



**POLITECNICO**  
MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE  
E DELL'INFORMAZIONE

EXECUTIVE SUMMARY OF THE THESIS

## Aeroacoustic Optimization of Propellers

LAUREA MAGISTRALE IN AERONAUTICAL ENGINEERING - INGEGNERIA AERONAUTICA

Author: CARLOS FERNÁNDEZ PEREIRA

Advisor: PROF. ALBERTO MATTEO ATTILIO GUARDONE

Co-advisor: LUCA ABERGO

Academic year: 2021-2022

---

*This work presents the progress carried out in the coupled CAA-CFD optimization framework inside SU2 for cases with axial symmetry. The pressure fluctuations at the body are obtained using CFD and propagated to the farfield using the FW-H formulation, then an AD-based discrete adjoint computes the sensitivities. To reduce the computational cost the unsteady simulation is replaced by a steady one and a rotating reference frame (RRF). Moreover, the application of periodic boundary conditions in the CFD simulation opens the possibility of studying multi-blade propellers by just analyzing a single blade. These techniques are applied to an optimization case in which the shape of a propeller blade is modified to reduce the noise signature while keeping the thrust constant. The proposed framework strongly reduces the average sound pressure level (SPL) for a mic positioned at a distance of ten times the radius.*

### 1. Introduction

Urban air mobility (UAM) is expected to arrive in the coming years. Multi-propeller vehicles will become an alternative transportation

method to commute or travel in between nearby cities [2]. These vehicles will fly over residential areas, thus the noise they produce must meet the requirements posed by the authorities and be low enough to improve the public acceptance. Obtaining an accurate prediction of the noise is challenging due to its 3-D, unsteady and turbulent nature. Ffowcs-Williams and Hawkings studied the noise produced by aerodynamic bodies in arbitrary motion creating the FW-H formulation. Di Francescantonio [4] combined the benefits of the FW-H and the Kirchoff formulations to develop a model known as the permeable FW-H formulation to distinguish it from the original one. Local blade modifications based on a trial-and-error approach are inefficient and time-consuming. Due to the large number of parameters involved in aerospace design, gradient-based optimization methods using a discrete adjoint formulation are a suitable choice because the computational cost of evaluating the sensitivities is independent of the number of design variables [1]. The aeroacoustics community usually employs scale-resolving simulations such as the detached-eddy simulation (DES). These simulations require a lot of computational power, so they are not practical for optimization. Over the past few years, a lot of interest has been dedicated to assess whether

lower fidelity alternatives such as the URANS equations are feasible for optimization [5].

## 2. Coupled CFD-CAA Optimization Framework

The coupled CFD-CAA optimization framework inside SU2 couples the RANS CFD solver present inside SU2 with a CAA solver based of the FW-H formulation. A discrete-adjoint formulation is used to compute the sensitivities.

### 2.1. CAA Acoustic Solver

The acoustic solver implemented in SU2 is based on the Di Francescantonio formulation [4]. It is an extension of the Formulation 1A developed by Farassat [3], which is a solution of the FWH equation for subsonic cases that can be numerically integrated. Di Francescantonio [4] proposed a formulation which joined the benefits of the Kirchoff and the FWH formulations shown in eq. 1. The discontinuity surface is chosen arbitrarily outside of the aerodynamic body. Therefore, the non penetration condition does not hold along this fictitious surface, so the local fluid velocity normal to the surface is not null ( $u_n \neq 0$ ) and the local normal velocity of the source surface is not null either ( $v_n \neq 0$ ), so it cannot be assumed that  $(u_n - v_n)$  is null anymore.

$$\square^2[c^2(\rho - \rho_0)] = \frac{\partial}{\partial t}[\rho_0 U_n \delta(f)] - \frac{\partial}{\partial x_i}[L'_{ij} n_j \delta(f)] + \frac{\partial^2 T_{ij} H(f)}{\partial x_i \partial x_j} \quad (1)$$

The parameters  $U_i$  and  $L_{ij}$  can be interpreted as a modified velocity and stress tensor respectively. If the discontinuity surface is chosen to be surface of the aerodynamic body, the equation turns into the classical FWH equation.

$$U_i = u_i + [(\rho/\rho_0) - 1](u_i - v_i) \quad (2a)$$

$$L_{ij} = P'_{ij} + \rho u_i (u_j - v_j) \quad (2b)$$

The Di Francescantonio formulation has the advantage that if the turbulent structures are properly resolved inside the permeable discontinuity surface, the effect of the quadrupole term can be neglected, so it does not need to be computed. To obtain the integral formulation of the problem shown in eq. 3, a Green function is used following the approach of Farassat [3] to

obtain a permeable version of the Formulation 1A. This formulation assumes that the contribution quadrupole terms outside the discontinuity surface is negligible, hence the pressure fluctuations at an observer location are the sum of the thickness and loading noise. If the impermeable surface model is used, this hypothesis is only reasonable for low Mach problems, in cases with transonic phenomena such as shocks this assumption is not valid anymore. In this work, only the impermeable surface model (classical FW-H) is used.

$$4\pi p' = \int_S \left[ \frac{\rho_0 (\dot{U}_i n_i + U_i \dot{n}_i)}{r|1 - M_r|^2} \right]_{ret} dS + \int_S \left[ \frac{\rho_0 U_i n_i K}{r^2 |1 - M_r|^3} \right]_{ret} dS + \frac{1}{c} \int_S \left[ \frac{\dot{F}_i \hat{r}_i}{r|1 - M_r|^2} \right]_{ret} dS + \int_S \left[ \frac{F_i \hat{r}_i - F_i M_i}{r^2 |1 - M_r|^2} \right]_{ret} + \frac{1}{c} \int_S \left[ \frac{\dot{F}_i \hat{r}_i K}{r^2 |1 - M_r|^3} \right]_{ret} dS \quad (3)$$

$$K = \dot{M}_i \hat{r}_i r + M_r c - M^2 c \quad (4a)$$

$$F_i = L_{ij} n_j \quad (4b)$$

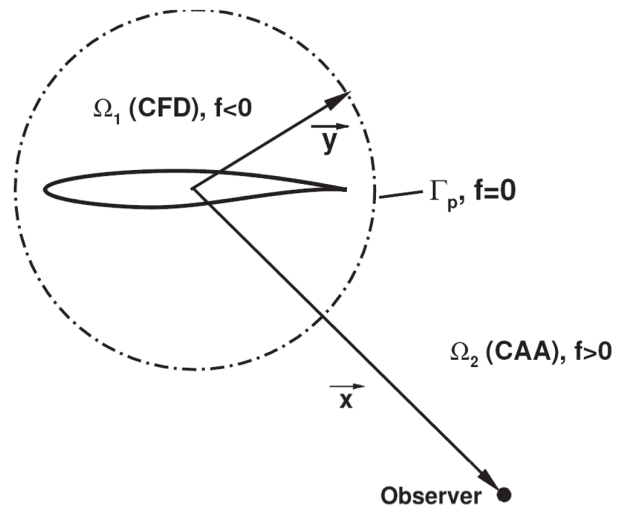


Figure 1: FWH Formulation Schematic [5]

### 2.2. Computational Cost Reduction Techniques

#### 2.2.1 Rotating Reference Frame (RRF)

In cases which have axial symmetry, a rotating reference frame moving with the body can be used to reduce the computational cost of the simulation. First, a steady flow simulation

which accounts for the movement of the body with respect to the freestream is conducted. Then, a rotation is applied to the steady solution to obtain the state variables at each time step. This results in a significant reduction of the computational cost because the unsteady simulation is replaced by a steady simulation and a set of rigid rotations. The rotated surface coordinates, grid velocity and normals at each time step are indicated by the hat superscript ( $\hat{\cdot}$ ), where  $\mathbf{R}$  is the rotation matrix.

$$\hat{\mathbf{x}} = \mathbf{R} \cdot \mathbf{x} \quad (5a)$$

$$\hat{\mathbf{u}}_{grid} = \mathbf{R} \cdot \mathbf{u}_{grid} \quad (5b)$$

$$\hat{\mathbf{n}} = \mathbf{R} \cdot \mathbf{n} \quad (5c)$$

### 2.2.2 Multi-blade Aeroacoustic Analysis using Periodic BC's

Conducting the CFD simulation of a multi-blade propeller usually results in a grid with several million elements. Since propellers have axial symmetry, the domain can be divided in sectors limited by the planes of radial symmetry. If periodic boundary conditions are applied along them, it is possible to analyze one blade and to account for the effect that the other blades have on the flow. Analyzing this new grid requires a fraction of the cost of the original one. Figure 2 shows an example in which periodic boundary conditions are applied to a two-bladed propeller. The input for the acoustic solver is the flow at the discontinuity surface. If periodic boundary conditions are used for the CFD simulation, only a fraction of the input data is available. However, the original surface can be reconstructed by placing a copy of the periodic solution in the correct position.

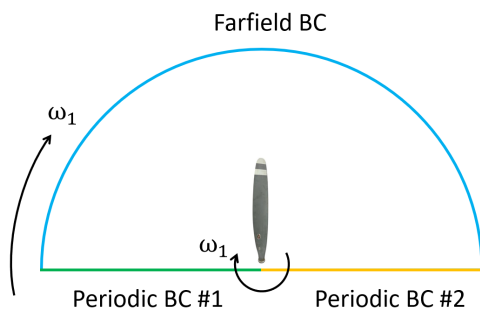


Figure 2: Periodic BC's for a 2-bladed propeller

### 2.3. Sensitivity Analysis

The sensitivity analysis consists on computing the derivatives of the outputs with respect to the inputs. These derivatives are used in gradient-based optimization to improve the system's performance. In the case of automatic shape optimization using CFD, the target is to improve a performance parameter called the objective function  $J$  by modifying the shape of the immersed body, which is parametrized using design variables  $\alpha$ , which are under control of the optimizer. The state variables of the flow field  $U(\alpha)$  are analyzed by the solver to assess the convergence criterion. To handle the surface and mesh deformation, the following constraint is added  $X = M(\alpha)$ . Therefore, the optimization problem takes the form:

$$\underset{\alpha}{\text{minimize}} \quad J(U(\alpha), X(\alpha)), \quad (6a)$$

$$\text{subject to} \quad U(\alpha) = G(U(\alpha), X(\alpha)), \quad (6b)$$

$$X(\alpha) = M(\alpha), \quad (6c)$$

By using the augmented Lagrangian and following the steps presented in [1], the following expression for the sensitivities is obtained.

$$\frac{dL^T}{d\alpha} = \frac{dJ^T}{d\alpha} = \frac{d}{d\alpha} M^T(\alpha) \bar{X} \quad (7)$$

## 3. Verification & Validation

The reference case used in this work is a two-bladed propeller. Each blade has a NACA4412 section and a radius  $R = 0.114m$ . The propeller is investigated in a forward flight condition in which it rotates around the x-axis at  $\omega = -523.598 \text{ rad/s}$ . In standard conditions for pressure and temperature the propeller has a tip Mach number of  $M = 0.15740$ , the free-stream has a velocity of  $U = 5m/s$ .

### 3.1. Verification of the Periodic BC's for Aeroacoustic Analysis

In order to verify the use of periodic boundary conditions for multi-blade aeroacoustic analysis, the results obtained with the periodic one-blade simulation are compared to the solution obtained using the original two-blade baseline propeller. Table 1 shows the thrust and moment coefficients for each case along with their relative error. It can be noted that the error is

smaller than 3% in both cases, so there is a good agreement between both simulations.

Table 1: Coefficients Verification

Param.	Baseline	Periodic	Rel.Err. %
$C_{F_x}$	$2.791 \times 10^{-2}$	$2.710 \times 10^{-2}$	2.898
$C_{M_x}$	$3.221 \times 10^{-3}$	$3.150 \times 10^{-3}$	2.198

Regarding the acoustic simulation, the impermeable Di Francescantonio formulation is used, so the acoustic discontinuity surface coincides with the propeller blade surface. An array of 10 microphones is placed at a distance  $d = 10R$ , with  $R$  being the radius of the propeller at an incidence angle from  $45^\circ$  to  $135^\circ$ . The the root mean square pressure ( $p_{rms}$ ) shown in equation (8) is compared at each observer location.

$$p_{rms} = \sqrt{\frac{1}{N - N_*} \sum_{n=N_*+1}^N (p'_{n,m})^2} \quad (8)$$

Figure 3 shows the relative error of the  $p_{rms}$  at each observer location. It can be observed that the error of the  $p_{rms}$  is around 2% at every observer which is of the same order as the relative errors for the thrust and torque coefficients ( $C_{F_x}, C_{M_x}$ ) shown in Table 1.

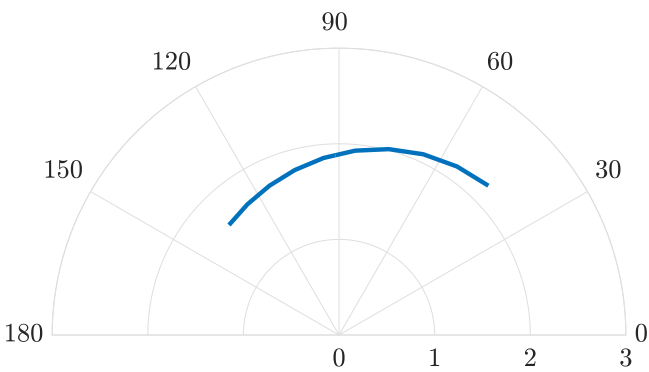


Figure 3:  $p_{rms}$  Relative Error % at 10R

### 3.2. Adjoint Sensitivities Validation

The sensitivities in the coupled CFD-CAA framework are computed using the discrete adjoint formulation, they are validated by comparing them to the ones obtained using finite differences. The geometry is parametrized using one FFD box with 24 design variables that can move along the freestream flow direction (x-axis). Figure 4 shows the comparison of the

noise sensitivity for each design variable using the discrete adjoint formulation and forward finite differences with two different steps. It can be observed that there is a very good agreement among the three methods.

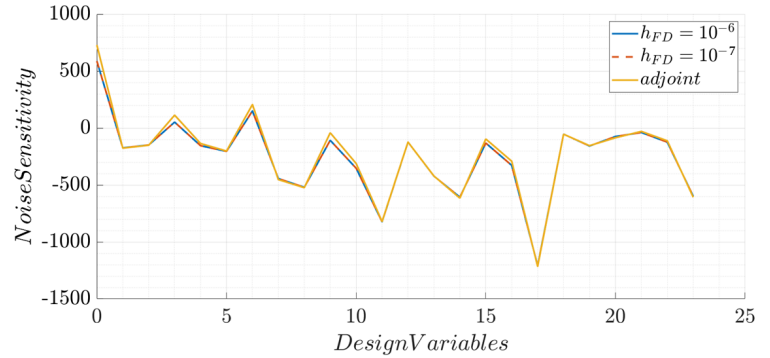


Figure 4: Noise Sensitivity for each DV

## 4. Results

In this section the baseline 2-blade propeller presented in section 3 is optimized to reduce its noise while keeping the thrust constant. The free-stream has a velocity of  $U = 34.3m/s$ . The objective function  $J$  shown in eq. 9 is the sound pressure level (SPL). Only one observer is used to analyze the pressure fluctuation, it is located at a distance  $d = 10R$ , with  $R$  being the radius of the propeller at an incidence angle of  $45^\circ$ .

$$J = 20 \log_{10} \left( \frac{p_{rms,avg}}{p_0} \right), \quad p_0 = 20 \mu Pa \quad (9)$$

The geometry is parametrized using one FFD box with 56 design variables that can move only along the freestream flow direction (x-axis). The optimization is carried out for 18 design updates. Figure 5 shows the difference between the baseline and the optimized designs. It can be observed that the optimizer reduces the incidence angle of the section while increasing the camber.

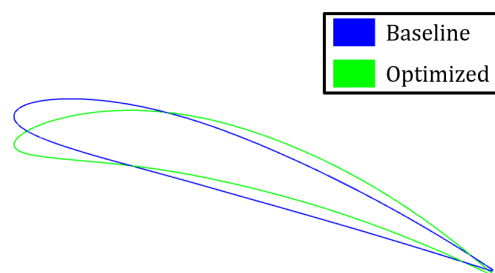


Figure 5: Propeller Sections Change 98 % of R

The results of the optimization are summarized in Table 2. The optimization achieves a 55% reduction in the objective function at the observer location. Moreover, the  $C_{F_x}$  and  $C_{M_x}$  modulus are reduced by 3% and 72% respectively. The thrust coefficient ( $C_{F_x}$ ) has decreased because the constraint is enforced in a weak manner by using a penalty function method.

Table 2: Optimization Results

Param.	Baseline	Optimized	Change %
SPL [dB]	52.42	23.18	-55.67
$p_{rms}$	$8.352 \times 10^{-3}$	$2.883 \times 10^{-4}$	-96.55
$C_{F_x}$	$2.573 \times 10^{-2}$	$2.494 \times 10^{-2}$	-3.07
$C_{M_x}$	$2.354 \times 10^{-3}$	$6.553 \times 10^{-4}$	-72.16

Figure 6 shows the  $SPL$  for both the baseline case and the optimized one at for an array of 10 microphones placed at a distance of  $d = 10R$ , with  $R$  being the radius of the propeller at an incidence angle from  $45^\circ$  to  $135^\circ$ . The average  $SPL$  was reduced by an 18.6% (10.9 dB). Since the microphone used for the optimization was placed at  $45^\circ$ , this is the position where the noise signature has been reduced the most. Therefore, it must be noted that the microphone position affects significantly the noise directivity of the optimized blade.

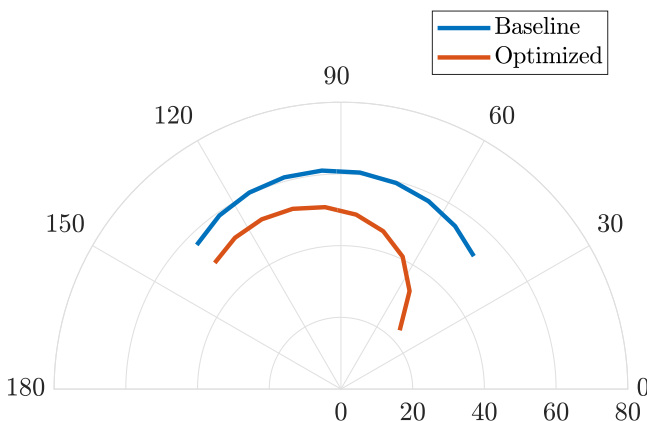


Figure 6:  $SPL$  [dB] at  $10R$

## 5. Conclusions

This work presented the progress carried out in the coupled CAA-CFD optimization framework inside SU2 for cases with axial symmetry. The RANS equations were used to solve

the flow around the body using a rotating reference frame (RRF), lowering the computational cost by replacing the unsteady simulation by a steady simulation and a set of rigid rotations. The FW-H formulation propagates the pressure fluctuations to the farfield and the AD-based discrete adjoint computes the sensitivities. The successful application of periodic boundary conditions opens the possibility of studying multi-blade propellers by just analyzing a single blade. An optimization case was conducted in which the shape of a propeller blade was modified to reduce the noise signature at a single observer location while keeping the thrust constant. The proposed framework reduced the average  $SPL$  by an 18.6% (10.9 dB) for a mic array positioned at a distance of ten times the radius at an incidence angle from  $45^\circ$  to  $135^\circ$ . It has also been noted that the position of the observer location used in the optimization affects significantly the noise directivity of the design.

## References

- [1] Tim A. Albring, Max Sagebaum, and Nicolas R. Gauger. Efficient aerodynamic design using the discrete adjoint method in su2. *17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2016.
- [2] Lukas Asmer, Henry Pak, Prajwal Shivaprakasha, Bianca Schuchardt, Peter Weiand, Frank Meller, Christoph Torens, Dennis Becker, Chen Zhu, Karolin Schweiger, Andreas Volkert, and Roman Jaksche. Urban air mobility use cases, missions and technology scenarios for the horizonum project. 08 2021.
- [3] Kenneth S. Brentner and F. Farassat. Modeling aerodynamically generated sound of helicopter rotors. *Progress in Aerospace Sciences*, 39(2):83–120, 2003.
- [4] P. di Francescantonio. A new boundary integral formulation for the prediction of sound radiation. *Journal of Sound and Vibration*, 202(4):491–509, 1997.
- [5] Beckett Zhou, Tim A. Albring, Nicolas R. Gauger, Carlos Ilario, Thomas D. Economon, and Juan J. Alonso. *Reduction of Airframe Noise Components Using a Discrete Adjoint Approach*.