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PUT-CALL-PUT
An alternative Option Selling Strategy

TESI DI LAUREA MAGISTRALE IN
MANAGEMENT ENGINEERING

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Abstract

Due to the turbulence experienced by major stock indexes, many investors have changed their expectations, becoming less concerned about going long to take advantage of the possible next bull market in equities and focusing more on a derivatives instrument. Insofar as expressed, our focus of has been in empirically validating some important studies according to which options listed in the markets, are systematically overpriced. After presenting the different pricing models, placing particular emphasis on the Black & Scholes model and of the role played by Implied Volatility within it, we investigated in more detail its significance. In fact, our study bases its existence on the interpretation that the financial world assigns to Implied Volatility during option pricing since it considers the latter as a good and reliable forecast of the future volatility that the underlying will actually have. Therefore, if we take this view of Implied Volatility as valid, we have focused on validating this assumption, since any mismatch between actual volatility and Implied Volatility is a symptom of actual mispricing in option prices.
In order to validate these studies, we constructed a strategy involving alternating sales of Cash Secured Put and Covered Call. We then carried out a portfolio analysis by first comparing this strategy (which we call PCP) in which option values on the SP500 index are derived from the markets using the Black and Scholes model and volatility is estimated with the VIX; in a second step, on the other hand, we downloaded the actual option data on an ETF that replicates the performance of the SP500 index – SPY - and applied our strategy by conducting an analysis on the returns comparing them with the benchmark.

Key-words: Implied Volatility; Black and Scholes; Cash Secured Put; Covered Call; Volatility Smile

Abstract in lingua italiana

A causa delle turbolenze registrate dai principali indici azionari, molti investitori hanno modificato le loro aspettative, diventando meno preoccupati di andare long per approfittare del possibile prossimo mercato toro delle azioni e concentrandosi maggiormente su strumenti derivati. Per quanto espresso la nostra attenzione di è focalizzata nella validazione empirica di alcuni importanti studi secondo i quali le opzioni quotate sui mercati, risultano sistematicamente sovraprezzate. Dopo aver presentato i diversi modelli di pricing, ponendo particolarmente risalto al modello di Black & Scholes e del ruolo compiuto dalla Volatilità Implicita all'interno dello stesso, abbiamo indagato più nel dettaglio sul suo significato. Infatti, il nostro studio basa la sua esistenza sull'interpretazione che il mondo finanziario assegna alla Volatilità Implicita durante le quotazioni delle opzioni, poiché considera quest'ultima come attendibile stimatore della futura volatilità che il sottostante effettivamente avrà. Pertanto, se diamo per valida questa visione dell'Implied Volatility, ci siamo concentrati a validare tale assunzione, poiché un eventuale mismatch fra l'effettiva volatilità e l'Implied Volatility è sintomo di un effettivo mispricning nelle quotazioni delle opzioni.

Al fine di validare tali studi, abbiamo costruito una strategia che prevede la vendita alternata di Cash Secured Put e Covered Call. Abbiamo successivamente effettuato una analisi di portafoglio comparando in un primo momento questa strategia (che definiamo PCP) in cui i valori delle opzioni sull' indice SP500 vengono derivati dai mercati utilizzando il modello di Black and Scholes e la volatilità viene stimata con il VIX; in un secondo momento invece abbiamo scaricato i dati reali delle opzioni su un ETF che replica l'andamento dell'indice SP500 – SPY - e abbiamo applicato la nostra strategia conducendo una analisi sui rendimenti comparandoli con il benchmark di riferimento.

Parole chiave: Volatilità Implicita; Black and Scholes; Cash Secured Put; Covered Call; Volatility Smile

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Introduction

Stock market predictability, portfolio allocation and derivative pricing are three prominent topics in modern finance. Since the inception of financial markets, many researchers have tried to carry out very extensive analyses and studies of their performance in order to identify possible links between the three variables. At the same time, investors have tried to structure investment strategies that seek to take advantage of the latter studies so that they can cope with the continuous evolution of financial markets, which becomes more and more unpredictable as time goes by.

To this end, over the past decades, many of the studies have focused on attempting to build mathematical models and algorithms that would try to predict the price movements of securities in the markets. In particular, given the great complexity in structuring a security and the great randomness to which it is subject due to the performance of the underlying asset, a large portion of these studies have focused on the same. The purpose of the latter is in fact, to analyze the pricing of derivatives in order to avoid any possibility of arbitrage, i.e., any "free-launches" that might occur in the markets.

Our study fits neatly into this context, as it seeks to combine the investor's and scholar's perspectives through empirical validation of theoretical results obtained through the development of an investment strategy on derivatives. Specifically, our study stems from the discussion in academia that options listed in the markets are not correctly priced because they are overpriced. This issue arises from the perception and consideration that the parameter of implied volatility has assumed over time. Since today it is widely accepted that the implied volatility - computed from the market price of an option through the inversion of Black & Scholes formula - is a good estimate of the market's expectations of the underlying asset's volatility, scholars perceive any discrepancy between the real actual volatility occurring on the market and the implied volatility as a sign of possible option's under- or over-estimation.

In order to empirically validate this hypothesis, we developed an investment strategy involving the sale of options, so that any mispricing arising from option prices could be exploited as much as possible. Specifically, the one we decided to propose with this study consists of a dynamic allocation since it involves the alternating sale of cash secured put or covered call options, depending on their actual exercise by the counterparty. Therefore, once the benchmark market was chosen, we proceeded to implement our proposed strategy over a time horizon of about 10 years, having as a reference that any increase in the performance of our strategy compared to the benchmark strategy, is a symptom of mispricing in option prices, and therefore an empirical confirmation to the issue.

Our strategy, stems from the more passive portfolio strategies was already proposed in the market, i.e., "Buy-Write" (covered call option writing) and "Put-Write" (cash collateralized shorting of put option) which had been applied to broad stock market indexes, S&P 500 Index (SPX). In 2002, Chicago Board of Option Exchange (CBOE) introduced S&P 500 Monthly Buy-Write Index (BXM) which can be used as a performance benchmark for related mutual funds, exchange-traded-funds (ETF) and other investment products. Subsequently, the CBOE S&P 500 Monthly Put-Write Index (PUT) was launched in 2007, followed by Ungar and Moran detailed analysis [1]. By capitalizing on a negative volatility risk premium by Bakshi and Kapadia through mechanical index option writing, standard Buy-Write and Put-Write strategies achieved on average better returns and reduced risks compared to the underlying S&P 500 Index [2].

Specifically, our strategy consists in the 'alternating selling of Put Options and Call Options according to their exercise and uses as a starting point the selling of the Put, which will therefore be followed by a series of alternating the two selling cycles. As can be deduced, the alternation of cycles (also called switching) also coincides with the alternation of the possession of the underlying and, therefore, in order to partially protect ourselves from market risk, we decided to use the proceeds from the sale of options.

The thesis work will be structured into four chapters, each of which will be the cog in a line of reasoning that will start with a description of the instruments used, proceed with a massive study of the case literature, and conclude with strategy definition and empirical testing on real data.

Proceeding in order, in *Chapter 1* we will discuss derivative instruments extensively and in detail, starting from their origin to how they are currently classified and used. During their presentation, we will particularly focus on the various models used to define their value, dwelling in depth on the Black and Scholes model and other more advanced pricing models.

In *Chapter 2*, on the other hand, we will accurately describe the role of Implied Volatility and its significance, having the copious literature present as a guiding canvas. Specifically, we will try to understand how the volatility smile works and the role it plays for option pricing, finally focusing on the problems of option pricing and how these mis-pricings can be used to construct selling strategies.

In *Chapter 3* we will get into the heart of our analysis by first studying option-selling strategies created by large financial institutions and represented by stock indices found on exchanges, such as the CBOE. We will then define and construct our proposed strategy, which will be called Put-Call-Put (CFP), deriving its operating formulas in two versions: one that we have called conservative in which we mitigate option selling by buying the underlying and a more aggressive one in which we sell as many options as possible.

Finally, in the *fourth* and final *Chapter*, we will go operationally to apply our strategy in the markets. We will first see how our strategy performs over a time horizon of about 10 years and having as source data, first option data on the SP500 deduced through the Black and Scholes model, then real data on an ETF that replicates the performance of the SPX index. Once the performance is obtained, we will go on to perform a series of evaluations and analyses considering risk-adjusted performance.

Finally, we will draw conclusions from our work trying to understand the critical points and try to give insights so that we can improve the strategy in order to optimize it. As such, a small part will be devoted to some possible avenues of analysis using simulators that can evaluate its performance considering real-time data and real transaction costs.

1. Derivatives

The use of Derivatives in Finance has been a well discussed topic among investors and academics since the 90s, entailing significant findings, consequences and benefits.

In the financial world, a derivative can be defined as *"a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. Very often the variables underlying derivatives are the prices of traded assets"*[3]. In other words, a derivative *is a financial instrument whose value depends on the performance of the price of the underlying asset, which may be real such as commodities and raw materials or financial such as stocks, bonds or financial indices but nowadays, it is possible to find derivative contracts on other types of asset class as Crypto Assets* [4].

The aim of this chapter is to provide a general overview of the different types of derivatives currently traded on the market and their main benefits, as well as the main pricing models used to assess their value. In fact, after a brief historical *excursus* on the main stages of their history, in the first paragraph we will present the main risks against which derivatives help to protect. Subsequently, in the second paragraph we will exhibit the main types of derivatives currently traded on the market and their main characteristics, distinguishing discrete-time models from continuous-time ones and analyzing European and American-style options. In particular, we will focus on the Black & Scholes model and on its usage for the computation of the Implied Volatility, proposing a brief overview of the main steps for its computation and the subsequent problem arising from it. Lastly, in the third paragraph, we will present the main pricing models used in order to value an option contract, highlighting the main differences between the commonly used Black & Scholes reference model and more advanced pricing models.

1.1 Overview

The derivatives market has its roots in the times of Ancient Greece in which it is said that a disciple of Aristotle thanks to forecasting atmospheric events was able to predict an exceptional harvest of olives and so bought the product before being harvested, being able to get big profits: this is one of the first forward contracts in history.

The emergence of the derivatives, as a financial instrument, dates back during the nineteenth century in America when farmers realized that finding buyers for the commodities had become a problem and to solve this it, they created a joint market called the "Chicago Board of Trade". A few years later, CBOT evolved into the first ever derivatives market where very customized contracts were traded directly by buyers and sellers which, due to their great success, became more standard contracts listed on the exchange which could be bought and sold by anyone. Such idea proved to be so successful that it led to derivatives being one of the most traded financial instruments in the market today. For this reason, soon Chicago Board of Trade had to create a spinoff called Chicago Mercantile Exchange to handle the growing business and in 2006 Chicago Board of Trade and Chicago Mercantile Exchange have been merged to form the CME Group which is still one of the most important derivatives markets in the world. *The massive success witnessed by the Chicago Board of Trade led to the creation of many similar exchanges around the world. However, during the era of the Chicago Board of Trade, derivatives trading was limited to commodities only and other financial instruments were largely outside of the realm of such trading* [5].

Despite this growth, some critics on such market remained. In 2002, the "Oracle of Omaha" Warren Buffet described the derivatives contract as *"financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal."* Such concept was reinforced by the same Warren Buffett a few years later, devoting a long section to the subject of derivatives in his 2008 annual letter. He has bluntly stated: *"Derivatives are dangerous. They have dramatically increased the leverage and risks in our financial system. They have made it almost impossible for investors to understand and analyze our largest commercial banks and investment banks."* Supporting such view is the fact that derivatives, specifically derivatives in the mortgage market, caused one of the greatest financial crises of all time in 2008 [6].

Although this financial instrument continues to be widely discussed and is difficult to understand, especially for retail investors, the most recent data from the Bank for International Settlements (BIS) estimated, for the first half of 2021, the total notional amounts outstanding for contracts in the derivatives market at $610 trillion which represents a growth of +34% year-on-year and +230% on a ten-year basis.[1]

Such fast growth in the value of derivatives is mainly due to their increased use among retail investors in their trading activity and an increase in the number of assets on which derivatives can be built. In fact, the introduction and use of derivatives essentially serve a twofold purpose: to provide investors with a flexible instrument for taking a position on the evolution of the underlying security, and to provide versatile instruments for hedging against risk. Although these two purposes appear contradictory at first glance, in reality, derivatives meet the needs of both investors seeking speculative instruments and those seeking protection against market risk. Indeed, there are three types of traders in derivatives markets: hedgers, arbitrageurs and speculators.

- Hedgers are those who are interested in a transaction to hedge against a business or market risk. The risk could exposure to a commodity, interest rate or currency;
- Arbitrageurs are those who are looking forward to an arbitrage transaction to take advantage of an incorrect price relationship that exists between a derivative and its underlying asset to which it relates;
- Speculators aim to profit from fluctuations in market prices. Speculators will take the opposite side of a hedging or arbitrage transaction.

All three of these groups come together to trade in derivatives markets. They all have different interests, market views and financial risk tolerances and, consequently, use derivatives to increase or decrease exposure to four common types of risk:

1. **Risk Coverage (hedging) or Risk Transfer** - it is intended to protect the value of a position from unwanted changes in market prices. The use of the derivative instrument allows neutralizing the adverse market trend, balancing the losses or gains on the position to be hedged with the gains or losses on the derivatives market;
2. **Speculation** - *strategies aimed at making a profit based on the expected evolution of the price of the underlying asset* [7];
3. **Market Efficiency** - the usage of derivatives implies fewer transaction costs (i.e., commission costs, trading costs). The derivatives market's success constitutes one of the key reasons which makes the financial markets more efficient. Indeed, borrowing and lending occur at a lower cost when derivatives

[1] https://www.bis.org/

are used, resulting in lower transaction costs. Large firms will have lower transaction costs in the securities market due to the large trade volume that is being undertaken [8];

4. **Arbitrage** – when a momentary misalignment between the price trend of the derivative and that of the underlying is exploited, it is possible to take advantage by selling the overvalued instrument and buying the undervalued one in order to gain a risk-free profit.

1.2 Typologies

Once presented the main reasons why derivatives are traded on the market, we will now look at the main types of derivatives currently traded on the markets and their chief characteristics. A first major distinction can be made about their symmetrical nature, since in the markets it is possible to find *symmetrical* and *asymmetrical* derivatives. In the first type, both contracting parties (buyer and seller) undertake to perform a service on the maturity date; vice versa, in asymmetric derivatives, only the seller is obliged to satisfy the buyer's will. In the latter category, therefore, the buyer paying a price (called premium), acquires the right to decide at a future date whether to buy (or sell) the underlying asset depending on what happens in the market. The buyer of a contract is commonly referred to as the Long Position, while the seller is the Short Position [9].

Next, it is possible to divide derivatives according to the type of their contract, so we will have three macro families:

1. Forward & Future
2. Swaps
3. Option

1.2.1 Forward & Future

A forward contract is an agreement between two parties who agree that at a certain date (expiration date) one of them will transfer a financial asset to the other at a fixed price. At the time the contract is signed, no money is transferred among the parties, but only the price that will be paid at maturity is agreed. There is no optional feature in the contract: both parties have the right and the obligation to respect the agreement. For this reason, a Forward is a symmetrical agreement[10].

The valuation problem for this type of contract is the determination at the time of entering into the contract of the price to be paid at maturity by the party acquiring the underlying. This price is determined based on the non-arbitrage principle. To be more precise, arbitrage in derivatives markets means the certainty of profiting from a price difference between a derivative and a portfolio of assets that replicates the derivative's cashflows [11]. Therefore, as mentioned above, derivatives are generally priced using the no-arbitrage or arbitrage-free principle: the price of the derivative is set at the same level as the value of the replicating portfolio, so that no trader can make a risk-free profit by buying one and selling the other. If any arbitrage opportunities do arise, they quickly disappear because traders, taking advantage of the arbitrage, will push the derivative's price up to match the value of replicating portfolios.

Futures are based on the same functioning as forwards in terms of payoff but the main difference concerns the market where they are traded. Indeed, forward contracts are not traded in the markets, but they are generally privately negotiated between counterparties and are therefore classified as Over-The-Counter (OTC) products. Futures, on the other hand, are widely traded on markets due to their highly standardized characteristics and have superior collateral requirements, particularly regarding counterparty insolvency, for such reasons they are considered less risky. Since futures' quantity and quality of the assets traded must be explicitly established, they are highly standardized derivatives, which is why they can be easily traded on exchanges; while Forwards are not standardized but are just a simple agreement between two parties on the trade of certain goods in the future. There is a great risk of default within the forward agreement, as one party may simply find itself in financial trouble and not be ready to meet its obligations. This risk is diminished in futures, as Clearing Houses[2] monitor and guarantee the financial arrangements between the parties making it almost impossible for one of the parties to default.[12].

The Future contract must always have a null value during its life. This is achieved through a complex mechanism based on the use of "guarantee deposits" known as "mark to market", which requires the party that is at a loss on the security deposit to pay the amount necessary to restore the zero value of the contract on a daily basis. Unlike the forward contract, the gain or loss relating to a future position is liquidated daily rather than in a single final solution. At maturity, the underlying is delivered, and the agreed price is paid, taking into account the debit / credit balance of the guarantee deposit. In practice, the closure out of futures positions seldom takes place at natural maturity with the delivery of the underlying: investors usually prefer to close positions earlier by trading the opposite sign of the original contract.[10].

[2] Market organ that is an automatic and specular counterparty (seller to the original buyer and buyer to the original seller) of all contracts entered in a market, in order to limit the risk of non-fulfillment of transactions – from Borsa Italiana Glossario

In the graph below, it is possible to see the payoff-path of two futures for a Long or Short position investor. Along the horizontal axis is reported the trend of the price of the futures contract, which changes according to the market condition, while along the vertical axis are represented the potential payoff based on the price. As can be seen from the graph in the Long position case, the gain will be positive if the price of the future (and therefore of the underlying) will rise while it will be negative in the opposite case; a completely specular reasoning can be considered in the case of the short position of the contract.

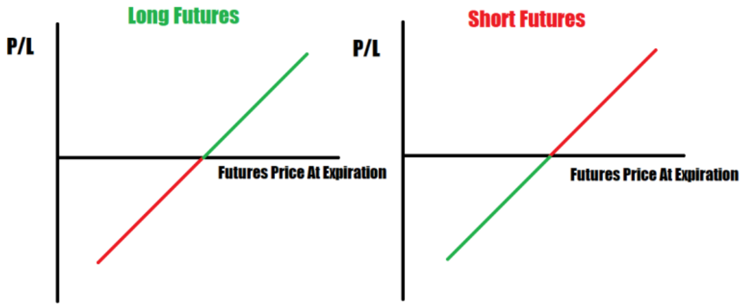

Figure 1.1 Futures' Payoff

The chart represents the trend of the futures' payoff according to the different level of price at maturity. Indeed, the chart is obtained putting on the abscissa axis the price at maturity of the underlying asset and on the ordinate axis the profit or loss.

1.2.2 SWAP

A swap is a contract that allows two counterparties to exchange cash flows in the future according to certain predetermined rules, which govern the classification by type of swap contracts. Swaps are usually drawn on a set of reference dates that are periodic, e.g., weekly, or monthly. A swap can, therefore, be shown as a portfolio of forward contracts, so a forward contract is a single swap contract with a single payment date. The most common swap is the Interest Rate Swap (IRS) in which two counterparties decide to agree to exchange, on predetermined future dates and up to a certain maturity, cash flows calculated by applying different interest rates to a predetermined sum. A party of the contract may be interested in an IRS to eliminate the uncertainty of a debt contracted at variable rates (hedging).

Currency Swap, instead, is similar to the IRS but introduces an additional variable: the currency one. In fact, in this contract, the two counterparties exchange cash flows in different currencies calculated using interest rates applied to two notional capitals denominated in the two currencies. In such case, this type of contract is used to have a currency hedge as not to be exposed to "exchange risk" i.e., the risk that one currency will be depreciated compared to the other one.

1.2.3 Option

When the New York Stock Exchange opened in 1791, it wasn't long before a market for stock options began to emerge among savvy investors, but a centralized marketplace for options didn't exist. Indeed, options were only traded OTC facilitated by broker-dealers who tried to match option sellers with option buyers. Each underlying stock strike price, expiration date and cost had to be individually negotiated and broker-dealers began placing advertisements in financial journals.

After the stock market crash of 1929, the US Congress decided to intervene in the financial marketplace. They created the Securities and Exchange Commission (SEC), which became the regulating authority under the Securities and Exchange Act of 1934.

In 1968, low volume in the commodity futures market forced CBOT to look for other ways of expanding its business. It was decided to create an open-outcry exchange for stock options, modeled on the futures trading method. Thus, the Chicago Board Options Exchange (CBOE) was created as a spin-off entity of the CBOT.

On opening day, the CBOE only allowed trading of call options on a 16 underlying stocks. However, a respectable 911 contracts changed hands and by the end of the month, the CBOE's average daily volume exceeded that of the over-the-counter option market.

The next major event was in 1983, when index options began to be traded. This development proved crucial in helping to fuel the popularity of the options industry. *The first index options were traded on the CBOE 100 index, which was later renamed the S&P 100 (OEX). Four months later, options began to be traded on the S&P 500 Index (SPX). Today, there are more than 50 different index options and more than 1 billion contracts have been traded since 1983* [13].

An option is a contract which allows one of two parties, without being obliged, to buy or sell the underlying security at a price set at the time of entering into the contract at a predetermined time or within a predetermined period. The other party is obliged to comply with the agreement. For such reason, options are asymmetric financial instruments since the parties involved haven't the same rights. Various types of option contracts are traded all over the world and one of the most common distinctions is between Plain Vanilla Option and Exotic Option where the main difference lies in their ability to be tailored to the investors' needs. Exotic Options are more customizable since they have more complex features and are generally traded OTC like Barrier Options, Asian Options and Digital Options. They can be combined into complex structures in order to reduce the net cost or increase leverage. Due to their high complexity and customization, such instruments won't be necessary for the purposes of our study.

Plain vanilla options, on the other hand, are the most basic version of an asymmetrical financial instrument as they do not have any special features as exotic ones, and for such reason they are often associated with low risk compared to exotics. Thus, they just give the right to their owner to buy (call option) or sell (put option) certain underlying assets at a prearranged price (exercise or strike price) at some point in the future. If we say that the owner can execute their right in every moment before the expiration date (maturity), then we are talking about American Style options. On the other hand, if the option right can be executed only at the maturity date, they are called European Style Options.

Unlike the futures contract, which is a symmetrical contract, an option is an asymmetrical contract since the option holder has the right, but not the obligation, to buy (or sell) an asset according to his convenience. *Consequently, in order to re-establish the symmetry of the agreement, the party that is long in the contract (i.e. the party that can exercise the right) must pay a premium, so a compensation, to the other party* [14].

The Options can be Call, if the long party in the contract acquires the right to buy a specific asset at a predetermined price on the expiry date or can be Put, if the party is buying the right to sell the underlying asset at the expiration date. In addition to the strike price (i.e., the price that will be paid in the future for the purchase/sale of the underlying asset) and spot price (current price of the underlying on the market), we can identify another relevant parameter for the study of options: Moneyness.

Moneyness describes the intrinsic value of an option in its current state, since it tells option holders whether exercise will lead to a profit if the option is exercised immediately [11]. There are many forms of moneyness, including in-, out- or at-the-money, and there are different ways of calculating moneyness according to the parameters employed. The most used method by academics, which will be the same as the one used in our study, is to consider the ratio between Strike Price (K) and Spot Price (S). In this case, we will have three possible moneyness according to the result of their relationship and whose nomenclature will depend on the type of option that we are analyzing. In fact, in the case of Call option, we will have that the moneyness will result:

- At-the-Money (ATM) when $\frac{K}{S} = 1$
- Out-the-Money (OTM) when $\frac{K}{S} > 1$
- In-the-Money (ITM) when $\frac{K}{S} < 1$

It is also typical to consider options with strikes around the Spot Price "near-at-the-money", while the further away from the strike we may find in "deep-in or out-of-the-money" situations, respectively.

Call OTM options are usually bought in bullish markets as the price of the underlying is expected to rise, while Call ITM options in bear markets as a lower future price is expected.

In the case of Put options, since the long position in the contract has the opposite right to buy, i.e., the right to sell a specific asset at a pre-determined price, the signs of the inequalities for ATM and OTM will be reversed with respect to call options; in fact, we will have that the moneyness in the case of put options will be:

- At-the-Money (ATM) when $\frac{K}{S} = 1$
- Out-the-Money (OTM) when $\frac{K}{S} < 1$
- In-the-Money (ITM) when $\frac{K}{S} > 1$

Obviously, in the financial markets on options you can also have a short position (or write option): in this case, the investor sells the right to buy (or sell) certain underlying asset at the expiration date. As the selling party, the investor no longer has the right but the obligation to fulfill the contract in case of exercise by the long counterparty.

The main difference in the position of the option contract concerns the future payoff. While in the case of Long option position, the loss is limited to the premium paid, in the case of Short option position there are no loss limits, so the loss could potentially be "infinite" depending on how the price of the underlying moves; in the case of a gain on the other hand, while in short position the maximum payoff is the premium received, in long position the investor is positively exposed to risk since his gain, given by the difference between the strike price and the spot price at expiration date, could potentially be equal to the strike price.

$$\pi_{Long\ Call} = Max[0; S - K]$$

$$\pi_{Short\ Call} = Min[0; K - S]$$

$$\pi_{Long\ Put} = Max[0; K - S]$$

$$\pi_{Short\ Put} = Min[0; S - K]$$

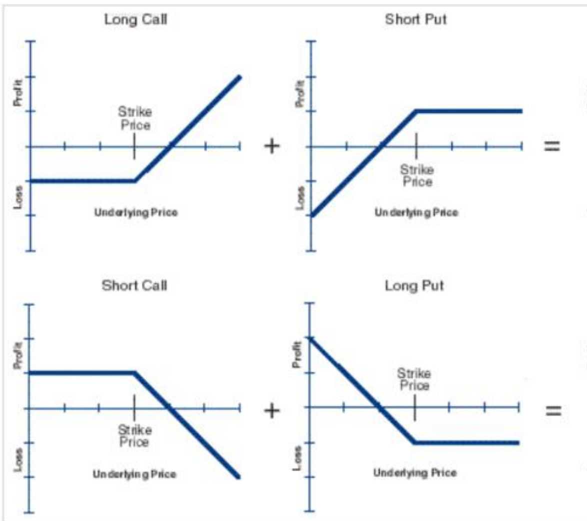

Figure 1.3 Options' Payoff

The graph represents how the return on options varies, depending on the different values of the underlying at expiry, the type of option and the position assumed in the contract. On the x-axis there is the price parameter of the underlying at expiration, while on the y-axis is the corresponding gain or loss according to the letter. We therefore see that the top left-hand graph represents the case of a call option in a long position. At the bottom, on the other hand, we always find call options, but in a short position, while on the right the two case histories of puts

1.3 Option Pricing Models

The central problem in the valuation of derivatives is to quantify the compensation (price) that one party owes to the other at the time the contract is concluded in such a way that neither party can engage in arbitrage. While in the case of forward or future contracts the principle of non-arbitrage is sufficient to determine the price, in case of options the identification of the non-arbitrage price is a more complex operation.

The mere assumption of the absence of arbitrage opportunities defines some restrictions on the price, but not the precise value at which the option should be traded on the market. From this hypothesis it is possible to obtain constraints which are valid independently of the model used for the evolution of the underlying security. *In particular, the first non-arbitrage relationship is defined "Call Pricing Relationships" and states that if there are no dividends prior to expiration, then to prevent arbitrage opportunities, the call price should never fall below the maximum between zero and the difference given by the current underlying spot-price less the present value of the strike*[15]. That from a mathematical point of view, it becomes:

$$C \geq \; max[S(0) - Ke^{-rT}; 0]$$

Evidence of this is deductible by considering two portfolios, A and B. A contains one European call option and K pure discount bonds with a face value of $1 each and maturity T. B contains a long position in the stock. Table 1.1 illustrates the prices of the two portfolios at the expiration date of the option. Note that the future value of portfolio A is never lower than the future value of portfolio B

Portfolio	*Current Value*	$S_T < K$	$S_T > K$
A	$C + Ke^{-rT}$	$0 + K$	$(S_T - K) + K$
B	S_0	S_T	S_T
Value		$V_A(T) > V_B(T)$	$V_A(T) = V_B(T)$

Table 1.2 Call Pricing Relationships Portfolios

If an investor bought portfolio A and sold portfolio B, then, at the expiration date, the combined portfolio, P, would have value $V_p(T)$, given by $V_p(T) = V_A(T) - V_B(T)$, where $V_A(T)$ and $V_B(T)$define the values of the portfolios A and B at time T.

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If the call option expired in the money, then $V_A(T) = V_B(T)$ and, hence, $V_p(T) = 0$. However, if the call expired worthless, then, $V_p(T) = V_A(T) - V_B(T) \geq 0$. The portfolio, P, thus, can never lose money and has a chance of making money. Let us now consider the initial cost of the portfolio, $V_p(0)$. Since this portfolio has a nonnegative terminal value, it must be worth a nonnegative amount now. Hence $V_p(T) = V_A(T) - V_B(T) \geq 0$ equivalently, $V_A(T) \geq V_B(T)$. Substituting for $V_A(0)$ and $V_B(0)$, leads to the final inequality:

$$C \geq S(0) - Ke^{-rT}$$

In the case of a Put a completely analogous reasoning can be made. Therefore, starting from the requirement that put price should never fall below the maximum, it is possible to obtain the non-arbitrage "Put Relationship":

$$P \geq Ke^{-rT} - S(0)$$

The absence of arbitrage opportunities is therefore sufficient to define certain restrictions that the price of an option must satisfy, but in order to define its precise value, it is necessary to choose a model which describes the evolution of the price of the underlying security. To this end, we will look at some of the most commonly used pricing models on the markets.

The main pricing methods can be divided into two macro categories: continuous models, such as Black & Scholes, and discrete models, such as Binomial Tree. The former replace that the price of the underlying undergoes continuous variations, while the latter change that the variations occur in precise time frames and that price is the unchanged price between two successive instants. Even though the two types of models differ conceptually, the fundamental elements considered by both for option pricing are the same and are:

- *Maturity* – the residual life of contract - T;
- *Volatility* - σ;
- *Underlying Price* - S;
- *Exercise Price* – K;
- *Interest Rate* - r;
- *Dividends* - D.

These factors have a different impact on options, an impact that can be positive or negative depending on whether it is a call or a put. These factors have a different impact on options, an impact that can be positive or negative depending on whether it is a call or a put. In the following pages, the different types of pricing methods and their characteristics will be explained.

1.3.1 Binomial Model

For the purpose of determining the price of a contract in a precise way, it is necessary to introduce a model for the evolution of the price of the underlying. A particularly interesting finite-state market model is the one that predicts the evolution of the price according to a **binomial tree**.

This model has the advantage of being easily extensible over several instants of time and of providing an approximation of the lognormal model, which is the basis of the Black & Scholes analysis that we will address later.

We consider two stocks: a risky one (Stock), and a risk-free one (Bond), and their market value at time t is $S(t)$ and $B(t)$ respectively. Both securities have a market value that varies over time, but while the value of the bond varies in a deterministic way, Stock is characterized by a stochastic evolution.

Initially we assume two instants of time $t = 0, 1$: the prices will be $S(0)$, $S(1)$, $B(0)$ and $B(1)$. The bond is characterized by a risk-free return r_f: the price goes from $B(0)$ in $t = 0$ to $B(0)r_f$ in $t = 1$; the stock at $t = 1$ can take only two values $S(1) = S(0)u$ and $S(1) = S(0)d$ with probability equal to p and *(1-p)* respectively. u stands for UP and d for DOWN and they represent an increase and decrease in the percentage price:

$$d < 1 < u$$

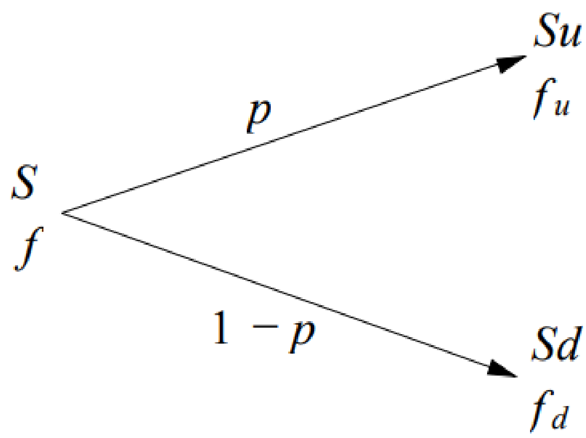


Figure 1.3 Binomial Tree Model: First Fork

In the following we set $B(0) = 1$. The absence of arbitrage opportunities leads to restrictions on the r_f, d and u parameters of the binomial model.

In fact, the model does not allow arbitrage opportunities if and only if the model parameters satisfy the condition:

$$d < r_f < u$$

If it is true than it means that a risk-less investment can be better, worse or even as well as a risky investment. If this is not true, then the risky asset is not risky at all.

In fact, if absurdly $d < u \leq r_f$, one would always prefer the risk-free asset to the risky asset because the yield of the bond is better than that of the stock in both states of the world. Let's assume that $S(0) = B(0) = 1$ an arbitrage can be constructed simply by buying a risk-free stock and short selling the stock:

Portfolio	$t = 0$	$t = 1\ up$	$t = 1\ down$
	$B(0) - S(0)$	$r_f - u$	$r_f - d$
Value	$V(0) = 0$	$V(1) \geq 0$	$V(1) > 0$

Table 1.2 First arbitrage portfolio construction

In the case $r_f \leq d < u$ an arbitrage could be created just changing operations' sign buying Stock and Short Selling the bond:

Portfolio	$t = 0$	$t = 1\ up$	$t = 1\ down$
	$S(0) - B(0)$	$u - r_f$	$d - r_f$
Value	$V(0) = 0$	$V(1) > 0$	$V(1) \geq 0$

Table 1.3 Second arbitrage portfolio construction

Therefore, we can say that the market is complete if the condition $d < r_f < u$ is satisfied and therefore there is a single **risk-neutral probability** measure:

$$\pi_u = \frac{r_f - d}{u - d} \ ; \pi_d = \frac{u - r_f}{u - d}$$

Therefore, using the risk-neutral probability measure, it is easy to verify that the price of the security, in absence of arbitrage opportunities, as the expected value of its payoff discounted through the risk-free interest rate:

$$E[S(1)] = \pi_u S(0) u + \pi_d S(0) d$$

Which with the necessary calculations, we obtain

$$E[S(1)] = r_f S(0)$$

Considering the problem of evaluating a derivative security which in $t = 1$ returns a payoff equal to $F(S(1))$ we will have:

$$F(S(0)) = \frac{1}{r_f} E[S(1)] = \frac{1}{r_f}\left[F(S(0)u)\frac{r_f - d}{u - d} + F(S(0)d)\frac{u - r_f}{u - d}\right]$$

The value of the derivative in $t = 0$ is nothing more than the expected value discounted according to the probability measure neutral to the risk of $F(S(1))$.

The non-arbitrage price (**rif. Formula .)** and related risk-neutral probabilities (**rif. Formula .)** can also be constructively obtained in two ways using market strategies:

- **Delta-Hedging Strategy**: for an individual who has a derivative, we determine the quantity of Stock to hold (or to short sell) that protects him from the fluctuations of the Stock, we obtain a portfolio consisting of a derivative security and a quantity of the Stock that is not affected by fluctuations of the Stock (market risk).
- **Replicating Portfolio**: this path allows an individual, who has undertaken to deliver a derivative from the $F(S(1))$ payoff in $t = 1$, to replicate it and then obtain it synthetically through a portfolio consisting of Stocks and Bonds.

Binomial Model in Multiperiod

Anything we have analyzed up to now is the case in which we have a starting state and a single bifurcation of the binomial tree in the following period; the model can be generalized to the case in which the time interval between the moment in which the contract is stipulated and the moment in which it expires consists of several periods of time, in each of which the two titles that make up our model market share as in the previous case, i.e. the stock follows a Bernoulli probability distribution.

Let's start by considering a binomial model in two time periods: $t = 0,1,2$. At each step, the risky security S can increase by a factor u or decrease by a factor d; We always assume that the return on the risk-free security is r_f. The evolution of the stock can be represented through the binomial tree as in Figure 1.3, while the evolution of the Bond is

$$B(t) = r_f^t, t = 0,1,2$$

The absence of arbitrage on this market requires that the condition will be satisfied at every step and therefore the existence and uniqueness of the risk-neutral probability measure provided that $d < r_f < u$.

To extend the non-arbitrage valuation to the multiperiod case, we build a recursive algorithm, i.e., we start from the final maturity $t = 2$.

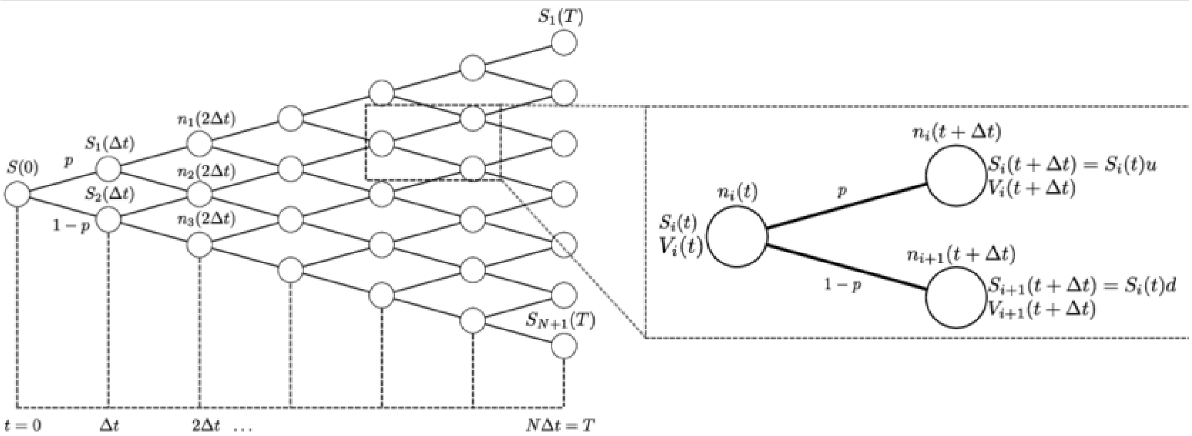


Figure 1.4 Binomial Model Multiperiod

The figure represents the extension of the binomial model to several periods, so the bifurcations grow exponentially. Specifically, a zoom on a generic node is shown in order to highlight the general writing of the model in its multi-period form

In $t = 2$ the value of the derivative is defined by its "payoff": starting from this value, we calculate the value of the derivative in the previous instants, proceeding backwards in time one step at a time. In $t = 2$, the price of the stock can assume three possible values depending on whether the price has risen twice in a row, whether it has fallen twice in a row and whether it has risen once and fallen the other (the order does not matter, since the final value will be the same in both cases): $S\,(0)uu, S(0)dd, S(0)ud$. Consequently, the derivative can take on three values: $F\,(S\,(0)uu)$, $F\,(S\,(0)dd)$, $F\,(S\,(0)ud)$. The non-arbitrage price of the $t = 1$ derivative can be obtained starting from the final values proceeding as in the one-period case:

$$F(S(0)u) = \frac{1}{r_f}[F(S(0)uu)\pi_u + F(S(0)ud)\pi_d]$$

$$F(S(0)d) = \frac{1}{r_f}[F(S(0)ud)\pi_u + F(S(0)dd)\pi_d]$$

Given derivative's value in $t = 1$ $F\,(S\,(1))$, the non-arbitrage value in $t = 0$ can be obtained by proceeding in the same way

$$F(S(0)) = \frac{1}{r_f}[F(S(0)u)\pi_u + F(S(0)d)\pi_d]$$

and replacing the values $F(S(0)u)$ *and* $F(S(0)d)$ obtained in the previous step

$$F(S(0)) = \frac{1}{{r_f}^2}{\pi_u}^2[F(S(0)uu) + F(S(0)dd){\pi_d}^2 + 2F(S(0)ud)\pi_u\pi_d]$$

Since ${\pi_u}^2$ is the risk neutral probability that the value of the underlying increases twice, ${\pi_d}^2$ decreases twice and $\pi_u\pi_d$ decreases and then increases, which is equivalent to first increasing and then decreasing, even in the case of two periods we obtain a valuation formula risk neutral. The price of the derivative can be written as follows:

$$F(S(0)) = \frac{1}{{r_f}^2} E[F(S(2))]$$

The initial value of the derivative is represented once again as the discounted expected value with respect to the risk-neutral measure. Proceeding in this way by means of the recursive algorithm, we obtain by induction a formula that provides the non-arbitrage value of the derived title in $t = 1$ $F(S(0))$ for a binomial model with T periods of unit length. The binomial model in the case of a European derivative with payoff $F(S(T))$ and with generic T we have that

$$F(S(0)) = \frac{1}{{r_f}^T} E[F(S(T))] = \frac{1}{{r_f}^T}\left[\sum_{k=0}^{T}\binom{T}{k}{\pi_u}^k{\pi_d}^{T-k}F(S(0)u^k d^{T-k}\right]$$

Where T are the periods involving k moves of the underlying stock up and T-k down.

American-Style Options

After the valuation of the European Options with the multi-period model, an important aspect is to try to evaluate the American Options whose value is necessarily not lower than those of the European Options since they guarantee the holders the right to exercise before the expiry date.

The fundamental difficulty that arises in evaluating this type of options is that, since the instant in which they are exercised is not known, it is not possible to know ex-ante the discounted payoff to be included in the expected value formula. The risk-neutral valuation principle, a consequence of the absence of arbitrage opportunities, can also be used in this case, for which the value of the option is still equal to the expected discounted value calculated with respect to the risk-neutral probability measure:

$$F(S(0)) = \frac{1}{r_f} E[S(\tau)]$$

where τ is the instant of exercise of the option; the problem is that this time τ , $t \leq \tau \leq T$ is not known at the time of valuation. In the case of an American call option with an underlying that does not pay dividends, early exercise is never optimal. This result implies that the price of an American Call is identical to that of a European Call; therefore, under the same hypotheses the problem arises only in the case of a Put.

The method for the valuation of an American contract proceeds backwards in time: in each instant of time t_k, assuming we know the price of the derivative in the two states of the world t_{k+1}, we proceed to calculate the value of an arbitrage in the hypothesis in if the American option is not exercised, this value is compared with the payoff obtained in the case of early exercise. The decision whether or not to exercise the option in t_k arises from which of the two options chosen guarantees the higher value. The algorithm works backwards as the contract payoff in T is known as $([K - S(T)]^+)$.

The value of the option in any case in t_k will therefore be given by the following recursive expression defined backwards in time:

$$P_k^A(i) = max[K - S(0)u^i d^{k-i}; \frac{1}{r_f^{\Delta t}}(\pi_u P_{k+1}^A\,(i+1) + \,\pi_d P_{k+1}^A(i))]$$

$i = 0,1,2,\dots,k$ where we have agreed to indicate with the index i the position within the tree, given i establishes how many times the stock has undergone an upward variation: if at the time t_k we had i upward variations, at the time t_{k+1} $i+1$ upward changes or i upward changes can be observed.

Using this notation, the value of a European Put in t_k can be written recursively as follows:

$$P_k^E(i) = \frac{1}{r_f^{\Delta t}}(\pi_u P_{k+1}^E\,(i+1) + \,\pi_d P_{k+1}^E(i))$$

The expression for the calculation of the value of the American option evaluates the opportunity of early exercise that, if this exercise is not advantageous, the price of the option is equal to that which the scheme provides for the corresponding European option; if the early exercise is instead convenient, it assigns the value of its payoff to the option. Therefore, the value at the final instant $k = M - 1$ proceeds recursively backwards in time until the value at the initial instant $k = 0$ is obtained [10].

1.3.2 Black and Scholes (Merton) Model

The most famous and general option pricing model was developed in the early 1970s by Fisher Black and Myron Scholes. Originally this model was formulated to price European-type financial options, and, from the first version, it has contributed and influenced all subsequent pricing models. An important contribution to the definite development of the Black and Scholes model goes undoubtedly to Merton who, based on the 1973 version, made changes and improvements [16].

The valuation of a derivative security requires making assumptions about the evolution of the price of the underlying security. If using the alternative binomial tree model, it considers an evolution of the price in the discrete, in the Black & Scholes can be considered a time continuous model and therefore the evolution of the underlying security's price is described through a stochastic process that has Wiener process as its basic element, with mean and variance known and constant over time.

The origin of such theoretical notions under Black and Scholes come from the physics world since the Brownian motion consists in a mathematical model used to describe the random movement of particles which is taken as a reference and comparison to describe the asset price evolution.

By constructing a Wiener process as the limit of a binomial random walk and in its standard form we obtain:

$$X(T) = \sigma W(t)$$

$X(t)$ is distributed as a Normal Distribution with zero mean and variance $\sigma^2 t$, $X(t) \sim N(0, \sigma^2 t)$. Alongside the process in the Standard form, Brownian motion with drift is usually introduced:

$$X(T) = \mu t + \sigma W(t)$$

In this case, a linear deterministic component in time is added to the Brownian motion ($X(t) \sim N(\mu t, \sigma^2 t)$).

The use of a Wiener process to model the evolution of prices has some drawbacks. In the first place there is a problem of sign: the process can assume negative values. The hypothesis that the price dynamics of the underlying asset is mathematically modeled through a geometric Brownian motion is fundamental, as it prevents the price from becoming negative. Furthermore, the fact that the logarithm of the ratio is normally distributed, rather than the difference between successive prices, can be seen as a way of describing the dynamics of the price not in terms of absolute changes, but rather in terms of relative changes. In order to simplify, the percentages of variation, instead of the price variations, are normally distributed.

$$ln\left(\frac{S(t)}{S(0)}\right) = X(T) = \mu t + \sigma W(t)$$

In addition to this assumption on the wiener process, Black and Scholes is based on the Perfect Market hypothesis, therefore it assumes a perfectly competitive market. This means that operators are considered not able to influence the price of the securities with their operations and the presence of frictionless market so there are no transaction costs, taxes and it is possible to sell short without any penalty; it can be bought and/or sold in inhabited and infinitely divisible quantities at a constant interest rate r, which coincides with the rate of return of fully capitalized securities (zero coupon bond). Furthermore, as already mentioned, there is no risk arbitrage. Lastly, it is assumed that no dividends will be distributed during the life of the contract.

Having such assumption as base, we can derive

$$S(t) = S(0)e^{X(T)} = S(0)e^{\mu t+\sigma W(t)}$$

What follows, therefore, will jump directly to the now classic formula and the way in which it is applied, without developing the more complex issues that affect stochastic integration and the famous Ito formula.

The BS model assumes as a hypothesis that the risky security has a lognormal stochastic evolution. The market is made up of two securities, one risky and one risk-free, their dynamics are of the following type:

$$\begin{cases} S(t) = S(0)e^{\mu t+\sigma W(t)} \\ B(t) = B(0)e^{rt}, B(0) = 1 \end{cases}$$

The price of the risky security therefore satisfies the stochastic differential equation

$$dS(t) = \left(\mu + \frac{\sigma^2}{2}\right)S(t)dt + \sigma S(t)dW(t), S(0) = S_0$$

According to the fundamental theorem of asset pricing, the arbitrage price of a European derivative is given by:

$$C_{BS} = e^{-rt}E[max[S(T) - K; 0]]$$

Given these values, Black and Scholes show that, in the presence of a geometric Brownian stochastic process (the stochastic process that corresponds to the hypothesis of lognormality of the instantaneous distributions of the reference variable), the following result is obtained for a Call Option:

$$C_{BS} = S(t)N(d_1) - Ke^{-r(T-t)}N(d_2)$$

$Ke^{-r(T-t)}$ is the present value of the Strike Price while $N(d_i)$ indicates the distribution function of the Standard Normal and the arguments d_1 is the first parameter of probability i.e., ''the factor by which the present value of contingent receipt of the stock, contingent on exercise, exceeds the current value of the stock'' and d_2 is the second parameter of probability which represents the risk-adjusted probability of exercise:

$$d_1 = \frac{ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{ln\left(\frac{S(t)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

For European options, without arbitrage opportunity, the so called *put-call parity* relationship applies that assign the link between the prices of a pair of put and call options. Let's consider the following portfolios:

- Portfolio A – Long Call + Bond
- Portfolio B – Long Put + Underlying

Assuming both options are written on the same stock with price S, have strike price K and maturity T, and let r be the risk-free interest rate. It is also assumed that at maturity T the bond gives the right to a payment equal to K. Let us also assume, for simplicity, that the underlying does not pay dividends.

Both wallets are worth:

$$max[S(T); K]$$

upon expiry of the options. We denote with P_{BS} and C_{BS} the current price, respectively, of the put and call. As both options cannot be exercised before expiration, in the absence of arbitrage both portfolios must have equal value throughout the life of each option and, therefore, also at current time. The following relationship therefore holds:

$$C_{BS} + Ke^{-r(T-t)} = P_{BS} + S(t)$$

Such relationship shows that the value of a European put (call) can be deducted from the value of a European call (put) with the same maturity and strike price, and vice versa.

And applying BS we lastly deduct the price of a Put Option:

$$P_{BS} = Ke^{-r(T-t)}N(-d_2) - S(t)N(-d_1)$$

How we can notice the price of a European Call Option according to Black & Scholes depend by six parameters:

$$C_{BS} = C_{BS}(t, S, K, T, r, \sigma)$$

In the model t and S are considered exogenous which affects the model itself but is not affected by the relationships in it; variables, r and σ parameters of the model that must be estimated from market observations; the other two parameters (K and T) are defined in terms of the contract.

American-Style Options

With regard to the way in which Black & Scholes model behaves for the pricing of American Options, whose value is greater (or equal) to that of European ones, since it guarantees the holder greater powers.

If we assumed the cost of an American Put equal to a European type, for some values of S we would find ourselves in the presence of arbitrage given that the price predicted by Black & Scholes is lower than the payoff value derived from the exercise of the option ($P_{BS} < K - S$).

To frame the problem of valuation of American Options it is necessary to include the constraint that the option model is always greater than or equal to the exercise payoff:

$$P(S(t); t) \geq [K - S(t)]^{+}$$

It all boils down to tackling a differential problem known in the literature of partial derivatives as a "free boundary problem". We must deal with a free-boundary problem whenever the solution of a differential equation is subject to a constraint, in this case to space-time, for which the solution and the constraint come into contact and therefore the point where one or more than one boundary condition is to be imposed. The point at which the solution of our problem and the constraint required by the principle of absence of arbitration come into contact (S_f) is that at which the following equality holds:

$$P(S_f; t) = [K - S(t)]^{+}$$

Since P is the price of a derivative, and therefore always positive, the preceding condition is equivalent to

$$P(S_f; t) = [K - S(t)]$$

However, such condition is not sufficient to uniquely determine the price of the American Put Option, as the satisfaction point of this boundary condition is not known. To determine this further unknown, an additional condition is required, which involves the regularity of the solution itself at the point of contact. The condition requires that the solution and the contact assume not only the same value at the point where they "weld", but that they also have the same derivatives. This condition follows from qualitative considerations always linked to the hypothesis of absence of arbitrage.

Limits of the model and role of Volatility

The market model underlying the analysis carried out by Black and Scholes has been very successful in application due to its simplicity and the fact that it allows explicit valuation formulas for many derivative securities.

While the Black and Scholes model is widely used, there are not a few limitations to it. First, it assumes continuous and costless trading, ignoring the impact of liquidity risk and brokerage charges; Second it presumes stock prices to follow a lognormal pattern, e.g., a random walk (or geometric Brownian motion pattern), thus ignoring large price swings that are observed more frequently in the real world; thirdly supposes no dividend payout, ignoring its impact on the change in valuations.

Lastly, the main limitations of the model consisting of estimating some input parameters, indeed, the Black & Scholes' parameters are for the most part observable, but there are other parameters, r and σ, that are unobservable. But if the former turns out to be easily approximated by the interest rate of a risk-free bond, the second is more difficult to approximate. Unusually, historical volatility is used to approximate the σ value in the formula, but there is nevertheless a relevant practice that is closely

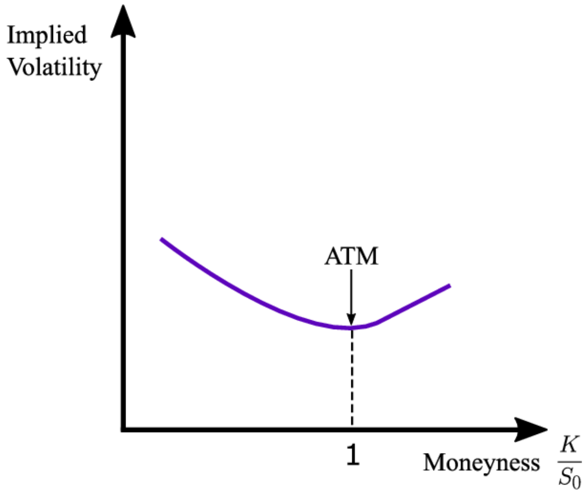


Figure 1.5 Implied Volatility- Moneyness Chart

The figure shows the development of implied volatility as moneyness changes.

related to this model, which is to derive volatility values from the prices of European options quoted in the market, obtaining a value that is called **Implied Volatility**.

In principle, it is always possible to invert the Black and Scholes formula and derive the volatility from the European option price because the latter is an increasing monotonic function of the volatility. The inversion of the formula is carried out through a numerical procedure, which makes it possible to obtain with the desired precision the value of the volatility which, once the other parameters of the formula are fixed, inserted into the Black & Scholes formula provides the value of the price observed on the market. In detail, the path that allows the computation of implied volatility is mainly composed of five steps:

1. It is necessary to collect the inputs of the Black and Scholes model, such as the market price of the underlying asset, the option's strike price, the expiration time, the risk-free rate and the market price at which the option is traded;
2. You then need to input the above data into the Black and Scholes model;
3. Once the data has been entered correctly, you need to start an iterative search based on trial and error;
4. You can also run interpolation on data close to implied volatility to get an estimate of the implied volatility close;
5. Once several attempts have been made, the nearest volatility is arrived at, i.e., such that when inserted into the Black and Scholes model, it returns as output the price of the option used as input in step1.

It is possible to calculate values of the implied volatility of European options tied to the same underlying asset for different values of the exercise price. However, the following phenomenon can be observed: the graph representing the implied volatility as a function of the exercise price (given the same values of the other parameters in the Black & Scholes formula), instead of being a horizontal line as in the lognormal model, presents an upward convexity. In the Black and Scholes model volatility is defined as a constant depending only on the underlying: according to such model, the value of σ should not change when the strike price or the expiry date of the options vary. In other words, according to Black & Scholes formula, the stock price evolves lognormally i.e., the stock price evolution over an infinitesimal time ∂t is described by the stochastic differential equation, keeping a constant local volatility σ at any time and market level [17]. This assumption of constant volatility in the return on the underlying asset has an important consequence because it implies that it is expected an implied volatility which would be largely constant across the moneyness and time to maturity.

However, in numerous empirical studies, the implied volatility shows sharp differences across moneyness and time to maturity, displaying either the so-called volatility smile or sneer [18] with reference to the "smile" which some quantitative analysts have believed to identify in the curve. As a rule, the curve has a minimum at the strike price equal to the value of the underlying (option at the money) and is often non-symmetrical with high volatility for options out of the money. If the model underlying the analysis conducted by Black & Scholes were correct, the volatility smile should be horizontal straight lines.

1.3.3 Advanced Option Pricing models with stochastic volatility

Stochastic volatility models have been introduced in an attempt to overcome the Black & Scholes limits. Specifically, in these models it is assumed that the price dynamics of the underlying are characterized by a diffusion coefficient that evolves according to a stochastic differential equation:

$$dS(t) = \mu S(t)dt + \sqrt{Y(t)}S(t)dW(t)$$
$$dY(t) = \gamma(Y(t))dt + \beta(Y(t))dZ(t)$$

Where γ, β are function of $Y(t), Z(t), W(t)$ are Brownian motions correlated with a correlation coefficient ρ. In this case the $Y(t)$ process represents the variance, which turns out to be a stochastic process while in the Black & Scholes model it is constant and equal σ^2.

The best-known stochastic volatility models are the following:

- $dY(t) = aY(t)dt + bY(t)dZ(t)$
- $dY(t) = k(\vartheta - Y(t))dt + \beta Y(t)dZ(t)$
- $dY(t) = \omega(\vartheta - Y(t))dt + \varphi\sqrt{Y(t)}dZ(t)$

The first model (**Hull & White)** describes the dynamics of variance by means of a geometric Brownian motion. It is the simplest model, and the non-arbitrage price of European Option can be obtained quite easily in closed form in the case in which the two Brownian motions are not correlated. The drift term in the second model (**Stein & Stein**) and in the third model (**Heston**) takes a form called mean-reverting: as soon as the variance value moves away from its "long-term equilibrium" value, this term tends to bring it closer to this value by means of a "recall" coefficient directly proportional to the difference of the two values. In fact, if $Y(t) > \vartheta$ then the drift is negative, while, on the contrary, in the case of $Y(t) < \vartheta$ the drift is positive. The drift then pushes the $Y(t)$ process towards the mean-reversion value ϑ, which is the long-term average of the variance. The drift is even more important in the dynamics of the k process, ω are high: these parameters are called speed of mean reversion.

The stochastic volatility models that we have presented offer undoubted advantages from the modeling point of view compared to the lognormal one: the tails of the distributions of the stock returns are thicker, the correlation between the Brownian motions allows to model the "leverage" effect (i.e. the negative correlation between volatility and the price of the underlying stock) and produces a Volatility Smile similar to that observed on the market in the case of not too close maturities.

The last of the three models is in some ways the most realistic and is very popular in the valuation of derivatives. Despite the complication due to the non-linear diffusion coefficient, it can be shown that, by imposing the constraint on the $\varphi^2 \leq 2\omega\vartheta$ parameters, it is possible to guarantee the positivity of the variance.

Furthermore, it is possible to explicitly derive the expression of the Fourier transform of the price of the European options and, with a simple numerical algorithm that inverts this transform, the prices of the European options can be obtained with the desired precision.

Considering this last model, that is the Heston model, it is possible to obtain the price of the European-type options in semi-closed form, which means that there is an explicit formula for the price of an option, but in its Fourier transform. Thanks to this result, the only numerical operation that involves an approximation of the determination of the price is represented by the inversion of the latter.

If, instead of referring to the variable $S(t)$, we consider its logarithm $X(t) = ln(\mathrm{S}(t)/(\mathrm{S}(0))$, the equations of the model, under the risk neutral probability, take the following form:

$$dX(t) = \left(r - \frac{Y(t)}{2}\right)dt + \sqrt{Y(t)}dW(t), X(0) = 0$$

$$dY(t) = \omega(\vartheta - Y(t))dt + \varphi\sqrt{Y(t)}dZ(t), Y(0) = Y_0$$

We define the following function:

$$f(t, x, y, u) = E[e^{iuX(t)}|X(t) = X, Y(t) = y]$$

That is the expected value of $e^{iuX(t)}$ conditional on values of $(X(T), Y(T))$. This function FFF is none other than the Fourier transform of $X(T)$, or rather the probability density associated with $X(T)$, and is called the **Characteristic Function**.

1.3.4 Jump-Diffusion Model for Option Pricing

A second class of models proposed in the literature to overcome the limitations of the lognormal model is that of stochastic processes with discontinuous trajectories to describe the dynamics of the underlying. Among these we have the **Lévy** processes. **Lévy**'s processes are fundamental processes of the lognormal model: Brownian motion is in fact a particular type of Levy process, the only one having continuous trajectories.

$$S(t) = S(0)e^{X(T)} = S(0)e^{\mu t+\sigma W(t)+\sum_{i=1}^{N(t)} X_i}$$

Where the number of addends appearing in the sum, *N(t)* is given by a Poisson process of intensity λ and the random variables X_i are independent and indentically distributed according to an assigned probability density.

The $\{N(t)\}_{t\geq 0}$ process is a Poisson process of λ intensity if, for each *t*, $N(t)$ is a random variable distributed according to a Poisson distribution of parameter λt. The $Y(t) = \sum_{i=1}^{N(t)} X_i$ stochastic process is called the compound Poisson process. In this model, the evolution of the underlying is described by an exponential process consisting of a Brownian motion with drift to which is added a compound Poisson process whose variables X_i are distributed according to a Normal: $X_i = N(\gamma, \delta^2)$. This model is called "diffusion model with jumps": the price of the underlying evolves as in a geometric Brownian motion and “sometimes” (with frequency $N(t)$) makes a jump, the amplitude of which depends on the chosen probability distribution and on the parameters that they define it.

The diffusion model with jumps is an incomplete market model in that the random source constituted by Brownian motion has been added to that constituted by jumps, while only one risky security is traded on the market: therefore, there is a source of uncertainty for which there is no possible to make a perfect coverage.

In this context, it is necessary to choose a risk-neutral probability measure from among the many allowed by the principle of absence of arbitrage opportunities that allows the derivative securities to be correctly valued. One way to do this is to have the following condition matched:

$$E[e^{X(T)}] = \frac{1}{S(0)}E[S(T)] = e^{rT}$$

Specifically, it proves that by imposing $\mu = r - \lambda k - \frac{\sigma^2}{2}$ represents the evolution of the price of the underlying under the risk-neutral probability measure. Note that the term λ corresponds to the average number of jumps per year, while k represents the average jump width measured as a percentage of $S(T)$.

The Merton model, like all Lévy processes, admits a characteristic function known analytically:

$$\varphi_T(u) = exp\left\{T\left(\left(-\frac{\sigma^2}{2} - \lambda k\right) iu - \frac{\sigma^2}{2} u^2 + \lambda\left(e^{-\frac{\delta^2}{2}u^{2+i\gamma u}} - 1\right)\right)\right\}$$

therefore, it is possible calculating the price of a European option using the Carr and Madan formula. It is also possible to obtain the non-arbitrage price of a European option in the form of a series; in the case of a Call, we have:

$$C_M(S(t), t) = \sum_{n=0}^{\infty} e^{-\psi(T-t)} \frac{[\psi(T-t)]^n}{n!} C_{BS}(n)$$

Where $\psi = \lambda(1 + k)$ and C_M is the Option price obtained by means “jump process” and C_{BS} the price of a European Call by Black and Scholes formula with variance:

$$\sigma_n^2 = \sigma^2 + \frac{n\delta^2}{T-t}$$

And risk-free interest rate:

$$r_f = r - \lambda k + \frac{n\left(\gamma + \frac{\delta^2}{2}\right)}{T-t}$$

The models with jumps based on Levy processes have a considerable ability to explain the behavior of the prices of the underlying and the options: they provide distributions for the "fatty queues" performance and produce realistic "smile" for close deadlines, which It does not happen for distant expiration dates.

In conclusion, both stochastic and jumps volatility models improve Black and Scholes' performance in describing price processes, but in complementary directions. Some models, such as the well-known Bates model, have initiated a synthesis of the two classes of models to combine the advantages they offer in predictive terms. These models are therefore called "jump stochastic volatility models".

2. State of the Art & Literature Review

In a colorful phrase popular in financial literature, Implied Volatility has been described as "*the wrong number which, when inserted into the wrong formula, gives the exact value of an option*". Indeed, in financial mathematics, the implied volatility (IV) of an option contract is that value of underlying instrument's volatility which, when fitted into Black and Scholes option pricing model, will return a theoretical value equal to the current market price of that option.

As presented in the previous chapter, one of the most attractive features of the Black & Scholes model is that its parameters are almost all observable except for the volatility of the underlying asset which must be predicted and therefore two different approaches can be used for this purpose. The first consists in computing the realized volatility over the recent past from the historical price data, instead the second one is to forecast and calculate the "implied volatility" from the current option prices in the market. However, the latter approach is the one that has become the prevalent in the academic finance profession because the implied volatility is considered the "market's volatility forecast" and as consequence, it is a better estimate than historical volatility. Indeed, researchers often use implied volatility in other models as an ex-ante measure of perceived asset price risk.[3]

Since it is widely accepted that the implied Black and Scholes volatility computed from the market price of an option is a good estimate of the "market's" expectation of the underlying asset's volatility and that the market's expectation is informationally efficient, it may be particularly useful for our analysis to investigate how reliable these expectations are.

In fact, any discovery of an under- or over-estimation of the IV with respect to the volatility that actually occurs in the market could indicate an under- or over-estimation of options' price in the market, and in the latter case, could be interesting to consider

[3] See Patell and Wolfson (1979, 1981) or Poterba and Summers (1986) for examples of the use of implied volatility as a proxy for the market's risk assessment. Implied volatility has also been used as a proxy for the true instantaneous price volatility of the underlying asset, as in Stein's (1989) study of the "term structure" behavior of implied volatility.

the performance of any strategies involving the sale of options. To this purpose, in the following chapter, we are going to report some empirical studies in order to understand if the market expectations included in the IV are actually realized i.e., whether there is a correct alignment or a possible misalignment between the IV and the volatility that actually arises in the market, and in the latter case if it is an overestimation or an underestimation of the realized volatility.

In the first paragraph will be reported a series of studies which show that the IV is an inefficient forecast of future volatility. Indeed, the authors point out the more we move away from ATM options, the more imprecise this forecast becomes. Consequently, a further study on the volatility smile phenomenon will be proposed.

Subsequently, in the following paragraph will be presented a series of plausible causes which could explain this misalignment between the IV calculated through the B&S inversion and the volatility actually realized.

Lastly, in the third paragraph, once this mismatch is understood, will be presented some insights on options' pricing error, which represents the main consequence of this phenomenon. Indeed, several empirical studies have pointed out, given for granted the IV as market expectations, such mismatch can be seen as an error in options' valuation, on which several authors agree to be an overvaluation.

2.1. Empirical evidence

In theory, *a fully rational investor should value options using a pricing model that deals with the stochastic nature of future volatility*, however *the high complexity of these advanced pricing models with stochastic volatility or with jumps models, make the constant-volatility Black & Scholes model the most used due to its easier implementation* [19].

Since the widely belief of implied volatility's superior information respect to historical volatility because it is the "market's" forecast, different academic researchers make numerous empirical studies to validate that idea.

However, the *empirical results show that implied volatilities appear to be neither unbiased nor efficient forecasts of future volatility because between the expected future volatility i.e., implied volatility, and the volatility that really occurs there is non-negligible discrepancy,* which several studies have shown to be dependent on the moneyness and time to maturity [20].

In this direction, among the authors who have been most concerned with analyzing and documenting this mismatch between volatilities are Linda Canina and Stephen Figlewski, who in their paper *The Informational Content of Implied Volatility* make a very deep analysis for S&P 100 index options, the most actively traded contract in the United States, and they *"…find implied volatility to be a poor forecast of subsequent realized volatility. In aggregate and across subsamples separated by maturity and strike price, implied volatility has virtually no correlation with future volatility, and it does not incorporate the information contained in recent observed volatility. "*

In order to validate and examine their proposition, L.Canina & S. Figlewski decided to take as samples over 17.000 call options of the most active options market in the United States, options on the Standard and Poors 100 Index (frequently called by their ticker symbol, OEX options). The data sample was built on the set of closing prices for all call options on the OEX index from 15 March 1983, shortly after index option trading opened, through 28 March 1987. They eliminated options with fewer than 7 or more than 127 days to expiration and those that were more than 20 points in- or out-of-the-money.

Among the data, some of the recorded option prices violated the lower arbitrage boundary, which is called "Call Relationship" and it states that call price should be greater than the current stock price minus the present value of the strike price plus future dividends. An option's price equals the boundary value if volatility is zero. In the case of a boundary violation, implied variance would have to be negative, so those options were also excluded from the sample. This left a total of 17,606 observations.

Maturity group (i)	Days to expiration	Number of obs	IV mean	Sample standard deviation
All		17,606	0.168	0.055
1	7–35	4,088	0.195	0.080
2	29–63	5,196	0.166	0.046
3	57–98	4,709	0.158	0.039
4	85–127	3,613	0.152	0.035

Table 2.1 Summary statistics of implied volatility by maturity group

The table shows the breakdown of implied volatilities for OEX call options between March 15, 1983, and March 28, 1987, into four maturity groups corresponding to the number of contract months to expiration. For example, the first group (i = 1) contains the near-month options. The rightmost columns give the mean and the standard deviation of implied volatilities within each group.

Table 2.1 shows that the average implied volatility in the sample is a decreasing function of time-to-option expiration. Indeed, the mean for the entire set of 17,606 observations is 0.168, while the averages for the four maturity groups decline monotonically from 0.195, for near-month options, to 0.152, for those expiring in the fourth month [19].

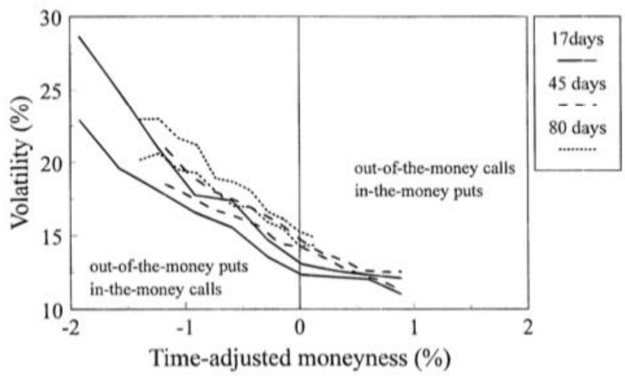
Table 2.1 result is particularly significant because it demonstrates that implied Black–Scholes volatilities strongly depend on the maturity and on the strike of the option under consideration, contrary to what we would expect on a theoretical level where it is expected that the implied volatility would be largely constant across the moneyness and the time to maturity, due to constant volatility Black–Scholes model. *In addition, several interesting regularities in the time pattern of implied volatilities drawn from OEX options, including a day-of-the-week effect. Call implied volatilities are low on Fridays and high on Mondays but puts do not show the same pattern* [21].

In this context, another relevant empirical study conducted to verify the veracity of the implied volatility as good estimator of the “market’s" expectation on the volatility of the underlying asset was made by Dumas, Fleming and Whaley [22].

The authors, using S&P 500 options from June 1988 through December 1993, examined the predictive performance of the deterministic volatility function option valuation model through Black & Scholes and they too have *found that Black-Scholes implied volatilities tend to differ across exercise prices and times to expiration*.

Going in depth, the goal of their study was to try to understand if the asset price behavior revealed by this method, was confirmed by the actual subsequent behavior of asset prices to endorse the time-series validity of assuming volatility as a deterministic function of asset price and time. As result of this study, the researchers found that when the Black-Scholes formula is used to imply volatilities from reported option prices, the volatility estimates vary systematically across exercise prices and times to expiration resulting in a constant mismatch between expected volatility and actual volatility.

Figure 2.1 Black-Scholes implied volatilities from June 1988 to April 1992

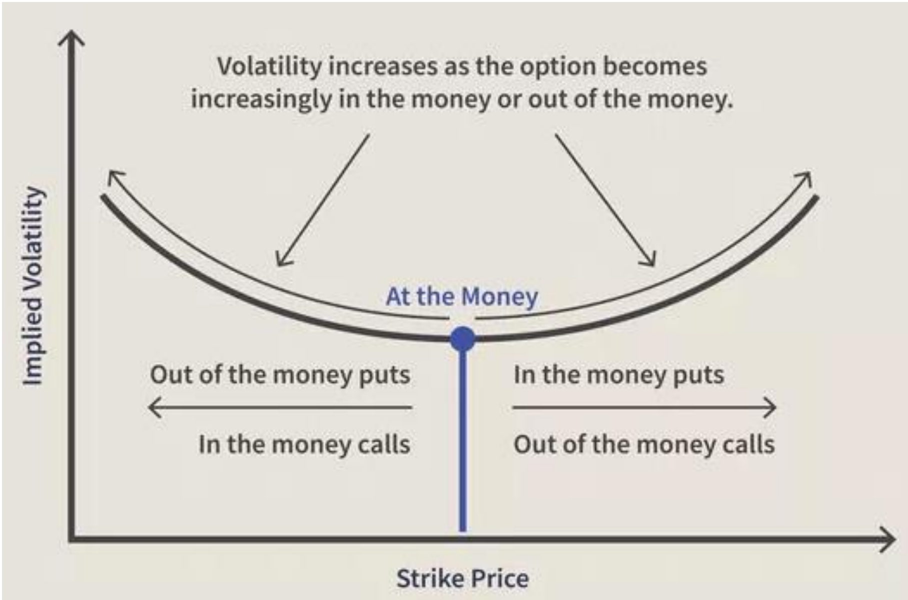


Implied volatilities are computed from S&P 500 index call option prices for the June 1988 to April 1992 option, with expirations at the end of the month and at the end of the following two months. The lower line of each pair is based on the option's bid price, and the upper line is based on the ask. Time-adjusted moneyness is defined as [X/(S - PVD) - 1]/ T, where S is the index level, PVD is the present value of the dividends paid during the option's life, X is the option's exercise price, and T is its number of days to expiration.

The Figure 1.1 confirms such deduction; indeed, it illustrates the typical pattern in the S&P 500 implied volatilities. Strikingly, the volatilities do not all lie on a horizontal line and for this reason, that volatility pattern is often called the volatility "smile" and constitutes evidence against the Black- Scholes model.

The importance of figure 1.1 lies in that it introduces the well know and already introduced volatility smile concept. Indeed, a volatility smile is a common graph shape that results from plotting the strike price and implied volatility of a group of options with the same underlying asset and expiration date, but with different strike prices. Since the graph shape looks like a smiling mouth, the volatility smile is so named. This U-shape comes from the fact that implied volatility rises when the underlying asset of an option is further out of the money (OTM), or in the money (ITM), compared to at the money (ATM).

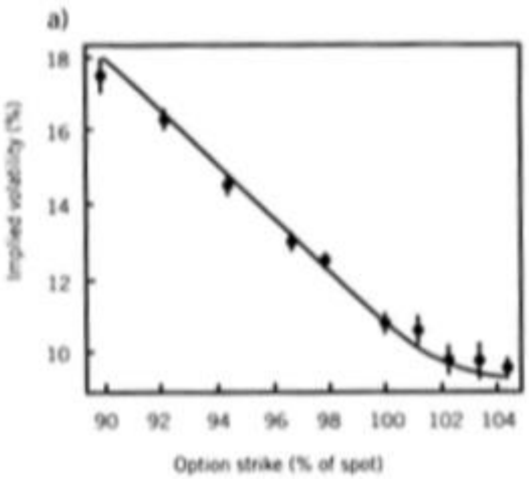
Figure 2.2 Volatility Smile Graph



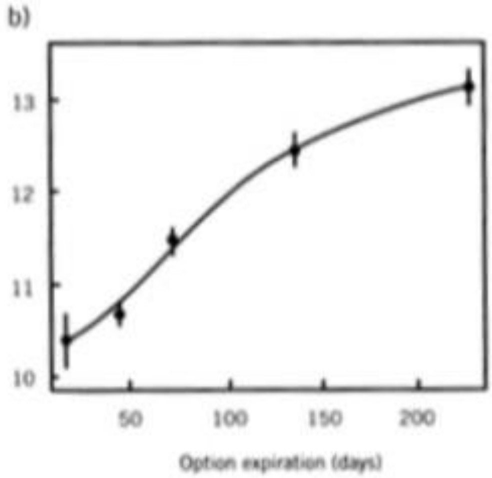
Volatility increases as the option becomes increasingly in the money or out of the money. This is evidenced by the increase in the curve as we move away from the 'equilibrium' situation in the strike price.

This volatility asymmetry phenomenon was also discovered by Derman. Indeed, the author, studying the consistency between market option prices with the Black & Scholes formula, found that though the exact shape and magnitude of the implied volatility vary from day to day, the asymmetry persists and belies the Black & Scholes theory, which assumes constant for all options [17].

Figure 2.3 Implied volatilities of S&P500 options (May 5, 1993)



(a) *Data for strikes above (below) spot come from call (put) prices.*



(b) *The average of at-the money call and put implied volatilities is use*

Figure 2.3 (a) shows the decrease of implied volatility with the strike level of options on the S&P500 index with a fixed expiration of 44 days, as observed on 5 May 1993, showing the volatility “skew” asymmetry. Instead, the figure 2.3 (b) shows the increase of implied volatility with the time to expiration of at-the money options, showing the so called “term structure”, which added to previous volatility “skew”, together determine the volatility “smile”.

As was shown by several empirical studies, the volatility smile is created by implied volatility changing as the underlying asset moves more ITM or OTM. The more an option is ITM or OTM, the greater its implied volatility becomes, instead implied volatility tends to be lower with ATM options [23].

However, in the Figure 1.1 the "smile" actually appears to be more of a "sneer" [22]. This little shape change, according to the authors, simply relates to a variation in label because according to them the smile label arose prior to the 1987 market crash when, in general, the volatilities were symmetric around zero moneyness, with in-the-money and out-of-the-money options having higher implied volatilities than at-the- money options, i.e. at higher volatilities discrepancy.

The sneer pattern displayed in Figure 1.1, however, is more indicative of the pattern since the crash, with call (put) option implied volatilities decreasing monotonically as the call (put) goes deeper out of the money (in the money), still showing that the sneer is influenced by the time to expiration of the underlying options.

From the previous analyzed reports, it is possible deducts that all empirical studies agree that contrary to expectations, implied volatility is not time-independent variable, but it varies according to the options’ time to expiration and for this reason, options expiring in different dates may reasonable be priced using different volatilities.

Furthermore, the implied volatility is strictly linked to the option’s moneyness, leading a regular dependency of the implied volatility’s structure to strike price of the options. Those two facts constitute evidence against the hypothesis that implied volatility is the market’s fully rational volatility forecast i.e., implied volatility is not a rational forecast of future volatility due to the systematic presence of volatility smile phenomenon.

This persistent volatility smile raises a question concerning the source of the Black & Scholes model's apparent deficiency since there is an obvious conflict in applying an approach that assumes the asset price process has a known constant volatility to a situation in which volatility must be forecast because it changes randomly over time. One possibility is that the constant volatility assumption is violated, or that the distribution of asset prices at expiration is not lognormal. However, it is commonly thought that the failure of the Black-Scholes model to describe the structure of reported option prices arise from the first one i.e., due to its constant volatility assumption because this mismatch can be explained by an increase in investors' probability assessment of downward or downward moves in the index level [22].

The sneers that are shown in the figure above in which IV-Moneyness and IV-Time to Maturity are plotted, in practice are "merged" and inserted into a single three-dimensional space in which IV-Moneyness-Time to Maturity is present, which manifests as a surface called the "Volatility Surface" (See next figure). For instance, options with lower strike prices tend to have higher implied volatilities than those with higher strike prices. As the time to maturity approaches infinity, volatilities across strike prices tend to converge to a constant level. However, the volatility surface is often observed to have an inverted volatility smile. Options with a shorter time to maturity have multiple times the volatility compared to options with longer maturities. This observation is seen to be even more pronounced in periods of high market stress. It should be noted that every option chain is different, and the shape of the volatility surface can be wavy across strike price and time. Also, put and call options usually have different volatility surfaces.

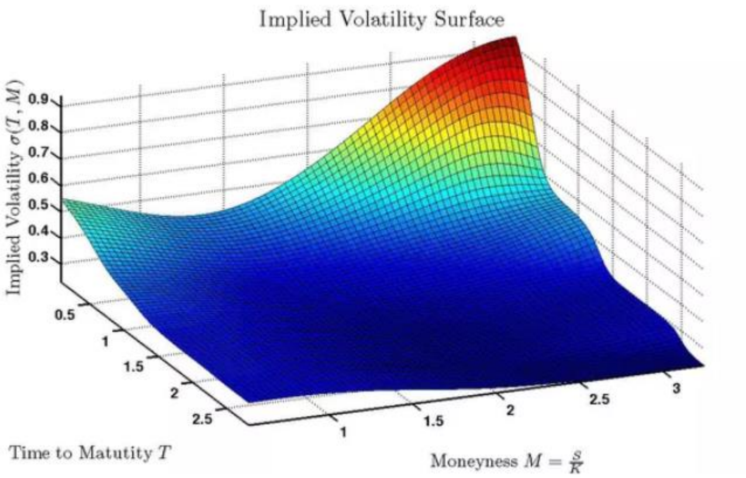


Figure 2.4 Implied Volatility Surface

Representation of the implied volatility surface, obtained from the previous Figure 2.2, by adding as a third axis the Time to Maturity T

In conclusion, such persistent volatility smile suggests a discrepancy between theory and the market, and it shows how the mismatch between volatilities is amplified the further we move away from ATM options. Accordingly, it recommends that *may be convenient to continue a quoting options price exploiting the Black & Scholes formula, but it is probably incorrect to consider the implied volatility, computed by inverting an option price* *via Black & Scholes, as the future expected volatility on the market* [17], even though accounting for nonconstant volatility within an option valuation framework is no easy task.

2.2. Volatility Smile Causes

As presented previously, the implied volatility is an inefficient and biased forecast of realized future volatility that does not impound the information contained in recent historical volatility. In fact, the statistical evidence shows *little or no correlation at all between implied volatility and subsequent realized volatility*. Therefore, given the interpretation of the IV as a good forecast of the future market volatility as valid, we will have that this mismatch results in a valuation error to which must be added the presence of the volatility smile, which further influences the consistency and relevance of the mismatch and thus of a possible option under- or over-valuation.
Even if research and trading experience have uncovered the fact that some, if not most, patterns are not literally smiles, because some are oblique and others are said to resemble a smirk, *whether the relationship between implied volatility and strike price resembles a smile, an objection or a smirk, any variation from the horizontal line is an indication of the existence of systematic factors that have led investors to price particular options high or low relative to others* [24].

In fact, since all option prices on the same underlying security with the same expiration date but different strike prices do not show the same implied volatility, the smiley volatility phenomenon suggests that the Black & Scholes formula tends to misprice deep in-the-money and deep out-of-the-money options.

In this paragrapher, we will report on the various studies that have been carried out in order to understand and highlight the main causes that lead to such volatilities' mismatch. More specifically, these studies can be divided in two main categories depending on which parameters the authors have indagated more:

- Some authors have traced the causes for external reasons, i.e., to variables that are not taken into account and do not enter into the Black & Scholes model, so factors that we could define exogenous;
- Other researchers, instead, have referred the reasons to the links between the variables and assumptions considered by Black & Scholes, therefore parameters that we could define endogenous in Black & Scholes model.

Authors who have contributed most to identifying possible exogenous causes of the volatility smile phenomenon are Ignacio Pena, Gonzalo Rubio and Gregorio Serna [25]. In their study, the authors analyze the underlying determinants of the well-known pattern of implied volatilities across exercise prices for otherwise identical options, employing a database comprised of all call and put options on the IBEX-35 Spanish index and on the S&P-500 US index traded daily from January 1994 to April 1996. Their results are significant because they suggest a strong seasonal behavior in the curvature of the volatility smile.

However, such seasonality tends to disappear when several economic variables in the analysis are included. In particular, transaction costs proxied by the bid-ask spread of the negotiated options and relative market momentum seem to be key variables in explaining the variability of the implied volatility function over the time, showing also a complex and nonlinear causality effects on the dynamic interrelations between these variables and the volatility smile. Put differently, trading costs, the degree of options market liquidity and market momentum conditions are widely accepted as exogenous parameters to the Black & Scholes model that strongly influence the occurrence of this volatilities mismatch.

In support of that authors' thesis, empirical results suggest a positive and significant relation between the degree of curvature of the volatility smile and transaction costs, which can be represented by the bid-ask spread. *Indeed, on average, whenever the bid-ask spread tends to increase, the degree of curvature of the volatility smile increases, so this means that when market makers tend to face higher adverse selection risks, out-of-the-money calls* (in-the-money puts) *and out-of-the-money puts* (in-the- money calls) *are more highly valued by the market relative to the Black & Scholes model*. This is a key result because it suggests that the option pricing model will not be correctly specified as long as it doesn't consider the transaction costs. *The compensation for market maker risks seems to be playing a crucial role in the behavior of the options market*, so excluding that compensation leads to an options' pricing error, *which will be more relevant the higher market maker risks*.

In this context, is possible to sustain that volatility smile's existence shows that *ITM and OTM options tend to be more in demand than ATM options and therefore, since demand drives prices because it affects the bid-ask spread, the option's market liquidity affects implied volatility* [23]. Indeed, it should be pointed out that the level of activity in the options market, as measured by the number of option contracts negotiated, is positively related to the at-the-money implied volatility, and negatively correlated with the (average) slope coefficient of the smile, therefore it means that the slope of the smile increases with volume. *Given that volume is significantly associated with the curvature of the smile, it is possible to conclude that a higher the liquidity i.e., higher volume in the option, market gives more value to out-of-the-money puts* (in-the-money calls*) relative to the values of the in-the- money puts* (out-of-the-money calls) [25].

Hence, transaction costs and market liquidity influence the valuation of out-of-money puts (in-the-money calls) and in-the-money puts (out-of-the- money calls) relative to at-the-money options. *Higher transaction cost or options' liquidity is associated with higher market values of extreme* (in term of moneyness) *options. These reasons might be an explanation for the apparent failure of Black and Scholes to explain the behavior of out-of-the-money puts* (in-the-money calls) *and in- the-money puts* (out-of-the-money calls). This idea is shared and sustained even by Longstaff because, given the evidence provided by his study in 1995, he sustains that *a serious candidate to explain the pronounced pattern of volatility estimates across exercise prices might be related to liquidity and trading costs* [26].

However, at the same time, the degree of uncertainty and relative market momentum also seem to be relevant factors associated with the shape of the volatility smile. Specifically, the empirical studies show that the implied volatility is positively correlated to market conditions i.e., the volatility increases whenever the current market conditions improve relative to the past. Hence, the relative momentum of the market seems to be weakly related to the degree of curvature of the smile since whenever the current level of the stock market is better than the past, it is found that, on average, the degree of curvature of the smile increases. This would have the consequences of increasing the tails of the underlying distribution, leading to skewness and kurtosis effects on option prices, the main consequence of which is a symmetric curvature in the volatility smile.

In conclusion, transaction costs, market liquidity and current market conditions play a simultaneous role in explaining the shape of the implied volatility pattern across exercise prices. Considering the significant and inverse relationship between time to expiration and degree of curvature, it is possible to conclude that market conditions and transaction costs are relatively more important whenever there is a short way to go in the life of the option. Furthermore, since Pena, Rubio and Ser have made a linear causality between the shape of the smile and transaction costs and no other economic variable seems to linearly cause the curvature of the smile from their linear causality tests, the authors claim that illiquidity costs are a crucial determinant of the magnitude of the volatility smile.

As mentioned above, there is another stream of researchers who have identified the causes of the volatile smile among the endogenous parameters i.e., among the input parameters and assumptions that are used by Black & Scholes model to price the options. In this regard, the papers by Corrado and Su (1996, 1997) sustain that *volatility smiles are a consequence of empirical violations of the normality assumption in the Black & Scholes model* [27]. In other words, skewness and kurtosis in the option-implied distributions of stock returns are the source of volatility smiles. Although this idea is widely shared, no studies on it will be reported in this article because an accurate discussion would be too complex and irrelevant.

Further endogenous parameters that has been much investigated and considered as a possible cause of volatility smile phenomenon are:

1. Time to Maturity
2. Real underlying volatility of the asset.

Especially relevant in this direction is the study carried out by João Duque. He studied to search for theoretical relations that should exist between the smile shape, time-to-maturity, and volatility, trying to establish statistically significant links between these two variables, which impact on option valuation, and the shape of the smile. In particularly, he focused on analyze empirically how implied volatility smiles vary with the approach of expiration as well as with changes in volatility and the author found empirical support for the smile intensification i.e., *the U-shape is more pronounced, as maturity approaches as well as when volatility rises*. However, this increase in the curvature is asymmetric. In general terms, they found that *as maturity approaches the implied volatility of out-of-the-money options tends to be higher than the implied volatility of in-the- money options and, as the volatility of the underlying increases, the implied volatility of in-the-money options tend to be higher than implied volatility of out-of-the-money options* [28].

As already pointed out, particularly interesting is the cause-effect relationship between expiration date and volatility smile shape. The maturity of the options tends to be associated with an increase of the exercise price bias for both in and out-of-the-money options, but out-of- the-money options tend to become more biased. More precisely, Duque notes that when observing longer time to maturity options, in-the-money implied volatilities becomes larger than out-of-the-money implied volatilities. But for shorter time periods, implied volatilities of in-the-money options become smaller than out-of-the- money options implied volatilities. In fact, the maturity approach changes the options smile asymmetry, changing the shape of the smile for long term options from expiring options, with a symmetric smile for middle term option

Although the empirical results found seem to differ from the pattern suggested by Hull and White, there are similarities may be observed. In fact, *as maturity approaches, the smile becomes deeper with out-of-the-money implied volatilities higher than in-the-money implied volatilities.* However, although the authors agree on *the persistence of the smile for* *longer term options, they do not report greater implied volatilities for in-the-money options than for out-of-the money, but equal* [29].

In the end, it is possible to conclude that as time to maturity reduces, the magnitude of the smile increases. Both the absolute and the relative difference between implied volatility of in and at-the-money options or out and at-the-money options rise as expiration approaches. Further, this maturity bias seems to be more evident for out-of-the-money options than for in-the-money options, showing an asymmetric pattern.

The second statistically significant link between a variable which impact on option valuation and the shape of the smile is the underlying asset' volatility. Indeed, many empirical studies have shown that *the degree of curvature of the volatility smile is negatively and significantly related to the historical volatility of the underlying asset. Specifically, high volatility periods tend to be associated with lower curvature of the smile* [25]. This idea is sustained by other several studies that find the similar statistically significant positive relation between the underlying stock volatility and the smile. *When recent historical volatility increases, the exercise price bias tends to rise, both for in and out-of-the-money option and as observed with the maturity effect, it is also possible to recognize an asymmetric behavior for in and out-of-the-money options.* For high volatilities, in-the-money options *tend to evidence higher estimated implied volatilities than for lower volatilities* [28].

These results are confirmed even by Beckers, who finds this pattern in his study of individual stock options. In his sample, *implied volatilities for deep-in-the-money near-to-expiration calls are as much as 10 times the implied volatility for the corresponding at-the-money options.* One explanation for this phenomenon is that in constructing the sample, he was obliged to exclude the calls that would have negative IVs because their prices violated the lower boundary "PUT/CALL Relationship". *Deep-in-the-money options are quite insensitive to volatility, meaning a large change in the implied volatility is produced by a small change in the option's price*. *At the same time, these options are rather illiquid, and they trade less frequently than those nearer to the money, so they have wider bid-ask spreads and nonsynchronous data is a larger problem*. Thus, there is relatively *more "noise" in the prices of these calls*, and apparent boundary violations are most frequent for them [30].

In summary, it is possible to conclude that the results regarding the correlations between volatility smile's shape and its main causes, both endogenous (market momentum, transaction costs and market liquidity) and exogenous (expiration date and historical underlying volatility of the asset), suggest that *periods which are relatively calm but at the same time have increasing current levels of the market stock exchange index tend to be associated with a higher degree of curvature of the volatility smile. Alternatively, the* *pattern across exercise prices becomes flatter whenever the volatility of the underlying asset goes up, and the relative market momentum gets worse. At these latter periods of time, out-of-money puts* (in- the-money calls) *and in-the-money puts* (out-of-the-money calls) *become more symmetrically valued by the market relative to at-the-money options* [25].

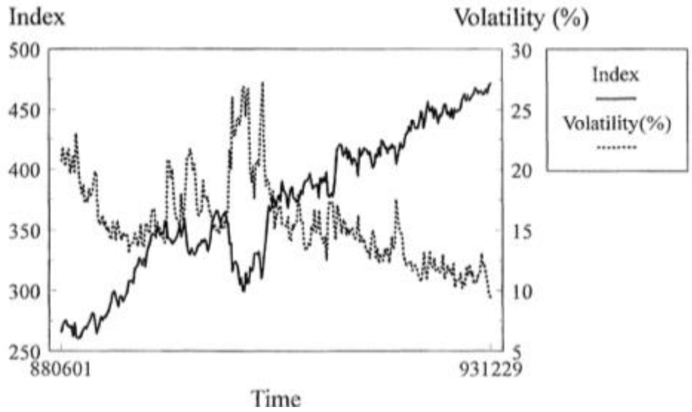
2.3. Pricing Error

Up to now, we have reported the existence of a mismatch between implied volatility (i.e., expected volatility) and the real occurred volatility and have presented the main parameters which are considered among the main determinants. Particularly, the volatility smile shows an amplified mismatch for OTM and ITM options. In the following section, we will mainly focus on understanding whether the mismatch is by default or by excess and its main consequences. In fact, we will report on several comprehensive empirical studies that show how such a mismatch implies an options' pricing error, particularly highlighting how this pricing error is mainly an overestimation of options' price since implied volatility tends to be always higher than real one.

As was presented in the previous paragraphs, soon after the Black & Scholes model was introduced, it was recognized that rather than infer option values from an estimated volatility, one can invert the process. Observed market prices for traded call options could then be inserted into the model to back out an estimate of the unobserved volatility of the underlying. In this manner, *one could infer the volatility of the underlying that was being used by market participants as they priced and traded options* [24] and, given the volatilities mismatch and volatility smile phenomenon, that could be interpreted as the error measure in option pricing.

In particular, it is possible to affirm that *there is an inverse relation between price market and implied volatility*. This inverse time-series relation between stock returns and volatility changes has been documented in several empirical studies. Most of the studies use stock returns to measure volatility, but the effect is also apparent when volatility is measured using option prices.

Figure 2.5 S&P 500 index level and Black-Scholes implied



The graph represents the S&P 500 index level and Black-Scholes implied volatility each Wednesday during the period of June 1988 through December 1993

Figure 2.5 shows the level of Black-Scholes implied volatility during the sample period of author's study, June 1, 1988 through December 31, 1993. As the S&P 500 index level trends up, *the level of implied volatility trends down, showing in this way a negative correlation among the two variables* [22].

In this regard, some researches have shown that: "*even if investors price options in accordance with the Black–Scholes–Merton model using a known volatility, the implied volatilities across strikes almost always vary and some quite notably*." Remarkably, they do so in the form of smiles, skews, and smirks that greatly resemble the patterns observed in practice using prices from markets that are far from perfect [24].

As can be inferred, taking the view of implied volatility as expectations, *misalignment of implied volatility is a symptom of incorrect option pricing*. One of the possible interpretations is based on the fact that along with *investors' volatility forecasts an option's market price also impounds the net effect of the many factors that influence option supply and demand, but they are not in the option model*. As already presented, option pricing theories ignore such factors because, in a frictionless market, unlimited no-arbitrage drives the price to the model value regardless of what trading strategies other market participants follow. In the real world, however, even if is possible to have arbitrage opportunities, *the arbitrage between an option and the underlying index is a difficult and very costly strategy*. In practice, there are *many factors can affect the price of an option without inducing arbitrage to offset them, and the implied volatility will impound the net price effect of all of them* [19].

Then it is possible to conclude that *the biases in implied volatilities lead to biases in observed prices because extreme events can occur, causing significant price shifts in option, therefore the potential for large shifts is factored into implied volatility*. As a result, one has to be careful when using implied volatilities to assess the performance of option pricing models [20].

To summarize, in the frictionless market of Black and Scholes, all prices are observed without error and every option price can be inverted to find the unique implied volatility consistent with the observed. However, *prices are observed with errors stemming from finite quote precision, bid-ask spreads and other measurement errors presented previously*. This seemingly innocuous difference turns implied volatility calculations into implied volatility estimations and raises questions about the precision of the estimates. *Implied volatility estimates are imprecise when large changes in volatility produce small changes in option prices, and conversely, small random independent errors in option prices or other option characteristics lead implied volatilities to be even more imprecise and large errors. This is especially true for options far from the money.*

Some of the authors who have studied most to understand the causal-effect link between pricing error and implied volatility are Song Xi Chen & Zheng Xu, who in their literature investigated in depth the nature and type of link between the two variables and found some interesting results, for this reason the main steps taken by the authors are outlined below in order to highlight the main outcomes [18].

Suppose that, in a given market time period, there are n option contracts with price Y_i at $X_i = (S_i; K_i; \tau_i; r_i)$ with $i = 1, \dots, n$. and let X be the covariate of the European call i.

Let $C(X)$ be the ''agreed'' price among the market participants. The form of $C(x)$ is likely to be unknown but is assumed to be ''rational'. Empirically, the observed price Yi is rarely exactly the underlying price $C\ (Xi\ )$ but rather

$$Yi = C(Xi) + \varepsilon i,$$

where εi is the pricing error and it needs to satisfy:

$$E(\varepsilon i | Xi) = 0.$$

The pricing error is largely present in the empirical prices of options, which is most reflected in the bid–ask spreads, the discreteness in the quoted prices as opposed to the continuous $C\ (Xi\ )$ and the market dis-synchronization between the derivative and the spot markets.

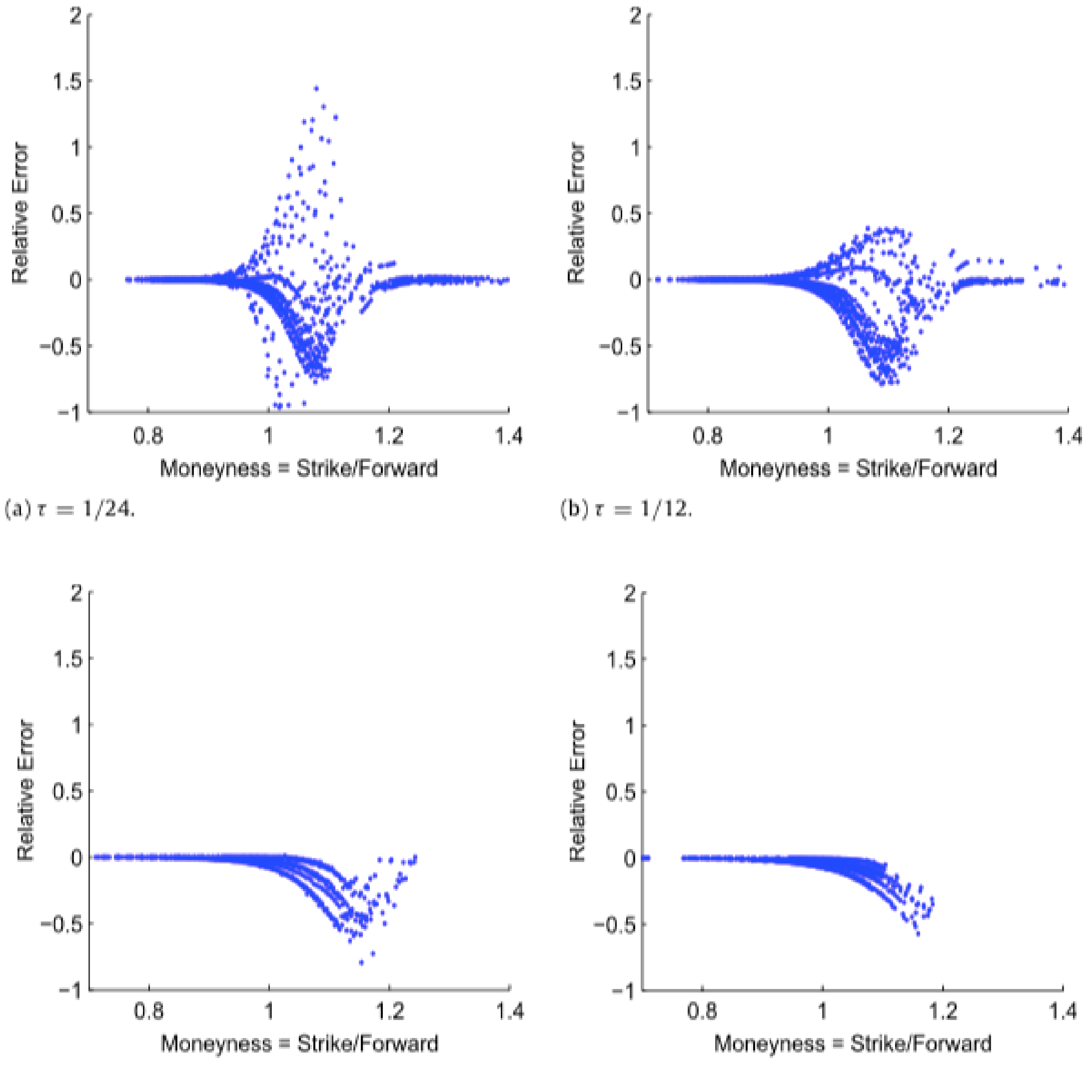
The error can also be understood as the natural fluctuations around the fair price $C(X)$, as reflected in the random movement of the bid and ask prices.

To demonstrate the presence of the pricing errors, they display the estimated relative pricing errors plotted against the moneyness at different maturities for S&P 500 call option data for the month before and the month after September 15, 2008, the day Lehman Brothers declared bankruptcy.

The authors also studied the estimated pricing errors for puts for the two months, which showed similar evidence of the errors. As can be seen from the empirical studies carried out in the figure 2.6 below, the pricing error determines four main aspects:

a. For both calls and puts, pricing errors was the biggest for the short maturity out-of-the-money options.
b. The relative errors became smaller but more negative as the time to maturity increased, indicating an underpricing for longer maturity out-of-the-money options.
c. We also observe that the pricing error was smaller for in-the-money options, partly because of higher underlying price $C(Xi)$.
d. Most importantly, we observed that the pricing errors for the short maturity at-the-money options were not negligible as commonly believed.

Figure 2.6 Relative pricing errors vs moneyness and



(a) $\tau = 1/24$.

(b) $\tau = 1/12$.

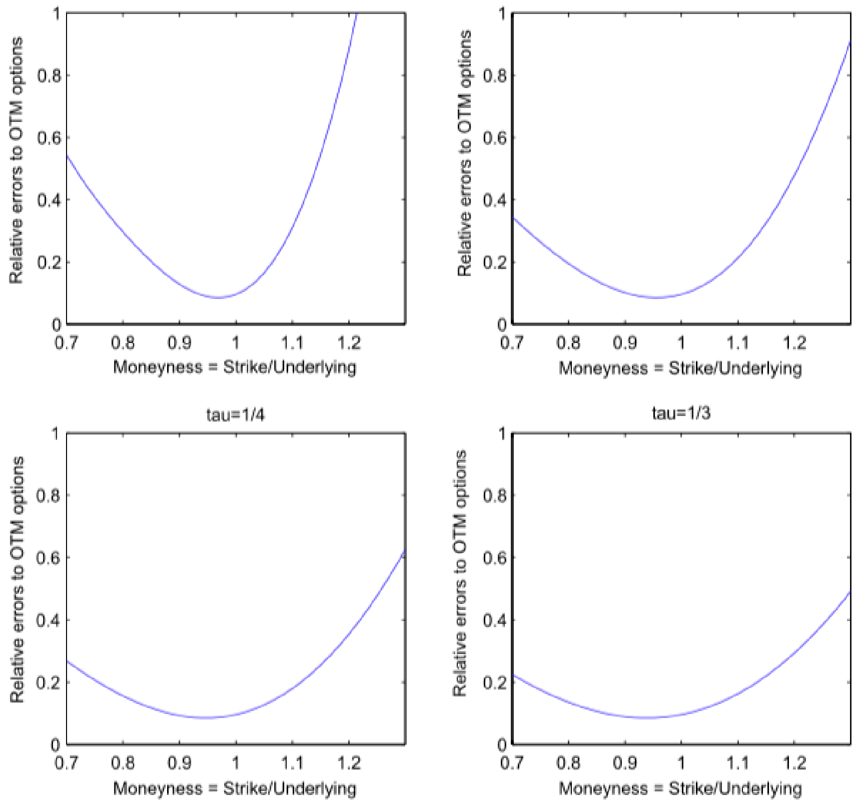
(c) $\tau = 1/6$.

(d) $\tau = 1/4$.

The graph shows the Nonparametrically estimated relative pricing errors vs moneyness and maturity (τ) for S&P 500 call option data in the month from September 15, 2008, for the moneyness and maturity.

These empirical pricing errors have some relevant impacts on the implied volatility computation which shows a very sensitivity to a price deviation. To illustrate the impacts of the errors, it is necessary to understand the baseline relative pricing error δ i.e., calibrated relative price deviation to OTM options, against moneyness for different time to maturity τ. This path can be seen in figure 2.6, where it is possible to observe how, given a certain moneyness, the pricing error becomes less relevant in proportion to the option's price as the time to expiration increases.

Figure 2.7 Relative price deviation to OTM options



The graph shows the Calibrated relative price deviation to OTM options, against moneyness for time to maturity τ = 1/12 (upper left panel), 1/6 (upper right panel), 1/4 (bottom left panel) and 1/3 (bottom right panel)

Once it is clear the calibrated relative price deviation, the authors show the impacts of the pricing error on the estimated implied volatility (IV). In particular, the IV curves display substantial deviation from the real underlying volatility ($\sigma = 0.3$) for the out-of-the money (OTM) options at all maturities. Although the IV curve deviates little from the underlying $\sigma = 0.3$ for ITM calls with longer maturity, at the short maturity of $\tau = 1/12$, there are also noticeable impacts of the pricing error for in-the-money (IMT) calls.

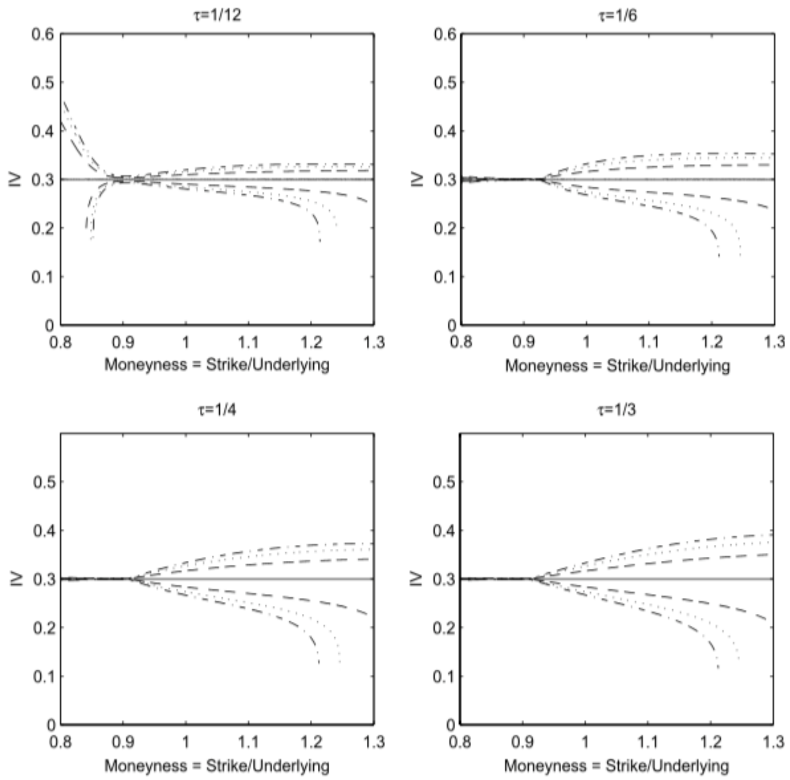


Figure 2.8 Sensitivity of implied volatility to the relative price deviation.

True volatility (solid line); implied volatility with relative pricing errors to OTM options, displayed in Fig. 1.6, amplified by ±0.5 (dashed lines), ±0.8 (dotted lines) and ±1 (dashed lines)

As can be deducted from the figure 1.7, the results of pricing errors on the estimated implied volatility are mainly three:

a. The IV curves display substantial deviation from the underlying volatility (σ = 0.3) for the out-of-the money (OTM) options at all maturities.
b. Although the IV curve deviates little from the underlying σ = 0.3 for ITM calls with longer maturity, (τ = 1/3)
c. at the short maturity of τ = 1/12, there are also noticeable impacts of the pricing error for in-the-money (ITM) calls.

The empirical studies show that the main determinants of pricing error's magnitude and its link to IV are time to expiration and moneyness. Indeed, considering the IV as an indicator of market expectations through which options are priced, the mismatch of volatilities is a symptom of a very relevant pricing error for all maturities of the

OTM options, instead for the ATM and ITM options, their IV deviation shrinks as the time to maturity increases, showing a lower pricing error. In other words, the OTM options are the ones with the greatest pricing error, followed by ITM options, and the latter becomes all the more relevant the nearer the expiration date, tending to be overpriced since IV is greater than the real occurred one.

Since IV is a very important issue for option pricing and since the relevance of the results, many scholars agree that it implies a significant positive pricing error for options. To this purpose, in the next chapter we will present what are the opportunities offered by this overestimation in option pricing and how they can be grasped through some investment strategies that involve buying and selling options, which will be actually applied to the financial markets in order to empirically validate the theoretical results presented until now.

3. Alternatives Strategies

As was presented in the previous chapter, the existing practice of inverting option price to get the implied volatility calculation assumes that the option prices are observed without errors. However, it is found that t*he implied volatility is subject to a systematic bias that results in presence of pricing errors, as it is inconsistent with the underlying volatility*. *Indeed, pricing errors are widely present in option data, which are due to a range of causes, including the bid–ask spread, non-synchronicity between the option and the spot markets, the discreteness in the quoted prices and other random errors.* As presented above, the pricing errors may also be as a *lack of consensus among the market participants on the value of options. This is especially the case for deep-in-the-money or deep-out-of-the-money options* [18].

In particular, it was found that for both calls and puts, pricing errors were the biggest for out-of-the-money options and they become more relevant the closer the expiration date is, leading in this way the long maturity OTM options to be generally underpriced and the short OTM options instead overpriced.

For these reasons, in the following chapter, we will mainly focus on presenting some viable alternative strategies that involve the selling of options in order to try to empirically confirm these results by comparing those strategies with a common buy & hold strategy.

For this purpose, the following chapter will be structured in four paragraphs; In the first, we will discuss the main reasons why we have chosen investment strategies based mainly on the sale of options, focusing our attention especially on options linked to indexes. Second section will propose the buy & hold strategy, i.e., the strategy which will be used as the primary reference for all the comparative analyses. In the third paragraph, will be proposed the functioning of the two strategies for the sale of cash secured put and covered call options that are currently traded on the market and are included in the CBOE INDEX. Lastly, in the fourth paragraph, we will propose our investment strategy consisting of alternating sales of cash-secured puts and covered call options according to their exercise.

3.1 Why to sell options?

Estimating implied volatility by inverting the Black-Scholes formula is subject to considerable error, leading to plausible mistakes in observed option characteristics. One of the most relevant aspects highlighted by the empirical studies is that, *especially for options away from the money, large changes in volatility produce changes in option prices. Conversely, even minor errors in option prices and other option characteristics produce large errors in implied volatilities* [20].

Indeed, as presented previously in figure 2.4, the differences in implied volatilities across exercise prices appear to be economically significant. *The bid-implied volatility for the short-term in-the-money call, for example, exceeds the ask-implied volatility for the short-term at-the-money call, implying the possibility of an arbitrage profit*. *However, a strategy of selling in-the-money calls and buying at-the- money calls to capture the "arbitrage profits" is more complex that merely spreading the options, however, and requires dynamic rebalancing through time* [22].

A relevant study in order to identify possible over/underpricing in the market was computed by Macbeth and Merville [31]. The authors, studying stock options listed on the CBOE, found evidence that *the Black and Scholes model systemically underprices in-the-money options and overprices out-of-the-money options*. However, Rubinstein [32], also studying stock options listed on the CBOE, found some confusing patterns. It seemed that *the Black and Scholes model was overpricing out-of-the-money options and underpricing in-the-money options for a time period between August 1976 and October 1977. However, the same model was overpricing in-the-money options and underpricing out-of-the-money options for a time period between October 1977 and October 1978* [28].

Another relevant aspect that can be pointed out on possible option mis-pricing is that a number of studies have found that *index options tend to be "richly" priced than individual options, in the sense that their implied volatility usually is higher than the subsequent realized volatility of the underlying index and as a result, that investors, who are consistent sellers of index options, have had the potential to generate relatively strong risk-adjusted returns* [1].

This phenomenon is perfectly explained by Bakshi, Kapadia and Madan [2] who complete a relevant empirical study based on nearly 350,000 option quotes based on the S&P 100 index (i.e. OEX) and its 30 largest individual equity components over the period January 1991 through December 1995. Their principal conclusion is that the *slopes of the individual equity smiles are persistently negative but are much less negative than* *the index*. The documented differences in the slope of index and individual smiles produces a substantial difference in the relative price of options: for the OEX's representative equity, the implied volatility of a deep OTM put is about 22%, as compared to at the money implied of 14%, whereas for the OEX index they are respectively 29% and 26%. Therefore, they make the important observation that *the pricing structure of individual equity options is flatter compared with that of the market index* [2].

Many explanations have been offered for this disparity, but the most reliable one refers to the huge buying pressure in index options by investors who use them to insure their equity portfolios. Correlations between individual equities tend to rise significantly during market pullbacks, so index options offer an effective way to insure diversified portfolios. Unlike with either commodity or fixed-income indexes, the vast preponderance of investment in equity indexes is long, and the consistent demand for portfolio protection by these investors represents an almost unlimited market for the far smaller number of natural options sellers.

Equity index buyers are insuring their portfolios against an unlikely event, not hoping to turn a profit on the trade. Option writers, on the other hand, tend to be speculators who receive a fixed amount of money—the premium—in return for providing portfolio protection to option buyers. Regardless of what happens later, the writer of an option keeps the premium received but can never make more than that amount on the position. However, if the S&P 500 Index fell far below the strike price of the option, the option writer could lose a much greater amount than the premium received. Therefore, as with anybody who provides insurance, the option writer can demand a premium for offering this protection. *Index option buyers tend to be hedgers who spend a relatively small premium to insure a much larger investment against precipitous loss. They do not seek to profit from their purchases and are usually better off not having to rely on the hedge at all. In this, they are comparable to flood-insurance policyholders, who are willing to renew policies ad infinitum without ever filing a claim. Buyers can never lose more than the premium paid; writers can never gain more than the premium received. This unequal relationship is compounded by the fact that the buyer can forget about his position, but the writer often incurs significant costs to hedge his book, typically in the futures market, a fact that further limits the pool of put writers* [1].

Therefore, even if theoretically one of the major achievements of financial economics is the no-arbitrage theory that determines derivative prices independently of investor demand, in the real-world, the demand-pressure has effects on option prices. Indeed, in contrast to the Black-Scholes-Merton structure, empirical results have demonstrated that, in the real world, options cannot be hedged perfectly. Consequently, since option market makers cannot perfectly hedge their inventories because of the impossibility of trading continuously, stochastic volatility, jumps in the underlying and transaction costs, option demand impacts option prices. In particular, *demand pressure in one option contract increases its price by an amount proportional to the variance of the unhedgeable part of the option. Similarly, the demand pressure increases the price of any other option by an amount proportional to the covariance of their unhedgeable parts* [33].

Given that the demand pressure of an option has a propagating effect, since the price increment is not only limited to the single option but also to the other options that have a non-zero covariance with the first one, this makes that index options tend to be "richly" priced than individual options. At this point, it is clear that there will always be a far greater number of option buyers than writers, and that buyers will always be far less price sensitive. Writers will continue to successfully demand a premium well above the expected value of the puts for the protection they provide, leading in this way options to be overpriced and, in particular, the options based on the index.

Lastly, a further aspect that is relevant to our study is the volatility of option prices. Indeed, as we can see in figure 3.1 below, the curve representing the value of an option will normally be concave upward. Since it also lies below the 45◦ line A, we can see that the option will be more volatile than the stock.

Figure 3.1 The relation between option value and stock price

Line A represents the maximum value of the option, since it cannot be worth more than the stock. Line B represents the minimum value of the option, since its value cannot be negative and cannot be less than the stock price minus the exercise price. Lines T1, T2, and T3 represent the value of the option for successively shorter maturities.

This means that a given percentage change in the stock price, holding maturity constant, will result in a larger percentage change in the option value. However, as was seen in the previous chapter, the relative volatility of the option is not constant, but it depends on both the stock price and maturity [16].

In conclusion, since those empirical studies show that normally options' prices are very volatile and overpriced, especially option indexes, could have sense consider strategies that involve the sale of options, in particular the sale of options based on indexes, in order to exploit and validate the presence of those possible pricing errors.

In the next paragraphs we are going to look specifically at strategies that consist of the Sale of Options such as the Cash Secured Put and Covered Call, we will also show some indices created ad hoc by the CBOE to replicate these strategies on the 'most traded index in the world that includes the 500 U.S. companies by market capitalization: the SP500.

Before delving into the more complex strategies in which options are involved, we will define the typical strategy involving the purchase and holding of the underlying asset, which is typically considered a benchmark of any other strategy on the underlying given its simplicity and efficiency.

3.2 Buy & Hold strategy

Equities have traditionally been regarded as risky assets. They may be attractive because of their high average returns, but these returns represent compensation for risk; thus, equities should be treated with caution by all but the most aggressive investors. In recent years, it has become a common view to argue that equities are relatively safe assets for investors who are able to hold for the long term. Many financial experts say a "buy-and-hold" strategy is the best investment strategy, especially during the weak stock market situation. An extreme version of this revisionist view is promoted by James Glassman and Kevin Hassett in their book Dow 36,000 [34].

A buy-and-hold strategy is a very conservative approach since it consists in an investment strategy where the investor buys a security and holds it for an extended period of time, regardless of the market's fluctuation. The investor actively selects stocks, but once in a position, is not concerned with technical indicators and short-term price movements. The meaning of "long-term holding" is not absolute or fixed and it is typically more than five years. Buy-and-hold strategy works best when all the proper research has been done to ensure that the stocks of a high-quality company are bought. *Conventional investing wisdom tells us that with a long-time horizon, equities render a higher return than other asset classes, such as bonds. The belief is that it is better to allow a security the opportunity to grow overtime. It rests upon the assumption that in a capitalist society, the economy will keep expanding, profits will keep growing as well as both the stocks prices and stocks dividends. There may be short-term fluctuations due to business cycles or rising inflation, but in the long term these will be smoothen out and the market as a whole will rise. A trader will ultimately be more successful over a multi-year timeframe.*

There is, however, some debate over whether a buy-and-hold strategy is superior to an active investing strategy. Both sides have valid arguments, but a buy-and-hold strategy has tax benefits because the investor can defer capital gains taxes on long-term investments [35]. *Therefore, a buy-and-hold strategy has tax benefit* (in certain countries where investors are taxed on the profit from the investment in stocks)*, since the trader is taxed at a lower tax bracket because buy-and-hold strategy often calls for a time horizon greater than one year. Trading commissions can also be reduced. It allows a trader to invest sizeable sums of money with minimal costs. By trading fewer stocks and not concerning oneself with every price movement, it makes it easier for a trader to follow their trading plan and stay the course. It is definitely less stressful for the trader.* The buy-and-hold strategy's killer is when the market is bearish. If a buy-and-hold trader purchases a stock prior to a swift market decline similar to the ones in '87 and '02, the trader may have to wait

5-10 years to breakeven on his/her initial investment. The other sour note for the buy-and-hold strategy is the fact that you have to buy-and- hold. *Making money in the market is not like working a job where more effort equals greater results. So, traders will have to fight the urge to over-trade, as the key to a successful buy-and-hold strategy is quality, not quantity* [36].

A buy-and-hold strategy has been used for a long time in financial markets. Amateur stock-market investors are commonly advised to adopt such a strategy. The standard justification by Rosenthal and Wang [37] was that *it saves on commissions and transactions costs. Secondly, buy-and-hold strategy pays on average in the long run because the stock market as a whole can be expected to generate superior long-run returns to investment (extrapolating from history). This justification relies vaguely on a hypothesis of market inefficiency that prevents future expected returns from being completely capitalized on current prices.* The auction aspect of market trading together with incompletely informed agents and illiquidity[4] produces a gap between the fundamental value of an asset and its equilibrium price, which renders the buy-and-hold strategy optimal.

Shen [38] described the buy-and-hold strategy as *a simple and crude investment strategy that buys a diversified stock market index and holds it. He assumed that investors would hold a security if and only if its expected return at the market price would provide an adequate tradeoff with the risk exposure the security brings. In other words, investors were assumed to make their own judgment on whether a security was worth holding.* It was only meant to identify the very rare times when the stock market seemed so pricey that investors might be better off to avoid it. So, the buy-and-hold strategy might suffer less from the potential data-snooping[5] problem. Lastly, his research suggested that it might be possible to use a simple rule-of-thumb to avoid some of the market downturns and to improve upon the widely preached buy-and-hold strategy. Besides Shen [38], Brozynski [40] also described buy-and-hold strategy *as a simplified notion of perfectly rational behavior. Those professionals who relied more on the buy-and-hold strategy behaved more like the arbitrageurs. So, they were more successful, more fundamentally oriented, and*

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[4] At each auction a new finite asset of individuals with private information about the asset's current-period income bids for the asset; the winning bidder acquires the asset, holds it, and consumes the income stream for as long as he desires or until some external circumstance forces him to sell at auction.

[5] Data snooping occurs when a set of data is used more than once for purposes of inference or model selection. [39]

less affected by behavioral anomalies. Buy- and-hold traders were comparatively risk averse and not self-confident, which motivated them to go along with the market. According to Wu [41], buy-and-hold strategy *had static and slow changing goals. It was always used to invest large pools of assets that were difficult and expensive to move and to eschew market timing*. It was also popular among those who were governed by boards with complicated decision making, such as pension funds, endowments, and foundations.

3.3 Cash Secured Put

Among the possible strategies involving the sale of options is certainly the well-known cash secured put, which is a strategy used by traders or investors to generate a certain income or buy stocks a predetermined price. The classic solution of buying put options to protect a portfolio of shares, precisely to protect against a drop in share prices, can easily be "overturned" in case the investor does not own shares and wants to accumulate positions on securities, taking advantage of a certain initial collection (the premium).

In particular, the Cash Secured Put involves selling a put option and simultaneously setting aside the cash to buy the stock if the option is exercised upon expiry. At the expiration date, if the put options are exercised, it allows an investor to buy the stock at a price below the agreed strike price, which reflects the investor's desired price. Conversely, if the option will not be exercised, the investor will have earned the premium received for their sale. As can be deducted, two main sub-steps can be identified for this strategy: the first is that Put options are sold on a concentrated portfolio of stocks or indexes to generate income on a cash position, where their expiration typically range from one to four months out in time; the second when the underlying is purchased due to the put options' exercises.

Cash Secured Put is primarily a stock acquisition strategy for a price-sensitive investor. Unlike a naked put writer whose only goal is to collect premium income, a cash-secured put writer actually wants to acquire the underlying stock via assignment. Therefore, we can see the cash-secured put as a variation of the naked put strategy. The fundamental difference is that the cash-secured put writer has set aside funds to buy the stock in case it is assigned, and he sees the assignment as a positive outcome. In contrast, the naked put writer hopes that the put will continue to lose value, so that the position won't be assigned and can be closed early with a profit because in this latter case, the investor would have to liquidate other assets quickly or borrow cash in order to honor an assignment notice.

The maximum loss is limited but substantial because the worst that can happen is for the stock to become worthless. In that case, the investor would be obligated to buy stock at the strike price. The loss would be reduced by the premium received for selling the put option. However, the maximum loss is lower than would have occurred if the investor simply purchased the stock outright rather than sell a put option.

At the same time, even the maximum gain from the put option is limited. However, the optimal outcome is not readily apparent in the expiration profit/loss payoff diagram, because it does not address to the possible developments after expiration. The best scenario for the short position investor, would be for the stock to dip slightly below the strike price at the put option's expiration, trigger assignment, and then rally immediately afterwards to record heights. In fact, the put assignment would have allowed our investor to buy the stock at the strike price just in time to participate in the following rally. From a strictly short-term perspective, the maximum potential gain occurs if the stock stays above the strike, causing the put option to expire without execution. The investor would keep the T-Bill cash originally set aside in case of assignment and would simply pocket the premium from the sale of the option.

However, this strategy exposes the investor to certain potential risks, which can be identified as follows:

- **Upside Risk**: Selling a put option does not allow the seller to participate in any upside appreciation of the underlying security. The maximum return is the option premium received for the put option sale.
- **Downside Risk**: The cash-secured put strategy buffers the downside risk of the underlying security but does not eliminate it. Indeed, once the put strike has been breached, the seller incurs all potential downsides below the put option strike price.

In the case of Bullish markets, a Cash Secured Put can be considered a winning strategy as the option is not exercised and, in any case, an excellent result is obtained especially if the underlying does not appreciate more than the premium collected.

A similar strategy to the Cash Secured Put, with the same payoff, but which takes into account the downside risk of the underlying, is the Covered Call which will be presented in the next paragraph.

3.4 Covered Call

As presented previously, even the covered call strategy is based on the short position in the options contract, but compared to the cash secured put, it is useful to protect the investor from the downside risk. For this reason, it is typically associated to a bearish market outlook.

Covered call strategy is an investing strategy that involves selling call options. It's the right to buy against stock that you already own or have recently purchased to generate additional income from those shares. Indeed, the options you sell is defined "covered" because the investor owns enough shares to cover the transaction at the delivery date of the call option sold as required by the contract in case of exercise.

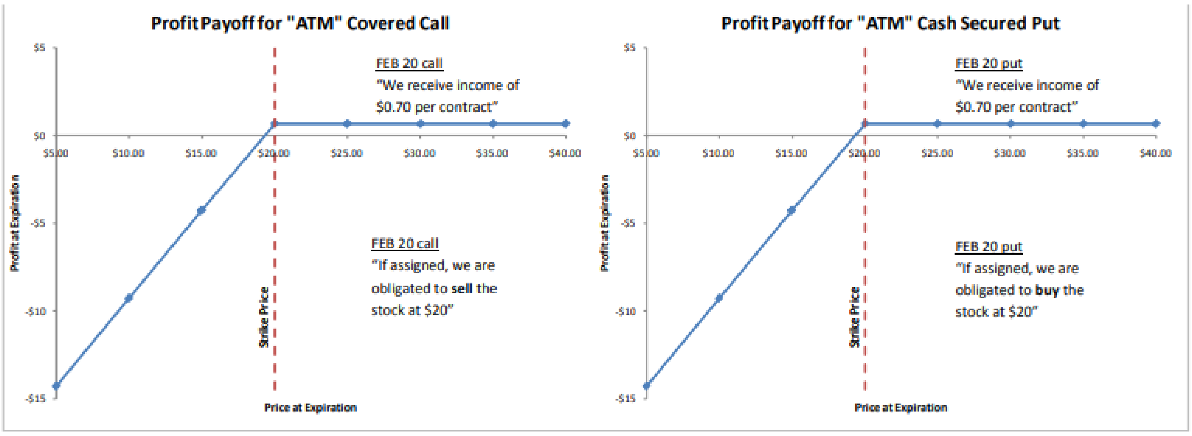
Covered calls are a neutral strategy, meaning the investor only expects a slight increase or decrease in the underlying stock's price for the life of the written call option. Indeed, this strategy is often employed when an investor has a short-term neutral view on the asset and, therefore, holds the asset long and simultaneously has a short position via the option to generate income from the option premium.

For this reason, the covered call strategy isn't useful for very bullish or very bearish investors because very bullish investors are typically better off not writing the option and just holding the stock.

As regards the risks of this strategy, they are the same as those of the cash secured put. In fact, even the sale of Covered Call presents an Upside Risk, as it doesn't allow the seller to participate in any upward appreciation of the security, since he is obliged to sell the underlying at a price lower than its market value.

However, covered call strategy offers to investors two potential benefits: first of all, many investors use covered calls to receive a premium on a regular basis, sometimes monthly other times quarterly, with the goal of adding several percentage points of cash income to their annual returns. Another reason to sell covered calls consists of helping investors to sell the underlying asset at a target price which is typically above the current price.

Figure 3.2 PayOff Covered Call and Cash Secured Put



Again, the trend in returns is obtained by entering the different values of the sottostnat at maturity as values along the X-axis, while the corresponding gain or loss is entered on the Y-axis

Looking at the profit payoffs of the two write strategies, we can see that the respective charts are equal. Therefore, this means that Covered Call and Cash Secured Put strategies (in the case of ATM moneyness) are expected to have, on average, the same performances. Indeed, the substantial difference came from the fact that in the first strategy, investor sells Call and owns the underlying asset, in the second investor sells Put and doesn't own the underlying but the capital required.

3.5 Indexes on Write Strategies

As presented in the first paragraph of the first chapter, SEC-regulated options contracts began with the launch of the Chicago Board Options Exchange (CBOE) in 1973, the same year the landmark Black-Scholes options pricing model was published. Since these write investment strategies became so common among the institutional and retail investors, they push the "index makers" to create real indices that replicate the *modus operandi* of all those who want to go into the "covered" sale of Options. To meet this need, the CBOE has devised two indices that replicate these strategies and in order to guarantee liquidity, it decided to build them on the largest and most traded stock index in the world i.e., the index including the 500 largest companies operating in the United States: The S&P500 (SPX).

3.5.1. BXM – BuyWrite Index

In April 2002, CBOE announced the S&P 500 BuyWrite Index (BXM), which is a benchmark index designed to track the performance of a hypothetical buy-write strategy on the S&P 500 Index. The BXM is a passive total return index based on buying an S&P 500 stock index portfolio, and "writing" the near-term S&P 500 Index (SPX) "covered" call option, generally on the third Friday of each month (the Roll Date). The SPX call written will have about one month remaining to expiration, with an exercise price just above the prevailing index level (i.e., slightly out of the money). The SPX call is held until expiration and cash settled, at which time a new one-month, near-the-money call is written. Many studies have exhibited that buy-write positions will generally have lower returns than stocks in times of rising stock markets, however another part of them have shown that in particular market condition, like during the mortgage's subprime crisis, the buy write position can present a comparable performance to the SPX. This phenomenon is mainly due to a covered call writer which does not participate in upside stock gains beyond the strike price plus the premium received.

Getting to the core of the index calculation, it is updated every 15 seconds according to the following formula:

$$BXM_t = BXM_{t-1}(1 + R_t)$$

Where:

- BXM_t is the current level of the BXM index;
- BXM_{t-1} is the level of the BXM index on the previous day;
- $(1 + R_t)$ is the return of the BXM index.

Now we need to distinguish the index in case of Non-Roll date and Roll-Date, which is the 3rd Friday of the month. According to Non-Roll Date, we calculated the return of the index as:

$$(1 + R_t) = \frac{(S_t - Div_t - C_t)}{S_{t-1} - C_{t-1}}$$

Where:

- S_t is the closing value of SP500 index on date t. For intraday calculations, the current reported value of SP500 Index is used;
- S_{t-1} is the closing value of SP500 at *t-1*;
- C_t is the arithmetic average of the last bid and ask price of the call option reported before 4:00 pm at time *t*;

- C_{t-1} is the arithmetic average of the last bid and ask price of the call option reported before 4:00 pm at time *t-1*;
- Div_t represent the ordinary cash dividends payable on the component stocks underlying the SP500 Index that trade "ex-dividend" at date t expressed in SP500 Index points.

On Roll-Date instead, the gross rate of return is compounded from three gross rates of return: the gross rate of return from the previous close to the SOQ[6] is determined and the expiring call is settled, the gross rate of return from the SOQ to the initiation of the new call position, and the gross rate of return from the time the new call options is deemed sold to the close of trading on the roll date. It can be expressed as the following:

$$(1 + R_t) = (1 + R_a) * (1 + R_b) * (1 + R_c)$$

Where:

- $1 + R_a = \frac{S^{SOQ} + Div_t - C_{settle}}{S_{t-1} - C_{t-1}}$
- $1 + R_b = \frac{S^{VWAV}}{S^{SOQ}}$
- $1 + R_c = \frac{S_t - C_t}{S^{VWAV} - C_{VWAP}}$

Where:

- R_a is the rate of return of the covered SP500 Index portfolio form the previous close of trading through the settlement of the expiring call option;
- S^{SOQ} is the Special Opening Quotation used in determining the settlement price of the expiring call option;
- $C_{settle} = Max(0; SOQ_t - K_{old})$ is the final settlement price of the expiring call option, where K_{old} is the strike of expiring option;
- R_b is the rate of return of the un-covered SP500 Index portfolio form the previous close of trading through the settlement of the expiring call option;
- S^{VWAV} is the Volume-Weighted average value of SP500 based on the same time and weights used to calculate the VWAP[7] in the new call option;

[6] SOQ stay for Special Opening Quotation and generally will be based on the opening values of the component stocks, regardless of when those stocks open on expiration day. However, if a stock does not open on that day, its last sale price will be used in the Special Opening Quotation. The Special Opening Quotation may or may not be within the cash index prices on expiration day.

[7] The Volume Weighted Average Price (VWAP) is a measure that represents the weighted average price at which a given day's trades on a given stock took place. The measure is used in particular by

- R_c is the rate of return of the covered SP500 Index portfolio from the time the new call option is deemed sold to the close of trading on the roll date;
- C_{VWAP} is the volume-weighted average trading price of the new call option between the 11:30 a.m. and 1:30 p.m.;
- C_t refers to the average bid/ask quote of the new call option reported before 4:00 on the roll date.

A similar index also proposed by the CBOE is the BXY. BXY tracks the value of a hypothetical portfolio that overlays a short 2% out-of-the-money call on an investment in S&P 500 stocks. The smaller premium of a 2% out-of-the-money call relative to an at-the-money call provides smaller buffer but also less give up on the uspside.

3.5.2. PUT – PutWrite Index

After CBOE introduced five buy-write indexes, some investors inquired as to the possibility of new benchmark indexes based on put options. For this reason, in mid-2007 CBOE introduced the first major benchmark index for the cash-secured ATM put sale strategy—the CBOE S&P 500 PutWrite Index (PUT)—and Ansbacher Investment Management, Inc. became the first money management firm to gain a license on the PUT Index. Daily historical prices on the PUT Index are available back to June 30, 1986. The PUT Index tracks the performance of a hypothetical portfolio of securities that yields a buffered exposure to S&P 500 stock returns. The PUT portfolio is composed of one- and three-month Treasury bills and of a short position in at-the-money put options on the S&P 500 index (SPX puts). The number of puts sold is selected to ensure that the value of the portfolio does not become negative when the portfolio is rebalanced. The PUT portfolio is rebalanced monthly, typically on the third Friday of the month, which is called roll-date and it coincides with the expiration of the SPX options. After that expiration, new number of SPX puts is then sold on the market.

The CBOE calculates the PUT in real-time every fifteen seconds during each trading day excluding roll days. On any given date, the index represents the mark-to-market value of the base date $100 invested in the PUT strategy. At the close of every business date, the value of the PUT is equal to the value of the Treasury bill account less the mark-to-market value of the puts:

$$PUT_t = M_t - N_{last}P_t$$

institutional investors as a reference for the execution of a sale and purchase transaction involving several exchanges.

Where M_t is the total Treasury Bill balance at close of date t, N_{last} is the number of put options sold at the last roll date, and P_t is the arithmetic average of the last bid/ask prices of the put option reported before 4:00 pm on date t.

On all but roll date, the Treasury bill balance is obtained by compounding the one and three-month Treasury balances at the previous business close at their respective daily rates.

$$M_t^i = (1 + r_{t-1}^i)M_{t-1}^i$$

Where $i = 1{,}3$ for 1- and 3-months T-Bill, and r_{t-1}^i is the corresponding T-Bill rate from the previous to the current close. The T-Bill rates between two roll dates are obtained by compounding the daily rates.

On the third roll date, the T-Bills are deemed to mature, the cash is used to pay for final settlement of the puts if they expire in-the-money, and new puts are sold. The net cash available for reinvestment is:

$$M_t = \sum_i (1 + r_{t-1}^i)M_{t-1}^i - N_{last}Max[0; K_{old} - SOQ_t] + N_{new}P_{vwap}$$

Where:

- K_{old} is the strike price of the put options sold at previous roll date;
- SOQ_t is the final settlement price on roll date t;
- N_{new} is the number of new puts sold;
- P_{vwap} is the volume-weighted average price at which the new options are sold.

The number of new puts sold on any roll date t is set such that the Treasury balance at the next roll date coverts the maximum put settlement loss:

The third Roll Date:

$$N_{new} = \frac{\sum_i (1 + r_{t-1}^i)M_{t-1}^i - N_{last}Max[0; K_{old} - SOQ_t]}{\frac{K}{(1 + R_1)} - P_{vwap}}$$

Others Roll Date:

$$N_{new} = \frac{M_{1\_Roll} + M_{3\_Roll}}{K - P_{vwap}(1 + R_1)}$$

Where:

$$M_{1\_Roll} = Max[0; (1 + r_{t-1}^1)M_{t-1}^1 - N_{last}Max[0; K_{old} - SOQ_t]] * (1 + R_1)$$

And

$$M_{3\_Roll} = (1 + r_{t-1}^{3})M_{t-1}^{3} + Min[0; (1 + r_{t-1}^{1})M_{t-1}^{1} - N_{last}Max[0; K_{old} - SOQ_t]] * (1 + R_3)$$

Where K_{new} is the strike price at which the new puts are sold, and R_1 and R_3 are the one- and three-month T-Bill rates to the next roll date.

As can be deduced, each CBOE Index represents only one particular write options strategy, which as such, will only perform satisfactorily in the case of a single market trend. To improve the performance of the strategies proposed by the CBOE, a different strategy will be proposed in the following paragraph, which envisages the dynamic alternation between the two different write options strategies. This strategy aims to try to improve performance, as it will be more suitable for common market swings that may occur.

3.6. Our strategy: Put-Call-Put

Over the past decades, many investors have changed their expectations, becoming less concerned about going long to take advantage of the possible next bull market in equities and more concerned about finding ways to reduce the volatility of their portfolio, increasing in this way their risk-adjusted return. This is mainly due to the turbulence experienced by major stock indexes from the earlier bursting of the dot-com bubble, the subprime mortgage and credit crises to the more recent Covid-19 pandemic.

For this reason, our intention is to propose a dynamic allocation approach to construct option writing strategy which tries to exploit the alleged options pricing errors in the market in order to provide a better risk-adjusted return than the possible buy-hold strategy applied on the same reference index. Specifically, our strategy involves the alternation of the sale of cash secured put or covered call options according to their eventual exercise. Specifically, it consists of switching between put write and buy write strategies through dynamic allocation in order to improve the performances they would have individually, by leveraging on bullish or bearish market expectations based on their options exercise.

Our strategy stems from the fact that stock market predictability, portfolio allocation and derivative pricing are three prominent topics in modern finance. Besides ample academic and empirical research, there are great practical interests to develop and implement investment strategies reflecting these theoretical underpinnings. Nowadays, most of the efforts made in this direction seek to exploit the digitalization to propose a dynamic allocation approach to construct portfolios instead of the more standard and traditional passive strategies. Among those new proposals, a relevant portion tries to combine also technical analysis using some of its indicators to support the switching moment, as in the *Active leveraged option overlay portfolio* (ALOOP)[8] proposed by George Yang [42].

Going in depth to our strategy, as presented above, it could be defined as **Put-Call-Put strategy** because it provides the alternating sale of cash secured put or covered call options, depending on their actual exercise by the counterparty. That is why, based on the effective exercise, our strategy could be divided in two main cycles: **Put cycle** and **Call cycle**.

Our starting point is represented by the **Put Cycle**. Since a country's economy can be considered as growing in the long run, it is therefore possible to assume that even the financial market is always expanding in the long run with slow but steady growth. Once this is expected, the initial capital available is used as collateral to sell as many cash secured puts as possible, since the put write is representative of bullish market view and therefore, in accordance with expectations, a lower probability of exercise is awaited. In addition, to preserve the value of our capital from a possible inflation, it is invested in the purchase of a bond with a maturity coinciding with that put options in order to have the possible liquidity requested at maturity.

However, with this bullish market outlook, we are limiting our gain to the proceeds from the sale of the put options. Thus, in order to hedge against a possible excessively bullish trend of the underlying, the proceeds gained from the sale of the put options are used as capital to buy the underlying asset. Once the options have been sold, on their expiration date we have two possible alternatives: the first consists of the options not being exercised, while the second consists of the put options exercise.

In the first case, a non-execution of the option means that the underlying asset at the expiration date has a higher price than the strike price, so we will have a total gain from the proceeds of the puts sold appreciated by an amount determined by the underlying asset's activity which was previously bought. Since the unexercised option is a sign of a raising market, our strategy is to repeat the put cycle i.e., we will sell the

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[8] Active leveraged option overlay portfolio (ALOOP) involves the switching between shorting call and writing put index options, having as market timing scheme to switch a technical analysis rule based on the popular "double cross-over method" i.e., Golden Cross and Black Cross signals of the moving averages indicators.

appreciated underlying asset and its proceeds will form part of the new capital available, which will be greater than the initial one. This increased capital will be reused as new collateral to sell a new quantity of cash secured put, which will be increased compared to the previous quantity and which in turn will provide greater proceeds from its sale, therefore a greater quantity of purchase of the underlying. This put cycle will be repeated until the options are non-exercised on their expiration date of the reference cycle.

However, as presented above, at the expiration date of the cash secured put cycle, a different situation can arise because the put options can be exercised. In this situation, since we are supposing a rational investor owns the option right, we will have that at the maturity date the spot price of the underlying asset will be less than the strike price. Due to our short position in the option contract, we will suffer a loss given by the difference in prices because we are forced to buy, at higher price than the market value, a quantity of the underlying asset proportional to the number of puts previously sold, plus a possible devaluation of the premium previously received from their sale, since it was invested in the underlying asset’s purchase.

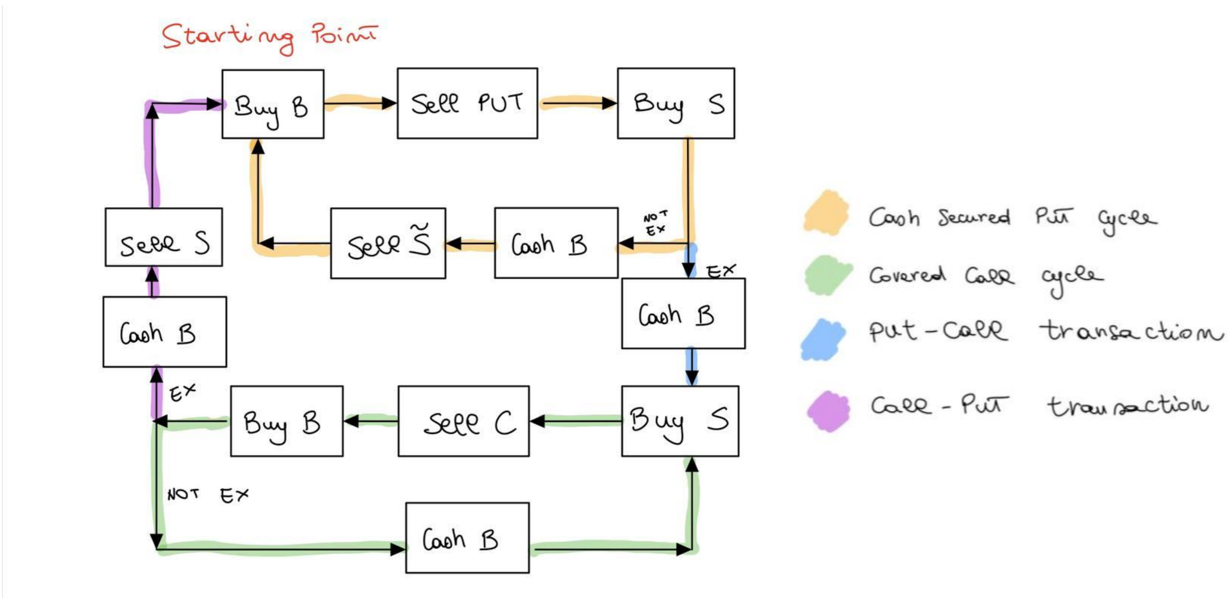
At this stage, our portfolio will be exclusively composed by the underlying asset, in a total quantity given by the sum of two components, represented by the exercise of the put and the quantity acquired through the premium received from the sale of the previous put. The overall quantity of the underlying asset will be required as collateral for the sale of a greater quantity of covered call options than the previous cash-secured puts, thus sanctioning entry into the **Call Cycle**. In fact, we consider the exercise of the put options as a swing signal in the market, suggesting possible entry into a bearish market trend. Since we physically own the underlying asset, our strategy involves selling covered call options because it fits with our market view, but in order to avoid a possible devaluation of the premium received, the latter will be invested in bonds with the same maturity of the covered call sold. As was the case for the put cycle, also in the call cycle we will have two possible cases to the expiration date of the covered call: a first one in which the covered call options aren’t exercised and a second one, in which the investor in a long position decides to exercise them.

In the first scenario, the covered call options will not be exercised on the expiration date because the spot price of the underlying asset will be lower than the strike price. Being short in the contract, our position will see a gain from the proceeds of the sale of the covered call, slightly increased by the yield on the bond. This gain will be used to buy the underlying asset at a discount since the price paid is lower than the agreed strike price, that is the maximum price at which we would be willing to buy the underlying asset. This new quantity of the underlying will be added to the previous quantity, allowing a larger quantity of covered call options to be sold. In the absence of covered call exercise, the persistence of a bearish market is suggested, so we will remain consistent with this view and proceed with the reiteration of the call cycle, but having, compared to the previous call cycle, an increased amount of asset, so amount of call sale and consequently an increased premium received for the sale.

If instead our bearish market's expectations are not realized, we would come in the second alternative of covered call options' exercise at the expiration date since we will have that the spot price of the underlying turns out greater of the strike price. This exercise will be taken by us as a further signal of a market swing, announcing entry into a bullish market and reporting a loss given by the difference in prices, since being short in the call, we will be forced to sell the owned underlying at a price below its market value. At this point, our portfolio will be made up exclusively of cash, which will be used as collateral for the sale of the largest number of cash secured puts, highlighting the switch in a new put cycle where the same steps described above will be taken.

All the steps defined so far can be summarized graphically in the following algorithm:

Figure 3.3 PUT-CALL-PUT Strategy Cycle



the image represents the algorithm of our strategy, which is basically based on the change of cycle based on the eventual exercise of options.

Once the working principle of our strategy has been presented, it is then possible to derive the analytical representation of the two different cycles.

PUT CYCLE

At the time instant $t + 1$, the capital value X_{t+1} will be computed by considering:

- The payoff gained from the PUT with strike price equals to K_t
- The capital X_t of the previous time instant t which is invested in a bond with same options' expiration date.
- The gain from the sale of the PUTs at time instant t revalued according to the performance of the underlying

$$X_{t+1} = X_t(1 + r_t) - N_{p,t}MAX(0; K_t - S_{t+1}) + N_{p,t}P_t \frac{S_{t+1}}{S_t} \quad (1)$$

Using as a constraint that the capital available at t invested in bonds is sufficient to cover any exercise of the PUT, we can derive the restriction:

$$X_t(1 + r_t) = N_{p,t}K_t \quad (2)$$

From the (2), we can deduce the number of puts which can be sold:

$$N_{p,t} = \frac{X_t(1 + r_t)}{K_t}$$

Where:

$N_{p,t} = Number\ of\ PUT\ sold\ at\ time\ t$

$P_t = Put\ Price\ in\ t$

$r_t = bond's\ yield$

$K_t = PUT's\ Strike\ Price$

$\frac{S_{t+1}}{S_t} = Appreciation\ or\ depreciation\ of\ the\ underlying\ from\ t\ to\ t + 1$

$X_t = Portfolio\ value\ in\ t$

Substituting $N_{p,t}$ into (1) we will have:

$$X_{t+1} = X_t(1 + r_t) - \frac{X_t(1 + r_t)}{K_t} MAX(0; K_t - S_{t+1}) + \frac{X_t(1 + r_t)}{K_t} P_t \frac{S_{t+1}}{S_t}$$

$$X_{t+1} = X_t(1 + r_t)\left[1 - \frac{MAX(0; K_t - S_{t+1})}{K_t} + \frac{P_t}{K_t}\frac{S_{t+1}}{S_t}\right]$$

CALL CYCLE

At the time instant $t + 1$, the capital value X_{t+1} will be computed by considering:

- The payoff gained from the Call options with strike price equals to K_t;
- The capital X_t of the previous time instant t, which is appreciated or depreciated according to the performance of the underlying, since it was invested to buy the underlying;
- The gain from the sale of the Calls, which is invested in to buy the bond with the same call options' expiration;

$$X_{t+1} = X_t\frac{S_{t+1}}{S_t} - N_{C,t}MAX(0; S_{t+1} - K_t) + N_{c,t}C_t(1 + r_t) \qquad (1)$$

This time there is no constraint, except that the available capital $K_t$coincides with the overall current value of the underlying asset $N_{C,t}S_t$ in our portfolio. From this relation, it is possible to directly obtain the number of call options we can sell:

$$N_{c,t} = \frac{X_t}{S_t}$$

$N_{c,t} = Number\ of\ CALL\ sold\ at\ time\ t$

$C_t = Price\ of\ CALL\ in\ t$

$r_t = bond's\ yield$

$K_t = Strike\ Price\ of\ the\ CALL\ option$

$\frac{S_{t+1}}{S_t} = Appreciation\ (or\ depreciation)\ of\ the\ underlying\ from\ t\ to\ t+1$

$X_t = Portfolio\ value\ in\ t$

Substituting $N_{c,t}$ into (1) we will have:

$$X_{t+1} = \frac{X_t}{S_t}[S_{t+1} - MAX(0; S_{t+1} - K_t) + C_t(1 + r_t)]$$

However, the strategy proposed so far is highly conservative since the proceeds from the sale of cash-secured puts or covered calls are used to hedge against an excessively bullish or bearish trend in the case of puts or calls respectively. For this reason, we have decided to propose a more aggressive strategy, which we will call Aggressive-Strategy. Conceptually it is identical to the previous one, with the difference that the premiums received from the sale of options is used to increase the position on the sale itself, thus allowing the sale of an increased quantum of options. Therefore, we will have that in the case of the put cycle, the premium from the sale of cash-secured puts is used as additional collateral to sell a larger quantity of puts; whereas in the case of the call cycle, the premium received will be used to immediately buy the underlying asset and sell an increased quantity of covered call.

PUT CYCLE-AGGRESSIVE

Within the put aggressive cycle, at time instant, the capital value will be computed by taking the gain from the sale of the puts $N_{p,t}P_t$ at time instant t, from the payoff $MAX(0; K_t - S_{t+1})$ accrued by the put options and from the capital X_t of the previous time instant t, which results invested in a bond with risk free equal to r_t. The final formula will be:

$$X_{t+1} = N_{p,t}P_t + X_t(1 + r_t) - N_{p,t}MAX(0; K_t - S_{t+1}) \quad (1)$$

This time, we will have as a constraint that the capital available as collateral for any future exercise of the options is increased by the premium from the sale equal to $N_{p,t}P_t$. Therefore, the final constraint formula will be:

$$N_{p,t}P_t + X_t(1 + r_t) = N_{p,t}K_t \quad (2)$$

From the (2), we can calculate the number of puts which can be sold:

$$N_{p,t} = \frac{X_t(1 + r_t)}{K_t - P_t}$$

As can be seen from the formula, the amount of put options that are sold in the Aggressive-Strategy is increased compared to the same in conservative strategy because the denominator is decreased from a quantity equal to the put price P_t.

Substituting $N_{p,t}$ into (1) we will have:

$$X_{t+1} = \frac{X_t(1 + r_t)}{K_t - P_t}P_t + X_t(1 + r_t) - \frac{X_t(1 + r_t)}{K_t - P_t}MAX(0; K_t - S_{t+1})$$

$$X_{t+1} = X_t(1 + r_t)\left[\frac{P_t}{K_t - P_t} + 1 - \frac{MAX(0; K_t - S_{t+1})}{K_t - P_t}\right]$$

$$X_{t+1} = \; X_t(1+r_t)\left[\frac{P_t + K_t - P_t - MAX(0; K_t - S_{t+1})}{K_t - P_t}\right]$$

$$X_{t+1} = \; X_t(1+r_t)\left[\frac{K_t - MAX(0; K_t - S_{t+1})}{K_t - P_t}\right]$$

CALL CYCLE-AGGRESSIVE

While in the case of the Put Cycle it was simple to calculate the largest number of options that could be sold, in the Call Cycle we assumed that there are two tranches of option sales: the first one whose proceeds are used to buy more underlying which allows us to sell the second call's tranche. At this point, we stop at the second stage since the premium from the sale of the second tranche of call options, will be used to purchase risk-free assets, albeit by a small percentage. So, the starting formula will be:

$$X_{t+1} = \; X_t\frac{S_{t+1}}{S_t} + N_{c,t}^1 C_t \frac{S_{t+1}}{S_t} - N_{c,t}^{tot} MAX(0; S_{t+1} - K_t) + N_{c,t}^2 C_t (1+r_t)$$

First tranche of call options sold

$$N_{c,t}^1 = \frac{X_t}{S_t}$$

After buying underlying with the proceeds of the sold Call Options, we sell additional calls and thus have a second tranche of sold call options:

$$N_{c,t}^2 = N_{c,t}^1 \frac{C_t}{S_t} = \frac{X_t}{S_t}\frac{C_t}{S_t}$$

Lastly, we will have a final total value of options sold:

$$N_{c,t}^{tot} = N_{c,t}^1 + N_{c,t}^2 = \frac{X_t}{S_t}\left(1 + \frac{C_t}{S_t}\right)$$

$N_{c,t}^1 = \; First\ tranche\ of\ CALL\ sold\ at\ time\ t$

$N_{c,t}^2 = \; Second\ tranche\ of\ CALL\ sold\ at\ time\ t$

$N_{c,t}^{tot} = Total\ \ Number\ of\ CALL\ sold\ at\ time\ t$

$C_t = Price\ of\ CALL\ in\ t$

$r_t = bond's\ yield$

$K_t = Strike\ Price\ of\ the\ CALL\ option$

$\frac{S_{t+1}}{S_t} = Appreciation\ or\ depreciation\ of\ the\ underlying\ from\ t\ to\ t+1$

$X_t = Portfolio\ value\ in\ t$

Substituting $N^1_{c,t}$, $N^2_{c,t}$ and $N^{tot}_{c,t}$ into (1) we will have:

$$X_{t+1} = X_t \frac{S_{t+1}}{S_t} + \frac{X_t}{S_t} C_t \frac{S_{t+1}}{S_t} - \frac{X_t}{S_t}\left(1 + \frac{C_t}{S_t}\right) MAX(0; S_{t+1} - K_t) + \frac{X_t}{S_t}\frac{C_t}{S_t} C_t(1 + r_t)$$

$$X_{t+1} = X_t \frac{S_{t+1}}{S_t}\left[1 + \frac{C_t}{S_t} - \frac{1}{S_{t+1}}\left(1 + \frac{C_t}{S_t}\right) MAX(0; S_{t+1} - K_t) + \frac{1}{S_{t+1}}\frac{C_t}{S_t} C_t(1 + r_t)\right]$$

$$X_{t+1} = X_t \frac{S_{t+1}}{S_t}\left[\left(1 + \frac{C_t}{S_t}\right)\left(1 - \frac{MAX(0; S_{t+1} - K_t)}{S_{t+1}}\right) + \frac{1}{S_{t+1}}\frac{C_t}{S_t} C_t(1 + r_t)\right]$$

As can be seen, even in this case, the number of call options sold via this strategy is increased compared to the case of the conservative strategy. Therefore, we are increasing our exposure to the market view suggested by the options.

In conclusion, the strategy we have decided to propose stems from an attempt to exploit the alleged overpricing of options present in the market in order to provide a better adjusted risk-return that one would have with a simple passive strategy such as buy and hold. Moreover, as can be deduced, it has two versions: a more conservative one that aims to protect against any excessive trends in the market, and the second more aggressive one that aims to maximize option selling.

Therefore, these two versions can be summarized with the formulas

Put-Call-Put

$$X_{t+1}\begin{cases} X_t(1 + r_t)\left[1 - \dfrac{MAX(0; K_t - S_{t+1})}{K_t} + \dfrac{P_t}{K_t}\dfrac{S_{t+1}}{S_t}\right]\ if\ from\ PUT \\ \dfrac{X_t}{S_t}\left[S_{t+1} - MAX(0; S_{t+1} - K_t) + C_t(1 + r_t)\right]\ if\ from\ CALL \end{cases}$$

Put-Call-Put Aggressive

$$X_{t+1}\begin{cases} X_t(1 + r_t)\left[\dfrac{K_t - MAX(0; K_t - S_{t+1})}{K_t - P_t}\right]\ if\ from\ PUT \\ X_t \dfrac{S_{t+1}}{S_t}\left[\left(1 + \dfrac{C_t}{S_t}\right)\left(1 - \dfrac{MAX(0; S_{t+1} - K_t)}{S_{t+1}}\right) + \dfrac{1}{S_{t+1}}\dfrac{C_t}{S_t} C_t(1 + r_t)\right]\ if\ from\ CALL \end{cases}$$

In order to assess the reliability of the proposed strategy and its more aggressive version, in the following chapter, we will present the empirical study we conducted, and the results obtained, highlighting their significance.

4. Empirical Results

The next chapter will be concerned with reporting the practical application of our previously proposed strategy in order to empirically assess the actual existence of market opportunities arising from option overpricing. As expounded in the previous section, if we take the interpretation of IV as reliable expectations of the future trend of the volatility of the underlying asset as valid, empirical studies show a deviation of the same from the actual volatility that actually occurs in the market. This discrepancy between volatilities results in the existence of errors in option pricing, and the subsequent occurrence of the volatility smile phenomenon, causes a further distortion of the pricing error. Since most studies agree that such pricing error is in fact overpricing, we have found it appropriate to use option selling as the leading criterion for our proposed investment strategy, which as presented in the previous chapter, involves the dynamic alternation of two different cycles.

In this chapter, we will be concerned with reporting what the performance of our strategy would have been over a 10-year time horizon if it had been concretely applied, in both of its two versions (conservative and aggressive), to an appropriately chosen benchmark index. Specifically in the first paragraph, we will report the main assumptions that will underlie our subsequent computations and the reason why they were made. In the part, we will report an initial application of our strategy to the S&P 500 index, in which the value of the options will be inferred by us through the use of the Black & Scholes model, due to a lack of historical data availability. After we will propose the computational application to the S&P500 of our strategy in both versions, which will be more accurate and timelier than the previous one, since the data used is an option chain history of a distribution ETF that replicates the index's performance. At the end, through the estimation of a series of indicators, a timely evaluation and interpretation of the performance obtained in the previous paragraphs will be carried out.

4.1 Assumptions

Distortions in Implied Volatilities are a symptom of biases in observed prices, the non-linear amplification of which is mainly determined by the volatility smile. Indeed, *"even if investors price options in accordance with the Black–Scholes– Merton model using a known volatility, the implied volatilities across strikes almost always vary and some quite notably*." As was exhibited previously, they do so in the form of smiles, skews, and smirks that greatly resemble the patterns observed in practice using prices from markets that are far from perfect [24].

In order to exploit this eventual behavior in the market and at the same time give reliability to our analysis, the first relevant assumption will be to consider a long-term time horizon. Although we would have liked to consider a time period starting as far back as the early years of the new millennium so as to keep track of the various black swans that have occurred in the markets, the period chosen will turn out to be roughly the last 10 years due to the lack of data availability. However, we still tend to consider our analysis reliable, since a decade is considered a sufficiently large time span to highlight any changes in the structural characteristics of the market.

In the presence of measurement errors, at-the-money options near expiration provide extremely noisy volatility estimates. Indeed, as evidenced by several studies reported above, the measurement error has a direct proportionality relationship with the expiration date and an inverse relationship with moneyness. We will therefore have that the further we move away from the ATM condition and the closer the option expiration is, the more significant this error will be. Since our goal is the identification of risk-adjusted return maximization, the second assumption will be to consider European options both ATM and with different percentages of ITM or OTM, all having as maturity 1 month since we believe this to be a sufficiently short time to have a relevant error, but at the same time sufficiently large to reflect the real market trend. Therefore, to assess the performance of our portfolio and decide to which type of equity risk of the underlying to expose ourselves through rebalancing between the three components (Underlying Asset S, Bond B, and Put or Call), we will use monthly option values. To keep *cetaris paribus* comparison between our performance and the option prices implied by the monthly option writing indices, such as BXM and PUT, we will also use the third Friday of each month as our roll-date. Therefore, we will have that the decision on whether to exercise the options and the value of the premium collected from the sale of the options, will be made based on the daily spot price and the daily closing value of the options in the roll-date, respectively. In this way, it is as if the operations of exercising the options of the previous cycle and selling them for the new one, occur in a single instant in time since there is no trading going on Saturday morning and therefore the price values can be considered stable during our rebalancing operations.

At this stage, the last assumption we have to make is the choice of the benchmark index on which our strategy will be concretely applied. In order to comply with the same conditions as the CBOE and since most of the studies that have been reported have focused on analyzing the U.S. financial market, we have decided to adopt our strategy on the Standard & Poor 500 Index, known as the S&P 500 (SPX) which represents the 500 largest companies by capitalization in the U.S. market. Since the U.S. market turns out to be the reference market at the global level, it turns out to be the most actively traded market, thus allowing us to minimize our liquidity risk and maximum freedom in the choice of option moneyness during the monthly selling or buying phases. Accordingly to the reference market, we will use Treasury bills as an approximation of the risk-free asset that will have a maturity of 4 weeks, so as to align its maturity with those options and thus provide liquidity if needed.

Next, having as our goal the development of an investment strategy suitable for all types of investors, both retail and institutional, we decided to consider an initial representative capital of $100, which theoretically represents a small enough amount to not affect the market book order even in the case of options far away from the ATM. Therefore, the option values considered will maintain a constant bid-ask spread during the roll date, and since in terms of performance evaluation it is irrelevant, we arbitrarily decided to consider selling fractions of options.

Once we have presented the framework of main assumptions that will guide all of our analysis, the following paragraphs will report in a detailed and timely manner, all of the steps we have taken.

4.2 PCP with option values derived via Black and Scholes formula

In the following section, our focus will be on applying our Put-Call-Put (PCP) Strategy, in both its conservative and aggressive versions, to the main U.S. index: S&P500 (SPX). Due to a difficulty of study material since we were unable to find the historical option price quotes written on the index, we decided to derive the option price independently through the Black & Scholes model. In fact, we proceeded to download historical data on daily prices of the SPX from January-2012, through February-2022, and aligning the study with the assumptions, we focused only on the index data of the third Friday of each month.

Once the daily price histories were available, we proceeded to calculate the theoretical value that options should have had according to the Black & Scholes model. As set out in previous chapters, the main attractive feature of B&S is the observability of most of its parameters except for the **risk-free rate** and **volatility**. While for the former, the approximation and findability was straightforward since it reflects the historical trend of the Yield-Curve of the four-week T-Bill (TB4W), the calculation of volatility was much more delicate. We approximate this volatility with the Market Volatility Index (VIX) constructed by the CBOE.

The choice of VIX as an approximation of volatility is more consonant by construction because it represents measures that averages implied volatilities from puts and calls options on the S&P500, having two separate maturities. [20] Indeed, the VIX Index is a financial benchmark designed to be an up-to-the-minute market estimate of the expected volatility of SP500 and is calculated by using the midpoint of real-time data S&P500 (SPX) option bid/ask quotes. More specifically, the VIX Index is intended to provide an instantaneous measure of how much the market expects the SPX will fluctuate in the 30 days from the time of each tick of the VIX Index. Intraday VIX Index values are based on snapshot of SPX option bid/ask quoted every 15 seconds and are intended to provide an indication of the fair market price of the expected volatility at particular points in time. Indeed, the VIX is an average of eight implied volatilities from near-the-money puts and calls for the S&P 500 option contract closest to expiration and the next- shortest maturity. For each maturity and strike price, the VIX averages implied volatilities from puts and calls. Next, for each maturity, the VIX linearly interpolates the volatilities from the high and low strike prices to an at-the-money volatility. Finally, the VIX linearly interpolates these two volatilities to a single volatility for a 30-day maturities. Ninety-five percent confidence intervals for the VIX are on the order of plus or minus 25 basis points. The three main sources of the VIX's precision are the focus on near-the-money options, the low weights assigned to implied volatility from options near expiration, and the averaging of implied volatilities from puts and calls to cancel errors in the underlying asset price. Therefore, we downloaded the daily data of the VIX index, focusing only on roll-date days, as we did for the historical prices of the S&P500.

Since the VIX is constructed based solely on the use of near ATM options, we decided to focus the use of B&S only on the calculation of only ATM options because of the use of the VIX within the model formula. Therefore, for the practical purposes of calculating the value of the options, we decided to use a strike price exactly equal to the spot price of the SPX and a maturity time expressed in terms of the fraction between the exact days between the two successive roll-date dates, and the total days of the base year.

Once we had highlighted the main assumptions useful in deriving the main data, we were able to proceed with the calculation of the call options by exploiting as a formula within the excel workspace, the generic B&S formula:

$$C_{BS} = S(t)N(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{ln\left(\frac{S(t)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

Where:

- $\sigma = \frac{VIX}{100}$ → poiché la volaitlity va espressa in termini percentuali
- $r = Treasury\ Bill\ 4\ Weeks\ yield$

To determine $N(d_1)$ and $N(d_2)$ the excel function =DISTRIB.NORM.ST.N($d_i; VERO$) was used where VERO (TRUE) will stand for the cumulative distribution function.

Put values, on the other hand, were obtained by the Put-Call-Parity principle:

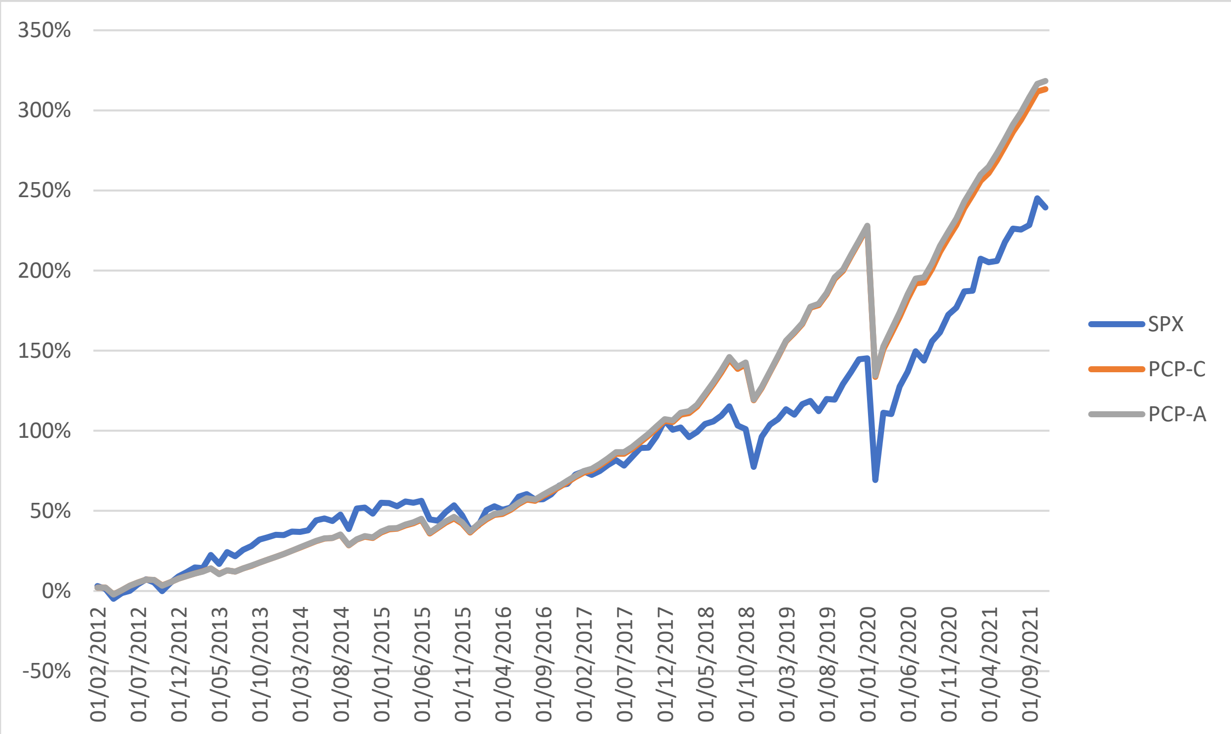
$$C_{BS} + Ke^{-r(T-t)} = P_{BS} + S(t)$$

Therefore, the values of the corresponding Put-Options were obtained with the following formula:

$$P_{BS} = C_{BS} + Ke^{-r(T-t)} - S(t)$$

Once the theoretical option values according to B&S were obtained, it was possible to apply our Put-Call-Put (PCP) strategy in both its versions, conservative (PCP-C) and aggressive (PCP-A), which brought out the cumulative performances shown in Figure 4.1.

Figure 4.1 Performance SPX and PCP strategy in the two version



The graph represents the cumulative performance trend over the analysis period (2012-2022) of the benchmark index (SPX) and our PCP strategy, in both its aggressive (PCP-A) and conservative (PCP-C) versions

As can be seen from the graph in Figure 4.1, our strategy over the chosen time frame shows an overall performance over 300%, which is increased, in both its versions, compared to the performance of the more passive Buy & Hold, represented by the performance of the SPX index, which instead reported an overall performance of approximately 250%. In order to have a more immediate and understandable reading of the performance, we subsequently calculated the annualized returns and we obtained:

	SPX	PCP-C	PCP-A
ANNUALIZED RETURNS	13.00 %	15.24 %	15.39 %

Table 4.2 Annualized Performances

In this way, the overall results previously set out could be translated into annualized returns of 15.24% for the PCP-C and 14.7 percent for the PCP-A, both of which are higher by 2.24% and 2.39% annually, respectively, in comparison with the annualized performance of the S&P500 index of 13%. As expected, the version that presented the greatest returns in terms of return on investment was the PCP-A, although the conservative PCP-C version was the strategy with the lowest volatility of monthly returns albeit slightly, at 3.50% versus 3.51% for the aggressive version and 5% for the SPX index, respectively. These results are particularly significant because, as suggested by Fischer and Myron, options tend to be more volatile than the underlying assets themselves. In fact, the conservative PCP-C version, due to the purchase of the underlying asset in the put cycles and the bonds in the call cycles through the premiums received from the sale of options, has a lower average volatility than the aggressive version, which instead uses the same premiums to sell additional options in the put cycles and buy the underlying asset in the call cycles, but at the same time has a higher volatility than the average volatility of the index itself, precisely because of the use of options.

	SPX	**PCP-C**	**PCP-A**
STD. DEV	5.00 %	3.50 %	3.51 %

Table 4.2 Standard Deviation of the analyzed strategies

4.2.1 Sensitive Analysis

As expressed above, the most "sensitive" assumptions for our strategy turn out to be the VIX as an approximation of σ and the use of TB4W returns as input data for r_f. Therefore, given the relevance of these two assumptions, we decided to perform an additional sensitivity analysis between our performance and these parameters in order to show how their path affected the performance of our strategy. This curiosity also stems from a desire to want to provide a rationale for the performance trend that is highlighted in the graph shown above.

As shown in Table 4.1, the time frame considered can be divided into 3 sub-periods in which 3 equally different things happen. Starting in 2013, our strategy begins to show a cumulative performance 3% lower than the SPX. This trend persists until August 2016, when our performance aligns with the market, and then diverges again in 2018, but in positive terms. In January 2018, there was a complete reversal in the cumulative performance of our strategy, as there was a marked deviation in positive terms from the trend of the SPX, which, as the years go by, tends to widen further until it reaches its maximum positive delta in February 2020, the month before the advent of the black swan due to the Covid-19 pandemic.

Table 4.3 Cumulative Performance for the different periods

RANGE PERIOD	SPX	PCP-C	PCP-A
2012-08/2016	60.43 %	57.14 %	57.97 %
08/2016-01/2018	106.45 %	106.28 %	107.28 %
01/2018-2020	239.45 %	313.23 %	318.36 %

The Table shows the cumulative values understood as Final Value/First Value (2012) -1 for the period under consideration

A particularly interesting aspect of our strategy was the greater sensitivity developed relative to the Covid-19 event, in that our strategy suffered a much more pronounced reduction in performance than the Buy & Hold strategy, however, our strategy continued to perform better overall to the latter, maintaining a reduced delta relative to the pre-pandemic phase, but still positive and constant to this day. Ultimately, comparing our PCP strategy-in both its two versions-with the more passive Buy & Hold strategy, represented by the performance of the benchmark SPX index, we can distinguish 3 relevant phases as shown in Table 4.1:

- 1stPhase: It represents the phase from the beginning of the study period in 2012 until August 2016. During this first phase, our strategy exhibited cumulative performances a bit below than the SPX performance
- 2ndPhase: Starting from 2016 and for the next two years, i.e., until January 2018, our strategy had cumulative performances aligned with the SPX benchmark index.
- 3rdPhase: phase in which starting from 2018 until the present day, our strategy presented significantly better cumulative performance than the SPX index.

As reported earlier, in order to give an interpretation of the occurrence of these 3 phases, we decided to report a *Sensitivity Analysis*: an analysis in which we have kept some parameters unaltered, while we have arbitrarily modified others to understand how the proposed strategy performed by setting them at different values. In particular, the values we have studied are **Volatility** and the **Risk-Free rate**.

VIX - For the Black and Scholes assumptions, we know that the estimation of volatility plays a crucial role (as we explained extensively in Chapter 2) and its correlation with options (both Call and Put) turns out to be positive, which can be easily verified by calculating the first derivative of the option price with respect to σ. Using an empirical approach, based mainly on simulations with some tests of the real data, we proceeded to calculate how the average annualized performance of our strategy, in both its conservative (PCP-C) and aggressive (PCP-A) versions, would change as the average volatility value changed over the entire time horizon. We therefore went to substitute in σ several values, which we held constant over the entire time horizon as they indicated the average volatility and kept all other conditions unchanged. Therefore, by varying the volatility values from the point values of the VIX according to the values in the first column, we obtain the following performances of annualized returns for the different strategies:

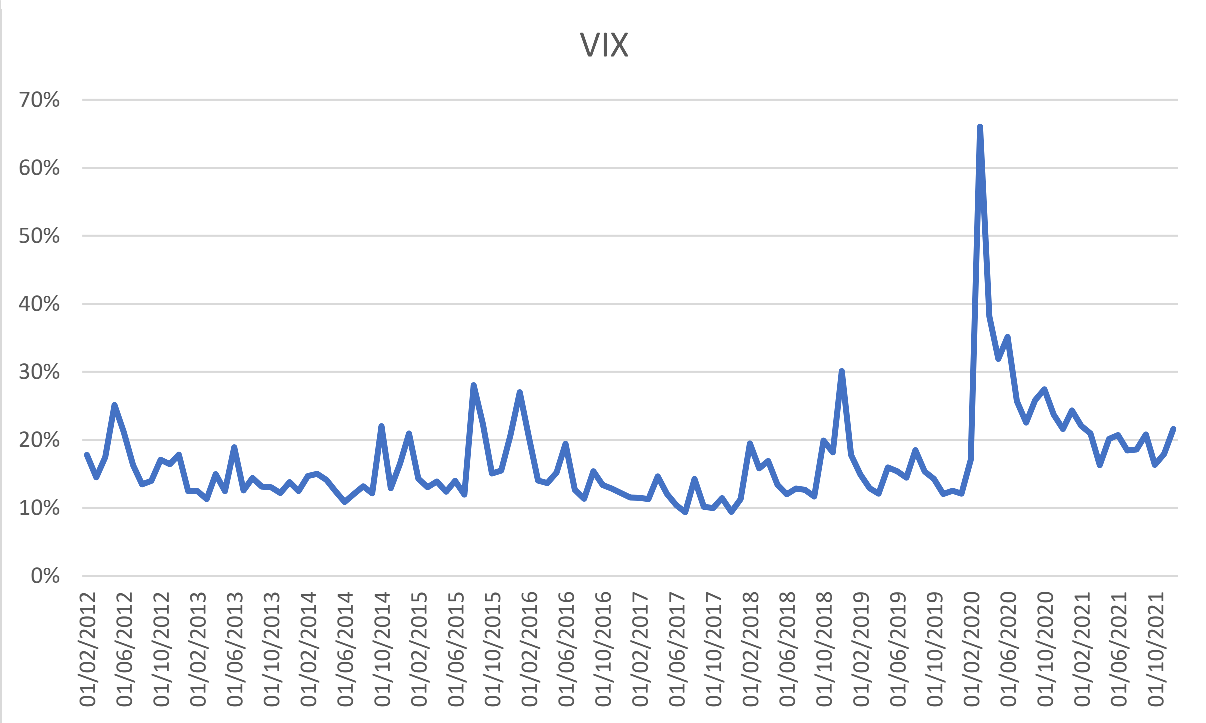
Table 4.4 Sensitivity Analysis setting Volatility parameter

VOLATILITY	SPX	PCP-C	PCP-A
REAL (VIX)	13.00 %	15.2 %	15.4 %
FIX 30	13.00 %	37.7 %	38.7 %
FIX 20	13.00 %	20.5 %	20.7 %
FIX 17	13.00 %	15.7 %	15.8 %
FIX 15	13.00 %	12.6 %	12.6 %

The table above shows the annualized returns of the SPX index and the annualized returns of the PCP-C and PCP-A strategies in the event that not the real volatility value (VIX) is used but fixed arbitrary average values is entered in the Black and Scholes formula.

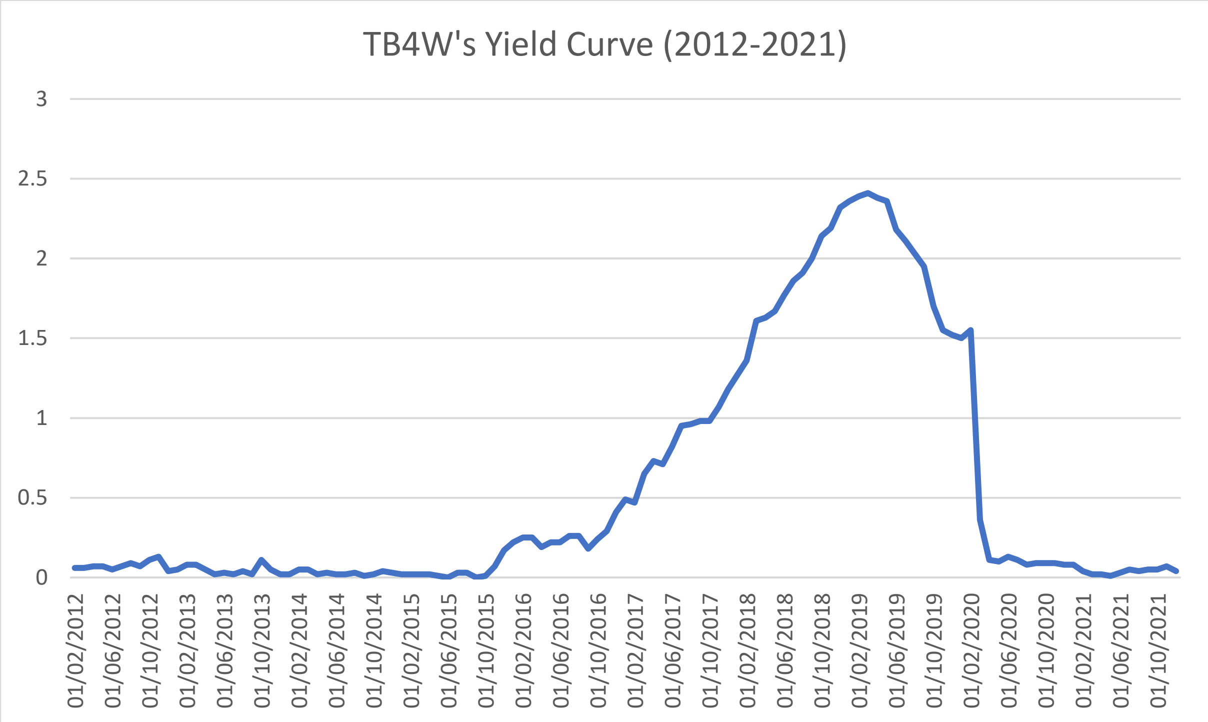
As expected, looking at Table 4.2, by bringing the volatility closer to the average real value of 13% of the VIX obtained during the entire time frame, the annualized values of our strategies tend to coincide with the actual returns obtained in case of using Real Volatility Value approximated with VIX. However, particularly interesting is the deviation in terms of annualized performance between the two versions PCP-C and PCP-A. In fact, as average volatility increases, the PCP-A strategy exhibits annualized returns that grow with a greater gradient than the PCP-C version. This result, however, does not surprise us, since by construction the aggressive version is more exposed to option pricing, since it involves the reuse of premiums collected to increase option sales, and therefore, an increase in average volatility implies an increase in its performance because of the positive correlation between volatility and option prices.

Figure 4.3 VIX Index Performances



$\boldsymbol{r_f}$ - As stated earlier, not only the VIX is the main assumption to determine the annualized return of the proposed strategy, which is better than the performance of the benchmark. In fact, the other crucial parameter for our performance is the risk-free rate. As can be seen from the Yield Curves of the last 10 years that is proposed here below, the FED since 2015 has been conducting a tight monetary policy, as the yields of 4-week T-Bills have seen their yield-range increase from 0-25 bps to peak yields of 242-243 bps reached in April 2019.

Figure 4.4 Yield Curve of U.S. Treasury Bills 4 weeks expiration



The gradual increase in interest rates by the FED since 2015 has macroeconomic origins and can probably be traced back to the end of an extraordinary period of government intervention in the financial markets that started at the height of the recession. After holding its benchmark federal-funds rate near zero for seven years, the FED increased rates a quarter-percentage point. The move signals the end of a monetary policy that began amid the worst financial crisis since the Great Depression. Yellen said the economy "has come a long way", though normalization "is likely to proceed gradually", and "inflation continues to run below our longer-run objective" [43]. However, given the difficulty in identifying the real reason and given the unimportance for the purpose of our study, we will not delve further into the macroeconomic reasons for the shape of the Yield Curve.

For the purpose of our studies, the most significant aspect turns out to be the strong correlation that can be inferred between the risk-free bond and the returns of our strategy. In fact, the results obtained show a positive correlation between yields and risk-free bond since by superimposing the graphs, it can be seen that our strategies started to provide better performance than buy & hold precisely at the same time as the FED raised interest rates. This correlation depends on the fact that the increase in interest rates influenced on the one hand the increase in the prices of Options (thus allowing to cash more from their sale although not significantly) and on the 'other hand allowed a substantial increase in the collateral in the PUT cycles, thus allowing a substantial increase in the number of options sold, especially in the case of an aggressive approach.

To better understand how the risk-free rate affects the annualized returns of our strategy, we proposed a sensitivity analysis, as in the previous case with the VIX.:

Table 4.5 Sensitivity Analysis setting Risk-Free rate parameter

INTEREST RATE (IN BPS)	SPX	PCP-C	PCP-A
REAL	13.00 %	15.2 %	15.4 %
FIX 0	13.00 %	10.07 %	10.26 %
FIX 25	13.00 %	12.31 %	12.48 %
FIX 50	13.00 %	14.60 %	14.74 %
FIX 100	13.00 %	19.29 %	19.39 %

The table above shows the annualized returns of the SPX index and the annualized returns of the PCP-C and PCP-A strategies in the event that not the real T-Bill 4-week expiration is considered but fixed arbitrary average values is entered in the Black and Scholes formula.

As we can see, a risk-free interest rate with a return of 100 bps compared to one with a return of 0 (which means keeping cash in cash) leads to annualized returns that increase from 10.07 percent to 19.97 percent in the case of PCP-C and from 10.26 percent to 19.39 percent; thus, we can say that the returns double.

This aspect can also make interesting reading by looking at the graph in Figure 4.1. In fact, in phase 1 of the period under analysis, which coincided with the period immediately following the great recession (2012-2016), average interest rates were 10 bps and in fact the performance of our strategy underperformed the benchmark index. In phase 2, i.e., the period between 2016-2018, interest rates began their gradual increase, bringing the performance of our strategy to values aligned with the benchmark. In contrast, in the 3rd and final phase, where there was an explosion in risk-free rates that peaked, our strategy strongly overperformed against the SPX. This correlation is further evidenced by the sharp drop due to the pandemic in 2019. In fact, a sharp drop from 200 bps to 0 bps in the risk-free rate due to the sudden outbreak of the pandemic, resulted in a contraction of our performance with -29% in 2020, which is less intense than the drop experienced by the markets, which instead stood at 31%.

To conclude and summarize this part of study, two other necessary factors to evaluate are the average monthly returns and the standard deviation of those returns: we note that the average returns are better by 18 bps (19 bps in the case of the aggressive strategy), and the standard deviation of 1.5 percent is significantly lowered.

	SPX	**PCP-C**	**PCP-A**
MEAN	1.17 %	1.28 %	1.29 %
DEV STD.	5.00 %	3.50 %	3.51 %

Table 4.6 Mean and Dev. Std. of Monthly Returns of SPX and proposed strategy

This may be partly since the premiums collected during the reporting period mitigate any rises or falls in the index although nevertheless our strategy has also suffered severe drawdowns, such as in the Covid-19 year, in which the loss was even increased relative to the SPX. Given the anomaly in the behavior of our strategy during strong market rally periods, we decided to provide an additional focus during those periods. In fact, the times when the strategy shows its most consistent flaw are between March and April 2020 at the height of the pandemic. In fact, the markets saw a drastic fall of 31 % as well as our portfolio, which maintained a 2 percentage point drop (-29%); however, the following month after there was a jump in the market that recovered 25 percent, our strategy saw a rise of only 7 % because although option prices rose tremendously due to the increase in the VIX due to the uncertainty in the markets at that time, this increase in option prices, however, was not enough to allow returns comparable to the buy-and-hold strategy whose return was of 25%.

4.3 Real Data Simulation

As previously explained, the next step in our analysis consists in replicating the same simulation as in the previous paragraph, which will however report more reliable results, since the actual historical option chains inherent to an ETF that replicates the trend of the market under analysis - the SPX - have been used as source data.

In this regard, the first aspect we dealt with was to identify the data provider from which the source data could be downloaded. Although the initial idea was to download as source data the option price history directly on the S&P500 index, we opted for options with an ETF replicating the SPX trend as the underlying. In fact, despite the large variety of data providers available for online consultation, to download the option values in real time, the most difficult aspect was to find a data provider that made available the entire option price history on the market given the large amount of data required. Since the amount of historical data to be stored grows exponentially with the passage of time, as it is enough to consider the fact that in a single day, providers make available option data with short-term expiry dates (1 to 3 days) of a few weeks, months or even years. Therefore, due to the complexity of the data required and after interacting with several data providers, the only platform from which it was possible to download the partially desired data - since it was still the history of options on ETFs - was the American Market Chamelon.

Once we had identified the data provider from which we could download the source data, we proceeded by choosing the ETF which best met our needs. Although the original idea was to opt for an ETF that had the characteristics of physical replication on the SPX and an accumulation distribution policy, however, due to the limited availability of the platform, the final choice fell on a distribution ETF replicating the performance of the S&P500 index and having the ticker SPY. Specifically, launched in January 1993, SPY was the very first exchange traded fund listed in the United States by SPDR Funds and has very low management costs, as its TER is 0.0945%.

Due to data availability constraints, we had to shorten the analysis period by a couple of years and so, study what the outcome of our strategy would be over an 8-year horizon, i.e., from January-2014 to February 2022. Despite the shortening of the analysis period, however, we believe that the choice of SPY as the reference ETF for the analysis is still a particularly reliable choice as it presents a sufficiently large track records on the markets to provide stability to its history of the last 10 years and presents a market capitalization of around $484,418.30 MLN, which determines sufficient daily liquidity to be able to analyze options even quite far from the ATM situation.

Figure 4.4 S&P500 Index (SPX) vs SPDR S&P500 ETF (SPY)

This chart downloaded directly from morningstar.com shows the deviation of the SPX index and the related ETF SPY.

As can be seen from the chart, comparing the performance of the ETF and the S&P500 Index over our period of analysis from 2012 to 2022, the ETF replicates the index very reliably, the difference in returns is mainly due to its distribution policy. Indeed, being a distribution ETF, the SPY distributes about 1.2 % of its value as dividends, which are allocated every quarter.

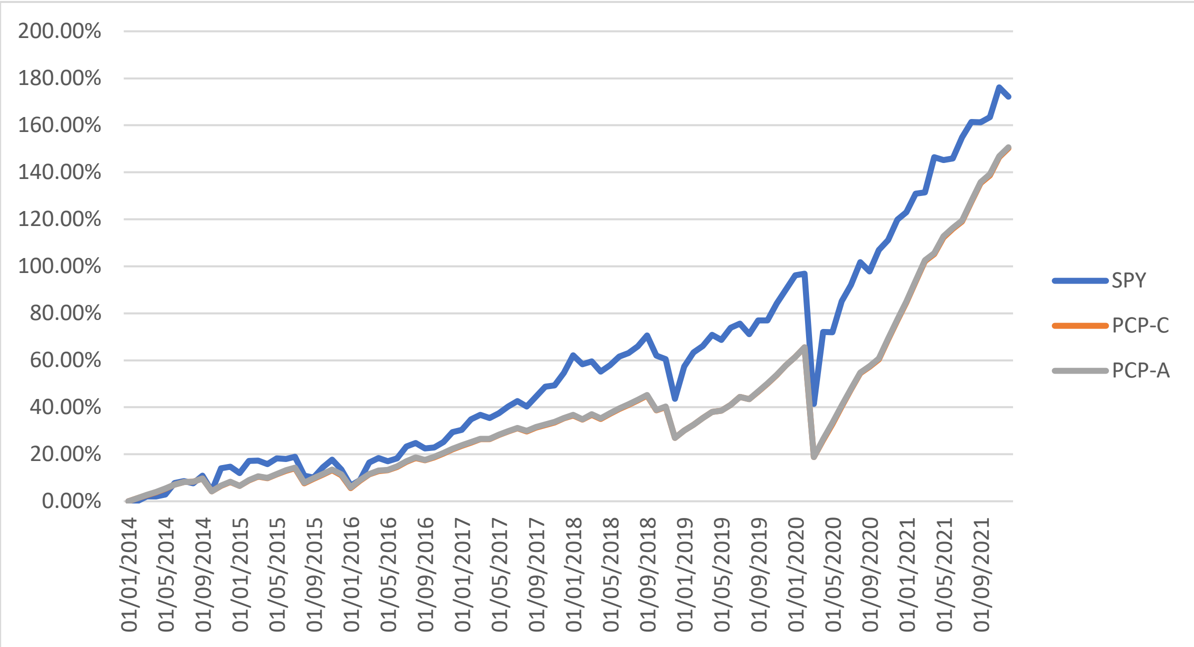
The latter is particularly significant, as the distribution of dividends alters the price trend of the SPY. In fact, in the month in which the dividends are detached, we will have that the price of the SPY will decrease by an amount exactly equal to the dividend just distributed, therefore, we will have a price fluctuation not determined by market dynamics, but to be attributed to the detachment of the dividend. In order to consider this variation purely technical and not real on the price trend of the SPY, we have decided to simply add the dividend to the ETF's valuation, so as to obtain the actual trend of the SPY and therefore, the actual yield of the Buy & Hold strategy represented by the SPY trend. Consequently, the value of the SPY price that will be used to see the performance of our strategies will be given by the following formula:

$$\bar{S}_{t+1} = S_{t+1} + DIV_{t+1}$$

Once we had established the main assumptions to be made when using the SPY as the underlying of the options, we dealt with the concrete implementation of the strategy under analysis in both its two versions, aggressive (PCP-A) and conservative (PCP-C). Unlike the previous implementation in which we had only studied the case of using ATM options - since the derivation of their values via Black & Scholes had as its main approximation of volatility the usage of the VIX index - in this case, thanks to the availability of data, we were able to carry out a broader analysis and move away from ATM conditions.

The first point of analysis consisted in comparing the trends in the cumulative performance of the Buy & Hold strategies - represented by the price trend of the ETF adjusted for the value of the dividend - and of the PCP under study (represented by both its two versions PCP-C and PCP-A), both having as underlying the use of ATM options and arbitrarily assuming the use of an initial capital of USD 100.

Figure 4.1 SPY vs PCP in ATM moneyness



As can be seen from the comparison chart above, our strategy during the initial years maintained a cumulative performance comparable to the more passive Buy & Hold strategy. However, starting from the beginning of 2016, our strategy - in both of its two versions - gradually started to underperform the benchmark strategy represented by SPY, reaching the maximum negative difference in the months prior to the outbreak of the pandemic. As can be seen from the cumulative performance trends in the table below, the respective performances of the strategies were 96.91% for the Buy & Hold (R_SPX) and 65.47% (R_PCP-C) in the conservative case or 65.46% in the aggressive case (R_PCP-A)

Table 4.7 Cumulative Performances of SPY vs PCP in the months near Covid19 crash

DATA_EXP	SPY_ADJ	PCP-C	PCP-A	R_SPX	R_PCP-C	R_PCP-A
	S+Div					
18/10/19	298,21	150,05	150,16	76,89%	50,05%	50,16%
15/11/19	311,70	153,69	153,74	84,24%	53,69%	53,74%
20/12/19	322,57	157,84	157,84	90,16%	57,84%	57,84%
17/01/20	332,05	161,40	161,38	96,18%	61,40%	61,38%
21/02/20	333,39	165,47	165,46	96,91%	65,47%	65,46%

The table shows actual SPY values (with dividends) and PCP values assuming an initial capital of $100. On the right the "R_" are the cumulative returns.

That implies a difference in cumulative returns until February 2020 of roughly 30% more in favor of Buy & Hold. However, a particularly interesting aspect is the reaction of the two different strategies to the market shock represented by Covid-19. In fact, while in the theoretical case of using Black and Scholes for option prices, our strategy was much more sensitive, as it reported a much more pronounced drop in performance in relative terms than the Buy & Hold, in this case the opposite situation occurs.

The Buy & Hold strategy is more sensitive to the occurrence of Covid-19, as it shows a relative decrease in March alone of -55.53% in cumulative terms, which is 9% lower than the losses suffered by both versions of our PCP strategy. This dynamic is probably due to the time to maturity of the call options sold, since these options, being sold with monthly expiry dates, were not already discounted in their pricing this catastrophe during the call cycle we were in. Although the delayed discounting of the Covid event in option prices favors our strategy in March, it reappears with opposite and decidedly more marked consequences in the following month of April. In fact, while the fall is more pronounced, the recovery of the Buy & Hold is much more marked and energetic than that of our strategy under analysis, since as shown by the data below, the former sees an increase in cumulative performance of 30% in the month of April alone, compared to a recovery of only 7% for the latter.

DATA_EXP	**DELTA PREVIOUS MONTHS**		
	SPY	**PCP-C**	**PCP-A**
21/02/20	0,73%	4,08%	4,09%
20/03/20	-55,53%	-46,66%	-46,62%
17/04/20	30,70%	7,25%	7,60%

Table 4.8 Delta cumulative returns near March 2020 of Covid19

This upswing in performance continues beyond April, as from that month onwards, the SPX index tends to grow in terms of cumulative performance at a higher rate than PCP, leading to a new delta maximum always in favor of Buy & Hold of 45% in October 2020. However, once this maximum distance is reached, there is again a reversal in growth speed, because our PCP strategy tends to grow at a much higher cumulative rate than the Buy & Hold strategy. This dynamic is also evidenced by the graph as we see a gradual rapprochement of the yield curves, resulting in the current 20% difference - still in favor of the SPX benchmark index - reflecting a trend in cumulative performance from January - 2014 to February 2022 respectively of:

	SPY	PCP-C	PCP-A
Cumulative Returns	172,13%	150,22%	150,65%

Table 4.9 Overall Cumulative returns over the entire period analyzed (2014-2022)

Once we had analyzed the performance under ATM conditions, we thought of repeating the same analysis, but using options that would loosen from that condition and opting for ITM options, so that we could collect a higher premium from their sale. The latter, given the higher probability of future exercise, guaranteed a greater premium from their sale and, therefore, allowed us to have greater liquidity to re-invest in order to increase our hedging position in the conservative version. Specifically, we have arbitrarily decided to consider ITMs first at 2%, then at 5% and finally at 10% moneyness, which is to be understood as the percentage of increase or decrease of the strike price from the ATM condition, depending on whether we are dealing with call or put options respectively. As mentioned above, these choices are particularly interesting as they allow a higher premium to be collected from option sales since, in *cetaris paribus*, ITMs at 2% are quoted twice as the same ATM options, which becomes even 6 times more expensive when compared with ITMs at 10%. All these values can be found in the tables in the last pages.

One aspect that should be particularly emphasized is the choice of the reference cycle switching signal, i.e., that indicator which suggests to our strategy the eventual change from a PUT cycle to a CALL cycle or vice versa. Until now, our main input sign for an eventual change of cycle has been the actual exercise of options, i.e., if at the expiry of a put or call cycle, the same options were exercised, automatically our strategy would sell call or put options respectively, while if they were not exercised, the cycle would reiterate. In this context, the main determinant of any change of cycle is therefore the choice of strike price of the options sold. In the ITM situation, although for consistency we should have always kept the actual exercise of the ITM options as the switching signal, we nevertheless decided to keep the actual exercise of the ATM options and not the ITM. Put differently, the ITM analysis of our strategy involves selling ITM options with varying degrees of moneyness but keeping exactly the same order of cycles as we had during the ATM situation. This relevant assumption stems from the desire to give greater stability to the cycle in place, as our PCP strategy becomes competitive through the collection of the premium from the sale of the options, without them subsequently being exercised. In fact, if we had kept as a switching signal the actual exercise of ITM options, - which by definition have strike prices with a higher probability of exercise than ATM options - we would have had an excessive alternation of cycles, which would have implied a more frequent exercise and, therefore, a more frequent loss.

Once we had clarified the main assumptions made in the ITM case, we proceeded to perform the same analysis as in the ATM case. Therefore, we proceeded to chart the comparison between the cumulative performance of the benchmark buy & hold strategy and our PCP strategy, in both versions, for all different levels of moneynesses, i.e., 2%, 5% and 10%.

Figure 4.6 SPY vs PCP - ITM 2%

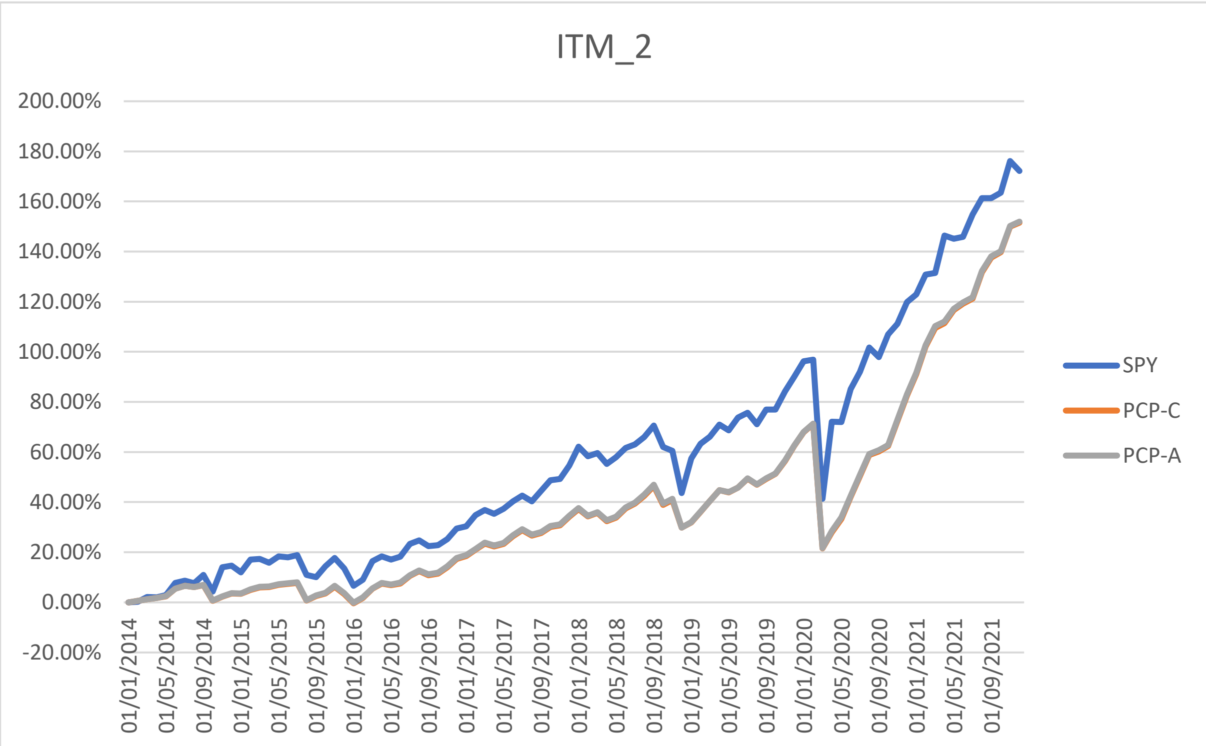


Figure 4.5 SPY vs PCP - ITM 5%

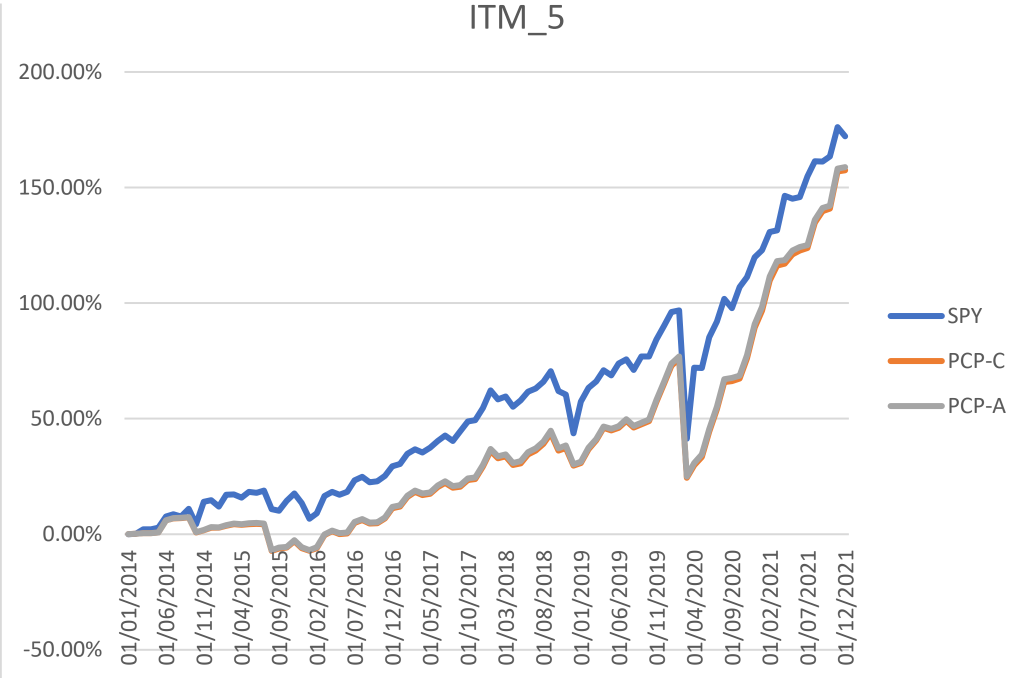
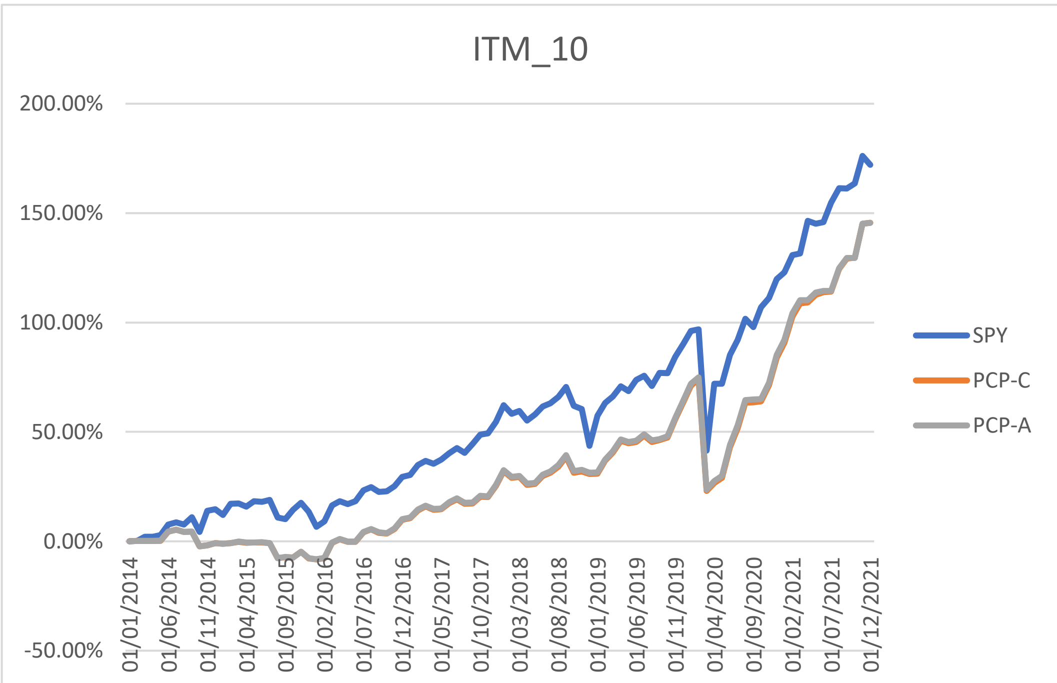


Figure 4.8 SPY vs PCP - ITM 5%



As can be seen from the three graphs above, although the 2% ITM is the only one of the three that initially appears to perform comparable and even sometimes better than the SPX benchmark index, at the end of the period under analysis our strategy performs worse, in both its two versions and for all the different levels of ITM moneyness. A further point of analysis is their sensitivity to the market shock represented by the Covid-19. As the chart suggests, unlike in the ATM case study where PCP was less sensitive to the market crash from Covid-19, in the ITM case study we have instead that our suggested strategy tends to follow more closely the trend of buy & hold. In fact, as the tables below show, all three levels of ITM moneyness in March were affected by a reduction in performance of around 51% in cumulative value, which is 5% higher than the -46% in the ATM case and thus, more in line with the -55% value of the buy & hold. This 5% greater sensitivity of the PCP in the ITM case compared to the same in the ATM case is not, however, maintained in April, since the PCP in the ITM case shows a cumulative performance of around 27%, perfectly in line with the 26% of the same in the ATM case, both of which are still well below the 72% of the Buy & Hold.

Table 4.10 Cumulative returns of strategies at moneyness ITM 2% - 5% - 10% during the Covid-19 period.

	SPX	PCP_C	PCP_A
		ITM 2%	
17/01/2020	96,18%	67,87%	67,96%
21/02/2020	96,91%	71,20%	71,30%
20/03/2020	41,39%	21,59%	21,68%
17/04/2020	72,08%	27,96%	28,40%
		ITM 5%	
17/01/2020	96,18%	72,93%	73,83%
21/02/2020	96,91%	75,95%	76,87%
20/03/2020	41,39%	24,38%	25,05%
17/04/2020	72,08%	29,81%	30,84%
		ITM 10%	
17/01/2020	96,18%	71,13%	71,95%
21/02/2020	96,91%	74,12%	74,95%
20/03/2020	41,39%	23,04%	23,65%
17/04/2020	72,08%	26,67%	27,51%

What is particularly evident from the respective charts is the strategy's performance in the early years of our analysis. In fact, while in the ATM case study the PCP exhibited performances comparable with the trend of the SPY until around the year 2017, in the ITM case study there is a much earlier divergence from the trend of the SP500 benchmark index. Specifically, the more moneyness increases, the earlier the divergence, as witnessed by the performance of the ITM 2% PCP, which begins to diverge substantially from 2016, i.e., 2 years later than the PCP in the ITM 10% case study. This different initial behavior, depending on the level of ITM moneynesses, is part of a much broader dynamic, since the more moneyness increases, the more the deviation of CFP from the Buy & Hold performance tends to remain constant. Specifically, the more moneyness we have, the more the dynamics shown in the ATM case tend to disappear. While in the ATM case we had dynamics such that initially the performance of the PCP was comparable to the performance of the SPY, which was followed by a progressive negative deviation until reaching a maximum negative delta in the weeks prior to Covid-19, only to reverse the trend again in a progressive healing of the performance until today, these dynamics tend to flatten out as moneyness increases. In fact, in the case ITM 2% the same dynamics are still detectable, which instead tend to disappear completely with options ITM 10%, in how much the course of the PCP tends to remain with a negative but constant deviation regarding the Buy & Hold for the entire period of analysis. These dynamics lead to cumulative performances over the period of analysis of:

CUM_RETURNS	SPX	PCP-C	PCP-A
ITM 2 %	172,13%	151,54%	151,95%
ITM 5 %	172,13%	157,46%	158,83%
ITM 10 %	172,13%	145,59%	145,56%

Table 4.11 Cumulative for the different level of ITM Moneyness

In order to verify the thesis expressed by Chen in the previous chapter, according to which OTM options turned out to be the most overpriced options, we proceeded with the analysis of our strategy by considering the sale of OTM options at the same percentage moneyness levels as before, i.e., 2%, 5% and 10%. To maintain the same specularity of assumptions and make the comparison as reliable as possible, we also considered the same switching parameters for OTM options as for the ATM case.

As for the previous case studies in which we had identified three phases, 2 pre-pandemic and one post, also in the OTM case - for all levels of moneyness - similar dynamics manifest themselves but with differences; from the beginning of the period considered until February 2016, the curves of the cumulative returns of the proposed strategies almost overlap with the Buy Hold: in fact while the cumulative returns of the SPY are around 9%, those of the 2% OTM strategy are a good 2 percentage points higher (11.42% to be exact), 15.6% in OTM 5% and even OTM10% touches 19% maintaining a certain dominance until May 2016. However, from February 2016 onwards, a real rally in the index began, which saw these resistance points of the proposed strategies fail. A reliable explanation for these values is to be found in the general trend-lessness of the markets, since from 2014 to the first months of 2016, the US benchmark index maintained a fairly flat trend without presenting excessive disruptions. Against this backdrop, the proposed strategy had very good returns because the options in most cases were not exercised and, thus, the proceeds contributed to capital growth.

The golden age of the proposed strategies was short-lived, as with the sudden and constant rise of the index, the returns from the sale of options were no longer sufficient to bring back comparable performances. Thus, the deviation becomes more and more evident as time goes by, mainly determined by a different speed of growth between the two curves.

This trend is persistent until 2020, when in March, during the pandemic, we can see that both the index and the strategy have a substantial decline. While the index falls by 30%, however, we can see that the decline resulting from the PCP is smaller; in fact, the OTM 2/5/10 values are -27%, -25% and -21% respectively, so we can see that the deeper-OTM we go, the more the strategy is able to absorb substantial market declines. Obviously, this positive aspect is totally cancelled out by what happens in April 2020, when the index recovers making a leap of no less than 23%, while the PCP fails to sustain this growth, although it grows in that month by 7% in OTM2, 8% in OTM5 and even 11% in OTM10. This phenomenon implies that more months are needed to recover the gains made pre-covid. In fact, while the SPX index - represented by the SPY - took about 5 months to recover and reach pre-covid levels, i.e., cumulative returns around 101% in August 2020, which are like the returns of 96.91% in February 2020, the same cannot be said of the PCP. The latter took a good eight months to reach its pre-pandemic performance of 59.55% in terms of cumulative returns.

Post-covid although it has had a slower recovery, PCP in all OTM versions is seeing a steady increase and a growth rate very similar to that of the index, which was not the case in the pre-covid period when the 'spread' of the difference in returns tended to open up.

Figure 4.6 SPY vs PCP - OTM 2%

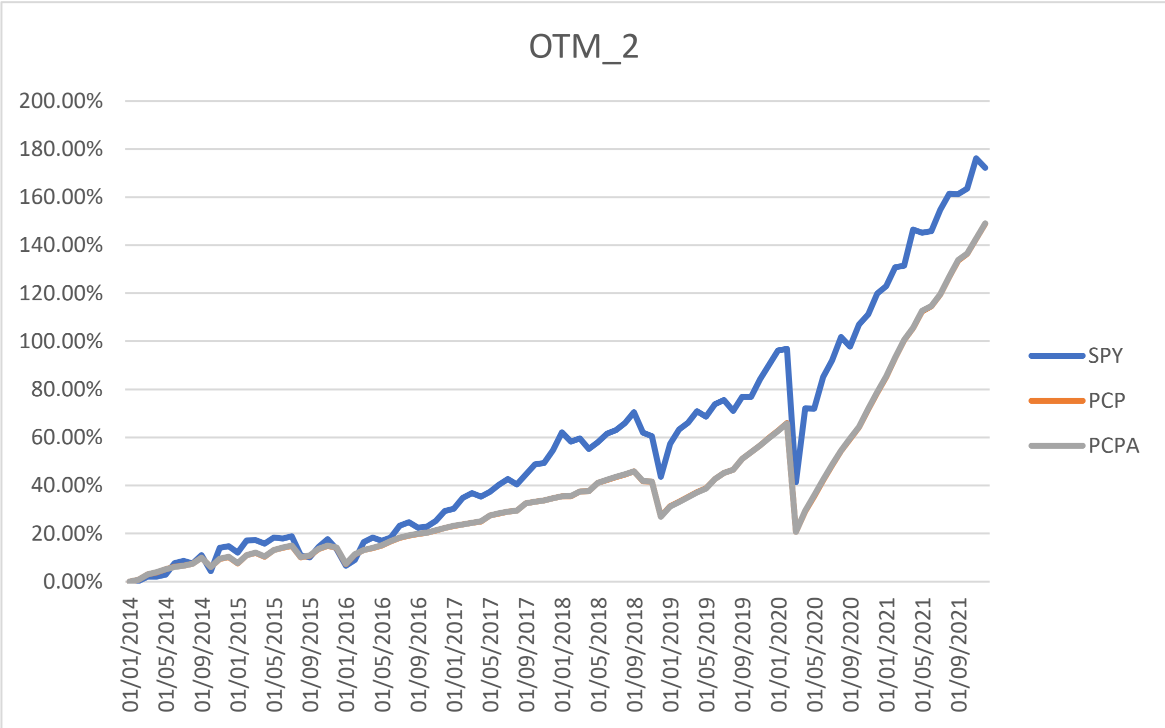


Figure 4.7 SPY vs PCP - OTM 5%

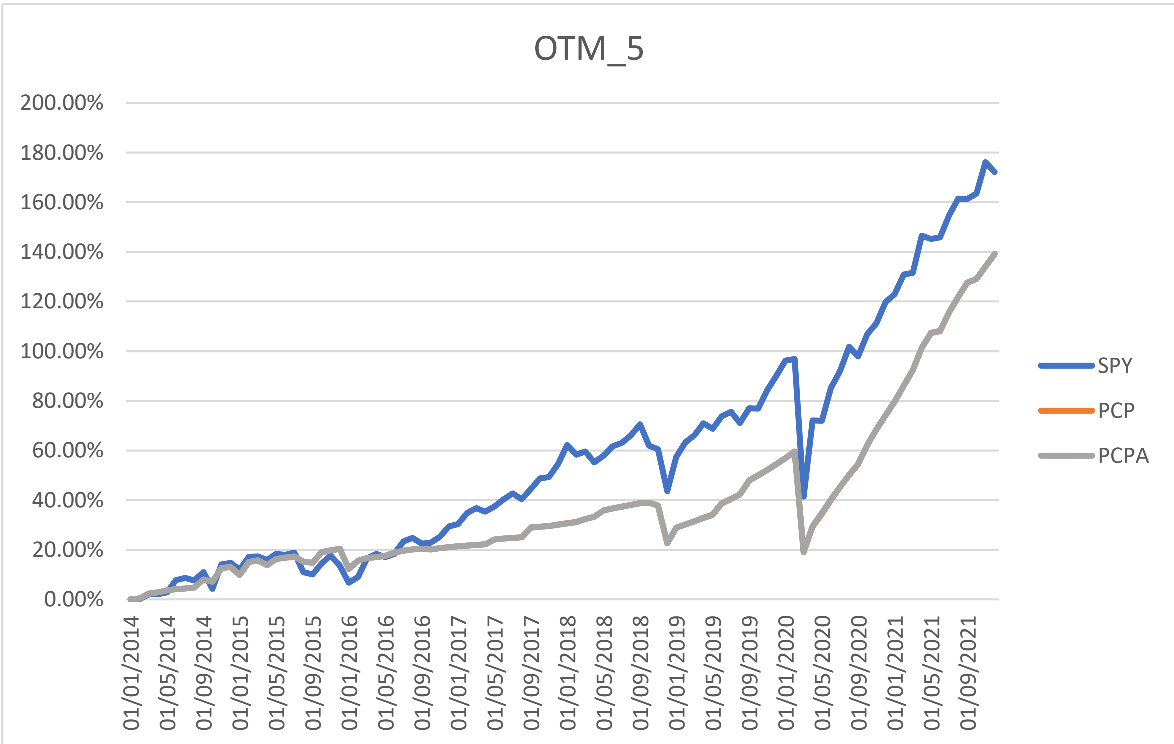
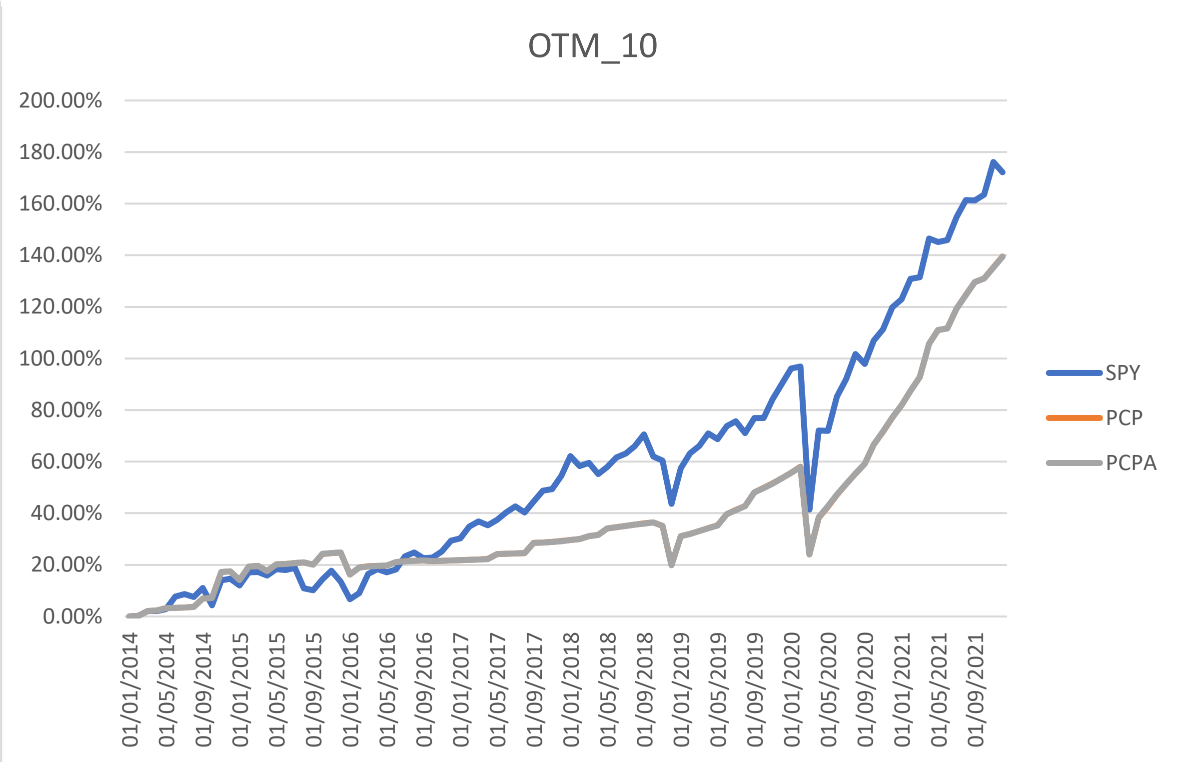


Figure 4.8 SPY vs PCP - OTM 10%



In conclusion, independently from the moneyness kind (ITM, OTM, ATM) and from the relative level (2%, 5%; 10%), the PCP strategy proposed and analyzed by us presents decidedly lower cumulative return performances than the more passive buy & hold benchmark strategy. This result can be extended to both the aggressive version and the conservative version of the PCP strategy under analysis, since, as can be easily verified by the comparison graphs, both the mentioned versions present practically identical performances over the time horizon considered in terms of cumulative returns.

The next step in our analysis was to look no longer at mere cumulative returns, but in comparing the adjusted risk-returns of the various strategies. Therefore, as shown in the table below, we took care of calculating several useful indicators for this purpose.

Analysis with transaction costs

The next step was to consider the performance of our strategy by considering market frictions and transaction costs, fees and all costs related to portfolio rebalancing; however, taxes, management fees and any potential market impact are not considered. We considered a cost ***f*,** expressed in basis points, that wears down capital over the years.

We assumed variable f (of 5,10 and 20 bps) to understand the impact of costs on capital and obtained the following table:

TRANSACTION COSTS IN BPS		SPX	ATM		ITM 2%		ITM 5%		ITM 10%	
			PCP	PCPA	PCP	PCPA	PCP	PCPA	PCP	PCPA
0	CR	172%	152%	152%	152%	152%	157.46%	158.83%	145.59%	145.56%
	AR	13.33%	12.15%	12.17%	12.22%	12.24%	12.55%	12.62%	11.89%	11.88%
5	CR		138.61%	139.02%	139.87%	140.26%	145.52%	146.82%	134.19%	134.16%
	AR		11.48%	11.51%	11.56%	11.58%	11.88%	11.96%	11.22%	11.22%
10	CR		127.53%	127.93%	128.73%	129.10%	134.12%	135.36%	123.32%	123.29%
	AR		10.82%	10.85%	12.15%	12.17%	11.22%	11.29%	10.56%	10.56%
20	CR		106.88%	107.24%	107.97%	108.31%	112.87%	114.00%	103.05%	103.03%
	AR		9.51%	9.54%	9.58%	9.61%	9.90%	9.98%	9.26%	9.26%

Table 4.12 Cumulative Return (CR) and Annualized Return (AR) in ATM and ITM

TRANSACTION COSTS IN BPS		SPX	OTM 2%		OTM 5%		OTM 10%	
F			PCP	PCPA	PCP	PCPA	PCP	PCPA
0	CR	172%	148.92%	149.08%	139.14%	139.14%	139.51%	139.44%
	AR	13.33%	12.07%	12.08%	11.51%	11.51%	11.54%	11.53%
5	CR		137.37%	137.52%	128.05%	128.04%	128.40%	128.33%
	AR		11.41%	11.42%	10.85%	10.85%	10.88%	10.87%
10	CR		126.35%	126.49%	117.46%	117.46%	117.79%	117.73%
	AR		11.51%	11.51%	10.20%	10.20%	10.22%	10.22%
20	CR		105.81%	105.94%	97.72%	97.72%	98.03%	97.97%
	AR		9.44%	9.45%	8.89%	8.89%	8.92%	8.91%

Table 4.13 Cumulative Return (CR) and Annualized Return (AR) in OTM

As expected, the performance obtained also considering transaction costs, decreased. This aspect is particularly significant since it highlights a further consideration. In fact, while the passive buy-and-hold strategy has the great advantage that transaction costs are almost completely absent because they are only incurred in the first month, our strategy suffers much more from them. Since the PCP provides for a dynamic monthly allocation, this implies that transaction costs are incurred monthly, i.e., they erode part of the capital and thus performance, leading in the long run to a further decrease in cumulative performance ranging from 10% (in the case of transaction costs of 5 bps) to 43% (in the case of 20 bps).

Performance Indicators

	SPX	ATM		ITM 2%		ITM 5%		ITM 10%	
		PCP-C	PCP-A	PCP-C	PCP-A	PCP-C	PCP-A	PCP-C	PCP-A
ANNUALIZED RETURN	13.33%	12.15%	12.17%	12.22%	12.24%	12.55%	12.62%	11.89%	11.88%
MEAN MONTLY RETURN	1.12%	1.05%	1.06%	1.07%	1.07%	1.11%	1.11%	1.05%	1.05%
STD. DEV. MONTLY RETURN	5.24%	3.82%	3.83%	4.02%	4.03%	4.36%	4.37%	4.27%	4.27%
SKEW	-1.66	-5.13	-5.10	-4.69	-4.68	-3.79	-3.77	-3.73	-3.70
EXCESS KURTOSI	13.92	33.92	33.57	30.12	29.91	22.07	21.87	24.08	23.83
SHARPE RATIO	8.70%	10.15%	10.18%	9.99%	10.03%	10.11%	10.23%	9.05%	9.03%
SORTINO RATIO	6.99%	5.47%	5.36%	5.63%	5.65%	6.84%	6.80%	7.01%	6.91%
BETA	1.00	0.61	0.61	0.64	0.65	0.68	0.68	0.63	0.63
TREYNOR RATIO	0.46%	0.63%	0.63%	0.62%	0.62%	0.65%	0.66%	0.61%	0.61%
MAX DRAWDOWN	-30.58%	-28.20%	-28.18%	-28.98%	-28.97%	-29.31%	-29.30%	-29.34%	-29.32%
JENSEN'S ALPHA	0.00%	0.11%	0.11%	0.78%	0.11%	0.80%	0.13%	0.76%	0.10%

Table 4.14 Performance Indicators in ATM and ITM

	SPX	OTM 2%		OTM 5%		OTM 10%	
		PCP-C	PCP-A	PCP-C	PCP-A	PCP-C	PCP-A
ANNUALIZED RETURN	13.33%	12.07%	12.08%	11.51%	11.51%	11.54%	11.53%
MEN. MONTLY RETURN	1.12%	1.04%	1.04%	0.99%	0.99%	0.99%	0.99%
STD. DEV. MONTLY RETURN	5.24%	3.69%	3.70%	3.60%	3.60%	3.53%	3.53%
SKEW	-1.66	-5.22	-5.18	-4.57	-4.54	-2.72	-2.68
EXCESS KURTOSI	13.92	34.59	34.21	28.67	28.41	16.22	16.12
SHARPE RATIO	8.70%	10.20%	10.20%	9.13%	9.12%	9.18%	9.17%
SORTINO RATIO	6.99%	4.37%	4.38%	3.73%	3.73%	4.30%	4.30%
BETA	1.00	0.60	0.60	0.58	0.58	0.58	0.58
TREYNOR RATIO	0.46%	0.63%	0.63%	0.56%	0.56%	0.56%	0.56%
MAX DRAWDOWN	-30.58%	-27.27%	-27.24%	-25.38%	-25.35%	-21.45%	-21.43%
JENSEN'S ALPHA	0.00%	0.77%	0.10%	0.73%	0.06%	0.73%	0.06%

Table 4.15 Performance Indicators in OTM

Going into the analysis, we can identify several aspects:

1. As expected of cumulative returns, the annualized returns of our PCP strategy, regardless of the type of version and its level of moneyness, are always lower than the SP500 benchmark. These returns are somewhat surprising when compared to the first derivation analysis during which, once the ATM option prices were calculated via Black and Scholes, the annualized performance of the

PCP strategy was higher than the buy & hold. The possible explanation for this can be found in the average value of the $\frac{Call\ or\ Put}{Index}$ ratio, which decreased significantly from 1.95 to 1.64 for CALLs and from 1.91 to 1.65 for PUTs. This reduction did not allow to maintain the expected returns suggested by the derivation case through B&S, therefore, there is a discrepancy between Derived Data and Actual Data.

However, this data discrepancy is not surprising since BS bases its application on the assumption of normality of returns. As suggested by the Skew value of -1.66, this assumption in the actual data of the SP500 does not occur, consequently the symmetry of the returns does not occur, leading to a Gaussian shifted to the right. To conclude, in relative terms, the version with the highest annualized returns turns out to be the conservative PCP-C version in both cases of moneyness, i.e., both in the in-the-money case with the 5% ITM and in the out-of-the-money case with the 2% OTM version.

2. The pivotal advantage of the proposed strategy is the significant decrease in the volatility of returns. In fact, in the in-the-money case, the version that reaches the lowest value in terms of volatility is the PCP ITM 5%. This strategy also coincides with the strategy with the highest returns, thus making it the best strategy in terms of risk-return adjusted. In the OTM case, on the other hand, volatility values fall further, reaching a minimum in the OTM 10 version with values of 3.53%. This positive value in terms of volatility, however, is countered by a 12-bps reduction in the average monthly return.

	ITM 5%		**OTM 10%**	
	PCP-C	**PCP-A**	**PCP-C**	**PCP-A**
MEAN	1.11%	1.11%	0.99	0.99
DEV. STD	4.36%	4.37%	3.53	3.53

Table 4.16 Performance Indicators in OTM

3. Performance comparison can be made in relative terms in terms of reward-to-variability ratio. This ratio represents the ex-post version of the Sharpe Ratio:

$$Sharpe\ Ratio: \frac{r - r_f}{\sigma_r}$$

where r is the sample mean of the fund return, rf is the risk-free return, and sigma is the volatility of the fund return. The higher the Sharpe Ratio, the higher the performance of the portfolio relative to the risk-free return compared with the risk of the portfolio. The indicator is not easy to apply because there is no such thing as a completely risk-free return. For consistency what we have calculated so far, T-Bills with a maturity of 4 weeks is considered risk-free return. As we can see, performance increases by about 2 percentage points as we move closer and closer toward ITM 5 reaching 10% so we can say that the relative performance of Options portfolios is better than the benchmark index. A similar value, exactly 10.2% is touched in case of OTM 2%.

4. With the Sortino Ratio, on the other hand, a modification of Sharpe is provided, and it penalizes returns below a given threshold level s by placing in the denominator the so-called semi-standard deviation of the return from s:

$$Sortino\ Ratio = \frac{r - s}{\sigma_r(s)}$$

Where $\sigma_r(s)$ is:

$$\sigma_r(s) = \sqrt{\frac{1}{V-1}\sum_{v=1}^{V}(r(v) - s)^2}$$

Where V are the observations (among T observations in the sample) such that r(v)-s<0 and in the summation, there are only observations such that the return is less than s. The denominator is called the **downside risk**. As a reference we placed s=rf therefore:

$$Sortino\ Ratio = \frac{r - r_f}{\sigma_r(s)}$$

Usually, this index is usually used to penalize managers with excessively lower returns than a risk-free benchmark.

In this case the relative return considering the downside risk of goes deteriorating at ATM and ITM 2%, but reaches levels like the benchmark at ITM 5%, while it manages to get better value at ITM 10% which is 7%; in OTM case Sortino rejects the PCP trend because is always below the benchmark.

5. A further modification of the Sharpe Ratio is the Treynor Ratio where in this case the denominator considers the β_{rm} parameter understood as market risk:

$$Treynor\ Ratio = \frac{r - r_f}{\beta'_{rm}}$$

Where β'_{rm} è is an estimator of $\beta_{rm} = \frac{cov(r, r_m)}{\sigma^2(r_m)}$ where r_m is the performance of the market portfolio. Thus, we can deduce that it is a more suitable to invest in PCP and PCPA at any level of moneyness because higher ratio indicates a more favorable risk/return scenario both in ITM and OTM case.

To conclude, we can say that the maximum drawdown (simply understood as the lowest negative monthly return) is always better in the case of PCP than the 30% of SPY that coincides with March 2020 at the height of the pandemic. The other side of the coin of this figure is the fact that in the period following the drawdown we do not see a rebound of the PCP as we do in the case of the SPY in fact the pre-drawdown recovery time is longer as explained above.

5. Conclusion

For over two decades, the average realized volatilities for S&P 500 index has been below the average implied volatilities for SPX index options. This is often quoted as the direct reason to pursue out-performance through index option writing strategies. However, passive buy-write or put-write portfolio strategies may not be fully effective to take advantage of the average negative volatility premium embedded in index options. [42]

To this end, we had decided to propose a strategy involving dynamic option allocation in order to take advantage of that average negative volatility premium highlighted in the early chapters of our study. While at first, the use of option quotes derived through the use of the Black & Scholes model, had indeed shown an over-perfomances in cumulative terms of the proposed PCP strategy compared to the benchmark buy & hold strategy, however, this result was later refuted with the use of real quotes in *cetaris paribus* on an ETF.

As shown by the studies reported in Chapter Two, the reason for this discrepancy is to be found in the assumptions made in the Black & Scholes model. The latter presumes stock prices to follow a lognormal pattern, e.g., a random walk (or geometric Brownian motion pattern) and assumes a constant volatility parameter over time. However, the empirical evidence suggested by our study seems to reject the latter assumption and, therefore, to suggest greater relevance to more advanced pricing models that see a stochastic evolution not only in stock prices but also in volatility itself. Thus, a first conclusion we feel to draw from our studies is the inadequacy of the view of implied volatility as a reliable and good forecast of the market's expectations of the underlying asset's volatility, since the opposite case would have implied similar performances of the two scenarios.

A further conclusion, which apparently seems to be derived from the analysis from the cumulative returns in the real case, is the lack of overpricing in option prices. Although there is a discrepancy between the volatility embedded in the options and that actually shown by the underlying, however, this discrepancy does not seem to benefit the pricing of the former. On the other hand, going into a more thorough analysis, our PCP strategy exhibits strongly comparable and, in some versions, even slightly better risk-return adjusted performances than the more passive buy & hold. These results are supported by the lower volatility that our strategy exhibits compared to the buy & hold. Therefore, the second conclusion we feel we can deduct from our studies is that contrary to what was argued in chapter three, the performance of option quotes tends to present less volatility than the underlying, although among the versions presented, the conservative one presents even less volatility supported by the alternating buy of the underlying or bond.

Finally, a third conclusion can be made about the manifestation of the volatility smile. As the empirical evidence shows, among all the levels of moneyness presented for CFP in the real case, the versions with higher returns are ITM 5 and OTM 2. This result thus leads us to assert that the presence of the volatility smile pushes options to consider higher intrinsic values than in the ATM case.

A relevant aspect to consider in our study is the possible presence of certain assumptions in the strategy construction phase, which may markedly influence the returns obtained and, therefore, any conclusions presented. Specifically, the most relevant assumption concerns the switching between PUT and CALL cycles; the assumption consists in considering as such, the eventual exercise of the options, i.e., starting from a PUT cycle, in the case of exercise of the put options at expiration, we would buy the underlying and switch accordingly to the next CALL cycle. Despite an efficiency in terms of transactions since the purchase (or sale) of the underlying would occur sequentially, there is a structuring aspect to them that is not considered. In fact, given what has been described, it is assumed that if the point value at expiration of the underlying in period t+1 is greater than the same in period t, the PCP continues to assume that the security continues its uptrend (or downtrend in case $S_{t+1} = S_t$) and therefore continues with a certain cycle or switches. This assumption does not consider the actual performance of the underlying stock throughout the month, in general it does not consider the performance of the stock having a broader view of the time frame to understand the cycle to be considered. Therefore, if we speculatively consider suitable PUT options during Bullish periods, it may be the case that the exercise of put options are purely due to a normal monthly market swing in the performance of the underlying, although in the general context it is in an uptrend. However, the desire to improve this hypothesis however would open the need for a technical analysis study, which was not intended to be the focus of this thesis study.

In fact, seeing the graph of the SPY over the period considered (2014-2022) we note that despite a horizontal period (or at least a period of slight growth), the index has subsequently seen a very substantial upswing that might suggest that the index is in a totally Bullish period and that therefore an alternation of switching as assumed earlier sees no logical foundation.

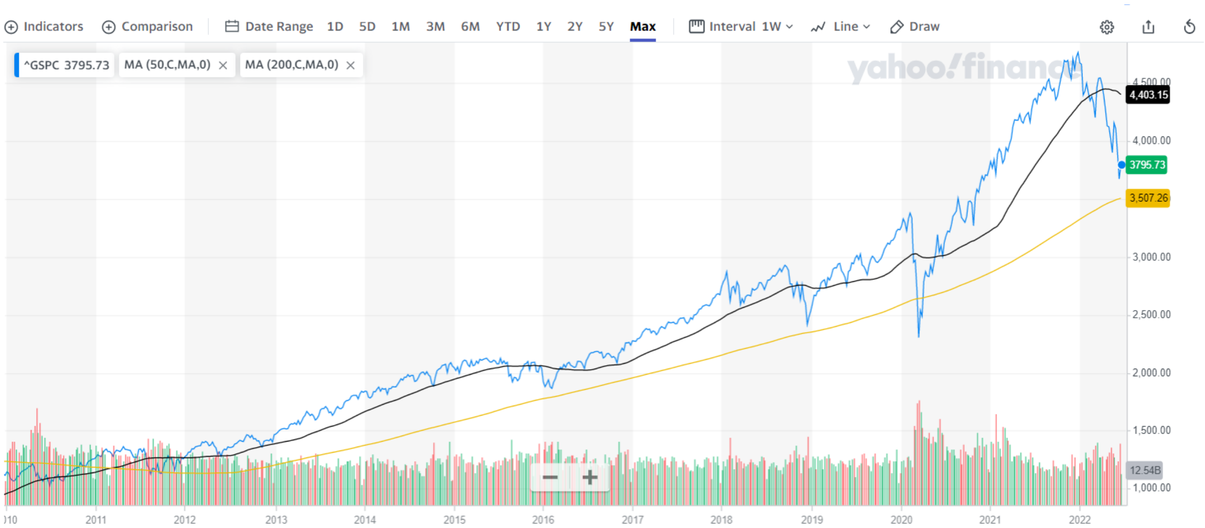


Figure 5.1 S&P500 Index Trend with Moving Averages

The figure depicts how the SP500 index has performed over the past 12 years, highlighting technical analysis indicators such as the 50-day (black) and 200-day (yellow) moving averages.

An improvement cue could be to use a switching hypothesis similar to the one proposed by Young. Indeed, the author, referring to the firm heads of technical analysis, uses the Black Cross and Golden Cross indicators for the detection of bear or bull market. According to these assumptions, it can be seen that in early 2011 there was a Golden Cross that saw the U.S. market come out of the housing bubble of 2007/2008 and started a bullish phase of the market and thus, according to Young's assumptions, saw the market in a Bullish phase. As can be deduced, the eventual use of these indicators as a switching signal in our CFP strategy, would have resulted in the use of PUT cycles alone, which would have been repeated in all periods, resulting in our strategy's returns, in both of its forms, certainly being different.

A second assumption that is significant to highlight is the absence of transaction costs. We could have assumed costs in terms of 1/5/10 bps but the result obtained would not have been truthful because only by considering a broker, thus a price list, can we actually assess the transaction costs of buying options, costs that we are aware would have steadily and persistently eroded the capital considered over the years lowering returns further.

Having said that we would like to conclude by saying that our study is not an end point with which we want to draw conclusions about an investment strategy that may or may not be adopted; but it should be seen as a continuation of a study that has seen its interest in the world of derivatives in general but particularly in the world of options. Therefore, we would like to conclude with two points of possible improvements that we have noted in our strategy.

Wearing the shoes of an investor, one aspect we have identified with wide room for improvement is the structuring of strategy switching. In fact, particularly valuable, it could consist of considering various entry signals from the various PUT or CALL cycles, signals that can be taken either from the field of technical analysis or from the field of fundamental analysis of macroeconomic factors that can give an idea of what the market trend might be.

The second improvement identified is the actual simulation of the strategy. In fact, we believe that it would be particularly interesting not only to backtest the strategy, but to apply it with special simulators in the markets so that we can have even more timely data that consider two aspects that we have not gone into much detail, such as transaction costs and option liquidity.

The usefulness of the latter lies in the possibility of doing a test with real data and with instruments in the markets much larger than those available to us. In fact, the actual implementation in the markets would make it possible not to consider a point value of the options, as done with the values of the options at the close of the roll date in our study, but it would be more appropriate to spread the purchase of options throughout the day so as to average the price of the option package purchased. This eventual implementation involves the use of a good amount of capital in order to provide for the purchase of both derivatives and the index, but at the same time it must provide enough capital so that there is no market manipulation. Carrying out such a simulation, therefore, would go beyond the liquidity problem of options especially going deep-in or out-of-the-money where the value of option volumes would drop significantly compared to the ATM or otherwise near-ATM case.

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