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A model for sight deposit liquidity and the impact of a central bank digital currency

Tesi di Laurea Magistrale in
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Abstract

This thesis employs a risk-factor based approach proposed by A. Castagna in 2019 to gauge the liquidity loss distribution of sight deposits issued by Italian banks, using data from the market and the Bank of Italy statistical database.
The risk factors are the risk-free rate, the deposit rates, the credit status of the Italian banking system and the introduction of a digital currency issued by the European central bank. Three designs for the central bank digital currency (CBDC) and various mechanisms modelling the competition between the CBDC and sight deposits are considered. In each scenario, a Monte Carlo method is employed to generate the deposits' liquidity distribution and the value-at-risk is computed as a risk metric.

Keywords: Sight deposits, liquidity risk, digital currency

Abstract in lingua italiana

Questa testi propone un approccio basato su dei fattori di rischio, proposto da A. Castagna nel 2019, con l'obiettivo di valutare ls distribuzione della perdita di liquidità dei depositi a vista emessi da banche italiane, usando sia dati di mercato che serie storiche provenienti dalla base dati statistica di Banca d'Italia.
Le sorgenti di rischio sono i tassi risk-free di mercato, i tassi dei depositi, lo status creditizio del sistema bancario italiano e l'introduzione di una moneta digitale da parte della banca centrale. Tre diversi meccanismi per la valuta digitale (CBDC, da *Central Bank Digital Currency*) e varie dinamiche di competizione tra CBDC e depositi vengono considerate. Per ogni alternativa, un metodo Monte Carlo viene utilizzato per generare la distribuzione della liquidità dei depositi e viene calcolato il *value-at-risk* (VaR) come metrica di rischio.

Parole chiave: Depositi a vista, rischio liquidità, valuta digitale

Contents

1 | Introduction

1.1. Overview of the approach:

Modelling sight deposits and other non-maturing liabilities is a crucial task for financial institutions, especially for banks focused on retail business. For these institutions, sight deposits represent one of the cheapest sources of funding, contributing significantly to the liquidity available for lending activity.
Before the 2008/2009 financial crisis these liabilities usually represented an arbitrage opportunity for commercial banks; indeed it was common to observe deposit rates which were lower than the corresponding rates offered by the central bank deposit facility, giving banks the opportunity for a *"free lunch"*.

However, the landscape has changed since the crisis: a liquidity shortage hit the interbank money market and a steep decrease in interest rates took place. Sight deposits are liabilities featuring a non-maturing nature, since the holder is allowed to withdraw the whole deposited amount without any notice, therefore it became at the same time harder -due to the low interest rate environment- and more crucial than ever -because of the general liquidity shortage- for the asset and liability management divisions of commercial banks to manage the maturity, interest rate and liquidity risks deriving from the issuance of sight deposits.

Forecasting the total volume invested in deposits became the object of renewed attention, since banks and regulatory authorities had to begin factoring in the possibility (or often reality) of a bank run as a response to negative news about the creditworthiness of the bank and/or the interest rate landscape.
It became clear that those volumes could be negatively affected by monetary policy decisions as well, especially since the introduction of negative policy rates, which banks have not passed on to their depositors, fearing a withdrawal of the deposited amounts and an increase of the cash held outside the banking system.

According to the most recent Basel standards, liquidity risk and interest rate risk are modelled separately: liquidity risk is assessed with tools like the *Liquidity Coverage Ratio (LCR)* and the *Net Stable Funding Ratio (NSFR)* and interest rate risk through the revised regulations regarding the bank management of the *Interest Rate Risk in the Banking Book (IRRBB)*. However, banks are allowed to use internally developed models once they are approved by the competent authorities.

We employ a model proposed in Castagna and Mistè [2019] in which it is assumed that cash flows directed towards and away from sight deposits show a behavioral trait that is not considered in the standard approach (see also Formenti [2019]). Including the behavioral component driven by predetermined factors allows to gain a more precise sensing of the maturity risk, which is particularly important for banks focused on the lending and deposit activity, since maturity transformation is a key task.
Castagna's approach enables us to obtain a forecast of future deposit liquidity, depending on the bank's own sight deposit rate, its credit situation and the market risk-free rate. These four building blocks are bound by qualitative assumptions regarding the behaviour of the clients (for example we expect the deposit volume to increase if the deposit rate increases and all the other quantities remain the same). In Castagna and Mistè [2019], an arbitrage-free evaluation formula for sight deposits developed in Jarrow and Van Deventer [1998] is cited and used. With these tools it is possible to manage interest rate risk and liquidity risk.

Sight deposits play an important role not only as investment opportunity available to the public but also as a payment method: together with banknotes they constitute the main instruments that facilitate the circulation of money in the economy.
This role might be sensibly impacted by the introduction of a central bank digital currency (***CBDC***), which is now one of the most discussed topics among central bankers, monetary policy researchers and members of the payment industry, for example Bindseil [2020], Bindseil and Panetta [2020], Kumhof and Noone [2018], Barrdear and Kumhof [2021] Barontini and Holden [2019] Pollock [2018], Juks [2018]. While at this time the public is enabled to access central bank money only in the form of banknotes, a CBDC would introduce a new payment method combining the advantages of central bank money with the convenience of modern electronic payments.
Although some arguments supporting the issuance of a digital Euro are beyond controversy, its features must be designed carefully in order to address some critical points.

One of the major discussions in the design of the digital currency regards its impact on the structure and scale of bank intermediation: advocates of central bank money see in bank disintermediation one of the goals of the CBDC, but others are concerned with the possibility of the CBDC inflating the central bank balance sheet at the expense of the deposit funding of banks.
The replacement of (a portion of) banknotes with CBDC is uncontroversial, as it simply translates one form of central bank money to another, with no effect on the rest of the financial system. On the other hand, a central bank digital currency replacing bank deposits is bound to affect the sight deposits industry (one of the cheapest and most stable sources of funding for banks), with negative consequences on the real economy. Indeed, it would force banks to rely more consistently on costlier sources of funding such as central bank credit and bond issuance, and to reflect the increased costs in their normal lending activity.
On the other hand, the increased funds in the central bank balance sheet would have to find their way back to the economy. Since the funds have been collected exploiting an unfair competitive advantage (the ability to issue the CBDC) which is not matched by a higher efficiency compared to the private sector, central bank investments into risk-intense portfolios should not be an option. One way to cushion the lengthening of the central bank balance sheet is the purchase of government and corporate bonds directly from banks and households, but this solution can only work as long as the total volume of CBDC is contained below a certain level, over which the central bank would instead be forced to become a financial intermediary, which is not desirable (see Bindseil [2020], Bindseil and Panetta [2020], Kumhof and Noone [2018], Pollock [2018]).
Another effect of the increased demand for central bank credit is that the central bank would be forced to revisit its collateral framework, considering as eligible collateral some asset classes it did not formerly accept.

One more important point that is being discussed regards the possibility of banks runs into CBDC. Unlike banknotes, which are riskier and more costly to store at home in large amounts, the same drawbacks do not apply to the CBDC, which is a liquid, riskless asset with no storage costs.
Usually, the possibility of a bank run towards other low-risk financial assets (like gold and highly rated bonds) in case of a crisis is balanced by the tendency of such prices to rise during those times, therefore investors reduce their default risk exposure in exchange for greater market and liquidity risk.
If investors were to be allowed to hold unlimited amounts of CBDC, in the case of a financial crisis the migration from bank deposits to CBDC would neither create physical

security nor be subject to scarcity-related price disincentives, resulting in the further aggravation of the crisis.
This may provoke unwanted cyclical effects, as demand for the CBDC increases in periods with a high perceived risk and decreases in better times.

The easiest way to address these two critical points is to find a way to limit the amount of CBDC available without affecting its availability and convenience as a payment method. Its use as a *store-of-value* asset must be avoided if the central bank wishes to keep its liabilities under control, and the easiest way to achieve that is with a remuneration system that can be controlled through monetary policy. Indeed, it is sometimes assumed that the *cash-like* properties of the CBDC would also include the zero nominal yield (i.e. the nominal yield of banknotes, regardless of the level of interest rates). However, ignoring the different implications that a null nominal remuneration has in different interest rate environments may be dangerous and cannot address the problems of bank disintermediation and/or a bank run by monetary policy means.
This is the reason why some CBDC advocates see the discontinuation of banknotes (and their zero nominal yield) as one of the main goals of the CBDC. Alternative remuneration systems are for example those proposed in Bindseil [2020] and Bindseil and Panetta [2020], in which the total volume of CBDC can be controlled through monetary policy thanks to a two-tiered remuneration system. This would be a mechanism that is already exploited in a variety of central bank deposit accounts, so there would be no need to build the necessary infrastructure or expertise.

Regardless of the remuneration type, we propose some general features for the behaviour of the CBDC, in particular regarding the competition with sight deposits and the influence of market factors; for example, in times when the perceived default risk is high, we will observe increased cash flows towards the CBDC, whereas during periods in which deposits are particularly convenient liquidity will tend to flow away from the CBDC.

As a new form of circulating money, the CBDC would undoubtedly add a new source of liquidity risk from the point of view of the banking system, giving the chance to citizens and businesses to transfer part of their money from their bank accounts to the newly introduced digital currency accounts.
In this thesis, we model both deposit and CBDC volumes, making forecasts as to what impact the CBDC may have on the banking system if it had been introduced in June 2021. Both CBDC and deposit volumes depend on market factors and other interest rates, and

the models for the single components are described in the following chapters.
For each of the remuneration systems and possible dynamics of the CBDC volume, we forecast the liquidity held in sight deposits and CBDC accounts using a Monte Carlo simulation.
To conclude, we compute the quantiles of the distribution of deposit liquidity, both as the volume evolves in time from the initial point and month-by-month. This enables us to assess what amount of liquidity the Italian banking system may expect to lose in favor of the CBDC in a number of different scenarios depending on market factors, sight deposits and CBDC features.

It must be clear that, although this model may be used by any commercial bank to assess its own risks, we are not treating the quantities as if they referred to a single specific bank, but rather as indicators representing the whole Italian banking system. The deposit rates are to be interpreted as average rates offered by banks throughout the country at a certain time, and similarly the volume represents the total of all banks. Also, the Italian 5-year CDS index indicates how creditworthy the market perceives the Italian banking system *as a whole*. For these reasons, we can consider the total volume of the deposits as a single deposit made at a generic Italian bank, paying a floating rate decided by the bank (based on the two market factors) and subject to a credit risk expressed through the CDS index.

1.2. Variable definition

In this thesis we make wide use of a vast variety of variables, which is why it seems necessary to preemptively define the rates and other important variables that we will use. Other quantities not included in this list might appear later and will be defined where they are introduced.

- ***Market factors***:
    1. The short rate $\boldsymbol{r(t)}$, which represents the EONIA rate and is therefore denoted also as "$EONIA(t)$" at times
    2. The CDS index $\boldsymbol{S(t)}$, representing the spread paid in the premium leg payments of a virtual credit-default-swap on the Italian banking system, with maturity 5 years.

- ***ECB deposit facility rate***: $ECBDF(t)$

- ***Deposit interest rate***: $I(t)$

- ***Deposit volume***: $V(t)$

- ***CBDC remuneration***: $I\_CBDC(t)$, it may contain also the tier (1 or 2) to which it refers

- ***CBDC volume***: $V\_CBDC(t)$, as the above it may contain also the tier to which it refers

1.3. Building blocks

The model used in this thesis was proposed in Castagna and Mistè [2019] and is based on four building blocks: deposit volumes (our target variable), deposit rates, market risk-free rate and an indicator of the creditworthiness of the Italian banking system.
The model enables us to forecast the deposit liquidity and to assess both interest rate and liquidity risk.

As for the risk-free market rate, a single factor model for the EONIA rate is proposed, and for the creditworthiness of the Italian banking system we model $S(t)$ (see 1.2) with an intensity-based approach. Both of these market models are described and calibrated in chapter 2.

It is assumed by Castagna that banks set their own deposit rate depending on the value of these market factors: indeed deposits have to compete with the short-term market rates to attract liquidity, and a bank with a worse credit situation will be forced to pay higher rates to its depositors than more creditworthy competitors. Castagna proposes to capture this dependence of deposit rates from the market factors qualitatively through a linear regression.

Finally, another time-series based model captures the dependence of the volumes from the three previous factors. The model is therefore structured in a nested fashion, with the fit of time-series based models being carried out in chapter 3.
The two models in chapter 2 are based on the risk-neutral measure and calibrated from market quotes, they therefore aim at representing the market-implied expectations for the risk-free rate and CDS index. On the other hand, the two models in chapter 3 aim at quantifying the coefficients that represent client behaviour from historical data and therefore they work in the real-world measure.

1.4. Risk management

The proposed dependence structure is capable of capturing how fluctuations in market factors impact the distribution of volumes of sight deposits and their profitability.
Jarrow and Van Deventer (Jarrow and Van Deventer [1998]) proposed an arbitrage-free pricing formula that allows us to treat deposits as an exotic *floating-for-floating* interest-rate swap, which enables us to compute the net present value of this contract and assess its sensitivities to changes in the market quotes. In particular, it provides an algorithmic approach for the assessment and management of interest rate risk in the banking book (for a description of the IRRBB management framework, see Zijderveld [2017]).
For example, if we wish to observe the sensitivity to a certain shift in the term structure of interest rates we can do so by simply computing the new coefficients and the difference in NPV before and after the shift.
The management of interest rate risk can be carried out by composing a portfolio of bonds with different maturities to replicate the notional value and deltas of the deposit with a procedure described in Kalkbrener and Willing [2004], Nyström [2008]. Additional greeks may be replicated by including in the portfolio other interest rate derivatives, as done in Ho [1992].

Although we will not carry out the procedure, the crucial result is that (unlike in the standard approach) it is possible to assess the different risks within the same analysis by identifying how market factors affect the profitability of sight deposits.

As for liquidity risk, having a way to obtain a Monte Carlo simulation for the paths of deposit volumes enables us to compute the term structure of liquidity, through which we can obtain all the quantiles of the volume distribution at any future time.

Being able to perform all these quantitative analyses by using a simple and intuitive model, based on the principles of behaviour of banks and customers is the main advantage of this approach. In order to carry out all the analyses in this thesis we compute the volume distribution up to a fixed *cut-off horizon* of five years.

1.5. Data and preliminary manipulation

The data needed to calibrate the interest rate model is the discount factor curve. We exploited a short rate model developed by Renne Renne [2016] based on a regime switching idea where short-term rates depend closely on deposit facility rates. This model allows us to describe the digital currency and market rates at the same time, and has the advantage of being able to coherently price a vast range of securities. However, to design of a parameter structure able to capture the market expectations regarding the future monetary policy decisions (from the implied volatilities of swaptions or caps and floors, for example) is beyond the purpose of this thesis, therefore we limit ourselves to a calibration from the yield curve and assume that the deposit facility rate switches among only three possible states: low (-0.5%), medium (1%) or high (3%). A detailed description of the Renne model, its pricing formulas and the calibration procedure can be found in chapter 2. The yield curve used is that of European bonds with at least an AAA rating, coming from the European Central Bank database[1], but only the maturities up to ten years have been included in the calibration.
Lastly, for a graph in chapter 2 we needed the time series of the ECB deposit facility rate, which we got from Eikon.

As for the CDS index representing the creditworthiness of the Italian banking sector

as perceived by the market, its simulated values are computed from an intensity-based model in which the intensity is assumed to evolve according to the dynamics of a *Shifted Square-Root Diffusion* stochastic process proposed in Brigo and Alfonsi [2003].
In principle, the parameters of such process can be calibrated from the market CDS quotes. However, as mentioned above, we are not referring to any specific bank but rather to the whole Italian banking system, therefore we have built such quotes using a combination of the CDS spreads of the three largest Italian banks (Intesa Sanpaolo, Unicredit and Mediobanca), weighted by market cap as of June 1^{st} 2021. The market data has been retrieved from the university's Refinitiv Eikon portal.

The dynamics of the last two building blocks, i.e. deposit rate and volume, have been estimated from historical data by fitting a linear regression and an ARX process, respectively. Different attempts have been made, the results of which (along with details of the model selection procedure) are contained in chapter 3.

[1] See https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves

The source of the data is the statistical database of the Bank of Italy [2], observations are monthly and the period ranges from October 2008 to May 2021. Specifically, we used the following time series:

- Volumes on current deposit accounts are taken from the dataset with code BSIB0300
- Deposit rates are taken from the dataset coded MIR0800

In order to fit the time series models we also needed the time series of EONIA and CDS spreads of Intesa Sanpaolo, Unicredit and Mediobanca. All these have been obtained from Eikon. While it is desirable that the range of observations be sufficiently large, we asked ourselves if the behaviour of deposit rates and volumes can be considered to have always had the same nature. As mentioned above, in the period prior to the subprime crisis it was common for sight deposits to represent an arbitrage opportunity for the banks issuing them. During the crisis, it was not immediately clear how the central bank was going to deal with the worsening situation and the speculation happening at the expense of some countries (among which Italy). It was only in May 2010 that the European Central Bank decided to begin a bulky purchasing phase for some of these sovereign bonds thanks to the establishment of the European Financial Stabilisation mechanism. Therefore, we eliminated from the dataset the observations antecedent to May 2010, assuming that the underlying factors driving the movements in rates and volumes may have noticeably changed at that time, while changes with the same importance have not happened later.

[2] The data are publicly available at https://www.bancaditalia.it/statistiche/basi-dati/bds/index.html.

2 | Market factor models

2.1. Overview

In this chapter, we describe the models adopted for the first two building blocks: the market risk-free rate (r_t) and the CDS index (S_t). Hereinafter, they will be referred to as ***market factors***.

They are observable on the market at the reference date and the calibration of the two models is performed by replicating the following market-implied curves:

- The discount factor curve up to a 10 year maturity
- The implied survival probability curve (relative to a combination of the major Italian banks, see 2.3.1

In 2.2 and 2.3 the two approaches are described in detail, the pricing equations are introduced and they are used to fit the market curves above.
Note that the final specification of the models is used to carry out the simulation of the market factors regardless of the presence or absence of the CBDC, the type of CBDC remuneration or its volume dynamic.

As for the interest rate, we rely on a single-factor model developed by Renne in Renne [2016], where the fluctuations in the short rate are mainly due to the movements in the ECB deposit facility rate.
On the other hand, the CDS index S_t evolves following the variations of the underlying stochastic intensity process, proposed in Brigo and Alfonsi [2003] ad thoroughly described in Brigo and Mercurio [2006], chapter 22.

2.2. Interest rate

2.2.1. Model description

We model the short-term interest rate (EONIA) and the deposit facility rate (ECBDF) with the approach proposed in Renne [2016].
The data on which we perform the calibration is the risk-free curve (given by European AAA rated bonds). We obtain the following discount factor curve:

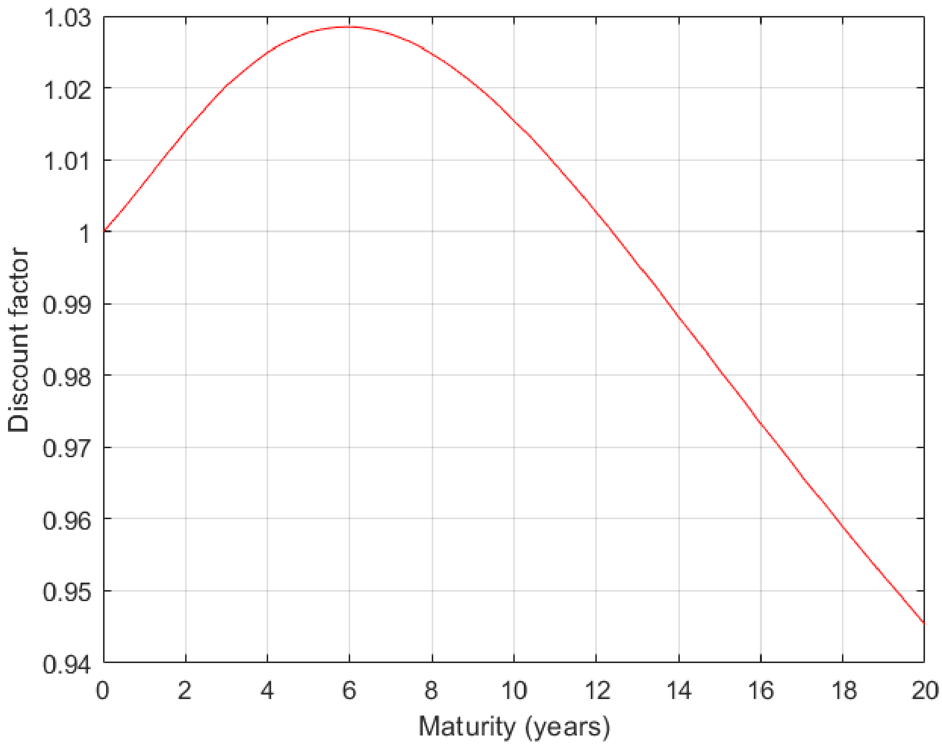


Figure 2.1: Market discount factor curve

As we will see in chapter 4, one of the features proposed by Bindseil for a central bank digital currency foresees a remuneration rate depending on the rate offered by the ECB deposit facility (ECBDF). This rate also happens to be one of the main sources of movement for interest rates, since through changes in monetary policy the central bank influences the short-end of the yield curve. This influence is neglected by standard models, where typically the short-term rate follows a Gaussian or square-root diffusion process, whose fluctuations are not related in any way to changes in monetary policy rates.

Contrarily, in Renne's approach this dependence is at the core of the description of movements in the short-term rates, which is convenient for our analysis since it allows us to model the market rates and CBDC rates at the same time.
It also has the advantage of including a closed formula for the pricing of interest rate derivatives that involves simple algebraic operations.

In order to grasp the link between monetary policy rates and the short end of the yield curve, let us observe how EONIA and ECBDF have evolved historically:

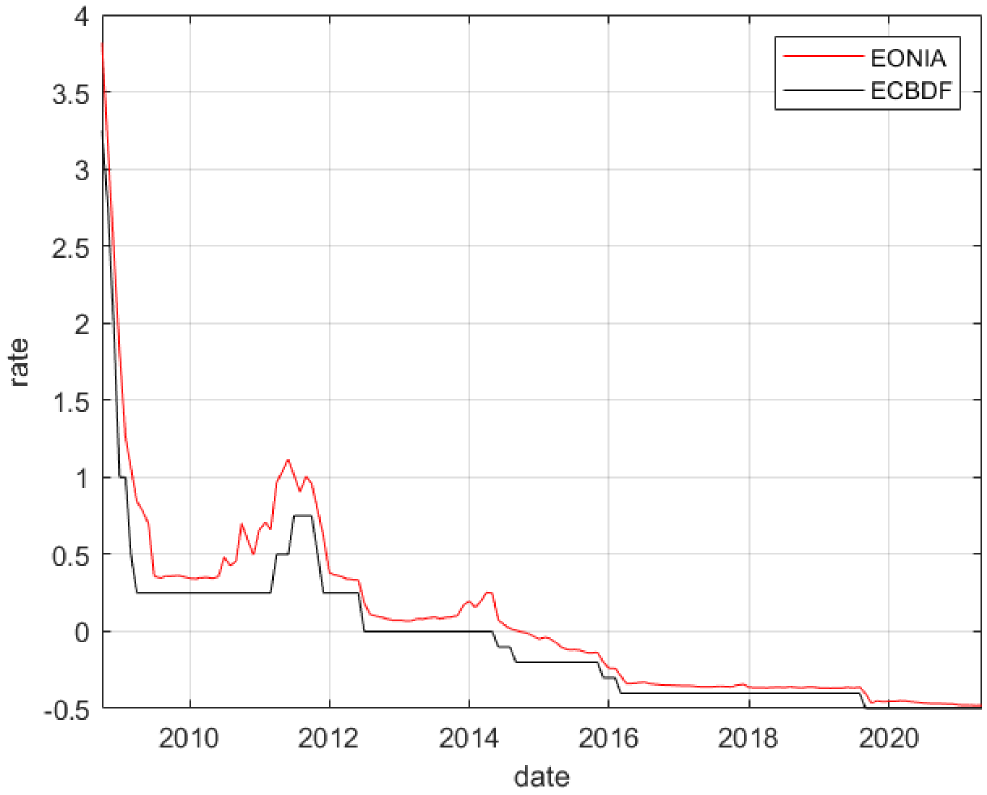


Figure 2.2: Overnight rates, deposit facility vs interbank

Although a study of its capability of capturing market expectations of future monetary policy changes (for example from caps, floors and swaptions volatilities) would be interesting and useful for our purposes, it would also require the design of an appropriate structure of the model parameters to efficiently capture information implied by the market (for example implied volatilities). For this reason, the model is calibrated directly from the discount factor curve.

In the approach proposed by Renne, the short rate dynamics are split into two components:

$$r_t = \Delta \cdot z_t + \xi_t \tag{2.1}$$

In the above we have:

- Δ: a vector containing the possible values of the ECBDF rate. The number of states is indicated by K.
- z_t: a K-dimensional "selection" vector, i.e. where all entries are set to zero except one, so that the scalar product $\Delta \cdot z_t$ represents the current ECBDF rate. It is referred to as ***regime vector*** in the rest of the section
- ξ_t: a serially uncorrelated random variable, the distribution of which does not need any specific assumptions, though it may depend on z_t. We only need to be able to compute its Laplace transformation conditional on z_t, using which we define the K-dimensional vector δ through the relation $\mathbb{E}[exp(-\xi_t)|z_t] = exp(-\delta)' \cdot z_t$, where by abuse of notation the exponentiation is carried out component-wise.

The switches between the different regimes contained in Δ are discrete, and only one state may be selected at a time. For these characteristics, we call this setup a *"regime switching framework"*. The selection vector follows a Markovian process governed by a transition probability that depends on the model parameters and whose specification will be the result of the calibration.

The persistence of rates is given by the term $\Delta \cdot z_t$, since ξ_t is serially uncorrelated. The vector z_t follows a Markovian process whose transition probability matrix may depend on time, though in a deterministic way. Thus, $\Delta \cdot z_t$ follows a step-like path.
Moreover, we assume that a change in the deposit facility rate is possible not at any moment but just monthly, in occasion of a meeting of the monetary policy committee. This is close to what happens in reality, although the frequency of such meetings might not be always regular.

2.2.2. Model specification

Historically, there is a positive difference (represented in the model by ξ_t) between EONIA and ECBDF rates, as we can observe from 2.2. Renne proposes to model it with a Beta distribution, with support in $[\,0\%, 1\%\,]$. The parameters of this distribution can be estimated from the time series by minimizing the difference between the sample and theoretical cumulative density function, and we obtain that the best-fitting Beta distribution is $B(\,0.9227,\ 6.6929\,)$.

In our specification of the model we make the following assumptions:

- Δ is made of three states: -0.5%, 1% and 3% corresponding to a low, medium and high monetary policy rate.
- z_t evolves through a Markovian process, whose transition probability matrix is constant in time.
- The distribution of ξ_t does not depend on z_t, therefore all components of the K-dimensional vector δ are equal to $\delta \simeq 0.0012$.

Switches from one of these regimes to the other are too extreme to represent plausible changes: for example, we do not expect to see a sudden jump from the current rate of -0.5% to 1%. However, we have defined the three regimes this way because it allows us to observe the differences in the behaviour of deposit volumes depending on the deposit facility rate and the features of the CBDC. This is our main goal and it is achievable already in this simplified scenario, with the regimes representing a low, medium and high interest rate environment, respectively. Furthermore, the calibration of a more extended model featuring a much larger state space results in a worse fit, and should therefore be designed more carefully by taking into account swaptions or caps/floors, which is beyond the purpose of the thesis.
The transition probability matrix governing the movements of the regime is defined as a function of the four parameters we have included:

$$\Pi = \begin{bmatrix} 1-p_1 & p_1 & 0 \\ p_2 & 1-p_2-p_3 & p_3 \\ 0 & p_4 & 1-p_4 \end{bmatrix}$$

where the four parameters are constrained so that all elements of the matrix above are contained in the interval $[\,0, 1)$.

2.2.3. Pricing and calibration

The pricing formula 2.2 obtained by Renne for zero coupon bonds needs a few introductory definitions. We will use:

- k: indicates the maturity of the zero coupon bond expressed in days
- $\Pi(i)$: denotes the matrix above if the date corresponds with the date of a monetary policy committee meeting, and the identity matrix otherwise (meaning that there is no possible switch on that day)
- γ is a diagonal matrix having as diagonal the elements of the vector $e^{-(\Delta+\delta)}$, where by abuse of notation we are denoting a K-dimensional vector where the exponentiation is performed component-wise

After converting all maturities in days (using the Act/365 day count convention) we have the expression for price of a zero-coupon bond with maturity of k **days**:

$$P(0, k) = \underline{1} \cdot \left(\left\{ \prod_{i=k}^{1} (\gamma \cdot \Pi(i)) \right\} \cdot \gamma z_t \right) \tag{2.2}$$

At this point what is left to do before proceeding with the calibration is identifying how many monetary policy committee meetings there are before every maturity and when they happen. Given that the first scheduled meeting after the reference date has been on June 9^{th} 2021 (i.e. eight days after), we assume a frequency of one meeting every 30 (calendar) days, and compute the dates of the meetings. Then, we are easily able to express the pricing formula 2.2 explicitly, since we have specified the term "$\Pi(i)$" for every i. The distance between theoretical and market prices is a function of the parameters defined as the sum of the squared differences.

Now we can perform the algebraic operations and minimize this distance, obtaining the transition matrix:

$$\Pi \simeq \begin{bmatrix} 0.08851 & 0.1149 & 0 \\ 0.0315 & 0.8780 & 0.0906 \\ 0 & 0.0200 & 0.9800 \end{bmatrix} \tag{2.3}$$

2.3 is used to generate the paths of the selection vector and obtain a Monte Carlo simulation of the deposit facility rate.
From that, we simulate also the CBDC remuneration rate and EONIA rate, which are all necessary ingredients in the simulation of deposit rates and volumes. Obtaining them at once with the Renne model is the reason why it was particularly handy.

2.3. Credit component

2.3.1. CDS index definition

We now need a way to model and simulate the CDS spread intended to represent the level of creditworthiness of the Italian banking industry, as perceived by the market. Such CDS is not related to a quoted instrument, so we have constructed a proxy for it by combining the CDS market quotes of the three major Italian banks (Intesa Sanpaolo, Unicredit and Mediobanca), as done by Castagna.
Given the CDS mid-market quotes of maturity 6 months to 10 years for the three banks, we combine the quotes of each maturity through a weighted average, with the weights being indicators of the fraction of the total market capitalization associated with each bank. At the reference date, Intesa Sanpaolo had a market capitalization of 57.11 billion Euros, Unicredit of 22.29 billion Euros and Mediobanca of 10.29 billion Euros, resulting in weights of 63.68%, 24.85% and 11.47% respectively. The outcomes of the combination of the three quotes for each maturity are reported below, measured in basis points:

Maturity	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
Spread (bp)	24.85	25.20	31.02	38.45	50.15	61.19	82.65	96.30

This is the set of "market quotes" we use to fit the credit model.

2.3.2. The Brigo-Alfonsi model

In intensity-based models, the default event corresponds to the first jump of a Poisson process with a certain intensity, which may be a positive constant, deterministic function or stochastic process. In the latter case we speak of *Cox process* rather than Poisson process. In our case, the intensity will evolve like a square-root diffusion process (Cox-Ingersoll-Ross) plus a deterministic shift.

First, we need to express the survival (risk-neutral) probability, expressed through a (non-decreasing) function called *hazard function* which is just the integral of the intensity. We have for every time t:

$$\Lambda^{market}(t) \;=\; \int_0^t \lambda(u)\,du \;=: \textbf{\textit{Hazard function}} \tag{2.4}$$

$$Q^{market}(\,\tau > t\,) \;=\; exp\,(-\Lambda(t)) \;=: \textbf{\textit{Implied survival probability}} \tag{2.5}$$

Notice that since the intensity takes non-negative values, the hazard function will be non-decreasing and consequently the survival probability will be non-increasing.
Using the CDS spreads, we first compute the piece-wise constant intensities that replicates implied survival probabilities. Then we obtain the hazard function as in 2.4, use it to define the deterministic shift of the stochastic intensity process in a way that reproduces exactly the survival probability curve and obtain the parameters.

Let us now begin with a more detailed description of the model proposed in Brigo and Alfonsi [2003] of the model equations and how they are related to each other.
The intensity is the stochastic process whose parameters we are trying to fit. As mentioned above, it is expressed as the sum of a Cox-Ingersoll-Ross (CIR) process and a deterministic shift, and in equation 2 β indicates the vector of model parameters. We have the following dynamic in the risk-neutral measure (which we indicate with Q):

$$\lambda(t) = \psi(t;\beta) + y^{\beta}(t) \qquad where \qquad \begin{cases} dy^{\beta}(t) = \kappa(\,\mu - y^{\beta}(t)\,)dt + \nu\sqrt{y^{\beta}(t)}dW(t) \\ y(0) = y_0 \end{cases} \tag{2.6}$$

In 2, the four parameters of the model are $(\,\kappa, \mu, \nu, y_0\,) \;=:\; \beta$ and $W(t)$ is a standard Brownian motion.
Notice that if we were to model also the short-term rate with a diffusion process, there may possibly be correlation between the two Brownian motions appearing in the dynamics of the interest rate and the intensity. The advantage of the Brigo-Alfonsi model is that the two calibrations can still be performed separately (assuming no correlation) thanks to a theoretical result (*"filtration switching formula"*) proved in Brigo and Mercurio [2006]. Even though this is not our case (the Renne model does not assume a diffusion process to be the driver of the interest rate fluctuations) it is still a crucial result as someone

possibly interested might reconsider the convenience of the model if the interest rate and credit parts had to be treated together.

This model is completely analogous to a (single factor) shifted CIR model for interest rates: if it was the spot rate that evolved according to the dynamic above instead of the intensity, we would have the same equations for the computation of survival probabilities (in the intensity case) and discount factors, since by definition:

$$\begin{aligned} Q(\tau > t) &= \mathbb{E}\Big[exp\Big(- \int_0^t \lambda(u)\, du \Big) \Big] \\ P(0, t) &= \mathbb{E}\Big[exp\Big(- \int_0^t r(u)\, du \Big) \Big] \end{aligned}$$

Some results that have been found for the short rate process can be easily transferred to the intensity process. In the remaining part of this section we use some of these available results, specifying wen we do so. The interested reader can find them in Brigo and Mercurio [2006], chapters 3, 21 and 22.

Given the yield curve and the survival probability curve, the definition of survival probability and 2 allows us to obtain the theoretical expression for the survival probability, which we impose to match the ones implied by the market for each maturity.

$$\begin{aligned} Q(\tau > t)^{model} &= \mathbb{E}\Big[exp\Big(- \int_0^t \big(\psi(u;\beta) + y^{\beta}(u)\big)\, du \Big) \Big] = \\ &= \mathbb{E}\Big[exp\Big(- \Psi(t;\beta) - Y^{\beta}(t) \Big) \Big] = \\ &= exp\Big(- \Psi(t;\beta) \Big) \mathbb{E}\Big[exp\Big(- Y^{\beta}(t) \Big) \Big] = \\ &= Q(\tau > t)^{market} \end{aligned} \tag{2.7}$$

Where we have defined:
$$\begin{cases} \Psi(\,t;\,\beta\,) \;:=\; \int_0^t \psi(u;\beta)\,du \\ Y^{\beta}(t) \;:=\; \int_0^t y^{\beta}(u)\,du \end{cases} \tag{2.8}$$

Taking the logarithm on both sides and rearranging the terms we obtain:

$$\Psi(\,t;\,\beta\,) = -log\Big(\,Q\,(\,\tau > t\,)^{market}\Big) + log\bigg(\mathbb{E}\Big[\,exp\Big(-Y^{\beta}(t)\Big)\Big]\bigg)$$

Thanks to 2.7 we are able to express the first term on the right hand side through the market-implied hazard function, while the second term on the right hand side has already an available analytical expression: indeed, if we think of Y as the integral of a **spot rate** evolving with a CIR dynamic, inside the logarithm we find precisely the price of a zero-coupon with maturity t, which is a known formula.

The final analytical expression for $\Psi(\,t;\,\beta\,)$ is therefore:

$$\Psi(\,t;\,\beta\,) \;= \Gamma(t)^{market} + \left(\frac{2\kappa\mu}{\nu^2}\right) log\left(\frac{2h \cdot exp((\kappa + h)\,t/2\,)}{2h \,+\, (\kappa + h\,)(e^{ht} - 1)}\right) - \\ - \; y_0\,\frac{2\,(e^{ht} - 1)}{2h \,+\, (\kappa + h)(\,e^{ht} - 1\,)}$$

In the above, $h \;:=\; \sqrt{\kappa^2 \,+\, 2\nu^2}$. Differentiating on both sides, we obtain:

$$\psi(\,t;\,\beta\,) \;=\; \gamma(t) - \frac{2\kappa\mu\,(\,e^{ht} - 1\,)}{2h \,+\, (\kappa + h)(\,e^{ht} - 1\,)} \;-\; y_0\,\frac{4h^2 e^{ht}}{(2h \,+\, (\kappa + h)(\,e^{ht} - 1\,))^2} \tag{2.9}$$

Where $\gamma(t)$ represents the piece-wise constant (alternatively also piece-wise linear) intensity function. Defining the deterministic shift this way, we are sure to fit the set of survival probabilities implied by the market.

The criterion proposed to compute the parameters is the minimization of the L^2 norm of the deterministic shift (the idea is that we want the accuracy of the fit to be due as much as possible to the stochastic component rather than the deterministic one). The parameters are therefore obtained minimizing the function:

$$F(\beta) := \int_0^{10} \psi^2(u; \beta)\, du \tag{2.10}$$

where the upper bound of the integration interval is 10 (years) because that is the longest maturity of the quoted CDS, which we set as the upper bound for the dominion of the function ψ defined in 2.9.

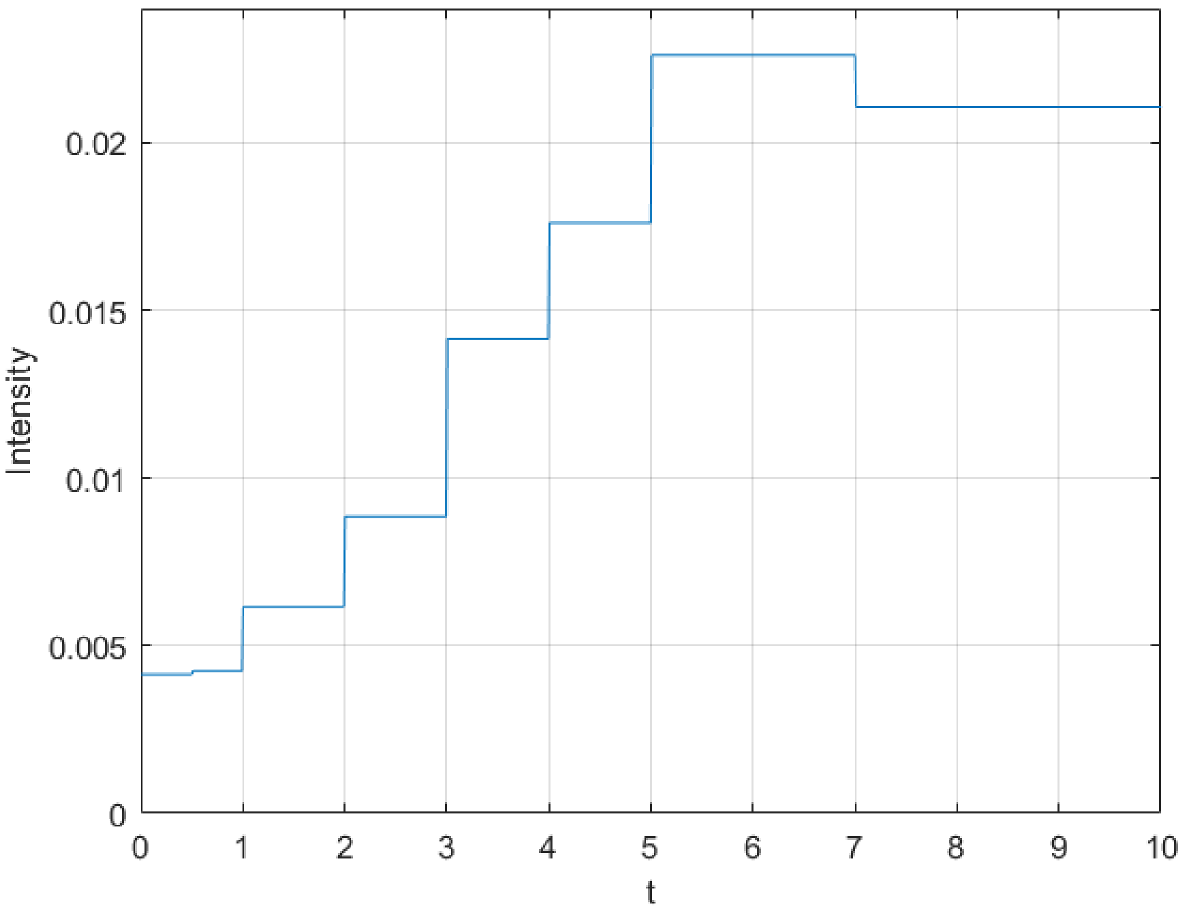
The values of the parameters have to satisfy some theoretical constraints, namely:

- All parameters must be positive
- $2\kappa\mu > \nu^2$ (Feller condition)
- They must be such that ψ defined in 2.9 is always positive
- $\gamma(0) = y_0 + \psi(0)$

The value of β that minimizes 2.10 must be sought in the region where these constraints are satisfied.

2.3.3. Model calibration

The goal of the calibration is to obtain an analytical expression for ψ. The only component of 2.9 that is missing is the intensity function, which we can retrieve from the CDS spreads.
Given the discount factor curve and the recovery (40%), we impose the equality of the net present values of the premium leg and contingent leg and obtain the piece-wise constant intensity function[1]. We obtain a step-like function:



Given this, we are able to express the deterministic shift (ψ) as in 2.9, in function of the parameters. After that, we minimize 2.10 and obtain:

$$\boldsymbol{\kappa \simeq 0.9338, \qquad \mu \simeq 0.35\%, \qquad \nu \simeq 8.03\%, \qquad y_0 \simeq 0.0020}$$

[1]See Brigo and Mercurio [2006], section 22.3 for the details of the procedure

Once the intensity parameters have been found, we are left with finding a relationship between intensity and CDS spread. From the intensity simulation we can (at each time step) obtain the corresponding 5-year survival probability by exploiting the properties of the CIR model and the fact that ψ is a known function. Indeed we obtain for every t:

$$\mathbb{Q}(\tau > t + 5years \,|\, \tau > t) = \mathbb{E}\Big[exp\Big(- \int_t^{t+5} \lambda_s \, ds \Big) \Big] \tag{2.11}$$

However, λ is the sum of a CIR process and a deterministic shift, therefore the integral on the right hand side can be divided into two components, one of which can be moved out of the expected value. The other component can be computed with an available formula, as it is equivalent to the price of a zero coupon bond for the CIR model having $\big(y_u\big)_u$ as short rate process, with starting value equal to y_t. We obtain:

$$\begin{aligned}
\mathbb{Q}(\tau > t + 5years \,|\, \tau > t) &= exp\Big(- \int_t^{t+5} \psi(s) \, ds \Big) \mathbb{E}\Big[exp\Big(- \int_t^{t+5} y_s \, ds \Big) \,|\, y_t = y \Big] = \\
&= exp\Big(- \int_t^{t+5} \psi(s) \, ds \Big) \cdot A^{CIR}(0, 5) e^{-B^{CIR}(0,5)\, y} \simeq \\
&\simeq exp\Big(- \int_t^{t+5} \psi(s) \, ds \Big) \cdot 0.9865 \, e^{-1.0573\, y}
\end{aligned}$$

From the 5-year survival probability we obtain the corresponding CDS spread through a Jarrow-Turnbull approximation:

$$R_t^{(5Y)} = - \frac{(1 - recovery) log(\mathbb{Q}(\tau > t + 5 \,|\, \tau > t))}{5} \tag{2.12}$$

3 | Time series models

Once the market factors have been properly described, we are left with the task of finding a way to use them as explanatory variables before carrying out the simulations. Although it is understandable that there exists some relationship between rates and total volumes of the deposits and the market factors, trying to model this dependence in an intuitive and statistically significant way is not an obvious task. We expect deposit rates to increase when market risk-free rates increase, given the underlying competitive nature between the two forms of investment: indeed, a bank will hardly maintain unchanged its liquidity in presence of far more convenient alternatives available on the market. We also expect the perceived creditworthiness of the bank (or in our case of the banking system) to play a role, since a bank that is perceived as riskier than its competitors will be forced to pay higher rates to avoid a migration of its clients.

Our objective in this chapter will be the development of two models, one for deposits interest rate and one for deposits volume, based on the data described in 1.5.
The approach proposed in Castagna and Mistè [2019] is based on the idea of quantifying client behaviour through time series analysis, which entails we will not be working in the risk-neutral measure any longer but rather in the *"real-world"* measure.

During the model selection, some different combinations of explanatory variables and dependence structures have been tried. Not all these attempts will be shown, but we report some that allowed us to gain important insights. In the equations, variables are denoted with the names defined in 3.1.

After the fit, the residuals obtained have also been observed to look for possible autocorrelation and non-normality. Finally, each of the two models has been tested to assess its statistical robustness: the dataset is divided into two subsets and the model calibrated on one subset is tested on the other. Ideally the partition of the total dataset should be 60/40 or 65/35, but in the deposit volume case we had to increase the proportion to 80/20 in order to pass the tests.

The first test regards residuals, the second is a forward predictive failure test. For the details about the test implementation and hypotheses, see Castagna and Mistè [2019]. The two tests have been passed by our final models for deposit rates and volumes.
All the fitting procedures and tests have been carried out using the R programming language.

3.1. Definitions

The **deposit rate** is indicated with **I**. To describe its evolution we employed a linear model, with explanatory variables being the market factors:

1. The short-term rate (EONIA), indicated by the name ***'eonia'*** in R outputs and measured in percentage points
2. The CDS index (S_t), i.e. spread of a Credit Default Swap on the Italian banking system with maturity 5 years, indicated in this chapter with ***'S_ bar'*** and measured in percentage points

As for the **volume** of the deposits, we define a transformation of it which is assumed to evolve as an ARX process (as in ref to Castagna). We will indicate the volume with V and the transformation on which we will perform the fit with λ. The latter is defined as:

$$\lambda(t) = 100 \cdot log\left(\frac{V(t)}{V(0)}\right) \tag{3.1}$$

In 3.1 the coefficient 100 has been inserted to make the coefficients in the R outputs more readable. The initial volume $V(0)$ is the last value assumed by the deposits volume time series, corresponding to May 2021.

Therefore, λ evolves following an ARX process, with one auto-regressive term and two exogenous variables. Some different attempts have been made when seeking the most statistically significant exogenous variables.

The best results have been obtained employing the following:

1. $\boldsymbol{C} := I - EONIA$. It represents the *'convenience'* of the deposits with respect to the risk-free assets, hence the name. It has a physical meaning and gives better results in the fit compared to using both $EONIA$ and I as separate regressors.
2. The CDS index, indicated with ***'S_bar'*** as above

Lastly, to obtain a higher significance of the regressors in the fit of deposit volumes we used the average of C over the current time and the previous, which led to a slight improvement (lower p-values of the t-test). The average operator is indicated with 'A' and is defined as:

$$A\big(C(t),\, p\big) \;=\; \frac{1}{p}\left(\sum_{i=0}^{p-1} C(t-i)\right) \tag{3.2}$$

3.2. Deposit rate

We begin by examining the observations of the variables I, *eonia* and S_bar and their empirical dependence.

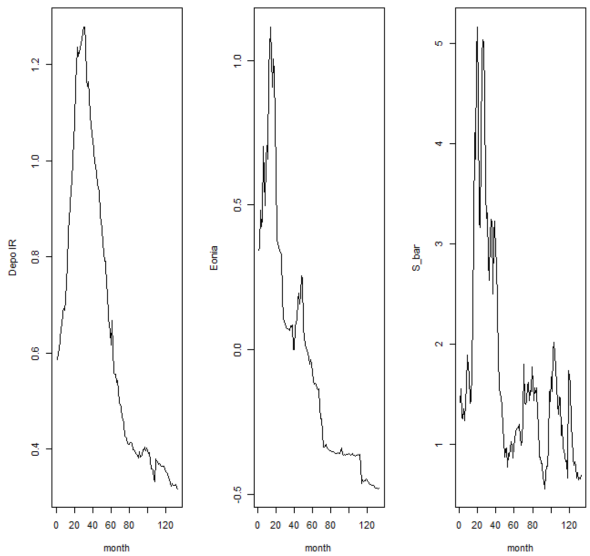


Figure 3.1: Time series of target variable and covariates

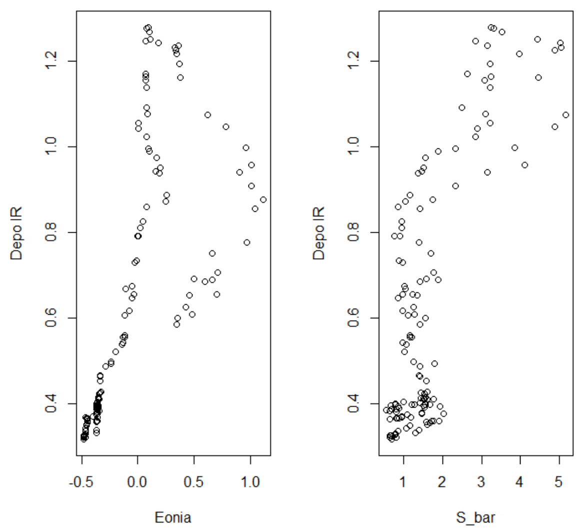


Figure 3.2: Plot of the empirical relations with the explanatory variables

From a first visual inspection, there seems to be a good hope for an explanatory linear model, indeed deposit rates are observed higher in correspondence of high market rates and CDS spreads. However, looking at a plot of the empirical observations, it is harder to detect an undoubtedly linear relation. Also, a lot of observations are clustered near the origin (see 3.2).

We will proceed to fit a linear model and then try to improve it if we were to discover that they are not statistically significant or in case that some coefficients are negative when expected positive (and vice versa), because they would be no longer representing the expected behaviour.

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.38883    0.03136  12.400  < 2e-16 ***
eonia        0.30478    0.04122   7.394 1.55e-11 ***
S_bar        0.15866    0.01554  10.207  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1663 on 130 degrees of freedom
Multiple R-squared:  0.7125,	Adjusted R-squared:  0.7081 
F-statistic: 161.1 on 2 and 130 DF,  p-value: < 2.2e-16
```

Figure 3.3: R output of model fit

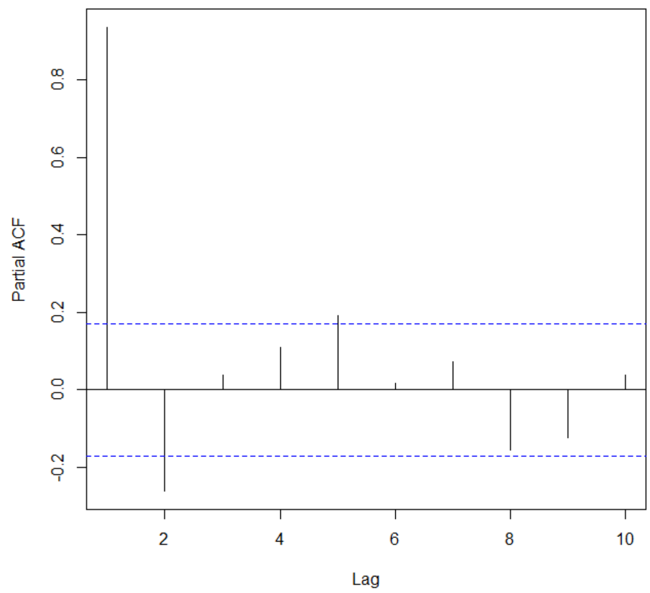
This is not the case with the output we have obtained: all coefficients are significant and their values are aligned with the features of the regressors that we would expect. The portion of variability that the model seems able to explain is just above 70%. We have therefore obtained a first equation for the deposit interest rates:

$$I(t) = \beta_0^I + \beta_1^I \cdot r(t) + \beta_2^I \cdot S(t) + \epsilon(t) \tag{3.3}$$

$$\text{with} \quad \beta_0^I \simeq 0.389, \quad \beta_1^I \simeq 0.305, \quad \beta_2^I \simeq 0.159$$

3.2.1. Autocorrelation of residuals

We now go on to test the residuals for autocorrelation, by using the Durbin-Watson test. The null hypothesis of the test (no serial correlation among residuals) is rejected under any level of confidence, and the partial autocorrelation function shows extremely high correlation with the first lag.



Partial Autocorrelation Function of Residuals

In this case, the option selected in Castagna and Mistè [2019] fits an AR on residuals, using a Cochrane-Orcutt procedure. This iterative method consists in estimating the correlation of the model residuals, then re-fitting the model coefficients assuming that estimate as the true correlation in the residual AR process, repeating until the change in the estimated correlation is smaller than a fixed threshold. However, even by carrying out this procedure one does not get residuals that are free from autocorrelation, and the quality of the fit decreases significantly. Indeed, after the procedure the values of the coefficients are very different.

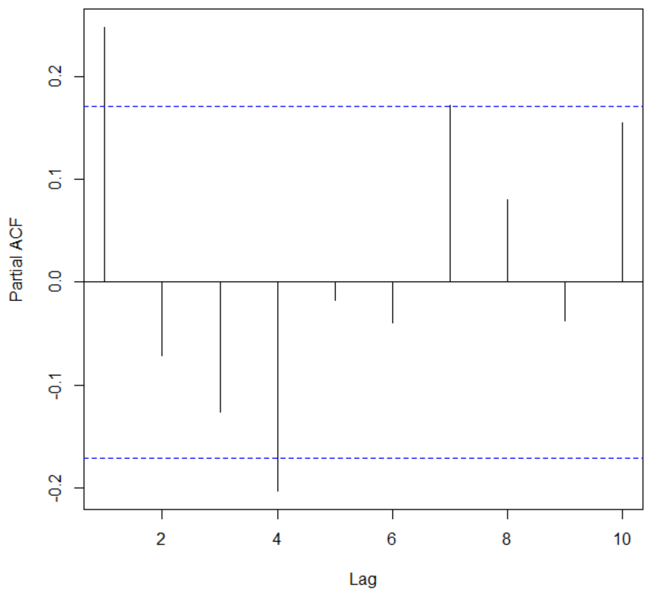
We therefore settled on estimating the autocorrelation and variance of the residuals directly, by fitting a standard AR model.

Our complete model is:

$$\begin{cases} I(t) = \beta_0^I + \beta_1^I \cdot r(t) + \beta_2^I \cdot S(t) + \epsilon(t) \\ \epsilon(t) = \rho^I \epsilon(t-1) + \eta(t) \end{cases} \tag{3.4}$$

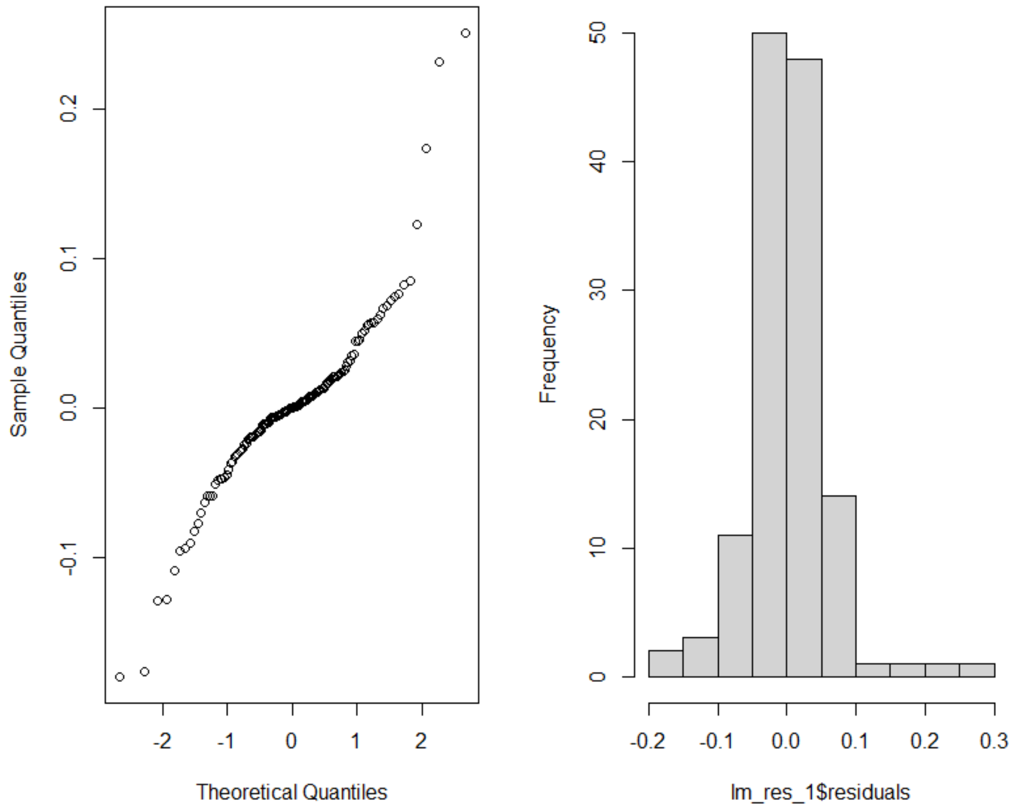
with the coefficients being the same as in 3.3, $\rho^I \simeq 0.934$ and $Var(\eta) \simeq 3.36 \cdot 10^{-3}$

Even though the null hypothesis of the Durbin-Watson test is still rejected below 99.5% confidence, the p-value of the test has increased by a factor of 10^{13}, and the partial autocorrelation is not as blatant as in the previous case. We will therefore accept this version of the model.



3.2.2. Normality of residuals

Lastly, we are left with the task of checking the normality of residuals. Given the time series $\{\eta(t)\}_t$, we will check the normality assumption visually with a histogram and a quantile plot and quantitatively with a Jarque-Bera test, which is frequently used for this purpose in time series analysis. In this test the sample skewness and kurtosis are compared to those of a normal distribution, and the null hypothesis is that the sample skewness and kurtosis are likely to come from a Gaussian distribution.



Normal Q-Q plot and histogram of η_t observations

The normal quantile plot does not appear to be aligned over a straight line, indicating possible non-normality. This is confirmed by the low p-value of the Jarque-Bera test. The model should therefore not be used for statistical inference and hypothesis testing.

3.3. Deposit volumes

In this section we will work with the variable λ defined in 3.1, in the same way as done by Castagna.

The first thing one is able to observe when looking at the plot of the two variables V and λ (3.4) is that a clear linear trend seem to be present, starting from around the 20^{th} observation (end of 2012/beginning of 2013). At the end we are even able to recognize a steepening of the trend, likely due to the Covid19 pandemic.

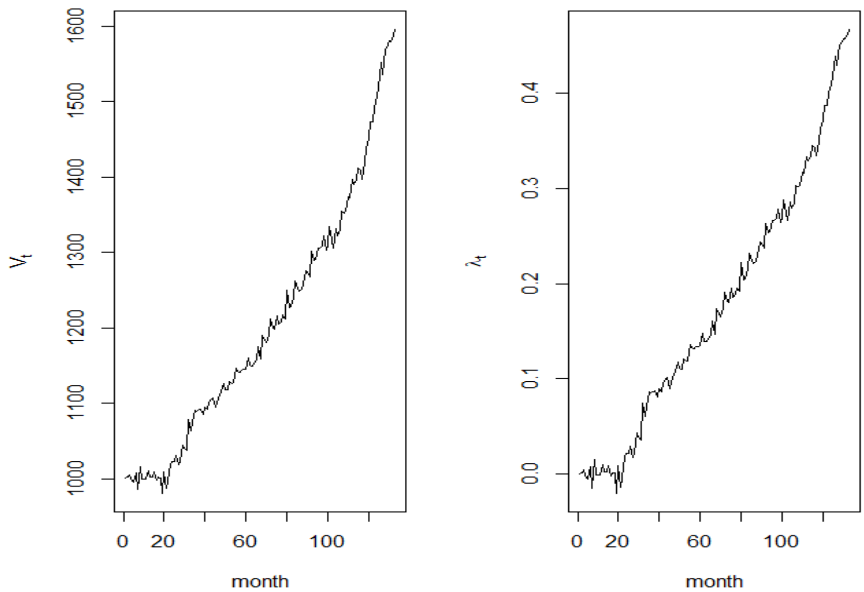


Figure 3.4: Monthly observations of V and λ

While we are not trying to understand the underlying motivations for this regular growth, we will see that it has an impact on our results. In particular, we will fit the model using Castagna's approach, and see that what we obtain is not useful: the vast majority of the variability is explained by time alone, so we will end up with a highly unstable model where the only significant term is the auto-regressive one.

Therefore, we will work with a de-trended version of the target variables, in which the market factors will be more significant.

As in the previous section, we report the graphs of the target variable and regressors:

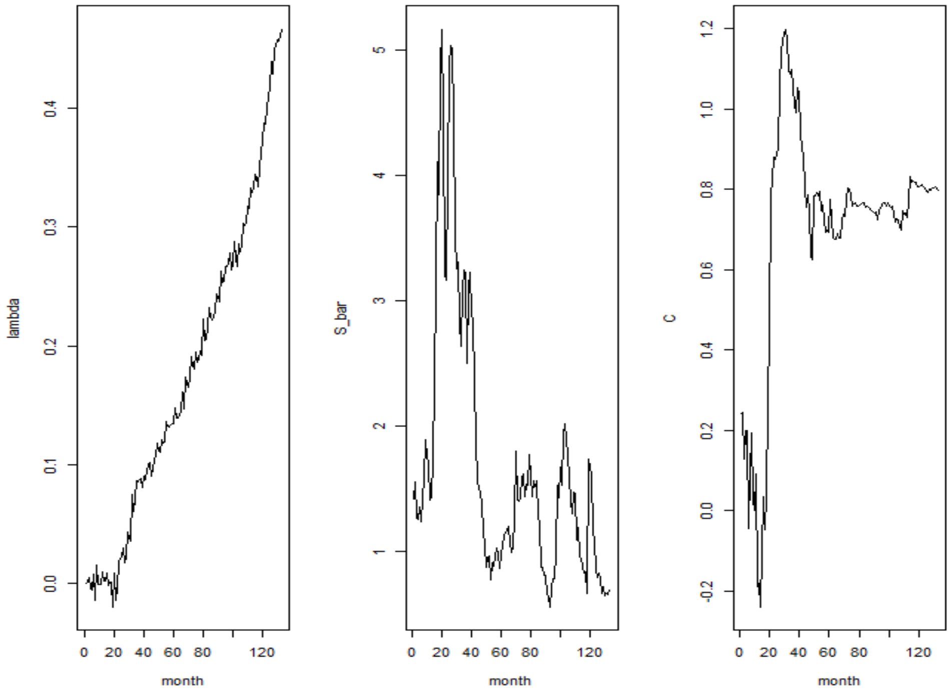


Figure 3.5: Monthly observations of λ and the regressors

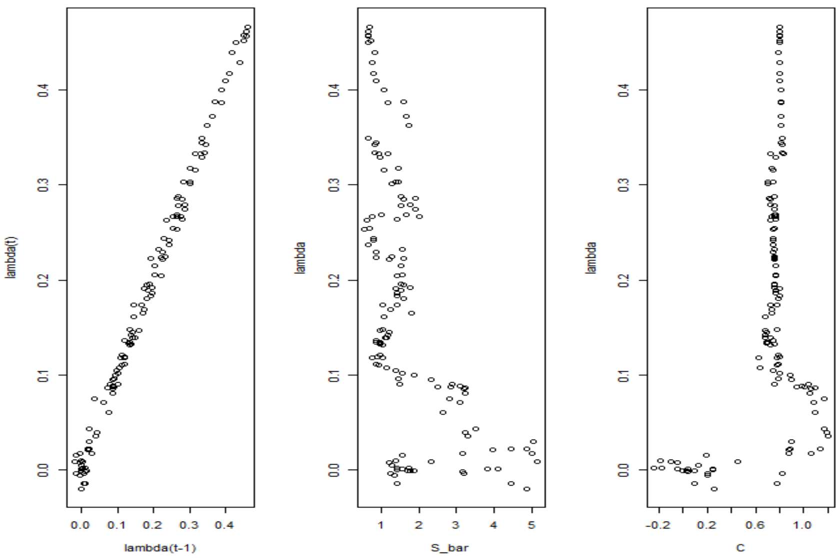


Figure 3.6: Plot of the empirical observations

It is quite evident that there is a close-to-linear relation with the lagged value, while dependence on the other covariates is harder to grasp.
A fit made using all the regressors gives birth to a model that explains more than 99.7% of the variability of the observations (implying the presence of an almost linear deterministic relation), even though S and C are not statistically significant.
With a backwards stepwise selection, we eliminate intercept and both regressors, obtaining:

```
Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
lambda_trend_tminus1 1.015682   0.004489   226.2   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01132 on 131 degrees of freedom
Multiple R-squared:  0.9974,    Adjusted R-squared:  0.9974
F-statistic: 5.119e+04 on 1 and 131 DF,  p-value: < 2.2e-16
```

Figure 3.7: R output of the model obtained through stepwise backward selection. The only regressor is the lagged value

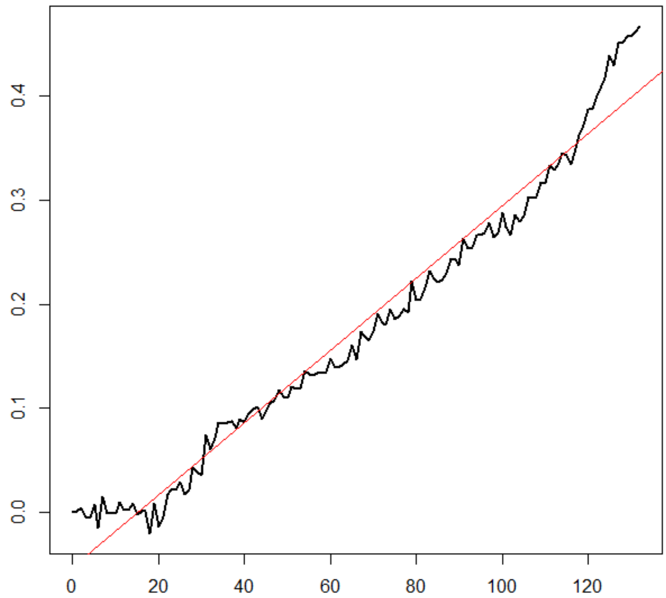
We can immediately recognize that there is an almost linear relation between λ and its lagged value, since it accounts for 99.74% of the variability. Furthermore, the value of the coefficient of this $AR(1)$ model is higher than 1, making it highly unstable and its asymptotic variance infinite. It is therefore useless for any prediction or simulation.

Given the disappointing results above, we wish to transform the variable in a way that gives a more central role to the regressors and a better fit overall. The simplest way is to just remove the trend by using time (measured in number of months since the reference date) as an explanatory variable and then fitting the remaining component using S, C and the lagged value. So, let us proceed with the linear trend estimation. By fitting a linear model to the time series, we obtain:

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.216e-02  4.074e-03  -12.80   <2e-16 ***
time         3.466e-03  5.335e-05   64.97   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02362 on 131 degrees of freedom
Multiple R-squared:  0.9699,    Adjusted R-squared:  0.9697
F-statistic:  4221 on 1 and 131 DF,  p-value: < 2.2e-16
```

It is a highly significant model, accounting for about 97% of the variability. Now it may be easier to find a statistically significant relationship between the regressors and the detrended variable. The values of λ when detrended are often negative (corresponding to the points in the graph where λ lies below the trend), unlike in the initial situation.



λ and estimated linear trend

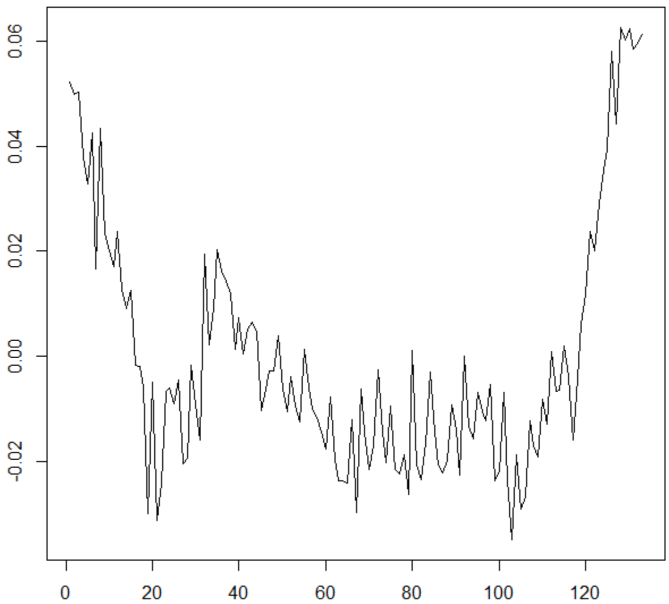


Figure 3.8: λ without trend

Once the time series has been relieved of the trend, we can hope for a better significance of the predictors C and S.

By fitting a linear regression with all the explanatory variables, we immediately observe the the intercept has a very high p-value and is therefore not significant, we proceed by removing it: we obtain a model that looks quite promising since the coefficients are as we expect and R^2 is high. Although the regressors are still not significant under a high level of confidence, it is a step forward, and the greatest improvement is obtained by using the average operator defined in 3.2 on C, with $p = 2$, i.e. the average of C over the current and previous month.

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
lambda_tminus1  0.88924    0.04201  21.168   <2e-16 ***
A(C_lagged, 2)  0.28890    0.20545   1.406    0.162    
S_bar_lagged   -0.11187    0.07656  -1.461    0.146    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.098 on 129 degrees of freedom
Multiple R-squared:  0.7792,	Adjusted R-squared:  0.7741 
F-statistic: 151.7 on 3 and 129 DF,  p-value: < 2.2e-16
```

Figure 3.9: R output of the best found model

The equation for the transformed deposit volume given by the fit in 3.9 is:

$$\lambda(t) = \beta_1^{(\lambda)} \cdot \lambda_{t-1} + \beta_2^{(\lambda)} \cdot A(\,C(t),\, 2\,) + \beta_3^{(\lambda)} \cdot S(t) + \epsilon^{(\lambda)}(t) \tag{3.5}$$
$$\text{with} \quad \beta_1^{\lambda} \simeq 0.889, \quad \beta_2^{\lambda} \simeq 0.289, \quad \beta_3^{\lambda} \simeq -0.112$$

3.3.1. Autocorrelation of residuals

As in the previous case, we must make sure that the residuals do not present a strong correlation between different time lags. The Durbin-Watson test strongly rejects the null hypothesis, therefore residuals are autocorrelated. The partial autocorrelation function indicates a sensible correlation with some time lags, although none are close to 1 (figure 3.10).

Trying a Cochrane-Orcutt procedure results in the same issues encountered in 3.2.1, so we fall back again on direct estimation from the time serie of the residuals. We fit an $AR(1)$ model and then check again whether the residuals are still autocorrelated or not.

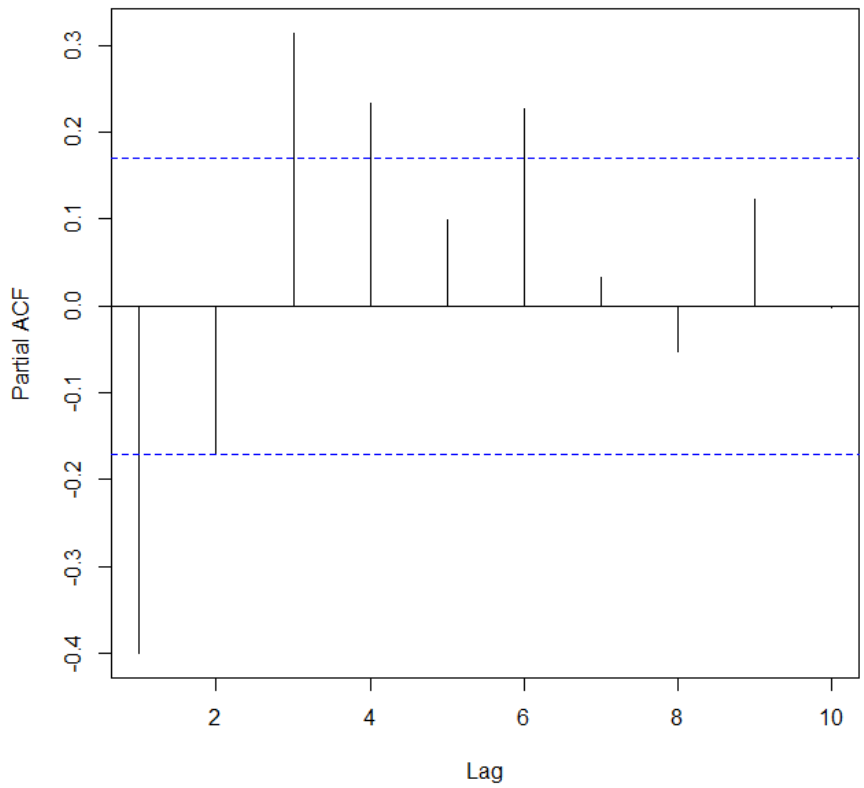


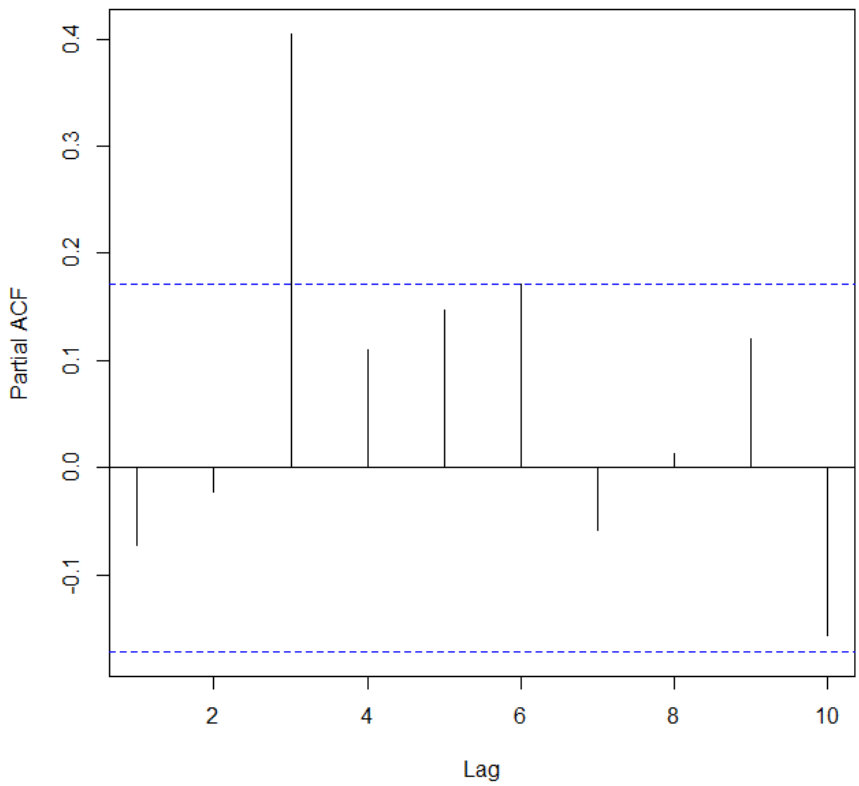
Figure 3.10: Partial Autocorrelation Function of Residuals

This gives us the final specification of the model:

$$\begin{cases} \lambda(t) = \beta_1^{(\lambda)} \cdot \lambda_{t-1} + \beta_2^{(\lambda)} \cdot A(\, C(t),\, 2\,) + \beta_3^{(\lambda)} \cdot S(t) + \epsilon^{(\lambda)}(t) \\ \epsilon^{(\lambda)}(t) = \rho^{(\lambda)} \cdot \epsilon^{(\lambda)}(t-1) + \eta^{(\lambda)}(t) \end{cases} \tag{3.6}$$

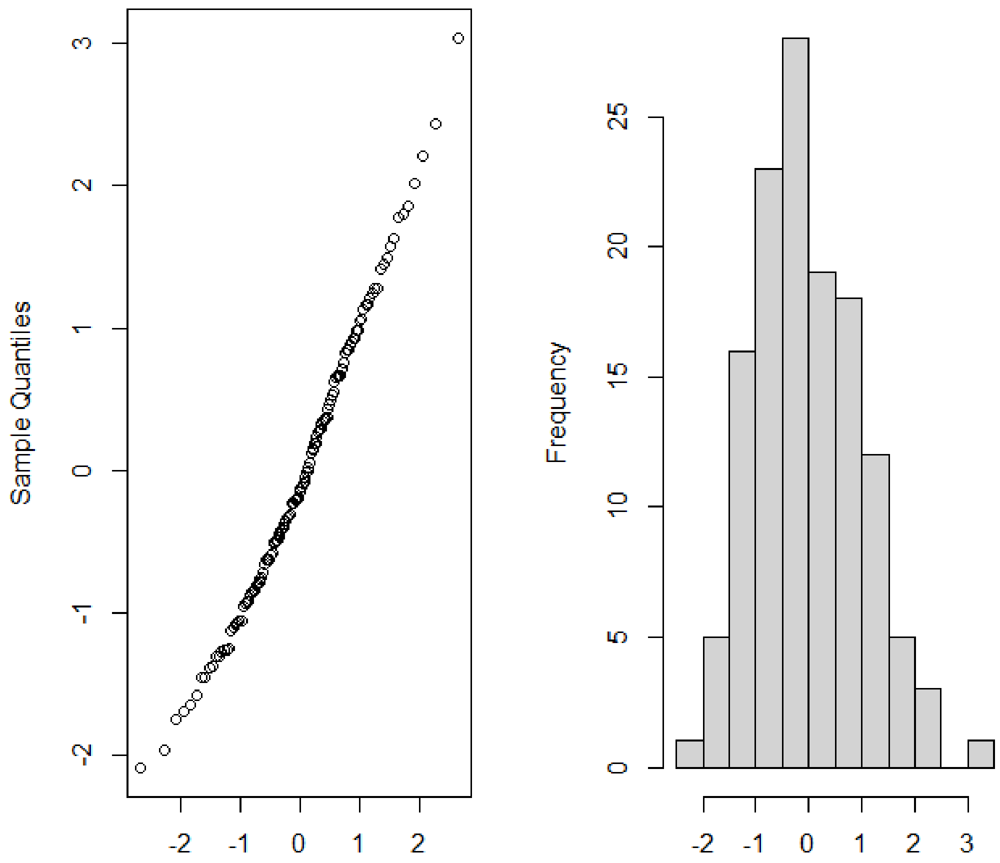
with the coefficients being the same as in 3.5, $\rho^{(\lambda)} \simeq -0.400$ and $Var\big(\eta^{(\lambda)}\big) \simeq 1.003$

In this case, the new graph of the partial autocorrelation function and the Durbin-Watson test (p-value = 0.37) indicate that the residuals obtained this way are actually free from autocorrelation.



3.3.2. Checking normality of residuals

We will again look for any evidence of non-normality by observing possible deviations from the quantiles of a normal distribution and by using the Jarque-Bera test. A histogram is also reported in order to give a more visual grasp.



There does not seem to be a great deviation from a normal distribution of the residuals of the $AR(1)$ residual process, denoted as $\eta^{(\lambda)}(t)$ in 3.6. In particular, the empirical quantiles look well-aligned with the theoretical ones. Moreover, the Jarque-Bera test has a p-value of 0.168, therefore there is no strong evidence of non-normality.

4 | Introduction of a central bank digital currency

The progress of new technologies and their nascent applications to the financial industry have led academics and central banks around the world to consider the merits and possibilities of the introduction of a central bank digital currency (CBDC). Among other publications and articles, we have based our assumptions and analyses on Bindseil [2020], Bindseil and Panetta [2020], Pollock [2018], Kumhof and Noone [2018], Juks [2018].
These sources indicate a wide spectrum of critical points, issues that may arise and possible macroeconomic consequences. Some also propose a solution to address the critical points and a possible design of the digital currency, for example Bindseil [2020] and Bindseil and Panetta [2020] present a design for the CBDC based on a tiered remuneration system, which would enable the central bank to control the total volume of CBDC through monetary policy decisions. Juks [2018] also provides a back-of-the-envelope estimation regarding what amount of CBDC we should expect as a base level upon introduction, due to its versatility as a payment method and risk management tool.

The sources thoroughly analyse the advantages and risks brought by the introduction of a digital currency, and in particular they try to address two critical points against CBDC, namely the disintermediation of commercial banks in the normal credit allocation process and the risk of facilitating a systemic run on banks in the event of a crisis.

4.1. Design of a digital Euro

The central bank digital currency would become a new form of payment that can be handled electronically and is accessible to the broad public, making available efficient, secure and modern central bank money to everyone while strengthening the resilience, availability and contestability of retail payments.
The underlying goal of the digital Euro is to overcome banknotes, or at least take over a portion of their common use. This also has the advantage of introducing a new monetary policy instrument. However, attention should be paid to the possibility of side effects on deposit collecting institutions (mainly banks), whose activity could be severely impacted by the competition of the central bank. Indeed, issuing a digital currency the central bank would benefit from an unfair competitive advantage that does not necessarily correspond to better efficiency in the credit allocation process.

The introduction of the CBDC would have as a consequence the creation of a third form of base money, along with banknotes and overnight deposits with the central bank (the latter being available only to banks and some other institutions). There are two different technical formats which could be used in order to implement this general use digital currency, namely:

1. **Deposit accounts with the central bank:** in this case, there would be no need for great innovations in the technological infrastructure of the central bank, but rather it would be a matter of scaling the number of accounts currently offered. This could be technologically challenging per se, as the number of accounts could skyrocket from about 10,000 to 500 million, a number representing inhabitants and firms in the Euro area satisfying certain basic legal and economic requirements.
   The technical maintenance and service may be assigned to third party providers to ensure that the allegedly less efficient public sector does not take over more tasks than needed. Commercial banks could provide the service of exchanging bank deposits against banknotes and digital currency, charging a competitive fee (similarly to what they already do today with ATMs).

2. **Digital token currency:** in this case, the problem is that these forms of payment based on a decentralized ledger are usually associated with anonymity, meaning that the bank would not know who holds the tokens issued, similarly to the case of banknotes. The addressing of this matter could generate a large number of issues,

as the bank would need to prevent facilitation of money-laundering and other illegal activities.
Some propose to deal with this through a Proof-of-Concept type of authentication. However, a well established and recognized technology is not yet available.

We will assume that the central bank digital currency is based on the first design (direct deposit to the central bank), as it relies on a procedure that allows the central bank to issue the CBDC and pay its remuneration in a way that is already widely used and known. The remuneration is a useful tool to address the negative impact that the digital Euro might have on the European banking sector and the central bank's balance sheet: by setting the remuneration at a low enough level (possibly negative) through monetary policy decisions, the central bank should be able to discourage the use of the CBDC as a store-of-value instrument. It is indeed crucial that the central bank does not allow its balance sheet to grow to such an extent that it becomes a financial intermediary, having to take a significant role in the credit allocation process.

This sort of control through monetary policy would not be available in the case of a non-remunerated CBDC, and this is a particularly critical point especially in a negative interest rate environment such as the one we are observing today. Indeed, allowing a risk-free, liquid asset with zero nominal yield to be purchased without constraints in a negative interest rate environment is not feasible as institutions that resort to the deposit facility in their normal activity would simply switch to the CBDC, as it is an equivalently risk-free and liquid asset, but one whose value is not eroded by negative rates. This issue would have to be addressed by some other means, therefore in the non-remunerated case we neglect the contribution in the CBDC volume generated by competition with the deposit facility. In general, we are trying to ignore the amounts of CBDC due to cash flow coming from the ECB deposit facility and banknotes.
More details about the critical points of the introduction of a digital currency discussed by Bindseil are illustrated in the next section.

4.2. What are the consequences of a CBDC?

When thinking of the desirable features of the digital Euro, we must understand how it might affect its competitors, namely banknotes and deposits made with commercial banks.
As for banknotes, there is really no drawback in substituting a portion of them with CBDC: it is as efficient and accessible as banknotes as a form of retail payment. Also, the diminishing demand for cash gives citizens ever less access to the central bank's balance sheet, resulting in citizens having to trust financial intermediaries to store their wealth. On the other hand, a major impact in the intermediation of banks must raise some concerns: if the volume of CBDC was to grow too far, it may cause severe losses of funds for banks focused on the retail business. Although advocates of sovereign money might see bank disintermediation as the precise goal of a CBDC, it must be taken into account that the central bank's balance sheet would grow far larger.

Liquidity raised by any commercial bank through sight deposits is then reinvested in the economy via the normal lending activity of the bank, but in in presence of a CBDC commercial banks would lose a portion of this funding source to the central bank. As this happens, the central bank may itself have to engage in lending activity to prevent damage to the economy, benefiting from an unfair competitive advantage in deposit collection which it would however ignore, leading to inefficiencies and losses that eventually citizens would have to bear, as argued in Pollock [2018], in a *Testimony to the Subcommittee on Monetary Policy and Trade of the Committee on Financial Services*.

Summarizing the effects of the CBDC on the balance sheets of commercial banks, the central bank and firms:

- **Banks rely more on central bank credit:** the loss of funding due to the switching of deposits from commercial banks to the central bank could trigger higher demand for central bank credit from commercial banks. The central bank might try to avoid this by purchasing corporate or government bonds directly from banks and households, but it cannot nullify the increased cost of funding, since both central bank credit and the issuance of bonds are more expensive than deposits as a funding source.

- **The ECB needs to match new liabilities:** as deposits are made with the central bank in exchange for (possibly remunerated) CBDC, the central bank would need to decide how to invest these funds. As mentioned above, it would make sense to purchase corporate and government bonds from private banks and households, also considering that investments by the central bank in risk-intense portfolios are not an option.

- **The central bank would need to accept new forms of collateral:** If the CBDC were to become a large-scale *store of value*, issues related to the centralization of credit would emerge: if we were to witness a full shift from bank deposits to CBDC, with private banks relying more on central bank credit, the ECB would have to provide around 4 trillion euros in credit of credit to banks (ceteris paribus), i.e. 20% of its balance sheet, as estimated in **?**. This would force the ECB to revisit its collateral framework, effectively compelling it to consider certain asset classes it did not formerly accept. Unless one one supports such exposure, it will be necessary to find solutions to contain the volume of CBDC, to make sure it does not lead to any centralization in the credit allocation process by becoming a store-of-value instrument.

- **Companies need to find new sources of credit:** The loss of funding sources by commercial banks would likely translate to a decrease of their loans to corporates, that may need to issue additional bonds to compensate for this loss of funding (and the central bank would likely play a role in purchasing these bonds).

The tool proposed in Bindseil [2020] and Bindseil and Panetta [2020] that allows to keep under control the total volume of CBDC is tiered remuneration, with rates not lower than zero for the first tier and not higher than zero for the second tier. Both of these rates would have to be functions of the deposit facility rate.
Bindseil argues this would be a safe design as rates with this feature are already common in deposits with the ECB. The first tier will only be made available to individuals who are citizens of the European Union, and Bindseil conjectures that the total amount available per individual could be capped at 3000 Euros, implying an amount of total tier one CBDC for households of around EUR 1 trillion (about EUR 180 billion in Italy). Every amount above this quote and all the CBDC held by corporates and other institutions is to be considered of tier 2.

The main purpose of tier 1 CBDC would be to substitute banknotes as a form of payment, and its introduction appears free from controversy, as its effect on both the central bank and commercial banks balance sheets is negligible.

On the other hand, tier 2 CBDC would compete precisely with sight deposits, leading to the consequences discussed above. It is therefore necessary, if the central bank wishes to avoid becoming a financial intermediary, to control the volume of tier 2 through monetary policy (or other means if the CBDC is designed non-remunerated), but the remuneration might be pushed arbitrarily low if discouraging the use of the CBDC as store of value becomes primarily important. As for non-remunerated CBDC, controlling its volume so that it does not become a store-of-value would be even more critical (especially in negative interest rate environments), but this limitation would have to be enforced in some other way.

Variable definition: To observe and model the dynamics of the two tiers we need to define their remuneration rate and volume:

- ***I_ CBDC1*** indicates the interest rate paid by the tier 1 CBDC
- ***I_ CBDC2*** indicates the interest rate paid by the tier 2 CBDC
- ***V_ CBDC1*** is the volume of the tier 1 CBDC, the evolution of which is modelled in 4.3.1
- ***V_ CBDC2*** is the volume of the tier 2 CBDC, the evolution of which is modelled in 4.3.2
- ***I_ CBDC*** and ***V_ CBDC*** are going to be used analogously to the notations above in the case of the non-remunerated CBDC, as there would be no tiers

Both CBDC rates depend directly on the ECB deposit facility rate $ECBDF(t)$ according to the design proposed in Bindseil [2020] and Bindseil and Panetta [2020], through the equations:

$$\begin{cases} I\_CBDC1(t) &= max\big(\,ECBDF(t) - 1\%,\, 0\,\big) \\ I\_CBDC2(t) &= min\big(\,ECBDF(t) - 1\%,\, 0\,\big) \end{cases}$$

$$\begin{cases} I\_CBDC1(t) &= max\big(\,ECBDF(t) - 2\%,\, 0\,\big) \\ I\_CBDC2(t) &= min\big(\,ECBDF(t) - 0.5\%,\, 0\,\big) \end{cases}$$

Instead, in the non-remunerated case they carry a null, fixed nominal rate, just like banknotes.

Assets			Liabilities		
Households, pension and investment funds, insurance companies					
Real Assets	20		Household Equity	40	
Sight deposits	5	-CBDC2			
Savings + time deposits	4		Bank loans	5	
CBDC		+CBDC1 +CBDC2			
Banknotes	1	-CBDC1			
Bank bonds	4				
Corporate/Government bonds	7				
Equity	8				
Corporates					
Real assets	13		Bonds issued	3	
Sight deposits	2		Loans	8	
Savings deposits	1		Shares / equity	5	
Government					
Real assets	11		Bonds issued	9	
			Loans	2	
Commercial Banks					
Loans to corporates	8		Sight deposits	7	-CBDC2
Loans to government	2		Savings + time deposits	5	
Loans to HH	5		Bonds issued	4	
Corp/state bonds	5		Equity	3	
Central bank deposits	0		Central bank credit	1	+CBDC2
Central Bank					
Credit to banks	1	+CBDC2	Banknotes issued	1	-CBDC1
Corp/Government bonds	0		Deposits of banks	0	
			CBDC		+CBDC1 +CBDC2

Figure 4.1: Financial accounts representation of CBDC and compensating securities purchases by the central bank (numbers in trillion euros broadly illustrating euro area accounts). It should give a qualitative idea of how the two tiers are going to impact the balance sheets of the central bank as well as households, commercial banks, corporates and governments.

4.3. How to include the CBDC

Introducing a central bank digital currency changes both the money base and the circulating money landscape: the money base that is currently composed by banknotes, bank reserves and sight deposits would have to accept the new competitor, a risk-free (guaranteed by the Central Bank), liquid asset that is also a new legally accepted form of payment, in addition to banknotes and sight deposits.
Its competition with sight deposits is modelled through its convenience and the current level of risk in the Italian banking system implied by the market.
The convenience is the difference in remuneration between deposit and CBDC rates, which depends on the tier and remuneration system. For our analysis, the modelling the cash flows from and towards sight deposits is the key point in order to assess the possible loss of liquidity (and funding) for the Italian banking system (represented by sight deposits) in favor of the CBDC. In order to gain liquidity, deposits issuers are able to make their products more attractive than the CBDC through convenient rates, whereas when the two rates get closer (or the CBDC becomes more attractive) we should observe cash flows from the banking system towards the CBDC.
The credit risk of the Italian banking system is measured with the metric represented by the CDS index defined in 2.3.1, and we assumed that (for equal convenience) higher values of the CDS index lead liquidity away from the deposits into the CBDC. This is due to the fact that the CBDC competes with the deposits as form of investment and payment, but it also represents a liquid risk-free asset, which therefore is more desirable than risk-bearing assets that perform the same function, like sight deposits.

As mentioned, the substitution of banknotes is quite free from controversy, as the more modern and secure CBDC substituting banknotes as payment method would negligibly affect the balance sheets of the central bank, households and other players. Summing up, the evolution of the CBDC volume in all three remuneration scenarios is driven by two factors:

1. **The difference between the deposits rate and the CBDC rate (of each tier)**. The deposit rate will be simulated according to the model developed in chapter 3, while the CBDC rates are functions of the ECBDF rate and are therefore simulated according to the Renne model, i.e. they will behave as a Markov chain with transition probability matrix given by 2.3.

2. **The perceived risk of the banking sector**. It is indicated by the simulated value of the CDS index.

We now describe the different mechanisms that are assumed for the cash flows between the deposits and CBDC.

4.3.1. Tier 1 vs deposits

This is the type of CBDC that is available to the broad public. Its total amount per person has a cap of 3000 Euros by definition (for a total of 180 billion Euros in Italy), so there is no risk of it becoming a store-of-value above its cap amount. Its main purpose is to substitute a large portion of banknotes and to function as a secure and modern payment method.
This feature is incorporated in our dynamic through the definition of a *base* level representing its adoption as form of payment. According to Bindseil[1], this base level consists in 10% of the current banknotes circulating in the economy (amount which we will ignore since it does not affect deposits) plus 2% of the deposit volume belonging to consumer households (i.e. those with access to tier 1).

Tier 1 will also pay a non-negative remuneration, so not only it is more appealing than banknotes but it may also attract liquidity from those citizens who see it as an investment opportunity, especially when its remuneration rate is close to or higher than that paid by deposits. This behaviour will be captured by the first driving factor.

Citizens might also transfer their liquidity from deposits to tier 1 according to the second driving factor (CDS index) when they perceive a higher than average risk in keeping their money in their deposit accounts because of potential difficult moments for the banking system.

In order to make equation 4.1 more readable, we preemptively define the quantities and functions described above.

- *Base1*: the base amount of 2% of household deposits, equal to 22 billion Euros

[1] Bindseil relies on Juks [2018], where a back-of-the-envelope estimation is developed

- *Cap1*: the maximum available total amount, equal to 180 Billion euros
- *spread1*: the difference between the deposit rate and the tier 1 rate, i.e. $I - I\_CBDC1$
- $f1$: the function modelling the exchanges between deposits and tier 1 due to the convenience as investment, it will have *spread1* as argument. See 4.2
- $f2$: the function modelling the exchanges between deposits and tier 1 due to the perceived risk, it will have the CDS index as argument. See 4.2
- w: a parameter defining the weight each of the two factors has on determining the exchanges of liquidity

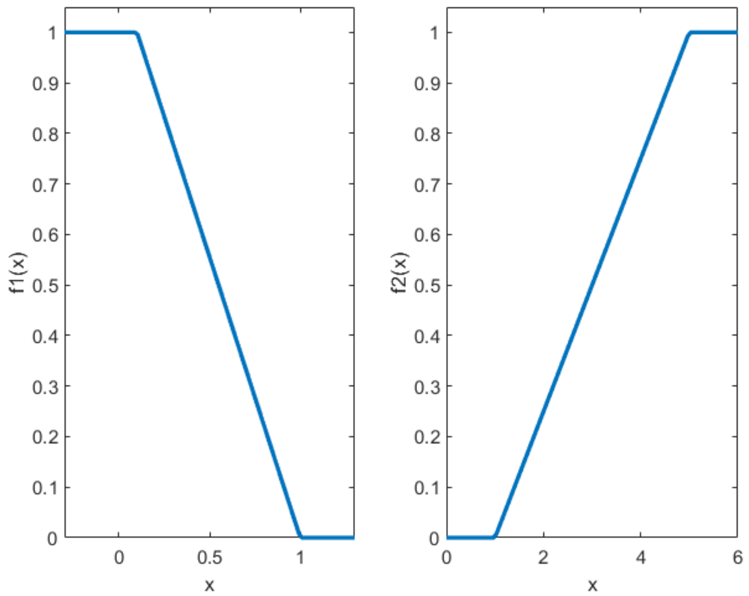


Figure 4.2: Graph of the functions governing the dynamics of tier 1 volume

The equation that will govern the evolution of tier 1 volume is:

$$V\_CBDC1(t;\, w) = Base1 + (\,Cap1 - Base1\,)\left[w \cdot f_1\big(spread1(t)\big) + (1-w)\cdot f_2\big(S(t)\big)\right] \tag{4.1}$$

Equation 4.1 describes the movements of tier 1 volume with respect to the driving factors we have identified. In particular, we will have no contribution to the total quantity of tier 1 by the first driving factor (convenience as form of investment) when the spread between the deposit rate and tier 1 rate is higher than 1%, and maximum contribution when it is lower than 0.1%[2], with linear degrowth in between.

As for the second driving factor (CDS index), we will assume it plays no role as long as its value stays below 100 basis points, and that the additional quantity of tier 1 CBDC reaches its maximum when the value of the CDS index reaches 500 basis points, with a linear growth in between.

Summing up, the two factors driving the changes between the base value and the cap value act through two functions (f_1 and f_2) defined as:

$$f_1(x) = \begin{cases} 1 & x \leq 0.1 \\ -\frac{10}{9}x + \frac{10}{9} & 0.1 < x < 1 \\ 0 & 1 \leq x \end{cases} \qquad f_2(x) = \begin{cases} 0 & x \leq 1 \\ \frac{1}{4}x - \frac{1}{4} & 1 < x < 5 \\ 1 & 5 \leq x \end{cases} \tag{4.2}$$

[2]Consider that historically it was only before the 2008/2009 crisis that deposit facility rates that were higher than sight deposits rates, allowing banks to enjoy a form of arbitrage

4.3.2. Tier 2 vs deposits

This is the tier that will be available to corporates, foreigners and private citizens when their digital currency accounts have reached the cap amount. It is the most delicate tier to implement, as the absence of a cap for its quantity is potentially troublesome in case of bank-runs. This is the reason why the central bank needs to control its volume (to avoid the disintermediation of banks) by imposing non-positive rates. Assuming therefore it will not serve as store-of-value asset, its main use will be that of a payment method and tool to manage credit risk.
Given the zero upper bound for I_CBDC2, we will assume that the main impact on the tier 2 volume is given by the second driving factor (CDS index), while the first will only have a limited effect.

The base level proposed by Bindseil reflects its standard use as a risk management tool, and is obtained following the back-of-the-envelope estimation given in Juks [2018]: we assume that the tier 2 CBDC will substitute a base level of 10% of real sector deposits (47 billion Euros) precisely with the purpose of lowering credit risk and enhancing risk management for firms.

The growth of the CDS index results in an increase in the volume of tier 2 because we assume that corporates and other institutions rely on it more heavily in times when intense risk is perceived in the banking system.

The "convenience" as an investment is assumed negligible because of the zero upper bound for I_CBDC2, but when the deposit rate is approached a certain amount can be added to the tier 2 quantity because of that.

As in the previous section, we give some definitions beforehand to make the equation of CBDC dynamics more readable:

- *Base2*: the base amount of 10% of the real sector deposits, equal to 47 billion Euros
- *spread2*: the difference between the deposit rate and the tier 1 rate, i.e. $I - I\_CBDC2$, always non-negative
- $g1$: the function modelling the exchanges between deposits and tier 2 due to the low value of $spread1$, which it will take as argument. See 4.3
- $g2$: the function modelling the exchanges between deposits and tier 2 due to the

perceived risk, it will have the CDS index as argument. See 4.3

- k: a parameter related to the maximum amount that we assume will be allowed to flow to the CBDC in risk-intense times

The equation that will govern the evolution of the tier 2 volume is:

$$V\_CBDC2(t;\, k) = Base2\, +\, 50\, g_1\big(spread2(t)\big) + k\, g_2\big(S(t)\big) \tag{4.3}$$

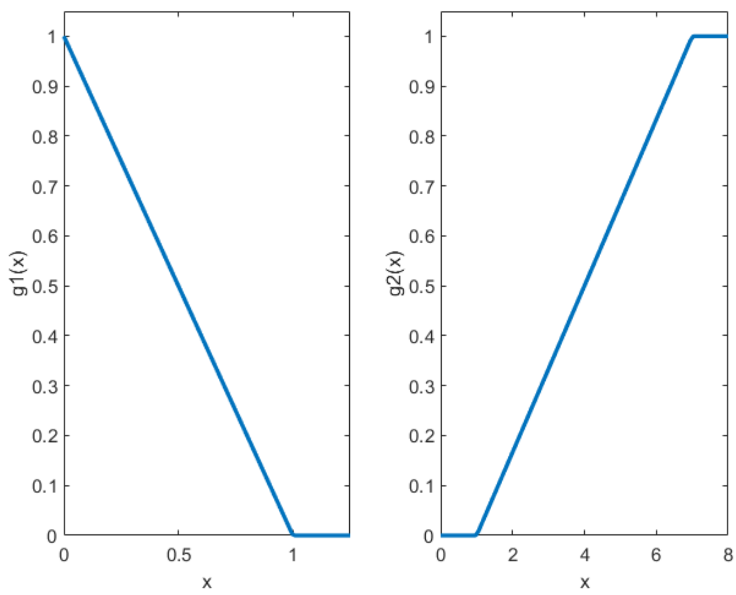


Figure 4.3: Graph of the functions governing the dynamic of tier 2 volume

Equation 4.3 describes the movements of tier 2 volume with respect to the driving factors. In particular, we will have no contribution to the total quantity of tier 2 from the first driving factor (remuneration spread between tier 2 and deposits) when it is higher than 1%, and maximum contribution at 0%. The value of *spread2* can reach zero only theoretically.

As for the second driving factor (CDS index), we assume it plays no role as long as its value stays below 100 basis points, and that the additional quantity of tier 2 CBDC reaches its maximum when the value of the CDS index reaches 700 basis points, with a linear growth in between.
The meaning of k is to set which percentage of the current deposit volume (about 1600

billion Euros) can switch to CBDC during extreme scenarios. For example, with $k = 753$ we are allowing tier 2 to substitute 50% of the current deposits when the CDS index reaches 700 basis points.
The amount that can be added to tier 2 volume thanks to the low value of $spread2$ can be adjusted in order to observe what happens with values that differ from 50 billion euros.

Summing up, the two factors driving the changes between the base value and the cap value act through two functions (g_1 and g_2) defined as:

$$g_1(x) = \begin{cases} -x+1 & 0.1 < x < 1 \\ 0 & 1 \leq x \end{cases} \qquad g_2(x) = \begin{cases} 0 & x \leq 1 \\ \frac{1}{6}x - \frac{1}{6} & 1 < x < 7 \\ 1 & 7 \leq x \end{cases} \tag{4.4}$$

4.3.3. Non remunerated CBDC

In this simpler case, the CBDC would not be distinguished into tiers, presenting a constant remuneration rate of zero.
We have already pointed out why special care should be paid to this case, especially in a negative interest rate environment, since having unlimited access to a liquid, risk-free asset would have important consequences on the deposit facility activity.
A CBDC with this design would have basically the same features as banknotes, but without the downsides of storage cost and poor maneuverability.
We assume that the this design of the CBDC would feature some aspects of both the other two types, namely:

- it would have a base amount equal to $Base1 + Base2$
- its volume would increase in a situation where sight deposits are perceived riskier as form of payment, in the same way as if those components of tier 1 and tier 2 volume were combined
- the "convenience" component due to the difference with deposit rates would behave as the same component in the second tier, although in this case such spread would be represented by the deposit rate itself

Summing up, we obtain the equation:

$$V\_CBDC\_N(t) = (Base1 + Base2) + 50\, g_1\big(I(t)\big) + \big(k + (cap1 - base1)(1-w)\big)g_2\big(S(t)\big) \tag{4.5}$$

5 | Simulations and results

In this chapter, we propose observing the effect on the Italian banking system of the different CBDC remuneration systems.
In particular, we use simulations of the deposit volume V and the CBDC volume V_CBDC to gain an insight into what percentage of the liquidity deriving from deposits might be lost due to the presence of the digital currency. The dynamics of the deposit volume are those presented in chapter 3, whereas the dynamics of the CBDC volume are those introduced in chapter 4, divided by each of the three remuneration systems (Bindseil, Bindseil-Panetta, no remuneration).

First, we analyze the case where there is no CBDC: by observing the distribution obtained through Monte Carlo simulation, we plot the quantiles (95%, 99% and 99.9%) at each time up to the cut-off horizon. At a fixed time step, we consider the minimum of the liquidity up to that time, then compute the quantiles of their distribution.
After that, considering the variations between single months as independent observations, we also compute the *month-by-month* relative loss distribution and its quantiles.
The same month-by-month analysis is then carried out for each remuneration system, and the quantiles are computed for different combinations of the parameters in 4.1 and 4.3.

With this procedure, we are able to assess the liquidity risk in different scenarios, it is sufficient to specify the remuneration system and the exchange of cash flows between deposits and CBDC as in 4.1 and 4.3. Note that it is not only the parameters that can be changed, but also the "triggers"; for example, one may want to observe how the quantiles change if the CDS index starts playing a role before or after reaching 100 basis points.
It is therefore a general method that allows us to assess the possible loss of liquidity for the banking system both in normal times and in the presence of a CBDC, once the features of the latter have been fixed.

5.1. The reference case

In this section we observe the liquidity distribution according to simulations made for the deposit volume only. The CBDC is not present, so the shifts in liquidity are only due to the four building blocks of Castagna's model.
The quantiles reported below are obtained computing (at an arbitrary time) the minimum volume for every path up to that time, and then considering the distribution of those across the simulation. This procedure is used to generate the term structure of liquidity as done in Castagna and Mistè [2019], ensuring that the function representing the quantiles is non-increasing with respect to time.

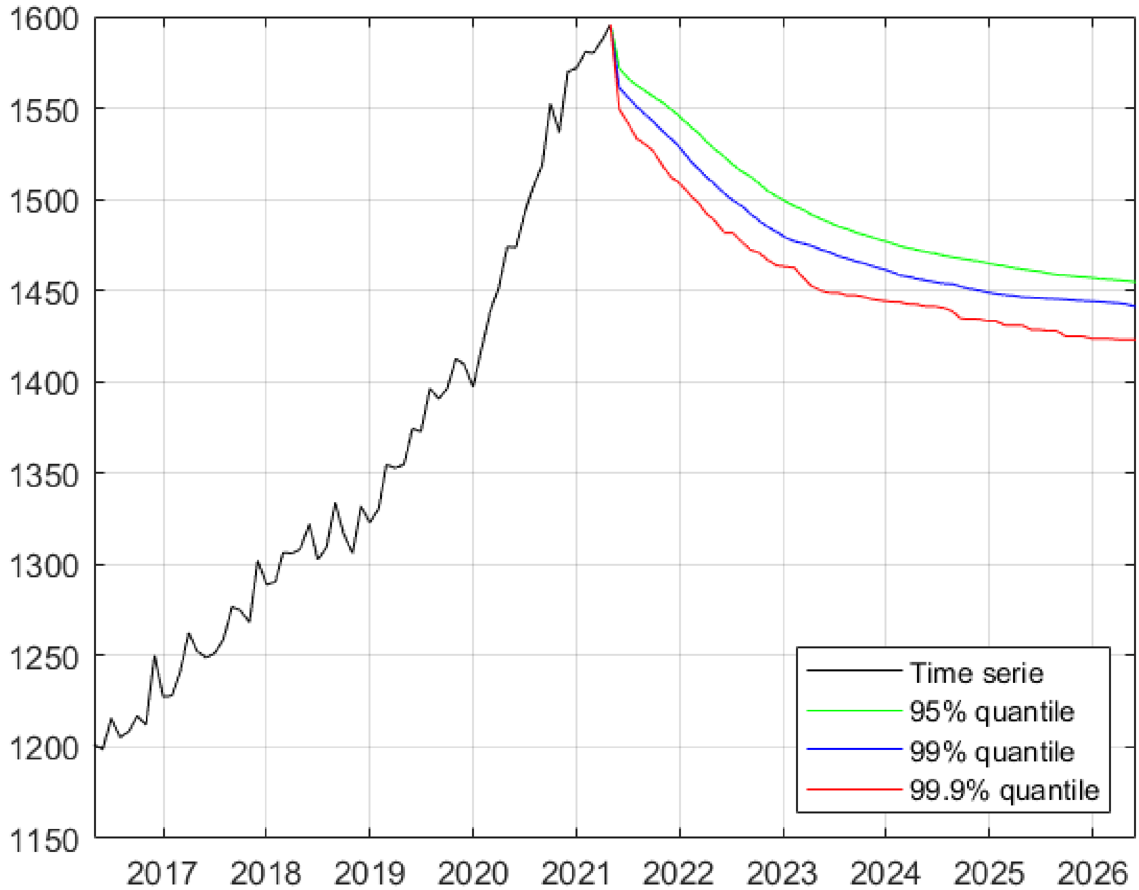


Figure 5.1: Historical evolution up to starting date and quantiles of the simulation

From the Monte Carlo simulation, we can also consider the variations of the volume in every path as single independent observations, obtaining a number of observations equal to the number of time steps (60, since the time step is one month) times the number of simulations. We express the losses as percentage of the total liquidity, and compute the *value-at-risk* (VaR) at the confidence levels of 95%, 99% and 99.9%:

Level	95%	99%	99.9%
VaR	1.99%	2.79%	3.69%

In the following sections we only report the second type of risk assessment (month-by-month), as our focus is on comparing the quantiles obtained with different rules for the volume dynamics.

Since the equations we have proposed in chapter 4 are designed with estimations that depend on underlying macroeconomical estimations rather than available data, we are extremely interested in evaluating the sensitivity of these risk metrics with respect to the design of the equations. Indeed, we expect that as this topic is explored further, having a way to assess the sensitivity with respect to changes in the dynamics (regardless of the specific design of the CBDC rates) will be desirable. Also, we assume observers of the banking system to be more interested in assessing the short term (30 calendar days) liquidity risk rather than having a picture of the VaR at different time horizons. Furthermore, interpreting the differences between graphs resembling **??** after changing the dynamics of the CBDC volume is a rather hard task, whereas observing the VaR gives an easy interpretation. Therefore, we do not focus on the representation of the evolution in time of the Var's as seen from the reference date, but rather on the liquidity risk at 30 days. For each different assumption regarding the dynamics described in 4.3, we report the three quantiles.

5.2. Zero remuneration

In this section, we consider the presence of a CBDC with no remuneration. This type of CBDC is substantially equivalent to banknotes. As explained in chapter 4, we do not take into account the amount of CBDC that might be generated from the competition with the deposit facility or banknotes, as we are limiting ourselves to factors that would take away liquidity from bank deposits. Therefore, while it is beyond doubt that in a negative interest rate environment a liquid, risk-free asset carrying null yield would undoubtedly attract large amounts of liquidity. However, this liquidity would come from banks and other institutions that have access to the deposit facility, and not from sight deposits holders who switch to the CBDC.
As for banknotes, the non-remunerated CBDC is bound to take away a portion of their use, as they are practically identical in their uses but the CBDC is more versatile and less costly/risky to hold in large quantity. This influence is neglected, too, as again it does not represent an amount of CBDC that flows from sight deposits directly.
We report the quantiles for different combinations of the parameters appearing in 4.1 and 4.3:

Parameters	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$
$w = 0.75, k = 273$	2.08%	2.93%	3.88%
$w = 0.75, k = 753$	2.11%	2.95%	3.91%
$w = 0.25, k = 273$	2.09%	2.94%	3.88%
$w = 0.25, k = 753$	2.11%	2.96%	3.91%

We can observe that the effect of the parameter k in 4.3 is somewhat light (a couple of basis points). As we will see in the following sections, this is common and is due to the fact that in the credit model we calibrated both the mean reverting level and the volatility of the CIR process are quite low, as are the values of the deterministic shift. This entails that the CDS index simulation never reach extremely high levels, for example 500 basis points. This is coherent with what was the perceived risk in the Italian banking system at the reference date. Furthermore, note that a level that high is reached usually in clearly difficult times (for example, the 5-year CDS spread was 570 basis points for Parmalat less than three weeks prior to its default).

There is no sensible difference for variations of w in 4.1

5.3. Bindseil remuneration

In this case, we observe also a direct competition between deposits and CBDC (especially tier 1), as they can both be considered financial assets that can be used as means of payment.
For this CBDC remuneration specification and our version of the Renne model, the image of the function evaluating I_CBDC1 is the set {0%, 0%, 2%}, which can be interpreted by saying that in times where the rate of the deposit facility is "high" (3%), the tier 1 CBDC becomes a quite profitable investment. Its volume is bounded by the cap of 180 billion euros, but it is still more than enough to be sensed by the deposits volume.

On the other hand, the CBDC is to be favored in moments when the perceived risk in the banking system (and therefore in the liquidity held in deposits) increases, if above a certain quota.

Parameters	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$
$w = 0.75, k = 273$	3.37%	7.65%	10.19%
$w = 0.75, k = 753$	3.39%	7.66%	10.23%
$w = 0.25, k = 273$	2.34%	3.37%	4.63%
$w = 0.25, k = 753$	2.36%	3.39%	4.66%

Here the variation with respect to the w parameter of 4.1 is more sensible, while k maintains a small effect.
The quantiles are much higher than those of the non-remunerated case. This is due to the positive remuneration of the tier 1, that adds a component (proportional to w) to its volume, which entails that the convenience of the tier 1 possibly hitting 2%, the tails of the V_CBDC1 simulated distribution are noticeably heavier.

5.4. Bindseil-Panetta remuneration

This third design for the remuneration of the two tiers entails a lower possible convenience of the tier 1, with a maximum remuneration of 1% in a high central-bank rate environment. The tier 2, on the other hand, becomes less expensive to hold even in an environment with a low deposit facility rate.
We have therefore an increase in the role played by the tier 2 and a decrease of the role played by the tier 1. However, since the tier 2 volume V_CBDC2 depends mainly on the credit risk part (whose simulation never exceeds 500 and 700 basis points, i.e. our thresholds), the tier 2 weight remains smaller, therefore we have an overall reduction of the VaR's with respect to the remuneration system proposed by Bindseil.

Parameters	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$
$w = 0.75, k = 273$	3.05%	5.88%	8.03%
$w = 0.75, k = 753$	3.07%	5.89%	8.04%
$w = 0.25, k = 273$	2.25%	3.17%	4.19%
$w = 0.25, k = 753$	2.27%	3.18%	4.22%

Here the impact of k remains low, but we still have a sensible impact given by w. The extreme scenarios bear a noticeably higher risk than the same scenarios in the non-remunerated case, but sensibly lower than the Bindseil remuneration scenario.

Conclusion and further developments

In this thesis, we presented a present-day object of discussion: the management of risks arising from the issuance of sight deposits, both in the current situation and in a hypothetical scenario where a central bank digital currency (CBDC) is introduced.
We modelled the liquidity of sight deposits in the Italian banking system in the same way as done in Castagna and Mistè [2019], focusing on their sensitivity with respect to interest rate and credit factors. In this model, some qualitative rules for the behaviour of deposit holders are assumed, and their influence is estimated with through time-series analysis.
We then introduced a central bank digital currency, designed with a few macroeconomic guidelines presented in various articles and we observed its behaviour in different scenarios assessing its impact on the liquidity of Italian sight deposits.

We initially estimated a model for the short-rate, closely tied with the ECB deposit facility rate. Moreover, a CDS index (built on a weighted average of the three largest Italian banks' CDS spreads) has been modelled with an intensity-based approach.
For the deposits' rate, a linear model has been estimated from time-series data. As for the deposits' volume, we assumed a dependency on deposit interest rates, risk-free rate, CDS index and an auto-regressive term, finding also that historical data regarding deposit volumes shows a noticeable linear trend. The de-trended component of the volume is affected by the difference between deposits rate and risk-free rate as well as by the CDS index, and the coefficients of its dynamics have been estimated. These coefficient have the purpose of quantifying the significance and influence of the building blocks on the deposit volumes.
These four building blocks have been used to obtain a Monte Carlo simulation of the deposits' volume.

We then hypothesized the introduction of a central bank digital currency (CBDC), described the critical points that must be addressed and introduced three different CBDC remuneration system that have been proposed in literature (Bindseil [2020] and Bindseil and Panetta [2020]), two of which focused on a tiered remuneration of the CBDC depending on the rate of the deposit facility, while the third assumes the CBDC to carry a null nominal yield, just as banknotes. After that, we modelled the cash flows between sight deposits and CBDC depending on their remuneration and credit status of the Italian banking system.

The proposed methodology has been used to observe the risk metrics variation caused by changes in the design of the CBDC remuneration and/or of the cash-flow exchanges between CBDC and deposits.
A Monte Carlo method has been employed to simulate the distribution of deposits' liquidity and the value-at-risk measures obtained with the simulation have been computed in all the different scenarios.

We found that a non-remunerated digital currency would have the least impact on the banking system, as it would never be particularly convenient assuming that deposit rates do not turn negative. On the other hand, the two systems with tiered remuneration present noticeable increases in the VaR's with respect to the CBDC-free case.

Future developments of the work include the extension and improvement of at least some of the models describing the building blocks, namely:

- The short-rate model proposed in Renne [2016] can be improved by relying on a far more extended state-space, so that the simulation of the deposit facility rate results in a set of more realistic paths.
  In order to achieve this, the interbank yield curve should be considered and the model parameter structure should be designed in a way that allows to capture the market expectations regarding future monetary policy changes
- The credit model could be substituted with another that takes into account more indicators regarding the credit situation of the banking system, rather than only the CDS spreads
- The models presented in chapter 3 could be improved by trying to capture the behavioral features with more sophisticated statistical tools

- The total liquidity held in CBDC, deposits, banknotes and deposit facility (as a whole) could be modelled more thoroughly, together with the dynamics of the competition between pairs of these forms of payment and/or base money, represented by the cash flows functions

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Extended summary

In this thesis we propose a model for the evaluation of sight deposit liquidity, the evolution of which is dependent on market and behavioral factors. This approach was proposed in Castagna and Mistè [2019] and it assumes that cash flows directed towards and away from sight deposits are driven by the convenience of deposits as a form of investment and by the magnitude of counterparty risk borne by deposit accounts.
Then, we study the possible introduction of a central bank digital currency (CBDC) and assess its impact on the sight deposit market in Italy. We gauge three different designs proposed in literature (in particular in Bindseil [2020] and Bindseil and Panetta [2020]) which are being considered for the digital currency and their impact on the Italian banking system. For each alternative we propose the dynamics driving the cash flows between CBDC accounts and the Italian banking system, in a broad picture meant to assess how much liquidity might be lost by Italian commercial banks to the new competitor.

For banks focused on retail business, sight deposits are an essential and cheap source of funding, making available the liquidity necessary for lending activity.
This was particularly true before the 2008/2009 financial crisis, indeed it was common to observe deposit rates that were below the overnight rate of the deposit facility, practically allowing banks to enjoy a form of arbitrage. The landscape has changed since the crisis: because of the low interest rate environment and contemporary liquidity shortage it became harder and yet more crucial than ever to manage the maturity, interest rate and liquidity risk linked to sight deposits.

Negative deposit facility rates and new regulatory policies compelled banks to renew their efforts in forecasting deposit volumes effectively. The new regulations require banks to comply with specific procedures to face liquidity risk (for example the computation of certain liquidity ratios) and interest rate risk, and provide standard formulas to do so. However, once approved by competent authorities the use of internal models is also permitted.
The model proposed in this thesis is similar to the standard approach in its employment of historical data, even though in the latter only the short term market interest rate is

considered as a risk factor, and no other variable related to the creditworthiness of the banking system is taken into account.

Castagna's approach foresees the behaviour of four variables (building blocks): the short term risk-free rate, a CDS index (representing creditworthiness), the deposit rate and deposit volumes. Here, the CDS index (like the risk-free rate) is assumed to influence the evolution of both deposit rates and deposit volumes. The first two are named *market factors*, and their specific models are calibrated from market data downloaded from Refinitiv Eikon on the reference date (June $1st$, 2021) while the dataset used in the fitting of deposit rate and volume models is the time series of their evolution, freely available on Bank of Italy's statistical database, with monthly observations beginning in May 2010. Lastly, the historical data for the deposit facility rate was also retrieved from Refinitiv Eikon and the yield curve from the ECB website.

A short description of the models employed for the four building blocks would be:

- **Short rate (r):** modelled with an approach proposed in Renne [2016], where the rate is composed by the sum of the deposit facility rate and a serially uncorrelated noise. The data needed in the calibration are the market discount factor curve and the deposit facility rate time series
- **CDS index (S)**: it is the weighted average of the 5-year maturity CDS spread of the three largest Italian banks (Intesa Sanpaolo, Unicredit and Mediobanca), with weights represented by the market capitalization. An intensity based model is calibrated on CDS quotes of the three banks, and the CDS index is simulated through the underlying intensity. The data needed in the calibration are the discount factor curve and the CDS market spreads
- **Deposit rate (I):** A linear regression with auto-correlated residuals is fitted from historical data, having the short rate and CDS index as covariates
- **Deposit volume (V):** an auto-regressive process of the first order, having $I - r$ and S as exogenous regressors, is fitted from historical data.

Through these models we can obtain a Monte Carlo simulation of the deposit volumes (with a cut-off horizon of five years), which enables us to compute the term structure of liquidity and other risk metrics, for example the VaR.
The sensitivities to interest rate buckets could be computed by exploiting a result obtained in Jarrow and Van Deventer [1998], that enables us to compute the present value of the

deposit volume by treating it like a floating-for-floating interest rate swap, and then the risks could be hedged with bonds and other interest rate derivatives, as explained in Ho [1992].

Finally, we consider the impact of a remunerated CBDC as if it were introduced on the reference date. We consider three designs proposed in literature, two of which (Bindseil [2020] and Bindseil and Panetta [2020]) are based on a tiered remuneration system, while the third is the non-remunerated case. The rate determining the CBDC remuneration is that of the deposit facility in both two-tiered mechanisms.
The issuance of the CBDC would happen in a centralized way through deposits made directly to the central bank in exchange for CBDC. Other than the number of such deposit accounts having to rise potentially from about ten thousand to 500 million, a deposit account with tiered remuneration is a mechanism already familiar to the central bank, whereas in case of a decentralized token currency the central bank would have to deal with anonymity and possibly illegal activities, as in the case of banknotes.
Therefore, we have deposits made directly to the central bank by the public or third-party providers (e.g. banks), and in the tiered remuneration case each account is considered tier 1 up to a certain amount and tier 2 over that amount.

The critical points to address in the design of the CBDC are the disintermediation of banks and the increased role of the central bank in the economy. Indeed, the ability to collect deposits (in exchange for a risk-free, remunerated and liquid asset) while providing a secure and digital payment method gives the central bank a competitive advantage over commercial banks. This is not desirable for several reasons:

- The central bank does not wish to lengthen its balance sheet too much while subtracting such a role in credit provision from commercial banks, as it would imply that the central bank becomes a financial intermediary
- If the CBDC were to become a store-of-value asset, in case of crisis it would facilitate a bank-run, further aggravating the weakened state of the banking system
- Commercial banks would have to rely more on central bank credit
- The central bank would have to revisit its collateral framework

Through tiered remuneration, Bindseil and Panetta assume the central bank will be able to control the total volume of CBDC. Tier 1 (whose remuneration has a zero-lower bound) would be available to individual citizens across the Euro area with a cap of 3000 euros, over which it is considered tier 2. Tier 2 remuneration would have a zero upper bound and also be available to non-European citizens, corporates and other institutions' accounts, but for these accounts the tier 1 cap would be set to zero.

Interest rate Deposit facility rates are not considered as driving factors in the traditional Gaussian or square root diffusion processes, but in this model (proposed in Renne [2016]) they are the main source of movement for the short-end of the yield curve, evolving like a Markov process in a discrete set of regimes.
The short rate dynamics are split into two components:

$$r_t = \Delta \cdot z_t + \xi_t \tag{1}$$

- Δ: a vector containing the possible values of the ECBDF rate. The number of states is indicated by K.
- z_t: a K-dimensional "selection" vector that indicates the current state.
- ξ_t: a serially uncorrelated random variable, plus we introduce the K-dimensional vector δ defined so that $\mathbb{E}[exp(-\xi_t)|z_t] = exp(-\delta)' \cdot z_t$, where by abuse of notation the exponentiation is carried out component-wise.

The process $\left(xi_t\right)_t$ is assumed to be a series of i.i.d random samples from a Beta distribution, whose parameters are estimated from historical differences, while the parameters to be estimated from the discount factor curve are the ones appearing in the transition probability matrix. Monetary policy changes can happen monthly.
Formula 2.2 is available for the pricing of any security and is made only of algebraic operations, allowing a fast calibration. We obtain the transition probability matrix 2.3.

CDS index The CDS index model is calibrated the weighted average of the CDS quotes for Intesa Sanpaolo, Unicredit and Mediobanca (weights 63.68%, 24.85% and 11.47% respectively).

Maturity	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
Spread (bp)	24.85	25.20	31.02	38.45	50.15	61.19	82.65	96.30

From these quotes, we fit a piece-wise constant intensity function replicating the market implied survival probabilities. Then, we are able to calibrate a stochastic intensity model, in which the intensity evolves like a CIR process, plus a deterministic shift. The shift (ψ) is defined in 2.9 so that the whole curve of the market implied survival probabilities is matched. The process governing the evolution of the intensity is:

$$\begin{cases} \lambda(t) = \psi(t;\beta) + y^{\beta}(t) \\ dy^{\beta}(t) = \kappa(\,\mu - y^{\beta}(t)\,)dt + \nu\sqrt{y^{\beta}(t)}dW(t) \\ y(0) = y_0 \end{cases} \tag{2}$$

with the parameters obtained minimizing function 2.10:

$$\kappa \simeq 0.9338, \qquad \mu \simeq 0.35\%, \qquad \nu \simeq 8.03\%, \qquad y_0 \simeq 0.0020$$

The CDS index can be computed at each time from the simulated value of λ by exploiting the properties of the CIR process through formula 2.12.

Deposit rate The linear regression is estimated directly from the time series of the deposit rate, EONIA rate and CDS index. We have found autocorrelation in the residuals, so they have been modelled with an AR(1) process.

$$\begin{cases} I(t) = \beta_0^I + \beta_1^I \cdot r(t) + \beta_2^I \cdot S(t) + \epsilon(t) \\ \epsilon(t) = \rho^I \epsilon(t-1) + \eta(t) \end{cases} \tag{3}$$

with the coefficients being $\beta_0^I \simeq 0.389$, $\beta_1^I \simeq 0.305$, $\beta_2^I \simeq 0.159$ $\rho^I \simeq 0.934$ and $Var(\eta) \simeq 3.36 \cdot 10^{-3}$.
The signs of the coefficients show how the rates tend to increase when the risk-free rate increases and decrease when the CDS index increases, which is the behavior we expect.

Deposit volume After removing a very significant linear trend, we fit an auto-regressive process of order 1 with $I - r$ and S as additional regressors, obtaining:

$$\begin{cases} \lambda(t) = \beta_1^{(\lambda)} \cdot \lambda_{t-1} + \beta_2^{(\lambda)} \cdot A(\,C(t),\,2\,) + \beta_3^{(\lambda)} \cdot S(t) + \epsilon^{(\lambda)}(t) \\ \epsilon^{(\lambda)}(t) = \rho^{(\lambda)} \cdot \epsilon^{(\lambda)}(t-1) + \eta^{(\lambda)}(t) \end{cases}$$

with $\beta_1^{\lambda} \simeq 0.889$, $\beta_2^{\lambda} \simeq 0.289$, $\beta_3^{\lambda} \simeq -0.112, \rho^{(\lambda)} \simeq -0.400$ and $Var\big(\eta^{(\lambda)}\big) \simeq 1.003$. A represents an average operator.

CBDC design We have 3 possible mechanisms for CBDC remuneration: non-remunerated, Bindseil, and Bindseil-Panetta. In the first case the CBDC carries a null yield, while in for the other alternatives we have respectively:

$$
\text{(Bindseil)} \begin{cases} I\_CBDC1(t) = max\big(ECBDF(t) - 1\%,\, 0 \big) \\ I\_CBDC2(t) = min\big(ECBDF(t) - 1\%,\, 0 \big) \end{cases}
$$

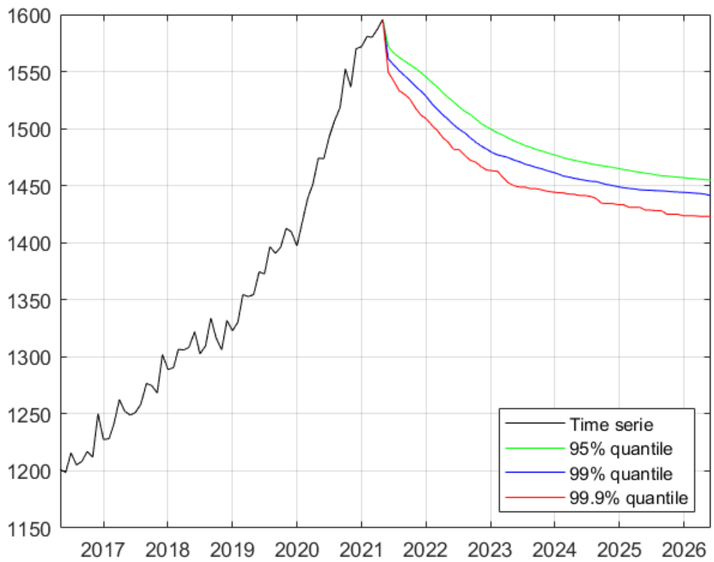
$$
\text{(Bindseil-Panetta)} \begin{cases} I\_CBDC1(t) = max\big(ECBDF(t) - 2\%,\, 0 \big) \\ I\_CBDC2(t) = min\big(ECBDF(t) - 0.5\%,\, 0 \big) \end{cases}
$$

In each case, we have a base amount of CBDC (estimated in Juks [2018]) representing the portion being immediately adopted as a payment tool; for tier 1 we also have a cap amount of 180 billion Euros (3000 Euros times 60 million Italians). We assume different scenarios for the impact on CBDC volumes of the driving factors (convenience with respect to sight deposits and default risk of the banking system), depending on two parameters (k and w). We have:

$$
\begin{aligned}
V\_CBDC1(t;\, w) &= Base1 \,+\, (\, Cap1 - Base1\,) \left[w \cdot f_1\big(spread1(t)\big) \,+\, (1-w) \cdot f_2\big(S(t)\big) \right] \\
V\_CBDC2(t;\, k) &= Base2 \,+\, 50\, g_1\big(spread2(t)\big) + k\, g_2\big(S(t)\big) \\
V\_CBDC\_N(t) &= (Base1 + Base2\,) \,+\, 50\, g_1\big(I(t)\big) + \big(k + (cap1 - base1)(1-w)\big)\, g_2\big(S(t)\big)
\end{aligned}
$$

with f_1, f_2, g_1 and g_2 defined in 4.2 and 4.4.

We can now compute a Monte Carlo simulation of all the variables: market factors, deposit rates and volumes, CBDC remuneration (from the deposit facility rate) and finally CBDC volumes. We can therefore compute the quantiles of the liquidity distribution:



However, we are most interested in assessing the month-by-month liquidity risk borne by the banking system. We can compute the monthly losses from the simulated paths for each remuneration case and parameter combination, considering them all as independent observations. We obtain a large sample of monthly losses, from which we compute the risk metrics (VaR), obtaining:

(Case without CBDC)

Level	95%	99%	99.9%
VaR	1.99%	2.79%	3.69%

(Zero remuneration)

Parameters	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$
$w = 0.75, k = 273$	2.08%	2.93%	3.88%
$w = 0.75, k = 753$	2.11%	2.95%	3.91%
$w = 0.25, k = 273$	2.09%	2.94%	3.88%
$w = 0.25, k = 753$	2.11%	2.96%	3.91%

(Bindseil)

Parameters	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$
$w = 0.75, k = 273$	3.37%	7.65%	10.19%
$w = 0.75, k = 753$	3.39%	7.66%	10.23%
$w = 0.25, k = 273$	2.34%	3.37%	4.63%
$w = 0.25, k = 753$	2.36%	3.39%	4.66%

(Bindseil-Panetta)

Parameters	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$
$w = 0.75, k = 273$	3.05%	5.88%	8.03%
$w = 0.75, k = 753$	3.07%	5.89%	8.04%
$w = 0.25, k = 273$	2.25%	3.17%	4.19%
$w = 0.25, k = 753$	2.27%	3.18%	4.22%

From our analysis we infer that the non-remunerated CBDC would not have much of an impact on the activity of sight deposits; however, it would affect the activity of the deposit facility and may have to be addressed specifically in a negative interest rate environment. Both Bindseil and Bindseil-Panetta remuneration schemes have a much greater influence on the risk metrics, especially at the 99% and 99.9% level. In particular, the biggest loss in liquidity occurs under the Bindseil remuneration scenario.
The main strength of the model proposed is that it enables us to compute the VaR under any assumption regarding CBDC volume and rate, allowing us to assess the risks in all scenarios.