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SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

TDOA Analysis and Probabilistic APIT Implementation

Laurea Magistrale in telecommunication Engineering - Ingegneria delle telecomunicazioni

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1. Introduction

Accurate node localization is a fundamental requirement in many wireless networking applications such as indoor navigation, asset tracking, emergency response, and industrial monitoring.For example in indoor buildings, warehouses, underground areas, and dense urban infrastructures [9, 12].

In these contexts, range-based localization techniques relying on time- or angle-based measurements often require specialized hardware, tight synchronization, and careful calibration, leading to high system cost and complexity [10]. As a result, range-free localization methods have gained significant interest, since they rely only on connectivity or relative RSSI information and are well suited to low-cost, large-scale wireless sensor networks.

Among range-free approaches, the Approximate Point-In-Triangle (APIT) algorithm is a widely studied solution. APIT estimates node location through inside/outside tests with respect to triangles formed by anchor nodes, using relative RSSI comparisons from neighboring devices [1, 2]. This geometric formulation avoids explicit distance estimation and makes APIT attractive for RSSI-based indoor localization.

However, classical APIT suffers from important limitations. Binary inside/outside decisions are highly sensitive to radio irregularities, multipath propagation, and node mobility, leading to misclassifications and unstable results in real indoor environments [3, 7]. Moreover, APIT performance strongly depends on deployment geometry, anchor placement, neighbor density, and the spatial scale of the scenario.

To mitigate these issues, several probabilistic and fuzzy extensions of APIT have been proposed, replacing hard decisions with soft confidence measures derived from RSSI statistics [4–6]. While these approaches improve robustness, most existing works mainly focus on average accuracy and provide limited insight into performance variability under changes in area scale and neighbor distribution.

This work addresses this gap by presenting a systematic experimental analysis of a probabilistic APIT formulation under controlled variations of deployment area and neighbor spatial distribution, considering both fully simulated scenarios and real RSSI measurements collected with an ESP32-based indoor testbed.

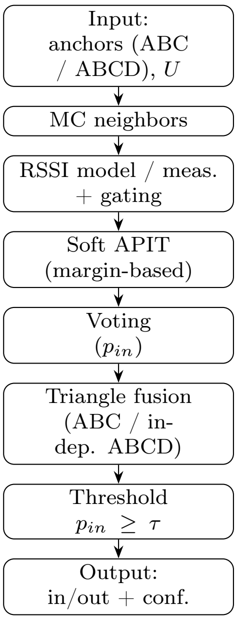

Figure 1: Compact probabilistic APIT pipeline.

2. Improved Probabilistic APIT Method Formulation

2.1. Method Overview

The proposed approach extends the classical APIT algorithm by replacing hard inside/outside decisions with a probabilistic formulation designed to handle RSSI uncertainty and neighbor variability.The probabilistic APIT framework is based on Monte Carlo neighbor generation, RSSI-gated selection, and soft inside/outside decisions with margin-based voting.

2.2. Monte Carlo Neighbor Generation

For each test position, a set of neighboring nodes is generated through local Monte Carlo sampling around the mobile node. Multiple Monte Carlo realizations are performed in order to reduce statistical fluctuations and to evaluate classification stability under different neighbor geometries.

2.3. RSSI Model and RSSI-Gated Selection

Received signal strength values are used to perform relative comparisons between anchors and neighbors. In simulated scenarios, RSSI is generated using a log-distance path-loss model with log-normal shadowing [7, 8]. In measurement-based scenarios, RSSI samples are collected and averaged to reduce fast fading and hardware-related fluctuations [7, 8]. The proposed method applies an RSSI-gated selection step to reduce the impact of unreliable comparisons caused by small RSSI differences. In addition to RSSI-based range-free localization, this thesis also analyzes Time Difference of Arrival (TDOA) multilateration as a representative example of geometry-based indoor localization techniques. The theoretical foundations of TDOA are discussed, together with its synchronization requirements and its sensitivity to multipath propagation and non-line-of-sight conditions. This analysis provides a reference framework to highlight the trade-offs between high-accuracy range-based methods and low-cost RSSI-based approaches, and motivates the focus on probabilistic enhancements of APIT in realistic indoor scenarios.

2.4. Soft Inside/Outside Decisions and Margin-Based Voting

For each anchor triangle, the APIT inside/outside test is repeated across multiple neighbor realizations. Instead of returning a binary decision, the algorithm computes an inside probability defined as the fraction of realizations classified as inside. A decision threshold τ is then applied when a binary output is required. Two thresholds are considered: a fixed value $\tau = 0.5$ and an optimized threshold τ^* obtained by maximizing classification accuracy on the considered dataset.

2.5. Independent Triangle Selection in the Four-Anchor Case

When four anchor nodes are available, four distinct triangles can be geometrically constructed. Using all four triangles would produce strongly correlated probability estimates, violating the independence assumption; therefore, only two non-overlapping diagonal triangles are selected. The two triangles are chosen along the two diagonals of the anchor quadrilateral, so that they do not share any edge and their corresponding probabilistic tests can be reasonably assumed to be statistically independent. The final inside probability is then obtained by combining only these two independent triangle estimates.

Using a larger number of highly correlated triangles does not necessarily improve localization performance and may lead to artificially over-

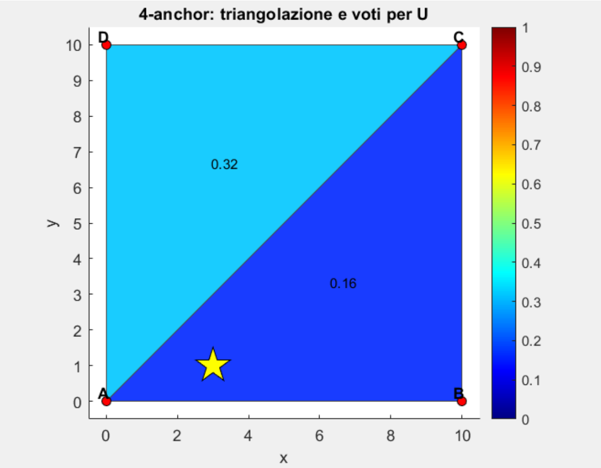

Figure 2: Example of independent triangle selection in the four-anchor case for a randomly extracted test position U in a Monte Carlo simulation. The mobile node position U (yellow star) is randomly drawn within the anchor hull. The two colored triangles correspond to the two non-overlapping diagonal configurations used for probabilistic APIT aggregation. The reported values represent the inside probabilities obtained independently for each triangle.

confident probability estimates [1, 3, 10].
Figure 2 illustrates a representative realization of this procedure. The position U is randomly extracted within the anchor hull during the Monte Carlo process. The two diagonal triangles produce independent inside probability values, which are then combined according to the probabilistic fusion rule. The absence of overlapping edges between the two selected triangles ensures that the corresponding RSSI comparisons rely on independent anchor triplets, thus justifying the proposed independence assumption.

3. Method Validation

3.1. Simulation-Based Validation

3.1.1 Simulation Setup

In this experiment, both anchor positions and mobile node locations are generated synthetically within a square deployment area. All neighbor nodes are obtained through local Monte Carlo sampling around each test position. The received signal strength values are generated using a log-distance path-loss model with log-normal shadowing [7, 8].
For each area scale factor, a fixed number of random test points is generated uniformly within the scaled region. For each test point, multiple Monte Carlo realizations are performed in order to reduce statistical fluctuations. Both three-anchor (ABC) and four-anchor (ABCD) probabilistic APIT configurations are evaluated. Two decision thresholds are considered: a fixed value $\tau = 0.5$ and an optimized threshold τ^* obtained by maximizing the classification accuracy on the same simulated dataset.

3.1.2 Accuracy Versus Area Scaling

The classification accuracy obtained in the fully simulated scenario is reported in Fig. 3. The results show that the localization accuracy varies significantly with the scale of the deployment area for both anchor configurations. For small area scales, the geometric diversity between anchors is limited, which reduces the discriminative capability of the probabilistic APIT test. As the area increases, accuracy improves due to a more favorable spatial separation between anchors and test points.
For larger area scales, a gradual saturation and partial degradation of performance is observed. This behavior is caused by the increasing average distance between the mobile node and the anchors, which amplifies the impact of path-loss and shadowing variability. The use of the optimized threshold τ^* consistently improves the accuracy with respect to the fixed threshold $\tau = 0.5$ for all considered area scales.
The four-anchor configuration does not exhibit a uniform advantage over the three-anchor case across all scales. While higher accuracy is obtained in some configurations when using four anchors, in other cases the three-anchor setup achieves comparable or superior performance. These results confirm that, in a purely simulated environment, the dominant factor affecting probabilistic APIT performance is the geometric configuration of anchors and test points rather than the sole number of anchors [1–3, 10].

3.2. Experimental Validation

3.2.1 Experimental Setup and Real RSSI Measurements

In the second set of experiments, the anchor positions and the mobile station locations are taken from the real indoor measurement campaign,

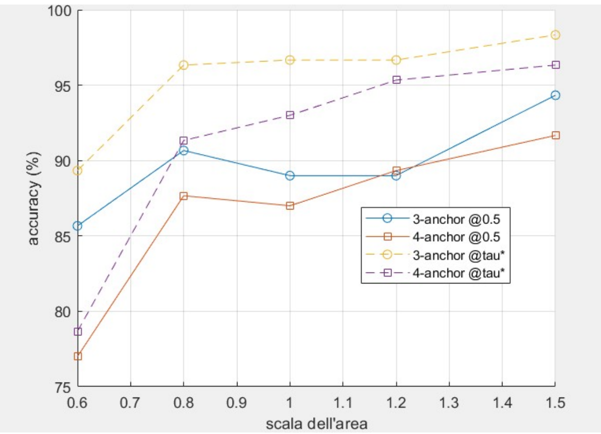

Figure 3: Classification accuracy versus area scale in the fully simulated scenario. Both anchor positions and mobile node locations are generated synthetically, while neighbors are obtained through local Monte Carlo sampling. Results are shown for three-anchor and four-anchor probabilistic APIT at fixed threshold $\tau = 0.5$ and optimized threshold τ^*.

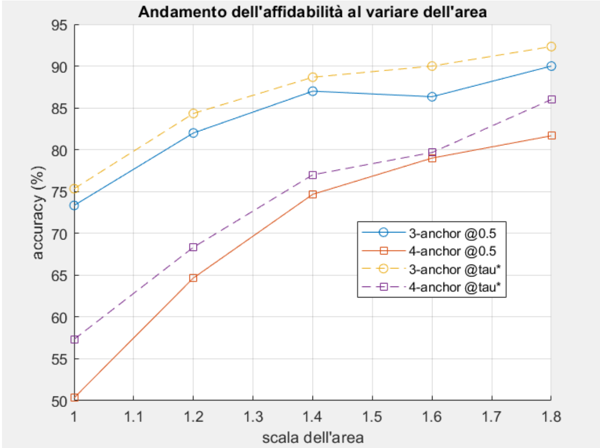

Figure 4: Classification accuracy versus area scale for the real-position scenario. Anchor and mobile station positions are taken from the experimental dataset, while neighbors are generated by local Monte Carlo sampling. Results are reported for three-anchor and four-anchor probabilistic APIT using a fixed threshold $\tau = 0.5$ and optimized threshold τ^*.

while neighbor nodes are still generated through local Monte Carlo sampling around each test point. For each real position, multiple RSSI samples are collected and averaged to reduce fast fading and hardware-related fluctuations [7, 8]. In this way, the probabilistic APIT algorithm is evaluated under realistic RSSI conditions, but the neighbour geometry remains statistically controlled.

The classification accuracy is computed as

$$\mathrm{Acc}(\tau) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\{\hat{y}_i(\tau) = y_i\}, \tag{1}$$

where N is the number of test points, $y_i \in \{0, 1\}$ is the geometric ground-truth label (inside/outside) and $\hat{y}_i(\tau)$ is the binary decision obtained by thresholding the probabilistic APIT output at level τ. The same definition is applied to the three-anchor (ABC) and four-anchor (ABCD) configurations.

3.2.2 Accuracy Versus Area Scaling

Figure 4 shows the accuracy as a function of the area scale when a fixed threshold $\tau = 0.5$ is used. Compared to the fully simulated case, the overall accuracy levels are lower and the curves are less smooth, as expected in the presence of real measurement noise and small calibration mismatches. Nevertheless, the same qualitative behavior observed in the synthetic scenario is preserved: accuracy depends strongly on the size of the area and exhibits a non-monotonic trend, with an intermediate range of scales that yields the best results. For very small scales, the limited geometric diversity between anchors and test points reduces the discriminative power of APIT; for very large scales, the average anchor distance increases and RSSI differences become less informative, degrading performance [9, 10].

The threshold τ^* is obtained by sweeping $\tau \in [0, 1]$ and selecting the value that maximizes (1) on the real dataset. As in the fully simulated scenario, the use of τ^* provides a clear improvement over the fixed decision rule. However, the optimal threshold is not constant across scales and differs between the three-anchor and four-anchor configurations, confirming that τ cannot be interpreted as a universal parameter independent of geometry and dataset.

A comparison between Figs. 3 and 4 highlights three main effects. First, moving from a fully simulated scenario to real RSSI measurements reduces the absolute accuracy and introduces additional variability, but it does not remove the strong dependence on area scale. Second, the four-anchor configuration does not exhibit a uniform advantage over the three-anchor configuration: in some scale ranges the additional anchor improves performance, while in others the three-anchor setup remains competitive or even preferable. This behavior is consistent with previous analyses of range-free localization sensitivity to deployment geometry and radio conditions [1–3, 10, 12]. Third, the optimal threshold τ^* must be calibrated on the specific scenario; applying a threshold tuned on simulated data to real measurements would not guarantee the same level of improvement.

Overall, the real-measurement experiments confirm that probabilistic APIT provides a meaningful probabilistic description of inside/outside classification under realistic RSSI conditions, but they also show that its performance remains strongly governed by the underlying geometry, the scale of the test area and the choice of the decision threshold.

3.3. Discussion on Threshold Calibration and Practical Applicability

The decision threshold τ plays a critical role in transforming the probabilistic APIT output into a binary inside/outside classification. As clearly observed in both the fully simulated and real-measurement scenarios, the optimal threshold τ^* is not constant and strongly depends on the geometric scale of the deployment, the anchor configuration and the spatial distribution of neighboring nodes.

A fundamental practical limitation emerges from this result: the computation of an optimal threshold τ^* requires the availability of ground-truth labels, i.e., the *a priori* knowledge of the true position of the mobile node. This implies that τ^* cannot be estimated online in standard localization problems, since the node position is precisely the unknown to be inferred. Consequently, although τ^* provides an upper bound on achievable classification accuracy, it cannot be directly exploited in real-time localization without an external calibration phase.

This observation highlights an important trade-off between optimal statistical performance and practical deployability. A fixed threshold such as $\tau = 0.5$ offers a fully unsupervised and deployment-independent solution, but at the cost of suboptimal accuracy. On the other hand, scenario-specific calibration of τ can substantially improve performance, but requires either a training phase with known reference positions or dedicated calibration measurements.

For this reason, the proposed probabilistic APIT formulation is particularly well suited to hybrid localization frameworks such as fingerprint-based localization and multipath-assisted positioning. In these scenarios, reference measurements with known positions are already available during the training phase, enabling an offline calibration of τ that can then be transferred to the online localization stage. Similar calibration strategies are commonly adopted in RSSI fingerprinting and multipath-based indoor positioning systems [9, 12].

Overall, these results indicate that probabilistic APIT can act as a robust geometric consistency layer within more complex localization architectures. While it cannot completely remove the sensitivity to deployment geometry and radio propagation conditions, the probabilistic formulation provides a principled way to quantify classification confidence and to integrate APIT into data-driven indoor localization systems. Overall, the thesis provides a unified view of indoor localization techniques, covering both geometry-based methods such as TDOA and range-free RSSI-based approaches. While the main experimental contribution focuses on probabilistic APIT, the broader analysis highlights how deployment geometry, radio conditions and algorithmic assumptions jointly affect indoor localization performance.

4. Conclusions

This Executive Summary has presented the main contributions of a thesis focused on indoor localization techniques under realistic radio propagation conditions. The work has combined a theoretical analysis of geometry-based localization, with particular attention to Time Difference of Arrival (TDOA), and an extensive experimental investigation of a probabilistic formulation of the APIT algorithm.

The proposed probabilistic APIT framework has been shown to provide a more robust and informative inside/outside classification compared to the classical binary APIT approach. Through simulation-based and measurement-based experiments, the analysis has highlighted the strong impact of deployment geometry, area scale, and decision threshold selection on lo-

calization performance. In particular, the results demonstrate that increasing the number of anchors does not guarantee improved accuracy, and that geometric configuration often plays a dominant role.
Although the optimized decision threshold provides an upper bound on achievable performance, its dependence on scenario-specific ground truth limits its direct use in fully unsupervised localization. Nevertheless, the probabilistic APIT formulation naturally lends itself to integration within hybrid and data-driven indoor localization systems, where offline calibration is feasible.
Overall, the thesis confirms that probabilistic extensions of range-free localization algorithms represent a valuable tool for improving robustness and interpretability in indoor positioning systems, while preserving low-cost and low-complexity deployment characteristics.

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