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EXECUTIVE SUMMARY OF THE THESIS

## A new particle filter application for estimating cutting forces and detecting regenerative instability in milling

LAUREA MAGISTRALE IN MECHANICAL ENGINEERING - INGEGNERIA MECCANICA

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### Introduction

In the last few years, high quality machining became the most crucial aspect in manufacturing, even if rising importance is dedicated to the design of environmentally friendly machines, which has the reduced energy consumption as a primary goal.

Machines are becoming smarter and sensorization is easier and can be implemented for an increasing number of purposes. In machine tools sector, data as cutting forces, vibrations, surface finish, temperature can be collected to control the production.

Between manufacturing processes, milling is one of the most important and widespread, thanks to its high material removal rate, precision and complexity of geometries which can be obtained.

### 1. Problem Statement

The primary goal of a manufacturing plant is the high productivity and reliability. In order to obtain it, cutting parameters such as axial depth of cut, radial immersion and feed have to be maximized, with the drawback of increasing vibrations and possible instability. During milling, it is possible to identify three main types of vibra-

tions:

- Free vibrations, which are damped over time with an oscillatory motion.
- Forced vibration, as a result of a continuous perturbation.
- Self-excited vibrations, which occur when the structure is not able to damp the energy introduced in the system from the interaction between workpiece and tool.

Knowing tooltip vibrations in real time helps with the estimation of workpiece quality, since they are linked. The regenerative instability, also known as "chatter", occurs when the oscillation of the tooltip in one pass of the tool leaves a waved mark on the machined surface, which will be regenerated by the following tooth. This oscillatory motion generates a modulation of the chip thickness, which leads to an indefinite increase of the amplitude of vibrations and cutting forces. [2]

There are two ways to measure cutting forces: direct methods (e.g. piezoelectric dynamometers) or indirect methods (relying on a model explaining the relationship between the force magnitude and the instrument reading). A solution of this kind is provided by state observers, systems able to estimate the internal states of the

real systems from the measurements of inputs and outputs.

Kalman filter is a state observer used by Albertelli et al. [1] for the estimation of tooltip vibrations and cutting forces of a milling machine, modelled as a state space system:

$$\dot{x} = Ax + Bu \quad (1a)$$

$$y = Cx + Du \quad (1b)$$

Marzatico [4] implemented a state observer based on the Riccati equation. The model improved performances in unstable conditions, thanks to a modified state equation:

$$x\dot{(t)} = Ax(t) + A_r x(t - \tau) + BF(t) \quad (2)$$

Torricella [5] developed a switching observer, in order to deal with the detachment of the tool from the workpiece which occurs in unstable conditions. The goal of this thesis is to design of a state observer based on a particle filter (a sequential Monte Carlo method) [3] for the estimation of vibrations and cutting forces, in stable and unstable conditions. The desired approach should improve previous methods in terms of robustness, while simultaneously providing an on-line chatter indicator.

## 2. Problem formulation

The starting point is a 2 degrees-of-freedom formulation of the milling process.

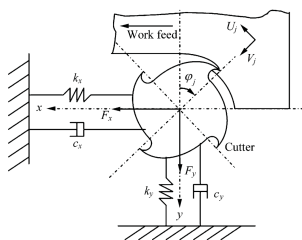


Figure 1: Two degrees of freedom model

The state space system is obtained from an experimental modal identification. State matrices are assembled starting from experimental FRFs and identifying the most relevant mode of vibrations along X and Y.

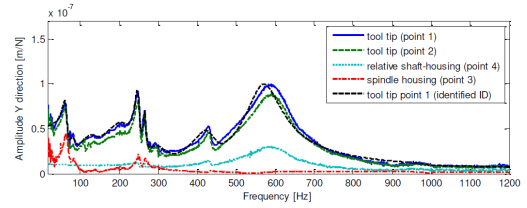


Figure 2: Experimental FRFs (amplitudes) along Y direction

The most important part of this section is the treatment of the two different formulations of the regenerative instability: Zero Order approach and detachment formulation. The starting point is the formulation of cutting forces as a function of the chip thickness  $h$ , comprehensive of both nominal and dynamic contribution:

$$h(\varphi_j) = [s_t \sin \varphi_j + (v_{j-1} - v_j)]g(\varphi_j) \quad (3)$$

From this common point, it is possible to identify two possible formulations:

- The zero order approach leads to the formulation already expressed in Equation (2). The matrix  $[A_{REG}]$  is obtained from the zero-order Fourier transform of the dynamic components of the chip thickness:

$$[A_0] = \frac{N}{2\pi} \int_{\varphi_{in}}^{\varphi_{out}} [A(\varphi)] d\varphi \quad (4)$$

- The detachment formulation relies on the opposite principle: the chip thickness is not averaged on one tooth pass, but it is computed for each tooth for each time step. The detachment is obtained by setting  $h = 0$  every time the dynamic component becomes higher but opposite in sign of the nominal one. The detachment can be seen as a limit to the instability.

Different formulations lead to different results, especially in unstable conditions.

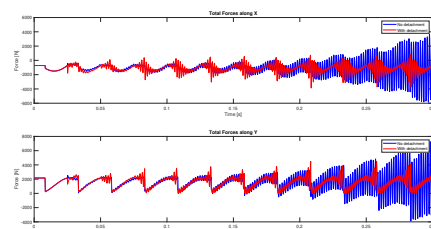


Figure 3: Cutting forces with the two formulations, unstable conditions

### 3. Particle filter design

The application of particle filter for the observation problem opens two main challenges: the first one is that the particle filter is conceived for discrete systems, while previous studies were all relying on continuous formulation. The required system becomes:

$$x_{k+1} = \mathbf{A}_d x_k + \mathbf{A}_{d,\text{REG}} x_{k-\kappa} + \mathbf{B}_d F_{NOM} \quad (5)$$

Discrete matrices are obtained starting from continuous ones:

$$\mathbf{A}_d = e^{\mathbf{A}T_s} \quad (6a)$$

$$\mathbf{A}_{d,\text{REG}} = \mathbf{A}^{-1} (\mathbf{A}_d - \mathbf{I}) \mathbf{A}_{\text{REG}} \quad (6b)$$

$$\mathbf{B}_d = \mathbf{A}^{-1} (\mathbf{A}_d - \mathbf{I}) \mathbf{B} \quad (6c)$$

The second challenge is the creation of extended matrices, since the input force is not known. In a particle filter algorithm, at each time step, the distribution of the state at the previous step is updated thanks to the *ProcessEquation*, exploiting the known mathematical model and introducing a stochastic process variance  $Q$  on the cutting forces. For each of the  $N$  particles, the process equation starts with:

$$\begin{aligned} Fx_k &= Fx_{k-1} + Q * \mathcal{N}(0, 1) \\ Fy_k &= Fy_{k-1} + Q * \mathcal{N}(0, 1) \end{aligned} \quad (7)$$

These two new values are introduced in the state vector and the system is updated with the following equation:

$$\begin{Bmatrix} q_{k+1} \\ F_{k+1} \end{Bmatrix} = [Ad] \begin{Bmatrix} q_k \\ F_k \end{Bmatrix} + [Ad_R] \begin{Bmatrix} q_{k-\kappa} \\ F_{k-\kappa} \end{Bmatrix} \quad (8)$$

The predicted measurements are computed through the *MeasurementEquation* and compared with the actual ones. A *weight* is assigned to each particle: particles which led to estimated measurements closer to the actual ones will obtain a higher weight. During the *resampling* stage, particles with higher weights are more likely to be saved and contribute at the identification of the posterior distribution of the internal states. The average of the aforementioned distribution is the estimated value for that time step.

The lone estimation of forces may not be enough for the complete comprehension of the cutting

process. An increasing profile of forces and vibrations can be either a consequence of a higher tool engagement required by the part program or of the rising instability. The chatter indicator  $\gamma$  is thought as a state variable to overcome this missing information. Its value can be only 0 if the system is stable and 1 when it is unstable. The process equation becomes:

$$\begin{Bmatrix} q_{k+1} \\ F_{k+1} \end{Bmatrix} = [Ad] \begin{Bmatrix} q_k \\ F_k \end{Bmatrix} + \gamma [Ad_R] \begin{Bmatrix} q_{k-\kappa} \\ F_{k-\kappa} \end{Bmatrix} \quad (9)$$

where  $\kappa$  is the number of discrete steps between two tooth passes. The process variance need to be as small as possible for a good estimation of the chatter indicator, but some steps of the simulation may need really high value of variance in order to reach a reasonable likelihood. An adaptive process variance cycle is implemented. The basic principle is to start from a small process variance, perform a step and calculate the residual weights. If the best weight is lower than a certain value, expressed as a percentage of the maximum achievable (estimated measurement coincident with the actual one), the process variance is increased and the time step repeated.

Particular attention was given at the end to the tuning of particle filter and variance parameters.

### 4. Results and validation tests

At first, the plant was simulated with the Zero Order Approach. Both tooltip vibrations and cutting forces were estimated well. (Figures 4 and 5)

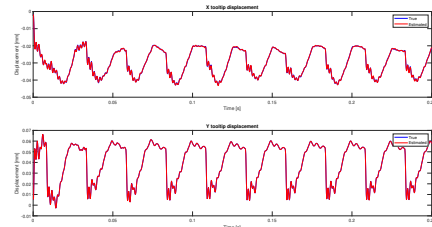


Figure 4: Tooltip displacements estimation, stable conditions

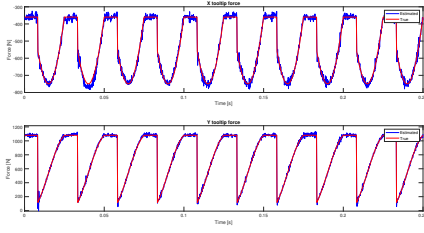


Figure 5: Cutting forces estimation, stable conditions

As regards the chatter indicator (Figure 6), after some time it stabilizes around 0.5. In stable conditions the contribution of  $[A_{REG}]$  is so small that its presence does not make important difference and particles with both values of  $\gamma$  are resampled.

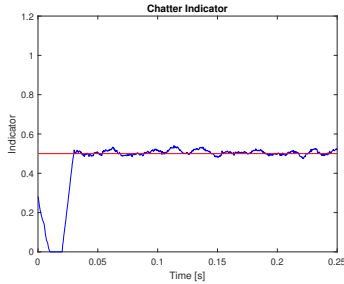


Figure 6: Chatter indicator, stable conditions

In unstable conditions, vibrations are estimated very well. Cutting forces estimation is again good and it is possible to differentiate the nominal and the regenerative contribution. (Figure 7)

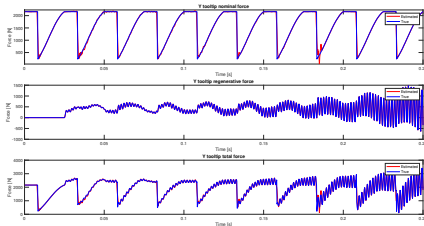


Figure 7: Cutting forces along Y, unstable conditions

The chatter indicator value (Figure 8) begins to rise with an oscillatory motion, whose frequency is the tooth passing frequency, but after some tooth pass, the chatter indicator reaches and stabilizes above 0.95, meaning that the system is fully unstable.

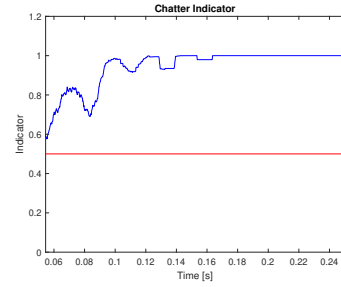


Figure 8: Chatter indicator, unstable conditions

When the plant is simulated with the detachment formulation, vibrations and forces are estimated very well. The most interesting difference can be observed in the chatter indicator (Figure 9). The detachment phenomenon is complex to be described because the instability is not irreversible: the system instability grows until the tool detaches from the workpiece ( $\gamma = 1$ ). Then the system goes back to stable conditions ( $\gamma = 0.5$ ) and begins again its ramp.

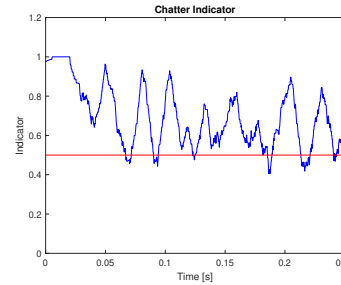


Figure 9: Chatter indicator, unstable conditions

Particle filter performances are now analyzed numerically and they are compared with previous works. The root mean square of the difference between real and estimated values proved to be a good indicator of the quality of the estimations.

Observer	Fx [N]	Fy [N]
Particle Filter	0.092	0.176
Delayed Observer	0.052	0.936

Table 1: Nominal conditions, Particle Filter vs Delayed Observer

Observer	F <sub>x</sub> [N]	F <sub>y</sub> [N]
<b>Particle Filter</b>	0.694	0.76
<b>Kalman Filter</b>	0.49	2.39

Table 2: Nominal conditions, Particle Filter vs Kalman Filter

In nominal conditions, the particle filter performed better than the previous methods. Off-design tests are performed with the cutting parameters of the observer slightly reduced or increased from the simulation ones. Particle filter showed once again better forces estimation.

Observer	F <sub>x</sub> [N]	F <sub>y</sub> [N]
<b>Particle Filter</b>	0.196	0.32
<b>Delayed Observer</b>	0.45	1.46

Table 3: Higher assumed engagement, Particle Filter vs Delayed Observer

Finally, the algorithm was tested in real cutting conditions, using experimental data.

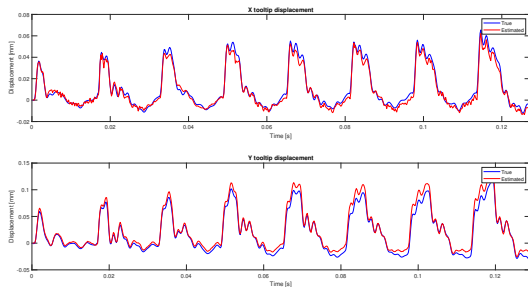


Figure 10: Tooltip displacements, unstable

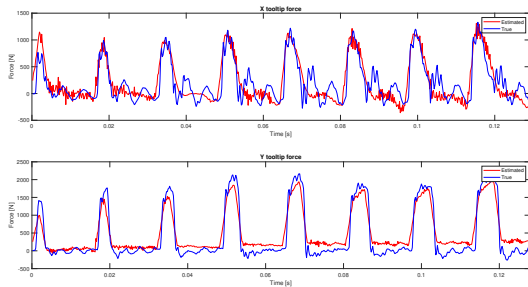


Figure 11: Cutting forces, unstable

The estimation of cutting forces and tooltip vibrations was really good in both stable and un-

stable conditions (Figures 10 and 11). Unfortunately, the chatter indicator was not performing as for the simulation, because in unstable conditions it was not increasing towards  $\gamma = 1$ , probably because the computed  $[A_{REG}]$  may be not enough to bring the system of the observer to instability and the particle filter algorithm matches the experimental measurements with a simple increase of forces.

## 5. Future developments

Future works may take two different directions. The first one is the optimization of the current algorithm, in order to reduce the computational cost of one single time step. Number of internal states, number of particles or sampling frequency are too high and only a reduction of them will result in the feasibility of an online observer. The other direction has the aim to obtain a robust functioning of the chatter indicator, thanks to the introduction of the cutting parameters as unknown variables, since the workpiece fixing may be not optimal. On the other hand, increasing the number of states is in clear disagreement with the goal of problem simplification.

## References

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