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Relative permeability models:
A systematic
sensitivity analysis

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ABSTRACT

La permeabilità relativa è una proprietà chiave dell'interazione tra roccia e fluido, necessaria per la modellazione alla scala del continuo della dinamica dei flussi multifase in mezzi porosi e fratturati. La quantificazione dei processi di flusso multifase che avvengono in rocce porose e fratturate ha una notevole rilevanza per la gestione e lo sfruttamento economicamente sostenibile di formazioni geologiche contenenti petrolio e gas naturale. Attualmente, sono disponibili diverse formulazioni empiriche per la caratterizzazione delle curve di permeabilità relativa di acqua e petrolio. La struttura di questi modelli è tipicamente guidata da osservazioni sperimentali, argomentazioni teoriche e/o concezioni euristiche (procedimenti non rigorosi che consentono di prevedere un risultato, ma che dovranno poi essere convalidati). A ciascun modello è associata una serie di parametri che sono normalmente stimati attraverso *fitting* di dati sperimentali, ovvero rispetto alle curve di permeabilità relativa disponibili. Una caratterizzazione affidabile delle curve di permeabilità relativa, inclusa la corretta quantificazione dell'incertezza, ci consente di valutare le prestazioni di un giacimento, prevedere il recupero finale di idrocarburi e studiare l'efficienza delle tecniche avanzate di recupero del petrolio.

In questo saggio, diverse tecniche di analisi di sensitività globale sono applicate in modo rigoroso ai modelli di permeabilità relativa di acqua e petrolio più comunemente adottati nell'industria, sia per singoli modelli, sia in un contesto multi-modello. L'obiettivo finale del lavoro è indagare come la variazione nell'*output* dei modelli di permeabilità relativa possa essere attribuita alle variazioni dei loro fattori di *input* e alla mancanza di conoscenza sulla struttura/formato dei modelli in scenari non informati (in cui nessuna informazione è disponibile a priori) nonché in scenari informati. I risultati ottenuti dai diversi metodi di analisi di sensitività vengono analizzati e discussi concentrandosi sulle loro eventuali concordanze e discordanze.

La tecnica dell'analisi di sensitività viene utilizzata sempre più frequentemente nell'ambito della modellazione ambientale per diversi scopi, tra cui la valutazione dell'incertezza, la calibrazione di modelli, la diagnostica dei modelli, l'analisi del controllo dominante e la sua funzione di supporto a un solido processo decisionale. Da un punto di vista pratico i risultati ottenuti in questo saggio puntano a guidare ulteriori azioni quali: ridurre l'incertezza dei parametri selezionati come i più rilevanti, programmare l'investimento di risorse dedicate all'acquisizione di dati più affidabili e guidare le attività di calibrazione dei modelli. Inoltre, l'analisi di sensitività offre anche spunti utili per la semplificazione dei modelli, per esempio, permettendo di identificare i parametri di *input* che hanno effetto trascurabile sull'*output* in esame. L'applicazione di numerose tecniche di analisi di sensitività mostra infine la necessità di non fare affidamento indiscriminatamente su un solo metodo di analisi.

ABSTRACT

Relative permeabilities are key rock-fluid properties, required for continuum-scale modelling of multiphase flow dynamics in porous and fractured media. Quantification of multi-phase flow processes taking place in natural porous and fractured rocks has a remarkable relevance to economically sustainable management and viable development of oil and gas-bearing geologic formations. Several empirical formulations are available to characterize observed water-oil relative permeability curves. The structure of these models is typically driven by experimental observations, theoretical arguments and/or heuristic concepts. Each model is associated with a set of parameters which are usually estimated through fits against experiments, *id est*, against available relative permeability curves. A reliable characterization of relative permeabilities, including proper quantification of uncertainty, enables us to assess reservoir performance, forecast ultimate oil recovery, and investigate the efficiency of enhanced oil recovery techniques.

In this essay, several global sensitivity analysis techniques are rigorously performed on the most widely adopted water-oil relative permeability models in the industry, both in a single-model and in a multi-model context. The ultimate goal of the work is investigating how the variation in the output of the relative permeability models can be attributed to the variations of their input factors and to the lack of knowledge about the models' structure/format in uninformed scenarios (no previous knowledge is available) as well as in informed scenarios. The outcomes of the different sensitivity analysis methods are analysed and discussed, focusing on eventual concordances and discordances.

Sensitivity analysis is increasingly being used in environmental modelling for a variety of purposes, including uncertainty assessment, model calibration and diagnostic evaluation, dominant control analysis and robust decision-making. From a practical point of view, the results obtained in this essay aim to guide further actions such as: reducing the uncertainty of parameters selected as more relevant, guiding investment of resources dedicated to acquiring more reliable data and guiding model calibration activities. Furthermore, sensitivity analysis also offers insights to guide model simplification, for example, by identifying model input parameters that have negligible effects on a target output. The application of many different sensitivity analysis techniques finally shows the need not to rely indiscriminately on just one sensitivity analysis method.

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1.1. THE SENSITIVITY ANALYSIS OF A MODEL OR OF A GROUP OF MODELS

1.1.1. WHAT IS SENSITIVITY ANALYSIS

Generally speaking, sensitivity analysis is a broad and multifaced collection of methods whose purpose is assisting the qualitative and quantitative characterization of the relevance of diverse model inputs affected by uncertainty with respect to a target output.

The main subject of sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system of models can be divided and allocated to the different sources of uncertainty belonging to its inputs. The first and more common source of uncertainty in the inputs consists of the non-deterministic knowledge of the model parameters value, so sensitivity analysis investigates how the variability of the output of a model can be attributed to variations of its input factors (Pianosi, et al., 2016). Furthermore, the quality and amount of data available in many practical situations justify the interpretation of the system under investigation through a collection of many interpretative models, leading to the second source of uncertainty: the uncertainty about the model structure/format. These two sources of uncertainty are combined when the parameters associated with each one of the different considered models are affected by uncertainty (Dell'Oca, Riva, & Guadagnini, 2020).
Accordingly, sensitivity analysis can be performed on any model or group of models describing the same phenomenon, when one or more of their input parameters are affected by uncertainty.

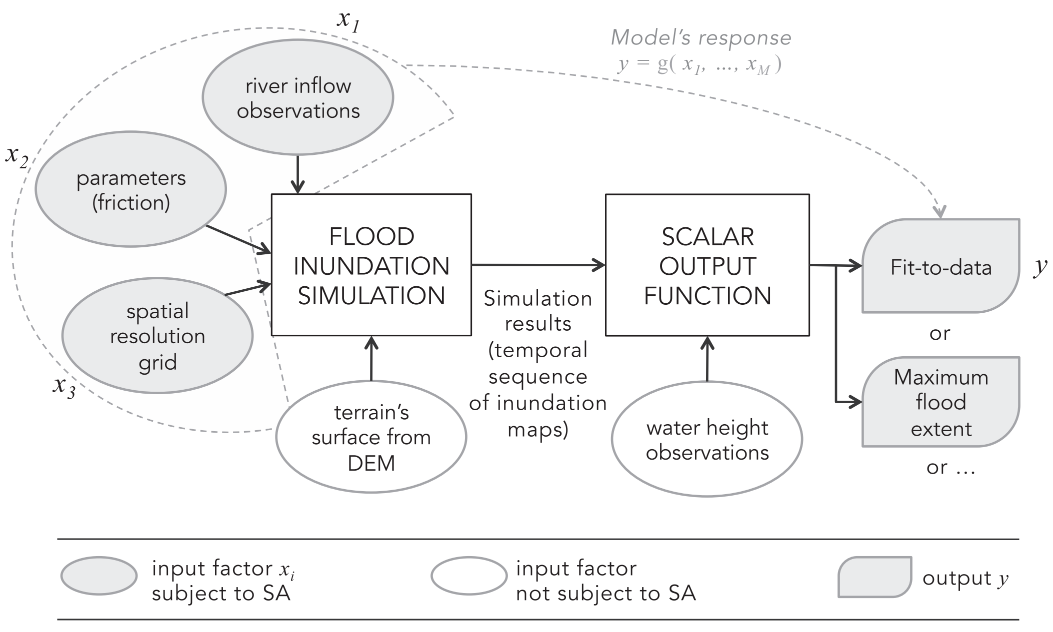

Figure 1: example of sensitivity analysis of a flood inundation model

1.1.2. CONCEPTUALIZATION: DEFINITION OF MODEL, INPUT FACTORS AND OUTPUTS

In this essay the term *model* will be used to refer to the diverse formulations available for the quantification of water-oil relative permeabilities (see chapter 3 for details). An *input factor* is any element that can be changed before the model execution, the *output* is what is obtained after the model execution. Examples of input factors are the parameters appearing in the model equation.

Given the above definitions, one can always resort to the general formula:

$$y = g(\boldsymbol{x}) = g(x_1, x_2, \dots, x_m) \tag{1.1}$$

Where y is the output, g is the model response function and $\boldsymbol{x} = [x_1, x_2, \dots, x_m]$ is the vector of the input factors, which belongs to the input variability space $\boldsymbol{X} = [X_1, X_2, \dots, X_m]$, where the terms $X_i = [x_{i,min}\ ,\ x_{i,max}]$ define the totality of the values which can be assumed by the input factors.

For simplicity, the output of a model obtained adopting a certain input factors vector $\boldsymbol{x}$ will be denoted as $y(\boldsymbol{x})$.

1.1.3. PURPOSE OF THE SENSITIVITY ANALYSIS OF A MODEL

Performing the sensitivity analysis of a model (or set of models) can be useful for a range of purposes (Pianosi, et al., 2016), (Saltelli, 2002):

- Testing the robustness of the model results in presence of uncertainty.
- Improving the understanding of the relationships between the input and output variables.
- Reducing the uncertainty through the identification of the model inputs that cause significant uncertainty in the output. These inputs should be made the focus of the attention in order to improve the model robustness.
- Looking for errors in the model, by encountering unexpected relationships between inputs and outputs.
- Simplifying the model, by fixing the model inputs which have negligible effect on the output.
- Identifying important relationships between observations, model inputs, and predictions, leading to the development of better models.

This broad variety of uses of the sensitivity analysis can be summarized in three ultimate goals:

1) *Ranking* (or factor prioritization): sorting of the input factors $(x_1, x_2, \dots, x_m)$ according to their relative contribution to the output variability.
2) *Screening* (or factor fixing): identification of the input factors, if any, which have negligible influence on the output variability and which can therefore be fixed.
3) *Mapping*: determination of the regions of the input variability space that produce significant (extreme) output values.

Sensitivity analysis is key to assist understanding and improvement of models, aiming at rendering the dynamics of any phenomenon or system. The need for a proper sensitivity analysis is exacerbated by the increasing complexity of conceptual models, in terms of model formulation and associated parametrization. The increasing complexity of the models is in turn sustained by the increased knowledge of the described phenomena and by the exponentially increasing computational power, available for numerical model simulations (Dell'Oca, Riva, & Guadagnini, 2020), (Ye & Dai, 2015).

1.1.4. TYPES OF SENSITIVITY ANALYSIS

In the previous paragraph, the typical questions addressed by sensitivity analysis have been presented. Different types of sensitivity analysis can be distinguished according on how these questions are formulated and answered. The different sensitivity analysis methods can be classified as follows (Pianosi, et al., 2016), (Saltelli, 2002):

- *Local* and *global* sensitivity analysis: local sensitivity analysis considers the output variability against variations of the input factors around a specific nominal value $\overline{\boldsymbol{x}} = [\overline{x_1}, \overline{x_2}, \dots, \overline{x_m}]$, while global sensitivity analysis considers variations of the input factors within their entire input variability space. Local sensitivity analysis is always used in combination with parameters estimation techniques, which provide the vector $\overline{\boldsymbol{x}}$.
- *Quantitative* and *qualitative* sensitivity analysis: quantitative sensitivity analysis refers to methods where each input factor is associated with a quantitative and reproducible evaluation of its relative influence, normally thorough a set of *sensitivity indices*. In qualitative sensitivity analysis, instead, sensitivity is assessed by visual inspection of model output's predictions. Often such visual tools are used complementary to more quantitative measurements.
- *One-factor-at-a-time* and *all-factors-at-a-time* sensitivity analysis: this distinction refers to the sampling strategy adopted to estimate the sensitivity indices. In fact, in general, sensitivity indices cannot be computed analytically due to the complexity of the input-output relationship, and thus they are numerically approximated from a sample of input factors and associated output evaluations. In *one-factor-at-a-time* methods, output variations are induced by varying one input factor at a time, while keeping the other fixed; in *all-factors-at-a-time* methods, output variations are induced by varying all the input factors simultaneously, and therefore the sensitivity to each input factor considers the direct influence of that factor as well as the joint influence due to interactions with other input factors.

The particular sensitivity analysis methodologies that are implemented and run for the purposes of the essay will be discussed later. All the sensitivity analysis methods implemented in the course of this work are so-called *sampling-based methods*, which proceed through these three logical steps (Pianosi, et al., 2016):

1) Sampling of the input factors.

2) Numerical evaluation of the model.

3) Processing of the results.

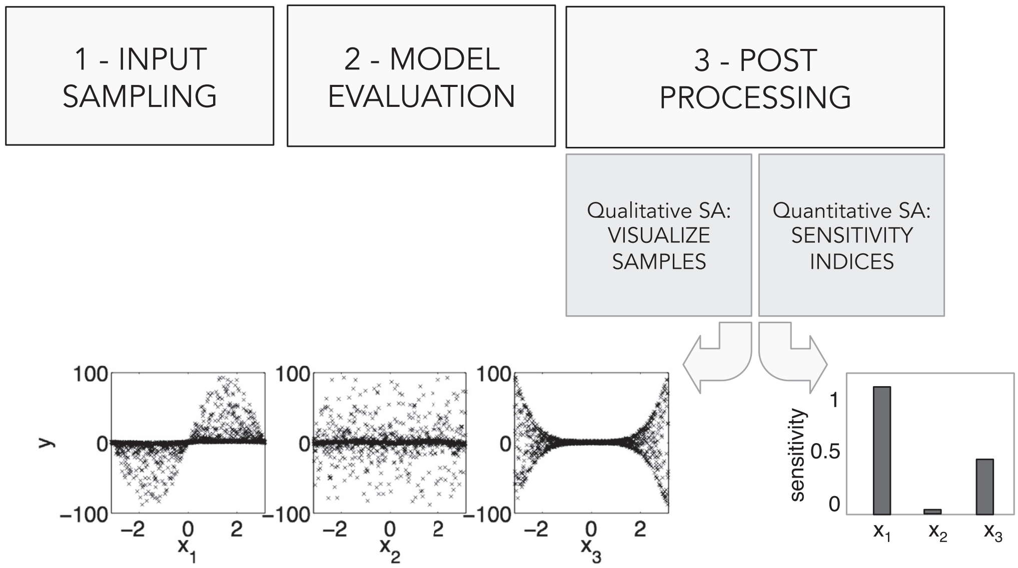

Figure 2: the three basic steps of sampling-based sensitivity analysis

1.2. RELATIVE PERMEABILITY: AN OVERVIEW

1.2.1. THE PERMEABILITY OF A POROUS MEDIUM

Permeability is the property of the porous medium that measures the capacity and the ability of the formation to transmit fluids. The rock permeability, k, is a subject of great interest because it controls the flow rate of the reservoir fluids in the formation.

In 1855 Henri Darcy, a French hydraulic engineer, oversaw a series of experiments aimed to understand the rates of water flow through sand filters and their relationship to pressure loss along the flow paths for the water purification purpose. Darcy's experiments consisted in a vertical steel column, with a water inlet at one end and an outlet at the other. The water pressure was controlled at the inlet and outlet ends of the column, using two reservoirs with constant water levels h_1 and h_2 (see Figure 3). The experiments included a series of tests with different packings of river sand, and a suite of tests using the same sand pack and column, but for which the inlet and outlet pressures were varied.

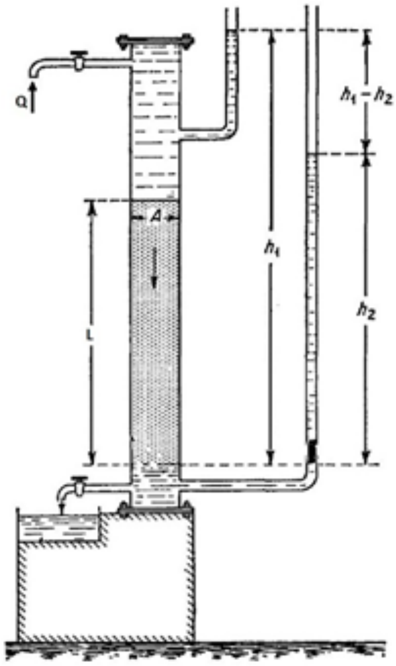

Figure 3: schematic of Darcy's experiment of water flow through sand

During his experiments Darcy developed a fluid flow equation that has since become one of the standard tools of petroleum engineering: Darcy's law (Darcy, 1856), (Whitaker, 1986).

$$Q = KA\frac{h_1 - h_2}{L} \quad (1.2)$$

With:

- Q: volumetric flow rate [m^3/s].
- K: hydraulic conductivity [m/s].
- A: cross section area [m^2].
- $(h_1 - h_2)$: height difference [m].
- L: length of the column [m].

Darcy's law can be extended to the multidimensional case in which the driving force of fluid motion is not only gravity, but also pressure, obtaining the equation (Muskat & Meres, 1936):

$$\boldsymbol{q} = -\frac{\boldsymbol{k}}{\mu}(\nabla P - \rho \boldsymbol{g}) \qquad (1.3)$$

With:

- $\boldsymbol{q}$: vector of the fluid volumetric flux [m^3/m^2s].
- $\boldsymbol{k} = K\frac{\mu}{\rho \boldsymbol{g}}$: permeability tensor (3x3 tensor) [m^2].
- μ: fluid's dynamic viscosity [Pa·s].
- ρ: fluid's density [kg/m^3].
- $\boldsymbol{g}$: acceleration due to gravity [m/s^2].
- ∇P: pressure gradient [Pa/m].

In this essay only 1-dimensional flow in isotropic porous media will be discussed, hence the terms k, q and g will be referred to as scalars.

1.2.2. MULTIPHASE EXTENSION OF DARCY'S LAW: THE RELATIVE PERMEABILITY

In 1936 Morris Muskat et al. developed the governing equation for multiphase flow in porous media as an extension of Darcy's law. For each liquid (or gas) phase α the extended form of Darcy's law assumes the following form (Muskat, Wyckoff, Botset, & Meres, 1937):

$$q_\alpha = -\frac{k_{r\alpha} \cdot k}{\mu_\alpha}(\nabla P_\alpha - \rho_\alpha \cdot g_{\parallel}) \qquad (1.4)$$

With:

- q_α: volumetric flux of the phase α [m^3/m^2s].
- $k_{r\alpha}$: ***relative permeability*** of the phase α [-].
- k: absolute permeability of the porous medium [m^2].
- μ_α: dynamic viscosity of the phase α [Pa·s].
- ρ_α: density of the phase α [kg/m^3].
- ∇P_α: pressure gradient in the phase α [Pa/m].
- $g_{\parallel}$: gravity acceleration component which is parallel to the flow direction [m/s^2].

The *relative permeability* $k_{r\alpha}$ of a certain fluid/phase α is defined as the *ratio* of the effective permeability of the phase α to some base permeability k, which in this case is the absolute permeability of the porous medium (*id est*, the permeability evaluated considering single-phase flow). It follows:

$$0 \leq k_{r\alpha} \leq 1 \qquad (1.5)$$

In this essay only the two-phase mixture of water and oil will be considered, and the corresponding relative permeabilities of the two fluids will be referred to as k_{rw} (for water) and k_{ro} (for oil).

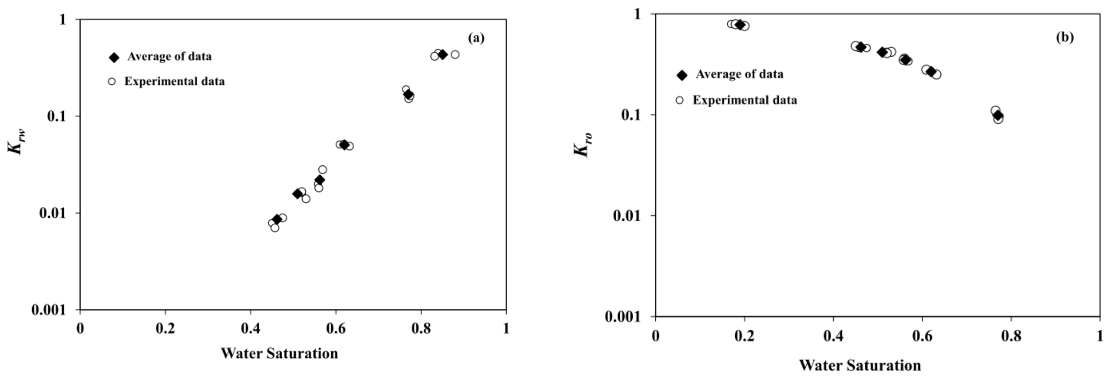

Figure 4: example of experimental measurements of water-oil relative permeabilities

2. SENSITIVITY ANALYSIS: METHODOLOGIES AND WORKFLOW

In this chapter the different sensitivity analysis methods which are adopted and implemented for the purpose of this essay are presented. Each method is discussed, focusing on the underlying concept, the assumptions, the specific purpose that the method can address, the mathematical workflow when applied to a model (or group of models) and the computational complexity.

2.1. SINGLE-MODEL, UNINFORMED GLOBAL SENSITIVITY ANALYSIS: METHODOLOGIES AND WORKFLOW

Single-model uninformed global sensitivity analysis investigates how the uncertainty in a defined model output y can be attributed to variations of its input variable factors x_i within their entire variability space. For this reason, the global uninformed approach is particularly useful and meaningful for the preliminary study of models, when no previous knowledge of the input factors (coming from experimental measurements or inverse modelling techniques) is available.

The single-model uninformed global sensitivity analysis techniques which are considered and used in this essay are:

- Multiple start perturbation method sensitivity analysis.
- Statistical moments-based (AMA and Sobol) sensitivity analysis.
- Cumulative probability density function-based (PAWN) sensitivity analysis.
- Variogram-based (VARS) sensitivity analysis.

2.1.1. MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

The simplest type of sensitivity analysis is the "perturbation and derivatives" method, which is based on the use of the partial derivatives evaluated at nominal values of the input variable factors as measurements of the output sensitivity (Devenish, Francis, Johnson, Sparks, & Thomson, 2012), (Paton, Maier, & Dandy, 2013), (Pianosi, et al., 2016):

$$S_i = \left| \frac{\partial y(x_1, \ldots, x_i, \ldots, x_m)}{\partial x_i} \right|_{x_{i,nominal}} \qquad (2.1)$$

Where S_i is the sensitivity index with respect to the variable input factor x_i and $y = y(\boldsymbol{x})$ is the model output. In the presented sensitivity analysis implementation partial derivatives are approximated by finite differences:

$$\left|\frac{\partial y}{\partial x_i}\right|_{x_i} \sim \left|\frac{y(x_1,\dots,x_i+\Delta_i,\dots,x_m)-y(x_1,\dots,x_i,\dots,x_m)}{\Delta_i}\right| \tag{2.2}$$

Where Δ_i is a small and constant fraction of the variability range of x_i:

$$\Delta_i = \frac{x_i}{1000} \tag{2.3}$$

This method only considers the local response of the model in the surroundings of the adopted input factors values and is therefore not suitable for a global sensitivity analysis.

A global extension of the perturbation approach is the "multiple start perturbation method", also called "Morris method" or "elementary effects test". According to this method, each sensitivity index S_i is evaluated not by a single partial derivative with respect to x_i, but by averaging a (large) number r of partial derivatives which are evaluated for different variable input factors combinations (Morris, 1991), (Saltelli, et al., 2008), (Pianosi, et al., 2016):

$$S_i = \frac{1}{r}\sum_{j=1}^{r}\left|\frac{y\left(x_1^j,\dots,x_i^j+\Delta_i,\dots,x_m^j\right)-y\left(x_1^j,\dots,x_i^j,\dots,x_m^j\right)}{\Delta_i}\right| = \frac{1}{r}\sum_{j=1}^{r} EE_i^j \tag{2.4}$$

Each partial derivative evaluation is called "elementary effect", EE. Besides the above sensitivity measure, also the standard deviation of the elementary effects is computed:

$$SD_i = stdv(EE_i^1, EE_i^2, \dots, EE_i^r) \tag{2.5}$$

The standard deviation SD_i provides information on the degree of interaction of the i^{th} input factor with the others. An input factor interacts with a second input factor when its associated sensitivity depends also on the value assumed by the second input factor. A high standard deviation of the elementary effects indicates that a factor is interacting with others because its sensitivity changes across the variability space.

According to this method, one sensitivity index S_i and one standard deviation SD_i are computed for each variable input factor x_i.

In the presented implementation of the method, the sampling strategy to select the evaluation points $\left(x_1^j,\dots,x_i^j,\dots,x_m^j\right)$, $j = (1,\dots,r)$ simply consists, for each value of j, in picking random values of the input factors in their variability space according to a uniform probability density function:

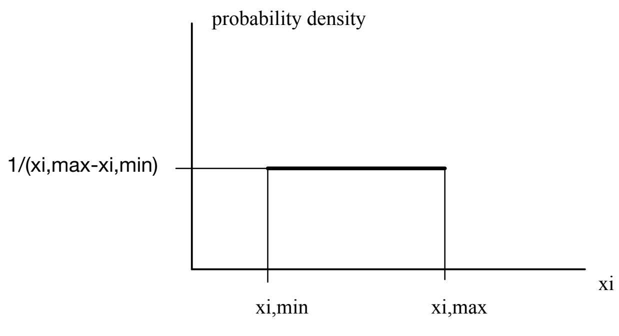

Figure 5: uniform probability density function

For a number r high enough the values of S_i and SD_i reach convergence and stay constant. The computation of the mean (and standard deviation) of the elementary effects of m input factors require $r(m + 1)$ model evaluations, a requisite that is far lower than the other considered global sensitivity analysis methods. Because of that, also extremely high values of r can be used without causing unacceptable computation times.

The multiple start perturbation method is particularly suitable for *screening* and for *ranking* the variable input factors.

2.1.2. STATISTICAL MOMENTS-BASED (AMA and Sobol) SENSITIVITY ANALYSIS

The statistical moments-based sensitivity analysis (Dell'Oca, Riva, & Guadagnini, 2017) studies the impact of the variable input factors on the main features of the probability density function of the model output, y. These features are the first four statistical moments:

- The mean value, $E(y)$.
- The variance, $V(y)$, measuring the spread around the mean of the output probability distribution function.
- The skewness, $Sk(y)$, measuring the asymmetry of the output probability distribution function.
- The kurtosis, $K(y)$, measuring the "tailedness" of the output probability distribution function.

This method can be seen as an extension of Sobol's analysis (typically referred as variance-based sensitivity analysis), where the output variance is taken as the only metric upon which sensitivity is quantified (Sobol, 1993). Relying solely on this criterion can provide an incomplete picture of a system response to model parameters, also considering that the relative permeability models output can be characterized by highly skewed and tailed distributions. The key idea at the basis of the statistical moments-based sensitivity analysis is that the joint use of sensitivity indices based on four different statistical moments of the model output leads to an improved understanding of the way a given uncertain model input can govern main features of the output probability density function. In the presented implementation of the method the calculation of the indices relies on Monte Carlo simulations of the models (in principle the indices could be calculated analytically).

Monte Carlo simulation is the key process underlying not only the statistical moments-based sensitivity analysis, but also any other sensitivity analysis method which is discussed in this essay, except for the multiple start perturbation method. It is therefore appropriate to define inputs, outputs and methodology of this procedure as it is applied to the relative permeability models in the following chapters.

The Monte Carlo simulation is a computational algorithm that relies on repeated random sampling of a model to obtain the results. For each considered model, the inputs of the Monte Carlo simulation consist of m (number of variable input factors) input variability spaces X_i in the form $[x_{i,min}, x_{i,max}]$, $i = (1, \dots, m)$ and a number n of required model evaluations.

The outputs of the Monte Carlo simulation as implemented for the purpose of this essay are two matrices:

- The unconditional output matrix.
- The conditional output matrix.

The *unconditional output matrix* is a (1xn) matrix containing n model evaluations obtained by considering random values of each variable input factor of the model, picking them from their variability space according to a uniform probability density function. In other words, the model outputs contained in the unconditional output matrix are obtained by considering each variable input factor of the model as a random variable uniformly distributed in its variability space.

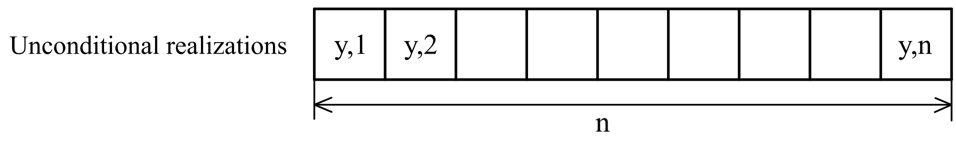

Figure 6: unconditional output matrix

The *conditional output matrix* is a $(m\text{x}n\text{x}p)$ 3-dimensional matrix, and it is structured according to the following criteria:

- The i^{th} row, $i = (1, \dots, m)$, contains model outputs obtained by conditioning the i^{th} variable input factor (x_i) of the model to a single and fixed value, while considering all the other input factors as random uniformly distributed variables. This model evaluation is repeated n times building a complete row.
- The 3^{rd} dimension of the matrix controls the specific value of the conditioned factors: each conditioned variable input factor x_i assumes values which range from $x_{i,min}$ in the cell (i, j, 1) to $x_{i,max}$ in the cell (i, j, p). The difference between the value of the conditioned factor x_i in the cell (i, j, k+1) and its value in the cell (i, j, k) results to be:

$$x_i^{(i,j,k+1)} - x_i^{(i,j,k)} = \frac{x_{i,max} - x_{i,min}}{p-1} \qquad (2.6)$$

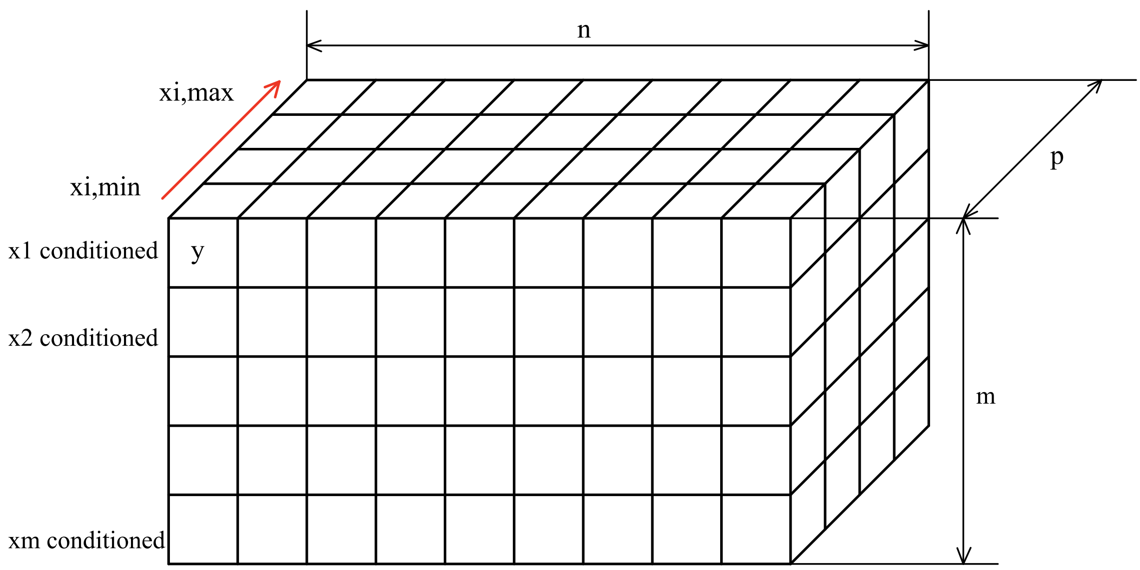

Figure 7: conditional output matrix

Now that the unconditional and conditional output matrices have been defined, the methodology of the statistical moments-based sensitivity analysis can be described in detail.

The conditional output matrix must be further elaborated in order to obtain the matrices of the conditional output statistical moments: $\boldsymbol{E}$, $\boldsymbol{V}$, $\boldsymbol{Sk}$, $\boldsymbol{K}$. These are $(p\text{x}m)$ 2-dimensional matrices obtained by computing the statistical moments of the conditional output matrix row-by-row, as shown in the figure below (where the computation of the matrix $\boldsymbol{E}$ is taken as an example):

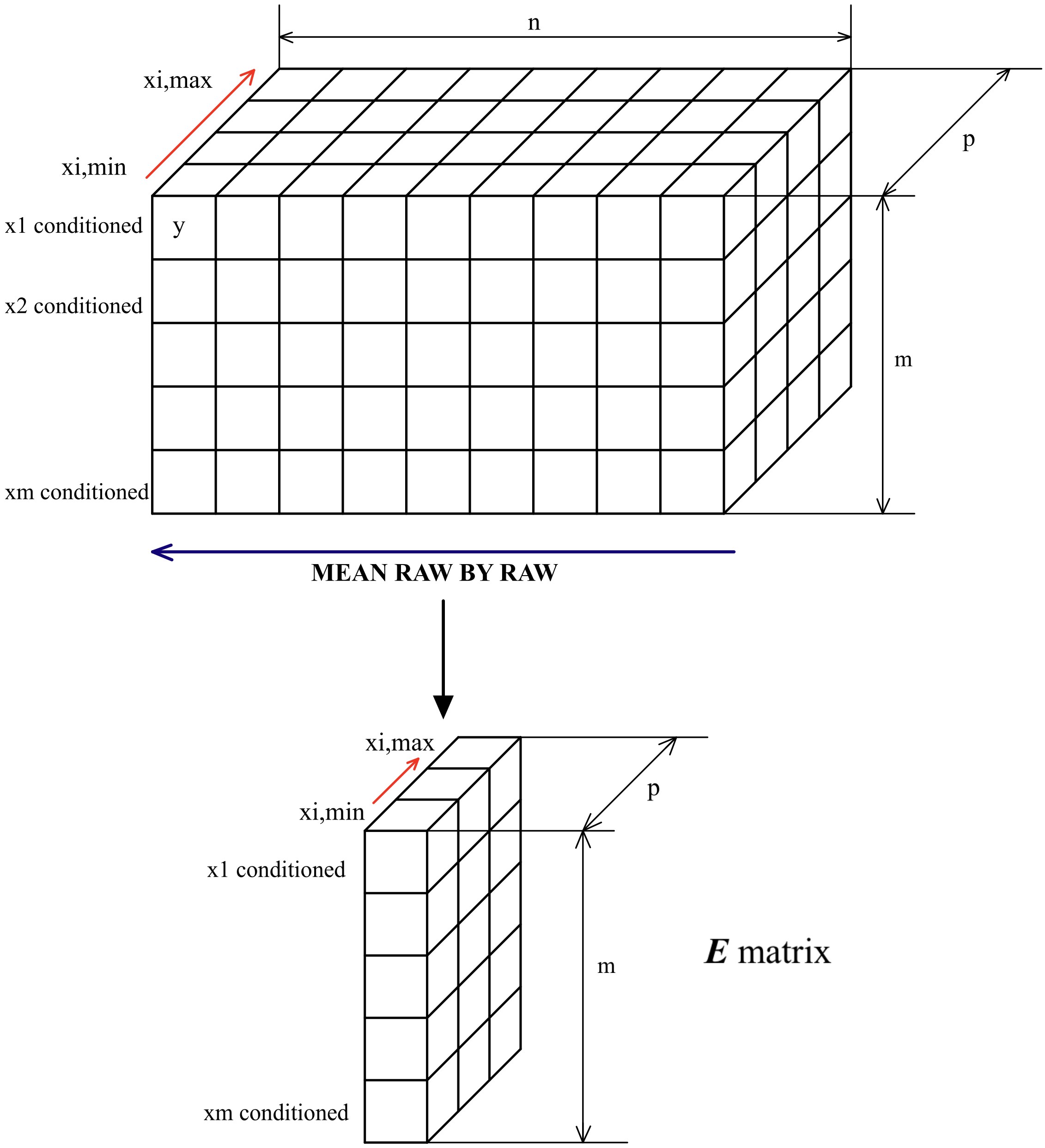

*Figure 8: calculation of the mean matrix **E** of the conditional output*

It should be remembered that the row i of any statistical moment matrix ($\boldsymbol{E}$, $\boldsymbol{V}$, $\boldsymbol{Sk}$, $\boldsymbol{K}$) is obtained conditioning the variable input factor x_i.

In the following, the complete i^{th} row of the statistical moment matrix (obtained conditioning x_i) will be referred to as "conditional mean/variance/(…) with respect to x_i" and as $\boldsymbol{E}(i,:)$/ $\boldsymbol{V}(i,:)$/...

Once $\boldsymbol{E}$, $\boldsymbol{V}$, $\boldsymbol{Sk}$ and $\boldsymbol{K}$ are calculated, six sensitivity indices are computed for each variable input factor x_i (Dell'Oca, Riva, & Guadagnini, 2017), (Sobol, 1993):

- $AMAE$ sensitivity index:

$$AMAE_i = E\left(\frac{|E(unconditional\ output) - \boldsymbol{E}(i,:)|}{|E(unconditional\ output|}\right) \quad (2.7)$$

The index $AMAE_i$ quantifies the expected relative change of the output mean value due to the variation/conditioning of the input factor x_i.

- $AMAV$ sensitivity index:

$$AMAV_i = E(\frac{|V(unconditional\ output) - \boldsymbol{V}(i,:)|}{|V(unconditional\ output|}) \quad (2.8)$$

The index $AMAV_i$ quantifies the expected relative change of the output variance due to the variation/conditioning of the input factor x_i.

- $Sobol$ first order sensitivity index:

$$Sobol_i^f = \frac{V(\boldsymbol{E}(i,:))}{V(unconditional\ output)} \quad (2.9)$$

The index $Sobol_i^f$ measures the expected reduction in the output variance that can be obtained conditioning the variable input factor x_i. It measures the contribution to the output variance from the individual input factors without considering their interactions. The sensitivity index $Sobol_i^f$ is equivalent to the index $AMAV_i$ only if the conditional variance $\boldsymbol{V}(i,:)$ is always (*id est*, for each value of x_i) smaller than or equal to its unconditional counterpart.

- $Sobol$ total order sensitivity index:

$$Sobol_i^t = \frac{E(\boldsymbol{V}_{\sim x_i})}{V(unconditional\ output)} \quad (2.10)$$

The matrix $\boldsymbol{V}_{\sim x_i}$ is a (m-1)-dimensional matrix where each cell contains the variance of n_t model realizations obtained conditioning all the input variable factors <u>except</u> x_i, which is considered a random variable uniformly distributed in its variability space X_i. Each dimension of the matrix is made of p_t cells, so as to include any possible combination of conditioned input factors given the discretization parameter p_t (p_t plays the exact same role of the discretization parameter p in the Monte Carlo simulations). The index $Sobol_i^t$ describes the relative contribution to the variance of the output due to the variability of the input factor x_i considering both its direct effect and its interactions with all the other input factors (which might amplify or mitigate the individual effects).

- $AMASk$ sensitivity index:

$$AMASk_i = E(\frac{|Sk(unconditional\ output) - \boldsymbol{Sk}(i,:)|}{|Sk(unconditional\ output|}) \quad (2.11)$$

The index $AMASk_i$ quantifies the expected relative change of the output skewness due to the variation/conditioning of the input factor x_i.

- $AMAK$ sensitivity index:

$$AMAK_i = E(\frac{|K(unconditional\ output) - \boldsymbol{K}(i,:)|}{|K(unconditional\ output|}) \quad (2.12)$$

The index $AMAK_i$ quantifies the expected relative change of the output kurtosis due to the variation/conditioning of the input factor x_i.

These sensitivity indices are particularly suitable for the *ranking* of the input factors. The Sobol total order index is particularly suitable for *screening* because a value of 0 of the corresponding total order index is a necessary and sufficient condition for a factor to be non-influential.

The Monte Carlo simulation is an extremely demanding process from the computational point of view: each simulation requires $(m \times n \times p)$ model evaluations and the simulation must be repeated many times. It should be noticed that the value of n must be extremely high to obtain the necessary stability (and accuracy). Because of that, any sensitivity analysis method which relies on Monte Carlo simulations requires rather long computation times, and a trade-off between stability of the results and code run-time is often necessary. The computation of the matrix $\boldsymbol{V}_{\sim \boldsymbol{x}_i}$ is extremely demanding as well: it requires $\left(n_t \times p_t^{(m-1)}\right)$ model evaluations, making the calculation exponentially slower when models with many input variable factors are considered.

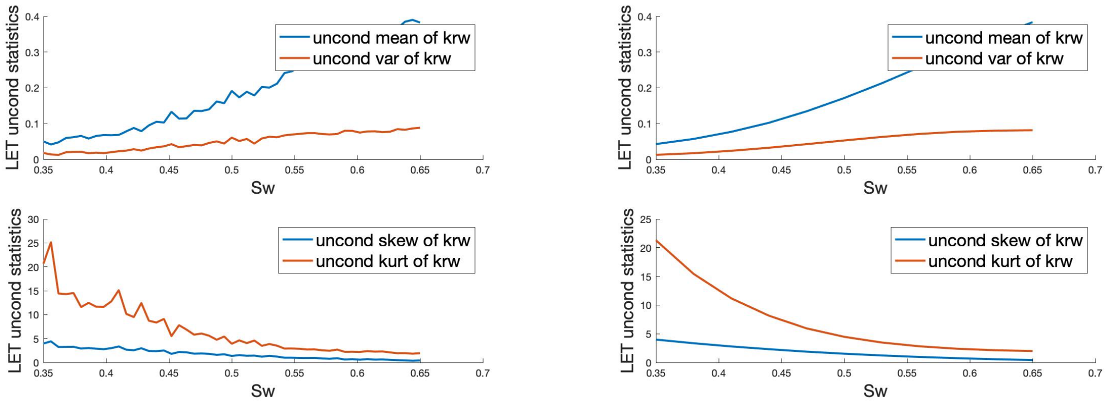

Figure 9: LET relative permeability model unconditional statistical moments for n=500 realizations (left) and n=100,000 realizations (right)

2.1.3. CUMULATIVE PROBABILITY DENSITY FUNCTION-BASED (PAWN) SENSITIVITY ANALYSIS

This sensitivity analysis technique is statistical moments-independent. It belongs to the family of the "density-based" sensitivity analysis: the entire probability distribution of the model output is considered rather than its statistical moments only (Pianosi & Wagener, 2015). The methods of this family measure sensitivity by estimating the variations that are induced in the output distribution when removing the uncertainty about one variable input factor (conditioning the factor). More specifically, the sensitivity to the variable input factor x_i is measured by quantifying the distance between the unconditional probability distribution of y, that is obtained when all inputs vary simultaneously, and the conditional distribution, that is obtained varying all inputs but x_i (*id est*, when x_i is conditioned). The cumulative probability density function-based approach, in particular, characterizes the conditional and unconditional distributions by their cumulative density functions. The main advantage of this approach is that the cumulative distribution functions vary between 0 and 1, regardless of the variation of the model output y, making the sensitivity indices absolute (Pianosi & Wagener, 2015).

The unconditional and conditional cumulative distribution functions are computed starting from the outputs of a Monte-Carlo simulation (unconditional output matrix and conditional output matrix).

In the following, the unconditional cumulative distribution function of the model output y will be denoted by $F(y)$, and the conditional cumulative distribution function with respect to x_i (when x_i is conditioned) will be denoted as $F_i(y)$. Since $F_i(y)$ accounts for what happens when the variability due to x_i is removed, its distance from $F(y)$ provides a measure of the effect of x_i on y. The limiting case occurs when $F_i(y)$ coincides with $F(y)$: in this case removing the uncertainty about x_i does not affect the output distribution and it can be concluded that x_i has no influence on y. Otherwise, if the distance between $F_i(y)$ and $F(y)$ increases, it means that the influence of x_i increases as well. As a measure of the distance between unconditional and conditional cumulative distribution functions, the Kolmogorov-Smirnov statistic is used:

$$KS_i = \max_y |F(y) - F_i(y)| \tag{2.13}$$

As KS_i depends on the value assumed by x_i, the PAWN sensitivity index T_i considers a statistic over all possible values of x_i.

According to this method, for each variable input factor (x_i) of the considered model, two sensitivity indices (PAWN sensitivity indices) are computed:

$$T_{max,i} = \max_{x_i}(KS_i) \tag{2.14}$$

$$T_{median,i} = \underset{x_i}{median}(KS_i) \tag{2.15}$$

$T_{max,i}$ quantifies the maximum possible sensitivity of the output to x_i, which happens only for a precise combination of input factors, while $T_{median,i}$ is the quantification of the most likely sensitivity of the output to the variable input factor x_i.

These PAWN sensitivity indices are suitable for *screening* and for *ranking* the variable input factors. The main associated drawback of the method is that it relies on Monte Carlo simulations, making it rather time consuming.

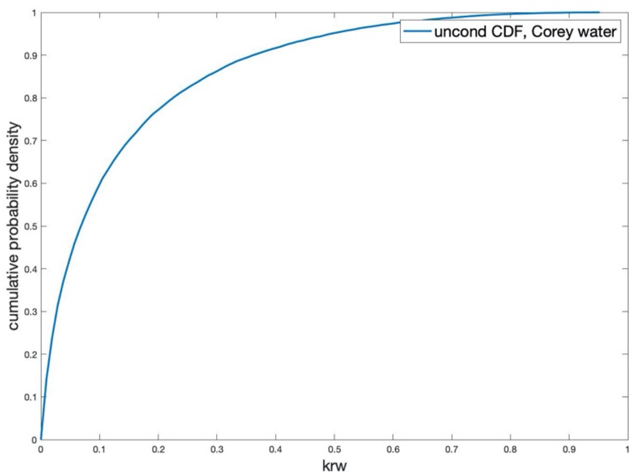

Figure 10: unconditional cumulative distribution function of the Corey relative permeability model for water, S_w=0.5

2.1.4. VARIOGRAM-BASED (VARS) SENSITIVITY ANALYSIS

In the VARS framework, the variograms of the model output are interpreted as a comprehensive manifestation of sensitivity (Razavi & Gupta, 2016).

In the field of spatial statistics, a variogram is a function that characterizes the spatial covariance structure of a stochastic process. The variogram of a mathematical model is defined as the variance of the differences between model output values computed at (a large number of) pairs of points at different locations across the input factors space, when these pairs of points are separated by the same difference.

Let $\boldsymbol{x} = [x_1, x_2, \dots, x_m]$ be a generic vector of variable input factors and the increment $\boldsymbol{h} = [h_1, h_2, \dots, h_m] = (\boldsymbol{x_A} - \boldsymbol{x_B})$ be the difference between two specific variable input factors vectors. The multi-dimensional variogram (γ) of the model output $y(\boldsymbol{x})$ is defined as:

$$\gamma(\boldsymbol{h}) = \frac{1}{2} V\big(y(\boldsymbol{x} + \boldsymbol{h}) - y(\boldsymbol{x})\big) \tag{2.16}$$

The variogram is only function of the increment $\boldsymbol{h}$. The relative permeability models respect the *constant mean assumption*, so the variogram can also be defined as:

$$\gamma(\boldsymbol{h}) = \frac{1}{2} E\left[\big(y(\boldsymbol{x} + \boldsymbol{h}) - y(\boldsymbol{x})\big)^2\right] \tag{2.17}$$

In order to compute the variogram-based sensitivity indices, in this essay, only mono-dimensional variograms are considered, where the increment affects only one variable input factor at a time and is of the form:

$$\boldsymbol{h}^1 = (h^1, 0, \dots, 0) \qquad \dots \qquad \boldsymbol{h}^m = (0, 0, \dots, h^m) \tag{2.18}$$

The j^{th} mono-dimensional increment relative to the i^{th} variable input factor (x_i) is referred to with the term h_j^i (scalar). The VARS sensitivity analysis of the relative permeability models is, again, based on Monte Carlo simulations: the matrices of the unconditional and conditional output of the model are needed for the computation of the variograms. This method requires a further input to evaluate the increments h_j^i: the ***conditioned input factors matrix***. This matrix (represented in the figure below) contains the values at which each parameter is conditioned when computing the conditional output matrix, and it is simply built by knowing the input variability space of the model and the size (p) of the conditional output matrix. Once all those inputs are defined, for each variable input factor the variogram can be calculated.

In the figure below it is also shown how the increments are defined from the conditioned input factors matrix:

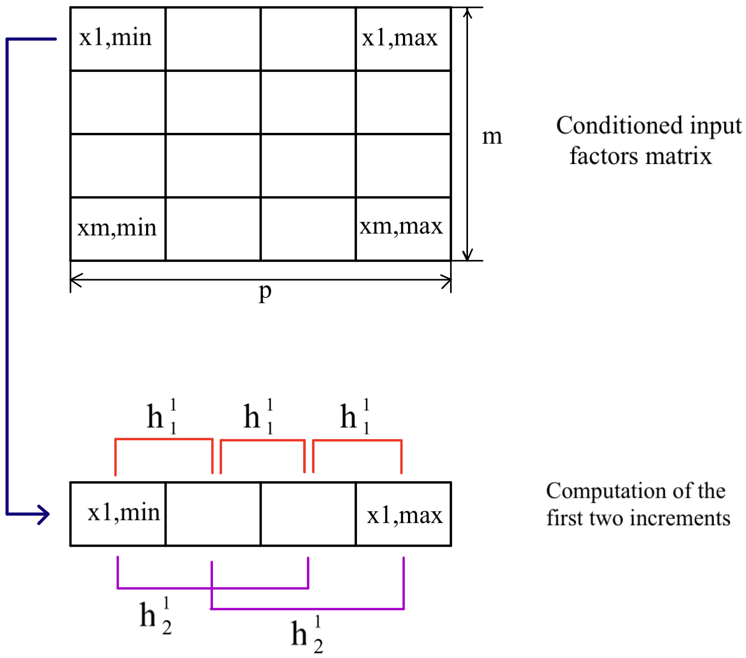

Figure 11: computation of the increments from the conditioned inputs matrix

By definition, h_2^i is two times h_1^i, h_3^i is three times h_1^i and so on. The maximum considered increment for the calculation of the variograms corresponds to 50% of the variability range of the factors, since it is the maximum meaningful increment for the variogram evaluation.

The variogram of the output with respect to each input factor is computed according to the formula:

$$\gamma_i\left(h_j^i\right) = \frac{1}{2}E\left[\left(y_i\left(x_1, \dots, x_i + h_j^i, \dots, x_m\right) - y_i(x_1, \dots, x_i, \dots, x_m)\right)^2\right] \quad (2.19)$$

Where y_i denotes the conditional output with respect to x_i (the variable input factor x_i is conditioned while the others are random variables) and γ_i denotes its variogram.

The figure below shows how the conditional output matrix is used to extract the pairs of points $[(y_i(x_1, \dots, x_i + h_j^i, \dots, x_m)\ ,\ y_i(x_1, \dots, x_i, \dots, x_m))]$ which are used to compute the variograms.

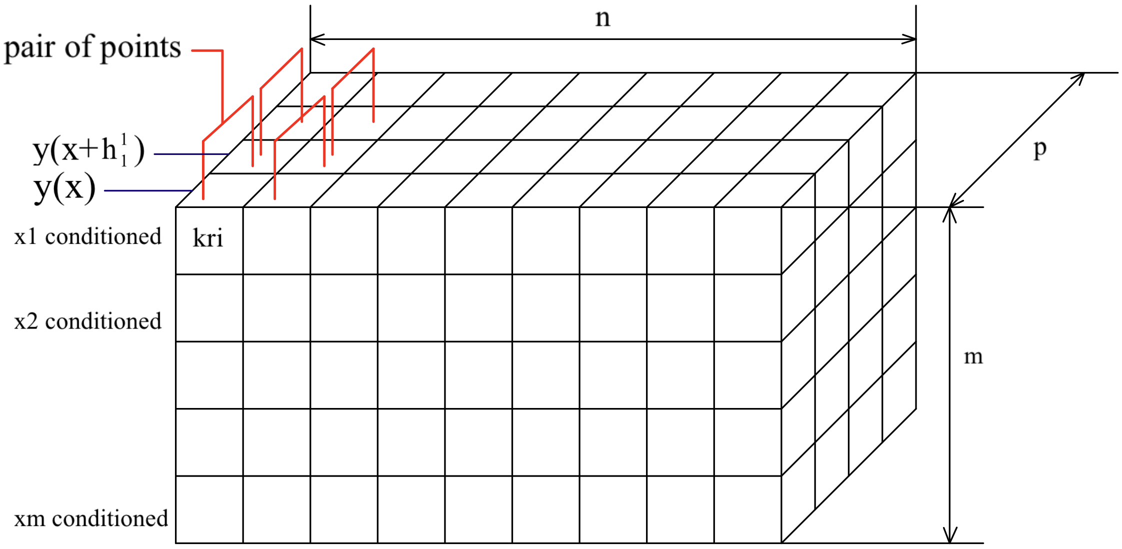

Figure 12: computation of the variogram from the conditional output matrix

The computed variograms are then normalized with respect to the increment h^i; in this way, even if the different input factors have different variability ranges, the value of $h^i_{normalized}$ always goes from 0 to 1, making the different variograms comparable and avoiding penalizing the quantification of the effect of factors with smaller variability ranges while computing the (integral) sensitivity indices.

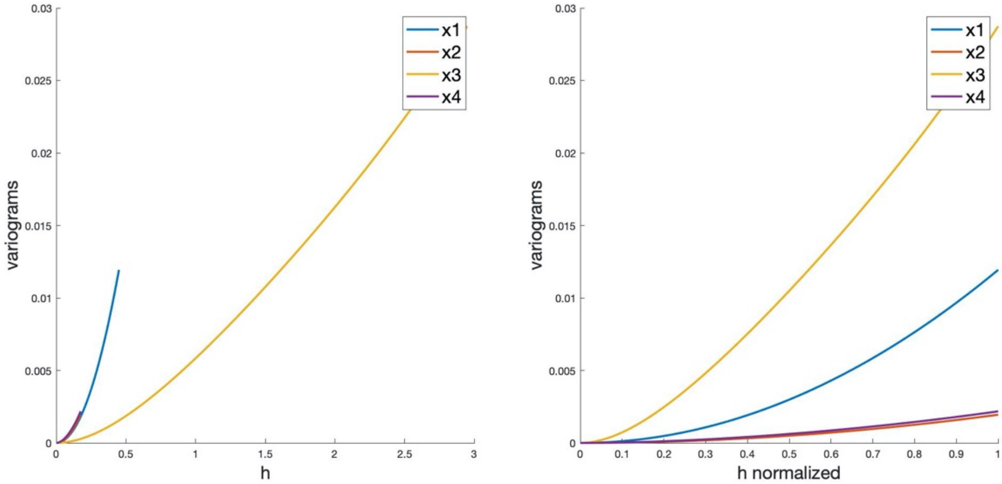

Figure 13: non-normalized (left) VS normalized (right) variograms of Corey relative permeability model for water relative permeability, S_w=0.5

According to the VARS sensitivity analysis method, for each variable input factor x_i two sensitivity indices are computed:

$$IVARS_{10,i} = \int_{h^i_{norm}=0}^{h^i_{norm}=0.2} \left(\gamma_i\left(h^i_{normalized}\right)\right) dh^i_{norm} \tag{2.20}$$

$$IVARS_{50,i} = \int_{h_{norm}^{i}=0}^{h_{norm}^{i}=1} \left(\gamma_i\left(h_{normalized}^{i}\right)\right) dh_{norm}^{i} \tag{2.21}$$

The sensitivity index IVARS_{10} is so called because the normalized increment $h_{normalized}^{i} = 0.2$ corresponds to 10% of the input variability space X_i; The sensitivity index IVARS_{50} is so called because the normalized increment $h_{normalized}^{i} = 1$ corresponds to 50% of the input variability space X_i. The sensitivity index IVARS_{10} is representative of the output sensitivity to small variations of the input variable factors (within 10% of the input variability space), while the index IVARS_{50} is representative of the output sensitivity to large variations of the input factors (within 50% of the input variability space). The variogram-based analysis can also study the sensitivity of the output against extremely small variations of the input factors, for instance by defining the index $IVARS_{0,1}$: in this hypothetical case the variogram-based sensitivity index would tend to be equal to the multiple start perturbation method index. In practice it is much more convenient to directly use the multiple start perturbation method to study the sensitivity of the output against small variations of the input factors.

Defining:

$$\Gamma_i(H_i) = \int_0^{H_i} \left(\gamma_i\left(h_{normalized}^{i}\right)\right) dh_{norm}^{i} \tag{2.22}$$

We can represent the IVARS (*Integrated Variogram Across a Range of Scale*) sensitivity indices as in the figure below:

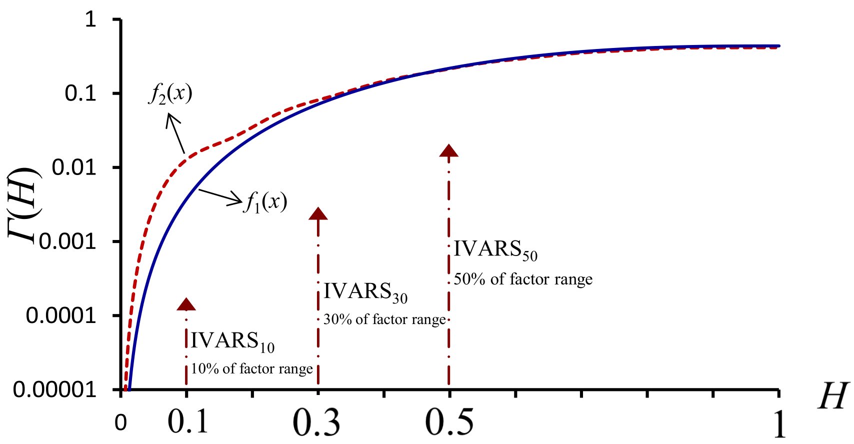

Figure 14: IVARS sensitivity indices

The IVARS sensitivity indices can provide a meaningful measure of the sensitivity of a model response to its input factors at different scales. In particular, IVARS-based sensitivity measures are normally used to *rank* the variable input factors according to their relative influence.

The main drawback of this sensitivity analysis method is its complicated workflow and the fact that it is extremely computationally intensive: both the Monte Carlo simulation and the computation of the variogram are highly time-consuming processes.

2.2. UNINFORMED GLOBAL SENSITIVITY ANALYSIS FOR MULTIPLE INTERPRETATIVE MODELS: METHODOLOGIES AND WORKFLOW

The quality and amount of data available in many practical situations justify the interpretation of the system under investigation through a collection of alternative interpretative models. This follows the observation that there is uncertainty about model structure/format when only empirical or semi-empirical interpretative models are available (as happens for the study of relative permeability). So, when studying a phenomenon through the simultaneous application of multiple models, two distinct sources of uncertainty can be identified (Dell'Oca, Riva, & Guadagnini, 2020), (Ye & Dai, 2015):

- Uncertainty affecting the model structure.
- Uncertainty affecting the input variable factors of each model.

In this context, quantification of the influence of these multiple sources of uncertainties on the output of interest is key to increase the understanding and confidence on model(s) functioning and guide further actions (including, for example, model calibration or collection of new data). The multi-model extension of global sensitivity analysis is not only preferable, but necessary when model uncertainty exists: identifying important parameters for a single model may be biased in that the important parameters identified for a single model may not be important to the processes that the models intend to simulate (Dell'Oca, Riva, & Guadagnini, 2020). The goal of this chapter is to provide the tools to quantify and allocate the uncertainty of the output obtained by the simultaneous application of many models in a completely uninformed scenario. It is appropriate to remind that a global uninformed sensitivity analysis is meaningful only if it is carried out when no previous knowledge (from experiments or model calibration) is available.

The multi-model sensitivity analysis techniques which are considered and used in this chapter are:

- Multi-model statistical moments-based global sensitivity analysis.
- Multi-model variance-based global sensitivity analysis.

2.2.1. MULTI-MODEL STATISTICAL MOMENTS-BASED GLOBAL SENSITIVITY ANALYSIS

This multi-model global sensitivity analysis allows investigating the sensitivity of the models output through diverse aspects of uncertainty, focusing on various statistical moments of the probability density function of the target output (Dell'Oca, Riva, & Guadagnini, 2020). Furthermore, it allows discriminating between the contribution to sensitivity due to the lack of knowledge in model format and the contribution due to the lack of knowledge of the input factors values. The single-model statistical moments-based sensitivity analysis method (see chapter 2.1.2.) defines sensitivity in terms of the average variation of the output's probability density function main statistical moments due to the model input factors uncertainty and proposes summary sensitivity indices (AMA indices) to quantify the concept. Here the AMA indices are extended to embed the effect of uncertainties both in the system model conceptualization and in the models input factors.

In the following, the q different models which are considered in a multi-model context are denoted as M^j, $j = (1, \dots, q)$. This analysis examines an unconstrained case, *id est*, when no data are available to constrain input factors uncertainty and to evaluate the relative plausibility of each considered model. When prior information is unavailable, equal prior probability (or a-priori model weight) w^j is assigned to each model M^j:

$w^1 = w^2 = \dots = w^q = {}^1/_q$ (2.23)

This sensitivity analysis method is based on Monte Carlo simulations of the relative permeability models: for each considered model M^j a Monte Carlo simulation is performed so as to obtain the corresponding unconditional output matrix ($uncond\_output^j$) and conditional output matrix. The conditional output matrices are then elaborated aiming to obtain the conditional statistical moments matrices $\boldsymbol{E^j}$, $\boldsymbol{V^j}$, $\boldsymbol{Sk^j}$ and $\boldsymbol{K^j}$, $j = 1, \dots, q$ (see chapter 2.1.2. for any additional information).

The unconditional output matrices are used to compute the *multi-model unconditional output statistics*:

- $E_{uncond}^{multimodel} = \sum_{j=1}^{q} w^j \{E(uncond\_output^j)\}$ (2.24)
- $V_{uncond}^{multimodel} = \sum_{j=1}^{q} w^j \{V(uncond\_output^j)\} + \sum_{j=1}^{q} w^j \{[E_{uncond}^{multimodel} - E(uncond\_output^j)]^2\}$ (2.25)
- $Sk_{uncond}^{multimodel} = \sum_{j=1}^{q} w^j \left\{ Sk(uncond\_output^j) \left[\frac{V(uncond\_output^j)}{V_{uncond}^{multimodel}}\right]^{\frac{3}{2}} \right\} + -\sum_{j=1}^{q} w^j \left\{ \frac{\left[E_{uncond}^{multimodel} - E(uncond\_output^j)\right]^3}{(V_{multimodel}^{uncond})^{\frac{3}{2}}} \right\} - 3\sum_{j=1}^{q} w^j \left\{ [E_{uncond}^{multimodel} + -E(uncond\_output^j)] \frac{V(uncond\_output^j)}{\left(V_{uncond}^{multimodel}\right)^{\frac{3}{2}}} \right\}$ (2.26)
- $K_{uncond}^{multimodel} = \sum_{j=1}^{q} w^j \left\{ K(uncond\_output^j) \left[\frac{V(uncond\_output^j)}{V_{uncond}^{multimodel}}\right]^{2} \right\} + \sum_{j=1}^{q} w^j \left\{ \frac{\left[E_{unconditional}^{multimodel} - E(uncond\_output^j)\right]^4}{\left(V_{uncond}^{multimodel}\right)^{2}} \right\} + 4\sum_{j=1}^{q} w^j \left\{ \left[\frac{E_{uncond}^{multimodel} - E(uncond\_output^j)}{V_{uncond}^{multimodel}}\right]^2 \left[\frac{3}{2} V(uncond\_output^j) + -\frac{Sk(uncond\_output^j)[V(uncond\_output^j)]^{\frac{3}{2}}}{E_{uncond}^{multimodel} - E(uncond\_output^j)}\right] \right\}$ (2.27)

Once these multi-model unconditional statistics have been calculated, it is possible to compute the sensitivity indices. The multi-model moments-based sensitivity indices quantify sensitivity by the average variation of the first four statistical moments of the output due to conditioning of the adopted model and of the variable input factors. These sensitivity indices are structured according to two key components:

- A *model-choice contribution* related to the conditioning of the adopted model.
- An *input factor-choice contribution* related to the conditioning of the input factors.

The sensitivity index corresponding to the variable input factor x_i^j (i^{th} input factor belonging to the model M^j) is referred to as AMA_i^j. It should be noticed that different models may have a different number of variable input factors.

According to this method, for each variable input factor belonging to each model, four sensitivity indices are defined and calculated:

- $AMAE_i^j = \frac{w^j}{d_E}\{|E_{uncond}^{multimodel} - E(uncond\_output^j)| + E(|\boldsymbol{E}^j(i,:) - E(uncond\_output^j)|\}$

$$\tag{2.28}$$

With

$$d_E = \begin{cases} |E_{uncond}^{multimodel}|, & E_{uncond}^{multimodel} \neq 0 \\ 1 & , otherwise \end{cases} \tag{2.29}$$

The first term in the curly brackets is the *model-choice contribution*, the second is the *input factor-choice contribution* (this statement stays true to all the sensitivity indices below). The terms d_V, d_{Sk} and d_K are calculated analogously to d_E for the sensitivity indices below, considering the corresponding statistical moment.

- $AMAV_i^j = \frac{w^j}{d_V}\{|V_{uncond}^{multimodel} - V(uncond\_output^j)| + E(|\boldsymbol{V}^j(i,:) - V(uncond\_output^j)|\}$

$$\tag{2.30}$$

- $AMASk_i^j = \frac{w^j}{d_{Sk}}\{|Sk_{uncond}^{multimodel} - Sk(uncond\_output^j)| + E(|\boldsymbol{Sk}^j(i,:) + -Sk(uncond\_output^j)|\}$ (2.31)

- $AMAK_i^j = \frac{w^j}{d_K}\{|K_{uncond}^{multimodel} - K(uncond\_output^j)| + E(|\boldsymbol{K}^j(i,:) - K(uncond\_output^j)|\}$

$$\tag{2.32}$$

These sensitivity indices are particularly suitable for *ranking* and for *screening*.

It is important to observe that, in this multi-model approach, each considered model retains its own identity and that different models do not share input factors: even if an input factor appears in more than one model its contribution to the output variability changes according to the model to which it is considered belonging (the same input factor appearing in two models behaves like two distinct input factors).

2.2.2. MULTI-MODEL VARIANCE-BASED GLOBAL SENSITIVITY ANALYSIS

The variance-based sensitivity analysis methods consider the output variance as the only metric upon which sensitivity is quantified. The multi-model variance-based method is an extension of Sobol's sensitivity analysis (in-depth presentation in chapter 2.1.2.): Sobol sensitivity indices are defined with consideration of only input factors uncertainty, while multi-model variance-based sensitivity analysis is centred on a hierarchical structure of input factors and model uncertainty, deriving new global sensitivity indices for multiple models (Ye & Dai, 2015). The multi-model variance-based sensitivity indices rely on the concept of model averaging. This (global) analysis considers an unconstrained case, *id est*, when no data are available to constrain input factors uncertainty and to evaluate the relative plausibility of each considered model, so equal prior probability w^j (or a-priori model weight) is assigned to each model M^j:

$$w^1 = w^2 = \cdots = w^q = {}^1/_q \tag{2.33}$$

This sensitivity analysis method is based on Monte Carlo simulations of the models: for each considered model M^j a Monte Carlo simulation is performed intending to obtain the corresponding unconditional output matrix ($uncond\_output^j$) and conditional output matrix. The conditional output matrices are then elaborated in order to obtain the statistical moments matrices $\boldsymbol{E}^j$, $j = 1, \dots, q$. Furthermore, with a focus on the total order effect of the input factors, for each model and for each input factor the matrix $\boldsymbol{V}^j_{\sim i}$ must be evaluated (see chapter 2.1.2. for further details). Before defining the sensitivity indices, it is important to understand that in this multi-model approach the models lose their identity as singular entities: the different models are considered acting as a single more complex model. It follows that if a certain input factor appears in more than one model, it is anyway considered as one single input variable factor (as opposed to what happens for the multi-model statistical moments-based approach). For this reason, the variable input factors belonging to this new more complex "hybrid model" must be redefined with respect to their single-model definition. Because of this behaviour of the variable input factors, each multi-model sensitivity index is only associated to the corresponding input factor x_i, not to a specific model.

The multi-model variance-based sensitivity indices are:

- Multi-model first order Sobol sensitivity index:

$$Sf_i^{multimodel} = \frac{\sum_{j=1}^{3} w^j V[\boldsymbol{E}^j(i,:)]}{\sum_{j=1}^{3} w^j V(uncond\_output^j)} \tag{2.34}$$

  This sensitivity index takes into account the influence of the variable input factor x_i under the individual models and provides a quantitative assessment of global sensitivity analysis with combined effects of uncertain input factors and models.

- Multi-model total order Sobol sensitivity index:

$$St_i^{multimodel} = \frac{\sum_{j=1}^{3} w^j E[\boldsymbol{V}^j_{\sim i}]}{\sum_{j=1}^{3} w^j V(uncond\_output^j)} \tag{2.35}$$

  This total order index considers also the effect of possible interactions between x_i and the other input variable factors. It should be remembered that a variable input factor x_i interacts with the others when its associated sensitivity depends on the value assumed by the other input factors.

It should be noticed that, if an input factor x_i does not belong to the model M^j, it follows:

$$V[\boldsymbol{E}^j(i,:)] = E[\boldsymbol{V}^j_{\sim i}] = 0 \tag{2.36}$$

So, the single-model Sobol (variance-based) sensitivity indices of the model M^j, corresponding to an input variable factor from which M^j does not depend, are equal to zero.

These multi-model sensitivity indices are particularly indicated for *ranking* and the total order index can be used for *screening*: a value of 0 of the corresponding total order index is a necessary and sufficient condition for a factor to be non-influential.

2.3. MULTI-MODEL, INFORMED GLOBAL SENSITIVITY ANALYSIS: METHODOLOGIES AND WORKFLOW

The aim of the informed global sensitivity analysis techniques is exactly the same of the standard (uninformed) global sensitivity analysis: investigating how the variability in the output y can be attributed to the uncertainty in the different input variable factors x_i and to the uncertainty in the interpretative model structure M^j (in a multi-model context). The only difference between the two approaches is that, while uninformed global sensitivity analysis considers variations within the entire space of variability of the input factors according to a uniform probability density function, the informed global sensitivity analysis considers the output variability against variations of the input factors around a specific reference input factors vector $\overline{\boldsymbol{x}} = [\overline{x_1}, \overline{x_2}, \dots, \overline{x_m}]$, which is obtained from model calibration or parameters estimation techniques. The informed global approach should not be confused with the local sensitivity analysis approach: while in the local techniques the input variability space is limited between specific values around the reference input vector $\overline{\boldsymbol{x}}$, in the informed global approach the input factors can vary within their entire variability space, but according to a probability density function which properly accounts for the higher probability of the input factors to lay close to their nominal (reference) values, rather than far from them. The use of an informed approach when performing a sensitivity analysis is not only preferable, but necessary, when previous knowledge of the model(s) input variable factors is provided by experimental measurements or by any inverse modelling technique: the indiscriminate use of a global approach when input factors have been correctly calibrated is not useful and can lead to deceiving conclusions. The application of informed global sensitivity analysis requires as extra inputs with respect to its unconditional counterpart the user-defined vector of the nominal input factors $\overline{\boldsymbol{x}}$ and the posterior probabilities w^j of the different models M^j (if a multi-model approach is adopted). The single-model and multi-model sensitivity analysis techniques described in sections 2.1. and 2.2. can be applied according to the exact same methodologies for informed global sensitivity analysis.

In this essay only multi-model techniques will be applied for informed sensitivity analysis purposes, because of how the models of interest are calibrated in the studied scenarios. The adopted sensitivity analysis methodologies are the ones described in chapter 2.2. In the studied cases a posterior probability w^j (or a-posteriori model weight) is assigned to each of the considered models M^j according to their relative skill to interpret the experimental observations (which is quantified by the models' posterior probabilities conditional to the observations in a maximum-likelihood estimation framework). The reference input factors are expressed by intervals $(\bar{x}_{i,min}, \bar{x}_{i,max})$ associated with their lower and upper limits identifying the Gaussian 95% confidence intervals around their estimated value. For each variable input factor x_i, its Gaussian distribution parameters (mean μ_i and standard deviation σ_i) can be derived from the provided confidence interval:

$$\mu_i = \frac{(\bar{x}_{i,min} + \bar{x}_{i,max})}{2} \quad (2.37)$$

$$\sigma_i = \frac{(\bar{x}_{i,max} - \bar{x}_{i,min})}{4} \quad (2.38)$$

According to these hypotheses each variable-input factor, when not conditioned (according to the sensitivity analysis workflow described in chapter 2.2.), is considered a random variable with Gaussian probability density function.

3. WATER-OIL RELATIVE PERMEABILITY MODELS

The relative permeability $k_{r\alpha}$ of a certain phase α is always a function of the phase saturation, *id est,* of the fraction of the fluid volume which is occupied by the phase α:

$$k_{r\alpha} = k_{r\alpha}(S_\alpha) \tag{3.1}$$

Because of the definition of the saturation of a phase, for a water-oil binary mixture it follows that:

$$S_w + S_o = 1 \tag{3.2}$$

Where S_w is the water saturation and S_o is the oil saturation.

In this essay the two-phase relative permeability models which are mainly employed in oil industry and for any industrial applications requiring water-oil relative permeability quantifications will be considered: the Corey model, the Chierici model, and the LET (Lomeland-Ebeltoft-Thomas) model.

In order to perform a global sensitivity analysis, for each of the considered models the vector of the variable input factors $\boldsymbol{x}$ and the input variability space $\boldsymbol{X}$ must be defined. While the definition of the variable input factors is trivial, the definition of the variability space is not. In fact, some of the relative permeability models' parameters are meant to be evaluated by model calibration starting from a set of experimental measurements of water-oil relative permeability. For this reason, their value lays in a range which is not limited by any mathematical definition or mathematical constraint and could therefore be infinitely wide. Hence, for each of the models, the variability space of each input factor is determined by two main criteria:

- Studying how the value of the considered input factor affects the relative permeability curve $k_{ri} = f(\boldsymbol{x}, S_w)$ and, accordingly, limiting the variability range of the parameter to the values which produce curves that can be considered representative of all the real-case scenarios.
- Trading-off the variability space of the input factors which can be in conflict (for example, the irreducible water saturation S_{wi} and the residual oil saturation S_{or}).

While studying the effect of a certain model input, all the other variable input factors are kept constant at the average value defined by their variability space.

3.1. THE COREY RELATIVE PERMEABILITY MODEL

The Corey model (Corey, 1954) is usually employed due to its simplicity, the limited amount of input data requirements, and the small number of parameters to be estimated. The mathematical structure of the model rests on capillary pressure concepts and is widely accepted to be fairly accurate for consolidated porous media. The model has also been proposed for unconsolidated sands through proper tuning of its parameters. Corey's equations for wetting (water) and non-wetting (oil) relative permeability read:

$$k_{rw} = k_{rw}^0 \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}\right)^{N_w} \tag{3.3}$$

$$k_{ro} = k_{ro}^0 \left(\frac{S_o - S_{or}}{1 - S_{wi} - S_{or}}\right)^{N_o} \tag{3.4}$$

With:

- $k_{rw}^0 = k_{rw}^{max}, k_{ro}^0 = k_{ro}^{max}$: end points of water and oil relative permeability curves.
- S_w: water saturation.
- $S_o = (1 - S_w)$: oil saturation.
- S_{wi}: irreducible water saturation, *id est* the water saturation for which $k_{rw} = 0$.
- S_{or}: residual oil saturation, *id est* the oil saturation for which $k_{ro} = 0$.
- N_w, N_o: parameters of the Corey model to be estimated through model calibration.

3.1.1. INPUT FACTORS AND VARIABILITY SPACE

Corey model for water relative permeability is now considered:

$$k_{rw} = k_{rw}(S_w) = k_{rw}^0 (\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}})^{N_w}$$

Observing the analytical expression of the model it is possible to identify four independent model parameters, which are the variable input factors x_i of the model.

The goal is now to determine the minimum and maximum value of each parameter, which define the Corey model input variability space. The minimum and maximum values of a certain input variable x_i are referred as $x_{i,min}$ and $x_{i,max}$. Before performing any calculations, some considerations can already be made:

- x_1: k_{rw}^0; it must be greater than 0 (otherwise, the curve represents a case of complete imperviousness to water) and it can be as high as 1 (because of the definition of relative permeability).
- x_2: S_{wi}; it can assume any value between 0 (no irreducible/initial water saturation) and 1 (the porous rock is completely full of water which can not be displaced).
- x_3: N_w; it can assume any value greater than 0 without producing non-sensical relative permeability results.
- x_4: S_{or}; it can assume any value between 0 (no residual oil) and 1 (the porous rock is completely full of oil which can not be displaced).
- Because of the definition of water and oil saturations there are constraining relationships between water saturation S_w, irreducible water saturation S_{wi} and residual oil saturation S_{or}:

    - $S_w \geq S_{wi}$
    - $S_w \leq (1 - S_{or})$
    - $(S_{wi} + S_{or}) \leq 1$

  These three conditions must always be satisfied, otherwise, non-sensical relative permeability curves will be obtained. It follows that the values of $S_{wi,max}$, $S_{or,max}$ and the values of water saturation S_w in which the model can be evaluated must be chosen according to a trade-off: having a wider variability range for S_{wi} and S_{or} leads to a narrower range of acceptable S_w values.

Now the input variability space of each factor can be fixed by trial and error, and the resulting output curves can be analysed. If the range of variability of a parameter produces a sufficiently wide change in the output (relative permeability curve), the variability space is considered satisfactory (*id est*, representative of the most of real case scenarios) and can be adopted; if not, the range of variability is made wider.

The adopted input variability space for Corey water relative permeability model is:

- $k^0_{rw,min} = 0.1$ $\quad k^0_{rw,max} = 1$
- $S_{wi,min} = 0$ $\quad S_{wi,max} = 0.35$
- $N_{w,min} = 0.1$ $\quad N_{w,max} = 6$
- $S_{or,min} = 0$ $\quad S_{or,max} = 0.35$

According to these assumptions, the model can be evaluated in $(0.35 \leq S_w \leq 0.65)$.

It should be remembered that while studying the effect of a certain model input, all the other variable input factors are kept constant at the average value defined by their variability space.

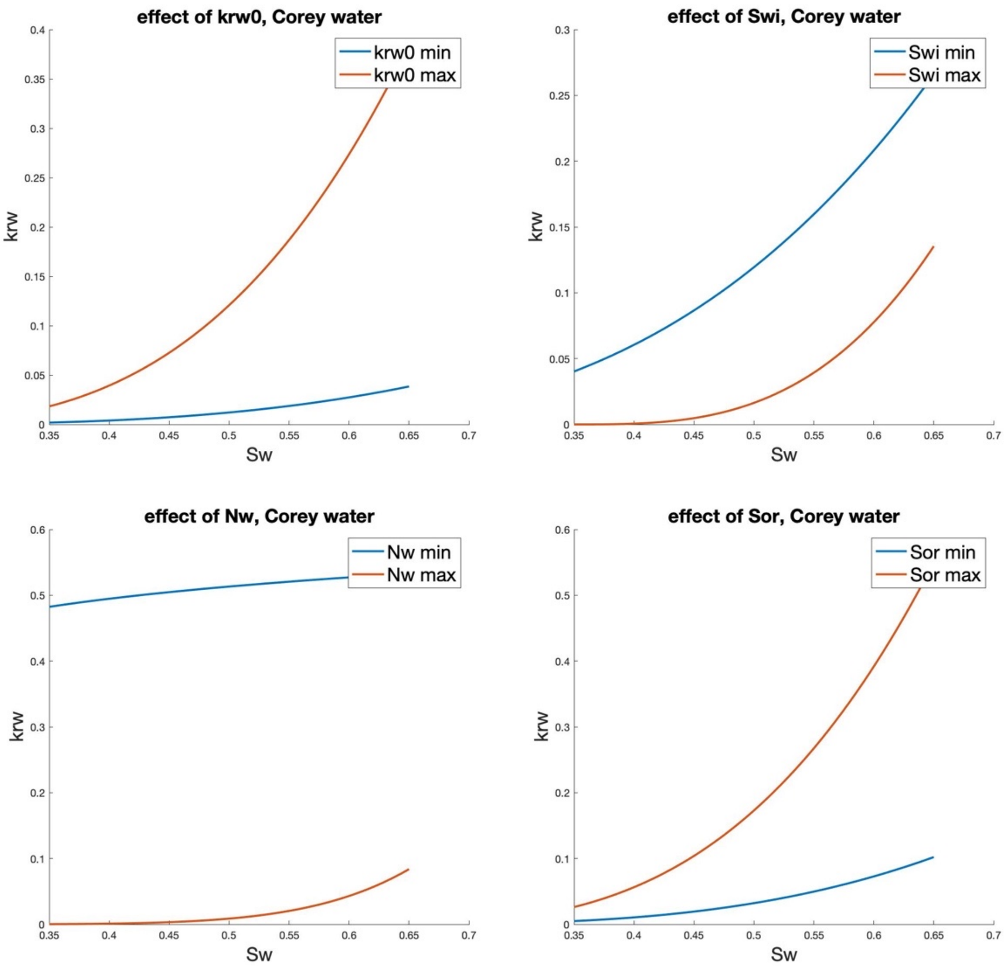

Figure 15: effect of the variable input factors on Corey model for water relative permeability

A qualitative observation of the resulting water relative permeability curves shows that the input factor N_w has by far the most dramatic effect on the output. It should be noted that, according to the assumptions, in the global sensitivity analysis approach the resulting relative permeability curve can be consequence of any allowed combination of input variable factors, *id est* of any input variable factors combination laying in the input variability space.

The exact same procedure must be repeated in order to study the input variability space of Corey model for oil relative permeability. The three considered water-oil relative permeability models have a structure for which the relative permeability curves obtained for oil are perfectly symmetrical to the ones obtained for water (symmetrical with respect to water saturation S_w), when the same input factors values are adopted. For all the considered models the input factors S_{wi} and S_{or} have inverted effects on water and oil relative permeability curves, due to the models' format. For these reasons, the results obtained when studying the input variability space of the models for oil relative permeability are simply symmetrical to the ones obtained for water, and the effects of the input factors S_{wi} and S_{or} are inverted with respect to the ones observed for water relative permeability.

The detailed study of the input variability space of Corey model for oil relative permeability is reported in appendix A, section A1.

3.2. THE CHIERICI RELATIVE PERMEABILITY MODEL

The Chierici model (Chierici, 1984) provides a reasonably good match against experimental relative permeability curves. It is considered to provide improved approximations at and near the initial and end points of these curves when compared against the Corey model and other polynomial approximations. The flexibility of the model is mainly due to the possibility of representing concave and/or convex relative permeability curves as a function of parameters values.

$$k_{rw} = k_{rw}^0 exp\left[-B_w\left(\frac{S_w - S_{wi}}{1 - S_{or} - S_w}\right)^{-M_w}\right] \qquad (3.5)$$

$$k_{ro} = k_{ro}^0 exp\left[-B_o\left(\frac{S_w - S_{wi}}{1 - S_{or} - S_w}\right)^{+M_o}\right] \qquad (3.6)$$

With:

- B_w, B_o, M_w, M_o: parameters of the Chierici model to be estimated through model calibration.

3.2.1. INPUT FACTORS AND VARIABILITY SPACE

$$k_{rw} = k_{rw}(S_w) = k_{rw}^0 exp\left[-B_w\left(\frac{S_w - S_{wi}}{1 - S_{or} - S_w}\right)^{-M_w}\right]$$

Observing the analytical expression of the model it is possible to identify five independent model parameters, which are the variable input factors x_i of the model:

- x_1: k_{rw}^0.
- x_2: S_{wi}.
- x_3: B_w; it can assume any value greater than 0 without producing non-sensical relative permeability results.
- x_4: M_w; it can assume any value greater than 0 without producing non-sensical relative permeability results.
- x_5: S_{or}.

The adopted input variability space for Chierici model for water relative permeability is:

- $k_{rw,min}^0 = 0.1$ $k_{rw,max}^0 = 1$
- $S_{wi,min} = 0$ $S_{wi,max} = 0.35$
- $B_{w,min} = 0.1$ $B_{w,max} = 8$
- $M_{w,min} = 0.1$ $M_{w,max} = 8$
- $S_{or,min} = 0$ $S_{or,max} = 0.35$

According to these assumptions, the model can be evaluated in ($0.35 \leq S_w \leq 0.65$).

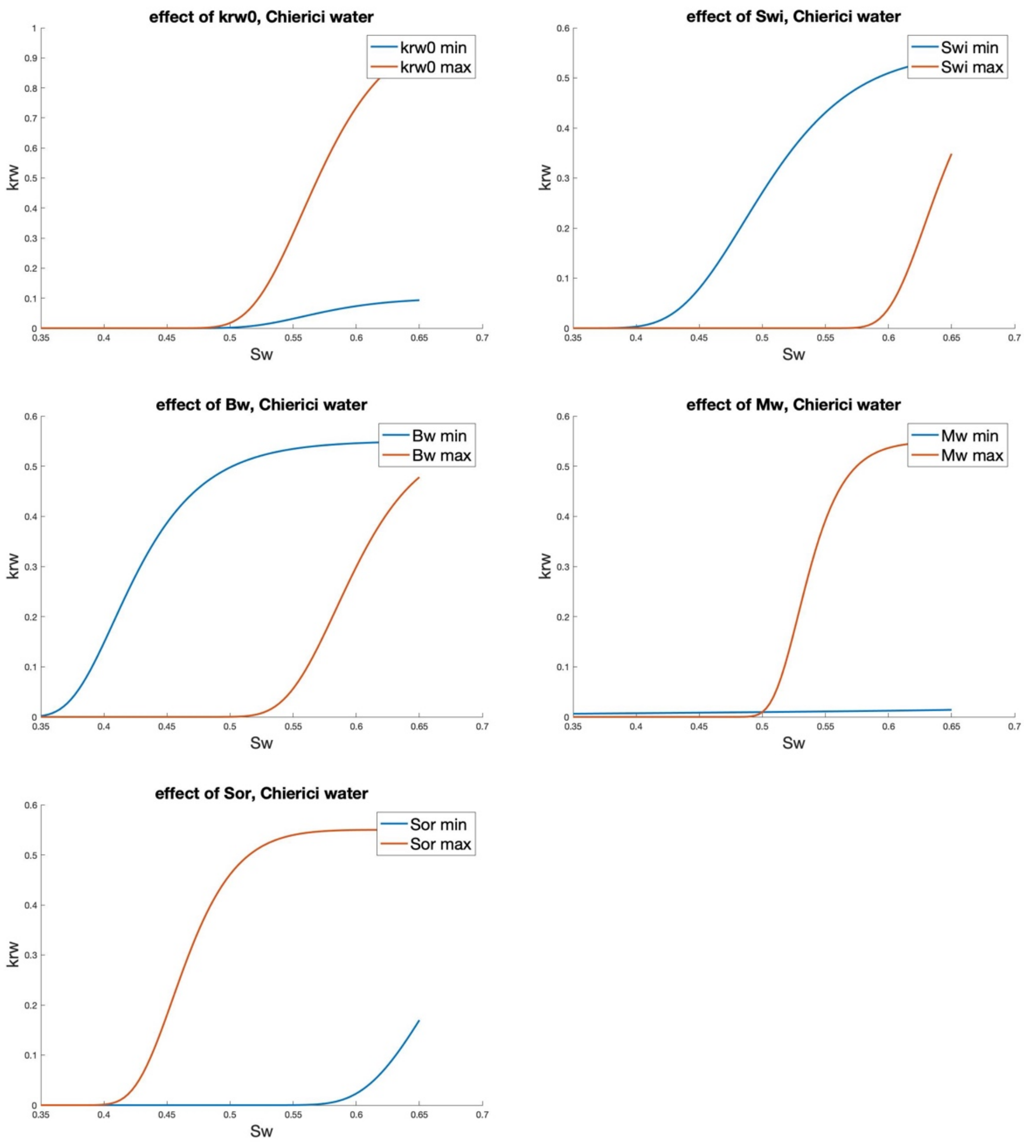

Figure 16: effect of the variable input factors on Chierici model for water relative permeability

The plots above show that all the input variable factors have a really strong effect on the output of Chierici model for water relative permeability. For this reason, it is appropriate to avoid any statement until a proper quantitative analysis is performed.

The detailed study of the input variability space of Corey model for oil relative permeability is reported in appendix A, section A2.

3.3. THE LET RELATIVE PERMEABILITY MODEL

The LET model (Lomeland, Ebeltoft, & Thomas, 2005) was proposed as a new versatile model. It is expressed in the form:

$$k_{rw} = k_{rw}^0 \frac{{S_w^*}^{L_w}}{{S_w^*}^{L_w} + E_w(1-S_w^*)^{T_w}} \tag{3.7}$$

$$k_{ro} = k_{ro}^0 \frac{(1-S_w^*)^{L_o}}{(1-S_w^*)^{L_o} + E_0(S_w^*)^{T_o}} \tag{3.8}$$

With:

- $S_w^* = \frac{(S_w - S_{wi})}{(1-S_{wi}-S_{or})}$: normalized water saturation.
- $L_w, L_o, E_w, E_o, T_w, T_o$: parameters of the LET model to be estimated through model calibration.

The values of T_α and L_α respectively drive the shape of the lower and upper part of the relative permeability curve, while E_α describes the slope and the elevation of the central portion of the curve. As such, the model is designed to include diverse parts of the relative permeability curve to capture variable behaviours across the entire water saturation range. The model has been shown to provide good interpretation of experimental data over a considerable range of saturations.

3.3.1. INPUT FACTORS AND VARIABILITY SPACE

$$k_{rw} = k_{rw}(S_w) = k_{rw}^0 \frac{{S_w^*}^{L_w}}{{S_w^*}^{L_w} + E_w(1-S_w^*)^{T_w}}$$

$$S_w^* = \frac{S_w - S_{wi}}{1-S_{wi}-S_{or}}$$

Observing the analytical expression of the model it is possible to identify six independent model parameters, which are the variable input factors x_i of the model:

- x_1: k_{rw}^0.
- x_2: S_{wi}.
- x_3: L_w; the inventors of the LET model suggest considering only values of $L_w \geq 0.1$. Despite of that, any value above 0 is mathematically acceptable (it does not produce non-sensical relative permeability curves).
- x_4: E_w; the inventors of the LET model suggest considering only values of $E_w \geq 0$.
- x_5: T_w; the inventors of the LET model suggest considering only values of $T_w \geq 0.1$. Despite of that, any value above 0 is mathematically acceptable.
- x_6: S_{or}.

The adopted input variability space for LET model for water relative permeability is:

- $k_{rw,min}^0 = 0.1$ $k_{rw,max}^0 = 1$
- $S_{wi,min} = 0$ $S_{wi,max} = 0.35$
- $L_{w,min} = 0.05$ $L_{w,max} = 10$
- $E_{w,min} = 0.005$ $E_{w,max} = 10$

- $T_{w,min} = 0.05$ $T_{w,max} = 8$
- $S_{or,min} = 0$ $S_{or,max} = 0.35$

According to these assumptions, the model can be evaluated in $(0.35 \leq S_w \leq 0.65)$.

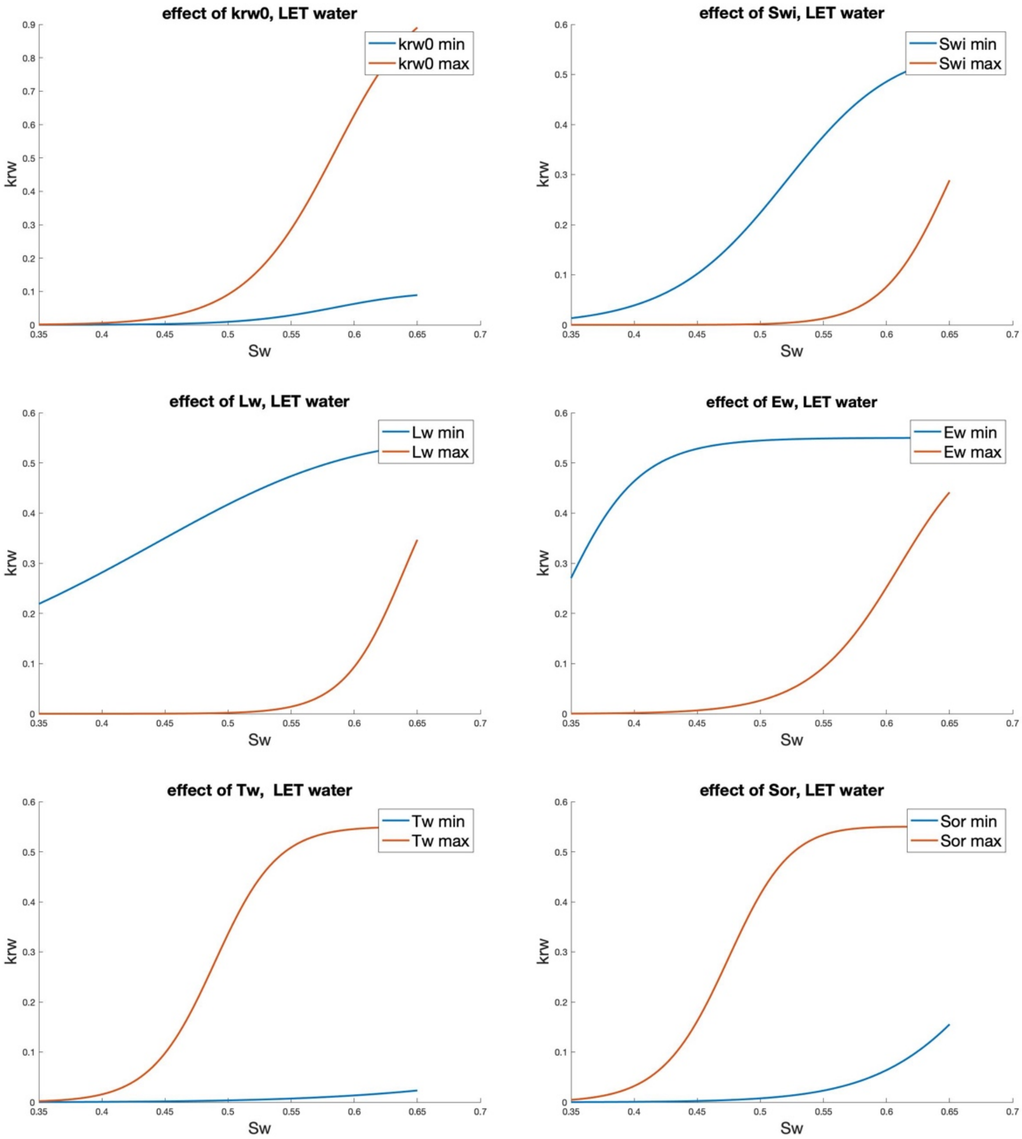

Figure 17: effect of the variable input factors on LET model for water relative permeability

The plots above show that all the input variable factors have an extremely strong effect on the output of LET model for water relative permeability. For this reason, it is appropriate to avoid any consideration until a proper quantitative analysis is performed.

The detailed study of the input variability space of Corey model for oil relative permeability is reported in appendix A, section A3.

4. RESULTS FOR WATER RELATIVE PERMEABILITY

4.1. SINGLE-MODEL UNINFORMED SCENARIO

In this section the four single-model global sensitivity analysis techniques presented in chapter 2.1. are applied to the three relative permeability models according to the stated assumptions and methodologies. The sensitivity indices are computed for 100 different values of water saturation S_w between $S_{w,min}$ and $S_{w,max}$ and they are plotted as functions of S_w. The results are presented, and their most significant aspects are discussed. The focus is mainly set on the quantitative *ranking* of the variable input factors of each model on the base of their relative importance with respect to the output variability. The rankings derived from different sensitivity analysis methods are studied to observe possible analogies and/or discordances. Eventually, for each relative permeability model the results obtained from the different sensitivity analysis techniques are summarized in a table for the sake of an easy visual inspection.

4.1.1. COREY MODEL

4.1.1.1. MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

$r = 500{,}000$ (number of elementary effects evaluations)

In the figure below the multiple start perturbation method sensitivity indices and the corresponding standard deviations of the elementary effects are reported as function of water saturation S_w for each of the Corey model input factors:

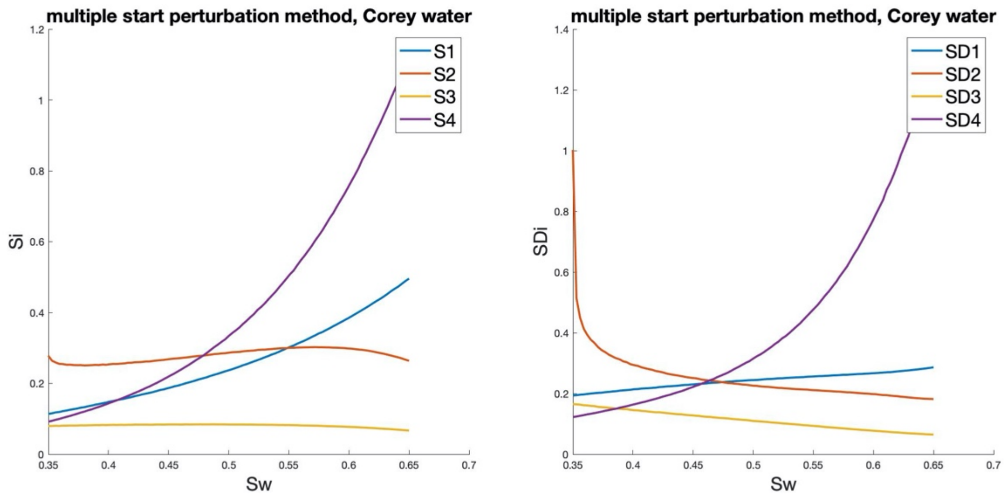

Figure 18: multiple start perturbation method global sensitivity analysis of Corey model for water relative permeability

An interesting aspect that can be observed from the sensitivity indices (S_i) plots is that the sensitivity associated to the input factors x_2 and x_3 is fairly constant on the whole water saturation domain, while the sensitivity associated to x_1 and x_4 strongly increases for increasing S_w.

The standard deviation of the elementary effects SD_i is an indicator of the interaction of the i^{th} variable input factor with the others: a high value of SD_i indicates that the sensitivity of the output with respect to x_i strongly depends on the values assumed by other variable input factors, while a value of SD_i equal to 0 indicates that the sensitivity to x_i is independent from the values assumed by the other variable input factors. From the plots above it can be observed that for high values of water saturation the sensitivity index S_4 (relative to the input factor x_4) strongly depends on the values assumed by the other factors.

For the multiple start perturbation method global sensitivity analysis applied to the relative permeability models the *ranking* of the variable input factors is determined by calculating and sorting from the highest to the lowest the values of the ranking indices $R_{S,i}$:

$$R_{S,i} = \sum_{S_{w,min}}^{S_{w,max}} S_i \quad (4.1)$$

In the table below the ranking of the variable input factors of Corey model for water relative permeability is reported as obtained from the multiple start perturbation method sensitivity analysis. In the brackets, the corresponding ranking indices values are reported in order to quantify the difference between the relative importance of the input factors.

Ranking according to $R_{S,i}$
x_4: S_{or} (42.86)
x_2: S_{wi} (28.11)
x_1: k^0_{rw} (26.22)
x_3: N_w (8.14)

In chapter (3.1.) it was qualitatively observed that the variable input factor x_3 seems to have the greatest effect on the model output. It is interesting to observe that, in clear opposition to the previous statement, according to the multiple start perturbation method sensitivity analysis x_3 has the smallest effect on the output variability. This result can be explained by looking at the methodology of this sensitivity analysis technique (see chapter 2.1.1.): the sensitivity indices S_i are numerical approximations of the output partial derivatives with respect to the input factors. This means that the sensitivity indices quantify the effect on the output variability only against extremely small variations of the variable input factors. This method is completely blind to the effect associated to the whole variability space of the input factors. From this analysis it results that small variations of the input factor x_3 produce minor variations on the model output, despite the huge potential effect of x_3 when considering its entire variability space. On the other hand, small variations of x_4 can have a strong effect on the output. This sensitivity analysis method is then particularly meaningful for the cases in which the variable input factors of a model are meant to be experimentally measured, and these measurements are affected by a reasonably small uncertainty: in these cases, the analysis successfully quantifies the effect of the uncertainty associated with the measurement errors, while considering other sensitivity analysis techniques would probably produce deceiving conclusions. It is important to be aware that any uninformed global sensitivity analysis technique is a tool for the a-priori study of a model, hence it is applied before any experimental measurement is available. If the input factors of the Corey model were hypothetically measured/calibrated with a small associated error, it could be stated that the uncertainty associated to x_3 is less significant and troublesome than the uncertainty associated to x_4.

The exact same procedure is repeated in order to study the sensitivity of Corey model for oil relative permeability. As mentioned in the previous section, the results obtained when studying the models for oil relative permeability are simply symmetrical to the ones obtained for water, and the effects of the input factors S_{wi} and S_{or} are inverted with respect to the ones observed for water relative permeability.

The detailed multiple start perturbation method global sensitivity analysis of Corey model for oil relative permeability is reported in appendix B, section B1.1.

4.1.1.2. STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

$n = 30{,}000$ (number of model evaluations for the AMA indices)

$p = 50$ (number of parameters' bins for the AMA indices)

$n_t = 2{,}000$ (number of model evaluations for the Sobol total-order index)

$p_t = 10$ (number of parameters' bins for the Sobol total-order index))

In the figure below the first four statistical moments of the unconditional output (k_{rw}) of Corey model are reported, *id est* the statistical moments of the model output when all its input factors are random variables uniformly distributed in their input variability space. A residual instability is present in the curves due to the limited number of Monte Carlo realizations: increasing n the amplitude of the oscillations decreases. Despite of that, $n = 30{,}000$ is an acceptable trade-off between accuracy and computation time for the purposes of the sensitivity analysis (adopting the described assumptions the computation time is 1255s).

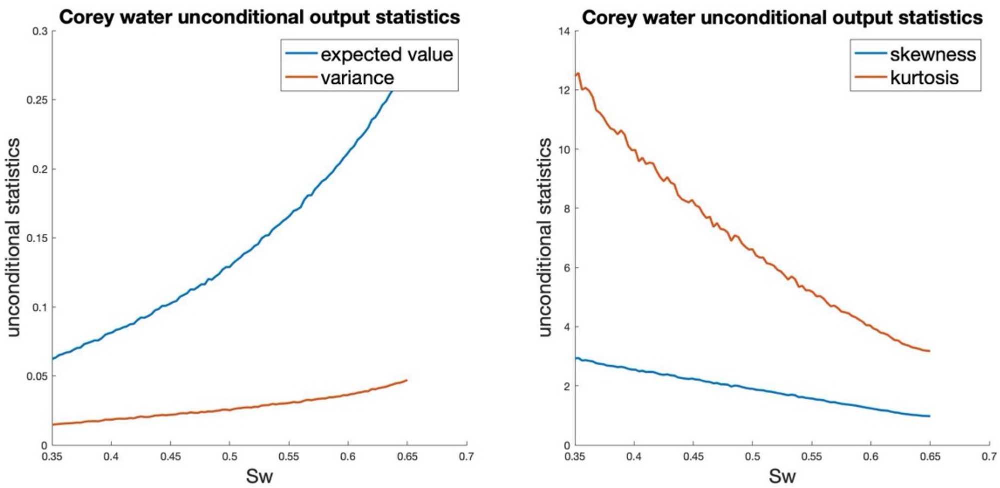

Figure 19: statistical moments of the unconditional output of Corey model for water relative permeability

The expected value of the unconditional output (water relative permeability k_{rw}), in agreement with the model structure, increases for increasing S_w. The unconditional variance of k_{rw} increases for increasing S_w, meaning that the spread around the mean, and hence the associated uncertainty, is higher for higher values of S_w. The unconditional skewness of k_{rw} decreases for increasing S_w, indicating that for higher values of S_w the unconditional probability distribution of k_{rw} becomes less asymmetrical. The unconditional kurtosis of k_{rw} decreases for increasing S_w, indicating that for higher values of S_w the unconditional probability distribution of k_{rw} becomes less tailed.

In the following figure the six statistical moments-based sensitivity indices are plotted as functions of water saturation S_w:

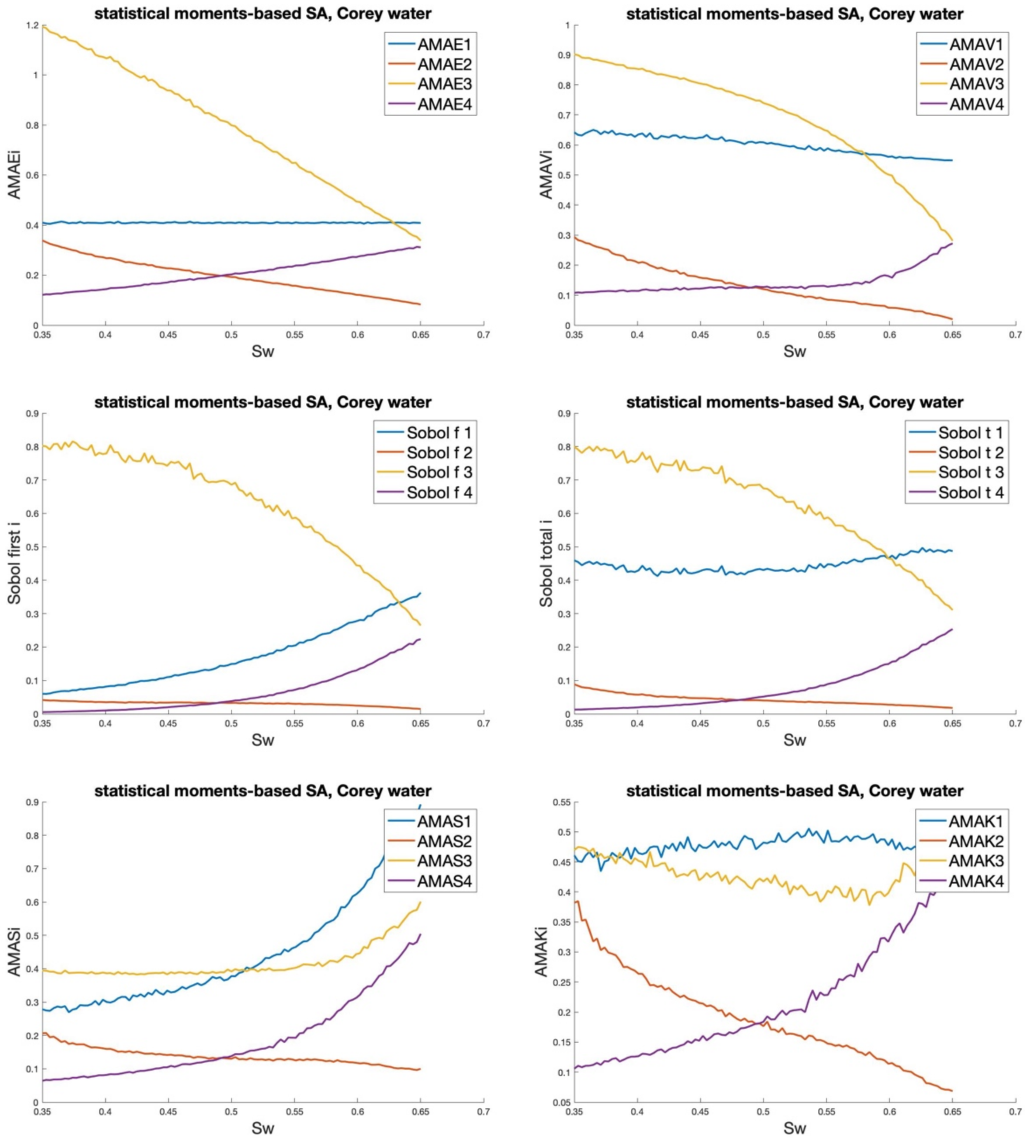

Figure 20: statistical moments-based sensitivity analysis of Corey model for water relative permeability

It can be observed that, generally speaking, the instability of the sensitivity indices curves increases when considering higher-order statistical moments. The plots above show that in general terms, as expected, the effect of a same input factor changes when considering different statistical moments of the output. As previously observed, the effect of some input factors is almost constant across all the values of S_w (the curve of $AMAE_1$ is, actually, perfectly horizontal), while the effect of some input factors depends strongly on S_w. Comparing the plots of $Sobol^f$ and $Sobol^t$ it can be observed that the interactions between the variable input factors can have a profound impact on the sensitivity of the output to the input factor x_1.

For the statistical moments-based global sensitivity analysis of the relative permeability models the ranking of the variable input factors is based on the four ranking indices:

$$R_{AMAE,i} = \sum_{S_{w,min}}^{S_{w,max}} AMAE_i \tag{4.2}$$

$$R_{AMAV,i} = \sum_{S_{w,min}}^{S_{w,max}} AMAV_i \tag{4.3}$$

$$R_{Sobol\ first,i} = \sum_{S_{w,min}}^{S_{w,max}} Sobol_i^f \tag{4.4}$$

$$R_{Sobol\ total,i} = \sum_{S_{w,min}}^{S_{w,max}} Sobol_i^t \tag{4.5}$$

The ranking index $R_{AMAE,i}$ quantifies the effect of the variable input factor x_i on the expected value of the output. The ranking index $R_{AMAV,i}$ quantifies the effect on the variance of the output. The ranking index $R_{Sobol\ first,i}$ quantifies the effect on the dispersion of the conditional output mean. The ranking index $R_{Sobol\ total,i}$ quantifies both the direct contribution of the variability of x_i and the contribution coming from the interactions between x_i and the other input factors. The higher order statistical moments are not used for ranking purposes, because in general the mean and the variance of the output are the main subjects of interest and the most evident manifestations of uncertainty.

From the statistical moments-based sensitivity analysis applied to Corey model for water relative permeability the resulting rankings are:

Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$
x_3: N_w (79.29)	x_3: N_w (69.74)	x_3: N_w (63.78)	x_3: N_w (63.61)
x_1: k^0_{rw} (41.32)	x_1: k^0_{rw} (60.67)	x_1: k^0_{rw} (17.39)	x_1: k^0_{rw} (45.00)
x_4: S_{or} (20.96)	x_4: S_{or} (14.18)	x_4: S_{or} (6.46)	x_4: S_{or} (7.92)
x_2: S_{wi} (19.77)	x_2: S_{wi} (13.33)	x_2: S_{wi} (3.12)	x_2: S_{wi} (4.32)

The rankings based on these four different indices return the exact same result: in this case these four sensitivity indices are equivalent for the ranking purpose. The rankings also show that the interactions do not change the relative importance of the input factors. The statistical moments-based sensitivity analysis technique confirms the qualitative observation made in chapter 3.1: the input variable factor x_3 is the most influential on the model output. These rankings are not in agreement at all with the ranking obtained from the multiple start perturbation method sensitivity analysis. This difference can be explained observing that the statistical moments based-sensitivity indices are defined in order to study the variability induced by the input factors, regardless of their variation amplitude. This approach is in complete opposition to the multiple start perturbation method, and it follows that in general the results coming from the two methods are discordant.

The detailed statistical moments-based global sensitivity analysis of Corey model for oil relative permeability is reported in appendix B, section B1.2.

4.1.1.3. CUMULATIVE PROBABILITY DENSITY FUNCTION-BASED SENSITIVITY ANALYSIS

$n = 30{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

In the figure below the unconditional cumulative density function of Corey model output is reported for water relative permeability, $S_w = 0.5$.

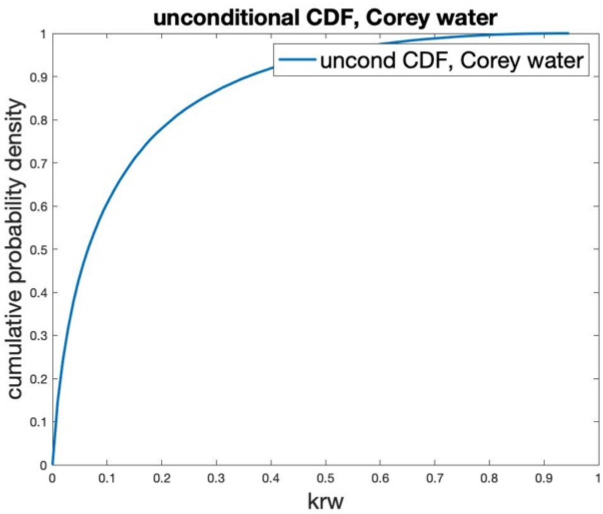

Figure 21: cumulative distribution of Corey model unconditional output for water relative permeability, S_w=0.5

In the figure below the two PAWN sensitivity indices are reported as function of S_w for each of Corey model variable input factors.

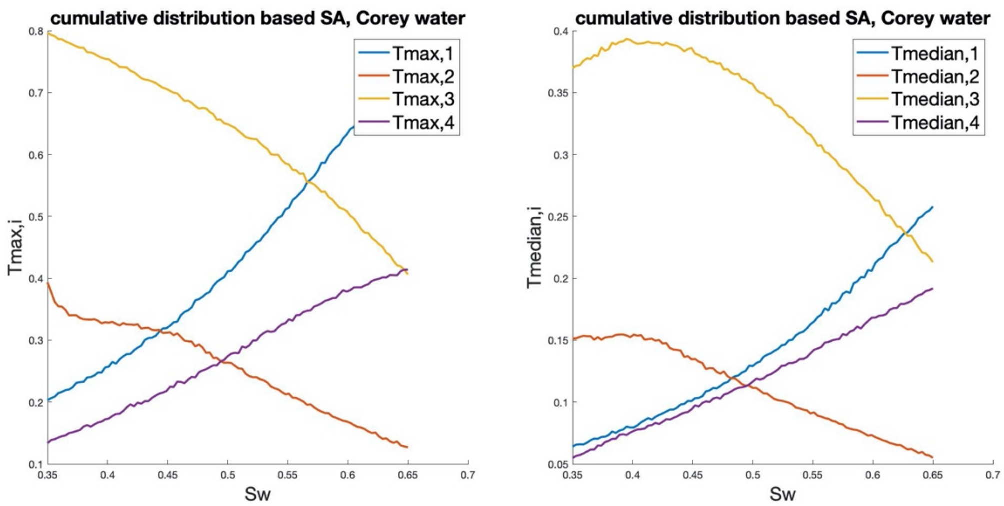

Figure 22: cumulative probability density function-based sensitivity analysis of Corey model for water relative permeability

From the sensitivity indices plots it can be observed that the numerical values assumed by the sensitivity indices $T_{max,i}$ are in general significantly larger than the values assumed by the indices $T_{median,i}$. This is in accordance with the definition of the two sensitivity indices, which are slightly different in their meaning: $T_{median,i}$ is the quantification of the most likely sensitivity of the output to the variable input factor x_i, while $T_{max,i}$ quantifies the maximum possible sensitivity of the output to x_i, which happens only for a precise combination of input factors. For this reason, normally, the index $T_{max,i}$ is a more conservative measure of sensitivity. Both the sensitivity indices related to any of the input factors appear to be strongly dependant on S_w. It is interesting to observe that according to the sensitivity index $T_{max,3}$ the sensitivity to the factor x_3 always decreases for increasing S_w, while according to the index $T_{median,3}$ the sensitivity to x_3 reaches a global maximum.

For the cumulative distribution function-based sensitivity analysis of the relative permeability models the ranking of the variable input factors is determined by two indices:

$$R_{Tmax,i} = \sum_{S_{w,min}}^{S_{w,max}} T_{max,i} \tag{4.6}$$

$$R_{Tmedian,i} = \sum_{S_{w,min}}^{S_{w,max}} T_{median,i} \tag{4.7}$$

From the cumulative probability density function-based sensitivity analysis applied to Corey model for water relative permeability, the resulting rankings are:

Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$
x_3: N_w (63.97)	x_3: N_w (33.74)
x_1: k^0_{rw} (43.97)	x_1: k^0_{rw} (14.27)
x_4: S_{or} (26.76)	x_4: S_{or} (12.08)
x_2: S_{wi} (24.68)	x_2: S_{wi} (11.27)

In this case, the rankings resulting from the two indices are equivalent. The rankings obtained from the cumulative distribution-based sensitivity analysis are equivalent to the ones obtained from the statistical moments-based sensitivity analysis. This is due to the characteristic that these two sensitivity analysis methods share: they both study the sensitivity of the model output regardless of the input factors variation amplitude. For this exact same reason, these rankings are completely different to the one obtained from the multiple start perturbation method sensitivity analysis.

The detailed cumulative probability density function-based global sensitivity analysis of Corey model for oil relative permeability is reported in appendix B, section B1.3.

4.1.1.4. VARIOGRAM-BASED SENSITIVITY ANALYSIS

$n = 30{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

In the figure below the normalized variograms of Corey model output conditional to each of the model input factors are reported for $S_w = 0.5$.

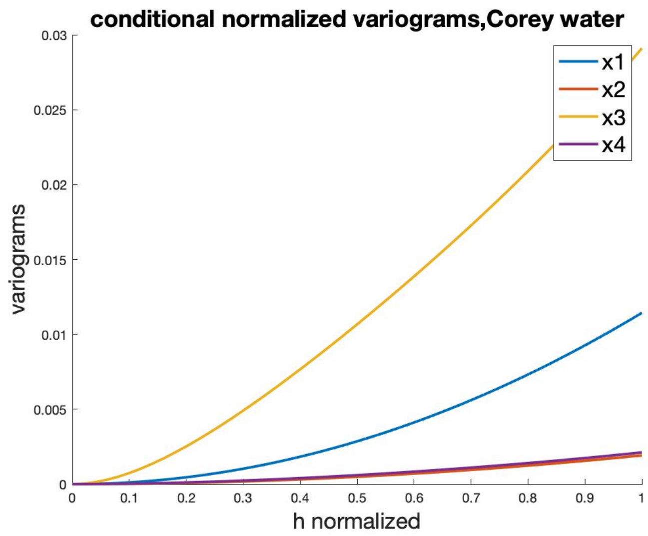

Figure 23: normalized variograms of Corey model conditional output for water relative permeability, S_w=0.5

In the figures below the IVARS sensitivity indices of Corey model for water relative permeability are reported as functions of S_w.

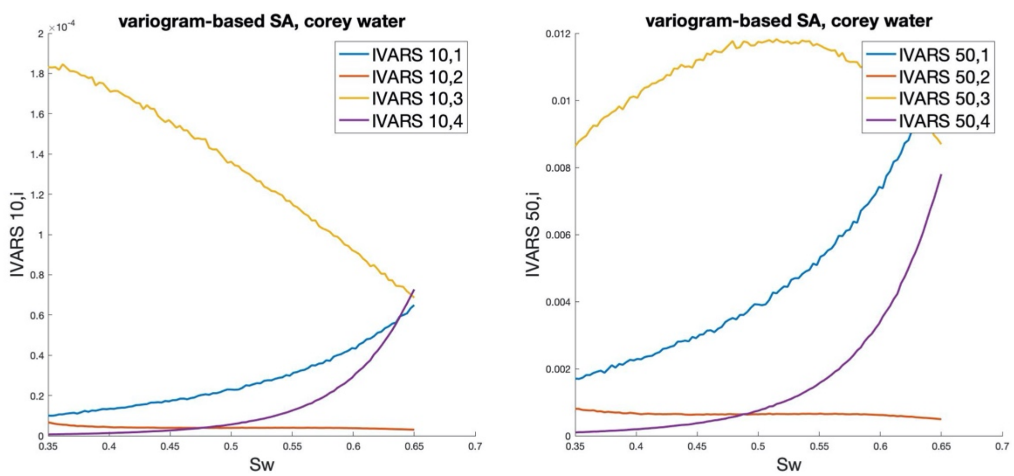

Figure 24: variogram-based sensitivity analysis of Corey model for water relative permeability

This sensitivity analysis method shows remarkable stability in the calculation of the sensitivity indices (even for lower numbers of Monte Carlo realizations), due to the fact that the $IVARS$ sensitivity indices are integral quantities. From the sensitivity indices plots it can be observed that the change of scale of the input factors variations has noticeable effects on their relative importance: according to the index $IVARS_{50}$ the sensitivity of the output to x_1 is always greater than the sensitivity to x_4, while according to the index $IVARS_{10}$ there are values of S_w for which the uncertainty associated to x_4 is more relevant than the uncertainty associated to x_1. It should be remembered that the index $IVARS_{10}$ quantifies the sensitivity of the model output to small variations of the variable input factors (up to 10% of the input variability space), while the index $IVARS_{50}$ quantifies the sensitivity to large variations of the variable input factors (up to 50% of the input variability space). Another interesting observation is that according to the index $IVARS_{10}$ the sensitivity to x_3 always decreases for increasing S_w, while according to the index $IVARS_{50}$ the sensitivity to x_3 reaches a global maximum.

For the variogram-based sensitivity analysis applied to the relative permeability models, the ranking of the variable input factors is determined calculating the two indices:

$$R_{IVARS\ 10,i} = \sum_{S_{w,min}}^{S_{w,max}} IVARS_{10,i} \tag{4.8}$$

$$R_{IVARS\ 50,i} = \sum_{S_{w,min}}^{S_{w,max}} IVARS_{50,i} \tag{4.9}$$

From the variogram-based global sensitivity analysis applied to Corey model for water relative permeability the resulting rankings are as follows:

Ranking according to $R_{IVARS\ 10,i}$	Ranking according to $R_{IVARS\ 50,i}$
x_3: N_w (0.011)	x_3: N_w (1.13)
x_1: k^0_{rw} (0.0020)	x_1: k^0_{rw} (0.47)
x_4: S_{or} (0.0010)	x_4: S_{or} (0.16)
x_2: S_{wi} (0.00030)	x_2: S_{wi} (0.064)

In this case, the rankings obtained by these two indices are equivalent. This follows the fact that even the sensitivity index $IVARS_{10}$ studies the output variability against rather large variations of the variable input factors (10% of their variability space). The rankings obtained from the variogram-based sensitivity analysis agree with the rankings obtained from the other methods except for the multiple start perturbation method.

The detailed variogram-based global sensitivity analysis of Corey model for oil relative permeability is reported in appendix B, section B1.4.

4.1.2. CHIERICI MODEL

4.1.2.1. MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

$r = 600{,}000$ (number of elementary effects evaluations)

The number of evaluations r is increased with respect to the number used for the Corey model sensitivity analysis, because the more complex a model is, the more unstable the calculated sensitivity indices will be (the complexity of a model is related to the number of its input factors). The only way to reduce the instability of the sensitivity indices curves is increasing the number of model evaluations. Even if this number is significantly high, this sensitivity analysis method is very computationally inexpensive, and its execution is remarkably quick (65s).

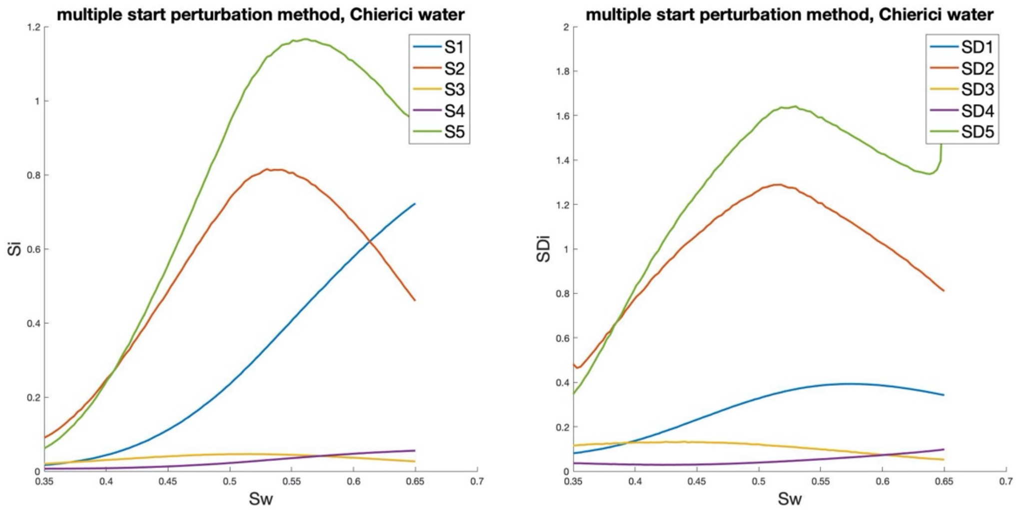

Figure 25: multiple start perturbation method global sensitivity analysis of Chierici model for water relative permeability

In this case the sensitivity to the input factors x_1, x_2 and x_5 varies widely across the considered values of water saturation, while the sensitivity to the input factors x_3 and x_4 is almost constant and much lower in value. From the standard deviation of the elementary effect (SD_i) plots it can be observed that the sensitivity to the input factors x_5 and x_2 strongly depends on the values assumed by all the other input factors (especially for medium values of S_w), while the sensitivity to the input factors x_3 and x_4 weakly depends on the other factors.

From the multiple start perturbation method global sensitivity analysis applied to the Chierici model for water relative permeability, the resulting ranking is:

Ranking according to $R_{S,i}$
x_5: S_{or} (75.91)
x_2: S_{wi} (54.35)
x_1: k^0_{rw} (29.28)
x_3: B_w (3.69)
x_4: M_w (2.62)

This ranking shows that the variable input factors x_3 (B_w) and x_4 (M_w) have only a minor effect on the output variability: their ranking indices are one order of magnitude smaller than the ranking indices of the other input factors. It should be remembered that the multiple start perturbation method, unlike all the other considered sensitivity analysis methods, quantifies the effect on the model output variability only against extremely small variations of the variable input factors. For this reason, this method is only meaningful for situations in which the variability of the input factors is likely to be extremely small, and the ranking resulting from the multiple start perturbation method is, in general, not in agreement with the rankings obtained by other techniques.

The detailed multiple start perturbation method global sensitivity analysis of Chierici model for oil relative permeability is reported in appendix B, section B2.1.

4.1.2.2. STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

$n = 50{,}000$ (number of model evaluations for the AMA indices)

$p = 50$ (number of parameters' bins for the AMA indices)

$n_t = 1{,}000$ (number of model evaluations for the Sobol total-order index)

$p_t = 10$ (number of parameters' bins for the Sobol total-order index)

The number of Monte Carlo realizations n has been increased for Chierici model with respect to the number used for Corey model, because the former is a more complex model and hence its output is subject to stronger instability when its input factors are random variables. The instability of the unconditional and conditional outputs gets further amplificated when computing the statistical moments-based sensitivity indices, especially the ones based on higher order statistical moments. A residual instability is still noticeable in the curves, but for the purpose of the essay $n = 50{,}000$ is a perfectly acceptable compromise between accuracy and required computation time. The number of Monte Carlo realizations n_t used for the computation of the Sobol total order indices has instead been reduced with respect to the number used for Corey model to avoid an unacceptable associated computation time (which increases exponentially with the number of the model input factors). Despite of the reduction of n_t, the plots of the $Sobol_i^t$ sensitivity indices keep showing remarkable stability.

In the figure below the first four unconditional statistical moments of Chierici model output are reported.

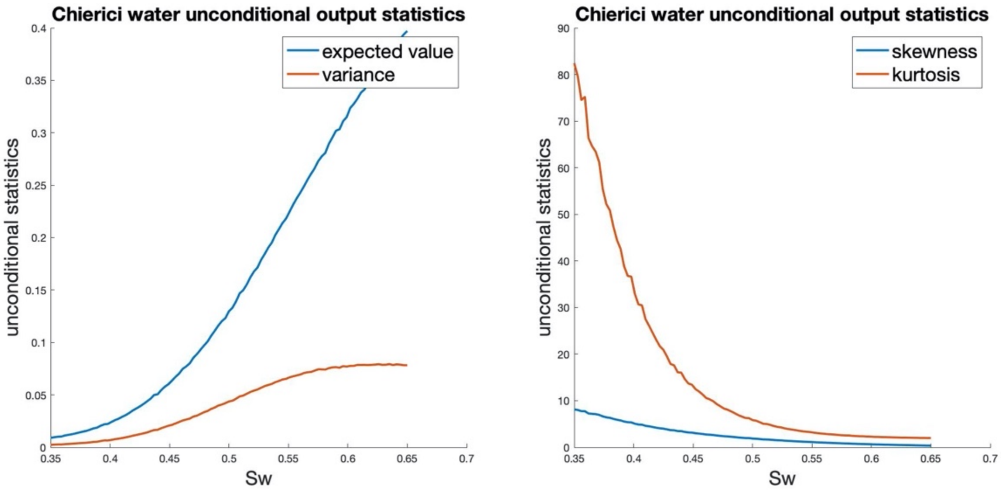

Figure 26: statistical moments of the unconditional output of Chierici model for water relative permeability

The unconditional variance of the output k_{rw} increases for increasing S_w with asymptotical behaviour, meaning that the spread around the mean, and hence the uncertainty, increases for increasing S_w until $S_w \sim 0.6$, and stays constant for higher values of water saturation. The unconditional skewness of k_{rw} decreases for increasing S_w, indicating that for higher values of S_w the unconditional probability distribution of k_{rw} becomes less asymmetrical (for high values of water saturation $Sk(k_{rw}) \sim 0$, meaning that it is almost perfectly symmetrical). The unconditional kurtosis of k_{rw} decreases for increasing S_w, indicating that for higher values of S_w the unconditional probability distribution of k_{rw} becomes less tailed.

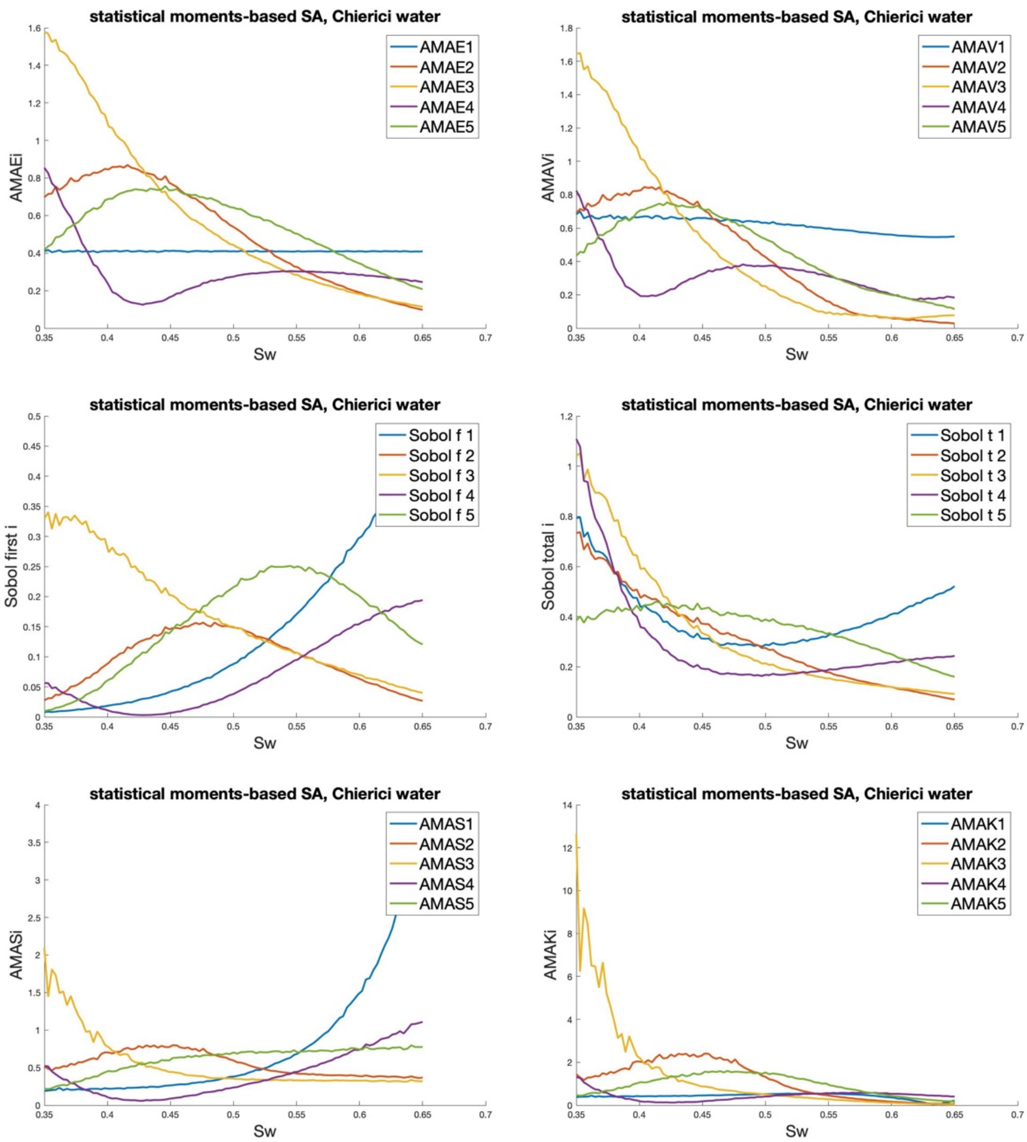

Figure 27: statistical moments-based sensitivity analysis of Chierici model for water relative permeability

It is interesting to observe that the sensitivity index $AMAE_1$ is perfectly constant across the entire range of water saturation values, while all the other sensitivity indices are affected by the considered value of S_w.

From the statistical moments-based global sensitivity analysis applied to Chierici model for water relative permeability the resulting rankings are:

Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\,first,i}$	Ranking according to $R_{Sobol\,total,i}$
x_3: B_w (59.67)	x_1: k^0_{rw} (62.56)	x_3: B_w (16.90)	x_1: k^0_{rw} (40.77)
x_5: S_{or} (54.54)	x_3: B_w (47.84)	x_5: S_{or} (15.72)	x_5: S_{or} (35.64)
x_2: S_{wi} (52.01)	x_5: S_{or} (46.50)	x_1: k^0_{rw} (14.15)	x_3: B_w (33.53)
x_1: k^0_{rw} (41.37)	x_2: S_{wi} (43.05)	x_2: S_{wi} (9.78)	x_2: S_{wi} (30.71)
x_4: M_w (30.08)	x_4: M_w (31.48)	x_4: M_w (7.23)	x_4: M_w (29.50)

The rankings based on these four different indices are not in agreement with each other. In the case of the sensitivity to the input factor x_1 they are even in direct opposition: x_1 is the most influential factor according to R_{AMAV} and to $R_{Sobol\,total}$, but it is nearly the least influential according to R_{AMAE}. This result shows how, even within the application of a same sensitivity analysis theory, the definition of the considered sensitivity index can affect the conclusions of the analysis. The rankings obtained from the indices $R_{Sobol\,first}$ and $R_{Sobol\,total}$ are not equivalent, meaning that in this case the interactions between the variable input factors can affect their relative importance. In particular, it can be observed that the relative importance of the input factor x_1 strongly increases due to its interactions with the others, while the relative importance of the input factor x_3 decreases due to the interactions.

The lack of agreement between the four rankings makes the situation rather complex: the only univocal statement which can be made is that the variable input factor x_4 (M_w) is the least influential.

These rankings are not in agreement with the ranking obtained from the multiple start perturbation method sensitivity analysis, due to the fact that the statistical moments based-sensitivity indices are defined in order to study the variability induced by the input factors regardless of their variation amplitude (unlike what happens for the multiple start perturbation method sensitivity index).

The detailed statistical moments-based global sensitivity analysis of Chierici model for oil relative permeability is reported in appendix B, section B2.2.

4.1.2.3. CUMULATIVE PROBABILITY DENSITY FUNCTION-BASED SENSITIVITY ANALYSIS

$n = 50{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

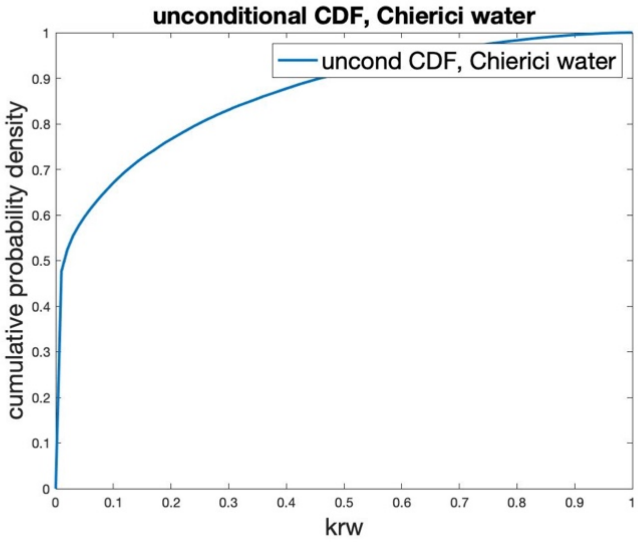

Figure 28: cumulative distribution of Chierici model unconditional output for water relative permeability, S_w=0.5

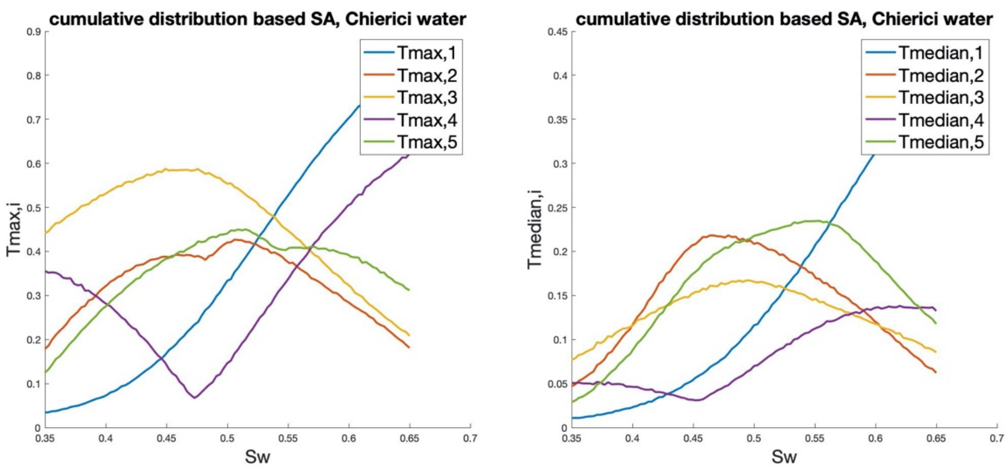

Figure 29: cumulative distribution-based sensitivity analysis of Chierici model for water relative permeability

In this case all the sensitivity indices are strongly dependant from S_w. There are significant differences between the plots of the sensitivity indices $T_{max,i}$ and $T_{median,i}$, meaning that the interactions between the variable input factors can have a noticeable effect on their relative importance ($T_{median,i}$ quantifies the most likely sensitivity of the output to the variable input factor x_i, while $T_{max,i}$ quantifies the maximum possible sensitivity to x_i, which verifies only for a precise combination of input factors). It is interesting to note that the sensitivity indices' curves relative to the input factor x_1 are the only monotonic ones (the sensitivity to the input factor x_1 always increases for increasing S_w).

From the cumulative probability density function-based sensitivity analysis applied to the Chierici relative permeability model for water relative permeability the resulting rankings are:

Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$
x_3: B_w (46.69)	x_5: S_{or} (16.48)
x_1: k^0_{rw} (37.60)	x_1: k^0_{rw} (15.47)
x_5: S_{or} (35.09)	x_2: S_{wi} (14.84)
x_2: S_{wi} (32.59)	x_3: B_w (13.19)
x_4: M_w (31.81)	x_4: M_w (8.23)

The rankings resulting from the two indices are not in agreement with each other. Specifically, it should be noticed that according to the ranking index R_{Tmax} the input variable factor x_3 is the most relevant, while according to the ranking index $R_{Tmedian}$ the input factor x_3 is the among the least relevant factors. For a more conservative sensitivity analysis the ranking based on $R_{Tmax,i}$ should be considered, even though the ranking based on $R_{Tmedian,i}$ is more likely to be close to reality due to its definition.

The two rankings agree with each other and with the rankings obtained from the statistical moments-based sensitivity analysis only on the fact that the variable input factor x_4 is the least influential.

The detailed cumulative probability density function-based global sensitivity analysis of Chierici model for oil relative permeability is reported in appendix B, section B2.3.

4.1.2.4. VARIOGRAM-BASED SENSITIVITY ANALYSIS

$n = 50{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

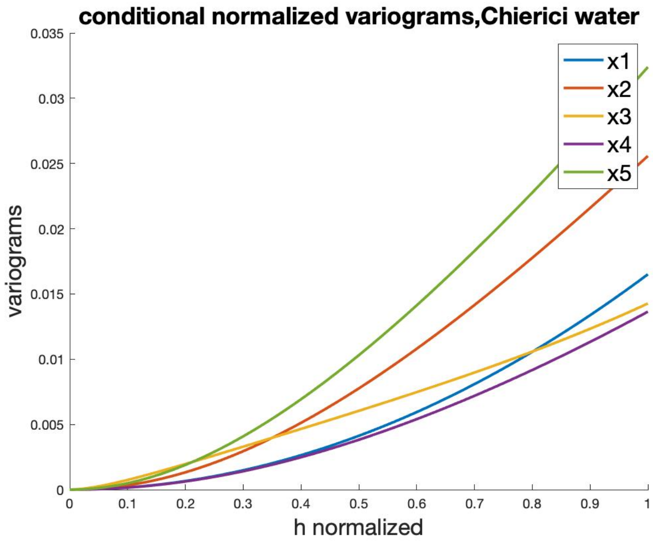

Figure 30: normalized variograms of Chierci model conditional output for water relative permeability, S_w=0,5

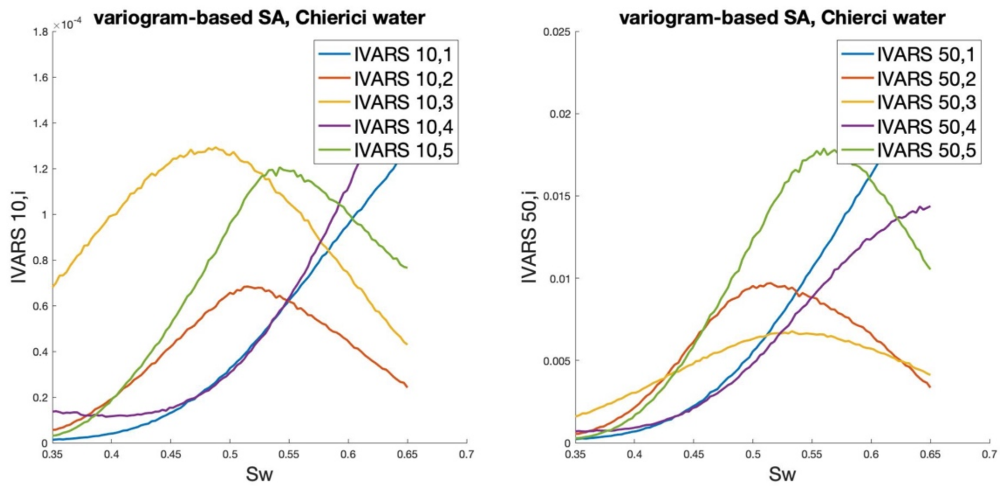

Figure 31: variogram-based sensitivity analysis of Chierici model for water relative permeability

By a qualitative observation of the sensitivity indices curves it can be clearly seen that the change of scale of the input factors variations has a strong effect on the relative importance of the different variable input factors. It should be remembered that the index $IVARS_{10}$ quantifies the sensitivity of the model output to small variations of the input variable factors, while the index $IVARS_{50}$ quantifies the sensitivity to large variations of the input factors. It is interesting to note that the sensitivity curves relative to the input factors x_1 and x_4 are the only monotonic ones.

For the variogram-based global sensitivity analysis applied to the Chierici model for water relative permeability the resulting rankings are:

Ranking according to $R_{IVARS10,i}$	Ranking according to $R_{IVARS50,i}$
x_3: B_w (0.0086)	x_5: S_{or} (0.99)
x_5: S_{or} (0.0051)	x_1: k^0_{rw} (0.78)
x_4: M_w (0.0040)	x_4: M_w (0.63)
x_1: k^0_{rw} (0.0033)	x_2: S_{wi} (0.59)
x_2: S_{wi} (0.0030)	x_3: B_w (0.53)

In this case, the rankings obtained by the two indices give deeply different conclusions about the relative importance of the model variable input factors. The most noticeable difference is that according to the ranking index $R_{IVARS10}$ the input factor x_3 is the most influential, while according to the ranking index $R_{IVARS50}$ the same input factor x_3 is the least influential. This shows again how the importance of a same input factor is tightly connected to the considered scale of the input factor variations. For this reason, the ranking based on the indices $R_{IVARS10,i}$ is the most trustworthy in situations in which the variations of the variable input factors are likely to be small (maximum 10% of the input variability space), while the ranking based on the indices $R_{IVARS50,i}$ is more suitable for situations in which a wide variability of the input factors is expected.

The detailed variogram-based global sensitivity analysis of Chierici model for oil relative permeability is reported in appendix B, section B2.4.

4.1.3. LET MODEL

4.1.3.1. MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

$r = 1{,}000{,}000$ (number of elementary effects evaluations)

The number of model evaluations r has been increased with respect to the number used for Corey and Chierici models, due to the higher complexity of the LET relative permeability model which makes the sensitivity indices evaluations more unstable. The higher complexity of the model follows its higher number of its input variable factors.

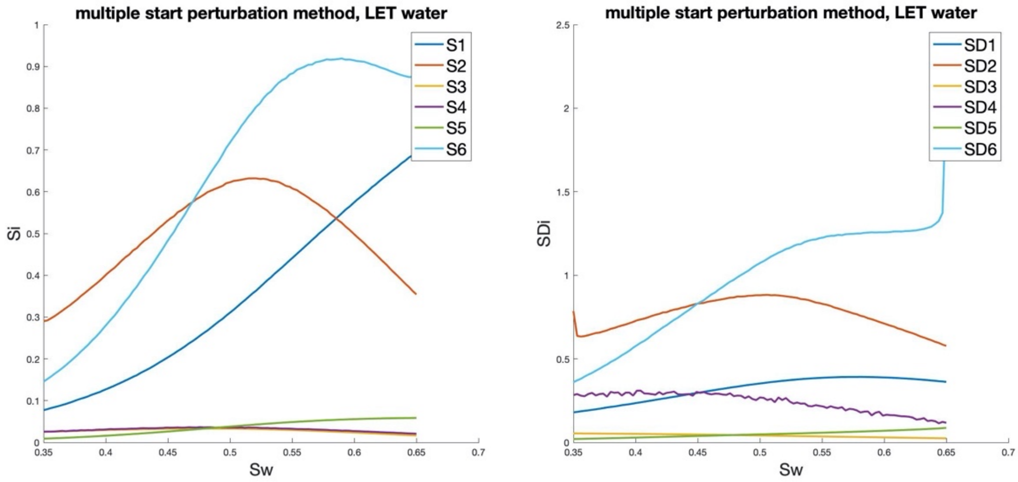

Figure 32: multiple start perturbation method global sensitivity analysis of LET model for water relative permeability

From the sensitivity indices plots it results that across all the possible values of water saturation S_w the variable input factors x_3 (L_w), x_4 (E_w) and x_5 (T_w) have only a minor effect on the output variability. It should be noted that the multiple start perturbation method, unlike all the other considered sensitivity analysis methods, quantifies the effect on the output variability only against extremely small variations of the variable input factors. From the plots of the standard deviation of the elementary effects it can be observed that, especially for the input factors x_2 and x_6, the interactions with the other input variable factors have an important role in the corresponding sensitivity indices.

From the multiple start perturbation method global sensitivity analysis applied to the LET model for water relative permeability, the resulting ranking is:

Ranking according to $R_{S,i}$
x_6: S_{or} (63.78)
x_2: S_{wi} (50.30)
x_1: k^0_{rw} (34.32)
x_5: T_w (3.67)
x_4: E_w (3.12)
x_3: L_w (2.84)

The ranking confirms that, according to this sensitivity analysis method, the variable input factors x_3, x_4 and x_5 have only a marginal effect on the output variability.

The detailed multiple start perturbation method global sensitivity analysis of LET model for oil relative permeability is reported in appendix B, section B3.1.

4.1.3.2. STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

$n = 70{,}000$ (number of model evaluations for the AMA indices)

$p = 50$ (number of parameters' bins for the AMA indices)

$n_t = 1{,}000$ (number of model evaluations for the Sobol total-order index)

$p_t = 6$ (number of parameters' bins for the Sobol total-order index)

The number of Monte Carlo realizations n has been increased with respect to the number used for Corey and Chierici relative permeability models, due to the higher complexity of the LET model, which makes the sensitivity indices evaluations more unstable. The value of the discretization parameter p_t used for the evaluation of the Sobol total order indices has instead been reduced with respect to the value used for the simpler models, due to its strong impact on the required computation time (further reducing the value of n_t would affect the stability of the $Sobol_i^t$ indices plots).

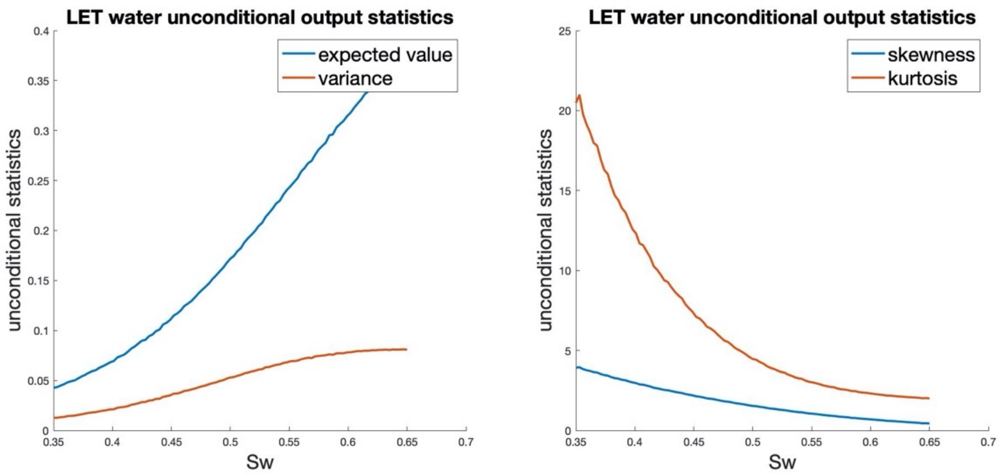

Figure 33: statistical moment of the unconditional output of LET model for water relative permeability

The unconditional variance of the output k_{rw} increases for increasing S_w, meaning that the spread around the mean, and hence the uncertainty, increases for increasing S_w. The unconditional skewness of k_{rw} decreases for increasing S_w, indicating that for higher values of S_w the unconditional probability distribution of k_{rw} becomes less asymmetrical. The unconditional kurtosis of k_{rw} decreases for increasing S_w, indicating that for higher values of S_w the unconditional probability distribution of k_{rw} becomes less tailed.

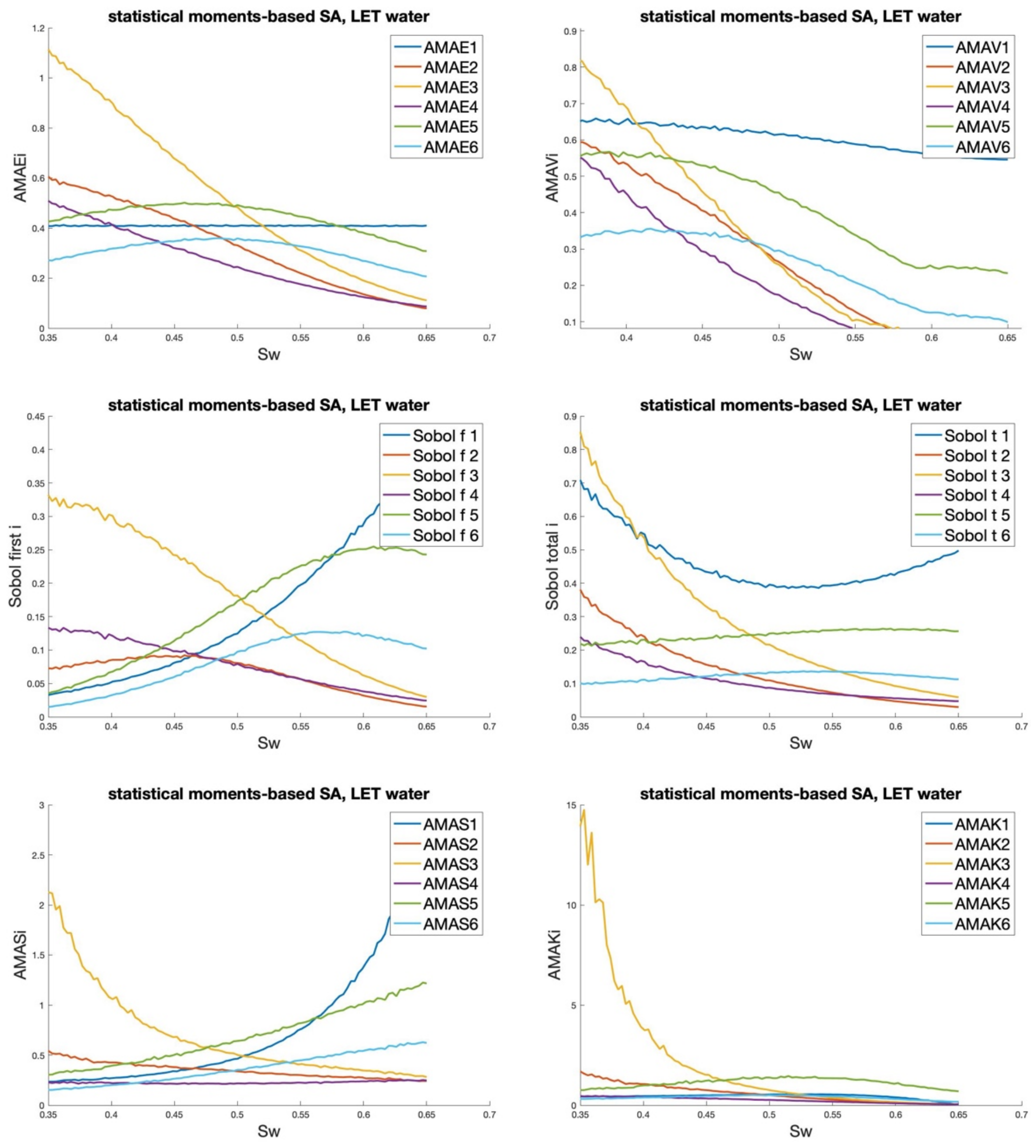

Figure 34: statistical moments-based global sensitivity analysis of LET model for water relative permeability

Observing the plots of the sensitivity indices $AMAE_i$ it is interesting to observe that the curves relative to the input factors x_2, x_3 and x_4 strongly decrease for increasing water saturation S_w, while the curve relative to the input factors x_1 is perfectly horizontal. Except for the index $AMAE_1$, all the represented sensitivity indices result to be dependant from S_w.

From the statistical moments-based global sensitivity analysis applied to LET model for water relative permeability the resulting rankings are:

Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$
x_3: L_w (53.22)	x_1: k^0_{rw} (61.38)	x_3: L_w (18.13)	x_1: k^0_{rw} (46.45)
x_5: T_w (44.72)	x_5: T_w (42.49)	x_5: T_w (16.39)	x_3: L_w (29.21)
x_1: k^0_{rw} (41.33)	x_3: L_w (34.13)	x_1: k^0_{rw} (16.17)	x_5: T_w (24.72)
x_2: S_{wi} (33.44)	x_2: S_{wi} (28.42)	x_6: S_{or} (8.35)	x_2: S_{wi} (13.69)
x_6: S_{or} (31.32)	x_6: S_{or} (25.82)	x_4: E_w (7.93)	x_6: S_{or} (12.31)
x_4: E_w (26.44)	x_4: E_w (22.59)	x_2: S_{wi} (6.55)	x_4: E_w (10.52)

The rankings based on these four different indices are not in agreement with each other. This result shows again how, even within the application of a same sensitivity analysis theory, the definition of the considered sensitivity index can affect the conclusions of the analysis (in particular, when considering complex models, like the LET). The differences between the ranking based on the index $R_{Sobol\ first,i}$ and the ranking based on the index $R_{Sobol\ total,i}$ show that in this case the interactions between the variable input factors can affect their relative importance. The lack of agreement between the four rankings makes the situation rather complex: the only univocal statement which can be made is that the variable input factor x_4 (E_w) is the least influential.

These rankings are not in agreement with the ranking obtained by the multiple start perturbation method sensitivity analysis, due to the fact that the statistical moments based-sensitivity indices are defined in order to study the variability induced by the input factors, regardless of their variation amplitude (unlike what happens for the multiple start perturbation method sensitivity index).

The detailed statistical moments-based global sensitivity analysis of LET model for oil relative permeability is reported in appendix B, section B3.2.

4.1.3.3. CUMULATIVE PROBABILITY DENSITY FUNCTION-BASED SENSITIVITY ANALYSIS

$n = 70{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

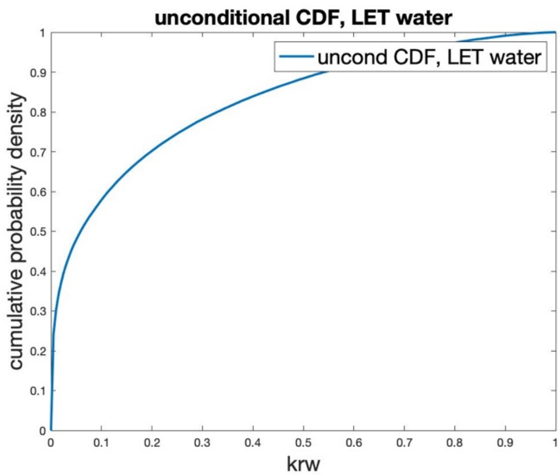

Figure 35: cumulative distribution of LET model unconditional output for water relative permeability, S_w=0.5

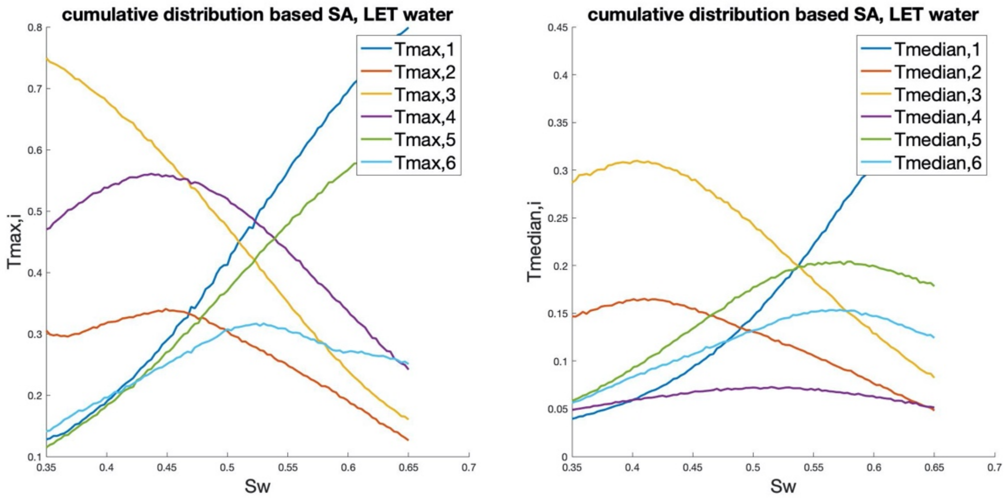

Figure 36: cumulative distribution function-based global sensitivity analysis of LET model for water relative permeability

There are noticeable differences between the plots of the sensitivity indices $T_{max,i}$ and $T_{median,i}$, indicating that the interactions between the variable input factors can give a significant contribution to their relative importance. It is interesting to observe that the T_{max} plots relative to the input factors x_3 and x_5 are monotonic, while the T_{median} plots relative to the same input factors reach a global maximum. Both the sensitivity indices relative to the input factor x_1 increase for increasing value of S_w.

From the cumulative distribution function-based sensitivity analysis applied to the LET model for water relative permeability the resulting rankings are:

Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$
x_3: L_w (46.75)	x_3: L_w (22.65)
x_4: E_w (46.22)	x_1: k^0_{rw} (17.65)
x_1: k^0_{rw} (44.11)	x_5: T_w (15.56)
x_5: T_w (37.70)	x_2: S_{wi} (12.32)
x_2: S_{wi} (26.10)	x_6: S_{or} (12.03)
x_6: S_{or} (24.99)	x_4: E_w (6.44)

In this case, the rankings resulting from the two indices are not in agreement with each other. Specifically, it should be noticed that according to the ranking index R_{Tmax} the input variable factor x_4 is the second most relevant, while according to the ranking index $R_{Tmedian}$ the input factor x_4 is the least relevant. The two rankings agree on the fact that the input factor x_3 is the most relevant.

For a more conservative sensitivity analysis the ranking based on $R_{Tmax,i}$ should be considered, even though the ranking based on $R_{Tmedian,i}$ is more likely to be close to reality due to its definition.

The detailed cumulative probability density function-based global sensitivity analysis of LET model for oil relative permeability is reported in appendix B, section B3.3.

4.1.3.4. VARIOGRAM-BASED SENSITIVITY ANALYSIS

$n = 70{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

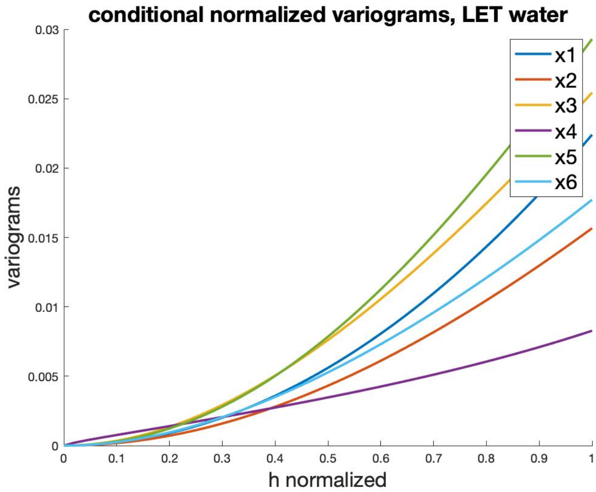

Figure 37: normalized variograms of LET model conditional output for water relative permeability, S_w=0.5

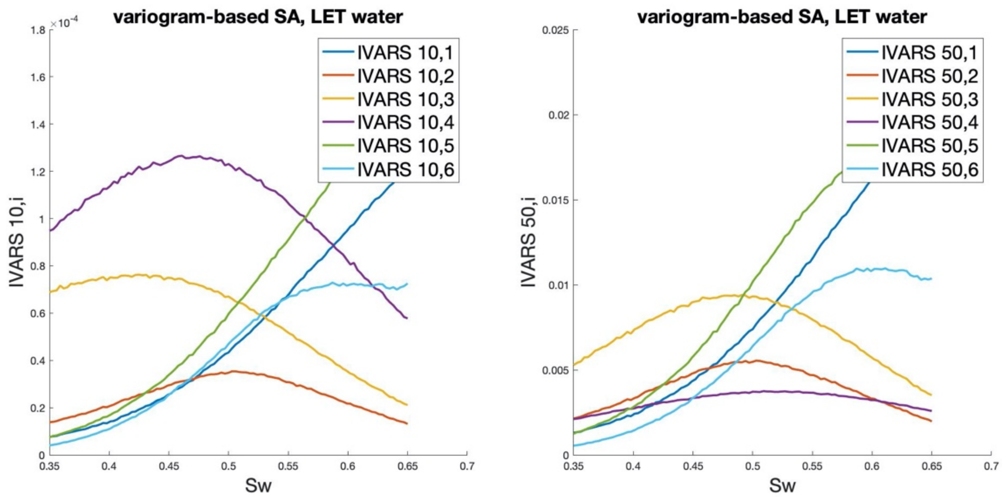

Figure 38: variogram-based global sensitivity analysis of LET model for water relative permeability

From the sensitivity indices plots it can be observed that the change of scale of the input factors variations has a clear effect on the relative importance of the different variable input factors. It should be remembered that the index $IVARS_{10}$ quantifies the sensitivity of the model output to small variations of the variable input factors, while the index $IVARS_{50}$ quantifies the sensitivity to large variations of the variable input factors.

From the variogram-based global sensitivity analysis applied to the LET model for water relative permeability the resulting rankings are:

Ranking according to $R_{IVARS10,i}$	Ranking according to $R_{IVARS50,i}$
x_4: E_w (0.0126)	x_5: T_w (1.04)
x_5: T_w (0.0050)	x_1: k^0_{rw} (0.89)
x_3: L_w (0.0043)	x_3: L_w (0.75)
x_1: k^0_{rw} (0.0038)	x_6: S_{or} (0.62)
x_6: S_{or} (0.0031)	x_2: S_{wi} (0.40)
x_2: S_{wi} (0.0018)	x_4: E_w (0.38)

In this case the rankings obtained by the two indices give highly different conclusions about the relative importance of the model variable input factors. The most noticeable difference is that according to the ranking index $R_{IVARS10}$ the input factor x_4 is the most influential, while according to the ranking index $R_{IVARS50}$ the same input factor x_4 is the least influential. This shows again how the importance of an input factor is tightly connected to the considered scale of the input factor variations. For this reason, the ranking based on the indices $R_{IVARS10,i}$ is the most trustworthy for situations in which the variations of the input factors are likely to be small (up to 10% of the input variability space), while the ranking based on the indices $R_{IVARS50,i}$ is more suitable for situations in which a wide variability of the input factors is expected.

The detailed variogram-based global sensitivity analysis of LET model for oil relative permeability is reported in appendix B, section B3.4.

4.1.4. DISCUSSION

4.1.4.1. COREY MODEL

In the table below the rankings obtained from the different global sensitivity analysis techniques when applied to Corey relative permeability model are summarized. A colour has been assigned to each variable input factor in order to help visualizing eventual concordances and discordances between the rankings obtained from different sensitivity analysis methods.

Rankings of the input factors of Corey model for water relative permeability								
multiple start	stat.moments-based				cumul.distrib.-based		variogram-based	
Ranking according to $R_{S,i}$	Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$	Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$	Ranking according to $R_{IVARS\ 10,i}$	Ranking according to $R_{IVARS\ 50,i}$
Sor (42.86)	*Nw (79.29)*	*Nw (69.74)*	*Nw (63.78)*	*Nw (63.61)*	*Nw (63.97)*	*Nw (33.74)*	*Nw (0.011)*	*Nw (1.13)*
Swi (28.11)	*k0rw (41.32)*	*k0rw (60.67)*	*k0rw (17.39)*	*k0rw (45.00)*	*k0rw (43.97)*	*k0rw (14.27)*	*k0rw (0.0020)*	*k0rw (0.47)*
k0rw (26.22)	*Sor (20.96)*	*Sor (14.18)*	*Sor (6.46)*	*Sor (7.92)*	*Sor (26.76)*	*Sor (12.08)*	*Sor (0.0010)*	*Sor (0.16)*
Nw (8.14)	*Swi (19.77)*	*Swi (13.33)*	*Swi (3.12)*	*Swi (4.32)*	*Swi (24.68)*	*Swi (11.27)*	*Swi (0.00030)*	*Swi (0.064)*

In this case, the different sensitivity analysis techniques show remarkable concordance, thanks to the numerical simplicity of Corey model. The only sensitivity analysis method which produces an outlier result is the multiple start perturbation method. This difference follows the fact that the multiple start perturbation method is the only "short range" sensitivity analysis technique: it studies the sensitivity of the model output only against extremely small variations of the variable input factors. All the other sensitivity analysis methods study the sensitivity of the output to the variable input factors regardless of their variations' amplitude (except for the variogram-based analysis, which anyway considers also relatively wide variations of the input factors).

The above-presented results indicate that the main criteria for choosing the most appropriate sensitivity analysis for the Corey relative permeability model is the likely amplitude of the input factors variations. In cases where the input factors variations are expected to be extremely narrow, the most suitable sensitivity analysis method is the multiple start perturbation method; for, in all the other cases, any different sensitivity analysis technique produces the same results.

It should be remembered that the global sensitivity analysis is a tool meant for the a priori study of a model, hence it must be applied before any model calibration or experimental measurement of the input factors is available. If the input factors have already been calibrated or measured, the indiscriminate application of a global sensitivity analysis technique could lead to deceiving conclusions.

4.1.4.2. CHIERICI MODEL

In the tables below the rankings obtained from the different sensitivity analysis techniques when applied to the Chierici relative permeability model are summarized.

Rankings of the input factors of Chierici model for water relative permeability								
multiple start	stat.moments-based				cumul.distrib.-based		variogram-based	
Ranking according to $R_{S,i}$	Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$	Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$	Ranking according to $R_{IVARS\ 10,i}$	Ranking according to $R_{IVARS\ 50,i}$
Sor (75.91)	*Bw (59.67)*	*k0rw (62.56)*	*Bw (16.90)*	*k0rw (40.77)*	*Bw (46.69)*	*Sor (16.48)*	*Bw (0.0086)*	*Sor (0.99)*
Swi (54.35)	*Sor (54.54)*	*Bw (47.84)*	*Sor (15.72)*	*Sor (35.64)*	*k0rw (37.60)*	*k0rw (15.47)*	*Sor (0.0051)*	*k0rw (0.78)*
k0rw (29.28)	*Swi (52.01)*	*Sor (46.50)*	*k0rw (14.15)*	*Bw (33.53)*	*Sor (35.09)*	*Swi (14.84)*	*Mw (0.0040)*	*Mw (0.63)*
Bw (3.69)	*k0rw (41.37)*	*Swi (43.05)*	*Swi (9.78)*	*Swi (30.71)*	*Swi (32.59)*	*Bw (13.19)*	*k0rw (0.0033)*	*Swi (0.59)*
Mw (2.62)	*Mw (30.08)*	*Mw (31.48)*	*Mw (7.23)*	*Mw (29.50)*	*Mw (31.81)*	*Mw (8.23)*	*Swi (0.0030)*	*Bw (0.53)*

In this case the discordances between the results from different sensitivity analysis techniques are dramatic. Even within the application of a same sensitivity analysis theory, different sensitivity indices return different rankings of the input factors.

The above-presented results show that when performing a global sensitivity analysis to the Chierici relative permeability model, the specific method and ranking index should be carefully selected case-by-case according to the ultimate goal of the analysis.

4.1.4.3. LET MODEL

In the tables below the rankings obtained from the different sensitivity analysis techniques when applied to the LET relative permeability model are summarized.

Rankings of the input factors of LET model for water relative permeability								
multiple start	stat.moments-based				cumul.distrib.-based		variogram-based	
Ranking according to $R_{S,i}$	Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$	Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$	Ranking according to $R_{IVARS\ 10,i}$	Ranking according to $R_{IVARS\ 50,i}$
Sor (63.78)	*Lw (53.22)*	*k0rw (61.38)*	*Lw (18.13)*	*k0rw (46.45)*	*Lw (46.75)*	*Lw (22.65)*	*Ew (0.0126)*	*Tw (1.04)*
Swi (50.30)	*Tw (44.72)*	*Tw (42.49)*	*Tw (16.39)*	*Lw (29.21)*	*Ew (46.22)*	*k0rw (17.65)*	*Tw (0.0050)*	*k0rw (0.89)*
k0rw (34.32)	*k0rw (41.33)*	*Lw (34.13)*	*k0rw (16.17)*	*Tw (24.72)*	*k0rw (44.11)*	*Tw (15.56)*	*Lw (0.0043)*	*Lw (0.75)*
Tw (3.67)	*Swi (33.44)*	*Swi (28.42)*	*Sor (8.35)*	*Swi (13.69)*	*Tw (37.70)*	*Swi (12.32)*	*k0rw (0.0038)*	*Sor (0.62)*
Ew (3.12)	*Sor (31.32)*	*Sor (25.82)*	*Ew (7.93)*	*Sor (12.31)*	*Swi (26.10)*	*Sor (12.03)*	*Sor (0.0031)*	*Swi (0.40)*
Lw (2.84)	*Ew (26.44)*	*Ew (22.59)*	*Swi (6.55)*	*Ew (10.52)*	*Sor (24.99)*	*Ew (6.44)*	*Swi (0.0018)*	*Ew (0.38)*

In this case, the discordances between the results from different sensitivity analysis techniques are dramatic. Even within the application of a same sensitivity analysis theory, different sensitivity indices return different rankings of the input factors.

The above-presented results show that when performing a global sensitivity analysis to the LET relative permeability model, the specific method and ranking index should be carefully selected case-by-case according to the ultimate goal of the analysis.

4.2. MULTI-MODEL UNINFORMED SCENARIO

In this section the different uninformed multi-model global sensitivity analysis techniques presented in chapter 2.2. are applied to the three relative permeability models, according to the stated assumptions and methodologies. The multi-model sensitivity indices are computed for 100 different values of water saturation between $S_{w,min}$ and $S_{w,max}$. The focus is set mainly on the ranking of the variable input factors on the base of their relative contribution to the output variability in an unconstrained multi-model context.

In the following, the different considered relative permeability models are denoted as M^j, with:

- M^1: Corey relative permeability model.
- M^2: Chierici relative permeability model.
- M^3: LET relative permeability model.

4.2.1. MULTI-MODEL, STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

It should be remembered that different relative permeability models have a different number of variable input factors, so:

$j = 1 \longrightarrow i = 1, 2, 3, 4$

$j = 2 \longrightarrow i = 1, 2, 3, 4, 5$

$j = 3 \longrightarrow i = 1, 2, 3, 4, 5, 6$

$n = 200{,}000$ (number of model evaluations)

$p = 50$ (number of parameters' bins)

The number of Monte Carlo realization n has been set to a very high value because the sensitivity indices' values result to be quite unstable in this multi-model context.

In the figure below the first four single-model and multi-model unconditional statistical moments are reported for each of the three relative permeability models for water relative permeability.

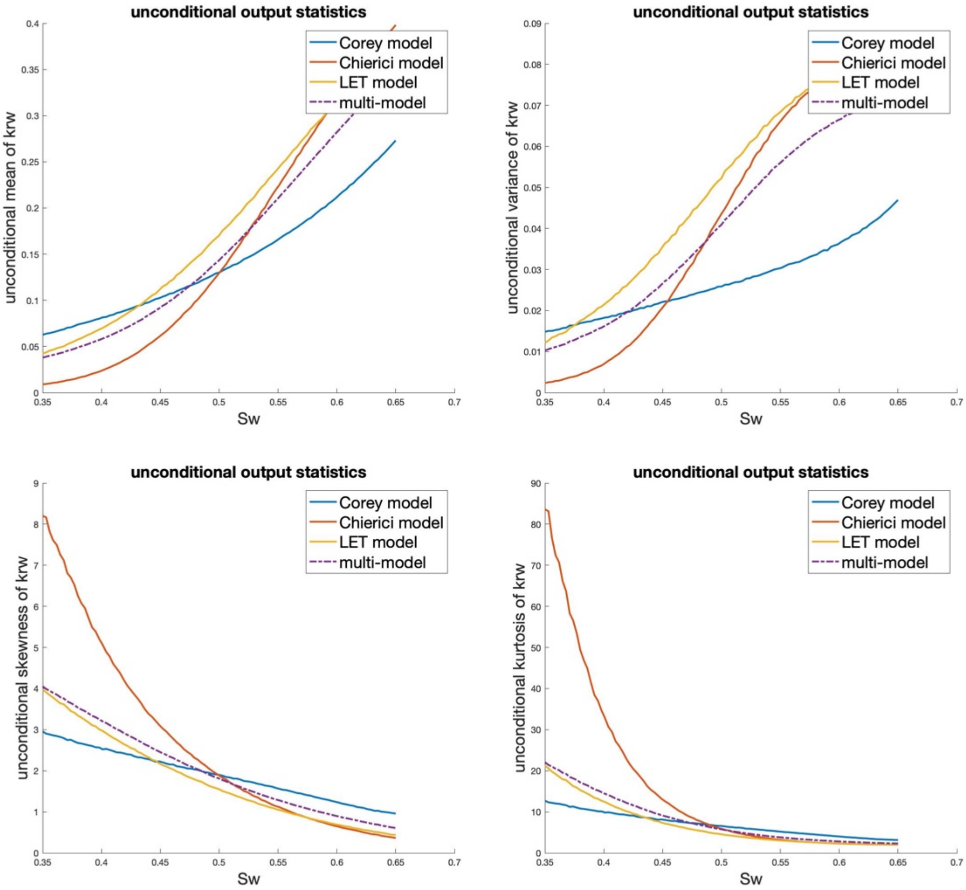

Figure 39: single-model and multi-model unconditional statistical moments of the three models for water relative permeability

The single-model unconditional statistics plots show the differences between the unconditional realizations of the three relative permeability models. From the plots of the unconditional variance, it can be observed that, for an unconditional scenario, the model with the lowest mean associated variance, and hence the lowest mean output uncertainty, is the Corey model followed by the Chierici model and then by the LET model. Apparently, the more complex a model (the higher the number of the model input factors), the greater the associated uncertainty. It should be remembered that the multi-model statistical moments are not the simple arithmetic mean of the single-model statistical moments (see chapter 2.2.1. for details).

In the figures below the four multi-model statistical moments-based sensitivity indices are reported for each variable input factor of each relative permeability model. The model-choice contributions to the sensitivity indices are plotted separately (dashed line) in order to allocate the contributions to sensitivity to the different sources of uncertainty (input factors uncertainty and model uncertainty).

The single-model statistical moments-based sensitivity indices are plotted too, with the purpose of visualizing the effect of the multi-model approach on the relative importance of the variable input factors:

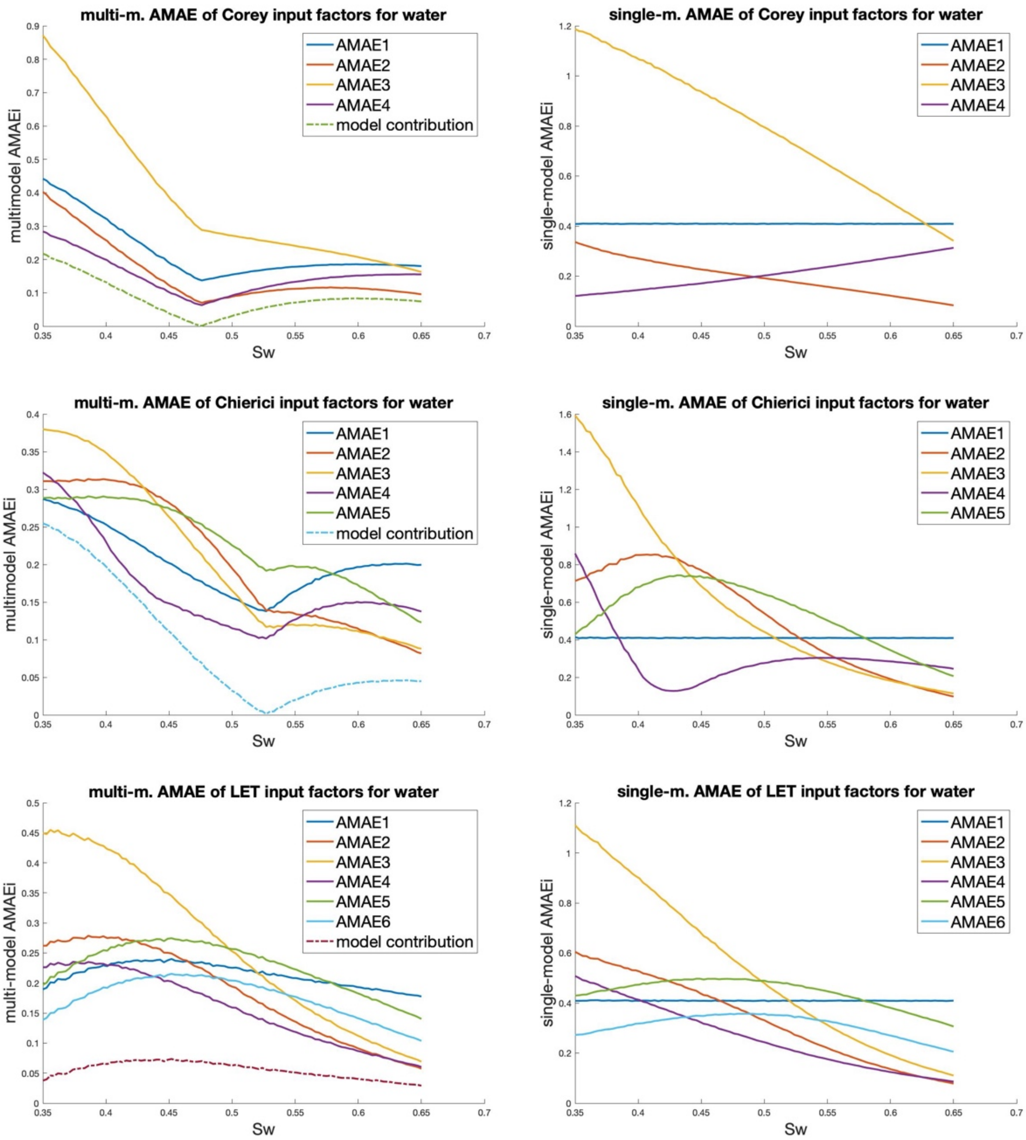

Figure 40: multi-model and single-model AMAE sensitivity indices for water relative permeability

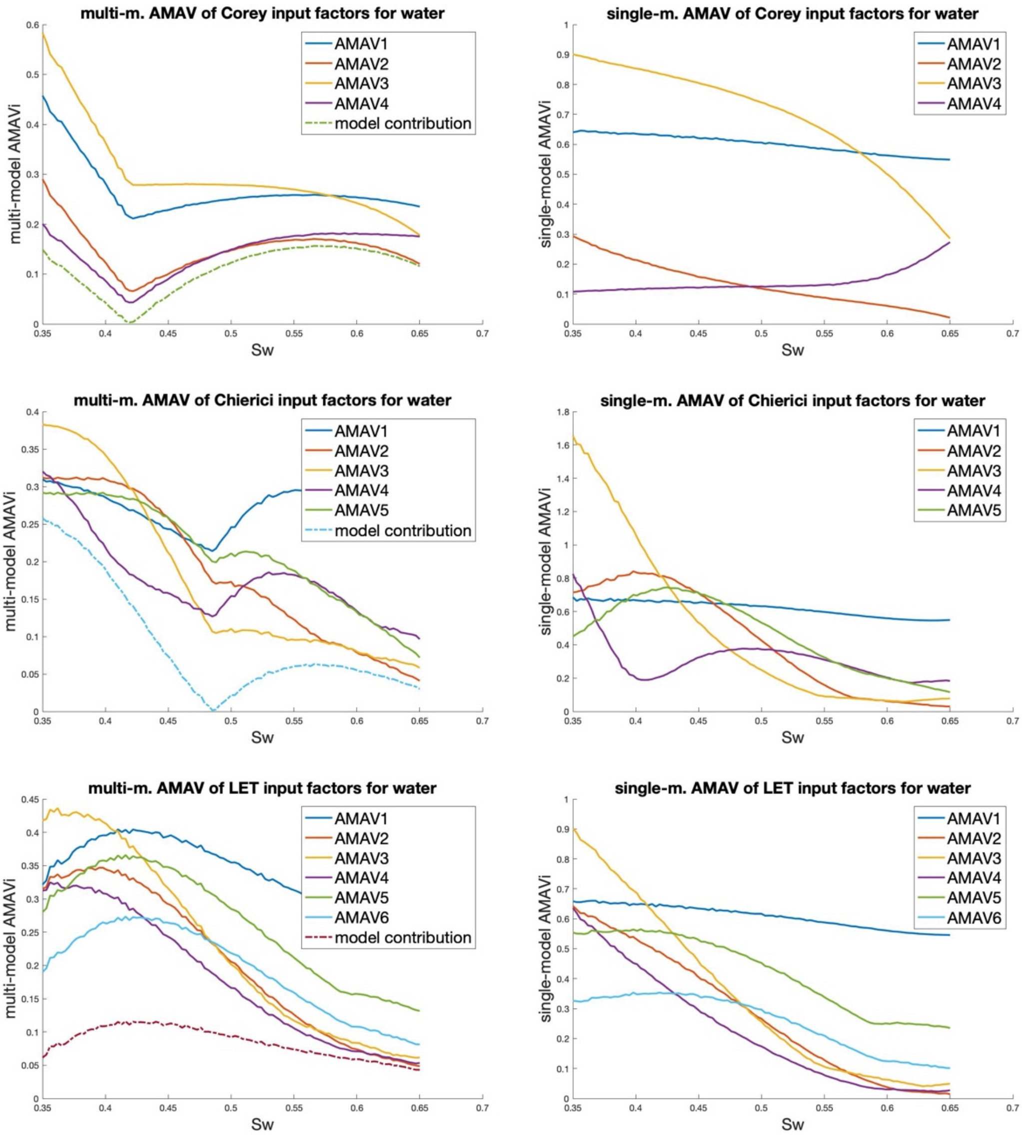

Figure 41: multi-model and single-model AMAV sensitivity indices for water relative permeability

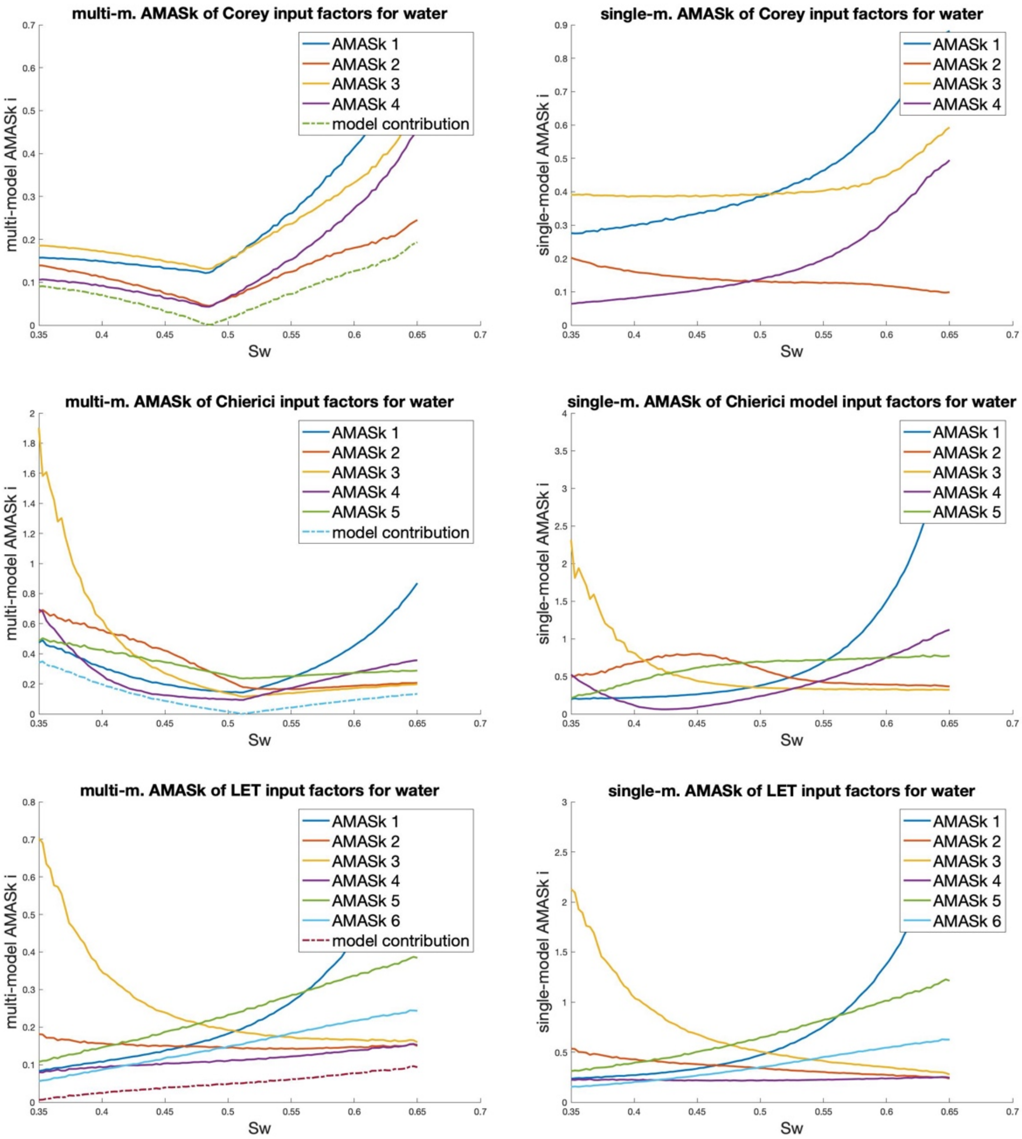

Figure 42: multi-model and single-model AMASk sensitivity indices for water relative permeability

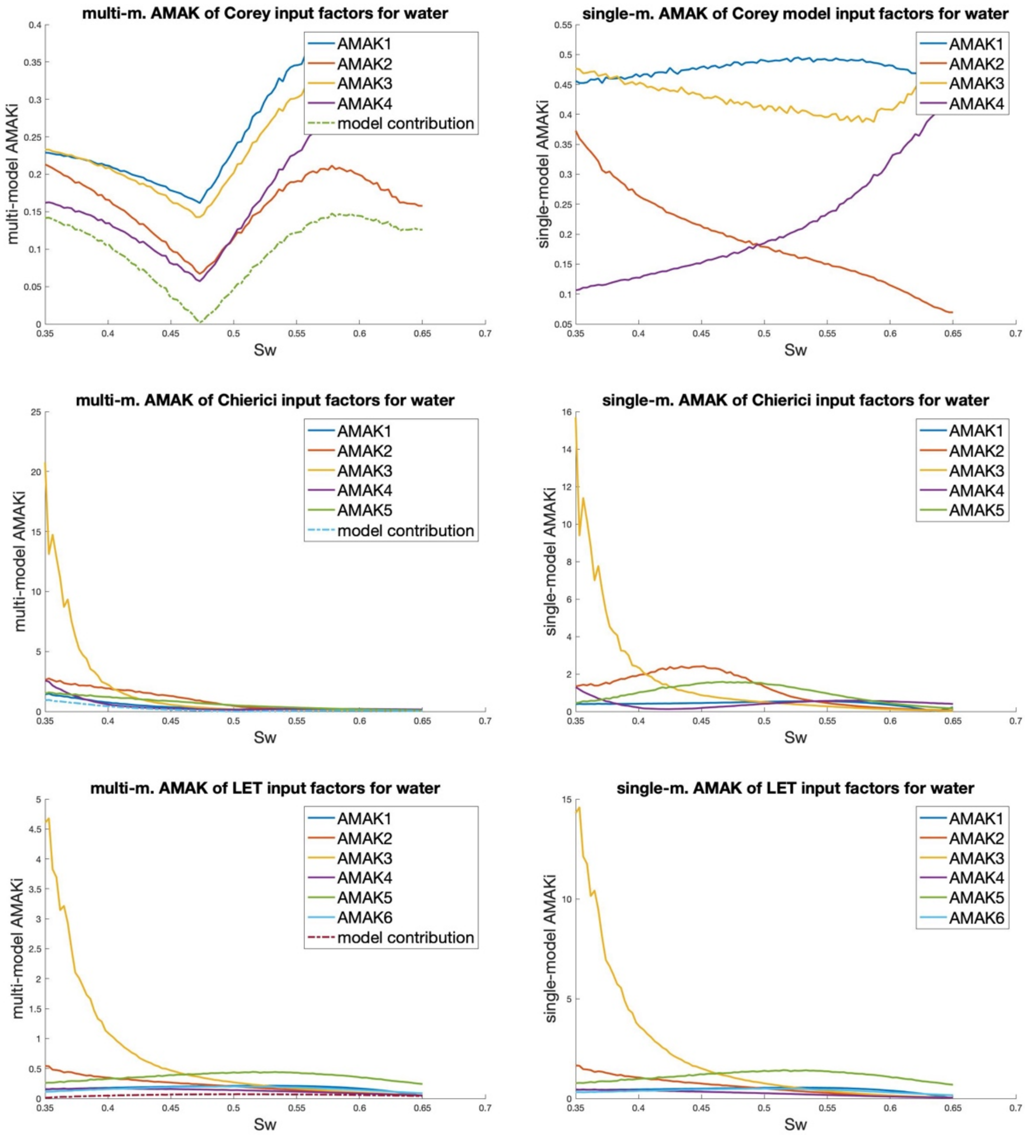

Figure 43: multi-model and single-model AMAK sensitivity indices for water relative permeability

From the sensitivity indices plots it can be observed that for some values of S_w the model-choice contribution to sensitivity is equal to 0: these values of S_w correspond to the points for which the single-model unconditional statistical moment of the model taken into account is equal to the multi-model unconditional statistical moment (as it can be observed in figure 39).

The ranking of the 15 variable input factors x_i^j (4 belonging to Corey model, 5 to Chierici model and 6 to LET model) is determined calculating and sorting the two ranking indices:

$$R_{AMAE_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAE_i^j \quad (4.10)$$

$$R_{AMAV_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAV_i^j \quad (4.11)$$

From the multi-model statistical moments-based sensitivity analysis of the three water relative permeability models the resulting rankings are shown in the table below. Each variable input factor is associated with a shade of the colour indicating the belonging model of the factor itself, for the sake of an easy visual inspection.

Input factors belonging to Corey model
Input factors belonging to Chierici model
Input factors belonging to LET model

Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
x_3^1: N_w^1 (37.13)	x_1^3: $k_{0,rw}^3$ (34.29)
x_3^3: L_w^3 (26.26)	x_3^1: N_w^1 (30.06)
x_5^2: S_{or}^2 (23.02)	x_1^2: $k_{0,rw}^2$ (27.06)
x_5^3: T_w^3 (23.01)	x_1^1: $k_{0,rw}^1$ (26.93)
x_1^1: $k_{0,rw}^1$ (22.42)	x_5^3 : T_w^3 (26.46)
x_1^3: $k_{0,rw}^3$ (21.61)	x_3^3: L_w^3 (23.18)
x_2^2: S_{wi}^2 (20.68)	x_5^2: S_{or}^2 (21.14)
x_3^2: B_w^2 (20.51)	x_2^3: S_{wi}^3 (20.76)
x_1^2: $k_{0,rw}^2$ (20.38)	x_6^3: S_{or}^3 (19.49)
x_2^3: S_{wi}^3 (18.68)	x_2^2: S_{wi}^2 (18.67)
x_6^3: S_{or}^3 (17.83)	x_4^3: E_w^3 (17.93)
x_4^2: M_w^2 (16.71)	x_3^2: B_w^2 (17.58)
x_2^1: S_{wi}^1 (15.68)	x_4^2: M_w^2 (17.55)
x_4^3: E_w^3 (15.38)	x_2^1: S_{wi}^1 (14.96)
x_4^1: S_{or}^1 (14.99)	x_4^1: S_{or}^1 (14.36)

The rankings obtained from these two multi-model indices are not in agreement with each other. This shows that in the multi-model context the effect of the variable input factors changes when considering different statistical moments of the output. Hence, the most appropriate ranking index must be chosen case-by-case according to the output's statistical moment of interest. The only aspect on which the two rankings agree is that the variable input factor x_4^1 (4^{th} input factor belonging to Corey model) is the less relevant in this multi-model scenario.

The detailed multi-model statistical moments-based global sensitivity analysis of the relative permeability models model for oil-relative permeability is reported in appendix C, section C1.

4.2.2. MULTI-MODEL, VARIANCE-BASED SENSITIVITY ANALYSIS

$n = 100{,}000$ (number of model evaluations for the first-order index)

$p = 50$ (number of parameters' bins for the first-order index)

$n_t = 1{,}000$ (number of model evaluations for the total-order index)

$p_t = 10$ (number of parameters' bins for the total-order index)

Before considering the sensitivity indices, it is important to understand that in this multi-model approach the models lose their identity as singular entities: the three relative permeability models are considered acting as a single more complex model. It follows that if a certain input factor appears in more than one model, it is anyway considered as one single input variable factor (as opposed to what happens for the multi-model statistical moments-based approach). For this reason, the variable input factors belonging to this new more complex "hybrid model" must be redefined.

When considering water relative permeability, the variable input factors of the models are:

- x_1: k_{rw}^0, end point of the water relative permeability curve.
- x_2: S_{wi}, irreducible water saturation.
- x_3: N_w, parameter belonging to Corey model for water.
- x_4: B_w, parameter belonging to Chierici model for water.
- x_5: M_w, parameter belonging to Chierici model for water.
- x_6: L_w, parameter belonging to LET model for water.
- x_7: E_w, parameter belonging to LET model for water.
- x_8: T_w, parameter belonging to LET model for water.
- x_9: S_{or}, residual oil saturation.

The input variability spaces of all these input variable factors have been defined in chapter 3.

It follows that, in this context:

- Corey model depends only on the input factors x_1, x_2, x_3, x_9.
- Chierici model depends only on the input factors x_1, x_2, x_4, x_5, x_9.
- LET model depends only on the input factors x_1, x_2, x_6, x_7, x_8, x_9.

In the figures below the multi-model first order and total order variance-based sensitivity indices are reported for each of the nine variable input factors. The single-model first order and total order Sobol (variance-based) sensitivity indices are also reported for the sake of comparison.

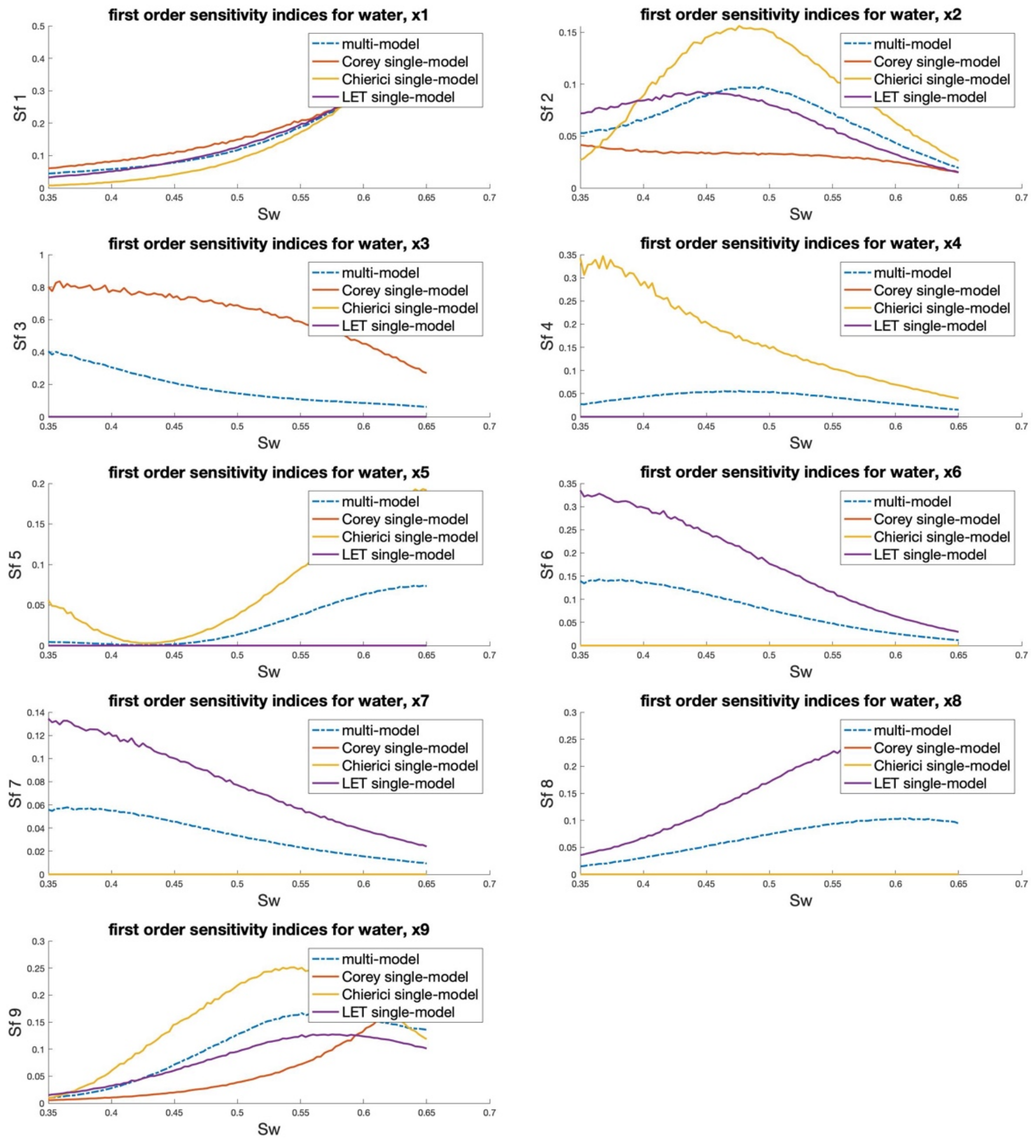

Figure 44: multi-model and single-model first order variance-based sensitivity indices for water relative permeability

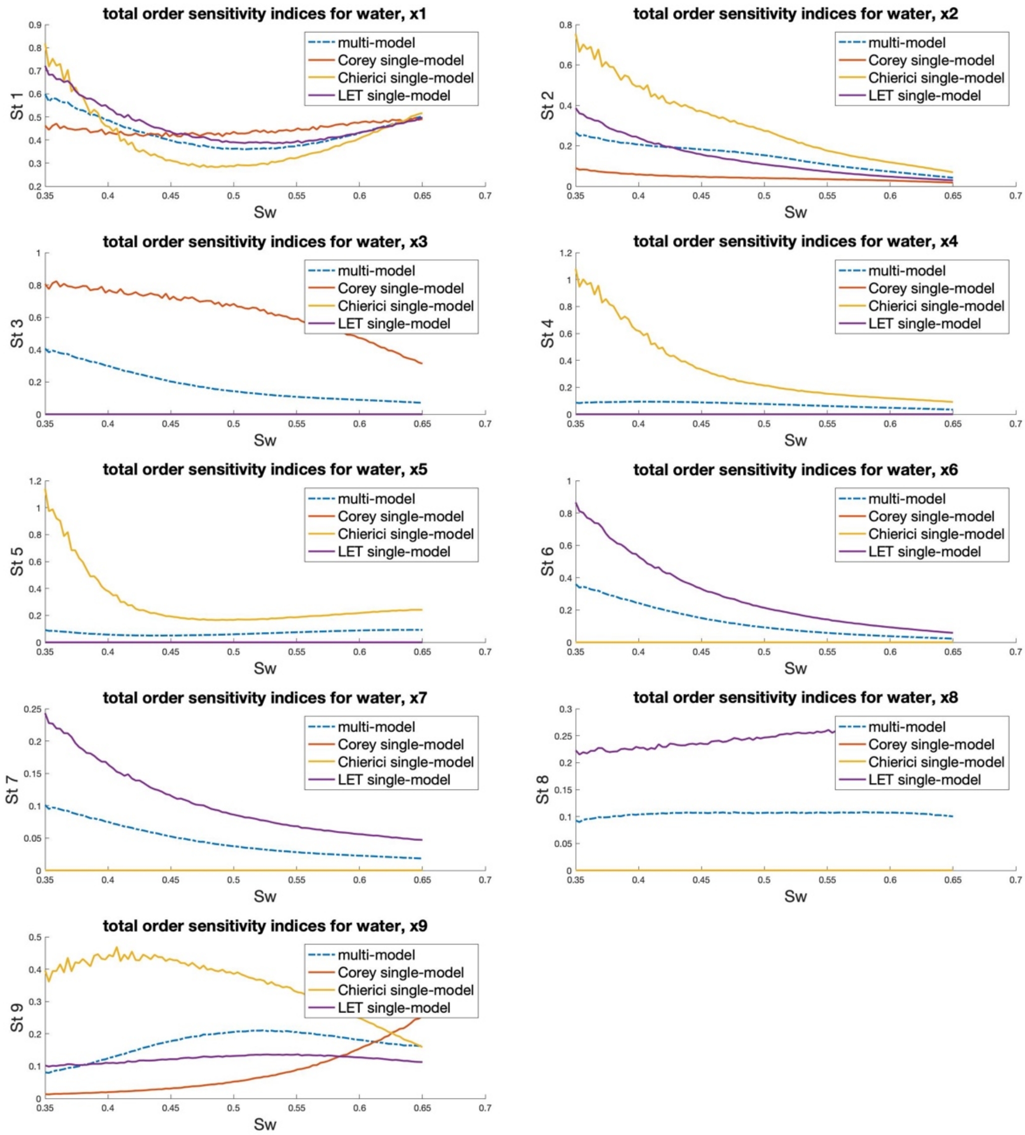

Figure 45: multi-model and single-model total order variance-based sensitivity indices for water relative permeability

The ranking of the 9 variable input factors x_i is determined calculating and sorting the two ranking indices:

$$R_{Sf_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} Sf_i^{multimodel} \quad (4.12)$$

$$R_{St_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} St_i^{multimodel} \quad (4.13)$$

For the multi-model variance-based sensitivity analysis of the three water relative permeability models the resulting rankings are shown in the table below. Each variable input factor is associated with a colour, to grant an easy visual inspection.

Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
x_3: N_w (18.27)	x_1: $k_{0,rw}$ (43.67)
x_1: $k_{0,rw}$ (16.16)	x_3: N_w (18.12)
x_9: S_{or} (10.43)	x_9: S_{or} (17.11)
x_6: L_w (8.00)	x_2: S_{wi} (14.65)
x_8: T_w (6.90)	x_6: L_w (12.92)
x_2: S_{wi} (6.78)	x_8: T_w (10.62)
x_4: B_w (4.09)	x_4: B_w (7.22)
x_7: E_w (3.48)	x_5: M_w (7.08)
x_5: M_w (2.66)	x_7: E_w (4.62)

The rankings obtained by these two multi-model variance-based indices are not in agreement at all with each other. It should be remembered that the sensitivity index $Sf_i^{multimodel}$ accounts only for the first order effect of the input factors on the output variability, while the sensitivity index $St_i^{multimodel}$ considers also the effect of the interactions between different variable input factors. In this case, the rankings show that in this multi-model context the interactions between the variable input factors play a significant role and can change the relative importance of the input factors.

The detailed multi-model variance-based global sensitivity analysis of the relative permeability models' model for oil relative permeability is reported in appendix C, section C2.

4.2.3. DISCUSSION

In the table below the rankings obtained from the different multi-model sensitivity analysis techniques, when applied to the three water relative permeability models, are summarized:

Multi-model statistical moments-based sensitivity analysis of the water relative permeability models		Multi-model variance-based sensitivity analysis of the water relative permeability models	
Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$	Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
$x_3^1: N_w^1$ (37.13)	$x_1^3: k_{0,rw}^3$ (34.29)	$x_3: N_w$ (18.27)	$x_1: k_{0,rw}$ (43.67)
$x_3^3: L_w^3$ (26.26)	$x_3^1: N_w^1$ (30.06)	$x_1: k_{0,rw}$ (16.16)	$x_3: N_w$ (18.12)
$x_5^2: S_{or}^2$ (23.02)	$x_1^2: k_{0,rw}^2$ (27.06)	$x_9: S_{or}$ (10.43)	$x_9: S_{or}$ (17.11)
$x_5^3: T_w^3$ (23.01)	$x_1^1: k_{0,rw}^1$ (26.93)	$x_6: L_w$ (8.00)	$x_2: S_{wi}$ (14.65)
$x_1^1: k_{0,rw}^1$ (22.42)	$x_5^3: T_w^3$ (26.46)	$x_8: T_w$ (6.90)	$x_6: L_w$ (12.92)
$x_1^3: k_{0,rw}^3$ (21.61)	$x_3^3: L_w^3$ (23.18)	$x_2: S_{wi}$ (6.78)	$x_8: T_w$ (10.62)
$x_2^2: S_{wi}^2$ (20.68)	$x_5^2: S_{or}^2$ (21.14)	$x_4: B_w$ (4.09)	$x_4: B_w$ (7.22)
$x_3^2: B_w^2$ (20.51)	$x_2^3: S_{wi}^3$ (20.76)	$x_7: E_w$ (3.48)	$x_5: M_w$ (7.08)
$x_1^2: k_{0,rw}^2$ (20.38)	$x_6^3: S_{or}^3$ (19.49)	$x_5: M_w$ (2.66)	$x_7: E_w$ (4.62)
$x_2^3: S_{wi}^3$ (18.68)	$x_2^2: S_{wi}^2$ (18.67)		
$x_6^3: S_{or}^3$ (17.83)	$x_4^3: E_w^3$ (17.93)		
$x_4^2: M_w^2$ (16.71)	$x_3^2: B_w^2$ (17.58)		
$x_2^1: S_{wi}^1$ (15.68)	$x_4^2: M_w^2$ (17.55)		
$x_4^3: E_w^3$ (15.38)	$x_2^1: S_{wi}^1$ (14.96)		
$x_4^1: S_{or}^1$ (14.99)	$x_4^1: S_{or}^1$ (14.36)		

It can be observed that, even within the application of the same multi-model sensitivity analysis technique, there is no accordance between the rankings obtained from different sensitivity indices.

The comparison between the rankings obtained from the two multi-model sensitivity analysis techniques does not make any theoretical sense, because of the different definition of the input variable factors (see chapter 2 for complete details). For this reason, the most appropriate multi-model sensitivity analysis technique (and the corresponding ranking index/indices) must be chosen case-by-case according to the output distribution features of interest and to the ultimate goal of the analysis. In general, as a first approach, the following guidelines can be taken into account:

- If a complete characterization of the output probability density function (through many statistical moments) is required, the statistical moments-based analysis should be chosen. For the quantification of sensitivity through the output variance only both the techniques are valid (but not equivalent!).
- If the models' input factors are meant to be calibrated model-by-model conserving the different models' identity (for example, by fitting each model to a set of measurements), the multi-model statistical moments-based sensitivity analysis should be adopted. If the models' input factors are meant to be calibrated according to their physical meaning only (for example, by experimentally measuring the factors or by inverse modelling in a multi-model context), the multi-model variance-based sensitivity analysis should be adopted.

Be aware: these few considerations are not exhaustive of all the possible cases and scenarios! So, for any specific application the theories, assumptions and mathematical workflows at the base of these two multi-model sensitivity analysis techniques should be carefully consulted and taken into account.

4.3. MULTI-MODEL INFORMED SCENARIO

In this section the multi-model informed global sensitivity analysis techniques described in chapter 2.3. are applied to the two different samples described in the research article "Interpretation of two-phase relative permeability curves through multiple formulations and model quality criteria" (Moghadasi, Guadagnini, Inzoli, & Bartosek, 2015). In this article, model identification criteria are employed to rank and evaluate a set of alternative models (Corey model, Chierici model, LET model) in the context of the interpretation of laboratory scale experiments yielding two-phase relative permeability curves. High quality two-phase relative permeability estimates are employed, which result from steady-state imbibition experiments on two diverse porous media: a quartz sand pack and a Berea sandstone core.

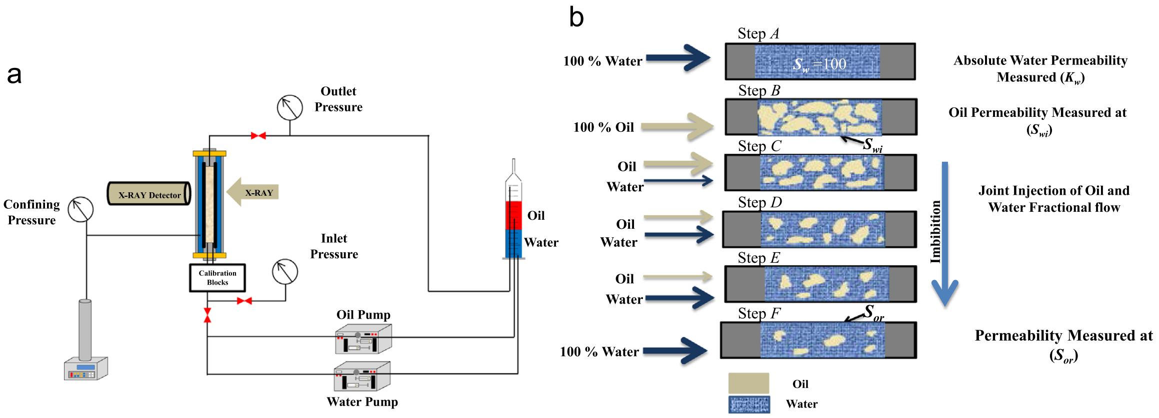

Figure 46: (a) sketch of experimental set-up; (b) steady state imbibition process

Model uncertainty is quantified through the model weights w^j which are rendered by model posterior probabilities conditional on the experimental observations. These weights are then employed to rank the models according to their relative skill to interpret the observations and obtain model averaged results which allow accommodating uncertainties arising from differences amongst model structures within a unified theoretical framework.

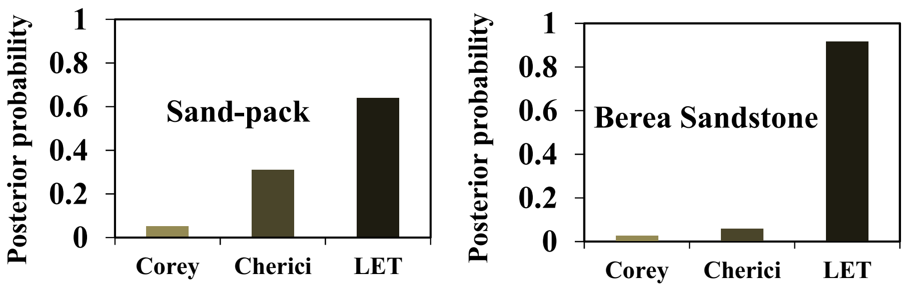

Figure 47: posterior probabilities associated with the models in the considered scenarios for water relative permeability

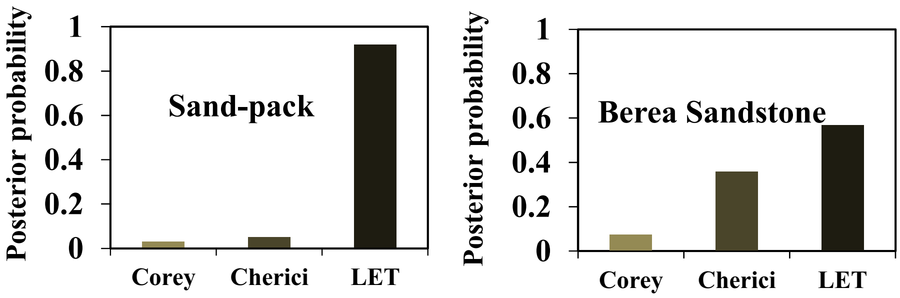

Figure 48: posterior probabilities associated with the models in the considered scenarios for oil relative permeability

The models input factors which are physical properties of the porous medium ($k_{r\alpha}^{0}$, S_{wi}, S_{or}) are experimentally measured multiple times, obtaining their estimated mean value and standard deviation: the uncertainty in the value of these input factors is a direct consequence of the measurement errors. The input factors which are model parameters ($N_{\alpha}, B_{\alpha}, M_{\alpha}, L_{\alpha}, E_{\alpha}, T_{\alpha}$) are estimated within a Maximum Likelihood framework, and for each of these variable input factors the lower and upper limits identifying the 95% confidence limits are provided.

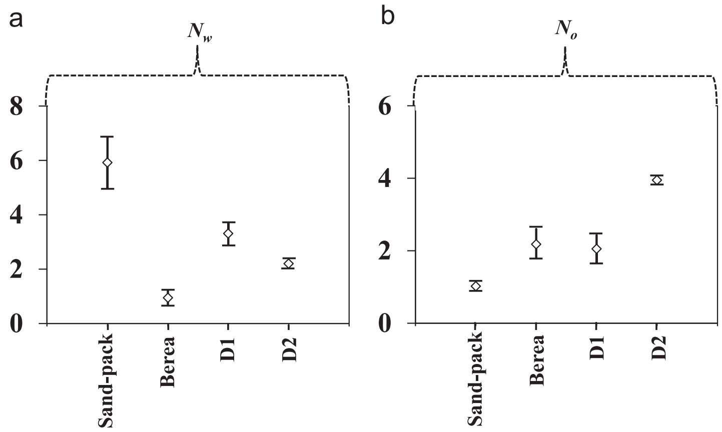

Figure 49: Estimated Corey model input factors values for (a) k_{rw} and (b) k_{ro} associated with the considered datasets. Intervals associated with the upper (U) and lower (L) limits identifying the 95% uncertainty bounds around the estimate are also depicted

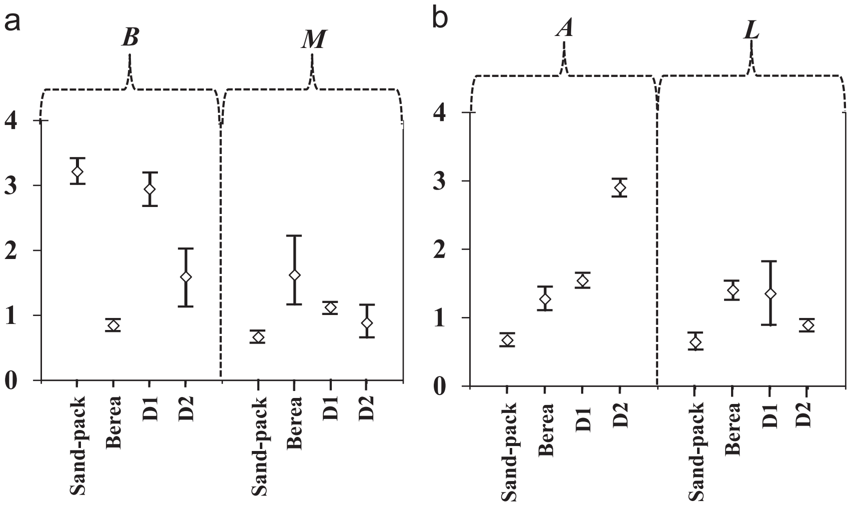

Figure 50: Estimated Chierici model input factors values for (a) k_{rw} and (b) k_{ro} associated with the considered datasets

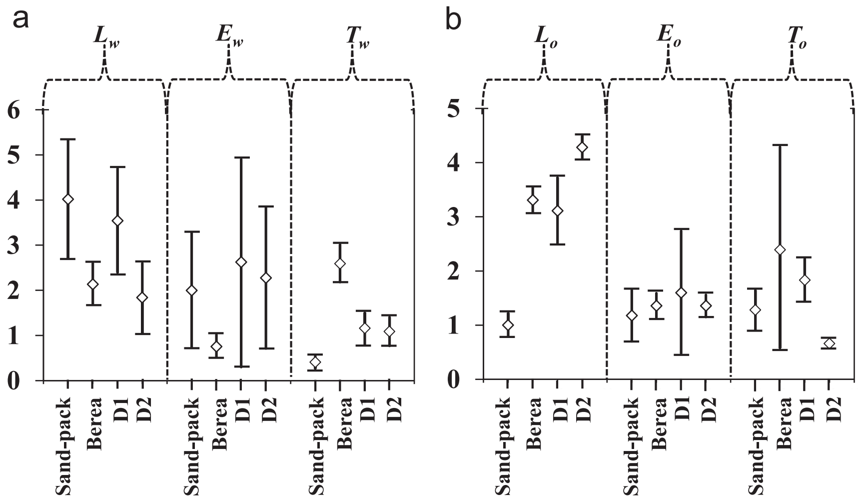

Figure 51: Estimated LET model input factors values for (a) k_{rw} and (b) k_{ro} associated with the considered datasets

The results obtained in the previously mentioned article highlight that in most cases the complexity of the problem appears to favour a model with a high number of uncertain parameters over a simpler model structure. Posterior probabilities reveal that in several cases, most notably for the assessment of oil relative permeabilities, the weights associated with the simplest models is not negligible. This suggests that, in these cases, uncertainty quantification might benefit from a multi-model analysis, including both low and high-complexity models. In most of the analysed cases, the model averaging leads to interpretations of the available data, which are characterized by a higher degree of fidelity than that provided by the most skillful model.

4.3.1. "SAND PACK" SAMPLE

4.3.1.1. MULTI-MODEL, INFORMED STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.05$
- $w^2 = 0.3$
- $w^1 = 0.65$

VARIABLE INPUT FACTORS

The models variable input factors are expressed in the form:

$$x_i^j = (x_{i,min}^j\ ;\ x_{i,max}^j)$$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the brackets define its 95% confidence limits.

- x_1^1: $k_{rw}^0 = (0.405\ ;\ 0.461)$
- x_2^1: $S_{wi} = (0.166\ ;\ 0.214)$
- x_3^1: $N_w = (4.9\ ;\ 6.8)$
- x_4^1: $S_{or} = (0.1\ ;\ 0.18)$

- x_1^2: $k_{rw}^0 = (0.405\ ;\ 0.461)$
- x_2^2: $S_{wi} = (0.166\ ;\ 0.214)$
- x_3^2: $B_w = (3.03\ ;\ 4.5)$
- x_4^2: $M_w = (0.62\ ;\ 0.7)$
- x_5^2: $S_{or} = (0.1\ ;\ 0.18)$

- x_1^3: $k_{rw}^0 = (0.405\ ;\ 0.461)$
- x_2^3: $S_{wi} = (0.166\ ;\ 0.214)$
- x_3^3: $L_w = (2.71\ ;\ 5.32)$
- x_4^3: $E_w = (0.73\ ;\ 3.28)$
- x_5^3: $T_w = (0.27\ ;\ 0.55)$
- x_6^3: $S_{or} = (0.1\ ;\ 0.18)$

Unacceptable combinations of input variable factors could be generated from the provided data, due to the fact that the Gaussian probability distribution function is not inferiorly or superiorly limited. This would lead to non-sensical model evaluations (like imaginary or negative relative permeabilities) during the Monte Carlo simulations of the models, which eventually cause lower accuracy and greater instability in the results.

For this reason, the input variable factors probability density functions are truncated to guarantee the respect of the following mathematical constraints:

- $N_\alpha, B_\alpha, M_{\alpha,} L_\alpha, E_\alpha, T_\alpha > 0$
- $0 \le k_r^0 \le 1$

- $S_{wi} \leq S_w$
- $S_{or} \leq (1 - S_w)$

To avoid excessive instability in the statistical moments' evaluations (and hence in the sensitivity indices' evaluations) it is also necessary to limit the range of water saturations S_w for which the models' output is evaluated: water relative permeability k_{rw} is evaluated for $0.4 \leq S_w \leq 0.82$.

$n = 200{,}000$ (number of model evaluations)

$p = 25$ (number of parameters' bins)

In the figure below the unconditional single-model and multi-model statistical moments are reported for water relative permeability for the "sand pack" sample:

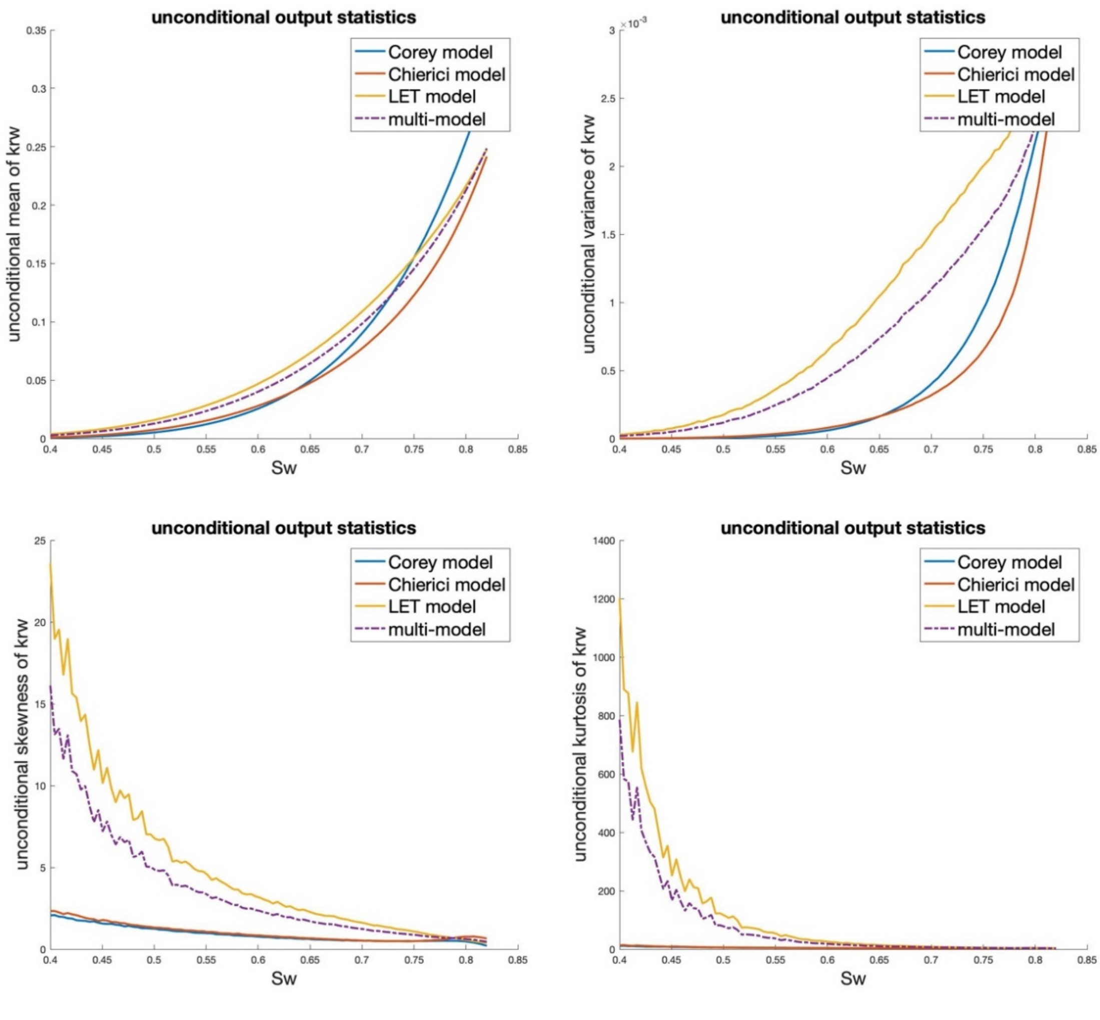

Figure 52: single-model and multi-model unconditional output statistics of the relative permeability models informed to the "sand pack" sample for water relative permeability

Despite of the measures adopted to prevent instability in the results, it can be observed that the skewness and kurtosis of the unconditional outputs are still strongly unstable, especially for lower values of water saturation S_w.

In the figures below the informed multi-model statistical moments-based sensitivity indices for the “sand pack” sample are reported for water relative permeability. The single-model statistical moments-based indices are reported too, in the attempt to highlight the differences between the multi-model and the single-model approach:

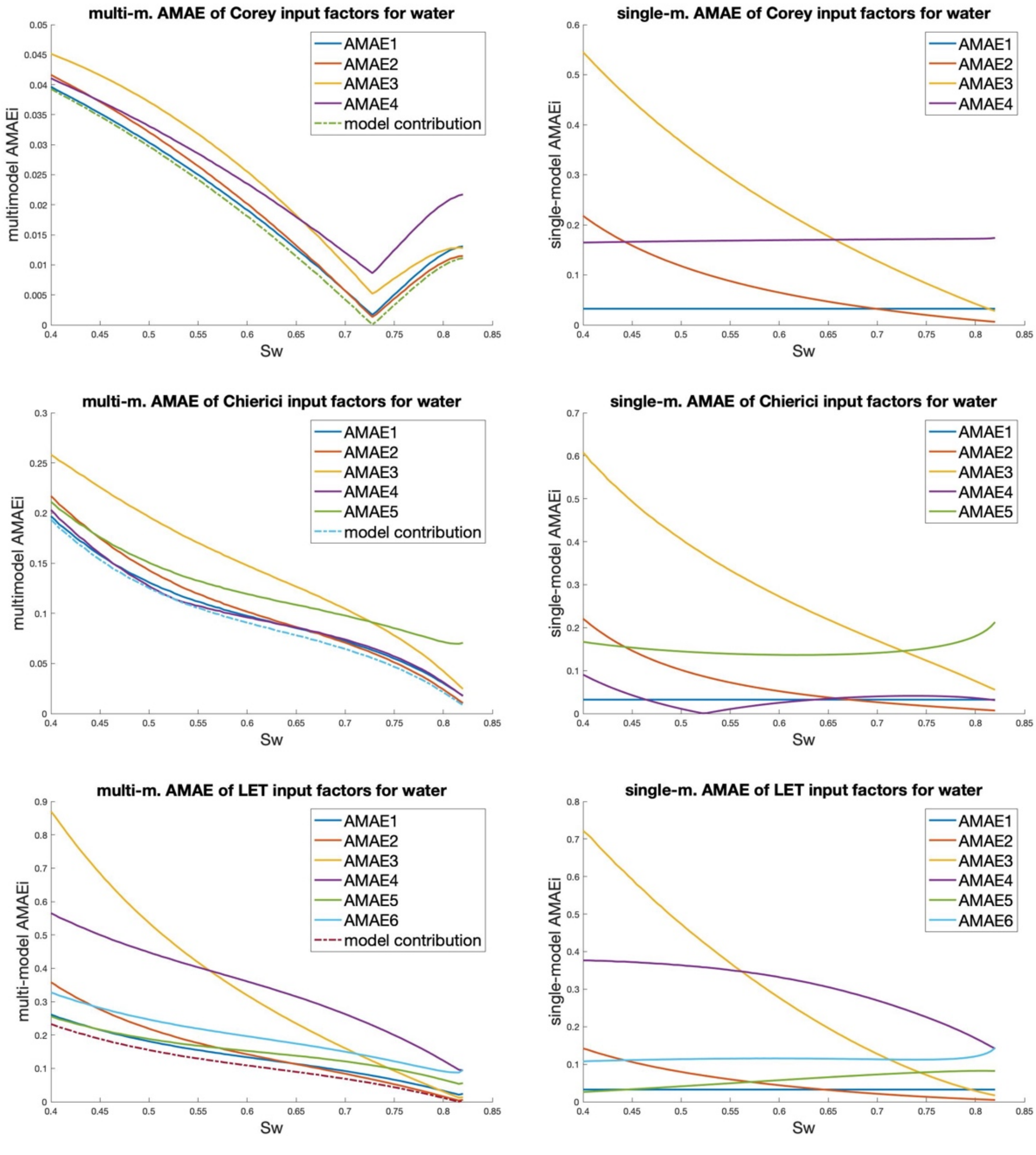

Figure 53: multi-model and single-model informed AMAE sensitivity indices of the “sand pack” sample for water relative permeability

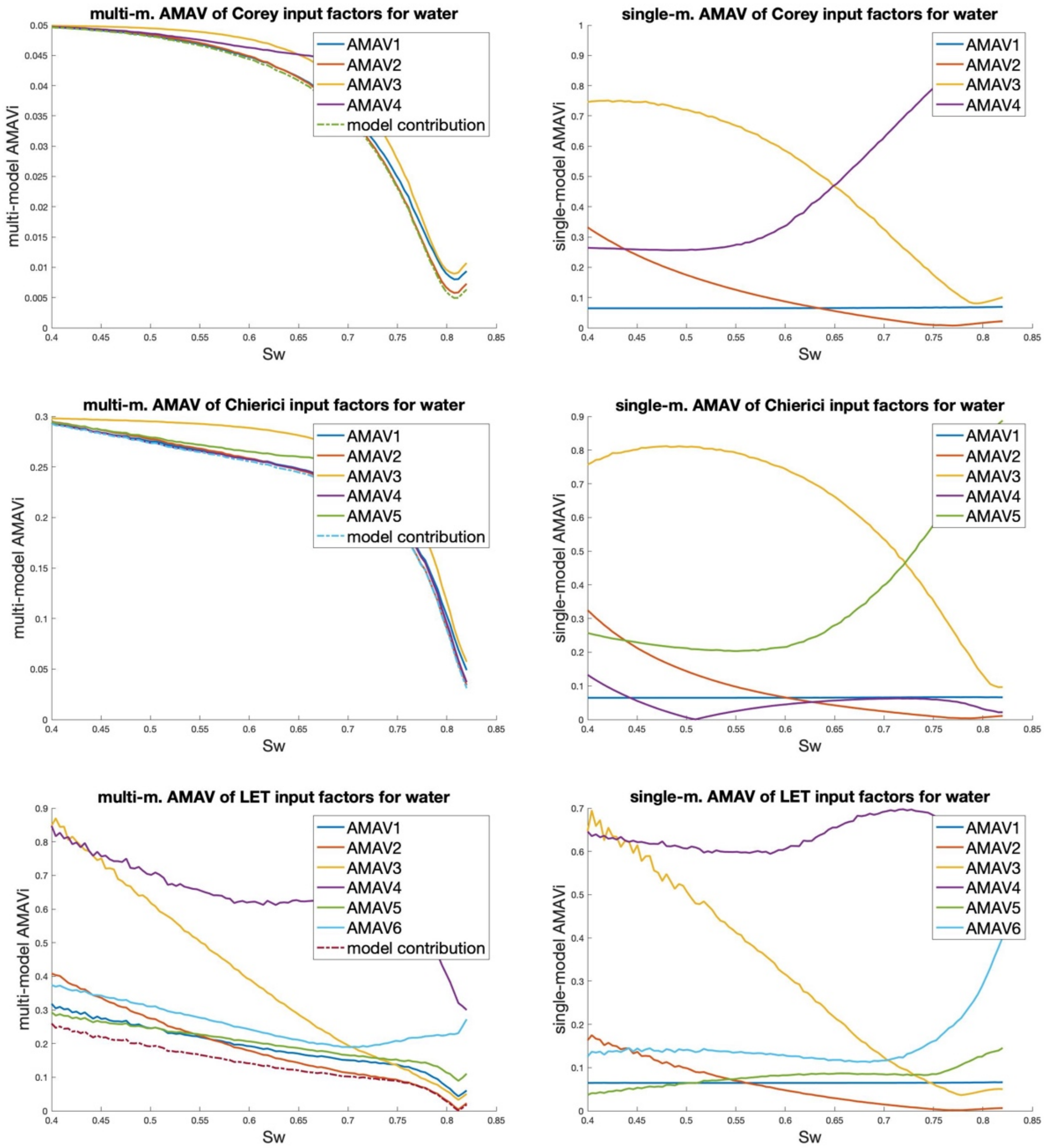

Figure 54: multi-model and single-model informed AMAV sensitivity indices of the "sand pack" sample for water relative permeability

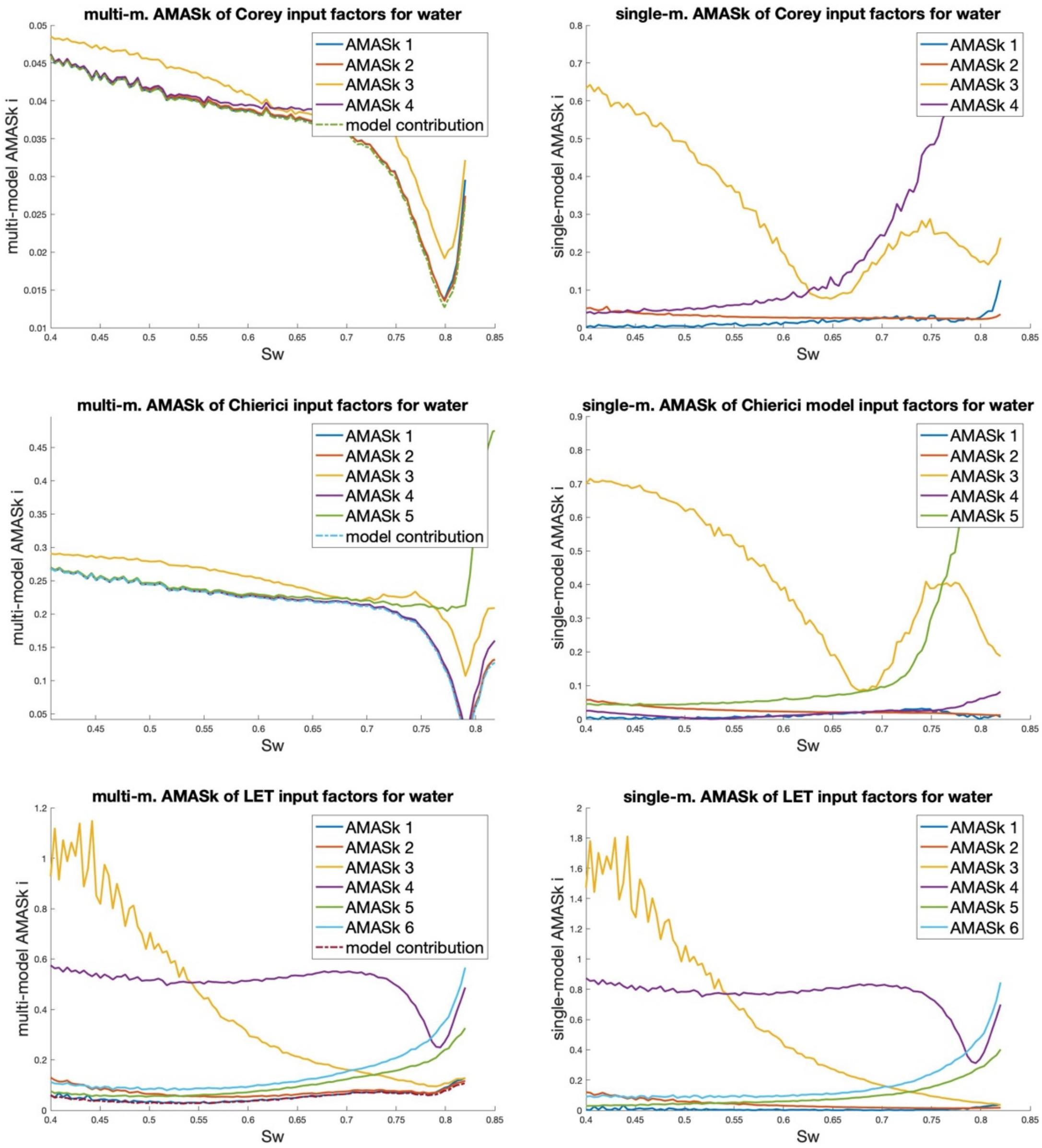

Figure 55: multi-model and single-model informed AMASk sensitivity indices of the "sand pack" sample for water relative permeability

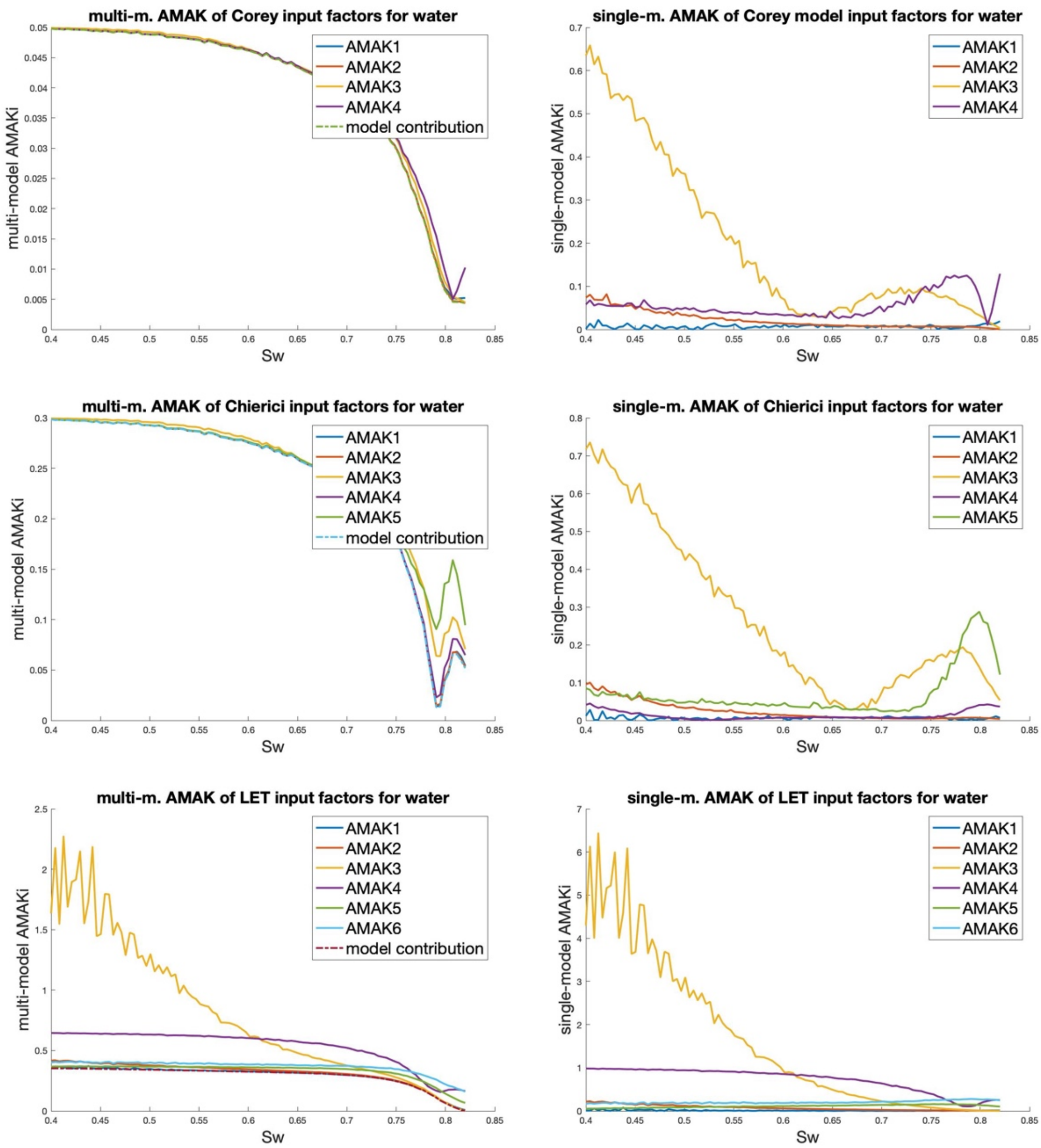

Figure 56: multi-model and single-model informed AMAK sensitivity indices of the "sand pack" sample for water relative permeability

The ranking of the 15 variable input factors x_i^j (4 belonging to Corey model, 5 to Chierici model and 6 to LET model) is determined calculating and sorting the two ranking indices:

$$R_{AMAE_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAE_i^j \quad (4.14)$$

$$R_{AMAV_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAV_i^j \quad (4.15)$$

For the multi-model statistical moments-based informed global sensitivity analysis of the "sand pack" sample for water relative permeability the resulting rankings are shown in the following table. Each variable input factor is associated with a shade of the colour indicating the belonging model of the factor itself, for the sake of an easy visual inspection:

Input factors belonging to Corey model
Input factors belonging to Chierici model
Input factors belonging to LET model

Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
x_4^3: E_w^3 (30.29)	x_4^3: E_w^3 (61.66)
x_3^3: L_w^3 (26.46)	x_3^3: L_w^3 (32.66)
x_6^3: S_{or}^3 (17.61)	x_5^2: S_{or}^2 (26.68)
x_5^3: T_w^3 (13.61)	x_3^2: B_w^2 (25.72)
x_3^2: B_w^2 (12.65)	x_6^3: S_{or}^3 (24.13)
x_2^3: S_{wi}^3 (11.91)	x_1^2: $k_{0,rw}^2$ (22.98)
x_1^3: $k_{0,rw}^3$ (11.33)	x_4^2: M_w^2 (22.82)
x_5^2: S_{or}^2 (11.21)	x_2^2: S_{wi}^2 (22.72)
x_2^2: S_{wi}^2 (8.79)	x_5^3: T_w^3 (18.85)
x_1^2: $k_{0,rw}^2$ (8.54)	x_1^3: $k_{0,rw}^3$ (17.26)
x_4^2: M_w^2 (8.50)	x_2^3: S_{wi}^3 (15.62)
x_4^1: S_{or}^1 (2.16)	x_4^1: S_{or}^1(4.66)
x_3^1: N_w^1 (2.09)	x_3^1: N_w^1 (3.93)
x_2^1: S_{wi}^1 (1.65)	x_1^1: $k_{0,rw}^1$ (3.67)
x_1^1: $k_{0,rw}^1$ (1.61)	x_2^1: S_{wi}^1 (3.62)

4.3.1.2. MULTI-MODEL, INFORMED VARIANCE-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.05$
- $w^2 = 0.3$
- $w^1 = 0.65$

VARIABLE INPUT FACTORS

The models variable input factors are expressed in the form:

$x_i = (x_{i,min}\ ;\ x_{i,max})$

The values in the brackets define its 95% confidence limits.

- x_1: $k_{rw}^0 = (0.405\ ;\ 0.461)$
- x_2: $S_{wi} = (0.166\ ;\ 0.214)$
- x_3: $N_w = (4.9\ ;\ 6.8)$
- x_4: $B_w = (3.03\ ;\ 4.5)$
- x_5: $M_w = (0.62\ ;\ 0.7)$
- x_6: $L_w = (2.71\ ;\ 5.32)$
- x_7: $E_w = (0.73\ ;\ 3.28)$
- x_8: $T_w = (0.27\ ;\ 0.55)$
- x_9: $S_{or} = (0.1\ ;\ 0.18)$

Water relative permeability k_{rw} is evaluated for $0.4 \leq S_w \leq 0.82$.

$n = 200{,}000$ (number of model evaluations for the first-order index)

$p = 25$ (number of parameters' bins for the first-order index)

$n_t = 700$ (number of model evaluations for the total-order index)

$p_t = 6$ (number of parameters' bins for the total-order index)

In the figures below the single-model and multi-model first order and total order variance-based informed sensitivity indices of the “sand pack” sample for water relative permeability are reported:

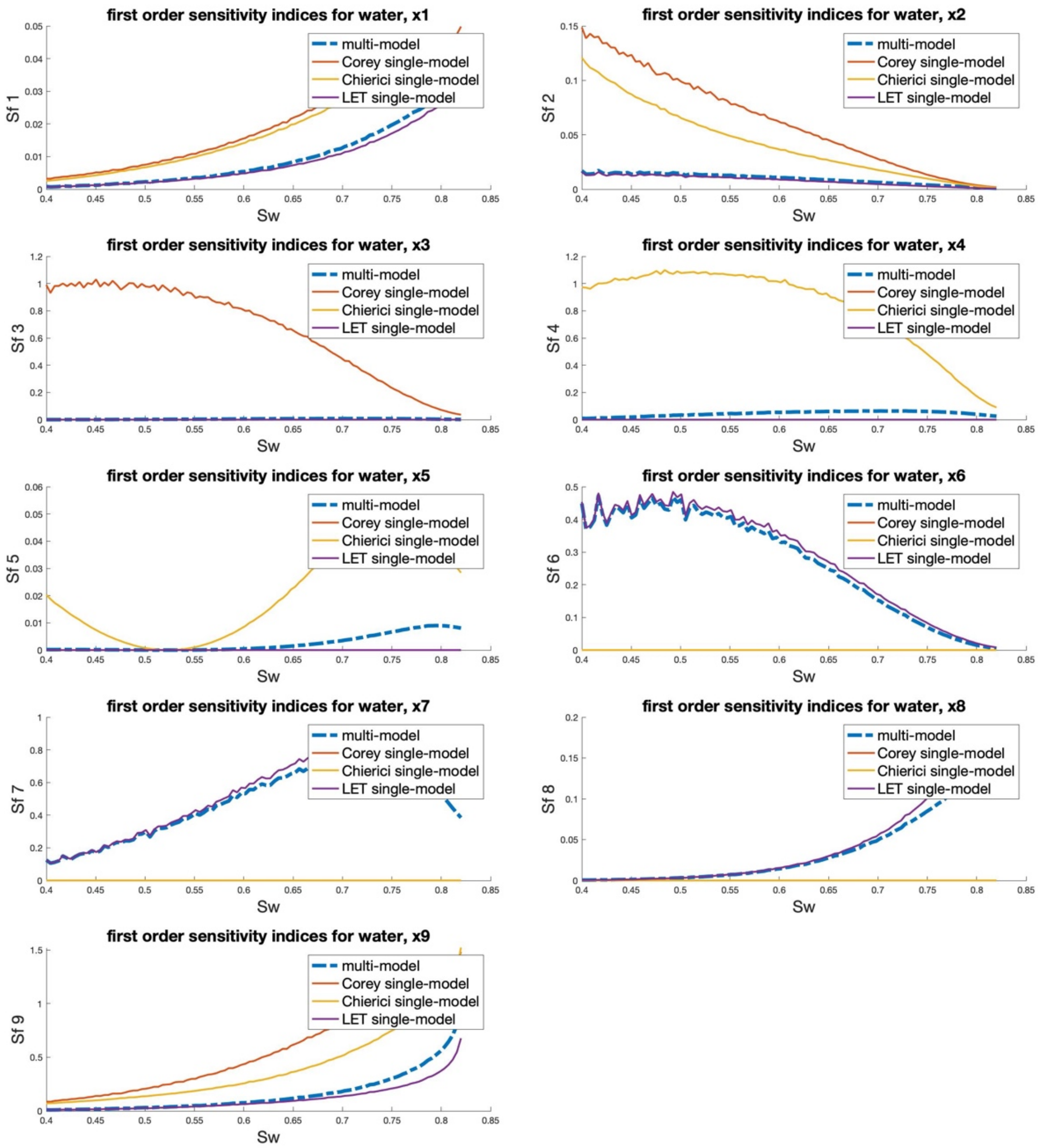

Figure 57: first order single-model and multi-model informed variance-based sensitivity indices of the “sand pack” sample for water relative permeability

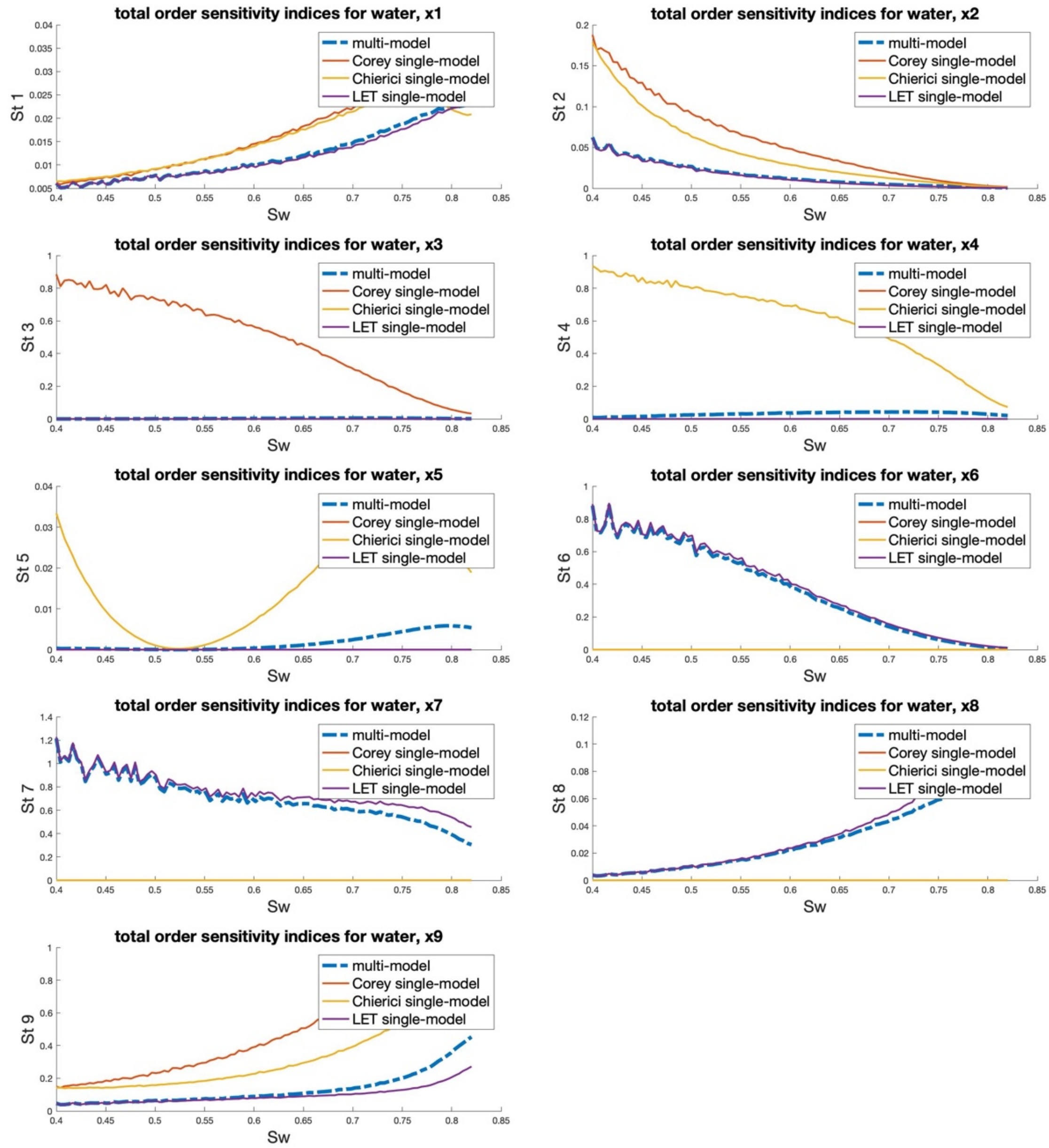

Figure 58: total order single-model and multi-model informed variance-based sensitivity indices of the "sand pack" sample for water relative permeability

The ranking of the nine variable input factors x_i is determined calculating the two ranking indices:

$$R_{Sf_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} Sf_i^{multimodel} \quad (4.16)$$

$$R_{St_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} St_i^{multimodel} \quad (4.17)$$

For the informed variance-based global sensitivity analysis of the "sand pack" sample for water relative permeability the resulting rankings are shown in the following table. Each variable input factor is associated with a colour, aiming to ease visual inspection.

Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
x_7: E_w (48.05)	x_7: E_w (70.78)
x_6: L_w (27.83)	x_6: L_w (34.47)
x_9: S_{or} (16.07)	x_9: S_{or} (12.91)
x_4: B_w (4.51)	x_4: B_w (3.16)
x_8: T_w (3.61)	x_8: T_w (3.03)
x_2: S_{wi} (0.97)	x_2: S_{wi} (1.59)
x_1: $k_{0,rw}$ (0.96)	x_1: $k_{0,rw}$ (1.21)
x_3: N_w (0.46)	x_3: N_w (0.32)
x_5: M_w (0.25)	x_5: M_w (0.17)

4.3.1.3. DISCUSSION

In the table below the rankings obtained from the different informed multi-model sensitivity analysis techniques when applied to the "sand pack" sample for water relative permeability are summarized.

Multi-model informed statistical moments-based sensitivity analysis of the "sand pack" sample for water relative permeability	
Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
$x_4^3: E_w^3$ (30.29)	$x_4^3: E_w^3$ (61.66)
$x_3^3: L_w^3$ (26.46)	$x_3^3: L_w^3$ (32.66)
$x_6^3: S_{or}^3$ (17.61)	$x_5^2: S_{or}^2$ (26.68)
$x_5^3: T_w^3$ (13.61)	$x_3^2: B_w^2$ (25.72)
$x_3^2: B_w^2$ (12.65)	$x_6^3: S_{or}^3$ (24.13)
$x_2^3: S_{wi}^3$ (11.91)	$x_1^2: k_{0,rw}^2$ (22.98)
$x_1^3: k_{0,rw}^3$ (11.33)	$x_4^2: M_w^2$ (22.82)
$x_5^2: S_{or}^2$(11.21)	$x_2^2: S_{wi}^2$ (22.72)
$x_2^2: S_{wi}^2$ (8.79)	$x_5^3: T_w^3$ (18.85)
$x_1^2: k_{0,rw}^2$ (8.54)	$x_1^3: k_{0,rw}^3$ (17.26)
$x_4^2: M_w^2$ (8.50)	$x_2^3: S_{wi}^3$ (15.62)
$x_4^1: S_{or}^1$ (2.16)	$x_4^1: S_{or}^1$ (4.66)
$x_3^1: N_w^1$ (2.09)	$x_3^1: N_w^1$ (3.93)
$x_2^1: S_{wi}^1$ (1.65)	$x_1^1 : k_{0,rw}^1$ (3.67)
$x_1^1: k_{0,rw}^1$ (1.61)	$x_2^1: S_{wi}^1$ (3.62)

Multi-model informed variance-based sensitivity analysis of the "sand pack" sample for water relative permeability	
Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
$x_7: E_w$ (48.05)	$x_7: E_w$ (70.78)
$x_6: L_w$ (27.83)	$x_6: L_w$ (34.47)
$x_9: S_{or}$ (16.07)	$x_9: S_{or}$ (12.91)
$x_4: B_w$ (4.51)	$x_4: B_w$ (3.16)
$x_8 : T_w$ (3.61)	$x_8: T_w$ (3.03)
$x_2: S_{wi}$ (0.97)	$x_2: S_{wi}$ (1.59)
$x_1: k_{0,rw}$ (0.96)	$x_1: k_{0,rw}$ (1.21)
$x_3: N_w$ (0.46)	$x_3: N_w$ (0.32)
$x_5: M_w$ (0.25)	$x_5: M_w$ (0.17)

It can be observed that, within the application of the statistical moments-based sensitivity analysis, there is no univocal accordance between the rankings obtained from the two different sensitivity indices. This result shows how, in this informed scenario, the relative importance of the input factors changes when considering different statistical moments of the output as measures of sensitivity.

On the other hand, the first-order and total-order sensitivity indices obtained from the variance-based analysis show remarkable unambiguity, leading to the conclusion that, in this informed case, the interactions between the variable input factors do not play a decisive role concerning their relative importance.

The colour shades of the $AMAE$-based ranking show that, except for the variable input factor x_3^2, the importance of the input factors belonging to a certain model follows the relative skill of the model to interpret the observations, given by the model's posterior probability.

The $AMAV$-based ranking shows how, despite the Chierici model's relatively low posterior probability, its variable input factors have a very strong effect on the output variance.

4.3.2. "BEREA SANDSTONE" SAMPLE

4.3.2.1. MULTI-MODEL, INFORMED STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.01$
- $w^2 = 0.04$
- $w^1 = 0.95$

VARIABLE INPUT FACTORS:

The models variable input factors are expressed in the form:

$x_i^j = (x_{i,min}^j \ ; \ x_{i,max}^j)$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the brackets define its 95% confidence limits.

- x_1^1: $k_{rw}^0 = (0.431 \ ; \ 0.435)$
- x_2^1: $S_{wi} = (0.39 \ ; \ 0.45)$
- x_3^1: $N_w = (0.69 \ ; \ 1.2)$
- x_4^1: $S_{or} = (0.32 \ ; \ 0.36)$

- x_1^2: $k_{rw}^0 = (0.431 \ ; \ 0.435)$
- x_2^2: $S_{wi} = (0.39 \ ; \ 0.45)$
- x_3^2: $B_w = (0.8 \ ; \ 0.9)$
- x_4^2: $M_w = (1.17 \ ; \ 2.2)$
- x_5^2: $S_{or} = (0.32 \ ; \ 0.36)$

- x_1^3: $k_{rw}^0 = (0.431 \ ; \ 0.435)$
- x_2^3: $S_{wi} = (0.39 \ ; \ 0.45)$
- x_3^3: $L_w = (1.7 \ ; \ 2.6)$
- x_4^3: $E_w = (0.52 \ ; \ 1.02)$
- x_5^3: $T_w = (2.18 \ ; \ 3.01)$
- x_6^3: $S_{or} = (0.32 \ ; \ 0.36)$

Water relative permeability k_{rw} is evaluated for $0.5 \leq S_w \leq 0.64$.

$n = 200{,}000$ (number of model evaluations)

$p = 25$ (number of parameters' bins)

In the figure below the unconditional single-model and multi-model statistical moments are reported for the "Berea sandstone" sample, for water relative permeability.

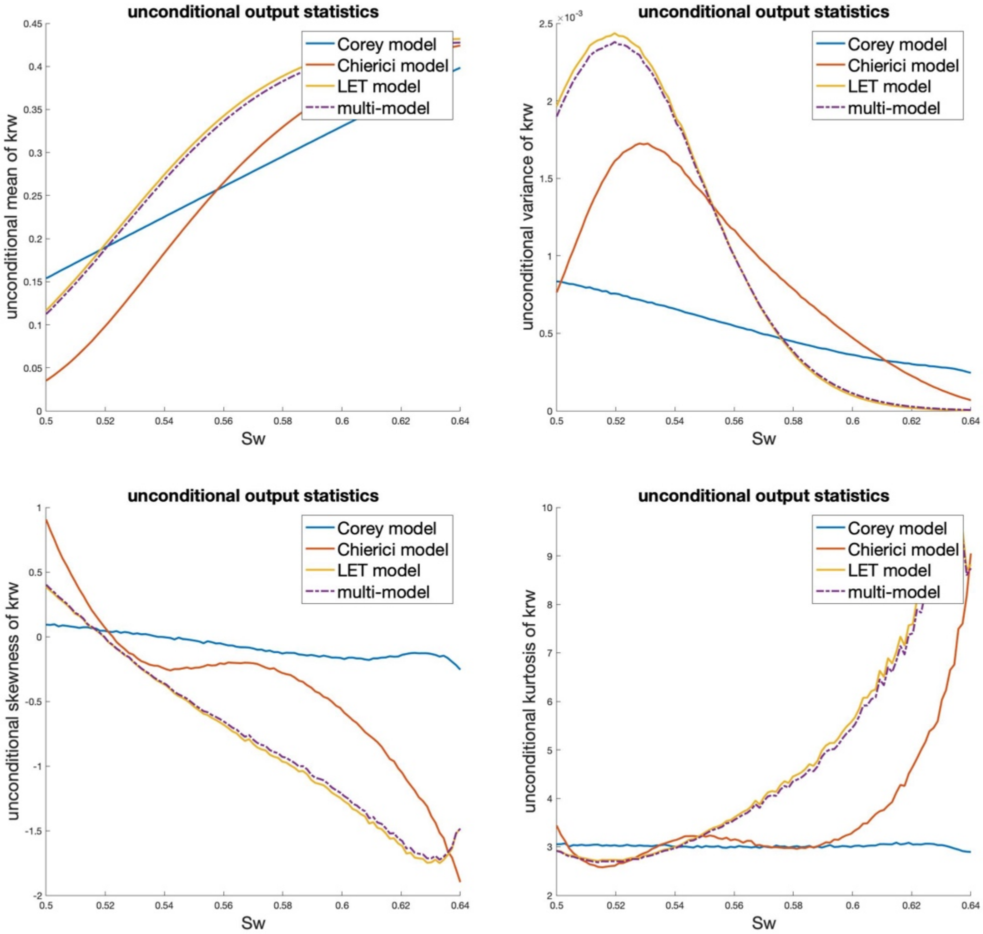

Figure 59: single-model and multi-model unconditional output statistics of the relative permeability models informed to the "Berea sandstone" sample for water relative permeability

For the "Berea sandstone" sample, the unconditional output statistical moments result to be particularly irregular and non-monotonic.

In the figures below the multi-model and single-model statistical moments-based informed sensitivity indices of the “Berea sandstone” sample for water relative permeability are reported:

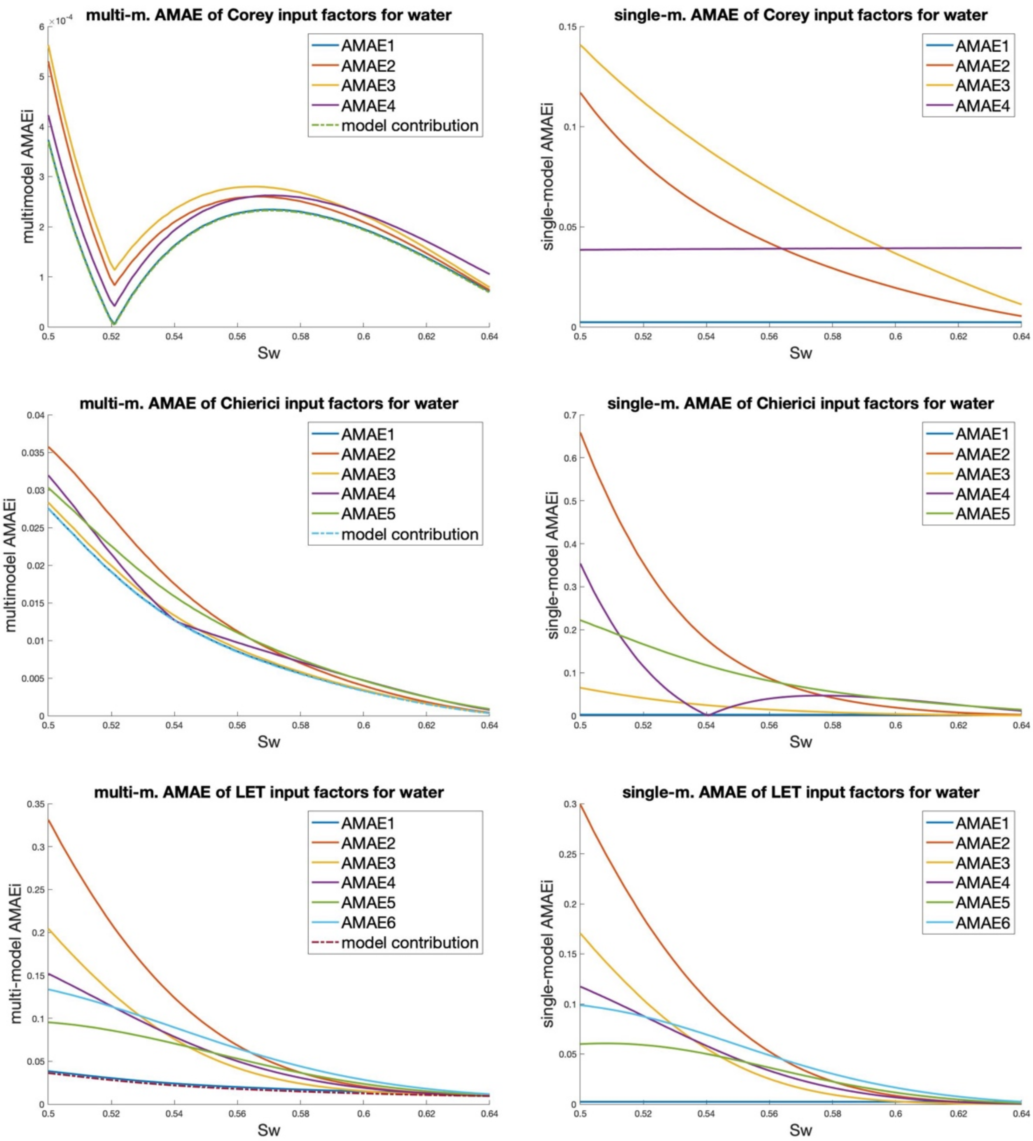

Figure 60: multi-model and single-model informed AMAE sensitivity indices of the “Berea sandstone” sample for water relative permeability

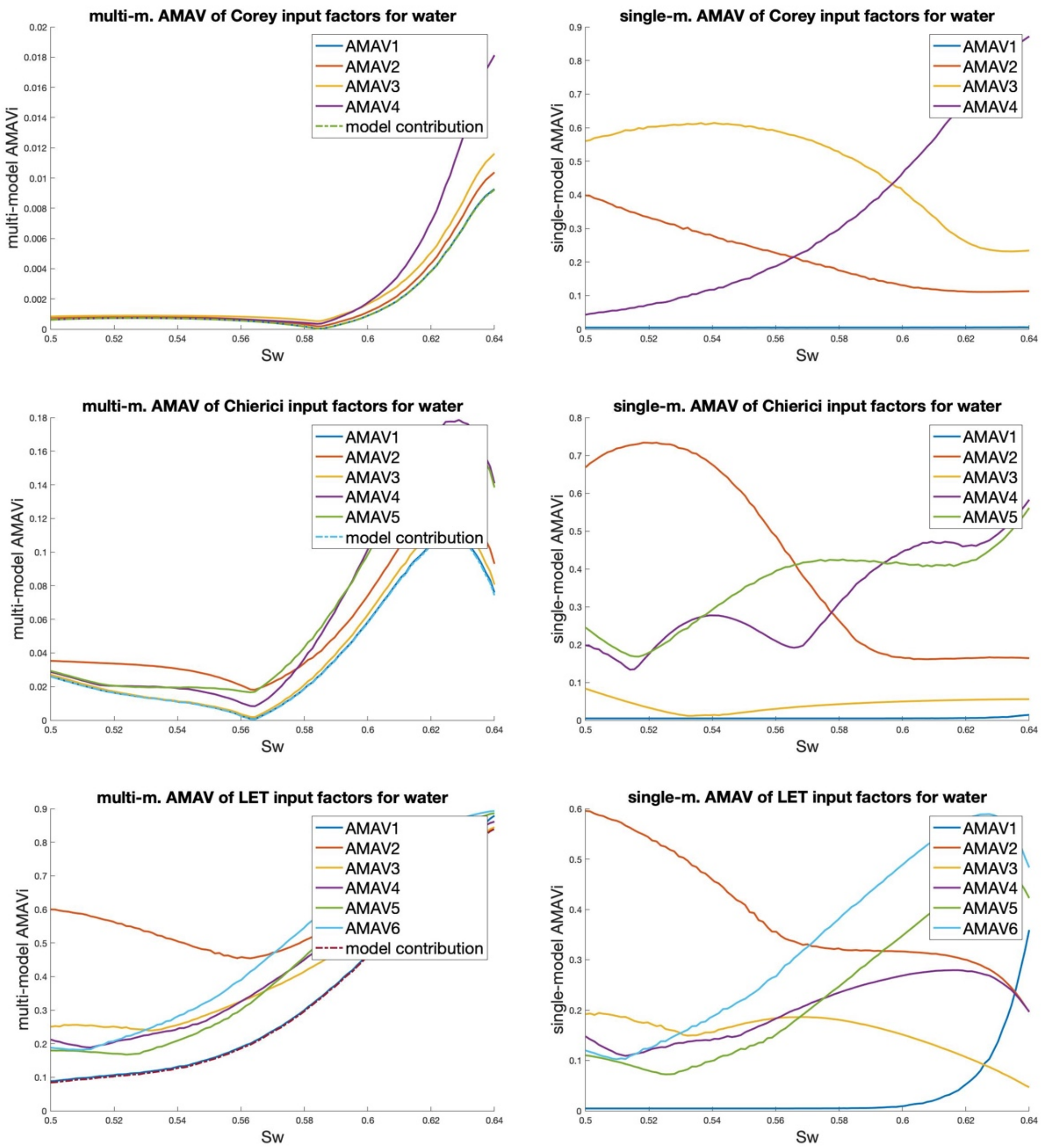

Figure 61: multi-model and single-model informed AMAV sensitivity indices of the "Berea sandstone" sample for water relative permeability

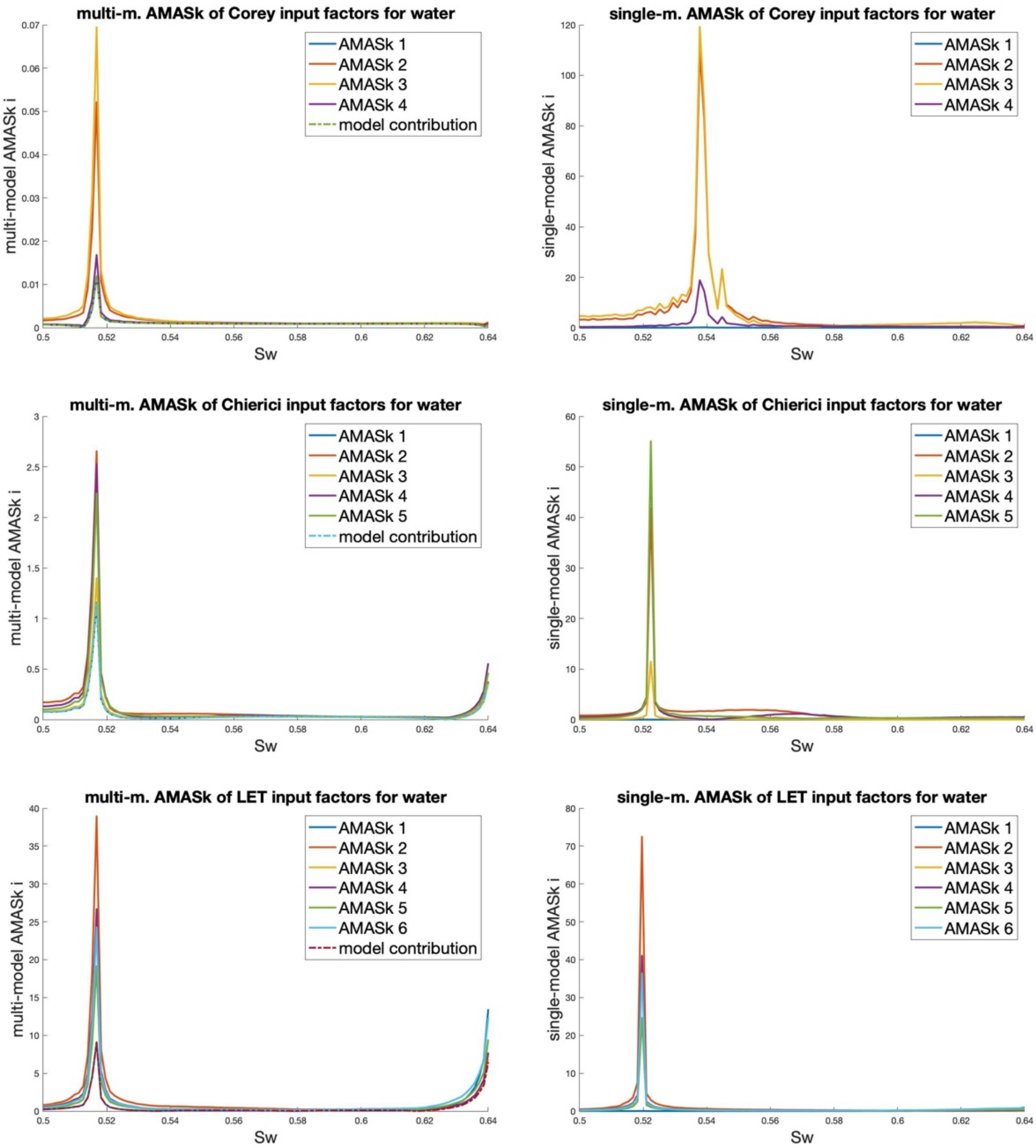

Figure 62: multi-model and single-model informed AMASk sensitivity indices of the "Berea sandstone" sample for water relative permeability

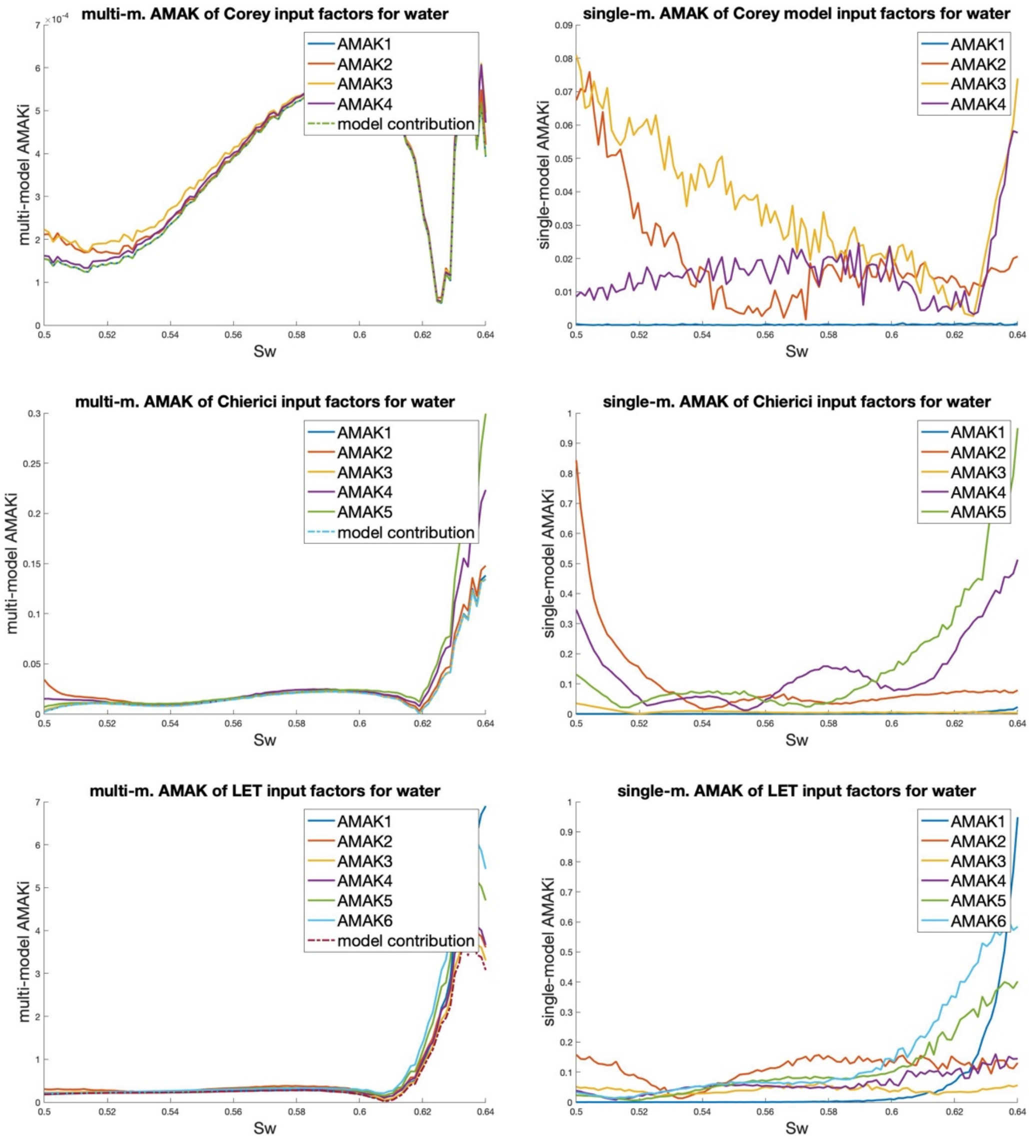

Figure 63: multi-model and single-model informed AMAK sensitivity indices of the "Berea sandstone" sample for water relative permeability

The ranking of the 15 variable input factors x_i^j (4 belonging to Corey model, 5 to Chierici model and 6 to LET model) is determined calculating and sorting the two ranking indices:

$$R_{AMAE_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAE_i^j$$

$$R_{AMAV_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAV_i^j$$

For the multi-model statistical moments-based informed global sensitivity analysis of the "Berea sandstone" sample for water relative permeability, the resulting rankings are:

Input factors belonging to Corey model
Input factors belonging to Chierici model
Input factors belonging to LET model

Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
x_2^3: S_{wi}^3 (8.38)	x_2^3: S_{wi}^3 (62.28)
x_6^3: S_{or}^3 (5.40)	x_6^3: S_{or}^3 (52.49)
x_3^3: L_w^3 (4.99)	x_4^3: E_w^3 (47.59)
x_4^3: E_w^3 (4.73)	x_5^3: T_w^3 (47.53)
x_5^3: T_w^3 (4.06)	x_3^3: L_w^3 (47.08)
x_1^3: $k_{0,rw}^3$ (1.30)	x_1^3: $k_{0,rw}^3$ (37.98)
x_2^2: S_{wi}^2 (1.26)	x_5^2: S_{or}^2 (5.31)
x_5^2: S_{or}^2 (1.17)	x_4^2: M_w^2 (5.23)
x_4^2: M_w^2 (1.10)	x_2^2: S_{wi}^2 (4.59)
x_3^2: B_w^2 (0.99)	x_3^2: B_w^2 (3.22)
x_1^2: $k_{0,rw}^2$ (0.96)	x_1^2: $k_{0,rw}^2$ (2.99)
x_3^1: N_w^1 (0.23)	x_4^1: S_{or}^1 (2.57)
x_2^1: S_{wi}^1 (0.21)	x_3^1: N_w^1 (1.99)
x_4^1: S_{or}^1 (0.21)	x_2^1: S_{wi}^1 (1.68)
x_1^1: $k_{0,rw}^1$ (0.17)	x_1^1: $k_{0,rw}^1$ (1.46)

4.3.2.2. MULTI-MODEL, INFORMED VARIANCE-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.01$
- $w^2 = 0.04$
- $w^1 = 0.95$

VARIABLE INPUT FACTORS:

The models variable input factors are expressed in the form:

$x_i^j = (x_{i,min}^j \, ; \, x_{i,max}^j)$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the bracket define its 95% confidence limits.

- x_1: $k_{rw}^0 = (0.431 \, ; \, 0.435)$
- x_2: $S_{wi} = (0.39 \, ; \, 0.45)$
- x_3: $N_w = (0.69 \, ; \, 1.2)$
- x_4: $B_w = (0.8 \, ; \, 0.9)$
- x_5: $M_w = (1.17 \, ; \, 2.2)$
- x_6: $L_w = (1.7 \, ; \, 2.6)$
- x_7: $E_w = (0.52 \, ; \, 1.02)$
- x_8: $T_w = (2.18 \, ; \, 3.01)$
- x_9: $S_{or} = (0.32 \, ; \, 0.36)$

Water relative permeability k_{rw} is evaluated for $0.5 \leq S_w \leq 0.64$.

$n = 200{,}000$ (number of model evaluations for the first-order index)

$p = 25$ (number of parameters' bins for the first-order index)

$n_t = 700$ (number of model evaluations for the total-order index)

$p_t = 6$ (number of parameters' bins for the total-order index)

In the figures below the single-model and multi-model first order and total order variance-based sensitivity indices informed to the "Berea sandstone" sample for water relative permeability are reported.

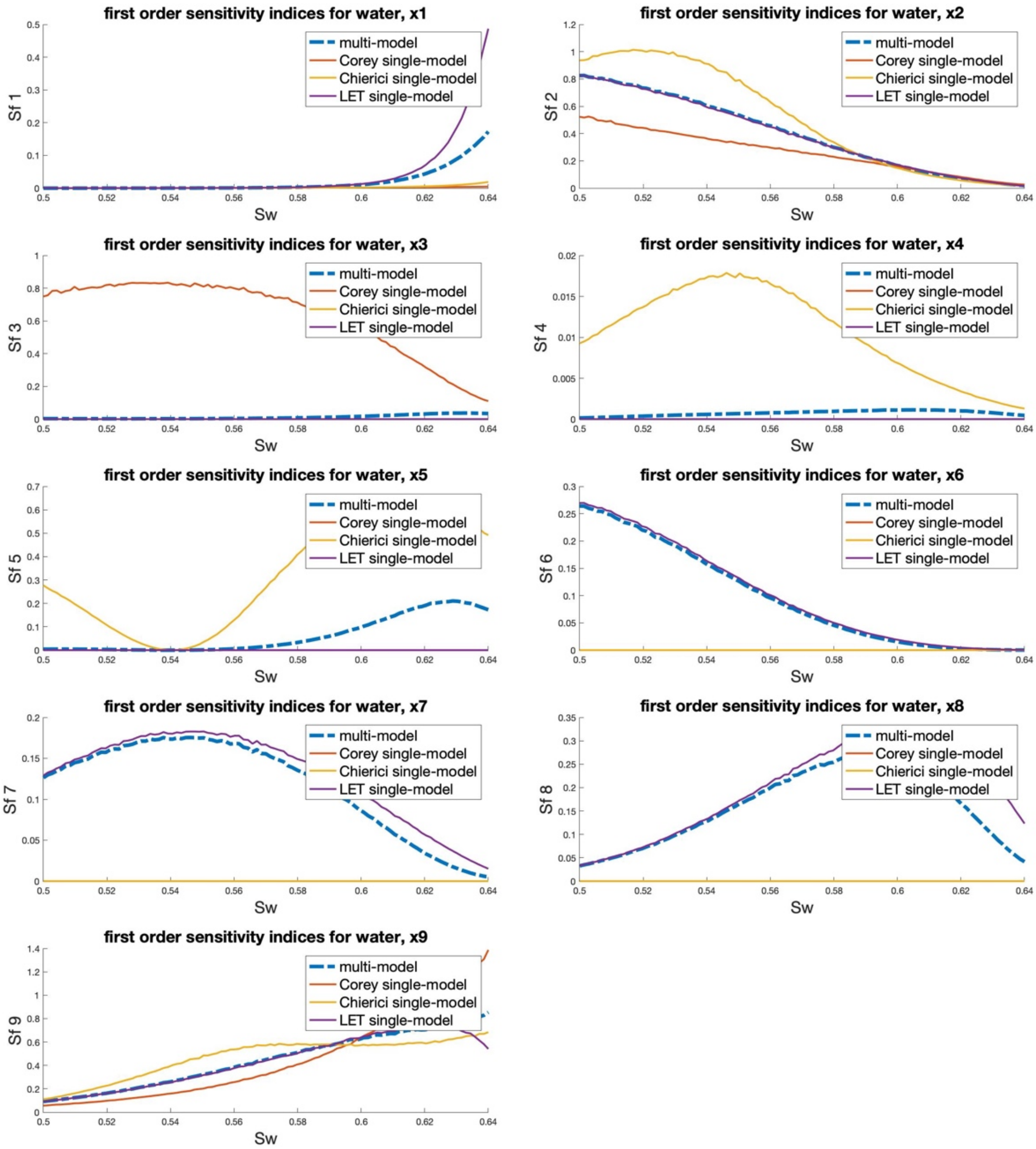

Figure 64: first order single-model and multi-model informed variance-based sensitivity indices of the "Berea sandstone" sample for water relative permeability

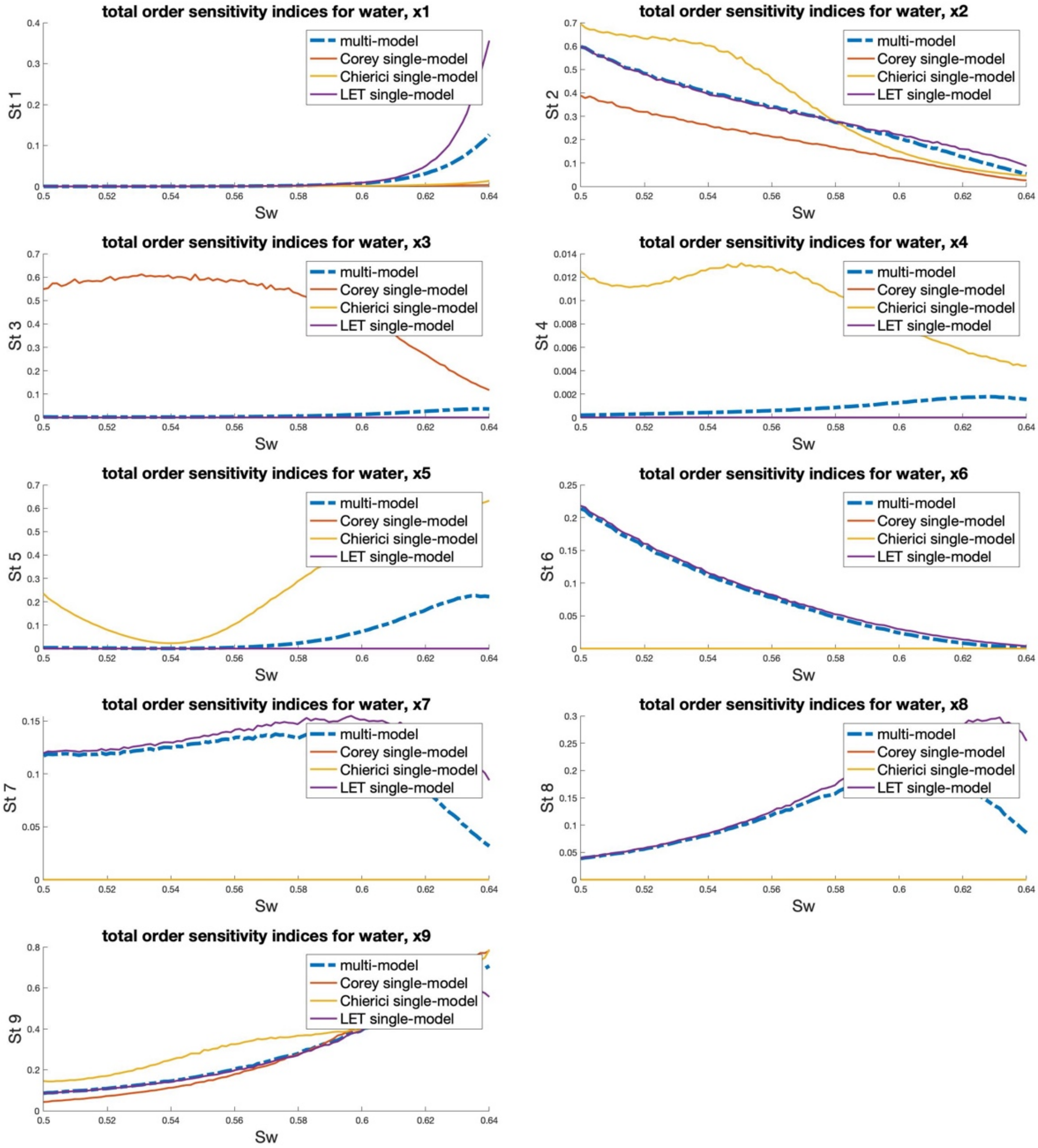

Figure 65: total order single-model and multi-model informed variance-based sensitivity indices of the "Berea sandstone" sample for water relative permeability

The ranking of the nine variable input factors x_i is determined calculating the two ranking indices:

$$R_{Sf_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} Sf_i^{multimodel}$$

$$R_{St_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} St_i^{multimodel}$$

For the variance-based informed global sensitivity analysis of the "Berea sandstone" sample for water relative permeability the resulting rankings are:

Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
x_9: S_{or} (44.98)	x_9: S_{or} (29.88)
x_2: S_{wi} (39.79)	x_2: S_{wi} (31.12)
x_8: T_w (16.14)	x_8: T_w (12.21)
x_7: E_w (11.89)	x_7: E_w (11.49)
x_6: L_w (9.68)	x_6: L_w (7.67)
x_5: M_w (6.25)	x_5: M_w (5.65)
x_1: $k_{0,rw}$ (1.89)	x_1: $k_{0,rw}$ (1.38)
x_3: N_w (1.26)	x_3: N_w (1.06)
x_4: B_w (0.075)	x_4: B_w (0.088)

4.3.2.3. DISCUSSION

In the tables below the rankings obtained from the different informed multi-model sensitivity analysis techniques, when applied to the "Berea sandstone" sample for water relative permeability, are summarized.

Multi-model informed statistical moments-based sensitivity analysis of the "Berea sandstone" sample for water relative permeability	
Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
$x_2^3: S_{wi}^3$ (8.38)	$x_2^3: S_{wi}^3$ (62.28)
$x_6^3: S_{or}^3$ (5.40)	$x_6^3: S_{or}^3$ (52.49)
$x_3^3: L_w^3$ (4.99)	$x_4^3: E_w^3$ (47.59)
$x_4^3: E_w^3$ (4.73)	$x_5^3: T_w^3$ (47.53)
$x_5^3: T_w^3$ (4.06)	$x_3^3: L_w^3$ (47.08)
$x_1^3: k_{0,rw}^3$ (1.30)	$x_1^3: k_{0,rw}^3$ (37.98)
$x_2^2: S_{wi}^2$ (1.26)	$x_5^2: S_{or}^2$ (5.31)
$x_5^2: S_{or}^2$ (1.17)	$x_4^2: M_w^2$ (5.23)
$x_4^2: M_w^2$ (1.10)	$x_2^2: S_{wi}^2$ (4.59)
$x_3^2: B_w^2$ (0.99)	$x_3^2: B_w^2$ (3.22)
$x_1^2: k_{0,rw}^2$ (0.96)	$x_1^2: k_{0,rw}^2$ (2.99)
$x_3^1: N_w^1$ (0.23)	$x_4^1: S_{or}^1$ (2.57)
$x_2^1: S_{wi}^1$ (0.21)	$x_3^1: N_w^1$ (1.99)
$x_4^1: S_{or}^1$ (0.21)	$x_2^1: S_{wi}^1$ (1.68)
$x_1^1: k_{0,rw}^1$ (0.17)	$x_1^1 : k_{0,rw}^1$ (1.46)

Multi-model informed variance-based sensitivity analysis of the "Berea sandstone" sample for water relative permeability	
Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
$x_9: S_{or}$ (44.98)	$x_9: S_{or}$ (29.88)
$x_2: S_{wi}$ (39.79)	$x_2: S_{wi}$ (31.12)
$x_8: T_w$ (16.14)	$x_8: T_w$ (12.21)
$x_7: E_w$ (11.89)	$x_7: E_w$ (11.49)
$x_6: L_w$ (9.68)	$x_6: L_w$ (7.67)
$x_5: M_w$ (6.25)	$x_5: M_w$ (5.65)
$x_1: k_{0,rw}$ (1.89)	$x_1: k_{0,rw}$ (1.38)
$x_3: N_w$ (1.26)	$x_3: N_w$ (1.06)
$x_4: B_w$ (0.075)	$x_4: B_w$ (0.088)

It can be observed that, within the application of the statistical moments-based sensitivity analysis, there is no perfect accordance between the rankings obtained from the two different sensitivity indices. This result shows how, in this informed scenario, the relative importance of the input factors can change when considering different statistical moments of the output as measures of sensitivity.

The first-order and total-order sensitivity indices obtained from the variance-based analysis show remarkable unambiguity, leading to the conclusion that, in this informed case, the interactions between the variable input factors do not play a decisive role concerning their relative importance.

The colour shades of the statistical moments-based rankings show that the importance of the input factors belonging to a certain model follows precisely the relative importance of the model given by its posterior probability.

5. CONCLUSION

The objective of this essay has been to demonstrate the need to adopt many different approaches and techniques when investigating the most widely used water-oil relative permeability models by the tool of global sensitivity analysis, both in single-model and multi-model applications, both in uninformed and informed scenarios.

This goal has been achieved by a preliminary study of the relative permeability models from a mathematical point of view, followed by the rigorous application of the state-of-the-art global sensitivity analysis techniques and by the in-depth analysis of the results, both from a qualitative and (most importantly) quantitative point of view.

The obtained results clearly show that relying indiscriminately on a single global sensitivity analysis method is in general a failure approach which can lead to deceiving conclusions when there is not a perfect theoretical understanding of the adopted method, especially when considering complex numerical models and when operating in multi-model contexts. The only case in which the global sensitivity analysis has proven forgiving (in terms of consistency between the results obtained from different methods) is when applied to very simple numerical models (Corey model). Despite of that, in light of the results, a multi-method sensitivity analysis investigation associated with a case-by-case identification of the most suitable sensitivity index/indices is always recommended.

The proper application of the different sensitivity analysis techniques and the analysis of the results have required a rich acquisition of information to allow the understanding of the theoretical foundations of the methodologies, and the numerical complexity of the sensitivity analysis procedures has led to an intense process of optimization (often carried out by a trial and error approach) of the MATLAB algorithms, aimed to find an optimal trade-off between computation time obtainable by home-available computers and accuracy (reliability) of the results. At their actual state, the developed MATLAB algorithms can provide the results of any sensitivity analysis described in the essay in less than an hour (on a modern computer).

The most significant and useful outcomes of the work carried out are the MATLAB codes themselves, which are specifically designed and ready to run for the global sensitivity analysis of the main water-oil relative permeability models. I personally invite any student, researcher or worker who can benefit from the use of these codes to download them from the link that I make available below.

Figure 66: link to the MATLAB scripts

6. APPENDICES

APPENDIX A: VARIABLE INPUT FACTORS AND INPUT VARIABILITY SPACE OF THE THREE RELATIVE PERMEABILITY MODELS FOR OIL RELATIVE PERMEABILITY

A1: COREY MODEL

$$k_{ro} = k_{ro}(S_w) = k_{ro}^0 (\frac{1-S_w-S_{or}}{1-S_{wi}-S_{or}})^{N_o}$$

Observing the analytical expression of the model it is possible to identify four independent model parameters, which are the variable input factors x_i of the model:

- x_1: k_{ro}^0; it must be greater than 0 (otherwise, the curve represents a case of complete imperviousness to oil) and it can be as high as 1.
- x_2: S_{wi}; it can assume any value between 0 (no irreducible/initial water saturation) and 1 (the porous rock is completely full of water which can not be displaced).
- x_3: N_o; it can assume any value greater than 0 without producing non-sensical relative permeability results.
- x_4: S_{or}; it can assume any value between 0 (no residual oil) and 1 (the porous rock is completely full of oil which can not be displaced).

The adopted input variability space for the Corey oil relative permeability model is:

- $k_{ro,min}^0 = 0.1$ $\quad k_{ro,max}^0 = 1$
- $S_{wi,min} = 0$ $\quad S_{wi,max} = 0.35$
- $N_{o,min} = 0.1$ $\quad N_{o,max} = 6$
- $S_{or,min} = 0$ $\quad S_{or,max} = 0.35$

According to these assumptions, the model can be evaluated in ($0.35 \leq S_w \leq 0.65$).

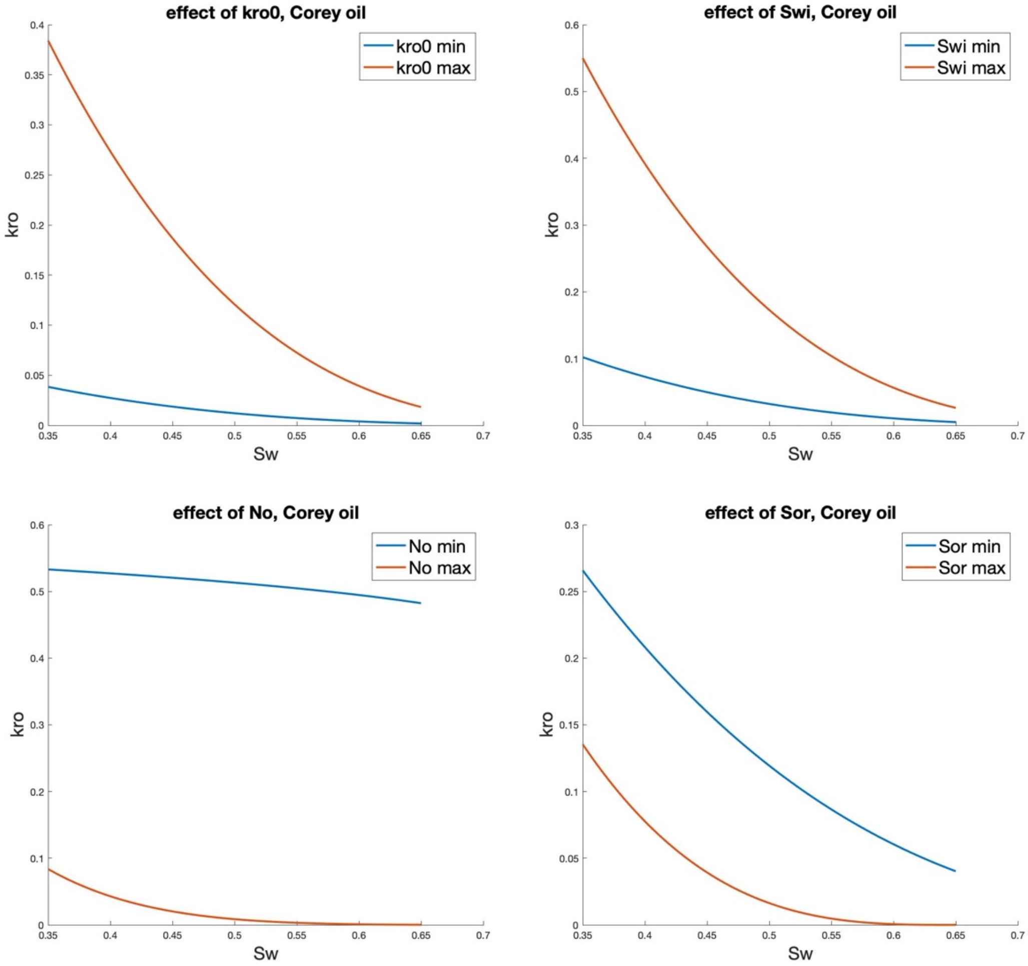

Figure 67: effect of the variable input factors on Corey model for oil relative permeability

Similarly to what happens for Corey model for water relative permeability, the variable input factor N_o has the most dramatic effect on the output. It should be noted that the input factors S_{wi} and S_{or} have inverted effects on water and oil relative permeability curves: the inversion of the roles of these two parameters, when considering the two phases, is effect of the symmetry of the relative permeability models for water and oil with respect to water saturation S_w.

A2: CHIERICI MODEL

$$k_{ro} = k_{ro}(S_w) = k_{ro}^0 exp\left[-B_o\left(\frac{S_w - S_{wi}}{1 - S_{or} - S_w}\right)^{+M_o}\right]$$

Observing the analytical expression of the model it is possible to identify five independent model parameters, which are the variable input factors x_i of the model:

- x_1: k_{ro}^0.
- x_2: S_{wi}.
- x_3: B_o; it can assume any value greater than 0 without producing non-sensical relative permeability results.
- x_4: M_o; it can assume any value greater than 0 without producing non-sensical relative permeability results.
- x_5: S_{or}.

The adopted input variability space for Chierici model for oil relative permeability is:

- $k_{ro,min}^0 = 0.1$ $k_{ro,max}^0 = 1$
- $S_{wi,min} = 0$ $S_{wi,max} = 0.35$
- $B_{o,min} = 0.1$ $B_{o,max} = 8$
- $M_{o,min} = 0.1$ $M_{o,max} = 8$
- $S_{or,min} = 0$ $S_{or,max} = 0.35$

According to these assumptions, the model can be evaluated in ($0.35 \leq S_w \leq 0.65$).

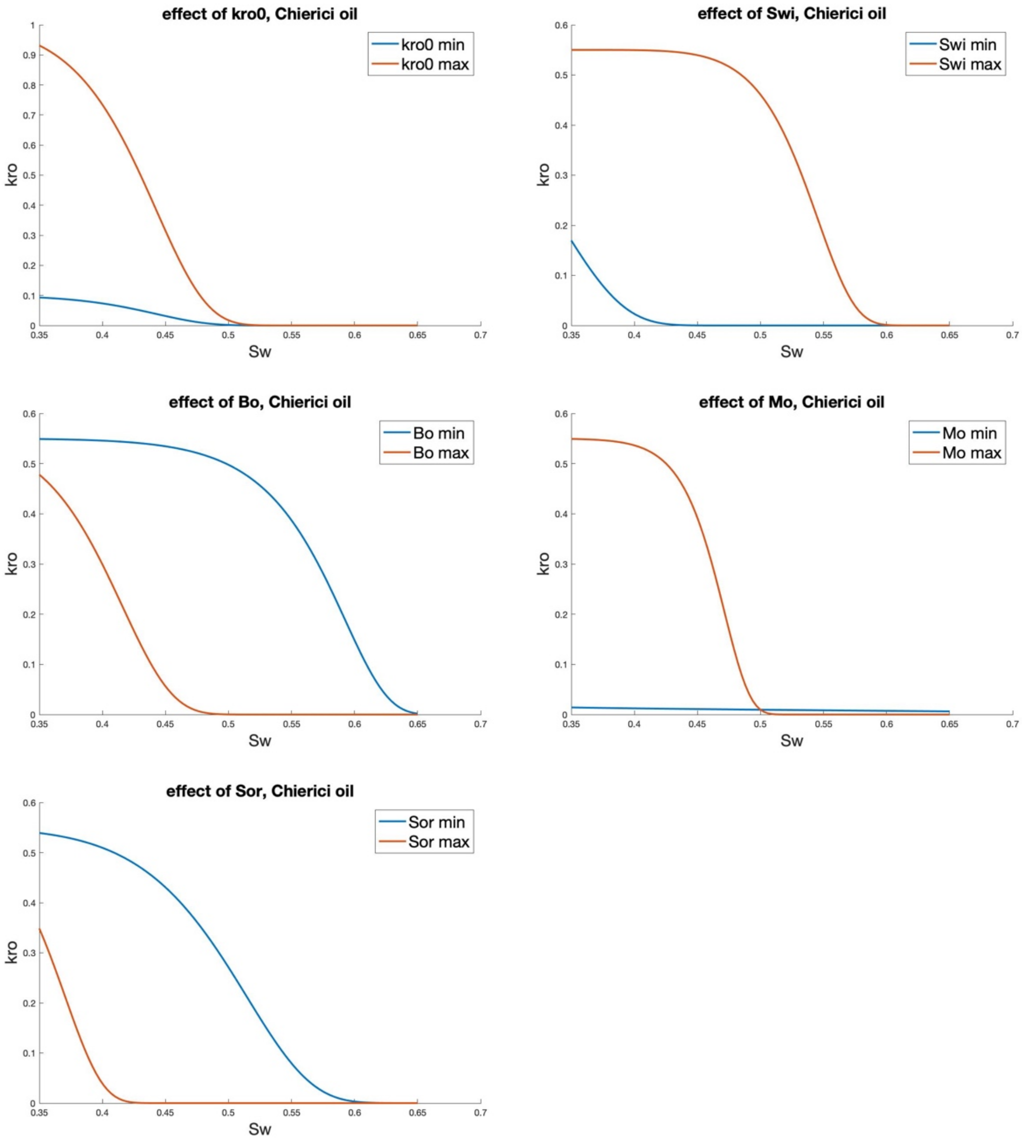

Figure 68: effect of the variable input factors on Chierici model for oil relative permeability

A3: LET MODEL

$$k_{ro} = k_{ro}(S_w) = k_{ro}^0 \frac{(1-S_w^*)^{Lo}}{(1-S_w^*)^{Lo} + E_0(S_w^*)^{To}}$$

$$S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}$$

Observing the analytical expression of the model it is possible to identify six independent model parameters, which are the variable input factors x_i of the model:

- x_1: k_{ro}^0.
- x_2: S_{wi}.
- x_3: L_o; the inventors of the LET model suggest considering only values of $L_o \geq 0.1$. Despite of that, any value above 0 is mathematically acceptable (it does not produce non-sensical relative permeability curves).
- x_4: E_o; the inventors of the LET model suggest considering only values of $E_o \geq 0$.
- x_5: T_o; the inventors of the LET model suggest considering only values of $T_o \geq 0.1$. Despite of that, any value above 0 is mathematically acceptable.
- x_6: S_{or}.

The adopted input variability space for LET model for oil relative permeability is:

- $k_{ro,min}^0 = 0.1$ $k_{ro,max}^0 = 1$
- $S_{wi,min} = 0$ $S_{wi,max} = 0.35$
- $L_{o,min} = 0.05$ $L_{o,max} = 10$
- $E_{o,min} = 0.005$ $E_{o,max} = 10$
- $T_{o,min} = 0.05$ $T_{o,max} = 8$
- $S_{or,min} = 0$ $S_{or,max} = 0.35$

According to these assumptions, the model can be evaluated in $(0.35 \leq S_w \leq 0.65)$.

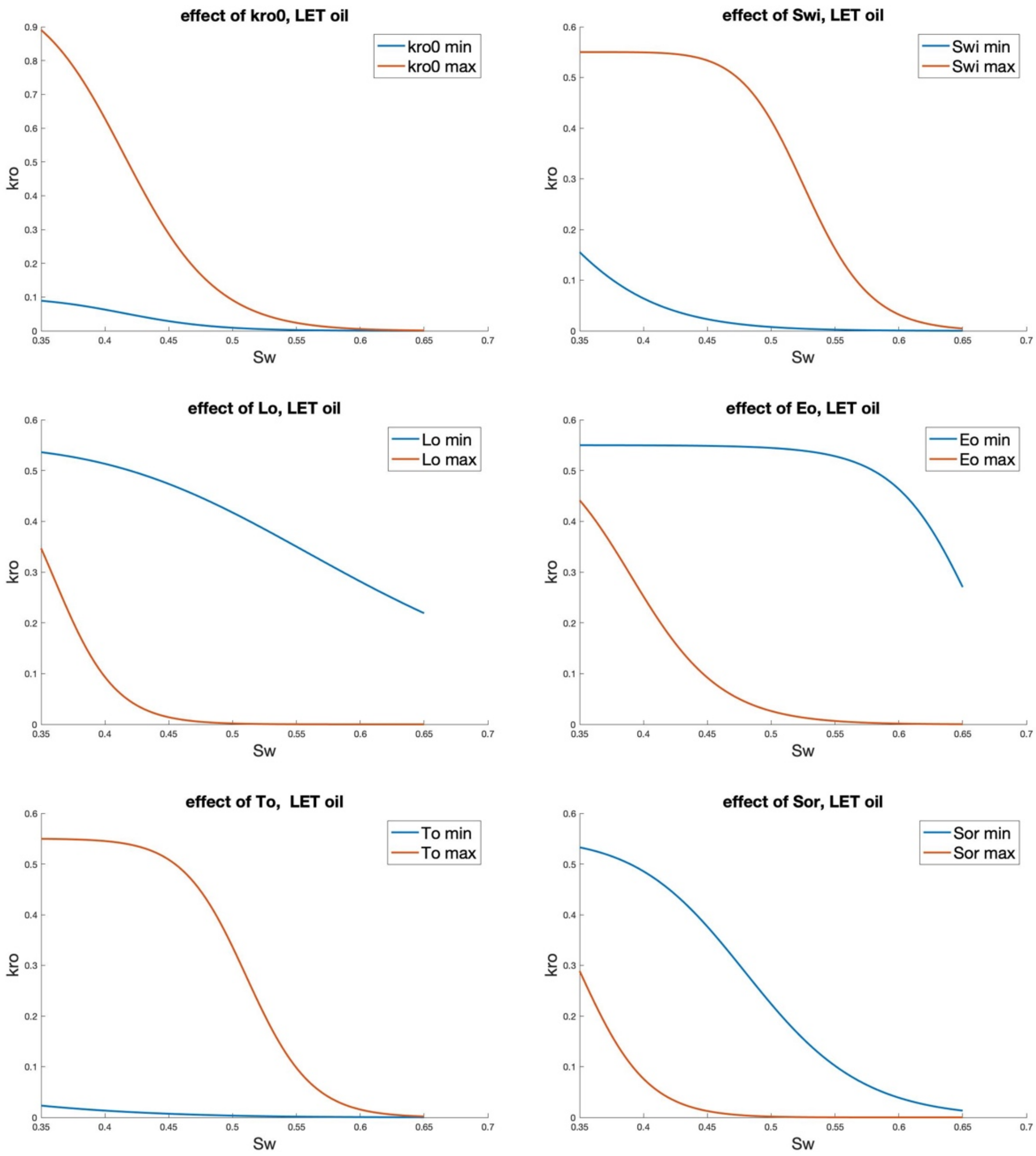

Figure 69: effect of the variable input factors on LET model for oil relative permeability

APPENDIX B: OIL RELATIVE PERMEABILITY, SINGLE MODEL UNINFORMED SCENARIO

B1: COREY MODEL

B1.1: MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

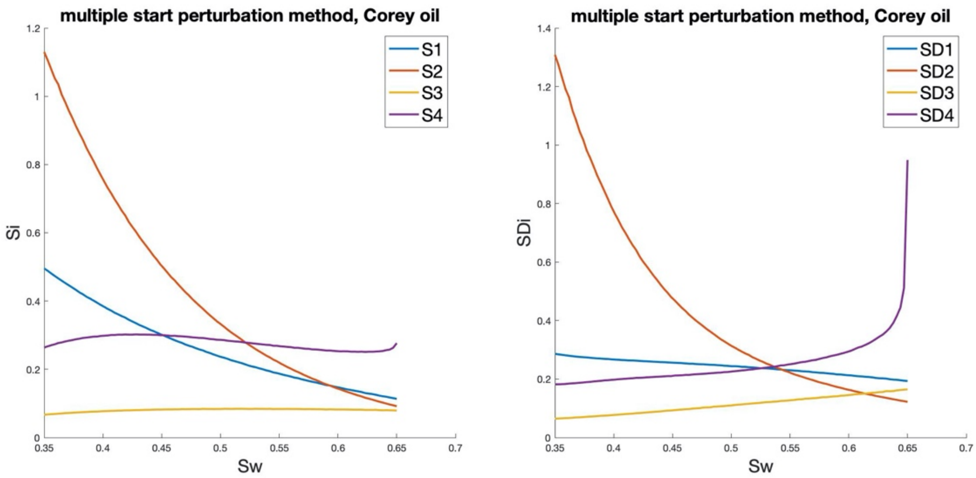

Figure 70: multiple start perturbation method global sensitivity analysis of Corey model for oil relative permeability

From the multiple start perturbation method global sensitivity analysis applied to Corey model for oil relative permeability, the resulting ranking is:

Ranking according to $R_{S,i}$
x_2: S_{wi} (42.85)
x_4: S_{or} (28.11)
x_1: k^0_{ro} (26.21)
x_3: N_o (8.14)

It is interesting to observe that, with respect to Corey model for water relative permeability, the input factors x_2 (S_{wi}) and x_4 (S_{or}) have an exactly opposite role in the model. This is consequence of the symmetry of the relative permeability curves of water and oil with respect to water saturation S_w (S_{wi} determines the point of nil water relative permeability k_{rw}, while S_{or} determines the point of maximum k_{rw}; the opposite happens for oil relative permeability k_{ro}).

B1.2: STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

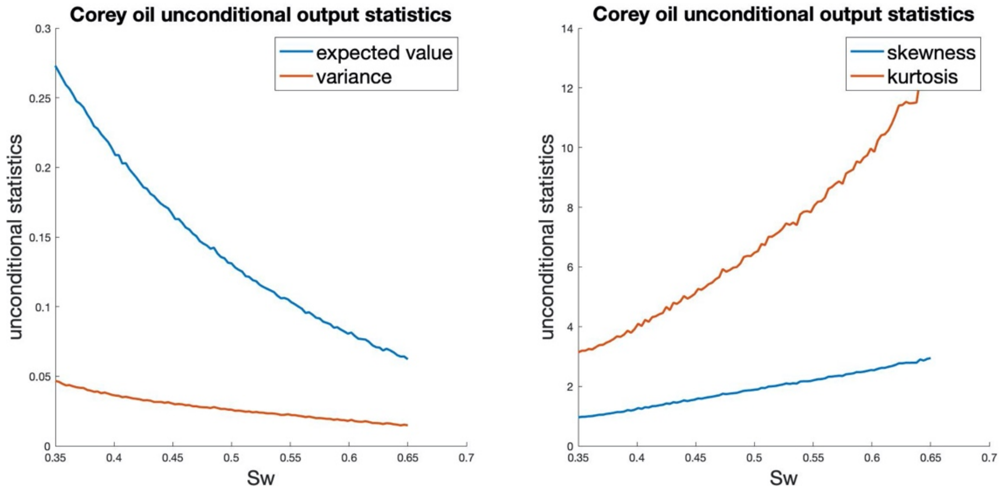

Figure 71: statistical moments of the unconditional output of Corey model for oil relative permeability

The unconditional statistical moments of oil relative permeability k_{ro} show exactly opposite trends with respect to the unconditional statistical moments of water relative permeability k_{rw}, due to the symmetry of Corey model with respect to water saturation S_w when considering the two fluids.

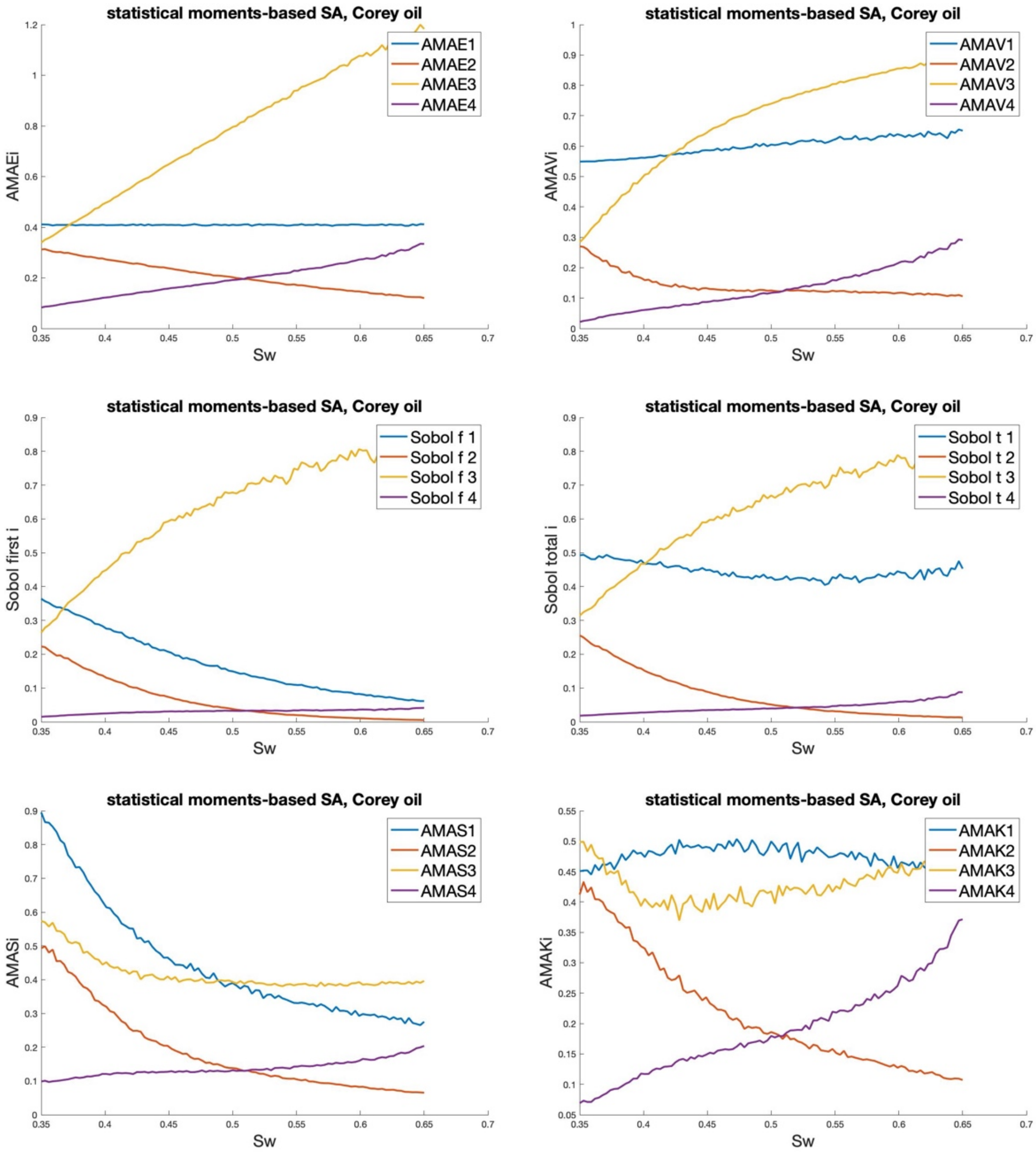

Figure 72: statistical moments-based sensitivity analysis of Corey model for oil relative permeability

The sensitivity indices plots are perfectly symmetrical to the curves obtained for water relative permeability (plus the inversion between the curves related to x_2 and x_4).

From the statistical moments-based sensitivity analysis applied to the Corey model for oil relative permeability the resulting rankings are:

Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$
x_3: N_o (79.25)	x_3: N_o (69.79)	x_3: N_o (63.66)	x_3: N_o (63.48)
x_1: k^0_{ro} (42.31)	x_1: k^0_{ro} (60.65)	x_1: k^0_{ro} (17.40)	x_1: k^0_{ro} (44.93)
x_2: S_{wi} (20.94)	x_2: S_{wi} (14.10)	x_2: S_{wi} (6.45)	x_2: S_{wi} (7.92)
x_4: S_{or} (19.76)	x_4: S_{or} (13.30)	x_4: S_{or} (3.12)	x_4: S_{or} (4.31)

It can be seen that (as it was observed from the multiple start perturbation method sensitivity analysis) when considering oil relative permeability, the relative importance of the input factors x_2 (S_{wi}) and x_4 (S_{or}) is inverted.

B1.3: CUMULATIVE PROBABILITY DENSITY FUNCTION-BASED SENSITIVITY ANALYSIS

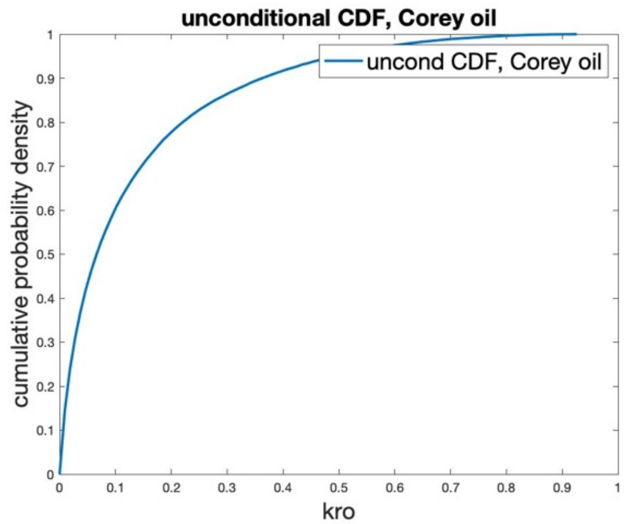

Figure 73: cumulative distribution of Corey model unconditional output for oil relative permeability, S_w=0.5

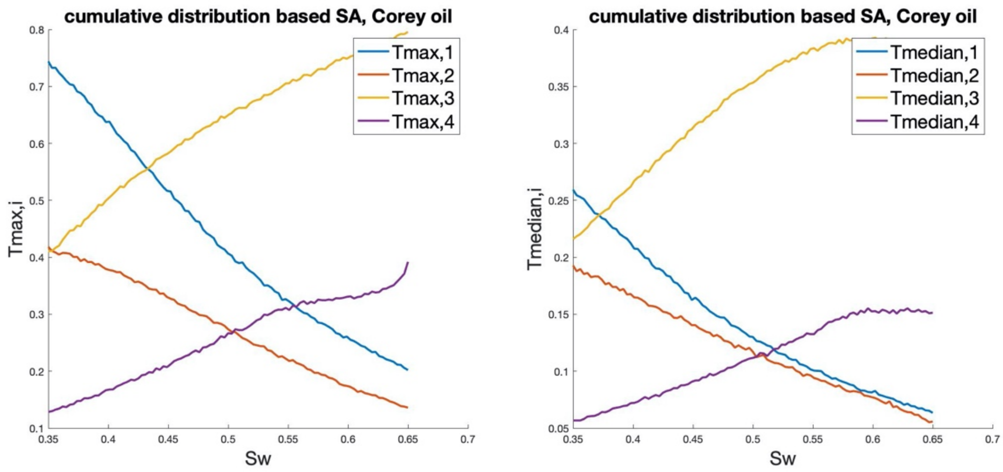

Figure 74: cumulative probability density function-based sensitivity analysis of Corey model for oil relative permeability

For the cumulative density function-based sensitivity analysis applied to the Corey model for oil relative permeability the resulting rankings are:

Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$
x_3: N_o (63.93)	x_3: N_o (33.74)
x_1: k^0_{ro} (43.98)	x_1: k^0_{ro} (14.29)
x_2: S_{wi} (26.69)	x_2: S_{wi} (12.09)
x_4: S_{or} (24.65)	x_4: S_{or} (11.25)

B1.4: VARIOGRAM-BASED SENSITIVITY ANALYSIS

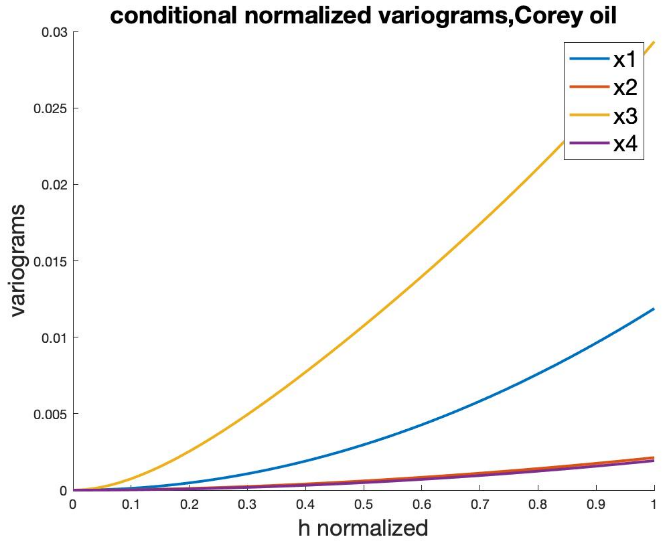

Figure 75: normalized variograms of Corey model conditional output for oil relative permeability, S_w=0.5

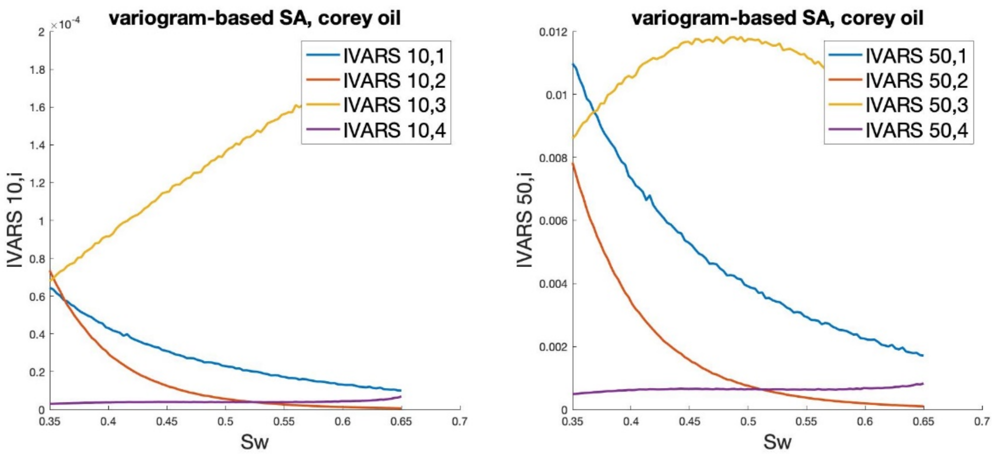

Figure 76: variogram-based sensitivity analysis of Corey model for oil relative permeability

For the variogram-based sensitivity analysis applied to the Corey model for oil relative permeability the resulting rankings are:

Ranking according to $R_{IVARS\ 10,i}$	Ranking according to $R_{IVARS\ 50,i}$
x_3: N_o (0.011)	x_3: N_o (1.13)
x_1: k^0_{ro} (0.0020)	x_1: k^0_{ro} (0.47)
x_2: S_{wi} (0.0010)	x_2: S_{wi} (0.16)
x_4: S_{or} (0.00030)	x_4: S_{or} (0.064)

B2: CHIERICI MODEL

B2.1: MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

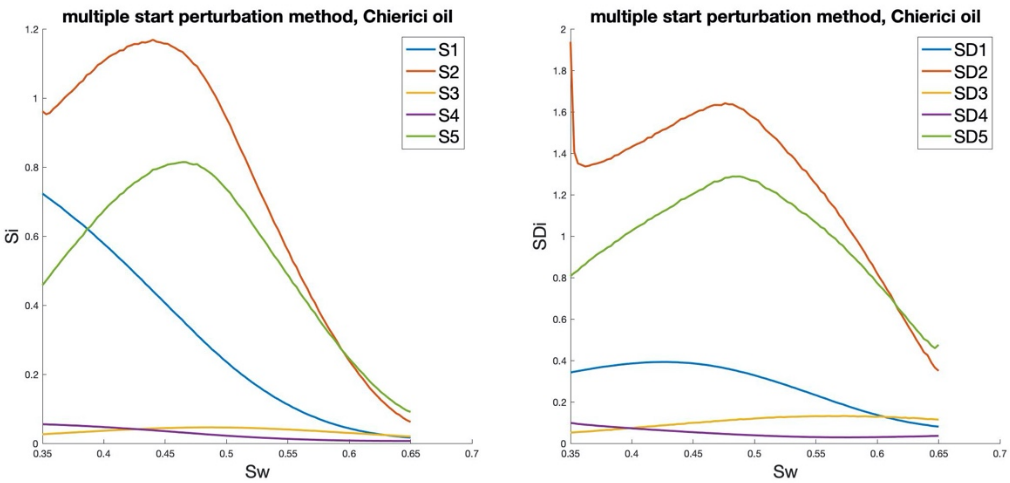

Figure 77: multiple start perturbation method global sensitivity analysis of Chierici model for oil relative permeability

From the multiple start perturbation method global sensitivity analysis applied to Chierici model for oil relative permeability, the resulting ranking is:

Ranking according to $R_{S,i}$
x_2: S_{wi} (75.91)
x_5: S_{or} (54.36)
x_1: k^0_{ro} (29.28)
x_3: B_o (3.70)
x_4: M_o (2.62)

With respect to the Chierici model for water relative permeability, the input factors x_2 (S_{wi}) and x_5 (S_{or}) have an opposite role, due to the symmetry of the relative permeability models for water and oil with respect to water saturation S_w. This symmetry reflects also on the sensitivity indices curves, which are perfectly symmetrical to the ones obtained for water relative permeability.

B2.2: STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

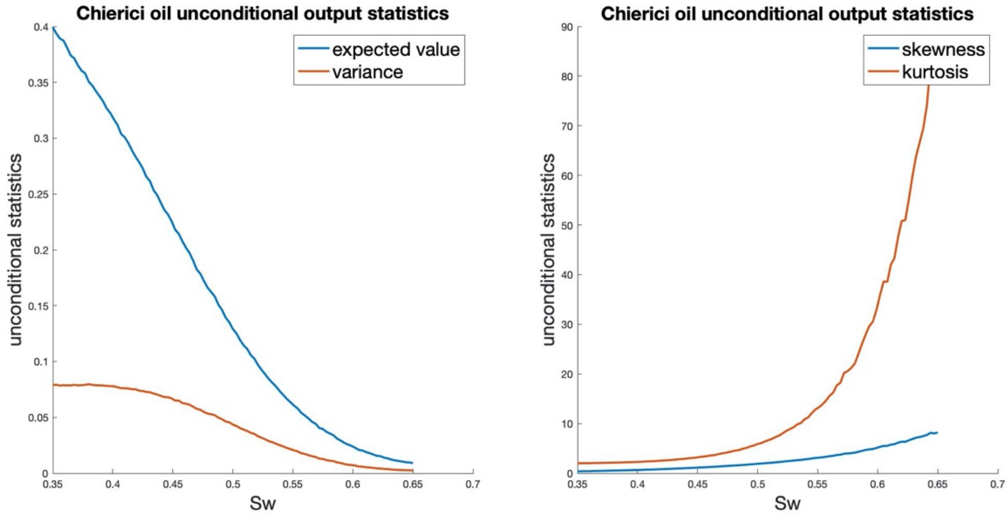

Figure 78: statistical moments of the unconditional output of Chierici model for oil relative permeability

The statistical moments of the unconditional oil relative permeability k_{ro} show exactly opposite trends with respect to the statistical moments of the unconditional water relative permeability k_{ro}, due to the symmetry of Chierici model with respect to water saturation S_w, when considering water and oil.

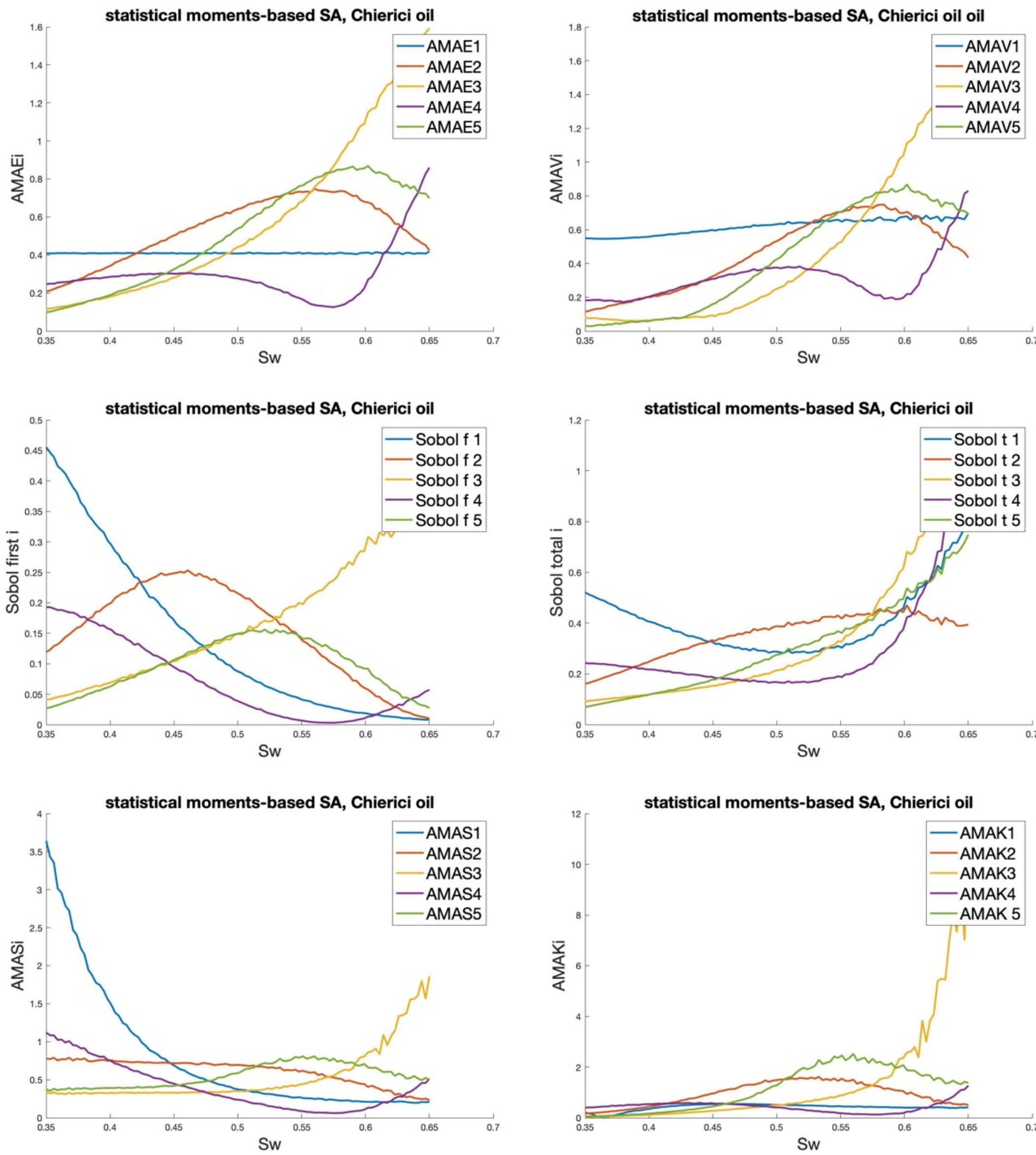

Figure 79: statistical moments-based sensitivity analysis of Chierici model for oil relative permeability

From the statistical moments-based sensitivity analysis applied to the Chierici relative permeability model for oil relative permeability the resulting rankings are:

Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$
x_3: B_o (59.58)	x_1: k^0_{ro} (62.55)	x_3: B_o (16.88)	x_1: k^0_{ro} (40.77)
x_2: S_{wi} (54.53)	x_3: B_o (47.82)	x_2: S_{wi} (15.70)	x_2: S_{wi} (35.60)
x_5: S_{or} (51.95)	x_2: S_{wi} (46.48)	x_1: k^0_{ro} (14.15)	x_3: B_o (33.56)
x_1: k^0_{ro} (41.34)	x_5: S_{or} (42.97)	x_5: S_{or} (9.77)	x_5: S_{or} (30.70)
x_4: M_o (30.09)	x_4: M_o (31.53)	x_4: M_o (7.24)	x_4: M_o (29.54)

B2.3: CUMULATIVE PROBABILITY DENSITY FUNCTION-BASED SENSITIVITY ANALYSIS

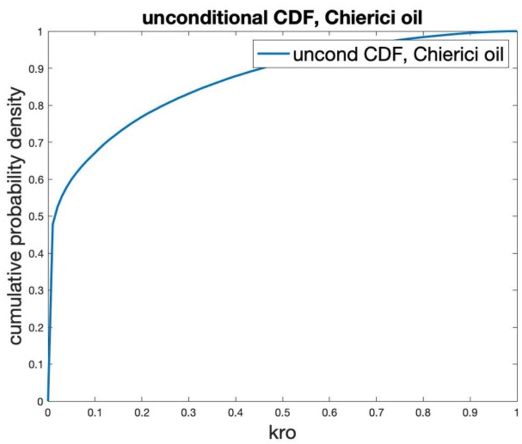

Figure 80: cumulative distribution of Chierici model unconditional output for water relative permeability, S_w=0.5

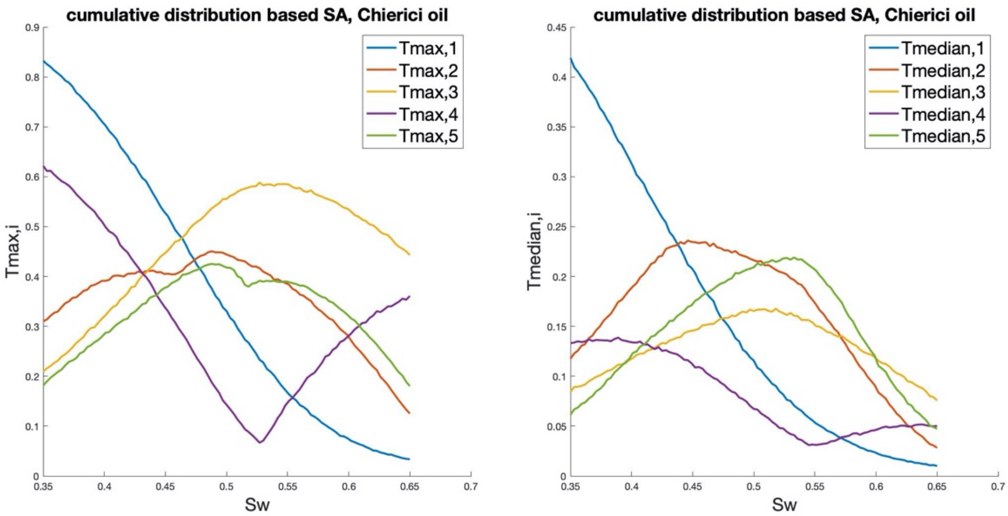

Figure 81: cumulative distribution-based sensitivity analysis of Chierici model for oil relative permeability

The sensitivity indices' plots for oil relative permeability k_{ro} are symmetrical to the ones for water relative permeability k_{rw} and the input variable factors x_2 and x_5 invert their relative contributions.

For the cumulative probability density function-based global sensitivity analysis applied to the Chierici model for oil relative permeability the resulting rankings are:

Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$
x_3: B_o (46.70)	x_2: S_{wi} (16.50)
x_1: k^0_{ro} (37.60)	x_1: k^0_{ro} (15.47)
x_2: S_{wi} (35.10)	x_5: S_{or} (14.81)
x_5: S_{or} (32.61)	x_3: B_o (13.17)
x_4: M_o (31.86)	x_4: M_o (8.23)

B2.4: VARIOGRAM-BASED SENSITIVITY ANALYSIS

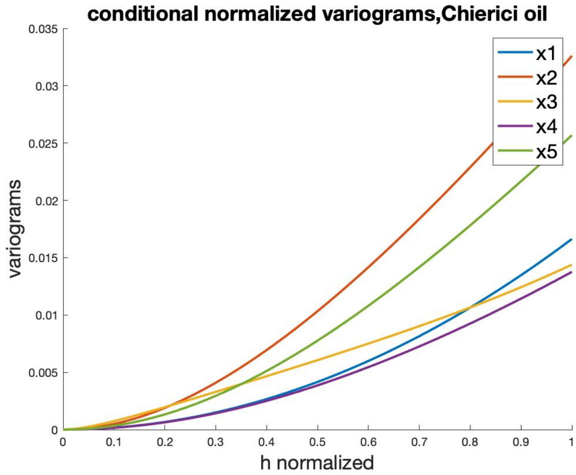

Figure 82: normalized variograms of Chierici model conditional output for oil relative permeability, S_w=0.5

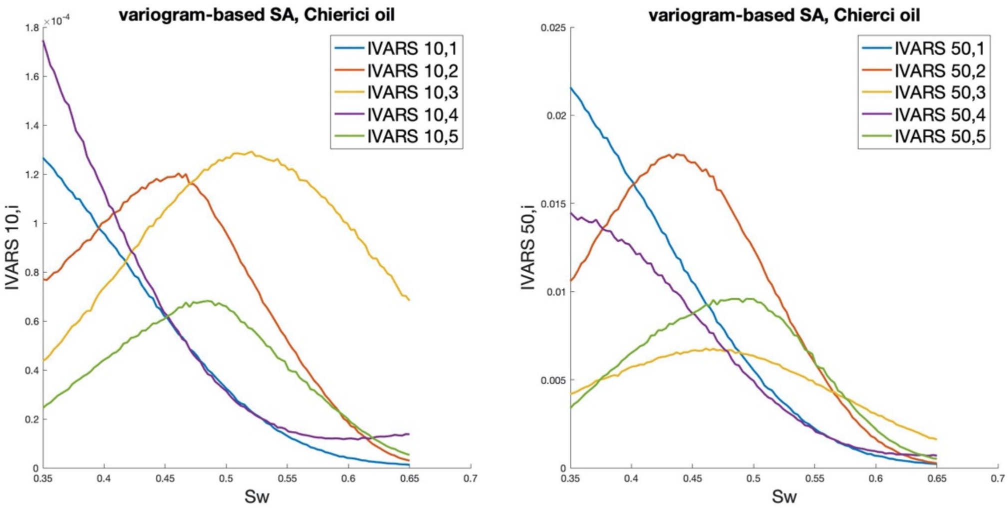

Figure 83: variogram-based sensitivity analysis of Chierici model for oil relative permeability

The sensitivity indices' curves are symmetrical to the curves obtained for water relative permeability; and the role of the variable input factors x_2 and x_5 is inverted.

For the variogram-based sensitivity analysis applied to the Chierici model for oil relative permeability the resulting rankings are:

Ranking according to $R_{IVARS10,i}$	Ranking according to $R_{IVARS50,i}$
x_3: B_o (0.0086)	x_2: S_{wi} (0.98)
x_2: S_{wi} (0.0051)	x_1: k^0_{ro} (0.78)
x_4: M_o (0.0040)	x_4: M_o (0.63)
x_1: k^0_{ro} (0.0033)	x_5: S_{or} (0.59)
x_5: S_{or} (0.0030)	x_3: B_o (0.53)

B3: LET MODEL

B3.1: MULTIPLE START PERTURBATION METHOD SENSITIVITY ANALYSIS

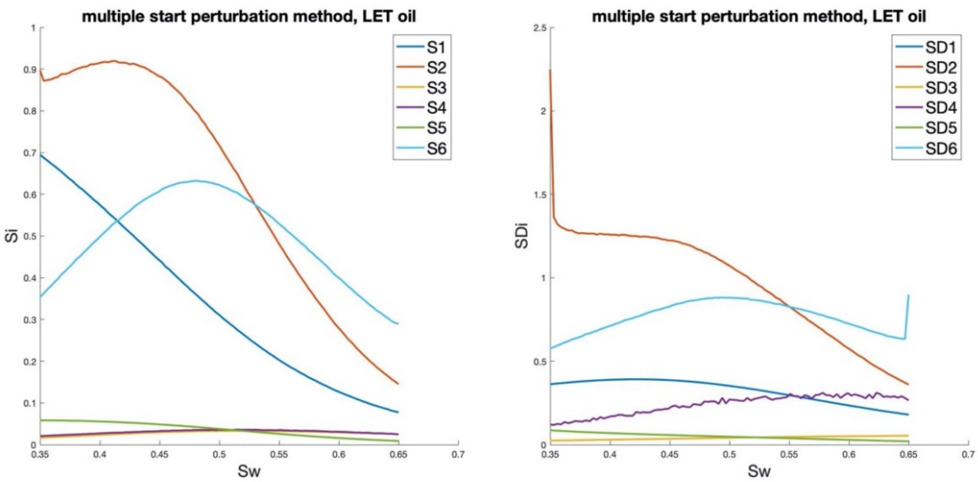

Figure 84: multiple start perturbation method global sensitivity analysis of LET model for oil relative permeability

With respect to the LET model for water relative permeability the input factors x_2 (S_{wi}) and x_6 (S_{or}) have an opposite role, due to the symmetry of the LET relative permeability model with respect to water saturation S_w, when considering the two phases. This symmetry also reflects on the sensitivity indices curves, which are perfectly symmetrical to the ones obtained for water relative permeability.

From the multiple start perturbation method global sensitivity analysis applied to LET model for oil relative permeability the resulting ranking is:

Ranking according to $R_{S,i}$
x_2: S_{wi} (63.77)
x_6: S_{or} (50.28)
x_1: k^0_{ro} (34.31)
x_5: T_o (3.67)
x_4: E_o (3.11)
x_3: L_o (2.84)

B3.2: STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

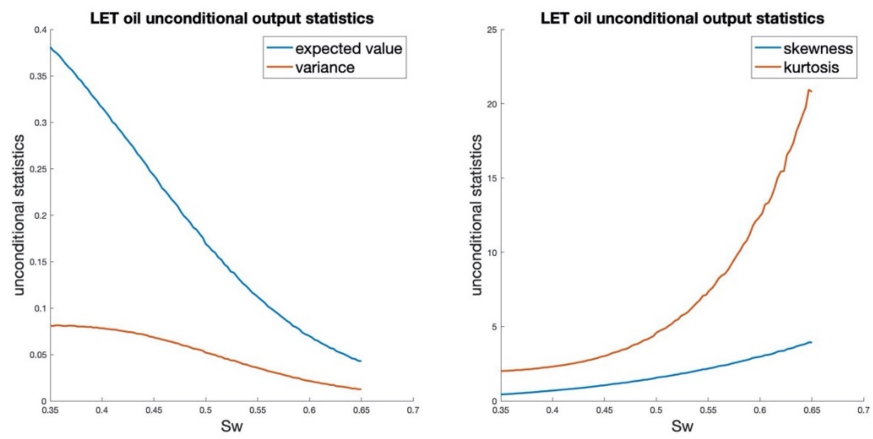

Figure 85: statistical moments of the unconditional output of LET model for oil relative permeability

The unconditional statistical moments of oil relative permeability k_{ro} show exactly opposite trends with respect to the unconditional statistical moments of water relative permeability k_{rw}, due to the symmetry of LET model with respect to water saturation, when considering water and oil.

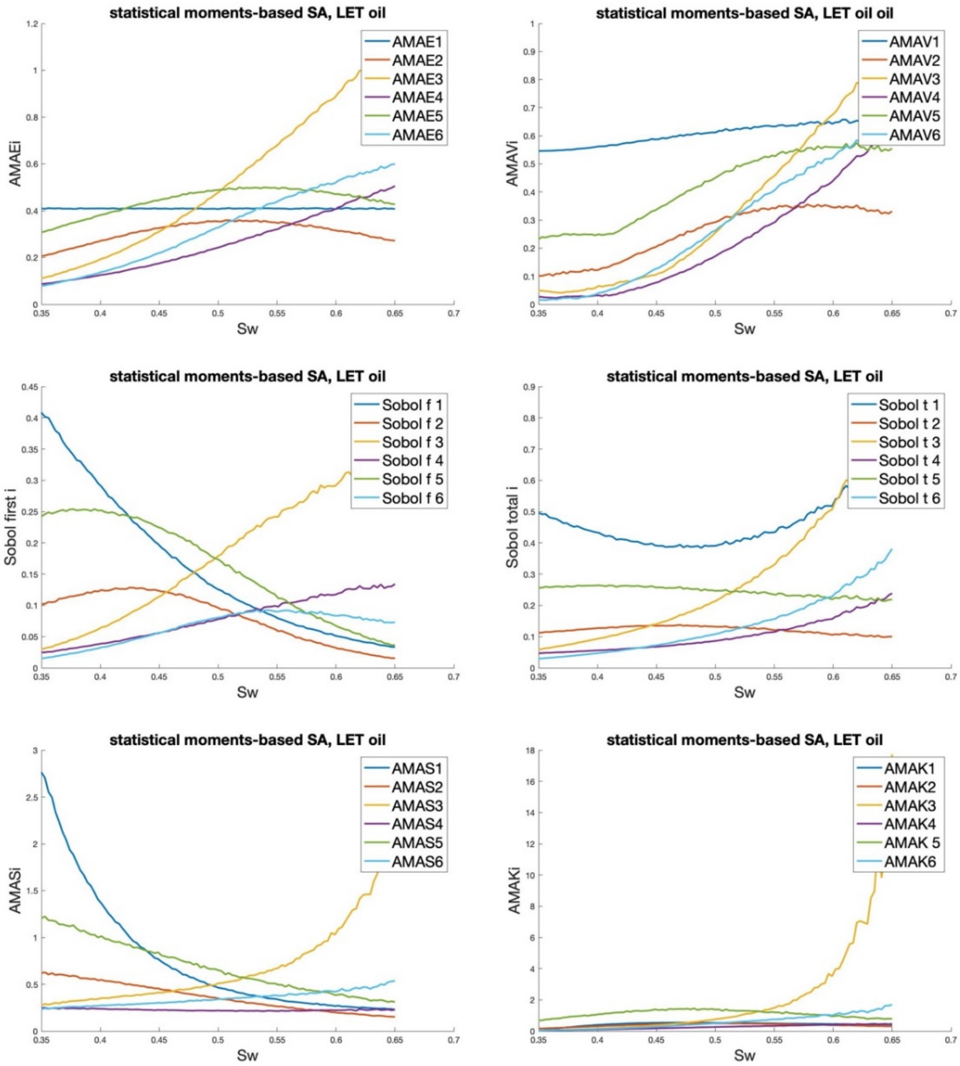

Figure 86: statistical moments-based global sensitivity analysis of LET model for oil relative permeability

From the statistical moments-based global sensitivity analysis applied to the LET model for oil relative permeability the resulting rankings are:

Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\ first,i}$	Ranking according to $R_{Sobol\ total,i}$
x_3: L_o (53.28)	x_1: k^0_{ro} (61.41)	x_3: L_o (18.17)	x_1: k^0_{ro} (46.73)
x_5: T_o (44.76)	x_5: T_o (42.53)	x_5: T_o (16.41)	x_3: L_o (29.27)
x_1: k^0_{ro} (41.33)	x_3: L_o (34.16)	x_1: k^0_{ro} (16.18)	x_5: T_o (24.76)
x_6: S_{or} (33.49)	x_6: S_{or} (28.44)	x_2: S_{wi} (8.36)	x_6: S_{or} (13.71)
x_2: S_{wi} (31.35)	x_2: S_{wi} (25.85)	x_4: E_o (7.94)	x_2: S_{wi} (12.32)
x_4: E_o (26.45)	x_4: E_o (22.56)	x_6: S_{or} (6.56)	x_4: E_o (10.55)

B3.3: PROBABILITY DENSITY FUNCTION-BASED SENSITIVITY ANALYSIS

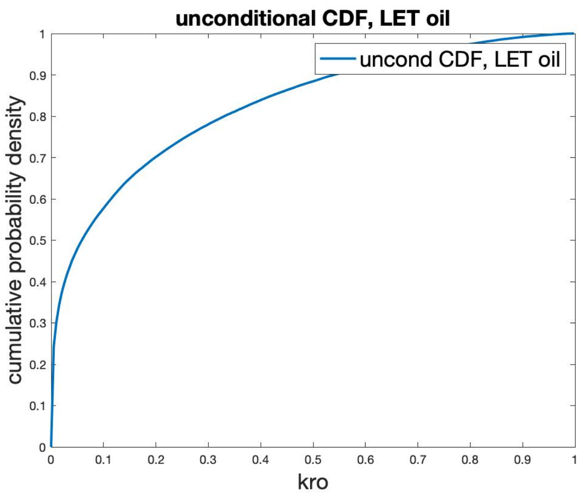

Figure 87: cumulative distribution of LET model unconditional output for oil relative permeability, S_w=0.5

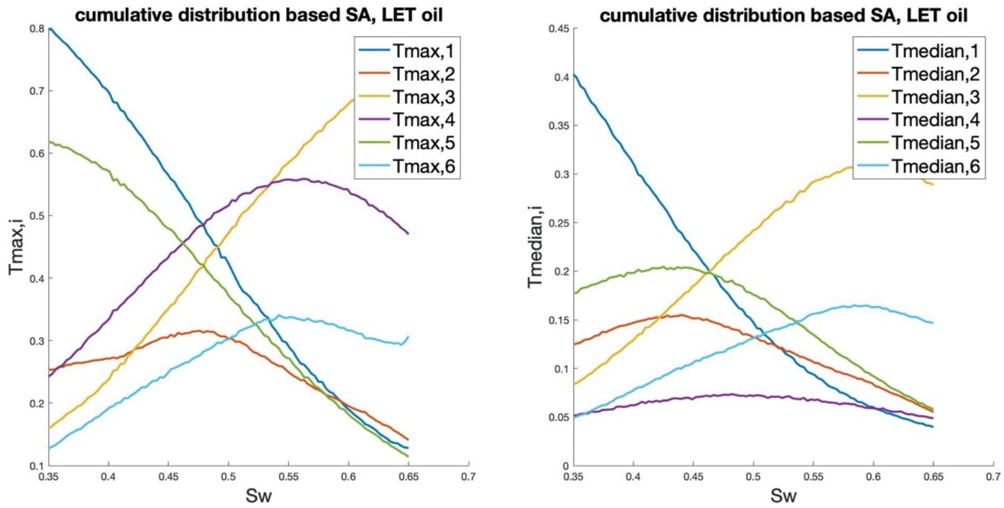

Figure 88: cumulative distribution function-based global sensitivity analysis of LET model for oil relative permeability

From the cumulative probability density function-based global sensitivity analysis applied to the LET model for oil relative permeability the resulting rankings are:

Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$
x_3: L_o (46.78)	x_3: L_o (22.65)
x_4: E_o (46.26)	x_1: k^0_{ro} (17.64)
x_1: k^0_{ro} (44.08)	x_5: T_o (15.57)
x_5: T_o (37.69)	x_6: S_{or} (12.31)
x_6: S_{or} (26.08)	x_2: S_{wi} (12.01)
x_2: S_{wi} (24.97)	x_4: E_o (6.45)

B3.4: VARIOGRAM-BASED SENSITIVITY ANALYSIS

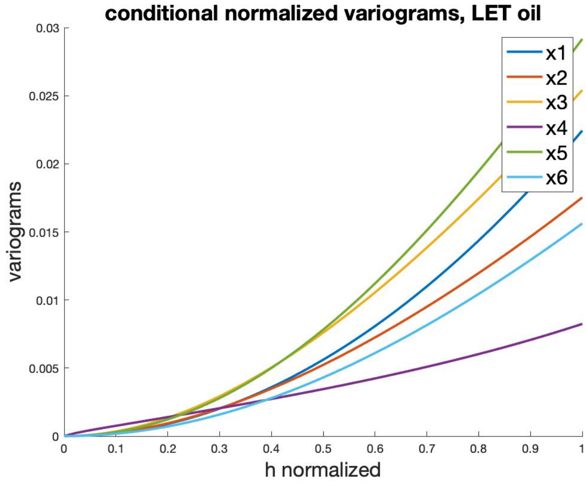

Figure 89: normalized variograms of LET model conditional output for oil relative permeability, S_w=0.5

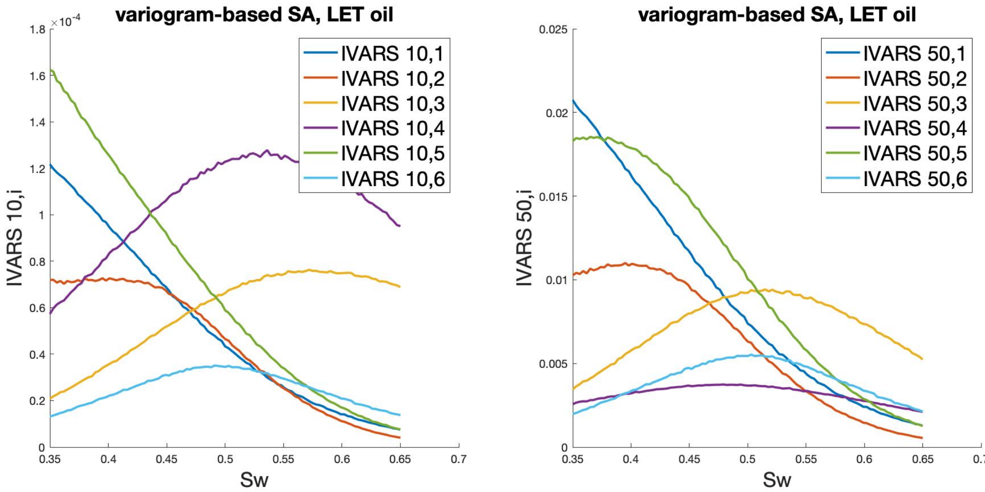

Figure 90: variogram-based global sensitivity analysis of LET model for oil relative permeability

From the variogram-based global sensitivity analysis applied to the LET model for oil relative permeability the resulting rankings are:

Ranking according to $R_{IVARS10,i}$	Ranking according to $R_{IVARS50,i}$
x_4: E_o (0.0126)	x_5: T_o (1.04)
x_5: T_o (0.0050)	x_1: k^0_{ro} (0.89)
x_3: L_o (0.0043)	x_3: L_o (0.75)
x_1: k^0_{ro} (0.0038)	x_2: S_{wi} (0.62)
x_2: S_{wi} (0.0031)	x_6: S_{or} (0.40)
x_6: S_{or} (0.0018)	x_4: E_o (0.38)

B4: RECAP

B4.1: COREY MODEL

In the table below the rankings obtained from the different global sensitivity analysis techniques when applied to Corey relative permeability model are summarized. A colour has been assigned to each variable input factor in order to help visualizing eventual concordances and discordances between the rankings obtained from different sensitivity analysis methods.

Rankings of the input factors of Corey model for oil relative permeability								
multiple start	stat.moments-based				cumul.distrib.-based		variogram-based	
Ranking according to $R_{S,i}$	Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\,first,i}$	Ranking according to $R_{Sobol\,total,i}$	Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$	Ranking according to $R_{IVARS\,10,i}$	Ranking according to $R_{IVARS\,50,i}$
Swi (42.85)	*No (79.25)*	*No (69.79)*	*No (63.66)*	*No (63.48)*	*No (63.93)*	*No (33.74)*	*No (0.011)*	*No (1.13)*
Sor (28.11)	*k0ro (42.31)*	*k0ro (60.65)*	*k0ro (17.40)*	*k0ro (44.93)*	*k0ro (43.98)*	*k0ro (14.29)*	*k0ro (0.0020)*	*k0ro (0.47)*
k0ro (26.21)	*Swi (20.94)*	*Swi (14.10)*	*Swi (6.45)*	*Swi (7.92)*	*Swi (26.69)*	*Swi (12.09)*	*Swi (0.0010)*	*Swi (0.16)*
No (8.14)	*Sor (19.76)*	*Sor (13.30)*	*Sor (3.12)*	*Sor (4.31)*	*Sor (24.65)*	*Sor (11.25)*	*Sor (0.00030)*	*Sor (0.064)*

B4.2: CHIERICI MODEL

In the tables below the rankings obtained from the different sensitivity analysis techniques when applied to the Chierici relative permeability model are summarized.

Rankings of the input factors of Chierici model for oil relative permeability								
multiple start	stat.moments-based				cumul.distrib.-based		variogram-based	
Ranking according to $R_{S,i}$	Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\,first,i}$	Ranking according to $R_{Sobol\,total,i}$	Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$	Ranking according to $R_{IVARS\,10,i}$	Ranking according to $R_{IVARS\,50,i}$
Swi (75.91)	*Bo (59.58)*	*k0ro (62.55)*	*Bo (16.88)*	*k0ro (40.77)*	*Bo (46.70)*	*Swi (16.50)*	*Bo (0.0086)*	*Swi (0.98)*
Sor (54.36)	*Swi (54.53)*	*Bo (47.82)*	*Swi (15.70)*	*Swi (35.60)*	*k0ro (37.60)*	*k0ro (15.47)*	*Swi (0.0051)*	*k0ro (0.78)*
k0ro (29.28)	*Sor (51.95)*	*Swi (46.48)*	*k0ro (14.15)*	*Bo (33.56)*	*Swi (35.10)*	*Sor (14.81)*	*Mo (0.0040)*	*Mo (0.63)*
Bo (3.70)	*k0ro (41.34)*	*Sor (42.97)*	*Sor (9.77)*	*Sor (30.70)*	*Sor (32.61)*	*Bo (13.17)*	*k0ro (0.0033)*	*Sor (0.59)*
Mo (2.62)	*Mo (30.09)*	*x4: Mo (31.53)*	*Mo (7.24)*	*Mo (29.54)*	*Mo (31.86)*	*Mo (8.23)*	*Sor (0.0030)*	*Bo (0.53)*

B4.3: LET MODEL

In the tables below the rankings obtained from the different sensitivity analysis techniques when applied to the LET relative permeability model are summarized.

Rankings of the input factors of LET model for oil relative permeability								
multiple start	stat.moments-based				cumul.distrib.-based		variogram-based	
Ranking according to $R_{S,i}$	Ranking according to $R_{AMAE,i}$	Ranking according to $R_{AMAV,i}$	Ranking according to $R_{Sobol\,first,i}$	Ranking according to $R_{Sobol\,total,i}$	Ranking according to $R_{Tmax,i}$	Ranking according to $R_{Tmedian,i}$	Ranking according to $R_{IVARS\,10,i}$	Ranking according to $R_{IVARS\,50,i}$
Swi (63.77)	*Lo (53.28)*	*k0ro (61.41)*	*Lo (18.17)*	*k0ro (46.73)*	*Lo (46.78)*	*Lo (22.65)*	*Eo (0.0126)*	*To (1.04)*
Sor (50.28)	*To (44.76)*	*To (42.53)*	*To (16.41)*	*Lo (29.27)*	*Eo (46.26)*	*k0ro (17.64)*	*To (0.0050)*	*k0ro (0.89)*
k0ro (34.31)	*k0ro (41.33)*	*Lo (34.16)*	*k0ro (16.18)*	*To (24.76)*	*k0ro (44.08)*	*To (15.57)*	*Lo (0.0043)*	*Lo (0.75)*
To (3.67)	*Sor (33.49)*	*Sor (28.44)*	*Swi (8.36)*	*Sor (13.71)*	*To (37.69)*	*Sor (12.31)*	*k0ro (0.0038)*	*Swi (0.62)*
Eo (3.11)	*Swi (31.35)*	*Swi (25.85)*	*Eo (7.94)*	*Swi (12.32)*	*Sor (26.08)*	*Swi (12.01)*	*Swi (0.0031)*	*Sor (0.40)*
Lo (2.84)	*Eo (26.45)*	*Eo (22.56)*	*Sor (6.56)*	*Eo (10.55)*	*Swi (24.97)*	*Eo (6.45)*	*Sor (0.0018)*	*Eo (0.38)*

APPENDIX C: OIL RELATIVE PERMEABILITY, MULTI-MODEL UNINFORMED SCENARIO

C1: MULTI-MODEL, STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

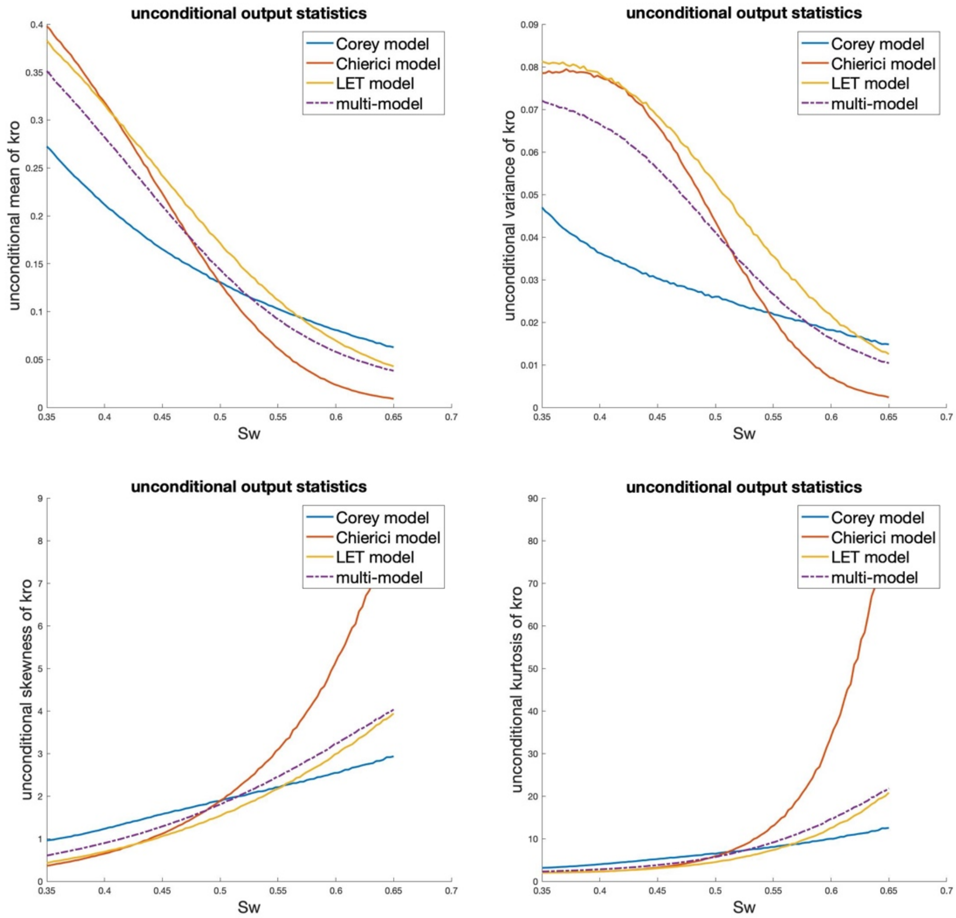

Figure 91: single-model and multi-model unconditional statistical moments of the three models for oil relative permeability

The plots of the multi-model unconditional statistical moments for oil relative permeability are symmetrical to the curves obtained for water relative permeability, with respect to water saturation S_w.

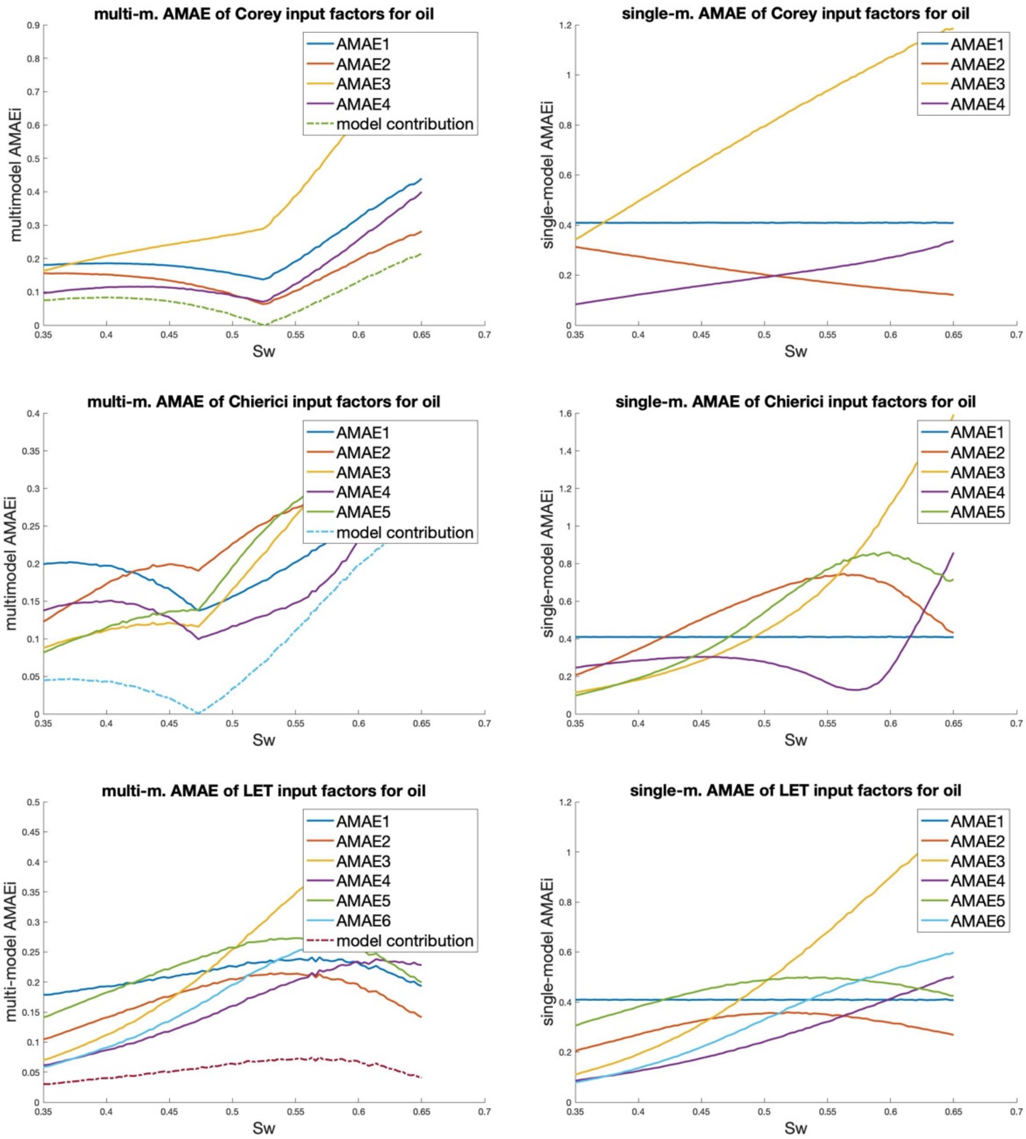

Figure 92: multi-model and single-model AMAE sensitivity indices for oil relative permeability

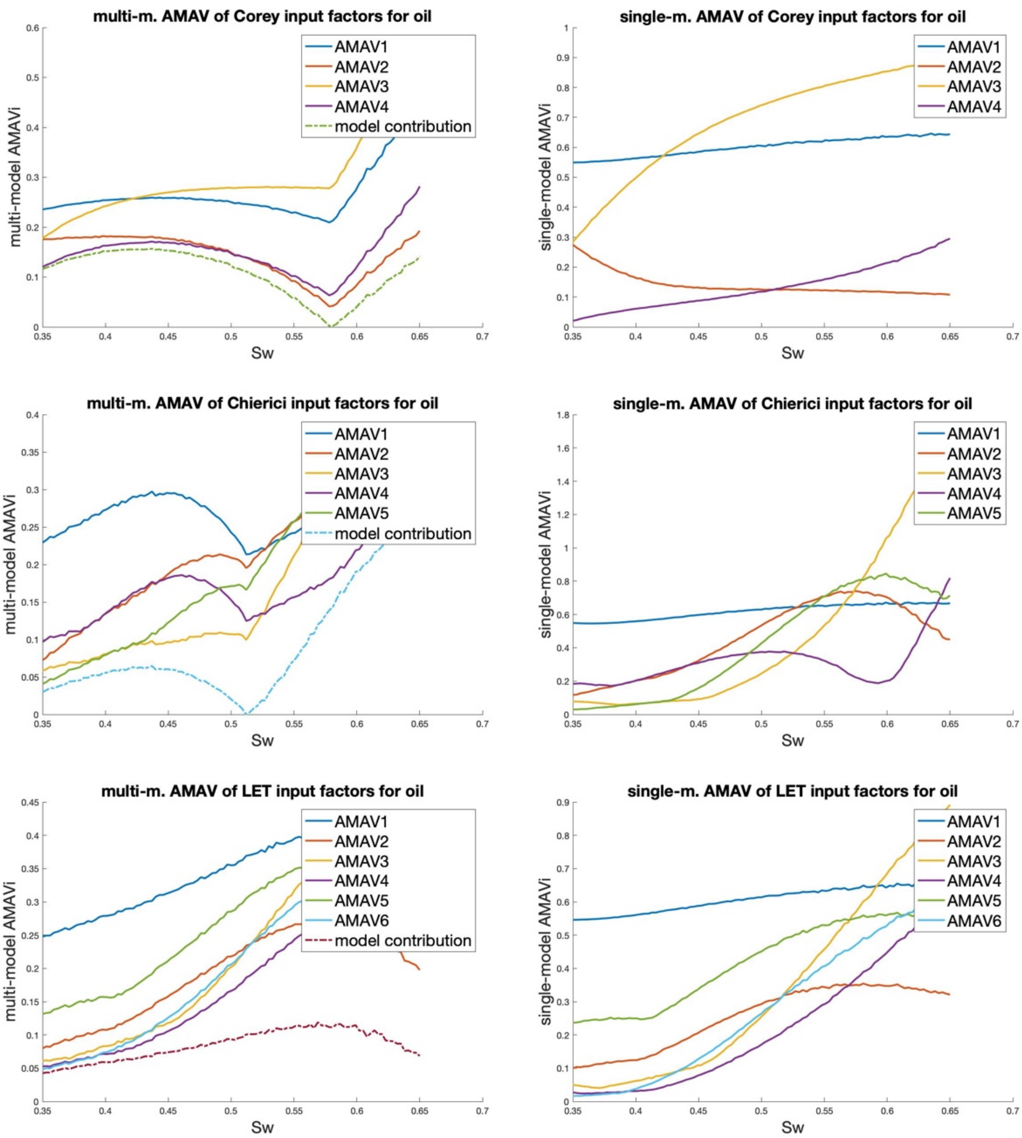

Figure 93: multi-model and single-model AMAV sensitivity indices for oil relative permeability

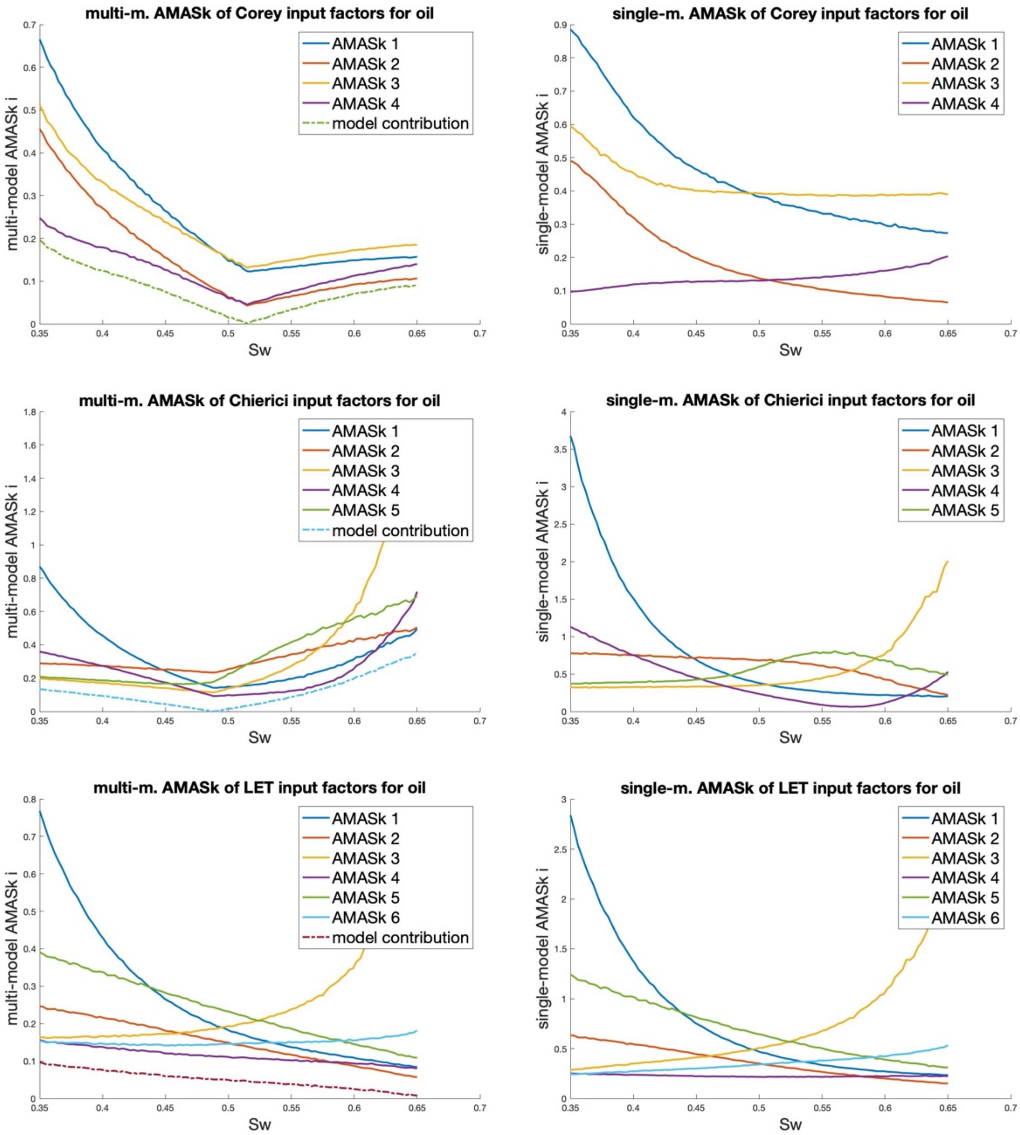

Figure 94: multi-model and single-model AMASk sensitivity indices for oil relative permeability

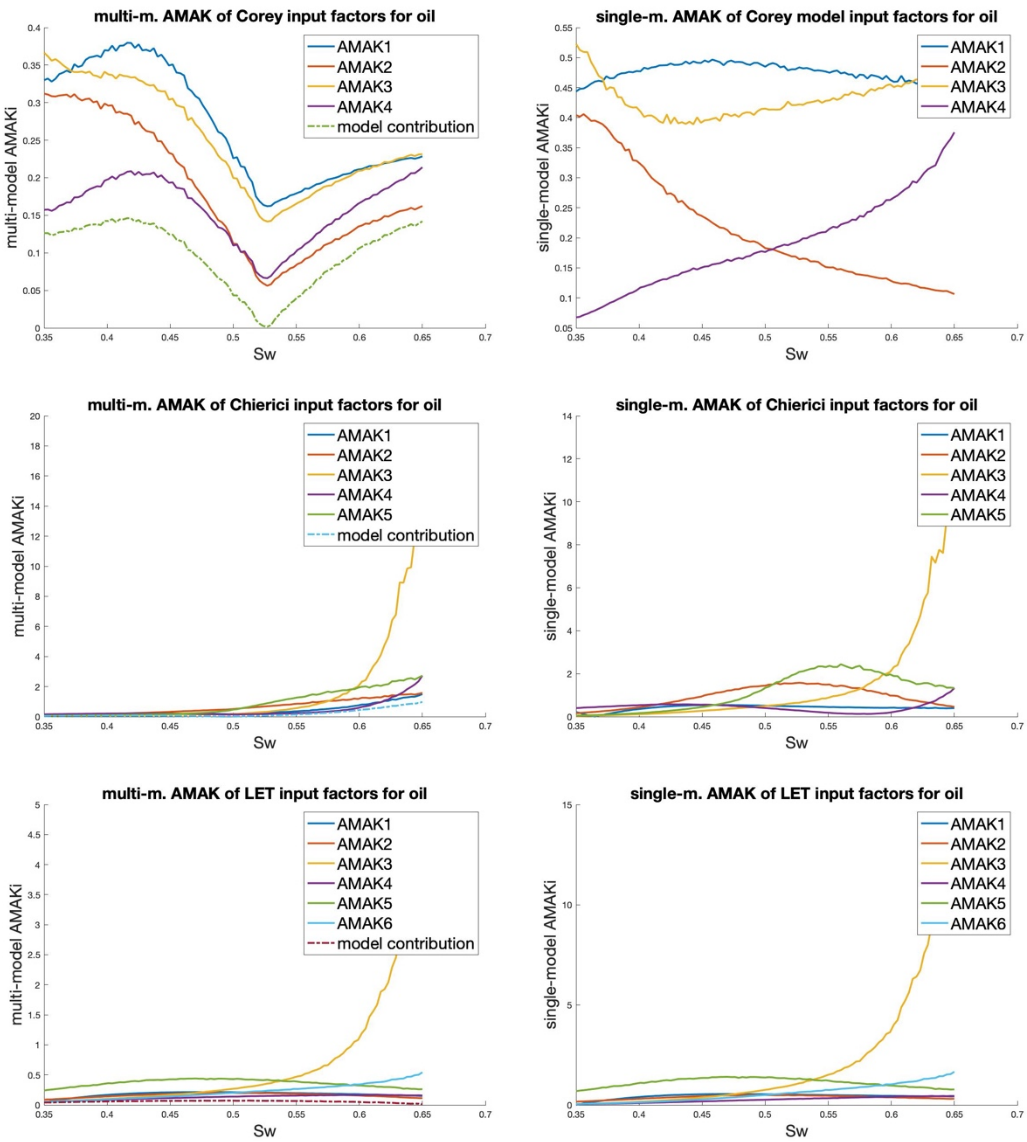

Figure 95: multi-model and single-model AMAK sensitivity indices for oil relative permeability

Considering oil relative permeability, the multi-model statistical moments-based sensitivity indices' plots are simply symmetrical to the ones obtained for water relative permeability and, for each model, the input factors corresponding to S_{wi} and S_{or} invert their roles.

For the multi-model statistical moments-based global sensitivity analysis of the three oil relative permeability models, the resulting rankings are:

Input factors belonging to Corey model
Input factors belonging to Chierici model
Input factors belonging to LET model

Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
x_3^1: N_o^1 (37.17)	x_1^3: $k_{0,ro}^3$ (34.92)
x_3^3: L_o^3 (26.26)	x_3^1: N_o^1 (30.08)
x_2^2: S_{wi}^2 (23.04)	x_1^2: $k_{0,ro}^2$ (27.10)
x_5^3: T_o^3 (23.00)	x_1^1: $k_{0,ro}^1$ (26.94)
x_1^1: $k_{0,ro}^1$ (22.42)	x_5^3: T_o^3 (26.48)
x_1^3: $k_{0,ro}^3$ (21.59)	x_3^3: L_o^3 (23.21)
x_5^2: S_{or}^2 (20.72)	x_2^2: S_{wi}^2 (21.19)
x_3^2: B_o^2 (20.53)	x_6^3: S_{or}^3 (20.79)
x_1^2: $k_{0,ro}^2$ (20.40)	x_2^3: S_{wi}^3 (19.50)
x_6^3: S_{or}^3 (18.68)	x_5^2: S_{or}^2 (18.74)
x_2^3: S_{wi}^3 (17.82)	x_4^3: E_o^3 (17.94)
x_4^2: M_o^2 (16.72)	x_3^2: B_o^2 (17.61)
x_4^1: S_{or}^1 (15.70)	x_4^2: M_o^2 (17.60)
x_4^3: E_o^3 (15.37)	x_4^1: S_{or}^1 (14.98)
x_2^1: S_{wi}^1 (15.00)	x_2^1: S_{wi}^1 (14.37)

As expected, these two rankings are equivalent to the ones obtained for water relative permeability, except for the inversion of x_2^1 with x_4^1, the inversion of x_2^2 with x_5^2 and the inversion of x_2^3 with x_6^3.

The rankings obtained from these two indices are not in agreement with each other. The only aspect on which they agree is that the variable input factor x_2^1 (2^{nd} input factor belonging to Corey model) is the less relevant in this multi-model scenario.

C2: MULTI-MODEL, VARIANCE-BASED SENSITIVITY ANALYSIS

When considering oil relative permeability, the variable input factors of the models are:

- x_1: k_{ro}^0, end point of the oil relative permeability curve.
- x_2: S_{wi}: irreducible water saturation.
- x_3: N_o: parameter belonging to Corey model for oil.
- x_4: B_o: parameter belonging to Chierici model for oil.
- x_5: M_o: parameter belonging to Chierici model for oil.
- x_6: L_o: parameter belonging to LET model for oil.
- x_7: E_o: parameter belonging to LET model for oil.
- x_8: T_o: parameter belonging to LET model for oil.
- x_9: S_{or}: residual oil saturation.

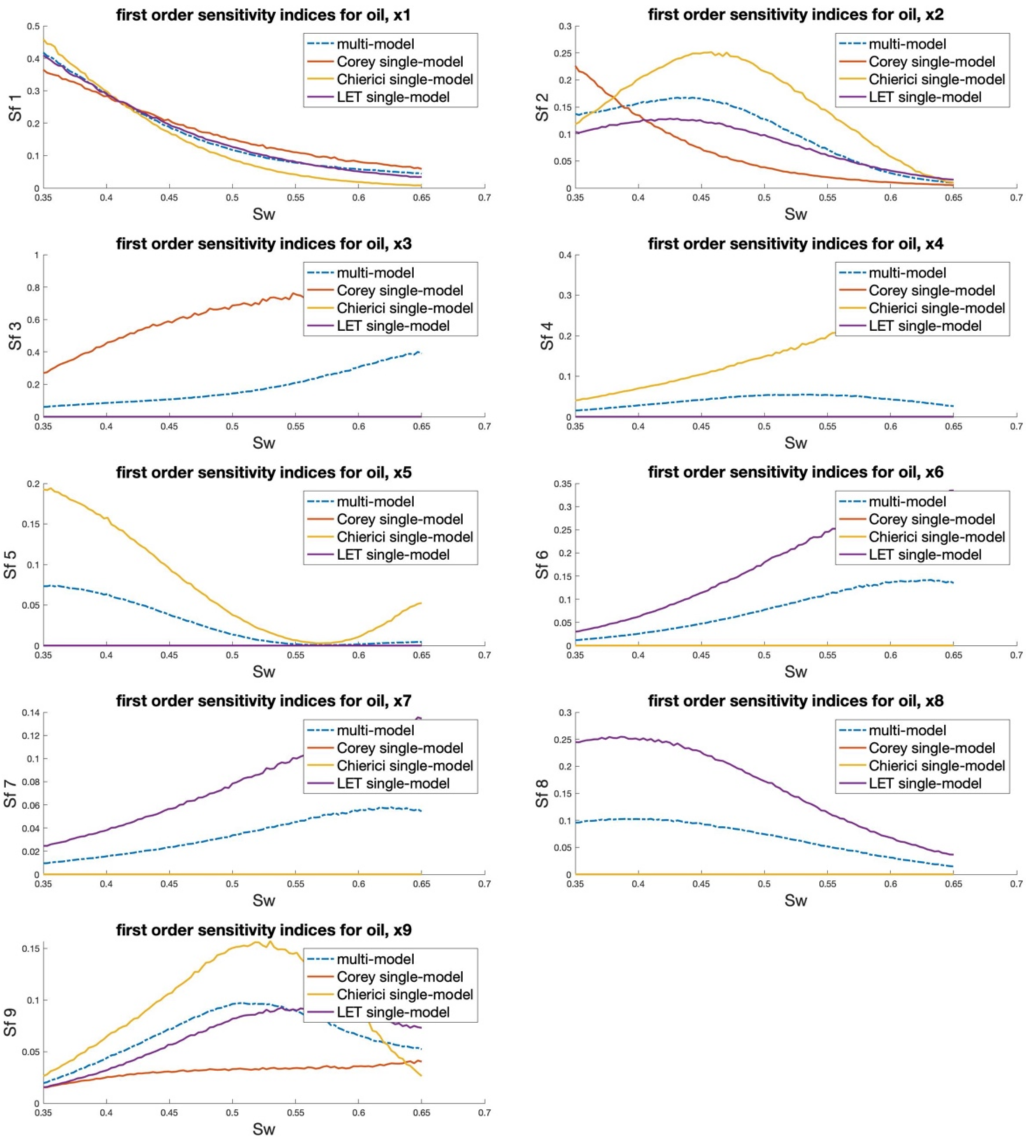

Figure 96: multi-model and single-model first order variance-based sensitivity indices for oil relative permeability

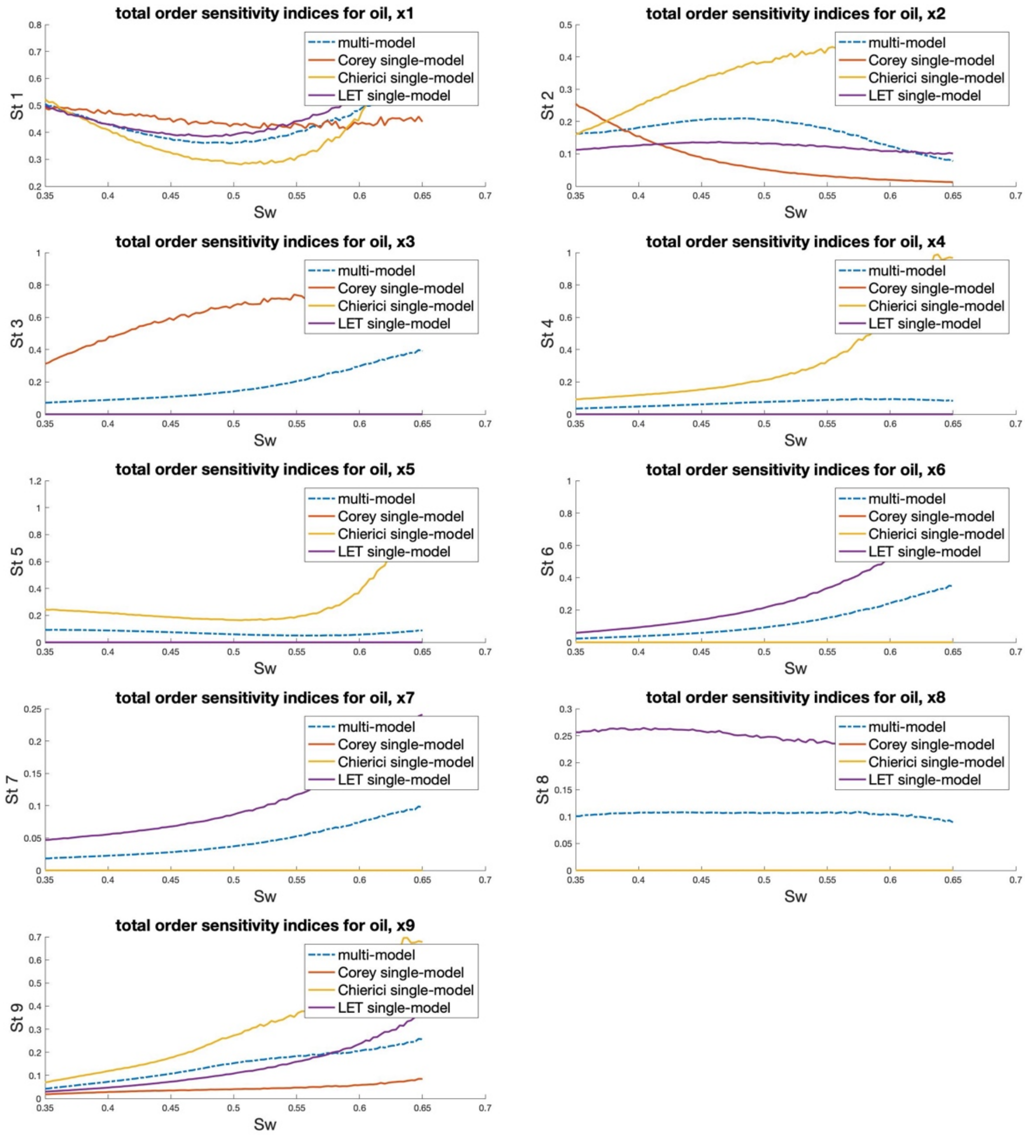

Figure 97: multi-model and single-model total order variance-based sensitivity indices for oil relative permeability

When considering oil relative permeability, the multi-model variance-based sensitivity indices' plots are simply symmetrical to the ones obtained for water relative permeability plus the inversion between the input factors corresponding to S_{wi} (x_2) and S_{or} (x_9).

For the multi-model variance-based sensitivity analysis of the three oil relative permeability models, the resulting rankings are:

Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
x_3: N_o (18.20)	x_1: $k_{0,ro}$ (43.55)
x_1: $k_{0,ro}$ (16.13)	x_3: N_o (18.04)
x_2: S_{wi} (10.43)	x_2: S_{wi} (17.07)
x_6: L_o (7.97)	x_9: S_{or} (14.60)
x_8: T_o (6.90)	x_6: L_o (12.87)
x_9: S_{or} (6.77)	x_8: T_o (10.59)
x_4: B_o (4.08)	x_4: B_o (7.20)
x_7: E_o (3.47)	x_5: M_o (7.06)
x_5: M_o (2.66)	x_7 (4.60)

As expected, these two rankings are equivalent to the ones obtained for water relative permeability, except for the inversion of x_2 with x_9.

C3: RECAP

In the table below the rankings obtained from the different multi-model sensitivity analysis techniques, when applied to the three oil relative permeability models, are summarized:

Multi-model statistical moments-based sensitivity analysis of the oil relative permeability models	
Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
$x_3^1: N_o^1$ (37.17)	$x_1^3: k_{0,ro}^3$ (34.92)
$x_3^3: L_o^3$ (26.26)	$x_3^1: N_o^1$ (30.08)
$x_2^2: S_{wi}^2$ (23.04)	$x_1^2: k_{0,ro}^2$ (27.10)
$x_5^3: T_o^3$ (23.00)	$x_1^1: k_{0,ro}^1$ (26.94)
$x_1^1: k_{0,ro}^1$ (22.42)	$x_5^3: T_o^3$ (26.48)
$x_1^3: k_{0,ro}^3$ (21.59)	$x_3^3: L_o^3$ (23.21)
$x_5^2: S_{or}^2$ (20.72)	$x_2^2: S_{wi}^2$ (21.19)
$x_3^2: B_o^2$ (20.53)	$x_6^3: S_{or}^3$ (20.79)
$x_1^2: k_{0,ro}^2$ (20.40)	$x_2^3: S_{wi}^3$ (19.50)
$x_6^3: S_{or}^3$ (18.68)	$x_5^2: S_{or}^2$ (18.74)
$x_2^3: S_{wi}^3$ (17.82)	$x_4^3: E_o^3$ (17.94)
$x_4^2: M_o^2$ (16.72)	$x_3^2: B_o^2$ (17.61)
$x_4^1: S_{or}^1$ (15.70)	$x_4^2: M_o^2$ (17.60)
$x_4^3: E_o^3$ (15.37)	$x_4^1: S_{or}^1$ (14.98)
$x_2^1: S_{wi}^1$ (15.00)	$x_2^1: S_{wi}^1$ (14.37)

Multi-model variance-based sensitivity analysis of the oil relative permeability models	
Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
$x_3: N_o$ (18.20)	$x_1: k_{0,ro}$ (43.55)
$x_1: k_{0,ro}$ (16.13)	$x_3: N_o$ (18.04)
$x_2: S_{wi}$ (10.43)	$x_2: S_{wi}$ (17.07)
$x_6: L_o$ (7.97)	$x_9: S_{or}$ (14.60)
$x_8: T_o$ (6.90)	$x_6: L_o$ (12.87)
$x_9: S_{or}$ (6.77)	$x_8: T_o$ (10.59)
$x_4: B_o$ (4.08)	$x_4: B_o$ (7.20)
$x_7: E_o$ (3.47)	$x_5: M_o$ (7.06)
$x_5: M_o$ (2.66)	$x_7: E_o$ (4.60)

APPENDIX D: OIL RELATIVE PERMEABILITY, MULTI-MODEL INFORMED SCENARIO

D1: "SAND PACK" SAMPLE

D1.1: MULTI-MODEL, INFORMED STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.01$
- $w^2 = 0.02$
- $w^1 = 0.97$

VARIABLE INPUT FACTORS:

The models variable input factors are expressed in the form:

$x_i^j = (x_{i,min}^j \, ; \, x_{i,max}^j)$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the brackets define its 95% confidence limits.

- x_1^1: $k_{ro}^0 = (0.751 \, ; \, 0.811)$
- x_2^1: $S_{wi} = (0.166 \, ; \, 0.214)$
- x_3^1: $N_o = (0.99 \, ; \, 1.05)$
- x_4^1: $S_{or} = (0.1 \, ; \, 0.18)$

- x_1^2: $k_{ro}^0 = (0.751 \, ; \, 0.811)$
- x_2^2: $S_{wi} = (0.166 \, ; \, 0.214)$
- x_3^2: $B_o = (0.62 \, ; \, 0.73)$
- x_4^2: $M_o = (0.53 \, ; \, 0.76)$
- x_5^2: $S_{or} = (0.1 \, ; \, 0.18)$

- x_1^3: $k_{ro}^0 = (0.751 \, ; \, 0.811)$
- x_2^3: $S_{wi} = (0.166 \, ; \, 0.214)$
- x_3^3: $L_o = (0.79 \, ; \, 1.22)$
- x_4^3: $E_o = (0.70 \, ; \, 1.65)$
- x_5^3: $T_o = (0.91 \, ; \, 1.64)$
- x_6^3: $S_{or} = (0.1 \, ; \, 0.18)$

Oil relative permeability k_{ro} is evaluated for $0.25 \leq S_w \leq 0.8$.

In the figure below the unconditional single-model and multi-model statistical moments are reported for the "sand pack" sample, for oil relative permeability:

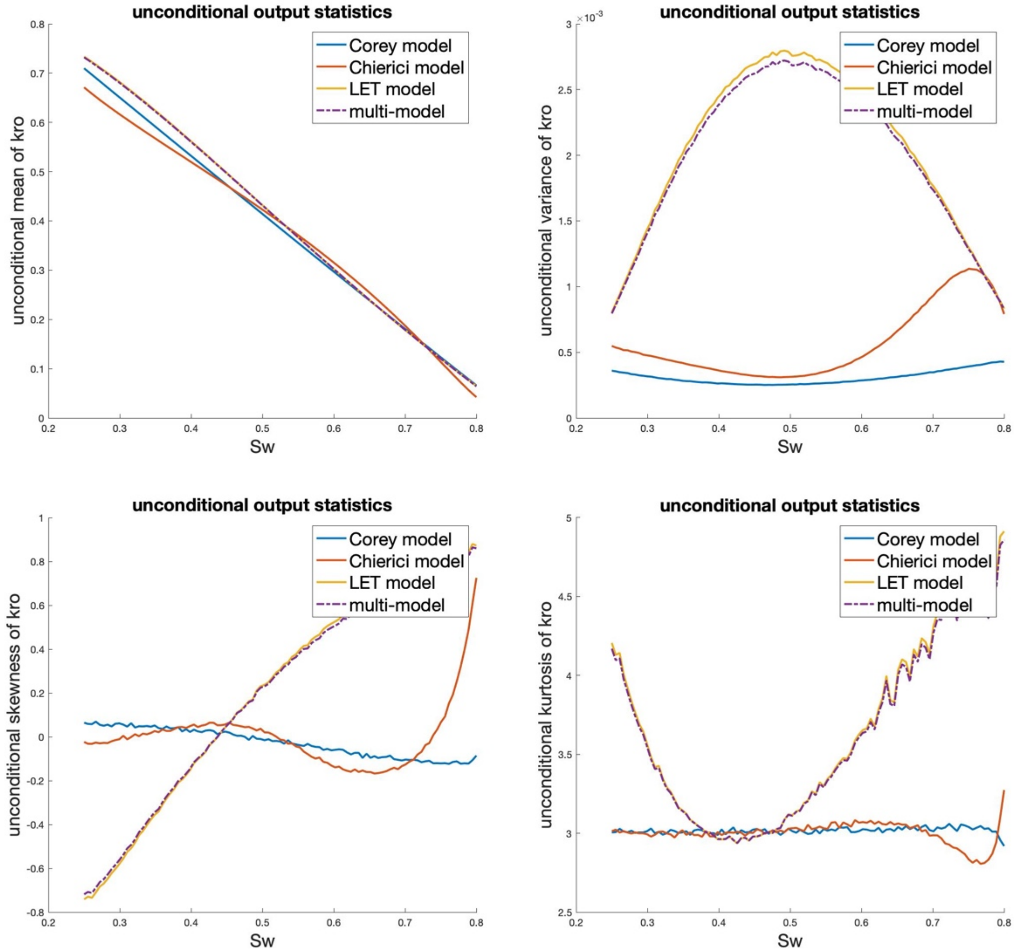

Figure 98: single-model and multi-model unconditional output statistics of the relative permeability models informed to the "sand pack" sample for oil relative permeability

In the figures below, the informed multi-model statistical moments-based sensitivity indices of the "sand pack" sample for oil relative permeability are reported. The single-model statistical moments-based indices are reported too, so as to highlight the differences between the multi-model and the single-model approach.

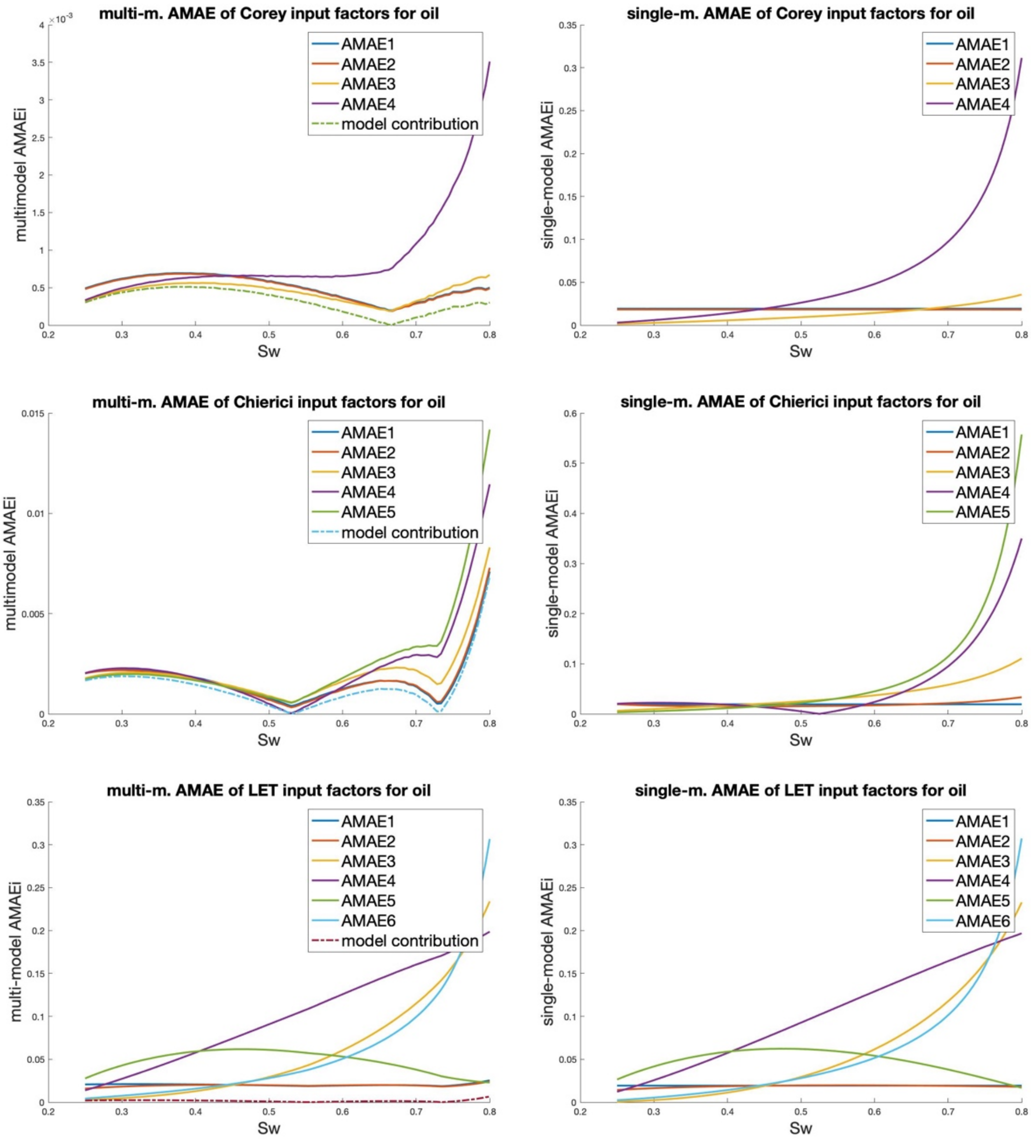

Figure 99: multi-model and single-model informed AMAE sensitivity indices of the "sand pack" sample for oil relative permeability

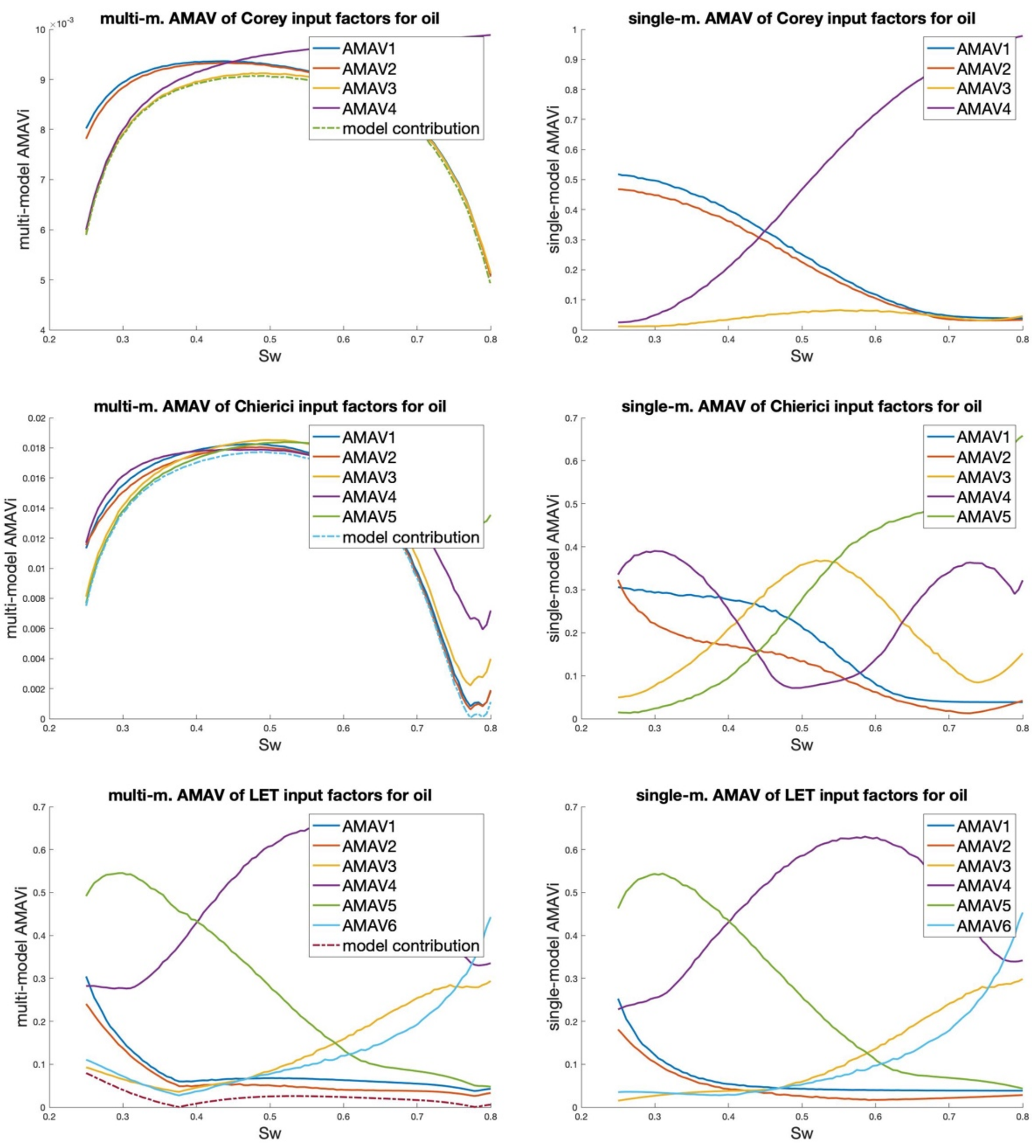

Figure 100: multi-model and single-model informed AMAV sensitivity indices of the "sand pack" sample for oil relative permeability

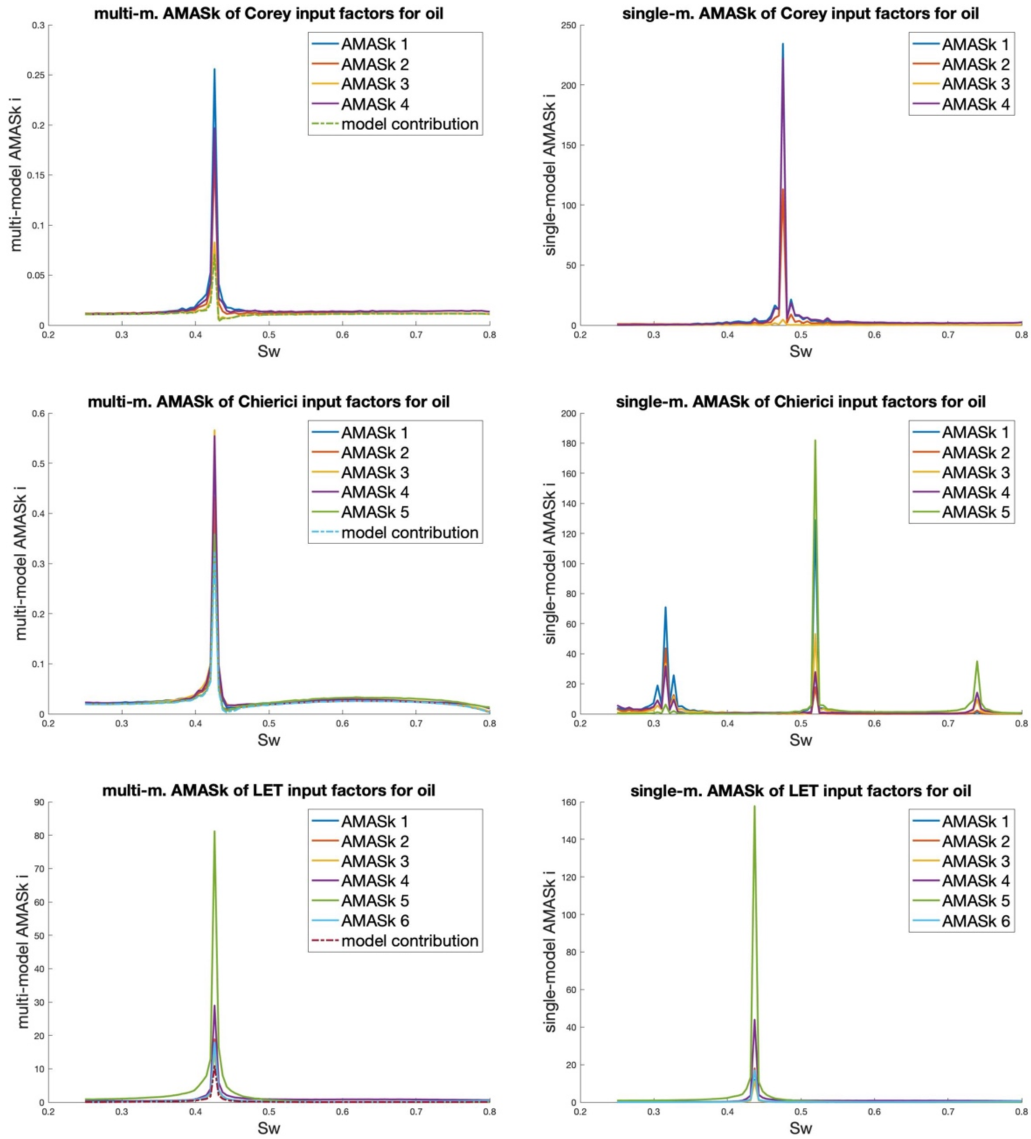

Figure 101: multi-model and single-model informed AMASk sensitivity indices of the "sand pack" sample for oil relative permeability

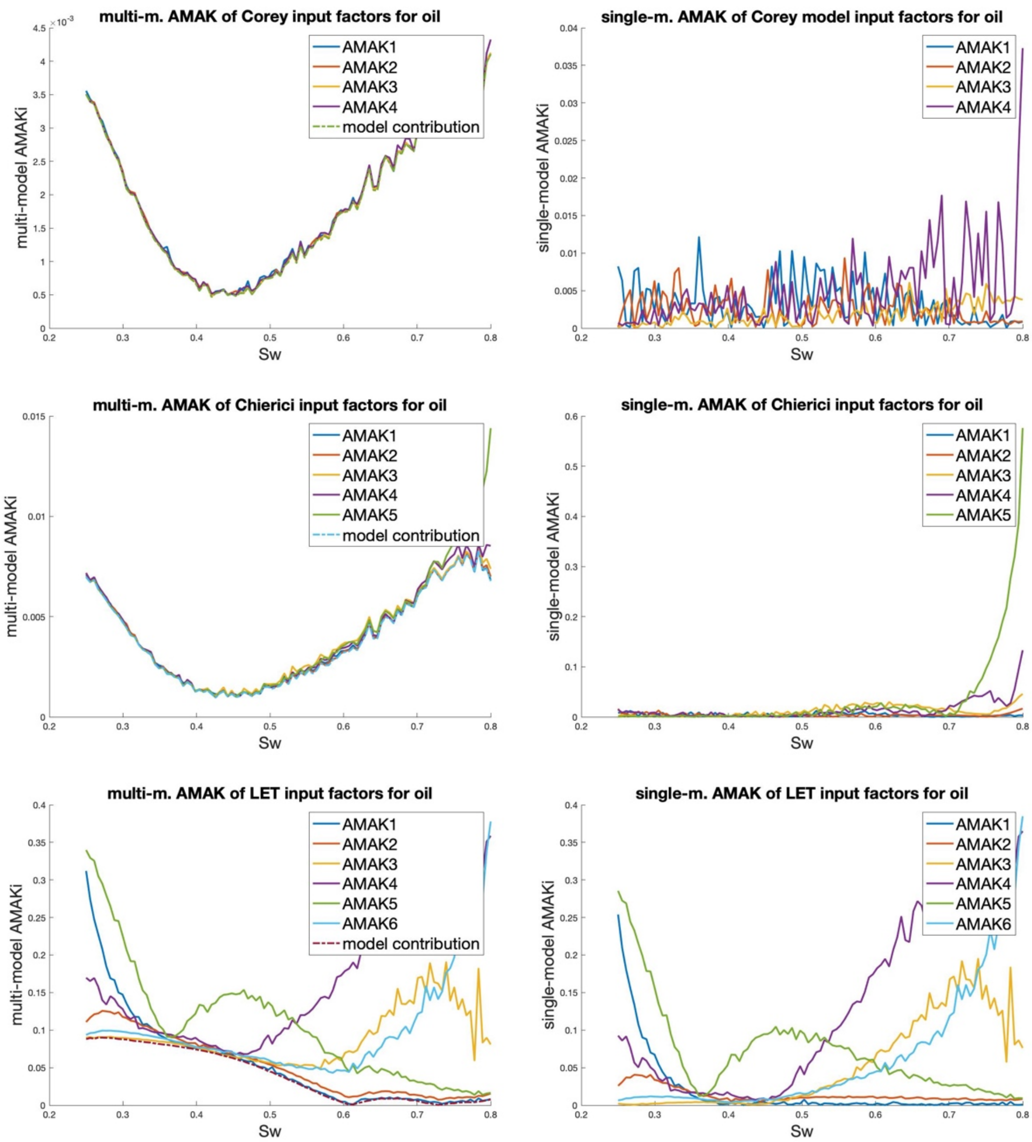

Figure 102: multi-model and single-model informed AMAK sensitivity indices of the "sand pack" sample for oil relative permeability

The ranking of the 15 variable input factors x_i^j (4 belonging to Corey model, 5 to Chierici model and 6 to LET model) is determined calculating the two ranking indices:

$$R_{AMAE_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAE_i^j$$

$$R_{AMAV_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAV_i^j$$

For the multi-model statistical moments-based informed global sensitivity analysis of the "sand pack" sample for oil relative permeability, the resulting rankings are:

Input factors belonging to Corey model
Input factors belonging to Chierici model
Input factors belonging to LET model

Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
x_4^3: E_o^3 (10.11)	x_4^3: E_o^3 (48.24)
x_3^3: L_o^3 (5.96)	x_5^3: T_o^3 (27.45)
x_6^3: S_{or}^3 (5.87)	x_3^3: L_o^3 (13.98)
x_5^3: T_o^3 (4.81)	x_6^3: S_{or}^3 (12.86)
x_1^3: $k_{0,ro}^3$ (2.04)	x_1^3: $k_{0,ro}^3$ (8.03)
x_2^3: S_{wi}^3 (1.96)	x_2^3: S_{wi}^3 (6.35)
x_5^2: S_{or}^2 (0.27)	x_5^2: S_{or}^2 (1.60)
x_4^2: M_o^2 (0.24)	x_4^2: M_o^2 (1.54)
x_3^2: B_o^2 (0.20)	x_3^2: B_o^2 (1.43)
x_1^2: $k_{0,ro}^2$ (0.17)	x_1^2: $k_{0,ro}^2$ (1.41)
x_2^2: S_{wi}^2 (0.17)	x_2^2: S_{wi}^2 (1.39)
x_4^1: S_{or}^1 (0.090)	x_4^1: S_{or}^1 (0.93)
x_1^1: $k_{0,ro}^1$ (0.051)	x_1^1: $k_{0,ro}^1$ (0.87)
x_2^1: S_{wi}^1 (0.050)	x_2^1: S_{wi}^1 (0.86)
x_3^1: N_o^1 (0.045)	x_3^1: N_o^1 (0.84)

D1.2: MULTI-MODEL, INFORMED VARIANCE-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.01$
- $w^2 = 0.02$
- $w^1 = 0.97$

VARIABLE INPUT FACTORS:

The models variable input factors are expressed in the form:

$x_i^j = (x_{i,min}^j \,;\, x_{i,max}^j)$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the brackets define its 95% confidence limits.

- x_1: $k_{ro}^0 = (0.751\,;\ 0.811)$
- x_2: $S_{wi} = (0.166\,;\ 0.214)$
- x_3: $N_o = (0.99\,;\ 1.05)$
- x_4: $B_o = (0.62\,;\ 0.73)$
- x_5: $M_o = (0.53\,;\ 0.76)$
- x_6: $L_o = (0.79\,;\ 1.22)$
- x_7: $E_o = (0.70\,;\ 1.65)$
- x_8: $T_o = (0.91\,;\ 1.64)$
- x_9: $S_{or} = (0.1\,;\ 0.18)$

Oil relative permeability k_{ro} is evaluated for $0.25 \leq S_w \leq 0.8$.

In the figures below, the single-model and multi-model first order and total order variance-based sensitivity indices informed to the "sand pack" sample for oil relative permeability are reported:

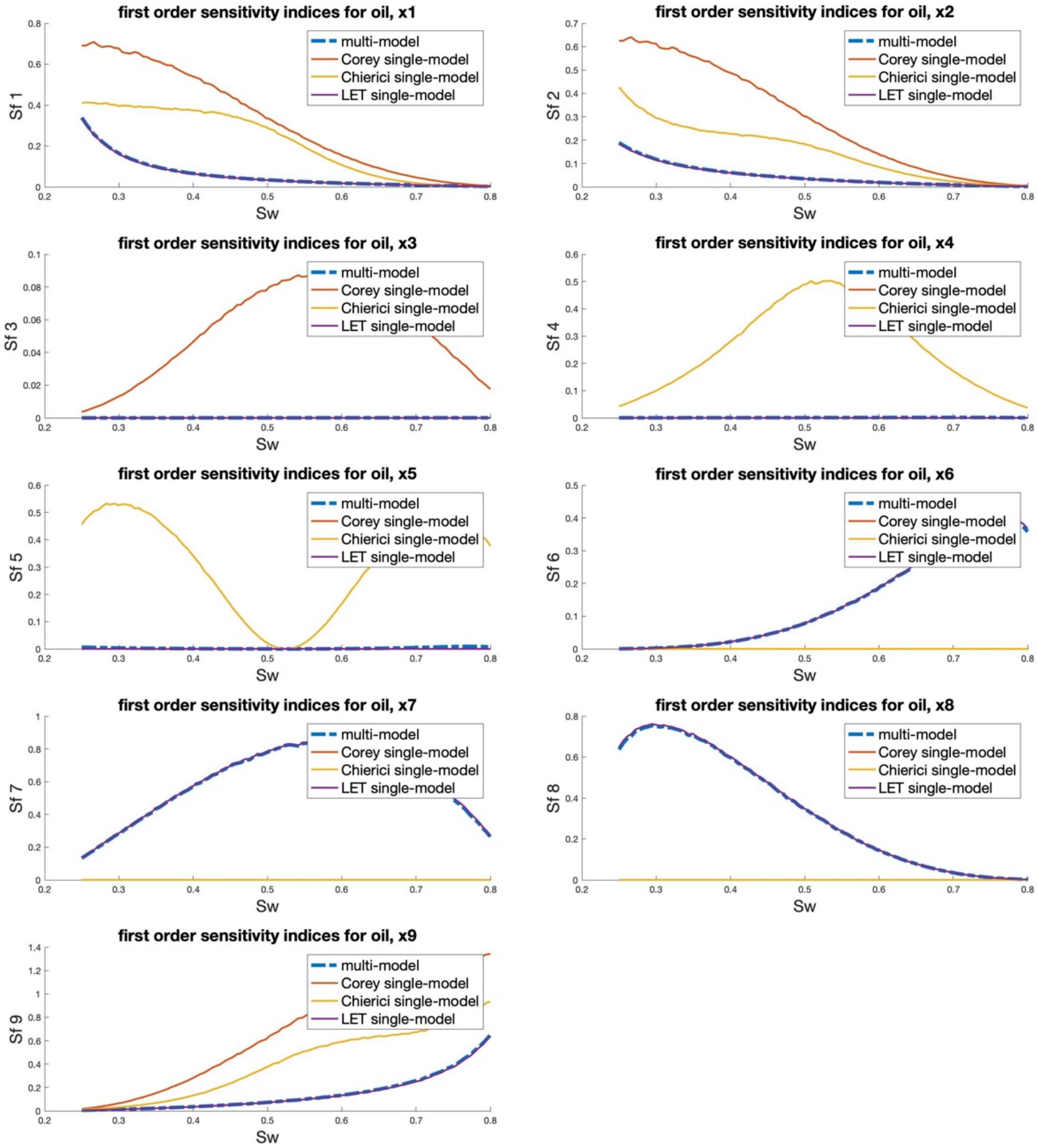

Figure 103: first order single-model and multi-model informed variance-based sensitivity indices of the "sand pack" sample for oil relative permeability

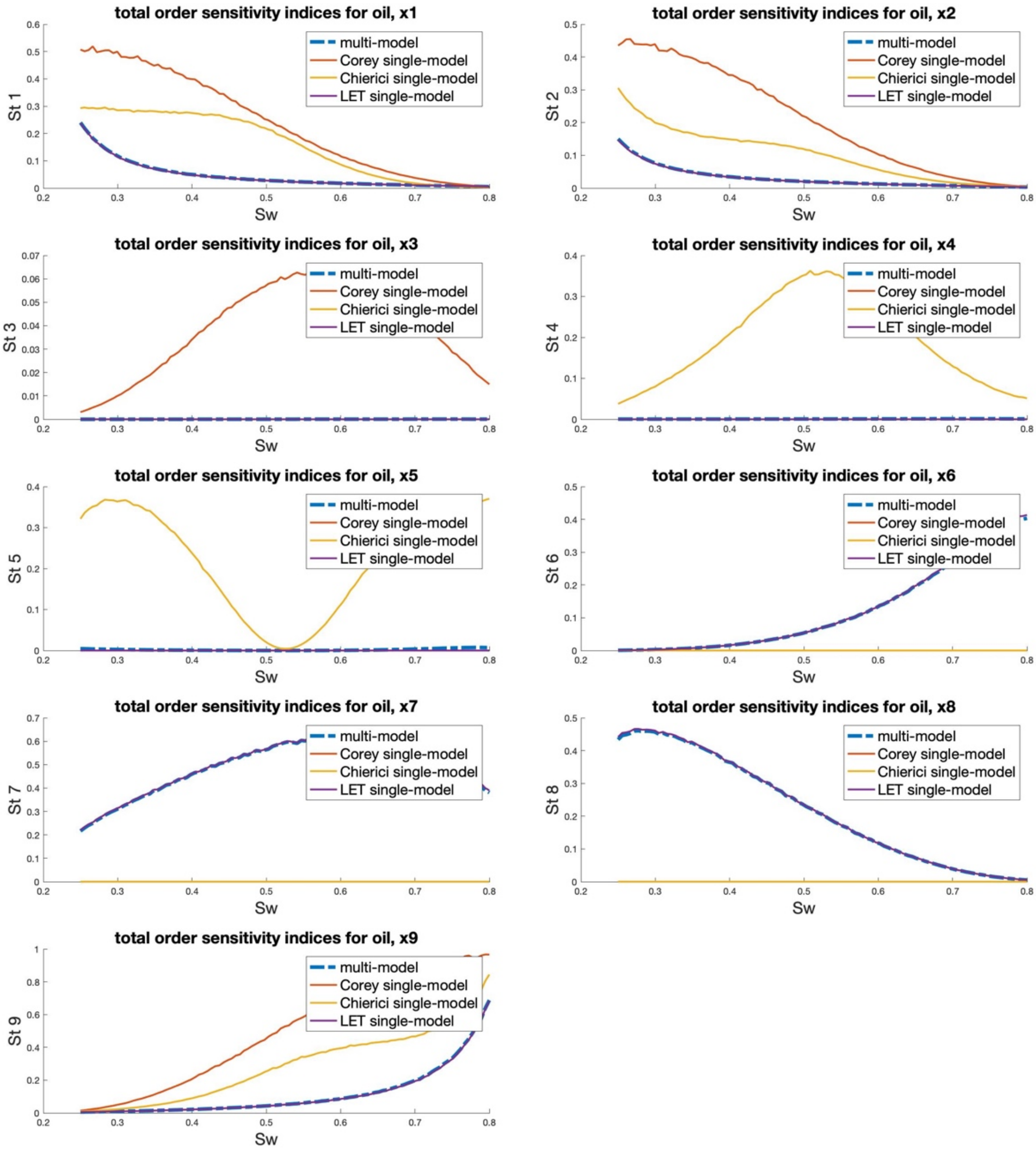

Figure 104: total order single-model and multi-model informed variance-based sensitivity indices of the "sand pack" sample for oil relative permeability

The ranking of the nine variable input factors x_i is determined calculating the two ranking indices:

$$R_{Sf_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} Sf_i^{multimodel}$$

$$R_{St_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} St_i^{multimodel}$$

For the variance-based informed global sensitivity analysis of the "sand pack" sample for oil relative permeability the resulting rankings are:

Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
x_7: E_o (60.17)	x_7: E_o (49.97)
x_8: T_o (34.13)	x_8: T_o (22.26)
x_6: L_o (15.08)	x_6: L_o (12.51)
x_9: S_{or} (14.46)	x_9: S_{or} (11.54)
x_1: $k_{0,ro}$ (5.98)	x_1: $k_{0,ro}$ (4.52)
x_2: S_{wi} (4.69)	x_2: S_{wi} (3.03)
x_5: M_o (0.28)	x_5: M_o (0.21)
x_4: B_o (0.12)	x_4: B_o (0.093)
x_3: N_o (0.0078)	x_3: N_o (0.0057)

D1.3: RECAP

In the table below the rankings obtained from the different informed multi-model sensitivity analysis techniques when applied to the “sand pack” sample for oil relative permeability are summarized:

Multi-model informed statistical moments-based sensitivity analysis of the “sand pack” sample for oil relative permeability	
Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
$x_4^3: E_o^3$ (10.11)	$x_4^3: E_o^3$ (48.24)
$x_3^3: L_o^3$ (5.96)	$x_5^3: T_o^3$ (27.45)
$x_6^3: S_{or}^3$ (5.87)	$x_3^3: L_o^3$ (13.98)
$x_5^3: T_o^3$ (4.81)	$x_6^3: S_{or}^3$ (12.86)
$x_1^3: k_{0,ro}^3$ (2.04)	$x_1^3: k_{0,ro}^3$ (8.03)
$x_2^3: S_{wi}^3$ (1.96)	$x_2^3: S_{wi}^3$ (6.35)
x_5^2 : S_{or}^2(0.27)	$x_5^2: S_{or}^2$ (1.60)
$x_4^2: M_o^2$ (0.24)	$x_4^2: M_o^2$ (1.54)
$x_3^2: B_o^2$ (0.20)	$x_3^2: B_o^2$ (1.43)
$x_1^2: k_{0,ro}^2$ (0.17)	$x_1^2: k_{0,ro}^2$ (1.41)
$x_2^2: S_{wi}^2$ (0.17)	$x_2^2: S_{wi}^2$ (1.39)
$x_4^1: S_{or}^1$ (0.090)	$x_4^1: S_{or}^1$ (0.93)
$x_1^1: k_{0,ro}^1$ (0.051)	$x_1^1: k_{0,ro}^1$ (0.87)
$x_2^1: S_{wi}^1$ (0.050)	$x_2^1: S_{wi}^1$ (0.86)
$x_3^1: N_o^1$ (0.045)	$x_3^1: N_o^1$ (0.84)

Multi-model informed variance-based sensitivity analysis of the “sand pack” sample for oil relative permeability	
Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
$x_7: E_o$ (60.17)	$x_7: E_o$ (49.97)
$x_8: T_o$ (34.13)	$x_8: T_o$ (22.26)
$x_6: L_o$ (15.08)	$x_6: L_o$ (12.51)
$x_9: S_{or}$ (14.46)	$x_9: S_{or}$ (11.54)
$x_1: k_{0,ro}$ (5.98)	$x_1: k_{0,ro}$ (4.52)
$x_2: S_{wi}$ (4.69)	$x_2: S_{wi}$ (3.03)
$x_5: M_o$ (0.28)	$x_5: M_o$ (0.21)
$x_4: B_o$ (0.12)	$x_4: B_o$ (0.093)
$x_3: N_o$ (0.0078)	$x_3: N_o$ (0.0057)

D2: "BEREA SANDSTONE" SAMPLE

D2.1: MULTI-MODEL, INFORMED STATISTICAL MOMENTS-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.05$
- $w^2 = 0.35$
- $w^1 = 0.6$

VARIABLE INPUT FACTORS:

The models variable input factors are expressed in the form:

$x_i^j = (x_{i,min}^j \, ; \, x_{i,max}^j)$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the brackets define its 95% confidence limits.

- $x_1^1: k_{ro}^0 = (0.779 \, ; \, 0.783)$
- $x_2^1: S_{wi} = (0.39 \, ; \, 0.45)$
- $x_3^1: N_o = (1.8 \, ; \, 2.6)$
- $x_4^1: S_{or} = (0.32 \, ; \, 0.36)$
- $x_1^2: k_{ro}^0 = (0.779 \, ; \, 0.783)$
- $x_2^2: S_{wi} = (0.39 \, ; \, 0.45)$
- $x_3^2: B_o = (1.11 \, ; \, 1.44)$
- $x_4^2: M_o = (0.89 \, ; \, 1.81)$
- $x_5^2: S_{or} = (0.32 \, ; \, 0.36)$

- $x_1^3: k_{ro}^0 = (0.779 \, ; \, 0.783)$
- $x_2^3: S_{wi} = (0.39 \, ; \, 0.45)$
- $x_3^3: L_o = (3.08 \, ; \, 3.54)$
- $x_4^3: E_o = (1.14 \, ; \, 1.59)$
- $x_5^3: T_o = (0.58 \, ; \, 4.29)$
- $x_6^3: S_{or} = (0.32 \, ; \, 0.36)$

In the figure below, the unconditional single-model and multi-model statistical moments are reported for the “Berea sandstone” sample, for oil relative permeability:

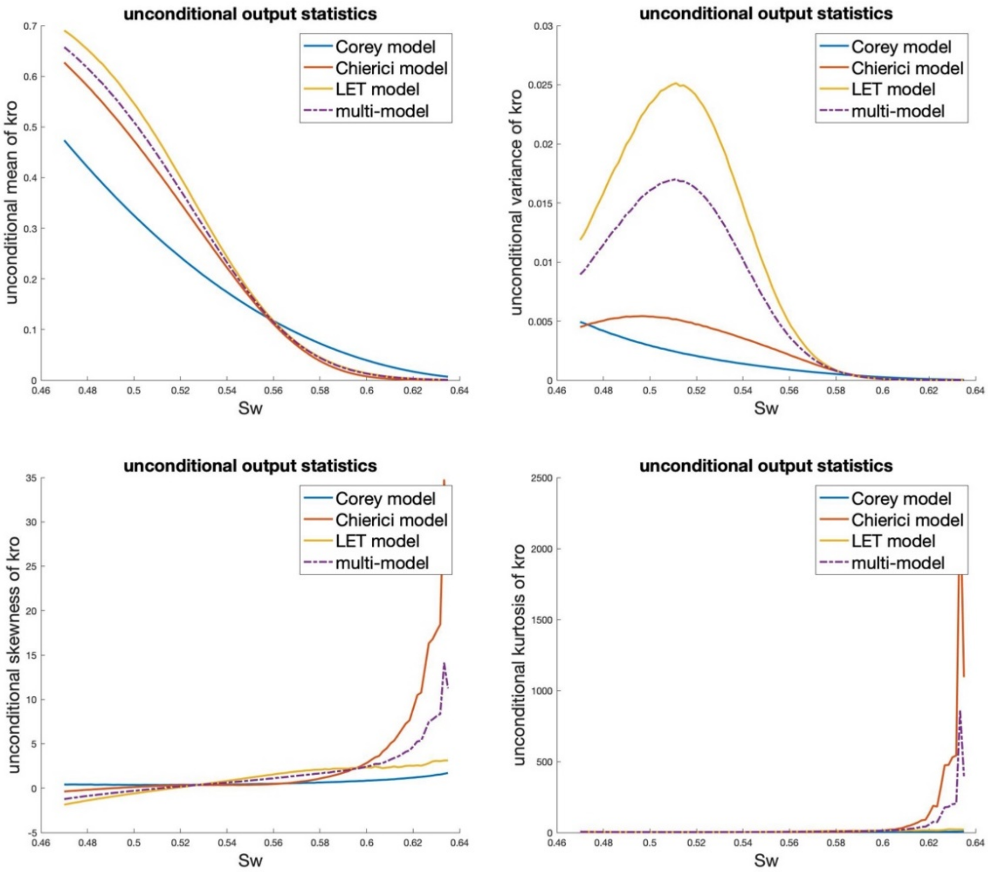

Figure 105: single-model and multi-model unconditional output statistics of the relative permeability models conditional to the “Berea sandstone” scenario for oil relative permeability

In the figures below, the multi-model and single-model statistical moments-based sensitivity indices informed to the “Berea sandstone” sample for oil relative permeability are reported:

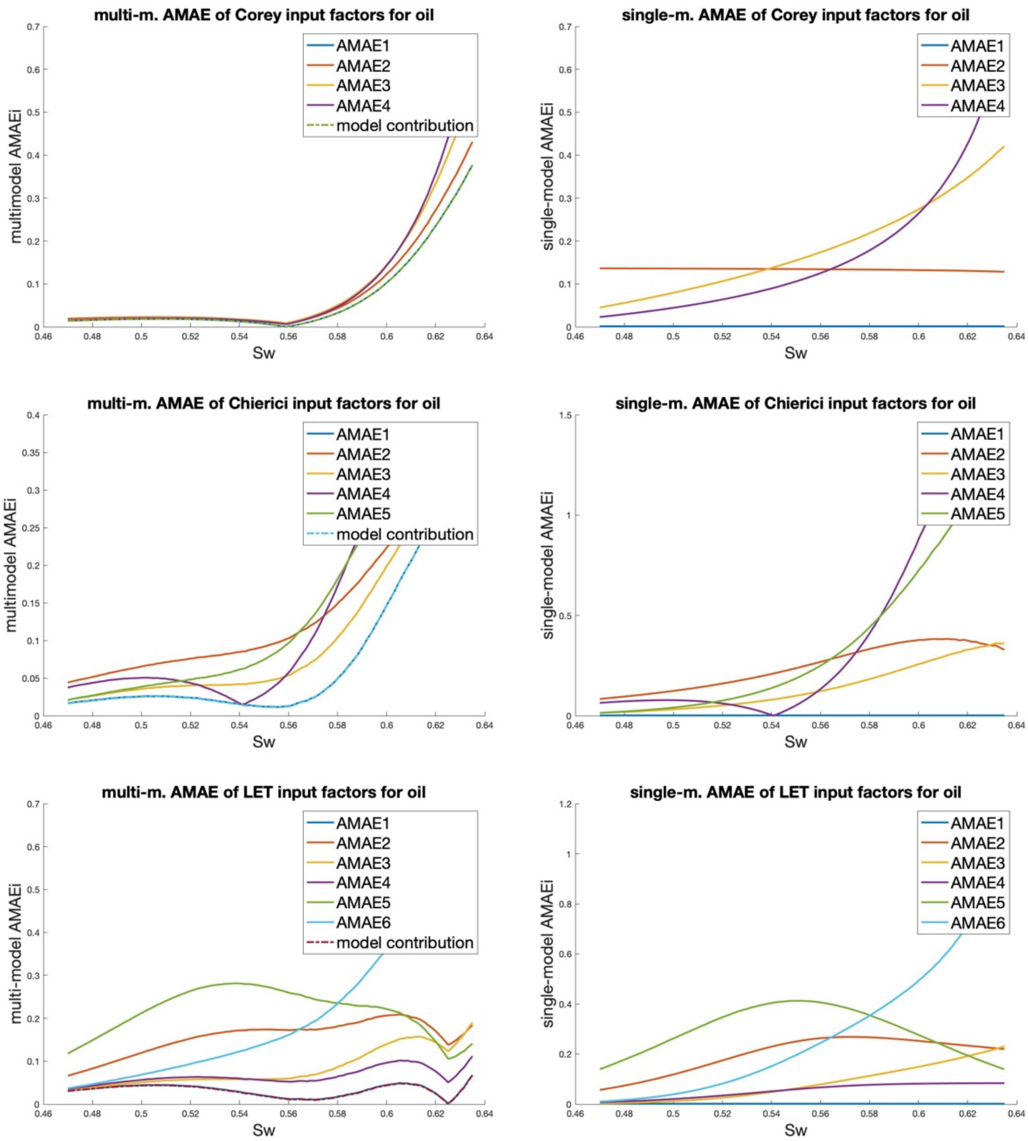

Figure 106: multi-model and single-model informed AMAE sensitivity indices of the “Berea sandstone” sample for oil relative permeability

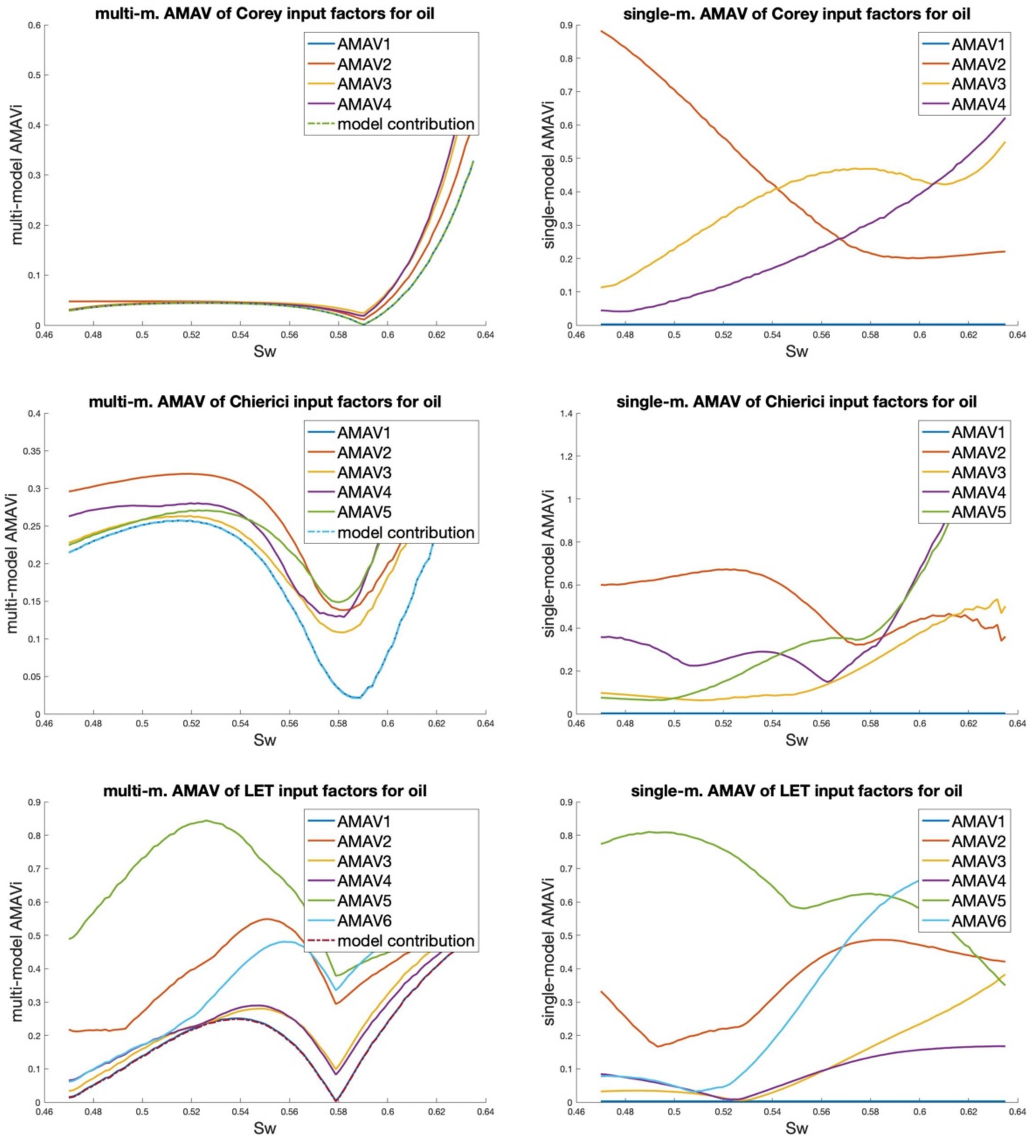

Figure 107: multi-model and single-model informed AMAV sensitivity indices of the "Berea sandstone" sample for oil relative permeability

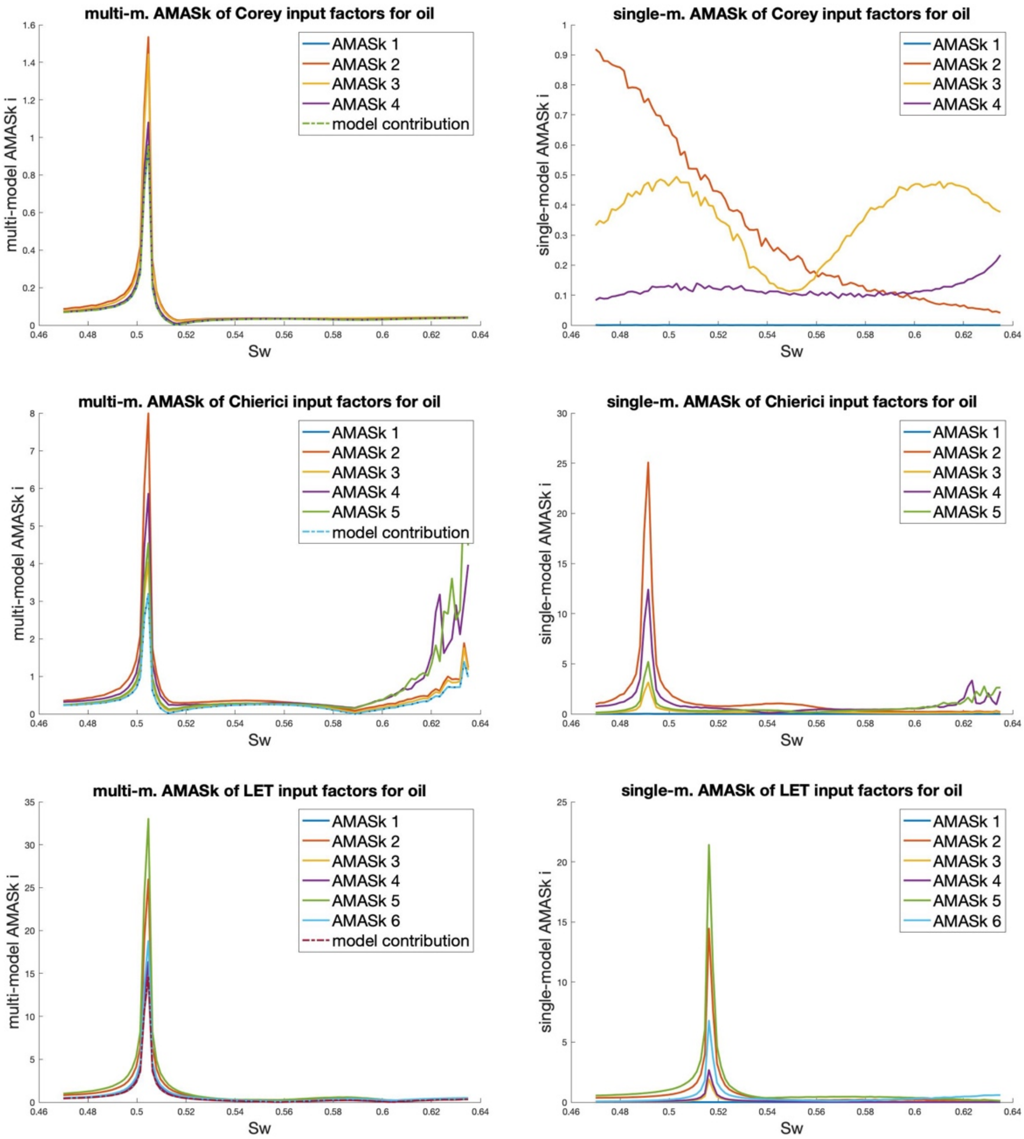

Figure 108: multi-model and single-model informed AMASk sensitivity indices of the "Berea sandstone" sample for oil relative permeability

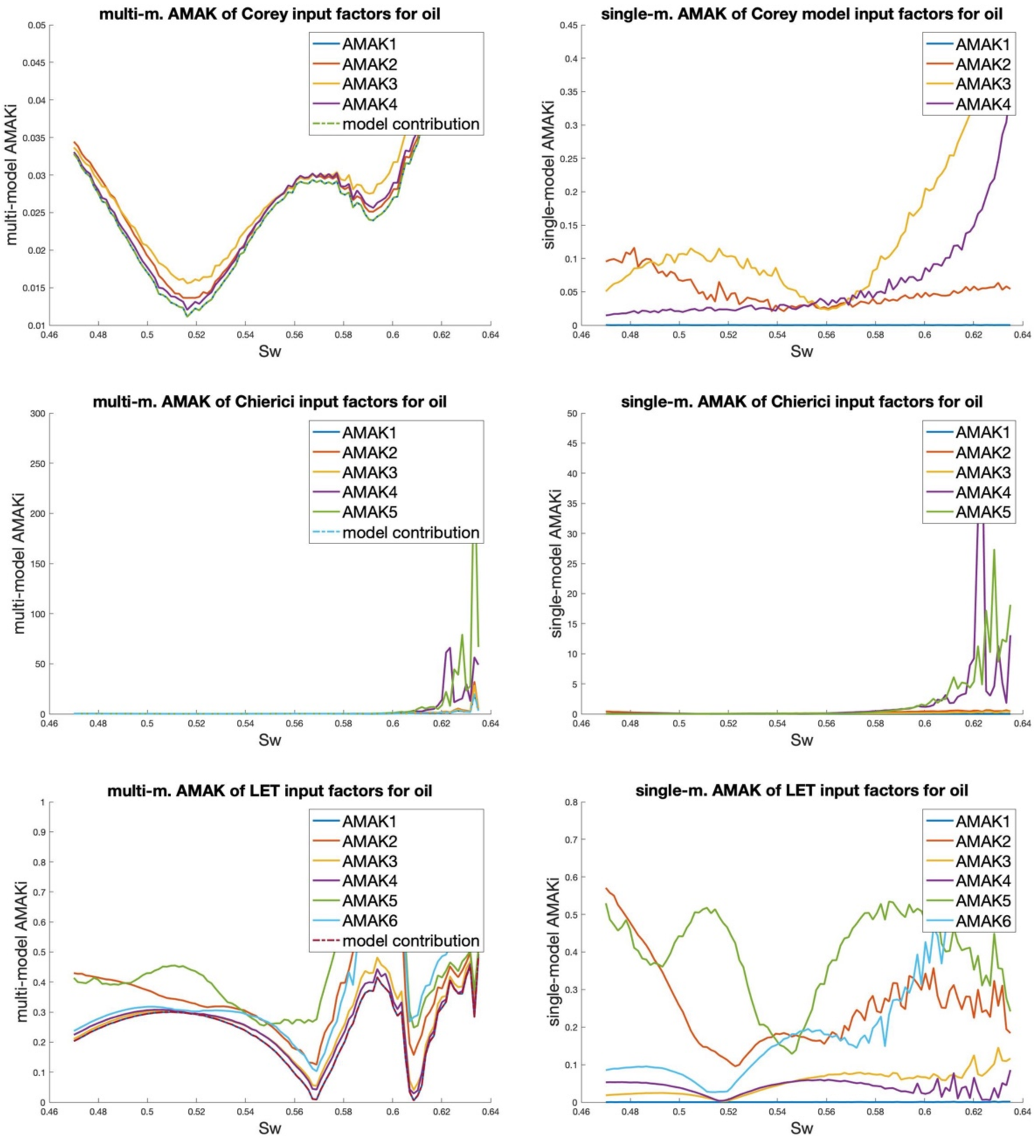

Figure 109: multi-model and single-model informed AMAK sensitivity indices of the "Berea sandstone" sample for oil relative permeability

The ranking of the 15 variable input factors x_i^j (4 belonging to Corey model, 5 to Chierici model and 6 to LET model) is determined calculating and sorting the two ranking indices:

$$R_{AMAE_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAE_i^j$$

$$R_{AMAV_i^j} = \sum_{S_{w,min}}^{S_{w,max}} AMAV_i^j$$

For the multi-model statistical moments-based informed global sensitivity analysis of the "Berea sandstone" sample for oil relative permeability, the resulting rankings are:

Input factors belonging to Corey model
Input factors belonging to Chierici model
Input factors belonging to LET model

Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
$x_5^3: T_o^3$ (21.93)	$x_5^3: T_o^3$ (61.67)
$x_6^3: S_{or}^3$ (20.82)	$x_2^3: S_{wi}^3$ (40.10)
$x_2^3: S_{wi}^3$ (15.93)	$x_6^3: S_{or}^3$ (36.08)
$x_5^2: S_{or}^2$ (14.54)	$x_2^2: S_{wi}^2$ (26.62)
$x_4^2: M_o^2$ (14.43)	$x_4^2: M_o^2$ (25.75)
$x_2^2: S_{wi}^2$ (13.87)	$x_5^2: S_{or}^2$ (25.59)
$x_3^2: B_o^2$ (10.69)	$x_3^3: L_o^3$ (25.02)
$x_4^1: S_{or}^1$ (9.95)	$x_4^3: E_o^3$ (24.69)
$x_3^1: N_o^1$ (9.44)	$x_3^2: B_o^2$ (22.41)
$x_3^3: L_o^3$ (8.30)	$x_1^3: k_{0,ro}^3$ (20.31)
$x_2^1: S_{wi}^1$ (7.98)	$x_1^2: k_{0,ro}^2$ (18.76)
$x_1^2: k_{0,ro}^2$ (7.81)	$x_4^1: S_{or}^1$ (8.77)
$x_4^3: E_o^3$ (6.54)	$x_3^1: N_o^1$ (8.68)
$x_1^3: k_{0,ro}^3$ (3.24)	$x_2^1: S_{wi}^1$ (7.51)
$x_1^1: k_{0,ro}^1$ (0.65)	$x_1^1: k_{0,ro}^1$ (6.09)

D2.2: MULTI-MODEL, INFORMED VARIANCE-BASED SENSITIVITY ANALYSIS

POSTERIOR MODELS PROBABILITIES

- $w^1 = 0.05$
- $w^2 = 0.35$
- $w^1 = 0.6$

VARIABLE INPUT FACTORS:

The models variable input factors are expressed in the form:

$x_i^j = (x_{i,min}^j \, ; \, x_{i,max}^j)$

Where x_i^j represents the i^{th} variable input factor of the j^{th} model, and the values in the brackets define its 95% confidence limits.

- x_1: $k_{ro}^0 = (0.779 \, ; \, 0.783)$
- x_2: $S_{wi} = (0.39 \, ; \, 0.45)$
- x_3: $N_o = (1.8 \, ; \, 2.6)$
- x_4: $B_o = (1.11 \, ; \, 1.44)$
- x_5: $M_o = (0.89 \, ; \, 1.81)$
- x_6: $L_o = (3.08 \, ; \, 3.54)$
- x_7: $E_o = (1.14 \, ; \, 1.59)$
- x_8: $T_o = (0.58 \, ; \, 4.29)$
- x_9: $S_{or} = (0.32 \, ; \, 0.36)$

In the figures below, the single-model and multi-model first order and total order variance-based sensitivity indices informed to the "Berea sandstone" sample for oil relative permeability are reported:

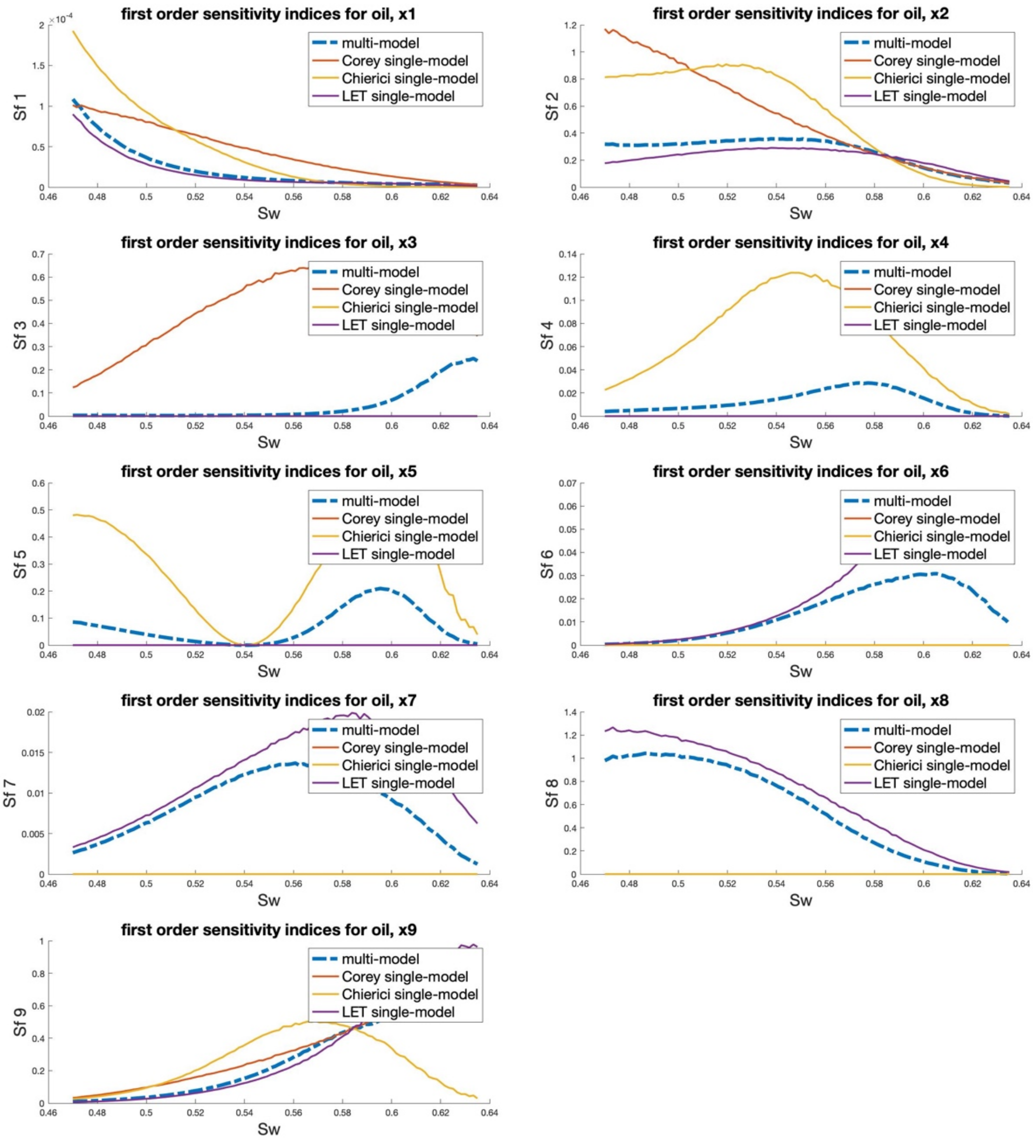

Figure 110: first order single-model and multi-model informed variance-based sensitivity indices of the "Berea sandstone" sample for oil relative permeability

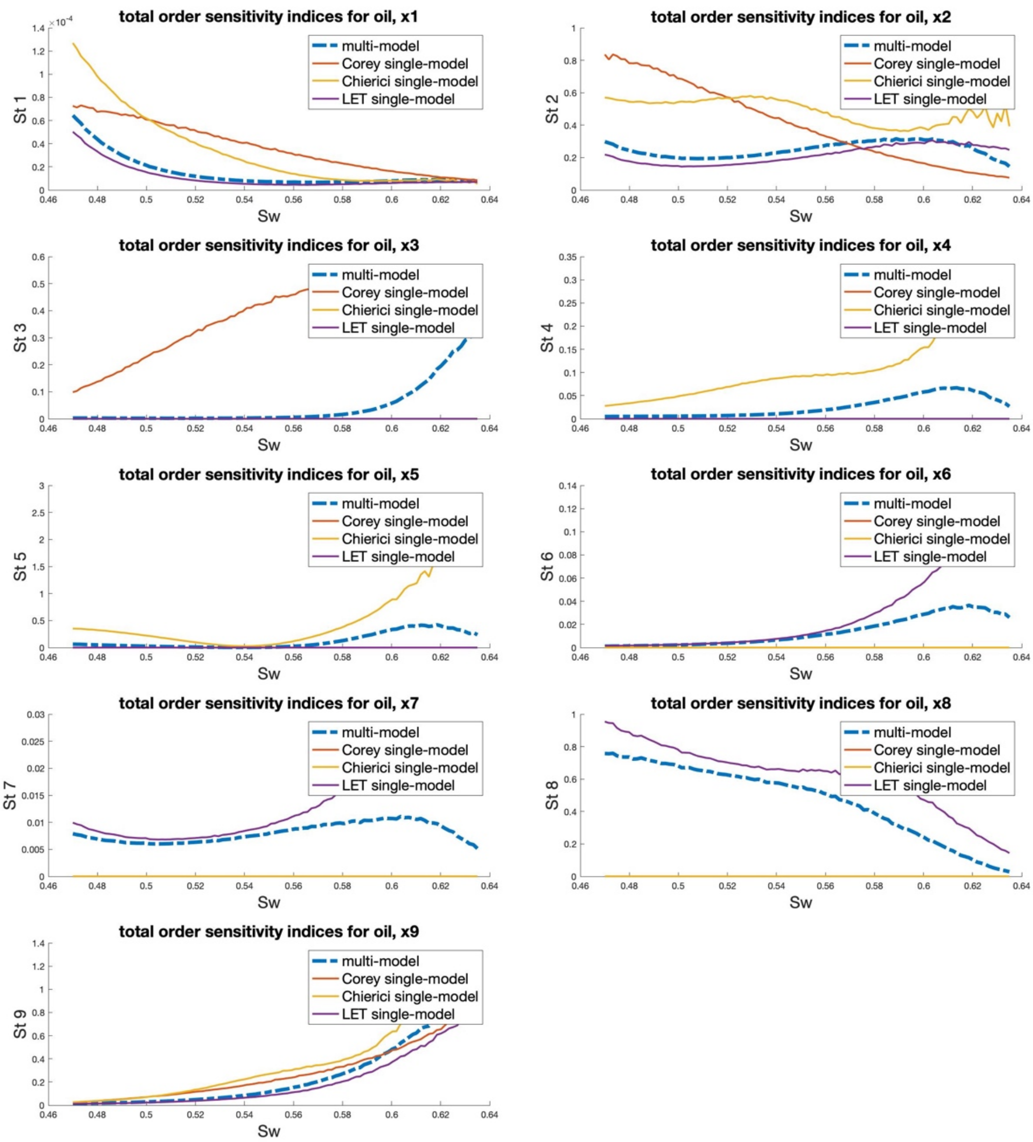

Figure 111: total order single-model and multi-model informed variance-based sensitivity indices of the "Berea sandstone" sample for oil relative permeability

The ranking of the nine variable input factors x_i is determined calculating and sorting the two ranking indices:

$$R_{Sf_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} Sf_i^{multimodel}$$

$$R_{St_i^{multimodel}} = \sum_{S_{w,min}}^{S_{w,max}} St_i^{multimodel}$$

For the variance-based informed global sensitivity analysis of the "Berea sandstone" scenario for oil relative permeability, the resulting rankings are:

Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
x_8: T_o (56.96)	x_8: T_o (47.20)
x_9: S_{or} (29.14)	x_9: S_{or} (25.33)
x_2: S_{wi} (26.19)	x_2: S_{wi} (25.16)
x_5: M_o (6.84)	x_5: M_o (12.53)
x_3: N_o (4.58)	x_3: N_o (4.62)
x_6: L_o (1.45)	x_4: B_o (2.54)
x_4: B_o (1.28)	x_6: L_o (1.40)
x_7: E_o (0.87)	x_7: E_o (0.81)
x_1: $k_{0,ro}$ (0.0021)	x_1: $k_{0,ro}$ (0.0015)

D2.3: RECAP

In the table below the rankings obtained from the different informed multi-model sensitivity analysis techniques, when applied to the “Berea sandstone” sample for oil relative permeability, are summarized:

Multi-model informed statistical moments-based sensitivity analysis of the “Berea sandstone” sample for oil relative permeability	
Ranking according to $R_{AMAE_i^j}$	Ranking according to $R_{AMAV_i^j}$
$x_5^3: T_o^3$ (21.93)	$x_5^3: T_o^3$ (61.67)
$x_6^3: S_{or}^3$ (20.82)	$x_2^3: S_{wi}^3$ (40.10)
$x_2^3: S_{wi}^3$ (15.93)	$x_6^3: S_{or}^3$ (36.08)
$x_5^2: S_{or}^2$ (14.54)	$x_2^2: S_{wi}^2$ (26.62)
$x_4^2: M_o^2$ (14.43)	$x_4^2: M_o^2$ (25.75)
$x_2^2: S_{wi}^2$ (13.87)	$x_5^2: S_{or}^2$ (25.59)
$x_3^2: B_o^2$ (10.69)	$x_3^3: L_o^3$ (25.02)
$x_4^1: S_{or}^1$ (9.95)	$x_4^3: E_o^3$ (24.69)
$x_3^1: N_o^1$ (9.44)	$x_3^2: B_o^2$ (22.41)
$x_3^3: L_o^3$ (8.30)	$x_1^3: k_{0,ro}^3$ (20.31)
$x_2^1: S_{wi}^1$ (7.98)	$x_1^2: k_{0,ro}^2$ (18.76)
$x_1^2: k_{0,ro}^2$ (7.81)	$x_4^1: S_{or}^1$ (8.77)
$x_4^3: E_o^3$ (6.54)	$x_3^1: N_o^1$ (8.68)
$x_1^3: k_{0,ro}^3$ (3.24)	$x_2^1: S_{wi}^1$ (7.51)
$x_1^1: k_{0,ro}^1$ (0.65)	$x_1^1: k_{0,ro}^1$ (6.09)

Multi-model informed variance-based sensitivity analysis of the “Berea sandstone” sample for oil relative permeability	
Ranking according to $R_{Sf_i^{multimodel}}$	Ranking according to $R_{St_i^{multimodel}}$
$x_8: T_o$ (56.96)	$x_8: T_o$ (47.20)
$x_9: S_{or}$ (29.14)	$x_9: S_{or}$ (25.33)
$x_2: S_{wi}$ (26.19)	$x_2: S_{wi}$ (25.16)
$x_5: M_o$ (6.84)	$x_5: M_o$ (12.53)
$x_3: N_o$ (4.58)	$x_3: N_o$ (4.62)
$x_6: L_o$ (1.45)	$x_4: B_o$ (2.54)
$x_4: B_o$ (1.28)	$x_6: L_o$ (1.40)
$x_7: E_o$ (0.87)	$x_7: E_o$ (0.81)
$x_1: k_{0,ro}$ (0.0021)	$x_1: k_{0,ro}$ (0.0015)

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