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Comparison of the interpolation methods on digital terrain models

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Abstract

Digital terrain models (DTM) have been used in many applications since they came into application in the late 1950s. It is a fundamental tool for applications such as hydrology, cartography, geology, geomorphology, landscape architecture and so on. Accuracy of DTM is quite important when putting it into applications. Based on the research on DTM, there are several factors related to the quality of DTM and the interpolation method is one of them. In this study, we focused on the interpolation methods for different terrain surface on the accuracy of interpolated heights.

Starting from a review of digital terrain modeling, in which interpolation is significant section. Data structure and visualization were also introduced since they related to our following study as well. In this study, comparisons were performed between Inverse Distance Weighted (IDW) and Radial Basis Functions (RBFs) to test their performances on different terrain surface such as mountainous, flat and real-world. A comprehensive comparison was also implemented not only on the aspect of Root Mean Square Error (RMSE), but also on the interpolation error and time consumption of each selected interpolation methods. Inverse multiquadric (IMQF) performed the best in all basis functions on the mountainous terrain surface also with the best time consumption. However, IMQF obtained a worst result in flat terrain compared to other basis functions. IDW was proved to be the poorest interpolators for all the terrain we tested.

The results obtained in this study allow us to observe the characteristics of each selected interpolation methods when they are applied to different terrain surface, which can show how the accuracy of digital terrain model is related to interpolation method and topography.

Keywords: digital terrain models, automation terrain generation, interpolation methods.

Abstract

I modelli digitali di terreno (DTM), inventati nel 1950, vengono utilizzati in molte applicazioni. Essi sono, infatti, uno strumento fondamentale per l'idrologia, la cartografia, la geologia e geomorfologia, etc. L'accuratezza delle DTM è una caratteristica spesso importante nelle applicazioni, per questa ragione sono stati sviluppati vari metodi che permettono di interpolare tali modelli di terreno al fine di aumentarne la qualità. In questo lavoro, vengono presentati e confrontati alcuni di questi metodi.

Il lavoro parte dall'analisi della letteratura relativa ai modelli digitali di terreno, alla loro interpolazione, visualizzazione e rappresentazione in termini di strutture dati. Vengono poi introdotti e confrontati, su tipologie differenti di terreni (pianura, montagna, etc.), due metodi di interpolazione: Inverse Distance Weighted (IDW) e Radial Basis Functions (RBFs). Detti metodi sono stati confrontati sia a livello dell'errore quadratico medio valutato sui dati ricostruiti che del tempo di calcolo necessario per eseguirli.

Dai risultati conseguiti si osserva che il metodo RBF basato su funzioni "inverse multiquadric" è il migliore, sia a livello di errore di interpolazione che di tempo di computazione, su terreni montani. Su terreni pianeggianti, invece, sono stati ottenuti risultati migliori con funzioni a base radiale differenti. Il metodo IDW, invece, è risultato il peggiore in tutte le tipologie di terreno provate.

In conclusione, quindi, i risultati mostrati in questo lavoro permettono di illustrare l'accuratezza di ciascun metodo di interpolazione in funzione della topografia del terreno considerato.

Keywords: modelli digitali di terreno, generatore automatic di terreno, metodi di interpolazione.

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To my family and friends

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Chapter 1

Introduction

Topography is one of the most important landscape controls acting on the terrain surface. It is a field of planetary science comprising the study of surface features of the earth, which allows us to see and study the shapes and landforms on a two-dimensional map. Topography maps can examine the geological environment, and how organisms, climate, soil, water, and landforms produce and interact. One of the most widely used data structures employed to store and analyze information on the topography in a Geographic Information System (GIS) environment is Digital Terrain Model (DTM). A DTM is a numerical data file which contains the elevation of the topography over a specified area. Different data collection techniques can be applied to generate DTMs such as satellite-based techniques, photogrammetric methods and laser scanning. Meanwhile, DTMs can be generated by contouring digitized from topographic maps. However, the generated DTMs is an approximation of the real world [4]. There will be errors in the process of generation. And since DTMs has a lot of applications, the quality of DTMs is particular significant.

The factors that have influence on the quality of the DTM can be separated into three classes [6]: The first factor is morphology such as flat, hilly or mountainous which can be seen as a matter of uncertainty [5] [7]. The second one is the data collection techniques, the accuracy of source data varies with techniques such as satellite-based techniques, photogrammetric methods and laser scanning. The remaining factor is interpolation methods which allow us to convert scatter data points to a continuous surface.

A series of researches was conducted on the relation between DTM accuracy and interpolation methods. Research of El Hassan [8] on the quality comparison of some spline interpolation methods and the test areas is in Cairo (Egypt) and Riyadh (Saudi Arabia). It shows that the pseudo-quintic spline algorithm gives the best accuracy of

DTM.

Chaplot [7] et al used some interpolation methods like kriging, inverse distance weighted, multiqudratic radial basis function and spline for generating DEM in various regions of France. The author has presented that for a high density of sample points, all of the interpolation methods perform similarly; and for a low density of sample points, Kriging and inverse distance weighted interpolation methods perform better than the others. However, the research carried out by Peralvo in the two watersheds of Eastern Andean Cordillera of Ecuador presents that the inverse distance weighted interpolation method produced the worst quality of DTM.

The goal of the study our study was to compare the quality of different interpolation methods on the derived DTMs with different terrain surface including flat, mountainous and a real-world terrain surface. The interpolation methods compared were Inverse Distance Weighted (IDW), Radial basis functions (RBF) with five basis functions, such as multiquadric function (MQF), Multilog (MLF), inverse multiquadric (IMQF), natural cubic splines (NCSF), and thin plate splines (TPSF). The accuracy of each interpolation method was quantified using the Root Mean Square Deviation (RMSE).

With respect to thr goald of our study, the whole work has been organized with the following structure:

Chapter 2 presents a broad technical background of DTM, it also briefly discusses the steps taken in digital terrain modeling. We also gave a detailed description on especially on the data structure and visualization techniques since they play a role in the following chapters.

Chapter 3 provides a description of the automation terrain generation, since two DTMs we used in the study were derived from automation terrain generation. It also describes in detail the process of generating a DTM based on different methods and parameter determination in each methods.

Chapter 4 and Chapter 5 introduce the principles of each selected interpolation methods , and the process of implementation in Matlab.

Chapter 6 describes the evaluation of the interpolation methods which includes the description of the study area, assessment method of the comparison, the results and the final conclusions.

Chapter 2

Digital terrain models and maps

2.1 Introduction

A digital terrain model (DTM) is a topographic model of the bare ground that can be manipulated by the computer programs, usually for earth, moon, or asteroid. DTMs have been used in the applications of geosciences since 1950s [10] and since then they have become a major constituent part in geographical information processing.

There are two general ways to represent a surface, one is from the Mathematic function: Z(X,Y)=f(X,Y, Coefficients).the other is the Image :Z(X,Y) for a given set of points. A DTM may be understood as a digital representation of a portion of the terrain surface, since overhanging cliffs are relatively rare on earth, topographic surfaces are often represented as ‘fields’, which means for the surface models, they have the unique Z-values over X and Y coordinates. Application-specific systems are provided to model cliffs when they are very crucially important.

Another digital model that should be mentioned is the digital elevation model， which stores continuously varying variables such elevation, groundwater depth or the buildings on earth. In contrast to digital elevation model (DEM), DTM is often used as a generic term for DTMs and only representing elevation information without any further definitions of the surface.

DTM provides a basis for a great number of applications in the earth and the engineering sciences. Compared to traditional representation of the terrain, the DTM has the following specific advantages:

- Greater feasibility of real-time processing. Data modification and updating are very flexible in digital form.
- DTM keeps its precision as time goes by.
- Terrain can be represented in a variety of forms such as topographic maps, cross

or vertical section when using DTM.

The process of the construction of a DTM surface is called digital terrain modeling, different techniques for the generation of DTMs have been developed since their inception more than fifty years ago [10] [24], the digital terrain modeling is introduced as following.

2.2 Digital Terrain Modeling

2.2.1 Structure of Digital Terrain Modeling

In this process of digital terrain modeling, the sampling points are obtained from the terrain, and the model is created with a certain density and distribution .In order to estimate elevations in the regions where data exist, an interpolation is applied to refine the surface. In general, digital terrain modeling encompasses the following tasks [11].

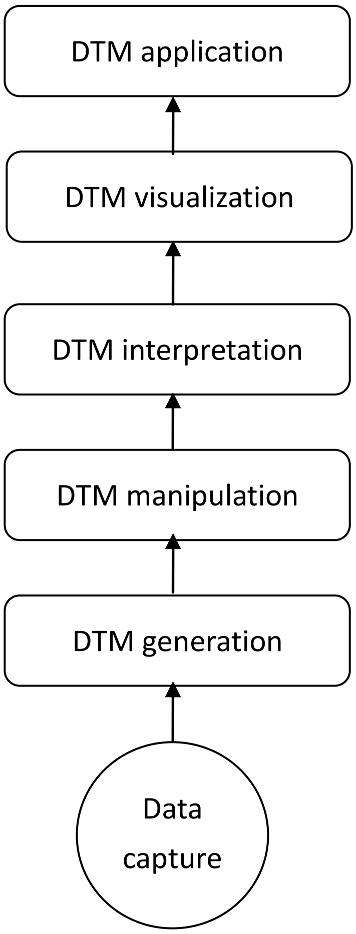


Figure2.1. The framework of a digital terrain modeling

As shown in Figure2.1, five tasks are defined in digital terrain modeling:

- DTM generation: Sampling of the original terrain data and model construction.
- DTM manipulation: Modification and refinement of the DTMs, and the derivation of intermediate models.
- DTM interpolation: Analysis of DTMs, information extraction from DTMs.
- DTM visualization: Display of elements and the derived information of DTMs.
- DTM application: Development of appropriate models for specific fields.

2.2.2 Data structure for digital terrain models

The original data must be structured in a form in order to handle the subsequent operations of terrain modeling. Today, there are two main structures for representing DTMS, one is rectangular grid, the other one is called triangular irregular network (TIN) [12], as shown in Figure2.2.

Rectangular grid is one of the simplest ways of representing the DTM, the terrain surface is represented as a set of elevations for points regularly distributed on the x and y coordinates. The simplicity of algorithms is the advantage of the grid structure. Besides, it will lead to memory saving. However, regular grids cannot be adapted to the complexity of the terrain surface. Thus, an excessive number of data points are needed to represent the terrain by interpolating to acquire a certain level of accuracy of the terrain.

TIN is a vector-based representation of the physical terrain surface, made up of irregularly distributed points and lines with three-dimensional coordinates that are arranged in a network of non overlapping triangles. Compared with rectangular grid, TIN needs larger storage and since the topological relations have to be computed or recorded, TIN is more complex and more difficult to handle than rectangular grid. But TIN can be more accurate to display the detail of the terrain [12] [15].

Considering the advantages and disadvantages of rectangular grids, TINs, It is clearly to note that no data structure is superior for all tasks of digital terrain modeling.

In our experiment, we prefer to use the rectangular grid since it is easier to handle and the evenly distributed points can be used to perform the evaluation better.

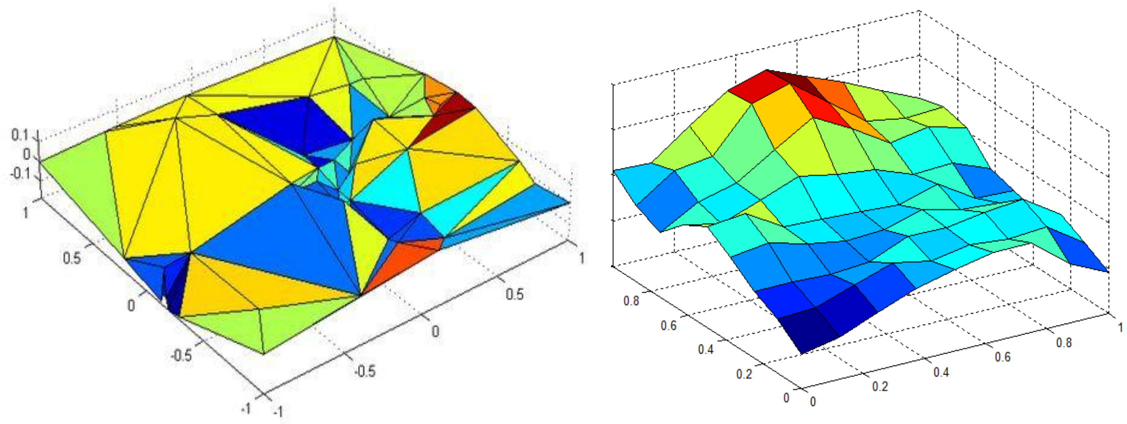


Figure2.2 The structures of triangular irregular network (on the right) and rectangular grid (on the left)

2.2.3 Visualization of the digital terrain models

In order to display DTMs in graphical form, visualization plays a vital role in the digital terrain modeling. It has a close relation with interpolation since results of interpolation steps need to be displayed, the interpolation operation may also make improvements in visualization.

Visualization mainly pursues two goals: one is interactive visualization, which helps the researcher explore models and refine hypotheses; the other is static visualization, which is used to compare the results and the concepts. Some traditional forms of visualization are:

- **Contours**

Contour lines are used to represent the elevation of the terrain surfaces, as shown in Figure2.3. At present, it is one of the most widely used techniques for displaying relief. However, the major drawback of contours is that they could not directly show a visual impression of topographic forms.

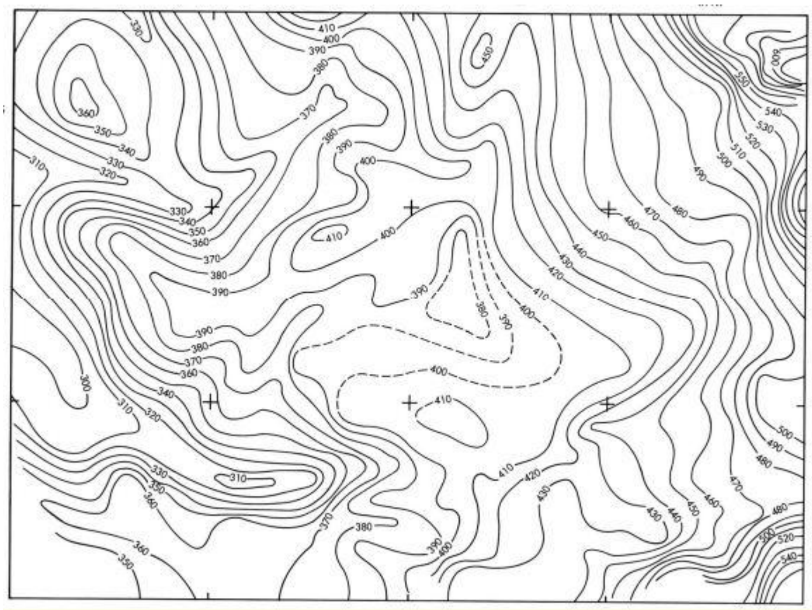


Figure2.3 Visualization of contour lines

- **Hill shading**

It displays the hypothetical illumination of a terrain surface according to a specified azimuth and altitude for the sun and gives maps a richly textured appearance (Figure2.4) .It provides a convenient way to qualitative cartographic relief depiction. The application of this method for cartographic purposes was first automated by Yoeli [13][14], and since that it has become a standard terrain display technique. Nevertheness, such maps may also have similar appearances, and generally it is hard to highlight the areas in which we are interested.

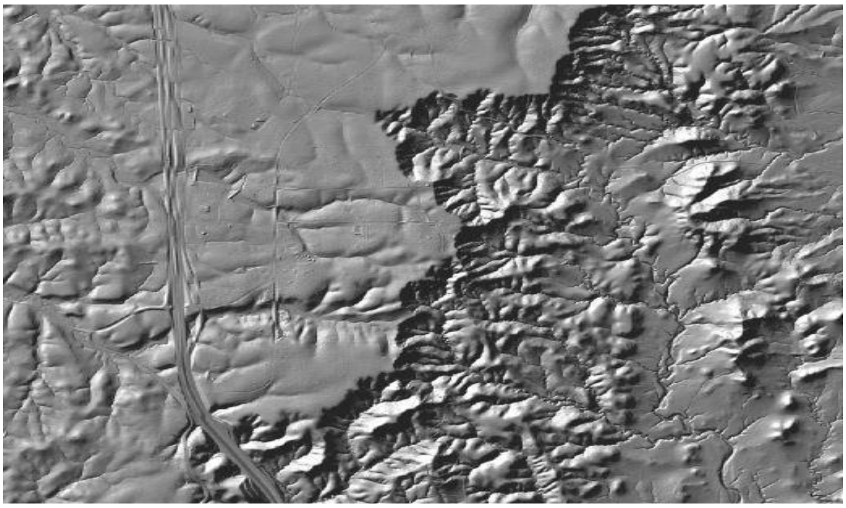


Figure2.4 Visualization of hill shading

- **Perspective views**

Since the advantage of hill shading (Figure2.5) and contours is that all parts of the terrain surface are visible. On the other hand, perspective views can provide more immediate visualization results. The main issues that to be carried out in the perspective display are the projection of the 3-D surface on to a 2-D medium, and the elimination of hidden sections from the display.

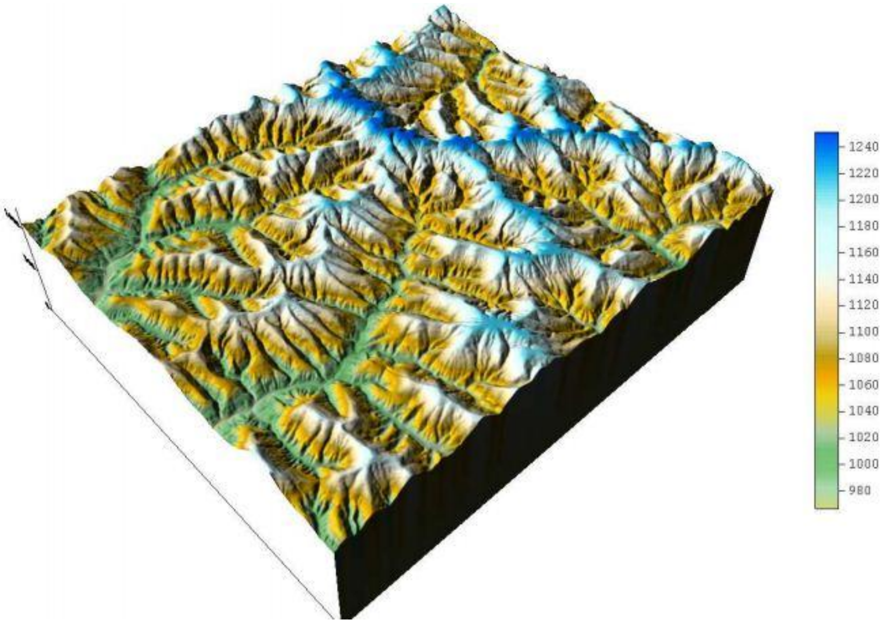


Figure2.5 Visualization of perspective views

2.3 The accuracy of the digital terrain models

In fact, the quality of DTMs is characterized by several issues, such as variables as, terrain ruggedness, sampling density and the interpolation method. In order to obtain a good DTM, we need to treat two phases with great care. The first phase is the technology employed to obtain sampling points. At present, several data sources are popular for digital terrain modeling:

- Contours digitized from topographic maps are a relatively cheap data source;
- Analytical photogrammetry is another technology that allows the measurement of selected points from a series of aerial images. However, it could not be applied to

wooded areas, for the reason that the visibility of the terrain surface is poor.

- Digital photogrammetery perform a regular data grid that consists of data points from automated selection.
- Laser scanning is relatively a new technique which can collect very dense terrain datasets, and it can also be used in wooded areas.

In this chapter, we paid much attention to the data structure and visualization in digital terrain modeling since the two sections related to our following experiment closely. Certainly, the interpolation process in digital terrain modeling is also the key point, because it could considerably affect the quality of the DTM generated, and we will explain this process in detail in the chapter 4 and chapter 5.

Chapter 3

Automatic Terrain Generation

3.1 Introduction

In our study, our researches on the interpolation methods are tested on the terrain generated by Automatic Terrain Generation [1]. The tool is a process which creates elevation values throughout a two dimensional grid and capable of producing an approximately realistic-looking terrain. To have a better understanding and application of the Automatic Terrain Generation, we prefer to introduce the algorithms in the following sections.

3.2 Outline of the Algorithms

Three basic algorithms are shown in the Automatic Terrain Generation tool package which includes algorithms, such as Generate_brownian_mesh, Generate_brownian_tri, Generate_terrain. Three of them are based on different principles, however ,the function of F=TriScatteredInterp (X, Y, V) is used in all of the algorithms, the function creates an interpolator F that fits a surface of the form V=F (X,Y) to the scattered data in (X, Y, V).In the process, three different principles are implemented to produce three dataset of scattered points, then the interpolator is applied to each of them in order to generate the data points distribute regularly on the mesh grids. All of the generated mesh grids can be predefined at the beginning except for the algorithm of Generate_terrain**.** Besides, a number of parameters can be set as the inputs to help the user control the terrain generated. For example, the parameter n is used in all algorithms, it defines how many iterations that the process will run. As a result, a big n will make the terrain surface become more complex, and correspondingly, a small n sometimes could not reflect a realistic-looking terrain that we expect. The details of the three algorithms are shown as following:

1. Generate_brownian_mesh.

The method（Figure 3.1）is also known as the midpoint displacement method. It can generate a mesh of points exhibiting ‘’fractal Brownian motion”, a random mesh that looks like terrain. The output “ height ”mesh maps over[0,0] to [1,1] with 2^n+1 points along each axis for(2^n+1)^2 total data points.

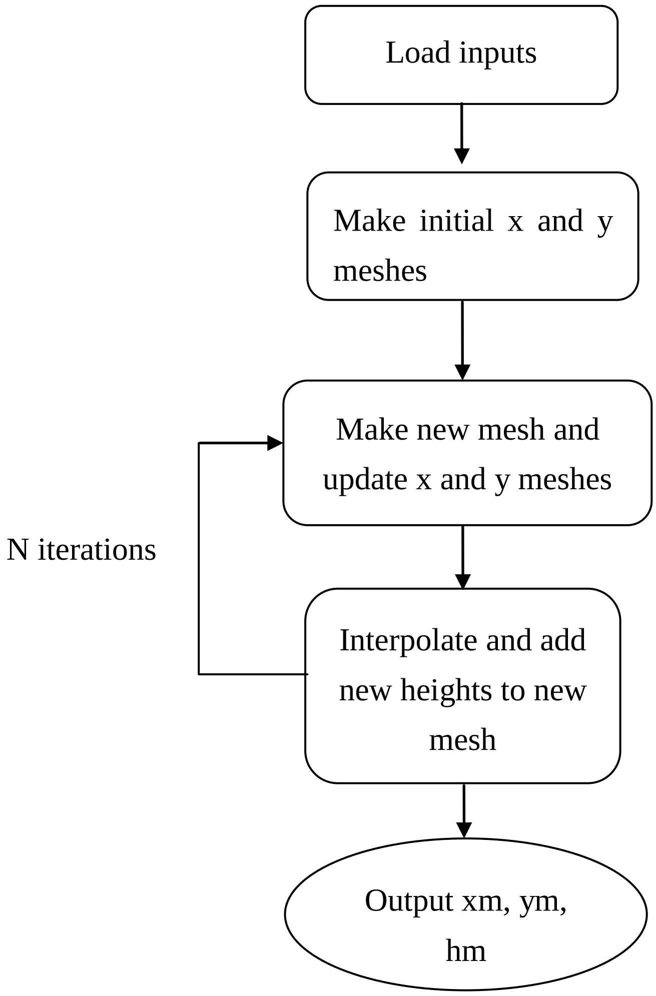


Figure3.1 The flow chart of the algorithm of generate_brownian_mesh

Function:

- [zm,xm,ym]=generate_brownian_mesh(n,zm)

Inputs:

- n -Number of iterations
- zm -Initial heights

Outputs:

- hm -“z” data corresponding to mesh

- xm -“x” data corresponding to mesh
- ym -“y” data corresponding to mesh

2. Generate_brownian_tri

It generates a random mesh that more-or-less looks like terrain using a Brownian fractal method.(Figure3.2)

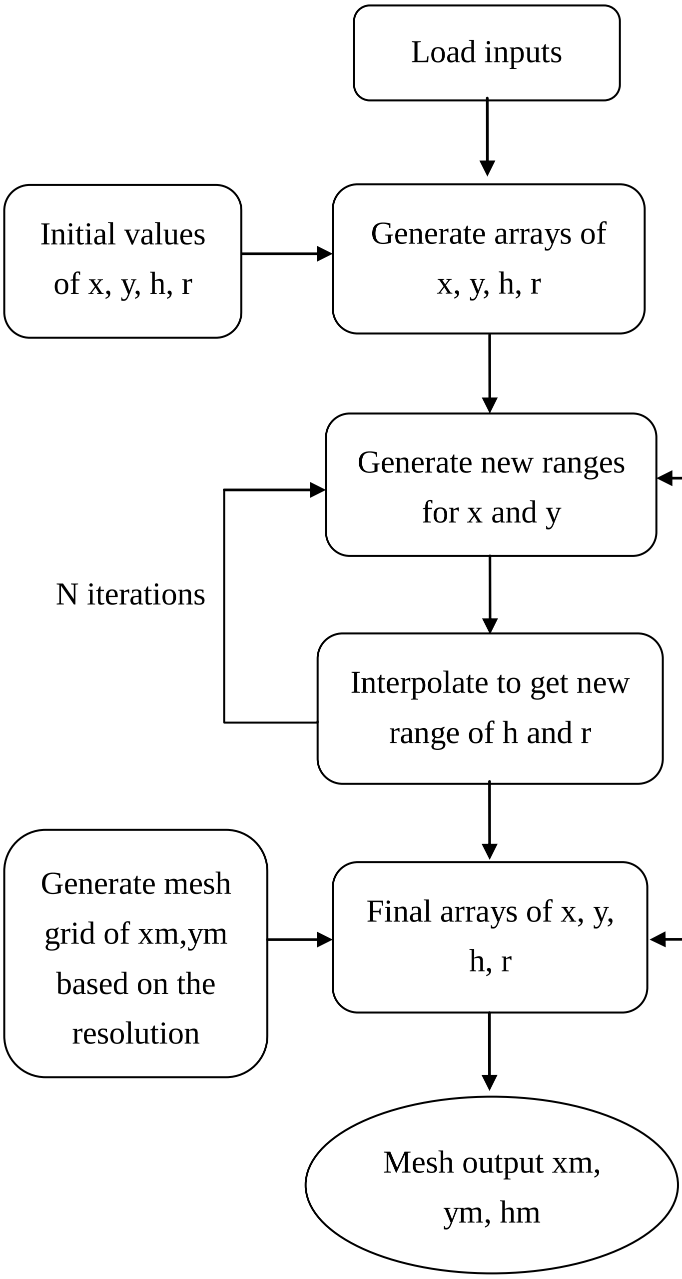


Figure3.2 The flow chart of the algorithm of generate_brownian_tri

Function:

- [hm,xm,ym,rm]=generate_brownian_tri (ni, nm, r0, el, rr)

Inputs:

- n -Number of iterations
- nm -Resolution of final map
- r0 -Initial roughness of terrain
- el -Initial elevation of terrain
- rr -Roughness of roughness over terrain(how much roughness changes)

Outputs:

- hm -Height mesh
- xm -“x” data corresponding to mesh
- ym -“y” data corresponding to mesh

3. Generate_terrain

This function generates a series of points that approximate terrain according to a simple algorithm and very few parameters (Figure3.3).

Function:

[x, y, h, hm, xm, ym]=generate_terrain(n, mesh_size, h0, r0, rr)

Inputs:

- n --Number of iterations of algorithm to perform.
- mesh_size-Size of the output mesh
- h0 -Initial elevation
- r0 -Initial roughness (how much terrain can vary in a step)
- rr -Roughness roughness(how much roughness can very in a step)

Outputs:

- x, y, and h -Vectors of the points comprising terrain
- hm -Height mesh
- xm -“x” data corresponding to mesh

- ym -“y” data corresponding to mesh

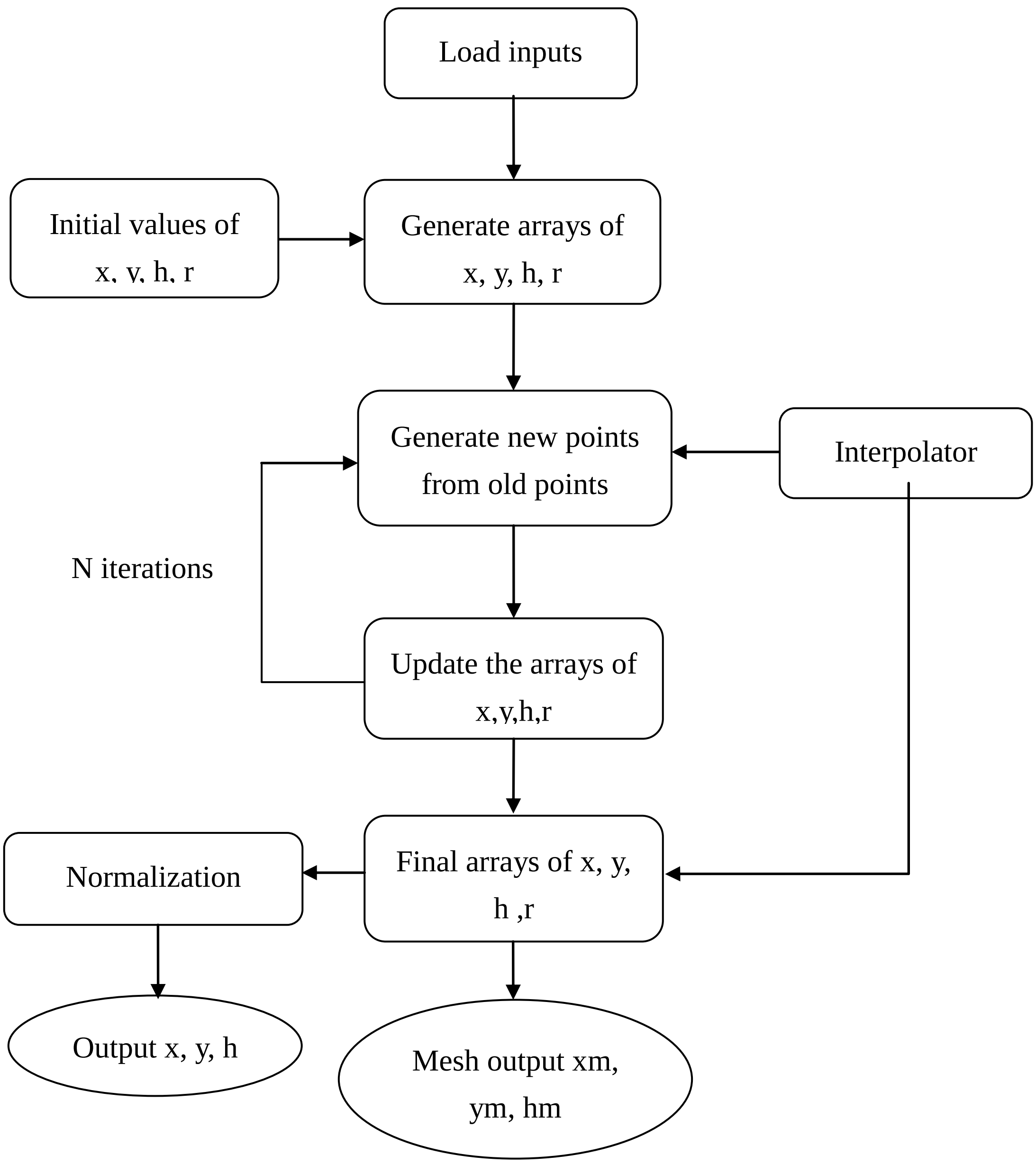


Figure3.3 The flow chart of the algorithm of generate_terrain

3.3 Comparison and Our option

The characteristics of the three algorithms are compared:

- Generate_brownian_mesh:Fastest, varies more along grid lines

- Generate_brownian_tri :Slower, but grid less
- Generate_terrain:Slowest, but best for terrain in particular

According to the comparison above, and in our experiment we want to generate the terrain surface with a good user control in the size of the mesh grid. Therefore, we selected the algorithm of Generate_brownian_tri and the meansurement unit for length was based on meter for all the terrain that used in the experiment. A example of this algorithm shows in Figure 3.4.

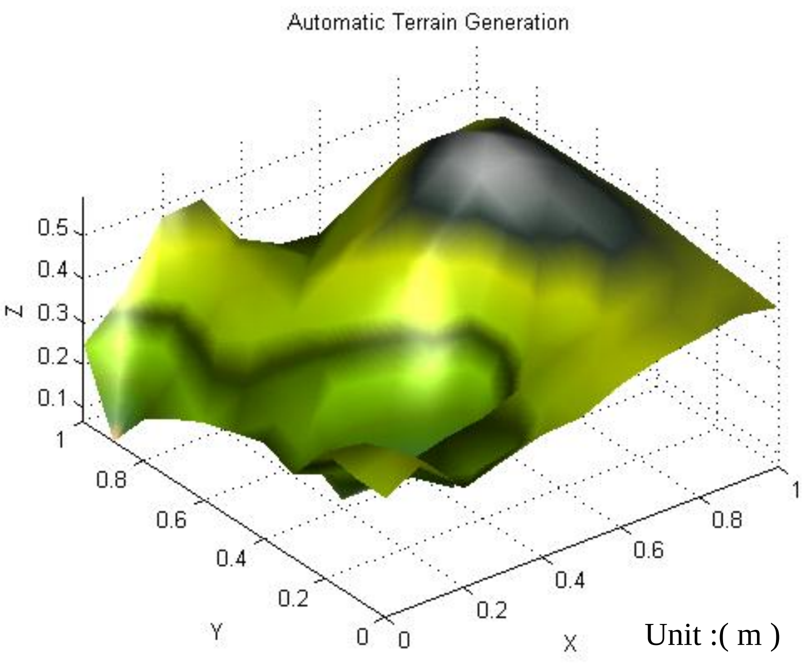


Figure3.4 A terrain generated by Generate_brownian_tr

Chapter 4

Inverse Distance Weighted (IDW)

4.1 Introduction

Interpolation is the process of predicting the value of elevations at un-sampled sites from measurements made at point locations on the same terrain. It is used to convert data from the point observations to the continuous surface so that the sampled spatial patterns by the measurements can be compared with the spatial patterns of other spatial entities. The principle behind spatial interpolation is that, on average, values at points close together have more correlations and similarities than the points further apart. In the following chapters, we will introduce two of the most popular interpolation methods. One is Inverse Distance Weighted (IDW), the other is Radial Basis Functions (RBF). The IDW interpolation is based on the concept of Tobler's first law from 1970.The IDW was developed by U.S. National Weather Service In 1972 and is classified as deterministic method. This is because of the lack of requirement in the calculation to get specific statistical assumptions, thus IDW differs from stochastic method like Kriging.

4.2 Method

4.2.1 Basis principle of IDW

Inverse Distance Weighted is an exact local deterministic interpolation technique, which is one of the most widely used methods for scattered data [16]. IDW assumes that the value at the interpolation location is a distance-weighted average of the values at sample points within a defined neighborhood surrounding the interpolation point [17], that means the closest observations must carry more weight in determining the interpolated value in one point. It estimates the value of variable Z in a non-sampling point x_0 from the following expression (4.1), which is called Shepard method [23] :

$$Z(x_0)=\frac{\sum_{i=1}^{n} w(d_i)\bullet z(x_i)}{\sum_{i=1}^{n} w(d_i)} \tag{4.1}$$

Where:

- $w(d_i)$: is the weight function on the n sampling points which is used in the calculation. The general form of w equals to d^{-u}.
- $z(x_i)$: is the elevation of every one of these n points.
- d_i : shows the distance between each point and x_0 .
- n: is the number of measured sample points within the neighborhood defined for x_0.

4.2.2 Power parameter

u in the weight function is called power parameter, it shows the significance of the surrounding points upon the interpolated value in the IDW. When the distance between the measured sample point and the interpolated point increases, the weight that the measured sample point has on the interpolated point will decrease, which means, a higher power will result in less influence from distant points. Generally, the typical value of power parameter is 2, and in our study, we find the interpolation method were not sensitive to the parameter when we moved in a 1 to 3 interval, finally we set the value to 2 for all of the experiments in our study.

4.2.3 The search neighborhood

In fact, the larger the relevant neighborhood is selected, the large the number of scattered frequency samples may be incorporated into the interpolation, and the more accurate interpolation results could be expected [27]. However, on the one hand, a large number of sample neighbors means also a higher computational expense, which will correspondingly slow down the reconstruction speed .One the other hand, the locations of sample points get farther away from the interpolation point, the measured values will have little influence with the value of the interpolation point, to improve the computational time, a trade-off between the reconstruction time and the interpolation accuracy has to be made. Therefore, it is practical to limit the number of measured values that are used to do the interpolation by specifying a search neighborhood. The following Figure4.1 shows a typical description of the search neighborhood, where R indicates the search radius and the sample points around the interpolation point within R will be selected [17].

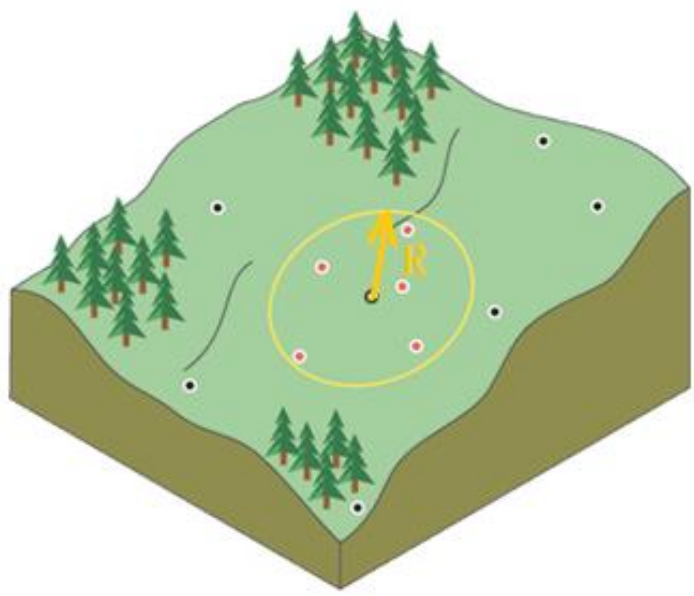


Figure4.1 The display of the search neighborhood

4.3 Implementation in Matlab

The IDW interpolation method was implemented in Matlab on the basis of a set of sample points data from a given terrain to produce more interpolation points .The Implementation flow is shown in Figure 4.2

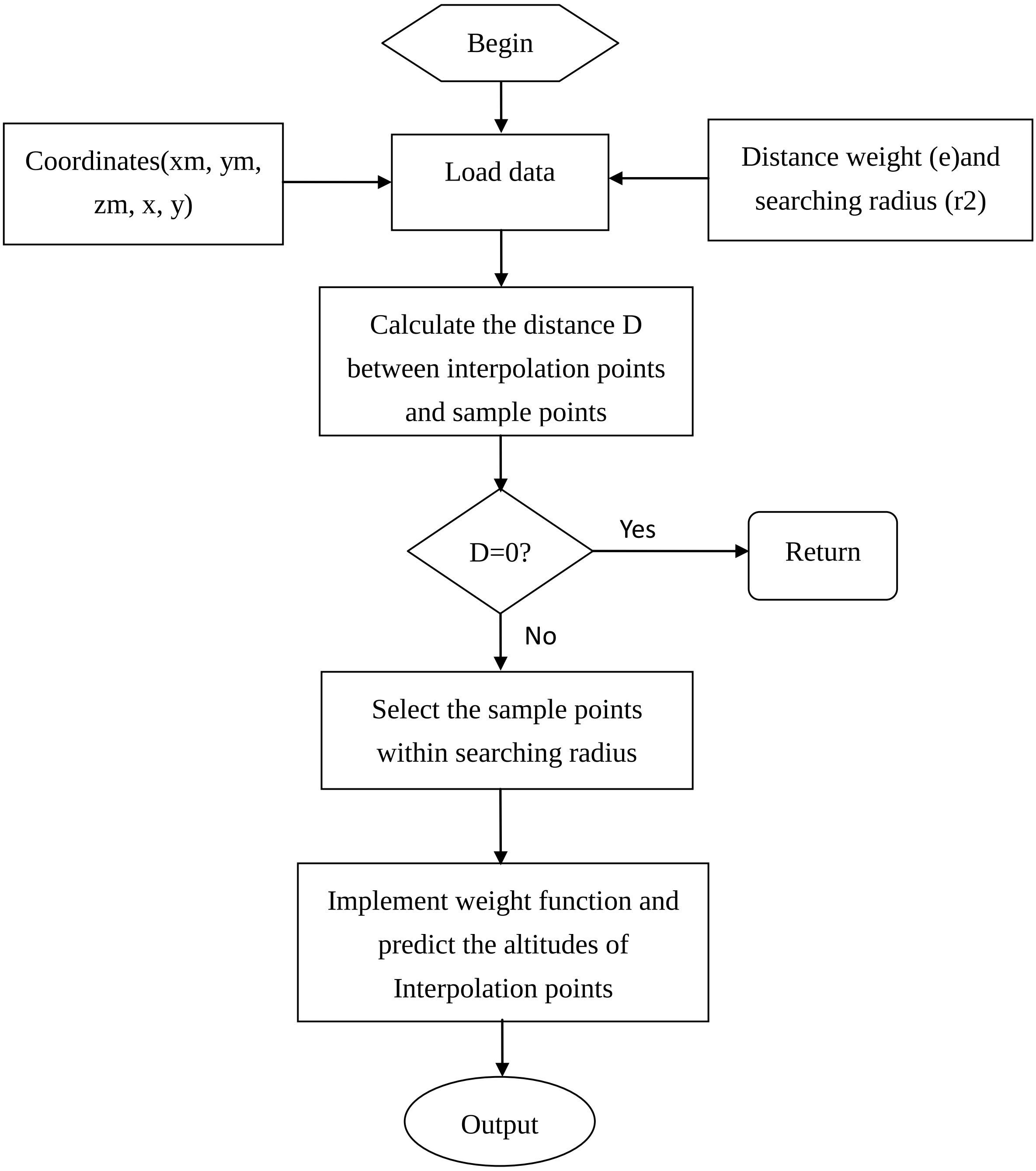


Figure4.2 Implementation flow of IDW

Chapter 5

Radial Basis Functions (RBF)

5.1Introduction to Radial Basis Functions

The history of Radial Basis Functions (RBF) approximations goes back to 1968,when multiquadric (MQ) RBFs were first used by Hardy to show topographical surfaces with sets of sparse scattered measurements [18]. Before the MQ, trigonometric and algebraic polynomials were used. The MQ is important because it allows for scattered data to be converted into a very accurate fit model of a graph or surface [20].

The next significant time in RBF history was in 1986 when an IBM mathematician Charles Micchelli proved that the system matrix for the MQ method was invertible [21]. After four years, physicist Edward Kansa first used MQ method to solve different equations [22] and since Kansa's discovery, research in RBF methods has grown rapidly. The RBF interpolation has become a powerful tool in nonlinear multivariate approximation theory through scattered data due to its great approximation properties [19].

5.2Method

5.2.1 Basis principle of RBF

Radial Basis Functions are a series of exact deterministic interpolators that include different basis functions dependent on the distance between the interpolated point and the sampling points. A Radial Basis Function is a function of form:

$$s\left(x\right)=p(x)+\sum_{i=1}^{n}\lambda_i\phi\left(\left\|x-x_i\right\|\right) \tag{5.1}$$

Where:

- s is the radius basis function.
- p is a low degree polynomial, typically linear.
- ϕ is a real valued function also called the basis function.
- λ_i 's are the RBF coefficients.
- x_c 's are the RBF central points.

The RBF consists of a weighted sum of radial basis function ϕ located at the centre is x_i and a low degree polynomial p .Given a set of n points x_i and values f_i, the process of finding an RBF, such that,

$$s(x_i) = f_i, \quad i = 1,2,\ldots.n \tag{5.2}$$

Is called fitting. The fitted RBF is defined by the coefficients of the basis function in a summation λ_i and the coefficients of the polynomial $p(x)$.

With respect to (5.1), we choose a linear function to represent the polynomial term $p(x) = k_0 + k_1(x)$ where, for the two-dimensional case, $k_0 \in R$ and $k_1 \in R^{2\times2}$, k_0 and k_1 represent the constant and linear terms of the polynomial $p(x)$, so we can rewrite the RBF expression as it follows:

$$s(x) = k_0 + k_1(x) + \sum_{i=1}^{n} \lambda_i \phi\left(\left\|x - x_i\right\|\right) \tag{5.3}$$

To solve the unknown coefficients k_0, k_1 and λ_i, we implemented the process of “real” interpolation on a terrain . In this way, defining the locations of the sample points $x = [x_{1,} \ldots\ldots x_n] \in R^{n\times2}$ and the elevations of the sampling points $h = [h_{1,} \ldots\ldots h_n] \in R^{n\times2}$, put them into the equation (5.3) and we can get:

$$s(x_i) = h_i, \forall i = 1, \ldots\ldots n \tag{5.4}$$

In order to calculate the unknown coefficients in (5.4), the following additional constraints are imposed, which can represent the conservation of total force, such that:

$$\sum_{i=1}^{n}\lambda_i = 0 \ , \sum_{i=1}^{n}\lambda_i x_{1,i} = \sum_{i=1}^{n}\lambda_i x_{2,i} = 0 \qquad (5.5)$$

The unknown coefficients are than determined from the additional constraints (5.5), which leads to the following symmetric linear expression:

$$\begin{bmatrix} A & 1_C & x \\ 1_C^T & 0 & 0 \\ x^T & 0 & 0 \end{bmatrix}\begin{bmatrix} \lambda \\ k_0^T \\ k_1^T \end{bmatrix} = \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix} \qquad (5.6)$$

Where $A \in R^{n\times n}$ represents the interpolation matrix $A_{i,j} = \phi\left(\left\|x_i - x_j\right\|\right)$, and $1_C \in R^{n\times 1}$ represents a unit vector. The unknown coefficients can be obtained by solving the equation (5.6).We can obtain a final interpolator after putting the coefficients back into the equation (5.3), which could be applied to the interpolation in the terrain.

5.2.2 Basis Functions

The ϕ basis functions is the form $\phi(\bullet)=\phi\left(\|\bullet\|_2\right)$, where ϕ: $\mathbb{R}^+ \to \mathbb{R}$ .Some of the most commonly used basis functions are:

1.Multilog Functions:

$$\phi(r) = \log\left(c^2 + r^2\right)$$

2.Inverse Multiquadric Functions:

$$\phi(r) = \frac{1}{\sqrt{\left(c^2 + r^2\right)}}$$

3.Natural Cubic Splines Functions:

$$\phi(r) = \left(c^2 + r^2\right)^{\frac{3}{2}}$$

4. Multiquadric Functions:

$$\phi(r) = \sqrt{\left(c^2 + r^2\right)}$$

5.Thin Plate Splines:

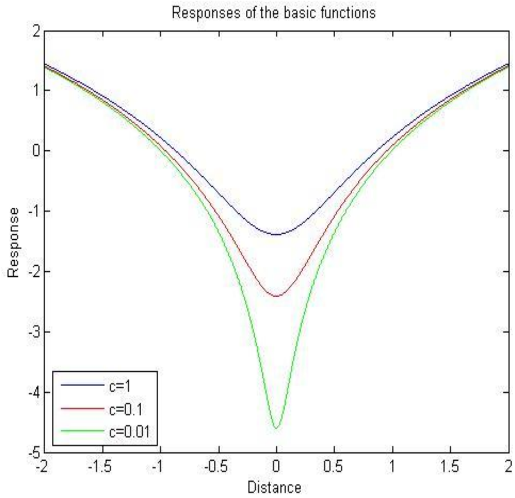
$$\phi(r) = \left(c^2 + r^2\right) \bullet \log\left(c^2 + r^2\right)$$

The choice of basis functions will determine which methods are available for solving equation (5.6), and whether such a solution even exists. If the interpolation matrix A is symmetric positive definite [19] then RBF interpolation problem always admits a unique solution. One property of a positive definite matrix is that all its eigenvalues are positive. Therefore, equation (5.6) can be solved since a positive definite matrix A is invertible. This could be guaranteed by making that every basis function are centered on each sample points, this results in $\|x_i - x_j\| = \|x_j - x_i\|$, which means $A_{ij} = A_{ji}$.

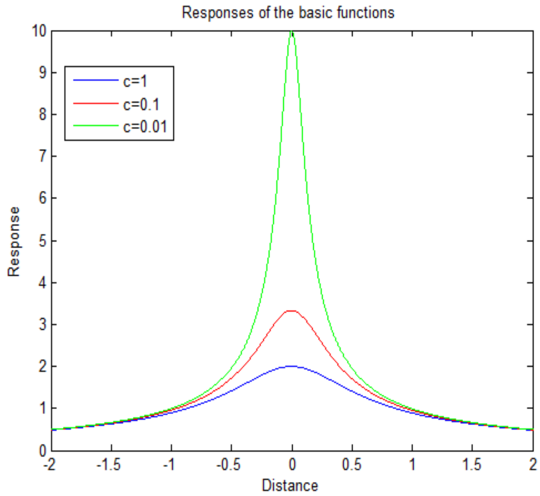
5.2.3 Selecting the smoothing factor c

Adding a smoothing factor c in the basis function was proposed by Hardy, since he found that it was an good method for approximating a topographic surface from sparse, scattered measurement. Besides, it was able to account for rapid variations of the topographic surface. Obviously, the smoothing factor c can have a marked influence on the interpolation results obtained.

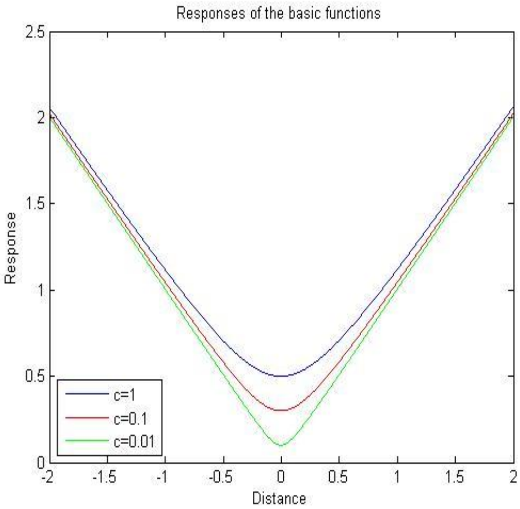
Figure5.1 five pictures show the response of each selected basis function basis on different smoothing factors. As we can see, a small smoothing factor corresponds to a ‘’sharp’’ basis function, and a bigger c corresponds to a ‘’flatter’’ basis functions. However, for Natural Cubic Splines function (NCSF), a small c leads to a corresponding to a ‘’flatter’’ response.



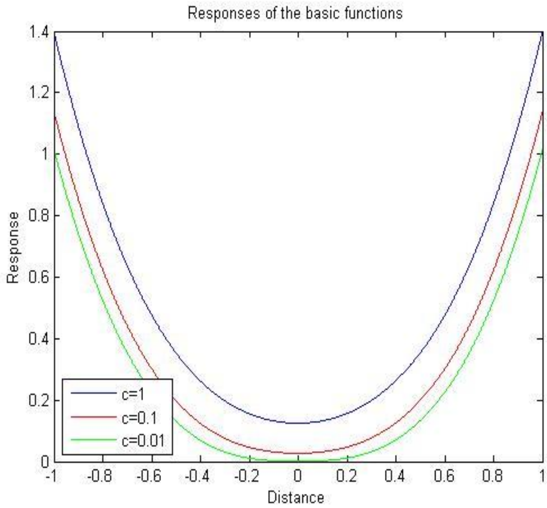
(a)



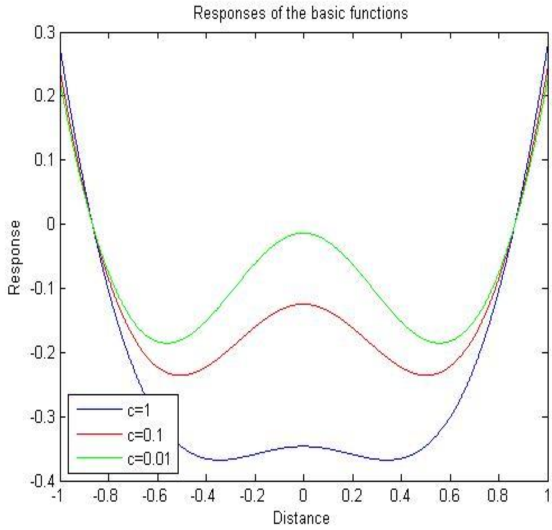
(b)



(c)



(d)



(e)

Figure5.1 Response of each selected basis function based on different smoothing factors. (a) MLF, (b) IMQF, (c) MQF, (d) NCSF, (e) TPSF

The characteristic feature of basis functions is that their response decreases or increases monotonically with the distance from a central point. For the selected basis functions, as shown in Figure5.1, all the response of each selected basis function are plotted, NCSF and Thin Plate Splines function (TPSF) have a global response, Multiquadric function(MQF) shows a smooth response for variation of the distance , the Multilog function (MLF) and Inverse Multiquadric function(IMQF) give a significant response only in the neighborhood near the central point, the difference is IMQF shows a positive response and MLF shows the opposite, a negative response.

Different smoothing factors correspond to different approximation resulting from RBF interpolation. Finding the optimal smoothing factor that will produce the most accurate approximation is a topic of current research, and selecting the constant value depends fundamentally on the number, elevation, and spatial distribution of the sampling points. There are various empirical type approximations, as those proposed by Hardy [18], together with others like recursive algorithms which try to work out the value of c which minimizes the global error of the interpolated surface [25]. In our study, we prefer to find a optimized smoothing factor by using Optimization Toolbox [2], and this will be presented in chapter6.

5.2.4 Radial Basis Function Networks

Basis functions are simply a class of functions which can be implemented in any sort of model (linear or nonlinear) and any sort of network (single-layer or multi-layer). However, radial basis function networks (RBF networks) have been assocated with basis functions in a single-layer network (Figure5.3) since Broomhead and Lowe's 1988 seminal paper [26].

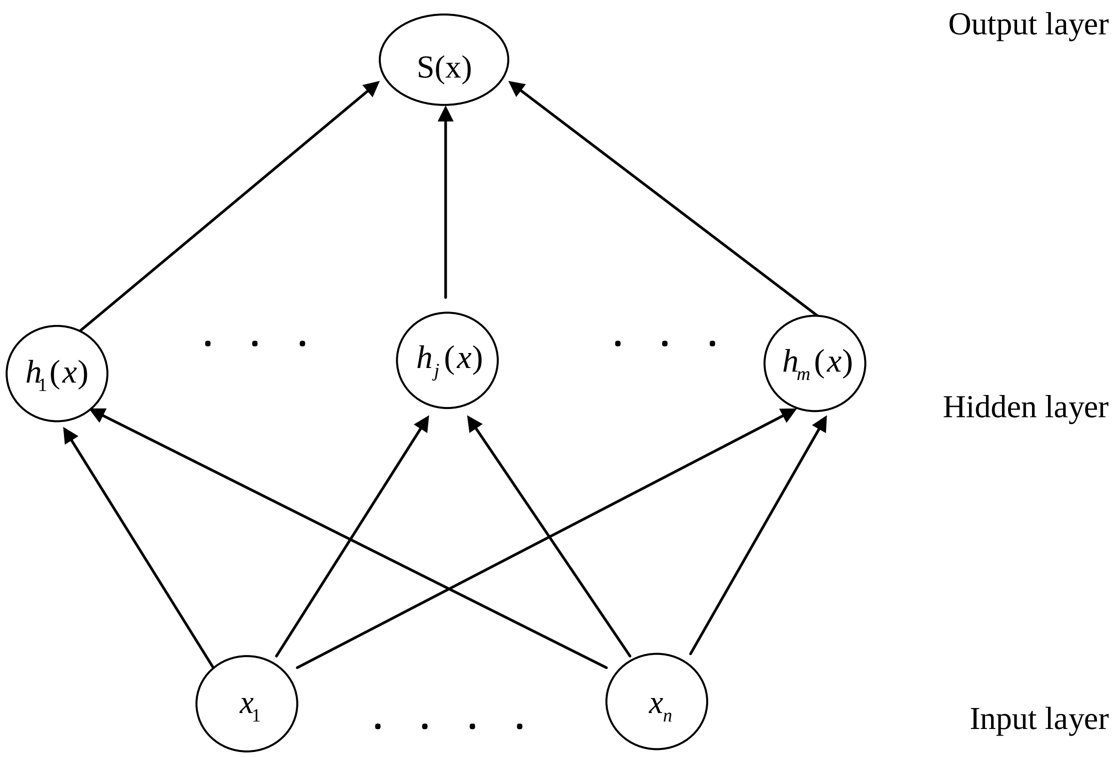


Figure5.2 The traditional radial basis function network

- The input layer serves as input distributor to hidden layer.

- The node in the hidden layer is a basis function $\phi\|x-x_i\|$ which is represented as $h(x)$ in Figure5.3. Then the hidden layer calculates the distance between the center and the network input vector and transfer the result to the radial basis function.

- The output is calculated by a linear combination weighted sum of the radial basis function plus the bias $s(x) = p(x) + \sum_{i=1}^{n} \lambda_i \phi(\|x - x_i\|)$.

5.3.Implementation in Matlab

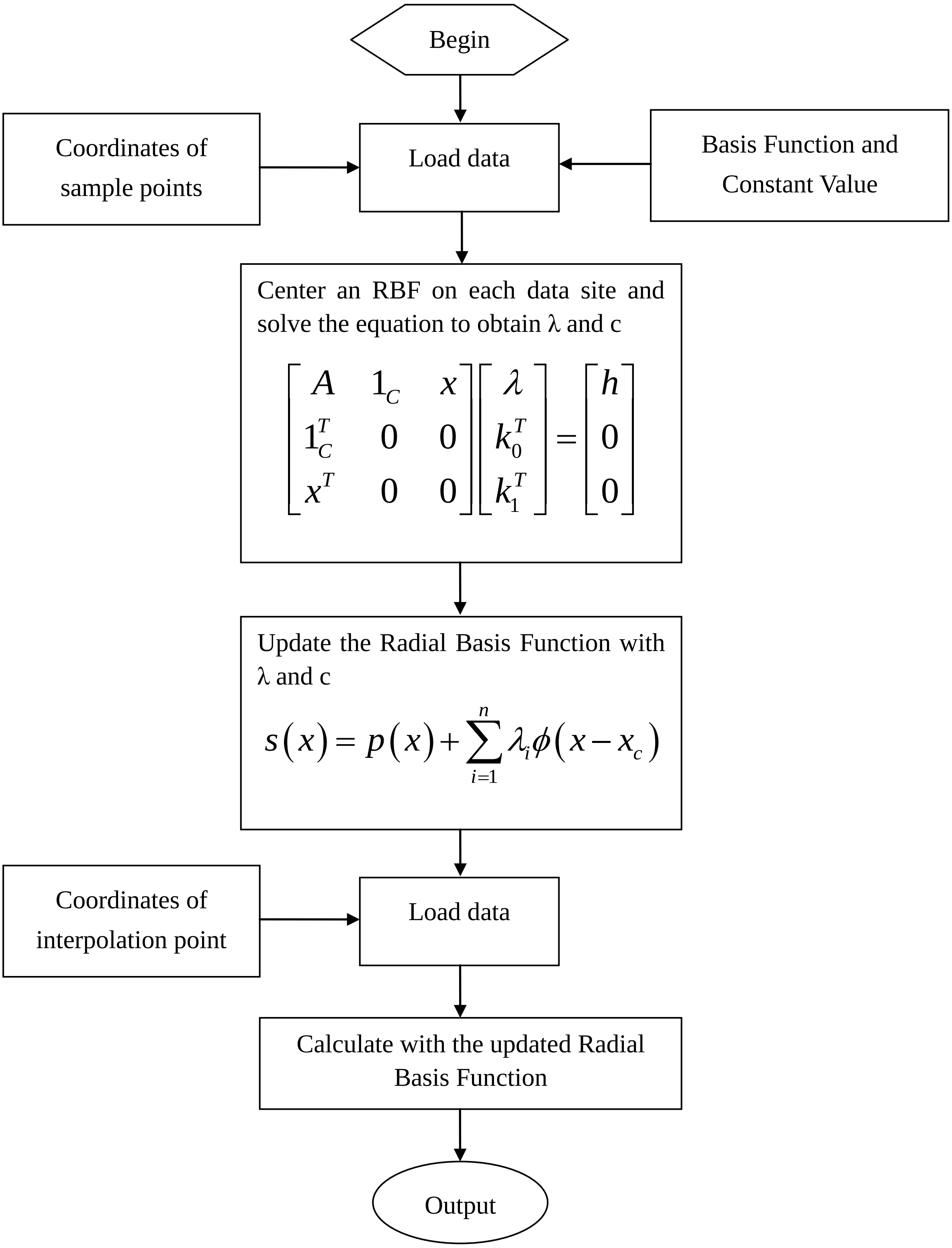


Figure5.3 Implementation flow of IDW

Chapter 6

Evaluation of the interpolation methods

6.1 Materials and Methods.

6.1.1 Data collection

It can be seen obviously that the inclusion of more check points to do the evaluation will lead to a more reliable results. However, it will be costly to include a large number of check points in a lot of practical applications. Therefore, in our study, we employed three terrain surfaces, the first two pre-defined terrain surfaces for testing and comparing selected interpolation methods were generated by using the tool of Automatic Terrain Generation, namely a flat surface and a mountainous surface, which are shown in Figure6.1.1. Both areas were 1*1, and 11*11 data points in total. Table 6.1 presents the values we selected for generating the terrains in the Automatic Terrain Generation.

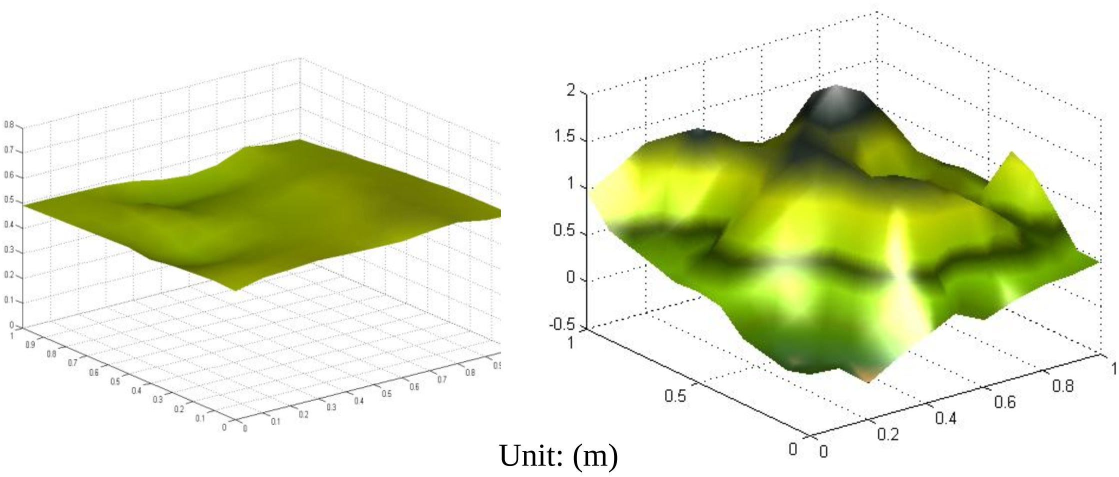


Figure6.1.1 Flat terrain (on the left) and mountainous terrain (on the right)

Type of relief	Number of iterations	Initial roughness	Initial elevation	Resolution of map	Roughness of roughness
Mountainous	5	0.3	1	11	0.9
Flat	5	0.1	0.5	11	0.2

Table6.1 Parameter determination in Automatic Terrain Generation

The ‘’true’’ data of the third DEM was derived from Salisbury, Maryland in the United States (Figure6.1.2), which is in a USGS 1-degree native format and downloaded from WebGIS website. The DEM of Salisbury provides coverage in 1 by 1-degree block and consists of 401*401 data points located in a mesh grid, the elevation ranges from 0 to 30 meters. Since the experiment scope of the district is big,

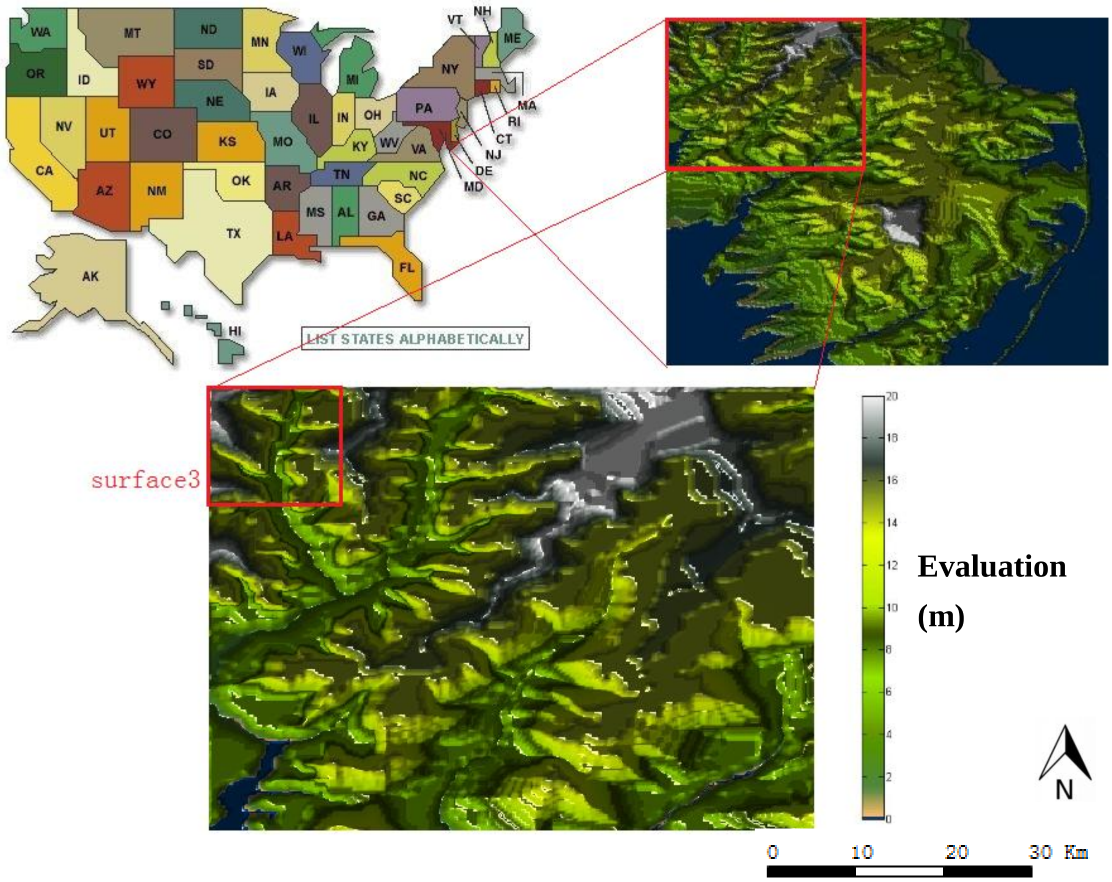


Figure6.1.2 The third study area

and not easy to present the difference of the interpolation results on the screen of computer, so we pick a part in the district, as shown in figure 6.1.2 and marked as surface3, which is a area consists of 41*41 data points. The area was chosen because it allows the evaluation of the interpolation methods under different topography conditions changing in relatively small distances. Besides, one should be mentioned that using of DEM has little difference with using of DEM in our experiment.

6.1.2 Data processing.

We divided the original data into two subsets in order to carry out the accurate assessment of the interpolation results. One subset is used to do the interpolation, the other is used to do the evaluation, since the interpolation results should reflect the closeness between the original data and the reconstructed data, which means the interpolated surface should as far as possible keep the characteristic of the original one

In figure6.1.3, we can observe the flowchart of the scheme that we employed for the Data processing. There are three main steps as following:

- **Step one**: The data points were divided into two subsets, which were named as subset1 and subset2. The subset1 took the first, the third …data points both in x and y directions, which consisted of 6*6 data points with a step of 0.2 between each point on the x and y coordinates, the subset2 took the rest data points which were used as the check points in the step three.

- **Step two**: Implement IDW and RBFs interpolation methods respectively in subset1 to reconstruct the data points which have same x and y locations corresponding to the data points in subset2

- **Step three**: Compare the elevations of between the data points in subset2 and the reconstructed data points in step two, and finally obtain the corresponding Root Mean Square Error value.

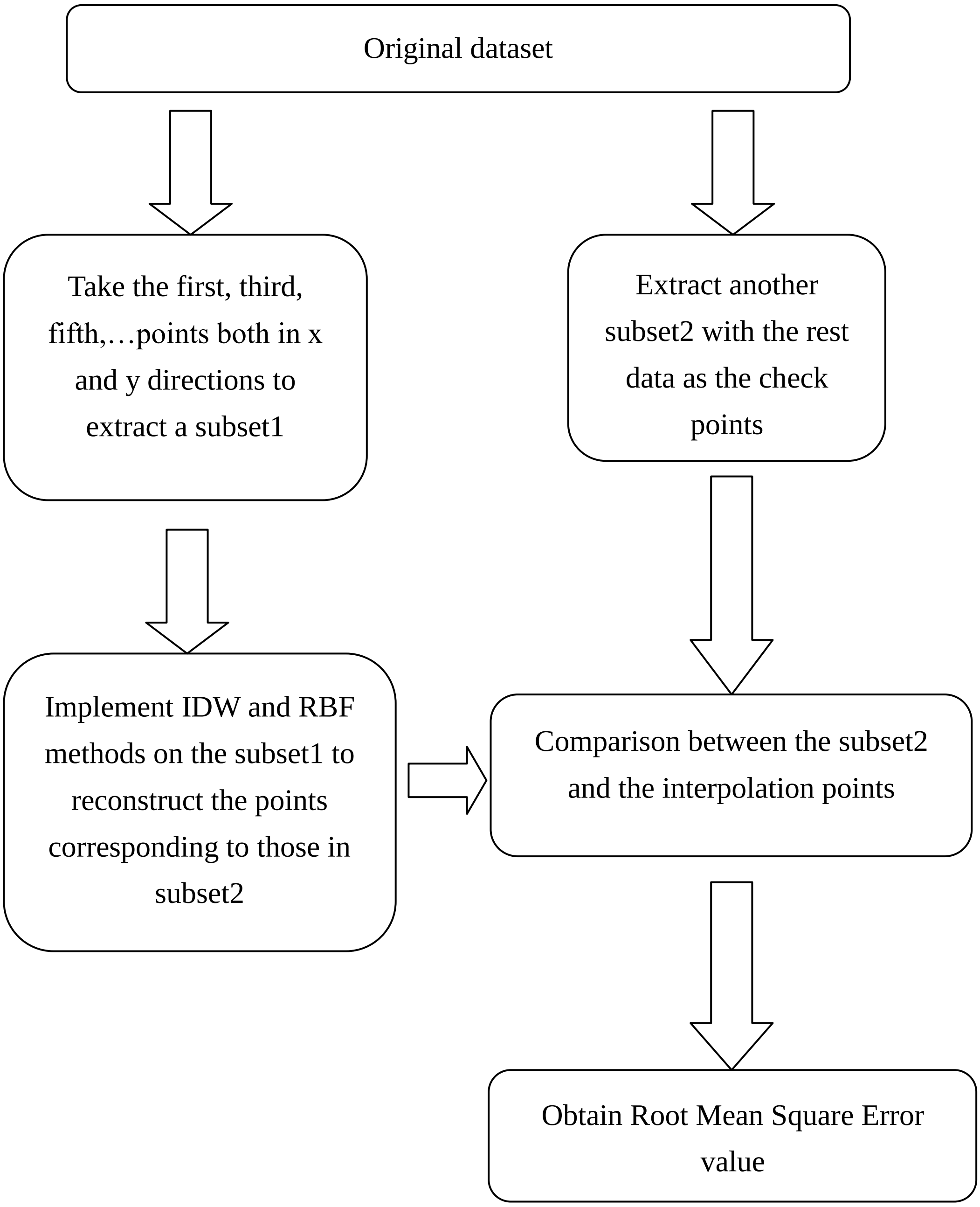


Figure6.1.3 Flowchart of scheme used to evaluate the interpolation accuracy

6.1.3 Organization of the comparison

In order to have a better understanding of the characteristics and the performance of IDW and RBF interpolation methods on different terrain, we organized the comparison by following this process (Figure6.1.4). There are three topographic surfaces using in our experiment, two of them were generated by the Automatic Terrain Generation, another was derived from WebGIS. At the beginning, the interpolation process was carried out using the methods of IDW and RBF respectively. And before implementing interpolation methods, we computed the optimal value of the smoothing factor by implementing the Optimization Toolbox and also set the optimal search radius. Then these values were applied to RBF and IDW to obtain the corresponding results of the interpolation. After it, we did a comparison to show how are the accuracies of the two interpolation methods on the same topographic surface. Finally, we gave a comprehensive comparison on all the results we obtained, in order to show the performance of the selected interpolation methods on different terrain.

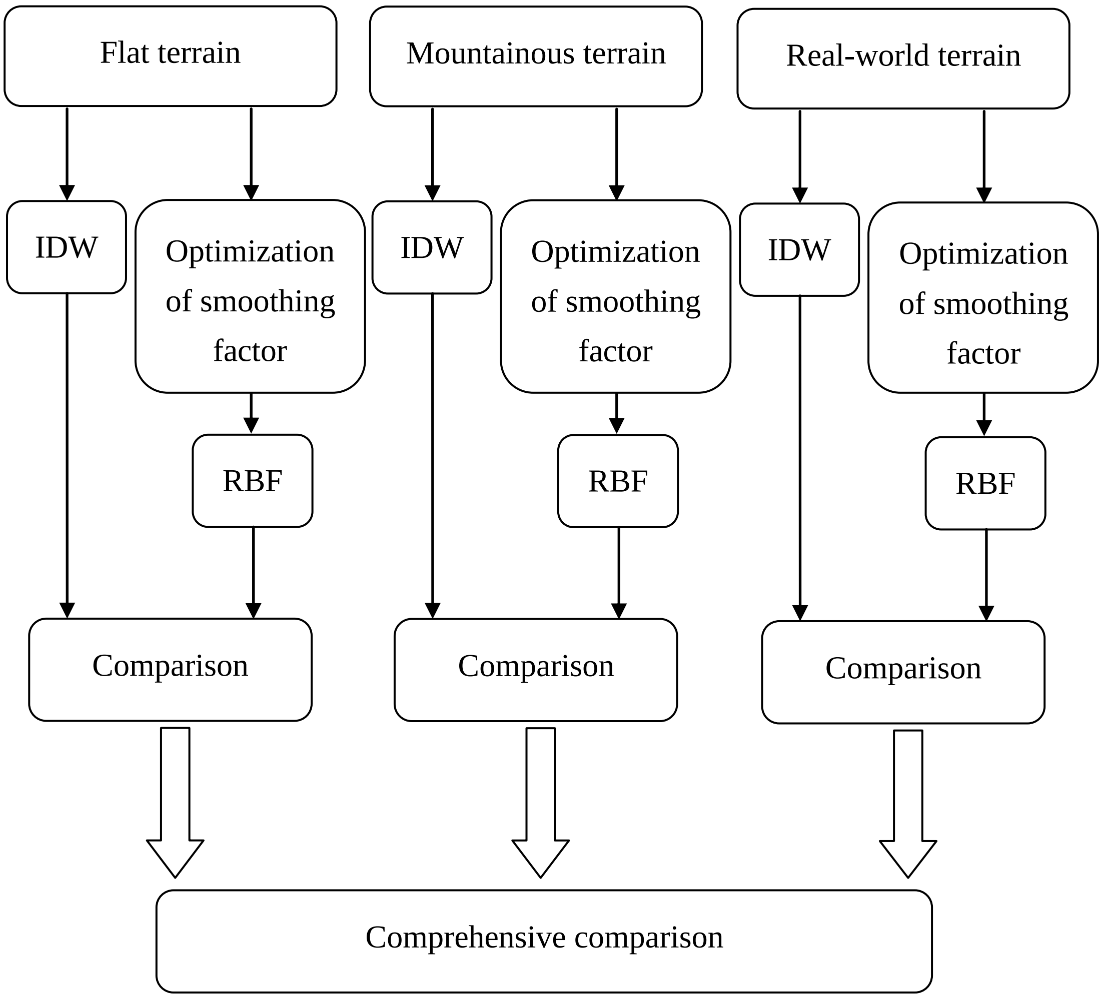


Figure6.1.4 Organization of the comparison

6.1.4 Assessment methods

The test of accuracy of the interpolation methods was based a measure, namely the Root Mean Square Error(RMSE) which is the mostly widely used global accuracy measure for evaluating the performance of DTM.The form of RMSE is as followed.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Z_i^{estimated} - Z_i^{real})^2}{n}} \tag{6.1}$$

Where:

- i is the number of each elevation.
- $Z_i^{estimated}$ is the altitude value of interpolation point.
- Z_i^{real} is the altitude value of the original point.
- n is the total number of points in the evaluation.

The error is derived by squaring the differences between real and estimated points, adding those together, dividing that by the number of test points, and then taking the square root of that result. The RMSE expresses an extent to which an interpolation value differs from a valid value. A higher value corresponds to a greater difference between two datasets.

In order to get the best performance of each basis functions in RBF, we introduced Optimization Toolbox and optimize the smoothing factor in each basis functions.

6.2 Results.

6.2.1 Optimization of the parameters

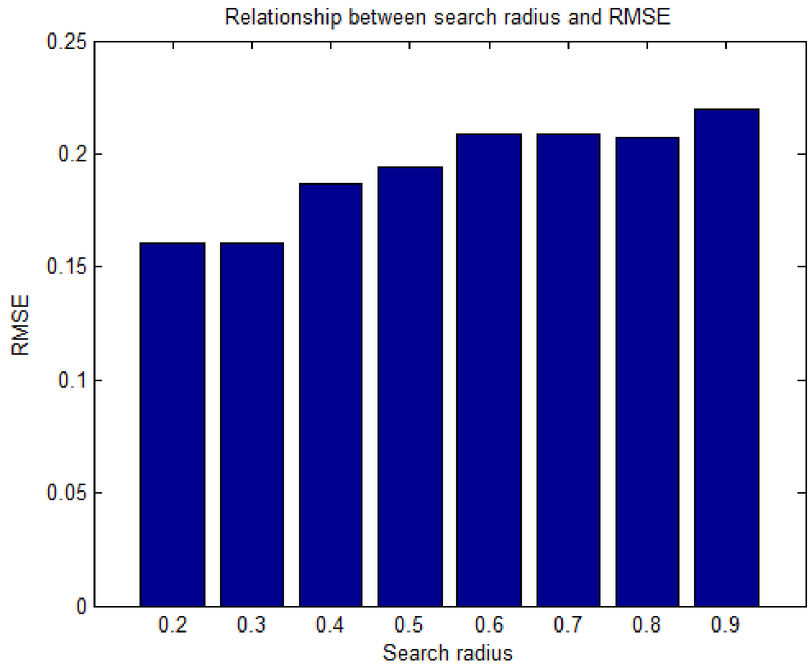
To achieve a best interpolation results, the smoothing factor was optimized by employing the Optimization Toolbox. After selecting solver of the constrained nonlinear minimization and setting the bounds between 0 and 2, and also a start point of 0.1, the minimum RMSE with the optimal smoothing factors was obtained, as shown in Table6.2.1**.**

We can also observe that when the topographic surface become rolling, the optimized smoothing factor trend to be smaller, since .the optimal smoothing factors on mountainous terrain are smaller than those on flat terrain, and the smoothing factors applied in NCSF basis function are always the smallest, with the values of 0.074 ,0.164 and 0.01 respectively on the three terrain. By contrast, the smoothing factors in IMQF are the biggest, which are 0.284, 0.512 and 0.046 on mountainous, flat and real-world terrain respectively.

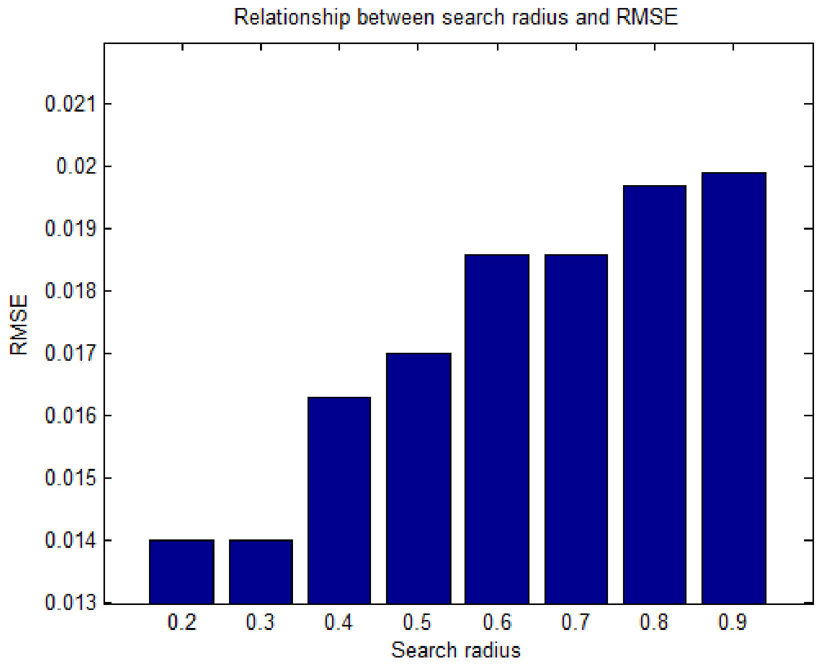
Basis function	IMQF	MLF	MQF	NCSF	TPSF
Smoothing factor on mountainous	0.284	0.244	0.2	0.074	0.148
Smoothing factor on flat	0.512	0.445	0.369	0.164	0.281
Smoothing factor on a real-world	0.046	0.038	0.028	0.01	0.011

Table6.2.1 Optimization of the smoothing factor in each basis functions

In our study on the IDW interpolation, it is also noticed that different search radius also have influence on the interpolation accuracy, in Figure6.2.2, we selected a range of the search radius from 0.2 to 0.9 since the resolution is 0.1 and the area is 1*1.For the mountainous terrain surface, the RMSE can reach its minimum value of 0.1605 when the search radius is set at 0.2 and 0.3. As the search radius increasing, this will also lead to observably increase in the RMSE value. The search radius raises to 0.9 where the maximum RMSE value exceeds 0.2, at 0.219.



(a)



(b)

Figure6.2.2 Relationship between search radius and RMSE (m) on mountainous terrain (a) and on flat terrain (b)

A similar tendency can be observed on the flat terrain surface Moreover, as presented on Figure6.2.2 (b), we can see a more significant increase on RMSE value when a larger search radius is selected. The influence of the variation of search radius will have more effect on the flat terrain. Nonetheless, the RMSE value still keeps at a very low level on flat terrain surface, with a minimum value of 0.014 and a maximum value of 0.0199.

We also applied different search radius to the real-world terrain to see the changes on the topographic surface. As we can see in Figure6.2.3, when the search radius increases, the features of the terrain become smoother, the unsampled peaks within the maximal value can also not be reconstructed. Because of this property, some features like valleys or ridges are obscured.

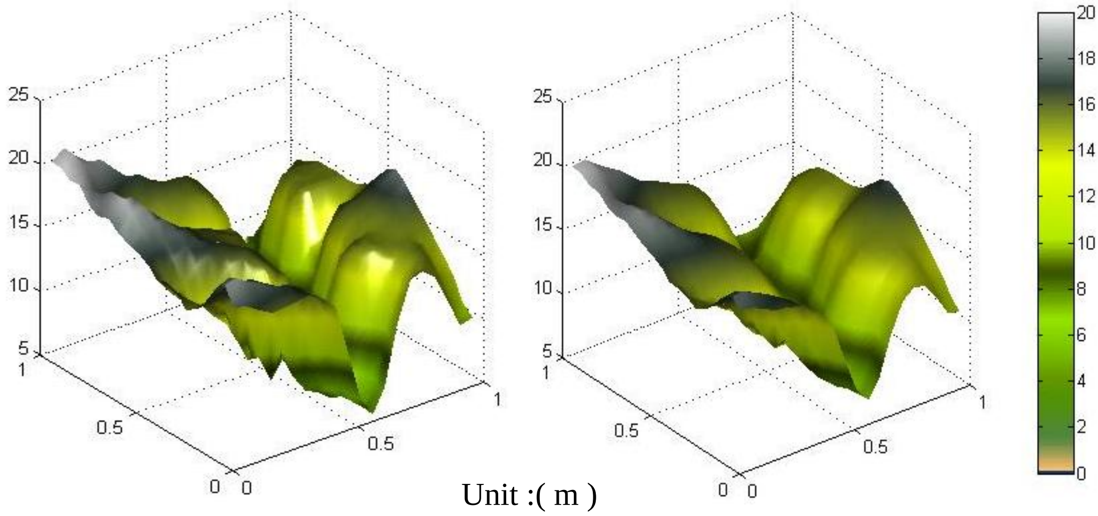


Figure6.2.3 Effect of different search radius(r=0.05 on the left and r=0.2 on the right)

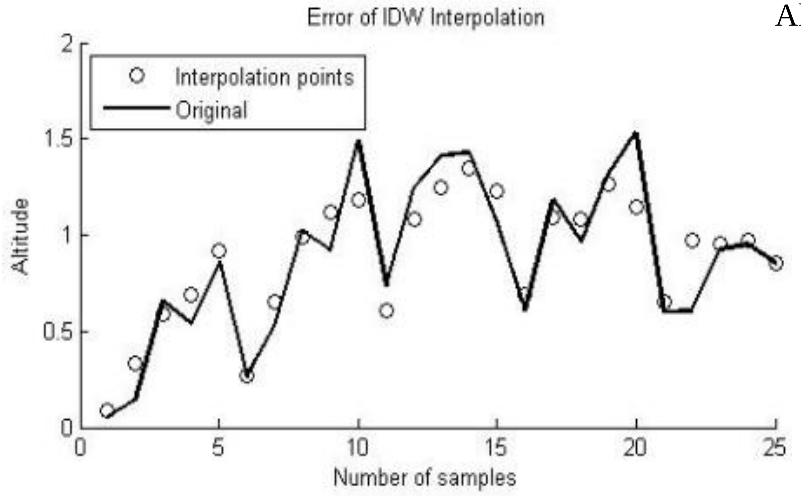
The solution to this drawback is to consider only data within a certain distance about the interpolation point. In our experiment, the search radius within double of the distance between the sampling points can lead to an optimal RMSE value.

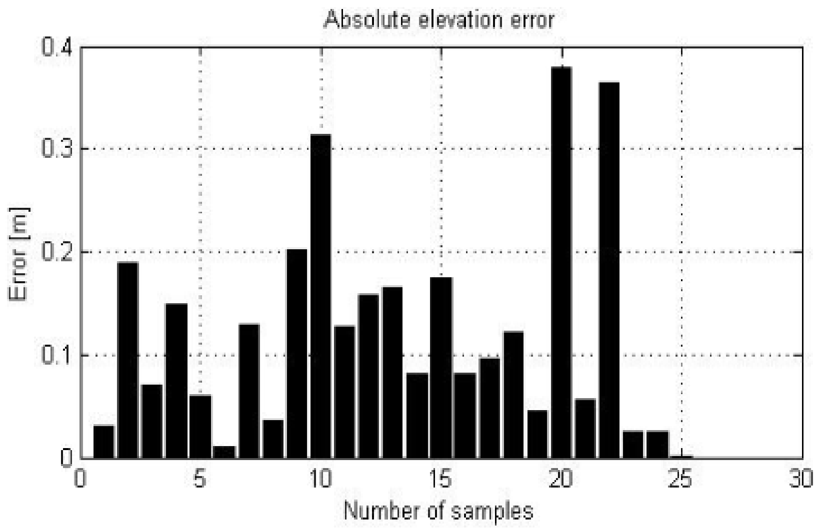
6.2.2 Analysis on the mountainous terrain surface

We can see the capability of reconstruction of each selected interpolation method from Figure 6.2.4

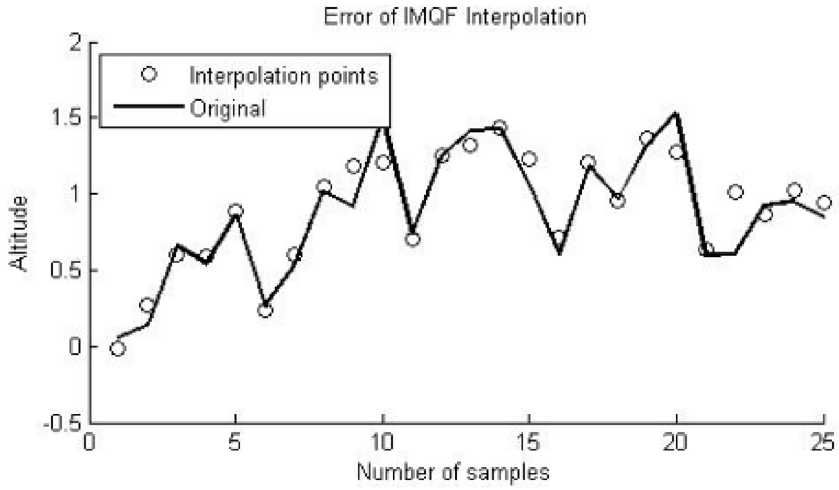
Figure6.2.4 Comparison of the interpolation errors between selected interpolation methods on mountainous for (a) IDW, (b)IMQF, (c)MLF, (d)MQF, (e)NCSF, (f)TPSF

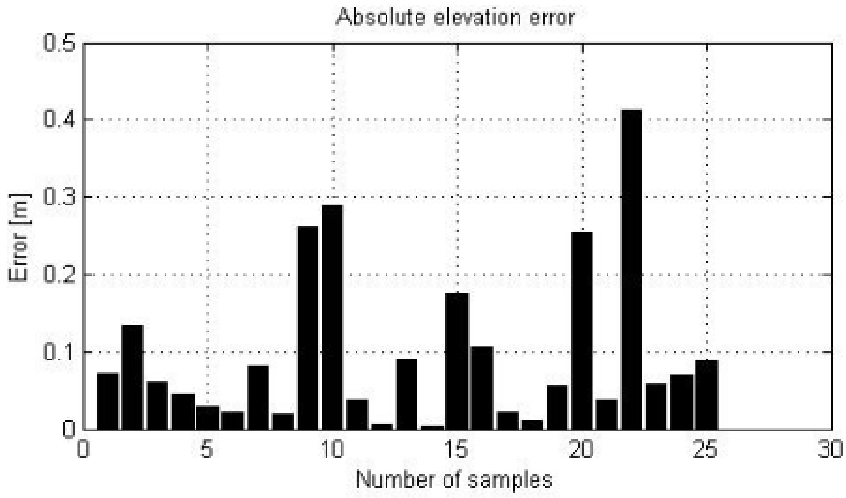
Altitude: (m)



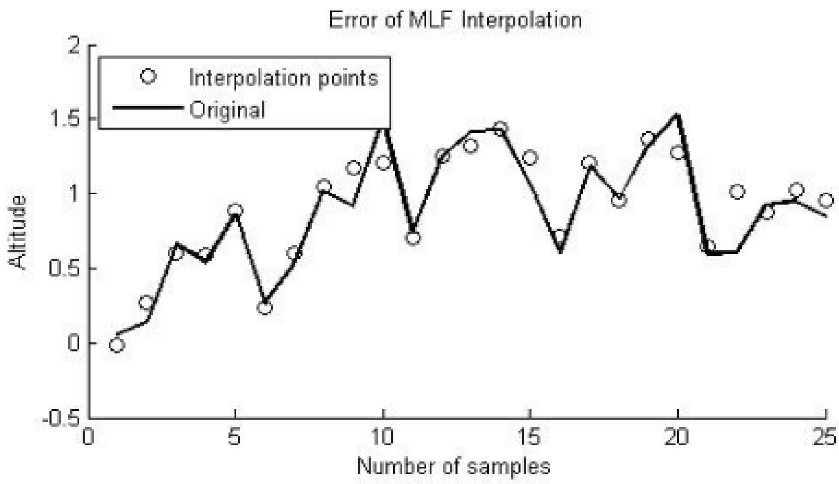


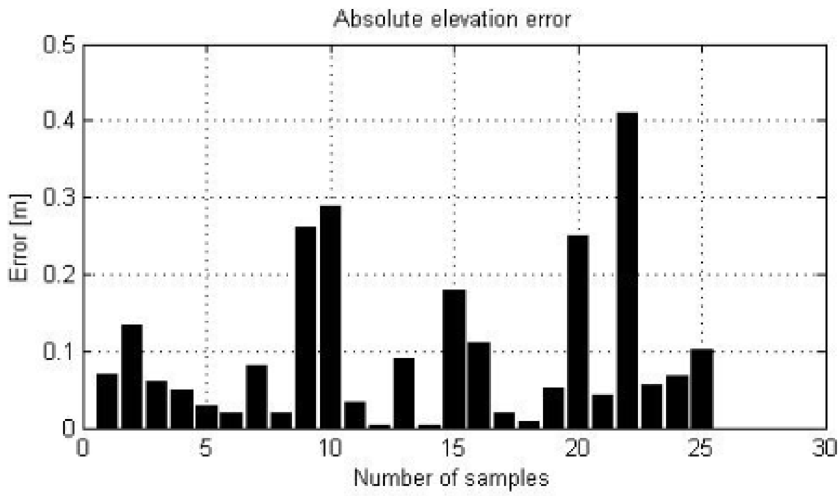
(a)


Error of IMQF Interpolation
Interpolation points
Original
Altitude
Number of samples

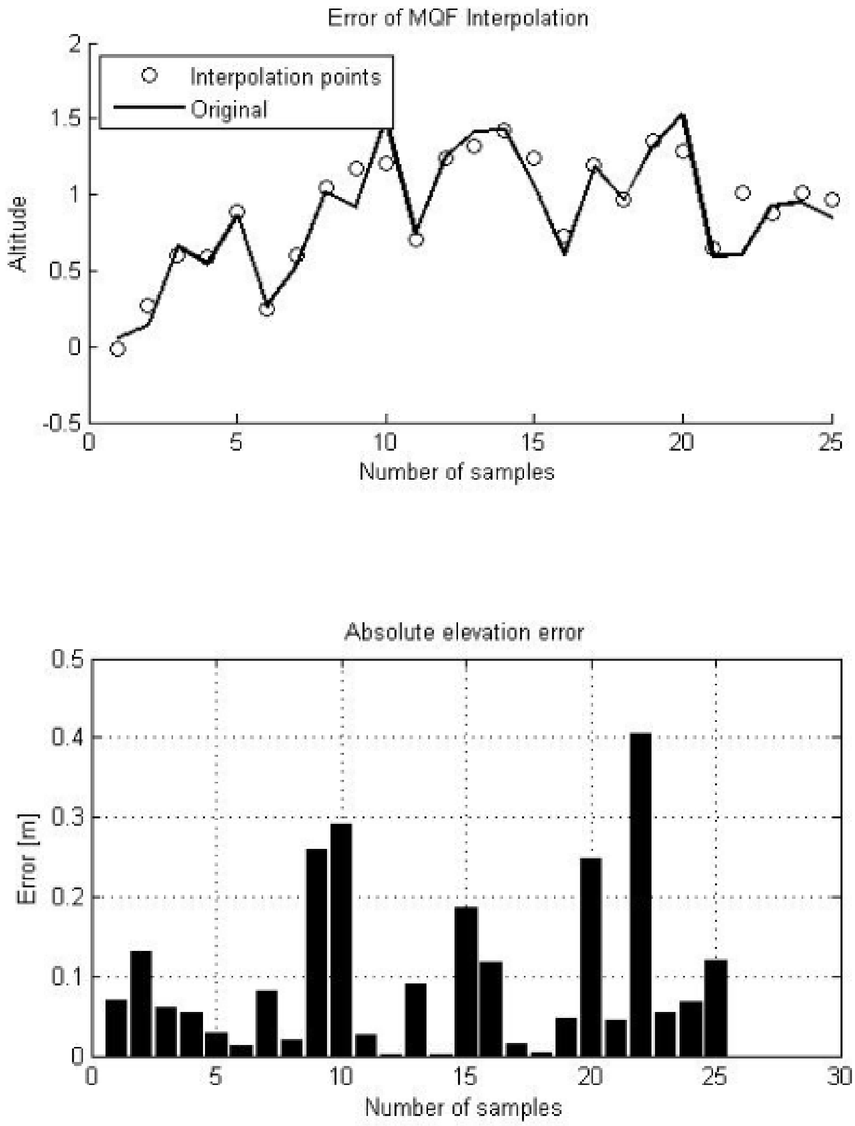

Absolute elevation error
Error [m]
Number of samples

(b)

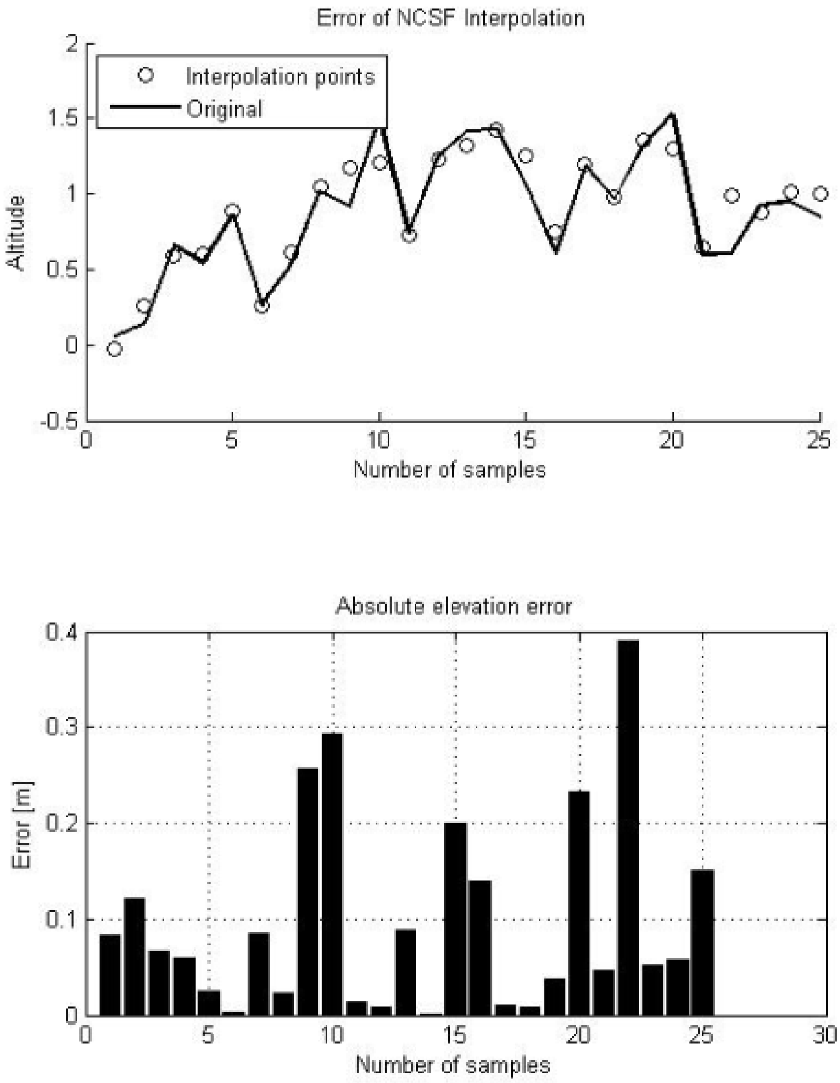

Error of MLF Interpolation
Interpolation points
Original
Altitude
Number of samples


Absolute elevation error
Error [m]
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
Number of samples

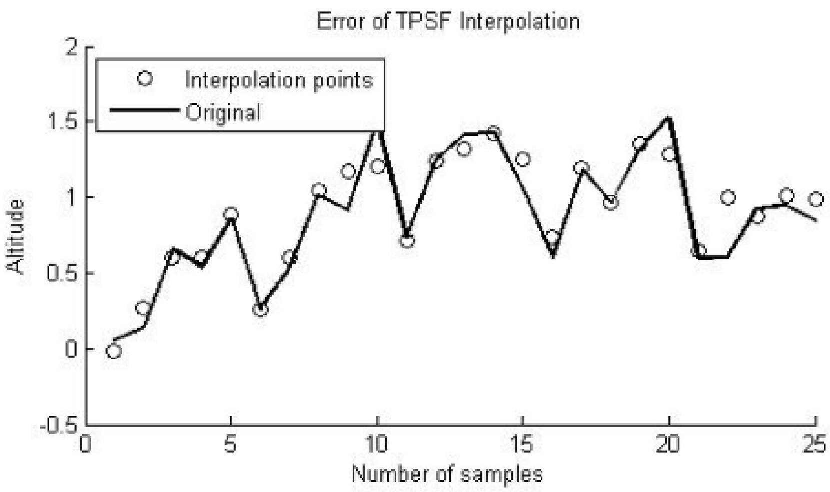
(c)

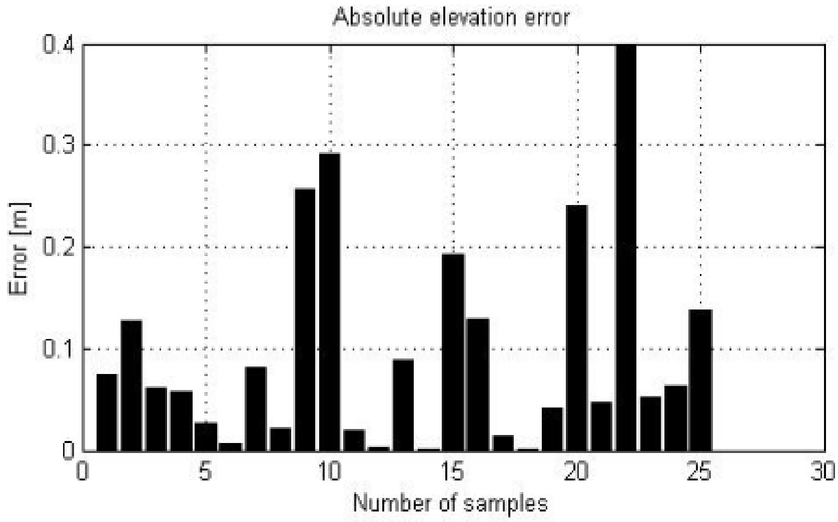

Error of MQF Interpolation
Interpolation points
Original
Altitude
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
Number of samples
Absolute elevation error
Error [m]
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
Number of samples

(d)


Error of NCSF Interpolation
Interpolation points
Original
Altitude
Number of samples
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
Absolute elevation error
Error [m]
Number of samples
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30

(e)

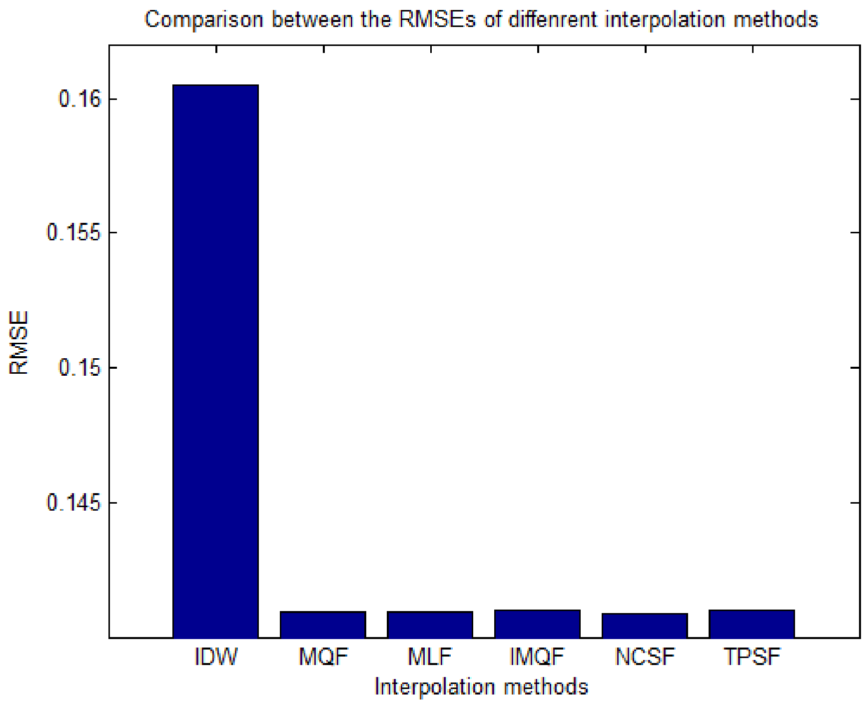

Error of TPSF Interpolation
Interpolation points
Original
Altitude
Number of samples
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25



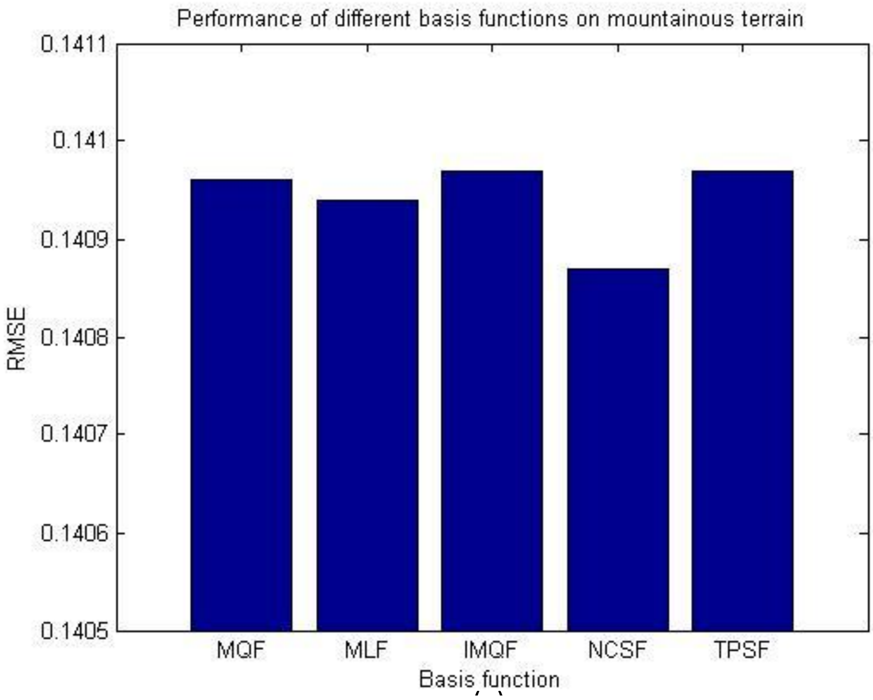
(f)

It has been demonstrated that interpolation accuracy can vary to a certain degree with different interpolation algorithms. The level of this error is important for various specific applications. Therefore, six of the most widely used interpolation algorithms were compared with different terrain surfaces.

Figure6.2.5 (a) presents the comparison on the mountainous terrain surface. From the tracking performance of the interpolation dataset on the original dataset, we can see all the interpolation algorithms can perform well on the points with corresponding



(a)



Unit: (m)

(b)

Figure6.2.5 (a) Comparison between IDW and RBFs on mountainous terrain
(b) Comparison among different basis functions on mountainous terrain

low rate of change of the slope, and it shows the distortion specifically on the tip of the terrain with a high rate of change of the slope. Besides, on the figures of the absolute elevation error, It can be noticed that there is a corresponding higher absolute error in the IDW interpolation, even on some sample points, all the methods have higher absolute error. This was verified when we compared the RMSE value of each interpolation algorithms.

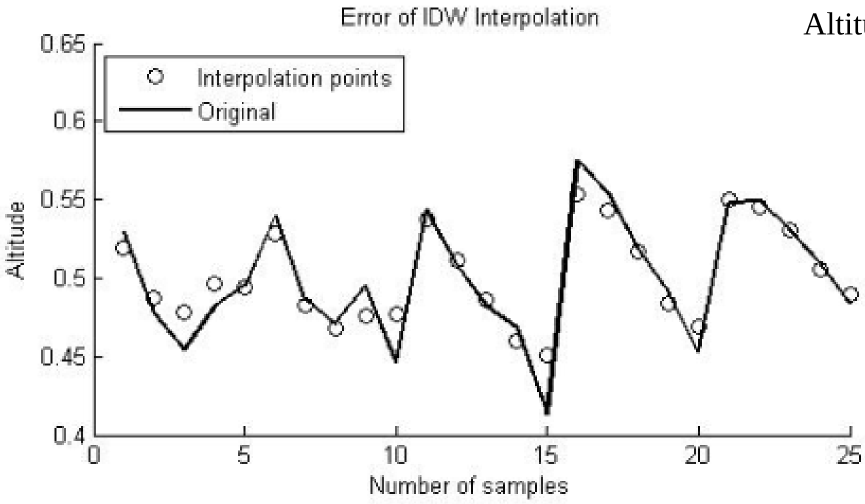
As we can see in Figure6.2.5 (b), from the comparison, the IDW algorithm provided the worst interpolation and produced the greatest RMSE value of all the methods, other five basis functions in RBF performed a corresponding lower RMSE, smaller than 0.145.The details of the comparison among five selected basis functions are shown in Figure 6.2.5(b). The MQF, IMQF, TPSF had nearly similar results, NCSF was the best interpolator with the smallest RMSE value for the mountainous terrain surface, and MLF was the second suitable interpolator for the mountainous terrain.

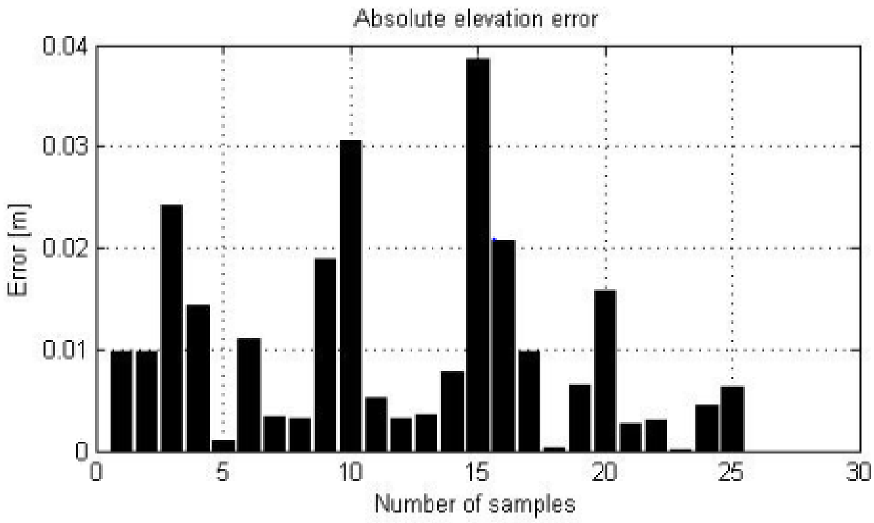
6.2.3 Analysis on the flat terrain surface

After a comparison on the mountainous terrain, we started to implement the evaluation on the flat terrain. The process was the same as before.

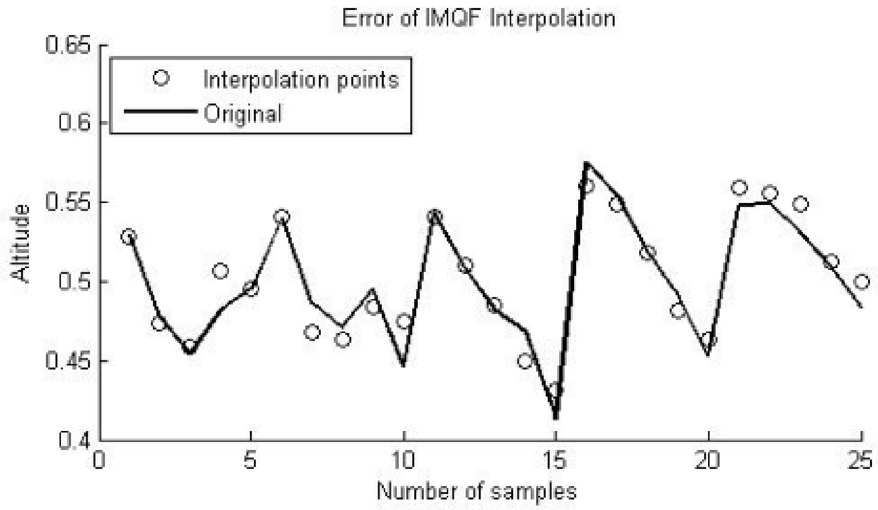
Figure 6.2.6 Comparison of the interpolation errors between selected interpolation methods on flat for (a) IDW, (b)IMQF, (c)MLF, (d)MQF, (e)NCSF, (f)TPSF.

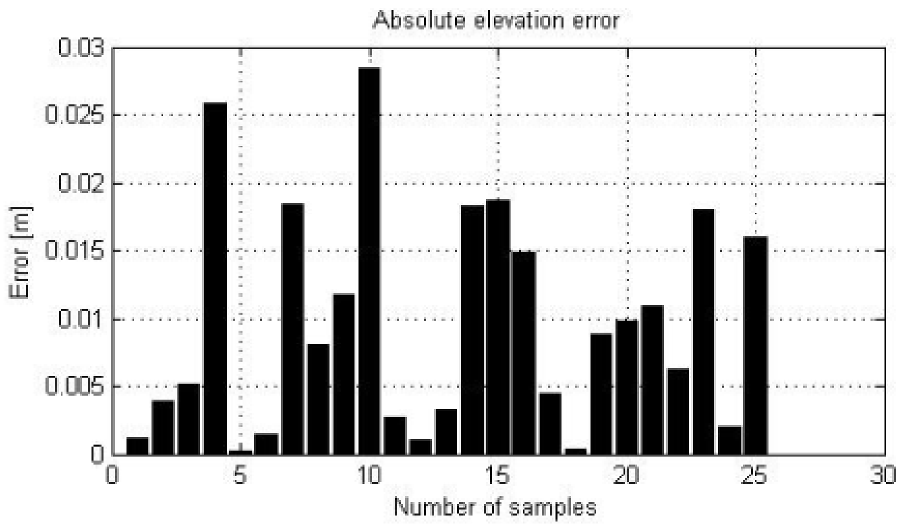
Altitude: (m)



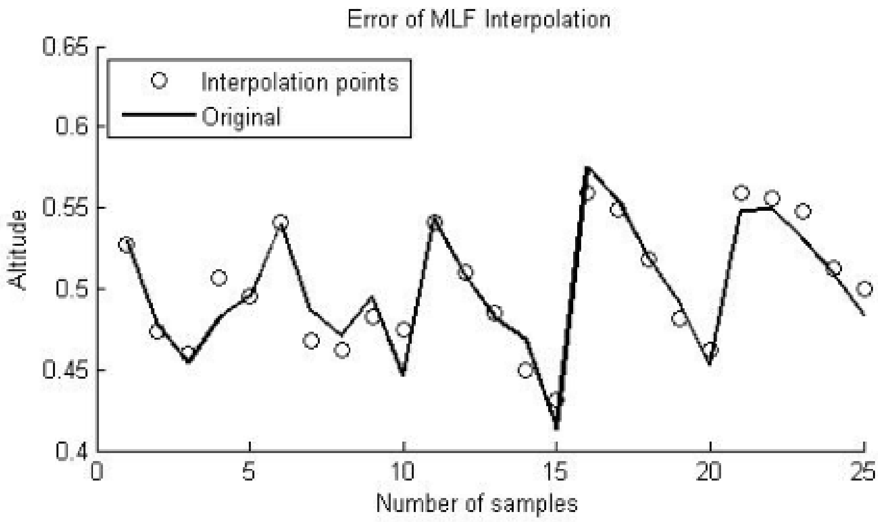


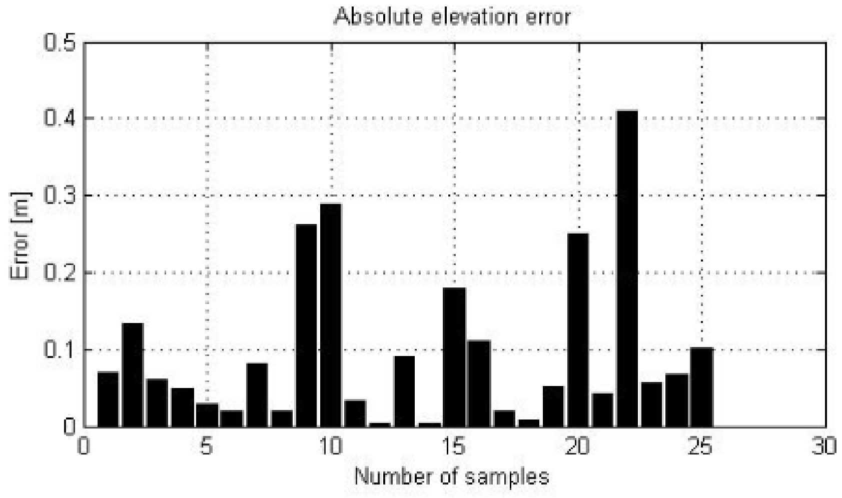
(a)


Error of IMQF Interpolation
Interpolation points
Original
Altitude
Number of samples
0.65
0.6
0.55
0.5
0.45
0.4
0
5
10
15
20
25

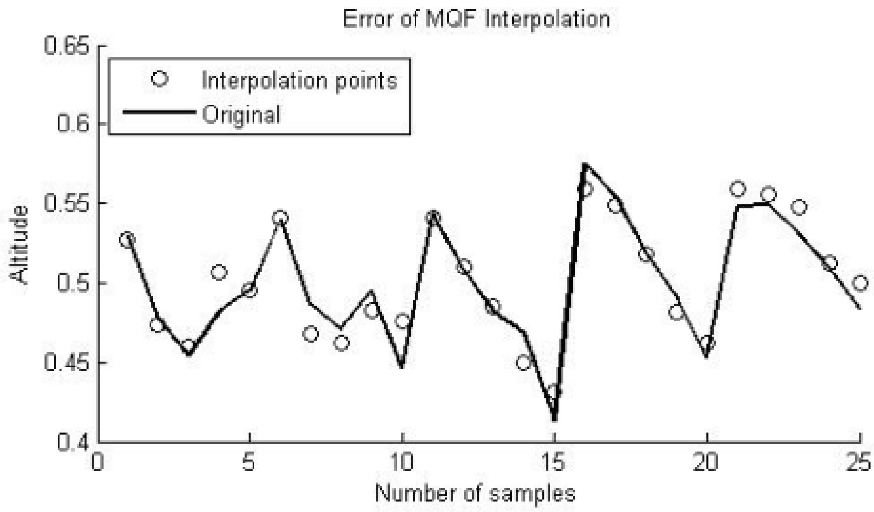

Absolute elevation error
Error [m]
Number of samples
0.03
0.025
0.02
0.015
0.01
0.005
0
0
5
10
15
20
25
30

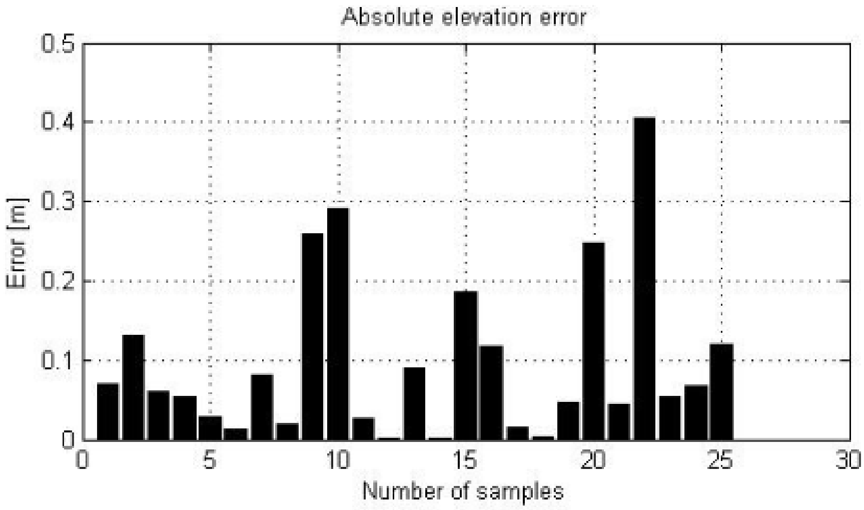
(b)


Error of MLF Interpolation
Interpolation points
Original
Altitude
Number of samples
0.65
0.6
0.55
0.5
0.45
0.4
0
5
10
15
20
25

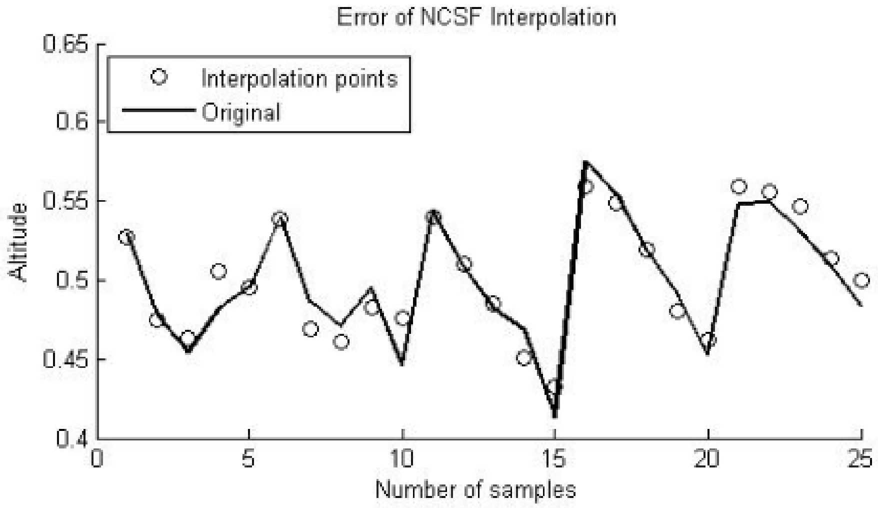

Absolute elevation error
Error [m]
Number of samples

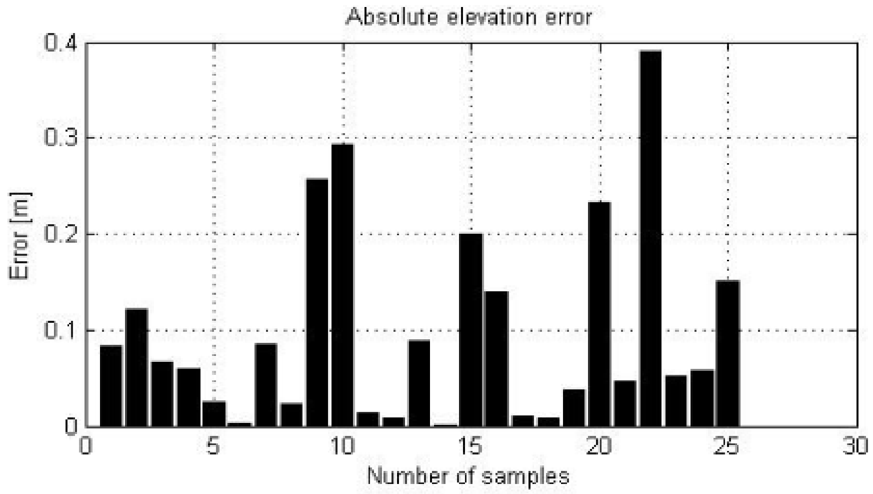
(c)


Error of MQF Interpolation
Interpolation points
Original
Altitude
Number of samples

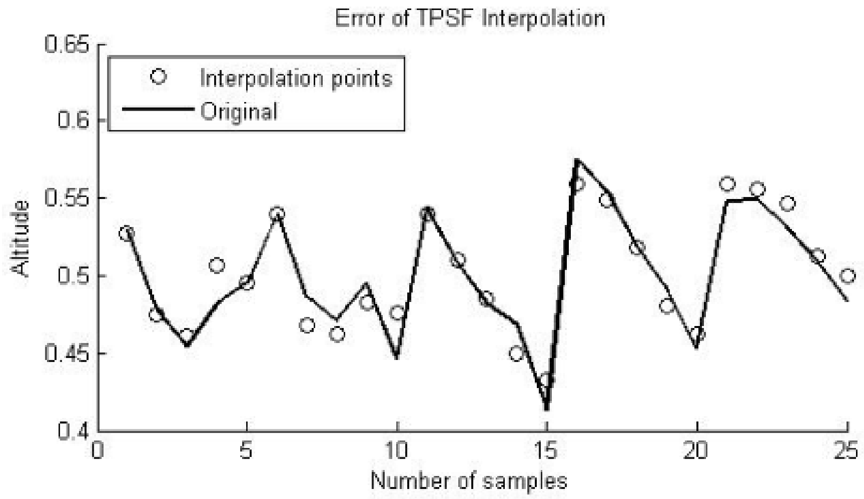

Absolute elevation error
Error [m]
Number of samples

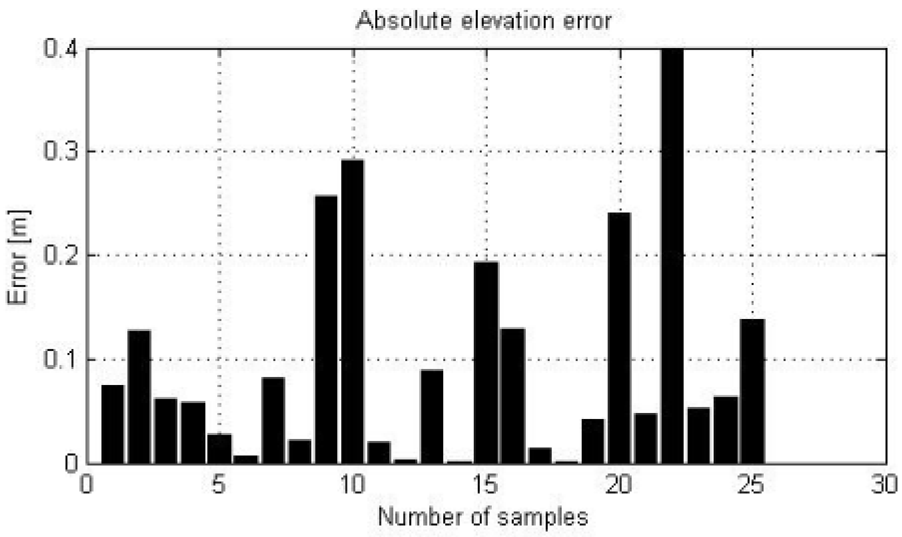
(d)


Error of NCSF Interpolation
Interpolation points
Original
Altitude
Number of samples
0.65
0.6
0.55
0.5
0.45
0.4
0
5
10
15
20
25


Absolute elevation error
Error [m]
Number of samples
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30

(e)


Error of TPSF Interpolation
Interpolation points
Original
Altitude
Number of samples
0.65
0.6
0.55
0.5
0.45
0.4
0
5
10
15
20
25

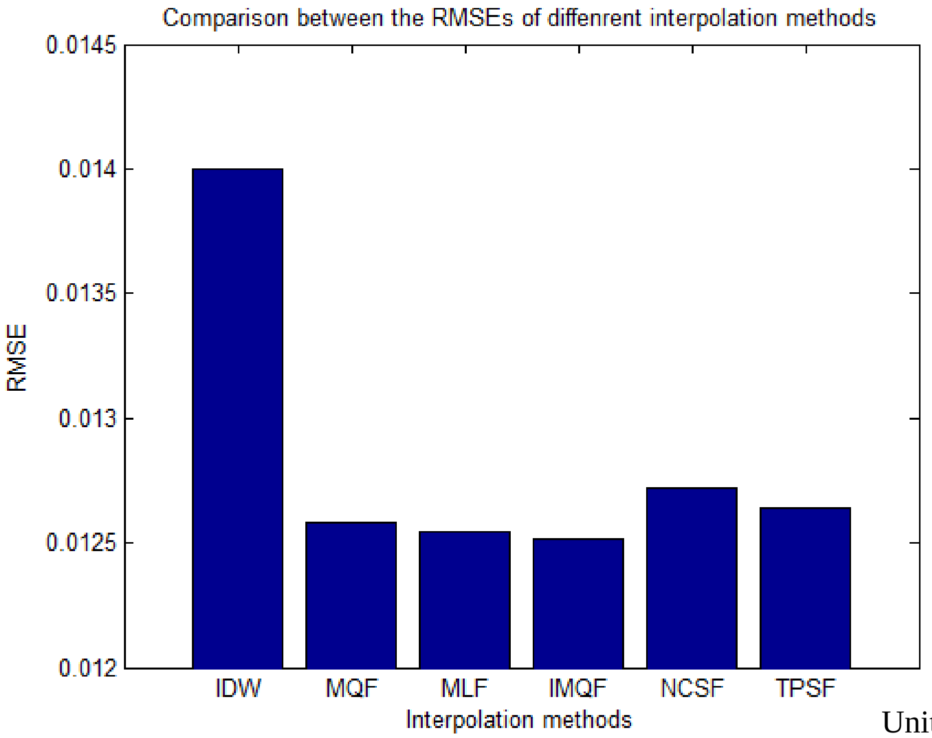


(f)

From Figure 6.2.6, we can find when it comes to the flat terrain surface, all the selected interpolation methods have dramatic improvements on the control of the interpolation errors, IDW have a better tracking on with small slopes, but it presents unexpectedly discrepancies between some of the check points elevations and the elevations interpolated from the sampling points datasets, especially for the points on the bottom. For the other four basis functions in RBF, all of them have a corresponding good performance on estimating the elevations of the tips and bottom, however, they also show a tendency to over-exaggerate the elevations on some check points.

In Figure6.2.7,we can observe there are the significant decrease on the values of RMSE for all the selected interpolation methods.IDW is still the poorest interpolator, with the highest RMSE about 0.014.The other interpolators, such as MQF, MLF,IMQF,NCSF and TPSF, keep at a corresponding lower level of the RMSE, around 0.0125.

Furthermore, when comparing the RMSE values among the five basis functions in RBFs, we can find they have different characteristics on flat terrain surface. The RMSE values of MQF, MLF, IMQF are smaller, and from the comparison, NCSF and TPSF are relatively larger, with the minimum belonging to IMQF and the maximum belonging to NCSF.

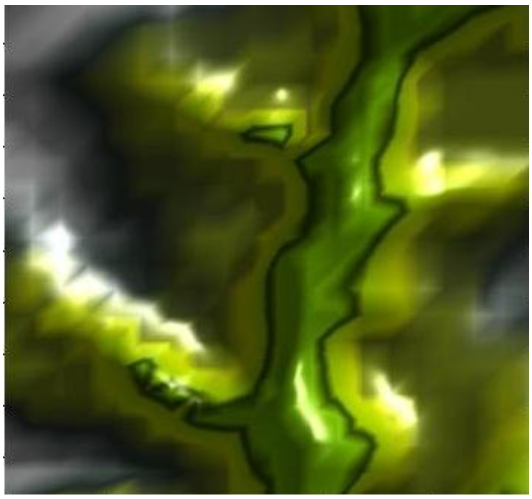


Unit: (m)

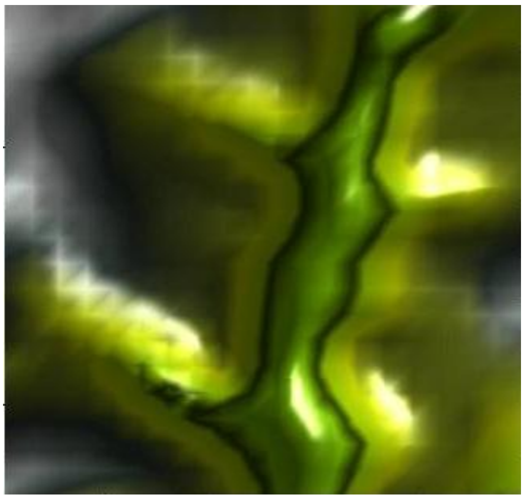
Figure6.2.7 Comparison between IDW and RBFs on flat terrain

6.2.3 Analysis on the real-world terrain surface

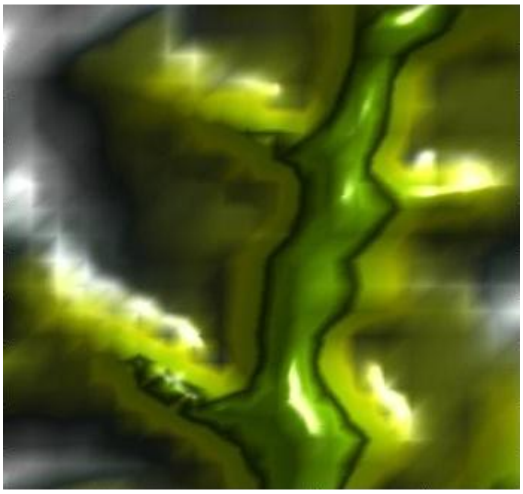
Figure6.2.8 Comparison between the original terrain surface (a) and the terrain reconstructed by (b) IDW, (c) IMQF, (d) MLF, (e) MQF, (f) NCSF, (g) TPSF.



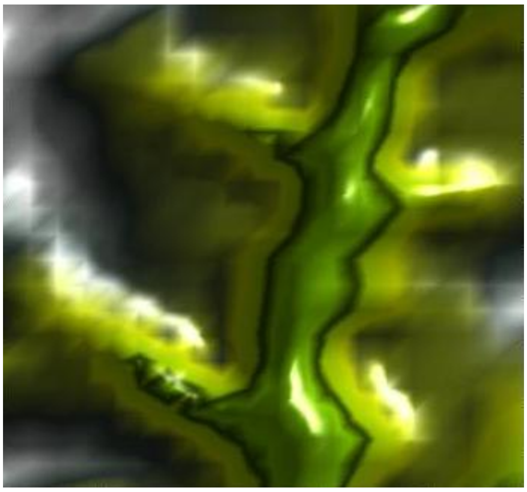
(a)



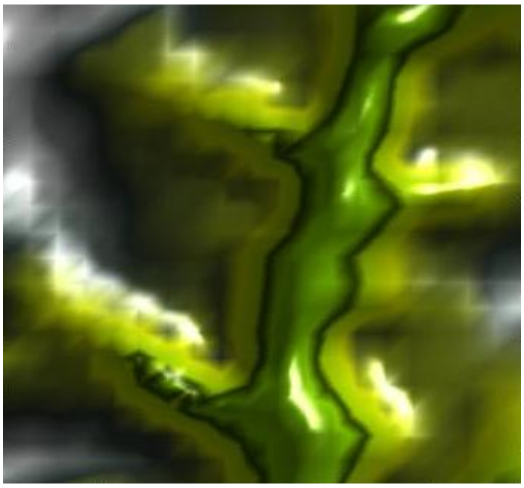
(b)



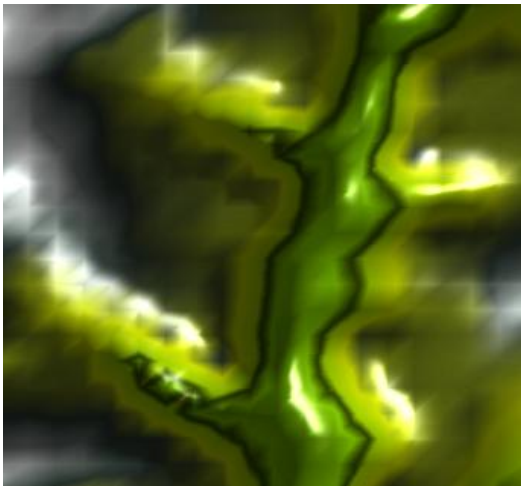
(c)



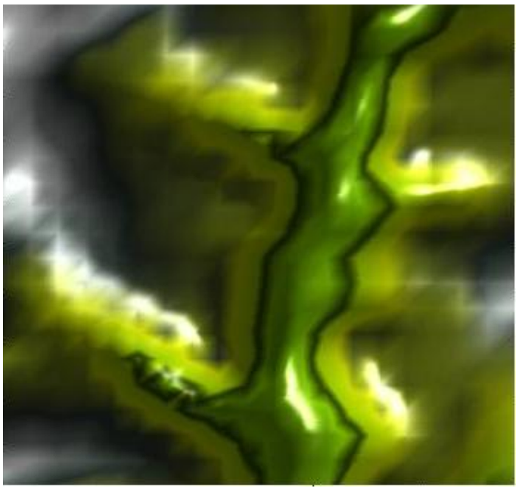
(d)



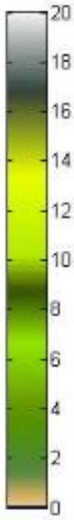
(e)



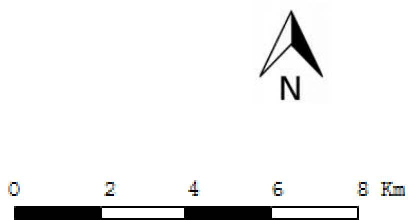
(f)



(g)



Evaluation (m)



In the real-world terrain surface, our study area reaches to a 41*41 squares, bigger than the terrain we used before. It is not easy to comparison the difference among the interpolation errors in a figure since the sample points increases. So in Figure6.2.8, we presented the results on a two-dimensional view from the top.

Overall, from Figure6.2.8, IDW produced the poorest result mainly in same small areas with big variation of slopes, which means IDW can predict the gentler or regular change of the slope, but steep changes of the slopes in small areas may be ignored by IDW interpolation. It is also a problem for the five basis functions in RBF from the comparison, but obviously, the terrain obtained from RBFs coincided better with the original terrain, especially for the ridge in the selected terrain surface. However, for all the methods there are some evident discrepancies on the shape of the mountain along the valley when we focus on the back contour line in the figure.

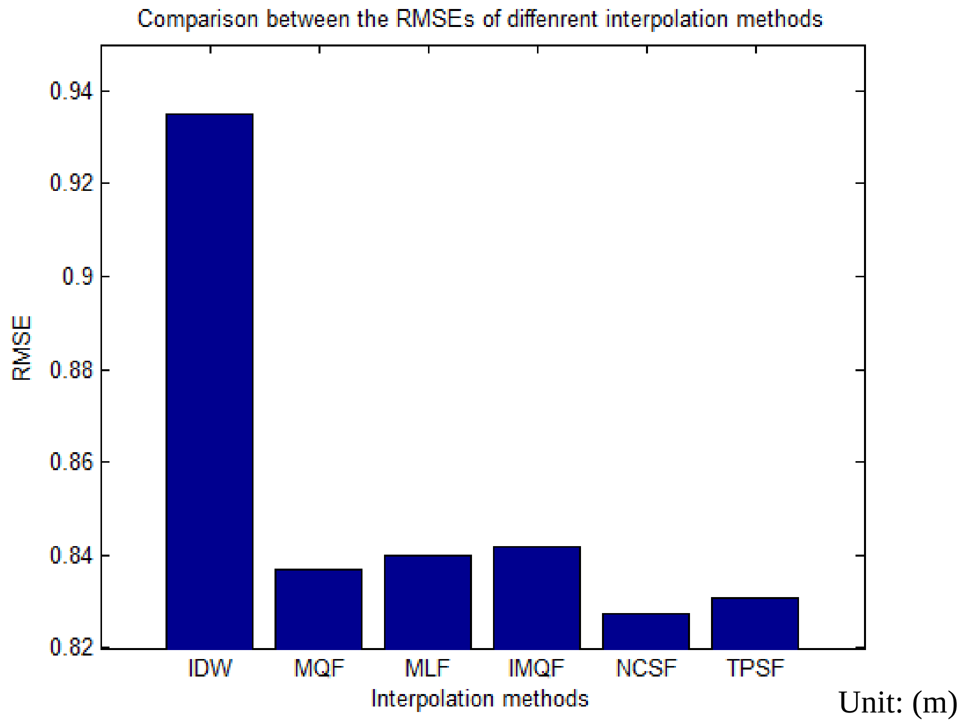


Figure6.2.9 Comparison between IDW and RBFs on the real-world terrain

A more intuitive result of the interpolation accuracy is presented in Figure6.2.9. IDW is the worst interpolator as we observed in this figure. All the selected basis functions in RBF provided the similar result. NCSF shows the best interpolation accuracy with the lowest RMSE value, and then followed by TPSF, MQF and MLF. The biggest RMSE value belongs to IMQF whose interpolation accuracy is the worst among the four selected basis functions.

6.2.4 Comprehensive evaluation

	IDW	MQF	MLF	IMQF	NCSF	TPSF
	RMSE(m)					
Mountainous	0.1605	0.14096	0.14094	0.14097	0.14087	0.14097
Flat	0.014	0.012581	0.012541	0.012512	0.012719	0.012637
Real	0.9348	0.8368	0.84	0.8418	0.8275	0.831

Table 6.2.2 Comparison of the RMSEs in all selected terrain surface

In table 6.2.2, we can see the comparison of RMSE for the selected interpolation methods working on mountainous, flat, and real-world terrain surface. IDW performs the worst in all the selected terrain surface, NCSF have the best performance on the mountainous and real-world terrain, however, it shows a worse result in the flat terrain among all the selected basis functions. For the flat terrain surface, IMQF is the most suitable one. It is also observed the RMSE values in real-world terrain are higher than those in mountainous and flat terrain. This is because the mean elevation in the real-world terrain is 13.8, higher than the mean values in mountainous and flat terrain which are 0.826 and 0.502 respectively.

	Error(m)	Mountainous	Flat	Real

IDW	Z min	-0.3651	0.0207	-3.75
	Z max	0.3796	-0.0386	5.25
MQF	Z min	-0.4058	-0.0322	-2.94
	Z max	0.2909	0.0179	5.1193
MLF	Z min	-0.4101	-0.0324	-3.0097
	Z max	0.2898	0.0168	5.1138
IMQF	Z min	-0.4129	-0.0323	-3.04
	Z max	0.2890	0.0174	5.1065
NCSF	Z min	-0.3905	-0.0302	-2.64
	Z max	0.2930	0.0179	5.03
TPSF	Z min	-0.3995	-0.0315	-2.84
	Z max	0.2921	0.0182	5.1065

Table 6.2.3 Comparison of the interpolation errors in all selected terrain

In table 6.2.3, we can find all basis functions have the problem of overestimating the elevation since absolute value of Z min is higher than that of Z max, but this phenomenon is reversed in the selected real-world terrain.

IDW	MQF	MLF	IMQF	NCSF	TPSF
Time consumption of the interpolation on real-world terrain (s)					
0.059	0.098	0.128	0.048	0.1185	0.1564

Table 6.2.4 Time consumption of the interpolation

Table 6.2.4 shows the time consumption of the interpolation on the selected real-world terrain, as we can see IMQF take the shortest time, only 0.048s, and the longest time belongs to TPSF, over three times the value of IMQF. IDW also takes a short time and one principal reason for this is we selected a small search radius.

6.3 Conclusions

The aim of our study was to compare the quality of different interpolation methods on the derived DTMs with different terrain surface. In Chapter 2 we gave a detailed description on especially on the data structure and visualization techniques since they play a role in the automatic terrain generation shown in Chapter 3 and the display of the results. In Chapter 4 and Chapter 5, we detailed the interpolation methods such as Inverse Distance Weighting, Multiquadric function, Multilog, Inverse Multiquadric, Natural Cubic Splines and Thin Plate Splines on three different terrain surfaces to give an overview of them . In Chapter 6,The data derived from each DTMs are divided into two subsets, the interpolation methods are applied in the first subset, and validation are done for the second subset. we also found the optimal smoothing factor in RBFs by means of Optimization Toolbox and decided a appropriate search radius in IDW from comparison, which means the obtained RMSE values that we compared were the best of each single interpolation method.

In the comparison, we have tested the interpolation methods on different terrain such as, a flat, a mountainous, and a real-world terrain respectively. .Furthermore, we have done a comprehensive comparison in order to find the performance of each single interpolation method on different terrain surface. According to the results, on the one hand, the results revealed that the magnitude and distribution of errors in a DEM were strongly related to the varying characteristics of the terrain, since with the variation of the elevations on one terrain surface, the errors of the interpolation points are varying. Besides, three selected terrain surface have big variations of the elevations and slopes among them, which corresponds to a significant variation on the RMSE values.

On the other hand, as we can see from the results, interpolation by NCSF proved to be more accurate than the other radial basis functions on mountainous terrain surface. On the contrary, NCSF performed the worst and IMQF performed the best on the flat terrain among all selected basis functions and IMQF also have corresponding best time consumption in the interpolation. From the comprehensive analysis, the classic Inverse Distance Weighted method proved to be less appropriate than all selected radial basis functions on the three terrain surface. MQF, MLF and TPSF have similar performance on three terrain surface.

The results obtained in this study allow us to observe the quality of the interpolation on DTM is related to such variables as terrain ruggedness and interpolation method, which can help us choose a appropriate interpolation method in order to obtain a good quality in the interpolation applied in digital terrain modeling. Concerning the future developments this works, further studies will be focused on the random sampling, and variation of sampling density, since in the process of data capture in digital terrain modeling, most of the data points are scattered and in this work the our data points were regularly distributed on a mesh grid. In addition, we were mainly focussed on the relationship between interpolation methods and morphology. With our further developments, the other factors related to the interpolation accuracy could be found which will lead to a better performance of the interpolation on DTMs.

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Appendices

Appendix A: List of symbols

Appendix B: List of acronyms

Appendix A

List of Symbols

Chapter 3:

Symbol	Description	Unite of measure
n	Number of iterations	-
zm	Initial heights	[m]
xm	“x” data corresponding to mesh	[m]
ym	“y” data corresponding to mesh	[m]
r0	Initial roughness of terrain	-
el	Initial elevation of terrain	[m]
rr	Roughness of roughness over terrai	-
hm	Height mesh	[m]

Chapter 4:

Symbol	Description	Unite of measure
$w(d_i)$	Weight function	-
$z(x_i)$	The elevation of all the points	[m]
d_i	The distance between each point and the interpolation point	[m]
n	Number of points within search radius	[m]
u	Power parameter	-
R	Search radius	[m]
x_0	Interpolation point	[m]
i	Number of sampling point.	-

Chapter 5:

Symbol	Description	Unite of measure
s	Radius basis function	-
p	A low degree linear ploynomial	-
ϕ	Basis function	-
λ_i	The RBF coefficients	-
x_c	The RBF central points	[m]
k_0, k_1	Constant and linear terms of polynomial	-
h	Elevations of sampling points	[m]
i	Number of sampling point.	-
A	The interpolation matrix	-
c	Smoothing factor	-

Chapter 6:

Symbol	Description	Unite of measure
i	Number of each elevation	-
$Z_i^{estimated}$	The altitude of interpolation point	[m]
Z_i^{real}	The altitude of original point	[m]
n	Total number of points	-
x_c	The RBF central points	[m]
Z min	Minimum error in the interpolation	[m]
Z max	Maximum error in the interpolation	[m]

Appendix B

List of Acronyms

Acronym	Meaning
DTM	Digital Terrain Model
GIS	Geographic Information System
DEM	Digital Elevation Model
IDW	Inverse Distance Weighted
RBF	Radial Basis Function
MQF	Multiquadric Function
IMQF	Inverse Multiquadric Function
MLF	Multilog Function
NCSF	Natura Cubic Splines Function
TPSF	Thin Plate Splines Function