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Executive Summary of the Thesis

# A FEM based study on wooden mechanical metamaterials

Laurea Magistrale in Music and Acoustic Engineering

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## 1. Introduction

The popularity of wood in stringed musical instruments is undeniable. Its workability, aesthetic, and its mechano-acoustic properties make it an ideal choice for their construction. Like many natural resources, wood is subject to variations even within samples taken from the same tree, making it a challenging task for instrument makers to maintain the exact same sound between one instrument and the next. Additionally, due to their unique properties, certain species of wood have become endangered because of their excessive use and the environment changes caused by global warming.
Researchers have been studying alternatives to the most used wood specimens considering not only their acoustic quality but also their environmental impact. The most popular solutions involve using either composite materials, or physically and chemically altered woods. This thesis instead aims to address these issues through the use of wooden mechanical metamaterials. These are specifically engineered materials whose unique properties are given by their structure rather than their chemical composition. In fact, previous studies have demonstrated that the mechanical parameters of a wooden plate can be tuned via its perforation with periodic patterns of holes [1]. Since wood is an orthotropic material, it is characterized by nine different elastic parameters as well as its density, making for ten independent parameters which can be tuned when designing a soundboard.
Until now, only metamaterials with homogeneous hole dimensions have been studied using the Caldersmith formulas to estimate their effective elastic constants [2]. In this thesis, we verify the accuracy of said formulas which were originally designed for solid rectangular plates when they are instead applied to metamaterials, and investigate the effect of heterogeneous distributions of the hole sizes on the vibrational and mechanical behaviour of wooden plates.

## 2. Methods

### 2.1. Reference plates

The CAD models of the investigated metamaterials are created on COMSOL. They are inspired from the ones studied in [1] and have the same $0.6 \times 0.24 \times 0.0035$ $[m]$ rectangular shape which is that of a regular cut for guitar soundboards. The plate represented in Figure 1 is the one studied to examine homogeneous hole size distributions. It is characterized by identical elliptical holes drilled at regular intervals.

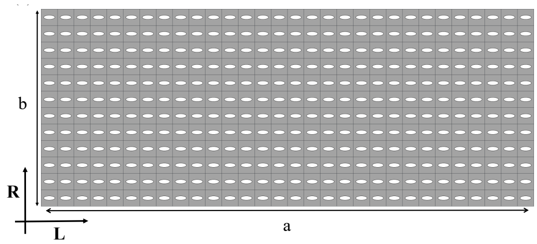



Figure 1: The geometry of the homogeneous plate is made of identical square cells.

For the heterogeneous configurations, 22 different patterns of circular holes are considered. They are divided into two groups. In the first one, the hole dimensions are symmetric with respect to the radial direction **R** while in the second they are symmetric to the longitudinal direction identified by **L** in Figure 2. An index k is used to identify the plates. The holes are larger near the center of the plate and smaller near to the sides for values of k close to 0 while the opposite is true for values of k close to 10.

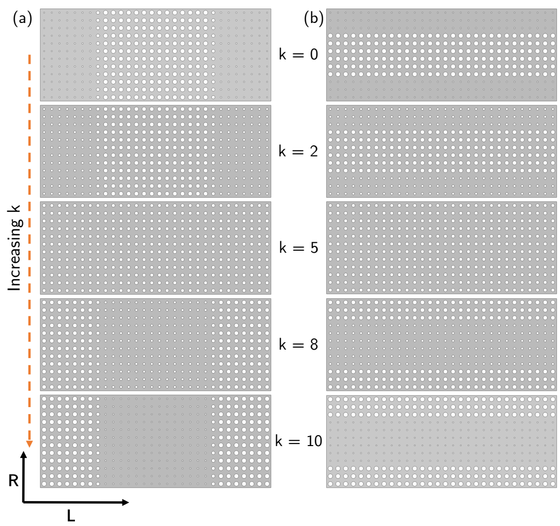



Figure 2: Heterogeneous reference plate configurations for radial symmetry (a) and longitudinal symmetry (b).

Figure 2 shows 10 of these patterns. The heterogeneous mass distributions could be an interesting tool for guitar makers who wish to make some parts of their soundboards lighter than others, or in the violin making process where the graduation of the plates is of high relevance.

## 2.2. Equivalent plates

The elastic constants of the investigated reference plates can be approximated through the study of a solid plate which is equivalent in its vibrational behaviour. In fact, the material parameters of this equivalent plate can be obtained by minimizing the difference between the eigenfrequencies and mode shape of the two plates. This approach allows us to study the metamaterials under the more familiar perspective of a solid plate.

Four different equivalent plate models are studied with the aim of finding the one which better fulfills this role. In particular, we consider a homogeneous mass model, one with variable thickness, another one with piece-wise constant density, and lastly one with linear density. They all share the same dimensions as the reference plates but have no holes drilled in them.

The first equivalent model is the simplest and is characterised by a homogeneous mass. Because we consider also heterogeneous hole dimension distributions, the other models have a varying mass distribution and are made to be applied to the heterogeneous configurations which have circular holes. For the variable thickness model, the profile height is calculated at the center of each cell represented in Figure 1 using equation 1. The general thickness profile is then interpolated as in Figure 3.

$$h_{eq} = h_{ref}\left(1 - \frac{\pi R^2}{l^2}\right) \tag{1}$$

$h_{eq}$ and $h_{ref}$ are the thickness of the equivalent and reference cells respectively, $l$ is the length of the side of the cells, and $R$ is the radius of the hole in the considered cell of the reference plate. For the piece-wise constant density model, the equivalent density of each hole file $(i)$ is calculated as in equation 2.

$$\rho_{eq}(i) = \rho_{ref}\left(1 - \frac{\pi R(i)^2}{l^2}\right) \tag{2}$$

Where $\rho_{eq}(i)$ is the equivalent density of file $(i)$. Finally, the linear density model uses a linear variation of the density expressed as in equation 3.

$$\rho_{eq}(x) = dx + \rho_0 \tag{3}$$

Where $d$ is the slope, $\rho_0$ the intercept, and x is the position along direction with varying hole

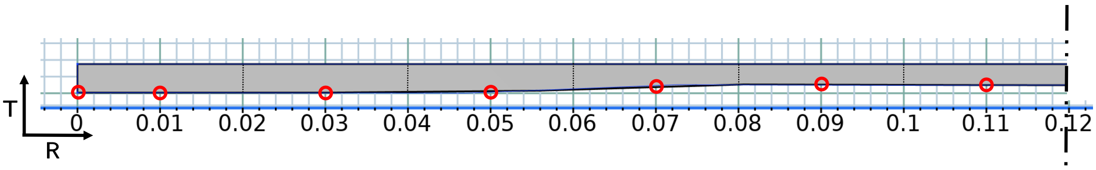



Figure 3: Profile section of the variable thickness equivalent plate of longitudinal configuration $k = 0$. The red dots correspond to the calculated points used to interpolate the thickness of the equivalent plate. At 0.12 m is located the axis of symmetry of the plate.

sizes. Figure 4 shows the density distribution for the piece-wise constant and linear density models for configuration k = 0.

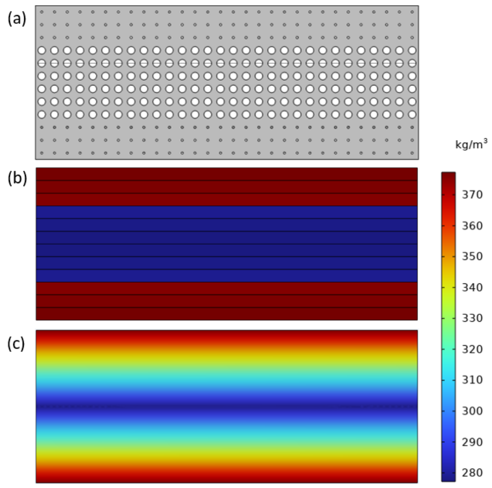



Figure 4: (a) Reference plate in longitudinal configuration $k = 0$. (b) Piece-wise constant density equivalent model. (b) Linearly varying density equivalent model.

## 2.3. Material parameters

For the reference plates, the considered material parameters are those of Engelman Spruce which was used also in [1]. It is a species of wood often used for soundboards because of its high stiffness and low density. Its material parameters are taken from the Wood Handbook [3].
The equivalent plate's material parameters are identified using the Finite Element Model Updating (FEMU) method which is used to calibrate a numerical model based on the actual behavior of a reference structure. In our case, this is done by altering the material parameters of the equivalent plate's numerical model in order to minimize the differences in eigenfrequencies and mode shapes between it and the reference. More specifically, the optimization is implemented in Python using the Nelder-Mead minimization algorithm to minimize the following objective function:

$$L=\sqrt{\sum_i\left(\frac{f_i^R-f_i^{Eq}}{f_i^R}\right)^2}+\alpha\sqrt{\sum_i(1-MAC_{ii})^2} \qquad (4)$$

To replicate the dynamic behavior of the reference plates, the objective function $L$ has two components that are summed and weighted by a coefficient $\alpha$. The first component measures the error in eigenfrequencies where $f_i^R$ and $f_i^{Eq}$ are the $i^{th}$ eigenfrequency of the reference and equivalent plate respectively. The second component evaluates the error in the modeshapes using the Modal Assurance Criterion commonly called MAC.

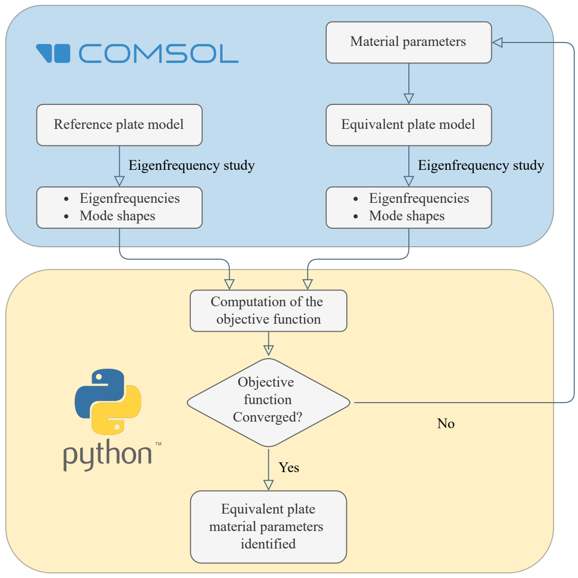



Figure 5: FEMU diagram of the optimization of the equivalent plate's material parameters.

The algorithm, as can be observed in Figure 5, takes as inputs the eigenfrequencies and mode shapes of the reference plate which are computed numerically on COMSOL using a physics controlled fine mesh and free boundary conditions.

The initial guess of the equivalent plate's material parameters is calculated using the Caldersmith formulas and is also given as input to the algorithm. Then, the optimization loop starts with the computation of the eigenfrequencies and mode shapes of the equivalent plate on COMSOL which are imported in Python and used to evaluate the objective function L. After that, if the objective function hasn't converged, new material parameters are defined for the equivalent plate and the process is repeated.

# 3. Results

## 3.1. Eigenfrequency study

Considering the homogeneous reference plate, various simulations have been made using the constant mass equivalent plate to verify the accuracy of the Caldersmith formulas. Because the coefficient used to approximate the value of $G_{LR}$ can be different in the literature, the equivalent plate's material parameters have been calculated first using the Caldersmith formulas as in [2], then using the coefficient proposed by McIntyre in [4] for the calculation of $G_{LR}$, and as an additional verification, using the proportionality coefficient given by the Wood Handbook linking the value of $G_{LR}$ to the longitudinal stiffness $E_L$. The error between the eigenfrequencies of these three equivalent plates and the reference are shown in Figure 6 and compared to the results obtained applying the implemented FEMU optimization.

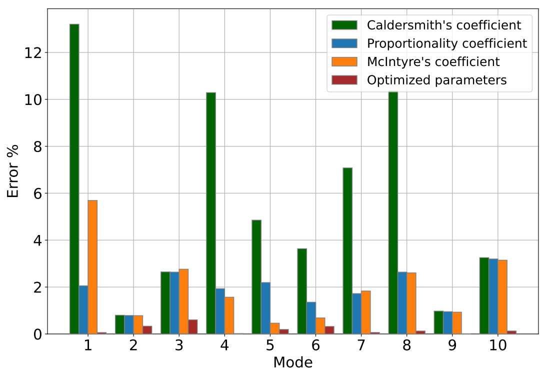



Figure 6: Eigenfrequency errors of the constant mass equivalent plates with respect to the reference homogeneous plate.

It is evident that when using the right coefficient, the Caldersmith formulas can give a good approximation of the elastic constants of homogeneous metamaterials. Using the optimization the mean error is further reduced to 0.3 %. Since most of the studied MACs show good correspondence between the mode shapes of the reference and equivalent plates, this executive summary will focus only on the eigenfrequency errors of the simulations.

Applying the same Caldersmith formulas to the heterogeneous reference plates is not as effective as can be seen from Figure 7.

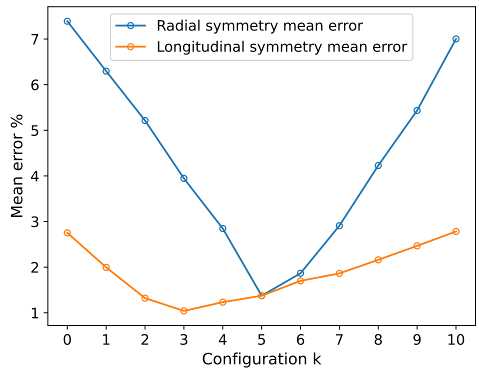



Figure 7: Mean error of the first ten eigenfrequencies for each configuration k using the Caldersmith formulas on the constant mass equivalent plate.

In fact, the error in the eigenfrequencies reaches a minimum for k = 5 which corresponds to the homogeneous distribution of hole sizes, and steadily increases as k gets closer to 0 and 10. This effect is particularly noticeable for radial symmetry configurations, clearly indicating that the Caldersmith formulas cannot be used when studying heterogeneous metamaterials since they do not take into account the mass distribution on the plate.

Because of this, the created equivalent models with variable mass are tested on longitudinal symmetry configuration 0. For each equivalent model, the optimized material parameters are computed and the eigenfrequency errors are plotted in Figure 8. The results show that using a linear density variation in the plates is the most accurate solution.

Using the linear density equivalent model and identifying the material parameters for all heterogeneous configurations k confirms that this method can offer a good approximation of the material parameters of the metamaterial with a total mean error in the eigenfrequencies for all configurations of 1.43 %.

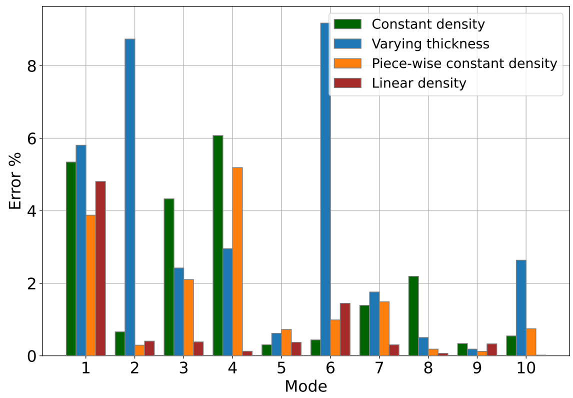



Figure 8: Eigenfrequency errors for the optimized equivalent models for the longitudinal symmetry configuration 0.

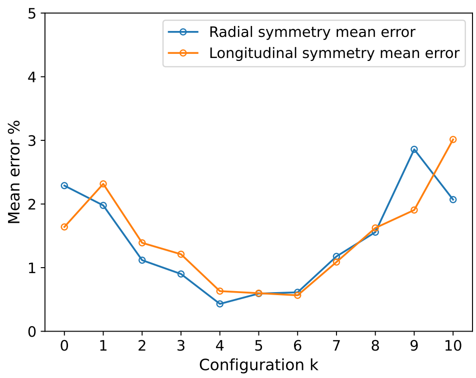



Figure 9: Mean error in the first 37 eigenfrequencies for each configuration using the optimized linear density equivalent model.

As can be seen from Figure 9, the error is still slightly higher for the most heterogeneous configurations suggesting that even though the equivalent plates method is still valid for heterogeneous metamaterials, their behaviour is different from homogeneous ones and is caused by more than a change in the material parameters of the plate.

## 3.2. Static Analysis

A static analysis of the obtained equivalent plates and their respective reference is conducted on COMSOL through the application of a stationary 10 N distributed force of the main surface of the plate. Figure 10 shows the Von Mises stress resulting from the application of such a force on the reference and equivalent plates of longitudinal configuration k = 10 while keeping their radial edges fixed.

These results, which have been performed on other heterogeneous configurations as well show a good correspondence of the stress distribution between the two plates.

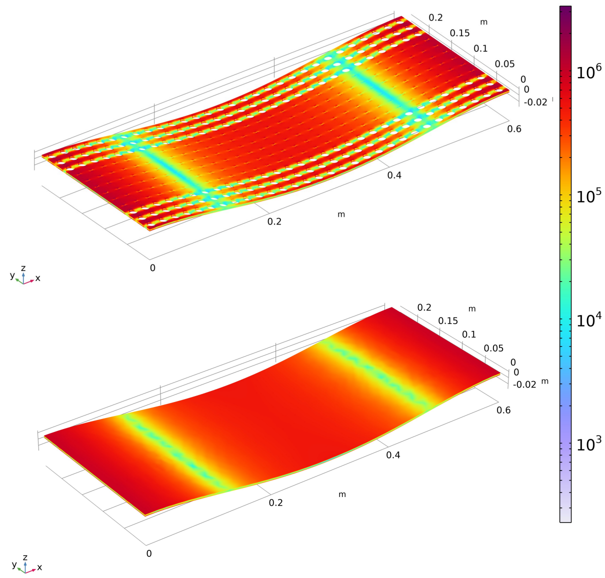



Figure 10: Von Mises stress measured for configuration k = 10 with longitudinal symmetry for a 10 N distributed force applied on the top surface of the plate. Fixed boundary conditions are applied on the radial sides of the plate. The reference plate is shown on top and the equivalent plate on bottom.

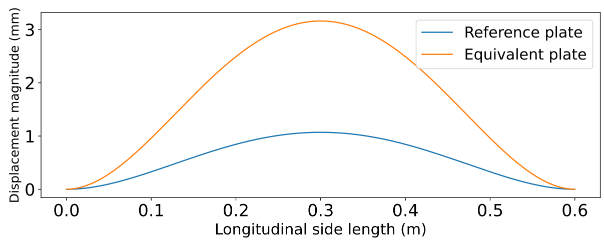
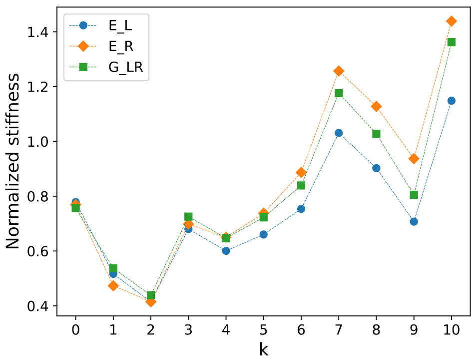



Figure 11: Displacement of the longitudinal edge in the tangential direction for longitudinal configuration k = 10.

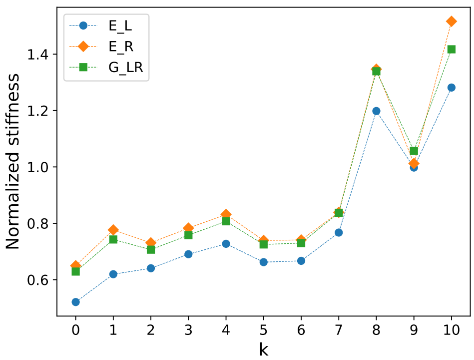
Figure 11 on the other hand reveals a discrepancy between the maximum displacement of the reference and equivalent plates. In fact, a larger displacement can be noticed for most heterogeneous configurations suggesting a lower stiffness under static conditions. This result is particularly interesting as it could be used to design bespoke instruments with different dynamic and static behaviours.

### 3.3. Material parameters

Analyzing the material parameters of the obtained equivalent plates reveals a correlation between the index k and the stiffness of the boards. Figure 12 shows the value of $E_L$, $E_R$, and $G_{LR}$ which are the most significant parameters as they can be used to calculate the acoustic radiation index and anisotropy ratio which are indicators of the acoustic quality of soundboards.




(a) Longitudinal configurations.




(b) Radial configurations.

Figure 12: Normalized value of $E_L$, $E_R$, and $G_{LR}$ to the respective stiffness of Engelman spruce for each heterogeneous configuration k obtained from the linear density optimized equivalent plate model.

It seems that having larger holes near the edges of the plate and smaller ones near the center increases the stiffness of the plate to the point of surpassing the original stiffness of Engelman spruce for values of k close to 10. This correlation can be used to manually control the stiffness and density of a plate through the application of specific patterns of holes. It could be particularly useful considering that one of the main requisites for achieving a good acoustic radiation with wooden a soundboard consists in having a high stiffness with a lower mass.

## 4. Conclusions

The objective of this thesis work was to investigate the effects of metamaterials with heterogeneous hole size patterns on the dynamic and static behaviour of rectangular wooden plates. To this end, a new approach has been used focusing on the study of solid plates with an equivalent vibrational behaviour. An optimization algorithm based on the FEMU method was implemented for the identification of material parameters.

Our results show that the investigated wooden metamaterials have striking differences between homogeneous and heterogeneous hole size distributions. In fact, the stiffness of the plate is observed to be correlated to its hole size distribution, and a different behaviour can be found as well when studying them under dynamic or static conditions. Proving that a lot still has to be discovered about metamaterials but also that they are a powerful tool to be used in musical acoustics and beyond.

## 5. Acknowledgements

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## References

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