therefore, a set of *remote failure modes* are introduced to represent the starvation of upstream machine and blocking of downstream machine.

Therefore, the key of the decomposition method is to model the behaviour of the machine fails in and recovers from remote failure modes and find out the unknown parameters of these remote failure modes.

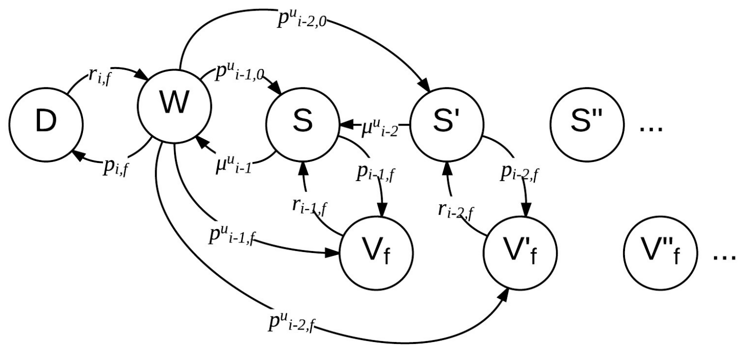

FIGURE 5.2: State-transition diagram for upstream pseudo machine $M^u(i)$

5.4 Pseudo machine modeling and parameters

According to the analysis carried out so far, the upstream pseudo machine $M^u(i)$ of the building block $l(i)$ is characterized by some real failure modes and some remote failure modes. While real failure modes are corresponding to the breakdown of the physical machine M_i with simple failure and repair rate, remote failure modes may have more complex behaviour. For example, if the upstream pseudo machine $M^u(i)$ is subject to a remote failure due to the breakdown of upstream machine M_{i-1}, it should first wait for machine M_{i-1} to get repaired, then wait until it produce enough pieces for a batch, after that the machine $M^u(i)$ could recover from the failure mode.

Considering the upstream pseudo machine $M^u(i)$, the state-transition diagram is shown in fig. 5.2 in which

- State W denotes that the machine is up for processing.
- State D denotes that the machine is subject to a real failure mode.
- State V_f denotes that the machine is subject to a remote failure mode due to the breakdown of the upstream physical machine M_{i-1} on failure mode f.

- State S denotes that the machine is subject to a remote failure mode due to the long process time of the upstream physical machine M_{i-1}.
- Similarly, state V' and V' denotes that the machine is subject to a remote failure mode the upstream physical machine M_{i-2}.
- $p_{i,f}$, $r_{i,f}$, $p_{i-1,f}$, $r_{i-1,f}$, etc. represent the real failure and repair rate of corresponding physical machine M_i, M_{i-1}.
- $p^u_{i-1,f}$ represents the remote failure rate due to the breakdown or starvation of upstream physical machine M_{i-1} and $p^u_{i-1,0}$ represents the remote failure rate due to the the long process time of machine M_{i-1}.
- μ^u_{i-1} represents the processing rate of machine M_{i-1} to produce enough pieces for its downstream batch.

Note that the processing rate to produce enough pieces μ^u_i is dynamic to buffer level and in this model an approximation is given for upstream and downstream machine:

$$\mu^u_i = \frac{\mu_i}{\lambda}, i = 1, \ldots, k \quad where\ \lambda = \sum_{k=0}^{c_{i+1}} p(c_{i+1} - k)\lceil \frac{k}{c_i} \rceil \tag{5.5}$$

$$\mu^d_i = \frac{\mu_{i+1}}{\lambda}, i = 1, \ldots, k \quad where\ \lambda = \sum_{k=0}^{c_i} p(N - c_{i+1} + k + 1)\lceil \frac{k}{c_{i+1}} \rceil \tag{5.6}$$

in which $\lceil x \rceil$ stands for round up x to nearest integer and $p(k)$ stands for the steady state probability to be in buffer level k.

However, with such transition diagram it would be hard to evaluate the performance of pseudo machine and build decomposition equations because it is dynamic with the number of stages in the line. Therefore a integration method is developed to obtain each remote failure modes in form of simple failure-repair as shown in fig. 5.3, which will be discussed in the next section. On the basis of the integrated state-transition

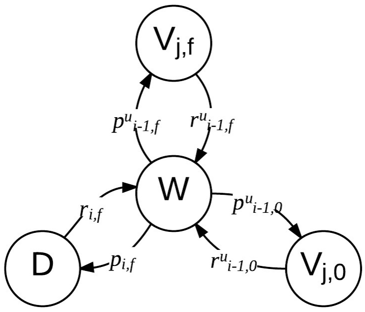

FIGURE 5.3: Integrated state-transition diagram for upstream pseudo machine $M^u(i)$

diagram , the parameters characterize the upstream pseudo-machine $M_u(i)$ of the two-machine line $l(i)$ is given as following:

- *processing rate* $\mu^u(i)$, representing the rate at which pseudo machine $M_u(i)$ processes material.
- *batch size* $c^u(i)$, representing the unit of workpieces pseudo machine $M_u(i)$ is able to process at once.
- *real failure modes* due to the breakdown of the physical machine M_i, with failure and repair rates respectively equal to $p_{i,f}$ and $r_{i,f}$, for $f = 1, \ldots, F_i$.
- *remote failure modes* due to the breakdown of the upstream physical machine M_j, with failure and repair rates respectively given by $p^u_{j,f}(i)$ and $r^u_{j,f}(i)$, for $j = 1, \ldots, i-1$ and $f = 1, \ldots, F_j$.
- *remote failure modes* due to the long processing time of the upstream physical machine M_j, with failure and repair rates respectively given by $p^u_{j,0}(i)$ and $r^u_{j,0}(i)$, for $j = 1, \ldots, i-1$.

Similarly, the parameters characterize the upstream pseudo-machine $M_d(i)$ of the two-machine line $l(i)$ is given as following:

- *processing rate* $\mu^d(i)$, representing the rate at which pseudo machine $M_d(i)$ processes material.
- *batch size* $c^d(i)$, representing the unit of workpieces pseudo machine $M_d(i)$ is able to process at once.
- *real failure modes* due to the breakdown of the physical machine M_i, with failure and repair rates respectively equal to $p_{i,f}$ and $r_{i,f}$, for $f = 1, \ldots, F_i$.
- *remote failure modes* due to the breakdown of the upstream physical machine M_j, with failure and repair rates respectively given by $p^d_{j,f}(i)$ and $r^d_{j,f}(i)$, for $j = i+1, \ldots, K$ and $f = 1, \ldots, F_j$.
- *remote failure modes* due to the long processing time of the upstream physical machine M_j, with failure and repair rates respectively given by $p^d_{j,0}(i)$ and $r^d_{j,0}(i)$, for $j = i+1, \ldots, K$.

In addition, it is implicit by definition that the remote failure modes could only happen at the moment when the machine finishes a batch, since the blocking and starvation states are only checked after a batch is finished. Therefore in the modelling of building blocks, upstream remote failure rate $p^u_{j,f}(i)$ represent the transition from state $\pi(n, 0, d_k)$ to state $\pi(n+c^u(i), u_{j,f}, d_k)$ in case that the upstream machine is not blocked and downstream remote failure rate $p^d_{j,f}(i)$ represent the transition from state $\pi(n, u_k, 0)$ to state $\pi(n-c^d(i), u_k, d_{j,f})$ in case that the upstream machine is not starved.

Given the modeling and parameters of the upstream and downstream pseudo machines, it is possible to obtain, by means of the exact analytical solution provided in previous section, an evaluation of the building block $l(i)$ performance. To be more specific, the performances which are of particular interest for the development of decomposition equations are

1. the *average production rate* $P(i)$ of the line
2. the *efficiency* of the upstream $E^u(i)$ and downstream $E^d(i)$ pseudo-machine
3. the *steady state probability* of downstream pseudo-machine of being starved $S_{j,f}(i)$ due to the failure mode j, f, for $j = 1, \dots, i$ and $f = 0, \dots, F_j$
4. the *steady state probability* of upstream pseudo-machine of being blocked $B_{j,f}(i)$ due to the failure mode j, f for $j = i + 1, \dots, K$ and $f = 0, \dots, F_j$.

5.5 Decomposition equations

Having on hand the modeling of pseudo machine, the next step consists in finding out the unknown parameters inside the model by means of a set of decomposition equations. Before addressing the derivation of the decomposition equations, it is necessary to indicate the notation we adopt to refer to the probability of the pseudo machines in certain states.

The upstream machine $M^u(i)$ could be in the following states:

- state that the machine is operational, with probability denoted by $W^u(i)$.
- state that the machine is down due to a real failure f, with probability denoted by $D^u_f(i)$, for $f = 1, \dots, F_i$.
- state that the machine is down due to the breakdown of upstream machine j in failure mode f, with probability denoted by $V^u_{j,f}(i)$, for $j = 1, \dots, i-1$ and $f = 1, \dots, F_j$.

- state that the machine is down due to the long processing time of upstream machine j, with probability denoted by $V^u_{j,0}(i)$, for $j = 1, \ldots, i-1$.
- state that the machine is blocked by downstream machine j fails in failure mode f, with probability denoted by $B_{j,f}(i)$, for $j = i+1, \ldots, K$ and $f = 1, \ldots, F_j$.
- state that the machine is blocked by downstream machine j due to long processing time, with probability denoted by $B_{j,0}(i)$, for $j = i+1, \ldots, K$.

Similarly, the downstream machine $M^d(i-1)$ could be in the following states:

- state that the machine is operational, with probability denoted by $W^d(i-1)$.
- state that the machine is down due to a real failure f, with probability denoted by $D^d_f(i-1)$, for $f = 1, \ldots, F_{i-1}$.
- state that the machine is down due to the breakdown of downstream machine j in failure mode f, with probability denoted by $V^d_{j,f}(i)$, for $j = i+1, \ldots, K$ and $f = 1, \ldots, F_j$.
- state that the machine is down due to the long processing time of downstream machine j, with probability denoted by $V^d_{j,0}(i)$, for $j = i+1, \ldots, K$
- state that the machine is starved by upstream machine j fails in failure mode f, with probability denoted by $S_{j,f}(i)$, for $j = 1, \ldots, i-1$ and $f = 1, \ldots, F_j$.
- state that the machine is starved by upstream machine j due to long processing time, with probability denoted by $S_{j,0}(i)$, for $j = 1, \ldots, i-1$.

Obviously, the batch size, the processing rates and the parameters of the real failures of the pseudo machines are known, since they are exactly the same of the corresponding machines in the original line. Conversely, for the parameters of the remote failures decomposition equations must be derived.

5.5.1 Resumption of flow equations

The resumption of flow equations provide the repair rates from the various remote failure modes in which pseudo machine $M^u(i)$ and $M^d(i-1)$ may fail. In deterministic case these parameters do not represent unknowns effectively since the recovery from a certain failure mode implies the removal of the cause, while in exponential model, the repair rate of remote failure modes become dynamic and is determined by the character of two machines as well as other parameters in between.

As has been noted, a integration method is proposed to transform remote failure modes into simple failure-repair model and reduce the complexity of model especially in case of long line. The basic idea of the method is to take an single exponential process with the same mean to approximate the multi-stage exponential processes. Although the multi-stage exponential processes could be better modelled by an Erlang distribution, the benefit of exponential approximated is that it facilitate the analysis in a Markov chain without losing much of accuracy.

Recall the state-transition diagram for upstream pseudo machine $M^u(i)$ noted in previous section, shown in fig. 5.4 p1. Note that as the first step we consider only remote failure from the upstream machine $i-1$ and local failure is hidden as they do not affect on the calculation. Since machine passes from state S to state V_f will finally move back to state S, it is possible to split the probability to go through this process into a separate node, denoted as T_f (shown in fig. 5.4 p2). Then since the only exit of state S is state W and state T_f is only connected to state S, state

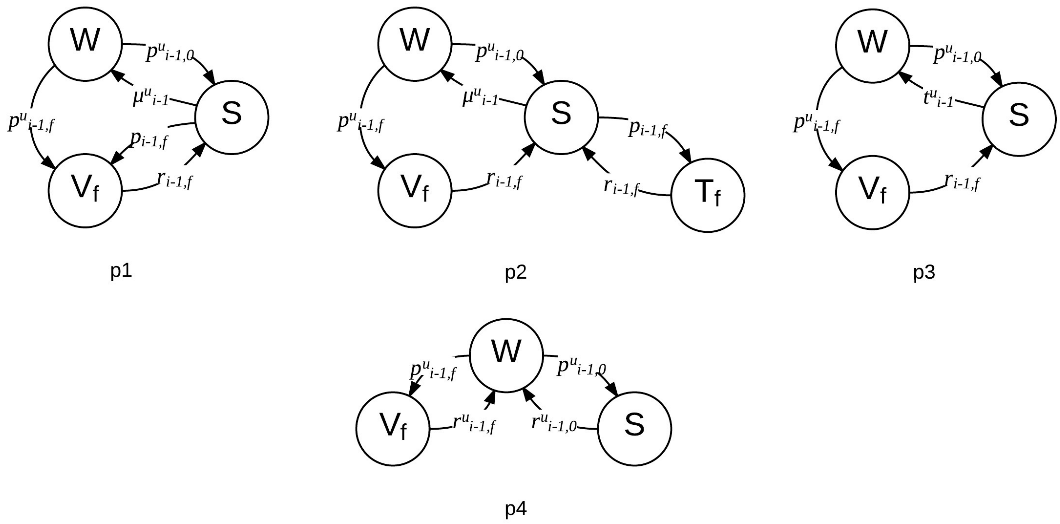

FIGURE 5.4: Steps to integrate the state-transition diagram of upstream pseudo machine $M^u(i)$

T_f could be consider as a slowdown for leaving state S (shown in 5.4 p3) with the delayed exit rate t^u_{i-1} given by:

$$t^u_{i-1} = \frac{\mu^u_{i-1}}{1 + p_{i-1,f}(i)/r_{i-1,f}} \tag{5.7}$$

Concerning the repair rate from remote failure mode $i-1, 0$ of a pseudo-macines $M^u(i)$, it can be seen that it is the same as the delayed exit rate t^u_{i-1}, which means:

$$r^u_{i-1,0}(i) = \frac{\mu^u_{i-1}}{1 + p_{i-1,f}/r_{i-1,f}} \tag{5.8}$$

Concerning the repair rate from remote failure mode $i-1, f$ of a pseudo-macines $M^u(i)$, it is a merge of transition from state V_f to state S then to state W since there is no alternative route. The repair rate $r^u_{i-1,f}$ can be given by

$$r^u_{i-1,f}(i) = \frac{t^u_{i-1} r_{i-1,f}}{t^u_{i-1} + r_{i-1,f}} = \frac{\mu^u_{i-1} r_{i-1,f}}{\mu^u_{i-1} + p_{i-1,f} + r_{i-1,f}} \tag{5.9}$$

The repair rate from remote failure mode of a pseudo-macines $M^d(i-1)$ can be derived similarly by:

$$r^d_{i,0}(i-1) = \frac{\mu^d_{i+1}}{1 + p_{i+1,f}/r_{i+1,f}} \tag{5.10}$$

$$r^d_{i,f}(i-1) = \frac{\mu^d_{i+1} r_{i+1,f}}{\mu^d_{i+1} + p_{i+1,f} + r_{i+1,f}} \tag{5.11}$$

The second step is to derive all repair rate $r^u_{j,f}$ and $r^d_{j,f}$ base on the equation obtained above. Note that the method can be applied recursively on upstream pseudo machine $M^u(j)$, for $j = 2, ..., i-1$ and downstream pseudo machine $M^d(j)$, for $j = K-1, ..., i+1$. Therefore the repair rate $r^u_{j,f}$ and $r^d_{j,f}$ can be obtained respectively by repeating the algorithm several times. As a result, the resumption of flow equations for upstream pseudo machine $M^u(i)$ can be given by:

$$\begin{aligned} r^u_{i-1,0}(i) &= \frac{\mu^u_{i-1}}{1 + \sum_{f=1}^{F_j} \frac{p^u_{j,f}(i-1)}{r^u_{j,f}(i-1)}} \\ r^u_{j,0}(i) = \frac{\mu^u_{i-1} r^u_{j,f}(i-1)}{\mu^u_{i-1} + p^u_{j,f}(i-1) + r^u_{j,f}(i-1)}&, \ for\ j = 1, \dots, i-2 \\ r^u_{j,f}(i) &= \frac{\mu^u_{i-1} r^u_{j,f}(i-1)}{\mu^u_{i-1} + p^u_{j,f}(i-1) + r^u_{j,f}(i-1)} \\ &, \ for\ j = 1, \dots, i-1, f = 1, \dots, F_j \\ & where\ r^u_{i,f}(i) = r_{i,f} \end{aligned} \tag{5.12}$$

Similarly, the resumption of flow equations for downstream pseudo machine $M^d(i-1)$ can be given by:

$$\begin{aligned} r^d_{i+1,0}(i-1) &= \frac{\mu^d_i}{1+\sum_{f=1}^{F_j}\frac{p^d_{j,f}(i)}{r^d_{j,f}(i)}} \\ r^d_{j,0}(i-1) &= \frac{\mu^d_i r^d_{j,0}(i)}{\mu^d_i + p^d_{j,0}(i) + r^d_{j,0}(i)},\ for\ j=i+2,\ldots,K \\ r^d_{j,f}(i-1) &= \frac{\mu^d_i r^d_{j,f}(i)}{\mu^d_i + p^d_{j,f}(i) + r^d_{j,f}(i)},\ for\ j=i+1,\ldots,K, f=1,\ldots,F_j \\ & where\ r^d_{i,f}(i) = r_{i+1,f} \end{aligned} \tag{5.13}$$

5.5.2 Interruption of flow equations

The interruption of flow equations provide the failure rates for the various failure modes affecting the pseudo machines $M^u(i)$ and $M^d(i-1)$. To derive this set of equations, it is necessary to observe that every time a machine breaks down in a certain mode it gets repaired. On the basis of this observation, it is possible to show, extending to the case of multiple failure modes the proof given by Gershwin, that *failure frequency equals repair frequency for each failure mode, real or remote.*

As a result, we can write for $M^u(i)$ the following equations:

$$p_{i,f}\cdot W^u(i) = r_{i,f}\cdot D^u_f(i),\ for\ f=1,\ldots,F \tag{5.14}$$

$$p^u_{j,f}(i)\cdot W^u(i) = r^u_{j,f}(i)\cdot V^u_{j,f}(i),\ for\ j=1,\ldots,i-1, f=1,\ldots,F_i \tag{5.15}$$

$$p^u_{j,0}(i)\cdot W^u(i) = r^u_{j,0}(i)\cdot V^u_{j,0}(i),\ for\ j=1,\ldots,i-1 \tag{5.16}$$

Similarly, for $M^d(i-1)$ te following equations hold:

$$p_{i,f} \cdot W^d(i-1) = r_{i,f} \cdot D^d_f(i-1),\ for\ f = 1, \ldots, F_i \tag{5.17}$$

$$p^d_{j,f}(i-1) \cdot W^d(i-1) = r^d_{j,f}(i-1) \cdot V^d_{j,f}(i-1),\ for\ j = i+1, \ldots, K, f = 1, \ldots, F_i \tag{5.18}$$

$$p^d_{j,0}(i-1) \cdot W^d(i-1) = r^d_{j,0}(i-1) \cdot V^d_{j,0}(i-1),\ for\ j = i+1, \ldots, K \tag{5.19}$$

Since the sum on all possible states of $M^u(i)$ must be equal to 1, we can write the following normalization equations:

$$\begin{aligned} W^u(i) + \sum_{f=1}^{F_i} D^u_f(i) + \sum_{j=1}^{i-1} \sum_{f=1}^{F_i} V^u_{j,f}(i) + \sum_{j=1}^{i-1} V^u_{j,0}(i) \\ + \sum_{j=i+1}^{K} \sum_{f=1}^{F_i} B_{j,f}(i) + \sum_{j=i+1}^{K} B_{j,0}(i) = 1 \end{aligned} \tag{5.20}$$

As has been defined, the efficiency in isolation of a machine, including also remote failures, can be obtained for the upstream pseudo machine $M^u(i)$ of the two-machine line $l(i)$:

$$e^u(i) = \frac{1}{1 + \sum_{f=1}^{F_i} \frac{p_{i,f}}{r_{i,f}} + \sum_{j=1}^{i-1} \sum_{f=1}^{F_i} \frac{p^u_{j,f}(i)}{r^u_{j,f}(i)} + \sum_{j=1}^{i-1} \frac{p^u_{j,0}(i)}{r^u_{j,0}(i)}} \tag{5.21}$$

Substituting equations (5.14), (5.15), (5.16) and (5.20) into equation (5.21), after some manipulations we obtain:

$$W^u(i) = e^u(i)(1 - Pb(i)) \tag{5.22}$$

which corresponds to the flow rate-idle time equation. Since $M^u(i)$, being the first machine, cannot be starved, we can conclude:

$$W^u(i) = E^u(i) \tag{5.23}$$

Similarly, the same set of equations can be derived for pseodu machine $M^d(i-1)$:

$$\begin{aligned} W^d(i-1) + \sum_{f=1}^{F_i} D_f^d(i-1) + \sum_{j=i+1}^{K} \sum_{f=1}^{F_i} V_{j,f}^d(i-1) + \sum_{j=i+1}^{K} V_{j,0}^d(i-1) \\ + \sum_{j=1}^{i-1} \sum_{f=1}^{F_i} S_{j,f}(i-1) + \sum_{j=1}^{i-1} S_{j,0}(i-1) = 1 \end{aligned} \tag{5.24}$$

with efficiency in isolation given by:

$$e^d(i-1) = \frac{1}{1 + \sum_{f=1}^{F_i} \frac{p_{i,f}}{r_{i,f}} + \sum_{j=i+1}^{K} \sum_{f=1}^{F_i} \frac{p_{j,f}^d(i-1)}{r_{j,f}^d(i-1)} + \sum_{j=i+1}^{K} \frac{p_{j,0}^d(i-1)}{r_{j,0}^d(i-1)}} \tag{5.25}$$

Finally, considering $M^d(i-1)$, being the last machine, cannot be blocked, we can derive:

$$W^d(i-1) = e^d(i-1)(1 - Ps(i-1)) = E^d(i-1) \tag{5.26}$$

The remote failure modes of $M^u(i)$ have been introduced to mimic the different causes of starvation of the physical machine M_i. Therefore, the steady state probability of $M^u(i)$ of being in a specific remote down state is supposed to be qual to the corresponding steady state probability of the starvation fo the previous two-machine line $l(i-1)$. But in our case, since the state-transition diagram has been modified, the probability to stay in remote failure state $V_{j,f}^u(i)$ and $V_{j,0}^u(i)$ should be modified correspondingly to match the repair rate. The modified steady state probability of the starvation of the previous two-machine line $l(i-1)$ can be given by (detailed prove given in appendix):

$$V^u_{j,f}(i) = (S_{j,f}(i-1) - S_{i-1,0}(i-1)\frac{p^u_{j,f}(i-1)}{r^u_{j,f}(i-1)})(1+\frac{p^u_{j,f}(i-1)+r^u_{j,f}(i-1)}{\mu^u_{i-1}})$$
$$,\ for\ j=1,\dots,i-1, f=1,\dots,F_j \tag{5.27}$$

$$V^u_{j,0}(i) = (S_{j,0}(i-1) - S_{i-1,0}(i-1)\frac{p^u_{j,0}(i-1)}{r^u_{j,0}(i-1)})(1+\frac{p^u_{j,0}(i-1)+r^u_{j,0}(i-1)}{\mu^u_{i-1}}) \tag{5.28}$$
$$,\ for\ j=1,\dots,i-2$$

$$V^u_{i-1,0}(i) = S_{i-1,0}(i-1)+\sum_{j=1}^{i-1}\sum_{f=1}^{F_j}(S_{j,f}(i-1)-V^u_{j,f}(i))+\sum_{j=1}^{i-2}(S_{j,0}(i-1)-V^u_{j,0}(i)) \tag{5.29}$$

Similarly, the modified steady state probability of the starvation of the previous two-machine line $l(i-1)$ can be given by:

$$V^d_{j,f}(i-1) = (B_{j,f}(i) - B_{i+1,0}(i)\frac{p^d_{j,f}(i+1)}{r^d_{j,f}(i+1)})(1+\frac{p^d_{j,f}(i+1)+r^d_{j,f}(i+1)}{\mu^d_{i+1}}) \tag{5.30}$$
$$,\ for\ j=i+1,\dots,K, f=1,\dots,F_j$$

$$V^d_{j,0}(i-1) = (B_{j,0}(i) - B_{i+1,0}(i)\frac{p^d_{j,0}(i+1)}{r^d_{j,0}(i+1)})(1+\frac{p^d_{j,0}(i+1)+r^d_{j,0}(i+1)}{\mu^d_{i+1}}) \tag{5.31}$$
$$,\ for\ j=i+2,\dots,K$$

$$V^d_{i+1,0}(i-1) = B_{i+1,0}(i)+\sum_{j=i+1}^{K}\sum_{f=1}^{F_j}(B_{j,f}(i)-V^d_{j,f}(i-1))+\sum_{j=i+2}^{K}(B_{j,0}(i)-V^d_{j,0}(i-1)) \tag{5.32}$$

Then, substituting equations (5.27), (5.28) and (5.29) respectively into equation (5.15) and (5.16), taking into account equation (5.23) and (5.12), we obtain the following expressions for the remote failure rates of $M^u(i)$:

$$p^u_{j,f}(i) = \frac{S_{j,f}(i-1)r^u_{j,f}(i-1) - S_{i-1,0}(i-1)p^u_{j,f}(i-1)}{E^u(i)}, \quad for\ j = 1,\dots,i-1, f = 1,\dots,F_j \tag{5.33}$$

$$p^u_{j,0}(i) = \frac{S_{j,0}(i-1)r^u_{j,0}(i-1) - S_{i-1,0}(i-1)p^u_{j,0}(i-1)}{E^u(i)}, for\ j = 1,\dots,i-2 \tag{5.34}$$

$$p^u_{i-1,0}(i) = \frac{V^u_{i-1,0}(i)}{E^u(i)} \cdot r^u_{i-1,0}(i) \tag{5.35}$$

while substituting equations (5.30), (5.31) and (5.32) respectively into equation (5.18) and (5.19), taking into account equation (5.26) and (5.13), we obtain the following expressions for the remote failure rates of $M^d(i-1)$:

$$p^d_{j,f}(i-1) = \frac{B_{j,f}(i)r^d_{j,f}(i) - B_{i+1,0}(i)p^d_{j,f}(i)}{E^d(i-1)}, \quad for\ j = i+1,\dots,K, f = 1,\dots,F_j \tag{5.36}$$

$$p^d_{j,0}(i-1) = \frac{B_{j,0}(i)r^d_{j,0}(i) - B_{i+1,0}(i)p^d_{j,0}(i)}{E^d(i-1)}, for\ j = i+2,\dots,K \tag{5.37}$$

$$p^d_{i+1,0}(i-1) = \frac{V^d_{i+1,0}(i)}{E^d(i-1)} \cdot r^d_{i+1,0}(i) \tag{5.38}$$

5.6 Solving algorithm

On the basis of the analysis carried out, an efficient algorithm to solve the decomposition equations has been obtained. The algorithm is based on the DDX scheme and consists of the following steps:

1. Initialization: for each two-machine line $l(i)$ in which the original line L is decomposed, the known parameters are introduced and an initial value α is set for the unknown parameters. Note that the repair rate of remote failure modes need to be derived recursively and μ_i^u/μ_i^d represents the processing rate to produce a batch of pieces for downstream/upstream machine defined in formula (5.5) and (5.6).

$$c^u(i) = c_i \tag{5.39}$$

$$\mu^u(i) = \mu_i \tag{5.40}$$

$$p_{i,f}^u(i) = p_{i,f}, \ \ f = 1, \dots, F_j \tag{5.41}$$

$$r_{i,f}^u(i) = r_{i,f}, \ \ f = 1, \dots, F_j \tag{5.42}$$

$$p_{j,f}^u(i) = \alpha, \ \ j = 1, \dots, i-1, \ \ f = 1, \dots, F_j \tag{5.43}$$

$$r_{j,f}^u(i) = \frac{\mu_{i-1}^u r_{j,f}^u(i-1)}{\mu_{i-1}^u + p_{j,f}^u(i-1) + r_{j,f}^u(i-1)}, \ \ j = 1, \dots, i-1, \ \ f = 1, \dots, F_j \tag{5.44}$$

$$p_{j,0}^u(i) = \alpha, \ \ j = 1, \dots, i-1 \tag{5.45}$$

$$r_{j,0}^u(i) = \frac{\mu_{i-1}^u r_{j,f}^u(i-1)}{\mu_{i-1}^u + p_{j,f}^u(i-1) + r_{j,f}^u(i-1)}, \ \ j = 1, \dots, i-2 \tag{5.46}$$

$$c^d(i-1) = c_i \tag{5.47}$$

$$\mu^d(i-1) = \mu_i \tag{5.48}$$

$$p_{i,f}^d(i-1) = p_{i,f}, \ \ f = 1, \dots, F_j \tag{5.49}$$

$$r_{i,f}^d(i-1) = r_{i,f}, \ \ f = 1, \dots, F_j \tag{5.50}$$

$$p_{j,f}^d(i-1) = \alpha, \ \ j = 1, \dots, i-1, \ \ f = 1, \dots, F_j \tag{5.51}$$

$$r^d_{j,f}(i-1) = \frac{\mu^d_i r^d_{j,f}(i)}{\mu^d_i + p^d_{j,f}(i) + r^d_{j,f}(i)},\ j = i+1,\dots,K, f = 1,\dots,F_j \quad (5.52)$$

$$p^d_{j,0}(i-1) = \alpha,\ j = 1,\dots,i-1 \quad (5.53)$$

$$r^d_{j,0}(i-1) = \frac{\mu^d_i r^d_{j,0}(i)}{\mu^d_i + p^d_{j,0}(i) + r^d_{j,0}(i)},\ j = i+2,\dots,K \quad (5.54)$$

2. For $i = 2,\dots,K-1$: evaluation of the two-machine $l(i-1)$ using for the parameters of $M^d(i-1)$ the most recent values from the previous iteration while updating the parameters of $M^u(i)$ with the results of the evaluation of the upstream building block $l(i-1)$:

$$\mu^u_{i-1} = \frac{u_{i-1}}{\lambda} \quad where\ \lambda = \sum_{k=0}^{c_i} p(c_i - k)\lceil \frac{k}{c_{i-1}} \rceil \quad (5.55)$$

$$r^u_{j,f}(i-1) = \frac{\mu^u_{i-2} r^u_{j,f}(i-2)}{\mu^u_{i-2} + p^u_{j,f}(i-2) + r^u_{j,f}(i-2)},\ j = 1,\dots,i-1,\ f = 1,\dots,F_j \quad (5.56)$$

$$r^u_{j,0}(i-1) = \frac{\mu^u_{i-2} r^u_{j,f}(i-2)}{\mu^u_{i-2} + p^u_{j,f}(i-2) + r^u_{j,f}(i-2)},\ j = 1,\dots,i-2 \quad (5.57)$$

$$p^u_{j,f}(i) = \frac{S_{j,f}(i-1) r^u_{j,f}(i-1) - S_{i-1,0}(i-1) p^u_{j,f}(i-1)}{E^u(i)},\ j = 1,\dots,i-1, f = 1,\dots,F_j \quad (5.58)$$

$$p^u_{j,0}(i) = \frac{S_{j,0}(i-1) r^u_{j,0}(i-1) - S_{i-1,0}(i-1) p^u_{j,0}(i-1)}{E^u(i)},\ j = 1,\dots,i-2 \quad (5.59)$$

3. For $i = K-2,\dots,1$: evaluation of the two-machine $l(i+1)$ using for the parameters of $M^u(i+1)$ the most recent values from the previous iteration while updating the parameters of $M^d(i)$ with the results of the evaluation of the downstream building block $l(i+1)$:

$$\mu^d_{i+1} = \frac{u_{i+2}}{\lambda}, i = 1,\dots,k \quad where\ \lambda = \sum_{k=0}^{c_{i+1}} p(N - c_{i+2} + k + 1)\lceil \frac{k}{c_{i+2}} \rceil \quad (5.60)$$

$$r^d_{j,f}(i) = \frac{\mu^d_{i+1} r^d_{j,f}(i+1)}{\mu^d_i + p^d_{j,f}(i+1) + r^d_{j,f}(i+1)}, \; j = i+1, \ldots, K, f = 1, \ldots, F_j \quad (5.61)$$

$$r^d_{j,0}(i) = \frac{\mu^d_{i+1} r^d_{j,0}(i+1)}{\mu^d_i + p^d_{j,0}(i+1) + r^d_{j,0}(i+1)}, \; j = i+2, \ldots, K \quad (5.62)$$

$$p^d_{j,f}(i-1) = \frac{B_{j,f}(i) r^d_{j,f}(i) - B_{i+1,0}(i) p^d_{j,f}(i)}{E^d(i-1)}, \; j = i+1, \ldots, K, f = 1, \ldots, F_j \quad (5.63)$$

$$p^d_{j,0}(i-1) = \frac{B_{j,0}(i) r^d_{j,0}(i) - B_{i+1,0}(i) p^d_{j,0}(i)}{E^d(i-1)}, \; j = i+2, \ldots, K \quad (5.64)$$

4. Steps 2 and 3 are repeated alternatively until the equations converge. A termination condition is set when the maximum difference between any two building blocks $E(i)$ and $E(j)$ is smaller than 10^{-5}.

Chapter 6

Numerical Results

In this chapter we confirmed all of the theoretical results in previous sections via numerical analysis. In order to evaluate the performance of the new decomposition method developed a set of numerical tests have been carried out to exam the methodology in two dimensions: accuracy and speed. We also carried out a set of numerical experiments to explore the characteristics of batch production lines and discussed on the allocation of buffer capacity.

All parameters used in these numerical analysis are generated using the methodology described by Gershwin [27] to ensure the effectiveness of the experiment. More specifically, the parameters are generated following some meaningful assumptions:

- We do not want to consider lines in which machines are very different from one another since it is not economic in real case. It also implies that machine efficiencies should not be unrealistically high or unrealistically low. (While still it is sometimes desirable to deliberately introduce bottlenecks);

- We want to test cases in which repair times are reasonable — neither too long or too short;
- We want the buffer size to be in a reasonable range, especially not too low to cause blocking too often.

With these assumptions, a set of constraints can be defined for which we require the parameters p_i, r_i and N_i should satisfy. These constraints can be written

$$R^{min} \leq r_i \leq R^{max}, \ i = 1, \ldots, k \tag{6.1}$$

$$\frac{r_i}{r_j} \leq \alpha, \ i = 1, \ldots, k, \ j = 1, \ldots, k \tag{6.2}$$

$$E^{min} \leq e_i = \frac{1}{1 + \sum \frac{p_i}{r_i}} \leq E^{max}, \ i = 1, \ldots, k \tag{6.3}$$

$$\frac{B^{min}}{\max_j r_j} \leq N_i \leq \frac{B^{max}}{\max_j r_j}, \ i = 1, \ldots, k-1, \ j = 1, \ldots, k \tag{6.4}$$

where R^{max} and R^{min} are predefined bounds on the mean time to repair r_i, E^{max} and E^{min} are predefined bounds on the efficiency of machine e_i and B^{max} and B^{min} are predefined bounds on the buffer capacity N_i.

It can be noticed that, as mentioned in Gershwin's paper [23], the system can be not ergodic when the batch size of upstream and downstream machine is not prime to each other. For example, if the upstream batch size is 2 and downstream batch size is 4 and the intermediate buffer size is 8, apparently the buffer level this subsystem can reach is limited to even numbers. Considering a initial buffer level of 0, the system have 5 possible buffer level 0,2,4,6,8 which means the buffer can store 4 batches of upstream machine or 2 batches of downstream machine, while when we consider a initial buffer level of 1, the system may just have 4 possible buffer level

1,3,5,7 which means the buffer can store 3 batches of upstream machine or 1 batch of downstream machine, which obviously reduce the productivity of the system. In our study, ergodicity is not considered to have a big impact on result generated because it is related to the initial state of the system while when compared with simulation you always starts from zero buffer level. In addition, when the buffer size is large to accept many batches, the impact of this phenomenon becomes less significant as well. Therefore here we take the assumptions that all buffers start at zero buffer level.

All computations are performed in MATLAB 2011b and are done in double precision while the results are presented in rounding up to 4 decimal points. All simulation model are built and run in Arena v14.5. The presented computational results were obtained on an HP Intel(R) Core 2 1.86 GHz laptop.

6.1 Numerical result: accuracy

In order to show the accuracy of the new decomposition method developed(TU), we compare the analytical results with those obtained running simulation experiments(SIM). For each simulation experiment, to ensure statistical significance 20 replications have been performed. Each run entails a simulations period of 1,000,000 time unit preceded by a warm-up period of 100,000 time units.

The parameters are taken using a program that generates variables completely in random and takes care of the rough balance of the whole line. In this case, the parameters that characterize the production line takes their values from a certain range. For example, buffer capacity N ranges from 6 to 24; batch size c ranges from 1 to 5; isolated efficiency ranges from 0.7 to 1; local failure rate ranges from 0.01 to

0.1 and local repair rate ranges from 0.1 to 1. We reproduce the same analysis for machine lines consists of 5, 6 and 7 machines. All parameters are given in appendix A. The parameters selected cover a wide variety of different configuration of production lines with balance or imbalanced line, different bottleneck location, various production batches and buffer capacity. Also different settings of failure mode parameters have been considered, with repair rates of different orders of magnitude. Therefore the set of parameter is able to test the accuracy of the proposed method under different circumstances. For detailed parameters refers to appendix A.

As shown in figure 6.1, the result shows a quite accurate performance estimation with respect to the simulation results (average %error is 0.512% and maxium %error is 1.364%). Meanwhile, the proposed method shows little increase in %error with the growth of length of production line. On the other hand, the accuracy of estimation towards average buffer level is not quite stable with an overall %error of 6.7% and the error seems to decrease with the growth of length of production line. We will discuss this phenomenon in the following experiment

# machine	% error TP	% error ABL
5	0.4719%	3.3172%
5	0.4476%	1.3603%
5	0.1517%	13.9751%
5	0.1524%	1.5373%
5	0.1818%	10.5872%
6	0.4552%	6.1332%
6	0.1745%	2.4833%
6	0.2312%	3.6363%
6	0.7397%	3.5833%
6	0.4073%	5.4491%
7	1.0177%	3.8873%
7	1.3643%	2.4644%
7	1.0746%	6.6367%
7	0.1116%	5.6092%
7	0.6927%	2.4583%

TABLE 6.1: Line results

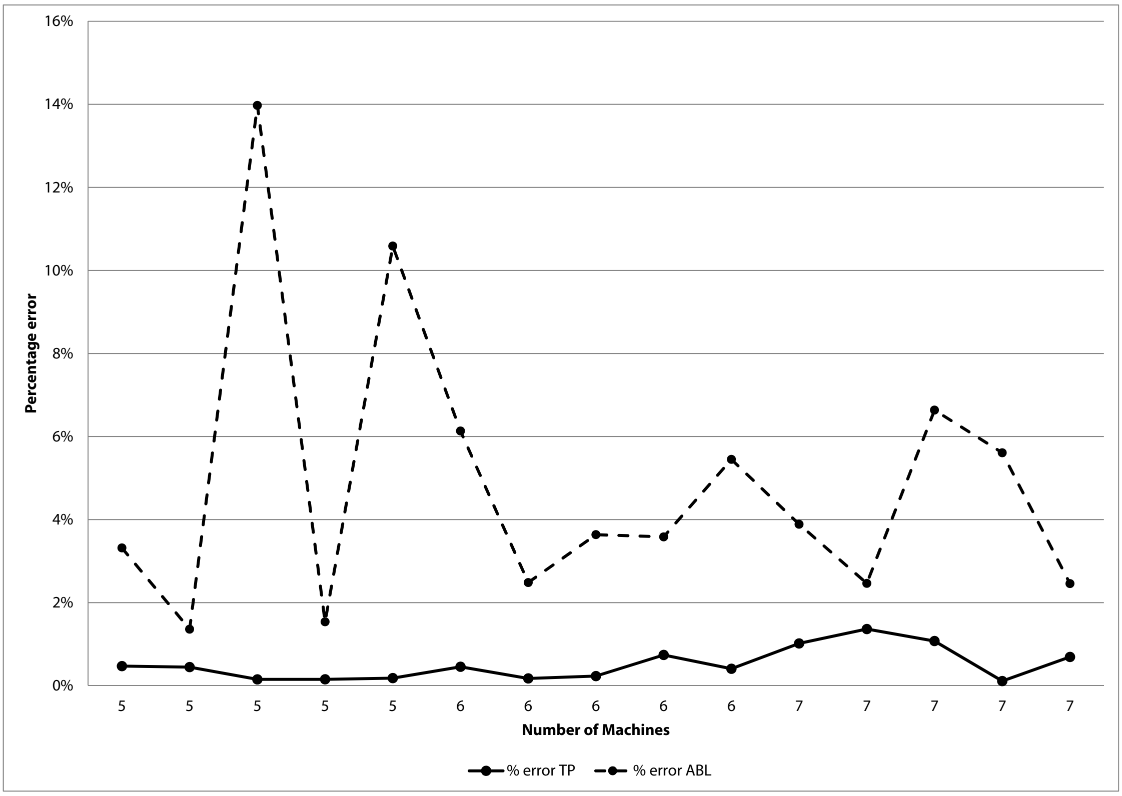

FIGURE 6.1: %error throughput and average WIP vs. length of production line see appendix A for data

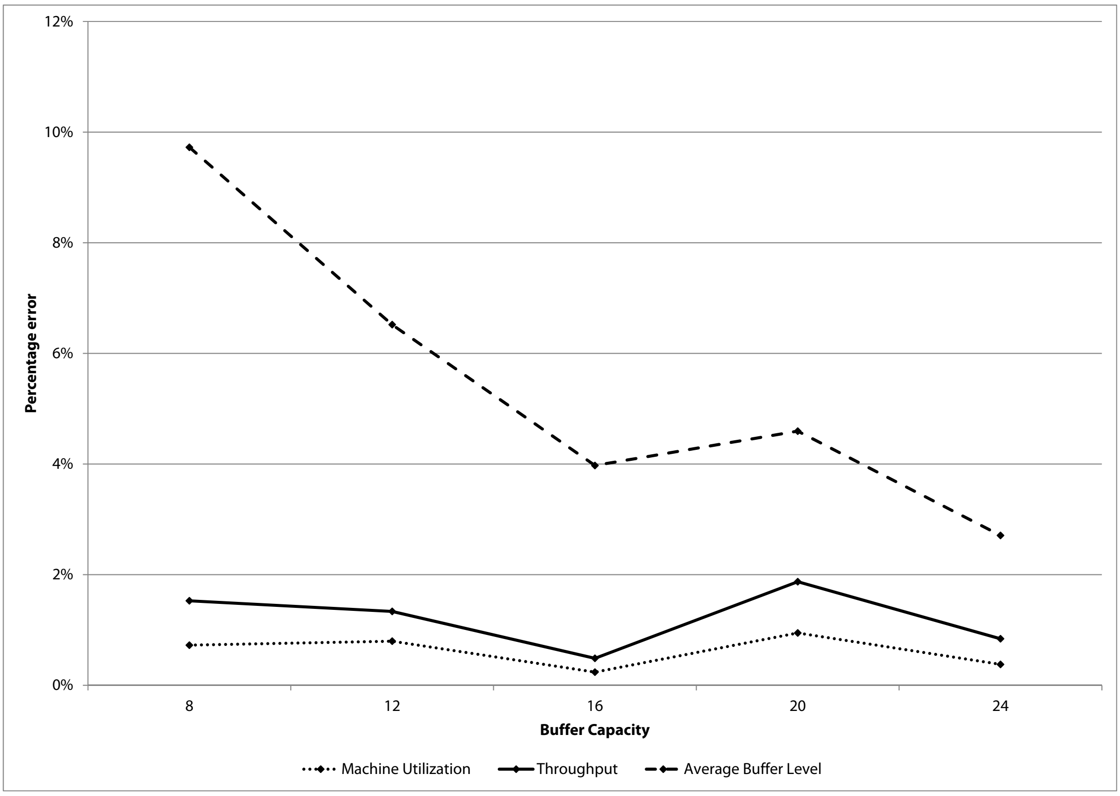

FIGURE 6.2: The impact of buffer capacity on evaluation accuracy

In the second set of experiment we exam the performance of the method under different buffer capacities to find out whether the performance evaluation error inflates or deflates with the growth of scale of the model. We took a five-machine line, increase the buffer capacity gradually and compare the result of analytical model and simulation. To eliminate the impact of machine efficiency and bottleneck, we set the parameters so that all machines has the same isolated efficiency around 4 and all buffers are set to be equal to each other.

Figure 6.2 shows how the accuracy of evaluation of throughput, utilization and average buffer level varies with the growth of buffer capacity. Note that the %error for utilization and average buffer level is displayed as the average error of all 5 machines and all 4 buffers. Observation shows that the throughput evaluation and utilization is quite stable among different scale of the model while the evaluation of average buffer level seems more accurate when the buffer capacity is high. Taking a close look at the data we are able to identify that the absolute error of average buffer level is still quite stable, the increase in accuracy is due to increase in the denominator

In the third set of experiment we compare the result of our model with that of Tolio's model [13], [14]. Since Tolio's model deals only with single-item machine, the parameters generated are restricted to batch size = 1. Simulation setting is the same as our previous analysis. For each simulation experiment, 20 replications have been performed. Each run entails a simulations period of 1,000,000 time unit preceded by a warm-up period of 100,000 time units. Different settings of failure mode parameters have been considered, with repair rates of different orders of magnitude. We reproduce the same analysis for machine lines consists of 5, 6 and 7 machines. Specifically, we considered model with relative low buffer capacity (therefore the effect of blocking and starvation are displayed). It should be noticed that Tolio's model takes a different assumption from our model: Tolio's model assumes that machines operates under blocking-after-service policy while we took the assumption that machines operations under blocking-before-service policy. Therefore in our test Tolio's method is modified to adopt blocking-before-service policy to be tested on the same basis. This switch should not affect the accuracy of Tolio's model because we can consider blocking-after-service model equivalent to blocking-before-service model with an additional buffer capacity equals to the batch size of downstream machine.

Figure 6.3 shows the comparison between our proposed method and Tolio's method on the capacity on estimating the production rate. Detailed parameters is shown on appendix B. In this case Tolio's method, not considering the behavior of remote

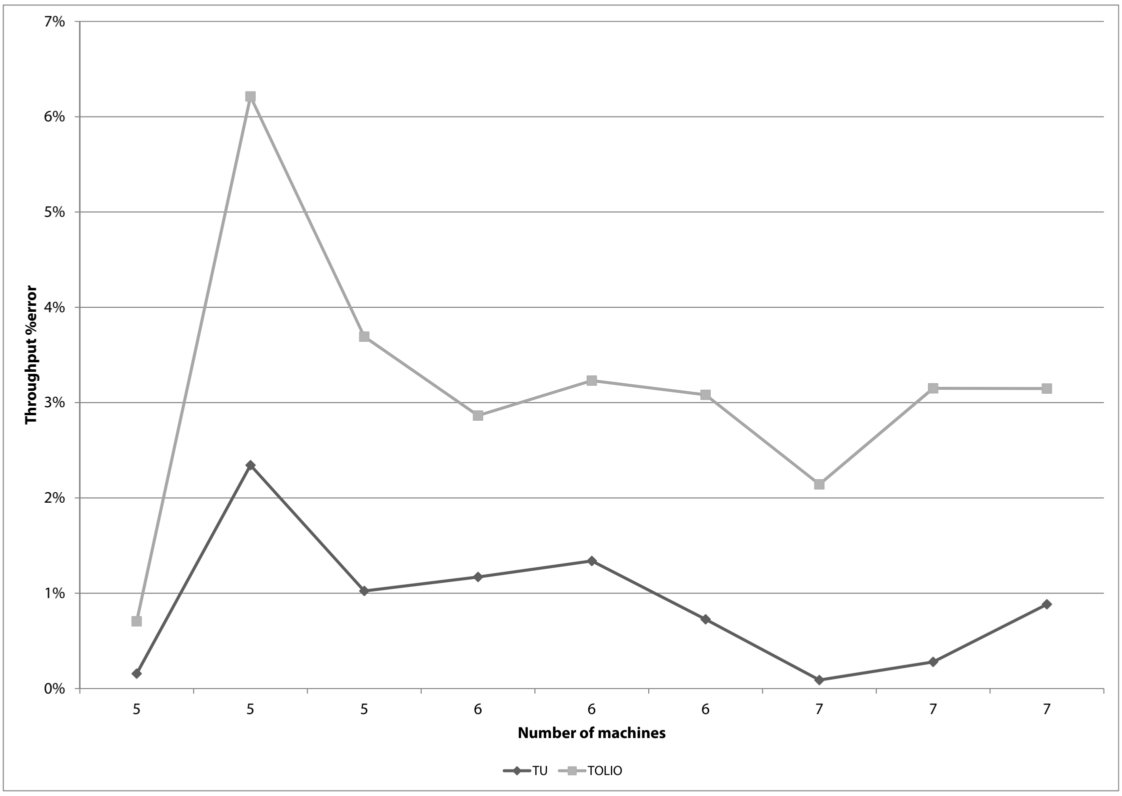

FIGURE 6.3: Line results: throughput error with respect to simulation see appendix B for data

failures, can lead to relatively large errors in performance estimation with respect to the simulation results (up to 7% as regard to the figure 6.3). On the contrary, our proposed method, being able to distinguish among the different behavior of local and remote failure modes, provides fairly accurate results.

On the other hand, Figure 6.4 shows the comparison between our proposed method and Tolio's method on the capacity on estimating the average buffer level. It can be seen that although the behavior of remote failures is considered, our proposed method does not outperform Tolio's model in term of estimation on average buffer level.

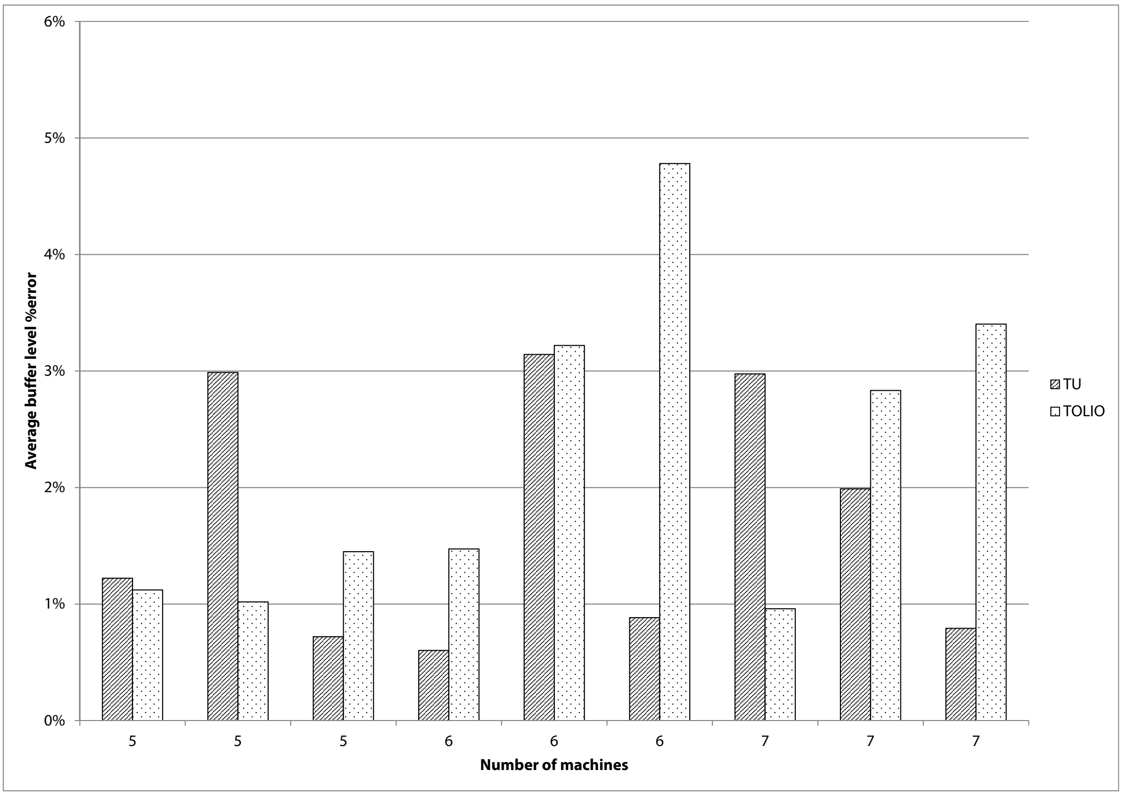

FIGURE 6.4: Line results: average WIP error with respect to simulation

6.2 Numerical result: speed

In this section we present the speed analysis of the method. Since the method is based on Markov chain and no additional mathematical analytic result is adopted, the computational time depends on the scale of the system. Therefore we study on the 3 variables which determine the scale of the system: number of machines, buffer capacity and number of failures to see their impacts on the computational time.

Table 6.2 presents the parameters taken for basic lines. All test in three dimensions are based on basic line and introduces some new parameters.

Figure 6.5 shows how the impact of number of failures on computational time. In this test we try to introduce new failure modes to the system (shown on table 6.3).

N	8	8	8	8	0
c	4	2	3	3	3
u	1.1	3.6	1.2	1.2	1.3
# failures	1	1	1	1	1

p	r
0.05	0.8
0.01	0.4
0.04	0.8
0.03	0.5
0.06	0.2

TABLE 6.2: Parameters of basic lines

p	r
0.01	0.3
0.03	0.7
0.03	0.5
0.03	0.3
0.04	0.9
0.03	0.8
0.04	0.5
0.02	0.5
0.05	0.5
0.02	0.4
0.05	0.6
0.02	0.6
0.03	0.8
0.04	0.8
0.05	0.7
0.01	0.5

TABLE 6.3: Parameters of additional failure modes

It can be found that the growth is quadratic.

Figure 6.6 shows the impact of buffer capacity on computational time. In this test we try to increase the buffer size simultaneously. It can be found that the growth is linear.

Figure 6.7 shows the impact of number of machines on computational time. In this test we try to add machines to the line (shown on table 6.4, failure and repair rates refer to the second test). It can be found that the growth is exponential.

In conclusion, the number of failures has the most impact on the speed of the

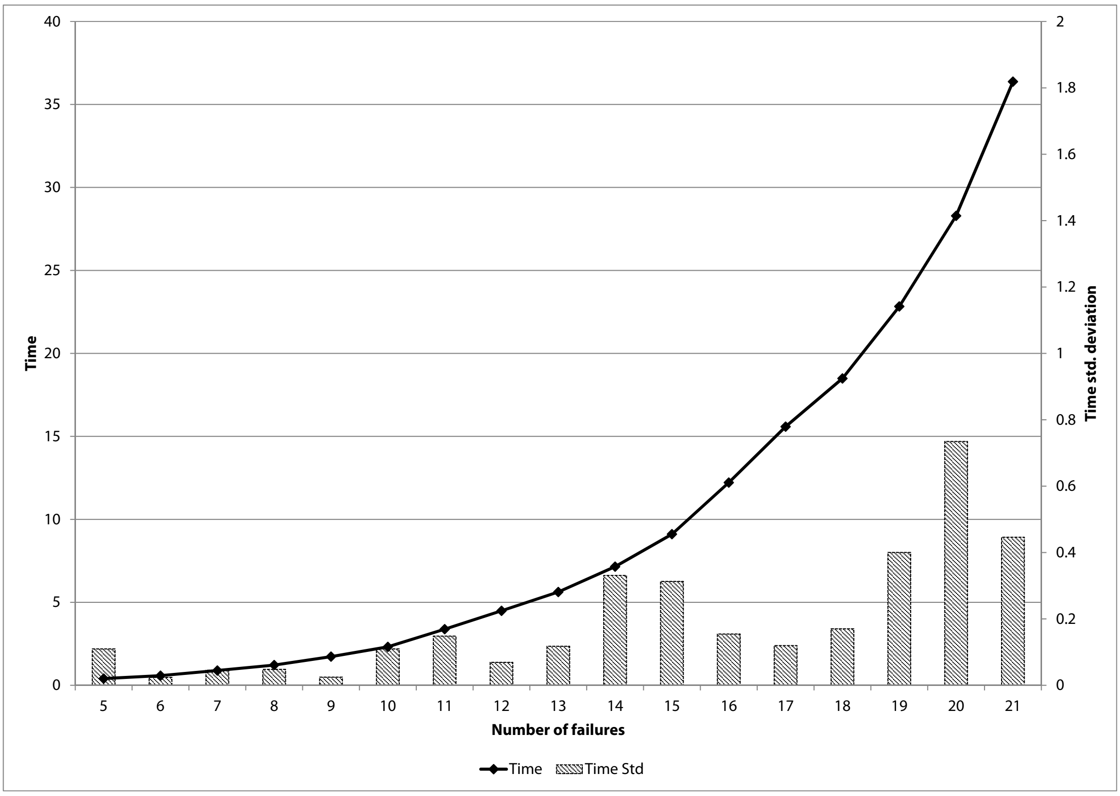

FIGURE 6.5: The impact of num. of failures on computational time

N	8	8	8	8	8	8	8	8	8	8	8	0
c	4	2	3	3	3	4	2	2	2	3	2	2
u	1.1	3.6	1.2	1.2	1.3	0.9	1.8	2	2.1	1.4	2	1.8
# failures	1	1	1	1	1	2	1	1	3	3	1	1
effe.	4.19	3.22	3.51	3.27	3.63	3.35	3.48	3.87	3.82	3.65	3.87	3.38

TABLE 6.4: Parameters of additional machines

algorithm, followed by number of machines and then buffer capacity. Since in real cases the number of failures are usually not very limited and number of machines are very likely to be predetermined, we can foresee that in real application the speed of the algorithm will be less variable with different configurations.

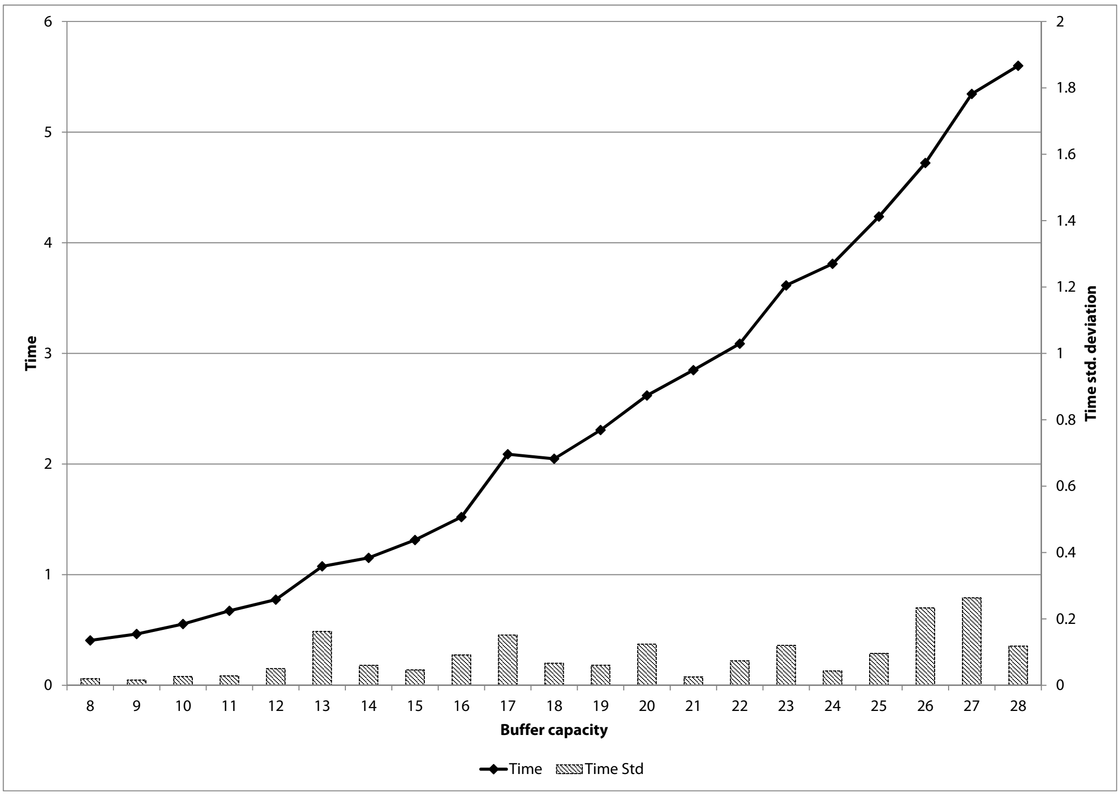

FIGURE 6.6: The impact of buffer capacity on computational time

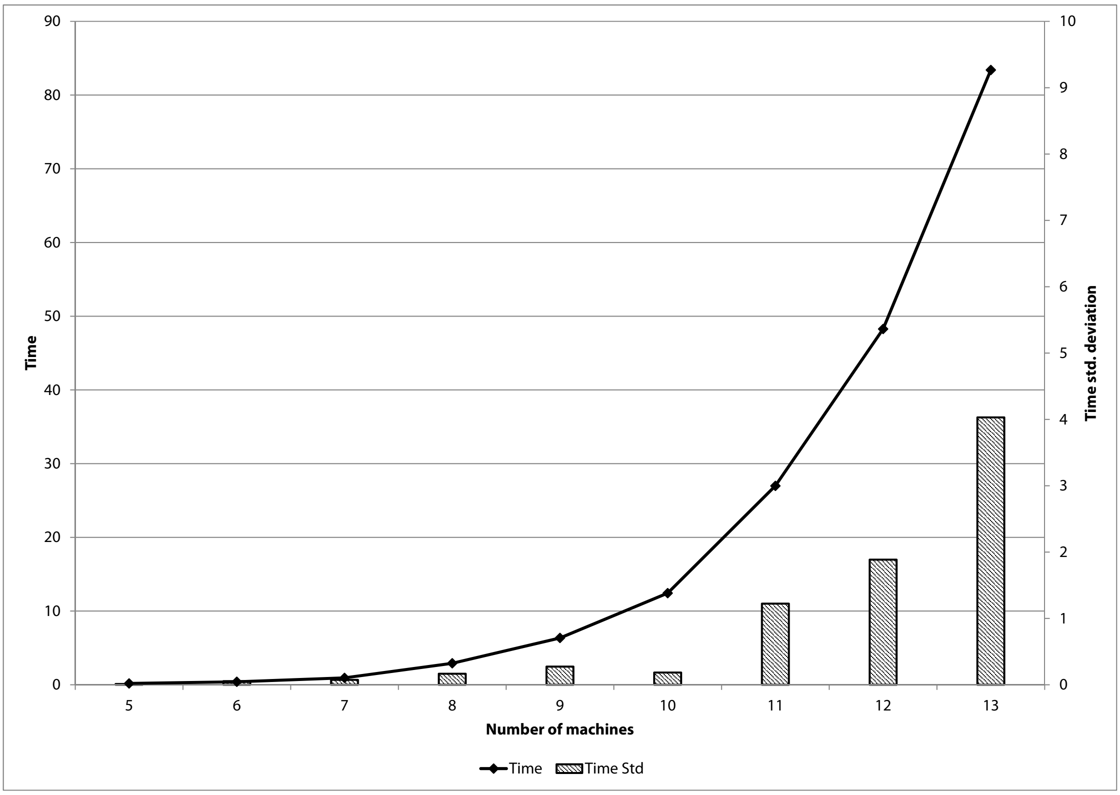

FIGURE 6.7: The impact of num. of machines on computational time

6.3 Other numerical results

In the first set of experiment, we try to increase simultaneously the buffer capacity for each buffer to track the change of line performance affected by the change of buffer capacity. Two set of systems of 5 machine line and all its parameters are generated for the experiment. The isolated efficiency is set to be around 2.5 and 4. Detailed parameters are given in appendix C. Since the maximum batch size is 4 in the line, we start the experiment at buffer capacity set to be 4 and increase gradually by 2 the buffer capacity of all buffer. The result we obtained is shown in figure 6.8 The reward of increasing in buffer capacity is decreasing with the growth of basic buffer capacity. Therefore to optimize the overall performance of buffer the cost-efficiency balance need to be studies carefully. It can also be observed that with buffer capacity growing, the ratio of the throughput of the two line are more close to their ratio of isolated efficiency, which means higher buffer level release the potential of the machine while lower buffer level constrains the machine at a relative low production rate.

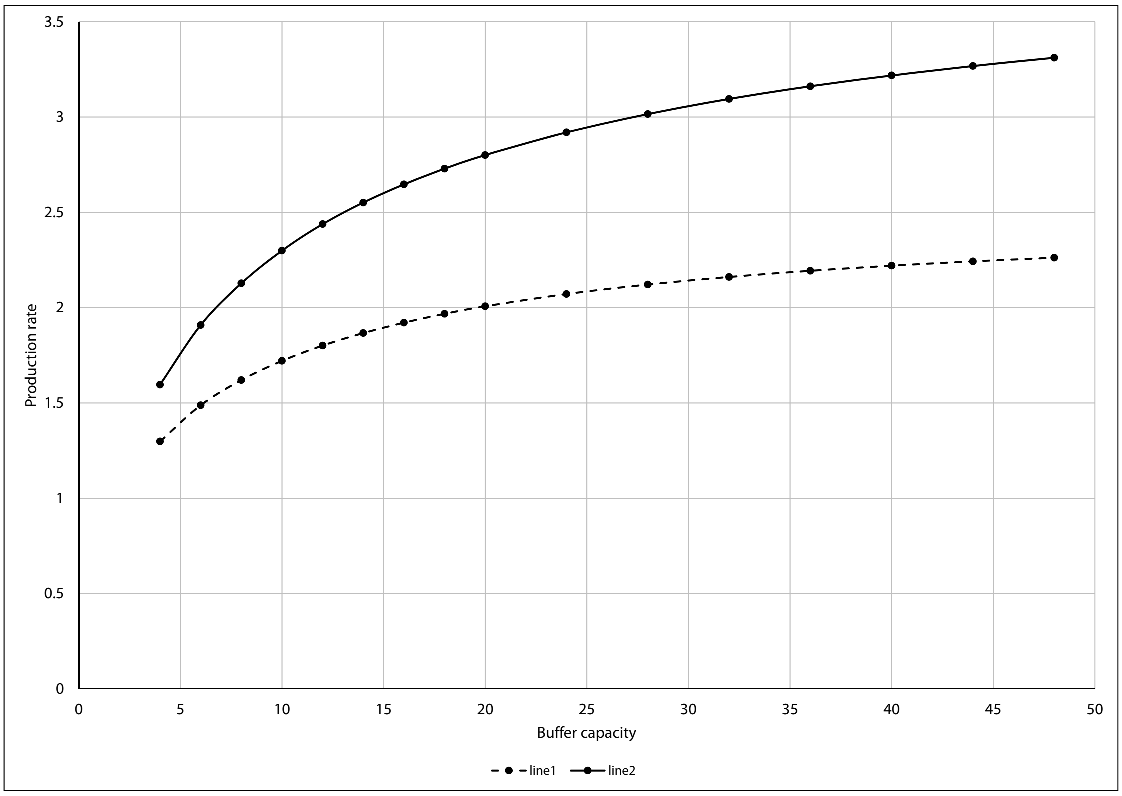

FIGURE 6.8: The impact of buffer capacity on the production rate

In the second set of experiment, we try to increase the buffer capacity in different position of the production line and with different basic buffer capacity to compare the change rate of throughput. A system of 5 machine line and all its parameters are generated for the experiment similar to the first experiment. In order to minimize the impact from different machine parameters we use the same machine configuration for M_1, M_3, M_5 and M_2, M_4. Detailed parameters are given in the table 6.5 (note that the failure mode identical machine is shown only once).

We tested on changing buffer capacity from 6 to 48 and on each buffer location with basic buffer capacity equals 12. We also repeat the experiment for basic buffer capacity equals 8 and 10 and it shows the similar behavior. The result we obtained is shown in figure 6.9

N	10	10	10	10	0	p	r
c	4	3	4	3	4	0.05	0.8
u	1.06	1.51	1.06	1.51	1.06	0.01	0.4
# failures	1	3	1	3	1	0.04	0.8
e	3.99	3.99	3.99	3.99	3.99	0.03	0.5

TABLE 6.5: Line parameters

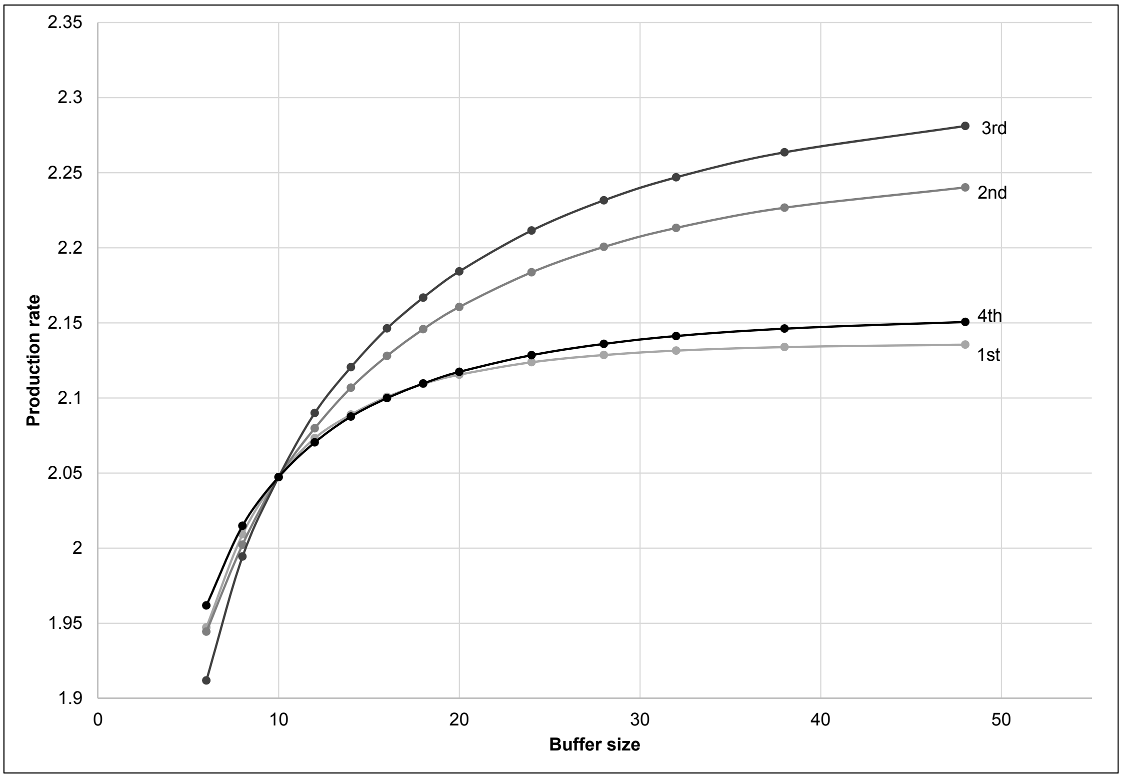

FIGURE 6.9: The effect of increasing buffer capacity in different position of the production rate

It is interesting that increasing buffer capacity in the middle of the line is much more effective than increasing it in the head and tail of the line in the same scale. Also we notice that increasing buffer capacity in the upper stream of the line is not so effective as increasing it in the lower stream of the line. The result offers an insight that in order to be most effective it is better start increasing buffer capacity of the middle-low stream of the line.

N	25	22	18	0
c	3	x	2	5
u	1.5	1	2.4	1
# failures	2	2	1	3
efficiency	4.18	1.75x	4.17	4.17

p	r
0.01	0.2
0.01	0.4
0.04	0.3
0.05	0.8
0.06	0.4
0.01	0.6
0.04	0.3
0.03	0.6

TABLE 6.6: Line parameters

In the third set of experiment, we study the effect of the size of one machine on the production rate of the line (shown in figure 6.10). Detailed parameters are given in the table 6.6. When the machine is the bottleneck, the continued increase in the size of the machine leads to an increase in the throughput. However, when the efficiency of the machine surpluses the other machine, the effect of increasing batch size on the production rate is more complicated. First it will increase with a decreasing rate until batch size equals to 15. Then the production rate begins to decrease with a slow rate until it suddenly drops to zero due to a deadlock (not shown on the figure). This is due to a too large batch size forces the machine to wait as long as there is not enough piece in the upstream buffer and there is not enough space in the downstream buffer. Both requirements need to be satisfied before the machine becomes operational, which makes the operational time of the machine to be very limited.

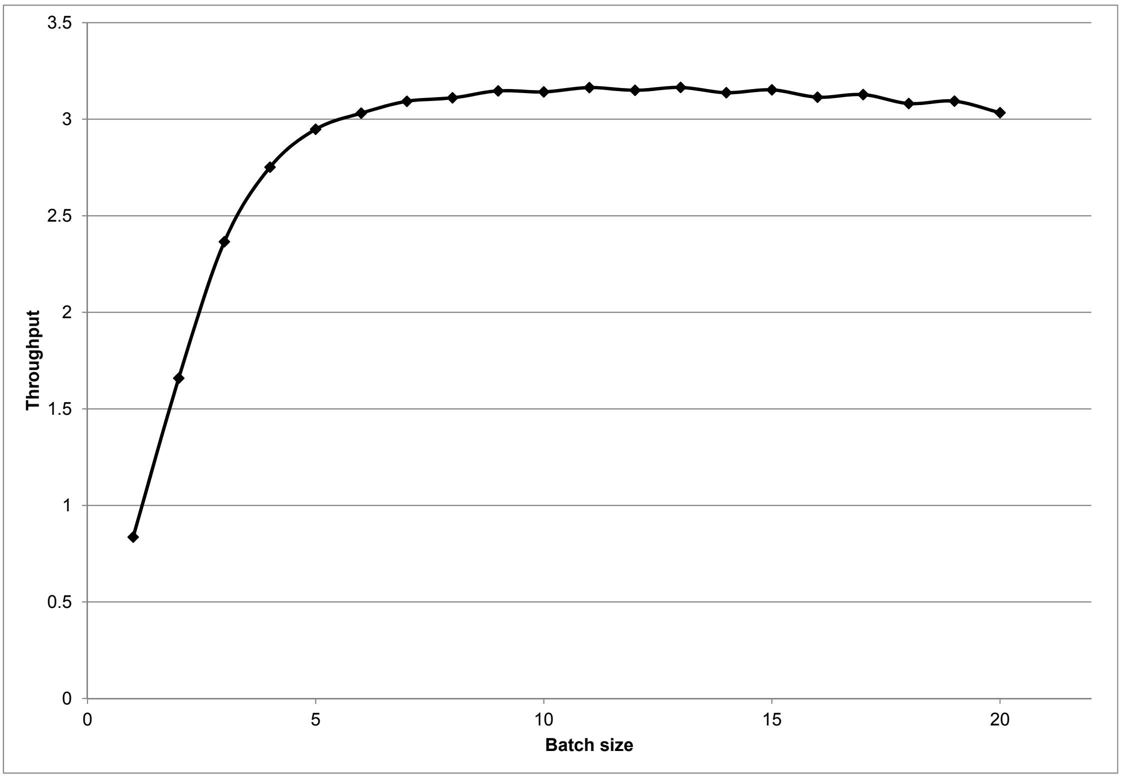

FIGURE 6.10: The effect of batch size on the production rate

It could be also noticed that the curve of the throughput is not so smooth. It is because, as mentioned in Gershwin's paper [23], the system can be not ergodic when the batch size of upstream and downstream machine is not prime to each other. When the batch size equals to $2, 4, 6, \ldots$ it is not prime to the downstream machine which has batch size equals 2 and when equals to $3, 6, 9, \ldots$ it is not prime to upstream machine which has batch size equals 3. Therefore different performance evaluation could be obtained with different settings for initial buffer level. Figure 6.11 further illustrate this issue by showing different results obtained with different initial buffer level settings. The buffer capacity is reduced and machine speed is increased to make the phenomenon more explicit (parameters given on table 6.7. It is clear on the figure that the production rate splits when the batch size is not

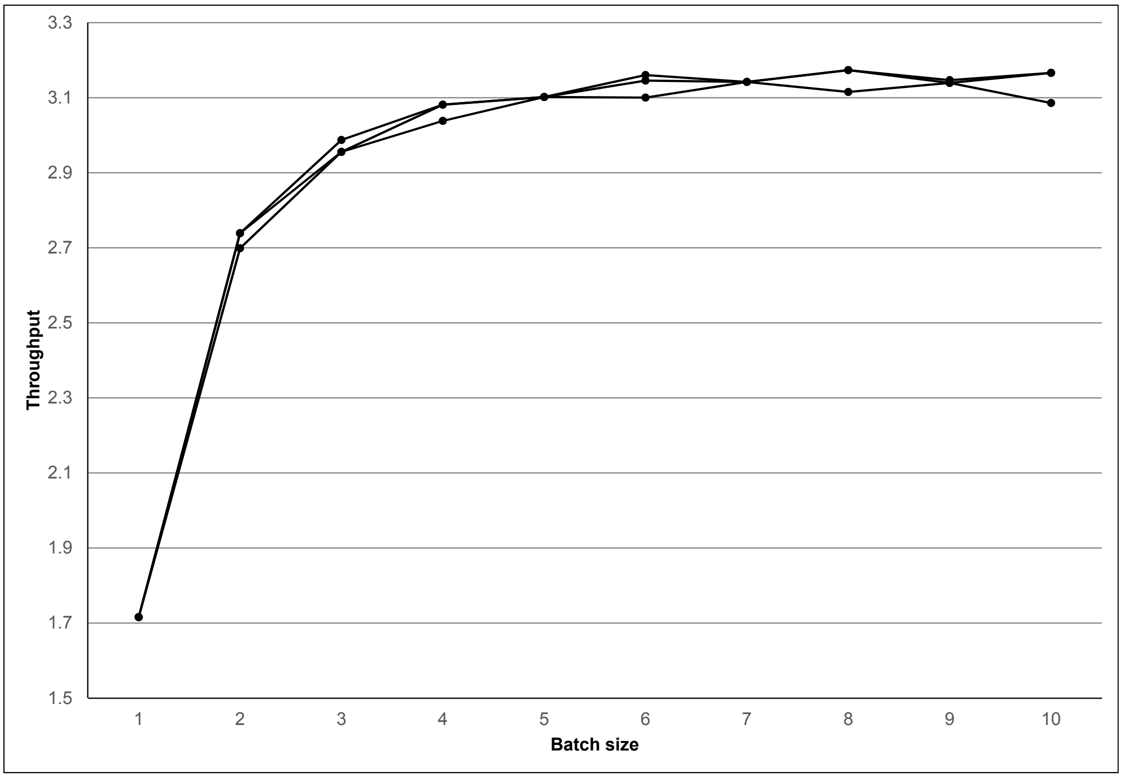

FIGURE 6.11: The effect of non-prime batch size on the production rate

N	15	12	18	0
c	3	x	2	5
u	1.5	2.1	2.4	1
# failures	2	2	1	3
efficiency	4.18	1.75x	4.17	4.17

TABLE 6.7: Line parameters

prime to upstream and/or downstream machines' batch size and merges when the batch size is prime (eg. when batch size equals 5 and 7).

Although the phenomenon is quite complex, it may not have large impact on practical applications because by nature a production line starts at zero buffer level and it can be seen from the results (table 6.8) that when starts with buffer level equals 0 always gets the best performance. But it does not mean that the result is meaningless. From a managerial point of view, we can understand that with linked machines produce in non-prime batches, the growth of line performance with the

batch size\initial buffer level	0	1	2
1	1.7156	1.7156	1.7156
2	2.739	2.6987	2.739
3	2.9873	2.9556	2.9556
4	3.0814	3.0382	3.0814
5	3.102	3.102	3.102
6	3.1604	3.1002	3.1457
7	3.142	3.142	3.142
8	3.1735	3.1153	3.1735
9	3.1469	3.1395	3.1395
10	3.1661	3.0859	3.1661

TABLE 6.8: Line results

growth of batch size is not smooth (sometimes even shrinks) and the growth is more significant when the buffer capacity is the least common multiple of the upstream and downstream batch. Therefore it is more economic to have buffer capacity equals a multiplier of the least common divider of the upstream and downstream batch when your line has many batch machines that have batch size not prime to each other.

To illustrate this effect, figure 6.12 shows how the production rate changes with the increase of buffer capacity when the line is composed of several batch machines with non-prime batch sizes. For such productions lines the performance is only improved when increase the buffer size by the least common divider of the upstream and downstream batch.

As a conclusion, the possibility to distinguish the different behavior of local and remote failure modes for each pseudo machine leads to a more accurate evaluation of the performance measures of building blocks while reasonable approximation of the remote failure modes behavior and physical machine mechanism results in analytical solution for long productions lines where machines are producing in batches with exponentially distributed processing time and may be subject multiple failure

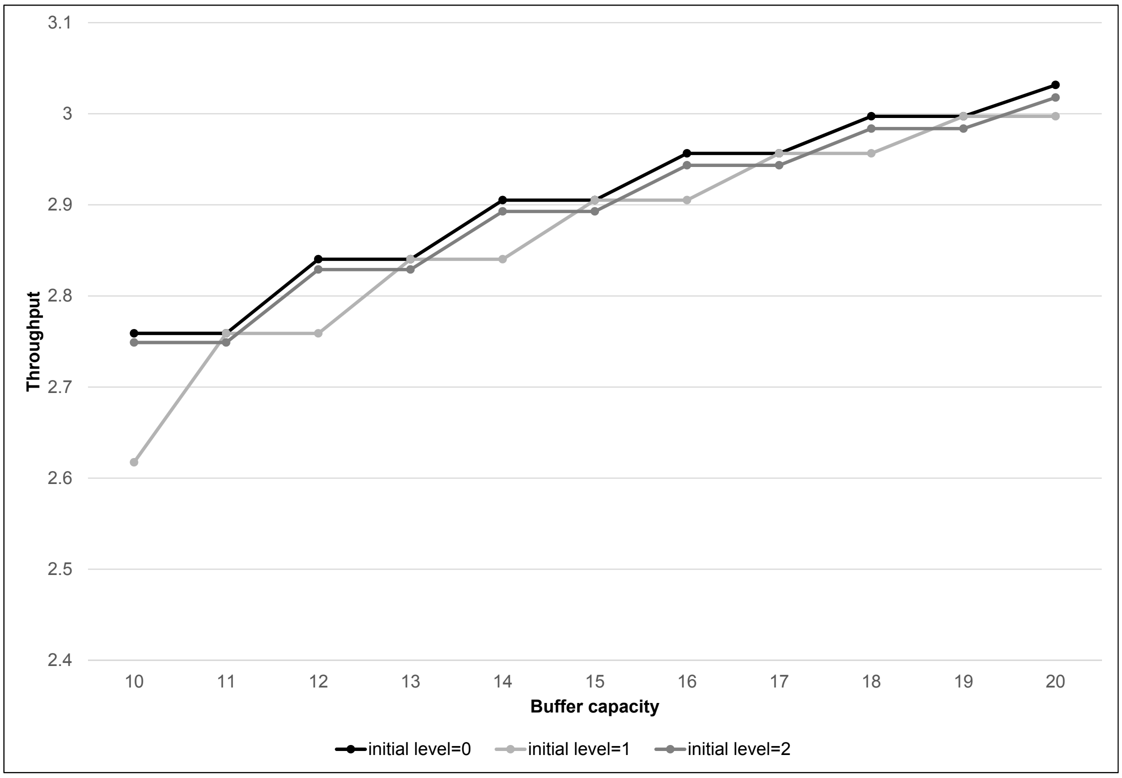

FIGURE 6.12: The effect of buffer capacity in production rate for lines composed of machines with non-prime batch sizes

modes. Some characteristics of the batch production line are also identified like (1) the more buffer capacity in a line, the less effective they are; (2) increase buffer capacity in middle of the line is more effective than that near the entrance or exits of the line; (3) increasing batch size of the machine does not always increase the throughput, buffer capacity and batch size need to be balanced to archieve best performance (4) the performance evaluation of batch machine line may not only be determined by the parameters of lines but also affected by the initial buffer level.

Chapter 7

Case Study

In this chapter, a real case will be studied and thus illustrate the application of the new approach in real scenario. The case comes from the process chain of a Bosch plant production electrical engines in Hildesheim. Indeed, to model a serial of automated machine line the exponential assumption is somehow not the best practice. But we will show that the approach is still able to reveal the characteristics of the produciton line. In this case study we have two focus: (1) obtain the performance evaluation of the production line (2) offer insights on how the performance of the line could be reconfigured and optimized with various production tasks.

7.1 Background

The case is based one the process chain of a Bosch plant in Hildesheim. The goal of this manufacturing system is the production of magnetic rotors, that find a potential use in the electrical vehicles. A schema of this plant is represented in Figure 7.1. Concerning the layout, the system is composed by two main branches,

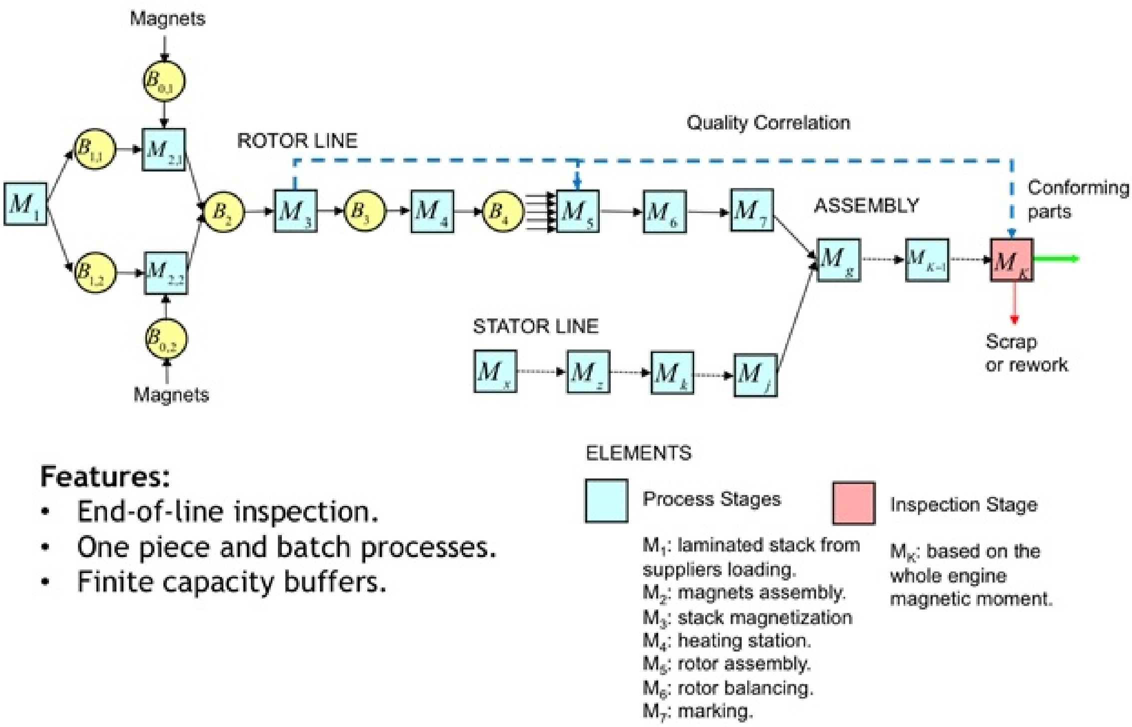

FIGURE 7.1: General model of the currently implemented production system

respectively dedicated to the assembly and magnetization of the rotor and to the stator production. The marriage between these two sub-components takes place at a downstream assembly stage and the complete engine is produced. Testing of the engine takes place at the end of the line (EOL testing). At this stage, the overall engine magnetic moment is measured and if it is observed to deviate more than 4% from its target value, the product is amenable to be considered as a defect to be scrapped. With the EOL testing solution, the defect is identified only when all expensive and time-consuming operations have been already performed on the product and all the possible defect sources have been accumulated, thus preventing from any possible repair operation. The e-mobility market is undergoing a rapid growth in recent years, Therefore, a new solution is expected to quickly evaluate the production capacity and reallocate production resources to market needs.

7.2 System layout

In detail, this line is composed of seven main stages, dedicated to the following operations:

1. M_1: loading of the stacks on the pallet.

2. $M_{2,1}$, $M_{2,2}$: two parallel stages assembling the magnets on the stacks. The stations are composed of a pick and place system for the positioning of the magnets in their locations. Moreover, the glue is dispensed at the interfaces between the magnets and the stacks surface. Finally, the glue is thermally treated in a single oven for both parallel stages.

3. M_3: Stack magnetization process. Each stack is centered in the magnetization device that is produced by a supplier and currently uncontrolled by Bosch.

4. M_4: heating station. A rotating table carrying 4 magnetized stacks moves the stack into a heating chamber for preparing the stack to the next assembly operation. Indeed the assembly principle is based on mechanical interference. Since it has 4 stacks position, it could be used in the future as a sequence decoupling stage.

5. M_5: assembly machine. The required number of stacks, normally varying from 5 to 10 for different product types, is taken from the heating machine and a pile of stacks in the z axis of the machine is formed by mounting each stack on the central shaft. This represents the core of the rotor. In the current production line, this is the only machine that produces in batches and this machine decides the types of engines manufactured by the whole line.

6. M_6: rotor balancing station.

7. M_7: rotor marking station.

8. M_x: processes of stator assembly, not treated in detail in this project.

9. M_g, M_z, M_l, M_j: assembling rotor and stator together.

10. M_k: End-of-line (EOL) testing of final motor where several motor properties are measured.

7.3 Model development

Having briefly mentioned the description of the Bosch production line together with current different types of defects, the following section is dedicated to evaluate the developed model for production logistics performance of the line. Since the proposed model discussed in previous sections is general, some assumptions based on the specific production line are considered. The reference system on which the assumptions below are referred to, is depicted in figure 7.1.

- Since the supply of both buffers $B_{1,1}$ and $B_{1,2}$ are provided by the same stacks from the upstream machine M_1, they are considered as one single buffer transferring the stacks to the downstream machine.

- In addition, according to the current configuration of Bosch production line, both machines $M_{2,1}$ and $M_{2,2}$ are performing the same operation, assembling the magnets on the stacks. Therefore they are considered as a batch machine M_2 in the model.

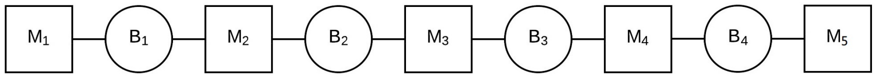

FIGURE 7.2: Approximation of the original Bosch layout with a multistage processing model

- Parallel machines $M_{2,1}$ and $M_{2,2}$ are fed by two out-of-line buffers in which magnets raw part are stored. The capacity of those buffers is considered infinite and therefore the magnets raw parts are always available.
- Having assembled the rotor at station M_5, all the other downstream machines are aggregated together, since there is no buffer after station M_5.
- The machine M_5 is, in theory, an assembly machine. But, since there is no buffer downstream, the approximation of the assembly operation is provided by dividing the throughput over the number of assembled stacks.
- The machines M_x, M_z, M_k and M_j, which create the stator assembly line, are not modeled in this project.

The figure 7.2 demonstrates the transformation of the original Bosch line model to an approximated equivalent production line, regarding the mentioned assumptions.

In order to analytically approximate the performance of the production model, the estimation of the reliability parameters of each machine is discussed in the subsequent section. Since the real data for estimation of the mentioned parameters are available from Bosch, a detailed description of the values used is provided. The details about the machines and buffers, are reported in table 7.1, 7.2 and 7.3.

1. M_1: Loading the stacks to the pallet is operated in this machine that can undergo to a unique operational failure. When the machine is operational, it

Machine	MTTF (min)	MTTR (min)	Cycle time (min)
M_1	480	0.15	0.15
M_2	150	2.5	1.8
M_3	160	1	0.69
M_4	175200	480	0.667
M_5	28800	150	0.8

Table 7.1: Line machine parameters

takes 0,15 minutes to process a part. Moreover, the machine fails on average every 480 minutes and takes 0,15 minutes to get repaired.

2. B_1: This buffer is obtained from the coupled buffers $B_{1,1}$ and $B_{1,2}$ from the original model in figure 7.1. The capacity of the buffer is considered finite and equal to 5.

3. M_2: As previously mentioned, machines $M_{2,1}$ and $M_{2,2}$ of Figure 7.1 are considered as batch machines with batch size equals to 2. Moreover, they are modeled as single failure mode machines, according with the suggestion obtained from Bosch. The processing rate of the machine is equal to 1,8 minutes per each stack. About the reliability, the machine fails on average every 150 minutes and gets repaired in around 2.5 minutes (this is the equivalent to two parallel machine system proposed by Burman [26]).

4. M_3, M_4 and M_5 are modeled as single failure mode in the system. According to the data provided by Bosch, the reliability parameters for these machines are reported in the table 7.1, as well as the processing rate when the machine is operational.

B_2, B_3 and B_4 are considered as finite buffers and the approximated values all buffer capacity are demonstrated in the table 7.2.

Buffer	Capacity
B_1	5
B_2	40
B_3	40
B_4	6 + batch size

Table 7.2: Line buffer capacities

Batch size	Throughput (stackes/min)	Throughput (rotors/min)
2	1.0892	0.5446
3	1.0897	0.3632
4	1.0900	0.2725
5	1.0901	0.2180
6	1.0902	0.1817
7	1.0903	0.1558
8	1.0904	0.1363
9	1.0904	0.1212
10	1.0905	0.1090

Table 7.3: Line buffer capacities

7.4 Result and analysis

Based on the prior analysis on the characterization of the line and the machines, the isolated efficiency of each machine in the line can be obtained respectively as 6.67 p/min, 1.09 p/min, 1.45 p/min, 1.49 p/min and 6.25 p/min. Apparently, machine M_2 is the bottleneck of the line. The throughput of the line obtained using our proposed approach is 0.9189 stackes/min which is equivalent to 4.5943 rotors/min.

Since the machine M_2 is a severe bottleneck of the line, with increase in batch size on machine M_5 you will not get much efficiency rewards, the respective throughput increasing production batch from 5 to 10 is shown in the table 7.3

Since the machine M_2 is a severe bottleneck of the line, a possible resolve of the constrained production capacity is to add a identical machine at the same position to form a three parallel machines system. Adopting the same Burman's equations

Batch size	Throughput (stackes/min)	Throughput (rotors/min)
2	1.4062	0.7031
3	1.4108	0.4703
4	1.4119	0.3530
5	1.4115	0.2823
6	1.4118	0.2353
7	1.4120	0.2017
8	1.4122	0.1765
9	1.4123	0.1569
10	1.4124	0.1412

TABLE 7.4: Line buffer capacities

to transform parallel machines into single machine, the machine fails every 100 min and can be repaired in 0.91 min. This will increase the isolated efficiency of the machine 2 to 1.65 p/min. The table 7.4 shows the respective throughput increasing production batch from 5 to 10. The production rate is greatly improved while there still not much different between different products because the bottleneck now transferred to the 3rd and 4nd machines (1.65 p/min compared with 1.45 and 1.49 p/min).

Chapter 8

Conclusion

In this thesis we presented the analytical modeling and exact analysis of production lines consists of multiple unreliable batch machines with finite buffers in-between. We have presented new conservation of flow and flow-rate-idle-time relationships for the building blocks considering the behavior of remote failures. Then we derived a new set decomposition equations which can be applied on batch machine with multiple failure modes. Finally we developed the new computational algorithm dealing with such production line on these basis. We have also presented various performance measures of interest such as production rate, probabilities of blocking and starvation, and expected in-process inventory. Numerical results and their qualitative interpretations have been presented. We demonstrated a set of characteristics of the batch production line. These new phenomena and insights are investigated and interpreted.

Numerical results revealed that the possibility to distinguish the different behavior of local and remote failure modes for each pseudo machine leads to a more accurate evaluation of the performance measures of building blocks while reasonable approximation of the remote failure modes behavior and physical machine mechanism results in analytical solution for long productions lines where machines are producing in batches with exponentially distributed processing time and may be subject multiple failure modes.

Furthermore, the important phenomena and insights observed can be summarized as follows.

1. the more buffer capacity in a line, the less effective they are;
2. increase buffer capacity in middle of the line is more effective than that near the entrance or exits of the line;
3. increasing batch size of the machine does not always increase the throughput, buffer capacity and batch size need to be balanced to archieve best performance;
4. the performance evaluation of batch machine line may not only be determined by the parameters of lines but also affected by the initial buffer level.

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Appendix A

Parameters and results for accuracy test

Case 5-1					
N	21	21	15	12	0
c	4	3	4	1	3
u	0.9	1.4	1.1	3.8	1.4
# failure	1	3	3	1	1
effe.	3.39	3.70	3.20	3.58	3.82
Case 5-2					
N	21	21	18	12	0
c	4	3	4	2	2
u	0.9	1.5	1	2.1	2.2
# failure	1	1	2	1	3
effe.	3.27	4.29	3.22	3.60	4.00
Case 5-3					
N	15	12	18	18	0
c	4	1	3	3	3
u	1.1	3.6	1.2	1.2	1.3
# failure	1	2	1	1	1
effe.	4.19	3.22	3.51	3.27	3.63
Case 5-4					
N	6	12	12	12	0
c	1	1	3	1	3
u	3.2	3	0.9	2.6	1
# failure	2	3	2	2	3
effe.	2.84	2.40	2.41	2.33	2.50
Case 5-5					
N	18	9	15	24	0
c	4	2	1	4	4
u	0.9	2.2	4.5	1	1.1
# failure	1	3	2	3	3
effe.	3.41	3.84	3.18	3.51	4.22

5-machine line									
p	r	p	r	p	r	p	r	p	r
0.05	0.8	0.02	0.2	0.01	0.2	0.02	0.2	0.04	0.7
0.01	0.4	0.03	0.6	0.06	0.9	0.01	0.4	0.01	0.5
0.04	0.8	0.03	0.4	0.02	0.4	0.05	0.5	0.02	0.3
0.03	0.5	0.05	0.3	0.01	0.4	0.02	0.4	0.03	0.5
0.06	0.2	0.05	0.3	0.02	0.2	0.03	0.3	0.05	0.3
0.01	0.3	0.01	0.9	0.03	0.4	0.05	0.8	0.05	0.2
0.03	0.7	0.02	0.7			0.03	0.5	0.05	0.8
0.03	0.5	0.03	0.5			0.05	0.9	0.02	0.6
0.03	0.3					0.03	0.5	0.04	0.9
						0.05	0.8	0.01	0.7
						0.05	0.5	0.01	0.6
						0.03	0.8	0.01	0.8

FIGURE A.1: Parameters generated for 5-machine line

Case 6-1						
N	18	21	18	12	9	0
c	3	3	4	2	2	1
u	1.3	1.4	1	2.2	1.9	3.6
# failure	3	2	2	2	2	2
effe.	3.49	3.83	3.46	3.76	3.12	3.15
Case 6-2						
N	12	21	21	12	6	0
c	1	3	4	3	1	1
u	3.8	1.4	0.9	1.2	4.2	4.1
# failure	3	3	2	1	3	2
effe.	2.89	3.65	2.67	3.43	3.64	3.47
Case 6-3						
N	21	12	9	12	18	0
c	4	3	1	2	2	4
u	1	1.2	4.1	1.9	1.9	1
# failure	1	3	2	2	1	2
effe.	3.91	3.11	2.99	3.10	3.35	3.03
Case 6-4						
N	21	18	18	21	12	0
c	3	4	2	4	3	1
u	1.3	1.1	1.9	1.1	1.3	3.5
# failure	2	2	3	1	2	2
effe.	3.27	3.77	3.31	4.11	3.50	3.23
Case 6-5						
N	12	12	6	12	15	0
c	1	3	1	1	3	2
u	3.3	1.1	2.8	3.1	1	1.5
# failure	2	2	3	2	2	2
effe.	2.91	2.87	1.94	2.70	2.71	2.73

6-machine line									
p	r	p	r	p	r	p	r	p	r
0.01	0.2	0.04	0.3	0.02	0.9	0.02	0.3	0.06	0.9
0.01	0.6	0.05	0.6	0.02	0.8	0.05	0.4	0.02	0.3
0.03	0.6	0.05	0.5	0.04	0.4	0.03	0.2	0.02	0.2
0.03	0.8	0.05	0.9	0.02	0.6	0.01	0.6	0.01	0.2
0.03	0.5	0.02	0.8	0.05	0.2	0.03	0.7	0.06	0.2
0.05	0.5	0.05	0.7	0.06	0.5	0.04	0.6	0.04	0.5
0.04	0.7	0.04	0.2	0.05	0.4	0.02	0.5	W	0.5
0.05	0.7	0.03	0.2	0.03	0.3	0.05	0.7	0.04	0.4
0.06	0.6	0.01	0.2	0.04	0.3	0.02	0.7	0.04	0.8
0.06	0.4	0.05	0.8	0.01	0.5	0.06	0.7	0.04	0.7
0.02	0.3	0.03	0.9	0.06	0.2	0.02	0.7	0.04	0.8
0.01	0.6	0.04	0.7			0.05	0.9	0.05	0.9
0.05	0.4	0.05	0.3					0.04	0.9
		0.01	0.7						

FIGURE A.2: Parameters generated for 6-machine line

Case 7-1							
N	12	15	15	18	15	15	0
c	1	3	2	3	3	2	3
u	3.1	1	1.5	0.9	1.1	1.3	0.9
# failure	2	1	2	3	3	1	1
effe.	3.01	2.65	2.69	1.82	2.55	2.39	2.08
Case 7-2							
N	18	12	9	12	15	12	0
c	4	2	2	2	3	2	2
u	0.9	1.8	2	2.1	1.4	2	1.8
# failure	2	1	1	3	3	1	1
effe.	3.35	3.48	3.87	3.82	3.65	3.87	3.38
Case 7-3							
N	24	15	15	21	12	9	0
c	4	4	1	4	3	1	2
u	1.1	0.9	3.9	1.1	1.4	4.5	2.1
# failure	2	3	3	1	3	3	2
effe.	3.36	2.81	3.14	4.23	3.78	4.05	3.32
Case 7-4							
N	18	18	15	12	15	18	0
c	2	4	2	3	1	4	2
u	1.9	1.1	1.9	1.4	3.7	1.1	1.9
# failure	2	3	3	3	2	1	1
effe.	3.39	3.74	2.98	3.44	3.25	4.16	3.75
Case 7-5							
N	18	9	15	18	9	9	0
c	4	2	1	4	2	1	2
u	1	2.2	3.5	0.9	1.8	4.1	2.1
# failure	1	1	3	3	2	1	1
effe.	3.83	4.24	2.87	3.09	3.31	3.83	4.12

7-machine line									
p	r	p	r	p	r	p	r	p	r
0.01	0.8	0.04	0.7	0.01	0.9	0.02	0.9	0.04	0.9
0.01	0.6	0.01	0.6	0.06	0.2	0.02	0.2	0.03	0.8
0.04	0.3	0.01	0.3	0.06	0.5	0.04	0.5	0.04	0.5
0.04	0.5	0.01	0.3	0.05	0.5	0.03	0.5	0.02	0.5
0.01	0.3	0.01	0.2	0.05	0.8	0.03	0.8	0.05	0.5
0.06	0.2	0.02	0.9	0.05	0.8	0.05	0.8	0.02	0.4
0.05	0.9	0.02	0.7	0.03	0.3	0.04	0.3	0.05	0.6
0.05	0.4	0.02	0.6	0.04	0.5	0.04	0.5	0.02	0.6
0.01	0.4	0.02	0.4	0.02	0.5	0.06	0.5	0.03	0.8
0.06	0.4	0.02	0.3	0.05	0.7	0.02	0.7	0.04	0.8
0.06	0.5	0.02	0.6	0.01	0.7	0.05	0.7	0.05	0.7
0.06	0.7	0.06	0.9	0.02	0.8	0.05	0.8	0.01	0.5
0.06	0.2			0.01	0.4	0.03	0.4		
				0.01	0.7	0.04	0.7		
				0.05	0.7	0.01	0.7		
				0.05	0.3				
				0.02	0.2				

Figure A.3: Parameters generated for 7-machine line

	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	2.3781	12.15	12.17	5.42	3.52	0.6613	0.5662	0.5398	0.626	0.5665
TU	2.3669	11.7174	11.6115	5.3193	3.5085	0.6575	0.5635	0.5379	0.6229	0.5635
%error	0.47%	3.56%	4.59%	1.86%	0.33%	0.57%	0.48%	0.35%	0.50%	0.53%
	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	2.4697	10.8892	12.4497	6.2414	2.7065	0.6891	0.5513	0.6202	0.5907	0.5638
TU	2.4808	10.4022	12.2693	6.3306	2.8455	0.6875	0.5500	0.6188	0.5893	0.5625
%error	0.45%	4.47%	1.45%	1.43%	5.14%	0.23%	0.23%	0.23%	0.24%	0.23%
	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	2.5272	9.2697	5.8987	6.92	4.3515	0.5743	0.7017	0.7026	0.7021	0.647
TU	2.531	9.1995	6.3521	8.4846	6.0987	0.5752	0.7031	0.7031	0.7031	0.649
%error	0.15%	0.76%	7.69%	22.61%	40.15%	0.16%	0.20%	0.07%	0.14%	0.31%
	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	1.7097	4.1265	7.0017	4.7213	3.5736	0.5349	0.5698	0.6335	0.657	0.5697
TU	1.7123	4.0379	6.5504	4.7015	3.8346	0.5351	0.5708	0.6342	0.6586	0.5708
%error	0.15%	2.15%	6.45%	0.42%	7.30%	0.04%	0.18%	0.11%	0.24%	0.19%
	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	2.3480	10.0019	4.6923	4.8447	3.5188	0.6531	0.5334	0.5215	0.5874	0.5334
TU	2.3437	10.1889	4.6032	5.6684	5.0384	0.651	0.5327	0.5208	0.5859	0.5327
%error	0.18%	1.87%	1.90%	17.00%	43.19%	0.32%	0.13%	0.13%	0.26%	0.13%

	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	2.3337	12.1549	14.2099	9.71088837	6.9518	3.1927	0.5986	0.5552	0.5853	0.53	0.614	0.6482
TU	2.3231	11.0656	12.8226	9.3886	6.864	3.2446	0.5957	0.5531	0.5808	0.528	0.6113	0.6453
%error	0.46%	8.96%	9.76%	3.32%	1.26%	1.63%	0.48%	0.38%	0.77%	0.38%	0.44%	0.45%
	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	2.0773	7.2922	12.233	7.5481	3.5226	1.5593	0.5467	0.4948	0.5778	0.5774	0.4947	0.5062
TU	2.0809	6.7297	11.9184	7.3539	3.6612	1.6935	0.5476	0.4954	0.578	0.578	0.4954	0.5075
%error	0.17%	7.71%	2.57%	2.57%	3.93%	8.61%	0.16%	0.12%	0.03%	0.10%	0.14%	0.26%
	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	1.9704	16.1809	6.6273	4.3866	3.9061	5.4745	0.4925	0.5463	0.4806	0.5178	0.5187	0.4926
TU	1.975	15.341	6.41	4.1575	3.7274	5.6095	0.4938	0.5486	0.4817	0.5197	0.5197	0.4938
%error	0.23%	5.19%	3.28%	5.22%	4.57%	2.47%	0.26%	0.42%	0.23%	0.37%	0.19%	0.24%
	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	2.4691	11.8033	8.78672763	6.997	9.4217	4.1827	0.634	0.5605	0.65	0.5612	0.6326	0.7051
TU	2.4874	10.9552	8.615	6.396	9.3352	4.414	0.6378	0.5653	0.6546	0.5653	0.6378	0.7107
%error	0.74%	7.19%	1.95%	8.59%	0.92%	5.53%	0.60%	0.86%	0.71%	0.73%	0.82%	0.79%
	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	1.5949	10.1231	7.5657	1.8023	3.3216	2.9308	0.484	0.4824	0.5691	0.5147	0.5309	0.532
TU	1.5884	9.2823	7.5038	1.7462	3.1207	2.6877	0.4813	0.4813	0.5673	0.5124	0.5295	0.5295
%error	0.41%	8.31%	0.82%	3.11%	6.05%	8.29%	0.56%	0.23%	0.32%	0.45%	0.26%	0.47%

	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	1.4747	10.3507	10.7324	10.3609	6.8035	6.1315	5.4372	0.4757	0.4905	0.4917	0.5453	0.4478	0.5673	0.5448
TU	1.4597	9.5614	9.9032	10.1738	7.107	5.6371	5.4972	0.4709	0.4866	0.4866	0.5406	0.4423	0.5614	0.5406
%error	1.02%	7.63%	7.73%	1.81%	4.46%	8.06%	1.10%	1.01%	0.80%	1.04%	0.86%	1.23%	1.04%	0.77%
	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	2.5426	9.4992	5.6619	3.7955	5.4321	5.0412	3.9196	0.7071	0.7065	0.6361	0.6045	0.6061	0.6347	0.707
TU	2.5079	9.4752	5.7567	3.5781	4.9378	4.7816	3.9982	0.6966	0.6966	0.627	0.5971	0.5971	0.627	0.6966
%error	1.36%	0.25%	1.67%	5.73%	9.10%	5.15%	2.01%	1.48%	1.40%	1.43%	1.22%	1.48%	1.21%	1.47%
	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	2.1404	16.3184	5.2986	4.0843	6.6268	3.4782	2.9495	0.486	0.5946	0.5487	0.4869	0.5099	0.4756	0.5099
TU	2.1174	14.9104	5.0304	3.697	5.5707	3.746	3.2292	0.4812	0.5882	0.5429	0.4812	0.5041	0.4705	0.5041
%error	1.07%	8.63%	5.06%	9.48%	15.94%	7.70%	9.48%	0.99%	1.08%	1.06%	1.17%	1.14%	1.07%	1.14%
	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	2.2789	12.9109	10.7583	6.7157	3.9515	3.4392	2.9323	0.6003	0.5172	0.599	0.5424	0.6158	0.5183	0.6007
TU	2.2764	11.8543	10.6016	6.2573	3.8951	3.4313	2.3849	0.5991	0.5174	0.5991	0.542	0.6152	0.5174	0.5991
%error	0.11%	8.18%	1.46%	6.83%	1.43%	0.23%	18.67%	0.20%	0.04%	0.02%	0.07%	0.10%	0.17%	0.27%
	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	2.3288	12.6823	6.3135	6.8968	6.0183	2.09643114	1.4779	0.583	0.5292	0.6653	0.6467	0.6466	0.5676	0.5545
TU	2.3127	12.108	6.0485	6.9621	5.4105	2.151	1.9328	0.5782	0.5256	0.6608	0.6424	0.6424	0.5641	0.5506
%error	0.69%	4.53%	4.20%	0.95%	10.10%	2.60%	30.78%	0.82%	0.68%	0.68%	0.66%	0.65%	0.62%	0.70%

FIGURE A.4: Results for accuracy test

Appendix B

Parameters and results for comparison test

Case 5-1					
N	5	6	5	6	0
c	1	1	1	1	1
u	1.4	1.1	0.8	1.6	1
# failure	3	3	2	2	1
effe.	1.16	0.89	0.68	1.49	0.88
Case 5-2					
N	3	3	4	4	0
c	1	1	1	1	1
u	1.4	1.6	1.5	1.4	1
# failure	2	3	1	2	3
effe.	1.12	1.34	1.25	1.20	0.71
Case 5-3					
N	3	5	3	4	0
c	1	1	1	1	1
u	1	1.1	1.1	0.8	1.3
# failure	2	3	1	1	1
effe.	0.96	0.73	1.05	0.62	1.20

5-machine line					
p	r	p	r	p	r
0.03	0.4	0.04	0.2	0.01	0.6
0.05	0.3	0.01	0.2	0.01	0.4
0.03	0.8	0.04	0.8	0.04	0.5
0.05	0.5	0.04	0.5	0.06	0.5
0.03	0.9	0.05	0.8	0.06	0.2
0.05	0.5	0.06	0.3	0.04	0.9
0.05	0.8	0.06	0.6	0.06	0.2
0.03	0.5	0.05	0.7	0.04	0.5
0.01	0.8	0.04	0.2		
0.02	0.8	0.06	0.6		
0.03	0.5	0.04	0.4		

FIGURE B.1: Parameters generated for 5-machine line

Case 6-1						
N	7	5	4	8	7	0
c	1	1	1	1	1	1
u	1.6	1.3	1	1.3	1.2	1.8
# failure	1	1	3	2	2	2
effe.	1.51	1.16	0.83	1.21	1.08	1.60
Case 6-2						
N	6	3	5	4	4	0
c	1	1	1	1	1	1
u	1	1.4	0.9	1.5	1.4	0.9
# failure	3	1	3	1	3	2
effe.	0.93	1.35	0.77	1.30	1.20	0.82
Case 6-3						
N	6	5	4	5	4	0
c	1	1	1	1	1	1
u	0.9	1.3	1.4	1	1.6	1.8
# failure	1	2	3	1	2	2
effe.	0.84	1.21	1.13	0.95	1.45	1.64

6-machine line					
p	r	p	r	p	r
0.04	0.7	0.01	0.6	0.06	0.8
0.05	0.4	0.01	0.2	0.04	0.9
0.05	0.5	0.01	0.8	0.02	0.7
0.03	0.5	0.03	0.8	0.05	0.7
0.04	0.8	0.05	0.9	0.02	0.3
0.04	0.7	0.04	0.9	0.06	0.6
0.01	0.5	0.04	0.6	0.04	0.8
0.04	0.7	0.06	0.4	0.04	0.5
0.05	0.9	0.01	0.2	0.02	0.9
0.03	0.8	0.06	0.6	0.01	0.2
0.06	0.7	0.01	0.6	0.02	0.4
		0.04	0.8		
		0.01	0.2		

FIGURE B.2: Parameters generated for 6-machine line

Case 7-1							
N	7	5	5	9	7	8	0
c	1	1	1	1	1	1	1
u	1.5	1.4	1.6	1.8	1.8	1.3	1.8
# failure	3	3	2	3	2	1	2
effe.	1.37	0.99	1.45	1.43	1.62	1.24	1.68
Case 7-2							
N	6	4	5	7	6	4	0
c	1	1	1	1	1	1	1
u	1.1	0.9	1	0.9	1.3	0.8	1.7
# failure	3	3	2	3	3	2	2
effe.	1.02	0.76	0.88	0.77	1.15	0.75	1.37
Case 7-3							
N	6	5	4	7	4	5	0
c	1	1	1	1	1	1	1
u	1	0.9	1.4	1.1	1.1	1.2	1.2
# failure	2	3	1	3	2	1	3
effe.	0.89	0.83	1.36	0.84	0.97	1.13	1.04

7-machine line					
p	r	p	r	p	r
0.01	0.7	0.04	0.8	0.03	0.4
0.01	0.3	0.01	0.9	0.04	0.8
0.02	0.4	0.01	0.5	0.02	0.8
0.04	0.3	0.02	0.3	0.02	0.6
0.04	0.3	0.03	0.3	0.01	0.4
0.06	0.4	0.01	0.6	0.02	0.7
0.04	0.9	0.01	0.8	0.05	0.3
0.03	0.5	0.05	0.4	0.02	0.5
0.04	0.5	0.03	0.5	0.05	0.5
0.05	0.3	0.05	0.7	0.05	0.6
0.01	0.9	0.03	0.9	0.05	0.9
0.05	0.5	0.03	0.3	0.05	0.8
0.01	0.8	0.01	0.7	0.02	0.9
0.03	0.6	0.01	0.6	0.02	0.3
0.02	0.5	0.01	0.5	0.04	0.6
0.03	0.9	0.04	0.9		
		0.02	0.5		
		0.06	0.3		

FIGURE B.3: Parameters generated for 7-machine line

	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	0.6739	6.0805	5.5822	0.5219	1.8648	0.4806	0.6119	0.8395	0.4209	0.6747
TU	0.6728	5.9951	5.5776	0.4468	1.8582	0.4806	0.6117	0.841	0.4205	0.6728
%error	0.16%	1.40%	0.08%	14.39%	0.35%	0.00%	0.03%	0.18%	0.10%	0.28%
TOLIO	0.6691	5.9046	5.4935	0.5665	2.2423	0.478	0.6083	0.8364	0.4182	0.6691
%error	0.71%	2.89%	1.59%	8.55%	20.24%	0.54%	0.59%	0.37%	0.64%	0.83%
	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	0.6386	1.8627	1.7394	2.2554	2.1557	0.4552	0.3984	0.4252	0.4568	0.6385
TU	0.6236	1.7855	1.7051	2.3866	2.3754	0.4355	0.3811	0.4065	0.4355	0.6097
%error	2.34%	4.14%	1.97%	5.82%	10.19%	4.33%	4.34%	4.40%	4.66%	4.51%
TOLIO	0.5989	1.8727	1.7107	2.1925	2.1557	0.4106	0.3593	0.3832	0.4106	0.5748
%error	6.21%	0.54%	1.65%	2.79%	0.00%	9.80%	9.81%	9.88%	10.11%	9.98%
	TH.	B1	B2	B3	B4	M1	M2	M3	M4	M5
SIM	0.5004	2.0004	2.4982	1.5746	0.3441	0.5007	0.455	0.4544	0.6264	0.3849
TU	0.5055	1.9245	2.3869	1.6476	0.4121	0.5055	0.4595	0.4595	0.6318	0.3888
%error	1.02%	3.79%	4.46%	4.64%	19.76%	0.96%	0.99%	1.12%	0.86%	1.01%
TOLIO	0.4819	1.9937	2.4098	1.5264	0.3944	0.5472	0.4975	0.4975	0.684	0.4209
%error	3.69%	0.33%	3.54%	3.06%	14.62%	9.29%	9.34%	9.49%	9.20%	9.35%

	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	0.7322	6.0333	3.3826	0.8595	1.4876	0.4787	0.459	0.5627	0.7321	0.562	0.6093	0.4064
TU	0.7236	5.9667	3.3548	0.8794	1.5441	0.5704	0.4522	0.5566	0.7235	0.5566	0.6029	0.402
%error	1.17%	1.10%	0.82%	2.32%	3.80%	19.16%	1.48%	1.08%	1.17%	0.96%	1.05%	1.08%
TOLIO	0.7112	5.8742	3.3498	0.9308	1.7017	0.5655	0.4445	0.5471	0.7112	0.5471	0.5927	0.3951
%error	2.86%	2.64%	0.97%	8.30%	14.39%	18.13%	3.16%	2.77%	2.85%	2.65%	2.72%	2.78%
	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	0.6476	4.0032	2.0254	1.44	1.5778	1.7121	0.6465	0.4623	0.7185	0.4316	0.4628	0.7196
TU	0.6563	3.7675	2.0114	1.2151	1.5444	1.8821	0.6563	0.4688	0.7292	0.4375	0.4687	0.7291
%error	1.34%	5.89%	0.69%	15.62%	2.12%	9.93%	1.52%	1.41%	1.49%	1.37%	1.27%	1.32%
TOLIO	0.6267	3.8195	1.9751	1.4437	1.5397	1.6342	0.6267	0.4476	0.6963	0.4178	0.4476	0.6963
%error	3.23%	4.59%	2.48%	0.26%	2.41%	4.55%	3.06%	3.18%	3.09%	3.20%	3.28%	3.24%
	TH.	B1	B2	B3	B4	B5	M1	M2	M3	M4	M5	M6
SIM	0.7405	2.3536	2.4069	1.7327	0.672	0.4326	0.8214	0.5688	0.529	0.7412	0.4629	0.411
TU	0.7459	2.1315	2.3711	1.813	0.7172	0.4979	0.8287	0.5737	0.5327	0.7458	0.4662	0.4144
%error	0.73%	9.44%	1.49%	4.63%	6.73%	15.09%	0.89%	0.86%	0.70%	0.62%	0.71%	0.83%
TOLIO	0.7177	2.4985	2.4564	1.7181	0.7618	0.5262	0.7974	0.5521	0.5126	0.7177	0.4485	0.3987
%error	3.08%	6.16%	2.06%	0.84%	13.36%	21.64%	2.92%	2.94%	3.10%	3.17%	3.11%	2.99%

	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	0.8475	5.0663	1.5092	1.196	1.3773	1.8275	0.6938	0.5647	0.6048	0.5299	0.4714	0.4707	0.6514	0.4715
TU	0.8467	4.9629	1.4376	1.1857	1.4936	2.0958	0.8417	0.5645	0.6048	0.5292	0.4704	0.4704	0.6513	0.4704
%error	0.09%	2.04%	4.74%	0.86%	8.44%	14.68%	21.32%	0.04%	0.00%	0.13%	0.21%	0.06%	0.02%	0.23%
TOLIO	0.8293	5.0112	1.542	1.2484	1.4372	1.8044	0.7389	0.5529	0.5924	0.5183	0.4607	0.4607	0.6379	0.4607
%error	2.14%	1.09%	2.17%	4.38%	4.35%	1.26%	6.50%	2.09%	2.05%	2.19%	2.27%	2.12%	2.07%	2.29%
	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	0.5860	4.6665	1.8334	2.0134	1.5707	2.4626	0.4193	0.5333	0.6502	0.5858	0.6486	0.451	0.7327	0.3446
TU	0.5876	4.5188	1.7308	1.9881	1.3715	2.6365	0.4623	0.5342	0.6529	0.5876	0.6529	0.452	0.7344	0.3456
%error	0.28%	3.17%	5.60%	1.26%	12.68%	7.06%	10.26%	0.17%	0.42%	0.31%	0.66%	0.22%	0.23%	0.29%
TOLIO	0.5675	4.5563	1.8062	1.9782	1.4583	2.366	0.4336	0.5159	0.6305	0.5675	0.6305	0.4365	0.7094	0.3338
%error	3.15%	2.36%	1.48%	1.75%	7.16%	3.92%	3.41%	3.26%	3.03%	3.12%	2.79%	3.22%	3.18%	3.13%
	TH.	B1	B2	B3	B4	B5	B6	M1	M2	M3	M4	M5	M6	M7
SIM	0.6573	3.6749	1.88	2.0664	2.205	0.9726	1.0641	0.6572	0.73	0.4689	0.5981	0.5978	0.5468	0.5474
TU	0.6631	3.49	1.6976	2.1952	2.3603	1.0122	1.2016	0.6631	0.7368	0.4736	0.6028	0.6028	0.5526	0.5526
%error	0.88%	5.03%	9.70%	6.23%	7.04%	4.07%	12.92%	0.90%	0.93%	1.00%	0.79%	0.84%	1.06%	0.95%
TOLIO	0.6366	3.7002	1.9702	2.1278	2.3323	1.0203	1.1158	0.6366	0.7074	0.4547	0.5788	0.5788	0.5305	0.5305
%error	3.15%	0.69%	4.80%	2.97%	5.77%	4.90%	4.86%	3.13%	3.10%	3.03%	3.23%	3.18%	2.98%	3.09%

FIGURE B.4: Results for accuracy test

Appendix C

Parameters and results for other numerical analysis

line1

N	6	6	6	6	0
c	1	1	3	1	3
u	2.9	3.1	0.9	2.8	1
# failure	2	3	1	2	3
effe.	2.5778	2.48	2.5412	2.51	2.454

line2

N	8	8	8	8	0
c	4	3	4	1	3
u	1.06	1.51	1.38	4.25	1.47
# failure	1	3	3	1	1
effe.	3.9906	3.9912	4.0111	4.0094	4.0091

p1	r1
0.02	0.2
0.01	0.4
0.05	0.5
0.02	0.4
0.03	0.3
0.05	0.8
0.03	0.5
0.05	0.9
0.03	0.5
0.05	0.8
0.05	0.5

p2	r2
0.05	0.8
0.01	0.4
0.04	0.8
0.03	0.5
0.06	0.2
0.01	0.3
0.03	0.7
0.03	0.5
0.03	0.3

FIGURE C.1: Parameters generated for buffer capacity analysis