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Shape reconstruction of Dimorphos with Milani's NavCam

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Abstract

The ESA Hera mission is the first in-situ follow-up of a planetary defense experiment, aiming to fully characterize the outcome of NASA’s DART kinetic impact on the near-Earth binary asteroid system Didymos–Dimorphos. A key objective of Hera is the accurate reconstruction of Dimorphos’ shape, which is essential to determine its volume and mass, and to correctly interpret the dynamical response induced by the impact. High-fidelity shape models are also crucial for optical navigation in a low-gravity environment. To address its objectives, Hera is supported by the Milani CubeSat, which will provide complementary close-range observations of the system, through its scientific payload and Navigation Camera (NavCam). In this context, this thesis investigates the accuracy of Dimorphos shape reconstruction achievable from Milani NavCam imagery using stereophotoclinometry (SPC). This technique combines photometric and geometric information from images acquired under varying illumination conditions to derive local surface slopes, albedo and topography. The SPC landmark-based processing chain for terrain estimation was complemented with additional modules, developed to adjust landmarks positions on the asteroid surface and to improve spacecraft pose knowledge. The SPC pipeline was applied to simulated images across different mission phases, ranging from far-range to experimental close-proximity operations. Reconstruction performance was evaluated by comparing the estimated Digital Terrain Models with a ground-truth reference shape. The newly implemented functionalities were validated under multiple scenarios, demonstrating a significant reduction in landmark position and spacecraft pose errors, essential to improve the shape reconstruction accuracy. The results indicate that SPC applied to Milani NavCam images can provide valuable information on the shape of Dimorphos, achieving meter- to decimeter-scale accuracy during close-range operations.

Keywords: stereophotoclinometry, shape reconstruction, bundle adjustment, Milani Cubesat

Abstract in lingua italiana

La missione Hera dell'ESA rappresenta il primo seguito in situ di un esperimento di difesa planetaria, con l'obiettivo di caratterizzare in modo completo l'esito dell'impatto cinetico della missione DART della NASA sul sistema binario di asteroidi Didymos–Dimorphos. Un obiettivo chiave di Hera è la ricostruzione accurata della forma di Dimorphos, fondamentale per determinarne volume e massa, e per interpretare correttamente la risposta dinamica indotta dall'impatto. Modelli di forma accurati sono inoltre cruciali per la navigazione ottica in un ambiente a bassa gravità. Per il raggiungimento di tali obiettivi, Hera è supportata dal CubeSat Milani, che fornirà osservazioni complementari del sistema tramite il proprio payload scientifico e la camera di navigazione (NavCam). In questo contesto, la presente tesi analizza l'accuratezza della ricostruzione della forma di Dimorphos ottenibile da immagini NavCam di Milani mediante l'impiego della stereofotoclinometria (SPC). Questa tecnica combina informazioni fotometriche e geometriche provenienti da immagini acquisite in condizioni di illuminazione variabili per derivare le pendenze locali della superficie, l'albedo e la topografia. La catena di processamento SPC basata su landmark è stata arricchita con moduli aggiuntivi, sviluppati per correggere le posizioni dei landmark sulla superficie dell'asteroide e migliorare la stima della posa del satellite. La procedura SPC è stata applicata a immagini simulate relative a diverse fasi della missione, dalle osservazioni a lunga distanza alle operazioni sperimentali di prossimità. Le prestazioni di ricostruzione sono state valutate confrontando i Modelli Digitali del Terreno stimati con un modello di riferimento dell'asteroide. Le nuove funzionalità sono state validate in diversi scenari, dimostrando una significativa riduzione degli errori nella posizione dei landmark e nella posa del satellite, con un conseguente miglioramento dell'accuratezza di ricostruzione della forma. I risultati indicano che l'utilizzo di SPC con le immagini di Milani può fornire informazioni rilevanti per la ricostruzione accurata della forma di Dimorphos, consentendo di raggiungere un'accuratezza dell'ordine del metro fino al decimetro durante le operazioni a corto raggio.

Parole chiave: stereofotoclinometria, ricostruzione della forma, aggiustamento del fascio, Cubesat Milani

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List of Acronyms

ADAM All-Data Asteroid Modeling.

AES Alignment, Extraction and Solve.

AFC Asteroid Framing Camera.

AIDA Asteroid Impact and Deflection Assessment.

AMICA Asteroid Multi-band Imaging Camera.

AO Adaptive Optics.

ASPECT Asteroid SPECTral imager.

AU Astronomical Unit.

BRDF Bidirectional Reflectance Distribution Function.

BVH Bounding-Volume Hierarchy.

CCD Charge-Coupled Device.

CRP Close-Range Phase.

DART Double Asteroid Redirection Test.

DoF Degree of Freedom.

DRACO Didymos Reconnaissance and Asteroid Camera for Optical navigation.

DTM Digital Terrain Model.

ESA European Space Agency.

EXP EXperimental Phase.

FOV Field Of View.

FRP Far-Range Phase.

GNC Guidance, Navigation and Control.

GRASS Gravimeter for small Solar System bodies.

GSD Ground Sampling Distance.

HST Hubble Space Telescope.

IAU International Astronomical Union.

IMU Inertial Measurement Unit.

ISL Inter-Satellite Link.

JAXA Japan Aerospace eXploration Agency.

JWST James Webb Space Telescope.

KOALA Knitted Occultation, Adaptive optics, and Lightcurve Analysis.

L-BFGS Limited-memory Broyden–Fletcher–Goldfarb–Shanno.

LICIACube Light Italian CubeSat for Imaging of Asteroids.

LiDAR Light Detection and Ranging.

LRO Lunar Reconnaissance Orbiter.

MASCOT Mobile Asteroid Surface Scout.

MBA Main Belt Asteroid.

MESSENGER MErcury Surface, Space ENvironment, GEochemistry, and Ranging.

MINERVA MIcro-Nano Experimental Robot Vehicle for Asteroid.

MPCD Multiresolution PhotoClinometry by Deformation.

MSI Multi-Spectral Imager.

NASA National Aeronautics and Space Administration.

NEA Near-Earth Asteroid.

NEAR Near Earth Asteroid Rendezvous.

NEO Near-Earth Object.

NEOMIR Near-Earth Object Mission in the InfraRed.

NIR Near InfraRed.

NLR NEAR Laser Rangefinder.

OCAMS OSIRIS-REx Camera Suite.

OLA OSIRIS-REx Laser Altimeter.

OSIRIS-APEX Origins, Spectral Interpretation, Resource Identification, Security, APophis EXplorer.

OSIRIS-REx Origins, Spectral Interpretation, Resource Identification, Security, Regolith Explorer.

PALT Planetary ALTimeter.

PCD PhotoClinometry by Deformation.

PDCO Planetary Defense Coordination Office.

PHA Potentially Hazardous Asteroid.

PHO Potentially Hazardous Object.

PSF Point Spread Function.

PSI Planetary Science Institute.

RMSE Root Mean Square Error.

SCI Small Carry-on Impactor.

SfS Shape from Silhouette.

SNR Signal to Noise Ratio.

SPC StereoPhotoClinometry.

SPG StereoPhotoGrammetry.

SPICE Spacecraft, Planet, Instrument, C-matrix, Events.

SS Solar System.

SSSB Small Solar System Body.

SSTO Self-Stabilized Terminator Orbit.

SWIR Short-Wavelength InfraRed.

TAG Touch-And-Go.

TIRI Thermal InfraRed Imager.

TNO Trans-Neptunian Object.

VIS Visible.

VISTA Volatile In Situ Thermogravimetry Analyser.

YORP Yarkovsky–O’Keefe–Radzievskii–Paddack.

1 | Introduction

1.1. Asteroids Context

1.1.1. Small Solar System Bodies

The Solar System is home to eight planets, several dwarf planets, and countless smaller celestial bodies, all orbiting around our star. Most of the mass of the system is concentrated in the Sun, while the remaining part is largely clustered in Jupiter, Saturn, and the other planets.

From the definition of the International Astronomical Union (IAU), a celestial body is considered a planet if it meets 3 conditions: orbit the Sun, be massive enough to be pulled into a nearly round shape by its own gravity, and clear its orbital path of other objects [32]. If the third condition only is not satisfied, the object is classified as dwarf planet. All the other Sun-orbiting entities, which are not planets, dwarf planets or natural satellites, are defined by IAU as Small Solar System Bodies (SSSB)[33]. This broad category includes asteroids, comets, trans-Neptunian objects, but also smaller fragments of these objects, as meteoroids and microscopic interplanetary dust.

Asteroids are a significant component of SSSBs. They are defined as small, rocky bodies, with a diameter comprised between few meters and several hundreds of kilometers. The majority of known asteroids reside in the Main Belt, a region located between the orbits of Mars and Jupiter. During the 18th century, the astronomers were observing this region to search for an additional planet, convinced of the apparent validity of Bode's law for predicting the locations of planets [34]. This research led to discovery of Ceres, by Giuseppe Piazzi in 1801, later reclassified as dwarf planet. In the following years further observations tracked also the 3 asteroids Pallas, Juno and Vesta, increasing the interest in this new category of celestial objects. Therefore, it became clear the presence of a belt of asteroids in that region, where Jupiter's strong gravitational influence prevented the material from aggregating into a single large planet [35].

Nowadays the technological advancements in astronomy allowed the identification of over a million new asteroids of smaller dimensions. The number of observed asteroids increased sharply, because modern techniques as photometry, spectroscopy and radar imaging allowed the detection of significantly smaller and faint objects.

1.1.2. Asteroids classification

Asteroids are rocky and metallic celestial bodies, typically irregularly shaped, as their limited mass is not sufficient to assume a spherical shape under the effect of their own gravitational attraction. They differ from comets as they are not icy and don't exhibit any observable activity due to sublimation when close to the Sun, generating a coma and tails [1]. Each asteroid receives a sequential number based on the order of its discovery, and sometimes also a name. Asteroids are commonly classified using two complementary criteria: their surface composition and their orbital location within the Solar System.

From a compositional perspective, asteroid classification relies primarily on reflectance spectroscopy and albedo measurements, which allow scientists to group them into different taxonomic classes [1]:

- **C-type (carbonaceous):** Dark, carbon-rich asteroids with low reflectivity, made of silicates mixed with organic compounds. They are considered primitive bodies from the early Solar System. They include Ceres, Pallas and most asteroids in the outer part of the belt, as Bennu and Ryugu.
- **S-type (silicate):** Stony or silicate-rich asteroids, for which the lack of dark carbon compounds determines a higher reflectivity and distinct silicate spectra. They are likely formed in a different region than C-types. The Didymos system and most of the inner belt asteroids belong to this group.
- **M-type (metallic):** Metal-rich asteroids, identifiable by radar due to their high reflectivity, but much less numerous than the previous. They are likely remnants of differentiated parent bodies with molten cores. Psyche is a notable example.

In addition to the 3 most common categories, other rarer D-, V-, P-, E-, and Q-types exist. Each class is defined by distinct surface compositions, spectral features and origins, further highlighting the diversity of bodies found across the Solar System.

On the other side, from an orbital perspective, the largest population is represented by the Main Belt Asteroids (MBAs), which occupy stable orbits between Mars and Jupiter at distances of roughly 2.1–3.3 AU from the Sun [36].

However, there are many less-populated regions where asteroids are also found. Notable orbital families include the Trojans, which share the orbit of a planet (mostly Jupiter, Mars and Neptune) and are located in stable regions around the L4 and L5 Lagrangian points. The Centaurs, instead, inhabit the region between Jupiter and Neptune and have unstable planet-crossing orbits. Smaller resonant groups, such as the Hungarias, Hildas and Cybeles, are trapped in specific orbital resonances with Jupiter [36].

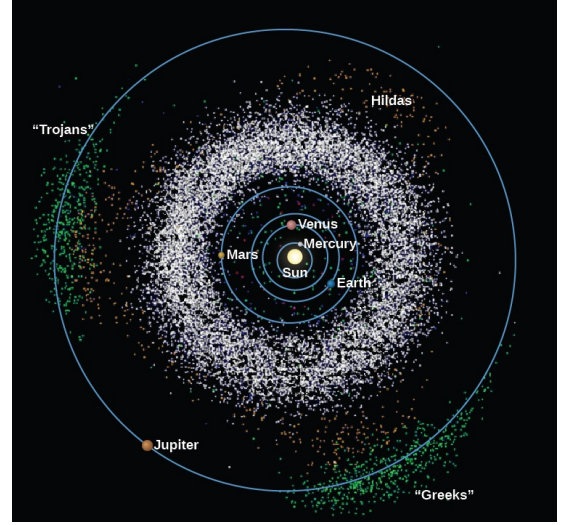


Figure 1.1: Asteroids in the Solar System [1].

Trans-Neptunian Objects (TNOs) are another category of icy-rocky bodies located beyond Neptune, in the Kuiper belt. They are considered within the broader category of SSSBs, although their properties often resemble comets more than classical asteroids.

A further important class of small bodies is represented by Near-Earth Objects (NEO), which include asteroids and comets whose orbits bring them within 1.3 AU of the Sun. Among them, a critical subset is represented by Potentially Hazardous Objects (PHO), defined as those larger than 140 meters in diameter with orbital paths that approach within 0.05 AU of Earth's orbit [2].

1.1.3. Near-Earth Asteroids

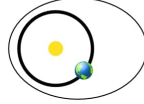
The majority of NEOs are Near-Earth Asteroids (NEAs). These are thought to originate mainly from the asteroid belt, where thermal forces such as the Yarkovsky effect slowly shift their orbits, until resonances with Jupiter or Saturn inject them into the inner Solar System [37]. Their orbital evolution, influenced by these gravitational perturbations, makes them dynamically unstable and potentially hazardous, as some might follow trajectories that intersect Earth's path. NEAs are classified into four principal groups based on their orbital parameters [2]:

- **Amor asteroids:** Remain outside Earth's orbit, with perihelion distances between 1.017 AU (Earth's aphelion) and 1.3 AU.
- **Apollo asteroids:** Cross Earth's orbit, with semi-major axes greater than 1 AU and perihelion distances less than 1.017 AU.

- **Aten asteroids:** Cross Earth's orbit, but with semi-major axes less than 1 AU and aphelion distances greater than 0.983 AU (Earth's perihelion).
- **Atiras asteroids:** Orbit entirely within Earth's path, with aphelion distances less than 0.983 AU.

Amors

Earth-approaching NEAs with orbits exterior to Earth's but interior to Mars' (named after asteroid (1221) Amor)



$$a > 1.0 \text{ AU} \\ 1.017 \text{ AU} < q < 1.3 \text{ AU}$$

Apollos

Earth-crossing NEAs with semi-major axes larger than Earth's (named after asteroid (1862) Apollo)



$$a > 1.0 \text{ AU} \\ q < 1.017 \text{ AU}$$

Atens

Earth-crossing NEAs with semi-major axes smaller than Earth's (named after asteroid (2062) Aten)



$$a < 1.0 \text{ AU} \\ Q > 0.983 \text{ AU}$$

Atiras

NEAs whose orbits are contained entirely within the orbit of the Earth (named after asteroid (163693) Atira)



$$a < 1.0 \text{ AU} \\ Q < 0.983 \text{ AU}$$

(q = perihelion distance, Q = aphelion distance, a = semi-major axis)

Figure 1.2: NEA orbital groups classification [2].

The study of NEAs is of great scientific and societal importance. On one side, their composition provides clues about Solar System and Earth formation, while on the other, their Earth-crossing potential requires careful monitoring for planetary defense.

1.1.4. The Importance of Asteroids in Space Exploration

The study of asteroids is of critical importance for several different reasons. Formed over 4.5 billion years ago from the same protoplanetary disk of gas and dust that gave rise to the planets, asteroids have largely escaped the intense geological processing that reshaped larger planetary bodies. As such, they can be considered remnant debris from the formation of the inner solar system, preserving essential clues about the processes that determined the evolution of the Solar System [38]. In this context, asteroid science plays the crucial role of exploiting the unique opportunity of studying the original materials and mechanisms that led to the formation of the terrestrial planets and the origin of life. In fact, asteroids may have had a strong interaction with Earth's biosphere, bringing water and organic substances on the Earth and changing the evolutionary path of life forms with catastrophic impacts.

Another important economic and strategic dimension of asteroid science is the prospect of asteroid mining. Beyond their scientific value, asteroids are a potential source of minerals for future space exploration and human civilization [39]. In-situ spacecraft missions can provide high-resolution, multi-wavelength data to complement ground-based observations, enabling the study of geology, surface evolution, regolith properties, and collisional history. Detailed measurements of shape, composition, thermal characteristics, and magnetic field can reveal insights into internal structure and evolutionary processes.

Understanding asteroid physical properties and dynamical behavior is essential also for assessing and mitigating the impact hazard posed by NEAs, occasionally crossing Earth's orbit. Even if most asteroids are small, a 50–100 m impactor could cause severe regional damage, while kilometer-scale impacts would be globally catastrophic [1]. The importance of implementing a planetary defense strategy is underlined by impact events in recent history: the Tunguska event in 1908 caused the fall of an estimated 30 million trees over a 2,150 km² area [40], while in 2013 a 19-meter asteroid exploded in the atmosphere above Chelyabinsk, Russia, causing a shockwave that injured more than 1600 people [41]. Therefore, characterizing asteroid orbits and properties is critical for refining impact prediction models and developing mitigation strategies. This challenge is further complicated by the fact that many small NEAs remain undetected until days or weeks before close approach.

1.1.5. Binary Asteroid Systems

Binary asteroid system is a special category of asteroids, widely diffused in NEAs population, accounting for about 15% of the total number [42]. The first known binary system was discovered in 1993, when the Galileo spacecraft imaged the small moon Dactyl orbiting the MBA (243) Ida, while performing a flyby around the latter [43]. Since then, hundreds of binaries and multiple asteroid systems have been identified across the Solar System through radar observations and lightcurve photometry.

Light curves are the most common technique, tracking the brightness of an object over time. As binary asteroids might pass in front of each other, their combined brightness can fluctuate predictably, indicating the presence of two asteroids [44]. These discoveries have transformed our understanding of small-body dynamics, making binary asteroids a central focus of asteroid science.

Binary asteroid systems are typically classified depending on the diameter of their primary D_p and secondary D_s . The most common systems are composed by asteroids smaller than 20 km and with a diameter ratio D_s/D_p ranging from 0.1 to 0.6 [45].

Several theories have been considered for the creation of small-body binaries, taking into account that different formation processes dominate for different asteroid sizes and in different parts of the Solar System [46]. The formation of binaries among the larger MBAs most likely occurs via collision events, followed by mutual capture and gravitational reaccumulation of fragments. Collisional processes typically tend to generate relatively wide binaries with slowly spinning primaries.

Other formation mechanisms are considered for the NEA binaries, generally characterized by closer separations and more rapidly spinning primaries. The prevailing formation mechanism for NEA binaries is rotational fission driven by the YORP effect [3, 47].

Solar radiation torques can gradually spin up small, rubble-pile asteroids to their critical rotation limit, resulting in the shedding of material that reaccumulates into a satellite. The spinning torque is generated by the anisotropic thermal re-emission of the absorbed solar energy from surface features as protrusions, craters, or asymmetric wedges, due to differential heating, as conceptually illustrated in Figure 1.3.

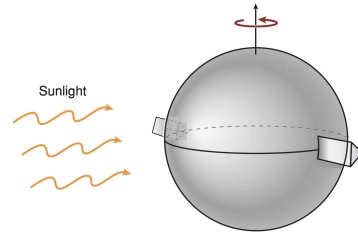


Figure 1.3: YORP effect [3].

These variations in thermal radiation pressure distribution across the asteroid’s surface lead to gradual changes in its rotational state [3]. This theory is supported by the observation of characteristic “spinning-top” shapes of many binary primaries, with pronounced equatorial ridges indicating past mass shedding events.

The scientific interest in binary asteroids extends beyond formation mechanisms. The determination of their mutual orbits allows measurements of mass and bulk density. Studies of surface morphology and material mobility also provide insight into the behavior of granular media in low gravity environments.

These binary systems are also of key interest for planetary defense: their complex dynamics makes them natural testbeds for deflection strategies. Understanding how binaries respond to external forces is crucial for designing effective mitigation techniques. For this reason the binary (65803) Didymos system was targeted by the Asteroid Impact and Deflection Assessment (AIDA) mission [48], a joint ESA–NASA effort, presented in detail in Section 1.3.

1.2. Past Asteroid Missions

Over the past few decades, numerous space missions have been devoted to the exploration of asteroids, providing fundamental insights into their physical properties, composition, and dynamical evolution. Each of these missions, conducted by different space agencies, has contributed to advance our understanding of small bodies, shaping current models of asteroid formation and evolution, as well as informing future planetary defense and resource utilization strategies.

Following the first close-up images of asteroids Gaspra and Ida, captured by NASA's Galileo spacecraft in 1991 and 1993 during its journey to Jupiter [49], subsequent missions as Deep Space 1 and Stardust only included asteroid flybys as part of broader multi-target mission profiles. The first mission specifically designed to orbit an asteroid for characterizing its properties was NASA's NEAR-Shoemaker [50], launched in 1996.

Asteroid exploration reached a new level of complexity with sample-return missions as Hayabusa [5] and OSIRIS-REx [51], designed not only to make contact with the surface but also to collect material for direct analysis on Earth. Instead, the most recent DART mission [52] was the first focused on planetary defense, aiming to test asteroid deflection techniques.

This section reviews the history and challenges of the most significant asteroid missions, with particular attention to imaging techniques and shape reconstruction, which are of interest for the following work.

1.2.1. NEAR - Shoemaker

The NEAR-Shoemaker (Near Earth Asteroid Rendezvous) mission was launched in 1996, inaugurating NASA's Discovery program. The selected target was the S-type NEA (433) Eros, a chondrite asteroid with no satellites nearby and a mean diameter of roughly 17 km. The main objectives were to collect images and scientific data for studying of the morphology, magnetic field, surface composition, internal mass composition and rotational characteristics [50].

In 1997 the spacecraft performed a fly-by of the C-class MBA (253) Mathilde, capturing 534 images with resolutions up to 160 m/pixel through its multispec-

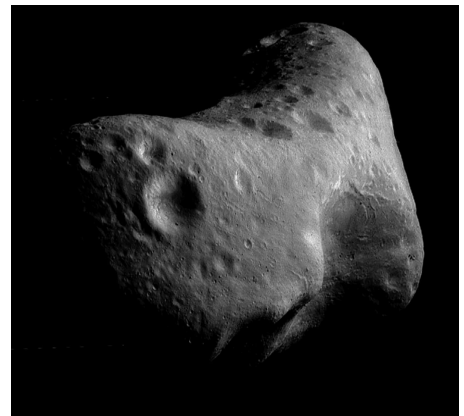


Figure 1.4: Eros imaged by NEAR-Shoemaker in 2000 [4].

tral imager camera (MSI). The observational geometry during the close encounter, characterized by a phase angle between 40-136 degrees of the visible side of the asteroid, allowed to roughly estimate Mathilde's albedo and shape, resulting in a mean diameter of 53 ± 2.6 km [53].

Then, the mission headed towards its main target, entering in orbit around Eros on 14 February 2000. During the approach phase, an intensive search for satellites was performed out to 100 asteroid radii, from which none larger than 20 *m* was found [38]. The spacecraft's radiometric tracking, optical and laser measurements collected from close orbit for over a year allowed to estimate the spherical harmonic and ellipsoidal gravity field of Eros, and the pole location. Despite the challenging elongated, curved and irregular shape of the asteroid, the imaging campaign permitted to construct a complete 3D model its shape [54]. Most of the images were obtained with a spatial resolution of 25 *m/pixel* from a 200 km terminator orbit, at a phase angle of about 90 degrees [4]. The determination of the mass and volume, with indicative dimensions of $34 \times 11 \times 11$ km³, resulted in a mean density of 2.67 ± 0.03 g/cm³. This ruled out a possible rubble pile structure and implied internal porosities, ranging from 20% to 30% [38].

In February 2001, NEAR executed a controlled descent, performing the first-ever landing on an asteroid, despite not being designed as a lander. The accurate models of shape and surface configuration, rich of boulders and craters, were fundamental for the final trajectory design and landing site selection. This landing manoeuvre also returned 70 ultra-high-resolution images down to centimeter scale [4].

1.2.2. Hayabusa and Hayabusa2

The JAXA mission Hayabusa, launched in 2003, was the first asteroid sample return mission. In 2005 the spacecraft reached the small S-type NEA (25143) Itokawa, at the time the smallest body ever explored by a spacecraft (~ 500 *m* in length) [5]. The spacecraft followed a hovering trajectory around the asteroid to perform remote sensing observations. The Asteroid Multiband Imaging Camera (AMICA) imaged the entire surface with a resolution of 70 *cm/pixel* [55]. Using optical navigation and a suite of instruments, Hayabusa conducted global mapping that revealed Itokawa's highly het-

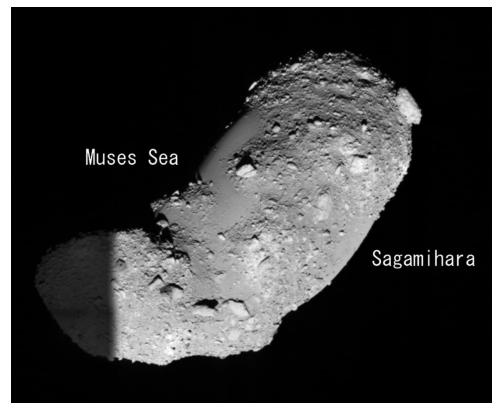


Figure 1.5: Itokawa imaged by Hayabusa in 2005 [5].

erogeneous surface, composed of smooth fine-grained regions and rough terrains covered in boulders. The mass of Itokawa was estimated from tracking and navigation data. Combined to the best volume estimate it yielded a bulk density of $1.95 \pm 0.14 \text{ g/cm}^3$ [38], which indicated a rubble-pile structure, loosely held together by gravity.

The spacecraft also released a microrover, but its landing onto the surface was unsuccessful. The preliminary global mapping of Itokawa drove the selection of the flat area called Muses Sea as landing site. Then Hayabusa performed Touch-And-Go (TAG) operations for terrain sampling through a sample horn technology, shooting a small projectile in the terrain and collecting the exposed material. A pin-point strategy was exploited, with Target Markers previously deployed on the surface. Despite major technical issues, Hayabusa successfully collected thousands of microscopic grains ($10\text{--}180 \mu\text{m}$) during the touchdown attempts [4]. These particles returned to Earth in 2010 and offered crucial insights into the composition, thermal history, and internal structure of rubble-pile asteroids.

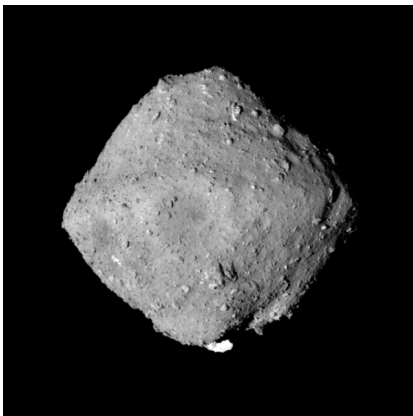


Figure 1.6: Ryugu imaged by Hayabusa2 in 2018 [6].

Hayabusa2 is JAXA's second asteroid sample return mission, launched in 2014 to investigate the C-type asteroid (162173) Ryugu, a carbonaceous NEA of about 900 *m* in diameter. Being a primitive body of 4.5 billion-year-old material that has been altered very little, it might contain precious information on the birth and evolution of the Solar System [56]. Built on its predecessor's success, the spacecraft also mounted the MASCOT lander and 2 MINERVA-II hopping rovers. After the arrival to Ryugu in 2018, these were successfully deployed on the surface, becoming the first mobile landers to operate and collect data on a small body [6]. The MASCOT mobile surface science package captured images during

its descent, bouncing and rest phases on the surface, allowing to study its interactions with the granular material and to infer some properties of the asteroid's surface [57, 58].

A major milestone was the Small Carry-on Impactor (SCI) experiment, which created an artificial crater to expose subsurface material. During its two touchdowns, Hayabusa2 collected samples both from the surface and from this freshly excavated region, which revealed material unaffected by space weathering [6]. In December 2020, the samples returned to Earth, unveiling the presence of hydrated minerals and organic compounds, and confirming Ryugu as a primitive carbonaceous body. The gravimetric analyses conducted have also shown that Ryugu, like Itokawa, has a highly porous rubble-pile structure.

1.2.3. Rosetta

Launched in 2004 under ESA's Horizon 2000 program, the Rosetta mission represented a milestone in the exploration of small bodies, becoming the first spacecraft to orbit a comet. 67P/Churyumov–Gerasimenko was reached in 2014 after a 10 years journey, including the flybys of the asteroids (2867) Steins and (21) Lutetia [4]. Even if the primary target was a comet, the mission should be included in the list, as it provided critical insights into small body dynamics and offered valuable comparisons between asteroids and other minor bodies.

Equipped with a comprehensive suite of instruments, including the deployable Philae lander, Rosetta performed long-term monitoring of the comet's activity as it approached and receded from the Sun. The two 5x5 degree FoV navigation cameras and the two science cameras, acquiring both narrow and wide angle images, provided the high-resolution imaging required for the estimation of 67P's complex shape, rotational state, and gravity field [59]. The Rosetta Radio Science Investigation (RSI) allowed to determine also the comet's mass, density, and internal structure, revealing a surprisingly homogeneous nucleus with no large voids [60].

In November 2014, Philae achieved the first-ever soft landing on a comet, despite complications with its anchoring system that caused multiple rebounds before settling in a shaded location [61]. Even with limited sunlight to power its instruments, Philae returned groundbreaking surface measurements, complementing Rosetta's orbital observations of the nucleus, coma, and plasma environment. The mission concluded with Rosetta's controlled descent onto the comet in 2016, allowing final close-up measurements.

1.2.4. OSIRIS-REx

Launched in 2016 as part of NASA's New Frontiers Program, OSIRIS-REx (Origins, Spectral Interpretation, Resource Identification, Security, Regolith Explorer) targeted the C-type NEA (101955) Bennu, a carbonaceous body with diameter of around 492 *m*. The mission's primary objective was to collect and return at least 60 grams of pristine surface material to Earth, providing direct insights into the composition of primitive SSSBs [51].

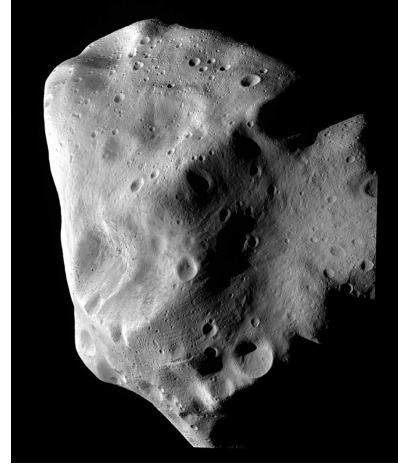


Figure 1.7: Lutetia imaged by Rosetta in 2010 [4].

Upon arrival in 2018, OSIRIS-REx conducted a comprehensive survey of Bennu, mapping its global shape, topography, and surface properties with the OCAMS camera suite and a laser altimeter (OLA). OCAMS comprised 3 cameras covering distances from 500 km to 2 m: MapCam (125 mm f , 4° FoV) for medium-range imaging, PolyCam (630 mm f , narrow FoV) for high-resolution mapping and landing site inspection, and SamCam (24 mm f , wide FoV) for documenting sampling operations [51]. The spacecraft also measured Bennu’s gravity field and mass through radiometric tracking, optical navigation and laser altimetry, revealing a heterogeneous internal structure and contributing to the understanding of its rubble-pile nature. Additionally, OSIRIS-REx observed the continuous ejection of small particles from Bennu’s surface, which offered a novel method to refine its gravity model [62].



Figure 1.8: Bennu imaged by OSIRIS-REx in 2018 [7].

In October 2020, the spacecraft performed a successful Touch-And-Go (TAG) maneuver, collecting 121.6 grams of regolith from the asteroid’s surface [63]. The samples returned to Earth in September 2023, providing precious data on the chemical composition, mineralogy, and history of carbonaceous NEA. The mission also improved our understanding of the Yarkovsky effect, the small force caused by anisotropic thermal re-emission of sunlight that gradually shifts an asteroid’s orbit around the Sun [47]. This is crucial for enhancing predictions of asteroid orbits and for the development of planetary defense strategies.

Additionally, the mission was further extended as OSIRIS-APEX, targeting the S-type NEA (99942) Apophis. The spacecraft will arrive shortly after Apophis’s close Earth flyby in 2029, conducting an 18-month survey of its shape, surface and interior, using thruster maneuvers to expose subsurface material [64]. This will provide the first close study of how a planetary encounter alters an asteroid.

1.2.5. DART

The Double Asteroid Redirection Test (DART) was NASA’s pioneering mission to test the kinetic impactor technique as a method for planetary defense. Launched on 24 November 2021, the spacecraft targeted Dimorphos, the small moonlet of the (65803) Didymos binary asteroid system.

A binary system was ideal for assessing this technique, as the Earth-based measuring of the mutual orbit change before and after the impact eliminates the need for a second spacecraft, dedicated to reconnaissance observations and assessment of the results [52]. The Didymos system was chosen among the known binary NEA systems for several reasons: it is one of the most accessible in terms of ΔV , the secondary is sufficiently small to measure a deflection caused by the impact of the spacecraft, has well characterized physical and orbital properties, and Dimorphos has an eclipsing orbit as viewed from the Earth, which makes it favourable for measurements of orbit changes via Earth-based observations of the system's brightness variations [52, 65].

Additionally, the diameter of around 165 m of Dimorphos is slightly above the minimum size of 140 m to be categorized as Potentially Hazardous Asteroid (PHA) [66]. However, Didymos system does not pose any threat as its orbit does not intersect with the Earth neither before nor after the DART impact. Moreover, remote observations shown that the asteroids are spectroscopically consistent with L/LL chondrites composition, representative of the most common Earth impactors [52].

On the 26 September 2022 DART successfully collided with Dimorphos at a speed of about 6.1 km/s, marking the first time humanity intentionally altered the trajectory of a celestial object. Post-impact observations revealed that Dimorphos' orbital period around Didymos was shortened by 33 ± 1 minutes [67], far exceeding the predicted minimum of 73 seconds and demonstrating the effectiveness of the technique [68]. The spacecraft was equipped with the narrow angle DRACO camera for navigation and impact targeting, while the Italian Space Agency's LICIACube cubesat, deployed 15 days before impact, documented the collision and the resulting ejecta plume with a high resolution monochrome camera and a color camera.

LICIACube performed a flyby of Dimorphos and reached its closest point of 58 km approximately 168 seconds after the DART impact [69]. The cubesat captured critical images of the impact event, observed the behavior of the ejecta in the low-gravity environment, and documented the hemisphere of Dimorphos not visible to DART, providing valuable data for the shape characterization.

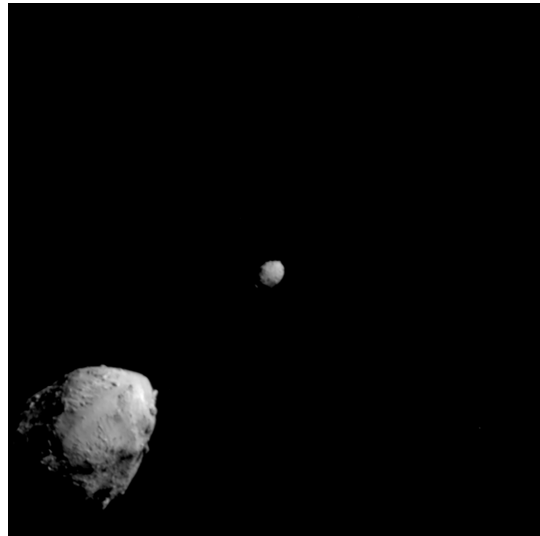


Figure 1.9: Didymos and Dimorphos imaged by DART in 2022 [8].

1.3. Mission Context

In the last decade, the growing awareness of the danger posed by Potentially Hazardous Objects has accelerated initiatives of space agencies for planetary defense. Although asteroid impacts with Earth are relatively rare, an object only a few hundred meters could cause devastating consequences. Therefore, detecting, tracking and characterizing PHOs is of key importance for risk assessment and planning of potential mitigation strategies.

After being engaged in NEO research since 1998, in 2016 NASA formalized its planetary defense efforts establishing the Planetary Defense Coordination Office (PDCO). This new NASA's Planetary Science Division oversees the research and observation of NEO, also coordinating activities with other government agencies and international participants [70]. The NEO discovery and characterization activities are expected to be furtherly accelerated by the NEO Surveyor space telescope, a specifically designed infrared space telescope which is planned for launch in late 2027.

Instead, ESA's Planetary Defense Office is developing FlyEye, a series of four automated telescopes spread across the globe, working together to carry out nightly sky surveys and automatically identify possible NEOs that could impact our planet. The first Flyeye telescope is undergoing testing and initial observations in Matera, from May 2025 [71]. In addition, ESA is currently conducting an early study for a possible space telescope mission called NEOMIR, to be dedicated for NEO detection and located at the first Lagrange point between Sun and Earth [72].

Despite this progress, key challenges still remain in the detection and mitigation of possible threats impacting Earth, but also in the international cooperation and interagency coordination. To address this need, NASA and ESA joined their efforts in the Asteroid Impact and Deflection Assessment (AIDA) collaboration, a planetary defense initiative representing the first international attempt to deliberately alter the trajectory of a NEA [48]. Its overarching objective is to validate the kinetic impactor technique as a viable method for deflecting potentially hazardous Near-Earth Objects (NEOs) and to quantify the resulting orbital change with high precision. The collaboration combines two independent but complementary missions:

- **DART** (Double Asteroid Redirection Test), led by NASA, with the objective of demonstrating the feasibility of asteroid deflection through kinetic impactor technique. Also carrying the LICIACube cubesat, developed by ASI;
- **Hera**, follow-up mission led by ESA, with the objective of executing a detailed in-situ analysis of the impact effects. Also carrying the cubesats Juventas and Milani.

The binary asteroid system (65803) Didymos was selected as target for the AIDA mission for various reasons, previously detailed in Section 1.2.5. A total of five spacecraft are part of the AIDA mission to Didymos, as shown in Figure 1.10.

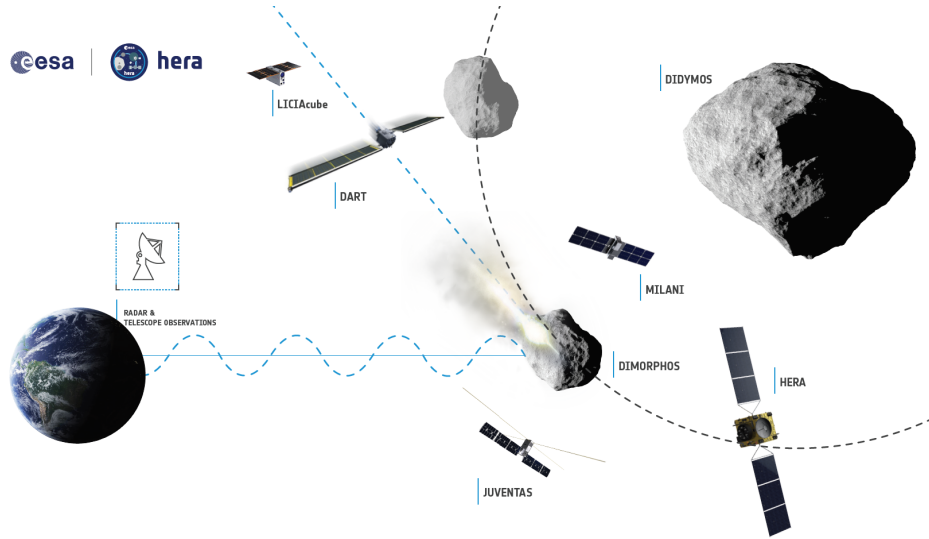


Figure 1.10: AIDA collaboration infographic [9].

1.3.1. Didymos Binary System

The NEA binary system of Didymos was discovered on April 11, 1996, by the Spacewatch survey at Kitt Peak Observatory. Initially only the primary body Didymos was observed, while the secondary Dimorphos remained undetected until 2003, when NASA's Goldstone Solar System Radar registered unusual echoes suggesting the presence of another body. This was confirmed by further radar and lightcurve analyses [73].

Didymos, meaning "twin" in Greek, from the visible and near-infrared spectroscopy observations appeared to be a silicate-rich S-type NEA. It shows a "spinning top" shape and a prominent equatorial ridge, common features among binary asteroid systems. The asteroid's rapid rotation period of 2.2593 hours likely redistributes material toward the equator, resulting in a characteristic equatorial bulge [73]. Its rotation rate is near the critical stability limit, at which surface material could begin to shed due to centrifugal forces overcoming the cohesive strength and gravity.

Didymos was thought to be a rubble-pile body, probably originating from the catastrophic disruption of a larger parent object, followed by the reaggregation of its fragments. Dimorphos' formation and physical characteristics, unknown before the DART mission, were assumed similar to the primary. After the impact, observations and numerical simulations provided valuable insights into its composition, confirming the rubble-pile structure,

likely formed from rotational mass shedding and reaccumulation of material ejected from Didymos.

The heliocentric orbit of the Didymos system is characterized by a semi-major axis of 1.64 AU and a moderate eccentricity of 0.38, crossing Mars' orbit and having a pericenter lying very close to Earth's orbit. This leads to occasional close approaches to our planet, as in 2003, when it reached a distance of 0.048 AU, sufficient to be classified a PHA [28]. The next closest approach will occur in November 2123. The complete heliocentric orbital parameters of the binary system are reported in Table 1.1.

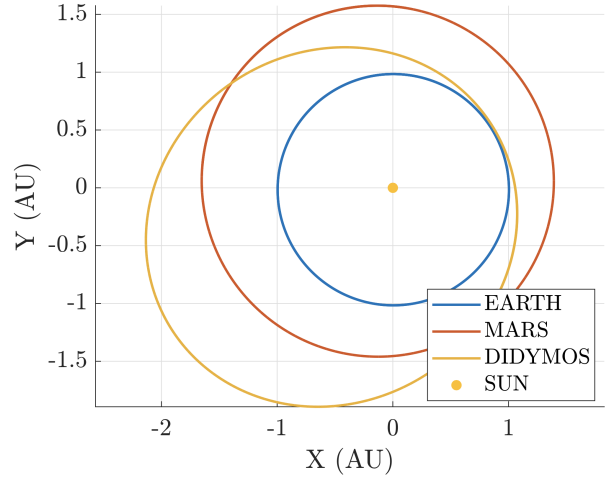


Figure 1.11: Earth, Mars and Didymos orbits, view from +Z @ECLIPJ2000.

Parameter	Value
Semi-major axis [AU]	$1.6426 \pm 2.95 \cdot 10^{-10}$
Pericenter distance [AU]	$1.0130 \pm 3.02 \cdot 10^{-10}$
Apocenter distance [AU]	$2.2721 \pm 4.09 \cdot 10^{-10}$
Eccentricity [-]	$0.383282 \pm 1.28 \cdot 10^{-10}$
Inclination [deg]	$3.4141 \pm 1.62 \cdot 10^{-8}$
Longitude of ascending node [deg]	$72.9849 \pm 2.22 \cdot 10^{-7}$
Argument of pericenter [deg]	$319.5949 \pm 2.50 \cdot 10^{-7}$
Orbital period [days]	$768.9266 \pm 2.07 \cdot 10^{-7}$
Epoch	5 May 2025 00:00:00

Table 1.1: Heliocentric orbital parameters of Didymos binary system [28].

The orbital motion of the Didymos system is retrograde with respect to its heliocentric orbit, and the equatorial plane is nearly inverted relative to the ecliptic plane, with rotation axis ecliptic coordinates $(\lambda, \beta) = (310^\circ, -84^\circ)$ [74]. Before the impact, Dimorphos was assumed to be tidally locked to the primary, with equal orbital and rotational period. This scenario is frequently observed in binaries. The spin axes of both asteroids were assumed oriented such that their north pole was near the ecliptic south direction.

The subsequent DART impact likely excited the rotation state of Dimorphos, also changing the binary system eccentricity and semi-major axis. After a transient period of approximately 70 days from the impact, the lightcurve measurements suggested a decrease in orbital eccentricity coinciding with Dimorphos entering a tumbling state, due to the conservation and transfer of angular momentum from the secondary's rotation to the mutual orbit [75]. The lightcurve analysis also indicated that Dimorphos remains on average tidally locked to Didymos, with its long axis generally oriented towards the primary.

Prior the DART arrival, Dimorphos was following a nearly circular orbit with a distance from Didymos of 1189 ± 17 meters and a period of 11.921 ± 0.0001 hours. From radar and ground telescopes observations, combined with the DRACO images, was measured a decrease in orbital period of approximately 33.24 ± 0.07 minutes, resulting in a new period of 11.367 ± 0.001 hours [75, 76]. Therefore, the distance between the two bodies consequently reduced to 1152 ± 18 meters, showing a decrease in the semi-major axis of approximately 37 ± 1 meters.

Parameter	Value
Impact speed [km/s]	6.1449 ± 0.0003
Impact angle [deg]	73 ± 7 from local horizontal 17 ± 7 from local normal
Impact site [deg]	Lat: 8.85 ± 0.45 S Lon: 264.30 ± 0.47 E
DART mass at impact [kg]	579.4 ± 0.7
Impact kinetic energy [GJ]	10.94 ± 0.01

Table 1.2: DART impact data [29].

A key parameter for characterizing DART's kinetic impact is the momentum enhancement factor, denoted β . It measures how much the ejecta produced during a deflection attempt contribute to the total momentum transferred to the target. Dynamical models indicate that the momentum enhancement resulting from DART's impact ranges from 2.4 to 4.9, assuming that Dimorphos has a bulk density between 1500 and 3300 kg/m³ [76]. The mass of Dimorphos remains the primary source of uncertainty. The fact that $\beta > 1$ indicates that the kinetic impact was highly effective, as the escaping impact ejecta imparted more momentum to Dimorphos than what was initially delivered by the spacecraft itself.

The high-resolution images obtained by the DART mission's DRACO camera, together with observations from LICIACube, offered the first detailed close-up views of this small NEA asteroid system. These observations played a crucial role in refining the physical and dynamical properties of both Didymos and Dimorphos, while supporting the shape reconstruction. On the surface of Didymos, a notable concentration of craters and boulders is present at high latitudes, with smoother equatorial regions. On the other hand, Dimorphos exhibits a more uniform surface, densely covered with boulders, and its topography features numerous asteroid-wide lineaments. The number of craters is significantly lower compared to the primary, indicating that Didymos is substantially older.

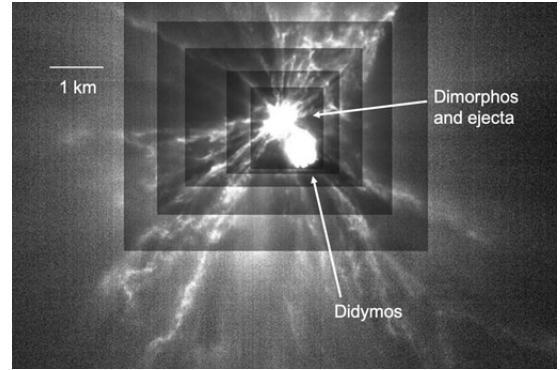


Figure 1.12: DART impact imaged by LICIACube [10].

Initial radar observations suggested nearly equal dimensions for Didymos, but the shape reconstruction by stereophotoclinometry from DRACO and LICIACube images revealed a more oblate shape with smaller overall size. Dynamical modeling of Dimorphos' orbit indicated even smaller dimensions of Didymos than those inferred from radar or spacecraft images [76].

The pre-impact shape model of Dimorphos was also obtained using stereophotoclinometry with DRACO images, revealing X, Y and Z dimensions of $179 \times 169 \times 115 \pm 1m$. These are expected to undergo changes after the impact, with the asteroid potentially entering a tumbling rotational state or transitioning to a prolate shape. In addition, Dimorphos' estimated post-impact equatorial axis ratio of 1.3 is significantly different from the pre-impact value of 1.02. This change could represent the formation of a large crater or a reshaping of the asteroid due to the impact, perturbing the gravitational fields [76].

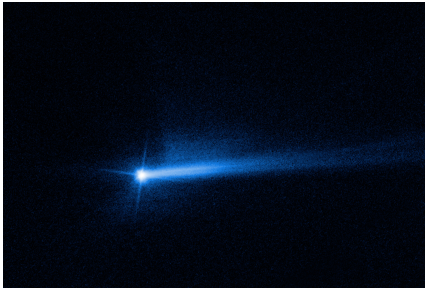


Figure 1.13: HST's image of Dimorphos' ejecta tail [11].

The total mass of the Didymos system was estimated to be $5.55 \times 10^{11}kg \pm 0.42 \times 10^{11}kg$. Observations after the impact allowed to estimate the mass of material ejected by DART's collision with Dimorphos, which is in the order of 10^7 kg [77]. Additional observations from both ground-based telescopes and space observatories, including Hubble and James Webb Space Telescope, have detected a persistent debris tail [78].

1.3.2. Hera

Following the success of NASA’s DART mission, ESA developed Hera as a dedicated follow-up investigation. Successfully launched on 7 October 2024, Hera is currently travelling to the Didymos binary system, with arrival scheduled for late December 2026.

Hera’s objective is to provide the first detailed characterization of a binary NEA system, while simultaneously assessing the consequences of DART’s impact on Dimorphos. By measuring key physical, compositional, and dynamical properties of both Didymos and its moonlet, the spacecraft will supply critical data for validating and refining asteroid deflection models. Importantly, Dimorphos represents the smallest asteroid ever to be visited by a spacecraft, making the mission a milestone in small-body exploration.

The mission’s scientific goals can be grouped into two categories: planetary defense and asteroid science. Regarding planetary defense, Hera mission objectives are [30]:

- Measure the mass of Dimorphos to accurately determine the momentum transfer efficiency of the DART impact, which is critical for evaluating the kinetic impact deflection technique;
- Investigate in detail the crater produced by DART to improve our understanding of the cratering process and the mechanisms by which the crater formation drives the momentum transfer efficiency;
- Detect and analyze subtle post-impact dynamical effects (e.g., libration imparted by the impact, orbital and spin excitation of Dimorphos) that are difficult to detect for remote observers;
- Characterize the surface and interior properties of Dimorphos, which have great influence on the impact response (in particular size, shape, volume, density and porosity), to allow the scaling of momentum transfer efficiency to different asteroids;

From a broader asteroid science perspective, the Hera mission aims to conduct the first comprehensive study of a binary near-Earth asteroid system. The mission will constrain key surface properties, including morphology, regolith composition and mobility, and material strength in ultra-low-gravity environments, while also providing in situ observations of freshly exposed subsurface material, fundamental for studying space weathering and meteorite impact effects [30].

In addition, Hera will investigate the dynamics of ejecta and test theoretical models of binary asteroid formation and evolution, in particular for Didymos that has a spin period close to the limit of structural stability.

Beyond its scientific objectives, the mission also offers significant technology demonstration opportunities for deep-space cubesat science [79]. In proximity to Dimorphos, the spacecraft will deploy two cubesats, Juventas and Milani. Hera and the two cubesats will establish an Inter-Satellite Link (ISL) network with ranging capabilities, enabling spacecraft-relayed cubesat operations in deep space. Additionally, the mission offers the opportunity to test collision avoidance and autonomous visual-based GNC techniques for low-altitude flybys in a low-gravity challenging environment.

To achieve these goals, Hera carries a sophisticated payload including two Asteroid Framing Cameras (AFCs), the Hyperscout-H spectral imager, a laser altimeter (PALT), and a thermal infrared imager (TIRI). The cubesats play a central role in complementing Hera's remote sensing with close-up investigations: Juventas will probe the interior structure using the low-frequency JuRa radar and GRASS gravimeter, while Milani will perform detailed mineralogical mapping and monitor space weathering effects.

The trajectory towards Didymos involves a Mars gravity assist in March 2025 and two deep-space maneuvers before rendezvous. Once inserted into orbit, Hera will carry out a sequence of operations, gradually reducing its altitude to enable increasingly detailed observations. These mission phases are briefly reported in Table 1.3.

Phase	Duration	Operations
Early characterization	6 weeks	Hyperbolic arcs at 20–30 km from Didymos. Short arcs of 3 days and long arcs of 4 days create a weekly cycle. Global shape, mass, gravity, thermal and dynamical properties are measured.
Payload deployment	2 weeks	Release of the two cubesats and support for their early operations.
Detailed characterization	4 weeks	Weekly hyperbolic arcs at reduced distance of 8–20 km. Partial autonomous attitude guidance.
Close observation	6 weeks	Pericenter distance reduced to 4 km. Full autonomous attitude guidance. Twelve close flybys for high-resolution mapping of Dimorphos.
Experimental phase	6 weeks	Innovative navigation techniques for flybys as low as 1 km. Decimeter-resolution observations, culminating with Hera landing on Didymos.

Table 1.3: Mission phases of Hera [30].

1.3.3. Milani

Milani is a 6U cubesat developed by Tyvak International and integrated as piggyback payload into the ESA Hera mission. Scheduled for deployment in March 2027, after Hera's arrival at the Didymos system, Milani is designed to conduct detailed in-situ observations of both Didymos and Dimorphos, from distances ranging between 2 and 10 km.

The mission serves both scientific and technological objectives, complementing Hera's measurements while demonstrating the capabilities of cubesat platforms in deep-space operations [30]. The primary scientific objectives for the detailed and comprehensive characterization of Didymos binary system are the following:

- Global mapping of Didymos and Dimorphos, analyzing compositional variations between the two bodies;
- Characterization of the morphology and surface properties of both the asteroids;
- Assessment of DART impact effects, such as ejecta distribution and surface alterations. Support the gravity field estimation through range and range-rate measurements exchanged with Hera via ISL;
- Characterization of the dust environment, detecting particles smaller than a few microns and analyzing inorganic materials, volatiles, and light organics within the system.

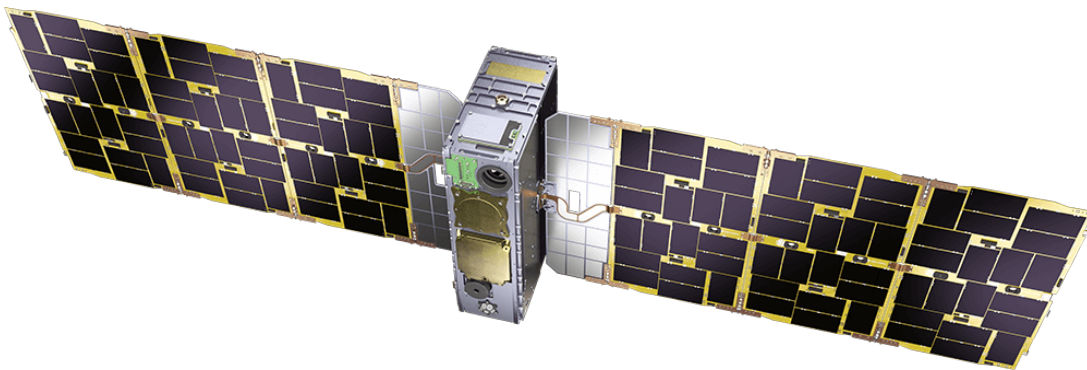


Figure 1.14: Milani cubesat [12].

Milani has a wet mass of 12 kg and features deployable solar arrays for power generation. Its cold-gas propulsion system allows six-degree-of-freedom (6-DoF) maneuvering, ensuring precise translational and attitude control in close-proximity operations [80].

The spacecraft carries a suite of instruments tailored to its scientific and operational goals. The ASPECT hyperspectral imager acquires data in the visible (VIS), near-infrared (NIR), and short-wavelength infrared (SWIR) ranges to perform global mapping, high-resolution imaging of Dimorphos, and the study of surface microstructures. The VISTA thermogravimeter is dedicated to detecting dust particles smaller than 5-10 μm and analyzing volatiles and light organics in the surrounding environment [81].

In addition, Milani carries a dedicated Navigation Camera (NavCam), which supports both scientific and operational tasks. The NavCam enables autonomous optical navigation, performing onboard image processing to derive precise navigation data. It is co-aligned with the ASPECT instrument, to guarantee effective synergy between navigation and scientific imaging operations.

The camera is based on an Onsemi AR0331 sensor, sensitive to wavelengths in the 400–700 nm visible range, and has custom optics with a focal length of 12.96 mm and a transmission coefficient above 90% [82]. Furthermore, the sensor integrates a Bayer filter in a GRBG layout, enabling the production of full-color RGB images. The NavCam is designed for operations over distances from about 30 km down to 200 m, making it suitable for close-proximity navigation while supporting Milani’s scientific objectives.

Milani is also equipped with a LiDAR, ensuring reliable distance measurements during navigation around Dimorphos. The cubesat mounts also a star tracker, two sun sensors, and an IMU [83]. Communication with Earth and inter-satellite positioning are achieved through dedicated ISL antennas, which allow to perform the radio science experiment.

Parameter	Value
FOV [deg]	19.72×14.86
Image size [pix]	2048×1536
Spectral range [nm]	400–700
F number [-]	2.8
Pixel size [μm]	2.2×2.2
Bits [-]	12
Full well charge [-]	100000
PSF width [-]	10
PSF sigma [-]	2.5

Table 1.4: NavCam parameters [13].

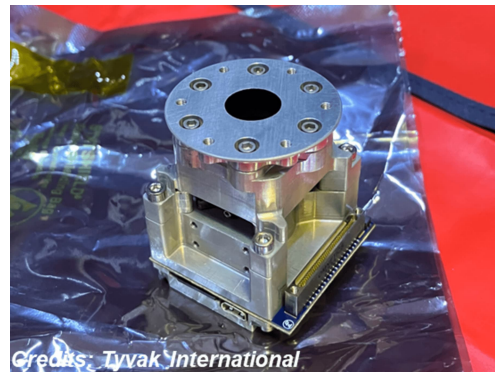


Figure 1.15: Milani Navigation camera (with baffle) [13].

Milani's mission analysis is overseen by Politecnico di Milano. The mission is structured in multiple sequential phases, designed to maximize the scientific return while respecting technical and operational constraints. During the 2.2-year cruise to Didymos binary system Milani will remain integrated within the Deep Space Deployer onboard Hera and mostly inactive, except for periodic health checks.

Following deployment from Hera at approximately 30 km from Didymos, Milani undergoes system detumbling, attitude acquisition and calibration, to ensure full operational capability. After this first commissioning, transfer maneuvers will position the spacecraft on its initial scientific orbit, to begin its operational phases [84]:

1. **Far-Range Phase (FRP):** The spacecraft performs global mapping of both asteroids at resolutions better than 2 m/pixel. This is achieved via a hyperbolic loop trajectory with six waypoints, repeated over three cycles of 3-4 days at distances of 8–14 km from Didymos, ensuring quasi-symmetric arcs relative to the Sun for optimal illumination;
2. **Close-Range Phase (CRP):** Focuses on high-resolution imaging of Dimorphos (better than 1m/pixel) and of the DART impact site (better than 0.5 m/pixel), operating as close as 2.78 km from the surface. Due to the tight resolution and illumination constraints, the trajectory is designed to optimize both imaging quality and operational efficiency. It follows an asymmetrical orbital design based on a keypoint strategy, consisting of a 7-4-7-7-3-7 maneuvering pattern [80]. In this scheme, the odd-numbered arcs are dedicated to imaging of surface and crater. The even-numbered arcs serve as recovery and repositioning segments, allowing the cubesat to phase its motion relative to the tidally locked rotation of Dimorphos. Additionally, the sequence gradually rotates the orbital plane toward the terminator plane, providing a smooth transition to the next phase's target orbit while ensuring observations at phase angles between 0–10 deg and 30–60 deg [85];
3. **Experimental Phase (EXP):** Milani will transition into a Self-Stabilized Terminator Orbit (SSTO) for close-proximity operations, progressively approaching the Didymos system. The baseline end-of-life plan involves attempting a landing on Dimorphos to gather high-value surface data. If a landing proves unfeasible, the spacecraft will be injected into a stable heliocentric graveyard orbit [83]. This phase represents a high-risk, high-reward operation: it could yield unique insights into surface properties and low-gravity interactions, but poses significant challenges, as bouncing at contact or survival post-impact, considering that Milani is not designed as a lander.

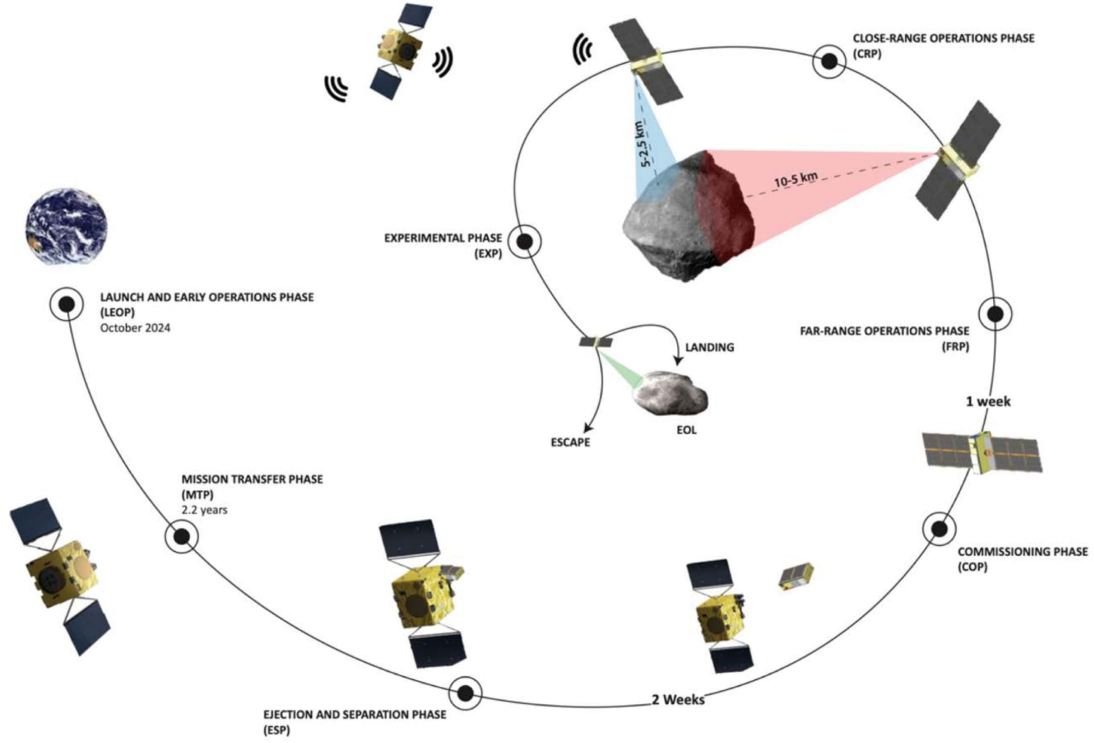


Figure 1.16: Schematics of the phases of the Milani mission [14].

1.4. Research Question

The accurate reconstruction of the asteroids global shape is fundamental for both scientific investigations and mission operations. The knowledge of the shape allows the estimation of critical parameters for the characterization of the body, enabling reliable navigation in the challenging low-gravity environment. In particular, in the Milani mission the reconstruction of Dimorphos' 3D shape is essential for achieving its primary scientific objectives, presented in Section 1.3.3.

The stereophotoclinometry (SPC) technique provides a way to integrate image data into a consistent 3D model by combining photometric and geometric information. This method has been successfully applied in previous missions, and its potential use for reconstructing the shape of Dimorphos can be important to support the scientific objectives of Hera and to evaluate the DART impact outcome. A precise global model will not only allow the determination of physical properties, such as mass and volume, but also enable the characterization of local topography, which is essential for performing optical navigation and for assessing potential landing sites.

Therefore, this thesis investigates the potential contributions of the Milani mission's Navigation Camera (NavCam) to the shape reconstruction of Dimorphos. The research is

conducted within the SPC framework to address the following question:

Which is the achievable accuracy for the global shape reconstruction of Dimorphos using the optical imaging data of Milani's NavCam in an SPC algorithm?

Milani's NavCam primary purpose is not the scientific investigation of Dimorphos, but the acquisition of images for optical navigation in the low-gravity environment. However, the camera is mounted coaxially with the ASPECT instrument, Milani's main payload for global asteroid mapping. In addition, the sensor of NavCam is designed for being sensitive to wavelengths that are not covered by other instruments carried by Hera, Juventas and Milani [13]. Consequently, shape reconstruction from NavCam images could provide valuable complementary information for the achievement of mission objectives, validating the global mapping of Dimorphos through comparison with shape models obtained from different data, and enhancing the overall characterization of the asteroid.

For this study, NavCam images are simulated based on the reference trajectory planned for the cubesat upon arrival at the Didymos system in 2027. It is important to consider that the spacecraft trajectory and the resulting image acquisition geometry have a direct impact on reconstruction quality, as the accuracy of the SPC algorithm strongly depends on the observation geometries. Therefore, from the primary research question, several specific inquiries arise:

- *Which SPC parameters, such as the number of landmarks and the maplet size, provide the best accuracy in topography reconstruction?*
- *How does the accuracy vary with different acquisition geometries and illumination conditions, depending on the spacecraft trajectory?*
- *Which surface regions are the most challenging to model accurately?*
- *With what level of accuracy can SPC reconstruct the impact site morphology of the DART mission?*
- *To what extent can the SPC-derived shape model support the identification of safe landing regions by excluding areas with large boulders or craters?*

These questions guide the analysis performed in this work and form the basis for evaluating the applicability and limitations of using SPC with the NavCam images for global shape modeling of Dimorphos.

1.5. Thesis Outline

This section presents the organization of the thesis, summarizing the content of each chapter. Chapter 1 introduced the background and research question, while the following chapters are structured as follows:

- **Chapter 2: State of the Art**

This chapter reviews the current state of the art in the techniques for shape reconstruction of asteroids. Firstly are presented the most relevant ground-based shape modeling methods (radar imaging and light-curve inversion). After are described the most important spacecraft-based shape modeling techniques (Shape from Silhouette, StereoPhotoGrammetry, StereoPhotoClinometry, laser altimetry and PhotoClinometry by Deformation). Particular emphasis is given to past asteroid missions that employed these methods, as well as to their advantages and intrinsic limitations.

- **Chapter 3: SPC Shape Reconstruction**

This chapter details the main blocks of the SPC algorithm for shape reconstruction. Firstly is introduced the image generation pipeline. Then is presented the topography reconstruction tool, with the additional SPC components implemented for correcting the landmark position vectors and the spacecraft's pose. Finally is explored the overlapping search tool, which is required for the generation of a global shape model.

- **Chapter 4: Results**

In this chapter are presented the results of the various SPC building blocks. The topography reconstruction is performed in several scenarios across different phases, with particular emphasis on the DART impact area. The two newly implemented SPC components are also tested and validated in different scenarios, demonstrating their functionality in an integrated SPC architecture.

- **Chapter 5: Conclusions**

This chapter summarizes the main results and achievements, while discussing the possible applications and also limitations of the use of NavCam imaging for SPC shape reconstruction of Dimorphos. Finally are provided potential future developments and improvements for the SPC algorithm.

2 | State of the Art

The shape reconstruction of an asteroid is a fundamental step in its characterization process, both for scientific and operational needs. Unlike planets, small asteroids are typically irregularly shaped, and their surface characteristics, as roughness, porosity, and composition, are not directly observable from Earth. The detailed characterization of these bodies is a very complex challenge, and the determination of their dimensions and shape of is essential for understanding their history, behavior and interactions with other solar system objects [86].

Accurate shape models are important for several reasons. They enable precise determination of an asteroid's size, mass, and density, which are crucial for assessing its gravitational influence and potential to affect nearby objects. For certain asteroids, the future orbital path cannot be reliably determined without shape and spin information. The Yarkovsky effect, for example, is responsible for the largest source of uncertainty in trajectory predictions for NEAs with sizes under 2 km, and it must be taken into account when evaluating impact probabilities [87, 88].

Shapes also affect the spin evolution during close planetary encounters and the orbital evolution of binary NEA systems as Didymos, in terms of tidal, gravitational, and non-gravitational interactions between the bodies [89]. Shape models are vital for planning space missions in low-gravity environments, including proximity operations as landing and sampling or deflection operations, because they provide essential information on surface topography and rotational dynamics [90].

Several methods have been employed for estimating the dimensions and shapes of asteroids, exploiting the improving capabilities of our ground-based telescopes and also the observations performed in dedicated space missions. In the following chapter are presented some of the most relevant techniques for the shape reconstruction of these small bodies, highlighting their peculiarities and intrinsic limitations.

2.1. Ground-based shape modeling techniques

Ground-based observations represent the first step in characterizing the physical properties of asteroids. They can provide essential information on their size, shape, spin state and surface properties. Among these, radar imaging and photometric light-curve analysis are the most widely used techniques. Over the years, the significant improvements in observational capabilities and reconstruction algorithms have greatly enhanced the fidelity of these methods for asteroid shape modeling, often providing the first 3D representations used for mission planning and scientific analysis.

2.1.1. Radar imaging

Earth-based radar has proven to be a powerful tool for gathering informations about bodies in the solar system. Radar observations consist in transmitting a radio signal towards the target and recording the echo. The distribution of echo power in delay time and Doppler frequency is related to the spin state and 3D shape of the target. Through radar observations is possible to obtain critical insights into the physical characteristics of small bodies, while significantly improving the estimation of their orbital dynamics.

A major advancement in recent decades has been the development of sophisticated inversion algorithms capable of deriving asteroid shapes directly from radar measurements. This problem is inherently complex, because the determination of an object's spin state and 3D shape are closely interdependent. Even if assuming perfect spin state information, the process of reconstructing the physical shape of the body from radar data is still very challenging and computationally intensive [86].

An example is the SHAPE software package, used since the 1990s to analyze delay-Doppler data and build shape models for a few dozen NEAs out of the hundreds observed with the Arecibo radio telescope. SHAPE projects an asteroid model, based on shape and spin, into the range-Doppler radar space, and iteratively adjusts the parameters to minimize residuals with the observed data.

The software progressively increases the complexity of its asteroid model, evolving from a simple ellipsoidal approximation to a spherical harmonic representation that captures finer surface details. SHAPE performance was firstly improved with multiple-parameter fitting algorithms, which allow to invert delay-Doppler datasets more efficiently, but still requiring extensive human oversight. Bayesian optimization and deep neural networks were later introduced to accelerate and automate the process [91], also incorporating occultation and Adaptive Optics data further constrained radar shape models.

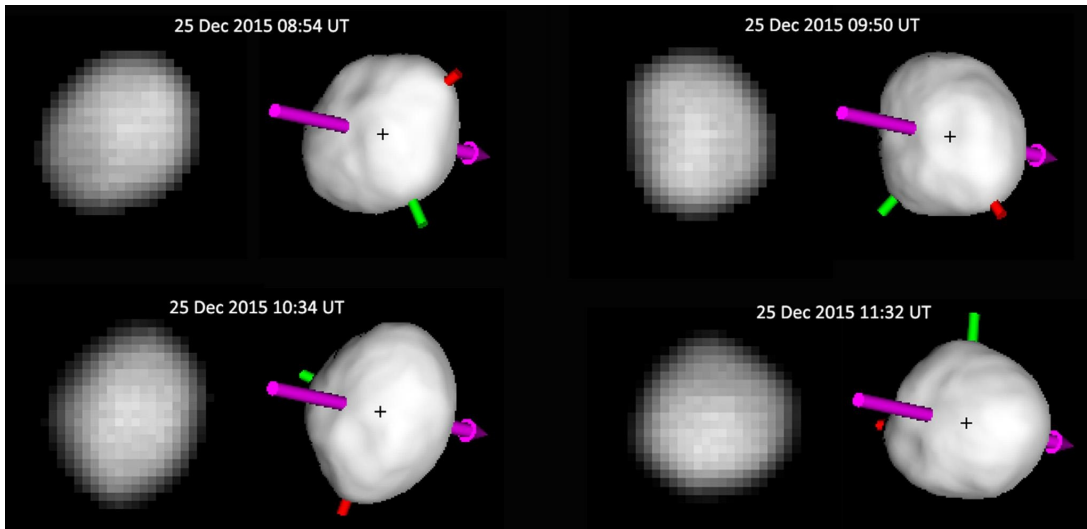


Figure 2.1: Psyche SHAPE model views, obtained from radar and AO images [15].

Despite these challenges in the reconstruction process, radar remains the most effective ground-based method to provide detailed 3D shape models of NEOs, due to the fact that its achievable spatial resolution is not constrained by optical diffraction limits [86]. Therefore, facilities as the now decommissioned Arecibo Observatory, formerly operating in S-Band at 2380 MHz with a beamwidth of about 2 arcminutes, have played a central role in producing some of the most detailed asteroid models to date.

The study conducted in 2015 on the M-type asteroid (16) Psyche, through Arecibo radar observations, shown that the shape model obtained with SHAPE is slightly larger but generally consistent with a convex model based primarily on lightcurve inversion. In addition, the radar shape model reveals large-scale features and depressions that are not evident in lightcurve models. In fact, the latter are typically constrained to be convex, resulting in an approximation of the actual body that excludes any concavities [15].

Furthermore, radar imaging does not rely on reflected sunlight. Its human-controlled illumination allows for greater flexibility in the observations. Radar has also the ability to probe an object's sub-surface properties, which can give important information, such as porosity, surface-ice content, and surface conductivity.

On the other hand, is important to consider the intrinsic limitations of this technique, like the inversion algorithm complexity and the frequent need for prior knowledge of the object's spin state. Being an active technology, radar's effectiveness depends critically on the object's reflectivity and on the proximity to Earth. The return signal drops off steeply with the distance (proportional to the inverse fourth power), limiting accurate observations only to close and radar-bright NEAs.

2.1.2. Photometric light-curve inversion

The photometric light-curve analysis was the first technique employed for obtaining informations about the shape of asteroids. Since the early 20th century astronomers began recording brightness variations of asteroids with telescopes, noticing that periodic changes in light were linked to the body's rotation period and shape.

Eros, discovered by G. Witt in 1898, was the first asteroid to show a light-curve [54]. A first determination of its rotation period dates back to 1901 by Theodore von Oppolzer, who detected its brightness variations and associated them to the asteroid's rotation and elongated shape [92]. Even today, photometric observations are the most abundant data source for determining the physical properties of asteroids. Of all the possible techniques, ground-based optical observation are the ones that encounter the least financial or technical hinders.

In the visible wavelengths, we observe the total amount of sunlight reflected from the surface of the asteroid at each epoch of observation. The intensity of reflected radiation depends on phase angle, the angle between Earth and Sun as seen from the asteroid. Surface scattering and shadowing cause more light to be reflected at small angles, with a sharp increase near opposition (zero phase angle), known as the opposition effect [93].

The observed brightness also depends on the asteroid's rotation. As its irregular surface turns, the illuminated and visible area changes, causing the reflected sunlight to vary. A series of such brightness measurements over one revolution is called a lightcurve. The more densely sampled the lightcurve points are, the more detailed information is obtained about the shape and albedo of the target.

The systematic analysis of light curves allows to estimate the rotation rate and approximate shape of an asteroid, by studying the variation in time of the target's integrated brightness. Formal light-curve inversion algorithms have been developed and improved during the years, allowing to derive 3D convex and also non-convex shape models [94, 95].

However, several limitations need to be considered. The small size and typically large distance from Earth makes asteroids generally appear star-like when observed with ground-based optical telescopes. Further restrictions come mostly from the long exposure times needed to achieve an adequate signal-to-noise ratio (SNR). This is typically not a problem for slowly moving MBAs, which are the majority of the observed asteroid population, but becomes relevant for NEAs, often moving fast in the telescope's field-of-view, making long exposure times useless.

Unlike radar imaging, photometric observations are also very sensitive to the weather, as they require a completely clear sky to have a sufficiently good SNR. Therefore, optimal planning of the observations is challenging. Another factor to consider is that absolute photometry requires the use of standard filters, which decreases the amount of incident radiation on the CCD, limiting the number of observable asteroids [93].

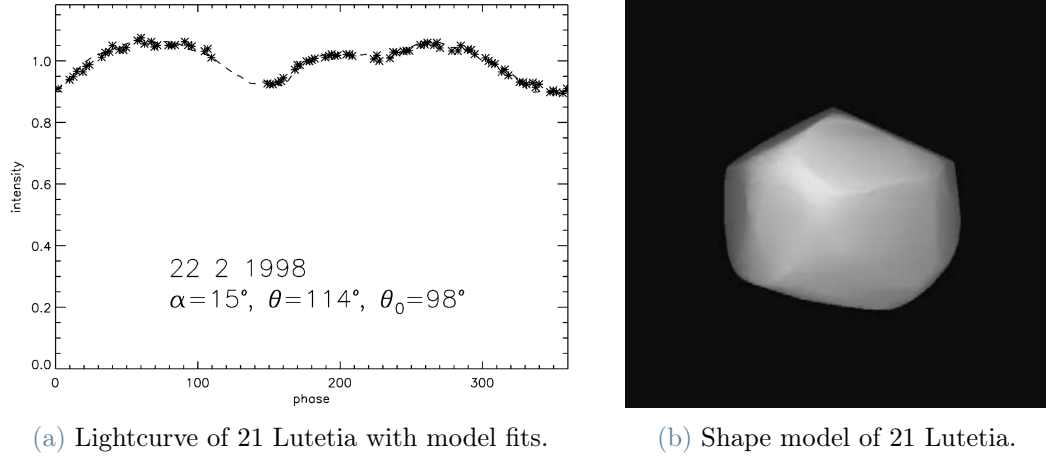


Figure 2.2: Shape model of 21 Lutetia derived from lightcurves inversion [16].

Beyond lightcurves, additional insights into asteroid shapes come from stellar occultations, occurring when an asteroid passes in front of a star casting a shadow on Earth. The duration of each disappearance, measured from different observing locations, traces straight lines across the asteroid’s silhouette, which can be combined to estimate its size and shape. However, occultations of bright stars by any asteroid occur rather infrequently.

Other important techniques are observations with Adaptive Optics, space telescopes and interferometry. The latter consists in combining light beams from separated telescopes and observing the resulting interference. Each of these methods provides a useful piece of information, but the best way to reconstruct an asteroid model is to use all the available data, mitigating their limitations. Therefore, lightcurve inversion methods are typically integrated with complementary data sources [96], as in the 3-D shape-modeling technique KOALA (Knitted Occultation, Adaptive-optics, and Lightcurve Analysis). This method was adopted to obtain the shape model of asteroid (21) Lutetia, then validated exploiting the images acquired by the ESA Rosetta mission during the flyby of the body [97].

Another method is ADAM (All-Data Asteroid Modeling), developed to combine disk-resolved data (AO images, interferometry, radar) with disk-integrated data (lightcurves, infrared) and occultation timings into a unified asteroid modeling framework [98].

2.2. Spacecraft-based shape modeling techniques

Ground-based observations have been fundamental in providing the first estimates of asteroid shapes, rotation states and spin-axis orientations. However, these techniques are inherently limited in spatial resolution due to the distance from the target, and the resulting models cannot reliably resolve the fine surface features and small concavities.

The arrival of dedicated spacecraft missions performing flybys and encounters with asteroids marked a major leap forward in shape reconstruction. The high-resolution imaging and mapping instruments employed in proximity operations allowed the derivation of 3D models with significantly higher accuracy, capturing local morphology, boulders, craters, and surface roughness at scales impossible to achieve from Earth.

A variety of techniques have been developed to exploit spacecraft data, depending on the observational conditions and mission objectives. Typically a multi-phased system approach is adopted to estimate the physical and dynamical parameters of a small body, as shown in Figure 2.3. During distant approach, when only a few hundred pixels cover the body, Shape from Silhouette (SfS) methods can exploit the sharp contrast of the silhouette against the dark background to constrain the initial shape and rotation axis [17]. Once surface features become resolved, Stereophotogrammetry (SPG) and Stereophotoclinometry (SPC) algorithms can be used to better reconstruct detailed digital terrain models. Furthermore, instruments like laser altimeters (LiDAR) can provide direct, high-accuracy ranging data, allowing the refinement of shape models by combining multiple data sources.

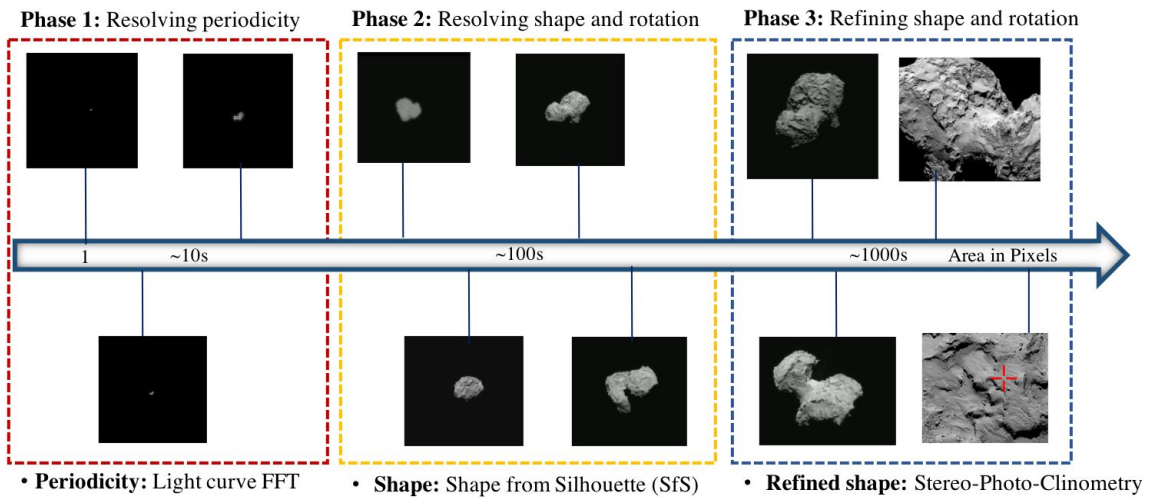


Figure 2.3: The multi-phased sequence of algorithms to estimate the physical and dynamical parameters of the small body [17].

2.2.1. Shape from Silhouette (SfS)

During the approach phase of an asteroid mission, images are taken from a very large distance and the small body's surface features are not discernible. Under these conditions, Shape-from-Silhouette algorithms are particularly useful for obtaining reliable asteroid models or for refining the previous ones, derived from radar or optical observations.

Various algorithms exist in literature for silhouette-based 3D shape reconstruction from multiple images. These methods typically rely on precise knowledge of spacecraft's and camera's position and orientation during image acquisition, assuming that the camera's pose has already been determined through a prior localization step [17]. The Sun's position relative to the spacecraft is also known.

A first type of algorithm is silhouette carving. This method was employed at the beginning of the characterization phase of Rosetta mission for carrying out a first coarse shape model reconstruction of 67P/Churyumov-Gerasimenko comet [59]. It uses a voxel representation of the shape to be estimated. Voxels are the 3D pixels that divide the initial bounding box of the small body, which is generated on initial estimates of its size. For each image, the voxels are projected in the image plane to check if they are inside the silhouette. Then, the voxels not belonging to the silhouettes are carved and removed, incorporating proper factors to handle erroneous images or projection errors in the camera views [17].

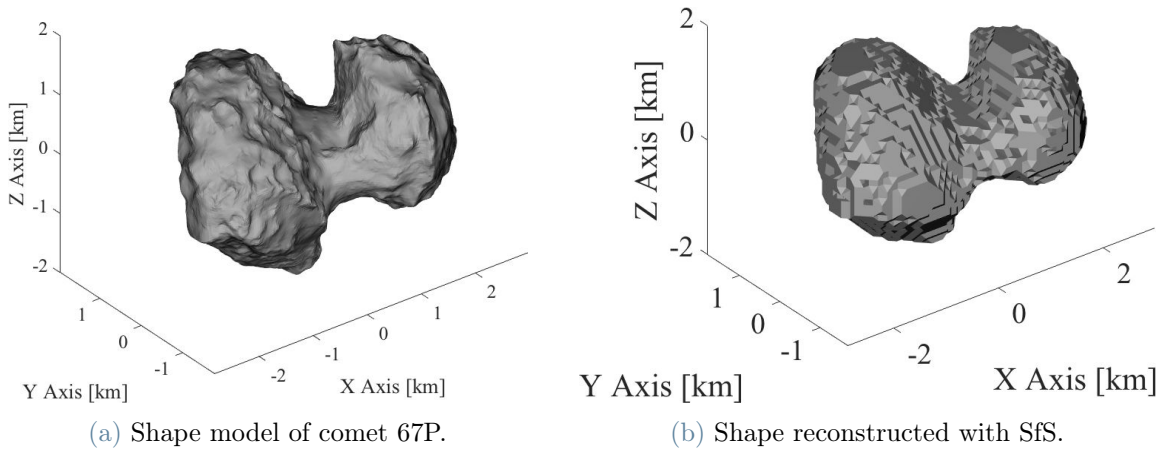


Figure 2.4: Shape model of comet 67P/Churyumov-Gerasimenko reconstructed with Shape-from-Shading from simulated images [17].

An alternative SfS method for shape reconstruction is based on the intersection of viewing cones [99, 100]. Each viewing cone is generated by the set of rays that originate from the camera's optical center and pass through the points outlining a given silhouette.

By projecting these cones onto the silhouettes obtained from other viewpoints, it is possible to identify the regions that consistently belong to the object. The intersection of these regions across multiple silhouettes provides a robust estimate of the three-dimensional shape of the target body [19].

A significant limitation of these techniques is the inability to recover concavities not seen in any silhouette. Since silhouettes only capture the object's outer limbs, they provide no information about interior points. Therefore, features not visible in the outline cannot be reconstructed. The closest approximation that can be achieved is the visual hull, which is the object obtained using all the viewpoints that belong to a viewing region completely enclosing the original object [18]. The concept of visual hull was formally introduced by Laurentini [101] as the envelope of all the possible circumscribed cones, and it can be seen also as the largest object that produces the same silhouettes as the given object.

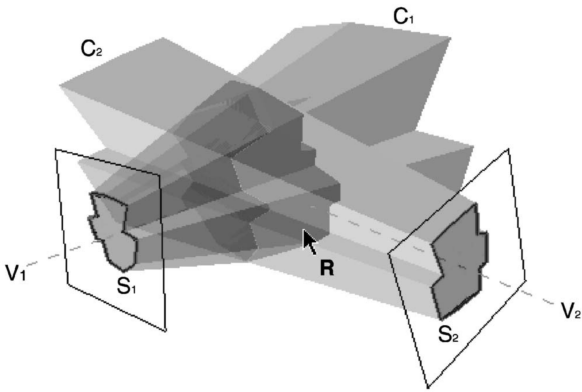


Figure 2.5: Volume intersection SfS [18].

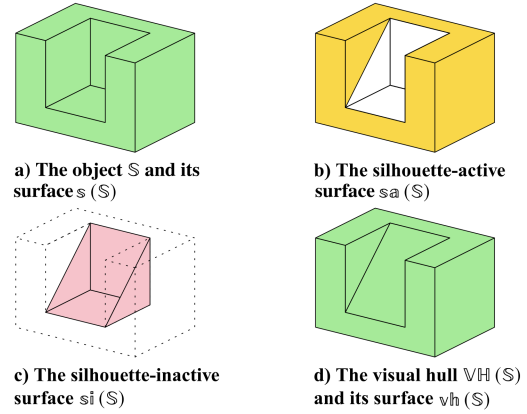


Figure 2.6: Visual Hull definition [19].

A further improvement of this algorithm was developed by Panicucci et al. [19] to account for shadowing effects. The silhouettes from raw images are modified to keep only limbs that are surely illuminated by the Sun and thus exploitable for the reconstruction of the visual hull.

The sunrays form two types of shadows on a concave shape: terminator-line shadows and self-shadowing. Self-shadowing boundaries are defined by tangential sunrays subsequently intersecting the small body after a concavity. In the standard algorithm, this self-shadowing could generate several contours that belong to the same object, but are perceived as isolated pieces in the image. Therefore, computing shadow-robust silhouettes the output of the algorithm becomes a polyhedral shape whose reconstruction is robust to shadowing.

2.2.2. Stereophotogrammetry (SPG)

Stereophotogrammetric (SPG) mapping is a widely used technique to reconstruct detailed three-dimensional models of planetary surfaces by exploiting stereo image data.

The method relies on combining multiple overlapping images, acquired from different viewing angles, together with precise spacecraft navigation and camera calibration data, to derive surface topography and generate ortho-rectified image products [20].

The processing pipeline typically involves two stages: systematic preprocessing of the raw spacecraft data (including decompression, radiometric calibration, and geometric corrections), and photogrammetric processing to extract surface points and build digital terrain models (DTMs). The whole procedure of DTM generation from stereo data, also reported in Figure 2.7, is carried out iteratively: initial estimates of spacecraft position and pointing, often provided in SPICE kernels, are refined through photogrammetric block adjustments, progressively improving the alignment of images to the surface model.

Firstly, the raw images are pre-rectified using the available spacecraft orbit and pointing information, aligning all stereo images to a common map projection and pixel scale. Then, a multi-image matching algorithm identifies corresponding features across all the overlapping stereo images at sub-pixel accuracy, through least-squares matching [102]. The matched image coordinates are transformed back to the original image geometry. Using these coordinates together with the camera calibration and refined spacecraft orientation, forward ray intersections are performed to compute 3D object points in body-fixed Cartesian coordinates.

Only points defined by at least three overlapping images are retained, reducing errors from occasional mismatches. These object points are subsequently interpolated to create regular-grid digital shape models (DTMs), which serve as the basis for ortho-rectifying the images. Finally, orthoimages from different orbits are mosaicked to produce high-resolution global or regional maps [103]. The quality of both DTMs and orthoimages depends strongly on the accuracy of spacecraft navigation and orientation data, as well as on local surface texture and illumination conditions.

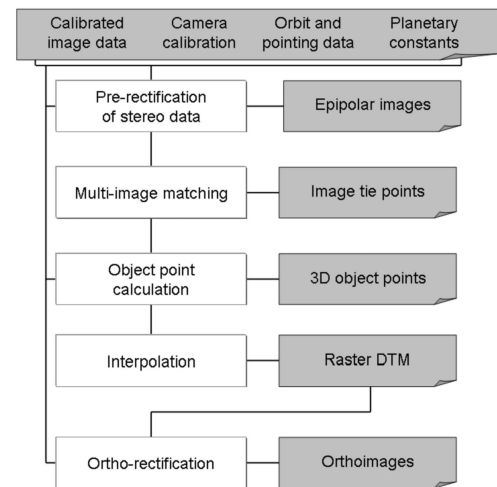


Figure 2.7: Processing flow for DTM generation from stereo data [20].

In addition, stereophotogrammetric methods typically struggle to reconstruct textureless areas and can only generate sparse surface models [104].

SPG has been successfully applied in several missions over the past decades, gaining considerable experience related to the photogrammetric processing of planetary data.

Examples include the mapping of Mars with Mars Express [105], the Moon with the Lunar Reconnaissance Orbiter [106, 107], Mercury with MESSENGER [108], and icy moons with Cassini and Galileo [109]. More recently, the Dawn mission used SPG extensively to generate global topographic models of Vesta and Ceres, enabling both scientific investigations and cartographic mapping [20]. The technique will be employed also in the topography and geomorphology investigation of the asteroid Psyche to derive its detailed shape [110].

The stereophotogrammetric method, apart from the generic requirement of overlapping images from different observation directions, does not depend on general imaging geometry. However, the quality and quantity of the DTM results are significantly affected by the viewing and illumination conditions [20]. Variable illumination within stereo datasets has a strong influence on the accuracy and robustness of the area-based image-matching techniques, while small stereo viewing angles reduce the quality of the finally reconstructed stereo image geometry [111].

To achieve the best possible quality (accuracy) and quantity (coverage) of the desired 3D data products, it is of central importance to consider these requirements during image-sequence planning under the given mission conditions (specific pointing, data volume, downlink capacities, etc.). Based upon substantial experience from stereo processing of other planetary image data, observation and illumination requirements for stereophotogrammetric processing can be defined for the specific mission in order to maximize the output quality. Examples of requirements for an optimal observing geometry are reported in Table 2.1.

Mission:	Rosetta [111]	Dawn [20]	Psyche [110]
Illumination Variation	0°–10°	0°–10°	<10°
Stereo angle	15°–45°	15°–55°	15°–65°
Incidence angle	5°–55°	5°–60°	10°–60°
Emission angle	0°–55°	0°–70°	< 55°
Phase angle	5°–80°	>2°	5°–90°

Table 2.1: SPG illumination and observation requirements for optimal imaging.

2.2.3. Stereophotoclinometry (SPC)

Stereophotoclinometry (SPC) is the most commonly used technique for the 3D shape reconstruction of small bodies using images acquired from spacecraft. It has been systematically applied to every small body mission since NEAR-Shoemaker, often combined with SPG and LiDAR for refining and validating the final models [7]. SPG still dominates the shape modeling of large planetary bodies with extensive imaging campaigns, because global geodetic consistency is crucial and stereo coverage is achievable. However, in the context of small irregular asteroids SPC is more robust to the imaging conditions, which typically involve limited stereo coverage and varying illumination on the rough surfaces.

SPC was developed by Robert Gaskell at the Planetary Science Institute (PSI) in support of optical navigation efforts for small body missions [112]. The software, written in Fortran, has evolved multiple times during the years to support accurate optical navigation and high-resolution DTM generation [7]. The technique is a fusion of stereophotogrammetry and photoclinometry, leveraging the strengths and mitigating the weaknesses of each. The stereo component provides absolute 3D position, anchoring the reconstructed topography in three-dimensional space, while photoclinometry generates a high-resolution topography through pixel-to-pixel slope estimation [113].

The method works by dividing the surface into small overlapping maplets, which are square patches of terrain defined around central landmarks. Each maplet is a local DTM containing height and albedo information [112]. These maplets can be used to identify the surface patches and locate their center in any image, under any illumination or viewing geometry [20]. For all the landmarks recognized in the images, the maplets are orthorectified as they are seen from above. In an iterative process, the slope and albedo at each maplet's pixel is determined by minimizing in the least squares sense the brightness residuals between the orthorectified images and the reilluminated maplet. For every pixel, SPC solves a system of three equations, corresponding to the two orthogonal surface slopes and albedo. This last component allows to render synthetic images that aid direct comparisons to the original input images [7]. The topographic reliefs are adjusted until the rendered brightness patterns match the images. The derived slopes are then integrated to produce a DTM. Maplet data can also

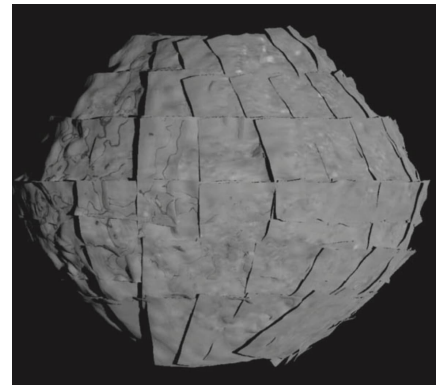


Figure 2.8: Initial stage of SPC model before topographic integration [21].

be combined to generate global or regional models.

A key advantage of SPC lies in its ability to integrate data from a wide variety of imaging conditions, gradually improving model accuracy as more images are incorporated.

The process begins with an a priori shape, often a coarse radar-based or ellipsoidal model, which is progressively refined by fitting maplets across successive iterations. Through this iterative approach, SPC is capable of producing highly detailed digital terrain models (DTMs), with spatial resolutions intimately tied to the resolution of input images and to the diversity of illumination and viewing geometries.

The extensive pre-launch test campaigns for OSIRIS-REx mission, detailed in [22, 114], demonstrated that SPC's final model quality typically matches the smallest pixel scale of the source images, provided that optimal imaging conditions are met. This SPC testing also highlighted that polar latitudes are represented less accurately due to poor illumination, and steep slopes tend to be underrepresented because of orthorectification errors [22]. Furthermore, being SPC based on photometric modeling, assumptions about surface reflectance properties can introduce systematic errors.

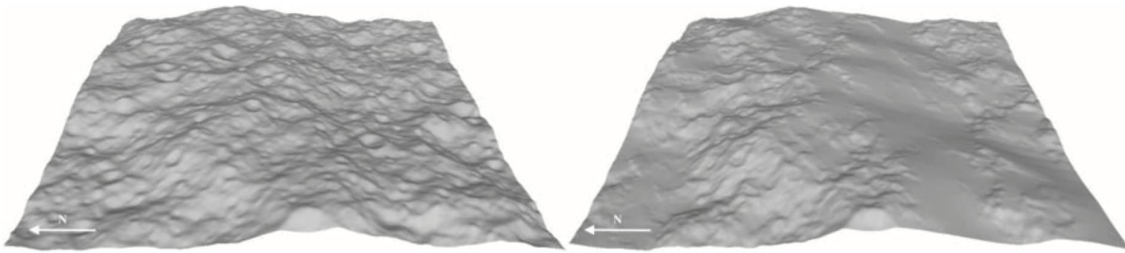


Figure 2.9: Reconstructed DTM using equatorial (left) and polar (right) images [22].

SPC has been successfully applied in several space missions dedicated to small-body exploration, including NEAR-Shoemaker [112], Hayabusa [115, 116], Dawn [20], Rosetta [117, 118] and OSIRIS-REx [119, 120]. A complete list is provided in [7]. The OSIRIS-REx mission provided the most comprehensive demonstration of SPC, employing it both for scientific investigations and for operational navigation in close proximity of Bennu. Two important SPC models were obtained [21]: the 75cm-GSD DTM (SPC v20), to support the transition from center finding to landmark-based navigation, and the 35cm-GSD DTM (SPC v42), to support global imaging, proximity operations and sample site selection. These models were both satisfying the mission's accuracy requirements, and were generated from images with resolution respectively of 12 cm/pixel and 5 cm/pixel.

2.2.4. Laser Altimetry (LiDAR)

Laser altimeters have been employed in four different missions to asteroids: NEAR-Shoemaker Laser Rangefinder (NLR) at Eros, the LIDAR of Hayabusa and Hayabusa2 missions at Itokawa and Ryugu, and the OSIRIS-REx Laser Altimeter (OLA) at Bennu. However, these instruments were designed with different objectives.

NLR's direct laser ranging provided millions of along-track measurements of the distance between the spacecraft and the surface of Eros, which contributed to the asteroid's characterization and estimation of properties as shape, volume, gravity, rotational state and surface roughness [121, 122].

The main purpose of the LIDAR mounted on both the Hayabusa missions instead was to aid the navigation around the asteroid during rendezvous, surveying and sample collection phases of the mission [123]. In addition, the data collected by the instrument, combined with imaging, were also used for scientific applications such as global topographic mapping, gravity estimation, and reflectance distribution observation [124]. In particular, the LIDAR data were used for improving the estimates of the asteroid shape and volume, but the failure of two of the three reaction wheels on Hayabusa dramatically reduced the hit ratio, making a systematic surface survey difficult [125].

The only mission that adopted laser altimetry as a specifically designed strategy for global shape reconstruction was OSIRIS-REx at Bennu. OLA was the world's first scanning laser rangefinder to fly on a planetary mission [126]. It differed from topographical LiDAR instruments used on previous asteroid missions for its improved ranging performance, the incorporation of a flexible scanning capability, and a much higher measurement rate. OLA operates at measurement rates of 100 Hz at ranges above 1 km and 10 kHz at lower ranges [127]. It generates a 3D point cloud of measurements, from which is derived the shape model. The initial estimate of each point's location, obtained using the calibrated instrument range and pointing, includes pointing uncertainties and spacecraft-position errors. These are corrected with a strip-adjustment process, which exploits a keypoint-matching of topographic features, to better align the scans and build a final shape model.

During the Orbital B mission phase, nearly 5 weeks of operations were dedicated to the global mapping of Bennu with OLA, obtaining a global coverage by creating overlapping high-density point clouds of the asteroid surface. From this dataset was generated the detailed 3D shape model OLA v21, which was used for comparisons with the final SPC v42 DTM, and for the creation of high-resolution digital elevation maps of areas of interest, as navigational landmarks and candidate sample sites [23, 128]. However, the OLA models were never used during operations for navigation.

The use of the laser altimetry model for the refinement of the main SPC model of Bennu was fundamental for demonstrating the high quality of the shape reconstruction obtained from the OSIRIS-REx observations, approaching centimeter scales during the sampling phase [129]. However, it is worth to consider that the instrument mass, power, and pointing budgets can strongly constrain the density of returns, representing important limitations for the use of laser altimetry in asteroid missions.

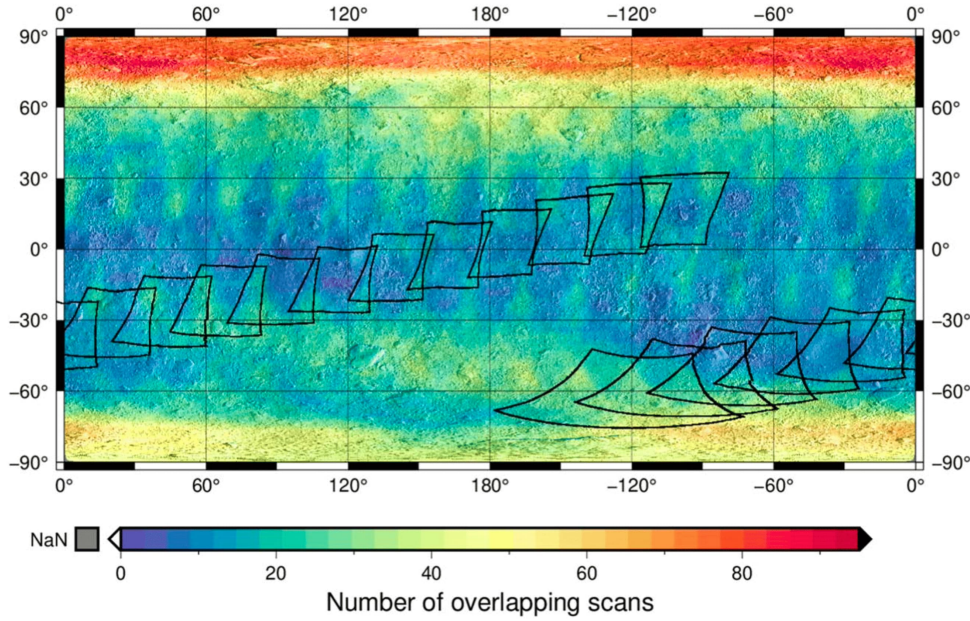


Figure 2.10: OLA scan coverage on the surface of Bennu during the Orbital B phase [23].

2.2.5. Photoclinometry by deformation (PCD)

Surface reconstruction from spacecraft imaging has traditionally relied on SPG and SPC. However, both methods have some limitations: SPG derives topography with a resolution comparable to the size of the patches used to build the stereo control points (5–10 times the pixel scale), while SPC often tends to smooth out small-scale features, interpreted as variations of the local reflectance properties when illumination and viewing conditions during imaging are unfavorable [24]. In addition, both methods require images in which the object spans at least ~ 100 pixels in diameter in order to reconstruct its 3D shape. Therefore SPG and SPC are often used in combination to improve the accuracy of the reconstructed models.

To overcome the intrinsic limitations of these techniques, photoclinometry by deformation (PCD) was introduced as an extension of SPC [20]. PCD operates by progressively refining an initial shape model, obtained from SfS, SPG, SPC, or as simple geometric surface

(sphere or ellipsoid). It also requires as input the pole orientation, photometric parameters, and spacecraft and camera position and orientation, derived from the navigation data [20].

The method is based on the deformation of a 3D triangular mesh, achieved through a non-linear optimization process in which the free parameters are the vertex displacements along their local normals. Synthetic images are generated from the 3D mesh, and the optimization minimizes a penalty function defined as the chi-square difference between observed and synthetic images [24]. PCD uses the quasi-Newton non-linear optimization algorithm called Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS).

The deformation is carried out within a multiresolution framework, where the mesh resolution and the image pixel scale evolve in parallel [130]. Furthermore, a smoothness constraint is imposed to avoid spurious artifacts, particularly near silhouettes. The optimization is firstly performed at coarse resolution, to reduce the risk of converging to local minima, and is then iteratively propagated to finer resolutions using a multigrid-like scheme. This strategy ensures that global shape features are well constrained before introducing higher-frequency details.

Iterative application allows PCD to enhance the accuracy of DTMs derived from SPG or SPC. This approach was applied to reconstruct the shape of asteroid (21) Lutetia [24, 130]. Compared to SPG and SPC, the MPCD method allows to model a larger portion of the surface and provides improved reproduction in images of both the object’s limb and the topography near shadowed regions. Craters and grooves are also reconstructed with a more accurate depth, as shown in Figure 2.11.

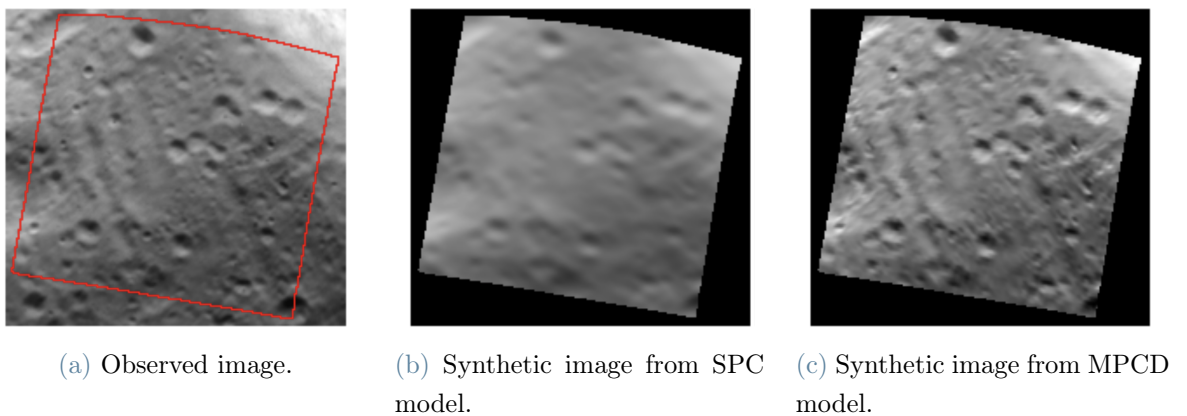


Figure 2.11: Comparison of SPC and MPCD shape reconstruction of (21) Lutetia [24].

3 | SPC Shape Reconstruction

This chapter describes the various steps required for the implementation and validation of the stereophotoclinometric algorithm for shape reconstruction of Dimorphos. In particular, each section presents one of the core components of SPC.

Firstly is presented the image generation tool, a custom-made raytracer which is fundamental for the simulation of the images that will be acquired by Milani's NavCam.

Then, in Section 3.2 is provided an overview of the fundamentals of the stereophotoclinometric procedure for asteroid shape reconstruction. The maplet-based algorithm implementation is described in its main steps, highlighting the pre-existing functionalities and the updated features.

Section 3.3 is dedicated to the detailed description of the implementation of another SPC core component, which allows to estimate and refine the landmark position vectors used to obtain the topography. Moreover, in Section 3.4 is also introduced a third module of SPC for the correction of spacecraft's position and orientation. The addition of these functionalities is crucial for developing an integrated shape estimation SPC architecture, capable of reducing the estimation errors and uncertainties.

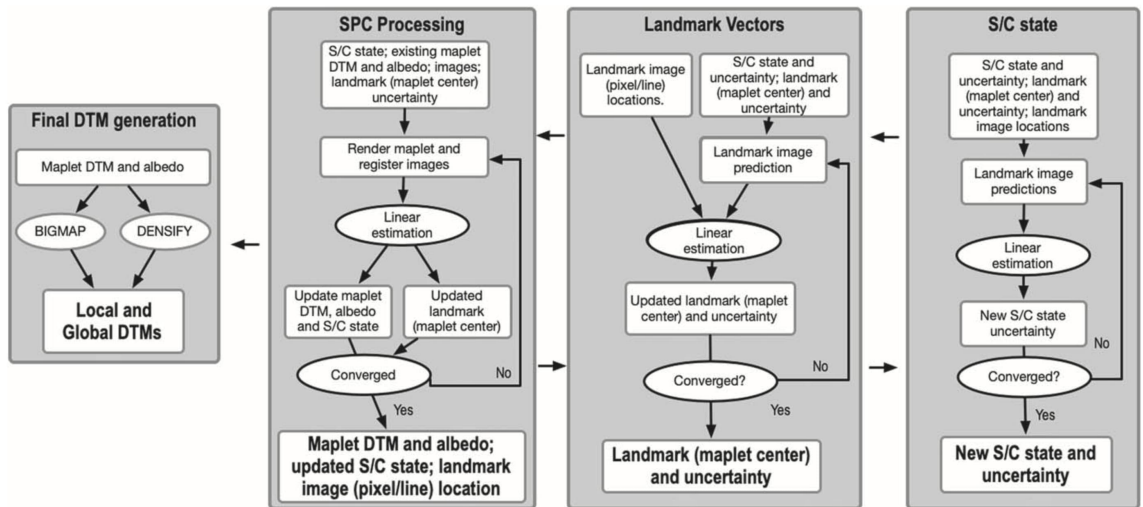


Figure 3.1: Three main loops of SPC algorithm, with inputs and outputs of each [25].

Finally, in Section 3.5 are presented the first steps of the pipeline for the optimization of the global model of the asteroid, starting from the maplet level data previously obtained. A scheme of the core modules of SPC is also provided in Figure 3.1.

3.1. Image Generation

In the following section is introduced the synthetic image generation tool employed for this thesis. The imaging scenarios' simulation is the first fundamental step for testing optical navigation methods and, in this case, for evaluating the performances of SPC for Dimorphos' 3D shape reconstruction with Milani's NavCam images. The development and validation of the raytracer architecture were detailed in [131]. The main characteristics and functionalities of the image generation technique are outlined, without delving into low-level implementation details, as the tool is not a direct contribution of this work.

3.1.1. Generation of the synthetic shape model

The shape models of Didymos and Dimorphos used in the image generation algorithm are described by *.obj* files, which provide a list of the vertices and triangular facets representing the shape of each body. These models were previously produced through the use of SPC, exploiting the DRACO imagery in DART mission. The *.obj* files are included in the SPICE kernel set for Hera mission [26].

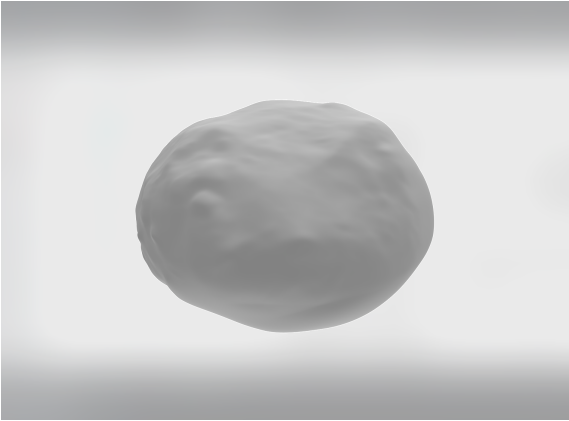


Figure 3.2: Didymos 3D model [26].

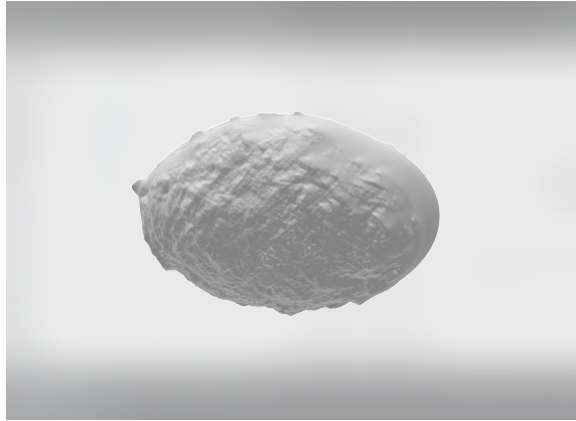


Figure 3.3: Dimorphos 3D model [26].

Due to the limited range of viewing geometries during the high-speed DART approach to Dimorphos, accurate shape reconstruction was possible only for a portion of the surface. As shown in Figure 3.3, high-resolution topography is resolved on the impact hemisphere imaged by DRACO, whereas the opposite side remains smooth due to the lack of imaging.

Being SPC based on photoclinometry, it requires a sufficient degree of topographic details spread across the entire model and generating shadowing effects. Therefore, in order to make the model suitable for SPC-based reconstruction across its full surface, synthetic topographic features were added to the unobserved hemisphere using Blender.

A procedural noise function was applied to the mesh vertices with a Hybrid Multifractal pattern, an advanced type of fractal noise, often used in terrain or natural surface modeling. By combining multiple noise functions, each associated with a frequency (octave), this pattern introduces small-scale roughness in a realistic manner, ensuring that shadowing effects would be present across the entire body. The resulting model of Dimorphos maintains fidelity to the observed regions while providing sufficient detail for shape reconstruction.

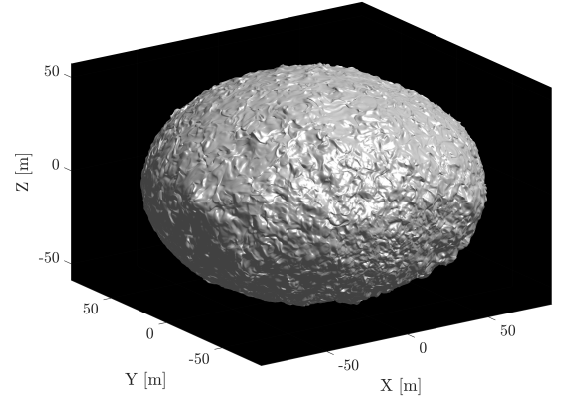


Figure 3.4: Dimorphos 3D model with simulated surface features.

3.1.2. Setup of camera scene

To generate realistic images, the raytracer requires precise information on the geometry of the observation scene, including Milani spacecraft's position and attitude, the relative motion of Didymos and Dimorphos, and the chosen reference frame. The Python code *spicedata.py* was used to extract the required data from the Hera mission SPICE kernels [26] and rearrange them in the proper *.yaml* format, needed as input for the raytracer main component *rt.cpp*, implemented in C++. This code sets as input the acquisition period and timestep, the target body of the observations and the reference frames.

The ECLIPJ2000 inertial frame centered at Dimorphos' barycenter is used as the global reference frame for the image acquisition. The spacecraft is approximated as a point, corresponding to the camera. The NavCam's boresight direction (Z-axis) is always taken as pointing at Dimorphos' barycenter, in order to place the moonlet at the center of the scene. Therefore, MILANI reference frame is defined as reported in Equations (3.1), where $\mathbf{r}_{S/C}$ is Milani's position vector in body-centered ECLIPJ2000.

$$\begin{cases} \mathbf{C}_x = [0; 0; -1] \times \mathbf{C}_z & (3.1a) \\ \mathbf{C}_y = \mathbf{C}_z \times \mathbf{C}_x & (3.1b) \\ \mathbf{C}_z = -\frac{\mathbf{r}_{S/C}}{\|\mathbf{r}_{S/C}\|} & (3.1c) \end{cases}$$

Didymos' position is extracted from SPICE in the ECLIPJ2000 Dimorphos-centered frame, previously defined. This representation of its orbit relative to Dimorphos allows the rendering process to focus on the moonlet while maintaining correct binary dynamics. In addition, the rotational states of the bodies are provided in the output *.yaml* file as quaternions at each time step, representing the rotations from the body-fixed frames (DIDYMOS_FIXED and DIMORPHOS_FIXED) to the ECLIPJ2000 reference frame.

It is noteworthy that the Z-axes of the body-fixed frames are inverted with respect to the global reference frame, because the spin axes of both Didymos and Dimorphos are oriented near the ecliptic south direction, as highlighted in Section 1.3.1. The position of the Sun is also obtained in terms of azimuth, elevation and distance from the scene origin, expressed in Astronomical Units.

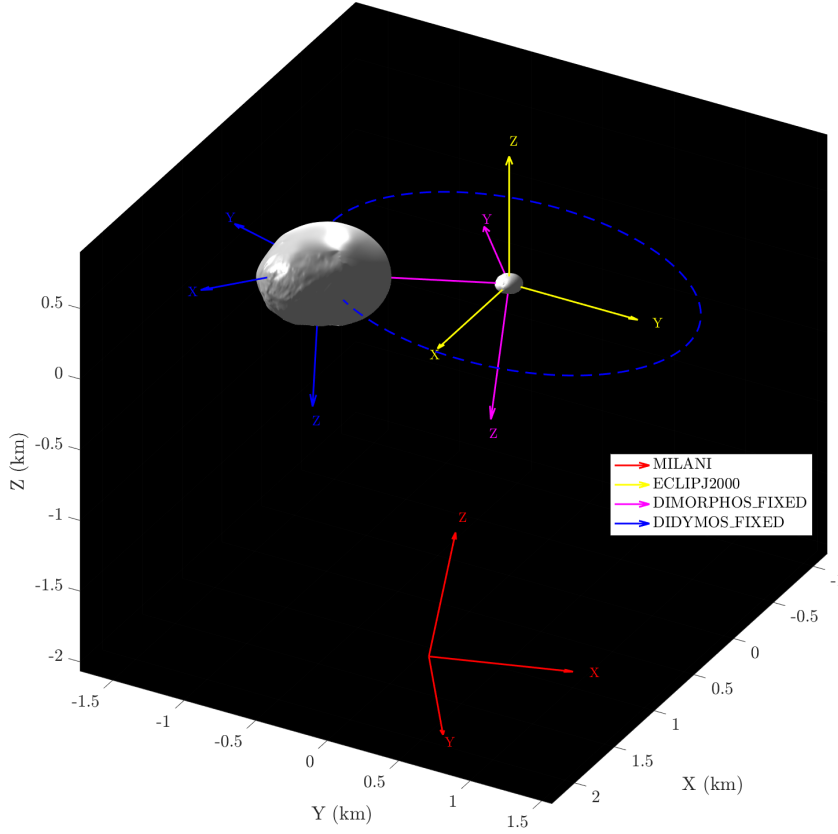


Figure 3.5: Acquisition scene and reference frames considered.

Finally, all the extracted quantities are stored in the output *.yaml* file. The NavCam parameters needed for its modelization, reported in Table 1.4, are also specified, together with the reflectance properties of the bodies. An exposure time of 5 *ms* was determined to yield the best image quality, preventing excessive brightness or darkness. Moreover, at this exposure, Milani's motion is sufficiently small to limit motion blur.

The Hapke model, reported in Equation (3.2), was chosen as Bidirectional Reflectance Distribution Function (BRDF) for the bodies, being widely used in planetary science for realistic asteroid's regolith image simulation [31, 132]. The μ and μ_0 parameters are the cosines of incidence and emission angles, while α is the phase angle. The functions $B(\alpha)$, $p(\alpha)$, and $H(\mu)$ represent, respectively, the shadow-hiding opposition effect, the single-scattering phase function, and the multiple-scattering function. They are governed by the parameters reported in Table 3.1, with typical values for S-type asteroids like Didymos and Dimorphos.

Hapke parameter	Value
B_{so}	1.6
h_s	0.08
w	0.23
g	-0.27

Table 3.1: Hapke values for S-type asteroids [31].

$$r(\alpha) = \frac{w}{4} \frac{\mu_0}{\mu_0 + \mu} \{ [1 + B(\alpha)] p(\alpha) + H(\mu_0) H(\mu) - 1 \} \quad (3.2)$$

$$B(\alpha) = \frac{B_{so}}{1 + \frac{1}{h_s} \tan\left(\frac{\alpha}{2}\right)} \quad (3.3)$$

$$p(\alpha) = \frac{1 - g^2}{(1 + 2g \cos \alpha + g^2)^{\frac{3}{2}}} \quad (3.4)$$

$$H(x) = \frac{1 + 2x}{1 + 2\sqrt{1 - \omega x}} \quad (3.5)$$

3.1.3. Raytracer architecture

The core rendering engine, implemented in C++, is based on a path-tracing algorithm that casts rays from the virtual camera through the scene and evaluates their interactions with the asteroid surfaces. In its original version, provided by DART Lab, the tool included standard features such as pinhole camera modeling, a Bounding-Volume Hierarchy (BVH) for efficient intersection tests in the raytracing process, and pixel over-sampling for image quality improvement.

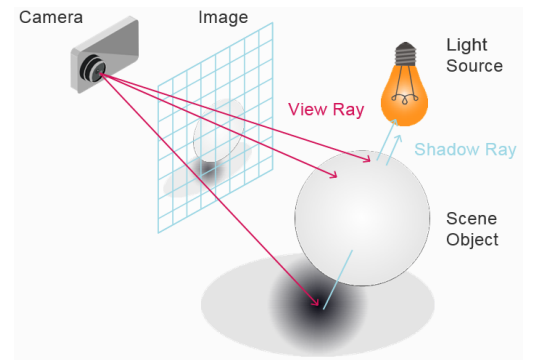


Figure 3.6: Raytracing principle.

The raytracing procedure contained in *rt.cpp* casts a ray for each image pixel and detects the intersections with the object. If a hit occurs, a shadow ray is traced from the impact point towards the Sun. The pixel's brightness is then computed using the BRDF model with the angles between observer, object and Sun, and the final 8-bit grayscale image is saved as a PNG file. Among the BRDF models integrated in the raytracer, also including Lambertian, McEwen, and Oren-Nayar, the Hapke model was chosen for rendering the images, as previously set for the body parameters in Section 3.1.2.

Several extensions were later introduced to better represent Milani's NavCam observations. The raytracer was enhanced to handle multiple bodies, allowing Didymos and Dimorphos to be rendered simultaneously, each with its own rotation and orbital state. This was done creating a list to store the BVH of all the n-bodies present in the scene, and rotating and translating the triangles composing the 3D models according to the body movement and rotation. Therefore, a new BVH is built for each image.

To improve shadow realism and surface brightness, the raytracer incorporates also a recursive multibounce raytracing, capturing multiple reflections of the light rays between the two bodies before reaching the camera. At each bounce on the surface of the asteroids the light is in part reflected and absorbed, according to the BRDF model.

Additionally, the raytracer was further extended to reproduce Milani's NavCam multispectral observations, accounting for the camera's Bayer filter. This allows the generation of raw mosaicked images from sensor pixels reactive to different ranges of wavelengths (red, green, or blue). The intensity recorded by each pixel (represented by a Digital Number DN) is calculated from the BRDF-based brightness I , adjusted by a pixel-specific gain G , that depends on filter's equivalent quantum efficiency q_{eq} and Sun's spectrum K_p integrated over the NavCam's spectral range. This gain also incorporates optical parameters such as the F-number f , pixel area A_p , exposure time t_e , and camera-Sun distance R . The following equations are used to compute numerically the quantum efficiency for each filter, which is applied after the multibounce raytracing, ensuring that each pixel's brightness correctly represents the intensity of a specific color.

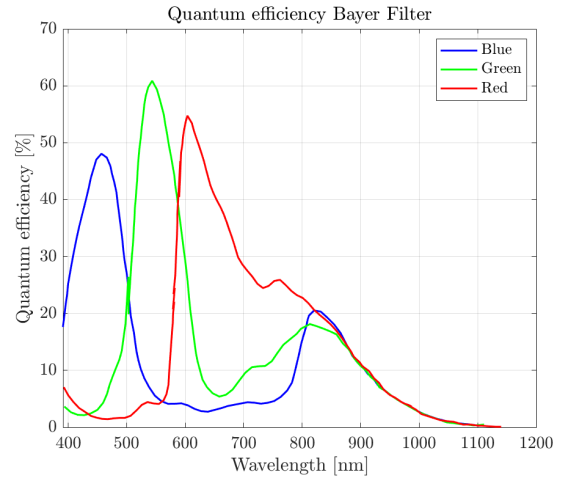


Figure 3.7: Quantum efficiency of the Bayer filter.

$$DN = I \cdot G \quad (3.6) \quad q_{eq} = \frac{\int_{\lambda_1}^{\lambda_2} q_e(\lambda) \frac{K_0}{\lambda^4 \left(e^{\frac{K_e}{\lambda}} - 1 \right)} d\lambda}{K_p} \quad (3.8)$$

$$G = \frac{K_p}{f^2 R^2} A_p t_e q_{eq} \quad (3.7) \quad K_0 = \frac{\pi^2 c R_{Sun}^2 10^{12}}{2 AU^2} \quad (3.9)$$

$$K_e = \frac{ch}{k_b T 10^{-9}} \quad (3.10)$$

In this formulation, λ denotes the wavelength, q_e the filter quantum efficiency, c the speed of light, h Planck's constant, k_b Boltzmann's constant, T the Sun's temperature (5800 K), R_{Sun} the solar radius, and AU the astronomical unit.

3.1.4. Demosaicing process

The multispectral rendering produces Bayer-patterned images, where each pixel encodes only one primary color. To obtain full-color output images, a demosaicing process is applied in MATLAB, exploiting the Image Processing Toolbox. Using a gradient-corrected linear interpolation [133], the *demosaicPhoto.m* algorithm reconstructs the missing color values by exploiting correlations among neighboring pixels. Since Milani's NavCam employs a GRBG filter layout, the reconstructed RGB images exhibit a slight predominance of green, consistent with the sensor's design. Finally the RGB images are converted to grayscale to facilitate the SPC processing, which relies on brightness and therefore requires a single intensity value rather than the three RGB channels.

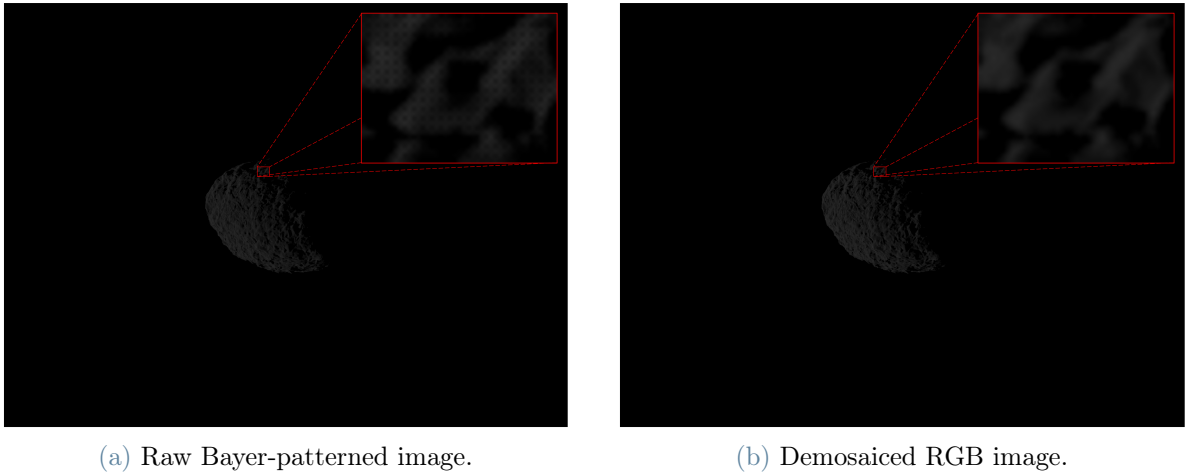


Figure 3.8: Image comparison before and after demosaicing process.

3.2. SPC topography reconstruction

The complete SPC architecture developed by R. W. Gaskell and the Planetary Science Institute comprises several modules, as described in [25] and shown in Figure 3.1. The three core components have the purpose of estimating respectively the surface topography and albedo, the positions of the maplet centers (landmarks), and the spacecraft pose at each image acquisition time. SPC solves for these quantities in three separate loops, which can be executed in parallel and with different frequencies. The first two loops, denominated as SPC-Processing and Landmark Vectors in Figure 3.1, can be run in a parallel process mode, while the third, referred as S/C state, is typically executed occasionally due to knowledge improvements in the spacecraft state or because new images are added to the SPC process during the course of the mission.

This section details the implementation of the core module of the SPC algorithm, for the estimation of the surface topography and albedo. The pre-existing functionalities, presented in [131], were further complemented and validated in several test cases, allowing to estimate the accuracy of the shape reconstruction of Dimorphos. A particular focus was given to the DART impact area on the asteroid, as it represents one of the fundamental objectives of the Milani mission.

The topography of the asteroid was firstly estimated under the assumptions of perfect knowledge of the landmark positions and the spacecraft pose, allowing to validate the implementation of the method. These hypotheses were later relaxed, for evaluating the impact of the respective errors in the shape reconstruction process. The influence of these parameters is explored in more detail in Section 3.3 and in Section 3.4, where the other two core components of SPC are introduced.

The SPC shape reconstruction process is initialized with a crude starting digital model of the asteroid, derived from radar imaging, light-curve analysis or observations of previous spacecraft missions [7]. A typical choice is a sphere or a triaxial ellipsoid. The SPC process employed in previous missions as OSIRIS-REx adopted a limb-based modeling during the approach phase, which allowed to refine the initial shape model before switching to a maplet-based modeling when the target becomes more resolved in the images [25]. Limbs are also used, together with data from other sources, to provide the fundamental constraining heights during the slope-to-height integration process [25]. Nevertheless, the shape reconstruction method developed in this thesis is focused only on the maplet-based modeling procedure, neglecting the limb-based modeling part. In the mission context, limb and LiDAR data are integrated also to solve the size/distance degeneracy.

The pole position and rotation rate are usually determined during approach or from previous ground-based observations. These parameters are taken as fixed during the SPC solution process, and updated only after convergence of several SPC solutions for different pole choices. In this thesis, they are taken as the actual ones, provided in the SPICE kernels of the mission.

The topography estimation procedure, schematically shown in Figure 3.10, begins with the generation of the landmarks on the surface of the asteroid, which are the central points of square maplets. Then the images obtained from the raytracing algorithm, demosaiced and converted in gray scale, are ingested and processed to identify the visible landmarks in the sunlit portion of Dimorphos. Photometric constraints are also imposed to the landmarks, in order to ensure optimal SPC reconstruction. The selected landmarks in each image are extracted with a rectification procedure. The rectified maplets are used to estimate the surface slopes and albedo. Finally, the terrain height can be obtained from integration of the slope field. This DTM generation process, called AES (Alignment, Extraction and Solve), is repeated iteratively until convergence is reached, reducing the mismatch between the observed and predicted brightness [7].

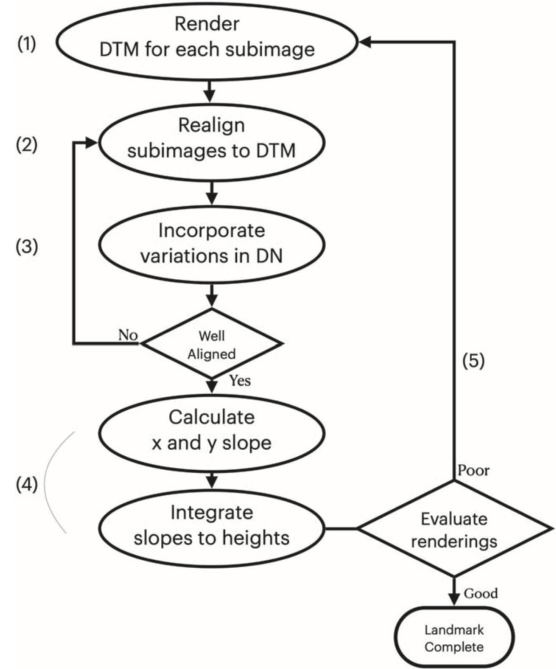


Figure 3.9: AES process [7].

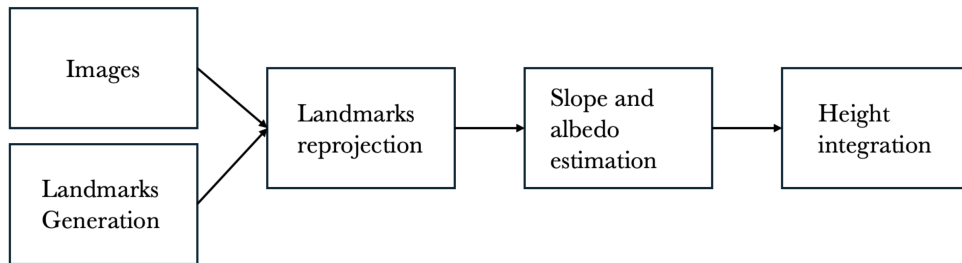


Figure 3.10: SPC topography reconstruction pipeline.

3.2.1. Landmarks generation

The landmarks are the central points of square grids of pixels, called maplets. They serve as reference points around which local maplets are constructed, each with its specific reference frame. They also represent control points, exploited for navigation purposes and to refine the estimate of the spacecraft's position, as better detailed in Section 3.3.

Landmarks are distributed across the entire surface of the asteroid to ensure adequate coverage of the target body. The SPC algorithm used in previous missions, as the latest OSIRIS-REx to the asteroid Bennu [119], adopted an approach that tiled the surface of the body with maplets of dimension 99×99 *pixels*, with a GSD ranging from 1.5 *m* in close-approach phase to 35 *cm* during scientific operations around the asteroid.

In the specific case of Milani mission to Didymos system, is important to consider several challenges and limitations to the application of SPC for shape reconstruction. Dimorphos represents an unprecedented scenario in terms of dimensions, as it exhibits a diameter of around 165 *m*, much smaller than the almost 500 *m* diameter of Bennu.

In addition, the NavCam of Milani was designed for navigation purposes rather than scientific observations, for which longer focal lengths are generally adopted because they yield a lower GSD and thus a better detail of surface features.

The GSD of the images acquired by the NavCam is relatively high, being Dimorphos completely contained in the FoV of the camera in all the mission scenarios. The GSD of the images represents an important parameter, as the SPC algorithm operates effectively when it is of the same order as the GSD of the maplet, ideally within a factor of three [59]. The standard 99×99 maplets of the standard SPC method developed by Gaskell are not compatible with the optimal maplet GSD, as the maplets would be covering excessively large terrain portions. For this reason, the maplet dimensions adopted in this thesis were significantly reduced, ranging from 19×19 to 29×29 *pixels*, to limit the effect of the global asteroid curvature into the height estimated in the single maplet.

Another fundamental SPC parameter is the number of landmarks. The definition of the amount of landmarks is coupled with the selection of the maplet dimension, as the combination of parameters needs to ensure a global coverage of the asteroid's surface. Tiling the body in its entirety with maplets allows to derive an extensive set of DTMs, which can be later combined in a regional or global model. Most importantly, a sufficient degree of overlapping between adjacent maplets is crucial for the imposition of the boundary conditions for the integration of the surface slopes into heights.

The distribution of landmarks over the asteroid surface can be achieved in different ways. In classical implementations, landmarks are uniformly positioned according to a latitude–longitude grid, ensuring regular coverage of the entire body. However, this approach tends to produce clustering near the poles due to the convergence of meridians. This effect is even enhanced in an ellipsoidal body as Dimorphos, resulting in excessive distribution of maplets at the poles but insufficient in equatorial regions.

Another way to distribute landmarks on the surface is through the Fibonacci lattice. This method, often applied in studies of satellite constellation coverage, relies on a generative spiral and provides an almost uniform distribution of points over a spherical surface [134]. The Fibonacci approach is particularly advantageous for reducing the number of landmarks required for achieving a sufficient coverage and overlap between maplets at the equator, lowering the computational demand.

The latitude and longitude coordinates of the landmarks are obtained with Algorithm 3.1, where $\phi = (1 + \sqrt{5})/2$ is the golden ratio and N is a natural number, which corresponds to a total amount of landmarks of $2N + 1$. The 3D positions $\mathbf{V}_i$ of the landmarks in the DIMORPHOS_FIXED frame are retrieved through the SPICE function *latsrf*, taking as input the corresponding geographic coordinates. The starting landmark positions can be either extracted from the real asteroid, model generated in Section 3.1.1, or from the approximated ellipsoidal model in the SPICE mission kernels, depending on the specific analysis and SPC loop.

Algorithm 3.1 Fibonacci lattice landmarks distribution

```

1: for  $i = -N, (-N + 1), \dots, (N - 1), N$  do
2:    $lat_i = \arcsin\left(\frac{2i}{2N + 1}\right) \times \frac{180^\circ}{\pi}$ 
3:    $lon_i = \text{mod}(i, \phi) \times \frac{360^\circ}{\phi}$ 
4:   if  $lon_i < -180^\circ$  then
5:      $lon_i = 360^\circ + lon_i$ 
6:   end if
7:   if  $lon_i > 180^\circ$  then
8:      $lon_i = lon_i - 360^\circ$ 
9:   end if
10:   $\mathbf{V}_i = \text{cspice\_latsrf}(lat_i, lon_i)$ 
11:   $\mathbf{U}_z = \text{cspice\_surfnm}(\mathbf{V}_i)$ 
12:   $\mathbf{U}_y = [0; 0; 1] \times \mathbf{U}_z$ 
13:   $\mathbf{U}_x = \mathbf{U}_y \times \mathbf{U}_z$ 
14: end for

```

Once the landmark positions are defined in body-fixed coordinates, they must be translated into inertial space for each observation epoch. This transformation relies on SPICE kernels, which provide the asteroid's rotation state and orientation relative to the inertial reference frame. The normal direction U_z of each maplet frame is obtained from the ellipsoidal model with *cspice_surfnm*. U_x and U_y are finally computed to complete the frame.

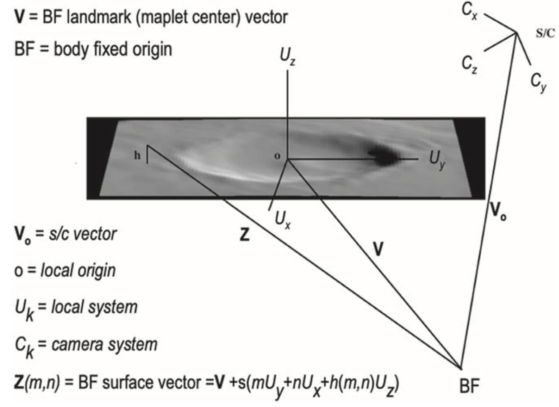


Figure 3.11: Local maplet frame [25].

In addition, another functionality was implemented to aid the research of overlapping regions between the maplets, detailed in Section 3.5. For each landmark is essential to identify which are the closest adjacent maplets, that might be subjected to overlapping with the considered one.

This is done with Algorithm 3.2, where for each landmark i is computed the angular separation $\theta_{i,j}$ with all the other landmarks j , defined as the angle formed at the asteroid's barycenter by a pair of landmark position vectors V_i and V_j . A landmark j is identified as adjacent if its angular separation $\theta_{i,j}$ is lower than a certain threshold θ_i^{lim} , which in terms of distance between landmarks on the surface corresponds to the diagonal of the maplet i . This ensures, with a consistent margin, that all the possibly overlapping maplets will be considered in the search for overlap regions, while giving an additional metrics for verifying that global coverage is achieved for the selected number of landmarks (n_{lmks}) and dimension of the maplets in pixels (*mapletDimension*).

In practice, this algorithm is implemented in matrix form, as MATLAB is optimized to perform vectorized and matrix-based computations more efficiently than iterative loops.

Algorithm 3.2 Adjacent landmarks search

```

1: for  $i = 1, \dots, n_{lmks}$  do
2:    $\theta_i^{lim} = \frac{\sqrt{2} \cdot mean(GSD) \cdot mapletDimension}{||V_i||}$ 
3:   for  $j = 1, \dots, n_{lmks}$  do
4:      $\theta_{i,j} = arccos\left(\frac{V_i}{||V_i||} \cdot \frac{V_j}{||V_j||}\right)$ 
5:     if  $\theta_{i,j} < \theta_i^{lim}$  then
6:        $j$  is adjacent maplet of  $i$ 
7:     end if
8:   end for
9: end for

```

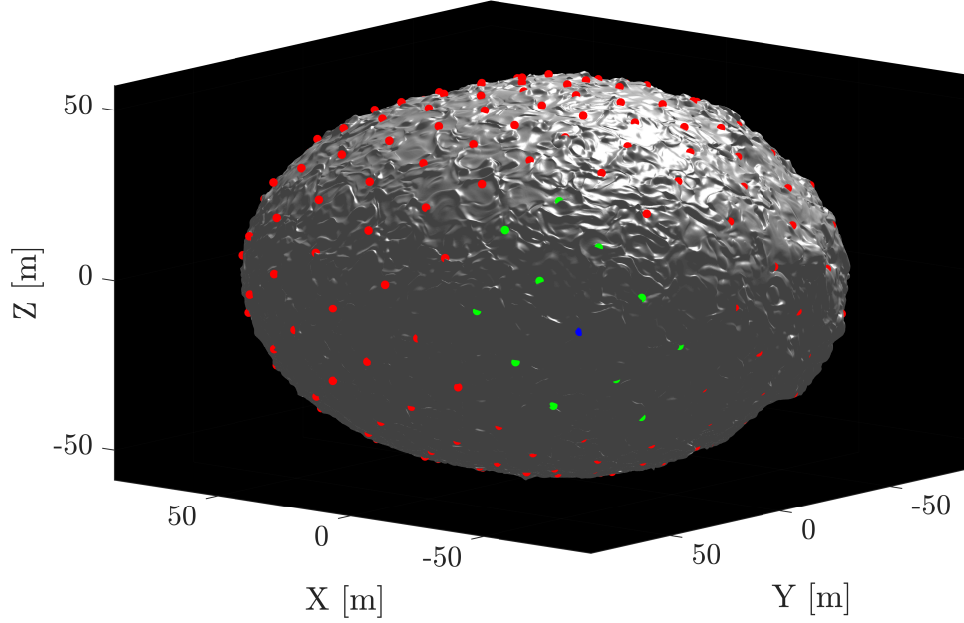


Figure 3.12: Adjacents of landmark 121 (CRP with 241 lmks, 29×29).

In the simulations, images are assumed to be captured at intervals of one hour. This acquisition frequency provides enough coverage of Dimorphos under different lighting conditions, while remaining compatible with Milani’s operational constraints. Consequently, the number of images per mission phase depends on their length, as seen in Table 3.2.

The GSD used for building the maplets in the different phases is the average value for each, except for CRP. During this phase, the image GSD fluctuates significantly between 2 m/pix and 0.25 m/pix , as shown in Figure 3.13, because the spacecraft remains mostly distant and performs brief close approaches to provide high-resolution views of the DART impact site. This challenging variability in the pixel scales of processed images is addressed selecting a GSD of 1 m/pix , slightly lower than the average value of 1.43 m/pix . Even if this choice excludes four images, not meeting the GSD requirement of falling within one-third to three times the maplet value [59], it represents a practical compromise between theoretical criteria and operational constraints.

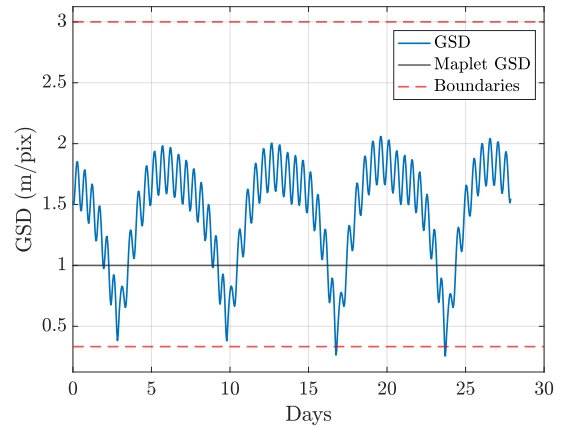


Figure 3.13: Image GSD during CRP.

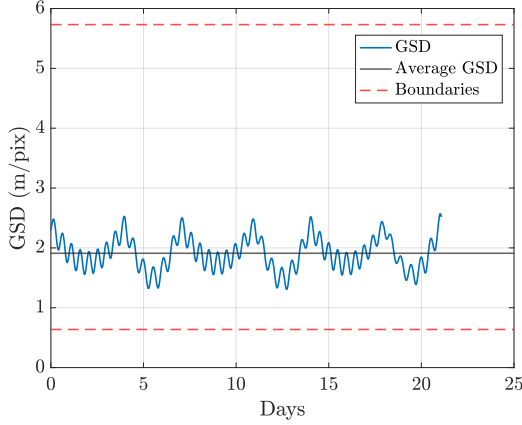


Figure 3.14: Image GSD during FRP.

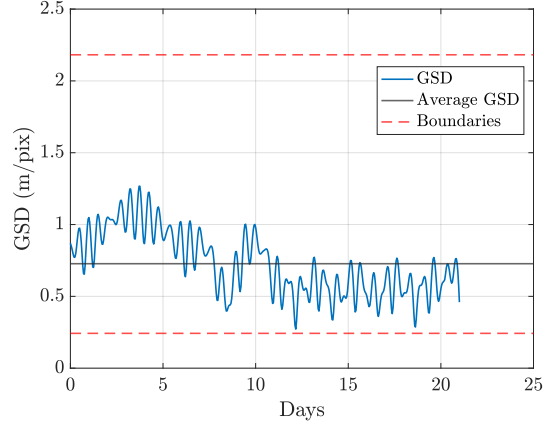


Figure 3.15: Image GSD during EXP.

Several combinations for number of landmarks and maplet dimension were explored. In Table 3.2 for each phase of the mission are reported three couples of values that guarantee complete coverage, as can be visually seen in Figure 3.16 and Figure 3.17. From the table can be noted that a reduction in GSD entails the need for a larger number of landmarks, while a decrease in the maplet dimension similarly results in more landmarks required.

Phase	Far Range	Close Range	Experimental
Start Date	2 APR 2027 16:00:00	27 APR 2027 21:00:00	28 MAY 2027 17:00:00
End Date	23 APR 2027 18:00:00	25 MAY 2027 18:00:00	18 JUN 2027 17:00:00
N. of images	507	670	505
Average GSD [<i>m</i>]	1.91	1.43 (1 for requirements)	0.73
Maplet dimension [<i>pixels</i>]	19 / 25 / 29	19 / 25 / 29	19 / 25 / 29
N. of landmarks	121 / 101 / 61	561 / 281 / 241 (GSD=1)	781 / 581 / 321

Table 3.2: Simulation scenarios parameters ensuring global coverage.

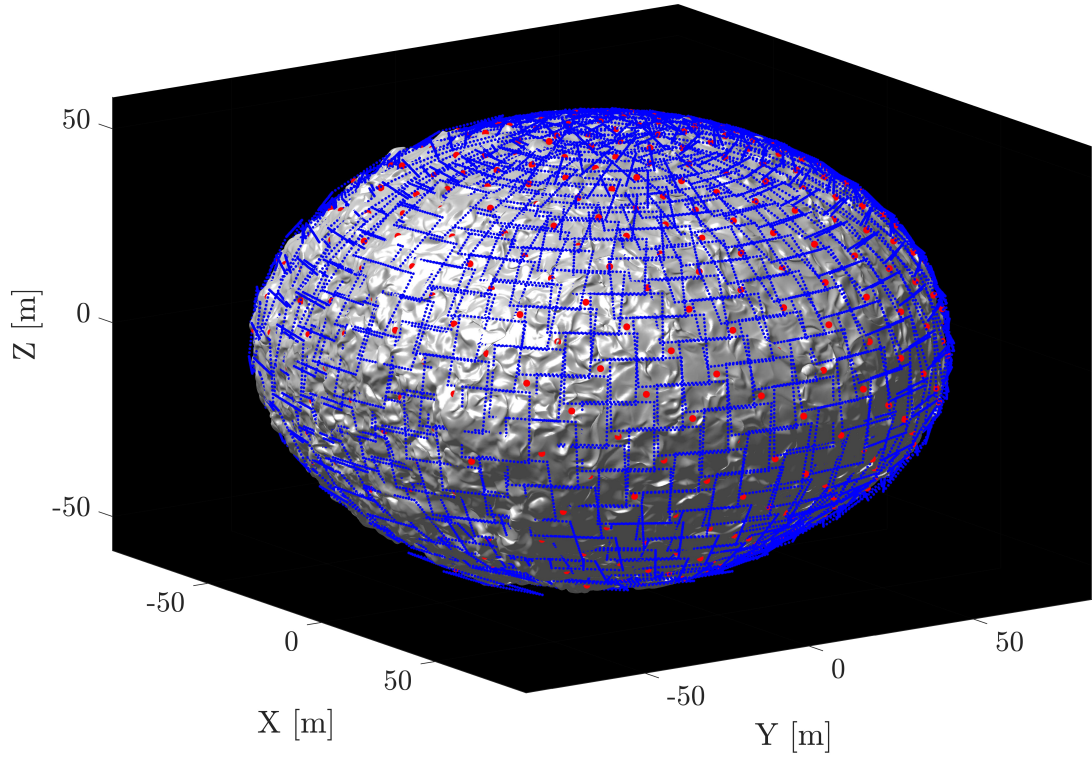


Figure 3.16: Global coverage during CRP (561 lms, 19×19).

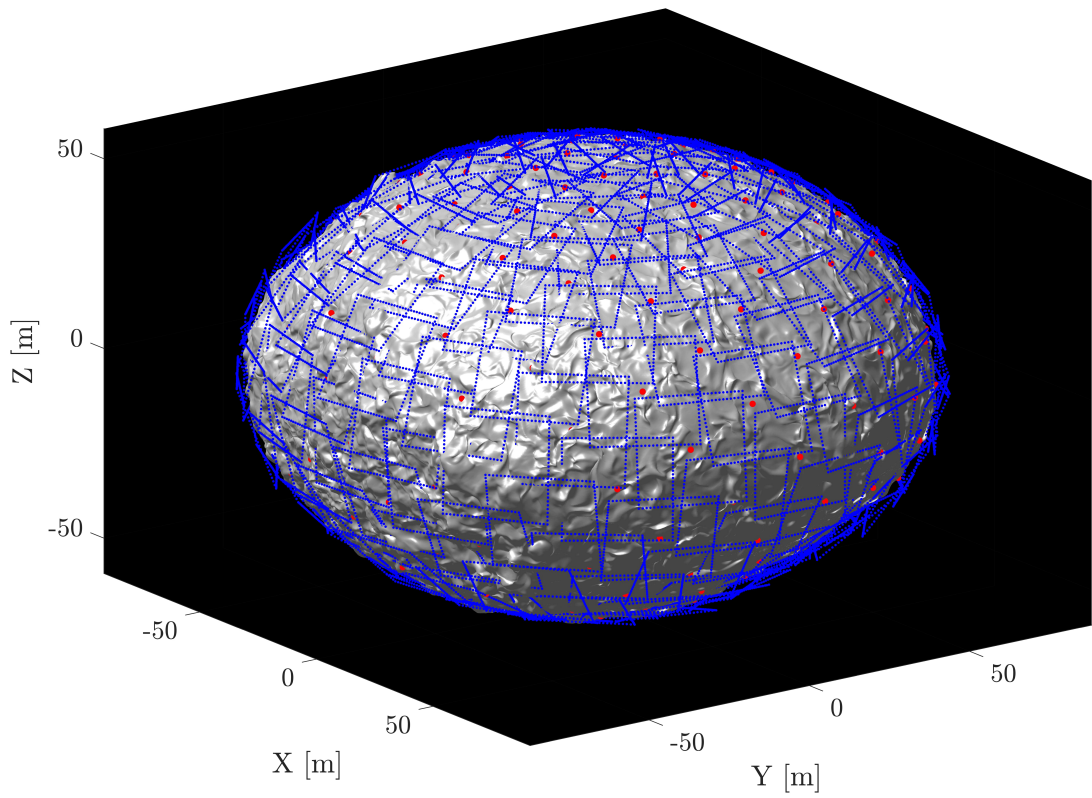


Figure 3.17: Global coverage during CRP (241 lms, 29×29).

For each maplet, a dedicated data structure is defined to store all the informations required for subsequent processing, together with the results that will be obtained in the following steps. Those include the geographic coordinates of the landmark, its 3D body-frame position, the base vector of the maplet's frame, the estimated slopes, albedo, heights, and the lists of images where the landmark is visible or observed under suitable photometric conditions. These last fields related to visibility will be filled in the following step.

3.2.2. Rectification process

After defining the landmarks and their maplets, they need to be identified in the images and properly rectified as they are seen from above, allowing to extract image information around them. This rectification process projects image pixels into the local coordinate frame of each maplet, creating a set of rectified subimages where the geometry of observation is normalized and comparable across different viewing conditions.

To identify which landmarks are visible in a given image, spacecraft's position and camera orientation are firstly extracted from SPICE kernels, reconstructing the acquisition scene. The pose of the spacecraft is initially taken as known. Later, a position and orientation Gaussian error will be added to the real values, simulating an inaccuracy in the initial knowledge of the acquisition geometry. This is essential in particular for validating the SPC component for spacecraft's pose estimation, detailed in Section 3.4.

The visibility check process begins by automatically discarding any images in which Didymos is completely or partially occulting Dimorphos. These are identified through the SPICE function *cspice_occult*. Then the angle γ between spacecraft position, landmark position and Dimorphos' baricenter is computed. A landmark is considered on the visible side of the asteroid if $\gamma > \pi/2$.

In order to exclude the landmarks that are on the visible side of Dimorphos but lying in a portion that is not directly exposed to the Sun, the incidence angle i is obtained through the SPICE function *cspice_ilumin*, and constrained to $i < \pi/2$ to be visible. The emission e and phase α angles are also retrieved. The limitation on i rules out the landmarks located in shadowed regions. However, the possible self-shadowing effects of large boulders are not considered, and the shadows from topography are assumed to be smaller than the maplet size.

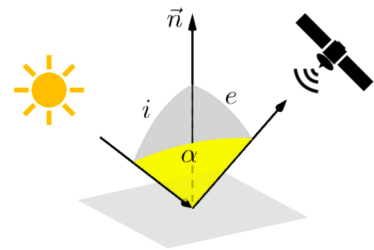


Figure 3.18: Incidence (i), emission (e) and phase (α) angles.

Finally, to filter out the landmarks lying within the shadow that Didymos might cast on Dimorphos, a ray is traced from the Sun to the target landmark on Dimorphos' surface, through the SPICE function *cspice_sincpt*. If the ray intersects Didymos before reaching the surface point, the landmark is classified as shadowed and disregarded.

Photometric angles	Ranges
i [deg]	5 – 65
e [deg]	5 – 65
α [deg]	20 – 165

Table 3.3: Photometric constraints.

Additional constraints are typically applied to the emission e and phase α angles. In particular, high emission angles are limited to prevent distortions during rectification, because for very oblique viewing geometries only a limited number of pixels contribute to the rectified image and interpolation dominates the result, significantly affecting its quality. Excessively low phase angles are also neglected, avoiding to include images lacking in shadows.

The photometric constraints, selected to obtain a good quality of the reprojected images while keeping a sufficient number of images assigned to each landmark, are reported in Table 3.3.

Similarly to landmarks, all the informations of the images are stored in proper data structures. Each image record contains the ephemeris time, positions of Milani, Didymos, and the Sun (including its azimuth and elevation), the camera frame's base vector, the surface point along the boresight, as well as the lists of visible landmarks and those observed under valid photometric conditions. All the parameters are expressed in the ECLIPJ2000 Dimorphos-centered reference frame.

After having filtered only the landmarks that can provide useful information for the shape reconstruction, the corresponding maplets can be properly rectified. Firstly, for each visible landmark identified in each image, its position has to be converted from the 3D coordinates $\mathbf{V}$ to the two-dimensional pixel coordinates (u, v) .

Under the assumption that Milani's NavCam operates as a pinhole camera, this can be done with the following equations, where u and v are respectively the column and row of the landmark's pixel location in the image. u_0 and v_0 are the pixel location of the principal point, equal to 1024 and 768 pixels. $\mathbf{r}_{Cam}$ is the position of the landmark in camera frame (MILANI), while $\mathbf{r}_{Body}$ is the vector from NavCam to landmark in DIMORPHOS_FIXED frame. The focal length f and pixel size p_{size} are the ones reported in Table 1.4.

$$\begin{cases} u = \frac{f}{p_{\text{size}}} \frac{x_{\text{Cam}}}{z_{\text{Cam}}} + u_0 \\ v = \frac{f}{p_{\text{size}}} \frac{y_{\text{Cam}}}{z_{\text{Cam}}} + v_0 \end{cases} \quad (3.11)$$

$$\begin{aligned} \mathbf{r}_{\text{Cam}} = \begin{bmatrix} x_{\text{Cam}} \\ y_{\text{Cam}} \\ z_{\text{Cam}} \end{bmatrix} &= \mathbf{R}_{\text{Body2Cam}} \cdot \mathbf{r}_{\text{Body}} \\ &= (\mathbf{R}_{\text{Inertial2Body}} \mathbf{R}_{\text{Inertial2Cam}})^T \cdot (\mathbf{V} - \mathbf{R}_{\text{Inertial2Body}} \mathbf{W}) \end{aligned} \quad (3.12)$$

As reported in Equation (3.12), a conversion is required from body-fixed to camera frame. The rotation matrix $\mathbf{R}_{\text{Inertial2Body}}$ is taken as fixed and extracted from SPICE kernels, as detailed in Section 3.2, while the vectors of the camera frame previously obtained from Equation (3.1) are used to build $\mathbf{R}_{\text{Inertial2Cam}} = [\mathbf{C}_x, \mathbf{C}_y, \mathbf{C}_z]$. The vector $\mathbf{W}$ represents Milani's position in the inertial Dimorphos-centered reference frame.

In a similar way, the image plane positions (u, v) of all the pixels belonging to the visible maplets can be retrieved with Equation (3.13). The additional terms in the numerators and denominators account for the displacement in the maplet plane relative to the landmark's position.

$$\begin{cases} u = \frac{f}{p_{\text{size}}} \frac{x_{\text{Cam}} + M_{11}x_{\text{map}} + M_{12}y_{\text{map}}}{z_{\text{Cam}} + M_{31}x_{\text{map}} + M_{32}y_{\text{map}}} + u_0 \\ v = \frac{f}{p_{\text{size}}} \frac{y_{\text{Cam}} + M_{21}x_{\text{map}} + M_{22}y_{\text{map}}}{z_{\text{Cam}} + M_{31}x_{\text{map}} + M_{32}y_{\text{map}}} + v_0 \end{cases} \quad (3.13)$$

$$\mathbf{M} = \mathbf{R}_{\text{Body2Cam}} \cdot [\mathbf{U}_x, \mathbf{U}_y] = (\mathbf{R}_{\text{Inertial2Body}} \mathbf{R}_{\text{Inertial2Cam}})^T \cdot [\mathbf{U}_x, \mathbf{U}_y] \quad (3.14)$$

The coefficients $M_{i,j}$ describe the projection of the camera axis i onto the maplet axis j , expressed in body frame. $\mathbf{M}$ can be derived by rotating the maplet frame into the camera frame, as in Equation (3.14). The maplet frame vectors $\mathbf{U}_x$ and $\mathbf{U}_y$ are obtained from Algorithm 3.1, as detailed in Section 3.2.1.

The axis $\mathbf{U}_z$ is omitted because the maplet is initialized as a flat surface with zero height. Therefore, z-axis rotation is unnecessary, as Equations (3.13) do not allow any out-of-plane displacement (z_{map}). x_{map} and y_{map} define the position of each maplet pixel, computed as the product of the pixel index and the maplet's GSD.

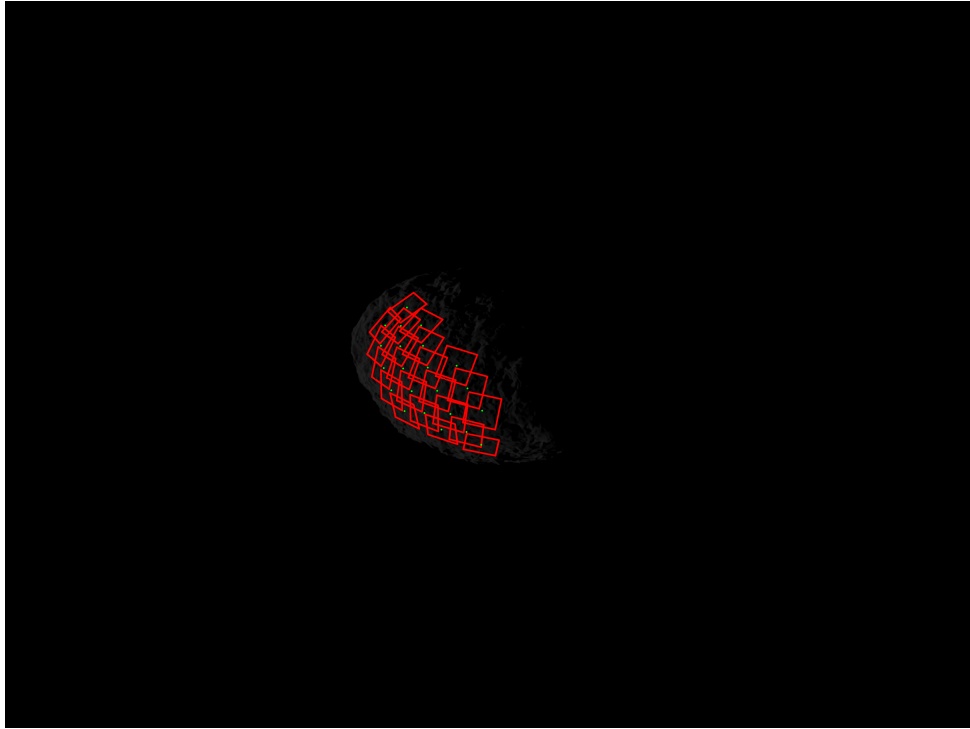


Figure 3.19: Visible maplets to be rectified (EXP, 321 landmarks, 29×29).

Once rectified the maplets on the image plane, the extraction process generates subimages around the landmarks in each picture by collecting the pixel brightness value within the maplet. This allows to produce subimages representing the rectified maplets as observed from nadir.

The same landmarks observed in different pictures might result in rectified maplets which appear darker or brighter, depending on the variety of illumination conditions. These multiple observations of the same features are essential for constraining both the topography and the reflectance properties of the maplet.

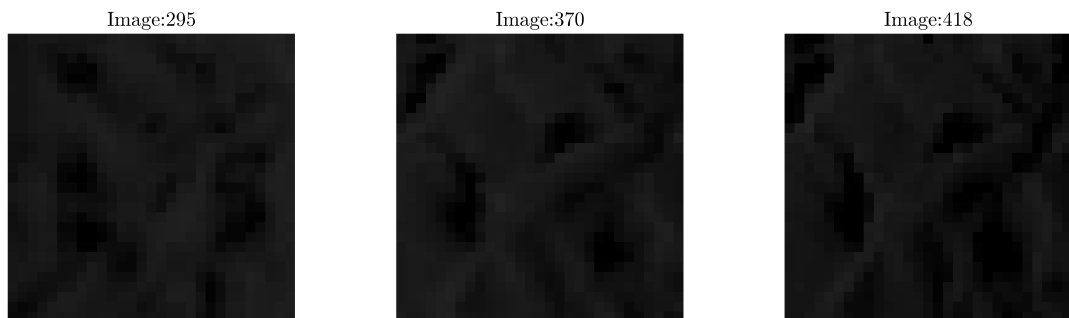


Figure 3.20: Landmark 173 reprojected in different images.

3.2.3. Slopes and albedo estimation

At the core of SPC's topographic solution lies the estimation of slopes and albedo. This step, along with all the other components of the SPC method, makes extensive use of linear estimation to determine the parameters of interest. The approach works under the fundamental assumption that from a starting point close enough to the solution there is a linear relationship between the observable O and the quantity $\mathbf{q}$ that we want to estimate [25]. The variable $\mathbf{q}$ can either be the maplet's DTM, as in this topography estimation component, but also the landmark vectors or the spacecraft position and orientation, as detailed in Section 3.3 and Section 3.4. A key feature of SPC is that the different quantities in $\mathbf{q}$ are solved independently in separate loops, while the formal uncertainties of one estimate are incorporated as weights in the estimation of the others [25]. This approach allows to break a big problem into many smaller ones.

The estimation process consists in determining a quantity q_i by comparing a predicted observable $P(\mathbf{q})$, computed from nominal q_i values, with the corresponding measured observable O . The q_i values are then adjusted by δq_i to minimize the sum of squared residuals, as expressed in Equation (3.15) [25]. σ is the uncertainty of each measurement.

$$\sum \frac{(O - P(\mathbf{q} + \delta \mathbf{q}))^2}{\sigma^2} \approx \sum \frac{\left(O - P(\mathbf{q}) - \delta q_i \frac{\partial P}{\partial q_i}\right)^2}{\sigma^2} \quad (3.15)$$

The sum in Equation (3.15) is over all of the observations. This corresponds to a sum over all the images of a visible landmark if solving for topography or landmark position, while when solving for spacecraft pose the sum is over all of the landmarks in a given image. The minimum of Equation (3.15) is found by setting to zero the derivatives with respect to δq_i , resulting in:

$$\sum \frac{\frac{\partial P}{\partial q_i} \left(O - P(\mathbf{q}) - \delta q_j \frac{\partial P}{\partial q_j}\right)}{\sigma^2} = 0 \quad (3.16)$$

$$\sum \frac{\frac{\partial P}{\partial q_i} (O - P(\mathbf{q}))}{\sigma^2} = \left[\sum \frac{\frac{\partial P}{\partial q_i} \frac{\partial P}{\partial q_j}}{\sigma^2} \right] \delta q_j \quad (3.17)$$

The bracketed term on the right-hand side corresponds to the symmetric information matrix M_{ij} , while the left-hand side is denoted as w_i .

By multiplying both sides of Equation (3.17) by the inverse covariance matrix M_{ki}^{-1} , is obtained the estimate for the corrections $\delta \mathbf{q}$ to $\mathbf{q}$.

$$\delta q_k = M_{ki}^{-1} w_i \quad (3.18)$$

In this thesis the slope estimation process, as well as the other linear estimation problems presented in the following SPC components, is implemented with a similar least squares approach. In the SPC component for topography, the objective function which needs to be minimized is obtained from the difference between the observed brightness I_o of the rectified maplet's pixels and the predicted brightness I_p .

$$\sum_k \frac{(I_o - I_p)^2}{\sigma^2} \quad (3.19)$$

The least-squares function is minimized for every pixel within each maplet, and the summation is carried out over all the k images containing that maplet. The predicted brightness I_p of a pixel is then computed with Equation (3.20).

$$I_p = \lambda_k (1 + t_z) \Phi(\cos(i), \cos(e)) \quad (3.20)$$

The function Φ represents the chosen reflectance law. A mixed Lambert–Lommel–Seeliger function [135], reported in Equation (3.21), was preferred to the more complex Hapke model, because its analytical gradient can be more easily computed, which is essential for the subsequent steps. In addition, within reasonable incidence and emission angles, the choice of Φ has little effect on the resulting maplet DTM [25]. This Φ was also used by Gaskell in [112].

$$\Phi(\cos(i), \cos(e)) = 0.35 \cos(i) + 0.65 \frac{\cos(i)}{\cos(i) + \cos(e)} \quad (3.21)$$

The coefficient λ_k in Equation (3.20) scales the reflectance into pixel brightness and is set to 255, corresponding to 8 bit images. The reflectance function Φ is a function of the slopes $t_x = -\frac{\delta h}{\delta x}$, $t_y = -\frac{\delta h}{\delta y}$. These are estimated, together with the term $(1 + t_z)$ denoting the albedo, by minimizing the residuals of the objective function (3.19).

Substituting the definition of the unit normal vector $\mathbf{N}$ of the maplet, given by Equation (3.22), into the definitions of the incidence and emission cosines, reported in Equations (3.23) and (3.24), is possible to explicit the dependence of Φ on the variables t_x and t_y .

In these equations, $\mathbf{C}$ and $\mathbf{S}$ are the camera and Sun directions. They are treated as constant across each maplet because of its limited terrain coverage.

$$\mathbf{N} = \frac{(t_x, t_y, 1)}{\sqrt{1 + t_x^2 + t_y^2}} \quad (3.22)$$

$$\cos(i) = \mathbf{N} \cdot \mathbf{S} = \frac{s_x t_x + s_y t_y + s_z}{\sqrt{1 + t_x^2 + t_y^2}} \quad (3.23)$$

$$\cos(e) = \mathbf{N} \cdot \mathbf{C} = \frac{c_x t_x + c_y t_y + c_z}{\sqrt{1 + t_x^2 + t_y^2}} \quad (3.24)$$

The terms s_i and c_i are obtained respectively from $\mathbf{S} \cdot \mathbf{U}_i$ and $\mathbf{C} \cdot \mathbf{U}_i$, with $i = x, y, z$.

Then, the least-squares function takes the form shown in Equation (3.25), which is minimized iteratively by solving for t_x , t_y , and t_z , starting from an initial flat surface with zero slope and albedo.

$$J = \sum_k (f_k(t_x, t_y, t_z))^2 = \sum_k \frac{(I_o - \lambda_k(1 + t_z)\Phi(t_x, t_y))^2}{\sigma^2} \quad (3.25)$$

The SPC topography algorithm processes each landmark by gathering all the rectified maplets extracted from the images in which it appears. The objective function J is minimized for each maplet pixel across the set of available images.

The least squares estimation is performed with MATLAB's least-squares solver *lsqnonlin*, using the Levenberg–Marquardt algorithm. The analytical gradient of the objective function J , reported in Appendix A, is also provided in the algorithm to aid convergence. This step yields two crucial outputs: a slope field, describing the gradient of the surface at each grid point, and an albedo map, capturing variations in surface reflectance.

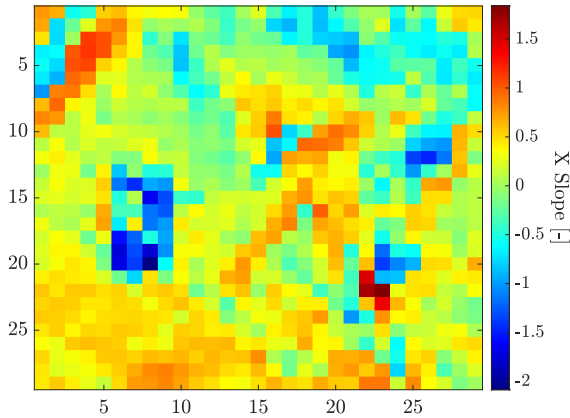


Figure 3.21: X slopes of landmark 173.

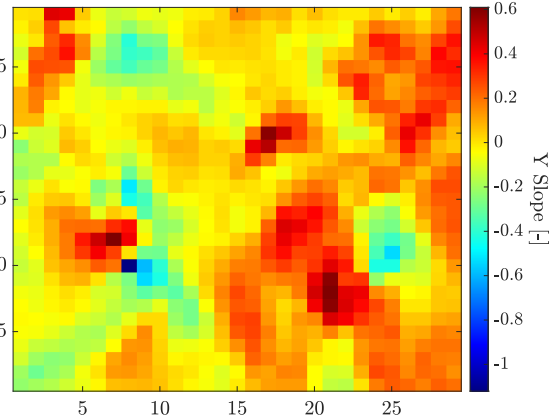


Figure 3.22: Y slopes of landmark 173.

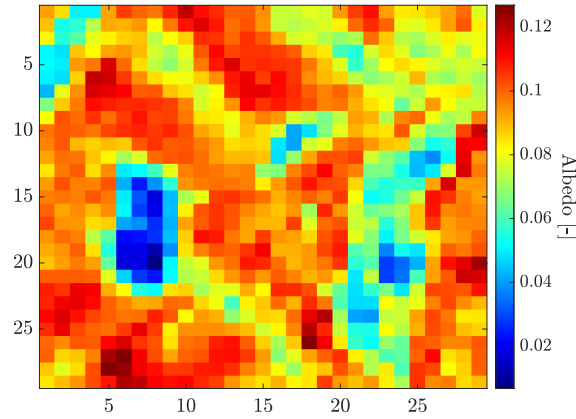


Figure 3.23: Albedo of landmark 173.

The redundancy of observations, with landmarks seen under different lighting conditions, ensures that slopes and albedo can be disentangled robustly. However, areas with poor coverage or extreme geometries may yield noisier estimates. The slope estimation process might be complicated for some pixels in shadowed areas [25], resulting in difficult convergence or unrealistic slope estimates. A proper normalization process is included to solve these problems before the integration.

3.2.4. Heights integration

The final step of SPC topography component is to reconstruct the actual surface heights from the estimated slopes. The estimated slope field defines partial derivatives of the surface height with respect to the maplet axes. Recovering the height values requires integrating these derivatives across the maplet. However, even if the slope solution reflects the brightness distribution in the images, it might not be necessarily consistent with an acceptable topography, which should satisfy the curl-free requirement $\partial_x t_y - \partial_y t_x = 0$ [112]. Therefore, direct integration might accumulate errors and produce inconsistencies.

In Gaskell’s SPC implementation, the relationship between heights at neighboring points and the average slope between them is used iteratively, in a relaxation procedure, to determine a height distribution for the maplet that is consistent with the slope solution [25]. These nearest-neighbor heights come from pixels located in a cross about the pixel considered. The approach is repeatedly applied to randomly chosen maplet points, until a converged height solution is reached.

This surface reconstruction procedure relies on the imposition of constraining heights at randomly selected positions, each with a proper small constraining weight. These heights can come from projected overlapping maplets, external maps derived from other

data sources, limb data, vector point clouds (typically from laser altimetry, often coupled with SPC in asteroid missions), cross-illumination interpolation in permanently shadowed areas, or differential stereography [25]. These constraining heights are crucial for an accurate height reconstruction and for ultimately setting the scale of the topography.

In this thesis, the approach used for surface reconstruction from the gradient field is different from the one employed by Gaskell. In the current SPC algorithm, only the core maplet-based modeling part was implemented, while the initial limb-based modeling during the approach phase was neglected. Therefore, constraining heights from limb data are not available in the current simulation setup. In addition, the other possible data sources on-board Milani mission are not considered in the analysis, as the focus of this thesis is on SPC method.

However, incorporating multiple sources and combining shape models derived from different methods, as SPC with laser altimetry, is a common practice in asteroid missions because it can significantly improve the results and solve the degeneracy between the object's size and the spacecraft distance [25]. Moreover, the iterative least-squares derivation of the terrain heights at random positions in the maplets, under the assumption of having previous knowledge of sufficiently distributed constraining heights, would significantly increase the computational demand to optimize a large number of variables.

Therefore, an alternative integration method was adopted, based on the solution of a Sylvester equation, as explained in detail in [136] and [137]. This approach, based on a least-squares problem, was chosen as it provided the best results and better aligned with the assumptions of the simulation. Given the derivative matrices $\mathbf{D}_x$ and $\mathbf{D}_y$, the partial derivatives of the surface height matrix $\mathbf{H}$ are expressed as:

$$\frac{\delta \mathbf{H}}{\delta x} = \mathbf{H} \mathbf{D}_x^T \quad (3.26) \quad \frac{\delta \mathbf{H}}{\delta y} = \mathbf{D}_y \mathbf{H} \quad (3.27)$$

$\mathbf{T}_x$ and $\mathbf{T}_y$ are used to represent the matrices of gradient fields previously obtained from the slope estimation step. The goal is to find the surface $\mathbf{H}$ whose gradients $\mathbf{D}_x$ and $\mathbf{D}_y$ best fit these estimates. This corresponds to minimizing the cost function J , reported in Equation (3.28).

$$J(\mathbf{H}) = \|\mathbf{H} \mathbf{D}_x^T - \mathbf{T}_x\|_F^2 + \|\mathbf{D}_y \mathbf{H} - \mathbf{T}_y\|_F^2 \quad (3.28)$$

Differentiating with respect to $\mathbf{H}$, is obtained the Sylvester Equation (3.29), representing a linear system in the unknown matrix $\mathbf{H}$.

$$\mathbf{D}_y^T \mathbf{D}_y \mathbf{H} + \mathbf{H} \mathbf{D}_x^T \mathbf{D}_x - \mathbf{D}_y^T \mathbf{T}_y - \mathbf{T}_x \mathbf{D}_x = 0 \quad (3.29)$$

When the surface boundary values are known (Dirichlet boundary conditions), the surface can be expressed as $\mathbf{H} = \mathbf{P}\mathbf{H}_I\mathbf{Q}^T + \mathbf{H}_B$, where $\mathbf{H}_I$ is the unknown interior portion of the surface, $\mathbf{H}_B$ defines the fixed boundary values, and $\mathbf{P}$ and $\mathbf{Q}$ are orthonormal permutation matrices that align the dimensions of $\mathbf{H}_I$ with $\mathbf{H}_B$. For square maplets, as in the considered case, $\mathbf{P} = \mathbf{Q}$. Substituting this boundary-constrained expression into the Sylvester equation, and pre and post multiplying respectively by $\mathbf{P}^T$ and $\mathbf{Q}$, yields the reduced Sylvester Equation (3.30) in the unknown $\mathbf{H}_I$, which can be solved directly to obtain a unique solution.

$$\begin{aligned} \mathbf{P}^T \mathbf{D}_y^T \mathbf{D}_y \mathbf{P} \mathbf{H}_I + \mathbf{H}_I \mathbf{Q}^T \mathbf{D}_x^T \mathbf{D}_x \mathbf{Q} + \mathbf{P}^T (\mathbf{D}_y^T \mathbf{D}_y \mathbf{H}_B \\ + \mathbf{H}_B \mathbf{D}_x^T \mathbf{D}_x - \mathbf{D}_y^T \hat{\mathbf{T}}_y - \hat{\mathbf{T}}_x \mathbf{D}_x) \mathbf{Q} = 0 \end{aligned} \quad (3.30)$$

In [131] was explored also an alternative method with modified boundary conditions. However, in this thesis the solution with the imposition of border heights is preferred, as it provides better results and ensures more consistency in the estimated heights between overlapping maplets, aiding the setup of a global shape optimization problem. This assumption does not limit the generality of the method, as in the original SPC implementation numerous constraining heights are imposed to the maplets [116], derived from several data sources as previously reported.

The outcome of the height integration step is a local digital elevation model for each maplet. An example is shown in Figure 3.25. In Figure 3.26 is also reported the height estimation error, as difference between the reconstructed DTM and the height of the terrain extracted from the reference asteroid model.

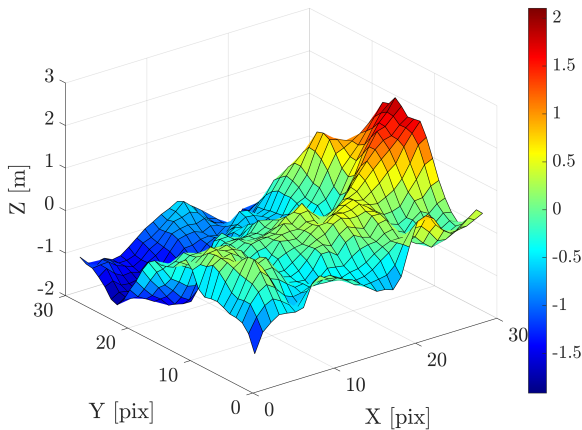


Figure 3.24: Reference real DTM of landmark 201.

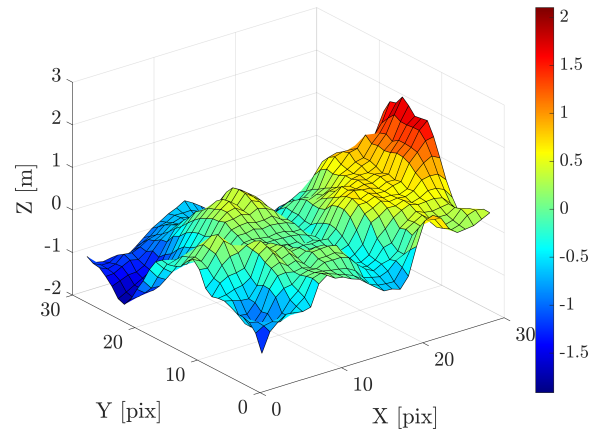


Figure 3.25: Reconstructed DTM of landmark 201.

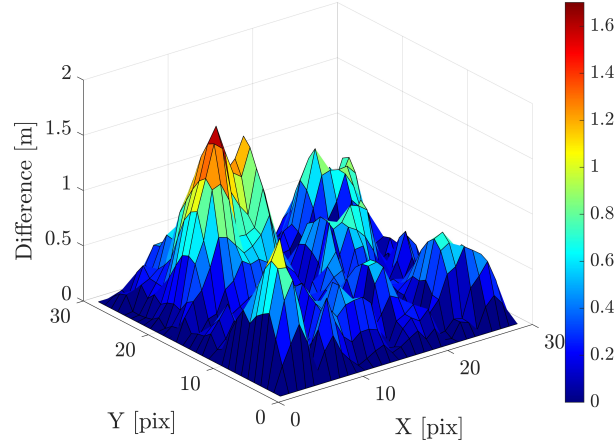


Figure 3.26: Estimation error of landmark 201.

An important improvement was made in the analysis of the results. The real heights used for the errors evaluation were previously obtained by projecting the undeformed points of the maplets in the radial direction, passing through Dimorphos' barycenter. In fact, the intersection points with the reference asteroid model were retrieved from the $[lat, lon]$ coordinates of each pixel. However, this approach was not taking into account the effect of the global curvature of the asteroid in the real heights extracted from the flat maplets.

For all the pixels of a maplet, except the central landmark, a small disalignment is present between the radial direction and the normal axis U_z of the maplet, which defines the direction of the heights estimated with the SPC algorithm. This effect, enhanced by the small dimensions of the asteroid, increases when moving towards the edges of the maplet, and is more evident for larger maplet sizes.

The effect also decreases during the mission phases, as the spacecraft moves closer to the asteroid and the GSD is reduced.

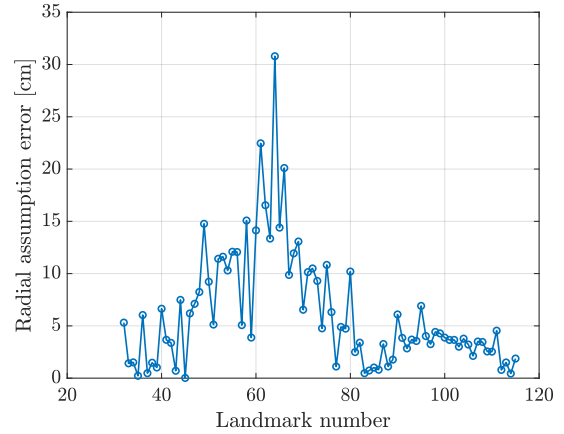


Figure 3.27: Radial height assumption error during FRP.

Therefore, the updated implementation extracts the real heights by projecting the flat maplet pixels in U_z direction and retrieving the intersection points with the reference shape model through the SPICE function *cspice_dskrv*. This more rigorous approach allows to slightly reduce the errors in the resulting heights, compared to the previous case neglecting the global curvature.

3.3. Landmark position vectors estimation

This section describes the implementation of the second core component of SPC algorithm, which is dedicated to the estimation and refinement of the landmark position vectors.

As previously mentioned, SPC solves for different quantities quantities $\mathbf{q}$ in three separate loops, iterating between them until the residual differences converge. In the second loop, indicated as Landmark Vectors in Figure 3.1, the variable $\mathbf{q}$ represents the position vector $\mathbf{V}$ of each landmark, in the Dimorphos-centered inertial reference frame.

The mathematical foundation of the algorithm is the same linear estimation introduced in Section 3.2.3 for the estimation of slopes and albedo. Assuming to start from a nominal value of $\mathbf{V}$ which is sufficiently close to the real solution, there is a linear relationship between the observed O and predicted value $P(\mathbf{V})$. Therefore, adjusting $\mathbf{V}$ with some values $\delta \mathbf{V}$, obtained from the minimization of squared residuals, as reported in Equations (3.15) to (3.18), is possible to accurately correct and improve the initial knowledge of the landmark vector positions.

3.3.1. Estimation procedure

The second and third block of SPC algorithm, dedicated respectively to the reconstruction of landmark position vectors and spacecraft's poses, locate common control points on the body's surface (landmarks) in the different images acquired during the mission [25].

Observations of many control points in a single image allow to retrieve the spacecraft position and orientation at the imaging time, assuming that the locations of these landmarks on the body's surface are known. This procedure is detailed in Section 3.4.

On the other hand, if a single control point can be found in many images, and the spacecraft pose when each image was acquired is known, then the location of the landmark on the body's surface can be determined from the stereo information. This result can also be combined with the estimates derived from the surface brightness variations in the topography estimation SPC component.

The whole SPC architecture iterates through solutions for spacecraft pose and landmarks position to arrive at a self-consistent solution for both. A simultaneous solution for spacecraft pose and landmark position could in principle be done by setting up a typical bundle adjustment problem. In photogrammetry and computer vision, bundle adjustment is a common nonlinear least-squares optimization process used to concurrently refine estimates of 3D structure and viewing geometry by minimizing the reprojection error between the observed image points and their predicted projections.

However, estimating at the same time the corrections for both the parameters in the SPC algorithm is usually impossible, due to the tens to hundreds of thousands of images and maplets commonly used [25]. Therefore, in SPC these two blocks are developed and executed in parallel, possibly with different frequencies. Then, the residuals from all the components of SPC are compared and minimized, until convergence to an optimal solution for all the parameters is achieved.

Differently from the previous topography reconstruction procedure, where the observable O is the pixel brightness, in both the second and third components of SPC the observed quantities are the pixel sample and line positions (u, v) of the landmarks in the images. The sum of the residuals in Equation (3.17) is still performed over all the images in which the considered landmark is located and visible, as previously done for each maplet pixel in the topography estimation.

The reconstruction of the landmark position vectors was implemented as a least-squares estimation. The pixel/line locations $[u, v]$ of the landmarks in the images can be obtained with Equation (3.11), reported in Section 3.2.2. In the same way, Equation (3.12) is used to obtain the position of each landmark $\mathbf{r}$ in camera frame, as shown in Figure 3.28.

The landmark vectors $\mathbf{V}$ are estimated in the DIMORPHOS_FIXED frame, as in this reference system the positions on the surface are fixed and decoupled from the rotational period of the asteroid around its rotation axis. The position vector $\mathbf{W}$ of Milani spacecraft is taken at each imaging time in the reference inertial frame. Therefore, the vectors reported in Figure 3.28 are used in three different frames in the SPC implementation, as it is more convenient.

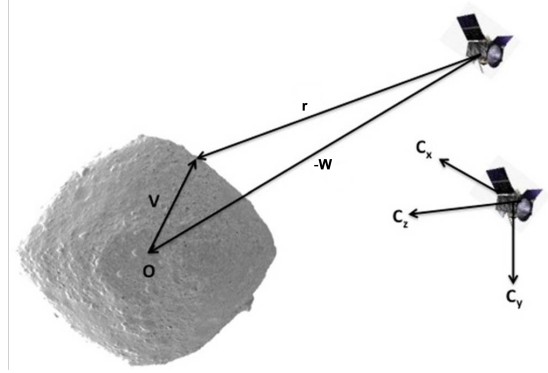


Figure 3.28: Considered vectors [25].

The position of the landmarks $\mathbf{V}$ at the beginning of the simulation is obtained from the ellipsoidal approximated model of the asteroid, using the same strategy employed in Algorithm 3.1. Instead, the inertial position of Milani is taken as fixed and equal to the nominal trajectory provided in the SPICE mission kernels. In the same way, the orientation of the spacecraft is assumed to be maintaining the perfect pointing towards Dimorphos, defined with Equations (3.1). These variable will be subsequently perturbed in Section 3.4, when estimating Milani's pose at each imaging time.

In order to translate these vectors $\mathbf{V}$ and $\mathbf{W}$ in the camera frame and obtain $[u, v]$, two rotation matrices are required, as seen in Equation (3.12). As previously done in the topography estimation, $\mathbf{R}_{Inertial2Body}$ is extracted from the SPICE mission kernels, assuming the rotational rate and pole position of the asteroid fixed and nominal. The estimation of these parameters is usually done in the approach phase, which is not considered in this thesis, and they can be subsequently updated after several complete SPC solutions if they clearly appear to be biased. The rotation matrix $\mathbf{R}_{Inertial2Cam}$ is obtained by concatenating the axes of the camera frame $[\mathbf{C}_x, \mathbf{C}_y, \mathbf{C}_z]$, obtained from Equations (3.1) because in this SPC component the orientation is taken as nominal.

The landmark estimation is implemented as a least-squares nonlinear optimization, through the MATLAB solver *lsqnonlin* using the Levenberg-Marquardt algorithm. The cost function to minimize is defined in Equation (3.31). The summed square residuals to be minimized are the differences between the observed and predicted image space location of the surface point, in the image coordinates (u, v) .

$$\sum_k \left[\frac{(u_o - u_p)^2}{\sigma^2} + \frac{(v_o - v_p)^2}{\sigma^2} \right] \quad (3.31)$$

σ is the uncertainty of each measurement, initially neglected for the validation of the algorithm, and then introduced as a Gaussian error on the values (u_o, v_o) measured by the NavCam's sensor.

An optimization is executed for each of the landmarks, and at each iteration, the sum in Equation (3.31) is performed over all the k images where the considered landmark is seen. Therefore, for each call to the solver the 3 components of the variable vector, corresponding to the spacial coordinates of a specific landmark in the body-fixed frame, are optimized to reduce the discrepancy in its position between the observed images and the ones predicted from the current knowledge of $\mathbf{V}$.

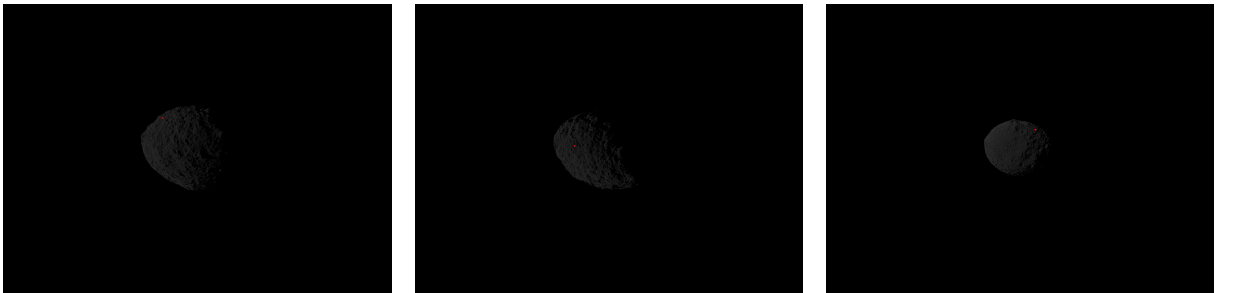


Figure 3.29: Landmark 321 seen during EXP in images 292, 447 and 505.

In the implemented objective function, the observed location (u_o, v_o) of a landmark in an image is computed from the nominal positions of both landmarks and spacecraft. Those are assumed to be equal to the real positions in this simulation, also assuming nominal pointing of the camera.

Instead, the predicted location (u_p, v_p) is estimated from the initially approximated landmark position and it is updated across the iterations, until convergence to a value closer to the observed one. An example of the discrepancy between the initial predicted and observed positions of landmark 239 is reported in Figure 3.30, which clearly shows that the ellipsoidal approximation overestimates the asteroid dimensions.

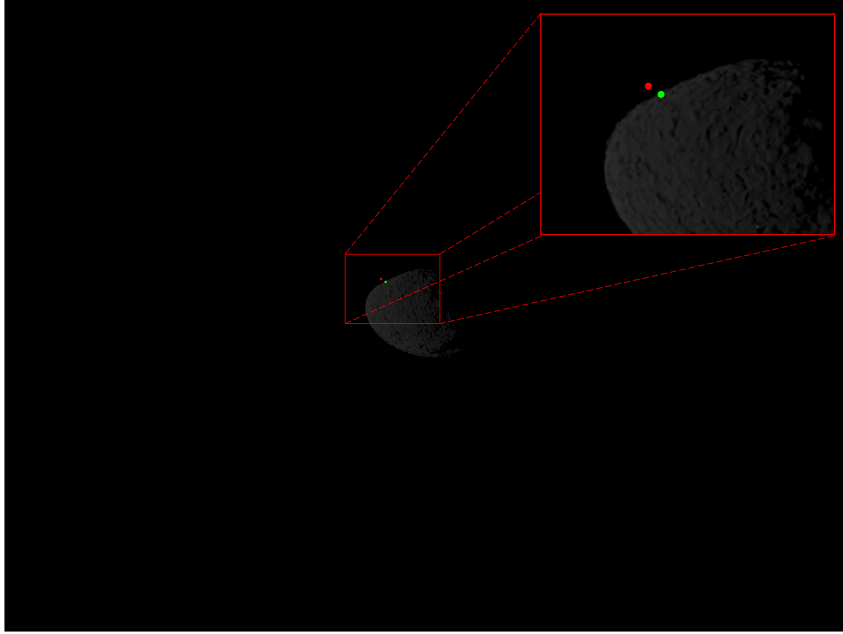


Figure 3.30: Predicted (red) and observed (green) landmark 239 in image 444 (EXP).

At each iteration, the objective function is also re-evaluating the visibility of the landmarks in the images. In fact, due to the changing position of the landmarks across different iterations, in some images the considered control point might not be in visibility anymore, while it might appear in other photos where it was not seen before.

The change in the number of images associated to the control point across different iterations might cause a dimension variation of the residual vector and of the Jacobian matrix, which is not supported by the solver employed. Therefore, the residual vector and Jacobian matrix are properly oversized, setting all their terms to zero at the beginning of each iteration. Then, only the fields corresponding to the images where the landmark is visible are filled with the computed residuals and derivatives.

The recognition of the visible landmarks in the images is executed with the same procedure of the topography estimation component, detailed in Section 3.2.2. However, in the landmark position estimation the additional photometric constraints on incidence, emission and phase angles are not applied, as they were introduced to guarantee optimal reconstruction only of the maplet topography. Removing these constraints, the number of images associated to each control point is increased. Nevertheless, the functionality to include these constraints is included in the algorithm, for comparison of results.

3.3.2. Objective function gradient

To improve the convergence of the least-squares estimation, the algorithm incorporates the analytical expression of the objective function's gradient. The correctness of the supplied derivatives was verified with the function *CheckGradients* of the MATLAB solver, compared to a centered finite differences approximation.

The analytical expression of the gradient was derived exploiting the Jacobian chain rule:

$$\frac{\partial(u, v)}{\partial \mathbf{V}} = \frac{\partial(u, v)}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{V}} \quad (3.32)$$

The results for the partial derivatives composing the previous expression are reported in Equations (3.33) and (3.34). The complete derivation procedure is given in Appendix B.

$$\frac{\partial(u, v)}{\partial \mathbf{r}_{Cam}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \quad (3.33)$$

$$\frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{V}} = (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam})^T = \mathbf{R}_{Body2Cam} \quad (3.34)$$

Finally, the complete gradient of the objective function is a 2×3 matrix expressed as in Equation (3.35). The gradient is also changed of sign in all its components in the algorithm, because the derivatives of the residuals correspond to the gradient of the predicted image locations (u_p, v_p) , which are negative in Equation (3.31).

$$\frac{\partial(u, v)}{\partial \mathbf{V}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam})^T \quad (3.35)$$

3.3.3. Regularization

The landmark position estimation component also includes a regularization process, that prevents the algorithm to wander too far from the initial knowledge of the position vectors. This is especially important when both spacecraft pose and landmark position errors are present, as an error in one can strongly affect the estimation of the other.

In order to penalize solutions that are very far from the initial guesses of landmark positions, additional terms are added to the residuals vector. Three regularization terms ($\mathbf{res}_{reg}$) are introduced in the optimization for each landmark estimation. Those values depend on the difference between the estimated landmark position in the current iteration and the initial guess position of that landmark. Equation (3.36) gives the total cost, computed in the solver as sum of the squared residuals. The formulation of the additional residual terms computed in the objective function is provided in Equation (3.37).

$$\begin{aligned} J_V &= \sum (\mathbf{res}_{(u,v)}^T \mathbf{R}^{-1} \mathbf{res}_{(u,v)} + \mathbf{res}_{reg}) \\ &= \sum (\mathbf{res}_{(u,v)}^T \mathbf{R}^{-1} \mathbf{res}_{(u,v)} + \lambda (\mathbf{V} - \mathbf{V}_{guess})^T \mathbf{P}_V^{-1} (\mathbf{V} - \mathbf{V}_{guess})) \end{aligned} \quad (3.36)$$

$$\mathbf{res}_{reg,lsqnonlin} = \sqrt{\lambda} \mathbf{P}_V^{-\frac{1}{2}} (\mathbf{V} - \mathbf{V}_{guess}) \quad (3.37)$$

$\mathbf{R}$ is the measurement noise covariance matrix, defined by the Gaussian noise σ added to the observations. The value λ is a tuning parameter used to increase or decrease the impact of the regularization terms in the optimization of the position of a certain landmark. The matrix $\mathbf{P}_V$ represents the initial covariance matrix of the landmark position vector.

In addition to the residual vector augmentation, the provided Jacobian matrix is enlarged to account for the regularization residuals. The derivatives of these terms with respect to the unknown vector $\mathbf{V}$ are given in Equation (3.38). Therefore, an additional 3×3 regularization part is appended to the analytical gradient reported in Equation (3.35).

$$\frac{\partial \mathbf{res}_{reg,lsqnonlin}}{\partial \mathbf{V}} = \sqrt{\lambda} \mathbf{P}_V^{-\frac{1}{2}} \quad (3.38)$$

The introduction of regularization terms in both the landmark position and spacecraft's pose estimation SPC components, respectively on the variables $\mathbf{V}$ and $(\mathbf{W}, \boldsymbol{\tau})$, gives an additional strategy to constrain the solution for both cases to be close to the initial guess, mitigating drastic changes of the variables in the iterated optimizations.

3.4. Spacecraft pose estimation

The following section describes the development of the third core component of SPC algorithm, dedicated to the estimation of the spacecraft's position and orientation at each imaging time.

In this third loop, indicated as S/C state in Figure 3.1, the variable $\mathbf{q}$ is a vector of six elements: the first 3 elements are the coordinates of Milani's position $\mathbf{W}$ in the inertial Dimorphos-centered reference frame, while the last 3 elements $\boldsymbol{\tau}$ are representing small rotation angles about the orthonormal camera vectors $\mathbf{C}_x$, $\mathbf{C}_y$ and $\mathbf{C}_z$, corresponding to the orientation error of the spacecraft [25]. Under the assumption of small rotations, the linearized changes in the camera frame vectors can be expressed as in Equation (3.39).

$$\begin{cases} \delta \mathbf{C}_x = \tau_z \mathbf{C}_y - \tau_y \mathbf{C}_z \\ \delta \mathbf{C}_y = \tau_x \mathbf{C}_z - \tau_z \mathbf{C}_x \\ \delta \mathbf{C}_z = \tau_y \mathbf{C}_x - \tau_x \mathbf{C}_y \end{cases} \quad (3.39)$$

As the previous SPC component, also the block to estimate the spacecraft's pose is based on the linear estimation foundation presented in Section 3.2.3. Therefore, starting from a nominal value of $\mathbf{W}$ and $\boldsymbol{\tau}$ sufficiently close to the real spacecraft's position and orientation, there is a linear relationship between the observed O and predicted value $P(\mathbf{W}, \boldsymbol{\tau})$. Applying proper corrections to these variables, resulting from the minimization of squared residuals as reported in Equations (3.15) to (3.18), is possible to accurately refine the initial knowledge of the spacecraft pose.

3.4.1. Estimation procedure

As introduced in Section 3.3.1, locating many control points in a single image allows to retrieve the spacecraft position $\mathbf{W}$ and orientation $\boldsymbol{\tau}$ at the time when the image is acquired. This works under the assumption that the locations of the landmarks on the surface of the asteroid are known. Iterating this estimation with the previous landmark estimation allows to correct both the variables and obtain a globally consistent solution.

Using the same approach adopted for landmark position estimation, the observables O are the pixel/line positions (u, v) of the landmarks in the images. However, differently from before, the sum of the residuals in Equation (3.17) is performed over all the visible landmarks k located in a given image, and no more over all the images where a landmark is seen.

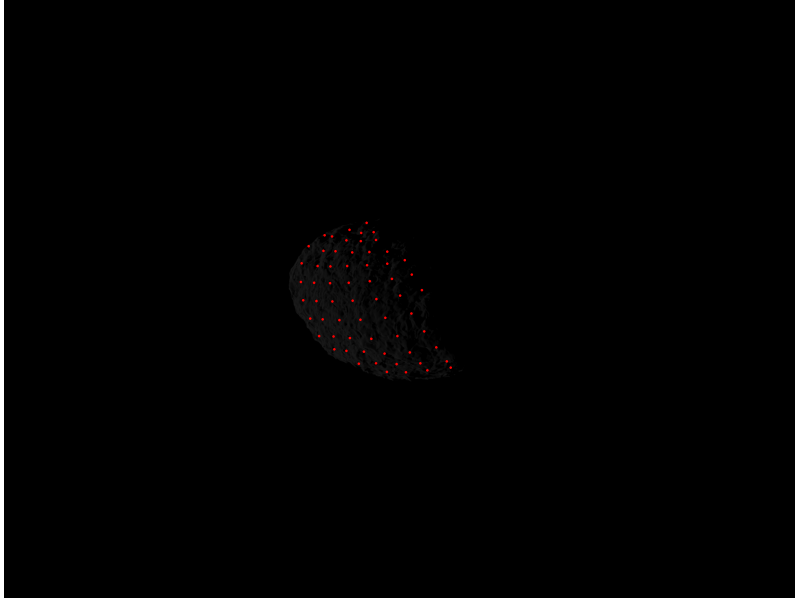


Figure 3.31: Control points (landmarks) visible in image 447 (EXP).

The spacecraft pose reconstruction was formulated as a least-squares estimation problem. Following the same approach previously described, the pixel and line coordinates $[u, v]$ of the landmarks in the images are computed from Equation (3.11), whereas Equation (3.40) provides the position of each landmark $\mathbf{r}$ in the camera reference frame. In this case, the rotation matrix $\mathbf{R}_{Inertial2Cam}^*$ is no more fixed, but changes with the spacecraft's orientation adjustments.

$$\mathbf{r}_{Cam} = \begin{bmatrix} x_{Cam} \\ y_{Cam} \\ z_{Cam} \end{bmatrix} = \mathbf{R}_{Body2Cam} \cdot \mathbf{r}_{Body} \quad (3.40)$$

$$= (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam}^*)^T \cdot (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W})$$

The variables $\mathbf{r}$, $\mathbf{V}$ and $\mathbf{W}$, appearing in Equation (3.40), are used in the same reference frames described in Section 3.3.1. The body-fixed position of the landmarks is taken as fixed and equal to the real position vectors extracted from the asteroid model, generated in Section 3.1.1.

The initial knowledge of Milani's inertial position $\mathbf{W}$ at each imaging time is defined as a perturbation of the nominal trajectory extracted from SPICE kernels, to simulate the position uncertainty during the mission. In the same way, a small Gaussian error in orientation is added through the small rotations vector $\boldsymbol{\tau}$, which are zero in nominal case. The assumption on the landmark positions $\mathbf{V}$ will be relaxed when the landmark position and spacecraft pose SPC modules are run iteratively to reduce both the uncertainties.

As in the landmark estimation case, the rotation matrix $\mathbf{R}_{Inertial2Body}$ is extracted from the SPICE mission kernels. The matrix $\mathbf{R}_{Inertial2Cam}$ is obtained by concatenating the orthonormal camera vectors. To account also of the small orientation errors introduced by $\boldsymbol{\tau}$, reported in Equation (3.39), a matrix approach is used, where $\mathbf{S}$ is the skew-symmetric matrix. Therefore, the perturbed rotation matrix $\mathbf{R}_{Inertial2Cam}^*$ is computed as follows:

$$\mathbf{R}_{Inertial2Cam}^* = (\mathbf{I} + \mathbf{S}(\boldsymbol{\tau})) \mathbf{R}_{Inertial2Cam} \quad (3.41)$$

$$\mathbf{S}(\boldsymbol{\tau}) = \begin{bmatrix} 0 & -\tau_z & \tau_y \\ \tau_z & 0 & -\tau_x \\ -\tau_y & \tau_x & 0 \end{bmatrix} \quad (3.42)$$

The nonlinear least-squares solver *lsqnonlin* was used with the Levenberg-Marquardt algorithm, as in the other SPC components. The minimized objective function follows the same formulation as in Equation (3.31), but the summation is performed over the k landmarks in the image.

At each imaging time, the solver is called and an optimization is executed. The least-squares algorithm minimizes the sum of the squared residuals of all the landmarks seen in the considered image, obtained as difference between their observed and predicted image locations. These are calculated by using respectively Milani's nominal (real) or perturbed (current knowledge) position and orientation.

For each iteration of the optimizer, the landmark visibility is updated with the same strategy and constraints detailed in Section 3.3.1. The residual vector and Jacobian matrix are also properly oversized to account for possible visibility variations.

Typically, in previous asteroid missions that used SPC as shape modeling strategy, most of the images were associated to more than 50 landmarks [7]. This means that at each imaging time a large number of control points can be used to accurately update the position and orientation of the spacecraft. Being more than the minimum three control points, the system has an overconstrained solution. By projecting the landmarks located on the image back into space, it is possible to calculate the position of the spacecraft when the image was taken, thus refining the initial knowledge.

In the specific case of Milani mission, most of the images have a consistent number of control points, allowing a correct reconstruction of the spacecraft pose. Only in some particular cases, when the target asteroid is eclipsed by its primary Didymos, the estimation is not possible as there are no landmarks in visibility of the NavCam.

3.4.2. Objective function gradient

To enhance the convergence also of the least-squares spacecraft's pose estimation, the analytical expression of the objective function's gradient was integrated into the algorithm, after being validated by comparison with a centered finite-difference approximation.

In this case the variable vector $\mathbf{g}$ is composed of two different types of parameters: the position $\mathbf{W}$ and the small rotations $\boldsymbol{\tau}$). By exploiting the Jacobian chain rule, the full Jacobian can firstly be divided in two Equations (3.43) and (3.44), separating the partial derivatives of (u, v) with respect to the parameters $\mathbf{W}$ and $\boldsymbol{\tau}$.

$$\left\{ \begin{array}{l} \frac{\partial(u, v)}{\partial \mathbf{W}} = \frac{\partial(u, v)}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} \\ \frac{\partial(u, v)}{\partial \boldsymbol{\tau}} = \frac{\partial(u, v)}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} \end{array} \right. \quad (3.43)$$

$$\left\{ \begin{array}{l} \frac{\partial(u, v)}{\partial \mathbf{W}} = \frac{\partial(u, v)}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} \\ \frac{\partial(u, v)}{\partial \boldsymbol{\tau}} = \frac{\partial(u, v)}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} \end{array} \right. \quad (3.44)$$

In these expressions, the partial derivative of (u, v) with respect to $\mathbf{r}_{Cam}$ is the same reported in Equation (3.33) in Section 3.3.1. The other partial derivatives composing the previous equations are given as follows:

$$\left\{ \begin{array}{l} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} = \mathbf{R}_{Inertial2Cam}^T (\mathbf{S}(\boldsymbol{\tau}) - \mathbf{I}) \\ \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} = \mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \end{array} \right. \quad (3.45)$$

$$\left\{ \begin{array}{l} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} = \mathbf{R}_{Inertial2Cam}^T (\mathbf{S}(\boldsymbol{\tau}) - \mathbf{I}) \\ \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} = \mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \end{array} \right. \quad (3.46)$$

In Equation (3.46), $\mathbf{s}$ is an auxiliary variable defined as:

$$\mathbf{s} = \mathbf{R}_{Inertial2Body}^T \mathbf{V} - \mathbf{W} \quad (3.47)$$

The complete expressions of the derivatives of (u, v) with respect to $\mathbf{W}$ and $\boldsymbol{\tau}$ are reported in Equations (3.48) and (3.49). These 2×3 matrices are then assembled in the algorithm to form a unique 2×6 matrix. The final gradient is also changed of sign in all its components, because the derivatives of the residuals correspond to the gradient of the predicted image locations (u_p, v_p) , which are negative in Equation (3.31). The complete derivation procedure is given in Appendix C.

$$\left\{ \begin{array}{l} \frac{\partial(u, v)}{\partial \mathbf{W}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \mathbf{R}_{Inertial2Cam}^T (\mathbf{S}(\boldsymbol{\tau}) - \mathbf{I}) \\ \frac{\partial(u, v)}{\partial \boldsymbol{\tau}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \end{array} \right. \quad (3.48)$$

$$\left\{ \begin{array}{l} \frac{\partial(u, v)}{\partial \mathbf{W}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \mathbf{R}_{Inertial2Cam}^T (\mathbf{S}(\boldsymbol{\tau}) - \mathbf{I}) \\ \frac{\partial(u, v)}{\partial \boldsymbol{\tau}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \end{array} \right. \quad (3.49)$$

3.4.3. Regularization

As previously done for the landmark position estimation in Section 3.3.3, a regularization process is applied to the spacecraft's pose estimation algorithm. Differently from before, 6 additional regularization terms are added to the residual vector, in order to penalize solutions that are very far from the initial guess of spacecraft's position and orientation at each imaging time. The first 3 components ($\mathbf{res}_{reg,W}$) are related to the spacecraft position, while the other 3 ($\mathbf{res}_{reg,\tau}$) to the orientation.

$$J_{(W,\tau)} = \sum (\mathbf{res}_{(u,v)}^T \mathbf{R}^{-1} \mathbf{res}_{(u,v)} + \mathbf{res}_{reg}) \quad (3.50)$$

$$\begin{aligned} \mathbf{res}_{reg} &= \mathbf{res}_{reg,W} + \mathbf{res}_{reg,\tau} \\ &= \lambda [(\mathbf{V} - \mathbf{V}_{guess})^T \mathbf{P}_W^{-1} (\mathbf{V} - \mathbf{V}_{guess}) + (\boldsymbol{\tau} - \boldsymbol{\tau}_{guess})^T \mathbf{P}_\tau^{-1} (\boldsymbol{\tau} - \boldsymbol{\tau}_{guess})] \end{aligned} \quad (3.51)$$

The components added in the *lsqnonlin* residual vector are the following:

$$\left\{ \begin{array}{l} \mathbf{res}_{reg,W}^{lsqnonlin} = \sqrt{\lambda} \mathbf{P}_W^{-\frac{1}{2}} (\mathbf{W} - \mathbf{W}_{guess}) \\ \mathbf{res}_{reg,\tau}^{lsqnonlin} = \sqrt{\lambda} \mathbf{P}_\tau^{-\frac{1}{2}} (\boldsymbol{\tau} - \boldsymbol{\tau}_{guess}) \end{array} \right. \quad (3.52)$$

The matrix $\mathbf{P}_W$ represents the initial covariance matrix of the spacecraft position vector, while $\mathbf{P}_\tau$ is the covariance matrix for the small rotation components.

The analytical gradient provided in the objective function was also enlarged with the 3×6 regularization components, reported in Equation (3.53).

$$\left\{ \begin{array}{l} \frac{\partial \mathbf{res}_{reg,W}^{lsqnonlin}}{\partial \mathbf{W}} = \sqrt{\lambda} \mathbf{P}_W^{-\frac{1}{2}} \\ \frac{\partial \mathbf{res}_{reg,\tau}^{lsqnonlin}}{\partial \boldsymbol{\tau}} = \sqrt{\lambda} \mathbf{P}_\tau^{-\frac{1}{2}} \end{array} \right. \quad (3.53)$$

3.5. SPC global model

The last step of the SPC architecture is the reconstruction of a global model for the asteroid. The SPC algorithm developed by R. W. Gaskell and the Planetary Science Institute is capable of generating global or regional (Bigmaps) models of the body surface at any point throughout the SPC process [7]. As shown in Figure 3.1, this operation is performed through the functions DENSIFY and BIGMAP, resampling all the topography and albedo data previously obtained for each maplet after several SPC iterations.

The obtained global model reflects the quality of the shape reconstruction in the single maplets at that time. Then, with the progressive ingestion of more images, acquired with different viewing geometries and illumination conditions, the accuracy of the estimation can significantly improve. The newly ingested images are also characterized by a lower GSD as the spacecraft approaches the asteroid and performs its orbiting strategy in the low-gravity environment. Therefore the detail of the final model is considerably improving during the mission.

In the SPC process is not uncommon to construct well over 100000 maplets of varying resolutions, usually much finer than what can be captured by even the highest-resolution shape model. This huge amount of data can be combined giving less weight to maplets with coarser resolution [25]. Maplets with different GSD can also be combined with a bilinear interpolation method to build higher-resolution terrain models. In addition, the comparison of estimations from different maplets can be very useful for error detection, in particular in the overlapping regions between adjacent maplets.

Having a sufficient level of overlapping between maplets is important not only for finding possible estimation errors, but also because these overlapping regions are used in Gaskell's algorithm to provide some of the fundamental constraints to the height integration process.

In this thesis was adopted a modified height estimation method, presented in Section 3.2.4. In Gaskell's implementation the constraining heights are derived from overlapping maplets, limb data, vector point clouds from laser altimetry, cross-illumination interpolation in permanently shadowed areas, differential stereography or other data sources. However, in the simulation scenarios considered in this thesis, these constraining heights are not provided.

In order to move towards a global model, and to a global optimization of the terrain heights, a crucial step required is the evaluation of the overlapping between adjacent maplets. For this purpose, an algorithm was developed to detect the overlapping squares of pixels and obtain constraining weights from a bilinear interpolation.

3.5.1. Maplet overlap search

The identification of overlapping maplets is a crucial step required to impose the integration boundary conditions and to move towards a global model. The detection of the overlapping areas is executed for each of the maplets that were previously reconstructed during the topography estimation process.

The landmarks that were not associated with any image respecting photometric constraints are excluded from the research. For each of these landmarks, the heights of the corresponding maplets are still equal to zero, as in the initial flat maplets. Therefore, the not imaged areas keep being approximated with the ellipsoidal model of the asteroid.

Firstly, the body-fixed position vector of all the maplet's pixels is calculated, and the geographic coordinates (lat, lon) of each are stored in the same data structures previously created for each landmark in Section 3.2.1. For each pixel of the estimated maplets, the position vector $\mathbf{V}$ is computed with the formula given at line 5 of Algorithm 3.3. The (lat, lon) coordinates are obtained from the SPICE function *cspice_reclat*.

Once defined the geographic coordinates, the algorithm iterates across all the possible squares of 4 pixels in each considered maplet i . The (lat, lon) coordinates of the vertices of these square regions are stored and properly rearranged to account for possible longitude discontinuities.

Then, the algorithm iterates across all the adjacent maplets j to the considered landmark i , previously identified during the landmark generation process through Algorithm 3.2. All the pixels of these adjacent maplets are tested with respect to the current square of pixels of the landmark i . The MATLAB function *inpolygon* is used to check if the (lat, lon) coordinates of the considered pixel of the adjacent maplet j is falling inside the square defined on the geographic coordinates plane by the 4 pixels of the landmark i . If that is the case, an overlap is identified.

When a pixel of an adjacent maplet j is found to be located inside the (lat, lon) region defined by 4 pixels of a landmark i , a bilinear interpolation scheme is introduced. Bilinear interpolation is a method for interpolating functions of two variables using repeated linear interpolation, in two directions, resulting in a quadratic interpolation in the sample location. It is usually applied to functions sampled on a 2D rectilinear grid, and allows to obtain weights for each of the 4 vertices. These can be used as constraints for the height of the internal overlapping pixel.

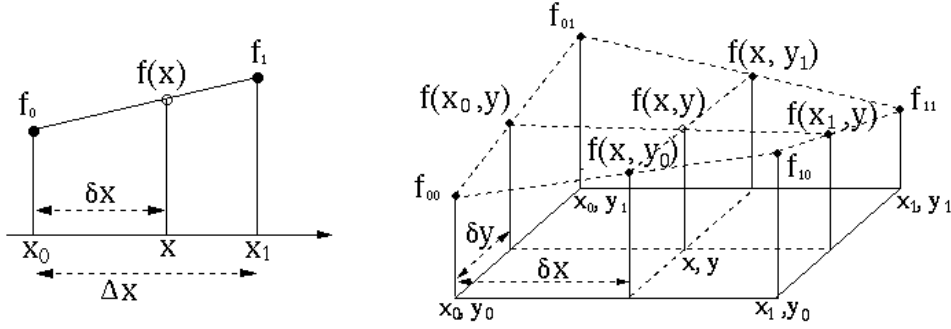


Figure 3.32: Bilinear interpolation scheme.

In standard bilinear interpolation, the value of a point inside a rectangular cell is estimated as a weighted combination of the four corner values, using linear interpolation along each axis. This assumes that the grid is axis-aligned and that the mapping between the parametric coordinates $(u, v) \in [0, 1]^2$ and the physical coordinates (lat, lon) is affine.

The bilinear interpolation is obtained with Equation (3.54), where the weights are computed as in Equation (3.55).

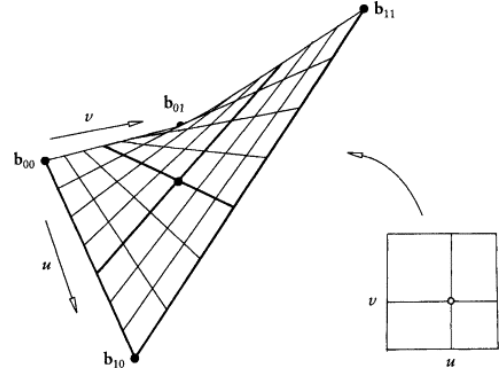


Figure 3.33: Bilinear interpolation parametric coordinates [27].

$$f(u, v) = w_{00}P_{00} + w_{10}P_{10} + w_{01}P_{01} + w_{11}P_{11} \quad (3.54)$$

$$\begin{cases} w_{00} = (1 - u)(1 - v) \\ w_{10} = u(1 - v) \\ w_{01} = v(1 - u) \\ w_{11} = uv \end{cases} \quad (3.55)$$

However, this mapping from parametric space (u, v) to geographic space (lat, lon) is affine only if the corners form a parallelogram. If the overlapping search process is run with the initial flat maplets, the pixel regions are squares, with dimensions equal to the GSD of the considered mission phase simulation. Instead, if the check for overlapping pixels is executed on already deformed maplets due to the SPC topography estimation, then the regions between 4 adjacent pixels might be non-rectangular or distorted quadrilaterals.

When the corners are arbitrary, the mapping becomes bilinear but nonlinear, and the inverse mapping cannot be solved analytically. Therefore, the parametric coordinates corresponding to a target point can be recovered iteratively, using Newton’s method to minimize the residual $P(u, v) - P(lon, lat)$. Once the solution converges, the standard bilinear weights can be computed, ensuring continuous interpolation even for skewed or curved cells. This modified approach generalizes classical bilinear interpolation to irregular quadrilaterals while preserving accuracy and smoothness.

To guarantee robustness, a fallback strategy is adopted for cases in which the Newton iteration fails to converge. In these situations, the quadrilateral can be divided into two triangles, and barycentric interpolation is applied within the triangle containing the target point. This alternative ensures stable interpolation weights and continuity across cell boundaries, even in degenerate or highly distorted grid configurations.

The information about the overlapping maplet pixels are properly stored in data structures. In particular, for each overlapping pixel of an adjacent maplet j , which is found to be overlapping to a certain landmark j , the IDs and weights of the square’s vertices are saved. Each pixel could be associated with more than one square region of different maplets, because in certain regions several maplets are overlapping.

The procedure used for the identification of overlapping pixels in adjacent maplets is detailed in its entirety in Algorithm 3.3. A test case result is reported in the following figure, where red and yellow pixels represent respectively single and multiple overlaps.

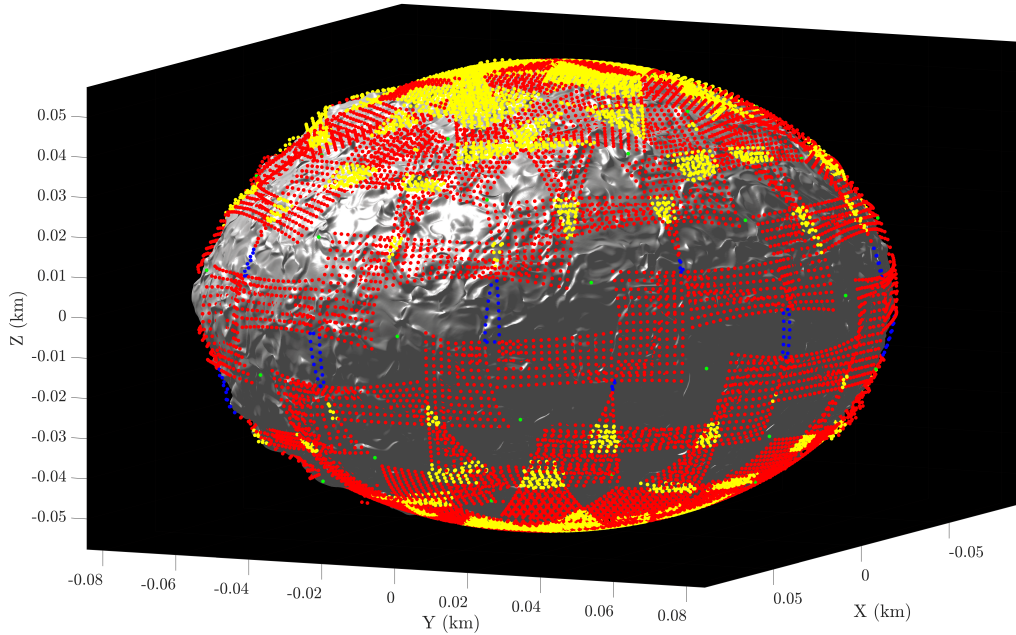


Figure 3.34: Single (red) and multiple (yellow) overlaps for 61 maplets (19×19 , FRP).

Algorithm 3.3 Overlapping maplets search

```

1:  $q = \frac{\text{mapletDimension} - 1}{2}$ 
2: for  $i = 1, \dots, n_{lmks}$  do
3:   for  $x = -q, \dots, q$  do
4:     for  $y = -q, \dots, q$  do
5:        $V_{i,x,y} = V_i + \text{mean}(GSD)(xU_x + yU_y) + h_{x,y}U_z$ 
6:        $[lat, lon]_{i,x,y} = \text{cspice\_reclat}(V_{i,x,y})$ 
7:     end for
8:   end for
9: end for
10: for  $i = 1, \dots, n_{lmks}^{estim}$  do
11:   for  $x_i = -q, \dots, q - 1$  do
12:     for  $y_i = -q, \dots, q - 1$  do
13:       consider  $[lat, lon]_{i,r,s}$  for  $r = x_i, x_i + 1$  and  $s = y_i, y_i + 1$ 
14:       if any  $|lon_{i,r,s} - lon_{i,r,s}| > \pi$  then
15:         if  $lon_{i,r,s} < 0$  then
16:           each  $lon_{i,r,s} = lon_{i,r,s} + 2\pi$ 
17:         end if
18:       end if
19:       for  $j = \text{adjacent maplet of } i$  do
20:         for  $x_j = -q, \dots, q$  do
21:           for  $y_j = -q, \dots, q$  do
22:             if  $lon_{i,r,s} < 0$  then
23:               if  $lon_{j,x_j,y_j} < 0$  then
24:                  $lon_{j,x_j,y_j} = lon_{j,x_j,y_j} + 2\pi$ 
25:               end if
26:             end if
27:              $flag = \text{inpolygon}([lat, lon]_{i,r,s}, [lat, lon]_{j,x_j,y_j})$ 
28:             if  $flag = \text{true}$  then
29:               pixel  $[lat, lon]_{j,x_j,y_j}$  overlaps in square  $[lat, lon]_{i,r,s}$ 
30:                $w_{r,s} = \text{bilinear\_weights}([lat, lon]_{i,r,s}, [lat, lon]_{j,x_j,y_j})$ 
31:             end if
32:           end for
33:         end for
34:       end for
35:     end for
36:   end for
37: end for

```

4 | Results

4.1. Topography reconstruction

In the following section are presented the results for the updated topography reconstruction SPC component. Several simulation scenarios were considered for the 3 different Milani mission phases, as previously reported in Table 3.2. As detailed in Section 3.2.1, the images for the simulation are acquired every hour, and the key SPC parameters (maplet GSD, total number of landmarks, dimension of maplets) were tuned for each scenario, to properly ensure full coverage of the asteroid.

4.1.1. Far Range Phase (FRP)

During the Far Range Phase, Milani is performing a 6 points hyperbolic loop trajectory, with a maneuvering pattern of 4-3 days repeated three times. The spacecraft is moving on quasi-symmetric arcs relative to the Sun, with distances from Dimorphos ranging from 8 to 14 km. This periodic translation can be seen also in the behavior of the GSD, reported in Figure 3.14. The selected average image GSD of 1.91 m, used for the generation of the maplets, is sufficient to meet the mission requirement of mapping the asteroid at resolutions better than 2 m/pixel.

A first mapping of Dimorphos' surface is performed by using the SPC algorithm with the images acquired in FRP. The photometric constraints, given in Table 3.3, are applied to further constrain the visibility of landmarks in the images. The resulting number of pictures associated to each landmark is depicted in Figure 4.1.

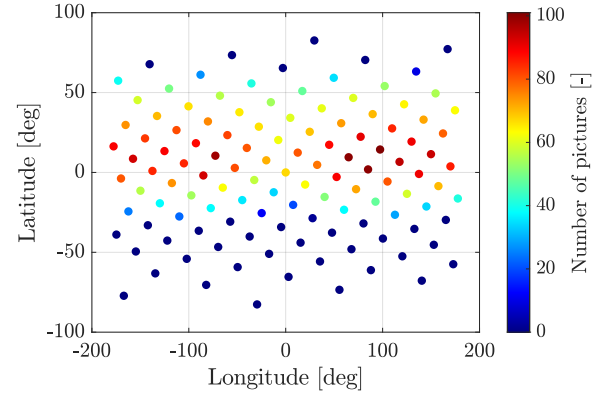


Figure 4.1: Visibility of landmarks (121) during FRP, with photometric constraints.

These constraints, applied to obtain a better topography reconstruction, allow the estimation of all the maplets located in the equatorial regions. However, the landmarks in polar regions are more challenging because they are associated to fewer images. Therefore, depending on the imaging conditions of the mission the topography estimation is not always possible. In Figure 4.4 some maplets located around the poles are not shown, as they were not estimated due to insufficient imaging.

The topography results obtained in FRP phase can be evaluated by using different metrics. The root mean square error (RMSE) between the observed and predicted brightness of a pixel, computed as follows, can be a useful parameter to identify problematic areas [25].

$$RMSE = \sqrt{\frac{\sum_{k=1}^N (I_o - I_p)^2}{N}} \quad (4.1)$$

The summation in Equation (4.1) extends over all the N images in which the landmark has been reprojected. The global RMSE result for FRP is shown in Figure 4.2, which presents the minimum, maximum and average RMSE for each estimated maplet.

Another useful metrics for evaluating the results is the difference between the maplet heights estimated in the SPC component for topography and the real heights of the DTM, extracted from the reference asteroid model through the updated procedure described in Section 3.2.4. The height discrepancies averaged on each maplet are reported in Figure 4.3, highlighting the maximum value for each landmark. The minimum height difference is not reported, because it is zero for the constraining heights applied during the height integration process.

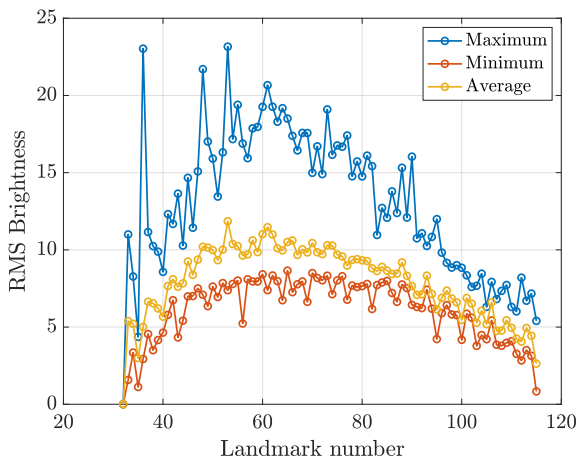


Figure 4.2: RMSE of predicted brightness during FRP.

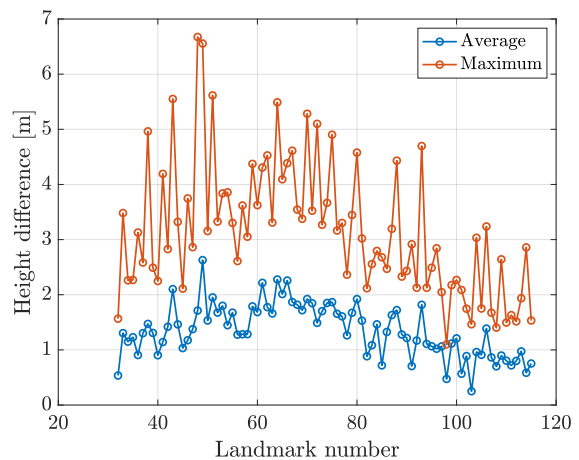


Figure 4.3: Height difference between estimated and reference DTM during FRP.

As for the RMSE, only the results for the estimated maplets are shown. In fact, the landmarks with highest and lowest number, corresponding to the two polar regions in the Fibonacci lattice generation algorithm, are not estimated because they have insufficient imaging that respects the photometric constraints. In asteroid missions this is a common problem, related to the challenging trajectory design in low-gravity environments. Typically, the not estimated regions can be approximated with an ellipsoidal model.

The RMSE of the predicted brightness shows a parabolic behaviour. This is directly related to the number of images in which the landmarks are visible, higher for those located in equatorial regions. In fact, for a greater number of associated pictures with different imaging geometries and shadowing effects, the least squares estimation is requested a higher effort to provide a solution of slopes and albedo that best fits all the observations. Instead, when the images associated to a landmark are only few, the matching of the observed brightness is inherently better, resulting in a lower RMSE.

These parameters provide a global perspective on the topographic results, mainly focused on the average and maximum discrepancies for each landmark. In order to have a more local overview of the topography reconstruction, the Hausdorff distance ϵ can be introduced as performance index, as done in [19]. It is defined as the minimal distance between a point of the reconstructed shape and the surface of the asteroid's reference model. The Hausdorff distance, depicted in Figure 4.4 for each estimated maplet pixel, is also normalized with respect to the small-body mean equatorial radius. Therefore, the height differences are displayed in the colormap as percentage of this radius.

The visual representation of all the height estimation errors on the asteroid model allows an easier identification of the most challenging areas for the shape reconstruction. As expected, the reconstruction error is typically higher in the central part of the maplets, which is the farthest from the imposed constraining heights. In addition, from a comparison with the asteroid reference model it can be noted that the red areas with the largest errors mainly correspond to sharp and large boulders, which SPC inherently tends to smooth.

The results obtained during FRP show that the GSD of the maplets, constrained by the camera characteristics, is still too high to provide a consistent surface reconstruction. Small and sharp details of the terrain cannot be detected with sufficient accuracy. However, this phase can be significantly valuable for a first surface reconstruction of the areas for which there is no previous knowledge, due to lack of DART imaging, improving the initial ellipsoidal approximation.

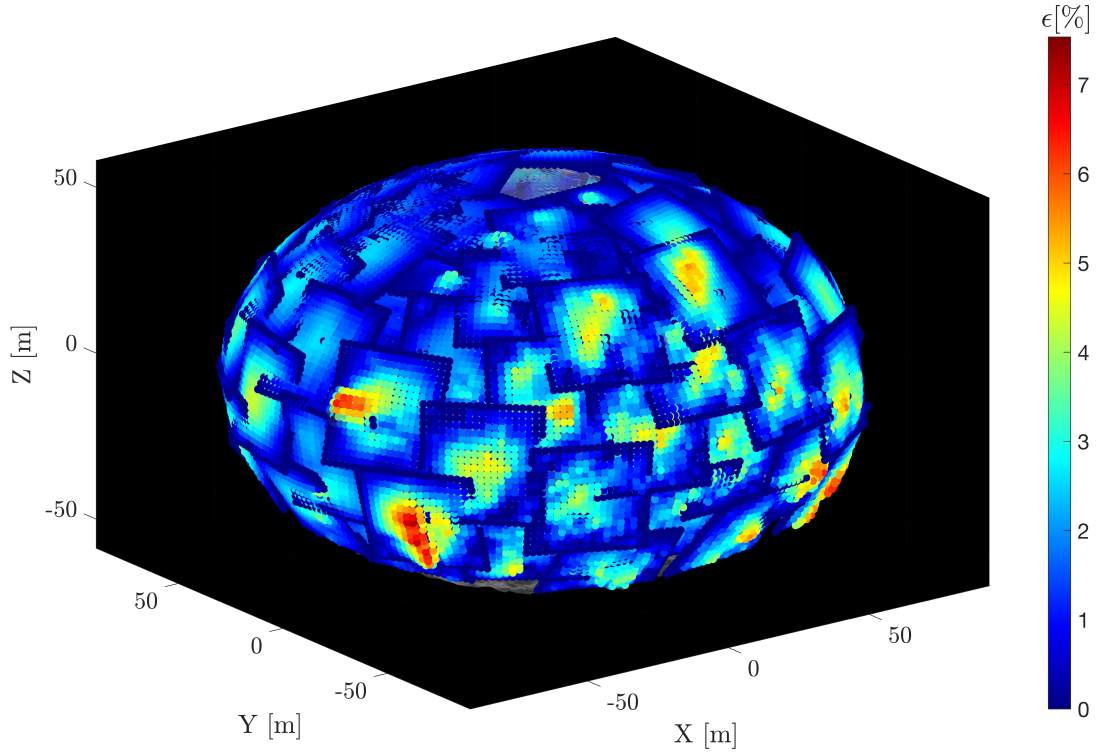


Figure 4.4: Height reconstruction error during FRP, represented on the asteroid model through Hausdorff distance (121 maplets, 19×19).

4.1.2. Close Range Phase (CRP)

The main objective of the Close Range Phase is the global mapping of Dimorphos at resolution better than 1 m/pixel, and the detailed study of the DART impact site through high-resolution imaging (better than 0.5 m/pixel). The trajectory described by Milani during CRP is characterized by very close approaches to the surface of the asteroid, followed by recovery and repositioning segments. This orbiting strategy, designed to optimize both imaging quality and operational efficiency, poses several challenges to the shape reconstruction.

As detailed in Section 3.2.1, the GSD selected for this phase is 1 m/pixel, slightly different from the average value in the whole phase. This represents a practical compromise between the SPC constraints and the large GSD oscillations caused by the brief proximity operations for detailed imaging of the DART impact site.

Being the longest phase of the mission, a very large number of images is available. As shown in Figure 4.5, the visibility of landmarks in the images is compatible with the photometric constraints in particular in 2 equatorial areas, with a maximum number of images per landmark for $\pm 90^\circ$ of longitude.

The DART impact site, whose coordinates in the DIMORPHOS_FIXED frame are given in Table 1.2, is correctly falling in one of these regions with most favorable imaging conditions. It can be seen in Figure 4.5 in magenta.

Since the detailed imaging and surface reconstruction of this area is a key objective of this phase, the following results are focused in that region. The same performance parameters introduced in Section 4.1.1 are used for the topography analysis.

The RMSE of the predicted brightness, reported in Figure 4.6, shows a significant improvement with respect to the previous FRP phase, despite some outliers in the polar region, for which the imaging conditions are less favorable.

The height difference between estimated and reference heights of the DTM exhibits a similar improvement, in particular in the average which is consistently less than 1 m.

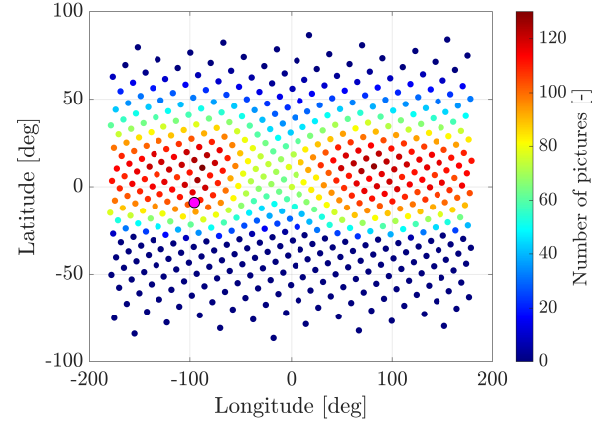


Figure 4.5: Visibility of landmarks (561) during CRP, with photometric constraints.

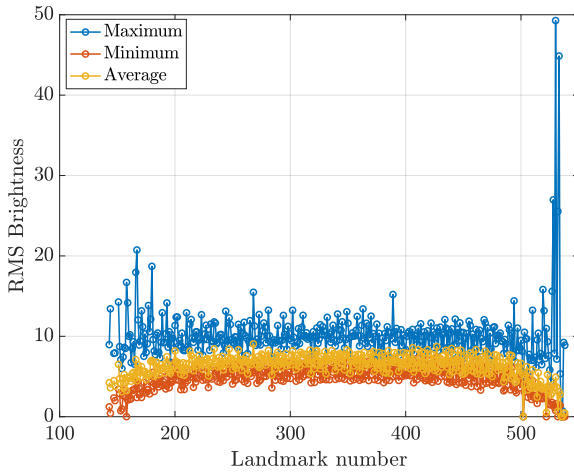


Figure 4.6: RMSE of predicted brightness during CRP.

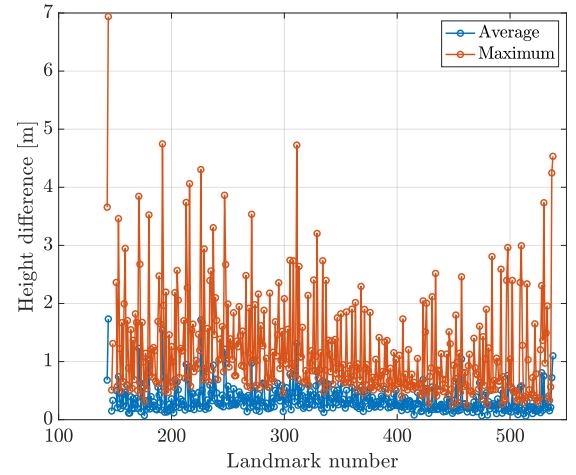


Figure 4.7: Height difference between estimated and reference DTM during CRP.

In the following figures are provided the results for the height reconstruction of the maplets surrounding the DART impact area. Hausdorff distance is used, as percentage of the mean equatorial radius.

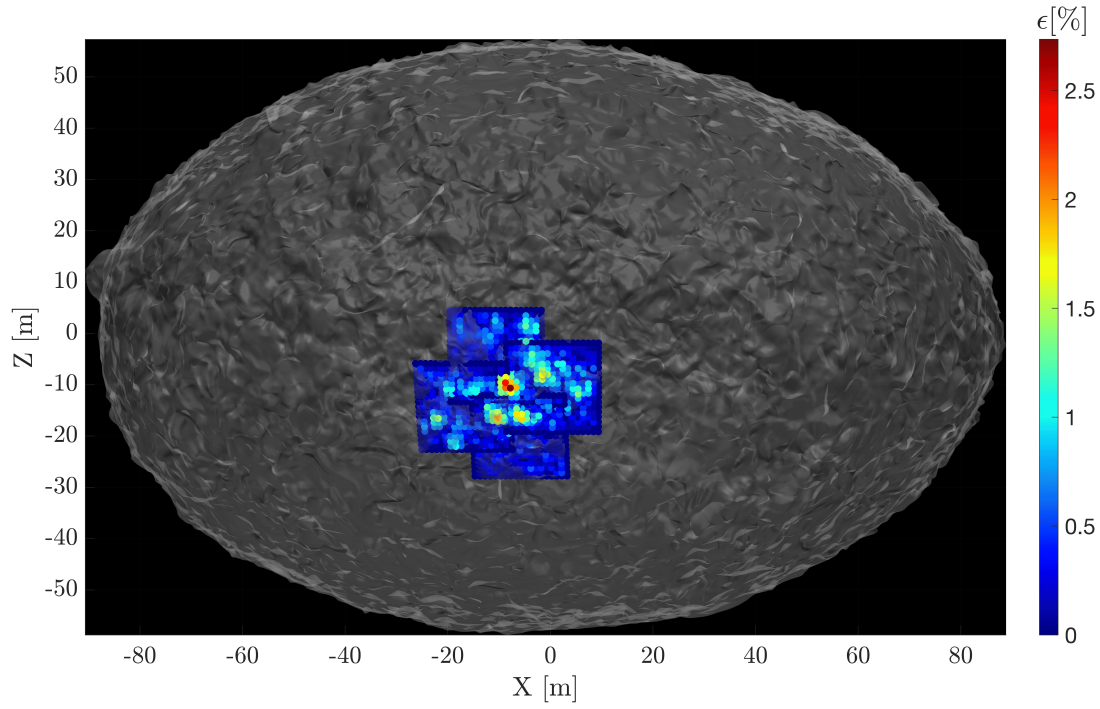


Figure 4.8: Height reconstruction error of DART impact area during CRP, represented on the asteroid model through Hausdorff distance (561 maplets, 19×19).

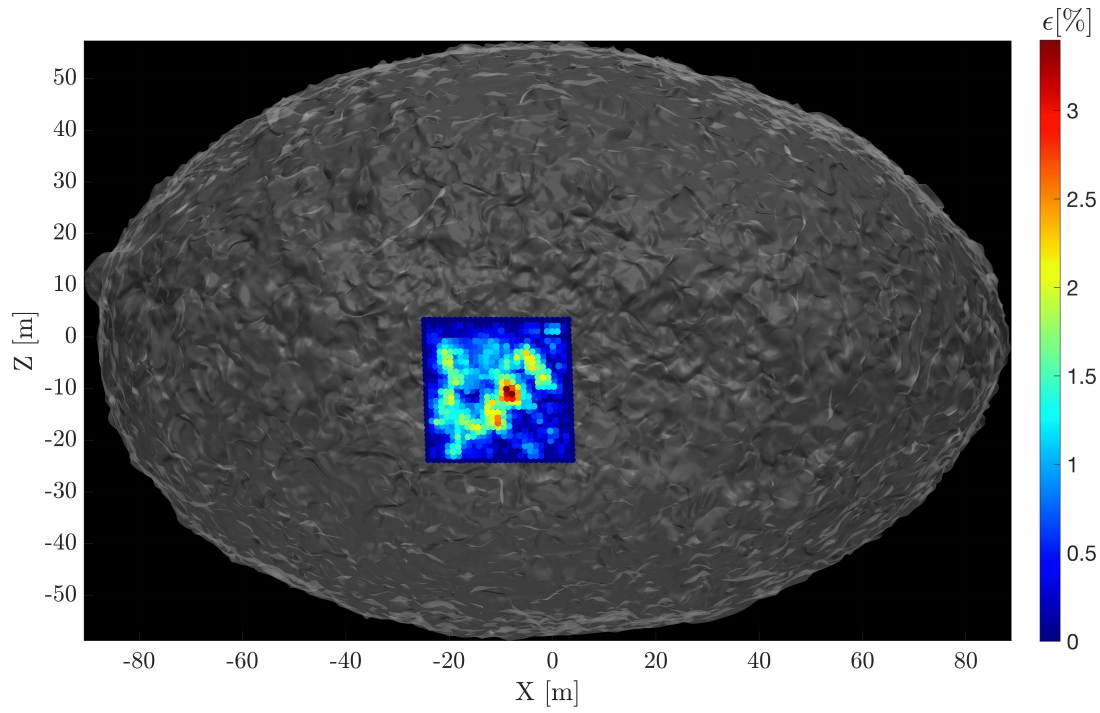


Figure 4.9: Height reconstruction error of DART impact area during CRP, represented on the asteroid model through Hausdorff distance (241 maplets, 29×29).

It is possible to see that the discrepancy between the estimated results and the reference model is lower if smaller maplets are used, because the distance from the constraining heights is smaller. The maximum height discrepancy is respectively of 2.48 m for maplets with 19×19 pixels, and 3.31 m for maplets with 29×29 pixels. For both the maplet dimensions, the maximum error is corresponding to the sharp boulder in the middle of the impact area, which tends to be smoothed out by the SPC algorithm.

A further improvement of the topography results can be achieved by properly modifying the SPC parameters for optimizing the reconstruction of only the DART impact site. This can be done selecting more strict photometric constraints for evaluating the visibility of the landmarks in the images, and using a lower GSD for the generation of only a restricted amount of landmarks in this area. In this way the details captured by the high resolution images acquired during the close approaches can be better exploited.

In Figure 4.10 is shown the height reconstruction error using a decreased GSD of 0.8 m to prioritize the images acquired when the spacecraft is closer to the asteroid, neglecting those taken during the repositioning orbital segments. The maximum estimation discrepancy in this case is significantly improved, corresponding to only 1.75 m.

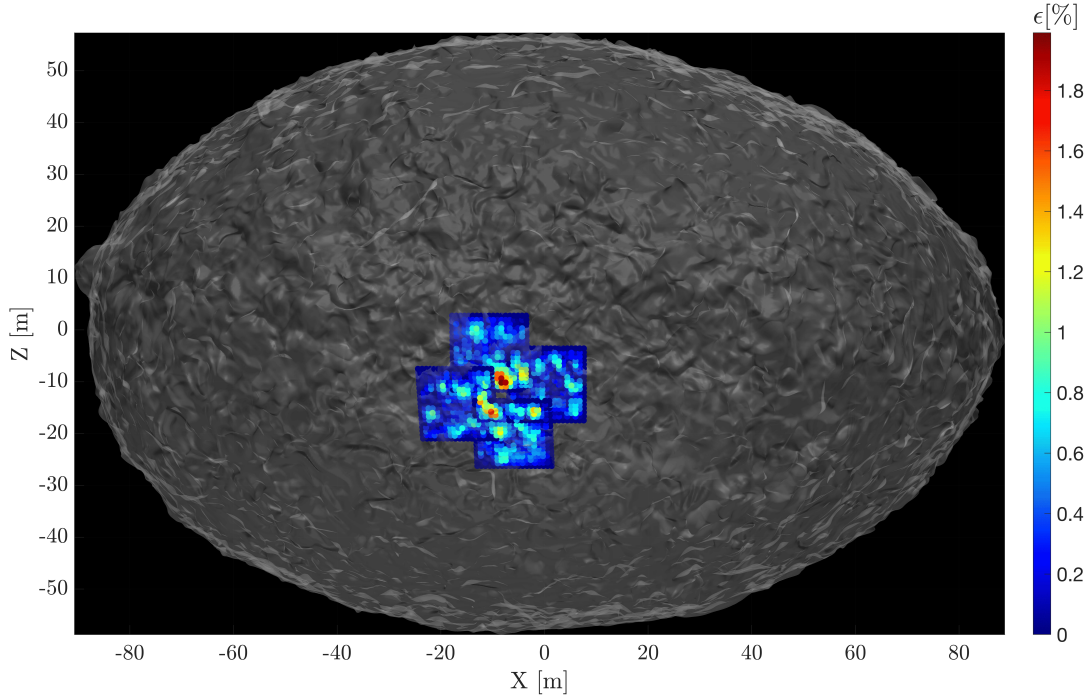


Figure 4.10: Height reconstruction error of DART impact area during CRP with modified parameters, represented through Hausdorff distance (241 maplets, 29×29).

4.1.3. Experimental Phase (EXP)

The last phase of the mission prepares the spacecraft for the possible final landing on the surface of Dimorphos. Therefore, Milani is following a SSTO which progressively reduces the distance from the asteroid, from 7 to around 2 km. This is also the phase that allows the most detailed topography reconstruction, due to its lower GSD of 0.73 m.

Coherently with the previous phases, also during EXP the visibility of landmarks respecting photometric constraints is higher in the equatorial region. Instead, the poles are not sufficiently imaged with the viewing geometries that guarantee a good topography reconstruction. For this reason, most of polar regions cannot be accurately estimated with the SPC algorithm. The RMSE of brightness and the height difference across the maplets are used again as performance parameters to evaluate the quality of the shape reconstruction.

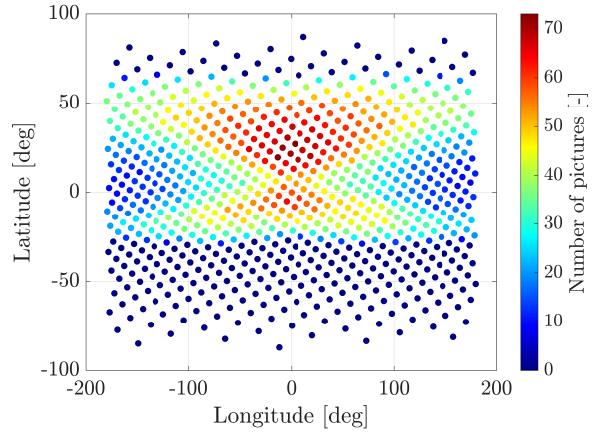


Figure 4.11: Visibility of landmarks (781) during EXP, with photometric constraints.

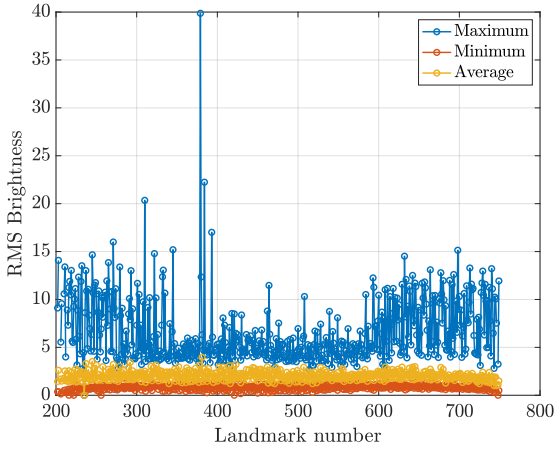


Figure 4.12: RMSE of predicted brightness during EXP.

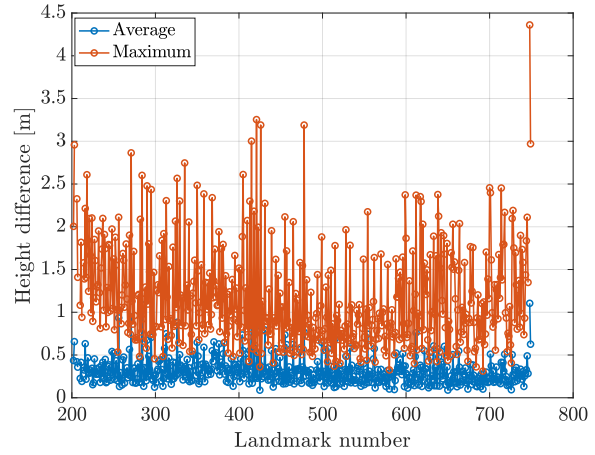


Figure 4.13: Height difference between estimated and reference DTM during EXP.

The RMSE and the height error results reflect the improved imaging and GSD of EXP. The average RMSE is consistently below 5, and the height difference is on average between 0.2 and 0.8 m. Some outliers are still present, due to the limited imaging in polar areas and due to occasional erroneous estimation of single pixels in the maplets.

Topography results are also evaluated using the Hausdorff distance for two cases with different numbers of landmarks and maplet dimensions.

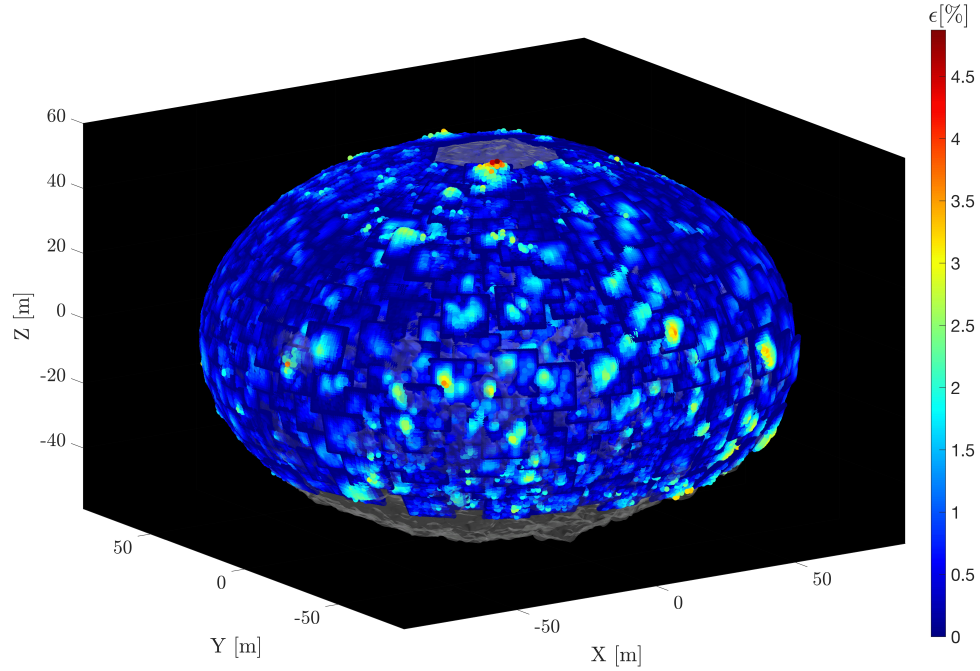


Figure 4.14: Height reconstruction error during EXP, represented on the asteroid model through Hausdorff distance (781 maplets, 19×19).

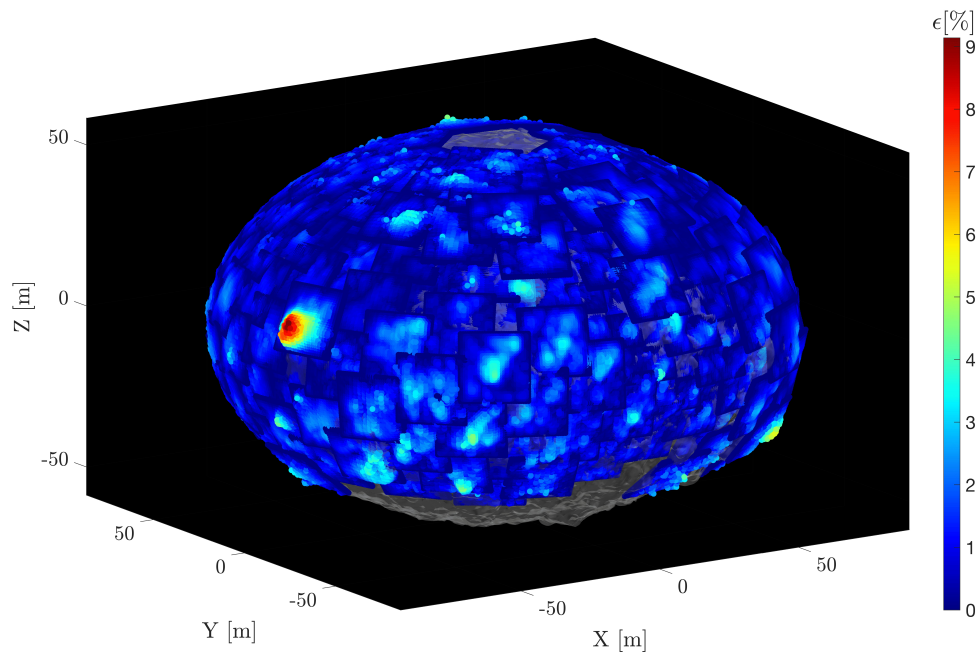


Figure 4.15: Height reconstruction error during EXP, represented on the asteroid model through Hausdorff distance (321 maplets, 29×29).

In the scenario illustrated in Figure 4.15, with 321 landmarks and maplets of 29×29 pixels, the errors in the estimated heights are clearly higher than the other case reported in Figure 4.14, which relies on a higher number of landmarks (781) and a smaller size of the maplets (19×19 pixels). In the first Figure the maximum reconstruction error is 4.36 m while with larger maplets the error reaches 8.18 m.

From a comparison with the asteroid model shown in Section 3.2.1, is possible to see that the highest estimation errors are in correspondence of the large and steep boulder located at a nearly equatorial latitude and clearly visible from the -X viewing direction in the plot. This terrain feature can be better seen on the left from the side view in Figure 4.8.

From a more detailed analysis it can be noticed that the algorithm is correctly detecting the presence of a large boulder in that area. However, its height is largely underestimated. This is a characteristic effect of the SPC algorithm, which struggles to represent steep boulders and tends to smooth their height. This effect can be seen also in Figure 2.9, related to the OSIRIS-REx mission to Bennu.

The smoothening effect is also enhanced by the fact that in the last scenario the large boulder is located in the central part of the maplet, for which no constraining heights from overlapping, limb analysis or other data are available during the height integration process. On the other hand, for the case with higher number of landmarks, in Figure 4.14, the boulder is reconstructed much more accurately, because it is located in an overlapping region, and some constraining heights are provided in the borders of those maplets.

This scenario can be considered more realistic, because constraining heights for large boulders as the considered one can be provided from a previous a limb analysis. Typically, the magnitude of this terrain feature can be better estimated from the limbs during the approach phase, and the SPC reconstruction can be complemented with data from laser altimetry, to mitigate its smoothening effect.

4.2. Landmark position vectors estimation

In this section is presented the validation process and the results obtained from the newly implemented SPC component for the estimation of landmark position vectors.

To obtain these first results, the initial landmark position is derived in each phase from the approximated ellipsoidal model of the asteroid. This is a good assumption in particular for the simulation of the first approach phase to the binary system and for FRP, when the complete mapping of the asteroid surface still has to be performed. However, for the following phases of Milani mission, taking the initial ellipsoidal positions of the landmarks represents a worse case assumption, because a previous knowledge of the asteroid surface should already be available in new refined body models developed in the previous phases.

Milani's position and orientation is taken as fixed during the execution of the landmark estimation SPC component. In fact, in order to validate the implementation of the algorithm to adjust the landmark positions, is important to decouple the effects of the uncertainties on the landmark position vectors from those of the spacecraft's pose. The spacecraft's pose is updated only in the following module, presented in Section 4.3. The initial values are taken from the nominal trajectory provided in the SPICE mission kernels.

The algorithm has been tested in several scenarios. Two scenarios are considered for a comparison, as they represent the two extreme situations in which the landmark estimation algorithm can be applied during the Milani mission.

Firstly, the algorithm is used with the images of the FRP, when the spacecraft is farther from the target. The selected number of landmarks is 61, for which a maplet dimension 29×29 is required to guarantee global coverage, as reported in Table 3.2.

In this scenario, the number of images associated to each landmark is illustrated in Figure 4.17. Important to notice, the visibility of the landmarks in this algorithm is different from the procedure previously applied in the topography reconstruction. In fact, in this case the photometric constraints are not applied, as detailed in Section 3.3.1.

Instead, in Figure 4.16 are illustrated the differences with respect to the real landmarks position vectors of respectively the initial ellipsoidal and the final updated landmark position vectors. The real landmark position vectors are assumed equal to the ones extracted from the reference asteroid model. The magnitudes of these vector differences are used to evaluate the performance of the algorithm, which is executed to correct the initial knowledge of each landmark position vector $\mathbf{V}$. In this specific case the position of the first 3 landmarks is not updated, because they are not associated to any image.

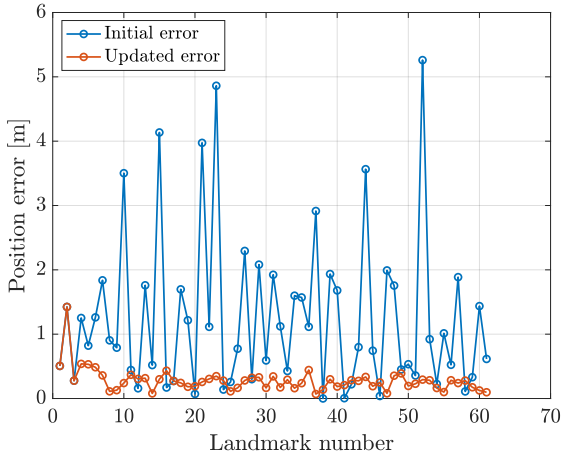


Figure 4.16: Landmark position error in FRP (61 landmarks).

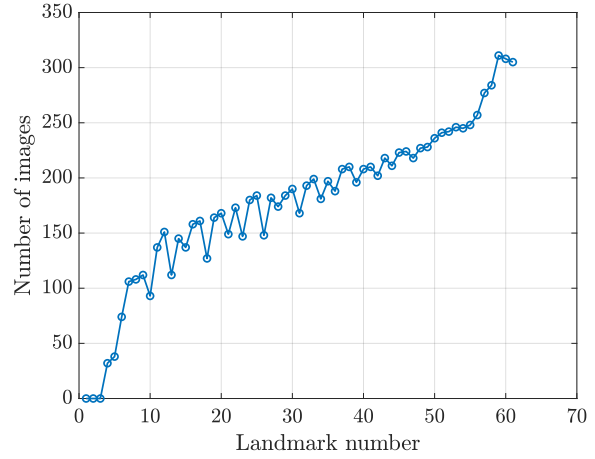


Figure 4.17: Number of images per landmark in FRP.

In the following figure are presented the results for the optimization of the position of each landmark, compared to the initial position extracted from the approximated ellipsoidal model. The position error is separated in its components. For each of them the error is given in absolute value.

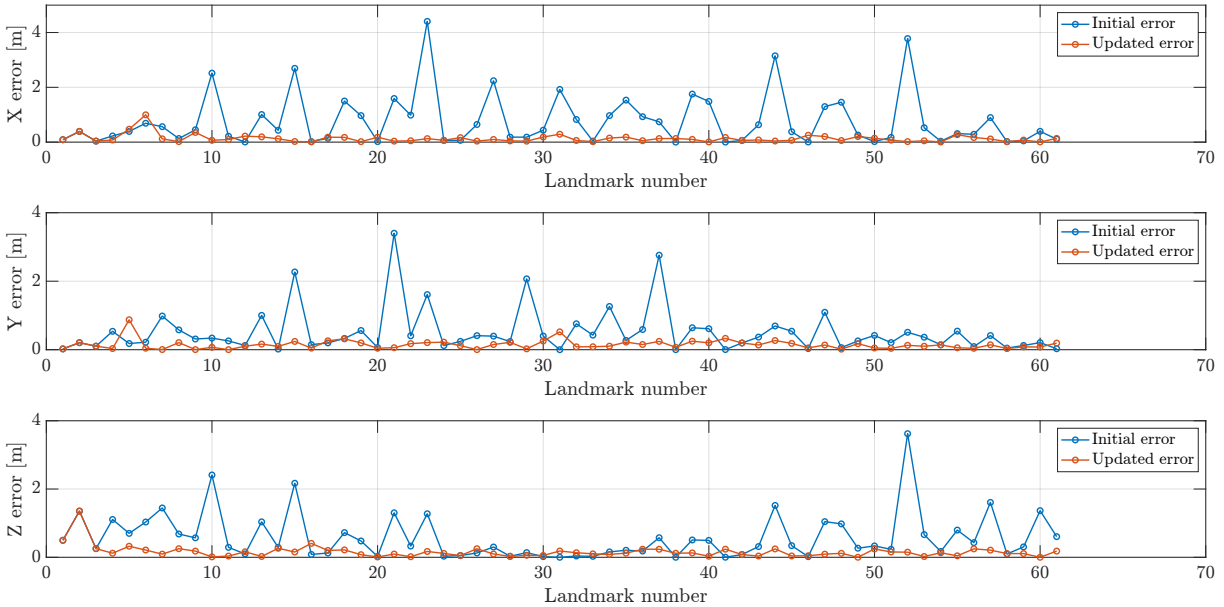


Figure 4.18: Landmark position error components during FRP (61 landmarks).

The results for the updated landmark positions show a significant improvement in the knowledge of the position vectors $\mathbf{V}$. The initial ellipsoid approximation, introducing a mean error of 1.25 m and reaching a maximum of 5.26 m, is drastically reduced to a mean final position error of 0.28 m.

In Figure 4.19 is represented the distance of each landmark from the barycenter of Dimorphos, for initial, updated and real landmark position. The characteristic parabolic shape is reflecting the oblate shape of the asteroid and the spiraling generation of landmarks with Fibonacci lattice, which proceeds from pole to pole.

It can be noted that the initial ellipsoidal approximation of the asteroid is on average overestimating the shape of Dimorphos. Instead, the updated positions of the landmarks are more closely matching the real positions of the asteroid model, successfully improving the initial knowledge. The only exceptions are the first three landmarks, for which the imaging conditions are not sufficient to update the positions.

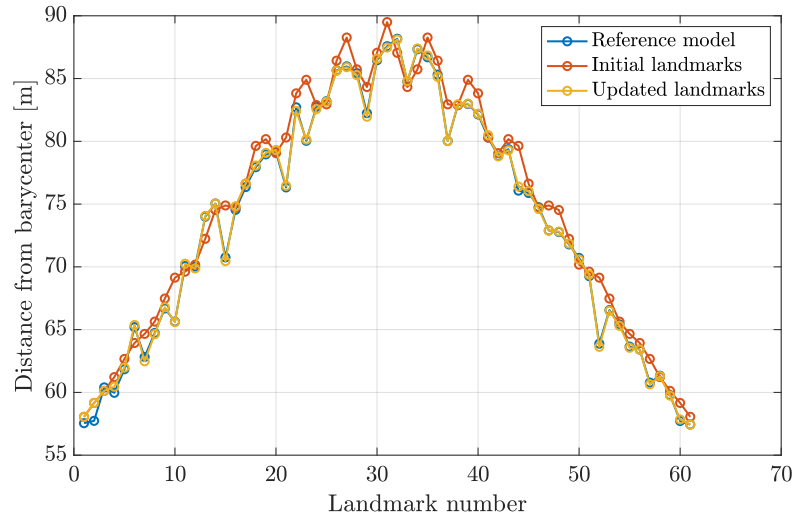


Figure 4.19: Initial, updated and real radius of the asteroid at each landmark position.

The second scenario considered is the more detailed observation of the asteroid during the experimental phase. The SPC parameters selected for this simulation are 781 landmarks, which would correspond to a maplet dimension of 19×19 pixels, as in Table 3.2. In EXP phase the accurate adjustment of the landmark position vectors $\mathbf{V}$ is crucial to provide precise control points for spacecraft navigation during the close proximity operations, leading to a possible landing attempt.

The number of images associated to each landmark are reported in Figure 4.21. Similarly to the FRP phase, the imaging conditions allow the estimation of most of the landmarks, except the first ones located close to the south pole of the asteroid. For those landmarks are not available sufficient images in which they appear visible. On the other hand, the removal of the photometric constraints to the visibility allows to see that the nominal trajectory is providing extensive imaging of the other pole. However, most of these images don't satisfy the constraints applied to obtain an optimal surface reconstruction.

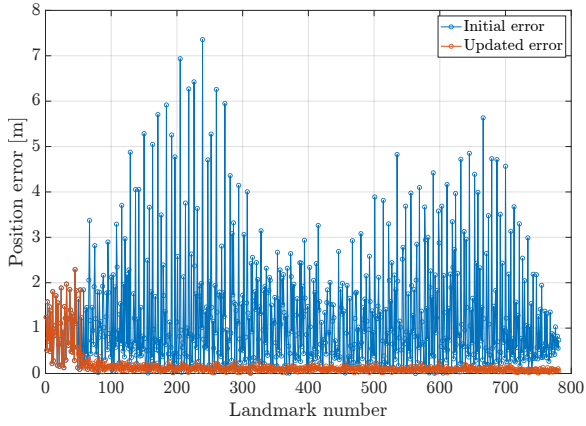


Figure 4.20: Landmark position error in EXP (781 landmarks).

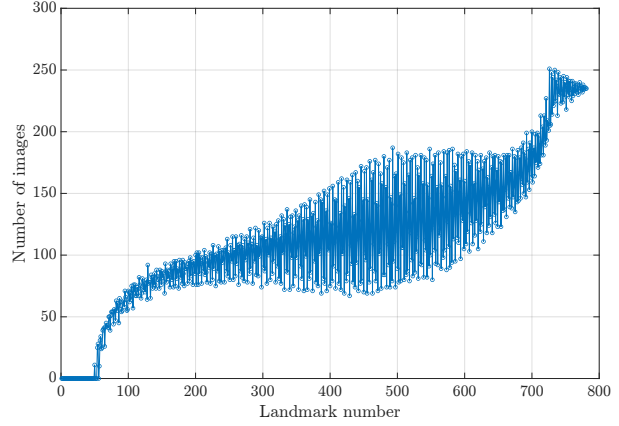


Figure 4.21: Number of images per landmark in EXP.

The magnitudes of the initial and updated position error for all the landmarks are reported in Figure 4.20. The initial ellipsoid approximation, which introduced a mean position error of 1.19 m with a maximum of 7.36 m, is significantly improved by the algorithm. The mean updated position error is only 0.16 m, considering the landmarks that were estimated.

For this second case, the position error components are kept with their sign, no more provided as absolute value as in the previous scenario.

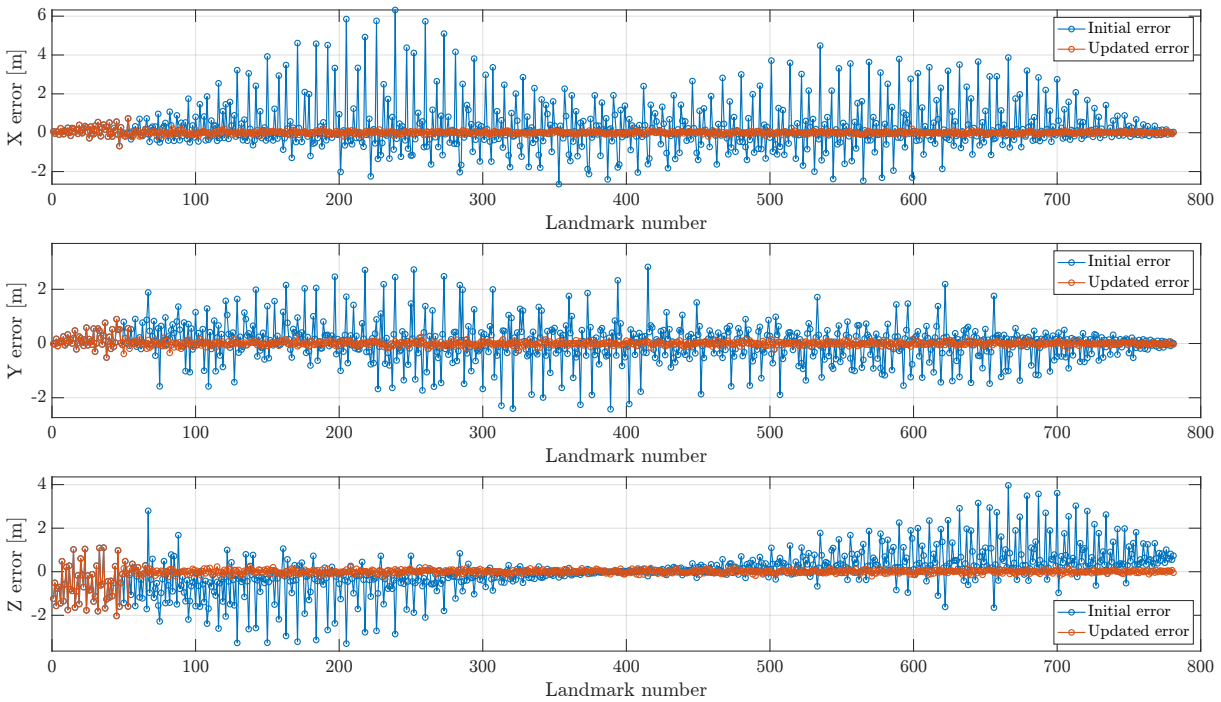


Figure 4.22: Landmark position error components during EXP (781 landmarks).

The different representation reveals a particular shape of the initial error along Z-axis, which is negative for low landmark numbers (negative latitudes) and positive for higher landmark numbers (positive latitudes). This translates in an overestimation of the polar radius in the initial ellipsoidal model of Dimorphos, which is not well reflecting its oblateness. In the central equatorial part instead the initial error of the ellipsoidal model is already well representing the real asteroid shape.

The results for the CRP are expected to be coherent to the ones previously detailed, because it is the longest phase that allowed the highest number of image acquisitions. Therefore, the number of images associated to each landmark is higher, and the solution of the problem is better constrained with more information.

4.3. Spacecraft pose estimation

The following section illustrates the validation procedure and the results obtained from the newly developed SPC module used to estimate position and orientation of Milani.

For all simulations, the real spacecraft's pose during the acquisition of images is assumed as the nominal for the mission, extracted from SPICE kernels, with pointing of Dimorphos defined as in Equations (3.1). The initial knowledge of Milani's position and orientation is different, due to the introduction of a Gaussian error.

The position vectors $\mathbf{V}$ of the landmarks are taken as fixed during the execution of this spacecraft's pose estimation SPC component, as detailed in Section 3.4.1. A perfect knowledge of the landmarks position is assumed, as they are extracted from the reference asteroid model.

As in the previous section, two test case scenarios are considered, respectively during FRP and EXP. The first scenario in FRP uses 61 landmarks while the second in EXP defines 781 control points on the asteroid surface.

For each spacecraft pose to be estimated, a different number of landmarks is visible at the correspondent imaging time. This parameter, reported in Figure 4.24 for the first case, is very important to guarantee a correct spacecraft pose reconstruction. A minimum of 3 control points is required in each image to accurately refine the initial knowledge of the position and orientation of the spacecraft [7]. It can be seen that the number of control points for almost all the imaging times is more than sufficient, ensuring an overconstrained solution for the spacecraft's pose. Only in some particular time instants, when in the image the target asteroid is eclipsed by its primary Didymos, the estimation is not possible because there are no landmarks in visibility of the NavCam. These eclipsing times can be seen having zero control points in Figure 4.24.

In Figure 4.23 are illustrated the magnitudes of the initial and updated Milani's position error vectors. These are computed as differences between the initial and updated spacecraft position vectors relative to Milani's nominal trajectory from SPICE kernels, assumed to represent the true imaging path. The magnitudes of these differences are used to evaluate the algorithm's performance in refining the spacecraft position vector $\mathbf{W}$ and orientation $\boldsymbol{\tau}$.

In the FRP case, a standard deviation of $\sigma_W = 100$ m is taken for each component of the spacecraft initial position vector $\mathbf{W}$. Instead, a standard deviation of $\sigma_O = 0.5$ deg is used for the initial orientation error, applied to each element of the small rotations vector $\boldsymbol{\tau}$.

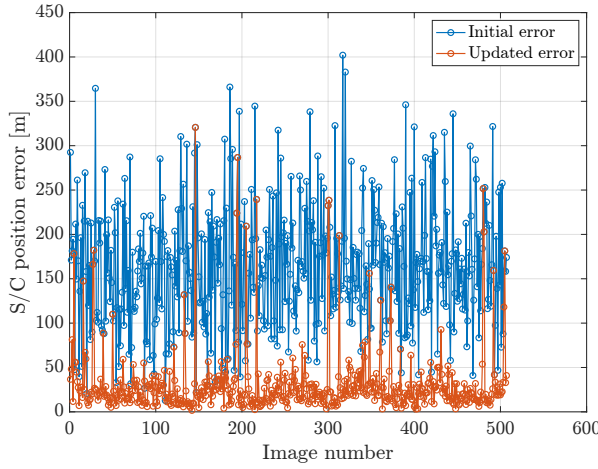


Figure 4.23: Spacecraft position error in FRP (61 landmarks).

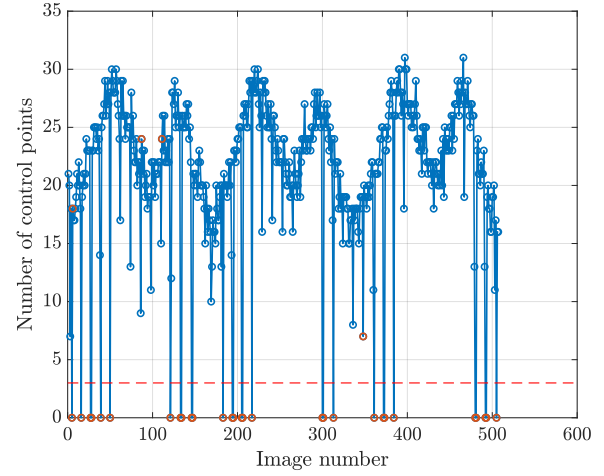


Figure 4.24: Number of control points (landmarks) per image in FRP.

From the previous figures, the algorithm successfully corrects the spacecraft position at most imaging times, aside from a few outliers (shown in red in Figure 4.24). Exceptions occur during eclipses, when no control points are visible and estimation is not possible. The initial mean position error of 164.3 m, with maximum value of 402 m, is significantly reduced to a mean updated error of 31.5 m. In Figure 4.25 are also provided the absolute initial and updated errors for each component of the spacecraft position vector, in logarithmic scale.

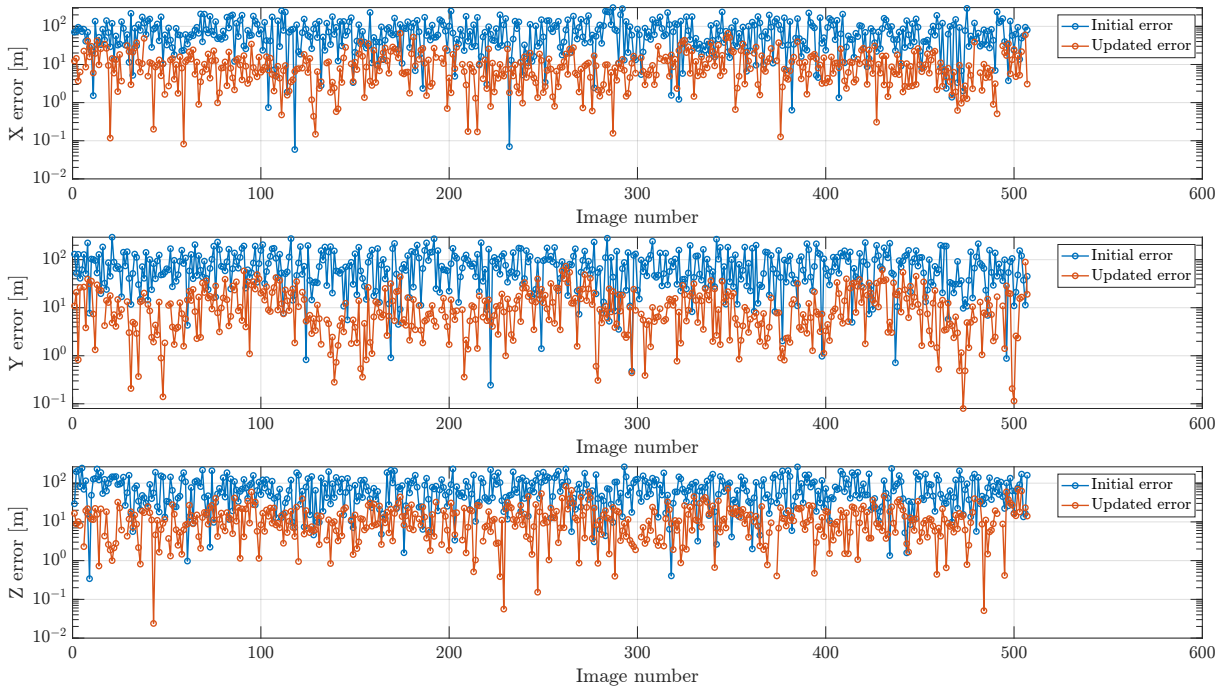


Figure 4.25: Absolute S/C position error components during FRP (61 landmarks).

To evaluate the capability of the implemented SPC component to adjust the orientation of the spacecraft, the 3 small rotation components τ_i are separately analyzed and represented in Figure 4.26. The nominal pointing corresponds to having these elements all equal to zero. In fact, they are initially introduced as a perturbation to the perfect pointing towards Dimorphos, as detailed in Section 3.4.1. Therefore, an improvement in orientation of the spacecraft corresponds to small rotation components τ_i converging to zero.

In the following figure, for all the rotation components the initial knowledge is successfully improved, significantly reducing the absolute orientation error.

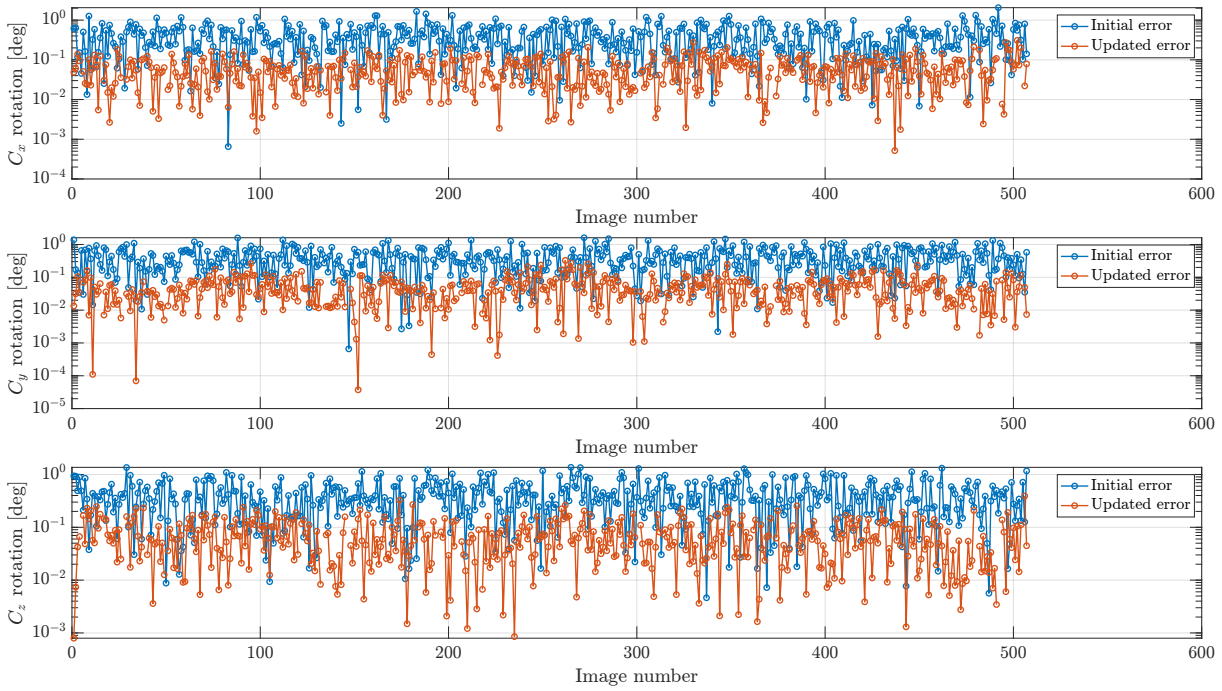


Figure 4.26: Absolute S/C orientation error components during FRP (61 landmarks).

The second scenario considered, during the experimental phase of the mission, is the most favorable for the algorithm of spacecraft's position estimation. In fact, in this case the higher number of landmarks corresponds to a significantly higher amount of control points for each imaging time, as shown in Figure 4.28. Therefore, the spacecraft's pose estimation is strongly overconstrained, and the algorithm is expected to provide a better performance. The initial and updated error in the spacecraft's position during EXP is given in Figure 4.27.

In this EXP case, a standard deviation of $\sigma_W = 10$ m is taken for each component of the initial position vector, while $\sigma_O = 0.05$ deg is used for the initial orientation error. These lower values reflect the better knowledge expected in this advanced phase of the mission.

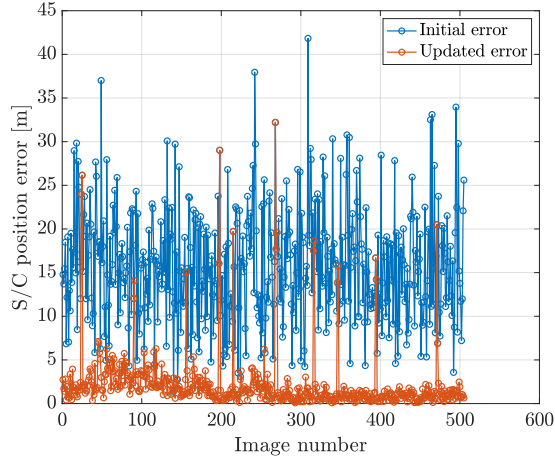


Figure 4.27: Spacecraft position error in EXP (781 landmarks).

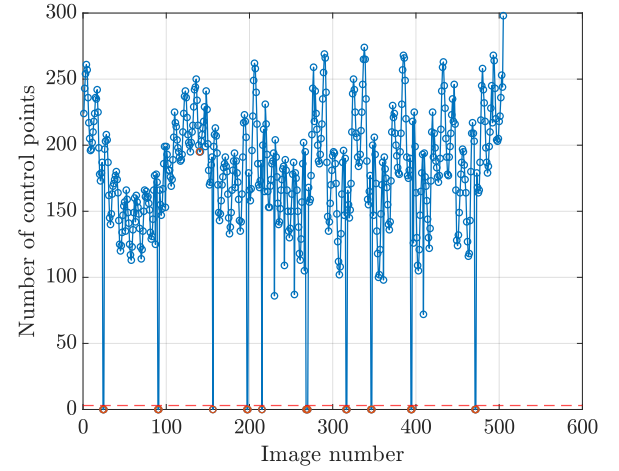


Figure 4.28: Number of control points (landmarks) per image in EXP.

Despite the cases when the eclipsing of Didymos is preventing the visibility of the landmarks in the image, in almost all the other imaging times the position error is drastically reduced. Only one isolated outlier can be seen in Figure 4.28, for which the estimation is not successful.

The initial mean position error of 15.77 m, with a maximum of 41.83 m, is reduced to an updated Milani's mean position error of only 2.24 m.

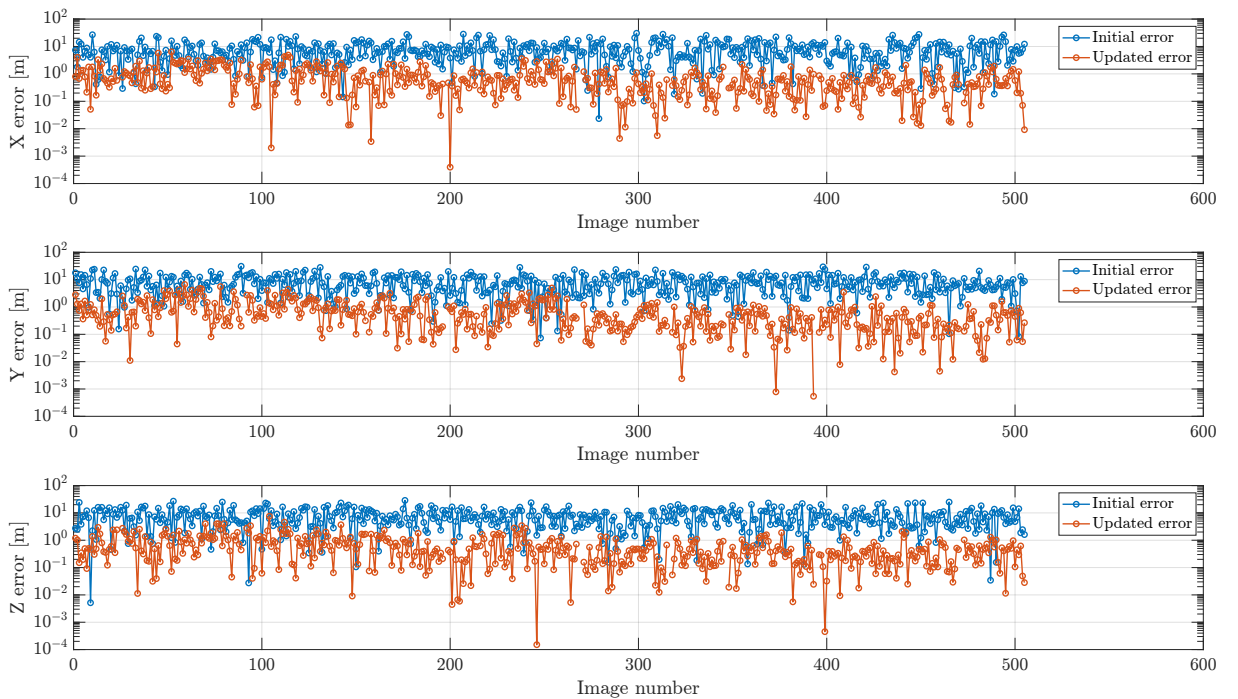


Figure 4.29: Absolute S/C position error components during EXP (781 landmarks).

Analogously to the previous scenario, the performance of orientation estimation is evaluated from the small rotation components τ_i . The errors in these parameters are reported in Figure 4.30, in absolute value and in logarithmic scale.

The algorithm is successfully adjusting the small shifts in orientation, obtaining an updated value of mean absolute error in the order of 0.01 deg.

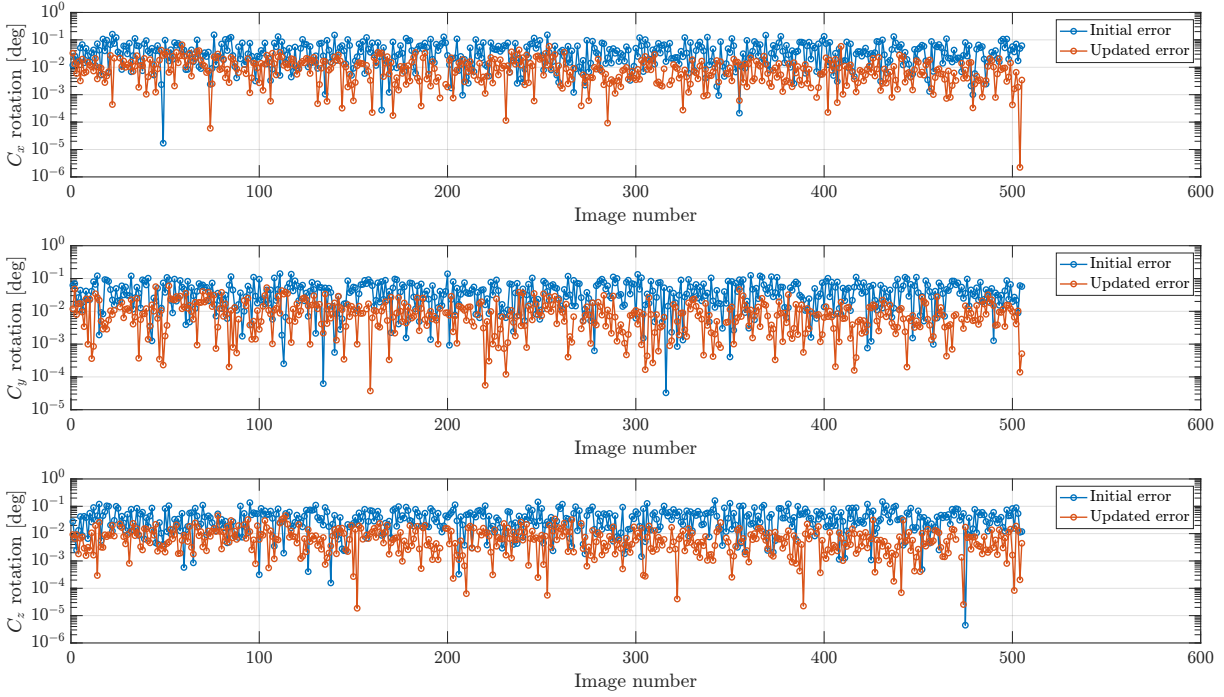


Figure 4.30: Absolute S/C orientation error components during EXP (781 landmarks).

Additional tests might be conducted on the spacecraft pose estimation algorithm, integrating it in a iterative approach with the landmark position estimation SPC component, previously presented and validated in Section 4.2.

In this approach, limitations in the correction of the parameters will arise due to the mutual interference between the discrepancies in spacecraft pose and landmark position. In fact, an error in the first inherently impacts the estimation of the other parameter, which is assuming a correct knowledge of the first. Therefore, to prevent the solution from falling too far from the real due to these uncertainties, the regularization terms might play a key role in ensuring still a good performance in both the SPC modules. The fine tuning of these allows a good estimation of both the modules, and will therefore guarantee a good topography estimation.

5 | Conclusions

In this thesis are detailed the implementation and results of the main blocks of SPC architecture. The algorithm is used for performing a shape reconstruction of asteroid Dimorphos from images acquired with the navigation camera of Milani mission.

The SPC component for topography reconstruction was improved and tested in different scenarios, to evaluate the capability of the NavCam to provide valuable information on the asteroid's terrain. Despite intrinsic limitations due to the characteristics of the camera, not specifically designed for shape reconstruction, processing the simulated images with SPC revealed the effectiveness of the algorithm in estimating the terrain shape with a half-meter level accuracy. In particular, during the experimental phase of the mission, this method allows to reconstruct the asteroid surface with a mean error of 0.35 m across all the maplets. The terrain estimation is not executed for some landmarks located on the polar regions, which have insufficient or not suitable imaging conditions. In addition, the high resolution imaging during the close approaches in CRP can be exploited by selecting proper SPC parameters for optimizing the reconstruction of the DART impact site, reaching a sub-meter accuracy with a maximum error of 1.75 m for sharp boulders.

These topography results can give crucial information to improve the shape model of Dimorphos, providing higher detail in the areas not previously imaged during DART mission. The implemented algorithm can guarantee a sufficient accuracy in reconstructing the largest meter-level boulders and craters on the asteroid surface, in particular at low latitudes, complementing the measurements of the other instruments on Milani mission, specifically dedicated to surface mapping.

The SPC component for the correction of the landmark position vectors was newly implemented and validated in different simulated scenarios. The results show a significant improvement of the initial knowledge, reducing the initial position error of the ellipsoidal approximated model to a mean value of only 0.28 m in FRP and 0.16 m in EXP.

The third block of SPC architecture was implemented and tested in different phases, reflecting the different uncertainties in the spacecraft pose during the mission. The validation of the algorithm revealed a consistent capability of reducing the position and

orientation errors. An initial mean position error of 164.3 m in FRP is decreased to 31.5 m, while an error of 15.77 m during EXP can be corrected to only 2.24 m. The orientation is also drastically reduced in both cases, successfully adjusting the spacecraft's pose for a better shape reconstruction.

The validation of these key SPC building blocks, integrated with the previous algorithm of topography reconstruction, provide a powerful tool for reducing the uncertainties during the mission. Considering the challenging low-gravity environment and the unique small dimensions of the asteroid, an integrated SPC architecture for shape reconstruction can provide invaluable data, and eventually support the generation of models for autonomous vision-based navigation.

Several future developments can be individuated to complement this work. Firstly, a global model estimation block of SPC can be developed, integrating the overlapping search algorithm to impose optimization constraints. This would allow to combine the information of the single maplets into a unique asteroid model representation.

In addition, a more comprehensive mission scenario could be considered. Additional data from a previous limb-based modeling could be simulated, together with point clouds obtained from laser altimetry, very common in those asteroid mission. These other measurements are crucial for the development of an optimized height integration process, as in Gaskell's SPC implementation. The height distribution of the maplets can be obtained from a relaxation procedure, iteratively imposing constraining heights at randomly selected positions until a converged height solution is reached, consistent with the estimated terrain slopes.

The integration of all the SPC components allows a more realistic simulation of the whole mission scenario, in which the uncertainties in the different parameters combine and influence the reciprocal estimation. The resulting shape model of the asteroid could also be tested for visual based navigation.

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A | Analytical gradient for topography estimation

In the following section is detailed the analytical derivation of the gradient of the objective function used in the SPC algorithm for topography estimation.

$$\begin{aligned} \frac{\partial f_k}{\partial t_x} = & -\lambda_k (1 + t_z) \left[\frac{7s_x}{20\sqrt{t_x^2 + t_y^2 + 1}} + \frac{13s_x}{20(c_z + s_z + c_x t_x + c_y t_y + s_x t_x + s_y t_y)} + \right. \\ & \left. - \frac{(c_x + s_x) \left(\frac{13s_z}{20} + \frac{13s_x t_x}{20} + \frac{13s_y t_y}{20} \right)}{(c_z + s_z + c_x t_x + c_y t_y + s_x t_x + s_y t_y)^2} - \frac{t_x \left(\frac{7s_z}{20} + \frac{7s_x t_x}{20} + \frac{7s_y t_y}{20} \right)}{(t_x^2 + t_y^2 + 1)^{3/2}} \right] \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \frac{\partial f_k}{\partial t_y} = & -\lambda_k (1 + t_z) \left[\frac{7s_y}{20\sqrt{t_x^2 + t_y^2 + 1}} + \frac{13s_y}{20(c_z + s_z + c_x t_x + c_y t_y + s_x t_x + s_y t_y)} + \right. \\ & \left. - \frac{(c_y + s_y) \left(\frac{13s_z}{20} + \frac{13s_x t_x}{20} + \frac{13s_y t_y}{20} \right)}{(c_z + s_z + c_x t_x + c_y t_y + s_x t_x + s_y t_y)^2} - \frac{t_y \left(\frac{7s_z}{20} + \frac{7s_x t_x}{20} + \frac{7s_y t_y}{20} \right)}{(t_x^2 + t_y^2 + 1)^{3/2}} \right] \end{aligned} \quad (\text{A.2})$$

$$\frac{\partial f_k}{\partial t_z} = -\lambda_k \Phi(t_x, t_y) \quad (\text{A.3})$$

B | Analytical gradient for landmark position estimation

In the following section is detailed the analytical derivation of the gradient of the objective function used in the SPC algorithm for landmark position estimation.

Consider the following equations for the coordinates of the landmarks in the image:

$$\begin{cases} u = \frac{f}{p_{\text{size}}} \frac{x_{Cam}}{z_{Cam}} + u_0 \\ v = \frac{f}{p_{\text{size}}} \frac{y_{Cam}}{z_{Cam}} + v_0 \end{cases} \quad (\text{B.1})$$

where the components of the camera-landmark vector $\mathbf{r}_{Cam}$ in camera frame are given in the following expression, in which auxiliary variables $\mathbf{R}$ and $\mathbf{b}$ are introduced to ease the subsequent derivations:

$$\begin{aligned} \mathbf{r}_{Cam} = \begin{bmatrix} x_{Cam} \\ y_{Cam} \\ z_{Cam} \end{bmatrix} &= \mathbf{R}_{Body2Cam} \cdot \mathbf{r}_{Body} \\ &= (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam})^T \cdot (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W}) \\ &= \mathbf{R} (\mathbf{V} - \mathbf{b}) = \mathbf{R} \mathbf{V} - \mathbf{R} \mathbf{b} \end{aligned} \quad (\text{B.2})$$

For the derivation of the analytical expression of the objective function's gradient, the Jacobian chain rule is employed:

$$\begin{cases} \frac{\partial u}{\partial \mathbf{V}} = \frac{\partial u}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{V}} \\ \frac{\partial v}{\partial \mathbf{V}} = \frac{\partial v}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{V}} \end{cases} \quad (\text{B.3})$$

$$\quad (\text{B.4})$$

Therefore, the partial derivatives composing the previous equations are derived singularly:

$$\frac{\partial u}{\partial \mathbf{r}_{Cam}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \end{bmatrix} \quad (\text{B.5})$$

$$\frac{\partial v}{\partial \mathbf{r}_{Cam}} = \frac{f}{p_{size}} \begin{bmatrix} 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \quad (\text{B.6})$$

$$\frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{V}} = \mathbf{R} = (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam})^T = \mathbf{R}_{Body2Cam} \quad (\text{B.7})$$

Finally, the complete gradient formulation can be assembled:

$$\frac{\partial(u, v)}{\partial \mathbf{V}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam})^T \quad (\text{B.8})$$

C | Analytical gradient for spacecraft pose estimation

In the following section is detailed the analytical derivation of the gradient of the objective function used in the SPC algorithm for spacecraft's pose estimation.

Consider the following equations for the coordinates of the landmarks in the image:

$$\begin{cases} u = \frac{f}{p_{\text{size}}} \frac{x_{Cam}}{z_{Cam}} + u_0 \\ v = \frac{f}{p_{\text{size}}} \frac{y_{Cam}}{z_{Cam}} + v_0 \end{cases} \quad (\text{C.1})$$

where the components of the camera-landmark vector $\mathbf{r}_{Cam}$ in camera frame are given in the following expression, in which the auxiliary variable $\mathbf{R}$ is introduced to ease the subsequent derivations:

$$\begin{aligned} \mathbf{r}_{Cam} &= \mathbf{R}_{Body2Cam} \cdot \mathbf{r}_{Body} \\ &= (\mathbf{R}_{Inertial2Body} \mathbf{R}_{Inertial2Cam}^*)^T \cdot (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W}) \\ &= (\mathbf{R}_{Inertial2Body} (\mathbf{I} + \mathbf{S}(\boldsymbol{\tau})) \mathbf{R}_{Inertial2Cam})^T \cdot (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W}) \\ &= \mathbf{R}(\boldsymbol{\tau})^T (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W}) \end{aligned} \quad (\text{C.2})$$

For the derivation of the analytical expression of the objective function's gradient, the derivatives with respect to $\mathbf{W}$ and $\boldsymbol{\tau}$ are separated. For each of them, the Jacobian chain rule is employed to split the derivation in simpler components:

$$\begin{cases} \frac{\partial u}{\partial \mathbf{W}} = \frac{\partial u}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} \\ \frac{\partial v}{\partial \mathbf{W}} = \frac{\partial v}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} \end{cases} \quad (\text{C.3})$$

$$\quad (\text{C.4})$$

$$\begin{cases} \frac{\partial u}{\partial \boldsymbol{\tau}} = \frac{\partial u}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} \\ \frac{\partial v}{\partial \boldsymbol{\tau}} = \frac{\partial v}{\partial \mathbf{r}_{Cam}} \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} \end{cases} \quad (\text{C.5})$$

$$\quad (\text{C.6})$$

The partial derivatives of u and v in the previous equations are derived as follows:

$$\frac{\partial u}{\partial \mathbf{r}_{Cam}} = \frac{f}{p_{size}} \begin{bmatrix} 1 & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ z_{Cam} & 0 & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \quad (C.7)$$

$$\frac{\partial v}{\partial \mathbf{r}_{Cam}} = \frac{f}{p_{size}} \begin{bmatrix} 0 & 1 & -\frac{y_{Cam}}{z_{Cam}^2} \\ 0 & z_{Cam} & -\frac{x_{Cam}}{z_{Cam}^2} \end{bmatrix} \quad (C.8)$$

Another auxiliary variable $\mathbf{s}$ was introduced in the derivation of $\mathbf{r}_{Cam}$ with respect to $\boldsymbol{\tau}$, to make the computations simpler:

$$\mathbf{s} = \mathbf{R}_{Inertial2Body}^T (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W}) = \mathbf{R}_{Inertial2Body}^T \mathbf{V} - \mathbf{W} \quad (C.9)$$

In addition, to obtain the partial derivatives of $\mathbf{r}_{Cam}$ with respect to $\mathbf{W}$ and $\boldsymbol{\tau}$, some properties of the skew-symmetric matrix were exploited:

$$\mathbf{S}(\boldsymbol{\tau})^T = -\mathbf{S}(\boldsymbol{\tau}) \quad (C.10) \quad \mathbf{S}(\boldsymbol{\tau})\mathbf{s} = -\mathbf{S}(\mathbf{s})\boldsymbol{\tau} \quad (C.11)$$

Therefore, the other partial derivatives are obtained as follows:

$$\begin{aligned} \frac{\partial \mathbf{r}_{Cam}}{\partial \mathbf{W}} &= \frac{\partial [\mathbf{R}(\boldsymbol{\tau})^T (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W})]}{\partial \mathbf{W}} \\ &= -\mathbf{R}(\boldsymbol{\tau})^T \mathbf{R}_{Inertial2Body} \\ &= -(\mathbf{R}_{Inertial2Body} (\mathbf{I} + \mathbf{S}(\boldsymbol{\tau})) \mathbf{R}_{Inertial2Cam})^T \mathbf{R}_{Inertial2Body} \\ &= -\mathbf{R}_{Inertial2Cam}^T (\mathbf{I} - \mathbf{S}(\boldsymbol{\tau})) \mathbf{R}_{Inertial2Body}^T \mathbf{R}_{Inertial2Body} \\ &= \mathbf{R}_{Inertial2Cam}^T (\mathbf{S}(\boldsymbol{\tau}) - \mathbf{I}) \end{aligned} \quad (C.12)$$

$$\begin{aligned} \frac{\partial \mathbf{r}_{Cam}}{\partial \boldsymbol{\tau}} &= \frac{\partial [\mathbf{R}(\boldsymbol{\tau})^T (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W})]}{\partial \boldsymbol{\tau}} \\ &= \frac{\partial [(\mathbf{R}_{Inertial2Body} (\mathbf{I} + \mathbf{S}(\boldsymbol{\tau})) \mathbf{R}_{Inertial2Cam})^T (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W})]}{\partial \boldsymbol{\tau}} \\ &= \frac{\partial [\mathbf{R}_{Inertial2Cam}^T (\mathbf{I} - \mathbf{S}(\boldsymbol{\tau})) \mathbf{R}_{Inertial2Body}^T (\mathbf{V} - \mathbf{R}_{Inertial2Body} \mathbf{W})]}{\partial \boldsymbol{\tau}} \\ &= \frac{\partial [-\mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\boldsymbol{\tau}) \mathbf{s}]}{\partial \boldsymbol{\tau}} \\ &= \frac{\partial [\mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \boldsymbol{\tau}]}{\partial \boldsymbol{\tau}} \\ &= \mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \end{aligned} \quad (C.13)$$

Finally, the complete gradient formulation can be assembled for each of the variables $\mathbf{W}$ and $\boldsymbol{\tau}$ in a 2×3 matrix. These can be concatenated to obtain the final 2×6 gradient matrix:

$$\left\{ \begin{array}{l} \frac{\partial(u,v)}{\partial \mathbf{W}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \mathbf{R}_{Inertial2Cam}^T (\mathbf{S}(\boldsymbol{\tau}) - \mathbf{I}) \end{array} \right. \quad (\text{C.14})$$

$$\left\{ \begin{array}{l} \frac{\partial(u,v)}{\partial \boldsymbol{\tau}} = \frac{f}{p_{size}} \begin{bmatrix} \frac{1}{z_{Cam}} & 0 & -\frac{x_{Cam}}{z_{Cam}^2} \\ 0 & \frac{1}{z_{Cam}} & -\frac{y_{Cam}}{z_{Cam}^2} \end{bmatrix} \mathbf{R}_{Inertial2Cam}^T \mathbf{S}(\mathbf{s}) \end{array} \right. \quad (\text{C.15})$$

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