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EXECUTIVE SUMMARY OF THE THESIS

A Numerical Study on Mechanical Metamaterials in Classical Guitars

LAUREA MAGISTRALE IN MUSIC AND ACOUSTIC ENGINEERING

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1. Introduction

The last few decades have seen significant progress in the field of mechanical metamaterials, i.e. composite materials which exhibit unique mechanical properties derived from their structure rather than their composition [1]. In particular, many of these composites have been shown to exhibit remarkable stiffness-to-density ratios [2], a property which is widely sought after in materials used in the construction of musical instruments, as it ensures efficient sound radiation.

When it comes to materials for musical instruments, indeed, one of the most widely adopted is wood, especially in the main radiating components of chordophones. Wood has many advantages for these applications, but it presents a high degree of variability in its mechanical properties [3]. This results in unpredictable and inconsistent mechanical response across different samples of the same wood species. Furthermore, the specific kind of woods that are most used in instrument making are prone to shortages due to deforestation or climate change-related habitat shrinking [4]. Having the possibility of deliberately engineering the vibrational behaviour of a material would be, therefore, of great interest for the design of musical instruments. This could allow for a better use of the supplies of already in use wood species, or for the adoption of different and more abundant types. Furthermore, it would also afford makers a new degree of control over their instruments, allowing for all kinds of innovative instrument designs.

Mechanical metamaterials, by their very nature, could represent a tool to achieve such a goal. This work is mainly inspired by recent findings which show that the elastic behaviour of thin rectangular wooden plates can be purposefully altered by carving them with specific patterns of holes [5]. These holes are elliptical and arranged periodically by repetition of a fixed unit cell. Effectively, this creates a wooden mechanical metamaterial, whose mechanical response can be tuned by changing the holes' size, orientation and aspect ratio.

In light of these results, we investigate the effect of similar wooden metamaterials in the soundboards of classical guitars. We set out to study how they would impact the sound of the instrument, as well as its structural integrity under the tension of the strings. Our goal is to find out whether these materials can produce better sounding instruments, and how much control they allow over the final tuning of the guitar.

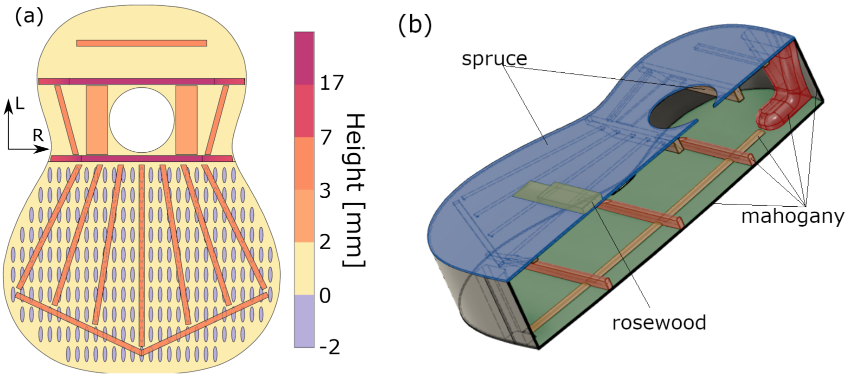

Figure 1: (a) Diagram of the underside of the soundboard with longitudinal holes ($p = 1$), showing the braces. The colors mark the height of each surface with respect to the lower surface of the soundboard. The arrows illustrate the direction of the longitudinal (L) and radial (R) axes of the soundboard's material. (b) 3D cut view of the model of the instrument's body, showing all the individual components. The choice of material for the main parts is also reported.

2. Methodology

All the studies in this work were carried out by means of finite element simulations (FEM). The reference 3D model of the guitar body was based on an 1884 instrument by Antonio de Torres, as described in [6], and it was realised with Autodesk Fusion 360®. This model includes a traditional bracing design, with 7 fan braces glued on the underside of the lower bout of the soundboard, all 3 mm thick. A simplified model of the bridge, used in the static load simulations (see next section), is also included. The soundboard, which has a uniform thickness of 2.5 mm, was then altered by cutting a pattern of elliptical cavities in the underside, to a depth of 2 mm. These holes aren't present throughout the entire extent of the plate, as they are instead limited to the lower bout of the instrument. The hole patterns are generated by juxtaposition of several copies of the same unitary cell. The cell is rectangular, with a size of 26×12 mm for all simulations, but we distinguish between two kinds of holes based on its orientation: when the long side is parallel to the longitudinal axis of the soundboard's wood, we'll talk about *longitudinal* holes, while when it's aligned with the radial axis, we will talk about *radial* holes. The orientation of the material axes of the wood can be seen in Fig. 1(a). The holes are centered with respect to the cell, and their size is varied while keeping the aspect ratio constant, by changing a dimensionless scaling parameter $p \leq 1$ that multiplies the sizes of the major and minor axes of the ellipses. In the cell with $p = 1$, the major semiaxis of the elliptical holes is 10 mm long, while the minor semiaxis is 2.5 mm long. If for example we consider instead $p = 0.6$, the major semiaxis will be 6 mm long, and the minor semiaxis will be 1.5 mm long. The results reported in the next section are limited, for brevity, to the cases with $p = 1$, with both orientations; the thesis contains results for various sizes, but in most cases the observed effects are qualitatively similar to those reported here, albeit less pronounced. Fig 1(a) shows a diagram of the underside of the soundboard for the case of longitudinal holes with $p = 1$. Simulations were also done for the reference case of the solid top plate, i. e. the traditional top plate without holes, which is the point of comparison for the metamaterial designs.

In Fig. 1(b) we report a cut view of the entire body of the instrument, showing the choice of wood for each component: namely, these are Engelmann spruce for the soundboard and the soundboard bracing, Honduran mahogany for the sides, back and back braces, and rosewood for the bridge. All the materials have been modeled as linear elastic and orthotropic, taking the mechanical properties for each particular wood (at 12% moisture content) from [7]. Appropriate care was taken to correctly define the material axes of each component of the model. The simulations are done using COMSOL Multiphysics® simulation software, in particular the Solid Mechanics interface from the Structural Mechanics module.

A useful metric to compare different soundboards is the *sound radiation coefficient* $R = \sqrt{E/\rho^3}$ [8, 9]. This quantity, when fixing a target modal density for a given plate, is proportional to the average level of its driving point mobility. Therefore, increasing R results in better acoustic radiation efficiency without drastic timbral changes in the resulting sound. As such, this quantity can be used as a measure of how loud we expect an instrument with a given soundboard to be. We will thus refer to this quantity when analyzing our results.

3. Results

3.1. Eigenfrequency studies

We have performed a series of eigenfrequency studies where we compared the eigenfrequencies and modal shapes of the unbraced soundboards first, then of the soundboards including the bracing, and finally of the complete body of the guitar. These simulations are aimed at studying the effect of the metamaterials in these different configurations, with the goal of finding out whether their impact remains relevant when connecting the soundboards to the braces first, and then to the rest of the body. For the simulations of the soundboards alone, the thesis also contains results obtained in free boundary conditions. For brevity, here we only report and discuss the results in simply supported boundary conditions, which provide a better approximation of the situation where the soundboard is glued to the ribs of the guitar.

Figure 2 shows a comparison between the eigenfrequencies in three different configurations: the unbraced simply supported soundboard, the braced simply supported soundboard, and the complete instrument body (without the bridge). Each plot shows the first 12 eigenfrequencies in the case of the solid top plate and those of the longitudinal ($p = 1$) and radial ($p = 1$) holes. In the unbraced plates, most eigenfrequencies are lowered, if slightly, when introducing the metamaterials. There seems to be a clear separation between the case of the longitudinal holes and that of the radial holes, however: the eigenfrequencies are indeed lowered far more in the latter case, with those for modes 5 and 12 showing a decrease of $\sim 8\%$. As for the braced plates, we found that some of the frequencies are clearly increased or decreased with varying hole size, but overall we can't identify collective upwards or downwards trends. However, the variations are less pronounced than in the unbraced plates, as the largest is $\sim 5\%$ for the longitudinal holes and $\sim 3.5\%$ for the radial holes. Finally, the eigenfrequencies of the complete body are barely altered by the introduction of the metamaterial: the changes relative to the frequencies of the body with the solid top do not exceed $\sim 3\%$.

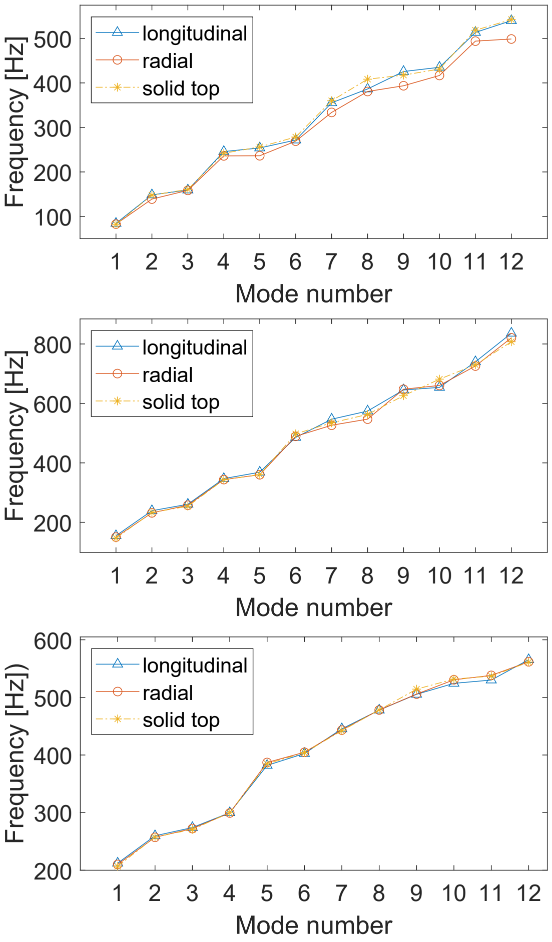

Figure 2: Eigenfrequencies of the unbraced (top) and braced (middle) soundboards in simply supported boundary conditions, and of the guitar body (bottom). For all configurations we report values for the case with the solid plate and with longitudinal ($p = 1$) and radial ($p = 1$) holes.

The relatively small impact of the metamaterials on the modal behavior of the instrument is confirmed by comparing the modal shapes in the various configurations with the Modal Assurance Criterion: the modal shapes are also minimally modified, especially in the case of the complete body.

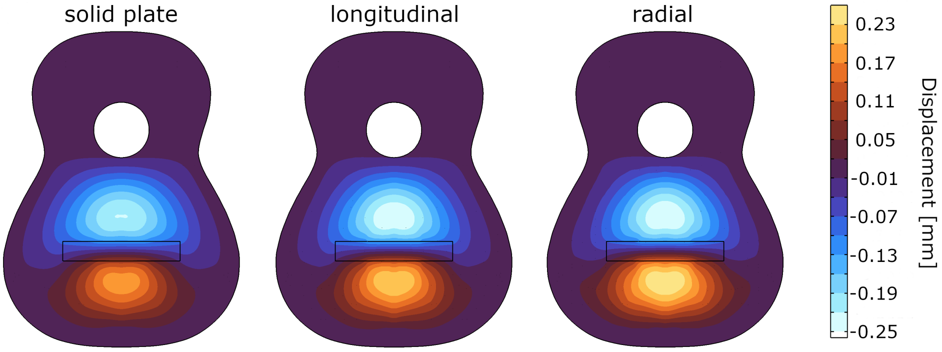

Figure 3: Normal displacement on the surface of the soundboard under the static load of the strings for the solid plate, longitudinal ($p = 1$) and radial ($p = 1$) holes.

3.2. Deformation Under Tension

The eigenfrequency studies were followed by static simulations aimed at assessing the effect of the metamaterials on the ability of the instrument to withstand the tension of the strings. Assuming a typical range of tensions of 50-80 N for nylon guitar strings [10], a load totaling 480 N was applied to six small circular areas with a 1 mm radius, located in the back of the bridge (which is included in the numerical model in these simulations), corresponding to the points where the strings are mounted on a real guitar. A fixed constraint was applied on the ribs.

Fig. 3 shows the resulting normal displacement on the surface of the soundboard for the solid plate and for the metamaterial plates with longitudinal ($p = 1$) and radial ($p = 1$) holes. The deformation is only slightly accentuated with the metamaterials, suggesting their viability with regards to the structural stability of the instrument. However, the deformation is also noticeably more pronounced with the radial holes, suggesting a sharper decrease of the longitudinal stiffness of the soundboard in this case, consistently with [5]. We have also compared the quadratic mean of the displacement in the top plate with the cubed effective density of the plate's lower bouts, in order to get a measure of how the sound radiation coefficient R changes with different hole sizes and orientations. A clear upwards trend with increasing hole size appears both for the longitudinal and radial orientations, with a steeper increment in the former case. Overall the results suggest that the longitudinal holes are a more suitable choice with regards to the capacity of the soundboard to withstand the tension of the strings.

We then varied the thickness d of the braces in the case of the longitudinal holes with $p = 1$, with the aim of trying to compensate for the added deformation. The braces are originally 3 mm thick, and they have been uniformly increased in thickness in 0.5 mm increments up to 5 mm total. These increments have been applied to the fan braces in the following combinations: referencing Fig. 1(a) and numbering the braces from left to right, the central brace (brace 4) was modified first, then the three pairs of braces 3 and 5, 2 and 6 and 1 and 7. Fig. 4 shows the quadratic average of the difference between the normal displacements in the metamaterial soundboard with the modified braces and in the solid soundboard with normal bracing. For each combination of modified braces, we also reported a best fit to a second degree polynomial in the thickness d. Clearly, intervening on the two outermost pairs of braces (1 and 7 and 2 and 6) has little to no effect. Results are quite good for braces 3 and 4 with $d = 4$ mm. However, the best compensation is seen by adding thickness to the central brace, with the optimal value of d lying between 4.5 and 5 mm. In this case, by increasing d up to 4.5 mm, we have added back as little as 0.88 g of material to the soundboard, thus obtaining a soundboard that can sustain the tension of the strings while being significantly lighter than in the solid case.

4. Mobility Frequency Response

Finally, we performed frequency-domain simulations to obtain the driving-point mobility of the complete body of the instrument. These

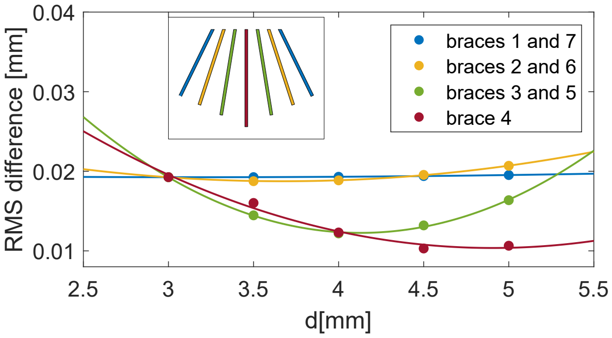

Figure 4: Quadratic average over the lower bout of the difference between the displacements in the metamaterial soundboards and the solid plate, for varying thickness of the braces. The solid lines represent the results of a best fit to a quadratic polynomial. A diagram of the 7 fan braces with the same color coding used in the plot is also shown.

were obtained by imposing a normal, harmonic load on a circular area with a 2 mm radius on the top surface of the soundboard, located under the bridge, in the middle of the lower bout, and then recovering the normal component of the velocity at the same place. Fig. 5 shows the magnitude of the bridge mobility for the solid top and the largest ($p = 1$) longitudinal and radial hole patterns, together with the first three modes of the instrument. In order to compare the responses in these three cases we compute the average of their magnitude. Indeed, we expect the sound radiation coefficient $R = \sqrt{E/\rho^3}$ to be modified with the inclusion of the metamaterials [5]. This should affect the average value of the driving-point mobility of the soundboard [8], as mentioned in Section 2. The average values are $0.120\,\mathrm{m\,N^{-1}\,s^{-1}}$ with the solid plate, $0.154\,\mathrm{m\,N^{-1}\,s^{-1}}$ with the longitudinal holes ($p = 1$), and $0.152\,\mathrm{m\,N^{-1}\,s^{-1}}$ with the radial holes ($p = 1$). Here we see a clear advantage in the use of the metamaterials: as we have seen in the eigenfrequency studies, the modal behavior of the instrument isn't substantially affected, and yet there is a substantial increase in the average levels of the mobility response. As such, we have obtained an instrument which has similar timbral characteristics as the traditional design, while being louder.

5. Conclusions

Our work has shown that the use of wooden mechanical metamaterials in the soundboards of classical guitars is feasible, and in many ways beneficial. We have found that the modal behavior of the instrument isn't radically modified by the metamaterial, meaning that the traditional timbre of the instrument is preserved. This is expected as the mechanical response of the instrument is determined by many material parameters related to its different components, and the elastic moduli of the soundboard have been shown not to be the most important in this respect [11]. At the same time, the metamaterial instruments have been shown to have higher average levels of the driving point mobility, so we expect them to be noticeably louder, a feature which is highly appreciated in classical guitars. The impact of the orientation of the holes on the dynamical behavior of the instrument aren't very clear-cut, which suggests that most of the observed effects are due to the reduction in mass, with the variation in the effective elastic moduli of the soundboard being compensated for by the bracing and the rest of the structure. At the same time however, the orientation seems to be relevant with regards to the structural stability of the guitar: in our static load studies, the soundboards with longitudinal holes showed consistently less deformation compared to those with radial holes. Overall, the longitudinal holes seem to be the most favourable choice.

There are many possibilities for future developments of the present work: from furthering the studies we presented by experimenting with different geometries of the hole patterns, to the extension of similar investigations to different instruments. The most obvious follow-up, however, is the experimental validation of our findings. Our virtual designs for the soundboards can be faithfully reproduced with a CNC machine, which would allow for a direct comparison of the numerical and experimental results. Planning for the realization of soundboards and instruments with metamaterials is currently ongoing, and this next experimental phase will be carried out by collaborators at Politecnico di Milano and Universidad de Chile.

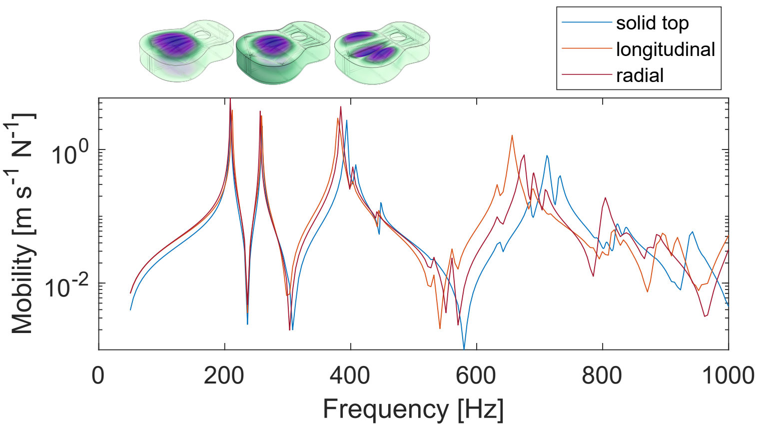

Figure 5: Driving point mobility at the bridge location, for the solid top and metamaterial plates with longitudinal ($p = 1$) and radial ($p = 1$) holes. Above the the first three peaks the shapes of the corresponding modes are reported.

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