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Executive Summary of the Thesis

Numerical Study of Electron Beam–Plasma Matching for AWAKE Run 2c

Laurea Magistrale in Nuclear Engineering - Ingegneria Nucleare

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1. Introduction and motivation

Plasma wakefield acceleration is investigated as a possible route towards accelerating gradients much larger than those achievable in conventional radiofrequency structures, offering a path towards more compact linear accelerators [1]. In the AWAKE experiment at CERN, high-energy proton bunches from the Super Proton Synchrotron are used to drive plasma wakefields over metre-scale plasmas, with the long-term goal of accelerating externally injected electron bunches to very high energies [2, 3].

As the AWAKE collaboration moves towards Run 2c, the next experimental campaign, the goal is to accelerate initially 150 MeV electron bunches through a 10 m-long plasma column while preserving their transverse quality [4, 5]. A key prerequisite for achieving this goal is the injection of the electron bunch into the plasma column under the so-called matching condition, in which the natural tendency of the beam to diverge is balanced by the transverse focusing force of the plasma wake.

The objective of this work is to investigate, through numerical simulations, whether beam–plasma matching can be studied in a simplified electron-only configuration, without including the SPS proton driver. This approach is motivated by the fact that the high-density electron witness bunch can itself generate a blowout cavity, corresponding to a nonlinear plasma regime that provides favourable conditions for acceleration. The final goal is to identify experimentally accessible post-plasma observables that can indicate whether the injected electron beam was matched to the plasma focusing channel.

2. Plasma wakefield and blowout regime

Plasma wakefield acceleration relies on the collective response of a plasma to the passage of a relativistic driver [6, 7]. In this work, the plasma is treated as cold and collisionless on the time scale of the beam–plasma interaction. This means that thermal motion and binary collisions are neglected, while the ions are assumed to remain approximately immobile because of their much larger inertia, compared with the plasma electrons. Therefore, the fast plasma response is mainly governed by the motion of the electron population. When an electron bunch propagates through the plasma at a velocity close to the speed of light, its electromagnetic field perturbs the initially neutral medium, displac-

ing plasma electrons. After being displaced, the plasma electrons experience the restoring force produced by the positive ion background. This gives rise to collective electron plasma oscillations, characterized by the electron plasma frequency

$$\omega_p = \left(\frac{n_p e^2}{m_e \epsilon_0}\right)^{1/2}, \tag{1}$$

where n_p is the background plasma density. The corresponding plasma wavelength is

$$\lambda_p = \frac{2\pi c}{\omega_p}. \tag{2}$$

For the AWAKE Run 2c plasma density considered in this work, $n_p = 7 \times 10^{14}$ cm^{-3}, $\omega_p \simeq 1.49 \times 10^{12}$ rad s^{-1} and $\lambda_p \simeq 1.26$ mm.
When the bunch density n_b becomes comparable to or larger than n_p, the response becomes nonlinear: the bunch can expel plasma electrons from its path, leaving behind an ion cavity, known as the blowout regime.

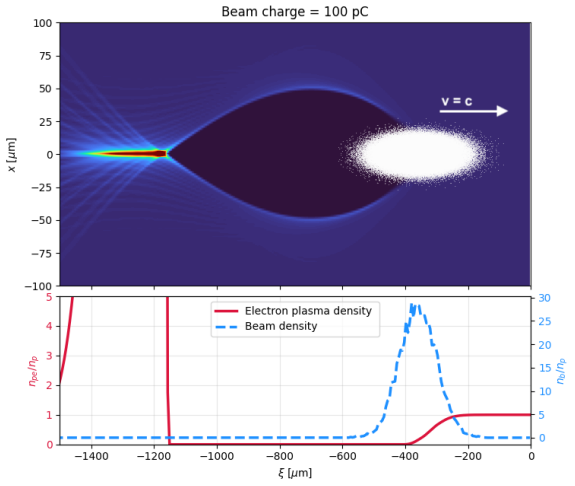


Figure 1: Nonlinear plasma response driven by a high-charge relativistic electron bunch. The bunch, shown in white, propagates to the right and expels plasma electrons, forming an ion cavity. The lineout shows the normalized plasma electron density n_{pe}/n_p and beam density n_b/n_p.

The blowout regime is favourable for beam-quality preservation because the ion cavity provides an approximately transverse-independent accelerating field and a linear focusing force [6, 8].

3. Envelope equation and matching condition

The transverse evolution of a relativistic electron beam is determined by the competition between its natural divergence and the external focusing force provided by the plasma. It is described by the envelope equation, [6, 9], which can be written as

$$\sigma_r'' + K^2 \sigma_r = \frac{\epsilon_n^2}{\gamma^2 \sigma_r^3}, \tag{3}$$

where σ_r is the r.m.s. beam radius, ϵ_n is the normalized emittance and γ is the relativistic Lorentz factor. Here, K^2 represents the focusing strength of the ion cavity, while $\frac{\epsilon_n^2}{\gamma^2 \sigma_r^3}$ represents the emittance divergence term.
The matched condition corresponds to a stationary solution of the envelope equation. In fact, by properly choosing the beam and plasma parameters, one can obtain a stationary envelope along the plasma column. Consequently, the matched transverse beam size is obtained as

$$\sigma_{r,0,\text{matched}} = \left(\frac{\epsilon_n^2}{\gamma^2 K^2}\right)^{1/4}. \tag{4}$$

For the AWAKE Run 2c parameters used in this work,

Parameter	Value
Initial beam energy, E_{in}	150 MeV
Normalized emittance, ϵ_n	2 mm mrad
Plasma density, n_p	7×10^{14} cm^{-3}

Table 1: Main physical parameters of AWAKE Run 2c [4, 5] used to estimate the matched beam size.

the matched r.m.s. transverse size is

$$\sigma_{r,0,\text{matched}} = 5.76\ \mu\text{m}. \tag{5}$$

Matching requires not only the correct beam size, but also the correct envelope slope at the plasma entrance, since Eq. (3) is a second-order differential equation. Therefore, the complete matching condition is

$$\sigma_{r,0} = \sigma_{r,0,\text{matched}}, \qquad \left.\frac{d\sigma_r}{dz}\right|_{z=z_0} = 0. \tag{6}$$

The second condition means that the beam waist must be located at the plasma entrance. If either the transverse size differs from the matched

value or the waist is displaced with respect to the plasma entrance, the beam is mismatched and its envelope oscillates during propagation through the ion cavity. These envelope oscillations can increase transverse losses and degrade the stability of the downstream beam properties.

4. Numerical method and simulation setup

The nonlinear nature of the blowout regime prevents the use of linear plasma theory for accurate quantitative predictions. Therefore, the beam–plasma interaction must be studied with numerical simulations. In this work, the simulations were performed with HiPACE++, a quasi-static particle-in-cell code designed for beam-driven plasma wakefield acceleration [10].
PIC codes describe the plasma and beam dynamics by representing the particle distributions with macro-particles, while the electromagnetic fields are computed on a spatial grid [11]. HiPACE++ further reduces the computational cost by exploiting the quasi-static approximation, which assumes a separation between the fast plasma response and the slower beam evolution [10, 12]:

$$\tau_p \ll \tau_b, \qquad \tau_p \sim \omega_p^{-1}, \qquad \tau_b \sim \omega_\beta^{-1}. \tag{7}$$

Under this approximation, the wakefields are computed slice by slice in the co-moving coordinate $\xi = z - ct$, while the beam is advanced over larger propagation steps. This makes metre-scale plasma-acceleration simulations computationally feasible while retaining the main kinetic and nonlinear features of the beam–plasma interaction.

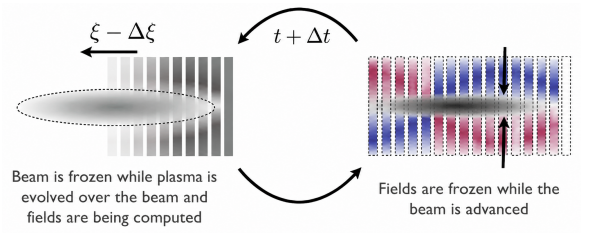


Figure 2: Staggered numerical scheme used in HiPACE++ for the separated treatment of beam and plasma macro-particles. Figure adapted from [12].

4.1. Simulation setup

The numerical study was based on two independent scans of the input beam parameters P_I, aimed at varying separately the initial transverse r.m.s. size and the envelope slope. The first scan varied the beam size at the plasma entrance, while keeping the waist at the entrance, $z_{\text{waist}} = 0$:

$$\sigma_{r,0} = \{2, 4, 5, 5.76, 7, 10, 15, 20\}\ \mu\text{m}. \tag{8}$$

The second scan kept the beam waist size equal to the matched value and varied the waist position with respect to the plasma entrance:

$$z_{\text{waist}} = \{-16.3, -11.75, -6.9, -3.4, 0, 3.4, 6.9, 11.75, 16.3\}\ \text{mm}. \tag{9}$$

The matched reference case is defined by $\sigma_{r,0} = 5.76\ \mu$m and $z_{\text{waist}} = 0$. For each simulation, output observables P_O were collected after propagation through the plasma column and analysed as functions of the initial beam parameters. The simulation domain covers 1 mm × 1 mm × 3 mm in (x, y, ξ), with a base grid of 511 × 511 × 1024 cells. A refined transverse region of 0.2 mm × 0.2 mm was added around the propagation axis, with 1023 × 1023 cells in the transverse plane, to better resolve the bunch region and the strongest plasma response. The time step was chosen adaptively by the code, with 20 time steps per betatron period.
The witness bunch was initialized with a Gaussian distribution, fixed macroparticle weights, and transverse symmetrization enabled. Approximately 10^7 macroparticles were used to represent the electron bunch.

5. Results and matching diagnostics

The simulations first confirmed that the electron bunch alone is able to drive a plasma blowout cavity. The portion of the beam located inside the ion cavity experiences the expected linear transverse focusing force (see Fig. 3)

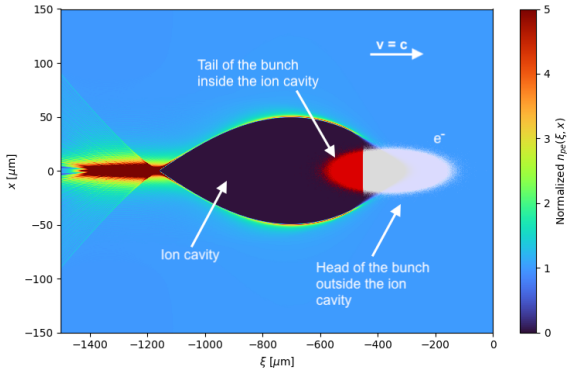


Figure 3: Matched electron bunch driving a blowout regime inside the plasma column at $z = 0$. The head of the bunch, shown in white, propagates outside of the ion cavity. The tail of the bunch, shown in red, lies inside the cavity. The ion cavity is shown in dark blue, corresponding to a normalized electron plasma density $n_{pe}(\xi, x) = 0$. The beam propagates to the right.

Several output parameters P_O were considered at the plasma exit, including the remaining beam charge, the transverse r.m.s. size, and energy-spectrum features. Among them, the remaining charge Q_{exit} and selected features of the beam energy spectrum, namely the minimum energy of the beam spectrum, evaluated as the 1% percentile of the energy distribution, $E_{\text{min,1.0\%}}$, and the charge contained in the interval 25–35 MeV, $Q_{25\text{–}35\,\text{MeV}}$, were found to provide the clearest signatures of matching.

Scan of $P_I = \sigma_{r,0}$. Figure 4 shows the charge remaining in the simulation domain $P_O = Q_{\text{exit}}$ after 10 m of plasma propagation as a function of the initial transverse beam size. The curve exhibits a maximum at $\sigma_{r,0} = 5.76$ μm, which corresponds to the matched beam size. This behaviour indicates that the matched beam undergoes the smallest transverse particle loss during propagation. Conversely, when the beam is injected with a transverse size smaller or larger than the matched value, envelope oscillations are excited and a larger fraction of particles reaches large transverse amplitudes, reducing the charge transported to the plasma exit. Therefore, Q_{exit} provides a direct matching signature in the $\sigma_{r,0}$ scan.

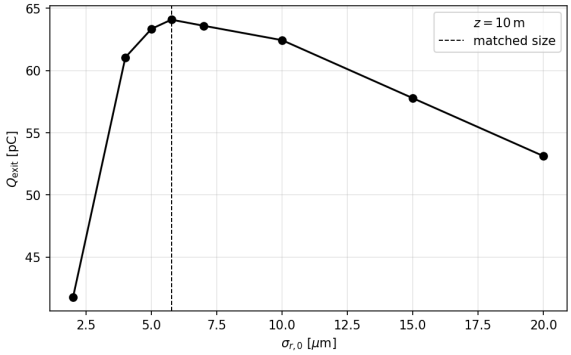


Figure 4: Electron beam charge (Q_{exit}) remaining in the simulation domain (dimensions 1000 × 1000 μm) after propagation in 10 m of plasma, as a function of the initial beam size $\sigma_{r,0}$.

The energy spectrum also contains information on the matching condition. The choice of the spectral observables $P_O = \{E_{\text{min,1.0\%}}; Q_{25\text{–}35\,\text{MeV}}\}$ considered here was motivated by a preliminary comparison between the matched case and the two extreme unmatched cases of the scanned interval. As shown in Figure 5, the energy spectra after propagation exhibit differences in both the low-energy tail and the charge accumulated in the energy range 25–35 MeV, motivating a finer scan of the $\sigma_{r,0}$ range.

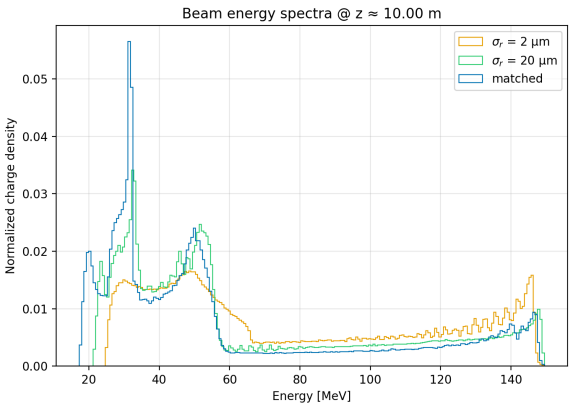


Figure 5: Beam energy spectrum at $z = 10$ m for three values of the initial transverse beam size $\sigma_{r,0}$: the matched case, $\sigma_{r,0,\text{matched}} = 5.76$ μm (blue), and two unmatched cases, $\sigma_{r,0} = 2$ μm (orange) and $\sigma_{r,0} = 20$ μm (green).

Their dependence on $\sigma_{r,0}$ is reported in Figure 6. The matched case is associated with a minimum of $E_{\text{min,1.0\%}}$ and with a maximum of $Q_{25\text{–}35\,\text{MeV}}$. This indicates that, for the matched configuration, a larger fraction of the bunch

reaches the low-energy region of the spectrum. These trends suggest that energy-resolved measurements could provide additional information on the matching condition, although their experimental use is expected to be limited by the available diagnostic.

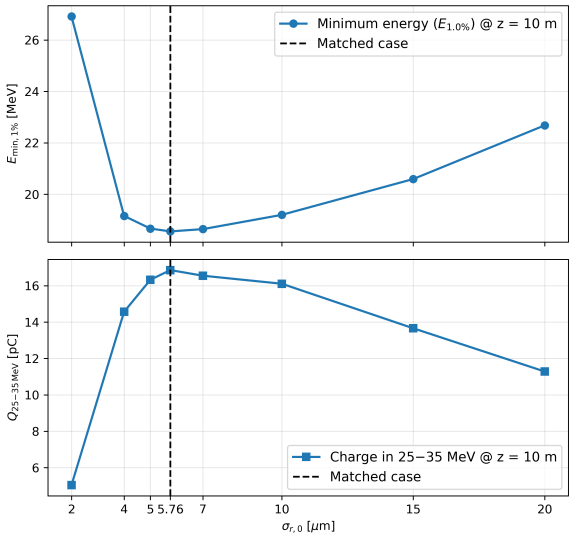


Figure 6: Top: minimum beam energy, evaluated as the 1% percentile of the electron energy distribution, $E_{\mathrm{min},1.0\%}$. Bottom: charge contained in the energy range 25–35 MeV, $Q_{25\text{–}35\,\mathrm{MeV}}$, at $z = 10$ m, as a function of the initial beam size $\sigma_{r,0}$. Vertical black dashed lines indicate the matched initial beam size.

Scan of $P_I = z_{\mathrm{waist}}$. The same observables P_O were then evaluated by varying the waist position z_{waist}, while keeping the beam size at the waist equal to the matched value. Figure 7 shows that the remaining charge again reaches its maximum for $z_{\mathrm{waist}} = 0$, corresponding to a beam waist located at the plasma entrance. When the waist is displaced upstream or downstream, the beam enters the plasma with a non-zero envelope slope. This produces transverse mismatch, enhances envelope oscillations during propagation, and increases particle losses. The maximum of Q_{exit} at $z_{\mathrm{waist}} = 0$ therefore confirms that the remaining charge is also sensitive to the longitudinal matching condition.

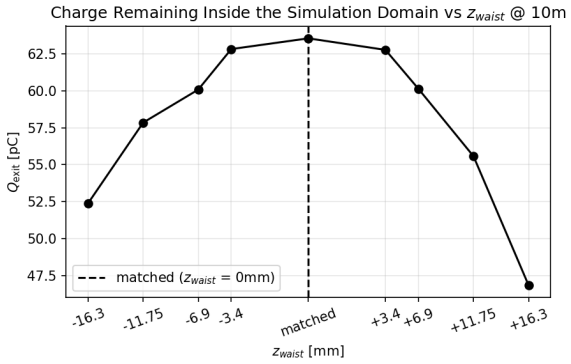


Figure 7: Electron beam charge remaining in the simulation domain after propagation in 10 m of plasma (Q_{exit}), as a function of the waist position z_{waist}.

Figure 8 shows the corresponding energy-spectrum observables as a function of z_{waist}. As in the $\sigma_{r,0}$ scan, the matched configuration is associated with a minimum of $E_{\mathrm{min},1.0\%}$ and a maximum of $Q_{25\text{–}35\,\mathrm{MeV}}$. The dependence on z_{waist} is approximately centred around the matched case, although it is not expected to be perfectly symmetric. Indeed, opposite values of z_{waist} can correspond to the same transverse beam size at the plasma entrance, but with opposite envelope slopes: one beam enters the plasma while still converging, whereas the other enters after passing its waist and is already diverging. These different transverse phase-space conditions can lead to residual asymmetries in the final observables.

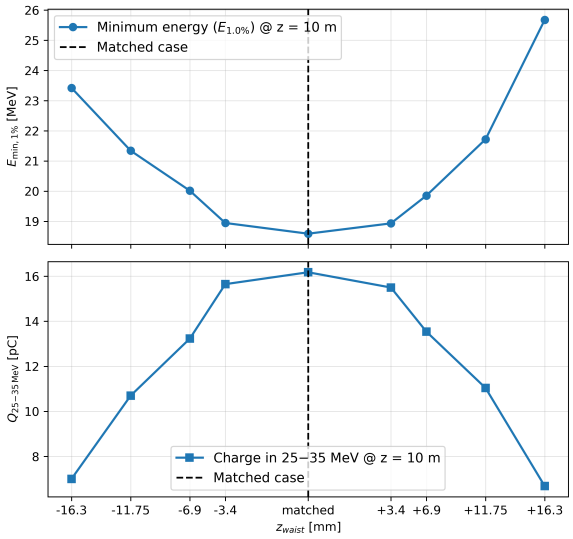


Figure 8: Top: minimum beam energy evaluated as the 1 % percentile of the electron energy distribution ($E_{min,1.0\%}$); Bottom: charge contained in the energy range of 25–35 MeV ($Q_{25–35\,\mathrm{MeV}}$), at $z = 10$ m, as a function of the waist position z_{waist}.

5.1. Experimental relevance

From an experimental point of view, the two scans have different practical implications. Varying the initial beam size $\sigma_{r,0}$ would require changing the optics of the 150 MeV electron beamline, making this scan relatively slow and operationally demanding. Conversely, varying z_{waist} is more practical, since it can be achieved by changing the timing of the ionizing laser pulse. This shifts the plasma entrance with respect to the pre-existing electron beam waist without requiring a full re-optimization of the beamline optics.

Among the considered possible observables P_O, the remaining charge Q_{exit} appears to be the most promising experimental diagnostic. The simulated charge variations are significantly larger than the expected resolution of the available charge diagnostics. In particular, for the 150 MeV line, an Integrated Current Transformer (ICT) can measure the bunch charge with an expected RMS noise of about 0.04 pC over a dynamic range compatible with the simulated transmitted charge. Therefore, the variations predicted by the simulations should be experimentally resolvable.

Energy-spectrum observables may provide complementary information, but they are less suitable as primary diagnostics. After the plasma, the beam propagates in vacuum before reaching the spectrometer, and this transport can broaden or distort the spectral features predicted at the plasma exit. In addition, the required spectrometer acceptance and resolution still need to be verified.

For these reasons, the waist-position scan shown in Fig. 7 is identified as the most realistic experimental strategy, with the downstream charge measurement proposed as the primary matching diagnostic.

6. Conclusions and future developments

This work shows that electron beam–plasma matching for AWAKE Run 2c can be investigated in a simplified electron-only configuration. The high-density 150 MeV electron bunch is able to generate a blowout cavity by itself, providing the linear transverse focusing force required for matching studies.

The most important result is that the charge remaining after propagation through the plasma column, Q_{exit}, provides a clear signature of the matched condition. In both scans considered in this work, the matched configuration corresponds to a maximum of the transmitted charge. Therefore, a charge-based diagnostic is proposed as the most robust and experimentally practical method for identifying the matching condition.

The most realistic experimental implementation would consist of scanning the waist position with respect to the plasma entrance by varying the timing of the ionizing laser pulse, and then identifying the configuration that maximizes the charge measured downstream.

A key future development concerns the connection between the charge remaining inside the simulation domain and the charge that would be experimentally measured by a downstream ICT. In the present analysis, particles leaving the transverse simulation domain before the plasma exit are removed from the simulation. A fully consistent comparison with experiment would require saving their phase-space coordinates at the moment of escape and propagating both the surviving and escaped particles through the downstream beamline. This would allow the effective ICT charge to be reconstructed and would provide a more direct validation of the

proposed charge-based matching diagnostic.

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