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SCUOLA DI INGEGNERIA INDUSTRIALE
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Executive Summary of the Thesis

Love Dynamics and The Transition to Parenthood: a Modeling Approach

Laurea Magistrale in Mathematical Engineering - Ingegneria Matematica

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Academic year: 2025-2026

1. Introduction

Parenthood represents one of the most significant transitions in adult life. It is often associated with joy and personal growth, but also with stress, fatigue, and relational strain. Empirical literature shows that this transition does not lead to a unique relational outcome [3]: while some couples experience a decline in relationship satisfaction, others maintain comparable levels of satisfaction or even strengthen their bond. One of the most influential factors shaping these heterogeneous outcomes is attachment style [4], which regulates how individuals perceive themselves and their partners, and how they manage emotional and physical intimacy [1]. Individuals differ in their capacity for closeness, emotional regulation, and responsiveness under stress. In this work, individuals are grouped into two categories based on attachment style: secure and insecure. Secure individuals are characterized by comfort with intimacy; insecure individuals, by contrast, may experience discomfort with closeness.

We propose an extension of the dynamical model introduced in [5], which describes the temporal evolution of romantic interaction through a system of two nonlinear ordinary differential equations. Each partner's emotional involvement evolves according to three fundamental mechanisms: oblivion (spontaneous decay of feelings), reaction to appeal (sensitivity to the partner's attractiveness), and reaction to love (response to the partner's emotional involvement, depending on attachment style). In addition, individuals may exhibit synergism, which amplifies the response to the partner's appeal. Depending on parameter values and personal characteristics, the model admits different qualitative regimes (stationary or cyclic) and couples may display robust dynamics, characterized by a single stable equilibrium, or fragile dynamics, characterized by two, or more, stable equilibria, so that long-term outcomes depend on initial conditions. Within this framework, we extend the original system by introducing two parameters representing the transition to parenthood: N, the total caregiving demands associated with the child, and ϵ describing how caregiving responsibilities are distributed between partners. These parameters modify the reaction to love mechanisms, reflecting the idea that caregiving competes with the emotional resources available for reciprocal involvement. The role of parenthood (ϵ, N) on the dynamic behaviour of the extended model is analyzed across different couple configurations (secure–secure, secure–insecure,...).

Moreover, caregiving demands are allowed to vary periodically in time, revealing more complex dynamical behaviors.

2. The Model

The model proposed in [5] describes the emotional interaction between two individuals through a system of two nonlinear ordinary differential equations. The state variables $x_1(t)$ and $x_2(t)$ represent, respectively, the emotional involvement of partner 1 toward partner 2 and vice versa. The temporal evolution of each variable is governed by:

$$\dot{x}_i = O_i + R_i^A + R_i^L(x_j),$$

where $i, j \in \{1, 2\}$ with $i \neq j$. The term O_i represents the oblivion mechanism and models the spontaneous decay of feelings in the absence of the partner. In particular:

$$O_i(x_i) = -\alpha_i x_i,$$

where $\alpha_i > 0$ denotes the individual rate of emotional fading. The term R_i^A captures the reaction to appeal and describes the sensitivity of individual i to the intrinsic attractiveness of partner j. It is modeled as a constant contribution proportional to the partner's appeal parameter A_j, weighted by the individual susceptibility γ_i:

$$R_i^A = \gamma_i A_j,$$

The reaction to love $R_i^L(x_j)$ describes how individual i responds to the partner's emotional involvement x_j and the functional form depends on the attachment style. For secure individuals, the response is assumed to be monotone and bounded, ensuring regulated emotional reactions:

$$R_i^L(x_j) = \frac{e^{x_j} - e^{-x_j}}{\frac{e^{x_j}}{R_i^+} - \frac{e^{-x_j}}{R_i^-}}, \tag{1}$$

where R_i^+ and R_i^- represent the asymptotic response levels. For insecure individuals, the reaction is non-monotonic, reflecting the possibility of emotional overactivation followed by withdrawal. For positive involvement levels, the response has the form:

$$R_i^L(x_j) = \beta_i x_j e^{-k_i x_j}$$

while for negative values has the same functional form as (1).
In addition to attachment style, individuals may exhibit synergism, which amplifies their response to the partner's appeal:

$$R_i^A(x_i) = (1 + s_i^A S_i^A(x_i))\gamma_i A_j,$$

The interaction between these components generates a two-dimensional dynamical system capable of exhibiting qualitatively different regimes, including convergence to a unique equilibrium (robust dynamics), bistability (fragile dynamics), and sustained oscillations [5].

2.1. Extension to Parenthood

In order to incorporate the transition to parenthood, two additional parameters are introduced: $N \geq 0$, representing the total caregiving demand, and $\epsilon \in [0, 1]$, describing the fraction of caregiving responsibilities assumed by partner 1, with $(1 - \epsilon)$ representing the fraction assumed by partner 2. The main assumption is that caregiving does not directly modify the perception of the partner (appeal remains unchanged), but competes with the emotional resources available for reciprocal involvement [3]. Parenthood therefore acts by reducing responsiveness. Formally, caregiving is incorporated into the reaction to love term by rescaling its amplitude. In particular, by denoting with $N_1 = \epsilon N$ and $N_2 = (1 - \epsilon)N$ the caregiving loads assigned to each partner, we have that for secure individuals, caregiving reduces the asymptotic reaction levels in (1):

$$R_i^{\pm} \to \frac{R_i^{\pm}}{1 + h_i N_i} \tag{2}$$

where $h_i > 0$ measures the sensitivity to caregiving burden. On the other hand, for insecure individuals, the introduction of caregiving demands affects both the height and location of the response peak, specifically:

$$k_i \to k_i(1 + h_i N_i),$$

while for negative involvement levels, caregiving modifies the reaction level as in Equation (2).
In the absence of synergism, we have either a unique stable equilibrium (robust regime) or bistability (fragile regime) and the introduction of caregiving demands only alters equilibrium

positions and regimes. When synergism mechanisms are included, oscillatory dynamics may arise. This distinction naturally motivates a separate analysis of non-synergic and synergic configurations.

3. Non-synergic Individuals

We first analyze the extended model under the assumption of non-synergic responsiveness. In this case, the qualitative behavior of the system is characterized through equilibrium analysis and nullcline geometry. Depending on parameter values and attachment configuration, the system exhibits either a robust regime, characterized by a unique stable equilibrium, or a fragile regime, in which two stable equilibria coexist.

For couples composed of secure individuals, those that are robust without a child remain robust under caregiving demands. Moreover, the introduction of caregiving demands N progressively flattens the nullclines, lowering the equilibrium levels of involvement for both partners. This reflects the competition between parental investment and reciprocal emotional interaction. The allocation parameter ϵ introduces asymmetry, shifting equilibrium values differently for each partner and modifying the relative position of the nullclines in the phase plane. Figure 1 shows a percentage variation of total involvement $(x_1 + x_2)$ with respect to the childless configuration in the (ϵ, N) parameter space. The black curve represents the locus of equal percentage loss for both partners, highlighting how different allocations of caregiving responsibilities affect individual involvement. As we can see we have a percentage loss in the whole space.

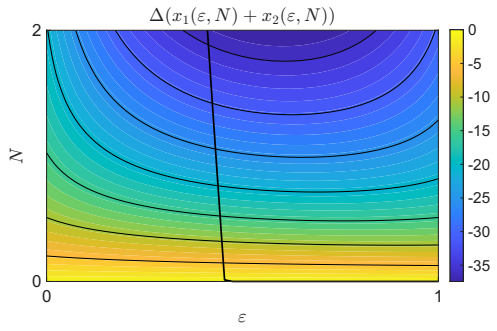

Figure 1: Percentage variation of total involvement with respect to the childless configuration in the (ϵ, N) parameter space. The black line identifies equal percentage loss for both partners.

In fragile configurations, caregiving significantly affects the dynamic regime. For fixed ϵ, increasing N implies the disappearance of one of the two stable equilibria (through a saddle-node bifurcation), thus eliminating bistability and restoring a single stable equilibrium. In this sense, caregiving demand suppresses fragility and promotes robustness. Conversely, for fixed N, different values of ϵ may either preserve bistability or yield a unique equilibrium, depending on the allocation of responsibilities. This is illustrated in Figure 2, where the saddle-node bifurcation in the (ϵ, N) parameter space separates regions of robust (R) and fragile (F) regimes.

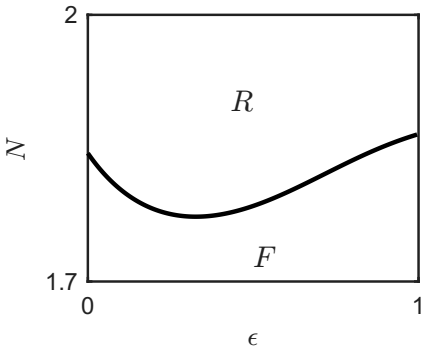

Figure 2: Saddle-node bifurcation diagram in the (ϵ, N) parameter space.

Caregiving also modifies the fragile region of alternative stable states (ASS) in both the appeal and reaction parameter spaces. As shown in Figure 3, the introduction of the child generally reduces the ASS region, facilitating the transition to robustness. Similar qualitative effects are observed in the (R_1^+, R_2^+) parameter space.

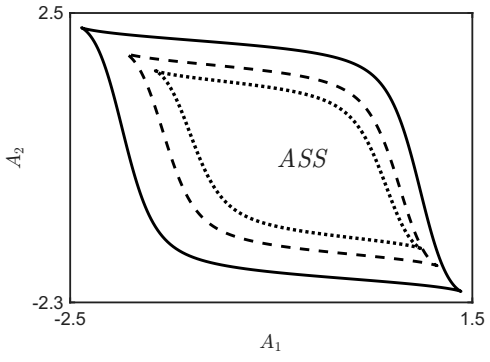

Figure 3: Alternative stable states regions in (A_1, A_2) for $\epsilon = 0.5$. In solid line $N = 0$, in dashed line $N = 0.5$, in dotted line $N = 1$

For fragile couples, the variation of involvement relative to the childless case in the (ϵ, N) param-

eter space shows results similar to those reported in Figure 1.

We now consider couples composed of one insecure and one secure individual. As in the secure-secure case, the childless system may exhibit either robustness or fragility depending on parameter values. Overall, the qualitative effects of caregiving are similar to those observed in the secure-secure configuration, with some differences. In particular, increasing the total caregiving demand N may eliminate bistability through a saddle-node bifurcation, restoring a robust configuration. For fixed N, the allocation parameter ϵ determines whether the system admits a unique equilibrium or two stable equilibria. As in the previous case, the introduction of a child generally reduces the region of alternative stable states (ASS) in both the appeal and reaction parameter spaces. A distinct behavior emerges in the equilibrium levels of involvement in the (ϵ, N) parameter space. As shown in Figure 4, which reports the comparison between the child and childless cases for the insecure partner, the latter may increase his equilibrium involvement by assigning the entire caregiving load to the secure partner, for sufficiently large N. Moreover, in this configuration it is also possible to increase the total involvement of the couple, as illustrated in Figure 5, when the insecure partner does not bear the caregiving burden. These findings suggest that, within the model, asymmetric caregiving distributions may lead to more favorable equilibrium outcomes in secure-insecure couples.

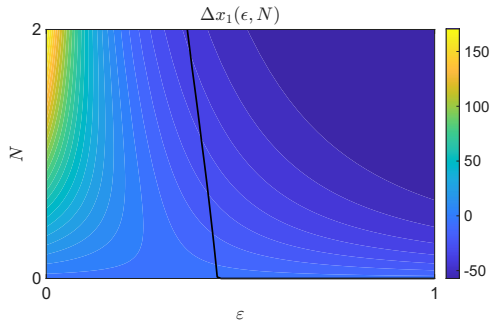

Figure 4: Comparison between the case with and without a child in the (ϵ, N) parameter space for partner 1.

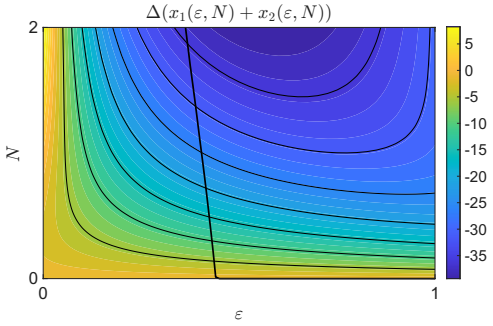

Figure 5: Percentage variation of total involvement with respect to the childless configuration in the (ϵ, N) parameter space.

4. Synergic Individuals

We now consider couples composed of one insecure individual and one secure synergic partner. In such configurations, the system may exhibit cyclic dynamics [2]. We investigate how caregiving intensity and its allocation affect this periodic behavior. In the (ϵ, N) parameter space, the system may undergo different bifurcations, so that increasing N and/or ϵ can induce transitions from cyclic (C) to stationary (E) dynamics, or from stationary behavior to cyclic regime and subsequently back to stationary, as illustrated in Figure 6.

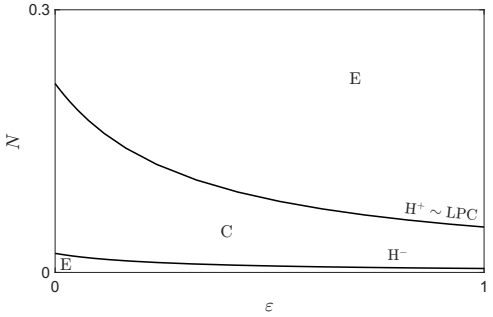

Figure 6: Bifurcation diagram in the (ϵ, N) parameter space showing a supercritical Hopf (below) and a subcritical Hopf almost coinciding with a fold of limit cycles (above).

Figures 7 and 8 report the involvement levels on the (ϵ, N) parameter space for the configuration shown in Figure 6. To account for the oscillatory variability, the involvement levels are computed as the average of feeling minus the standard deviation scaled by a parameter δ; in this case $\delta = 0$. The insecure partner attains higher involvement outside the cyclic region, even at the cost of higher caregiving demands. In contrast, the secure biased partner achieves larger

involvement levels within the cyclic region, with an abrupt decrease when crossing the subcritical Hopf boundary. For the couple as a whole (Figure 9), the cyclic region does not necessarily correspond to the lowest total involvement, suggesting that cyclic dynamics are not intrinsically unfavorable in terms of overall relational involvement.

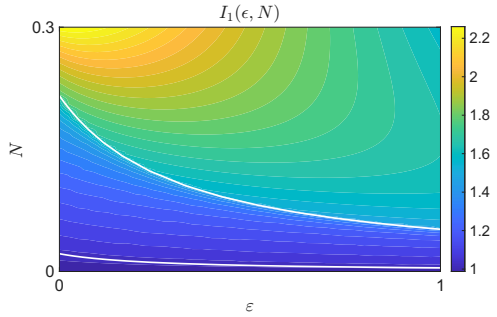

Figure 7: Averaged involvement levels on the (ϵ, N) parameter space for partner 1.

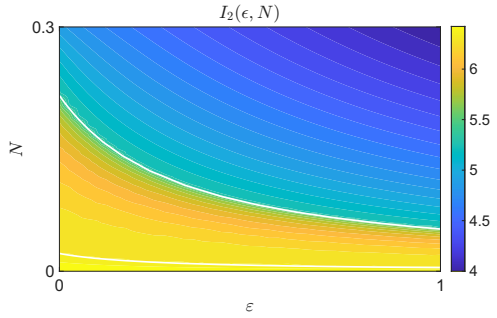

Figure 8: Averaged involvement levels on the (ϵ, N) parameter space for partner 2.

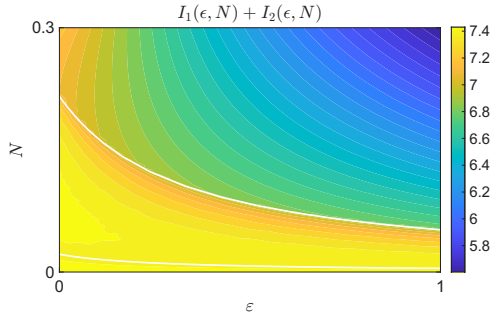

Figure 9: Averaged involvement levels on the (ϵ, N) parameter space for the couple.

4.1. Periodic Caregiving Demands

We now allow the caregiving intensity N to vary periodically in time, representing recurrent fluctuations in parental workload, such as school schedules, extracurricular activities, or seasonal commitments. The total caregiving demand oscillates around a mean value, introducing an exogenous periodic forcing into the system. In configurations that exhibit intrinsic cyclic dynamics, the interaction between the endogenous oscillation and the exogenous modulation, typically characterized by a different period, generates more complex behaviors. For small forcing amplitudes, quasi-periodic motion emerges from the coexistence of the endogenous limit cycle and the periodic signal. As the forcing amplitude increases, the system undergoes further qualitative transitions, eventually leading to chaotic attractors through torus destruction. The attractors corresponding to quasi-periodic and chaotic regimes are illustrated in Figure 10 and Figure 11, respectively.

Figure 10: Torus on the (x_1, x_2)-phase plane.

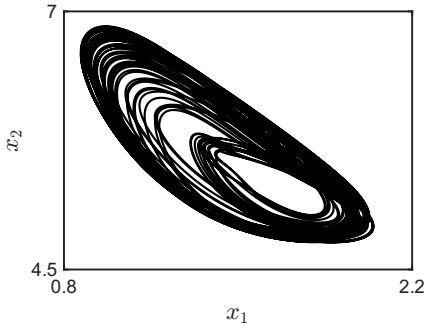

Figure 11: Strange attractor on the (x_1, x_2)-phase plane.

Moreover, even configurations that are stationary under constant caregiving may transition to chaotic dynamics when subjected to sufficiently strong periodic modulation. In this case, the presence of a strange attractor is confirmed by the structure of the associated Poincaré map, shown in Figure 12.

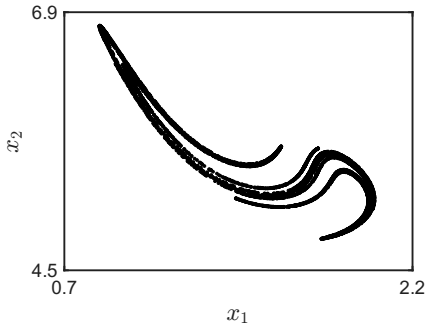

Figure 12: Poincarè map of a strange attractor on the (x_1, x_2)-phase plane.

5. Conclusions

This work extends the love dynamics model in [5] by incorporating caregiving intensity N and responsibility allocation ϵ as parameters modeling the transition to parenthood. By embedding these variables into the reaction mechanisms, the analysis moves to a framework in which external demands actively reshape emotional exchange. The results show that parenthood does not generate a unique relational outcome, accordingly to literature, e.g. [3]. Instead, caregiving intensity N and its distribution ϵ modify equilibrium levels, robustness properties, and bifurcation structures. In non-synergic configurations, caregiving primarily modifies the thresholds separating fragile and robust regimes, thereby enabling transitions between bistable and unique-equilibrium dynamics, often reducing involvement but, in insecure–secure couples, potentially enhancing involvement for the insecure partner under specific allocations. The redistribution of responsibilities therefore induces a structural trade-off between robustness, equity, and total emotional intensity. When synergism mechanisms are present, caregiving affects not only equilibria but also oscillatory regimes. Depending on parameter values, several bifurcations may occur, making parenthood either "stabilizing" (from C to E) or "destabilizing" (from E to C). Finally, periodic caregiving signals can generate quasi-periodic or chaotic dynamics. Parenthood thus acts as a control mechanism capable of reorganizing the behaviour of the couple. Overall, the transition to parenthood emerges not as inherently "stabilizing" or "destabilizing", but as a structural perturbation capable of producing stationary, cyclic, quasi-periodic, or chaotic relational regimes depending on attachment configuration and caregiving structure.

6. Future Developments

Despite its explanatory power, the model remains simplified: it is deterministic, limited to two state variables, and treats caregiving demands as externally imposed. Future developments could introduce stochastic perturbations, adaptive redistribution of responsibilities, or the explicit modeling of the child as an additional dynamic agent. Empirical validation represents a crucial next step. Longitudinal studies following couples across the transition to parenthood could provide data suitable for parameter estimation and qualitative comparison with predicted dynamical regimes. Repeated assessments of involvement and interaction quality may allow the identification of stationary, cyclic, or irregular relational patterns. From a theoretical perspective, further extensions could incorporate additional attachment profiles and contextual variables, such as socioeconomic conditions or demographic factors.

References

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