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EXECUTIVE SUMMARY OF THE THESIS

# Control Tuning of Autonomous Vehicles Considering Performance-Comfort Trade-offs

LAUREA MAGISTRALE IN AUTOMATION AND CONTROL ENGINEERING - INGEGNERIA DELL'AUTOMAZIONE

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## 1. Introduction

The progress of autonomous vehicles (AVs) has attracted a growing amount of attention in studies and industries recently. A number of successful path-following and planning strategies have been introduced and implemented in the last few decades. However, the comfort of passengers while driving is less discussed compared to other aspects of AVs. The comfort of passengers is a key component of increasing customer acceptance of AVs. However, The comfort definition is subjective and Several types of features can affect it: dynamic factors like acceleration, vibration, and shock. Ambient factors, such as thermal comfort, air quality, noise, pressure gradients, and spatial elements like the ergonomics of the passenger's position. On the basis of them, different measurement indices for comfort are provided. However, in this thesis, comfort has been considered when it comes to dynamic characteristics.

Studies have shown that acceleration and deceleration, as well as their temporal derivatives, jerk, significantly affect passenger comfort. Therefore, one of the ways to provide comfort for passengers is to keep these parameters within particular thresholds. The self-driving car can reach this goal by applying adequate driver

model.

The vehicle's driver model has two tasks. Firstly, the driver is responsible for controlling speed and braking (longitudinal control). As well, the driver should follow the desired path by adjusting the steering of the vehicle (lateral control). Therefore, steering and torque are the inputs of the vehicle in this case.

A vehicle's lateral and longitudinal controllers are then designed considering passenger comfort. In this thesis, during path following and planning, three aspects were taken into account. Firstly, passengers' comfort as acceleration and jerk minimization. Additionally, time optimization is needed to make the autonomous vehicle as fast as possible. Moreover, the vehicle has been kept within the limits of realistic real-world lanes during path following. To achieve all these goals together, a multi-objective optimization has been used. Multi-objective optimization involves tuning several key parameters of longitudinal and lateral controllers.

Lastly, the tuned closed-loop driver model is evaluated on real-world paths.

## List of Symbols

Variable	Description	unit
$\delta$	Steering angle	Rad
$d_{lookahead}$	Look ahead distance of the vehicle	m
$V_x$	Longitudinal velocity	$Kmh^{-1}$
$V_y$	Lateral velocity	$Kmh^{-1}$
$a_x$	Longitudinal acceleration	$ms^{-2}$
$a_y$	Lateral acceleration	$ms^{-2}$
$X$	Longitudinal positions of vehicle	m
$Y$	Lateral positions of vehicle	m
$\psi$	Yaw angle of vehicle	Rad
$\beta$	side slip angle of vehicle	Rad
$r$	yaw rate of vehicle	$Rads^{-1}$
$F_y^f$	Lateral force of front tire	N
$F_y^r$	Lateral force of rear tire	N
$\mu$	tire-road friction coefficient	
$F_z^r$	rear normal tire load	N
$F_z^f$	front normal tire load	N
$T_d$	traction Torque	Nm
$T_{driver}$	Driver reaction time	s
$\Delta\psi$	yaw angle error	Rad
$e_y$	Lateral error	m
$\kappa$	curvature of path	m
$\mu_y$	lateral tyre-road friction	
$j_x$	Longitudinal jerk	$ms^{-3}$
$j_y$	Lateral jerk	$ms^{-3}$

## 2. Vehicle Model and Reference Path

### 2.1. Vehicle Model

A nonlinear bicycle model is used to model the vehicle. Considering the forces acting on the vehicle as well as some kinematic relations in the vehicle. The model includes six states for the vehicle considering both lateral and longitudinal dynamics. [1].

$z(t) = [X(t), Y(t), V_x(t), \beta(t), \psi(t), r(t)]^T$  are the states of the model, and  $U(t) = [T_d(t), \delta(t)]$  is the input of the vehicle model. This nonlinear model is used in the multi-objective optimization part (4), while the above model is linearized in order to design controllers.

### 2.2. Reference Path model

Paths are predefined and the environment is static in this study. Therefore, given the path, some characteristics should be exploited. Paths can be defined in many ways, such as straight line segments, parameterized curves, and waypoints. This study uses GPS data from roads.

Therefore, waypoints are the preferred option to define the path.  $P_i = [x_i, y_i]$  is the vector of the points forming the path. As a consequence, moving along the vector means moving along the path. Then, three characteristics of the path: curvature, tangential angle, and total distance are considered. These characteristics are used for both lateral and longitudinal control of a vehicle.

## 3. Control Design

On straight lanes, the driver tries to accelerate the vehicle's speed. Before reaching the corner, the driver reduces speed in order to ensure passengers' safety and comfort. This is accomplished by a longitudinal speed controller.

Moreover, the driver changes the steering wheel of the vehicle to match the heading angle of the road and vehicle. As a result, the vehicle does not deviate much from the center line.

### 3.1. Lateral Control

The lateral controller is designed with cascade loops. The cascade controller consists of two loops: an inner loop and an outer loop. In general, cascade control is used to make sure that disturbances are quickly rejected before they propagate into other parts of the plant and may cause problems. In the inner loop, the purpose is to control the yaw rate, while in the outer loop, the purpose is to ensure that the desired yaw rate is generated as a setpoint for the inner loop. The vehicle position error and heading angle error are inputs to the outer control.

The path following model is designed in the Serret-Frenet frame:

$$\dot{e}_y = V_x \Delta\psi + d_{lookahead} r + V_y \quad (1a)$$

$$\Delta\dot{\psi} = r - V_x \kappa \quad (1b)$$

Where look-ahead distance is:

$$d_{lookahead} = V_x T_{driver} \quad (2)$$

$\Delta\psi$  Is the heading error, which is determined by the difference between the heading angle of the vehicle when viewed from a look-ahead distance and the tangent angle of the path when viewed from a look-ahead distance.  $e_y$  is the lateral position error which is the lateral difference between the position of the vehicle and the path at look ahead distance (see figure 1).

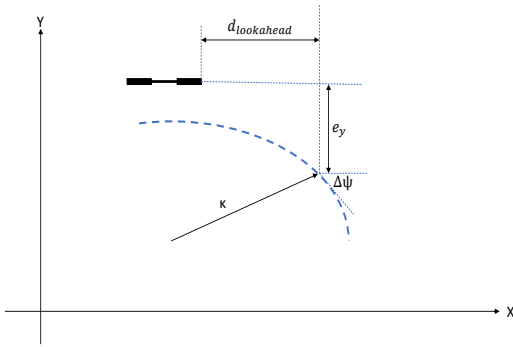


Figure 1: Path-following model

The equations (1) is then transformed into state space form. Where  $X = [e_y \quad \Delta\psi]^T$  is the state of the system,  $u = r$  yaw rate is the control input and the disturbance is  $w = [v_y \quad -v_x\kappa(s)]^T$ .

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu(t) + w \\ y(t) &= CX(t) \end{aligned} \quad (3)$$

A state feedback controller is designed for this system. It should be noted that a time varying parameter in the system model is longitudinal velocity, which varies between a minimum and a maximum  $[V_{xmin}, V_{xmax}]$ . Therefore,  $V_x$  can be written as:

$$V_x = h_1 V_{xmin} + h_2 V_{xmax} \quad (4)$$

Where

$$h_1(t) = \frac{V_x(t) - V_{xmin}}{V_{xmax} - V_{xmin}}, \quad h_2(t) = \frac{V_{xmax} - V_x(t)}{V_{xmax} - V_{xmin}} \quad (5)$$

As a result, the state feedback controller gain will be modified:

$$K = \sum_{i=1}^{i=2} h_i(t) K_i \quad (6)$$

Then, the desired yaw rate is as following:

$$r_d = \sum_{i=1}^2 h_i(t) K_i X \quad (7)$$

State feedback controller is designed using Linear Quadratic Regulator (LQR). Q and R are state and input weighting matrices respectively. If  $(A, C)$  in the system (3) is observable, then  $Q = C'C$  guarantees the stability of the system. Following that, R will be optimized in the section (4).

Inner loop setpoint is desired yaw rate, which is defined in (7). As a result, the inner loop controls the yaw rate of the vehicle. The steering angle of the vehicle is the output of this controller. Proportional-Integral (PI) controllers are used for this purpose.

It should be taken into consideration that according to rule of thumb, the inner loop should be at least 3-10 times faster than the outer loop. Hence, the cross-over frequency of inner loop should be at least 10 times higher than outer loop.

Here, the inner loop is chosen to be around 100 times faster than the outer loop in order to reject disturbances quickly. Therefore, the PI controller is designed with a cross-over frequency of 200.

### 3.2. Longitudinal Control

The longitudinal controller of the vehicle is composed of two parts. A speed profile design is first established and a cruise control system is used to follow the reference speed.

With a maximum known lateral acceleration, the speed based on the curvature of the path at each waypoint on the path ( $P_i$ ) can be defined as following:

$$V_{x,curv,i} = \sqrt{\frac{a_{y,max}\mu_y}{|\kappa_i|C}} \quad (8)$$

Where  $\kappa_i$  is the curvature of the path at position  $P_i$ , then  $a_{y,max}$  is the maximum lateral acceleration of vehicle, and  $\mu_y$  is the lateral tyre-road friction.  $a_{y,max}$  can be defined in different ways, however it is chosen to be  $3.70 m/s^2$  based on Battelle study.  $\mu_y$  is considered as 0.85 (dry asphalt). In this way the proper longitudinal velocity for each position based on its curvature is defined ( $V_{x,curv,i}$ ). The comfort parameter C is added to the reference speed equation (8). Using the proper value of C will reduce the reference speed and provide passengers with more comfort. C is one of optimization variables which will be discussed in section (4). On the other hand, it should be noted that every road has a maximum speed that is mandatory by law. Thus, the vehicle's reference speed cannot be greater than the law maximum speed. Finally, the reference speed is defined as follows:

$$V_{ref,i} = \max(V_{maximum}, V_{x,curv,i}) \quad (9)$$

Where  $V_{ref,i}$  refers to the reference speed of the  $i$ -th waypoint on the path.

This study uses the Butterworth low pass filter to smooth out the speed profile. Butterworth filters' cut-off frequencies  $f_c$  affect the final reference speed profile.  $f_c$  is one of the optimization variables that will be optimized in (4).

One of the most widely used control applications in autonomous vehicles is cruise control. Using torque commands, cruise control maintains the reference speed, as well as accelerates or decelerates to a new reference speed. Cruise control is designed based on Proportional-Integral (PI) controllers. First, the longitudinal dynamic of the system is linearized around  $40 \text{ km/h}$  in order to design the PI controller. The proportional and integral gains are designed in the frequency domain. They designed in a way to have a steady state error of less than 1% and a rise time in the step response of around 5 seconds, which is applicable to real-world cars.

#### 4. Multi-Objective Optimization

This study aims to design a trajectory and path planner that follows the planned path with the smallest possible error while prioritizing passenger comfort and travel time. In order to achieve the goal mentioned above, an optimization algorithm should be applied to the closed loop control system of a vehicle to tune some parameters of it.

This problem has three objectives, a comfort cost function, a time cost function, and a lateral error cost function. These cost functions are conflicting, therefore a multi-objective analysis should be performed on them.

According to ISO 2631-1:1997 [2], R.M.S acceleration values can reflect the discomfort that humans face. In [3] Different objectives are tested to see which one provides a smoother speed profile. The third-order temporal derivative of the position is called jerk. Finally, the results show that the metric which is based on the square of jerk minimization results in a smoother speed profile that means more passengers' comfort. Thus, the comfort cost function can be defined

as follows:

$$J_{comfort} = \sum_{i=1}^N j_x(i)^2 + j_y(i)^2 + a_x(i)^2 + a_y(i)^2 \quad (10)$$

Where  $N$  is the number of samples (in our case, waypoints on the path).

The other cost function is the vehicle's lateral error relative to the path. A lower lateral error results in more accurate path following. In this way, you will keep your vehicle as close as possible to the center line of the road.

$$e_y(t) = y_{ref}(t) - y(t) \quad (11)$$

$$J_{lateral} = \sum_{i=1}^N e_y(i)^2$$

Where  $e_y$  is the lateral error used in equation (1a) and  $N$  is the number of waypoints on the pre-defined path.

The other term refers to minimizing travel time. Aiming to maximize speed as much as possible will minimize travel time. Therefore,  $J_{speed}$  will be defined as:

$$J_{speed} = - \sum_{i=1}^N V_x(i)^2 \quad (12)$$

Three parameters are discussed as optimization variables during control design.  $f_c$ , the cut-off frequency in Butterworth filter, and  $C$  the comfort parameter in speed profile design (8), and  $R$ , the weight on input matrix in lateral control. The defined objective function is most affected by these three parameters.

As a result, the final objective function can be written as follows:

$$\begin{aligned} \min_X \quad & J = w_{speed} J_{speed} + w_{comfort} J_{comfort} \\ & + w_{lateral} J_{lateral} \\ \text{s.t.} \quad & \zeta(i+1) = f_\zeta(\zeta(i), u(i), \theta, X) \quad i = 1, \dots, N \\ & y(i) = g_\zeta(\zeta(i), u(i), \theta, X) \quad i = 1, \dots, N \\ & \underline{j_x} \leq j_x(i) \leq \overline{j_x} \quad i = 1, \dots, N \\ & \underline{j_y} \leq j_y(i) \leq \overline{j_y} \quad i = 1, \dots, N \\ & \underline{a_x} \leq a_x(i) \leq \overline{a_x} \quad i = 1, \dots, N \\ & \underline{a_y} \leq a_y(i) \leq \overline{a_y} \quad i = 1, \dots, N \\ & \underline{f_c} \leq f_c \leq \overline{f_c} \\ & \underline{C} \leq C \leq \overline{C} \\ & \underline{R} \leq R \leq \overline{R} \end{aligned}$$

Where  $N$  is the number of waypoints and  $w_{speed}$ ,  $w_{comfort}$ ,  $w_{lateral}$  are the weights of cost functions.  $f_{\zeta}$  and  $g_{\zeta}$  are the closed-loop non-linear driver model.  $X$  is the vector of optimization variables  $X = [f_c, C, R]^T$ . The minimum and maximum value of constraints are as follows:

minimum	variable	maximum
-0.9 $m/s^3$	$\dot{j}_x, \dot{j}_y$	0.9 $m/s^3$
-2 $m/s^2$	$a_x, a_y$	2 $m/s^2$
4	$f_c$	12
10	$C$	30
100	$R$	200

Table 1: Minimum and maximum value of constraints.

Taking into consideration the highly nonlinear nature of the vehicle as well as the control system design, it is a challenging to solve the optimization problem. Gridding approach is a simple solution to this problem. The cost function is thus minimized over the feasible set of optimization variables. Within the range of the optimization variables, the sets of variables are further divided into discrete values. The process can be summarized as follows:

#### Algorithm 1 Gridding Optimization

- 1: Consider feasible range of  $X \in (\underline{X}, \bar{X})$
- 2: Division of the selected range into discrete values  $(\underline{X}, X_2, \dots, X_i, \dots, \bar{X})$
- 3: Arrangement of different combination of variables (D)
- 4: Calculate the cost function using different arrangements ( $J(X)$ )
- 5: Select the minimum cost function value and corresponding optimization variable values

In this study, step 1) The feasible range of the optimization variables are defined in table (1). Step 2) These ranges are discretized into four values as follows:

$$\begin{aligned}
 R &= [100, 125, 150, 200] \\
 f_c &= [4, 6, 10, 12] \\
 C &= [10, 15, 25, 30]
 \end{aligned} \tag{14}$$

Moreover, the combination of these vectors results in the arrangement vector (D) with a size of 64. Step 3) each arrangement is then used

to calculate the cost function. Finally, the minimum cost function is selected.

When dealing with multi-objective functions, Pareto front is one method.

Different tests are conducted to build the Pareto front. In each test, the aim is to find the minimum objective function. In other words, for each combination of weights, optimization of control parameters is repeated and the Pareto front is built based on them.

	$w_{speed}$	$w_{comfort}$	$w_{lateral}$
<b>Test 1</b>	1	20	0.2
<b>Test 2</b>	1	7	3
<b>Test 3</b>	1	5	5
<b>Test 4</b>	1	3	7
<b>Test 5</b>	1	1	10
<b>Test 6</b>	1	0.2	20

Table 2: Different tests to obtain Pareto front

An average lateral error during the path is compared with an average of longitudinal and lateral accelerations to form the Pareto front.

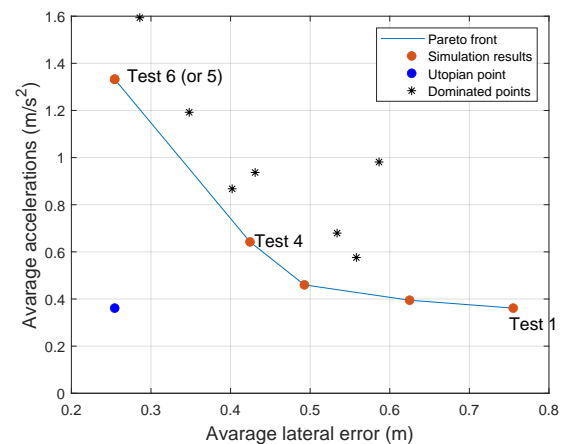


Figure 2: Pareto front with average lateral error during path-following and average accelerations.

It can be seen that the Pareto front is close to the Utopian point when Test 4 is carried out. Finally, The weight of objective functions for optimization problem are chosen  $w_{lateral} = 3, w_{comfort} = 7, w_{speed} = 3$  (Considering also other Pareto fronts that have not been mentioned).

Based on these weights, The optimal control parameters are  $f_c = 6$ ,  $R = 200$ ,  $C = 20$ .

## 5. Performance Evaluation

The designed controllers using the optimized parameters are applied to the following path in order to verify the results. The road is chosen from an area in northern Italy close to Aosta.

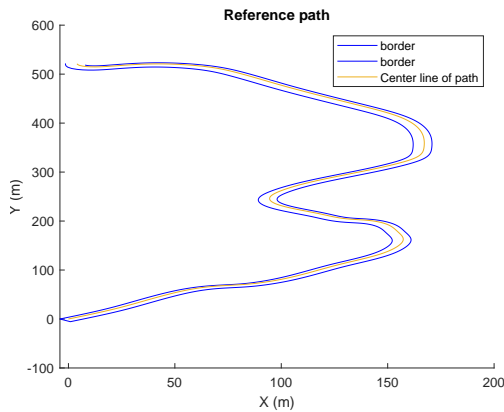


Figure 3: Road in the global coordinate. The starting point of the center line of the path is  $(0, 0)$ .

It has been observed that lateral and longitudinal jerks and accelerations remain within the threshold (table 1). Except for the initial part of the path, which is caused by the controllers' initialization.

While following the path, the speed varies between 20 and 60  $km/h$ , with an average speed of 24.5  $km/h$ . The maximum lateral error is 1.47  $m$ . However, the vehicle remains within the borders of the road. While the average lateral error is 0.36  $m$  during the entire path.

Additionally, the illness rating is 0.405, which means the passengers are comfortable during journey (illness ratings below 1 indicate comfort. Illness rating is an index to measure comfort based on R.M.S of acceleration).

## 6. Conclusions

In this study, longitudinal and lateral control for autonomous vehicles is designed to make trade-offs between passenger comfort and vehicle performance. Some key parameters of the controllers should be tuned to achieve our goal. However, the cost functions in the objective func-

tion are conflicting. Thus, a multi-objective analysis was carried out, followed by the development of a Pareto front. Finally, the closed loop system with tuned parameters was tested on real world paths.

## References

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