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EXECUTIVE SUMMARY OF THE THESIS

Self-corrected dual-comb spectroscopy in the near-infrared by 1-GHz solid state lasers

LAUREA MAGISTRALE IN ENGINEERING PHYSICS - INGEGNERIA FISICA

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1. Introduction

Optical spectroscopy is a powerful tool for the non-intrusive investigation of matter. Traditional broadband methods, such as Fourier Transform Infrared (FTIR) spectroscopy, are limited by incoherent light sources and movement of mechanical components, which restrict acquisition speed and sensitivity. The development of the Optical Frequency Comb (OFC) has bypassed these problems [4]. By emitting a train of ultrashort pulses that corresponds to a dense grid of evenly spaced laser lines, the OFC acts as a precise spectral reference. It combines broad emission bandwidth with high brightness and spatial coherence, enabling the simultaneous excitation of molecular transitions, particularly within the mid-infrared and visible fingerprint regions. Building upon this, Dual-Comb Spectroscopy (DCS) employs two optical combs with slightly detuned repetition rates to perform multi-heterodyne detection [3]. This approach down-converts the optical absorption information directly into the radio-frequency (RF) domain, recording broadband spectra without moving parts and reducing acquisition times. Consequently, DCS can be crucial in several applications, such as environmental remote sensing, breath analysis and high-speed LIDAR systems. However, residual phase and timing jitters rapidly destroy the mutual coherence between the sources, causing spectral smearing and preventing long-term coherent averaging. Traditional hardware-based stabilization loops are bulky and expensive. This thesis shifts the stabilization from hardware to software. First, an offline post-processing self-correction is developed in MATLAB. Second, the work explores the possibility of a real-time on-chip self-correction [3] using an AMD Xilinx RFSoC 4x2 platform, moving the first steps towards more compact dual-comb spectrometers.

2. Optical Frequency Comb Spectroscopy

Optical Frequency Combs (OFCs) generated via cavity mode-locking acts as precise "optical rulers" that establish a coherent bridge between hundred-terahertz optical oscillations and measurable radio-frequency (RF) signals [4].Indeed the discrete emission lines (ν_n) of an OFC are strictly governed by the fundamental comb equation:

$$\nu_n = f_{\text{ceo}} + n f_{\text{rep}} \tag{1}$$

where ν_n are the optical frequencies, n is an integer, f_{rep} is the RF-repetition rate and f_{ceo} is the carrier-envelope offset frequency, which is also in the RF-domain (typically 100 MHz − 1 GHz). To adapt these sources for spectroscopic applications, the pulse train requires power management and spectral optimization. This is experimentally achieved through Chirped Pulse Amplification (CPA), a process that safely boosts pulse energy without inducing non-linear optical damage, followed by supercontinuum (SC) generation to broaden the spectrum. Active hardware stabilization of f_{rep} and f_{ceo} is traditionally necessary to prevent spectral line broadening and enable long integration times. This is achieved through Phase-Locked Loops (PLLs) utilizing piezoelectric actuators and pump current modulation, combined with f-$2f$ self-referencing interferometers to isolate f_{ceo} via heterodyne second-harmonic generation (SHG). While traditional detection systems like mechanical Fourier Transform Infrared (FTIR) spectrometers or bulky Virtually Imaged Phased Array (VIPA) spectrometers are constrained by tracking limits or alignment complexities, it will be discussed now how Dual-Comb setups bypass these limits.

3. Self-Corrected Dual-Comb Spectroscopy

Dual-Comb Spectroscopy (DCS) exploits two optical frequency combs with slightly detuned repetition rates ($f_{\text{rep},1}$, $f_{\text{rep},2}$) to reconstruct molecular absorption features via multi-heterodyne detection [1]. In this work, Comb 1 propagates through the sample undergoing spectral amplitude attenuation and phase shifts, while Comb 2 acts as a local oscillator [1]. When combined at a fast photodiode, the non-linear mixing of the two optical fields down-converts the unresolvable optical frequencies into a detectable radio-frequency (RF) comb. The generated RF comb teeth are located at:

$$f_{\text{RF},n} = |\Delta f_0 + n\Delta f_{\text{rep}}| \tag{2}$$

where $\Delta f_0 = f_{0,1} - f_{0,2}$ is the carrier-envelope offset frequency difference and $\Delta f_{\text{rep}} = f_{\text{rep},1} - f_{\text{rep},2}$ is the repetition rate detuning.
In the time domain, this multi-heterodyne interference manifests as a periodic sequence of interferograms or bursts. Each burst occurs at intervals of $T_r = 1/\Delta f_{\text{rep}}$, which is when the pulses from the two lasers overlap at the detector. This is the temporal mechanism that compresses the absolute optical frequency scale to the RF domain by a scaling factor $a = \Delta f_{\text{rep}}/f_{\text{rep},1}$. To achieve an unambiguous frequency mapping, this down-conversion is bounded by the Nyquist-Shannon sampling theorem. To prevent spectral aliasing, where high-frequency components fold back onto themselves and overlap destructively, the entire RF spectrum must be contained within the first Nyquist zone ($\leq f_{\text{rep},1}/2$). For a target optical interrogation band $\Delta\nu$, this sampling condition imposes an upper limit on the allowable detuning [1]:

$$\Delta f_{\text{rep}} \leq \frac{f_{\text{rep},1}^2}{2\Delta\nu} \tag{3}$$

This quadratic dependence on the repetition rate demonstrates that operating with high-f_{rep} laser sources expands the accessible Nyquist bandwidth [1].

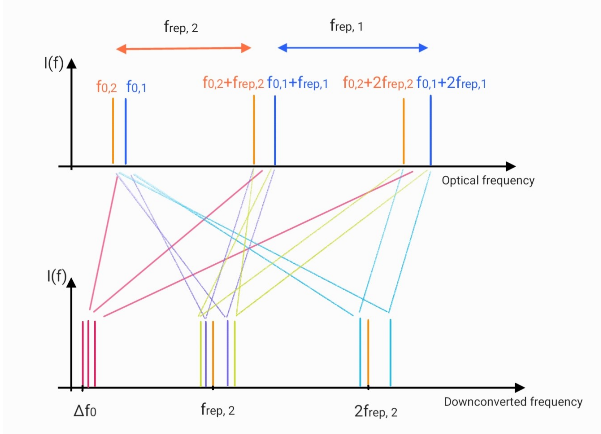

Figure 1: Spectrum of the two optical frequency combs and their beatings.

3.1. Impact of Frequency Noise

In real-world scenarios, continuous cavity fluctuations degrade the performance [3] of free-running dual-comb systems via two distinct mechanisms:

- **Timing Jitter:** Fluctuations in Δf_{rep} cause random variations in the burst repetition period T_r, dynamically distorting the temporal mapping axis.
- **Phase Jitter:** Fluctuations in Δf_0 destabilize the carrier oscillations, causing the in-

terference fringes to randomly slide beneath the pulse envelope from burst to burst. This noise destroys the mutual coherence between the lasers, preventing time-domain coherent averaging and blurring the comb lines. [1].

3.2. Self-Correction Algorithm

To avoid the complex hardware stabilization, this work begins by developing a software-based self-correction post-processing algorithm[3]. The idea behind this algorithm is to convert the real-valued experimental interferogram into its complex analytical representation via a continuous Hilbert transform to extract its instantaneous envelope and phase. Having these information, the signal can be processed to remove both timing and phase jitters. The first one is deleted by tracking the temporal bursts (interferograms) peak positions and realigning them onto an equidistant temporal grid . The second one instead is tracked by sampling the discrete phase values strictly at the bursts maxima. These points are interpolated to construct a continuous time-dependent phase drift vector $\Phi(t)$. Multiplying the analytical signal by the inverse of this extracted phase function achieves exact phase cancellation:

$$s_{\text{corr}}(t) = s(t) \cdot e^{-i\Phi(t)} \tag{4}$$

where $s(t)$ is the temporally realigned analytical signal and $s_{\text{corr}}(t)$ is the phase-stabilized output. This numerical correction restores full long-term mutual coherence.

3.3. Towards Real-Time Self-Correction

While the post-processing algorithm avoids bulky hardware stabilization, it is strictly limited by the memory storage capacity of data acquisition (DAQ) systems, preventing continuous data streaming. Overcoming this barrier requires a real-time stabilization framework where corrections are executed directly on the incoming data stream [3]. To do so, the physical tool chosen in this work is a Radio Frequency System-on-Chip (RFSoC) [2] featuring an embedded Field Programmable Gate Array (FPGA) [2]. Using a digital signal processing hardware implies a mathematical shift from the continuous-time Hilbert transform to the discrete-time one [3]. To implement this

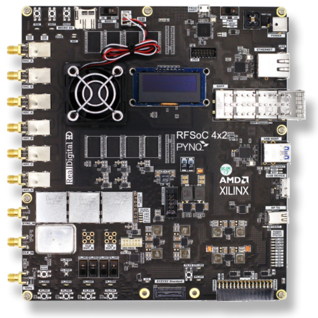

Figure 2: The AMD Xilinx Zynq UltraScale+ RFSoC 4x2 development board configured with a Gen 3 ZU48DR device.

operator on an FPGA, the Hilbert transform is modeled as a digital Finite Impulse Response (FIR) filter. The final expression for the discrete impulse response $\hat{x}_D[n]$ is given by:

$$\hat{x}_D[n] = \begin{cases} \frac{2}{\pi n} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \\ 0 & n = 0 \end{cases} \tag{5}$$

Therefore applying the Hilbert transform to the digitized interferogram $x[n]$ becomes equivalent to performing a discrete convolution between the input signal and this impulse response $\hat{x}_D[n]$. This FIR structure allows the FPGA to generate the complex analytical signal $z[n] = x[n] + i\hat{x}[n]$ sample-by-sample as the data streams, which is essential for a real-time self-correction algorithm. Another key element for a real-time correction, is the platform utilized in this work. A Radio Frequency System-on-Chip (RFSoC) extends the standard SoC architecture (System on Chip) by embedding Analog-to-Digital Converters (ADCs) directly onto the same silicon die alongside the processing core. For dual-comb spectroscopy, this integration allows the signal to be acquired and processed on a single device, eliminating external component latencies. The strength of this platform is an embedded Field Programmable Gate Array (FPGA), a reconfigurable hardware substrate composed of programmable logic blocks and flexible interconnects. By using a Hardware Description Language (HDL), the user can effectively "draw" a custom digital circuit directly onto the silicon. This reconfigurable nature makes this platform flexible and even more suited for real-time laser stabilization.

4. Dual Comb Spectroscopy Setup

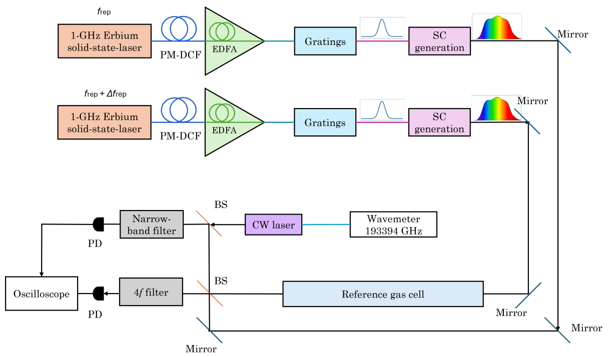

Figure 3: Scheme of the dual-comb spectroscopy setup.

The experimental setup integrates two independent 1-GHz Erbium solid-state lasers operating with a repetition rate detuning Δf_{rep} ranging from 1 to 50 kHz. The pulse trains undergo a sequence of temporal stretching via polarization-maintaining dispersion-compensating fibers (PM-DCF), amplification through Erbium-Doped Fiber Amplifiers (EDFA) and grating-based compression to trigger broadband supercontinuum generation. The broadened output of the first comb interrogates the molecular sample inside a reference gas cell ($^{13}CH_4$ and C_2H_2), while the second comb acts as a local oscillator. Absolute frequency calibration is verified via a co-propagating continuous-wave (CW) laser ($\nu_{\text{CW}} = 192234.9$). After undergoing $4f$ spatial filtering, the multi-heterodyne dual-comb interferogram is captured by a fast (InGaAs) photodiode. The down-converted electronic RF signal is directed to a high-definition digital oscilloscope (or eventually to an AMD Xilinx RFSoC 4x2 board).

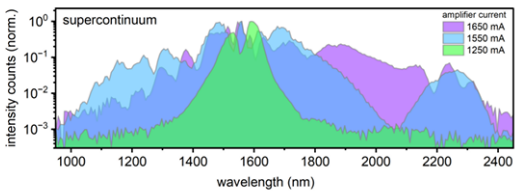

Figure 4: Experimental optical spectrum of the dual-comb supercontinuum at various EDFA currents. The octave-spanning emission (1000–2400 nm) covers the target molecular absorption bands, including the acetylene transitions around 1.55 μm ($\approx$ 6450 cm^{-1}).

5. Experimental Measurements and Results

The results on self-correction stabilization algorithms are organized into two parts. The first one addresses a post-processing algorithm, made with an offline software-based approach to correct free-running laser instabilities directly inside the digital domain from pre-recorded datasets. The second part details the transition toward embedded hardware architectures, starting to approach the possibility of real-time data-streaming on a reconfigurable system.

5.1. Post processing Data Analysis

The offline processing pipeline relies on the separation between data acquisition and processing, using a digital oscilloscope purely as a data recorder. Dual-comb interferograms are captured by a fast photodiode and stored at a sampling rate of 1.25 GSps over a 20 ms time window; the setup achieves a broad near-infrared optical bandwidth spanning approximately 230 nm without aliasing. The post-processing framework is implemented via four sequential scripts:

- **Script 1 (Self-Correction):** Subtracts the background offset, localizes the interferogram bursts via Hilbert envelope detection, and cancels out relative timing and phase fluctuations to enable coherent time-domain averaging.
- **Script 2 (Multi-Trace Averaging):** Aggregates and averages successive data files to maximize the signal-to-noise ratio.
- **Script 3 (Baseline & Normalization):** Extracts the upper envelope of the calibrated spectrum to decouple the non-flat supercontinuum intensity profile from the molecular absorption lines, yielding the final normalized transmittance spectrum.
- **Script 4 (Absolute Calibration):** Combines the continuous-wave (CW) reference laser beat-note with multi-heterodyne sampling theory to map the radio-frequency axis into an absolute optical frequency scale.

While Program 1 must strictly initiate the pipeline, the subsequent execution order is flexible.

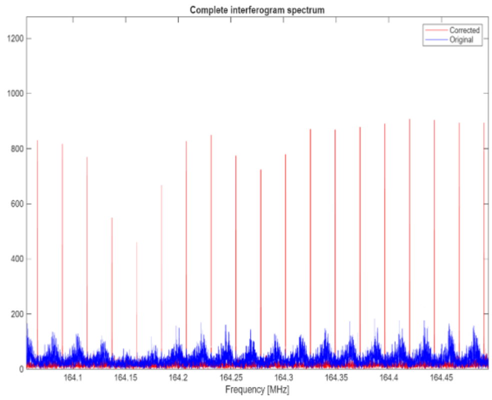

Figure 5: Multi-heterodyne frequency comb structure before (blue) and after (red) digital computational correction.

5.2. Towards Real-Time Analysis

To transition toward real-time stabilization, an adaptation of the algorithm based on the framework by Eber et al.[3] was evaluated. Parallelized Python simulations utilizing FIFO queues and a 101-coefficient FIR filter validated the real-time tracking of phase jitters for a fixed detuning of $\Delta f_{\text{rep}} = 20$ KHz, demonstrating dynamic time-domain resampling on a reference acetylene dataset. Physical hardware capabilities were verified on the AMD RFSoC 4x2 platform [2] using the PYNQ framework and the manufacturer's `base.bit` overlay. This successfully validated the board's RF input channels by acquiring external test waveforms. Finally,

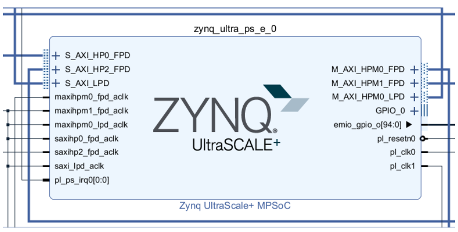

Figure 6: Detail of the Zynq UltraScale+ MPSoC processing system block.

the overlay within AMD Xilinx Vivado was investigated, highlighting the interconnected Processing System (PS), Programmable Logic (PL), and RF Data Converter (RFDC) domains of the board. This confirmed the possibility of implementing custom real-time correction algorithms directly on the chip.

6. Conclusions

This work demonstrates that a free-running dual-comb configuration combined with digital phase correction represents a highly advantageous alternative to heavy phase-locked architectures. By shifting the stabilization from electronic feedback loops to the digital domain, experimental complexity and costs are reduced without compromising the high spectral resolution and mutual coherence.
Moving forward, short-term improvements will focus on the current scripts optimization, for instance by merging the MATLAB post-processing scripts into a single and automated framework to enhance execution speed and user accessibility. In the long term future developments will focus on transitioning the current algorithm from offline post-processing to a fully real-time one. Using the AMD Xilinx RFSoC 4x2 platform, the next step involves implementing the self-correction algorithm directly into the programmable logic using custom Vivado IP cores. Ultimately, this digital framework aligns with the broader scientific effort to transition dual-comb spectroscopy from specialized laboratory setups toward compact and practical real-world applications such as environmental monitoring, LiDAR, and breath analysis.

References

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