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EXECUTIVE SUMMARY OF THE THESIS

Contracts as Coordination Mechanism between Brands and Retailers: The Mid-Season Reorder Model

TESI MAGISTRALE IN MANAGEMENT ENGINEERING – INGEGNERIA GESTIONALE

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1. Introduction

The **role of contractual design** in demand management in the high-end Jewellery industry warrants further investigation. In this industry, contracts must be drafted with accuracy and clarity because they have the potential to significantly influence supply chain effectiveness and direction. Contracts that clearly define roles, deadlines, and technical requirements encourage greater consistency and cohesiveness among the different parties involved, which lowers the possibility of opportunistic behaviour and operational inefficiencies.

To categorize contracts and create an environment where the parties are motivated to put long-term common benefits ahead of individual short-term interests, it is essential to analyse **contractual incentives to transfer payment**. The industry under review places a high value on quality and service, and this approach helps to maintain those standards while reducing the possibility of conflicts of interest.

Sharing inventory risk is a significant challenge, particularly considering products' high value and vulnerability to unpredictable demand swings. An effective way to lessen the possible damaging effects of overstocking or a scarcity of products on the market is to share it through the adoption of particular contractual clauses. This strategy is especially helpful in guaranteeing a more accurate match between supply and demand, helping in the reduction of waste and expenses related to sub-optimal inventory management.

There is a discernible trend among businesses to **outsource their manufacturing procedures**. It is clear that this strategy has some serious weaknesses, including the loss of control over the operational stages of production and retail distribution, which could lead to a decline in brand image dominance, even though it may initially bring benefits like lower production costs and market expansion. In an environment where brand image and exclusivity are critical, this kind of control loss can have detrimental effects that are not negligible. Many businesses in the industry are revaluating the **re-insourcing of production activities** because of the issues mentioned above. This reversal brings to light the intricate complexity of demand management within this industry.

In the high-end jewellery industry, **ineffective demand management** can have a significant impact on several key parameters. These include the capacity to adjust to changes in consumer demand, maintaining the uniqueness of the brand, reducing inventory management expenses, and guaranteeing product availability based on the time and location preferences of customers. The adoption of coordination contracts is an effective approach to addressing these challenges and establishing a supply chain management system based on fair cooperation and interest alignment. To ensure that coordination contracts meet the unique requirements of the luxury Jewellery market, they must be carefully crafted.

2. Objectives and Methodology

2.1 Research Objectives

Recent research has concentrated on analysing existing contractual forms and developing new contractual models to improve their efficiency. Nonetheless, a significant issue that has surfaced from this examination is the absence of a methodical strategy intended to combine several contractual forms to create a hybrid contractual structure that can take advantage of their inherent benefits. Moreover, the incapacity of current contracts to completely satisfy the requirements of distinct parties is another issue mentioned. This is because the implemented contract often does not match the industry and the participating companies' characteristics. These drawbacks open a significant avenue for further investigation.

The purpose of the **present study** is to introduce a novel contractual model in this context, thereby representing a substantial advancement in the contractual dynamics' optimization. The possibility of a mid-season reorder is introduced by the new model. This strategy might capitalize on the unquestionable advantages of equitable benefit distribution among participants while also ensuring brand management through the buyback of unsold products. The combination of these characteristics may lead to improved goal alignment and efficiency, as well as more effective handling of the difficulties arising from fluctuating demand and the unique peculiarities of luxury Jewellery manufacturing.

2.2 Research Questions

Four research questions will be answered along the dissertation:

1. *What are the predominant challenges and peculiarities associated with Demand Management (DM)? How can a mid-season product reorder model in the luxury Jewellery sector impact the resolution of these challenges? In this regard, are the dynamics of the new model more efficient compared to those of conventional procurement models?*
2. *How can game theory be applied to optimise contracts, taking into account variables such as price, quantity, and timing of orders?*
3. *How can a new contractual model maximise profits for both sides of the supply chain, considering the unique dynamics of the industry under consideration?*
4. *What could be the possible application scenarios and how could these be affected? Which stochastic and non-stochastic variables need to be considered besides seasonal changes, demand fluctuations and price variations?*

2.3 Research Methodology

We started our work by reviewing the **existing literature** to thoroughly analyse a sizable sample of studies that have been published since 2001 and nowadays. In terms of contractual agreements, this research assisted in identifying trends, best practices, and recurring problems in the luxury Jewellery sector. The examined publications involve a wide range of methods, including mathematical programming techniques and game theory, providing a variety of approaches to handling contractual and management concerns. In particular, the theoretical frameworks underpinning our research, on which the tools are used to develop the discussion, are the publications Cachon and Lariviere (2005) and Brun and Moretto (2012).

We selected the source of origin and keywords as our two selection criteria when looking for these articles.

The **research journals** that have been chosen are in the fields of operations research and management science (OR/MS/OM). These include INFORMS journals like Management Science, Operations Research, Manufacturing and Service Operations Management, Interfaces, Information Systems Research, Marketing Science, Service Science, Transportation Science, Mathematics of Operations Research, and INFORMS Journal of Computing. Our decision to concentrate on these publications is a sign of our concern for reliable and high-quality sources. This lends our work a high degree of credibility because their publications typically undergo thorough peer assessment. This variety of sources adds to our grasp of industry dynamics and potential contractual solutions, making it more comprehensive and in-depth.

By applying terms such as "fashion", "supply chain", "retail", "game theory", "luxury", and "contract", it was possible to perform a **targeted search** in the selection of publications, thus focusing only on contributions relevant to our field of study. Without the use of targeted keywords, we could have been overwhelmed by an excessive number of articles with the possibility that some of them were not relevant.

The chosen articles were then divided into **three macro-categories** based on the authors' objectives and the proposed material, which proved to be a particularly useful search technique:

- Articles related to the luxury industry;
- Articles on the several types of contracts currently in use;
- Articles on the employment of game theory in supply chain management.

This allowed for a clearer, more cohesive arrangement and organization of the literature review.

The new contract form was developed with careful consideration of **key variables for optimization**. Specifically, the time of reorder was included, along with the number of items the retailer ordered. This constitutes a substantial paradigm shift in comparison to previous studies. The introduction of models that are more closely aligned with the complexity of the high-end Jewellery industry results in better arrangement with demand fluctuations and contributes to a significant reduction in risk and cost.

With the help of **concepts from game theory**, we were able to develop a set of scenarios meant to test our model in various settings and determine whether it would be especially effective.

We then used Microsoft Excel software to generate an elaborate **simulation** to assess the usefulness and efficacy of our model and to gain a deeper understanding of the outcomes. The purpose of this simulation was to produce accurate data for one hundred distinct articles during a century-long observation period. A demand distribution with a mean of 100 and a standard deviation of 10 was assumed for every article. To differentiate between products with high and low demand, a threshold was established below which it would not be prudent for the retailer to incur the risk of placing a new large order, and a probability of this event occurring was linked to it. All the scenarios that were considered during the model development phase are covered by the simulation. Within this framework, we meticulously measured the outcomes derived from our model with what a hypothetical retailer or producer could have accomplished by utilizing the current models within the industry in question. This approach has several inherent benefits:

– The simulation provides realistic and reliable data that support or refute the theoretical claims made previously;
– It allows to evaluate the new model in various scenarios under controlled conditions, reducing risks and costly errors;
– Its scalability and repeatability make it easy to explore further scenarios or evaluate the effectiveness of strategies over time;
– It identifies areas where our model can be strengthened.

This work provides solid empirical basis for supply chain-related business decisions in the future.

3. Structure of the content

The **first chapter** opens with a foreword on the current state of the luxury industry, focusing on its historical evolution and future perspectives. In particular, it emerges that the sector under study is experiencing a period of considerable expansion. Increased demand from emerging Countries, technological advances and growing environmental awareness are identified as some of the main trends influencing the luxury Jewellery market. Although some challenges may arise, the outlook for this industry looks promising.

The **second chapter** provides a detailed summary of the potential problems within the area of demand management. Demand management for a luxury product involves complexities related both to the intrinsic characteristics of the product and to the external context of the market in which it is sold. The most recurring problems concern product customisation, the management of returns and repairs, disputes and conflicts, adaptation to changing consumer tastes, regulatory fluctuations and the specific nature of the items being traded (impulse purchases, volatility, and unpredictability of demand, ...). A further variable of relevance is the distinction between fashion items and carry-over items, as they require a differentiated approach. The resolution of these issues is identified in the correct formulation of a contract between producers and retailers. Through this practice, they can adapt to the needs of discerning consumers, ensuring that the Critical Success Factors (CSF) peculiar to luxury products are met. These include high-quality standards, the legacy of skilled artisanry, product exclusivity, emotional appeal, brand identification, distinctive style and design, belonging to a nation renowned for excellence, uniqueness, and the promotion of a distinctive lifestyle. This chapter provides the basis for the formulation of the first research question (*What are the predominant challenges and peculiarities associated with Demand Management (DM)? How can a mid-season product reorder model in the luxury jewellery sector impact the resolution of these challenges? In this regard, are the dynamics of the new model more efficient compared to those of conventional procurement models?*).

The subsequent three chapters offer a review of the contemporary literature pertaining to the topic under consideration.

The **third chapter** provides an overview of the several types of contracts in force and the criteria for their selection.

CONTRACT	CHARACTERISTICS
Wholesale price	At a fixed wholesale price, the producer sells the goods to the retailer, who then marks them up and sells them to end users
Buyback	The retailer pays a set price per unit purchased and the producer commits to buyback any unsold goods from the retailer at a set price and within a given time frame
Revenue-sharing	This is a sort of contract where the parties concur to split a portion of the profits made from a particular commercial activity
Quantity flexibility	The retailer and the producer determine an initial number of items to be supplied but also a range within which the quantity may be altered in response to predetermined circumstances. The producer is required to pay the retailer back for any unsold units that are higher than a predefined threshold at the same fixed price per unit that the retailer paid
Capacity reservation	A producer grants the retailer access to a specific quantity of products for a defined period. If: - The total number of units ordered is less than the agreed quantity, he must pay a late fee or the full amount

	- The total number of units ordered is higher than the agreed quantity, additional units are offered at a higher price
Risk based contract	It allocates risks and responsibilities based on a proper assessment of potential uncertainties and adverse events that could impact the collaboration
Linear cost sharing	It aims to jointly shoulder the costs of a project in direct proportion to their respective individual shares
Fixed rate cost sharing	The parties to this agreement decide on a predetermined fixed fee via which they will equally divide the expenditures incurred during the collaborative endeavour
Option contract	A retailer and a producer enter an arrangement whereby the retail business pays the producer a fee in exchange for having the only right to buy a certain amount of goods at a predetermined price within a predetermined time frame

Table 3.1: Comprehensive survey of the in force contractual mechanisms for coordinating actors in supply chain management literature

Evaluation criteria requiring appropriate consideration for the implementation of a specific contract include:

- Administrative costs;
- The impact on supply chain coordination in terms of efficiency in ensuring that each participant does not deviate from optimal decisions for the entire supply chain;
- The sharing of risks and rewards.

In the **fourth chapter**, a detailed analysis is conducted of some of the previously mentioned contracts, which have recently gained notoriety in the sector of interest. Special attention is paid to the investigation of their strengths, relative weaknesses, application scenarios and future prospects. In summary, the following emerges:

- Buyback contracts demonstrate superiority over wholesale price contracts;
- Revenue-sharing contracts provide a solution to the challenges of coordinating supply chains, ensuring that production and ordering decisions are in line with optimal decisions. Moreover, these contracts prove to solve problems that buyback contracts cannot coordinate, as they are independent of the retail price. However, it is important to note that they have limitations, including high administrative costs, considerable effort required of the retailer, the possibility of moral hazard, and the challenges of appropriate quota selection for profit sharing.

Although many of the current studies are predominantly theoretical in nature, it is possible to draw the conclusion that these contract types exhibit characteristics that are particularly aligned with the dynamics of the industry under review.

The **fifth chapter** outlines an analysis of game theory, recognized as a powerful analytical tool for negotiation. In this section, key concepts relevant to the investigation, such as strategy, objective function, Pareto optimality, Nash Equilibrium, and possible modes of cooperation between two actors, are examined. The previously presented contractual forms undergo a revision through this new analytical perspective, which has identified multiple advantages, such as intrinsic flexibility and effectiveness in coordinating dynamics while simultaneously achieving Pareto improvements. This explains the widespread adoption of such contracts in the contemporary business landscape. However, unresolved challenges persist in their practical implementation and their limitation in meeting the individual interests of each member. This chapter has laid the groundwork for the second research question (*How can game theory be applied to optimize contracts, taking into account variables such as price, quantity, and timing of orders?).*

The **sixth chapter** outlines the objectives, research methodology, literature gaps, and research questions that this document aims to address. This chapter highlights the challenges and opportunities that this field of research can offer. Moreover, it stands as the most pivotal chapter of the entire document, elucidating the pursued procedure and serving as the guiding thread that binds the entire work together.

In the **seventh chapter**, the development of the new hybrid contract form, based on the possibility of placing a second order mid-season, is

expounded. This innovative model supports improved order planning, ensuring that the company can adapt to changing market conditions and consequently maximize overall profit. Specifically, the assumptions underlying the model and the profit optimization formulas for both parties involved in the contract are presented in detail. The use of game theory has allowed for a more accurate evaluation of the choices and strategies available in this context, highlighting the potential implications of implementing the new model compared to the Newsvendor model.
While the Newsvendor model remains an effective tool for inventory management, its intrinsic limitations stemming from rigid conditions may not always be realistic or suitable for a dynamic and evolving business environment. This underscores the need to introduce more flexible inventory management models. This chapter answers to the third research question (*How can a new contractual model maximise profits for both sides of the supply chain, considering the unique dynamics of the industry under consideration?*).

Chapter eight examines the hypotheses previously formulated through the implementation of a simulation conducted using the Microsoft Excel tool. The empirical assessment of the real validity and effectiveness of the proposed model is carried out through the adoption of three distinct approaches:

- The first step carries out a preliminary comparative analysis of the financial performance of the proposed new model and the Newsvendor model to draw a broad picture of its overall cost-effectiveness. This section contains two different versions of the Mid-Season Reorder model, depending on when the retailer decides to place a new order.
- The second approach consists of presenting a summary of the potential financial results that the parties could achieve through the application of one of the two contractual models. This analysis is conducted in relation to specific scenarios and assumptions defined in the context of game theory, discussed in depth in the preceding chapter;
- The last approach aims to assess the impact of the model in terms of performance improvement when varying spread values between high and low demand products.

This chapter reveals valid and interesting results in response to the last research question (*What could be the possible application scenarios and how could these be affected? Which stochastic and non-stochastic variables need to be considered besides seasonal changes, demand fluctuations and price variations?*).

The **concluding chapter** outlines an exhaustive discussion of the results obtained, providing an analysis of the answers to the research questions, as well as an assessment of the implications and limitations inherent in the study. These considerations offer stimulating insights for potential future developments.

The chapter concludes with a list of Bibliographical references and a detailed Appendix documenting the commands performed and providing further background on the simulation conducted.

4. Mid-Season Reorder Model

Consider a two-part supply chain. The entire time horizon is divided into two distinct but interconnected periods. The producer must determine the production quantity for each period to maximize its overall profitability, and the retailer must decide the order quantity for each period to maximize his total expected profit. In this perspective, a significant portion of decision-making power lies in the hands of the retailer.

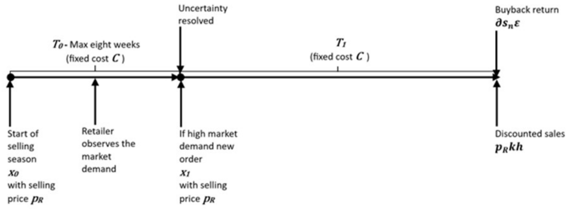

Figure 4.1: Graphical representation of the events occurring within the selling season in the Mid-Season Reorder model

After observing the stochastic demand in the period T_0, the retailer can deliberate on whether to place a further order based on actual demand trends. This multi-period problem is formulated as an inventory game between producer and retailer, with the possibility of deriving optimal decision policies for both parties and demonstrating the existence of a Nash equilibrium.

The retailer can structure its supply management as follows:

- At the first moment of purchasing goods (t_0), the retailer adheres to the Newsvendor model approach[1], covering the demand for items for the entire selling season. This satisfies the demand for low-demand products during the season and temporarily fulfils the demand for products with a higher demand;
- In the second period (t_1), the retailer can purchase another percentage of products, but only for those for which he has concrete evidence of strong market demand.

At the end of the last period, part of the inventories held by the retailer can be sold at a discount or returned to the producer (only those items purchased in the second period benefit from the buyback option).

It is expected that products with low market demand will follow the logic of the Newsvendor model, as the application of this model is sufficient to optimise the supply chain and maximise profits for these items. For products with high market demand, this new model is superior, thereby allowing both parties to better manage inventory.

To ensure product availability for the retailer during period T_1, the retailer has the option, upon payment of a fee, to reserve a maximum quantity of a specific product portfolio at the time of purchase in T_0, without the obligation to purchase the entire reserved quantity thereafter.

4.1. Problem Formulation

With the aim of providing a comprehensive understanding of the model, it is outlined some key assumptions for the context:

- It is considered two distinct product types within the supply chain: high and low demand products. They are procured independently of each other;
- The time frame being examined is divided into two successive periods, T_0 and T_1, which are intricately connected to each other;
- The distribution of these products is handled by a single distributor;
- The mean of the demand for the products is known and provided as input for each individual period. Across all individual periods within the entire time horizon, the demand can either remain constant or fluctuate;
- Assuming the retailer has a storage space large enough to accommodate the inventory for the entire selling season.

Additional assumptions within the model framework include:

- The possibility of stock-outs from previous periods;
- Prompt activation of the order and subsequent order fulfilment at the conclusion of the eight-week interval from the commencement of the sales season;
- The initial inventory is set at 0;
- At the end of the selling season the leftovers (R_n) are equal to 0;
- The selling season begins at t_0, when a certain quantity of 'x_0', purchased by the retailer in anticipation of the season's beginning, starts to be sold. The quantity 'x_0' has been chosen in accordance with the sales forecast for the entire selling season.

To make the understanding of the model clearer, a table summarizing the used symbology is provided below.

SYMBOL	DEFINITION
i	Range of products with high market demand
j	Range of products with low market demand

[1] With the exception of the case of products with a high demand.

T_0	Eight-week period from the beginning of the selling season to the end of the interval to carry out reordering
T_1	Period of 12 weeks starting from the end of the reorder interval and ending with the end of the selling season
t_0	Commencement of period T_0, corresponding to the start of the new selling season for products of both types i and j
t_1	Time of beginning of the period T_1
t_n	Moment that marks the end of the selling season
x_T	Quantity of products purchased by the retailer for the period T
b	Demand forecast referring to all selling season
b_T	Demand forecast for the period T
p_P	Procurement price set by the producer
p_R	Selling price set by the retailer
L	Storage costs (as a percentage) incurred by the retailer
s_t	Inventories present in the warehouse at the moment t subjected to buyback
s_n	Inventories at the end of the selling season subjected to buyback
k_T	Inventories at the end of period T not subjected to buyback
$R_{t,T}$	End-of-period inventory for period T, calculated at time t, encompassing both types of stocks present at that moment for that specific period
π	Profit functions
M	Quantity locked in t_0 for possible later reorder, i.e., maximum quantity that can be ordered by the retailer in t_1
μ	Binary variable indicating the occurrence of a second order
h	Percentage discount applied at the end of the selling season to all products for which the buyback option is not available
ε	Mutually agreed-upon value between the retailer and the producer for the return of unsold goods at the end of the season
∂	Percentage of product s_n, for which the buyback option is available, that the producer reclaims at the season's end
w	Percentage reduction from the selling price for the goods reordered at the beginning of the period T_1
F	Guaranteed fee to reserve M for the following period
Δk	Number of pieces of k_0 sold in the period T_1
C	Unitary costs incurred by the producer regarding all the selling period
U	Unitary revenue for the producer associated with the disposal of the unsold product units at the end of the selling season

Table 4.1: Notation system for Mid-Season Reorder model analysis

To calculate the profits of both parties involved in the upcoming contract, it is essential to highlight the various sources of revenues and cost items involved.

$$
\begin{aligned}
Maximise\ \pi_R = {} & p_R(x_0 + x_1\mu - R) + \partial s_n \varepsilon \\
& + p_R(1-h)[k_1 + (1-\partial)s_n] \\
& + \frac{F}{M}x_1\mu - [x_0 p_P + x_1\mu(1-w)p_P \\
& + F + Lp_P(x_o + \mu x_1)]
\end{aligned}
$$

Equation 4.1

Concerning the retailer, the revenue sources considered include sales during the high season, sales regulated by the repurchase agreement, discounted sales of unsold goods at the end of the season and the discount applied to the total amount of the second order. An interesting factor in this formula is the value that 'w' takes as it reflects the importance of the reorder time.

Regarding the costs to be borne by the retailer, it is important to keep in mind:

- The costs incurred by the retailer in the process of acquiring goods, services, or works from external producers. These costs include, among other things, shipping and transport costs;
- The costs incurred for the purchase of goods from the producer;
- The charge that ensures the availability of a specific quantity of goods 'M' until the end of the replenishment interval, which extends over eight weeks;

– The costs associated with storing the goods in the warehouse.

Now, the same approach is applied to the producer.

$$
\begin{aligned}
Maximise\ \pi_P = Total\ revenues_P - Total\ costs_P \\
= [x_0 + x_1\mu(1-w)]p_P + F \\
+ U(M - x_1 + \partial s_n) \\
- C(x_0 + max\{M, x_1\}) - \frac{Fx_1}{M}
\end{aligned}
$$

Equation 4.2

The revenue components contributing to the producer's profit calculation include proceeds from the sale of goods to the retailer, the deposit amount made by the retailer as a guarantee for the availability of merchandise 'M' for the T_1 period, from which a portion will be deducted and subsequently returned to the retailer, and the income generated by the disposal of merchandise at the end of the season.
In terms of costs, it is necessary to consider production costs and expenses associated with product storage.

4.2. Game Theory Application

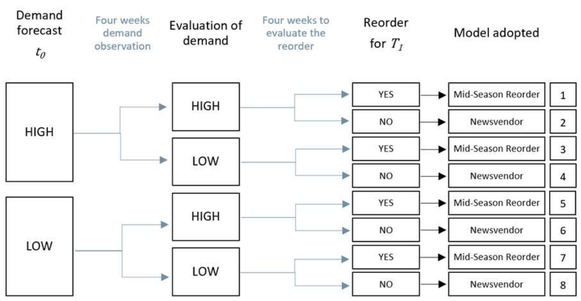

Table 4.2: Game theoretic scenarios for the Mid-Season Reorder model

Table 4.2 illustrates the retailer's decision-making process during the evolution of the selling season and its impact on the agents' objective function, with reference to the implemented model. The objective of this analysis is to determine the combination of decisions that maximizes profits for both parties involved. The outputs of this representation correspond to the producer's and retailer's profits in each of the eight scenarios developed.

In the case where the retailer does not place a new order, he automatically follows the Newsvendor model. Therefore, the newly developed model is not involved in the profit calculation. It is crucial to consider the impact of this choice, especially in relation to the type of product in question, as it may give rise to costs due to overstocking or loss of potential sales. Such inefficiencies affect the supply chain and have implications for end-consumer service and the objective functions of the parties involved.

The two emerging Pareto-optimal solutions are the *"high demand - high demand - yes reorder"* and *"low demand - low demand - no reorder"* combinations. Products with low demand do not require a second replenishment because the quantity required by the retailer is limited, making the Newsvendor model appropriate and efficient for managing this product category. In contrast, for products with high market demand, using the new model is more advantageous for both producer and retailer. This dynamic can translate into competitive advantages for both parties over market competitors. For example, they could enable better customer service, avoid out-of-stock situations, improve delivery times, and optimize overall costs. This underscores the importance of targeting the choice between the Newsvendor model and the new model based on the nature of demand and specific product characteristics.

4.3. The Simulation

In the first stage of the simulation experiment, the demand levels for each of the 100 items included in the simulation are randomly generated by the software. This casual generation is conducted on the basis of statistical parameters, including mean and standard deviation. An average of 100 products and a standard deviation of 10 products are used. It is important to note that the casual generation process is carefully designed to ensure that demand can never take on negative values. Each randomly generated demand scenario is assigned its associated probability of occurrence.

The algorithm at the heart of this simulation takes inspiration from game theory, which means that it is designed to represent and evaluate the decisions of the various actors involved in the supply chain according to principles of rationality and strategy.

The 100-year simulation period is significant because it allows one to observe and evaluate financial performance over the long term.
The main objective of simulation is to observe and compare financial performance. Analysing this financial performance helps evaluate the effectiveness of supply chain management strategies and identify best practices.

A crucial element of the simulation is the creation of different scenarios based on the desired spread percentages within a given product range. This concept is important because it emphasises the heterogeneity of demand, which can differ considerably across products.

However, it is important to emphasise that the luxury referred to is not too extreme but accessible to a considerable part of the population. This type of segmentation may be important to understand the target market and the positioning of the retailer.

The initial order calculation for the entire season is a process that seeks to strike a balance between satisfying customer demand for the entire season and managing the risks associated with fluctuations in demand. The initial order is made at the beginning of the selling season must be sufficient to cover the expected demand for the entire season, with the exception of the case of products with a high demand, because in such a case the quantity x_0 is given by the expected demand for the T_0 period only plus a number of standard deviations equal to the 10% of the inverse normal distribution of the critical ratio value. In all the other cases, the initial order is equal to the entire forecast for the selling season plus a number of standard deviations equal to the inverse normal distribution of the critical ratio value. This addition to the demand forecast mitigates the risk of out-of-stock due to unforeseen demand variability.

Classifying demand into high or low is an important step in inventory management and sales activity planning. To do this, it is necessary to define a threshold or cut-off point that allows decisions to be made based on the amount of demand.

The simulation has been organized into several mirrors. Each annual mirror is divided into three distinct parts, each representing a specific time in the selling season. The first part constitutes a starting point where decisions must be made based on historical estimates and forecasts, since current data may not yet be available. Subsequent parts refer to later moments in the selling season, which allow planning to be adjusted and updated based on changing market conditions and actual data as they become available during the selling season.
Decisions made during this period are better informed than decisions made at the beginning of the season, and this is also reflected in more targeted strategy adoption.

The process for calculating the retailer's and producer's profit applies the formulas derived from the two models, the Newsvendor model and the Mid-Season Reorder model. These expressions consider various parameters analysed in detail in Chapter 8 of the dissertation. The calculation is performed separately for the retailer and the producer. After calculating the individual profits for the retailer and producer in each simulation period for 100 items, the total supply chain profit was calculated by summing up the profits of the two agents. Using this approach, it is possible to examine how the decisions of the individual agents influence the overall results of the supply chain in different situations.

The initial stage of analysis of the models considered offers a key opportunity to explore and compare the financial implications of the strategic choices made by the retailer, and consequently by the producer and the entire supply chain. To ensure a fair comparison between the three models, it was essential to establish the same initial value for the random variable in the three contexts.

	Advanced Mid-Season[2]	Newsvendor	Mid-Season
Average producer	17.925 €	16.441 €	13.096 €
Average retailer	43.699 €	33.905 €	35.092 €
Average margin	43 %	43 %	45 %

Table 4.3: Summary of the three approaches comparison

The financial performance resulting from the implementation of the Advanced Mid-Season Reorder model significantly exceeds that of the other models, highlighting benefits for all the agents. This result underlines the inherent effectiveness of the model in optimally managing a diverse range of market conditions, giving this approach an edge in supply chain management strategies.

For the second analysis, an algorithm was implemented to select the most cost-effective model, given certain input values.
In the first period of the simulation, which covers a total of 8 weeks, the algorithm tests the two demand management models to determine which one of them is able to minimize the probability of stock most effectively out or overstocking. Initial sales performance plays a decisive role at this stage, as it is a significant indicator for predicting future demand and making informed decisions on managing reorder strategies in the long term.

When setting up the simulation, a special cell is included in the spreadsheet. This cell shows the number of the game theory case that is automatically generated by the simulation. These numbers follow the numbering logic previously illustrated in Table 4.2.

The function AVERAGE allowed to recap the average results obtained from the simulation, offering a clear identification of the most profitable situations within the simulation.

Subsequently, the eight scenarios developed in relation to the retailer's decision to make or forgo additional reorder were compared in pairs, highlighting how the decision to reorder can affect the final profit.

ANALYSIS	COMMENTS	MODEL APPLIED
Case 1 vs. Case 2	The retailer would do well to place a reorder if the initial estimate of high demand proves to be accurate. In this way, the possibility of out-of-stock is reduced, guaranteeing the producer and the retailer the highest possible profit	Mid-Season Reorder model
Case 3 vs. Case 4	Although demand was overestimated in the first period, the stock at the end of that period is not enough to meet the expected units required for the following period. Re-ordering is therefore necessary to avoid a stockout situation	Mid-Season Reorder model
Case 5 vs. Case 6	As a result of an initial underestimation of demand, an additional order must be placed to avoid serious stockouts and loss of opportunities for both parties involved	Mid-Season Reorder model
Case 7 vs. Case 8	The first case concerns the purchase and maintenance of stocks in excess of actual market demand. This could expose the retailer to several risks. The second case concerns the decision not to re-order. In situations where the retailer can look forward to a future increase in demand, it might seem a good idea to place new orders to meet this growing demand. However, it is essential to consider that, given the initial assumptions, there is no guarantee that the market will be willing to accept all additional units. This leads to uncertainty regarding the profitability of such new orders	Mid-Season Reorder model / Newsvendor model

Table 4.4: A thorough examination of the outcomes derived from the simulation for each game-theoretic scenario

In general, inventory management and reordering decisions are of crucial importance for the retailer and the producer. The key to success in this context depends largely on the ability to respond effectively to fluctuations in demand and real-time market dynamics.

When market demand is high, the best choice for the retailer seems to be using the Mid-Season Reorder model, which allows him to respond promptly to demand. On the other hand, when demand is low, the Newsvendor model remains a convenient choice for retailers. Under these

[2] This refers to the Mid-Season Reorder model approach without an a priori decision to re-order during the season.

circumstances, placing additional orders may only increase costs and entail unnecessary risks.

Subsequently, summaries obtained from the simulation for each actor (retailer, producer, and supply chain) and for each scenario are provided. In particular, the comparisons previously examined refer to the scenario in which the replenishment process takes place in week eight.

WEEK 8				
Average GT case	Producer profits	Retailer profits	Retailer margin	Supply chain profits
1	22.487 €	63.725 €	44%	86.212 €
2	-	-	-	-
3	23.601 €	50.703 €	39%	74.304 €
4	-	-	-	-
5	18.456 €	52.337 €	45%	70.793 €
6	13.311 €	26.671 €	40%	39.982 €
7	12.178 €	26.567 €	42%	38.744 €
8	14.251 €	33.529 €	47%	47.780 €

Table 4.5: Average profits and contribution margin in relation to the reorder week

It is noteworthy that through simulation, it was possible to identify the probability density associated with the various scenarios under consideration. Cases 1, 7, and 8 emerge as the most probable. In contrast, Cases 2, 4 and 6 exhibit the lowest probability of occurrence.

After a meticulous assessment of the outcomes derived from the AVERAGE analysis, it is deduced that a shift in perspective was imperative to attain a more thorough and comprehensive understanding of the examined context. This novel approach is characterized by its orientation toward a more intricate and nuanced understanding of the market demand distribution, surpassing the analysis concentrated on the specificity of individual cases. Central to this revitalized perspective is the notion of 'spread', a pivotal indicator assuming a critical role in delineating the percentage disparity between the quantities of two distinct product types.
The assigned spread values encompass a broad range, ranging from a minimum of 50% to a maximum of 400%.

While analysing the results, an interesting trend emerges. In light of distinct strategies employed for the two products, it becomes evident within the context of 50% spread that the optimal choice for the comprehensive management of low-demand products is the Newsvendor model, for the high-demand, instead, the Mid-Season Reorder model is preferable.

Furthermore, upon exploring subsequent scenarios, this dynamic becomes more evident. Indeed, under conditions of heightened demand, the retailer's profits derived from the implementation of the new model markedly surpass those achievable through the utilization of the Newsvendor model. Conversely, in scenarios of low demand, the profits realized with the Mid-Season Reorder model exhibit a marginal increase over the Newsvendor model.

The Mid-Season Reorder model emerges as the most lucrative option for high-demand products, while the Newsvendor model resulted more suitable for low-demand products, and this disparity in profit expands with an escalation in the spread percentage.

SPREAD 50%		
DEMAND in T_0	**30**	**45**
HIGH/LOW in T_0	LOW	HIGH
x_0	81	46
DEMAND in T_1	24	46
HIGH/LOW in T_1	LOW	LOW
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	17.010 €	11.776 €
π_P Mid-Season	18.203 €	25.359 €
π_P	**17.010 €**	**25.359 €**
π_R Newsvendor	42.525 €	17.912 €
π_R Mid-Season	26.799 €	57.780 €
π_R	**42.525 €**	**57.780 €**
π_S	**59.535 €**	**83.139 €**
Margin	**50 %**	**40 %**

Table 4.6: Summary of a hypothetical scenario with a 50% spread between high and low-demand products

SPREAD 400%		
DEMAND in T_0	**19**	**95**
HIGH/LOW in T_0	LOW	HIGH
x_0	54	96
DEMAND in T_1	42	81

HIGH/LOW in T_1	LOW	HIGH
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	13.176 €	23.424 €
π_P Mid-Season	13.176 €	35.097 €
π_P	**13.176 €**	**35.097 €**
π_R Newsvendor	28.365 €	38.064 €
π_R Mid-Season	29.646 €	95.770 €
π_R	**28.365 €**	**95.770 €**
π_S	**41.541 €**	**130.867 €**
Margin	**43 %**	**45 %**

Table 4.7: Summary of a hypothetical scenario with a 400% spread between high and low demand products

The new model proves to be more functional and elastic in dealing with variations in demand, and this flexibility translates into better economic performance for the entire supply chain.
These results confirm the importance of selecting the appropriate model according to the specific dynamics of the market and the characteristics of the products involved. Understanding them is key to optimising business decisions and ensuring optimal economic performance within the supply chain.

4.4. Results

The simulation conducted denotes a significant step forward in the understanding and validation of supply chain management models.
It allowed to examine and evaluate the performance of the Newsvendor model and Mid-Season Reorder model under a diversified range of demand conditions, making an important contribution to filling the gap in the supply chain management literature.

The Mid-Season Reorder model clearly demonstrates its advantage in situations where flexibility is required, unlike the Newsvendor model which is best suited to handle constant and predictable demands. This analysis confirms the previously mentioned theoretical assumption on the Pareto optimality of Cases 1 and 8.

However, more complex and realistic situations require more in-depth analysis, and the third phase of the simulation addressed this need. This analysis was crucial to further explore the performance of the two models under more changeable and volatile demand conditions, so in a more realistic representation of the challenges that supply chains often face. The results that emerged from this phase of the analysis clearly confirm the advantage of the Mid-Season Reorder model in high-demand contexts, especially when the spread percentage is significant. In these scenarios, the model demonstrates its ability to maximise profits and ensure efficient supply chain management. Its flexibility in dealing with deviations in demand results in superior performance in dynamic contexts. On the other hand, the Newsvendor model continues to prove to be an optimal choice in cases of low demand and when product requirements are more stable. This result agrees with the theories underlying the model, which suggest that it is best suited for situations where demand is constant and predictable.

One of the key conclusions that emerged from this analysis is the importance of adapting the model according to the specific characteristics of the market and products involved. The choice between the Newsvendor model and the Mid-Season Reorder model must be carefully weighed, taking into account key variables such as demand variability, forecast accuracy and the goal of maximising profits in supply chain management.

In conclusion, this research embodies an important contribution to the supply chain management literature by providing an empirical evaluation of the examined models under real operating conditions. It also highlighted the effectiveness of the new model compared to the current one in certain contexts and how it is able to significantly enhance the financial performance of all involved participants and the supply chain as a whole. Therefore, the need to thoroughly evaluate its implementation in the market is empirically confirmed.

5. Conclusions

The dynamics and difficulties of applying a contractual model for dynamic reordering in the luxury Jewellery industry were thoroughly examined in this dissertation. New insights have emerged through critical examination of traditional models and thorough analysis of initial queries. These insights significantly advance our understanding of supply chain management in the context of variable and changing demand.

First, it became evident how important precise demand forecasting is. The luxury market is one where trends can shift quickly, so being able to predict customer preferences and market influences with precision is essential.

One key component that has been identified is supply chain flexibility. Optimizing production and delivery schedules and working effectively with producers proved to be crucial.

Its ability to react quickly to changes in the market guarantees a more efficient inventory management procedure, reducing negative consequences on the financial statements and reputation of the business.
The Mid-Season Reorder turned out to be a practical solution for handling this process as well as a cost-effective way for businesses looking to hold onto their competitive edge.

Game theory has emerged as an extremely useful tool for contract optimization in supply chain management. This approach opened new avenues for collaboration between retailers and producers, allowing the supply chain's overall value to be maximized thanks to a depth analysis of the model to be applied.

The findings highlight the importance of the Mid-Season Reorder model's adoption in the luxury industry's supply chain management optimization, and the simulation phase's profits, which were determined using the newly created formulas and newly determined parameters, provide a compelling illustration of this. This new contract model's innovation gives businesses in the industry a competitive edge by enabling them to more effectively respond to the changing needs of their customer base. The Mid-Season Reorder model's demonstrated efficacy validates its applicability and appropriateness as a crucial instrument to tackle the difficulties of contemporary supply chain management within the ever-changing luxury industry.

In conclusion, this study introduces an advanced contractual model as a response to market challenges. The new insights provided not only enrich existing theory, but also offer practical guidelines for industry players wishing to improve the flexibility, adaptability, and efficiency of their supply chain.

5.1. Novel Contributions

Notwithstanding its strength as an inventory management tool, the Newsvendor model has many inherent drawbacks stemming from the presumption that orders are only placed at the start of the season and cannot be changed. But in the actual world of business, there might be instances where orders need to be adjusted to better accommodate shifting demand, shifting delivery schedules, or other unanticipated events.
The Mid-Season Reorder model considers several factors, it adds new benefits to get around these restrictions and offers greater flexibility in stock optimization. These benefits include:

- Updating forecasts. This allows for more informed decision-making;
- The possibility of placing second orders during the selling season based on market dynamics. This implies that they can gradually adjust to variations in demand rather than being forced to commit to large orders up front;
- Reordering mid-season gives both players the option to better manage variability in a world where volatility has become the norm;
- Reduction of waste and obsolescence and its impact on profit. This model gives players more control over excess stock and obsolete items. This lessens losses brought on by having to discard unsold products at the end of the season;

- Enhance the utilization of monetary resources. The new model enhances corporate liquidity by enabling a more effective division of inventory and reorder-related expenses;
- Improved customer satisfaction. The flexibility provided by the new model enables retailers to reply to end-user requests faster and guarantees that the right products are available when needed. From the retailer's perspective, the same applies to the producer. This can lead to stronger customer loyalty and cross-selling opportunities;
- Market competitiveness. Retailers can increase their market share or hold a leading position by being able to react quickly to changing conditions.

In summary, the Mid-Season Reorder model improved inventory management by better meeting the needs of the fast-paced business world of today. It boosts performance and customer satisfaction by enabling producers and retailers to maximize their resources, cut waste, and keep better control over their inventory.

5.2. Limits

The introduction of the contractual model for dynamic reordering in the luxury Jewellery sector offers a significant theoretical contribution to the dynamics of supply chain management. However, it is crucial to examine the implications and limitations arising from this strategic transition to fully understand the context in which the answers provided can be relied upon.

The fundamental role that the partnership between retailers and producers' plays is highlighted by the necessity of effective communication and collaboration with producers. Dynamic reordering becomes dependent on close integration and prompt information sharing.

Capabilities for data analysis are equally important. Setting aside money for sophisticated analytical tools as a top priority suggests that, to reap the full benefits of the model, businesses should think about providing staff with analytical skill training.

The difficulty of organizational adaptation is one of the study's most significant practical implications. An organizational structure that is adaptable and agile is required to allow for quick adjustments in response to shifting market conditions. To maintain flexibility over time, a company needs to foster a culture of adaptation and ongoing development. Organizational flexibility puts businesses in a better position to handle uncertainty, grab new opportunities, and increase their overall competitiveness in the market.

Moreover, the universal applicability of the contract model for dynamic reorganization is questioned, suggesting that not all industries or products will profit from this strategy in the same way. According to the theoretical contribution, supply chain complexity and demand variability have a direct impact on how effective the suggested model is. It highlights the necessity of carefully assessing the situations in which the model can be used with success.

The practical implications of the study suggest that companies in the luxury Jewellery industry should wisely consider the transition to dynamic reordering as it is a strategic decision that requires targeted investment and a change of mindset. The limitations of the study indicate the need for further research exploring other industries and contexts to generalise and refine the conclusions reached here.

5.3. Future Evolutions of the work

The model's development has revealed important obstacles to efficient communication amongst all parties involved and a clear shortcoming in precisely predicting market demand. Taking these factors into account, future improvements might include putting in place mechanisms and incentives that are intended to improve alignment between the parties, fostering better communication and more efficient responsiveness to market dynamics. Furthermore, a comprehensive examination of efficient tools to support forecasting analysis ought to be investigated.

To improve the Mid-Season Reorder model's effectiveness and flexibility in response to the target market's unpredictability, future research efforts might focus on rethinking the model to strengthen its resilience in situations where the initial demand projections turn out to be incorrect. These endeavours could potentially aid in the development of innovative tactics and methods that are more suitable for the unique obstacles that businesses face in the field of supply chain management.

The results obtained from the game theory-constructed scenarios that were then analysed through simulation provide a strong basis for further research into the viability and relevance of supply chain management models in various business contexts. Considering that the model works well in situations where demand fluctuates a lot, one of the most compelling directions for future research and development would be to perform a benchmark analysis to find industries that have characteristics that align with the model's principles and thus make it advantageous to apply.

Additionally, a fascinating direction for future investigation entails carrying out additional study to dive into a more comprehensive evaluation of the ideal week for reordering. In order to produce even more accurate scenarios and solutions, this might require adding a third demand forecast.

Finally, one potential area of improvement could be to examine the financial effects of matching the delivery week with the reordering week. During the current time frame, inventories are expected to be directly impacted by this alignment.

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List of Executive Summary Tables and Figure

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