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Executive Summary of the Thesis

CBDC and Bank Runs in an open economy context

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1. Introduction

Since digitalization is reaching all areas of our lives and the digital payments are prevailing on the physical ones, it seems natural to introduce a digital version of the physical currency. The central banks answer to this demand with the Central Bank Digital Currency (CBDC), this is not yet a reality but a certain and immediate goal, something we will reach in the next years. The literature is dealing with both the possible construction characteristics of such a currency and the impact it could have on the economy. Since we can suppose that in the next future financial crisis, we could already have to deal with this kind of currency, we can analyze how a financial crisis will behave in this situation, and it is within this context that our work falls. Starting from the already existing models of a crisis, the Diamond and Dybvig models, we adapt it to a digital currency by exploiting the Fernandez-Villaverde et all model. Finally, we modify this last model to introduce a foreign central bank and understand the market behaviour when a consumer can choose between a domestic central bank and a foreign risk free deposit. This paper contributes to the literature on international implications of CBDCs on financial stability by analyzing the behaviour of an economy composed by a domestic central bank and a foreign central bank, both providing a CBDC.

2. CBDC presentation

A Central Bank Digital Currency is a digital form of central bank money, a virtual form of a fiat currency. It is issued and regulated by a nation's monetary authority or central bank.
In recent years lots of innovations involving digital coins high on the agenda. Starting with Bitcoin and the cryptocurrencies that followed it, going through the discussion of stablecoins and ending with the entry of large technology firms into payment services and financial services more generally, it became necessary to introduce a digital currency. However, the technologies used by cryptocurrencies can only provide a basis for the construction of CBDC, but they need improvement and changes to make it safer and more controlled.
As described in the BIS [6], there's a variety of reasons driving the central bank's research to CBDC. Currently, the focus is on providing a CBDC for payments, enabling broad access to central bank money and providing resilience, but there are other motivations such as financial stability risks and enhancing monetary policy tools. According to the Atlantic Council [2], 105 countries, representing over 95% of global GDP, are exploring a CBDC. A little over two years ago,

in May 2020, just 35 countries were considering this. So far, 10 countries already launched a CBDC (Nigeria, the Bahamas, Jamaica and seven countries in the Eastern Carribean), while 15 countries are on pilot.

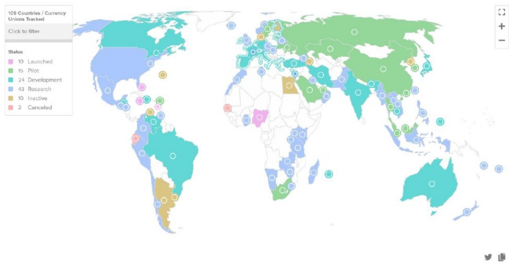

Figure 1: Central banks at varying stages of developing digital currencies (26 June 2022) [2]

Let's focus on the characteristics. First, it must be decided whether it will be retail or wholesale, account-based or token-based. A retail CBDC would be used like a digital extension of cash by all people and companies, whereas a wholesale CBDC could be used only by permitted institutions as a settlement asset in the interbank market. An account-based CBDC ties a CBDC to an identity system where a sender first verifies whether the receiver has an account and then verifies the identity of the receiver before making a payment or transfer. A token-based CBDC is not tied to an identity system but it ties a CBDC to an access technology based on digital tokens. It is a digital version of the physical cash. Both can be used for retail payment. But unlike account-based, the liabilities are not stored at accounts at a central bank but they are stored through digital wallets.
Moreover, there are three common foundational principles for a central bank's consideration of CBDC issuance that flow from their common objectives[6].

- **Do no harm** : new forms of money supplied by the central bank should continue supporting the fulfilmnet of public policy objectives and should not interfere with or impede a central bank's ability to carry out its mandate for monetary and financial stability. That means to not have a negative impact.
- **Coexistence**: different types of central bank money should complement one another and coexist with robust private money to support public policy objectives.
- **Innovation and efficiency**: a continuous updating of technology, innovation and competition must be guaranteed, in order to drive efficiency in a jurisdiction's payment system.

Moreover, central banks have common public policy objectives that allow common principles to be agreed, therefore they must satisfy some core features:

- **Cash-like payment system**: a CBDC must work as a banknote or a credit cards, it must be "peer-to-peer".
- **Convenient**: CBDC payment should be as easy as using cash.
- **Resilient and robust**: a CBDC system should be resilient to operational failure and disruptions, natural disasters and, electrical outages and other issues. CBDC system should have the ability to recover from potential hardware or software failures.
- **Accessible**: a CBDC must be accessible to everyone to ensure to be used as a payment.
- **Interoperability**: such dimension must be taken into consideration in order to guarantee the possibility of interaction between different CBDC systems, therefore it impacts a higher-level layer of decision for the CBDC design.
- **Privacy-protected**: any CBDC would need to strike an appropriate balance between safeguarding the privacy rights of consumers and affording the transparency necessary to deter criminal activity.
- **Intermediated**: under an intermediated model, the private sector would offer accounts or digital wallets to facilitate the management of CBDC holdings and payments.
- **Transferable**: for a CBDC to serve as a widely accessible means of payment, it would need to be readily transferable between customers of different intermediaries.

Besides the characteristics, the architecture a CBDC economy must have is another important feature to describe. Currently, three different architecture are designed[1]:

- **Indirect CBDC model**: the consumer has a claim on an intermediary, with the central bank keeping track only of wholesale accounts.

- **Direct CBDC model**: the CBDC represents a direct claim on the central bank, which keeps a record of all balances and updates it with every transaction.
- **Hybrid CBDC model**: is an intermediate solution providing for direct claims on the central bank while allowing intermediaries to handle payments.

The technology best suited to offer the required functionality is distributed ledger technology (DLT), of which the best known example is blockchain. Moreover, in order to be convenient it must be available on modern smartphones and, for users without smartphones, central banks should provide devices designed for this purpose. To be resilient, it must be similar to current payment system and operate a 24/7/365 services. In order to provide security, modern technology used in credit cards and smartphones must be a basis for the CBDC security. Another important feature it must satisfy is the ability to meet volume and velocity of modern way of payments, feature that is not already met in the cryptocurrencies world.

In addition, they need to understand whether CBDC will be remunerated or not.

The advantages of introducing a CBDC are innumerable, but so the risks associated with this technology. The first risk to consider, is the cyber attacks. A more specific risk is that the malfunctioning of the IT infrastructure underlying the digital currency could cause loss and damage to individual users, raising questions about the responsibility of the central bank. CBDCs would have implications for financial intermediation and would need careful design and implementation.

3. Diamond and Dybvig model for a bank run

The introduction of a CBDC will affect every aspect of financial life. Our goal is to study the effect related to bank runs, a phenomenon related to the fear that the bank will become insolvent. We therefore start by analyzing how commercial banks behave during a crisis, exploiting Diamond and Dybvig's model [3].

What drives the possibility of a run in the model is demand for liquidity, that is, a desire on the part of savers to be able to retrieve their funds at any time.

Focusing on the model, it develops on three periods time with a single homogeneous good. At the initial time there is the investment, the good is moved from the agent portfolio to the bank. Then the agent can choose whether to withdraw its good at time 1, without earning anything, or leaving it to the bank and withdraw at time 2 when an yield $R>1$ is added to it. This is caused by the the fact agents are rewarded if they decide to leave their money at the bank's disposal. But each agent can't know at time 0 if he will withdraw at period 1 or at period 2, it depends on random factors, on events that are not predictable since life is so unpredictable.

What is desirable is to study the equilibrium this kind of model can achieves, where as equilibrium we refer to the Nash equilibrium. There are two different equilibrium, the good equilibrium, is the case where type 1 agents withdraw at time $T=1$ while type 2 agents prefer to wait, and the bad equilibrium, where all agents get scared and try to withdraw their deposits at $T=1$. The authors also propose two possible solutions to prevent bank runs: the suspension on convertibility and the government deposit insurance.

Diamond and Dybvig's model has been widely used to understand bank runs and other types of financial crises, as well as to prevent such crises. It has been a workhorse of banking research over the last years and during the recent financial crisis it has been one that researchers and policymakers consistently turn to when interpreting financial market phenomena.

4. Central Bank run

The Diamond and Dybvig model can be applied to a central bank as well. There are only few details to be changed, as descibred by Fernandez-Villaverde et all [5][4]. The differences are in the contracts that must be nominal and in the role of the central bank as a powerful financial intermediary, that means it can manipulate the price level, moreover, it cannot be forced into liquidation.

A central bank run acts differently from a commercial bank run. Indeed, it cannot liquidate long-term investment directly since they are situated in an investment bank. Therefore, to fulfill the agent withdraw in a bank panic, only short term investment can be liquidated. Mathematically, a run on central bank happens if and only if:

$$\alpha > \lambda \tag{1}$$

where α is the fraction of agent that withdraw, and λ is the fixed threshold coming from the probability of an agent to be impatient.

The model is similar to the Diamond and Dybvig one, the difference is that the demand deposit contracts offered to depositors are nominal rather than real.

Mathematically speaking, at time 0 agents have a zero balance on their CBDC account and a unit of real good that wanted to invest in $M > 0$ units of CBDC, where M is a quantity set by the central bank.

At time 1 agents learn their type and decide whether to spend all their digital currencies or to roll over. Naturally, to simplify the model, we are imposing agents cannot maintain only a part of their CBDC, and they cannot sell it to other agents. At this time period the central bank decides the fraction $y(\alpha)$ of real goods to be liquidated to fulfill the agents 1 request, and sell this quantity to the spending agents at a market clearing price P_1. Moreover it decides the nominal interest rate $i(\alpha)$ to be paid at period 2 on the CBDc balances remaining at the end of period 1.

In period 2, the type 1 agents have zero CBDC, while patient agents have $(1 + i)M$ digital currency. This means that the nominal contract is given by $(M, M(1 + i))$. The agents type 2 can spend their remaining balance on the remaining real goods $(1 - y)R$ held to maturity by the central bank and sold to the agents at a market clearing price P_2.

Let's start the discussion on the equilibrium. As in the commercial bank case, a central bank can have two different equilibrium: the "good" one and the run. The first one lead to the social optimum that is the point on the utility possibility frontier that maximizes social welfare. We must underline that a run in the presence of CBDC does not mean the central bank is running out of digital money, is not running out of the item it has promised to agents and that can produce freely, which is a difference with respect to the classical bank run. As in the classical case, a run happens only if patient agents believe the expected real consumption in t=1 will exceeds the consumption in t=2, this means $x_1 > x_2$.

Let's explain all the cases:

- $\alpha > \lambda$: if the number of withdraw is bigger than the threshold then it can happen a run that is always an equilibrium. Let's analyze in detail the cases that can cause a run:
  - $\alpha = 1$: all the agents withdraw at t=1. This is an exhaustive run equilibrium, it can happen if and only if $x_1(1) \geq x_2(1)$.
  - $\lambda < \alpha < 1$: only some patient agents withdraw in t=1. In this case there is a partial run on the central bank, this can happen if and only if $x_1(\lambda) = x_2(\lambda)$.
- $\alpha = \lambda$: if the number of withdraw is equal to the threshold then no runs can happen. Only the good equilibrium can be reached, this is the case $x_1(\lambda) \leq x_2(\lambda)$.
- $\alpha < \lambda$: a run cannot happen, therefore there is a single equilibrium, the good one.

Moreover, we can spend a moment on the case of the optimal equilibrium.

Proposition 4.1. *The central bank policy implements the social optimum (x_1^*, x_2^*) in equilibrium if:*

- *Given that only impatient agents spend, i.e. $\alpha = \lambda$, the central bank policy satisfies $y(\lambda) = y^*(\lambda) = x_1^*\lambda > \lambda$, implying $x_1(y, \alpha) = x_1^* = \frac{y^*}{\lambda}, x_2(y, \alpha) = x_2^* = \frac{R(1-y^*)}{1-\lambda}$ and $P_1^* < M$.*
- *Given that also patient agents spend, i.e. $\alpha > \lambda$, the central bank sets a liquidation policy that implies $x_1(\alpha) < x_2(\alpha) \quad \forall \alpha > \lambda$*

Indeed, since central bank observes aggregate spending behaviour before liquidating real assets, it is not committed to liquidate y^* if it observes that also impatient agents are spending. To deter patient agents from spending, the central bank can threaten them to implement a liquidation policy that makes spending non optimal ex post.

5. An open economy framework

Let's focus on the real goal of this paper: how a digital run might behave in an open economy context. In this contest, a CBDC can be issued both by a domestic and a foreign central bank, and both be available to domestic investors. In our model, we suppose the foreign central bank offers a global safe asset. We modify the

Popescu [7] analysis introducing a domestic central bank.
The actors of the model are a domestic economy populated by agents and a central bank that issues a CBDC and could be susceptible to runs, and a highly credible foreign central bank which issues a CBDC accessible to domestic residents in the form of interest-bearing deposits.
The model will generate competition for deposits between the domestic central bank and the foreign one, indeed consumers can choose to deposit in both the central banks according to their preferences and the possibility of a greater gain.

5.1. The model

As in previous models, we have a 3-time period, agents can be type 1 if they withdraw in t=1 or type 2 if they withdraw in t=2. α is the fraction of agents that actually withdraw earlier, meaning the type 1 agents plus a possible fraction of type 2 agents. It does not depend on the choice an agent made about where to invest. λ is probability of being of type 1. We have a strictly increasing, strictly concave and continuously differentiable utility function $u : R_{++} \rightarrow R$ such that:

$$U(x_1, x_2) = \begin{cases} u(x_1) & \text{with probability } \lambda \\ \\ u(x_2) & \text{with probability 1 -}\lambda \end{cases} \tag{2}$$

Moreover we have a real investment technology: both the domestic and the foreign economies allow for a short-term and a long-term investment opportunity. In the short term technology they invest 1 unit in the first period and return r units in t=1 (r can be 1 or bigger than 1, according to the investment strategy, usually is set at 1). The long term investment invests one unit in t=0 and receive $R > 1$ units in t=2.
Then we must add a variable related to the fraction of agents who choose the foreign central bank, i.e $f \in [0, 1]$. $0 < f < 1$ only if consumers are indifferent between depositing at the domestic central bank or foreign central bank, therefore both central banks offer the same contract. If $f = 0$, the agents prefer the domestic asset as it gives them a higher expectation. If $f = 1$, all the agents choose the foreign asset, meaning that the domestic central bank is completely abandoned.
Let's move on to the nominal contract, we denote with an $'$ the quantity related to a foreign currency. M/M' is the quantity of domestic/foreign CBDC an agent receives if he sells his endowment to the domestic/foreign central bank. y(α)/y'(α) is fraction of real goods to be liquidated in period 1 by the domestic/foreign central bank in order to meet the withdrawal requests. The central bank sell this quantity to the spending agents at a market clearing price P_1/P_1'. i(α)/i'(α) is the nominal interest rate chose in period 1 by the domestic/foreign central bank, and paid in period 2 on the CBDC domestic/foreign balances remaining at the end of period 1.
At the beginning of t=0 an agent has a zero balance on his CBDC account and an endowment of one unit of the real good. With a probability of f, he sells his endowment to the foreign central bank, receiving a balance of $M' > 0$ of digital foreign currency, otherwise he sells his endowment to the domestic central bank receiving $M > 0$ domestic CBDC. At time t=1, the agent learns his type and decides whether to spend his CBDC balance or not. In period t=2, the agent type 1 have a zero CBDC balance while the agent type 2 increased his balances by a factor term that depends on the corresponding nominal interest rate.
Turning to the real contract (x_1, x_2): in t=1, a fraction y of the real goods must be liquidated to fulfill the demand of the agents' type 1 in the domestic central bank, then this quantity must be equally distributed across all spending agents in this period who have a domestic central bank deposit, i.e. $(1 - f)\alpha$. The same happens in the foreign central bank, the bank liquidates a fraction y' of real the goods and equally distributes it across all spending agents in this period who have a foreign central bank deposit, i.e $f\alpha$. At t=2, the remaining fraction of real goods is liquidated and equally distributed across the agents that didn't withdraw in the previous period.
But the foreign central bank can implement a run deterring policy, so type 2 agents have no incentive to withdraw earlier.

Proposition 5.1. *The foreign central bank is* *run free if its liquidation policy is run-deterring, that means:*

$$x'_1 < x'_2 \iff y' < \frac{\alpha R}{1-\alpha(1-R)} \quad \forall \alpha \in (\lambda, 1] \tag{3}$$

Therefore, it implements the socially optimum in the unique equilibrium.

Therefore, each agent behaves according to its own type, i.e. $\alpha = \lambda$ for the foreign central bank. This means the real contract is given by

$$\begin{aligned}(x_1, x_2) &= \left(\frac{M}{P_1}, \frac{M(1+i)}{P_2}\right) \\ &= \left(\frac{y}{(1-f)\alpha}, \frac{1-y}{(1-f)(1-\alpha)}R\right)\end{aligned} \tag{4}$$

$$(x'_1, x'_2) = \left(\frac{M'}{P'_1}, \frac{M'(1+i')}{P'_2}\right) = \left(\frac{y'}{f\lambda}, \frac{1-y'}{f(1-\lambda)}R\right) \tag{5}$$

The consumer must decide whether to deposit with the domestic central bank, consuming either x_1 upon withdrawal in period 1 or x_2 in period 2 but with an uncertainty, or with the foreign central bank, consuming surely either x'_1 upon withdrawal in period 1 or x'_2 in period 2.

5.2. Equilibrium

Our analysis started from the cases where there is only a good equilibrium. This is the case where all agents behave according to their type. Of course, we must underline that, in this case, the socially optimal contract is offered either by the domestic central bank or the foreign central bank, or both. But, since central banks can choose their interest rates, there could be also a case where the foreign central bank offers a better socially optimum bank deposit contract.

Proposition 5.2. *The foreign central bank can replicate the socially optimum domestic bank deposit contract, if the interest rate on the foreign CBDC is such that $x'_1 = x^*_1$ and $x'_2 = x^*_2$. But it can also offer a better bank deposit contract if the interest rate on the foreign CBDC is such that $x'_2 > x^*_2$.*

In this cases, agents prefer to move to the foreign central bank and this leads to a partial run in the domestic central bank into the foreign central bank. Indeed, at t=1 the foreign central bank chose its interest rate and, if it leads to a higher consumption for type 2 agents with respect to the domestic consumption, agents could choose to migrate to the foreign central bank causing a domestic run.

Let's now focus on the situation in which agents do not behave according to their type, i.e. $\alpha > \lambda$. The domestic central bank can implement the social optimum if it imposes $x_1(\alpha) < x_2(\alpha)$, otherwise it experiences a run situation. Even if the domestic bank offers the socially optimum contract, agents will prefer the foreign bank if they fear the domestic bank will fail, unless the contract it offers gives a too low return. This agents' preference to choose the foreign asset can give to the foreign central bank a market power. This means it can offer a lower payoff contract with respect to the socially optimum one.

Proposition 5.3. *If the foreign central bank offers a riskless deposit contract which guarantees a payoff equal to the ones offered by the domestic socially optimal contract, then it will attract all deposits up to the capital account constraint, i.e. $f = k$. Moreover, the foreign central bank can offer a lower payoffs contract with respect to the socially optimal one up to a fixed payoff, and still attract the highest possible amount of deposits, i.e. $f = k$.*

In this cases, there is a partial run in the domestic central bank into the foreign central bank.

Finally, the case where the domestic central bank does not offer a socially optimal contract is not significant, indeed, the agents' behaviour will be the same both if we are in a closed economy or in an open one. A run happens anyway.

6. Conclusion

We find that in an open economy model where domestic central bank is subject to run and competes with a foreign central bank that offers risk-free deposits, new episodes of run are generated. This are partial if a limit is imposed on the number of agents that can move to the foreign bank, exhaustive otherwise.

Therefore, the financial stability will be reduced, this type of economy is risky, and must impose a collaboration between the central banks to prevent the possible negative outcome of these events.

Clearly, we have analyzed a simplified case. Further analyzes must be done if we assume the foreign central bank does not offer a safe asset but could be susceptible to runs.

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