

**POLITECNICO DI MILANO**

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***INDIRECT ESTIMATION of CUTTING FORCE in  
MILLING MACHINE TOOLS***

A Thesis Submitted in the Partial Fulfillment of the requirements

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## Abstract

This work investigates the indirect and online estimation of the cutting forces from spindle integrated displacement sensor (SIDS) in addition to current of feed drive by two approaches. In the first approach, an FEM model of the spindle integrated sensor is used to measure relative displacement between the sensor head and the rotating spindle shaft under cutting load. In order to increase the bandwidth of the indirect force measurement and compensation for the spindle dynamics, the design of a state estimator, namely “Kalman filter” scheme, has been used.

The second approach is the indirect estimation of cutting forces from feed drive of servo motors. Current of the machine tool feed drive has direct contributes to the torque generated by the motor and the external resistant loads. The total external load torque to the servo motor consists of the torque induced from the cutting force and that from the friction force in the guide way. The main idea is to estimate the disturbance load which is exerted to the feed drive system due to the cutting force. Hence, a Kalman filter is implemented as a model based disturbance observer for indirect estimation of cutting forces in milling process.

The estimation of cutting forces is performed based on Matlab/Simulink software. The simulation results of the spindle integrated displacement sensor model are reported by applying step and simulated real cutting forces to the system. Nevertheless, the simulation results of the feed drive models are presented based on closed loop and open loop systems model, applying step force input.

**Keywords:** Smart machine tools, State estimation, Monitoring, Machining process, Cutting force, Kalman filter, Spindle integrated displacement sensor, Feed drive, Productivity



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## List of Acronyms

ACR	Acronyms
ARE	Algebraic Riccati equation
CARE	Continuous ARE
DARE	Discrete ARE
EKF	Extended Kalman filter
LTI	Linear time-invariant
LTV	Linear time-varying
MIMO	Multiple input, multiple output
Pdf	Probability density function
PDF	Probability distribution function
RMS	Root mean square
RV	Random variable
SISO	Single input, single output
TF	Transfer function
FRF	Frequency response function
PIF	Parallel information filter
KF	KF
DIF	Distributed information Kalman filter
DKFCF	Distributed Kalman filter with consensus filter
DKFWA	Distributed Kalman filter with weighted averaging
CKF	Centralized Kalman filter
DKF	Decentralized Kalman filter
SIDS	Spindle integrated displacement sensor
OBE	Plant Input Based Estimator
IBE	Plant Input Based Estimator

IOBO      Input and Output Based Observer

## List of Symbols

$T_m$	Torque at the produced by motor (NM)
$T_d$	Linear approximation for viscous friction (NM)
$T_c$	Cutting Torque (NM)
$k_t$	Armature constant
$i_q$	Quadrature current (Ampere)
$F$	Cutting force on feed drive (N)
$R_t$	Conversion gain of force to torque
$\omega$	Angular velocity (rad/s)
$\theta$	Angular position (rad)
$J_{eq}$	Inertia of the ball screw shaft (Kg.m <sup>2</sup> )
$G_n$	System noise matrix
$w$	Zero-mean white Gaussian process noise
$v$	Zero-mean white Gaussian measurement noise
$Q$	Covariance matrix of ( $w$ )
$R$	Covariance matrix of ( $v$ )
$G_s$	Static compliance ( $\mu\text{m}/\text{N}$ )
$K_s$	Static stiffness (N/ $\mu\text{m}$ )
$F_a$	Actual cutting force on SIDS (N)
$\Phi(s)$	Dynamic compliance ( $\mu\text{m}/\text{N}$ )
$\delta_F$	Displacement ( $\mu\text{m}$ )
$\zeta$	Damping factor
$f_N$	Natural frequency (Hz)
$W$	Observability matrix
$W_t$	Weight matrix
$\hat{\delta}_F$	Estimated displacement ( $\mu_m$ )
$\hat{x}$	Estimated state

$x_e$	Expanded state
$K$	Kalman gain
$E$	Expected value
$P$	Error covariance matrix
$G$	Input matrix for discrete-time
$H$	Measurement matrix in discrete-time
$K_{re}$	Radial cutting force coefficients
$K_{rc}$	Radial cutting force coefficients
$K_{te}$	Tangential cutting force coefficients
$K_{tc}$	Tangential cutting force coefficients

# 1 Introduction

## 1.1 Machining process monitoring and machine tools

Manufacturing technology has undergone significant developments over the last decades aiming at improving precision and productivity. The development of numerical control (NC) technology in 1952 made a significant contribution to meeting these requirements. Today, thanks to the significant developments in sensor and computer technologies, it can be said that the necessary tools are available for achieving the adaptive control of manufacturing processes, assisted by monitoring systems, which was a dream in the 1950's.

For the following reasons, monitoring technology with reliable sensors is becoming more and more important in modern manufacturing systems [1]:

- Machine tools operate with speeds that do not allow manual intervention. However, process failures may cause significant damage.
- Increase of labor costs and the shortage of skilled operators call for operation of the manufacturing system with minimum human intervention; this requires the introduction of advanced monitoring systems.
- Ultra-precision manufacturing can only be achieved with the aid of advanced metrology and process monitoring technology.

### 1.1.1 CNC machine tools and adaptive control

Given the importance of machining to most industries, machine tools have often led the way in the development of automation technology. A significant development in machine tool automation was the introduction of Computer Numerical Control (CNC) in the early 1970s where a dedicated computer replaced most of the electronic hardware and punch cards of the NC machines. Increased processor speed, user-friendly programming tools, and increased sensor resolution have all contributed to the great strides in the areas of servomechanism control and interpolation.



However, process control, which is not commonly integrated into today's machine tools, is the automatic adjustment of process parameters e.g., feeds, speeds in order to increase operation productivity and part quality [2].

Machining process control strategies are classified into two main groups, namely adaptive control constraint (ACC) and adaptive control optimization (ACO).

In the former, ACC strategies, a process variable (as an example the cutting force) is kept constant and under control through the real-time, in-process regulation of a process parameter (as an example the feed rate), with the aim of cutting force control and self excited vibration suppression [3].

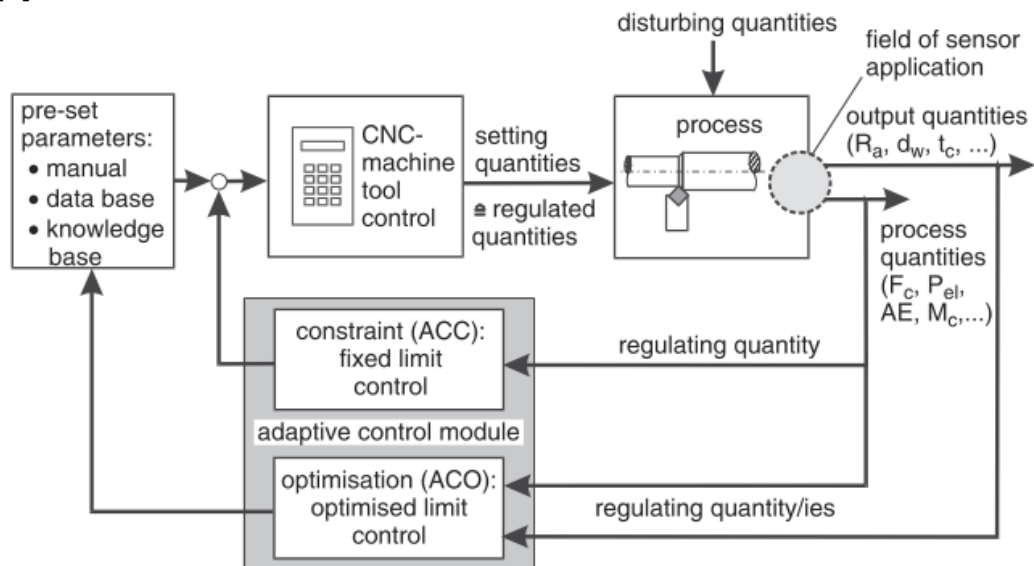


Figure 1-1: Schematic set-ups of adaptive control systems [1]

### 1.1.2 New generation of NC machine tools

Machining process monitoring and control is a core concept on which to build up the new generation of flexible self-optimizing intelligent NC machines. In-process measurement and processing of the information provided by dedicated sensors installed in the machine, enables autonomous decision making based on the on-line diagnosis of the correct machine, workpiece, tool and machining process condition, leading to increase machine reliability towards zero defects, together with higher productivity and efficiency.

The expected characteristics of the next generation of machine centres are described as follows [4]

- Integration: development of an integrated machine tool being capable of performing both conventional and non-conventional processes in one platform.
- Process control loop: development and CNC integration of robust and reliable real-time strategies for the in-process tool, part, and process condition monitoring and control.
- Autonomous optimization: development of a self-configuring and self-optimizing control system for autonomous manufacturing, based on the in-process monitoring, characterization and management of process knowledge.

### **1.1.3 Intelligent Systems for machining Processes**

There is no clear definition of an intelligent system. Many authors have used this term to describe an unattended machining process, where the tool cuts the workpiece while the process is monitored and controlled by the aid of suitable sensors. Also, 'intelligent tools' have been presented which consist of a specific sensor as an integral part of the tool design. Furthermore, authors sometimes refer to 'intelligent machining operations, which means a model-based cutting simulation for pre-process cutting parameter optimization, followed by an adaptive controlled machining operation [1].

Here in Figure 1-2, Hierarchical order of machining control loops in intelligent machine tools, including, Internal control loop, post process control loop, and superior control loops are going to be explained.

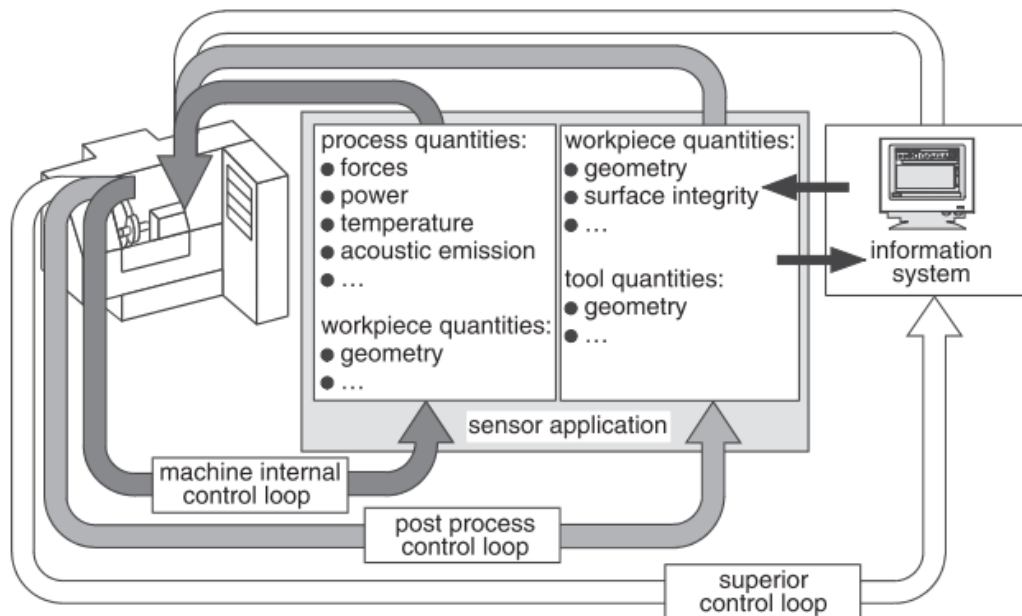


Figure 1-2: Hierarchical order of control loops in manufacturing [1]

Sensors for measuring process quantities and sensors for any kind of in-process measurement are related to the machine internal control loop.

However, Sensors which are not installed in the working space of the machine tool or which only measure in auxiliary times belong to the post-process control loop.

In the superior control loop, the direct use of sensors is not required. In this highest level, any kind of intelligent information system is used either to download control tasks to the sensor systems or to collect measuring data for further processing.

Although the sensor application is effected at a lower level according to the hierarchy of the control loops, it can still be regarded as an essential part of a whole so-called intelligent system. The major task is to obtain as much information as possible from the current process. With single-phenomenon monitoring this aim often cannot be met, so the application of multiple sensors in one process is part of many activities to achieve an intelligent system [1].

### 1.1.4 Machining process monitoring (direct and indirect method)

Process monitoring is the measurement and estimation of process variables. A broad spectrum of on-line sensors has been implemented that use acoustic, optical, electrical, thermal, magnetic, etc. sensing systems. In the recent decades, there were special attention on monitoring strategies for part condition monitoring (surface roughness, surface integrity and dimensional accuracy), tool condition monitoring (the so-called TCM for wear and breakage detection), process condition monitoring (chatter onset and collision detection).

The measuring techniques for the monitoring of machining operations have traditionally been categorized into two approaches: direct and indirect [Figure 1-3].

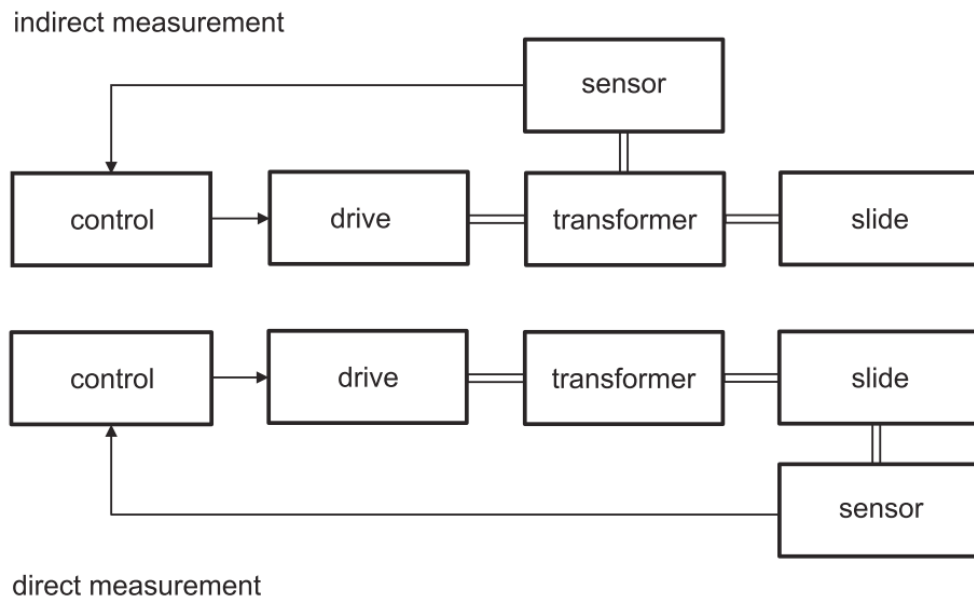


Figure 1-3: Placement of position sensors [1]

Direct and in-process measurement is not generally possible due to the aggressive environment in the cutting zone. The main research effort over the last decades for part and tool monitoring has been focused on indirect measurement techniques (process condition-based) in which cutting process characteristics (i.e. cutting forces and power, vibrations, cutting temperature, acoustic emission, etc.) are measured in order to indirectly infer the part and tool condition [4]. Indirect sensing in general

has the advantage that there is no need for cost-effective measuring devices so that simple and reliable seals can be used.

It has the disadvantage that errors of the transmission system are introduced in the measured quantity. These can be, for instance, thermal or elastic deformations of the ball screw or geometric and kinematical aberrations of the transmission system in robots. Figure 1-4 depicts the sensors that are commonly used for online measurement [5].

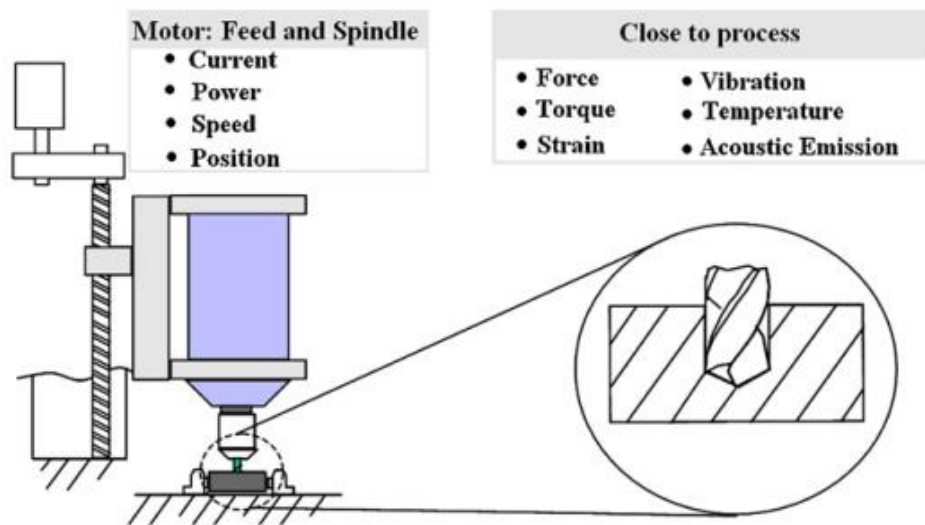


Figure 1-4: Measurable phenomena for online sensor monitoring [5].

### 1.1.5 Monitoring and estimation of cutting force

Torque and force are widely used for detection of tool and monitoring purpose. This is due to the high sensitivity and rapid response of force signals to changes in cutting states. Torque is similarly to the force consist but in this case the applied load is torsional. Force and torque sensors (as example piezoelectric sensors) generally employ sensing elements that convert the applied force or torsional load into deformation of very rigid element that is the quartz itself [5].

## 1.2 State of art

Here in this sub-chapter, the cutting force monitoring, estimation and measurement compensation by external and internal sensors has been reviewed.

### 1.2.1 Cutting force monitoring by internal sensors

Power and current measurement are an economical monitoring solution for many machining operations. Motor current sensing uses the motor itself as an indirect sensor of cutting forces. Thus, when using sensor systems based on motor current, it is crucial that the relationship between input current and output force is linear and understood [5].

The performance of indirect sensors, such as motor current/power, can be improved by developing a model of the distortion introduced by the sensor within the mechanical/electro-mechanical system. A number of studies provided a better understanding of signal features for various spindles and drive systems [6].

Commercial numerically-controlled machine tools use current sensors for the motoring control of servo motors for both axis and spindle. The current have direct contributes to the torque generated by the motor and the external resistant loads. The total external load torque to the servo motor consists of the torque induced from the cutting force and that from the friction force in the guide way. The main idea is to estimate the disturbance load which is exerted to the feed drive system due to the cutting force. so cutting torque or force calculated from a motor's armature current is probably the cheapest way to monitor cutting loads because no extra sensors are needed [6].

Altintas [7] discussed tool breakage detection based on cutting force calculation by servo motor current. Armature current is influenced by dynamic characteristics of the drive system. In a feed drive, particularly, the compensation of the dynamics of the moving mass is important for the separation of the cutting force influence. A disturbance estimator for a feed drive can be applied for this purpose [8],[9].

Jeong and Cho [10] estimated cutting force from rotating and stationary feed motor currents in milling process, while increasing the bandwidth dramatically.

M. Dolen et al. [11] presented a cutting force estimator topology for feed drives of CNC vertical machining centers to compute the machining forces. They developed an accurate (and high-bandwidth) force estimator for CNC machine tools with the utilization of advanced FD physical models that includes the major nonlinearities of such systems.

Kim et al. [8] studied indirect cutting force measurement method in contour NC milling processes by using current signals of servo motors.

Z. Jamaludin et al. [12] studied accurate motion control of xy high-speed linear drives using friction model feedforward and cutting forces estimation. They discussed design and experimental validation of friction and cutting force compensation for a linear drive xy table.

### **1.2.2 Cutting force monitoring by external sensors**

Commercially available dynamometers measure cutting force using quartz piezoelectric transducers [13]. Table and spindle dynamometers are commercially available. These dynamometers have sufficient resolution to be used in micro machining using a non-rotating tool or a miniature rotating tool up to several dozen microns in the diameter [6]. Despite the accuracy and reliability of commercial dynamometers, their high cost may limit their use in machining process control [14]. Moreover, they suffer from limited workpiece sizes and mounting constraints.

Developments like the integration of force sensors into the machine structure have taken place over the last 10 years with concepts developed for milling. S.S Park and Altintas[14] , [15] used Piezo-electric force sensors that are integrated into the stationary spindle housing, named Spindle Integrated Force Sensor (SIFS), to measure cutting forces, [Figure 1-5] They also presented a method for measuring cutting forces from the displacements of rotating spindle shafts using capacitance displacement sensors [16].

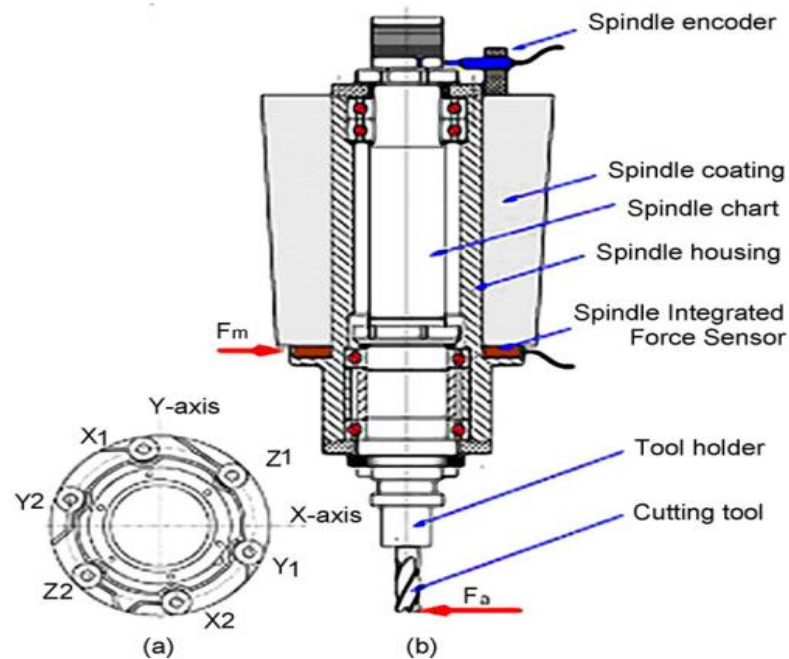


Figure 1-5: Integrated force sensor in motor spindle [5]

Kim and Chang [17] used Cylindrical Capacitive Displacement sensor (CCDS) to estimate cutting forces by Measuring Spindle Displacement. They used CCDS and non-contact magnetic to identify the dynamic characteristic of the spindle tool system during spindle rotation.

Sarhan et al. made cutting force calculation using spindle-integrated displacement sensors [18],[19]. They used four inexpensive, contamination-resistant eddy-current displacement sensors (S1-S4). Calculating cutting force from spindle displacement, however, involves two major issues, thermal influence and spindle stiffness.

### 1.2.3 Methods for online compensation of the measured force

A potentially critical issue with sensors integrated in spindles is the heat generated by the spindle motor. Jun et al. [20] studied the subject suggesting that temperature compensation was needed to reliably monitor torque. Motor heat may critically deform or displace a rotating spindle, which must be separated from displacement caused by cutting force.



Sarhan et al. [18] used multiple thermocouples installed in the spindle to calculate thermal displacement. The thermal growth increases bearing preload that shifts modal frequencies. Many studies have reported thermal-mechanical spindle modeling [21], [22].

Another potential problem is the spindle stiffness. The dynamic stiffness differs significantly from static stiffness. If the cutting frequency is within the natural modes or higher the measured forces will be distorted.

Rantatalo et al. [23] developed a magnetic exciter to laterally vibrate a spindle to measure its dynamic response. Park et al [15] has compensated the dynamic stiffness problem by model based state estimator.

Armature current is influenced by dynamic characteristics of drive system. In feed drive particularly compensating for the dynamics of moving mass is important in separating out the influence of cutting force. A disturbance estimator for a feed drive can be applied for this purpose [8].

An important outcome of the models developed by researchers is the quantification of the sensitivity and dynamic bandwidth of the motor power/current sensing loop. Stein and Wang [24] related the sensitivity of the current measured from the spindle and feed drive motors to the cutting forces and noted that the currents were very sensitive to the presence of Coulomb and viscous friction. The bandwidth of the spindle drive was between 2 and 18 Hz. Kim et al [5] has reported the bandwidth for the feed drive up to 130 Hz [5]. The dynamic characteristics of the current feedback control loop of the feed drive system determine the bandwidth of the current sensing system for indirect cutting force measurements [2], [6].

### **1.3 Thesis objectives and structure**

To extend the previous discussion (state of art), there should be particular attention to estimate the cutting forces based on the numerical model of the systems before performing the experimental tests. Thereby, we have implemented the state estimator Kalman filter (will be introduce

in Chapter 2.3.1) in combination with two above-mentioned models (external sensor: spindle integrated displacement sensors and internal sensor: current of feed drive servomotors), described as follows.

### 1.3.1 Thesis structure

This master thesis describes estimation of cutting force used for detecting of machining process anomalies such as self excited vibration in machining process, cutting tool failure, and surface roughness, etc. In the conventional method, the mentioned machining failures were detected and compensated by a machine tool operator which cause interruption in production process and have an adverse effect of productivity of operations. Hence, it is necessary to develop autonomous smart machine tools to conduct feedback control while minimizing human intervention [Figure 1-6]. In this context, the measurement of cutting forces is the key information needed to monitor, troubleshoot, or control the machining operations.

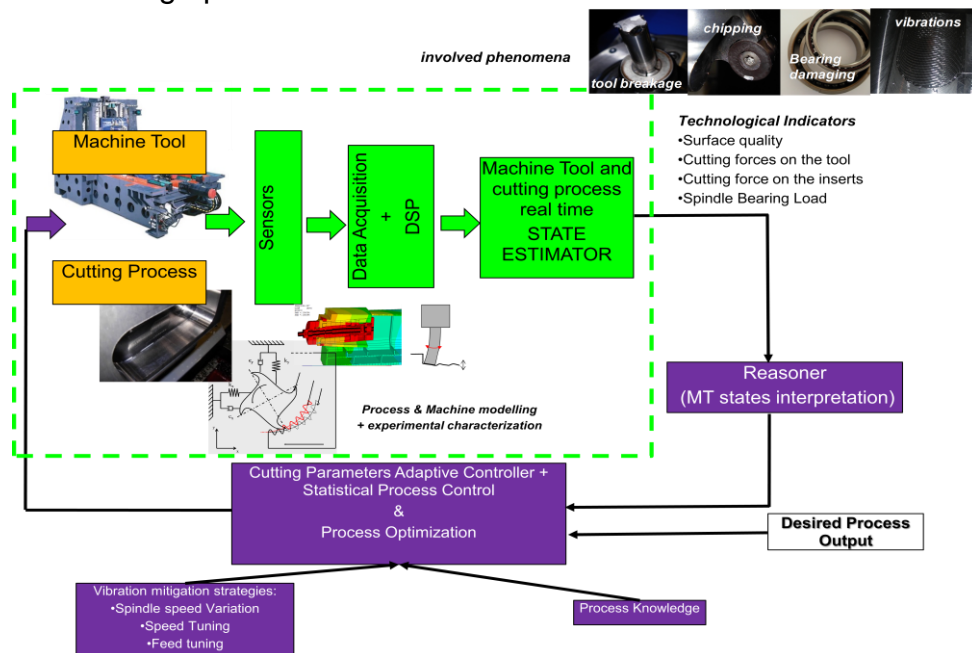


Figure 1-6: Schematic concept of monitoring and adaptive control system in machine tools

In order to measure the cutting force the force dynamometers are commonly used. They consist of piezoelectric sensors, mounted on the

machine tools axes. Although table dynamometers provide accurate and effective force measurements, they are more suitable for laboratory use and have limited applications on production machines due to the workpiece size and mounting constraints, etc. Therefore, there have been strong interests in developing a force-sensing mechanism built into a machine tool structure or cutting tool in order to measure the force indirectly.

In the chapter 3, a method of estimation of cutting forces from the displacements of rotating spindle shafts has been studied. In this scenario, the FEM model of the spindle has been utilized to find the modal parameters. Moreover, the dynamic model of the displacement sensor to the applied force to the tool tip has been developed. Additionally, a state estimator namely Kalman filter have been utilized to estimate the cutting force. Finally, the results of the simulation were reported based on continuous and discrete time domains.

In the chapter 4, indirect estimation of the cutting force from the current of feed drive has been studied. In this method, the idealized closed loop model of the feed drive was considered as a plant model. Additionally, the state estimator Kalman filter was used in order to estimate the cutting force. Finally, the results of the simulation were reported based on continuous and discrete time domains.

## 2 State and Disturbance Observer

Observers or estimators can be used to augment or replace sensors in a control system. Observers are algorithms that combine sensed signals with other knowledge of the control system to produce observed signals.

### 2.1 Observers overview

Control systems are used to regulate an enormous variety of machines, products, and processes. They control quantities such as motion, temperature, heat flow, fluid flow, fluid pressure, tension, voltage, and current. Most concepts in control theory are based on having sensors to measure the quantity under control. In fact, control theory is often taught assuming the availability of near-perfect feedback signals. Unfortunately, such an assumption is often invalid. Physical sensors have shortcomings that can degrade a control system.

There are at least four common problems caused by sensors [25].

1. Sensors are expensive.
2. Sensors and their associated wiring reduce the reliability of control systems.
3. Some signals are impractical to measure.
4. Sensors usually induce significant errors such as stochastic noise, cyclical errors, and limited responsiveness.

These observed signals can be more accurate, less expensive to produce and more reliable than sensed signals [Figure 2-1].

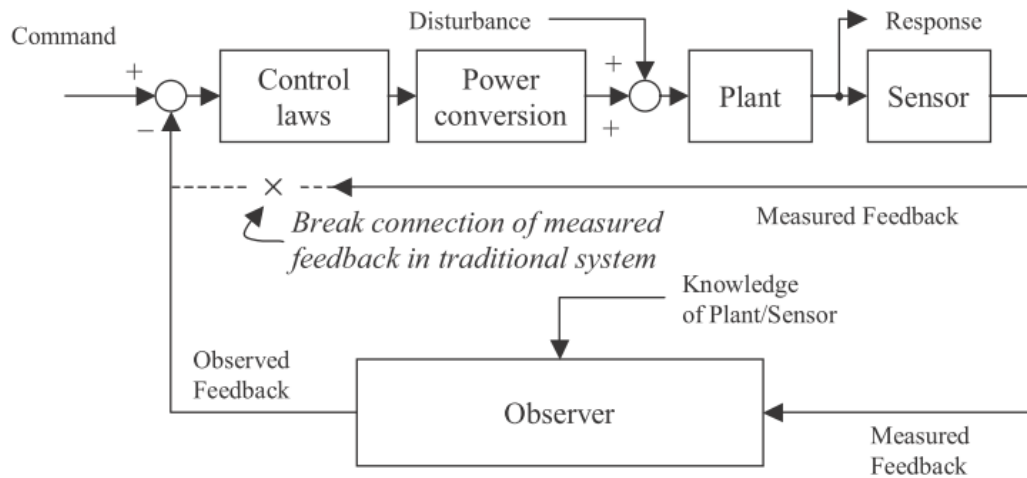


Figure 2-1: Role of an observer in a control system [25]

### 2.1.1 Roles of observers in Control Systems

Here are given some common roles of observers in control systems, explained as follows [25]:

- **Competitively Priced:** Control systems, like almost all products in the industrial market, must be delivered at competitive prices. Arguments for observer-based methods can be at either end of the cost–value spectrum. For example, if an observer is used to help replace an existing sensor with one that is less expensive, the argument may be that for a modest investment in computational resources, sensor cost can be reduced. In other cases, it can be argued that observers increase value; for example, value could be increased by providing a more reliable feedback signal.
- **Highly reliable:** Control systems must be reliable. A proven way to enhance reliability is by reducing component count, especially connectorized cables. Electrical contacts are among the least reliable components in many systems. Observers can increase reliability when they are used to eliminate sensors and their cables. Since observers combine the sensed signals (which may have high noise content) with the model signals (which are nearly

noise free), they can remove noise from the calculated output, greatly extending the range of sensorless operation.

- **Stability:** Observers can improve stability by reducing the phase lag within the control loop. For example, the process of converting a sensor signal often involves filtering or other sources of phase lag. In the motion-control industry, it is common to use the simple difference of two position samples to create a velocity signal. Such a process is well known to inject a time delay of half the sample time. By using an observer this phase delay can be removed.
- **Disturbance Rejection:** Disturbance rejection is a measure of how well a control system resists the effect of disturbances. As with command response, higher gains help the system reject disturbances, but they reduce margins of stability. Observers can help disturbance rejection in two ways. As with command response, disturbance response can be improved incrementally through higher control-law gains when the observer allows the removal of phase lag. Second, as discussed in Section 2.1.3, observers can be used to observe disturbances, allowing the use of disturbance decoupling where it otherwise might be impractical.
- **Robustness:** Robustness is a measure of how well a system maintains its performance when system parameters vary. The most common variations occur in the plant. The control system must remain stable and should maintain consistent performance through these changes. One challenge of observer-based techniques is that robustness can be reduced by their use. This is because observers rely on a model; when the plant changes substantially and the model is not changed accordingly, instability can result. Thus, robustness should be a significant concern any time observers are employed.

## 2.2 A historical review of observers design

Radke and Gao [26] presented unified and historical review of observer design for the benefit of users. A critical question that is concerned with the practitioners is selecting among many candidates, an appropriate observer for a particular problem. For this purpose, observers have been reviewed in terms of the applicable dynamic structure of the plant, required sensors, plant knowledge, and implementation.

Without a doubt, “observers”, also known as “estimators” or “filters” are indispensable tools for engineering. Over the years, two classes of design methods for observers have emerged. One is concerned with state estimation based on a mathematical plant model; the other is concerned with disturbance estimation based on input/output data. Here are introduced some of the observers classified based on their evolution in history. The following sections use the:

- 1) Plant description,
- 2) Input and estimate, and
- 3) Implementation characteristics, to survey the early, modern, and disturbance estimation based observers, respectively.

### 2.2.1 Early Estimators

Early on engineers discovered internal values, could be extracted from input output data. The mechanism used for this purpose is known as a state estimator. Unmeasured internal values can be extracted from input, output and plant dynamic information. The following discusses the development of early estimators as a popular and important base set.

#### 2.2.1.1 Plant Output Based Estimator (OBE)

This estimator simply extracts information from the output of a plant or signal; for this reason, it is called an Output Based Estimator (OBE). Some common types of OBE’s are the low pass noise filter, approximate differentiator [26] and  $\alpha\beta\gamma$  filter [27]. The OBE represented in terms of (1), (2) and (3) is:

$$\dot{x} = Ax \quad 2.1$$

$$y = Cx$$

$$OBE: \{y, A, C\} \xrightarrow{yields} \{\hat{x}\} \quad 2.2$$

$$\hat{\dot{x}} = A\hat{x} + K(y - C\hat{x}) \quad 2.3$$

Where, K is chosen such that the estimation error is driven to zero. This filter is useful for common applications that only have an output. Although simple, the information is often delayed and corrupted by disturbances and sensor noise.

### 2.2.1.2 Alpha Beta Gamma Filter

A special case of the OBE is the Alpha Beta Gamma ( $\alpha\beta\gamma$ ) filter since the output is the only information used for estimation. The ( $\alpha\beta\gamma$ ) filter [28], [29] was a very early sampled data filter used as a practical radar estimation algorithm for velocity and acceleration when only position is available.

$$y^{(n)} = f(x, t, w_f) \quad 2.4$$

$$x = [y \ \dot{y} \ \dots \ y^{(n-1)}]^T$$

$$\alpha\beta\gamma : \{y, n\} \xrightarrow{yields} \{\hat{x}\} \quad 2.5$$

$$\Phi_{i,j} = \begin{cases} \frac{T^{j-i}}{(j-i)!}, & i \leq j; \\ 0, & \text{else} \end{cases} \quad 2.6$$

$$\hat{x}_{k+1} = \Phi[A\hat{x}_k + K(y_k - \hat{x}_{1k})] \quad 2.7$$

Here T is the discrete time sampling period. Design is simplified since (2.4) requires a specified structure that is a special case of the famous Kalman filter [27] and other equivalent forms. Although design is simplified, the problem of all OBE's still exists for plants with excessive noise, delay, and output disturbances.

### 2.2.1.3 Plant Input Based Estimator (IBE)

One way to get around sensor noise and output disturbances is not using them.



$$\begin{aligned} \dot{x} &= Ax + Bu & 2.8 \\ y &= Cx \end{aligned}$$

$$IBE: \{u, A, C, x_0\} \xrightarrow{\text{yields}} \{\hat{x}\} \quad 2.9$$

$$\dot{\hat{x}} = A\hat{x} + Bu \quad 2.10$$

If the plant model in the observer is accurate, inputs, and initial conditions,  $x_0$ , are available then internal system states can be determined from inputs alone. This can be thought as attempting to estimate internal plant information by running a simulated plant in parallel.

However, initial conditions must be given. For example, to estimate velocity a noisy position output could be differentiated using the OBE or acceleration input could be integrated with the IBE. This method is also applicable if  $y$  is not measurable.

#### 2.2.1.4 Input and Output Based Observer (IOBO): Luenberger Observer

The real workhorse began with the IOBO, popularly known as the Luenberger Observer [25].

$$\begin{aligned} \dot{x} &= Ax + Bu & 2.11 \\ y &= Cx \end{aligned}$$

$$IOBE: \{u, y, A, C, x_0\} \xrightarrow{\text{yields}} \{\hat{x}\} \quad 2.12$$

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}) \quad 2.13$$

This is a simple combination of the OBE (2.3) and IBE (2.9). By feeding back the estimated state along with measured data, it eliminates the IBE requirement for accurate initial conditions [27]. Since the estimate is fed back through the estimator it is also often called a Closed Loop Observer. The key advantage of the IOBO is the ability to use both the input and output data along with plant information to reduce noise and phase lag without the knowledge of initial conditions.

The Luenberger observer established the structure on which most estimators are based today. The difference lies in the method of choosing  $K$ .

## 2.2.2 Modern estimators

From this base set of observers, there have been a few key advances. The advance in modern control theory has been made by formulating the problem with disturbances in mind. These methods minimize a cost function based on mathematical assumptions about disturbances [27]. However, the design complexity has substantially increased.

### 2.2.2.1 Kalman filter (KF)

The Kalman filter [27] was one of the first estimators to include the formulation of disturbances and provide optimal solutions.

$$\dot{x} = Ax + Bu + w_{N(0,Q)} \quad 2.14$$

$$y = Cx + v_{N(0,R)}$$

$$KF : \left\{ \begin{array}{l} u, A, B, C, \\ cov(w_N), cov(v_N) \end{array} \right\} \xrightarrow{yields} \{\hat{x}\} \quad P. \min \|x - \hat{x}\|_2 \quad 2.15$$

$$\dot{P}(t) = P(t)A^T + AP(t) + Q - P(t)C^T R^{-1}C P(t) \quad 2.16$$

$$K(t) = P(t)C^T R^{-1}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x})$$

In order to derive an optimal estimator, a few mathematical assumptions are made about the unknown disturbance. First, the model must be “sufficiently accurate” [27]. Secondly, disturbances are stochastic, zero mean and Gaussian (no time correlation) with known input and output noise covariance  $Q=E[w w^T]>0$ ,  $R=E[v v^T] >0$  and  $E[w v^T]=0$ .

The KF can be viewed as an optimal balance between the IBE and OBE [27] since the final equation of (2.11) is an IOBO with a time varying  $K$ . Since the 1960 seminal paper [30], many KF variations have been made. It was originally defined in discrete time and has been extended to continuous time [27]. It has also been formulated for non-Gaussian noise, and applications have spread to parameter estimation. The KF

has not been widely applied to industrial applications, probably, due to the complexity of the implementation.

### 2.2.2.2 Extended Kalman filter (EKF)

The EKF was the first major effort to adapt the Kalman filter for nonlinear systems [27].

$$\dot{x} = f(x, u, w_{N(0,Q)}, t) \quad 2.17$$

$$y = h(x, u, v_{N(0,R)}, t)$$

$$EKF : \left\{ \begin{array}{l} u, y, f, h, \\ cov(w_N), cov(v_N) \end{array} \right\} \xrightarrow{yields} \{\hat{x}\} \quad 2.18$$

$$A(t) = J_f = \frac{\partial(f(x, u, 0, t))}{\partial(x)} \quad 2.19$$

$$A(t) = J_h = \frac{\partial(h(x, 0, t))}{\partial(x)} \quad 2.20$$

$$\dot{P}(t) = P(t)A^T + AP(t) + Q - P(t)C^T R^{-1}C P(t) \quad 2.21$$

$$K(t) = P(t)C^T R^{-1}$$

$$\dot{\hat{x}} = f(\hat{x}, u, 0, t) + K(t)(y - h(\hat{x}, 0, t))$$

At each point in time, f and h are linearized to  $A(t)$  and  $C(t)$  to then be used in the standard Kalman filter. One of the most recent Kalman filter modifications for nonlinear systems is the Unscented Kalman Filter (UKF). It moves beyond the EKF by additionally passing intermediate values through the known nonlinear equations [27]. Although the UKF is derived for an algorithmic implementation, the complexity and model information required make it impractical for the majority of practical applications.

$$\dot{x} = f(x, u, t) + w_{N(0,Q)} \quad 2.22$$

$$y = h(x, t) + v_{N(0,R)}$$

$$UKF : \left\{ \begin{array}{l} u, y, f, h, \\ cov(w_N), cov(v_N) \end{array} \right\} \xrightarrow{yields} \{\hat{x}\} \quad 2.23$$

(The UKF implementation is too involved to include in this survey, however it is an important recent modern extension to include for historical reference.)

### 2.2.2.3 H<sub>∞</sub> estimator

Another significant tool in the modern direction is the H<sub>∞</sub> estimator.

$$\dot{x} = Ax + Bu + B_w w_f \tag{2.24}$$

$$y = Cx + D_w w_f \tag{2.25}$$

$$H_\infty : \left\{ \begin{matrix} u, y, A, B, C, \\ B_w, D_w, \gamma \end{matrix} \right\} \xrightarrow{\text{yields}} \{\hat{x}\} \quad P. \min \left\| \frac{x - \hat{x}}{w_f} \right\|_\infty < \gamma \tag{2.26}$$

$$\dot{Q}(t) = Q(t)A^T + AQ(t) + B_w B_w^T - Q(t)(C^T C - \gamma^{-2} C^T C) Q(t) \tag{2.27}$$

$$K(t) = Q(t)C^T \tag{2.28}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + K(t)(y - C\hat{x}) \tag{2.29}$$

This also optimizes a cost function based on an assumption about the disturbance. This formulation is significant because it uses a unique characterization of the disturbance. Kalman minimizes the minimum squared error because it is a mathematically manageable optimization problem. Using infinity norms, the H<sub>∞</sub> estimator is able to minimize the maximum or worst case disturbance [27]. In (2.25), W<sub>F</sub> is unknown but not necessarily random or stochastic. The estimator is guaranteed to be optimal under a user defined upper bound γ.

Figure 2-2 depicts a summary of observer's techniques, complexities, and evolution through the history.

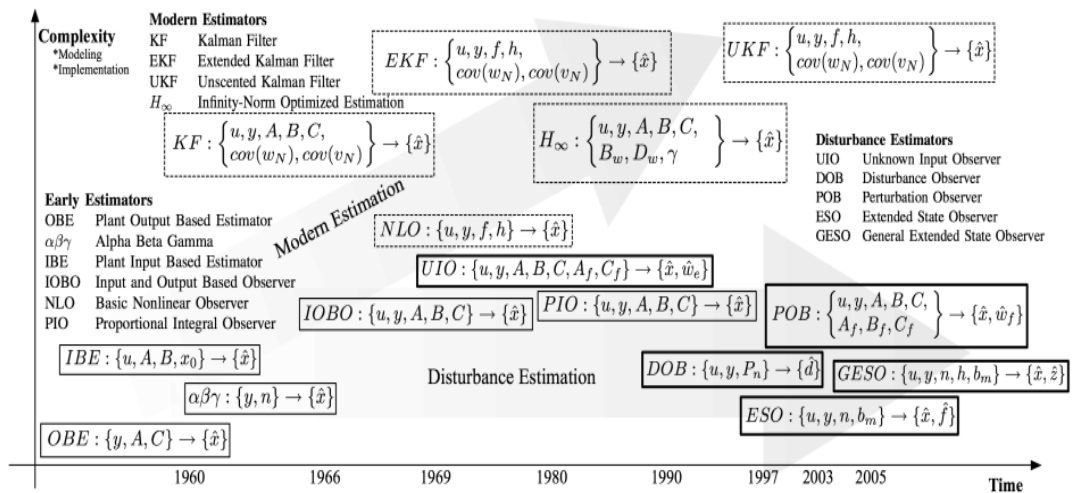


Figure 2-2: A summary of observers' techniques [26]

## 2.3 Kalman Filter

In this sub-chapter, there are short descriptions to the Kalman filter characteristics and follows with an introduction to the concepts. Generally, Kalman filter,

- Estimates states and disturbances based on input, output, and dynamic model of the plant,
- Is computationally efficient due to its recursive structure,
- Incorporates all information that can be provided to it,
- Copes with variable **A**, **C**, **Q** and **R** matrices,
- Copes with the large uncertainty of the initialization phase,
- Needs only the latest best estimate (not a history of states),
- Provides a convenient measure of estimation accuracy (via the covariance matrix **P**),
- Fuses information from multiple-sensors,
- Minimize a cost function based on mathematical assumptions about disturbances.

### 2.3.1 An introduction to concepts

A Kalman filter is simply an optimal recursive data processing algorithm. One aspect of this optimality is that the Kalman filter incorporates all information that can be provided to it. It processes all available measurements, regardless of their precision, to estimate the current value of the variables of interest, with use of [31],

- Knowledge of the system and measurement device dynamics,
- The statistical description of the system noises, measurement errors, and uncertainty in the dynamics models, and
- Any available information about initial conditions of the variables of interest.

The word recursive in the previous description means that, unlike certain data processing concepts, the Kalman filter does not require all previous data to be kept in storage and reprocessed every time a new measurement is taken. This will be of vital importance to the practicality

of filter implementation. The “filter” is actually a data processing algorithm.

A Kalman filter combines all available measurement data, plus prior knowledge about the system and measuring devices, to produce an estimate of the desired variables in such a manner that the error is minimized statistically.

The Kalman filter was derived as the solution to the Wiener filtering problem using state space model for dynamic and random processes. It can estimate the state of a linear dynamic system, perturbed by Gaussian white noise using measurements that are linear fractions of the system state but corrupted by additive Gaussian white noise.

A Kalman filter performs based on conditional probability density propagation for problems in which the system can be described through a linear model and measurement noises are white and Gaussian. Under these conditions, the mean, mode, median, and virtually any reasonable choice for an “optimal” estimate all coincide, so there is in fact a unique “best” estimate of the value of states. Under these three restrictions, the Kalman filter can be shown to be the best filter of any conceivable form [31].

### 2.3.2 The discrete-time Kalman filter

Here we present the discrete-time Kalman filter in the following steps [27].

a) The dynamic system is given by the following equations:

$$\begin{aligned} x_k &= Ax_{k-1} + Bu_{k-1} + \omega_{k-1} \\ y_k &= Hx_k + v_k \end{aligned} \quad 2.30$$

b) The Kalman filter is initialized as follows:

$$\hat{x}_0^+ = E(x_0) \quad 2.31$$

$$P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \quad 2.32$$

c) The Kalman filter is given by the following equations, which are computed for each time step,  $k = 1, 2, \dots$ :

$$P_k^- = A P_{k-1}^+ A^T + Q \quad 2.33$$

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \quad 2.34$$

$$\hat{x}_k^- = A \hat{x}_{k-1}^+ + G u_{k-1} \text{ a priori state estimate} \quad 2.35$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y - H \hat{x}_k^-) \text{ a posteriori state estimate} \quad 2.36$$

$$P_k^+ = (I - K_k H) P_k^- \quad 2.37$$

Figure 2-3 depicts the discrete-time recursive Kalman filter operation, considering the above mentioned algorithms.

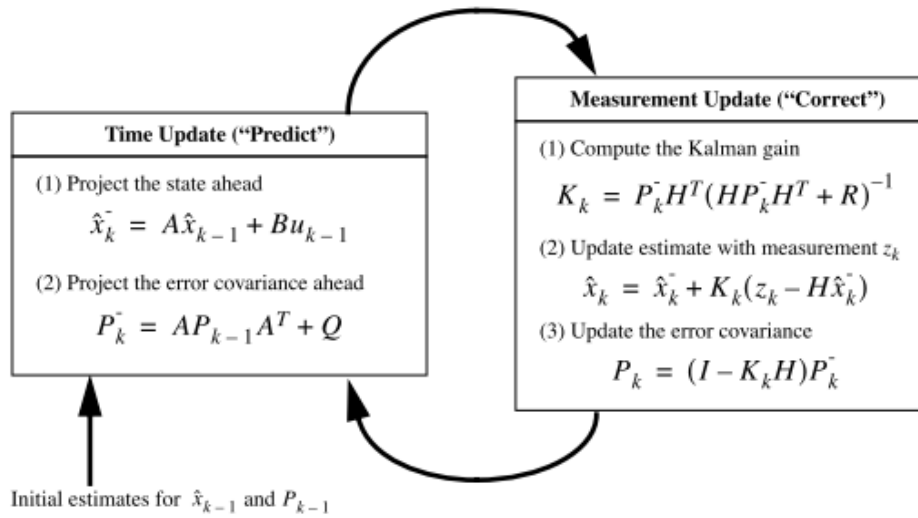


Figure 2-3: A complete picture of the operation of the discrete-time Kalman filter [27]

### 3 Cutting force estimation by Spindle Integrated Displacement Sensor (SIDS)

This chapter presents a method of estimation of cutting forces from the displacements of rotating spindle shafts under cutting load. In this way, the displacement sensor is utilized as an indirect force sensor.

In order to overcome the experimental issues, the instrumental hammer tests are emulated by FEM software. The Frequency Response Function (FRF) of relative displacement/force is achieved on a spindle-machine FEM model [32] (Figure 3-1).

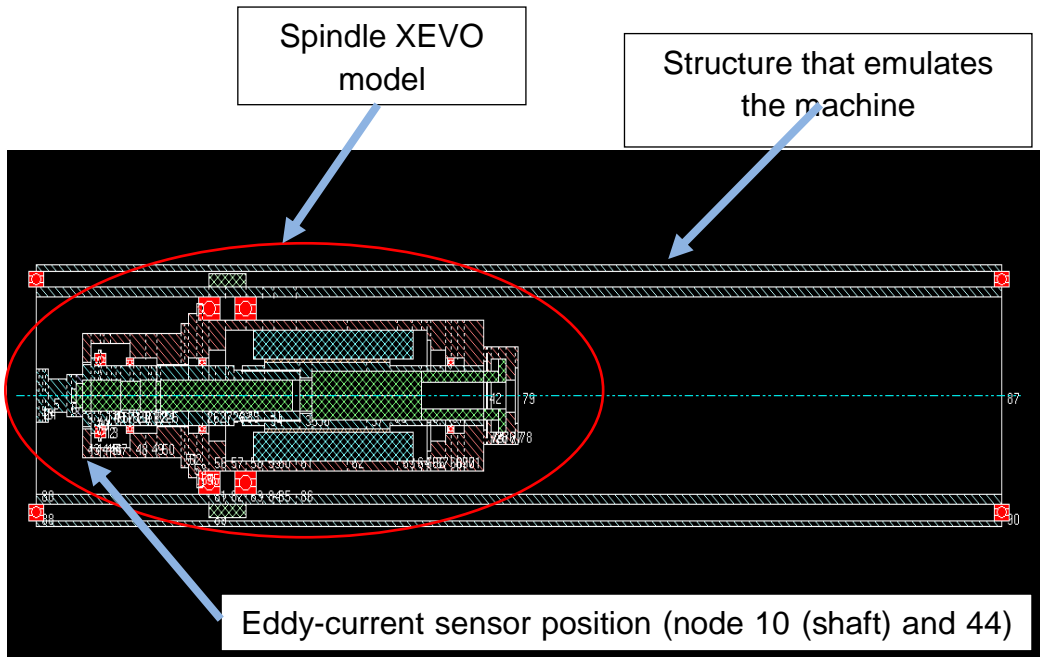


Figure 3-1: Spindle FEM model

In this technique, the bandwidth of FRF of relative displacement/force is limited by the natural modes of the spindle structure. If cutting force frequency content is within the range of the natural modes of the spindle



structure or higher, the measurements will be distorted due to the dynamic characteristics of the spindle system. In order to increase the bandwidth of the indirect force sensor, the design of a Kalman filter scheme is used for compensating the spindle dynamics. Figure 3-2 shows the graphical representation of spindle integrated displacement sensors [15].

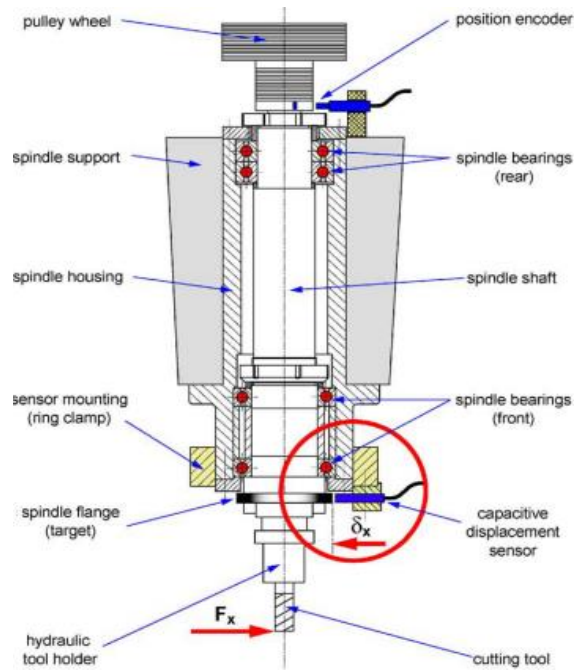


Figure 3-2: Graphical representation of spindle integrated displacement sensors [16]

### 3.1 Identifying the modal parameters

Figure 3-3 shows the FRFs of displacements of spindle shaft and flange (point 10 and 44) due to the force applied to the tool tip. Here, according to the figure, the first two modes are related to the low-frequency machine tool behaviour.

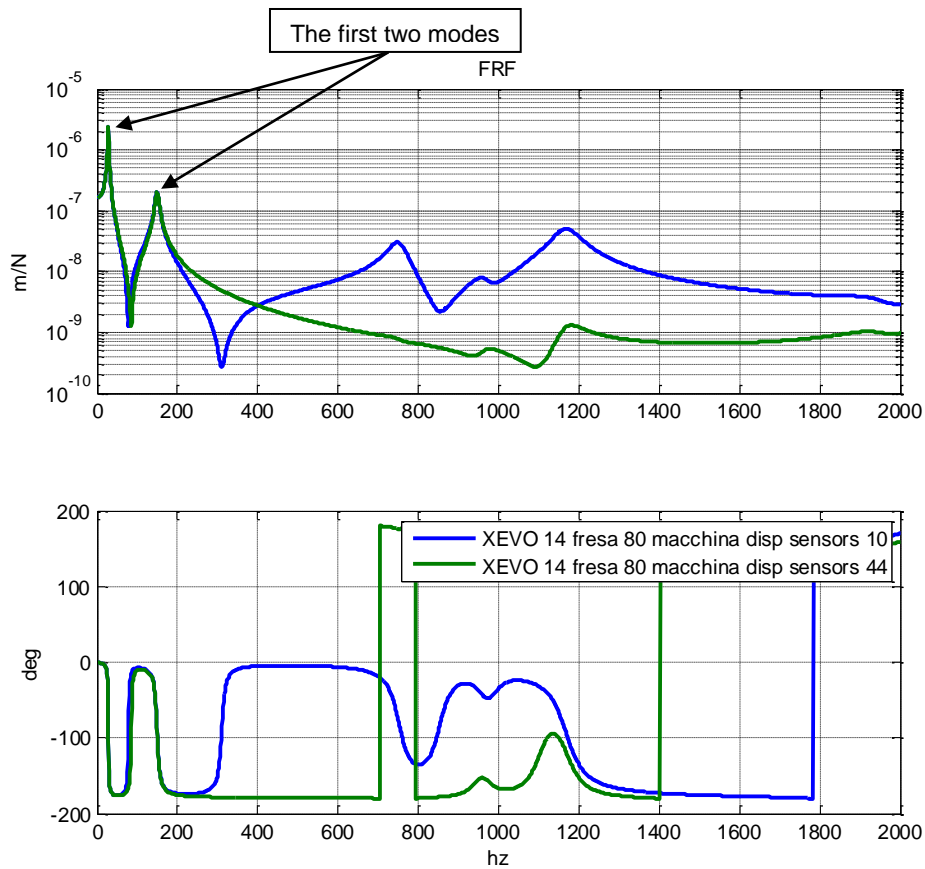


Figure 3-3: Displacement (point 10 and 44)/tool tip force FRFs

It is possible to make a FRF representation of relative displacement to the applied force by making the difference between the two FRFs the spindle shaft and the spindle flange as is depicted in the Figure 3-4.

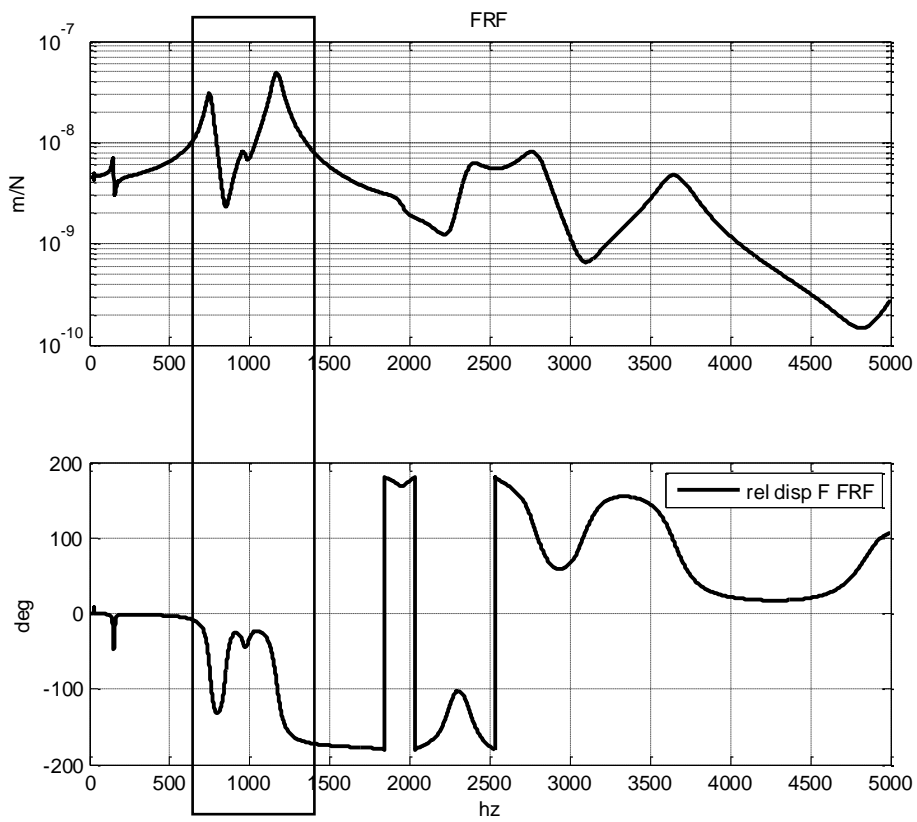


Figure 3-4: Relative displacement/tool tip force

Starting from the above mentioned FEM model the modal parameters of the spindle have been obtained, and tabulated in Table1.

Table 1: Modal parameters of the spindle FEM model

k	$F_n$ (Hz)	$\zeta$	$\alpha$ (1/kg)
1	753	0.0297	25.756
2	965	0.0288	130.369
3	1168	0.0303	6.284

### 3.2 Dynamic compliance of the spindle structure as system plant

After finding the modal parameters, it is necessary to represent the dynamic compliance of the spindle structure in transfer function form using the three natural modes indicated in Figure 3-4. The process is performed by a modal curve fitting technique as,

$$\begin{aligned}\Phi(s) = \frac{\delta_F}{F_a} &= \sum_{k=1}^3 \frac{\alpha_k}{s^2 + 2\xi_k \omega_{n,k} s + \omega_{n,k}^2} = \\ &= \sum_{k=1}^3 \frac{\alpha_k}{s^2 + 2\xi_k (2\pi f_{n,k}) s + (2\pi f_{n,k})^2}\end{aligned}\quad 3.1$$

Where  $k$  is the number of modes,  $\Phi(s)$  are the displacements measured by the spindle integrated sensor, and  $F_a$  is the actual force acting on the tool tip. The modal equation can be expanded in polynomial form as:

$$\Phi(s) = \frac{\delta_F}{F_a} = \frac{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}\quad 3.2$$

### 3.3 Model representation in state space form

In order to design the Kalman filter, the transfer function of the spindle dynamics given in Eq. (3.2) is mapped into state space form that yields to:

$$\begin{aligned}\dot{x} &= A_n x + B_n u \\ y &= C_n x\end{aligned}\quad 3.3$$

Where  $x$  is the state vector;  $u = F_a$  is the input vector or the actual force applied to the tool; and,  $y = \delta F$  is the measurement vector or the displacement sensor reading.

The observability matrix  $W$  is found to be full rank, which guarantees the observability of the system:

$$W = [C_n^T A_n^T C_n^T \dots (A_n^{n-1})^T C_n^T] \quad 3.4$$

### 3.4 Disturbance model expansion for Kalman filter

The aim of dynamic compensation is to reconstruct the actual force,  $F_a$ , exerted on the tool, which is the system input,  $u$ , in Eqs. (3.3). Since the Kalman filter only yields estimates for state vector  $\hat{x}$  and output,  $\hat{z} = \hat{\delta}_F$ , the balanced system in Eq. (3.3) is expanded with the actual force,  $F_a$ , as an additional unknown state in the state vector. It is assumed that the cutting force signals are piece-wise constant and both the actual force and the displacement signal are contaminated with system noise,  $w$ , and measurement noise,  $v$ , yielding,

$$\begin{aligned} \dot{x}_{e(7 \times 1)} &= A_{(7 \times 7)} x_{e(7 \times 1)} + G_{(7 \times 1)} w_{(1 \times 1)} \\ z_{(1 \times 1)} &= C_{e(1 \times 7)} x_{e(7 \times 1)} + v_{(1 \times 1)} \end{aligned} \quad 3.5$$

Where  $G$  is the system noise matrix. The expanded state vector is depicted as below; the former input vector  $u_{(1 \times 1)}$  has disappeared.

$$x_e = [x_{n(1 \times 6)}^T F_{a(1 \times 1)}]^T, \quad u_e = [0] \quad 3.6$$

The expanded and noise contaminated state space model (denoted by e) given by Eq. (3.6) can be rewritten as:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{n(6 \times 1)} \\ \dot{F}_{a(1 \times 1)} \end{bmatrix} &= \begin{bmatrix} A_{n(6 \times 6)} & B_{n(6 \times 1)} \\ 0_{(1 \times 6)} & 0_{(1 \times 1)} \end{bmatrix} \begin{bmatrix} x_{n(6 \times 1)} \\ F_{a(1 \times 1)} \end{bmatrix} + G_{(7 \times 1)} w_{(1 \times 1)} \\ z_{(1 \times 1)} &= [C_n \quad 0_{(1 \times 1)}] \begin{bmatrix} x_{n(6 \times 1)} \\ F_{a(1 \times 1)} \end{bmatrix} + v_{(1 \times 1)} \end{aligned} \quad 3.7$$

In this form, the cutting force can be estimated through a disturbance Kalman filter designed for the expanded model of the spindle integrated force sensor system as,

$$\begin{aligned} \dot{\hat{x}}_e &= A_e \hat{x}_e + K(z - \hat{z}) = \\ &= A_e \hat{x}_e + K(z - C_e \hat{x}_e) = (A_e - KC_e) \hat{x}_e + Kz \end{aligned} \quad 3.8$$

For digital signal processing, Eq. (2.13) is transformed into an equivalent discrete transfer function using continuous-to-discrete transformation and zero-order-hold:

$$\hat{x}_e(k+1) = \exp\{(A_e - KC_e)t_d\}\hat{x}_e(k) + \left(\int_0^{t_d} \exp\{(A_e - KC_e)t\}K dt\right)z(k) \quad 3.9$$

Where, in this application, the discrete sampling time is  $t_d = 0.1$  ms.

### 3.5 Kalman filter implementation

The Kalman filter gain matrix is identified by minimizing the state estimation error covariance matrix  $P = E[\tilde{x} \tilde{x}^T]$ . Based on the assumption that system and measurement noise are uncorrelated zero-mean white noise signals, the covariance matrices are  $Q = E[w w^T] > 0$ ,  $R = E[v v^T] > 0$ ,  $E[w v^T] = 0$ . The noise covariance matrix  $Q$  is tuned to accommodate the compensations. For this case  $R$ ,  $Q$ , and  $G$  are:

$$R = [\delta_{F \text{ noise}}]; \quad Q = [F_a \text{ noise}]; \quad G = [0_{(1 \times 6)} \quad 1]^T \quad 3.10$$

#### 3.5.1 Continuous-time Kalman filter gain

The minimum covariance matrix  $P$  for the state estimation error is evaluated by solving the following time variant Riccati equation

$$\dot{P} = A_e P + P A_e^T + G Q G^T - P C_e^T R^{-1} C_e P \quad 3.11$$

Using the solution of Eq. (3.11), the optimal Kalman filter gain matrix is obtained as:

$$K = P C_e^T R^{-1} \quad 3.12$$

### 3.6 Model representation in Simulink for SIDS

Figure 3-5 shows the Simulink model of cutting force estimation by spindle integrated displacement sensor (SIDS). In this model Kalman filter estimator is utilized to estimate the cutting force based on continuous-time state space models.

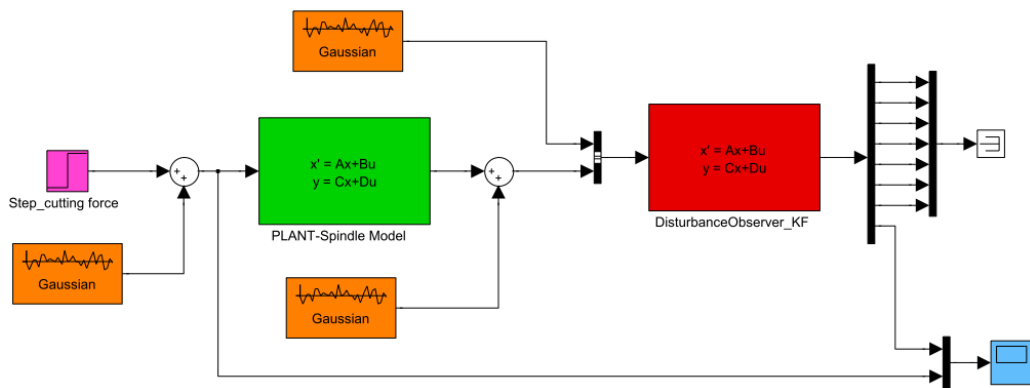


Figure 3-5: Simulink model of cutting force estimation from SIDS

Figure 3-6 depicts the Input-Plant-Output model of spindle integrated displacement sensor (SIDS) in state space representation. The expanded matrices,  $A_e$ ,  $B_e$ ,  $C_e$ ,  $D_e$  have been utilized in defining of our plant model. The step force input  $F_a$  and the measurement (as displacements) are contaminated by process and measurement white Gaussian noises as represented in the figure.

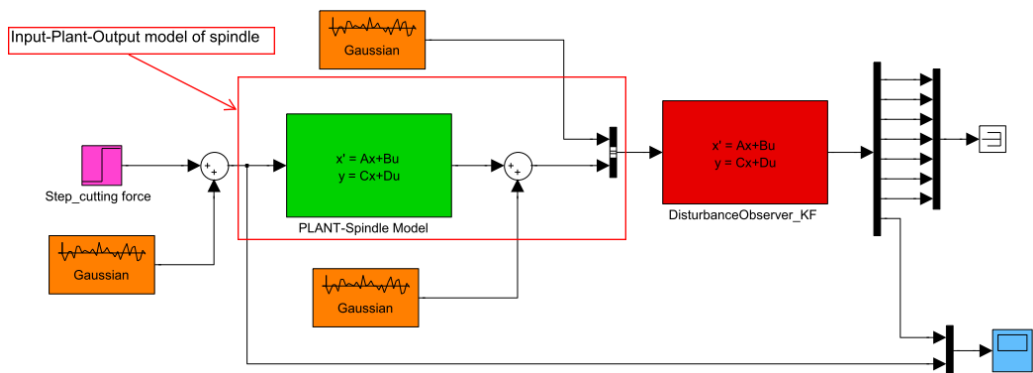


Figure 3-6: Input-Plant-Output model of SIDS

Figure 3-7 represents the Kalman filter implementation in the system. The inputs to the Kalman filter are displacements and the white Gaussian noise, and the outputs are estimation of displacements and cutting force. According to the figure, the measured and estimated values of the displacement as well as the applied and the estimated cutting forces have been compared with each other to evaluate the estimation accuracy.

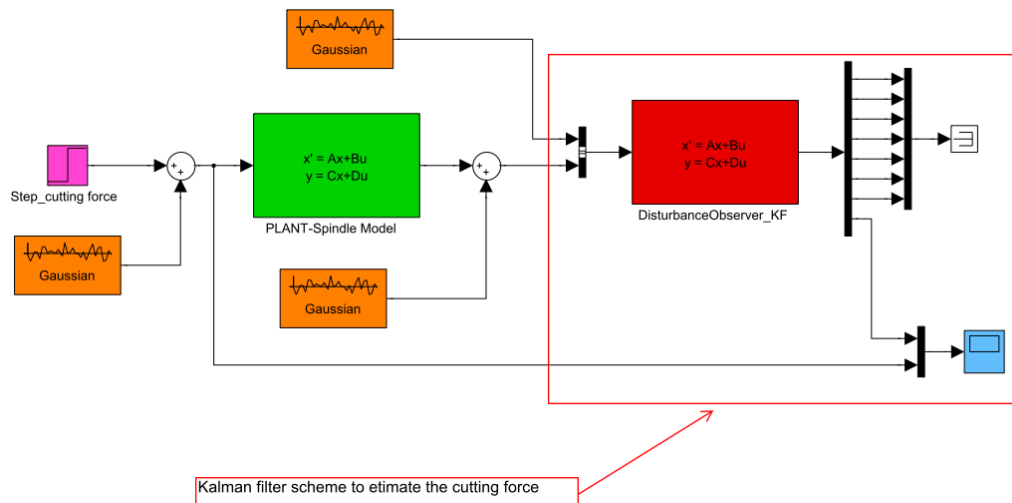


Figure 3-7: Model representation of Kalman filter implementation

The same procedure is utilized for the discrete-time system after discretizing of the plant and of the Kalman filter state space matrices.

### 3.7 Results of Cutting force estimation by SIDS

In this chapter the results of the force estimation by Kalman filter for SIDS is going to be presented base on two approaches, in continuous-time and discrete-time forms. In the first approach, the results applying input step force and the simulated real force considering the dynamics of the milling process are going to be taken to the account. The simulated real force is explained in the sub chapter 3.7.1.



In the meanwhile, the second approach (discrete-time Kalman filter) has been implemented considering the real condition of the milling process in which the plant and Kalman filter model must be discretized.

### 3.7.1 Simulated real force

Figure 3-8 shows the milling model which has been prepared by CNR ITIA, explained in [33]. According to the figure, the blue box represents the kinematics of the milling process, while, the orange box deals with the dynamics of the cutting. The milling model is applied as an input force to the spindle integrated displacement sensor system. Then we have estimated the force by help of Kalman filter.

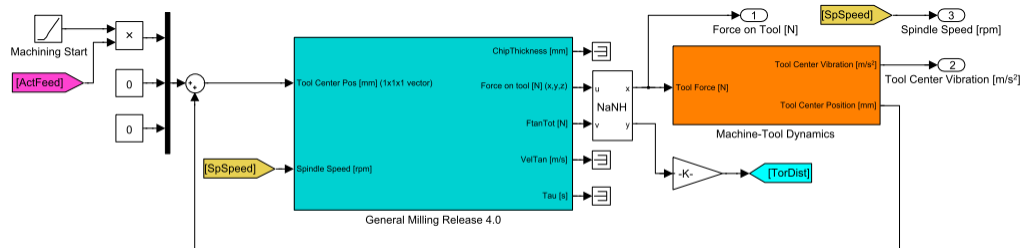


Figure 3-8: Milling model [courtesy to CNR ITIA]

### 3.7.2 Continuous-time Kalman filter

#### 3.7.2.1 Simulation of continuous-time Kalman filter to input step force

Figure 3-9 shows the Estimation of cutting force by Kalman filter for SIDS. According to the figure, the input true force (in red color) to the system is a step force and the amplitude is 100 N. The estimated force (in blue color) precisely follows the true value of the force after tuning the process and measurement covariance noise matrices,  $Q$  and  $R$  respectively. The estimated value is contaminated by the white Gaussian process noise,  $w$  and measurement noise  $v$ .

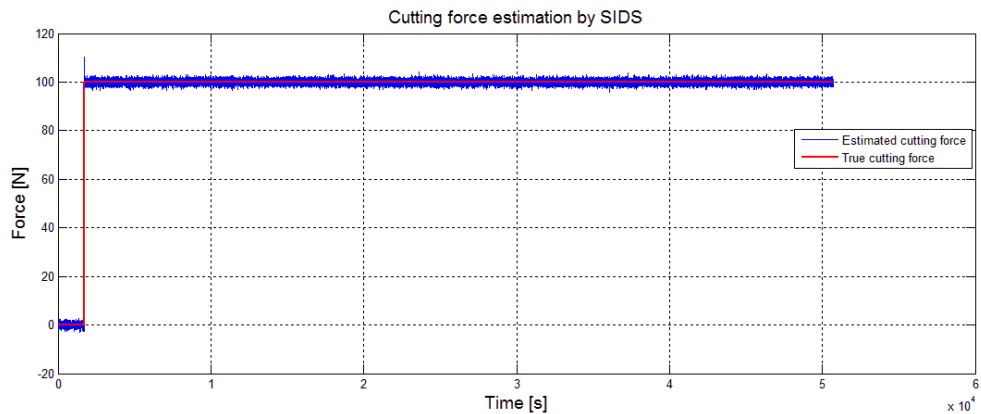


Figure 3-9: Estimation of cutting force by Kalman filter for SIDS

### 3.7.2.2 Simulation of continuous-time Kalman filter to simulated real force

Figure 3-10 and Figure 3-11 show the Cutting force estimation as a result of the simulated real input force in machine tool X and Y axes. The process parameters which have been selected for this study have been tabulated in the Table 2.

Table 2: Process parameters of simulated real milling process

	Values	Units
Workpiece Material	C40 steel	-
$K_{re}$	5	N/mm
$K_{rc}$	1309	Mpa
$K_{te}$	9	N/mm
$K_{tc}$	2598	Mpa
Milling Operation	Half immersion, Down Milling	-
Dept of Cut	0.1-0.2	mm
Cutting tool Dia.	80	mm
Feed per Tooth	0.25	mm/rev
Spindle Speed	650	rpm
Simulation Time	5	s
No. Of teeth	5	-

According to Figure 3-10 and Figure 3-11, the estimated force (in blue color) precisely follows the true value of the force (in red color) after tuning the process and measurement covariance noise matrices,  $Q$ , and  $R$ , respectively. The estimated value is contaminated by the white Gaussian process noise,  $w$  and measurement noise  $v$ .

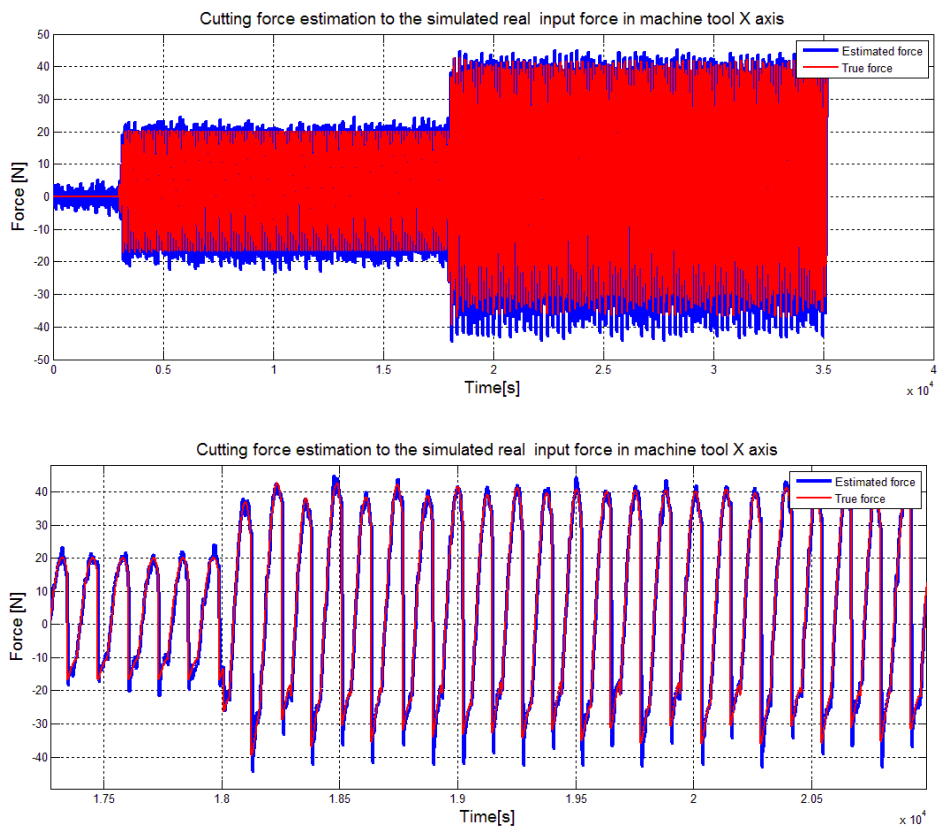


Figure 3-10: Cutting force estimation to the simulated real input force in machine tool X axis

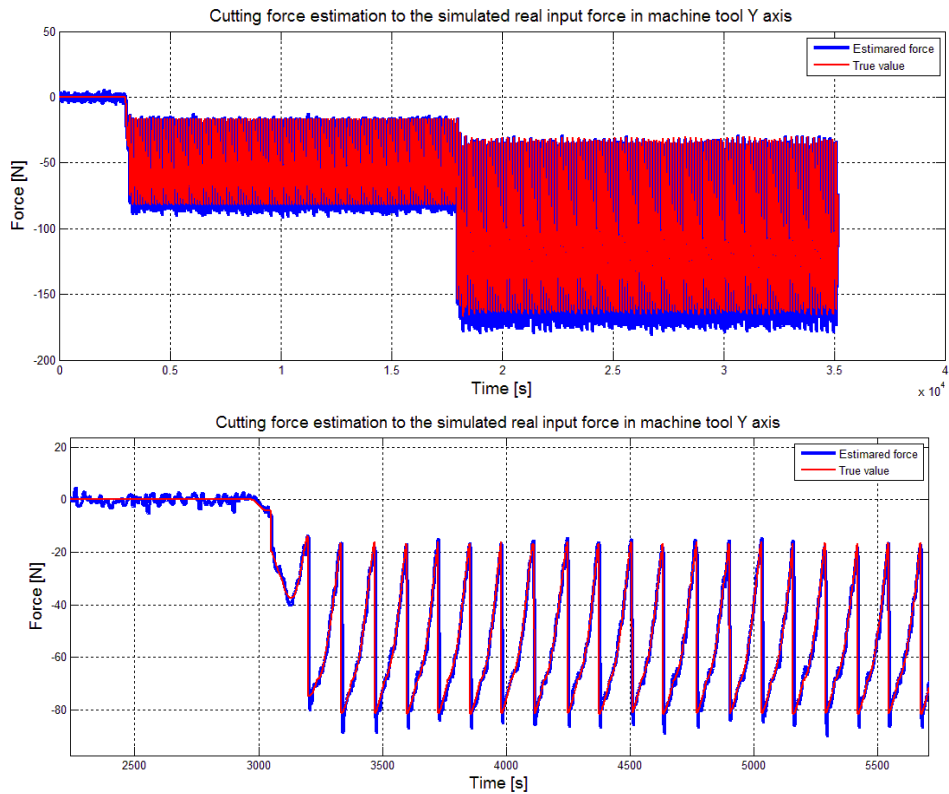


Figure 3-11: Cutting force estimation to the simulated real input force in machine tool Y axis

### 3.7.3 Discrete-time Kalman filter

In order to implement the Kalman filter in the real application of cutting force estimation, the model must be discretized based on the sampling time. The sample time in this work was selected  $t_d = 1 \text{ ms}$ .

Figure 3-12 shows the cutting force estimation by discrete time Kalman filter for SIDS. According to the figure, the estimated cutting force (in blue color) follows the true value of the cutting force (in red color).

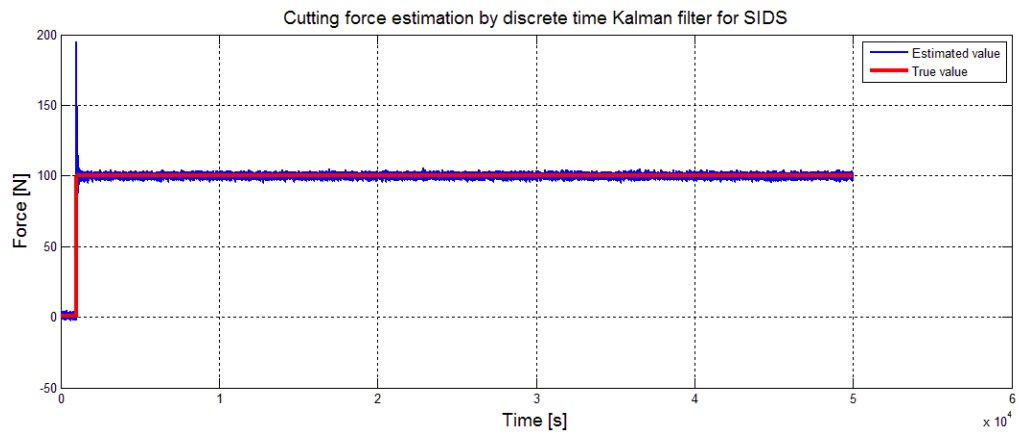


Figure 3-12: Cutting force estimation by discrete time Kalman filter for SIDS

## 4 Cutting force estimation from feed drive current

In-process cutting force measurement is one of the most important sensor systems for cutting condition monitoring and control in machining. CNC machine tools servomotors have current sensors for motion control. The current have direct contributes to the torque generated by the motor and the external resistant loads.

The total external torque to the servo motor consists of the torque induced from the cutting force and that from the friction force in the guide way. Here in this chapter, the main idea is to estimate the disturbance load which is exerted to the feed drive system due to the cutting force. Hence, a Kalman filter is implemented as a model based disturbance observer for indirect measurement of cutting forces in milling process through the current.

### 4.1 Closed loop and open loop model of feed drive (plant model)

Figure 4-1 represents the simplified closed loop control model of the feed drive. In this figure, the model can be dived in two parts: electrical and mechanical. In the electrical part, the velocity and current regulators are present, while the mechanical part consists of global inertial  $J_{eq}$  and integrators to achieve angular velocity  $\omega$  and angular position  $\theta$ .

The inputs to the closed loop system are angular position  $\theta$  as well as the disturbance cutting force  $F_c$  as a disturbance. In addition, the outputs are velocity  $\omega$  and angular position  $\theta$ .

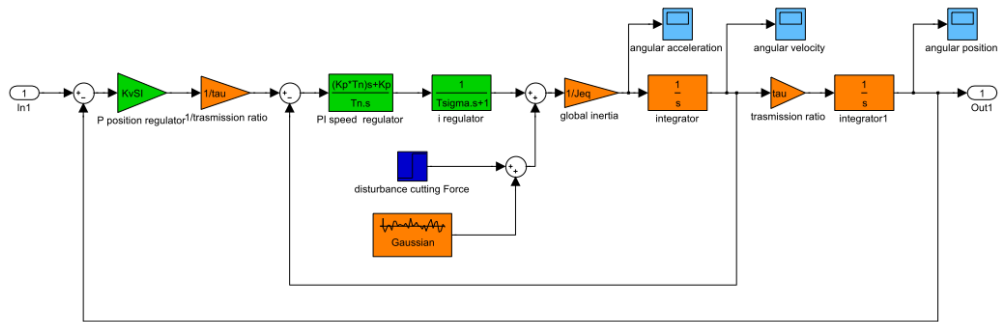


Figure 4-1: Simplified closed loop control model of the feed drive

Figure 4-2 shows the mechanical part of the feed drive along with the current regulator, as an open loop model. The inputs to the model are the current and the disturbance cutting force whereas the outputs are angular velocity  $\omega$  and angular position  $\theta$ .

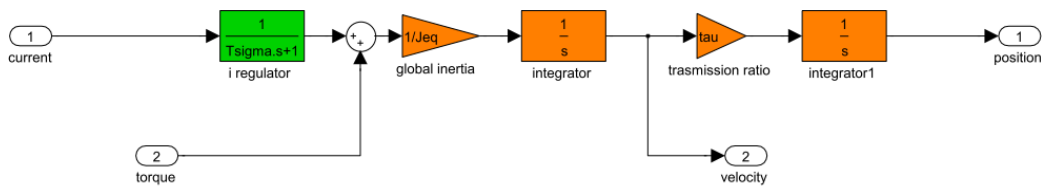


Figure 4-2: Mechanical part of the feed drive along with the current regulator

#### 4.1.1 Closed loop model verification

In order to verify the closed loop model whether it reacts to the input step force properly or not, we emulate the feed drive condition when it is in a stationary situation. Thus, the angular reference position is set to zero, whereas, we apply a step cutting force as input.

From the Figure 4-3, Figure 4-4, and Figure 4-5, it can be seen that the system response to the step cutting force yields to the reference position value after few seconds.

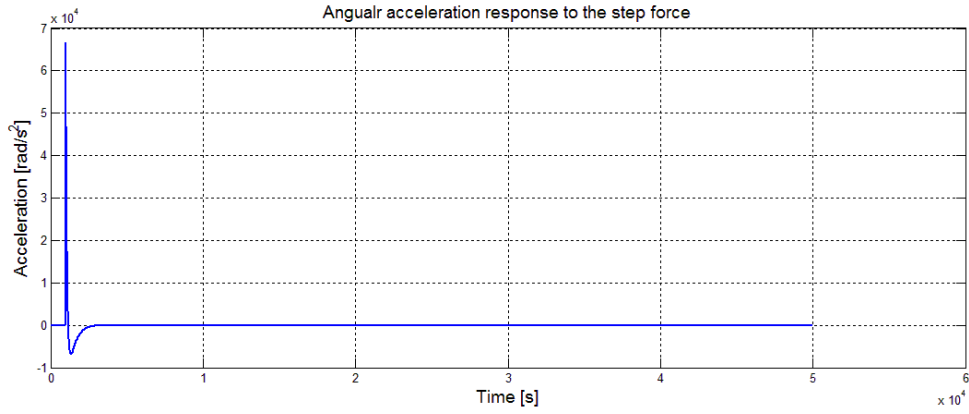


Figure 4-3: Angular acceleration response to the step force

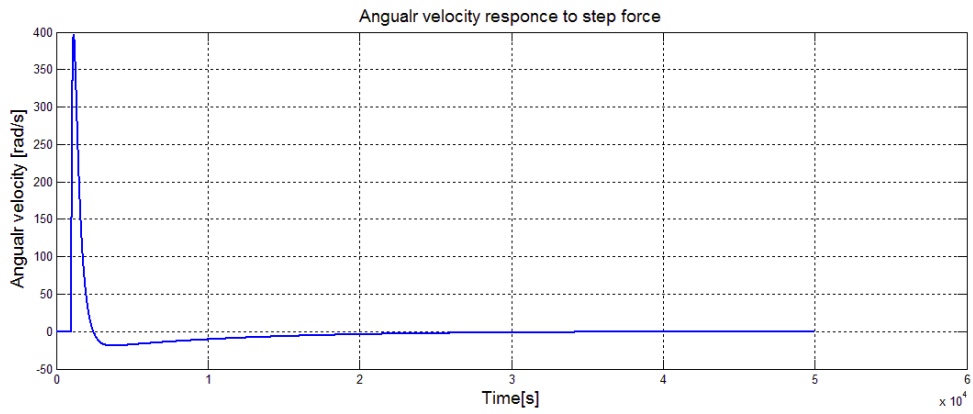


Figure 4-4: Angular velocity response to the step force

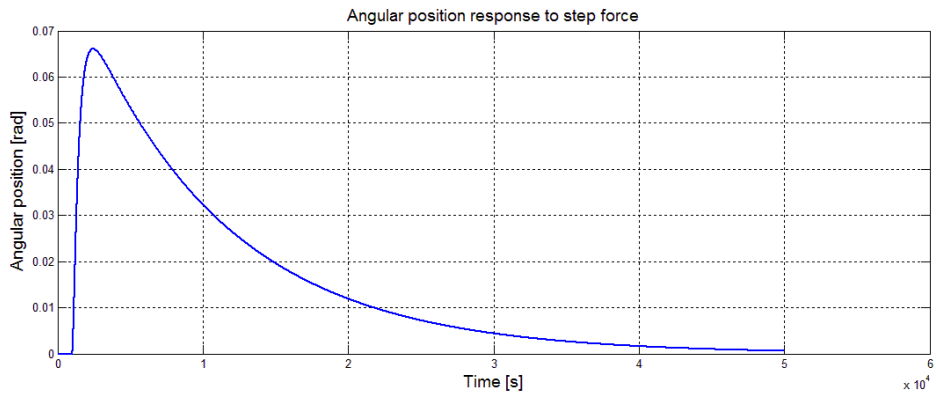


Figure 4-5: Angular position response to step force



## 4.2 Implementation of Kalman filter into the feed drive models

The closed loop plant model of feed drive in combined with the Kalman filter (Figure 4-6) in order to estimate the cutting forces. In this scheme, the inputs to the Kalman filter are the output of the plant model (Figure 4-7) current  $i$ , angular position  $\theta$ , and angular velocity  $\omega$ . The output of the Kalman filter is the estimated cutting force compared with the “disturbance” cutting force  $F_c$ .

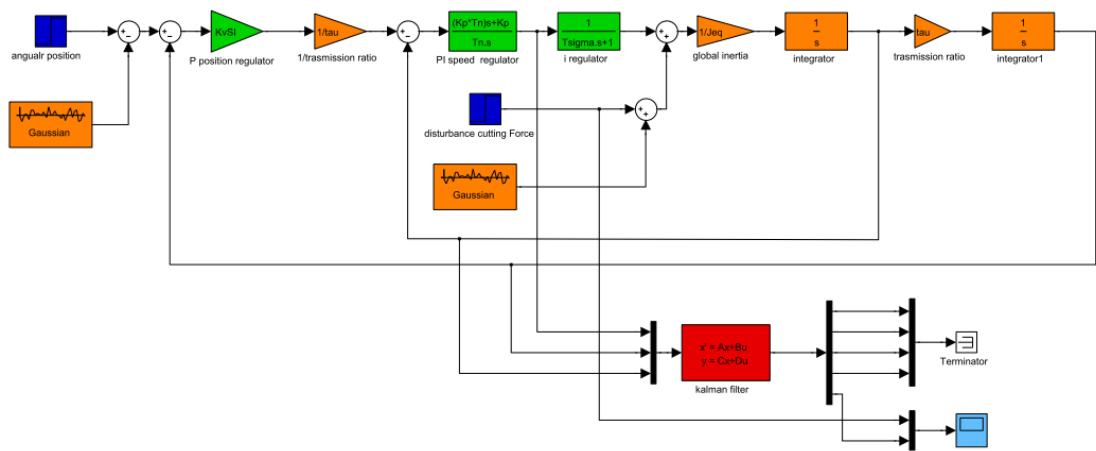


Figure 4-6: Feed drive model combined with the Kalman filter

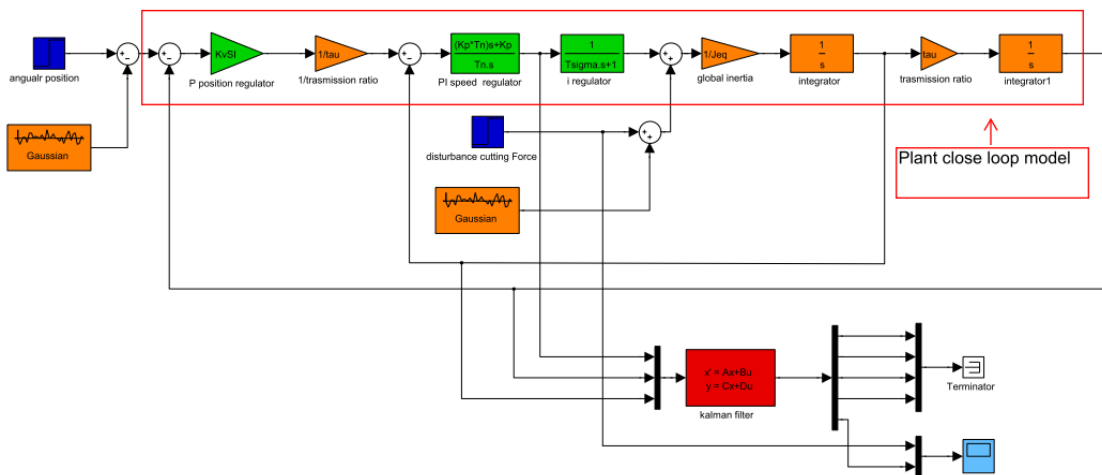


Figure 4-7: Plant closed loop model

In addition, Figure 4-8 represents the open loop plant model of feed drive combined with the Kalman filter. In this figure, the inputs to the plant are the current  $i$  and the cutting force  $F_c$ , whereas the outputs from the plant are the angular position  $\theta$  and the angular velocity  $\omega$ . Moreover the inputs to the Kalman filter are current  $i$ , angular position  $\theta$  and angular velocity  $\omega$ , while the output of the Kalman filter is the estimated cutting force to be compared with the true one.

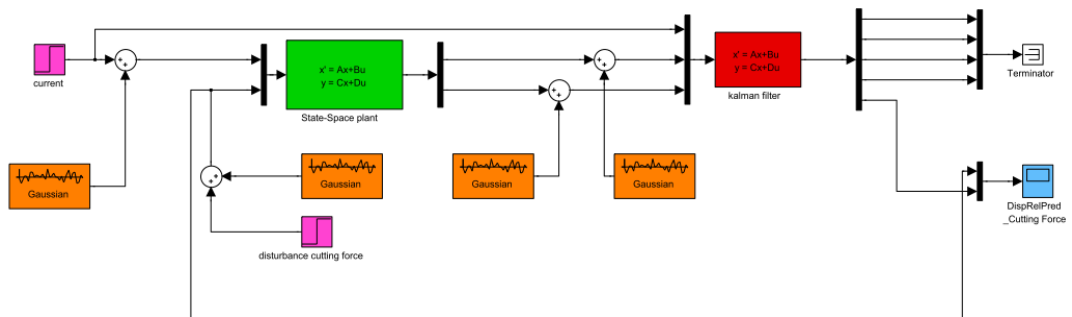


Figure 4-8: Open loop plant model of feed drive combined with the Kalman filter

### 4.3 Design of the Kalman filter

Figure 4-9 shows the Kalman filter scheme in the Simulink model of the cutting force estimation from feed drive. The filter is designed in state-space representation, where the A, B, C, and D matrices have been obtained from the mechanical part of the feed drive model (open loop model) Figure 4-2.

Furthermore, we assume that the current (input) and the angular velocity and position (outputs) are contaminated by the system noise  $w$  and measurement noise  $v$ . Both  $w$  and  $v$  are assumed zero-mean white Gaussian noise inputs whilst  $Q$  and  $R$  are their covariance matrices, respectively.  $G$ ,  $w$  and  $v$  can be written as:

$$w = i_{noise} \tag{4.1}$$

$$v = [\omega_{noise} \ \theta_{noise}] \tag{4.2}$$

$$Q = \sigma_w^2 \tag{4.3}$$

$$R = \sigma_v^2 \tag{4.4}$$

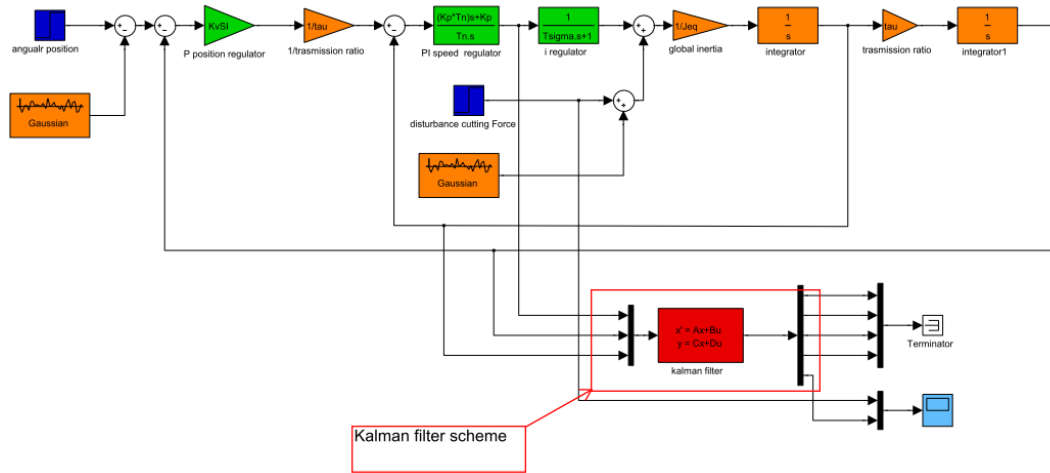


Figure 4-9: Kalman filter scheme

## 4.4 Results of cutting force estimation from feed drive

In this chapter the results of the force estimation by Kalman filter from feed drive is going to be presented base on two approaches, open loop and closed loop plants. In the first approach, the results are shown applying step force and current as inputs.

The second approach (closed loop plant) is implemented considering the cutting force as the only input whereas the angular position is set to zero.

### 4.4.1 Kalman filter for closed loop plant

Figure 3-9 shows the estimation of cutting force by Kalman filter from feed drive closed loop model. According to the figure, the input true force (in blue color) to the system is a step force and the amplitude is 100 N. the estimated force (in red color) precisely follows the true value of the force after tuning the process  $Q$  and measurement  $R$  covariance noise matrices. The estimated value is contaminated by the white Gaussian process noise  $w$  and measurement noise  $v$ .

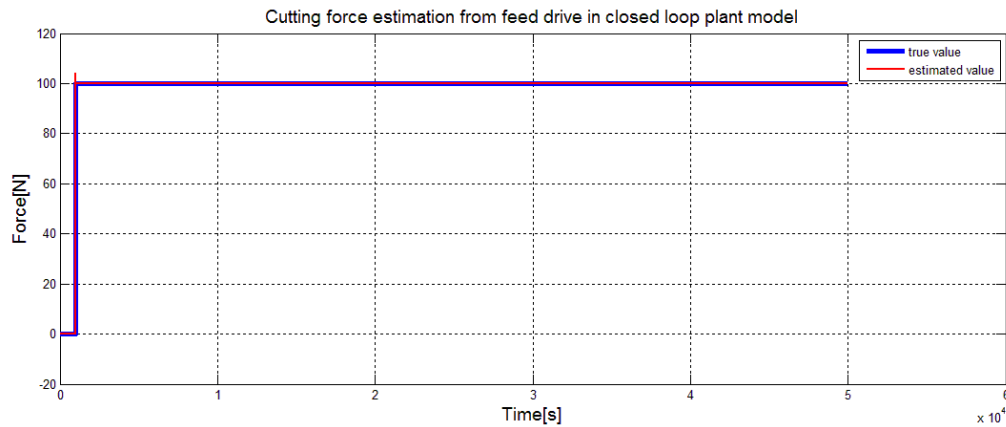


Figure 4-10: Cutting force estimation from feed drive for closed loop model

Figure 4-11 and Figure 4-12 show the estimation of angular position and velocity by Kalman filter for the closed loop model. According to the figures the estimated values exactly follow the true values of the angular position and the velocity.

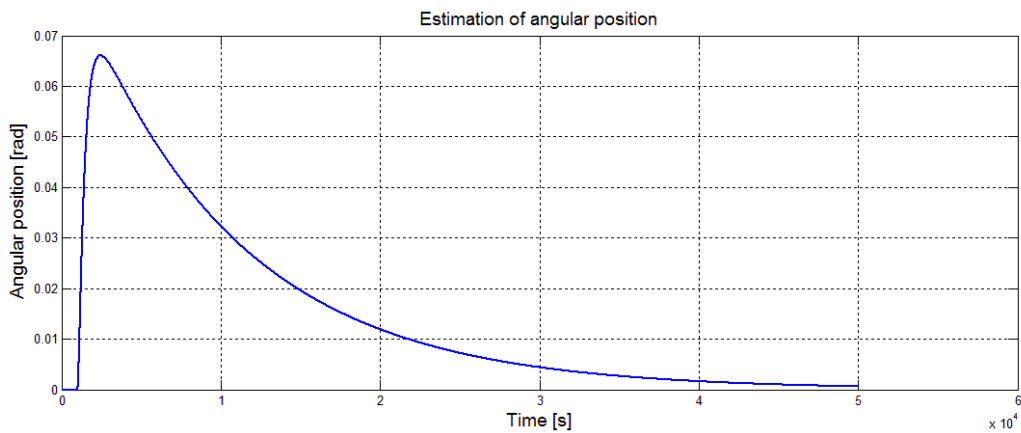


Figure 4-11: Estimation of angular position

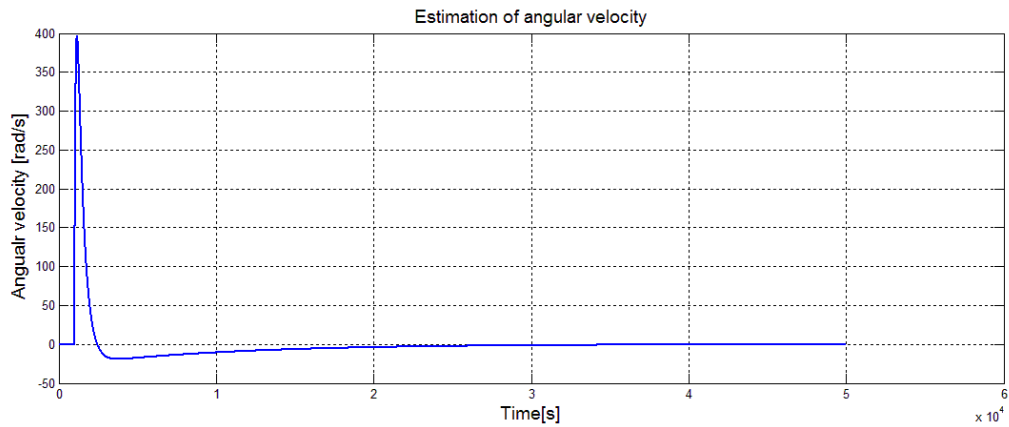


Figure 4-12: Estimation of angular velocity

#### 4.4.2 Kalman filter for open loop plant

Figure 4-13 depicts the cutting force estimation by Kalman filter from open loop plant model. According to the figure, the input true force (in blue color) to the system is a step force and the amplitude is 100 N. the estimated force (in red color) precisely follows the true value of the force after tuning the process  $Q$  and measurement  $R$  covariance noise matrices. The estimated value is contaminated by the white Gaussian process noise  $w$  and measurement noise  $v$ .

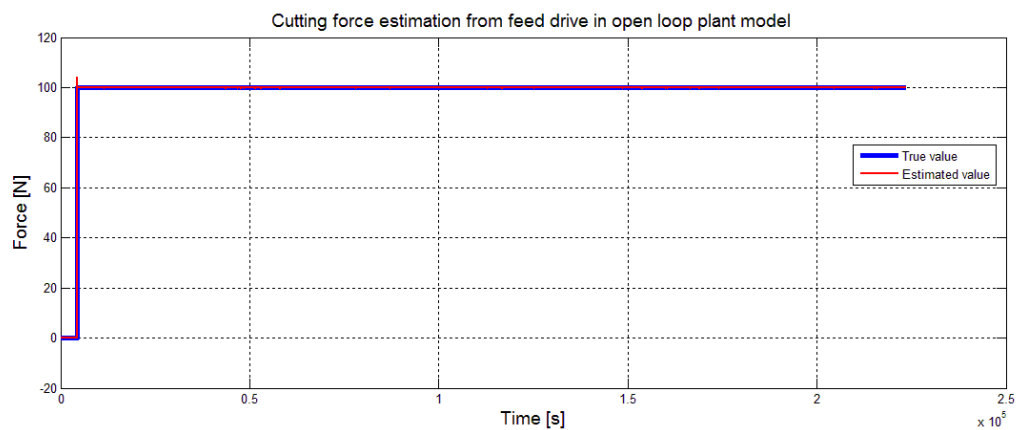


Figure 4-13: Cutting force estimation by Kalman filter from open loop plant model

#### **4.4.3 Comparison between the results of the closed loop and open loop model**

By making comparison between the closed loop and open loop systems results (Figure 4-10 and Figure 4-13), it can be seen that the estimated cutting forces by Kalman filter have same values.



# 5 Conclusion and Future Work

## 5.1 Completed work

In the present work, the estimation of cutting force is performed from the model of the spindle integrated displacement sensors in addition to the one of the current of the feed drive. The approaches that are used for the estimation were based on numerical methods in Matlab/Simulink software in order to replicate the experimental tests.

In the chapter 3, a method for the estimation of the cutting forces from the displacements of rotating spindle shafts is studied. In this scenario, the FEM model of the spindle is utilized to find the modal parameters. Moreover, dynamic relationship between the displacement and the force at the tool tip has been modeled. Additionally, a state estimator, namely Kalman filter, is utilized to estimate the cutting force. Finally, the results of the simulation are reported based on continuous and discrete time domains. In this approach the applied force to the system is selected based on two levels. The first level is step input force, since its response has a lot of dynamic content and is simple to study. The second level of the force is based on the simulated real force that has considered the dynamics and kinematics of the milling process. The obtained results show that the estimated force well predicts the real one.

In the chapter 4, indirect estimation of the cutting forces from the current of feed drive has been studied. In this method, the idealized closed loop and open loop models of the feed drive are considered as plant models. Additionally, the state estimator Kalman filter is used in order to estimate the cutting force. Finally, the results of the simulation are reported based on open loop and closed loop plant model in continuous time domain. The outcome results depict that the estimated force follows the applied force properly.



## **5.2 Sensor fusion as future works**

### **5.2.1 Sensor fusion- overview**

With a specific focus to monitoring, researchers have developed over the years a wide variety of sensors and sensing strategies, each attempting to predict or detect a specific phenomenon during the operation of the process and in the presence of noise and other environmental contaminants. Although able to accomplish the task for a narrow set of conditions, these specific techniques have almost uniformly failed to be reliable enough to work over the range of operating conditions and environments commonly available in manufacturing facilities.

Therefore, researchers have begun to look at ways to collect the maximum amount of information about the state of a process from a number of different sensors (each of which is able to provide an output related to the phenomenon of interest although at varying reliability) [1]. The strategy of integrating the information from a variety of sensors with the expectation that this will 'increase the accuracy and ... resolve ambiguities in the knowledge about the environment' is called sensor fusion.

Sensor fusion is able to provide data for the decision-making process that has a low uncertainty owing to the inherent randomness or noise in the sensor signals, includes significant features covering a broader range of operating conditions and accommodates changes in the operating characteristics of the individual sensors (due to calibration, drift, etc.) because of redundancy.

The most advantageous aspect of sensor fusion is the richness of information available to the signal processing/feature extraction and decision-making methodologies employed as part of the sensor system. Sensor fusion is best defined in terms of the 'intelligent' sensor as introduced in [34] since that sensor system is structured to utilize many of the same elements needed for sensor fusion.

### 5.2.2 Centralized and decentralized Kalman filter

Current advances in sensor technology enable the design of small-scale low-cost sensing devices (sensor nodes) endowed with embedded computing and communication capabilities. A number of these sensor nodes can be connected to each other following a certain topology. The collection of such connected sensors is called a sensor network.

There are numerous applications of sensor networks, for instance, in military, health care, or agriculture [35]. An example of the use of sensor networks in the control field is networked control [36]. The use of wireless sensors for feedback has also been reported [37]. An attractive feature of sensor networks is their capability to perform sensing and state estimation in environments with spatially distributed parameters. In this case, sensor nodes are placed at specified locations of the environment to collect measurements that serve as inputs to the filter.

To this end, decentralized variants of Kalman filter can be used. One approach for using the measurements is treating them as one measurement matrix. The measurements from the nodes are then sent to a central processor in which global estimates are computed. In this case the sensor nodes do not have an active role. This approach is called centralized Kalman filter.

As opposed to the centralized approach, the estimate computations can be decentralized to the nodes where local estimates are computed. The local estimates can subsequently be transmitted to a central processor to get global estimates.

Alternatively, local estimates are communicated between two or more nodes using an algorithm to get the estimates of the global state. In this case the sensor nodes have an active role to estimate the states. There are several decentralized Kalman filter methods that have been proposed in the literature. The most representative methods are: parallel information filter [38], distributed information filter [39], distributed Kalman filter with consensus filter, and distributed Kalman filter with weighted averaging [40].

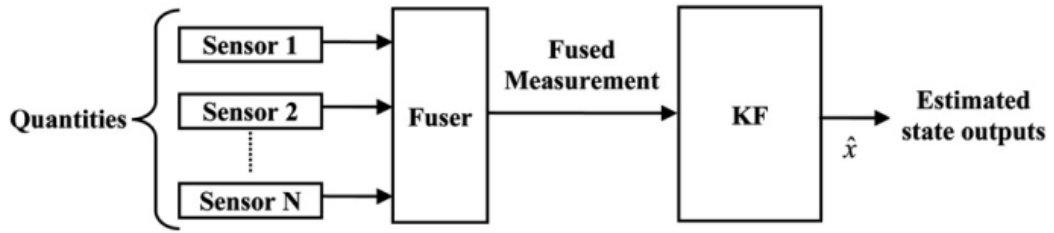


Figure 5-1: Centralized integration architecture [40]

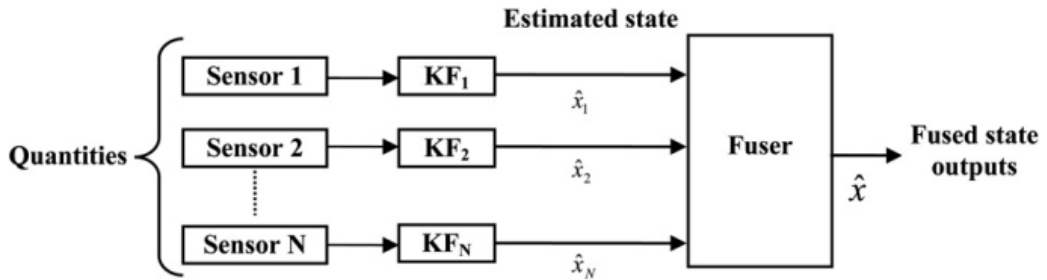


Figure 5-2: Decentralized integration architecture[40]

### 5.2.2.1 Centralized Kalman filter

The Kalman filter equations consist of two parts: time update equations and measurement update equations. The following time update equations compute estimates at time step,  $k$ , based on the process model and the previous estimates to get *a priori* estimates:

$$P^-(k) = A P^+(k-1) A^T + Q \quad 5.1$$

$$\hat{x}^-(k) = A \hat{x}^+(k-1) + G u(k) \quad 5.2$$

*a priori state estimate*

Where  $P^-(k)$  is the estimation error covariance matrix, and the superscripts “-” and “+” respectively indicate the *a priori* and *a posteriori* estimates and the error covariance matrix. This step is also referred to as the prediction step.

Once the measurements at time step,  $k$ , are available, the measurement update corrects the *a priori* estimates to get *a posteriori* estimates:

$$K(k) = P^-(k) H^T (H P^-(k) H^T + R)^{-1} \quad 5.3$$

$$\hat{x}^+(k) = \hat{x}^-(k) + K(k)(z - H \hat{x}^-(k)) \quad 5.4$$

*a posteriori state estimate*

$$P^+(k) = (I - K(k)H)P^-(k) \quad 5.5$$

$$\hat{x}_0^+ = E(x_0)P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)]^T \quad 5.6$$

Where  $K(k)$  is the Kalman gain matrix. The initial conditions are  $\hat{x}^+(0) = x_0$  and  $P^+(0) = P_0$ , where  $x_0$  and  $P_0$  are respectively the initial guesses of the estimate and estimation error covariance matrix.

The above mentioned Kalman filter is called centralized Kalman filter. It is because the measurements are treated in one measurement matrix. The estimates from the centralized Kalman filter are called global estimates.

Besides the form in (5.1) to (5.7), there is another form of the Kalman filter that uses the inverse of the estimation error covariance matrix that is called information matrix, denoted by  $I$  and defined as  $I = P^{-1}$ . The filter that uses this information matrix is called the information filter. The *a priori* estimate equations of the information filter are equal to (5.1) and (5.2). The measurement update computations are the following:

$$\Delta s(k) = H^T R^{-1} z(k) \quad 5.7$$

$$\Delta I = H^T R^{-1} H \quad 5.8$$

$$\hat{x}^+(k) = \hat{x}^-(k) + \Delta s(k) \quad 5.9$$

$$I^+(k) = I^-(k) + \Delta I \quad 5.10$$

Where  $\Delta s$  and  $\Delta I$  are respectively the information vector and matrix update. The information filter avoids the need of matrix inverse computation that is preferable from a numerical point of view.

In the application of the decentralized Kalman filter with sensor networks for distributed parameter systems, each node,  $i$ , has a capability to compute its own estimates  $\hat{x}_i(k)$  and the corresponding estimation error covariance matrix  $P_i(k)$ . The estimate and/or the error covariance matrix are communicated to other nodes based on the network topology. In our case, consider a sensor network consisting of  $N$  sensor nodes.

The nodes are connected to each other, following a specified network topology. In the network, nodes,  $i$ , and  $j$ , are neighbors if there is a direct link between them. The set of neighbors of node,  $i$ , and including node,  $i$ , itself is denoted by the set  $N_i$ . We assume each node has an identical process model (2.26) and the corresponding process noise  $Q$ , but a different measurement matrix. Since each node measures one or more

state components of the system and since no state component is measured by two or more nodes, we can assume that each local measurement matrix  $H_i$  is one block row of the global measurement matrix  $H$ . In other words, the global measurement matrix  $H$  is the stack of all local measurement matrices  $H_i$ ,

$$H = \begin{bmatrix} H_i \\ \vdots \\ H_N \end{bmatrix} \quad 5.11$$

So the local measurement in node,  $i$ , is expressed as

$$z_i(k) = H_i x_i(k) + v_i(k) \quad 5.12$$

It is assumed that measurement noise between node  $i$  and  $j$  is uncorrelated. In this work, measurement updates and the resulting estimates in each node are called local updates and local estimates respectively.

### 5.2.2.2 Parallel information filter (PIF)

The parallel information filter computes local a posteriori estimates  $\hat{x}_i^+(k)$  and the corresponding estimation error covariance matrix  $P_i^+(k)$  in parallel in each node. Then  $\hat{x}_i^+(k)$  and  $P_i^+(k)$  are sent to a central processor in which the estimates are combined to get the global estimate  $\hat{x}(k)$  [41]. The time and measurement update equations for node,  $i$ , are:

- The local time update:

$$P_i^-(k) = A P_i^+(k-1) A^T + Q \quad 5.13$$

$$\hat{x}_i^-(k) = A \hat{x}_i^+(k-1) + G u(k-1) \quad 5.14$$

- And the local measurement:

$$K_i(k) = P_i^-(k) H_i^T (R_i)^{-1} \quad 5.15$$

$$(P_i^+(k))^{-1} = (P_i^-(k))^{-1} + H_i^T R_i^{-1} H_i \quad 5.16$$

$$z_i(k) = H_i x_i(k) + v_i(k) \quad 5.17$$

In the central processor, estimates from all nodes are combined into one estimate. It is desired that the estimate is as certain as possible, or

in other words, an estimate with a low uncertainty is preferable. In case of estimates and uncertainties from  $N$  measurements, where the measurement of node,  $i$ , is independent to that of node  $j$  for,  $i \neq j$ , estimates with lower uncertainty should be given larger weights. With such consideration, the weight for each measurement can be calculated as [41].

Once the weights have been determined, the global a posteriori estimate and its estimate error covariance matrix can be expressed as follows,

$$P_i^-(k) = \sum_{i=1}^N w_i(k) (P_i^+(k))^{-1} \quad 5.18$$

$$\hat{x}(k) = \sum_{i=1}^N w_i(k) P(k) (P_i^+(k))^{-1} \hat{x}_i^+(k) \quad 5.19$$

This method relies on the central processor to get the global estimates. Hence, it is necessary that all sensor nodes are neighbors of the central processor to assure that all local measurements can be combined into global ones.

### 5.2.2.3 Distributed information Kalman filter (DIF)

The decentralized information filter was proposed by Rao and Durrant-Whyte [39] to eliminate the need of a central processor in the decentralized Kalman filter. Using a central processor creates a hierarchy in the network.

Furthermore, the network is highly dependent on the central processor. Eliminating the central processor makes that all nodes are at the same level and removes dependency on a single component. The key idea of this method is expressed in the relation between information vectors and matrix updates, respectively, for the global estimates of the centralized method and local estimates in each node  $i$ .

$$\Delta s(k) = H^T R^{-1} z(k) = \sum_{i=1}^N H_i^T R_i^{-1} z_i(k) \quad 5.20$$

$$\Delta I = H^T R^{-1} H = \sum_{i=1}^N H_i^T R_i^{-1} H_i \quad 5.21$$

Local updates are computed in each node and sent to the neighboring nodes. Node,  $i$ , adds all information updates from its neighbors to its own updates and then computes the updated estimations and error covariance matrix.

Estimates after the communications of the nodes are called communication update estimates. The time and measurement update equations for node  $i$  are

- The local time update equations

$$P_i^-(k) = A P_i^+(k-1) A^T + Q \quad 5.22$$

$$\hat{x}_i^-(k) = A \hat{x}_i^+(k-1) + G u(k-1) \quad 5.23$$

- The information vector and matrix update

$$\Delta S_i(k) = H_i^T R_i^{-1} z_i(k) \quad 5.24$$

$$\Delta I_i = H_i^T R_i^{-1} H_i \quad 5.25$$

- The communication update

$$\hat{x}_i^+(k) = P_i^+(k) [(P_i^-(k))^{-1} \hat{x}_i^-(k) + \sum_{j \in N_i} \Delta S_j(k)] \quad 5.26$$

$$(P_i^+(k))^{-1} = (P_i^-(k))^{-1} + \sum_{j \in N_i} \Delta I_j \quad 5.27$$

This method decentralizes the computations of global estimates to every node without the need of a central processor.

If all nodes are fully connected, then (5.20) shows that the performance of this method is equal to that of the centralized Kalman filter.

#### 5.2.2.4 Distributed Kalman filter with consensus filter (DKFCF)

The distributed Kalman filter with consensus filter is proposed by Olfati-Saber [40]. The main feature of this method is the use of the consensus algorithm to obtain the communication update estimates. The consensus algorithm at node  $i$  is performed as follows: for each consensus step, node  $i$  receives estimates from its neighbors.

Node,  $i$ , subtracts its estimate from the estimate of each of its neighbors, weights the result with factor  $\gamma$  and adds the obtained value to its estimate. Another function of this method is availability of stability analysis. Olfati-Saber et al. in [40] presented stability analysis of the consensus algorithm using algebraic graph theory. The time and measurement update equations for node  $i$  are:

- The local time update equations:

$$\begin{aligned} P_i^-(k) &= A P_i^+(k-1) A^T + Q & 5.28 \\ \hat{x}_i^-(k) &= A \hat{x}_i^+(k-1) + G u(k-1) & 5.29 \end{aligned}$$

- The information vector and matrix update

$$\begin{aligned} \Delta s_i(k) &= H_i^T R_i^{-1} z_i(k) & 5.30 \\ \Delta I_i &= H_i^T R_i^{-1} H_i & 5.31 \end{aligned}$$

- The measurement update

$$\hat{x}_i^+(k) = P_i^+(k) [(P_i^-(k))^{-1} \hat{x}_i^-(k) + \sum_{j \in N_i} \Delta s_j(k)] \quad 5.32$$

$$(P_i^+(k))^{-1} = (P_i^-(k))^{-1} + \sum_{j \in N_i} \Delta I_j \quad 5.33$$

- Consensus step, iterated  $S$  times

$$\hat{x}_{i,l}^+(k) = \hat{x}_{i,l-1}^+(k) + \gamma \sum_{j \in N_i} (\hat{x}_{j,l-1}^+(k) - \hat{x}_{i,l-1}^+(k)) \quad 5.34$$

For  $l = 1, \dots, s$  where  $s$  is the number of iterations.

Up to the consensus step, this method is identical to the distributed information filter.

### 5.2.2.5 Distributed Kalman filter with weighted averaging (DKFWA)



The distributed Kalman filter with weighted averaging has been proposed in [42], [43]. A feature of this method is the reduction of computation and communication load. The reduction is because the nodes only compute and send the estimates, without the error estimation covariance matrix.

Different from the previous methods, this method consists of two parts: on-line and off-line. The on-line part computes and communicates estimates. In the off-line part, Kalman gains and weights are computed for each node. In this method, the Kalman gain and the weight  $W$  are computed once and used during the entire operation.

The idea of this method is as follows: node  $i$  receives estimates from its neighbors and weights them with a weight matrix  $w$ . Then the weighted estimates are added to the estimate of node  $i$ . The on-line steps of the distributed Kalman filter with weighted averaging at node  $i$  are as follows:

- The time update equation:

$$\hat{x}_i^{1-}(k) = A \hat{x}_i^{1+}(k-1) + G u(k-1) \quad 5.35$$

where  $\hat{x}_i^{1-}(k)$  denotes the local estimates at node  $i$ .

- The measurement update equation

$$\hat{x}_i^{1+}(k) = \hat{x}_i^{1-}(k) + K_i (z(k) - H_i \hat{x}_i^{1-}(k)) \quad 5.36$$

- The information exchange equation

$$\hat{x}_i^+(k) = \sum_{j \in N_i} w_{i,j} \hat{x}_j^{1+}(k) \quad 5.37$$

Where  $w_{i,j}$  is the weight of the estimate of node  $j$  that is used to compute the global estimates in node  $i$ . The value of  $w_{i,j}$  is zero if node  $i$  is not connected to node  $j$ .

The off-line computations are performed to minimize the trace of estimation error covariance matrix  $P^+(k)$  that is defined as:

$$P^+(k) = E[(x(k) - \hat{x}^+(k))(x(k) - \hat{x}^+(k))^T] \quad 5.38$$

$$x(k) = [x_1(k)^T \dots x_N(k)^T]^T \text{ and } \hat{x}^+(k) = \quad 5.39$$

$$[\hat{x}_1^+(k)^T \dots \hat{x}_N^+(k)^T]^T \text{ using (5.37) and} \quad 5.40$$

$$\sum_{j \in N_i} w_{i,j} = I \quad 5.41$$

To get unbiased estimates, we obtain the following relation,

$$P^+(k) = w \left( x(k) - \hat{x}_i^{1+}(k) \right) \left( x(k) - \hat{x}_i^{1+}(k) \right)^T w^T = w P^{1+}(k) w^T \quad 5.42$$

The covariance  $P^{1+}(k)$  in the last equation can be written as

$$P^{1+}(k) = \begin{pmatrix} I & \tilde{K} \end{pmatrix} \Phi(k) \begin{pmatrix} I & \tilde{K} \end{pmatrix} \quad 5.43$$

With

$$\Phi(k) = \begin{pmatrix} I \\ -\tilde{H} \end{pmatrix} P^-(k) \begin{pmatrix} I \\ -\tilde{H} \end{pmatrix}^T + \begin{pmatrix} 0 & 0 \\ 0 & R \end{pmatrix} \quad 5.44$$

And

$$\tilde{K} = \text{blockdiag}(K_1, \dots, K_N) \quad 5.45$$

$$\tilde{H} = \text{blockdiag}(H_1, \dots, H_N) \quad 5.46$$

It should be noted that for this method, it is possible to have a non-diagonal measurement covariance matrix R. In order to get an optimal filter, it is necessary to find the values of  $w$  and  $\tilde{K}$  that minimize (5.42). Instead of direct minimization of (5.42), the Kalman gain  $k$  and weight  $w$  are computed by solving the following optimization problem:

$$\min_{\tilde{K}, w} \text{tr} [w \begin{pmatrix} I & \tilde{K} \end{pmatrix} \Phi(k) \begin{pmatrix} I & \tilde{K} \end{pmatrix} w^T] \quad 5.47$$

$w_{i,j} = 0$  if node  $i$  and  $j$  are not connected and (5.40)

Table 3: Characteristic comparison of different filters

	CKF	PIF	DIF	DKFCF	DKFWA
Central processing	yes	yes	no	no	no
Connectivity	full	full	partial	partial	partial
Communication	single	single	single	multi	single
Global estimates	yes	yes	no	no	no

The obtained gain  $\tilde{K}$  and weight  $w$  are employed in the on-line computation of the states. Details on how to solve the optimization problem (3.43) are given in [41]. Basically, this method is a consensus filter but with only one information exchange. The characteristics comparison of the Kalman filter methods presented in this section is shown in Table 1.

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