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EXECUTIVE SUMMARY OF THE THESIS

Design of robust and stochastic model predictive control strategies for collision-free trajectory tracking of multiple mobile robots

LAUREA MAGISTRALE IN AUTOMATION AND CONTROL ENGINEERING

Author: ERNESTO CERAMELLI

Advisor: PROF. PATRIZIO COLANERI

Co-advisor: PROF. GIAN PAOLO INCREMONA, CHRYSTIAN P. E. YUCA HUANCA

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1. Introduction

Mobile robotics provides a large spectrum of opportunities for research and applications ranging from classical tasks such as real-time navigation and mapping to more involved ones requiring multidisciplinary expertise, as in the healthcare and finance fields. However, due to the complex nature of nonholonomic constraints, even the most simple robot structure requires the inclusion of these nonlinearities in the design of the controllers. In the multi-robot system case, challenges such as collective motion planning, a dynamic environment with obstacles and uncertainties, and a constrained communication topology naturally lead to demanding advanced control strategies. For instance, in [5], the concept of control barrier function is exploited to achieve collision-free motion, while in [1], a behaviour-based approach is exploited to tackle several tasks. Among all strategies developed in the literature, model predictive control (MPC) approaches have shown to be particularly powerful tools due to their tasks and constraints assignment flexibility, and the possibility of handling robust frameworks [2, 3].

In this work, some solutions to the trajectory tracking problem for a multiple mobile robot system are proposed, with the assumption that the robot dynamics is constrained to a finite number of motions. Furthermore, the position and dimension of the obstacles are assumed to be available. Different stabilizing switching model predictive control approaches, able to handle the presence of uncertainty and disturbances in both the robust and stochastic frameworks, have been developed. Based on the design of stabilizing switching laws, an original switching MPC approach that relies on the disturbances reachable set is proposed, highlighting the main advantages with respect to other approaches.

2. Modelling and problem formulation

This section introduces the robotic platform model and the problem formulation.

Differential wheeled robot model

This work considers a differential wheeled robot (DWR) as a robotic platform, constituted by two main wheels, individually controlled, and a third passive castor wheel, for the stabilization of the robot. Figure 1 depicts the real setup and a planar schematic view of a DWR, its main physical parameters, and the global $(X_G - Y_G)$ and body $(X - Y)$ frames.

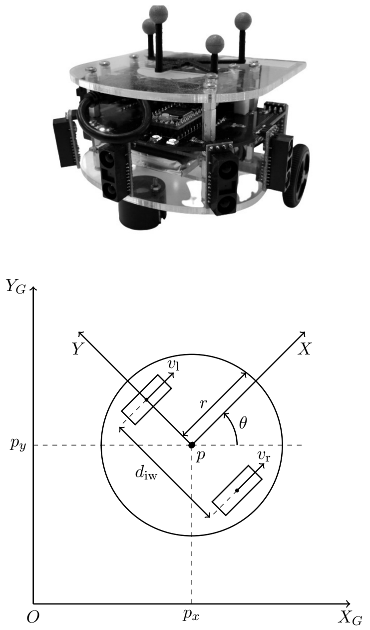

Figure 1: Differential wheeled mobile robot.

Letting v_l and v_r be the left and right wheel linear velocities, respectively, and assuming that the robot wheels roll without slipping, it is possible to derive expressions for the linear (v) and angular (w) velocity of the robot, i.e.,

$$\begin{aligned} \omega &= \frac{1}{d_{iw}}(v_r - v_l) \\ v &= \omega R = \frac{v_r + v_l}{2}. \end{aligned} \tag{1}$$

Then, by considering as state variable the robot posture (position of the robot and orientation) in the reference global frame $p = [p_x,\ p_y,\ \theta]^\top$, and as input action the linear and angular velocity of the robot $u = [v,\ \omega]^\top$, the kinematic model of the mobile robot can be defined as

$$\begin{aligned} \dot{p}_x &= v\cos\theta \\ \dot{p}_y &= v\sin\theta \\ \dot{\theta} &= \omega. \end{aligned} \tag{2}$$

Switched model and disturbances

In order to introduce a switching property into the mobile robot, the DWR dynamics are considered, for the rest of this work, to be constrained to a finite motion set. The first motion is defined by the fixed parameter values $v = 0$ and $\omega = \omega_0$, corresponding to a rotation on the spot of the robot. The second motion describes the roto-translation of the robot and is defined by the parameter pair $v = v_1$ and $\omega = \omega_1$. These particular motion configurations are retrieved from [4, 6], where the self-aggregation control problem is considered. In [4], an optimal control law is obtained using a limited amount of environment information, while in [6], a similar switched formulation of the system exploiting a control Lyapunov function is proposed.
Let us introduce the switching signal $\sigma(t) \in \{0, 1\}$, which indicates the active vector field of the system. Then, the switching formulation of system (2) can be written as follows

$$\dot{p}(t) = f_{\sigma(t)}(p(t)), \tag{3}$$

where $f_{\sigma(t)}$ belong to the set of vector fields $\{f_0, f_1\}$ with

$$f_0 = \begin{bmatrix} 0 \\ 0 \\ \omega_0 \end{bmatrix}, \quad f_1 = \begin{bmatrix} v_1\cos\theta \\ v_1\cos\theta \\ \omega_1 \end{bmatrix}. \tag{4}$$

Analogously to [4], we obtain the parameters ω_0, v_1, and w_1 by applying the vector $[\ v_{l1}\ ,\ v_{r1}\ ,\ v_{l0}\ ,\ v_{r0}\ ] = [-0.7, -1, 1, -1]v_{\max}$ to (1), where $v_{\max}$ is the maximum linear velocity of the robot wheels.
On the other hand, a relevant focus point of the work is the analysis and design of switching laws in the presence of uncertainties and disturbances in the mobile robot dynamics. In particular, multiplicative and additive characterizations of the uncertainties are considered. The system expression of (2) with multiplicative disturbances is formulated as follows

$$\begin{aligned} \dot{p}_x &= ((1 + d_1)\cos(\theta) - d_2\sin(\theta))v_\sigma \\ \dot{p}_y &= ((1 + d_1)\sin(\theta) + d_2\cos(\theta))v_\sigma \\ \dot{\theta} &= (1 + d_3)\omega_\sigma. \end{aligned} \tag{5}$$

Note that, for the remainder of the work, the term d_3 will be considered null or negligible, compared to d_1 and d_2, assuming its effect lumped in the overall contribution. Furthermore, it is possible to consider the multiplicative uncertainty as a mode-dependent disturbance acting on the system, i.e.,

$$\begin{aligned} \dot{p}_x &= v_\sigma\cos(\theta) + d_x(\sigma) \\ \dot{p}_y &= v_\sigma\sin(\theta) + d_y(\sigma) \\ \dot{\theta} &= \omega_\sigma. \end{aligned} \tag{6}$$

It is worth highlighting that by considering a multiplicative description of the uncertainty acting on the system, it is implicitly assumed that the effect of the uncertainty, in the case of $\sigma(t) = 0$, is null. This may be a stringent condition on the disturbances acting on the system, e.g., it prohibits the presence of external influences on the system dynamics. In order to account for this type of disturbance, a model with additive disturbances is defined as follows

$$\begin{aligned}\dot{p}_x &= v_\sigma \cos(\theta) + d_x \\ \dot{p}_y &= v_\sigma \sin(\theta) + d_y \\ \dot{\theta} &= \omega_\sigma\end{aligned} \tag{7}$$

The main difference between models (6) and (7) resides in the fact that in the multiplicative disturbance case, the uncertainties are independent of the value of the switching signal σ; indeed, when $\sigma = 0$, disturbances do not affect the system. Moreover, if instead of using p_x and p_y as state variables, we consider as state variables the deviation e_x and e_y from a time-varying reference position, the model expression (7) is obtained and the disturbances can be interpreted as the rate of change of the desired reference position.

Problem statement

This thesis considers the so-called parking problem, which consists in the stabilization of each robot composing the system around a given desired reference position while avoiding collisions among agents and obstacles, for a network of robotic agents, with constrained dynamics as in (3).

3. Proposed control approaches

This section presents several control strategies based on model predictive control for the nominal system (3) and in the presence of disturbances (6) and (7). To this aim, the components of the finite horizon optimal control problem (FHOCP) are presented in the following.

3.1. Nominal design

We start by illustrating the proposed approach for a nominal scenario with no disturbances acting on the system.

Discrete model

In order to apply the receding horizon strategy, a discrete system model has to be derived. Following a standard discretization technique, the discrete model for the ith robot, with sampling time T, is written as

$$p^{[i]}(k+1) = f_{\sigma(k)}(p^{[i]}(k)), \tag{8}$$

where

$$f_0 = \begin{bmatrix} p_x^{[i]}(k) \\ p_y^{[i]}(k) \\ \theta^{[i]}(k) + T\omega_0 \end{bmatrix},$$

$$f_1 = \begin{bmatrix} p_x^{[i]}(k) + v_1 \frac{\sin(\theta^{[i]}(k)+T\omega_1) - \sin(\theta^{[i]}(k))}{\omega 1} \\ p_y^{[i]}(k) + v_1 \frac{-\cos(\theta^{[i]}(k)+T\omega_1) + \cos(\theta^{[i]}(k))}{\omega_1} \\ \theta^{[i]}(k) + T\omega_1 \end{bmatrix}.$$

Cost function

Our case study considers a reference tracking problem of a system of N robots over a prediction horizon of length N_p. Therefore, a quite natural cost function is of the following form

$$\begin{aligned} J &= \sum_{k=0}^{N_p} l(p(k), \bar{p}), \\ l(p(k), \bar{p}) &= \sum_{i=1}^{N_{rob}} (p_x^{[i]} - \bar{p}_x^{[i]})^2 + (p_y^{[i]} - \bar{p}_y^{[i]})^2, \end{aligned} \tag{9}$$

where $\bar{p}^{[i]} = [\bar{p}_x^{[i]}, \bar{p}_y^{[i]}]$ is the reference position of the ith robot.

Constraints

In order to achieve the parking objective in a challenging environment, two types of constraints, obstacle and inter-robot collision avoidance, need to be introduced.
Regarding obstacle avoidance, without loss of generality, let us consider circular obstacles of radius R_{obs}, and center p_{obs}.
Therefore, the obstacle avoidance constraint for the robot i can be formulated as a constraint on the distance between the robot and the obstacle. As described in [3], this type of constraint can be relaxed as a set of n linear inequalities constructing an outer polytope approximation. The collision checking is performed by the evaluation of n inequalities of the type

$$H_n p^{[i]}(k) \geq S_n, \tag{10}$$

where H_n and S_n contain the information of the polytope and the minimum distance between a robot and an obstacle, respectively. If at least

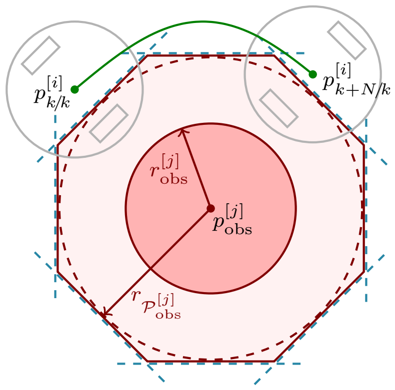

Figure 2: Polytope $\mathcal{P}^{[j]}_{\text{obs}}$ (blue dashed lines) associated to the jth obstacle (red circle). The pink area represents the linear approximation of the dashed red circle, while the green line depicts the predicted trajectory of the ith robot (gray circle)

one of the inequalities is satisfied, it implies that the ith robot lies in the exterior of the polyhedron, and the collision is avoided. Note that the described procedure does not offer an advantage in the nominal case (no disturbances) since the complexity of the optimization problem to be solved is not affected by the nonlinear nature of the constraint, being a pure integer programming problem due to the discrete nature of the switching signal.
Moreover, for a centralized implementation, it is possible to consider a robot as an obstacle with a time-varying position. Therefore, by exploiting the same procedure for obstacle avoidance (with a slight modification due to the time-varying nature of the constraint), it is possible to formulate constraints for inter-robot collision avoidance with the matrix $H_n(k)$ and $S_n(k)$. In principle, these procedures must be applied to all robot-obstacle and robot-robot pairs. However, it is convenient to consider constraints only between the robots and obstacles that are sufficiently close to each other in order to reduce the complexity of the optimization problem.

3.2. Robust design

In this section, a robust formulation of the previous approach is presented.

Discrete robust model

The robust controller design uses the multiplicative disturbances model defined in (6). Furthermore, a norm-bound constraint on the disturbances will be assumed, i.e., $d_x^2 + d_y^2 < D_M^2$, and the nominal evolution of the trajectories can be computed as in (8). It is also useful to evaluate the set containing all possible trajectories. In particular, the prediction error of the Cartesian position $e(k) = \begin{bmatrix} e_x(k) & e_y(k) \end{bmatrix}$ is contained in the set

$$\begin{aligned} &E(k) = E(k-1) \oplus \sigma^{[i]}(k)\mathcal{B},(0, D_M) \\ &E(0) = \{0\}, \end{aligned} \tag{11}$$

where $\oplus$ is the Minkowski sum[1], and $B(0, D_M)$ is the set that defines a ball centered at the origin with radius D_M. Then, it is easy to derive that the set E is equal to

$$\begin{aligned} &E(k) = B(0, R_t(k)) \\ &R_t(k+1) = R_t(k) + D_M \\ &R_t(0) = 0 \end{aligned} \tag{12}$$

Cost function

As common in robust MPC formulation, the worst-case cost function will be considered, i.e.,

$$\begin{aligned} J_w &= \sum_{i=1}^{N} \bar{J}_w^{[i]}, \\ \bar{J}_w^{[i]} &= \sum_{k=0}^{N_p} \bar{l}_w(p^{[i]}(k), \bar{p}^{[i]}, R_c^{[i]}(k)), \end{aligned} \tag{13}$$

where l_w is the maximum distance from the desired set to a point located on a ball centered on $p^{[i]}(k)$ with radius $R_t^{[i]}(k)$.

Constraints

Regarding robust constraint satisfaction, we can consider a progressive tightening of the nominal constraints defined in Section 3.1. In particular, it is possible to ensure the fulfillment of the robustness property at a generic time instant $\bar{k}$ of the prediction horizon by enforcing the constraint

$$H_{\bar{n}} p^{[i]}(k) \geq S_{\bar{n}} + R_t^{[i]}(k) \tag{14}$$

[1]The Minkowski sum of two sets A and B is formed by adding each vector in A to each vector in B

This constraint ensures a margin of $R_t^{[i]}(k)$ before the violation of the constraint, and therefore, the original constraint is automatically satisfied for each possible disturbance realization. In case of an inter-robot constraint, it is necessary to consider the uncertainty related to all the robots involved, and therefore the considered tightening for two robots, i and j, will be $R_t(k) = R_t^{[i]}(k) + R_t^{[j]}(k)$.

3.3. Stochastic design

It can be noticed that the robust approach of the previous section is quite conservative. A stochastic formulation has been developed and presented in the following to improve this limitation.

Discrete stochastic model

The controller design will consider the multiplicative disturbance model (6) as in the robust case. In addition, it will be assumed that the disturbances are zero-mean white noise with covariance Σ_D. Therefore, being the considered disturbances white noise signals, the variance can be computed as

$$\begin{aligned}\Sigma_x^{[i]}(k+1) &= \Sigma_x^{[i]}(k) + \sigma^{[i]}(k)\Sigma_D,\\ \Sigma_y^{[i]}(k+1) &= \Sigma_y^{[i]}(k) + \sigma^{[i]}(k)\Sigma_D,\end{aligned} \tag{15}$$

where $\Sigma_x(0)$ and $\Sigma_y(0)$ depends on the initial uncertainty of the robot position.

Cost function

As usual in stochastic MPC formulations, the proposed cost function is the expectation of the original cost function (9) over the distribution of the disturbance.

$$J_s = \mathbb{E}[J] = \mathbb{E}[\sum_{k=0}^{N_p} l(p(k), \bar{p})], \tag{16}$$

which can be easily expressed in terms of expected value $\mu_x^{[i]}, \mu_y^{[i]}$ and variance $\Sigma_x^{[i]}, \Sigma_y^{[i]}$ of p_x, p_y.

Constraints

The constraints defined in Section 3.1 have been reformulated in terms of chance constraints, in other words, constraints that bound the probability of violation,

$$\Pr(H_n p^{[i]}(k) \leq S_n) \leq \epsilon, \tag{17}$$

where ϵ is a design parameter representing the maximum probability of violation allowed and tuned to obtain a trade-off between performance and constraint satisfaction. Constraints of this type can be reformulated in a deterministic framework resorting to the Cantelli inequality as

$$H_n \mu_p^{[i]}(k) \geq S_n + f(\epsilon)\sqrt{H_n \Sigma_p^{[i]}(k) H_n^T}, \tag{18}$$

where

$$f(\epsilon) = \sqrt{\frac{1-\epsilon}{\epsilon}}. \tag{19}$$

The inter-robot collision avoidance constraint can be considered as the total uncertainty related to the robots; therefore, the joint variance expression for two robots, i and j, is $\Sigma_p(k) = \Sigma_p^{[j]}(k) + \Sigma_p^{[i]}$.

3.4. Tube based design

Differently from the previous section, the MPC control law in this section takes advantage of the knowledge of a stabilizing switching law and considers the model (7). In particular, the switching signal to be applied is calculated based on the auxiliary switching law $\bar{\sigma}(p^{[i]}(k), \bar{p}(k))$, while the reference $\bar{p}(k)$ is computed in a receding horizon fashion.

Auxiliary switching law and 0-reachability

To this scope, a brief analysis of the underlying switched system has to be presented. In order to simplify the analysis of the system, let us perform a change of coordinates such that one obtains

$$\begin{aligned} z_1 &= p_x \sin(\theta) - p_y \cos(\theta)\\ z_2 &= p_x \cos(\theta) + p_y \sin(\theta)\\ z_3 &= \theta. \end{aligned} \tag{20}$$

With respect to the above change of coordinates, which corresponds to a rotation of the reference frame, the equation describing the evolution z_1 and z_2 is governed by the following switched affine system (SAS)

$$\begin{aligned} z(k+1) &= A_{d\sigma} z + B_{d_\sigma} \\ A_{d\sigma} &= e^{(A_\sigma T)} \\ B_{d\sigma} &= \int_0^T e^{A_\sigma \tau} B_\tau d\tau, \end{aligned} \tag{21}$$

where

$$A_{d\sigma} = \begin{bmatrix} \cos(\omega_\sigma T) & \sin(\omega_\sigma T) \\ -\sin(\omega_\sigma T) & \cos(\omega_\sigma T) \end{bmatrix}, \tag{22}$$

$$B_{d\sigma} = \begin{bmatrix} \frac{-v_\sigma(\cos(\omega_\sigma T)-1)}{\omega_\sigma} \\ \frac{v_\sigma \sin(\omega_\sigma T)}{\omega_\sigma} \end{bmatrix}. \tag{23}$$

Note that the state variable z_3 does not affect the subsystem dynamics, that is composed by the state $[z_1,\ z_2]$ only. Furthermore, the equilibrium $\bar{p} = [0,\ 0,\ \theta]^\top$ is mapped to $[\bar{z}_1,\ \bar{z}_2] = [\ 0,\ 0]$. Thus, the problem of Cartesian regulation is equivalent to the regulation of the subsystem $[z_1,\ z_2]$.
Moreover, it is possible to find a practically stabilizing switching law. To this scope, consider the candidate control Lyapunov function $V(z) = z_1^2 + z_2^2$ and its variation with respect to the time $\Delta V = V(z(k+1)) - V(z(k))$. Therefore, it can be shown that the switching law

$$\bar{\sigma} = \arg\min_{\sigma} \ \Delta V_\sigma$$

ensures global practical asymptotically stability. The next main step consists of analyzing the perturbed closed-loop system

$$z(k+1) = A_i z(k) + B_i + D_z \ \forall z(k) \in \Omega_i. \tag{24}$$

Systems of this form are called piecewise affine systems (PWA) and are common models used to describe hybrid and nonlinear systems. The tube-based control law is based on the computation of the 0-reachable space from the disturbances $\mathcal{R}$, assuming a norm bound on the disturbances $d_x^2 + d_y^2 < D_M^2$. It is important to define the quantity R_M, defined as

$$R_M = \max\{r \mid [z_1, z_2] \in \mathcal{R}, z_1^2 + z_2^2 \leq r^2\} \tag{25}$$

that represents the maximum deviation from the origin considering the defined set of disturbances.

Cost function

Again a natural cost function to consider is the deviation of the reference position of the auxiliary switching law from the desired reference position, i.e,

$$J = \min_{\Delta\bar{p}(k)} \sum_{i=1}^{N_{rob}} \sum_{k=\bar{k}}^{N_p+\bar{k}} \bar{p}^{[i]\top}(k)\bar{p}^{[i]}(k). \tag{26}$$

Constraints

Obstacle avoidance constraints and inter-robot collision avoidance constraints can be included as done in Section 3.1, considering as the robot radius the quantity $R_M + R_{rob}$. Furthermore, it has to be ensured that the rate of change of the reference position, $\Delta\bar{p}(k)$, of the switching auxiliary law is compatible with the assumed quantity R_M. In practice, once the maximum value D_d on the norm of the disturbances is defined, the maximum norm change rate of the reference position D_p must satisfy $D_p^2 \leq D_M^2 - D_d^2$. This type of nonlinear constraint can be managed with polytopic approximation.

3.5. Comments

Different from the approaches in the previous section, the tube-based design leads to a quadratic programming problem, which is much easier to solve than the integer programming problem of the other approaches.

4. Simulation results

In order to assess the performance of the proposed algorithms, several simulations have been performed. For the sake of brevity, a very simple scenario is presented. A network composed by $N_{rob} = 3$ mobile robot is proposed, initial state $p_1 = \begin{bmatrix}-1.6 & -1.6 & 0\end{bmatrix}$, $p_2 = \begin{bmatrix}1.2 & 1.4 & 0\end{bmatrix}$, $p_3 = \begin{bmatrix}-1.6 & -0.4 & 0\end{bmatrix}$, reference position $r_1 = \begin{bmatrix}1.2 & 1.3\end{bmatrix}$, $r_2 = \begin{bmatrix}-1.6 & -0.6\end{bmatrix}$, $r_3 = \begin{bmatrix}1.2 & -0.8\end{bmatrix}$, sampling time $T = 0.1$ s, and as physical parameters for the robot: Robot radius $R_{rob} = 0.055$ m, inter-wheel distance $d_{iw} = 0.105$ m, wheel radius $R = 0.016$ m, maximum linear velocity $v_{\max} = 0.2$ m/s. In the following, red circles represent robot position, blue circles represent obstacles in the environment and squares represent the desired reference position.

Nominal design

As expected, the nominal algorithms are able to steer the network to the desired reference position while avoiding collision (Figure 3).

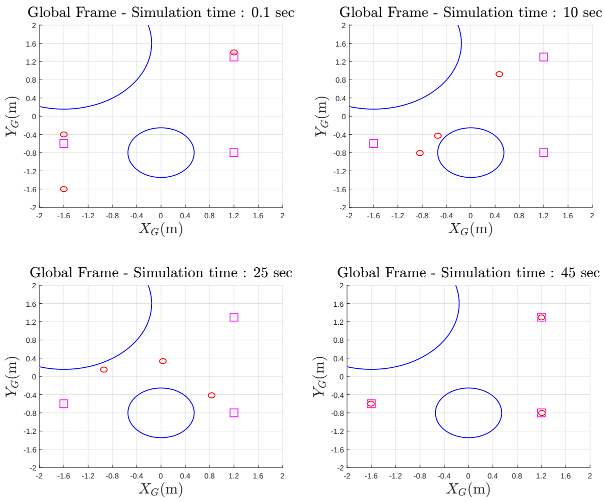

Figure 3: Simulations for the nominal MPC

Robust design

For the robust algorithm have been considered disturbances acting on each robot characterized by $D_M = 0.4Ta_1$.

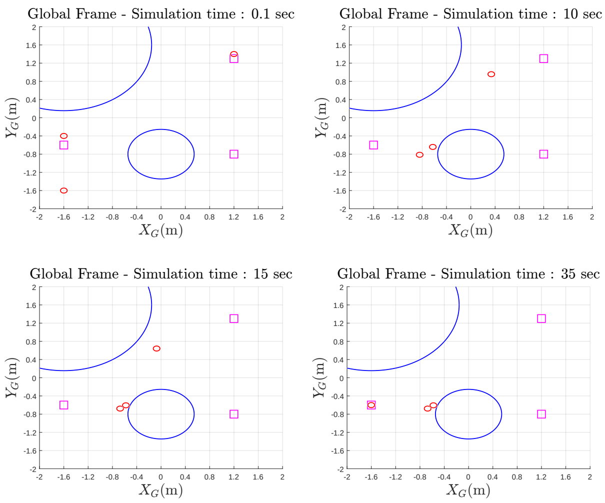

Figure 4: Simulations for the robust MPC

The overall conservativism of the robust formulation makes the network fail to converge to the desired reference position (Figure 4).

Stochastic design

For the stochastic formulation, a standard deviation of the disturbances equal to $\frac{0.4Ta_1}{3}$ has been considered to better compare with the robust formulation. Furthermore, $\epsilon = 0.1$ has been considered.

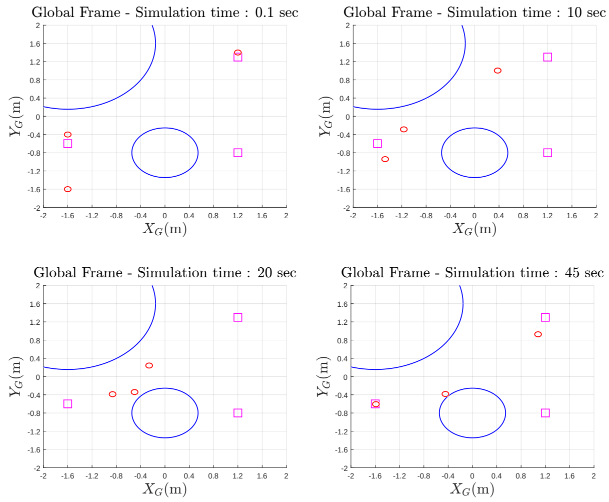

Figure 5: Simulations for the stochastic MPC

The reduced conservativism of the stochastic formulation allows the network to reach the desired reference position (Figure 5).

Tube based design

For the tube-based approaches, it is considered a scenario with no disturbances and a maximum rate of change of reference for the auxiliary law equal to $D_p = 0.4Ta_1$. Black circles of radius R_M represent the area where the robots can lie (Figure 6).

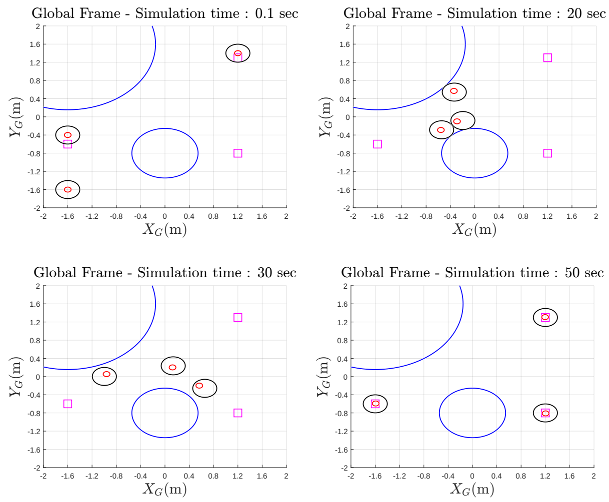

Figure 6: Simulations for the tube MPC

5. Conclusions

This work aimed to develop control strategies for the trajectory tracking of a multiple mobile robot system in situations where disturbances are present. The approaches developed in this work rely on robust and stochastic reformulation of switching model predictive control (SMPC) strategy and on the computation of disturbances reachable set. Future developments can be done in many directions, such as considering more complex robot models or a greater number of allowed motions or integrating the design of the proposed approaches with controllers located in an inner loop to limit the effect of disturbances.

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