POLITECNICO MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Executive Summary of the Thesis

Optimization of Multi-Impulse Earth–Moon Transfers in High-Fidelity Models

Laurea Magistrale in Space Engineering - Ingegneria Spaziale

Author: Guido Grossi

Advisor: Prof. Francesco Topputo

Co-advisors: Dr. Carmine Giordano, Carmine Buonagura

Academic year: 2022-2023

1. Introduction

The Artemis program is just the beginning of a new era of lunar exploration, wherein mankind will not only be engaged in discovering new frontiers but also exploiting the resources from space. Within this framework, improving lunar transfer trajectories is a fundamental measure towards achieving more economical and efficient missions. Moreover, as space missions become more demanding, high-fidelity astrodynamical models are necessary to design unique trajectories. Nevertheless, simplified models offer a deeper grasp of the dynamics of the cislunar environment, and are more suitable to perform extensive optimization searches. Hence, the transition from easier formulations to more complex ones is a crucial step in planning future missions. This thesis is divided into two parts. The first addresses the possibility of reducing the fuel cost of two-impulse Earth–Moon transfers in a four-body model with the addition of an intermediate impulse, by means of a semi-analytical approach. Instead, the second concerns the transformation of multi-impulse trajectories from the four-body model to a full-ephemeris model of the solar system. If the first part is intended to have a more theoretical orientation, dealing with basic research, the second part is more geared towards practical real-world applications.

2. Trajectories Optimization

Lawden's Primer Vector theory is applied to identify Earth–Moon transfers which could benefit from the introduction of an additional intermediate impulse. Subsequently, three-impulse transfers are designed with a direct numerical optimization method.

2.1. Earth–Moon Transfers

The starting point of the thesis is the collection of two-impulse Earth–Moon transfers found by Topputo in the Planar Bicircular Restricted Four Body Problem (PBRFBP), with the Sun, the Earth, and the Moon as primaries [5]. On these *reference trajectories* the spacecraft is transferred from an Earth-circular orbit of altitude $h_i = 167$ km, to a lunar circular orbit of altitude $h_f = 100$ km. Each transfer is achieved by means of an initial and a final impulse, both parallel to the local velocities. The total number of solutions found by Topputo is almost 300 000 and ranges from simple interior-direct transfers, to exterior low-energy transfers with ballistic capture at the Moon.

2.2. Primer Vector Theory

Primer Vector theory, formulated by Lawden in [3], is a collection of first-order necessary conditions for fuel-optimal impulsive trajectories.

These conditions are expressed in terms of a particular vector comprised of the adjoin variables associated with the velocity, called *primer vector* and indicted by $\mathbf{p}$. However, Lawden's theory was derived in the framework of the Restricted Two-Body Problem. Following the analysis of Hiday, who adapted the principle to the Elliptic Restricted Three-Body Problem in [2], the theory is extended to the PBRFBP. The first-order Lawden's necessary conditions (LNC) for a fuel-optimal impulsive transfer in the PBRFBP, with fixed terminal times, positions and velocities, are obtained by means of Pontryagin's Maximum Principle. Particularly they require that

1. $\mathbf{p}$ and $\dot{\mathbf{p}}$ are continuous and satisfies

$$\ddot{\mathbf{p}} = G_r \mathbf{p} + G_v \dot{\mathbf{p}} \ ,$$

   where G_r and G_v are the derivatives of the gravitational acceleration with respect to the position and the velocity, respectively;
2. $\|\mathbf{p}\| \leq 1$, with impulses occurring at those instants when $\|\mathbf{p}\| = 1$;
3. at all impulses, the primer is a unit vector aligned in the thrust direction;
4. at all interior impulses $\mathbf{p} \cdot \dot{\mathbf{p}} = 0$.

2.3. Methodology

2.3.1. Non-Optimal Trajectory

Among the thousand reference solutions, some of them already meet LNC, but others can be further optimized. The litmus paper is the history of the primer vector associated with each trajectory, which also gives a clear indication of how a trajectory can be improved [2]. The addition of a midcourse maneuver is a potential way of decreasing the cost of an impulsive mission. Therefore, each non-optimal reference trajectory, indicated with $\overline{\gamma}$ [1], is compared with a neighboring three-impulse trajectory γ, named *perturbed trajectory*. The two share the same terminal conditions, but γ has an additional intermediate impulse.

Let J represent the total *characteristic velocity*, or simply *cost*, of each transfer, defined as the sum of the magnitudes of all the maneuvers.

[1] Barred letters indicate quantities evaluated on the reference trajectory.

The following analytical criterion is used to determine if the addition of an intermediate impulse can reduce the total cost of the transfer.

A perturbed trajectory with a cost J_γ lower than $J_{\overline{\gamma}}$ exists if $\|\overline{\mathbf{p}}\|$ exceeds unity at any intermediate time. Moreover, the cost difference between the two is, to the first-order:

$$\delta J = \kappa \left(1 - \overline{\mathbf{p}}_m^\top \hat{\boldsymbol{\kappa}}\right) \ ,$$

where κ denotes the magnitude of the intermediate maneuver, and $\hat{\boldsymbol{\kappa}}$ its direction.

If δJ can be made negative, then γ represents an improvement in the cost over $\overline{\gamma}$. The optimal location and timing for the new impulse are computed in two steps: 1) identifying a first initial guess for the position $\mathbf{r}_m$ and time t_m of the third impulse; 2) iterating on these values until a local minimum of the cost is achieved.

2.3.2. Perturbed Trajectory

The time of maximum primer magnitude $\overline{t}_m$ is selected as an initial estimate for t_m. Let δ symbolize the difference between a quantity evaluated on the perturbed and reference trajectory; the position of the impulse is computed as $\mathbf{r}_m = \delta\mathbf{r}_m + \overline{\mathbf{r}}_m$, where $\overline{\mathbf{r}}_m$ is the position on $\overline{\gamma}$ at $t = \overline{t}_m$. A first-order analysis results in

$$\delta\mathbf{r}_m = \kappa K^{-1} \frac{\overline{\mathbf{p}}_m}{\|\overline{\mathbf{p}}_m\|} \ ,$$

where

$$K \equiv \overline{\Phi}_{vv}(t_{f^-}, t_{m^+})\overline{\Phi}_{rv}^{-1}(t_{f^-}, t_{m^+}) - \overline{\Phi}_{vv}(t_{i^+}, t_{m^-})\overline{\Phi}_{rv}^{-1}(t_{i^+}, t_{m^-}) \ .$$

The quantity $\overline{\Phi}$ is the *state transition matrix* associated with the flow φ_4 of the PBRFBP and evaluated on $\overline{\gamma}$. If the value of κ is selected small enough to justify a first-order analysis, the previous choice for $\delta\mathbf{r}_m$ ensures that $\hat{\boldsymbol{\kappa}}$ and $\|\overline{\mathbf{p}}_m\|$ are aligned at the time t_m [2]. Once the location in time and space of the midcourse impulse is determined, it is needed to build a trajectory connecting the prescribed initial and final states with the newly computed position of the intermediate maneuver. Therefore, a forward-backward

[2] This choice is compliant with the third LNC.

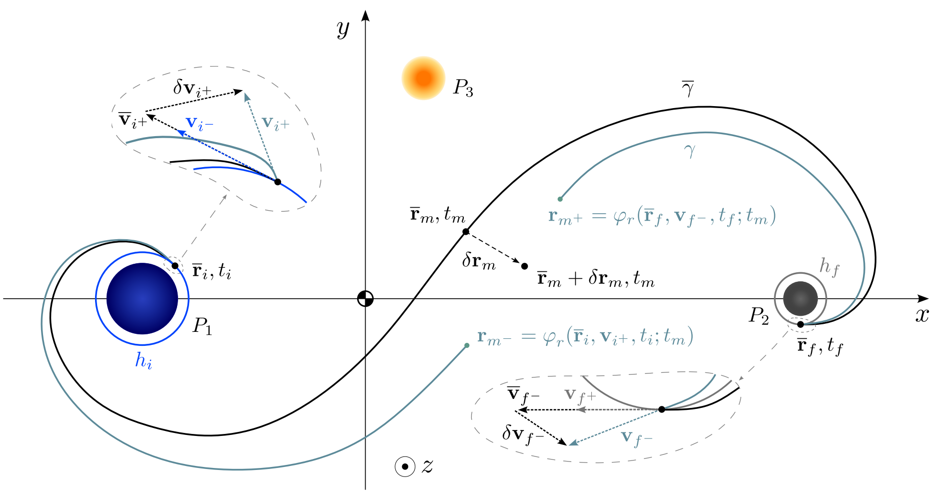

Figure 1: Double-shooting problem solved to compute the perturbed trajectory γ from the reference one $\overline{\gamma}$; the PBRFBP is described in a synodic frame which co-rotates with the Earth (P_1) and the Moon (P_2) and is centered at their barycenter; the distances and the sizes of the Sun (P_3) and the two primaries are not to scale; the transfers lie on the (x, y)-plane

shooting problem is set up, which consists in solving two Lambert's problems, connecting:

- $\{\bar{\mathbf{r}}_i,\, t_i\}$ with $\{\bar{\mathbf{r}}_m + \delta\mathbf{r}_m,\, t_m\}$;
- $\{\bar{\mathbf{r}}_f,\, t_f\}$ with $\{\bar{\mathbf{r}}_m + \delta\mathbf{r}_m,\, t_m\}$.

The idea of the solution method, represented in Fig. 1, consists in guessing the values of $\mathbf{v}_{i+}$ and $\mathbf{v}_{f-}$ and propagating the two states (forward and backward in time) until the continuity on the position is satisfied at $\mathbf{r}_m$. Concerning κ, its value is initially set to 1 m/s, but it is progressively reduced until the Matlab®'s built-in function `fsolve` reaches convergence in a prescribed number of iterations.

2.3.3. Refined Trajectory

The inclusion of an additional impulse (as it was discussed up to this point) does not necessarily yield an optimal three-impulse trajectory. The time history of the primer vector on γ must be examined again, in order to verify the compliance with LNC. When some conditions are not met, then a neighboring three-impulse solution $\tilde{\gamma}$ with a lower cost must exist. This shares the same terminal conditions with $\overline{\gamma}$ and γ, but, differently from the perturbed trajectory, the mid-course impulse is allowed to be located at a time $\tilde{t}_m \neq t_m$ and a position $\tilde{\mathbf{r}}_m \neq \bar{\mathbf{r}}_m$. Therefore, $\tilde{\gamma}$, named *refined trajectory*, consists of two ballistic arcs connecting the states

- $\{\bar{\mathbf{r}}_i,\, t_i\}$ with $\{\tilde{\mathbf{r}}_m\, \tilde{t}_m\}$,
- $\{\tilde{\mathbf{r}}_m\, \tilde{t}_m\}$ with $\{\bar{\mathbf{r}}_f,\, t_f\}$.

The idea now is to iterate on the position and time of the mid-course impulse, until a local minimum of $J_{\tilde{\gamma}}$ is reached. Thus, a nonlinear programming (NLP) problem is stated, where the unknowns are $\tilde{\mathbf{r}}_m$ and $\tilde{t}_m$. However, the two arcs of the solution must meet at the position of the second impulse and this depends on the velocities just after the initial impulse, and before the final one. Therefore, to avoid solving a shooting problem at each iteration of the NLP problem, both $\tilde{\mathbf{v}}_{i+}$ and $\tilde{\mathbf{v}}_{f-}$ are considered among the unknowns of the problem. These are collected in a vector of NLP variables $\mathbf{y} \equiv \left(\tilde{\mathbf{v}}_{i+},\, \tilde{\mathbf{v}}_{f-},\, \tilde{\mathbf{r}}_m,\, \tilde{t}_m\right)$ and Problem 1 is solved by means of Matlab®'s suite `fmincon`. For numerical efficiency, the variations of the cost $J_{\tilde{\gamma}}$ and the nonlinear equality constraints $\mathbf{h}_{m^-}$, $\mathbf{h}_{m^+}$ with respect to $\mathbf{y}$ are computed analytically.

Problem 1 (Trajectory Optimization). — *The refined trajectory is computed by solving:*

$$\min_{\mathbf{y}} \; J_{\tilde{\gamma}}(\mathbf{y}) := \|\Delta\tilde{\mathbf{v}}_i\| + \|\Delta\tilde{\mathbf{v}}_m\| + \|\Delta\tilde{\mathbf{v}}_f\|$$

subject to the constraints

$$\begin{cases} \mathbf{h}_{m^-}(\mathbf{y}) := \varphi_r(\mathbf{r}_i, \tilde{\mathbf{v}}_{i+}, t_i; \tilde{t}_m) - \tilde{\mathbf{r}}_m = \mathbf{0} \ , \\ \mathbf{h}_{m^+}(\mathbf{y}) := \varphi_r(\mathbf{r}_f, \tilde{\mathbf{v}}_{f-}, t_f; \tilde{t}_f) - \tilde{\mathbf{r}}_m = \mathbf{0} \ , \\ t_i < \tilde{t}_m < t_f \ . \end{cases}$$

2.4. Results

More than 10 000 trajectories are improved automatically following the aforementioned procedure. A global picture of the results obtained in the first part of the thesis, is presented in Fig. 2. Here, three-impulse transfers are reported in the $(\Delta t, \Delta v)$ plane[3], with the color of each point in-

[3] Δt is the time of flight of the transfer, and Δv its total cost, i.e., J.

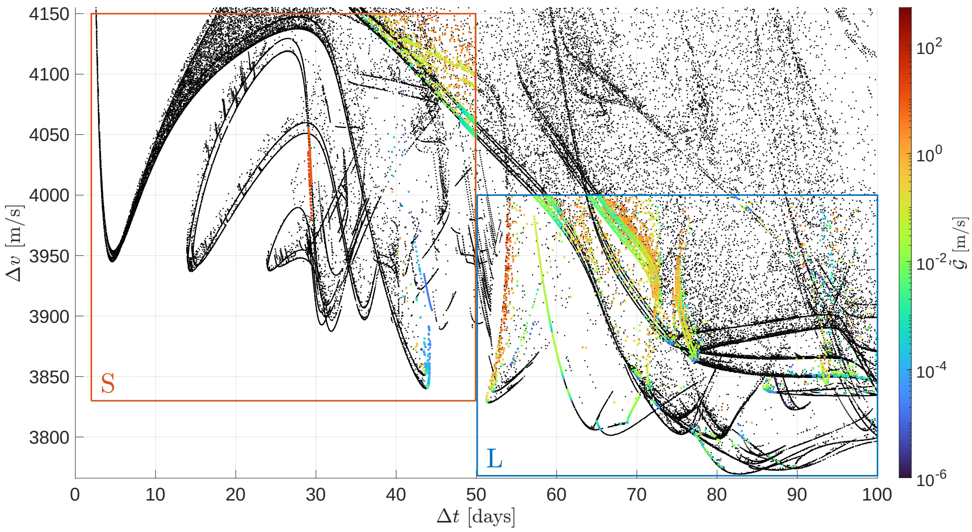

Figure 2: Multiple-impulse Earth–Moon transfers in a $(\Delta t, \Delta v)$ graph [5]. Black dots represent two-impulse solutions, while colored dots correspond to three-impulse refined trajectories; the color is related to the improvement in the cost provided by the additional impulse with respect to the reference two-impulse transfer

dicating the improvement achieved by adding a midcourse impulse: $\tilde{\mathcal{G}} = J_{\gamma} - J_{\tilde{\gamma}}$. Instead, black dots represent two-impulse transfers that are not optimized further, either because they are already optimal, or their trajectories are deemed irrelevant for a real mission to the Moon. Generally, the incorporation of a third impulse leads to a small percentage variation in the total cost of the final solutions. Nonetheless, intermediate maneuvers are beneficial for transfers with a moderate time of flight (e.g., 50 days) that involve an initial lunar flyby. In such scenarios, the additional degrees of freedom allow to take full advantage of the lunar gravity assist, and still achieve a feasible transfer by modifying the path after the Moon encounter. As a result, the largest improvement is obtained, which is in the range of tens m/s (red dots in Fig. 2).

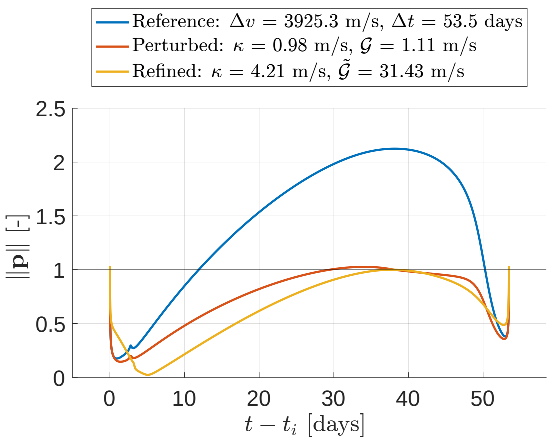

Figure 3: Primer vector magnitude on a reference, perturbed and refined trajectory

Figure 3 shows the primer vector magnitude on a reference, perturbed and refined transfer of this class. Note that the refined trajectory is the only one compliant with LNC, and thus optimal.

3. High-Fidelity Continuation

In the second part of the thesis, a procedure is developed to facilitate the transition of multi-impulse Earth–Moon transfers from a four-body model to a complete model of the solar system (this is what is intended for "continuation").

3.1. Background

3.1.1. Rotopulsating n-Body Problem

The astrodynamical model selected to describe the solar system is the Rotopulsating Restricted n-Body Problem (RPRnBP) [1]. Here, the motion of the spacecraft is described under the gravitational attraction of $n-1$ celestial bodies, in a local reference frame that rotates and pulsates with the Earth and the Moon, named rotopulsating frame (RPF). The coordinate axes and the adimensionalization scheme are such that the Earth and the Moon always occupy a fixed position in the RPF. SPICE toolkit is used to read the states of the celestial bodies from the JPL ephemeris data DE432 at any epoch. Moreover, the oblateness of the celestial bodies and the effects of solar radiation pressure are taken into account. Adimensional time, position, and velocity are computed with the transformation between sidereal coordinates (inertial), and synodic ones (non-inertial) reported in [1].

3.1.2. Low-Energy Transfers

The focus of the second part is on ballistic low-energy Earth–Moon transfers, due to their numerous advantages [4]. In short, these trajectories involve putting the spacecraft beyond the orbit of the Moon, and taking advantage of the Sun's gravity to reduce the transfer fuel cost.

3.2. Methodology

Let γ_4 be an Earth–Moon transfer in the PBRFBP that embeds either two or three impulsive maneuvers. The final transfer γ_n in the RPRnBP should connect two circular orbits with the same altitudes as those in the four-body model. However, while the orientation of the initial parking orbit is not constrained, the arrival orbital plane is fixed in a Moon-Centered Moon-Equatorial frame. Particularly, it is possible to select in advance the values of right ascension of the ascending node and inclination. Furthermore, also the case of a generically oriented polar orbit is accounted for. The methodology presented consists of two steps:

1. the generation of an initial seed trajectory;
2. the recursive correction with a modified multiple-shooting strategy.

3.2.1. Initial Seed Trajectory

The first guess solution for the multiple-shooting problem is obtained by transforming the states of γ_4 into the RPRnBP. To this aim, a suitable epoch must be computed to define the RPF. Given an *alignment time* in the PBRFBP, the *Frames Alignment problem* consists in finding an *alignment epoch* such that the configurations of the three primaries (Earth, Moon, and Sun) in the two models are as close as possible. A careful choice is to set the alignment time on each transfer γ_4 when the Sun's effects on the trajectories are more relevant, i.e., when spacecraft distance from the Earth–Moon barycenter is maximum. Then, a grid-search algorithm is implemented to find the best alignment epoch within a given time window (e.g, a solar year). In Fig. 4, the trajectories of the Sun in the two models are depicted during the time of a transfer. In the four-body model the Sun moves on a circular orbit, while in the n-body model its trajectory is read from the ephemerides. The solution to the Frame Alignment problem is indicated with an asterisk, while the positions at the departure are spotted with a dot. The more precise the overlap of the two models, the closer the initial seed orbit is to a feasible trajectory in the PBRFBP (with positive consequences on the quality of the guess solution). Subsequently, a series of transformations, through a non-rotating frame in the PBRFBM, the International Celestial Reference Frame, and finally the RPF, are used to convert the states of γ_4 to the nondimensional quantities of the RPRnBP.

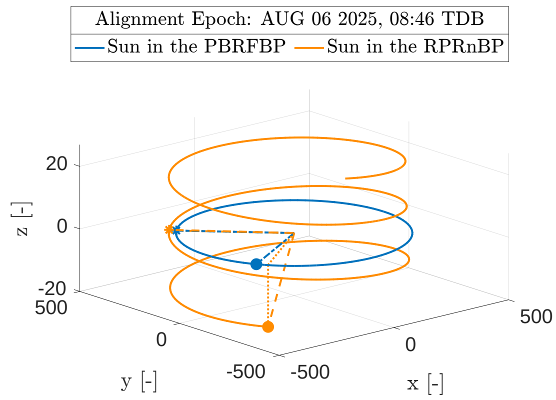

Figure 4: Trajectories of the Sun in the two models during the time of a transfer; departure and alignment times are spotted with a circle and an asterisk, respectively; coordinates are adimensional according to the definition of each model

3.2.2. Multiple-Shooting Strategy

The initial seed orbit is then refined with a direct transcription and multiple-shooting technique, which maps the current optimal control problem to a NLP problem [1]. The trajectory is sampled at a finite number of points evenly spaced in time, called nodes, where the state vector (position and velocity) is taken as an unknown variable. The overall NLP variables vector comprises the collection of the states at each node, the initial and final states of the trajectory, and the time of each maneuver. The problem to be solved is to minimize the cost of the trajectory J, while preserving the continuity of the states at each node and constraining the initial and final states to the parking orbits around the Earth and the Moon. Moreover, when a midcourse impulse is present, an additional constraint makes the velocities before and after the maneuver aligned, to prevent any singularity of the cost function. Again, the problem is solved by means of Matlab®'s suite `fmincon`, provided with the analytical expressions for the derivatives of the objective function and the constraints with respect to the NLP variables.

3.3. Results

Figure 5 shows a low-energy transfer targeting a polar orbit around the Moon without any prescribed right ascension of the ascending node. The spacecraft path is made of an initial lunar flyby, a corrective maneuver, and a final ballistic capture at the Moon. The continuation algorithm preserves both the geometrical features of the trajectory as well as its time of flight and cost. Furthermore, in this case, the cost of the final solution is lower than the one of the initial transfer in the four-body model. The algorithm succeeds also in continuing other families of transfers. For instance, exterior weak stability boundary transfers with a time of flight of up to 90 days, and interior transfers, both direct and with an Earth gravity assist.

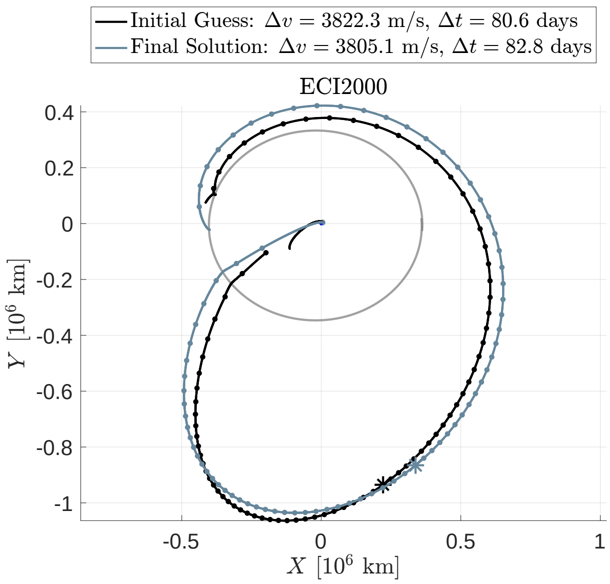

Figure 5: Low-energy transfer successfully adapted to the RPRNBP

4. Conclusions

With the beginning of a new phase of lunar exploration, finding optimal trajectories in high-fidelity astrodynamical models is a crucial step for maximizing the scientific and exploratory potential of future missions.

In the first part of the thesis, Primer Vector theory is applied to improve time-fixed, two-impulse, Earth–Moon transfers. For the first time, a set of first-order optimality necessary conditions is derived in a four-body model. These are employed to design more than 10 000 three-impulse trajectories, which result in lower fuel consumption compared to the original two-impulse transfers. Even if the incorporation of a third impulse typically produces a little variation in the total cost of the transfer, some specific trajectories with an initial lunar flyby are improved by tens of m/s.

The second part of the thesis describes a procedure to adapt a multi-impulsive transfer targeting any lunar circular orbit, from a four-body model to an ephemerides-based model of the complete solar system. Obtaining a feasible final trajectory heavily relies on the quality of the initial guess. For this reason, an algorithm based on the Sun's positions in the two models is developed to find a proper epoch when to plant the initial seed orbit in the real solar system. Then, a direct multiple-burn, multiple-shooting method is applied to refine the orbit until a desired transfer is achieved. The methodology described successfully generates various families of Earth–Moon transfers starting from solutions obtained with a simplified, but still meaningful, astrodynamical model.

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