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Contracts as Coordination Mechanism between Brands and Retailers: The Mid-Season Reorder Model

TESI DI LAUREA MAGISTRALE IN
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Abstract

The enquiry into contractual design for demand management within the high-end jewellery industry is a crucial issue that deserves further investigation.

Contracts, in this context, play an extraordinarily important role as they can have a significant impact on the effectiveness and operational dynamics of supply chains. Consequently, the need to devise and implement contractual forms in line with the peculiar characteristics of the luxury jewellery industry and the parties involved emerges imperatively.

A pivotal criterion, which guides the categorisation of the agreements between the parties, concerns the incentives that facilitate the transfer of payments. This is of utmost importance, as it holds the potential to shape the intricate dynamics inherent in the supply chain. In particular, this can act as a catalyst for promoting collaboration between suppliers and brand owners, thereby incentivising behaviour aimed at greater efficiency and better alignment of objectives.

A further important element is the fair sharing of inventory risk, a substantial element in the context of luxury jewellery. Given the inherent characteristics and susceptibility to unpredictable product demand fluctuations in this industry, inventory management is a primary challenge. Its strategic management through the implementation of contractual clauses emerges as a measure to mitigate the negative consequences resulting from an excess or shortage of products. This strategic approach strives to ensure a better convergence of supply and demand, minimising waste and costs associated with inadequate inventory management.

In addition, an emerging threat could undermine the effectiveness of demand management. Market signals indicate an increasing inclination of businesses in the field to outsource production processes and adopt indirect marketing channels. Although initially advantageous for reducing production costs and accessing new markets, this strategy is proving to be permeated by significant weaknesses.

The main disadvantage of this approach is the loss of control over production and retail activities, potentially leading to a decrease in brand image management. In an industry where brand image and exclusivity are dominant elements, this lack of control could lead to negative consequences. Faced with these challenges, many companies are reconsidering their strategies, re-evaluating the use of internal resources for production activities. This shift underlines the complexities of demand management in this specific industry. It is, therefore, essential that the demand management process takes centre stage to ensure that brand owners can effectively oversee both the production and distribution of their products, while simultaneously maintaining strict control over brand image.

Ineffective demand management, thus, could reverberate on several critical success factors, including the ability to meet customer demand, preserving brand exclusivity, reducing inventory management costs, and ensuring product availability in line with customer preferences.

In light of these considerations, the adoption of effective coordination contracts emerges as an essential factor in addressing these challenges and establishing a collaborative supply chain. Coordination contracts facilitate the equitable sharing of costs, risks, and rewards between different actors in the supply chain. Such a methodological approach mitigates potential conflicts of interest between the various parties involved, thus contributing to a fruitful cooperation environment. Moreover, given the inclination of each entity involved to inherently pursue maximum individual benefit, sometimes even to the detriment of collective success, they serve as a remarkably useful tool to identify and correct potential behavioural misalignments between actors, promoting closer alignment with common supply chain objectives. In this way, contracts provide a key vehicle for promoting genuine collaboration between the parties involved, helping to strengthen the operational and relational network of the supply chain under consideration.

Contemporary literature has mainly focused on the analysis of existing contractual forms and the development of new contractual models aimed at improving their efficiency. However, a glaring gap that has emerged from this analysis is the absence of a systematic effort to combine different contract types with the goal of developing a hybrid contractual form capable of exploiting the advantages inherent in different contractual forms. This gap not only represents a challenge, but also constitutes a significant opportunity that this research proposes to explore in depth.

To this end, the objective of this paper is to thoroughly investigate the contractual aspect, clarifying the crucial function that contracts play in coordinating supply chain operations and reducing the difficulties associated with demand management. The expected result is an advanced framework aimed at fostering fruitful collaboration, greater adaptability and, consequently, the achievement of more robust financial results.

For the development of this work, a meticulous analysis of a series of articles published from 2001 to the present was undertaken. The primary goal of this analysis is to examine the trends and challenges of contract design within the industry of interest to draw a clear picture of the current situation. The use of tools belonging to the branch of game theory provided an innovative perspective, bringing to light several scenarios in which hypothetical retailers and producers might find themselves in the dynamic reality of the luxury industry. The validity and effectiveness of the formulated statements were carefully investigated with Microsoft Excel software. In this context, it was possible to model a quantitative simulation of the distribution of demand for the products examined, as well as to identify best practices that retailers should adopt in specific contexts. The financial performance of the supply chain agents was subsequently compared with that which could be achieved by adopting two competing policies. The proposed new model differs from such policies by recognising the innumerable financial benefits of judiciously evaluating key variables such as reorder interval, updated demand forecasting, accurate management of unsold goods and the inherent variability in market demands.

Further comparisons and observations were conducted through two more detailed analyses of the proposed new model.

The results obtained highlight the limitations of the model currently in use. Given the multiple external variables influencing the demand for luxury products, such as economic conditions, fashion trends and social dynamics, forecasting demand is a complex task, and a model such as the Newsvendor model, designed for scenarios with stable and predictable demand, may have difficulty adapting to this volatility, potentially leading to overstocking or product shortages. This, in turn, can negatively impact the retailer's performance, resulting in missed sales opportunities and dissatisfaction from customers, who, in the luxury context, expect exclusivity and immediate availability.

Against this backdrop, the Mid-Season Reorder model emerges as a promising solution to address these challenges. By accommodating mid-season adjustments and adopting a more flexible approach to inventory management, this model allows luxury brands to respond more effectively to the changing nature of their market. In turn, the inherent flexibility encourages greater synchronisation between supply and demand, thereby mitigating the risks associated with depleted or excess inventory. The profits achieved during the simulation phase, determined using the new formulas and parameters, are a clear demonstration of this.

However, it is crucial to emphasise that the introduction of a contractual model allowing for dynamic reordering may introduce additional challenges in supply chain management. Effective communication and close cooperation with producers are prerequisites, as is the ability to analyse data in real time and to adapt promptly to changing market conditions. Furthermore, it should be noted that not all products or sectors lend themselves to the implementation of a contractual model allowing for dynamic reordering. The effectiveness of such a model depends on the variability of demand and the complexities of the supply chain. Therefore, it is essential to conduct a targeted assessment of the contexts in which this model can be successfully applied.

In conclusion, this paper has meticulously explored the central role of contracts in demand management in the luxury jewellery industry. It emphasised the need to design industry-specific contracts and presented an innovative hybrid contract model, which exploits the advantages of existing models and seeks to address the challenges and problems they fail to address. Moreover, the combination of concepts such as the possibility of reordering, the presence of updated demand forecasts, the revaluation of unsold products and the introduction of new key variables, such as the reorder interval, not only highlights the differences of the new model compared to existing models, but also underlines the innovative impact it has on the current contract literature in the high-end jewellery industry. The improvement that this new contract model brings offers companies in the sector a competitive advantage, as it enables them to respond more effectively and efficiently to changing customer needs.

The proven validity of the Mid-Season Reorder model confirms its applicability and suitability as a tool to address the difficulties of current supply chain management in the industry under review. The newly acquired insights not only enrich the existing literature, but also offer practical guidelines for industry agents wishing to improve the flexibility, adaptability, and efficiency of their supply chain.

Key-words: Fashion; Luxury; Jewellery; Supply chain management; Contract; Retail; Game theory.

Abstract in Italiano

L'indagine sulla progettazione contrattuale per la gestione della domanda nell'ambito dell'industria della gioielleria di alta gamma costituisce un nodo cruciale che merita ulteriori approfondimenti.

I contratti, in questo contesto, svolgono un ruolo di rilevanza straordinaria, in quanto possono esercitare un impatto significativo sull'efficacia e sulle dinamiche operative delle catene di fornitura. Di conseguenza, emerge con imperatività la necessità di concepire e attuare forme contrattuali in linea con le peculiari caratteristiche del settore gioielliero di lusso e delle parti coinvolte.

Un criterio cardine, che orienta la categorizzazione degli accordi tra le parti, riguarda gli incentivi che agevolano il trasferimento dei pagamenti. Tale aspetto si configura come di massima importanza, in quanto detiene il potenziale per plasmare le intricate dinamiche insite nella catena di fornitura. In particolare, ciò può fungere da catalizzatore per la promozione della collaborazione tra i fornitori e i titolari dei marchi, incentivando così comportamenti finalizzati a una maggiore efficienza e un miglior allineamento degli obiettivi.

Un ulteriore elemento di rilievo è l'equa condivisione del rischio di inventario, un elemento sostanziale nel contesto della gioielleria di lusso. Date le caratteristiche intrinseche e la suscettibilità alle fluttuazioni imprevedibili della domanda dei prodotti in questo settore, la gestione delle scorte si configura come una sfida primaria. La sua gestione strategica attraverso l'implementazione di clausole contrattuali emerge come una misura per mitigare le conseguenze negative derivanti da un eccesso o una carenza di prodotti. Questo approccio strategico mira a garantire una migliore convergenza tra domanda e offerta, riducendo al minimo gli sprechi e i costi associati a una gestione inadeguata delle scorte.

In aggiunta, una minaccia emergente potrebbe compromettere l'efficacia della gestione della domanda. Segnali da parte del mercato indicano una crescente inclinazione delle aziende del settore a esternalizzare i processi produttivi e ad adottare canali di commercializzazione indiretti. Benché inizialmente vantaggiosa per la riduzione dei costi di produzione e l'accesso a nuovi mercati, questa strategia si sta rilevando permeata da significativi punti deboli.

Il principale svantaggio derivante da tale approccio è la perdita di controllo sulla produzione e sulle attività di vendita al dettaglio, con conseguente potenziale decremento della gestione dell'immagine del marchio. In un settore in cui l'immagine del marchio e l'esclusività costituiscono elementi dominanti, questa mancanza di controllo potrebbe comportare conseguenze negative. Di fronte a queste sfide, numerose imprese stanno riconsiderando le proprie strategie, rivalutando il ricorso a risorse interne per le attività produttive. Questo cambiamento di tendenza sottolinea le complessità della gestione della domanda in questo specifico settore. Risulta, dunque, essenziale che il processo di gestione della domanda assuma un ruolo centrale al fine di garantire che i proprietari dei marchi possano sovrintendere in modo efficace sia alla produzione sia alla distribuzione dei loro prodotti, mantenendo simultaneamente un controllo rigoroso sull'immagine del marchio.

Una gestione inefficace della domanda, dunque, potrebbe riverberare su diversi fattori critici di successo, compresa la capacità di soddisfare la domanda dei clienti, la preservazione dell'esclusività del marchio, la riduzione dei costi legati alla gestione delle scorte e l'assicurazione della disponibilità dei prodotti in linea con le preferenze della clientela.

Alla luce di tali considerazioni, l'adozione di contratti di coordinamento efficaci emerge come un fattore essenziale per fronteggiare le sfide emerse e stabilire una catena di fornitura collaborativa. I contratti di coordinamento agevolano l'equa condivisione di costi, rischi e ricompense tra i diversi attori della catena di fornitura. Tale approccio metodologico consente di attenuare i potenziali conflitti di interesse tra le varie parti coinvolte, contribuendo così a instaurare un ambiente di cooperazione proficuo. Inoltre, data l'inclinazione di ogni entità coinvolta a perseguire intrinsecamente il massimo vantaggio individuale, talvolta anche a discapito del successo collettivo, essi si configurano come uno strumento di notevole utilità per identificare e correggere potenziali disallineamenti comportamentali tra gli attori, promuovendo un allineamento più stretto con gli obiettivi comuni della catena di fornitura. Così facendo, i contratti costituiscono un veicolo chiave per promuovere un'autentica collaborazione tra le parti coinvolte, contribuendo a rafforzare il tessuto operativo e relazionale della supply chain in esame.

La letteratura contemporanea si è concentrata prevalentemente sull'analisi delle forme contrattuali esistenti e sullo sviluppo di nuovi modelli contrattuali volti a migliorarne l'efficienza. Tuttavia, una lacuna evidente emersa da questa analisi è l'assenza di uno sforzo sistematico per combinare diversi tipi di contratto, con l'obiettivo di sviluppare una forma contrattuale ibrida in grado di sfruttare i vantaggi insiti nelle diverse forme contrattuali. Tale lacuna non solo rappresenta una sfida, ma costituisce altresì un'opportunità di rilievo che questa ricerca si propone di esplorare approfonditamente.

A tal fine, il presente lavoro si prefigge l'obiettivo di indagare a fondo l'aspetto contrattuale, chiarendo la funzione cruciale che i contratti rivestono nel coordinamento delle operazioni della supply chain e nel ridurre le difficoltà associate alla gestione della domanda. Il risultato atteso è un framework avanzato, mirato a favorire una proficua collaborazione, una maggiore adattabilità e, conseguentemente, il conseguimento di risultati finanziari più robusti.

Per lo sviluppo di questo lavoro, è stata intrapresa un'analisi meticolosa di una serie di articoli pubblicati dal 2001 ad oggi. L'obiettivo primario di questa analisi è esaminare le tendenze e le sfide della progettazione dei contratti all'interno dell'industria di interesse, al fine di tracciare un quadro chiaro della situazione attuale. L'utilizzo di strumenti appartenenti alla branca della teoria dei giochi ha fornito una prospettiva innovativa, portando alla luce diversi scenari in cui ipotetici rivenditori e produttori potrebbero ritrovarsi nella dinamica realtà del settore del lusso. La validità e l'efficacia delle affermazioni formulate sono state attentamente indagate attraverso l'impiego del software Microsoft Excel. In tal contesto, è stato possibile modellare una simulazione quantitativa della distribuzione della domanda dei prodotti esaminati, nonché individuare le migliori pratiche che i rivenditori dovrebbe adottare nei contesti specifici. Le performance finanziarie degli agenti della catena di approvvigionamento sono state successivamente confrontate con quelle ottenibili tramite l'adozione di due politiche concorrenti. Il nuovo modello proposto si differenzia da tali politiche in quanto riconosce gli innumerevoli vantaggi finanziari derivanti da una valutazione oculata di variabili chiave come l'intervallo di riordino, l'aggiornamento della previsione di domanda, una gestione accurata delle merci invendute e la variabilità insita nelle richieste da parte del mercato.

Ulteriori confronti e osservazioni sono stati condotti mediante due analisi più dettagliate del nuovo modello proposto.

I risultati ottenuti mettono in luce le limitazioni del modello attualmente in uso. Date le molteplici variabili esterne che influenzano la domanda di prodotti di lusso, quali le condizioni economiche, le tendenze della moda e le dinamiche sociali, la previsione della domanda si configura come un compito complesso, e un modello come quello del Newsvendor, progettato per scenari con una domanda stabile e prevedibile, può avere difficoltà ad adattarsi a questa volatilità, causando potenzialmente un eccesso di scorte o una carenza di prodotti. Ciò, a sua volta, può impattare negativamente sulle performance del rivenditore, traducendosi in opportunità di vendita mancate e insoddisfazione da parte dei clienti, i quali, nel contesto del lusso, si attendono esclusività e disponibilità immediata.

In questo scenario, il modello Mid-Season Reorder emerge come una soluzione promettente per fronteggiare tali sfide. Accogliendo aggiustamenti a metà stagione e adottando un approccio più flessibile alla gestione delle scorte, questo modello consente ai marchi di lusso di rispondere in modo più efficace alla natura mutevole del loro mercato. La flessibilità intrinseca incoraggia, a sua volta, una maggiore sincronizzazione tra domanda e offerta, attenuando così i rischi associati a scorte esaurite o in eccesso. I profitti conseguiti durante la fase di simulazione, determinati utilizzando le nuove formule e i nuovi parametri, ne sono un'evidente dimostrazione.

Tuttavia, è fondamentale sottolineare che l'introduzione di un modello contrattuale che consente il riordino dinamico può introdurre ulteriori sfide nella gestione della catena di approvvigionamento. Una comunicazione efficace e una stretta collaborazione con i produttori sono requisiti indispensabili, così come la capacità di analizzare i dati in tempo reale e di adattarsi prontamente all'evoluzione delle condizioni di mercato. Inoltre, è opportuno notare che non tutti i prodotti o i settori si prestano all'implementazione di un modello contrattuale che consente il riordino dinamico. L'efficacia di tale modello dipende dalla variabilità della domanda e dalle complessità della catena di fornitura. Pertanto, è indispensabile condurre una valutazione mirata dei contesti in cui questo modello può essere applicato con successo.

In conclusione, questo elaborato ha esplorato meticolosamente il ruolo centrale dei contratti nella gestione della domanda nell'industria della gioielleria di lusso. Ha sottolineato la necessità di progettare contratti specifici per il settore e ha presentato un modello contrattuale ibrido innovativo, che sfrutta i vantaggi dei modelli esistenti e cerca di affrontare le sfide e i problemi a cui essi non riescono a dare una soluzione. Inoltre, la combinazione di concetti quali la possibilità di riordino, la presenza di previsioni di domanda aggiornate, la rivalutazione dei prodotti invenduti e l'introduzione di nuove variabili chiave, come l'intervallo di riordino, non solo evidenzia le differenze del nuovo modello rispetto ai modelli vigenti, ma sottolinea anche l'impatto innovativo che ha sull'attuale letteratura in materia di contratti nel settore della gioielleria di alta gamma. Il miglioramento che apporta questo nuovo modello contrattuale offre alle aziende del settore un vantaggio competitivo, in quanto consente loro di rispondere più efficacemente ed efficientemente alle mutevoli esigenze della clientela. La validità dimostrata del modello Mid-Season Reorder ne convalida l'applicabilità e l'idoneità come strumento per affrontare le difficoltà della gestione attuale della catena di fornitura nell'industria in esame. Le nuove padronanze acquisite, non solo arricchiscono la letteratura esistente, ma offrono anche linee guida pratiche per gli agenti del settore che desiderano migliorare la flessibilità, l'adattabilità e l'efficienza della loro catena di approvvigionamento.

Parole chiave: Fashion; Lusso; Gioielleria; Gestione della catena di approvvigionamento; Contratto; Vendita al dettaglio; Teoria dei giochi.

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EXECUTIVE SUMMARY OF THE THESIS

Contracts as Coordination Mechanism between Brands and Retailers: The Mid-Season Reorder Model

TESI MAGISTRALE IN MANAGEMENT ENGINEERING – INGEGNERIA GESTIONALE

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1. Introduction

The **role of contractual design** in demand management in the high-end Jewellery industry warrants further investigation. In this industry, contracts must be drafted with accuracy and clarity because they have the potential to significantly influence supply chain effectiveness and direction. Contracts that clearly define roles, deadlines, and technical requirements encourage greater consistency and cohesiveness among the different parties involved, which lowers the possibility of opportunistic behaviour and operational inefficiencies.

To categorize contracts and create an environment where the parties are motivated to put long-term common benefits ahead of individual short-term interests, it is essential to analyse **contractual incentives to transfer payment**. The industry under review places a high value on quality and service, and this approach helps to maintain those standards while reducing the possibility of conflicts of interest.

Sharing inventory risk is a significant challenge, particularly considering products' high value and vulnerability to unpredictable demand swings. An effective way to lessen the possible damaging effects of overstocking or a scarcity of products on the market is to share it through the adoption of particular contractual clauses. This strategy is especially helpful in guaranteeing a more accurate match between supply and demand, helping in the reduction of waste and expenses related to sub-optimal inventory management.

There is a discernible trend among businesses to **outsource their manufacturing procedures**. It is clear that this strategy has some serious weaknesses, including the loss of control over the operational stages of production and retail distribution, which could lead to a decline in brand image dominance, even though it may initially bring benefits like lower production costs and market expansion. In an environment where brand image and exclusivity are critical, this kind of control loss can have detrimental effects that are not negligible. Many businesses in the industry are revaluating the **re-insourcing of production activities** because of the issues mentioned above. This reversal brings to light the intricate complexity of demand management within this industry.

In the high-end jewellery industry, **ineffective demand management** can have a significant impact on several key parameters. These include the capacity to adjust to changes in consumer demand, maintaining the uniqueness of the brand, reducing inventory management expenses, and guaranteeing product availability based on the time and location preferences of customers. The adoption of coordination contracts is an effective approach to addressing these challenges and establishing a supply chain management system based on fair cooperation and interest alignment. To ensure that coordination contracts meet the unique requirements of the luxury Jewellery market, they must be carefully crafted.

2. Objectives and Methodology

2.1. Research Objectives

Recent research has concentrated on analysing existing contractual forms and developing new contractual models to improve their efficiency. Nonetheless, a significant issue that has surfaced from this examination is the absence of a methodical strategy intended to combine several contractual forms to create a hybrid contractual structure that can take advantage of their inherent benefits. Moreover, the incapacity of current contracts to completely satisfy the requirements of distinct parties is another issue mentioned. This is because the implemented contract often does not match the industry and the participating companies' characteristics. These drawbacks open a significant avenue for further investigation.

The purpose of the **present study** is to introduce a novel contractual model in this context, thereby representing a substantial advancement in the contractual dynamics' optimization. The possibility of a mid-season reorder is introduced by the new model. This strategy might capitalize on the unquestionable advantages of equitable benefit distribution among participants while also ensuring brand management through the buyback of unsold products. The combination of these characteristics may lead to improved goal alignment and efficiency, as well as more effective handling of the difficulties arising from fluctuating demand and the unique peculiarities of luxury Jewellery manufacturing.

2.2. Research Questions

Four research questions will be answered along the dissertation:

1. *What are the predominant challenges and peculiarities associated with Demand Management (DM)? How can a mid-season product reorder model in the luxury Jewellery sector impact the resolution of these challenges? In this regard, are the dynamics of the new model more efficient compared to those of conventional procurement models?*
2. *How can game theory be applied to optimise contracts, taking into account variables such as price, quantity, and timing of orders?*
3. *How can a new contractual model maximise profits for both sides of the supply chain, considering the unique dynamics of the industry under consideration?*
4. *What could be the possible application scenarios and how could these be affected? Which stochastic and non-stochastic variables need to be considered besides seasonal changes, demand fluctuations and price variations?*

2.3. Research Methodology

We started our work by reviewing the **existing literature** to thoroughly analyse a sizable sample of studies that have been published since 2001 and nowadays. In terms of contractual agreements, this research assisted in identifying trends, best practices, and recurring problems in the luxury Jewellery sector. The examined publications involve a wide range of methods, including mathematical programming techniques and game theory, providing a variety of approaches to handling contractual and management concerns. In particular, the theoretical frameworks underpinning our research, on which the tools are used to develop the discussion, are the publications Cachon and Lariviere (2005) and Brun and Moretto (2012).

We selected the source of origin and keywords as our two selection criteria when looking for these articles.

The **research journals** that have been chosen are in the fields of operations research and management science (OR/MS/OM). These include INFORMS journals like Management Science, Operations Research, Manufacturing and Service Operations Management, Interfaces, Information Systems Research, Marketing Science, Service Science, Transportation Science, Mathematics of Operations Research, and INFORMS Journal of Computing. Our decision to concentrate on these publications is a sign of our concern for reliable and high-quality sources. This lends our work a high degree of credibility because their publications typically undergo thorough peer assessment. This variety of sources adds to our grasp of industry dynamics and potential contractual solutions, making it more comprehensive and in-depth.

By applying terms such as "fashion", "supply chain", "retail", "game theory", "luxury", and "contract", it was possible to perform a **targeted search** in the selection of publications, thus focusing only on contributions relevant to our field of study. Without the use of targeted keywords, we could have been overwhelmed by an excessive number of articles with the possibility that some of them were not relevant.

The chosen articles were then divided into **three macro-categories** based on the authors' objectives and the proposed material, which proved to be a particularly useful search technique:

- Articles related to the luxury industry;
- Articles on the several types of contracts currently in use;
- Articles on the employment of game theory in supply chain management.

This allowed for a clearer, more cohesive arrangement and organization of the literature review.

The new contract form was developed with careful consideration of **key variables for optimization**. Specifically, the time of reorder was included, along with the number of items the retailer ordered. This constitutes a substantial paradigm shift in comparison to previous studies. The introduction of models that are more closely aligned with the complexity of the high-end Jewellery industry results in better arrangement with demand fluctuations and contributes to a significant reduction in risk and cost.

With the help of **concepts from game theory**, we were able to develop a set of scenarios meant to test our model in various settings and determine whether it would be especially effective.

We then used Microsoft Excel software to generate an elaborate **simulation** to assess the usefulness and efficacy of our model and to gain a deeper understanding of the outcomes. The purpose of this simulation was to produce accurate data for one hundred distinct articles during a century-long observation period. A demand distribution with a mean of 100 and a standard deviation of 10 was assumed for every article. To differentiate between products with high and low demand, a threshold was established below which it would not be prudent for the retailer to incur the risk of placing a new large order, and a probability of this event occurring was linked to it. All the scenarios that were considered during the model development phase are covered by the simulation. Within this framework, we meticulously measured the outcomes derived from our model with what a hypothetical retailer or producer could have accomplished by utilizing the current models within the industry in question. This approach has several inherent benefits:

- The simulation provides realistic and reliable data that support or refute the theoretical claims made previously;
- It allows to evaluate the new model in various scenarios under controlled conditions, reducing risks and costly errors;
- Its scalability and repeatability make it easy to explore further scenarios or evaluate the effectiveness of strategies over time;
- It identifies areas where our model can be strengthened.

This work provides solid empirical basis for supply chain-related business decisions in the future.

3. Structure of the content

The **first chapter** opens with a foreword on the current state of the luxury industry, focusing on its historical evolution and future perspectives. In particular, it emerges that the sector under study is experiencing a period of considerable expansion. Increased demand from emerging Countries, technological advances and growing environmental awareness are identified as some of the main trends influencing the luxury Jewellery market. Although some challenges may arise, the outlook for this industry looks promising.

The **second chapter** provides a detailed summary of the potential problems within the area of demand management. Demand management for a luxury product involves complexities related both to the intrinsic characteristics of the product and to the external context of the market in which it is sold. The most recurring problems concern product customisation, the management of returns and repairs, disputes and conflicts, adaptation to changing consumer tastes, regulatory fluctuations and the specific nature of the items being traded (impulse purchases, volatility, and unpredictability of demand, ...). A further variable of relevance is the distinction between fashion items and carry-over items, as they require a differentiated approach. The resolution of these issues is identified in the correct formulation of a contract between producers and retailers. Through this practice, they can adapt to the needs of discerning consumers, ensuring that the Critical Success Factors (CSF) peculiar to luxury products are met. These include high-quality standards, the legacy of skilled artisanry, product exclusivity, emotional appeal, brand identification, distinctive style and design, belonging to a nation renowned for excellence, uniqueness, and the promotion of a distinctive lifestyle. This chapter provides the basis for the formulation of the first research question (*What are the predominant challenges and peculiarities associated with Demand Management (DM)? How can a mid-season product reorder model in the luxury jewellery sector impact the resolution of these challenges? In this regard, are the dynamics of the new model more efficient compared to those of conventional procurement models?*).

The subsequent three chapters offer a review of the contemporary literature pertaining to the topic under consideration.

The **third chapter** provides an overview of the several types of contracts in force and the criteria for their selection.

CONTRACT	CHARACTERISTICS
Wholesale price	At a fixed wholesale price, the producer sells the goods to the retailer, who then marks them up and sells them to end users
Buyback	The retailer pays a set price per unit purchased and the producer commits to buyback any unsold goods from the retailer at a set price and within a given time frame
Revenue-sharing	This is a sort of contract where the parties concur to split a portion of the profits made from a particular commercial activity
Quantity flexibility	The retailer and the producer determine an initial number of items to be supplied but also a range within which the quantity may be altered in response to predetermined circumstances. The producer is required to pay the retailer back for any unsold units that are higher than a predefined threshold at the same fixed price per unit that the retailer paid
Capacity reservation	A producer grants the retailer access to a specific quantity of products for a defined period. If: - The total number of units ordered is less than the agreed quantity, he must pay a late fee or the full amount - The total number of units ordered is

	higher than the agreed quantity, additional units are offered at a higher price
Risk based contract	It allocates risks and responsibilities based on a proper assessment of potential uncertainties and adverse events that could impact the collaboration
Linear cost sharing	It aims to jointly shoulder the costs of a project in direct proportion to their respective individual shares
Fixed rate cost sharing	The parties to this agreement decide on a predetermined fixed fee via which they will equally divide the expenditures incurred during the collaborative endeavour
Option contract	A retailer and a producer enter an arrangement whereby the retail business pays the producer a fee in exchange for having the only right to buy a certain amount of goods at a predetermined price within a predetermined time frame

Table 3.1: Comprehensive survey of the in force contractual mechanisms for coordinating actors in supply chain management literature

Evaluation criteria requiring appropriate consideration for the implementation of a specific contract include:

– Administrative costs;
– The impact on supply chain coordination in terms of efficiency in ensuring that each participant does not deviate from optimal decisions for the entire supply chain;
– The sharing of risks and rewards.

In the **fourth chapter**, a detailed analysis is conducted of some of the previously mentioned contracts, which have recently gained notoriety in the sector of interest. Special attention is paid to the investigation of their strengths, relative weaknesses, application scenarios and future prospects. In summary, the following emerges:

– Buyback contracts demonstrate superiority over wholesale price contracts;
– Revenue-sharing contracts provide a solution to the challenges of coordinating supply chains, ensuring that production and ordering decisions are in line with optimal decisions. Moreover, these contracts prove to solve problems that buyback contracts cannot coordinate, as they are independent of the retail price. However, it is important to note that they have limitations, including high administrative costs, considerable effort required of the retailer, the possibility of moral hazard, and the challenges of appropriate quota selection for profit sharing.

Although many of the current studies are predominantly theoretical in nature, it is possible to draw the conclusion that these contract types exhibit characteristics that are particularly aligned with the dynamics of the industry under review.

The **fifth chapter** outlines an analysis of game theory, recognized as a powerful analytical tool for negotiation. In this section, key concepts relevant to the investigation, such as strategy, objective function, Pareto optimality, Nash Equilibrium, and possible modes of cooperation between two actors, are examined. The previously presented contractual forms undergo a revision through this new analytical perspective, which has identified multiple advantages, such as intrinsic flexibility and effectiveness in coordinating dynamics while simultaneously achieving Pareto improvements. This explains the widespread adoption of such contracts in the contemporary business landscape. However, unresolved challenges persist in their practical implementation and their limitation in meeting the individual interests of each member. This chapter has laid the groundwork for the second research question (*How can game theory be applied to optimize contracts, taking into account variables such as price, quantity, and timing of orders?*).

The **sixth chapter** outlines the objectives, research methodology, literature gaps, and research questions that this document aims to address. This chapter highlights the challenges and opportunities that this field of research can offer. Moreover, it stands as the most pivotal chapter of the entire document, elucidating the pursued procedure and serving as the guiding thread that binds the entire work together.

In the **seventh chapter**, the development of the new hybrid contract form, based on the possibility of placing a second order mid-season, is

expounded. This innovative model supports improved order planning, ensuring that the company can adapt to changing market conditions and consequently maximize overall profit. Specifically, the assumptions underlying the model and the profit optimization formulas for both parties involved in the contract are presented in detail. The use of game theory has allowed for a more accurate evaluation of the choices and strategies available in this context, highlighting the potential implications of implementing the new model compared to the Newsvendor model.
While the Newsvendor model remains an effective tool for inventory management, its intrinsic limitations stemming from rigid conditions may not always be realistic or suitable for a dynamic and evolving business environment. This underscores the need to introduce more flexible inventory management models. This chapter answers to the third research question (*How can a new contractual model maximise profits for both sides of the supply chain, considering the unique dynamics of the industry under consideration?*).

Chapter eight examines the hypotheses previously formulated through the implementation of a simulation conducted using the Microsoft Excel tool. The empirical assessment of the real validity and effectiveness of the proposed model is carried out through the adoption of three distinct approaches:

- The first step carries out a preliminary comparative analysis of the financial performance of the proposed new model and the Newsvendor model to draw a broad picture of its overall cost-effectiveness. This section contains two different versions of the Mid-Season Reorder model, depending on when the retailer decides to place a new order.
- The second approach consists of presenting a summary of the potential financial results that the parties could achieve through the application of one of the two contractual models. This analysis is conducted in relation to specific scenarios and assumptions defined in the context of game theory, discussed in depth in the preceding chapter;
- The last approach aims to assess the impact of the model in terms of performance improvement when varying spread values between high and low demand products.

This chapter reveals valid and interesting results in response to the last research question (*What could be the possible application scenarios and how could these be affected? Which stochastic and non-stochastic variables need to be considered besides seasonal changes, demand fluctuations and price variations?*).

The **concluding chapter** outlines an exhaustive discussion of the results obtained, providing an analysis of the answers to the research questions, as well as an assessment of the implications and limitations inherent in the study. These considerations offer stimulating insights for potential future developments.

The chapter concludes with a list of Bibliographical references and a detailed Appendix documenting the commands performed and providing further background on the simulation conducted.

4. Mid-Season Reorder Model

Consider a two-part supply chain. The entire time horizon is divided into two distinct but interconnected periods. The producer must determine the production quantity for each period to maximize its overall profitability, and the retailer must decide the order quantity for each period to maximize his total expected profit. In this perspective, a significant portion of decision-making power lies in the hands of the retailer.

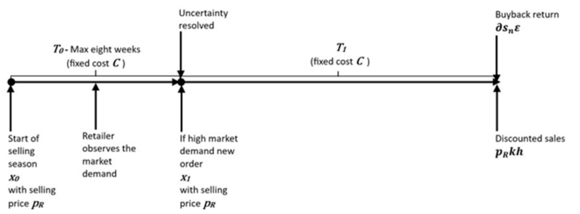

Figure 4.1: Graphical representation of the events occurring within the selling season in the Mid-Season Reorder model

After observing the stochastic demand in the period T_0, the retailer can deliberate on whether to place a further order based on actual demand trends. This multi-period problem is formulated as an inventory game between producer and retailer, with the possibility of deriving optimal decision policies for both parties and demonstrating the existence of a Nash equilibrium.

The retailer can structure its supply management as follows:

- At the first moment of purchasing goods (t_0), the retailer adheres to the Newsvendor model approach[1], covering the demand for items for the entire selling season. This satisfies the demand for low-demand products during the season and temporarily fulfils the demand for products with a higher demand;
- In the second period (t_1), the retailer can purchase another percentage of products, but only for those for which he has concrete evidence of strong market demand.

At the end of the last period, part of the inventories held by the retailer can be sold at a discount or returned to the producer (only those items purchased in the second period benefit from the buyback option).

It is expected that products with low market demand will follow the logic of the Newsvendor model, as the application of this model is sufficient to optimise the supply chain and maximise profits for these items. For products with high market demand, this new model is superior, thereby allowing both parties to better manage inventory.

To ensure product availability for the retailer during period T_1, the retailer has the option, upon payment of a fee, to reserve a maximum quantity of a specific product portfolio at the time of purchase in T_0, without the obligation to purchase the entire reserved quantity thereafter.

4.1. Problem Formulation

With the aim of providing a comprehensive understanding of the model, it is outlined some key assumptions for the context:

- It is considered two distinct product types within the supply chain: high and low demand products. They are procured independently of each other;
- The time frame being examined is divided into two successive periods, T_0 and T_1, which are intricately connected to each other;
- The distribution of these products is handled by a single distributor;
- The mean of the demand for the products is known and provided as input for each individual period. Across all individual periods within the entire time horizon, the demand can either remain constant or fluctuate;
- Assuming the retailer has a storage space large enough to accommodate the inventory for the entire selling season.

Additional assumptions within the model framework include:

- The possibility of stock-outs from previous periods;
- Prompt activation of the order and subsequent order fulfilment at the conclusion of the eight-week interval from the commencement of the sales season;
- The initial inventory is set at 0;
- At the end of the selling season the leftovers (R_n) are equal to 0;
- The selling season begins at t_0, when a certain quantity of 'x_0', purchased by the retailer in anticipation of the season's beginning, starts to be sold. The quantity 'x_0' has been chosen in accordance with the sales forecast for the entire selling season.

To make the understanding of the model clearer, a table summarizing the used symbology is provided below.

SYMBOL	DEFINITION
i	Range of products with high market demand
j	Range of products with low market demand
T_0	Eight-week period from the beginning of the selling season to the end of the interval to carry out reordering

[1] With the exception of the case of products with a high demand.

T_1	Period of 12 weeks starting from the end of the reorder interval and ending with the end of the selling season
t_0	Commencement of period T_0, corresponding to the start of the new selling season for products of both types i and j
t_1	Time of beginning of the period T_1
t_n	Moment that marks the end of the selling season
x_T	Quantity of products purchased by the retailer for the period T
b	Demand forecast referring to all selling season
b_T	Demand forecast for the period T
p_P	Procurement price set by the producer
p_R	Selling price set by the retailer
L	Storage costs (as a percentage) incurred by the retailer
s_t	Inventories present in the warehouse at the moment t subjected to buyback
s_n	Inventories at the end of the selling season subjected to buyback
k_T	Inventories at the end of period T not subjected to buyback
$R_{t,T}$	End-of-period inventory for period T, calculated at time t, encompassing both types of stocks present at that moment for that specific period
π	Profit functions
M	Quantity locked in t_0 for possible later reorder, i.e., maximum quantity that can be ordered by the retailer in t_1
μ	Binary variable indicating the occurrence of a second order
h	Percentage discount applied at the end of the selling season to all products for which the buyback option is not available
ε	Mutually agreed-upon value between the retailer and the producer for the return of unsold goods at the end of the season
∂	Percentage of product s_n, for which the buyback option is available, that the producer reclaims at the season's end
w	Percentage reduction from the selling price for the goods reordered at the beginning of the period T_1
F	Guaranteed fee to reserve M for the following period
Δk	Number of pieces of k_0 sold in the period T_1
C	Unitary costs incurred by the producer regarding all the selling period
U	Unitary revenue for the producer associated with the disposal of the unsold product units at the end of the selling season

Table 4.1: Notation system for Mid-Season Reorder model analysis

To calculate the profits of both parties involved in the upcoming contract, it is essential to highlight the various sources of revenues and cost items involved.

$$\begin{aligned} Maximise\ \pi_R = {} & p_R(x_0 + x_1\mu - R) + \partial s_n \varepsilon \\ & + p_R(1-h)[k_1 + (1-\partial)s_n] \\ & + \frac{F}{M}x_1\mu - [x_0 p_P + x_1\mu(1-w)p_P \\ & + F + Lp_P(x_o + \mu x_1)] \end{aligned}$$

Equation 4.1

Concerning the retailer, the revenue sources considered include sales during the high season, sales regulated by the repurchase agreement, discounted sales of unsold goods at the end of the season and the discount applied to the total amount of the second order. An interesting factor in this formula is the value that 'w' takes as it reflects the importance of the reorder time.

Regarding the costs to be borne by the retailer, it is important to keep in mind:

– The costs incurred by the retailer in the process of acquiring goods, services, or works from external producers. These costs include, among other things, shipping and transport costs;
– The costs incurred for the purchase of goods from the producer;
– The charge that ensures the availability of a specific quantity of goods 'M' until the end of the replenishment interval, which extends over eight weeks;
– The costs associated with storing the goods in the warehouse.

Now, the same approach is applied to the producer.

$$Maximise\ \pi_P = Total\ revenues_P - Total\ costs_P = [x_0 + x_1\mu(1-w)]p_P + F + U(M - x_1 + \partial s_n) - C(x_0 + max\{M, x_1\}) - \frac{Fx_1}{M}$$

Equation 4.2

The revenue components contributing to the producer's profit calculation include proceeds from the sale of goods to the retailer, the deposit amount made by the retailer as a guarantee for the availability of merchandise 'M' for the T_1 period, from which a portion will be deducted and subsequently returned to the retailer, and the income generated by the disposal of merchandise at the end of the season.

In terms of costs, it is necessary to consider production costs and expenses associated with product storage.

4.2. Game Theory Application

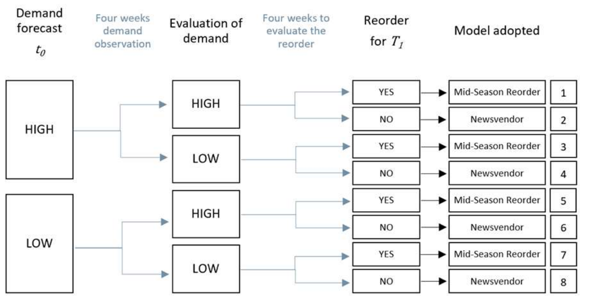

Table 4.2: Game theoretic scenarios for the Mid-Season Reorder model

Table 4.2 illustrates the retailer's decision-making process during the evolution of the selling season and its impact on the agents' objective function, with reference to the implemented model. The objective of this analysis is to determine the combination of decisions that maximizes profits for both parties involved. The outputs of this representation correspond to the producer's and retailer's profits in each of the eight scenarios developed.

In the case where the retailer does not place a new order, he automatically follows the Newsvendor model. Therefore, the newly developed model is not involved in the profit calculation. It is crucial to consider the impact of this choice, especially in relation to the type of product in question, as it may give rise to costs due to overstocking or loss of potential sales. Such inefficiencies affect the supply chain and have implications for end-consumer service and the objective functions of the parties involved.

The two emerging Pareto-optimal solutions are the *"high demand - high demand - yes reorder"* and *"low demand - low demand - no reorder"* combinations. Products with low demand do not require a second replenishment because the quantity required by the retailer is limited, making the Newsvendor model appropriate and efficient for managing this product category. In contrast, for products with high market demand, using the new model is more advantageous for both producer and retailer. This dynamic can translate into competitive advantages for both parties over market competitors. For example, they could enable better customer service, avoid out-of-stock situations, improve delivery times, and optimize overall costs. This underscores the importance of targeting the choice between the Newsvendor model and the new model based on the nature of demand and specific product characteristics.

4.3. The Simulation

In the first stage of the simulation experiment, the demand levels for each of the 100 items included in the simulation are randomly generated by the software. This casual generation is conducted on the basis of statistical parameters, including mean and standard deviation. An average of 100 products and a standard deviation of 10 products are used. It is important to note that the casual generation process is carefully designed to ensure that demand can never take on negative values. Each randomly generated demand scenario is assigned its associated probability of occurrence.

The algorithm at the heart of this simulation takes inspiration from game theory, which means that it is designed to represent and evaluate the decisions of the various actors involved in the supply chain according to principles of rationality and strategy.

The 100-year simulation period is significant because it allows one to observe and evaluate financial performance over the long term.
The main objective of simulation is to observe and compare financial performance. Analysing this financial performance helps evaluate the effectiveness of supply chain management strategies and identify best practices.

A crucial element of the simulation is the creation of different scenarios based on the desired spread percentages within a given product range. This concept is important because it emphasises the heterogeneity of demand, which can differ considerably across products.

However, it is important to emphasise that the luxury referred to is not too extreme but accessible to a considerable part of the population. This type of segmentation may be important to understand the target market and the positioning of the retailer.

The initial order calculation for the entire season is a process that seeks to strike a balance between satisfying customer demand for the entire season and managing the risks associated with fluctuations in demand. The initial order is made at the beginning of the selling season must be sufficient to cover the expected demand for the entire season, with the exception of the case of products with a high demand, because in such a case the quantity x_0 is given by the expected demand for the T_0 period only plus a number of standard deviations equal to the 10% of the inverse normal distribution of the critical ratio value. In all the other cases, the initial order is equal to the entire forecast for the selling season plus a number of standard deviations equal to the inverse normal distribution of the critical ratio value. This addition to the demand forecast mitigates the risk of out-of-stock due to unforeseen demand variability.

Classifying demand into high or low is an important step in inventory management and sales activity planning. To do this, it is necessary to define a threshold or cut-off point that allows decisions to be made based on the amount of demand.

The simulation has been organized into several mirrors. Each annual mirror is divided into three distinct parts, each representing a specific time in the selling season. The first part constitutes a starting point where decisions must be made based on historical estimates and forecasts, since current data may not yet be available. Subsequent parts refer to later moments in the selling season, which allow planning to be adjusted and updated based on changing market conditions and actual data as they become available during the selling season.
Decisions made during this period are better informed than decisions made at the beginning of the season, and this is also reflected in more targeted strategy adoption.

The process for calculating the retailer's and producer's profit applies the formulas derived from the two models, the Newsvendor model and the Mid-Season Reorder model. These expressions consider various parameters analysed in detail in Chapter 8 of the dissertation. The calculation is performed separately for the retailer and the producer. After calculating the individual profits for the retailer and producer in each simulation period for 100 items, the total supply chain profit was calculated by summing up the profits of the two agents. Using this approach, it is possible to examine how the decisions of the individual agents influence the overall results of the supply chain in different situations.

The initial stage of analysis of the models considered offers a key opportunity to explore and compare the financial implications of the strategic choices made by the retailer, and consequently by the producer and the entire supply chain. To ensure a fair comparison between the three models, it was essential to establish the same initial value for the random variable in the three contexts.

	Advanced Mid-Season[2]	Newsvendor	Mid-Season
Average producer	17.925 €	16.441 €	13.096 €
Average retailer	43.699 €	33.905 €	35.092 €
Average margin	43 %	43 %	45 %

Table 4.3: Summary of the three approaches comparison

The financial performance resulting from the implementation of the Advanced Mid-Season Reorder model significantly exceeds that of the other models, highlighting benefits for all the agents. This result underlines the inherent effectiveness of the model in optimally managing a diverse range of market conditions, giving this approach an edge in supply chain management strategies.

For the second analysis, an algorithm was implemented to select the most cost-effective model, given certain input values.
In the first period of the simulation, which covers a total of 8 weeks, the algorithm tests the two demand management models to determine which one of them is able to minimize the probability of stock most effectively out or overstocking. Initial sales performance plays a decisive role at this stage, as it is a significant indicator for predicting future demand and making informed decisions on managing reorder strategies in the long term.

When setting up the simulation, a special cell is included in the spreadsheet. This cell shows the number of the game theory case that is automatically generated by the simulation. These numbers follow the numbering logic previously illustrated in Table 4.2.

The function AVERAGE allowed to recap the average results obtained from the simulation, offering a clear identification of the most profitable situations within the simulation.

[2] This refers to the Mid-Season Reorder model approach without an a priori decision to re-order during the season.

Subsequently, the eight scenarios developed in relation to the retailer's decision to make or forgo additional reorder were compared in pairs, highlighting how the decision to reorder can affect the final profit.

ANALYSIS	COMMENTS	MODEL APPLIED
Case 1 vs. Case 2	The retailer would do well to place a reorder if the initial estimate of high demand proves to be accurate. In this way, the possibility of out-of-stock is reduced, guaranteeing the producer and the retailer the highest possible profit	Mid-Season Reorder model
Case 3 vs. Case 4	Although demand was overestimated in the first period, the stock at the end of that period is not enough to meet the expected units required for the following period. Re-ordering is therefore necessary to avoid a stockout situation	Mid-Season Reorder model
Case 5 vs. Case 6	As a result of an initial underestimation of demand, an additional order must be placed to avoid serious stockouts and loss of opportunities for both parties involved	Mid-Season Reorder model
Case 7 vs. Case 8	The first case concerns the purchase and maintenance of stocks in excess of actual market demand. This could expose the retailer to several risks. The second case concerns the decision not to re-order. In situations where the retailer can look forward to a future increase in demand, it might seem a good idea to place new orders to meet this growing demand. However, it is essential to consider that, given the initial assumptions, there is no guarantee that the market will be willing to accept all additional units. This leads to uncertainty regarding the profitability of such new orders	Mid-Season Reorder model / Newsvendor model

Table 4.4: A thorough examination of the outcomes derived from the simulation for each game-theoretic scenario

In general, inventory management and reordering decisions are of crucial importance for the retailer and the producer. The key to success in this context depends largely on the ability to respond effectively to fluctuations in demand and real-time market dynamics.

When market demand is high, the best choice for the retailer seems to be using the Mid-Season Reorder model, which allows him to respond promptly to demand. On the other hand, when demand is low, the Newsvendor model remains a convenient choice for retailers. Under these circumstances, placing additional orders may only increase costs and entail unnecessary risks.

Subsequently, summaries obtained from the simulation for each actor (retailer, producer, and supply chain) and for each scenario are provided. In particular, the comparisons previously examined refer to the scenario in which the replenishment process takes place in week eight.

WEEK 8				
Average GT case	Producer profits	Retailer profits	Retailer margin	Supply chain profits
1	22.487 €	63.725 €	44%	86.212 €
2	-	-	-	-
3	23.601 €	50.703 €	39%	74.304 €
4	-	-	-	-
5	18.456 €	52.337 €	45%	70.793 €
6	13.311 €	26.671 €	40%	39.982 €
7	12.178 €	26.567 €	42%	38.744 €
8	14.251 €	33.529 €	47%	47.780 €

Table 4.5: Average profits and contribution margin in relation to the reorder week

It is noteworthy that through simulation, it was possible to identify the probability density associated with the various scenarios under consideration. Cases 1, 7, and 8 emerge as the most probable. In contrast, Cases 2, 4 and 6 exhibit the lowest probability of occurrence.

After a meticulous assessment of the outcomes derived from the AVERAGE analysis, it is deduced that a shift in perspective was imperative to attain a more thorough and comprehensive understanding of the examined context. This novel approach is characterized by its orientation toward a more intricate and nuanced understanding of the market demand distribution, surpassing the analysis concentrated on the specificity of individual cases. Central to this revitalized perspective is the notion of 'spread', a pivotal indicator assuming a critical role in delineating the percentage disparity between the quantities of two distinct product types.
The assigned spread values encompass a broad range, ranging from a minimum of 50% to a maximum of 400%.

While analysing the results, an interesting trend emerges. In light of distinct strategies employed for the two products, it becomes evident within the context of 50% spread that the optimal choice for the comprehensive management of low-demand products is the Newsvendor model, for the high-demand, instead, the Mid-Season Reorder model is preferable.

Furthermore, upon exploring subsequent scenarios, this dynamic becomes more evident. Indeed, under conditions of heightened demand, the retailer's profits derived from the implementation of the new model markedly surpass those achievable through the utilization of the Newsvendor model. Conversely, in scenarios of low demand, the profits realized with the Mid-Season Reorder model exhibit a marginal increase over the Newsvendor model.

The Mid-Season Reorder model emerges as the most lucrative option for high-demand products, while the Newsvendor model resulted more suitable for low-demand products, and this disparity in profit expands with an escalation in the spread percentage.

SPREAD 50%		
DEMAND in T_0	**30**	**45**
HIGH/LOW in T_0	LOW	HIGH
x_0	81	46
DEMAND in T_1	24	46
HIGH/LOW in T_1	LOW	LOW
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	17.010 €	11.776 €
π_P Mid-Season	18.203 €	25.359 €
π_P	**17.010 €**	**25.359 €**
π_R Newsvendor	42.525 €	17.912 €
π_R Mid-Season	26.799 €	57.780 €
π_R	**42.525 €**	**57.780 €**
π_S	**59.535 €**	**83.139 €**
Margin	**50 %**	**40 %**

Table 4.6: Summary of a hypothetical scenario with a 50% spread between high and low-demand products

SPREAD 400%		
DEMAND in T_0	**19**	**95**
HIGH/LOW in T_0	LOW	HIGH
x_0	54	96
DEMAND in T_1	42	81
HIGH/LOW in T_1	LOW	HIGH
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	13.176 €	23.424 €
π_P Mid-Season	13.176 €	35.097 €
π_P	**13.176 €**	**35.097 €**

π_R Newsvendor	28.365 €	38.064 €
π_R Mid-Season	29.646 €	95.770 €
π_R	**28.365 €**	**95.770 €**
π_S	**41.541 €**	**130.867 €**
Margin	**43 %**	**45 %**

Table 4.7: Summary of a hypothetical scenario with a 400% spread between high and low demand products

The new model proves to be more functional and elastic in dealing with variations in demand, and this flexibility translates into better economic performance for the entire supply chain.
These results confirm the importance of selecting the appropriate model according to the specific dynamics of the market and the characteristics of the products involved. Understanding them is key to optimising business decisions and ensuring optimal economic performance within the supply chain.

4.4. Results

The simulation conducted denotes a significant step forward in the understanding and validation of supply chain management models.
It allowed to examine and evaluate the performance of the Newsvendor model and Mid-Season Reorder model under a diversified range of demand conditions, making an important contribution to filling the gap in the supply chain management literature.

The Mid-Season Reorder model clearly demonstrates its advantage in situations where flexibility is required, unlike the Newsvendor model which is best suited to handle constant and predictable demands. This analysis confirms the previously mentioned theoretical assumption on the Pareto optimality of Cases 1 and 8.

However, more complex and realistic situations require more in-depth analysis, and the third phase of the simulation addressed this need. This analysis was crucial to further explore the performance of the two models under more changeable and volatile demand conditions, so in a more realistic representation of the challenges that supply chains often face. The results that emerged from this phase of the analysis clearly confirm the advantage of the Mid-Season Reorder model in high-demand contexts, especially when the spread percentage is significant. In these scenarios, the model demonstrates its ability to maximise profits and ensure efficient supply chain management. Its flexibility in dealing with deviations in demand results in superior performance in dynamic contexts. On the other hand, the Newsvendor model continues to prove to be an optimal choice in cases of low demand and when product requirements are more stable. This result agrees with the theories underlying the model, which suggest that it is best suited for situations where demand is constant and predictable.

One of the key conclusions that emerged from this analysis is the importance of adapting the model according to the specific characteristics of the market and products involved. The choice between the Newsvendor model and the Mid-Season Reorder model must be carefully weighed, taking into account key variables such as demand variability, forecast accuracy and the goal of maximising profits in supply chain management.

In conclusion, this research embodies an important contribution to the supply chain management literature by providing an empirical evaluation of the examined models under real operating conditions. It also highlighted the effectiveness of the new model compared to the current one in certain contexts and how it is able to significantly enhance the financial performance of all involved participants and the supply chain as a whole. Therefore, the need to thoroughly evaluate its implementation in the market is empirically confirmed.

5. Conclusions

The dynamics and difficulties of applying a contractual model for dynamic reordering in the luxury Jewellery industry were thoroughly examined in this dissertation. New insights have emerged through critical examination of traditional models and thorough analysis of initial queries. These insights significantly advance our understanding of supply chain management in the context of variable and changing demand.

First, it became evident how important precise demand forecasting is. The luxury market is one where trends can shift quickly, so being able to predict customer preferences and market influences with precision is essential.

One key component that has been identified is supply chain flexibility. Optimizing production and delivery schedules and working effectively with producers proved to be crucial.
Its ability to react quickly to changes in the market guarantees a more efficient inventory management procedure, reducing negative consequences on the financial statements and reputation of the business.

The Mid-Season Reorder turned out to be a practical solution for handling this process as well as a cost-effective way for businesses looking to hold onto their competitive edge.

Game theory has emerged as an extremely useful tool for contract optimization in supply chain management. This approach opened new avenues for collaboration between retailers and producers, allowing the supply chain's overall value to be maximized thanks to a depth analysis of the model to be applied.

The findings highlight the importance of the Mid-Season Reorder model's adoption in the luxury industry's supply chain management optimization, and the simulation phase's profits, which were determined using the newly created formulas and newly determined parameters, provide a compelling illustration of this. This new contract model's innovation gives businesses in the industry a competitive edge by enabling them to more effectively respond to the changing needs of their customer base. The Mid-Season Reorder model's demonstrated efficacy validates its applicability and appropriateness as a crucial instrument to tackle the difficulties of contemporary supply chain management within the ever-changing luxury industry.

In conclusion, this study introduces an advanced contractual model as a response to market challenges. The new insights provided not only enrich existing theory, but also offer practical guidelines for industry players wishing to improve the flexibility, adaptability, and efficiency of their supply chain.

5.1. Novel Contributions

Notwithstanding its strength as an inventory management tool, the Newsvendor model has many inherent drawbacks stemming from the presumption that orders are only placed at the start of the season and cannot be changed. But in the actual world of business, there might be instances where orders need to be adjusted to better accommodate shifting demand, shifting delivery schedules, or other unanticipated events.

The Mid-Season Reorder model considers several factors, it adds new benefits to get around these restrictions and offers greater flexibility in stock optimization. These benefits include:

- Updating forecasts. This allows for more informed decision-making;
- The possibility of placing second orders during the selling season based on market dynamics. This implies that they can gradually adjust to variations in demand rather than being forced to commit to large orders up front;
- Reordering mid-season gives both players the option to better manage variability in a world where volatility has become the norm;
- Reduction of waste and obsolescence and its impact on profit. This model gives players more control over excess stock and obsolete items. This lessens losses brought on by having to discard unsold products at the end of the season;
- Enhance the utilization of monetary resources. The new model enhances corporate liquidity by enabling a more effective division of inventory and reorder-related expenses;
- Improved customer satisfaction. The flexibility provided by the new model enables retailers to reply to end-user requests faster and guarantees that the right products are available when needed. From the retailer's perspective, the same applies to the producer. This can lead to stronger customer loyalty and cross-selling opportunities;
- Market competitiveness. Retailers can increase their market share or hold a leading position

by being able to react quickly to changing conditions.

In summary, the Mid-Season Reorder model improved inventory management by better meeting the needs of the fast-paced business world of today. It boosts performance and customer satisfaction by enabling producers and retailers to maximize their resources, cut waste, and keep better control over their inventory.

5.2. Limits

The introduction of the contractual model for dynamic reordering in the luxury Jewellery sector offers a significant theoretical contribution to the dynamics of supply chain management. However, it is crucial to examine the implications and limitations arising from this strategic transition to fully understand the context in which the answers provided can be relied upon.

The fundamental role that the partnership between retailers and producers' plays is highlighted by the necessity of effective communication and collaboration with producers. Dynamic reordering becomes dependent on close integration and prompt information sharing.

Capabilities for data analysis are equally important. Setting aside money for sophisticated analytical tools as a top priority suggests that, to reap the full benefits of the model, businesses should think about providing staff with analytical skill training.

The difficulty of organizational adaptation is one of the study's most significant practical implications. An organizational structure that is adaptable and agile is required to allow for quick adjustments in response to shifting market conditions. To maintain flexibility over time, a company needs to foster a culture of adaptation and ongoing development. Organizational flexibility puts businesses in a better position to handle uncertainty, grab new opportunities, and increase their overall competitiveness in the market.
Moreover, the universal applicability of the contract model for dynamic reorganization is questioned, suggesting that not all industries or products will profit from this strategy in the same way. According to the theoretical contribution, supply chain complexity and demand variability have a direct impact on how effective the suggested model is. It highlights the necessity of carefully assessing the situations in which the model can be used with success.

The practical implications of the study suggest that companies in the luxury Jewellery industry should wisely consider the transition to dynamic reordering as it is a strategic decision that requires targeted investment and a change of mindset. The limitations of the study indicate the need for further research exploring other industries and contexts to generalise and refine the conclusions reached here.

5.3. Future Evolutions of the work

The model's development has revealed important obstacles to efficient communication amongst all parties involved and a clear shortcoming in precisely predicting market demand. Taking these factors into account, future improvements might include putting in place mechanisms and incentives that are intended to improve alignment between the parties, fostering better communication and more efficient responsiveness to market dynamics. Furthermore, a comprehensive examination of efficient tools to support forecasting analysis ought to be investigated.

To improve the Mid-Season Reorder model's effectiveness and flexibility in response to the target market's unpredictability, future research efforts might focus on rethinking the model to strengthen its resilience in situations where the initial demand projections turn out to be incorrect. These endeavours could potentially aid in the development of innovative tactics and methods that are more suitable for the unique obstacles that businesses face in the field of supply chain management.

The results obtained from the game theory-constructed scenarios that were then analysed through simulation provide a strong basis for further research into the viability and relevance of supply chain management models in various business contexts. Considering that the model works well in situations where demand fluctuates a lot, one of the most compelling directions for future research and development would be to perform a benchmark analysis to find industries that have characteristics that align with the model's principles and thus make it advantageous to apply.

Additionally, a fascinating direction for future investigation entails carrying out additional study to dive into a more comprehensive evaluation of the ideal week for reordering. In order to produce even more accurate scenarios and solutions, this might require adding a third demand forecast.

Finally, one potential area of improvement could be to examine the financial effects of matching the delivery week with the reordering week. During the current time frame, inventories are expected to be directly impacted by this alignment.

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1. Premise

The luxury market has traditionally been associated with class, excellence and social standing.

Finding a universally accepted definition of what luxury implies in the modern world is getting harder and riskier. Over time, the idea of luxury has evolved. Luxury has traditionally been defined as a combination of beauty and high quality pursued through handwork and associated with tangible things (Cappellieri et al., 2020).

Nowadays, the idea of luxury is increasingly associated with intangible qualities, the preciousness of values taking the place of the preciousness of commodities, so much so that luxury itself is defined as the ability to transform the essence of one's time into a commodity. It is intangible, remote from demands yet near to each person's ambitions and goals.

One of the most renowned and sought-after segments in this industry, due to its intrinsic exclusivity and uniqueness, is jewellery retailers.

It is impossible to give a clear definition of jewellery because of how context-dependent its value and significance are.

Materials are no longer the only defining factor used today to determine if a piece of jewellery falls under the category of luxury. Instead, design decisions that help tell a story are made regarding the materials and methods.

The market for luxury jewellery has undergone significant changes recently as a result of several factors, including globalisation, technological advancements, rising demand from emerging markets and shifting customer preferences.

In this brief introduction, it will look at the luxury and jewels industries while assessing their historical context, present state and future perspectives.

1.1. Historical context

The history of luxury jewellery dates back thousands of years. Jewellery was already a sign of social prestige and power in ancient Egypt and other ancient civilisations. Jewellery has evolved significantly over time, both in terms of design and the materials employed (La Storia E L'evoluzione Dei Gioielli - Capri Kronos - Mario Ruocco, n.d.). For instance, jewellery during the Renaissance was mostly composed of gold and embellished with valuable stones, whereas, during the Baroque era, it was distinguished by more ornate and sophisticated forms.

With the rise of industrialization in the 19th century, jewellers started utilising novel materials like silver and platinum. The new style of Art Nouveau, which emerged around the turn of the century, was marked by flowing lines and organic curves.

The Global Wars in the 20th century had a big impact on the luxury jewellery industry because many jewellers had to shut down due to of the financial crises.

1.2. Present state

The market for luxury jewellery is now expanding substantially. The global market for luxury jewellery is anticipated to expand at a compound annual growth rate (CAGR) of 6.7% between 2021 and 2028, according to a report released by Research and Markets in 2023[3]. This is mostly a result of rising demand from developing nations like China and India, where an expanding

[3] Luxury Goods: Global Strategic Business Report https://www.researchandmarkets.com/reports/1244796/luxury_goods_global_strategic_business_report

middle class is showing an increasing interest in these types of luxury goods (Ltd, n.d.).

The luxury jewellery market faces additional challenges. Many companies are making efforts to reduce their influence on the environment, for example by using recycled or sustainably sourced materials, as a result of growing environmental awareness and the demand for sustainable products.

Furthermore, the development of technology has enabled jewellers to design distinctive and customised jewellery using modern methods such as 3D printing (La Stampa 3D Aiuta Le Start-up Della Gioielleria a Cavalcare L'onda Della Personalizzazione Di Massa | 3D Systems, 2021).

1.3. Future perspectives

The booming demand for luxury goods from emerging countries, particularly China, seems to be one of the main drivers of the sector's expansion. According to an analysis by McKinsey[4], China currently holds 35% of the global luxury jewellery market and this figure is expected to rise to 40% by 2025.

Another significant trigger for the expansion of the luxury jewellery market seems to be the digitisation of the industry. For making online jewellery purchases easy and secure for customers, many jewellery stores are starting to invest in digital solutions such as websites and apps.

In addition, jewellers are increasingly using augmented reality to allow customers to virtually see and try on items.

[4] The state of Fashion 2023: Holding onto growth as global clouds gather. https://www.mckinsey.com/industries/retail/our-insights/state-of-fashion

However, the high-end jewellery market will also face some difficulties in the future (Muret, n.d.). New jewellery companies and producers are increasing the level of competition among jewellers by selling their products at lower prices and increasingly environmentally conscious consumers may favour companies that use sustainable materials and minimise their influence on the environment.

1.4. Summary

In conclusion, the high-end jewellery market is a constantly growing industry. Increasing demand from emerging countries, technological advances and growing environmental awareness are some of the key trends influencing the luxury jewellery market.

Luxury jewellery companies have made numerous investments in innovation to better adapt their products to consumer needs.

Although there may be some challenges, the prospects for this industry seem to be favourable.

2. Introduction

2.1. Overview

The existence of several sellers specialised in the creation of rare and expensive pieces defines the luxury jewellery market. These frequently rely on shops to market and sell their products. However, tight collaboration between producers and retailers is necessary due to the intrinsic nature of the items, the high consumer standards and the necessity to ensure the highest possible quality.

Producers and retailers work together through supply and sales contracts to ensure the luxury jewellery market operates as efficiently and competitively as possible. These agreements define the terms and conditions of product supply and sale as well as the obligations of producers, distributors and retailers regarding product quality, delivery and payment.

The requirement of product customisation is a key component of coordination agreements between producers and retailers in the luxury jewellery sector. In order to meet particular customer requirements, luxury jewellers often must produce unique, customised items. In this situation, the supply contract must specify how the product has to be customised, delivery terms, cost and payment terms.

Relationship management between producers and retailers is a crucial component of coordination contracts in the luxury jewellery industry. Mutual trust and cooperation are the basis of these partnerships, which is why coordination contracts must be open and provide a mechanism for the resolution of potential conflicts, and explicitly detail how returns and repairs will be handled.

To achieve optimum efficacy and market competitiveness, contracts are often an essential coordination tool between producers and retailers in the luxury jewellery sector (Brun et al., 2020).

Producers and retailers can satisfy even the most demanding consumers, guaranteeing the achievement of the CSFs[5] typical of luxury products, such as high levels of quality, the heritage of skilful craftsmanship, product exclusivity, emotional appeal, brand identification, recognition of style and design, affiliation to a nation of origin known for excellence, uniqueness and lifestyle development (Brun & Moretto, 2012).

Achieving high quality standards and ensuring maximum customer satisfaction is only possible through this close cooperation.

2.2. Demand Management issues

Managing the supply chain is a crucial concern when working with the fashion sector. Demand Management is an area that needs research while managing retail because retail is frequently the primary point of interaction between the company and its clients.

For this reason, is important to comprehend how the agents handle both their retail channel and the DM[6] procedure, and to identify the key factors that shape their behaviours.

The demand management of a particular luxury product may involve several complexities, related either to its characteristics or to the external market environment in which it is sold. The following are the common ones:

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[5] Critical Success Factors
[6] Demand Management

1. Product customisation: one of the distinctive features of the luxury jewellery market is the possibility to design customisable items. However, as it may lead to higher prices and longer delivery times, customisation can be difficult to deal with for contracts.
2. Management of returns and repairs: jewellery returns and repairs require special handling due to the nature of the items.
3. Disagreements and conflicts: coordination contracts may give rise to disagreements and conflicts between producers and traders, e.g., regarding quality, prices or delivery of products. Such disputes could damage brand reputation and cause delays in the production and delivery of goods.
4. Consumer tastes are constantly evolving, for this reason producers and retailers in the luxury jewellery industry must be able to adapt to these changes. Otherwise, contract management and production of items may be challenging.
5. Regulatory modifications: the administration of contracts between producers and retailers may be impacted by modifications to laws and regulations governing the manufacture, sale, and distribution of jewellery.
6. Demand management: a number of other issues are related to the nature of the bargaining object, such as impulse buying, volatility and unpredictability of demand, make it increasingly difficult to forecast demand in line with the real executive demand.
   a. High demand fluctuation and a high rate of product mix rotation make it difficult to have an effective forecasting process, necessitating the stressing of speed and responsiveness (Fisher et al., 2000).
   b. Time-based competition: improving product availability, simplifying the forecasting process, and subsequently lowering inventory all depend on minimising lead times. (Jacobs, 2006; Fisher et al., 2000; Christopher & Peck, 1997).

c. Choosing between a Make To Stock (MTS) or a Make To Order (MTO) strategy that is consistent with demand and supply features (Jacobs, 2006).
d. Systems for gathering data and information are the foundation for creating a responsive system and accelerating the demand chain (Fisher et al., 2000).

Another aspect that must be taken into account is the distinction between fashion items and carry-overs[7] as they require different attention.

A luxury jewel has a short and volatile life cycle. The caducity of the life cycle influences the following levers:

– Levers for the timing of demand and supply: the brand-owning company responds to retailers according to the product's lifetime. Fashion products usually followed a Make to Order strategy while carryover products are maintained using a Make to Stock strategy, even though a 10% inventory is occasionally kept at the central level.
– Demand forecasting levers: in accordance with the MTO strategy, forecasting for fashion products tries to foresee the needs for essential raw materials and components. Such seasonal forecasting is based on qualitative techniques that are appropriate for high demand variability and limited data availability (Mentzer et al., 1984). It is done once per season.
  Typically, every piece in a seasonal collection would be unique from the ones that came before. In contrast, forecasting carry-over products (often managed using the MTS[8] approach) is done at the item level and is based on

[7] The former are marketed for only one seasons, while the latter are offered for several one.
[8] Make to Stock

data from prior seasons' sales that is frequently updated using fresh data gathered throughout the ongoing lifecycle.

- Levers for order and replenishment management: retailers often place one seasonal order for fashion items, with extremely few options for restocking. Orders and replenishments for carry-overs are distributed more regularly throughout the seasons.

The features of retail are also important drivers that the brand-owner must consider before entering a contract with it. For example, the size and total sales volume (units) of a company is an appropriate assessment of its investment capacity (Christopher & Peck,1997).

In conclusion, coordination agreements are crucial tools for ensuring optimum effectiveness and market competitiveness in the luxury jewellery industry. However, it is essential to properly handle the concerns mentioned above and offer suitable solutions to solve them in order to guarantee the success of such contracts.

2.3. Objectives of the study

This thesis seeks to explore the role of contract design in addressing critical Demand Management issues within the high-end jewellery industry. Indeed, the choice of contract by the BO[9] significantly influences and at times dictates crucial challenges, leading to suboptimal actions. Crafting effective mechanisms can assist companies in aligning their competitive objectives while distributing risks, costs, and rewards in collaborative endeavours (Narayanan and Raman, 2004; Chen, 2011).

[9] Brand Owner

Therefore, it becomes imperative to thoroughly analyse the business landscape in order to identify the contract elements that should be tailored to align with the luxury industry's competitive priorities. The ultimate aim is to initially identify the primary challenges encountered in the Demand Management process due to contractual limitations and subsequently investigate the pivotal role of contractual clauses in resolving these issues.

The present study provides results of relevant interest, identifying critical areas of the demand management process and demonstrating how the formulation of a new contract form can improve the efficiency of the negotiation phases between producer and retailer. In addition, the cost-effectiveness of the proposed solution has been numerically tested, underscoring the importance for businesses in the industry to optimize the contract phase through the use of models adapted to specific contexts.

The structure of the thesis is organized as follows. After a foreword on the current state of the luxury sector, with a focus on its historical evolution and future perspectives, chapter 2 provides an introduction to the possible problems found in the area of demand management. This chapter also outlines the objectives of the study. Subsequent chapters offer a review of the contemporary literature pertaining to the topic under consideration. Chapter 3 provides an overview of the different types of contracts in force and potential contracting issues. In the next section, those contracts are analyzed in detail, paying particular attention to their strengths, weaknesses, application contexts, and their future evolutions. Chapter 5 lays out key concepts related to game theory, which is a powerful analytical tool for contracting arrangements. Here, the contractual forms presented in previous chapters are revisited from the perspective of game theory. In addition, possible modes of cooperation between two actors are listed. Chapter 6 reports the objectives, the research methodology, the gaps in the literature, and the research questions that this thesis aims to answer. This dissertation proceeds to report the results, in particular the new hybrid contract form developed to meet the needs of both parties of the contract by including the possibility of mid-season rearrangement, in chapter 7.

This model is subsequently validated through a simulation analysis performed using Microsoft Excel software, the results of which are detailed in chapter 8.

The paper concludes with a discussion of the results obtained while conducting the study; it also includes the implications of the study in terms of theoretical contribution, the answers to the research questions, the limitations of the model, and its possible future developments.

3. Literature analysis

3.1. Introduction

Contracts play a pivotal role in orchestrating the coordination within a supply chain. In the realm of contract analysis, two key criteria are often utilized to classify contracts: transfer payment contractual incentives and inventory risk sharing.

When delving into the theory of supply chain coordination, it becomes evident that it encompasses a multitude of dimensions and interactions among supply chain participants. Arshinder et al. (2008) developed a classification method for coordinating literature, as depicted in Figure 3.1. This diagram underscores the diverse perspectives that literature adopts when exploring coordination, encompassing discussions on the function of coordination within supply chains, coordination across various functions within the supply chain, coordination at interfaces, empirical case studies, and numerical examples. Furthermore, coordination can be achieved through various mechanisms that encourage decentralized system participants to engage in optimizing supply chain networks. In the context of this study, the focus is on coordination through contracts, which is one of the four mechanisms discussed in the literature, alongside information technology, information exchange, and collaborative decision-making.

As Tsay (1999) aptly defines, the supply chain contract is a "coordination mechanism that provides incentives to all of its members so that the decentralized supply chain behaves almost exactly like the integrated one". Contracts are instrumental in enhancing the relationships between producers and buyers by explicitly specifying crucial contract elements, such as quantity, price, quality, and timelines.

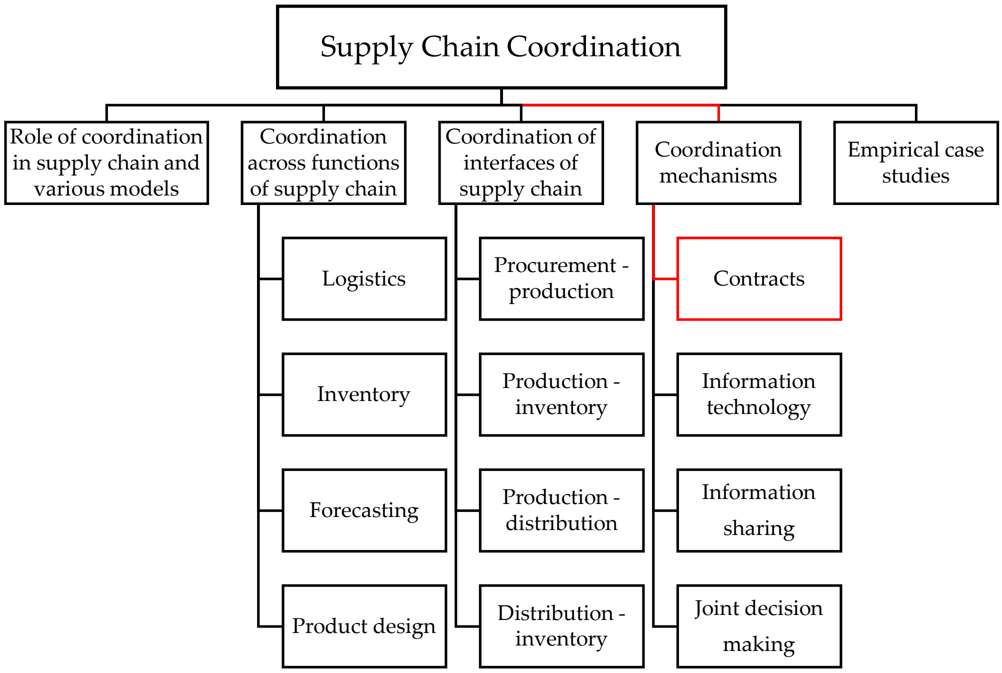

Figure 3.1: Classification scheme of the coordination literature
Source(s): Arshinder, Arun Kanda, Deshnukh, S.G., 2008, Supply chain coordination: Perspectives, empirical studies and research directions, International Journal of Production Economics 115, 316-335

According to the classification by Arshinder et al. (2008), these coordinating contracts have several overarching objectives:

- Maximizing the overall profit across the supply chain;
- Mitigating the costs associated with inventory, including shortages (goodwill costs) and overstocks (salvage costs);
- Equitably distributing risks among all involved parties.

Within the literature, various forms of coordination contracts have been thoroughly examined, and it will delve into the details of these agreements in Section 3.2. These contracts, originally conceived to foster collaboration within the forward supply chain, hold the potential to be extended and adapted to promote coordination among participants within the reverse supply chain as well.

3.2. Overview articles

There are various models that have been suggested in the general SCM[10] literature. Some models (Mentzer and Moon, 2005) only concentrate on forecasting, however these models are not appropriate for the luxury market since, when the product lifespan is brief, a quick response to demand is more efficient than forecasting improvements. Other models, such those by Crum and Palmantier (2003) and Croxton et al. (2001), take the entire DM process into account.

In the following sections, the main contract theories that emerge from an analysis of the most recent literature will be analysed in detail.

It will discuss and compare the PROs and CONs of the individual models, their possible evolutions and implications, in order to define which is the best solution to align the parties' interests.

A compilation of the types of contracts in force is proposed in the following table.

CONTRACT	CHARACTERISTICS
Wholesale price	The products are offered at a defined price that is less than the retail price under the terms of a commercial agreement between a producer and a retailer. At a fixed wholesale price, the producer sells the goods to the retailer, who then marks them up and sells them to

[10] Supply Chain Management

	end users. Retailers can make money by selling the products to customers at a higher retail price because they can buy things in bulk at a lower cost per unit. Both parties may benefit from this type of agreement: the producer receives a dependable, large-scale customer, while the retailer gains access to products at a reduced price, thereby raising their profit margin.
Buyback	It is a written agreement between the retailer, who pays a set price per unit purchased, and the producer, who commits to buyback any unsold goods from the retailer at a set price and within a given time frame. Such a deal can assist the retailer reduce the risks of overstocking and unsold inventory while also guaranteeing a specific level of demand for the producer's goods.
Revenue-sharing	This is a sort of contract where the parties concur to split a portion of the profits made from a particular commercial activity. The producer sets a fixed price that is less than the wholesale cost for products sold to retailers. A revenue-sharing agreement's main goals are to bring the parties' interests into alignment, encourage cooperation, and give drive for the company's success. However, it should be noted that in order to ensure that the terms are reasonable, well-defined, and

	favourable for all parties concerned, such agreements necessitate rigorous negotiation and definition of details.
Quantity flexibility	It is a commercial contract that, under certain circumstances, permits fluctuations in the number of products to be supplied. In a quantity flexibility contract, the retailer and producer determine an initial amount of items to be supplied but also a range within which the quantity may be altered in response to predetermined circumstances (real demand, market fluctuations, seasonal changes, etc.). The producer is required to pay the retail back for any unsold units that are higher than a predefined threshold at the same fixed price per unit that the retailer paid. In industries with unpredictable demand patterns, they are helpful. This kind of agreement can save inventory costs, cut waste, and boost supply chain effectiveness.
Capacity reservation	This is a business practice in which a producer ensures a retailer's access to a specific quantity of products for a defined period. The retailer is required to pay a late fee (pay to delay) or the full amount (take or pay) for units that fall short of the originally ordered quantity. If the total number of units ordered is greater than the

agreed upon amount, additional units are offered at a higher price. This agreement ensures that the retailer will always have easy access to the reserved capacity, allowing them to satisfy operational needs and better control demand.

Risk based contract

A risk-based contract refers to a contractual deal between the parties that allocates risks and responsibilities based on a proper assessment of potential uncertainties and adverse events that could impact the collaboration. This type of contract aims to manage and mitigate risks by setting out specific actions and responsibilities should certain predefined risks occur.

These agreements demand serious discussion and a complete comprehension of all the possible outcomes that might occur throughout the business relationship.

Linear cost sharing

It is an agreement between two or more parties with the aim to jointly shoulder the costs of a project in direct proportion to their respective individual shares.

In cooperative projects when the parties participating want a clear approach to allocate costs based on their proportionate engagement or interest in the project, linear cost sharing contracts are frequently used.

The agreement enables the parties to promptly recover costs for the agreed-upon activities. The final outcomes are high efficiency, increased resource coordination,

	and economies of scale. Such agreements are created to cover usage of research and development as well as rights to intangible assets.
Fixed rate cost sharing	An agreement of this type is made between two or more parties in order to share costs related to a tangible or intangible asset. The parties to this agreement decide on a predetermined fixed fee via which they will equally divide the expenditures incurred during the collaborative endeavour. Fixed-rate cost sharing agreements are frequently used in situations where the parties share an interest in the project's success but want to reduce financial risk. When project expenses are reasonably predictable or when the parties want to assure a balanced investment over the course of the project, this sort of contract may be especially advantageous.
Option contract	A retailer and producer enter into a legally binding arrangement known as an option contract whereby the retail business pays the producer a fee in exchange for having the only right to buy a certain amount of goods at a predetermined price within a predetermined time frame. Due to the flexibility afforded by this contract, the retail company may choose whether or not to exercise the option to acquire the items depending on the needs for inventory, market demand, and consumer preferences.

	When dealing with limited-edition or seasonal luxury goods, this can be very helpful since the retail business can determine consumer demand and interest before making a purchase. Both the retailer and the producer profit from the option contract: the retailer receives exclusivity and the capacity to manage inventory well, and the producer secures a commitment and possible revenue stream without having to provide the items right now. This structure aids both parties in navigating uncertainty and market changes and is consistent with the dynamic character of the luxury market.

Table 3.1: Comprehensive survey of the in force contractual mechanisms for coordinating actors in supply chain management literature

Contracts are especially necessary in decentralised SC[11], because the double marginalisation effect prevents achieving the ideal SC profit (Spengler, 1950). This consequence is the result of the transfer price imposed in the SC having an impact on each actor's cost structure, preventing the gain of the optimal profit (Chen and Xiao, 2009).

[11] Supply Chain

3.3. Evaluation criteria for contract implementation

Cachon (2003) has put forth a set of evaluation criteria that provide valuable insights into the implementation of contracts, aiding in the decision-making process regarding which contract form to adopt. These criteria serve as practical guidelines for assessing the viability of various contract options:

- Administrative Costs: one crucial consideration is the administrative costs associated with a contract. This metric suggests that the effectiveness of a coordination contract is closely linked to the level of detail specified within it. In simpler terms, the more intricate the contractual provisions, the higher the administrative costs tend to be;
- Supply Chain Coordination: another pivotal aspect is ensuring that the contract is designed in a manner that discourages any participant from deviating from the optimal supply chain decisions and actions. In essence, the contract should create incentives that align with the broader objectives of supply chain coordination;
- Sharing of Risks and Rewards: equitable distribution of risks and rewards is a fundamental requirement for any contract. It should enable a fair allocation of risks and, ultimately, the overall supply chain profit among the involved parties.

It is worth noting that a substantial body of evidence supports the notion that most contracts indeed foster cooperation and an equitable sharing of risks and profits among participants. When it comes to assessing administrative expenses, contracts that involve only a single transaction tend to be simpler to understand and consequently less costly to administer. In this context, contracts featuring wholesale prices and quantity discounts are somewhat on par in terms of administrative expenses. However, contracts that involve revenue-sharing, buybacks, and quantity flexibility require a more substantial investment. This is primarily due to the increased complexity of these contract types, which necessitates a higher degree of administrative effort and additional material and informational flows to ensure their successful implementation.

In sum, Cachon's evaluation criteria offer valuable insights for decision-makers seeking to choose the most suitable contract form. These criteria help guide organizations in making informed choices that align with their specific supply chain needs and objectives.

3.4. Bargaining issues

Brun and Moretto (2012) show in detail different types of issues related to different perspectives between producer and retailer:

1. Long-term oriented;
2. Forecast oriented;
3. Operational oriented.

The long-term management of the cooperative relationship between BO and retailer is the focus of the "Long-term oriented" group.

1. A different range of products. The product selection is seen differently by the BO and the store. To increase the chance of spontaneous purchases and to draw the retailer's attention to his wares, the BO needs as much shelf space as feasible. The store, on the other hand, increases the possibility that one customer will be satisfied by offering a wider variety of items (different goods from multiple brands at various price points) in an effort to lower the risk of lost sales.

2. Creating and managing the product portfolio. When defining the product range, the BO and retailer must decide which product categories to include at the POS[12]. The retailer wants to select from a selected group of product categories, while the BO likes to display the entire collection because he is more interested in selling a complete set than a single item because an incomplete collection is viewed as a missed opportunity.
3. There is no performance measurement system (PMS). The BO typically doesn't assess the retailer's operational effectiveness. The retailer does a self-evaluation without taking into account the standards that were set in conjunction with the BO (retailers, for instance, are not interested in assessing their ability to satisfy the CSFs of the BO).

The second area of concern is "Forecast oriented" and is specifically focused on all the factors affecting forecast errors.

1. Limited information exchange on demand projections. The BO frequently disregards the information from the shop (such as the sell-out, consumer wants, retailer perception, etc.). The contract between the actors has less to do with this problem than it does with the cultural views of some businesses. The inventory level is also not disclosed, which makes it completely useless for predicting future demand.
2. The responsibilities for forecasting operations are not clearly outlined. As a result, the retail sector tends to underestimate final demand or avoid from developing personal projection plans.
3. A request for the order to continue. Many BOs require the retailer to maintain order continuity that conflicts with the retailer's requirements (for

[12] Point of Sale

example, at least one order per month or a minimum number of orders annually). However, due to the size of the store and its clientele, small shops, in particular, cannot guarantee order continuity throughout the year. This could indicate a discrepancy between BO and retailers, which might lead to lesser orders being placed early in the season. According to the SC, this behaviour does not provide the highest benefits because it is more likely to lead to stock shortages.

The "Operations-oriented" area concerns the management of interactions between parties in relation to the flow of products.

1. To handle inventories and unsold goods, there is no established policy. Overstocking may happen as a result of the BO's practice of omitting return policies from the contracts. Many well-known companies just consider new orders and ignore difficulties like lost sales or out-of-date merchandise. The retailer, who must sell the products at a lesser margin, and the BO, who lowers the exclusivity of his wares, both stand to lose money because of this trend. Frequently, the BO is ignorant of this phenomenon and disregards the sales of earlier products. This matter is related to the contract that the BO has adopted, and from the SC's perspective, it may have significant effects on the financial standpoint.
2. Examining the variation in delivery timeframes. This subject is particularly crucial when dealing with impulse purchases since missed sales may occur if a product is not present at the point of sale. Many times, BOs are more focused on producing high-quality goods than on the importance of offering clients’ high-quality services. The enormous sales losses caused by the lack of products that were formerly advertised and highlighted in periodicals have gone unnoticed by most shops. This situation has grown especially significant in big cities, where a high percentage of lost sales due to tourists

who can't come back to the store later if a product isn't accessible has been noted in recent years.

DM SUB-PROCESSES	DM CRITICAL ISSUES
Determine DM goals and strategy	– Different perception of the product range. – Defining and managing the product portfolio. – No clear definition of the roles during forecasting activities.
Determine forecasting procedures	– Lack of information sharing about demand forecasting.
Plan information flow	– Lack of information sharing about demand forecasting: no communication of plans to retailers. – Request of order continuity.
Determine synchronization procedures	– Lack of policy for the management of unsold products and stock management.
Develop contingency management system	– Management of lead time variability.
Develop framework metrics	– Absence of a performance measurement system.

Table 3.2: Identification of the sub-process of demand management influenced by each critical issue.
Source(s): Brun, Alessandro, and Antonella Moretto. "Contract Design and Supply Chain Management in the Luxury Jewellery Industry." International Journal of Retail & Distribution Management, vol. 40, no. 8, Emerald Publishing Limited, June 2012, pp. 607–28. https://doi.org/10.1108/09590551211245416

All the problems highlighted by the three groups can have an impact on the success of the DM process and the CSFs, which provide a competitive advantage to the BO. The relationship between the main DM sub-processes and the significant problems described above is illustrated in Table 3.2. The analysis of the strategic sub-processes was carried out using the model presented by Croxton et al. (2001)[13].

3.5. The origins

The majority of businesses in the last years have outsourced their production processes and use indirect channels to market their goods.
However, this business strategy has shown some drawbacks:

- The lack of control over production operations for the BO;
- The lack of control over retail, which suggests a lesser level of control over brand image.

These factors have prompted businesses to gradually alter their strategies: in the recent years, re-insourcing of production activities has been strongly revaluated as a successful strategy.
The impact of poor DM process management on numerous CSFs highlights the value of carefully controlling this operational process for luxury brands and jewellery businesses.

[13] The model provided by Croxton et al. (2001) offers a framework for examining the supply chain management's strategic sub-processes. It describes how different aspects of supply chain operations can be divided into three key phases: strategic planning, operational planning, and implementation.
It stresses how these phases are interconnected and how strategic decisions and operational execution must be in line. To optimize supply chain performance and accomplish strategic goals, all phases of the process must effectively communicate with one another and coordinate their efforts.

The correlations between DM key concerns and the affected CSFs are summarised in Table 3.3. It shows that for the jewellery sector, the production of flawless and extremely accurate products may not be sufficient to guarantee the success and full execution of the firm plan. Operational procedures and DM could have a significant impact on a company's success, affecting things like the brand's reputation or the degree of exclusivity (Brun & Moretto, 2012).

Critical issue	Exclusivity	Emotional appeal	Brand reputation	Uniqueness	Creation of a lifestyle
Different perception of the product range			X		
Defining and managing the product portfolio	X	X	X		
Absence of a PMS		X			
Lack of information sharing	X	X		X	X
No clear definition of roles	X	X	X		
Request of order continuity		X			
Lack of a policy for the management of unsold products and stock management	X	X	X	X	

Table 3.3: Examination of the impact on the CSF
Source(s): Brun, Alessandro, and Antonella Moretto. "Contract Design and Supply Chain Management in the Luxury Jewellery Industry." International Journal of Retail & Distribution Management, vol. 40, no. 8, Emerald Publishing Limited, June 2012, pp. 607–28.

The coordination contract is the most efficient tool for trying to establish a cooperative SC. The sharing of costs, risks, and rewards as well as the detection and correction of potential behavioural misalignments between the actors are all made possible by the use of powerful contract systems. To be effective, a contract must be drafted in accordance with the particular requirements of the concerned industry. The parties involved should be protected by specific clauses in a new contract that is made.

The major difficulties discussed above must be resolved through a hypothetical contract, emphasizing the significance of:

- Encouraging retailers to comprehend and raise brand awareness of particular products, possibly by implementing incentives;
- Promoting inter-actor cooperation and communication for growth based on a single strategy and a long-term viewpoint. Perhaps by including explicit provisions that incentivize shopkeepers to take particular training classes. The issue of not having a performance measurement system may be resolved by doing this;
- Increasing the emotional appeal associated with the level of service, enabling a better alignment between the customer's and the retailer's view of the brand;
- Increasing the overall process' efficiency and lowering predicting errors by having each party assume a certain level of responsibility;
- The BO recovering unsold goods (or at least a specific amount);
- Reducing variability in the management of the lead-time variability by specifying lead-time and delivery should occurs in time.

These clauses in the contract serve to equalize the significance of the same CSF for the BO and the retailer. Additionally, this might enhance how customers see your brand.

The BO would gain a lot of advantages if these observations were taken into consideration, to name a few:

– Ensured product availability in retailers;
– A better comprehension of consumer preferences;
– Better performance thanks to centrally managed vital supplies;
– The DM process being streamlined (lowering the likelihood of stock outs and overstocks).

In the sections that follow, it will be examined in depth how each type of contract seeks to align its clauses with the aforementioned guidelines and observations.

4. Contract

The following section will take a closer look at some of the above-mentioned contract types that have become more successful in the industry of our interest in recent years.

4.1. Wholesale price contract

In this particular type of contract, a retailer and a producer enter into an agreement that involves the purchase and sale of items. The retailer commits to buying these items at an expected unit price, while the producer offers them at a wholesale price. Within this arrangement, the retailer is afforded the flexibility to determine their optimal stock level and subsequently sells the goods during the designated selling season. An important characteristic of this contract is that the retailer does not have the option to return unsold merchandise, meaning they retain the entire profit generated from the sales. The transfer payment mechanism within this contract is represented as

$$T_w(q, w) = wq$$

Equation 4.1

where 'w' represents the wholesale price and 'q' denotes the quantity of items purchased. It's important to note that this wholesale price contract effectively coordinates the supply chain only when the wholesale price is equal to or greater than the producer's production cost.

While Dong and Zhu (2006) concentrate on the availability of stocks within the supply chain, Sabbaghi, Sheffi, and Tsitsiklis (2007) delve into a similar scenario wherein capacity constraints influence the wholesale price, Chen and Li (2007) approach the wholesale price contract from a different angle by focusing on aspects like double marginalization and demand distribution.

The two-party tariff contract is a specific type of wholesale contract. In this arrangement, the producer's wholesale price aligns precisely with their production cost. What sets this contract apart is the introduction of a franchise fee 'F', which is determined before the demand is observed. This fee effectively transfers all demand-related risks to the retailer. The transfer payment structure in this case is represented as

$$T_{w2p}(q, w) = wq + F$$

Equation 4.2

Further research by Fauli-Oller and Sandonis (2007) and San Martin and Saracho (2010) delves into the realm of patent licenses and royalties within the licensing mechanism concerning the two-party tariff contract. These studies shed light on how this specific type of contract can be adapted and utilized within different contexts, exploring the intricacies of licensing arrangements and their implications for the supply chain.

4.2. Buyback contracts

In a buyback contract, a retailer engages in a unique arrangement with a producer. Before the selling season commences, the retailer commits to purchasing 'q' units at a price of 'w' per unit. However, the retailer retains the option to return any unsold items, up to a maximum of 'q', to the producer at the conclusion of the selling period (Cachon & Lariviere, 2005; Jokar & Hosseini- Motlagh, 2020; Yue & Raghunathan, 2007).

This unsold merchandise is repurchased by the producer at a slightly lower rate, termed the buyback price ('b' where $b < w$). This arrangement gives rise to an intriguing transfer payment structure, represented as:

$$T_b(q, w, b) = bS(q) + (w - b)q$$

Equation 4.3

The buyback contract is a widely adopted practice in the realm of supply chain management. What makes it particularly significant is its ability to provide an invaluable safety net to retailers when market demand follows a stochastic pattern. In situations where there's an excess of stock, the option to repurchase unsold inventory becomes exceptionally valuable to the retailer. This buyback mechanism effectively mitigates the potential costs associated with surplus inventory, as the retailer has the assurance of returning unsold goods. Consequently, when a buyback option is on the table, the retailer may feel more inclined to place larger orders with the producer compared to situations where no such option exists. The producer, through thoughtful management of repurchase terms, can harmonize the interests of the retailer with the broader supply chain objectives.

Over the years, many have become interested in studying the topic. Donohue (2000) embarks on a mission to enhance demand forecast accuracy within the dynamic fashion industry, all while orchestrating the complexities of a buyback contract. Hou et al. (2010) take a comprehensive approach, analysing the multifaceted effects of supply and demand uncertainty, supply disruptions, and the intricate decision-making processes involved in buyback contracts. Meanwhile, Wang and Zipkin (2009) delve into the curious phenomenon known as "channel stuffing", shedding light on the intricacies of managing inventory within the context of buyback contracts.

When it comes to deciding the buyback price, there are two distinct approaches. The buyback price can be established ex ante, which means it's determined before the uncertain market demand is realized. At this point, neither the producer nor the retailer knows the exact quantity of unsold inventory because the ambiguous demand has not yet been satisfied. Alternatively, the buyback price can be decided ex post, after the uncertain demand has unfolded. In this scenario, both the producer and the retailer are aware of the exact quantity of unsold inventory when making the buyback price decision. In the domain of stochastic decision-making, these contingent decisions fall under the category of "wait-and-see" solutions, as coined by Birge & Louveaux (2011).

In the grand scheme of supply chain management, nurturing long-term producer relationships is of paramount importance. Theoretical research suggests that, for the supply chain and the producer, the ex-ante buyback contract tends to outperform the ex-post buyback contract but performs less favourably for the retailer. However, experimental data indicates that both the retailer and the producer, and consequently the overall supply chain, fare worse with the ex-ante buyback contract compared to the ex-post variant. To bridge the gap between theory and experimental findings, researchers have developed behavioural models that emphasize human behavioural preferences for reciprocity and fairness.

Ex-post buyback pricing appears to encourage greater reciprocity among producers and greater trust among retailers, aligning more effectively with the interests of all supply chain partners. These findings suggest that, in practice, ex-post buyback pricing might be a more competitive and viable option, considering that supply chain managers typically make the majority of the decisions.

In summary, the research underscores the superiority of repurchase contracts over wholesale price contracts. To address the challenges associated with aligning initial and replenishment order quantities and to facilitate mass customization, decisions on inventory replenishment must be made based on early demand information for fashion products (Fisher et al., 2001). Furthermore, production choices are made following an analysis of demand realization, as elucidated by Zipkin (2001).

4.2.1. Standard theory

In this study, there is just one producer and one retailer in the supply chain (Zhu et al., 2022). Products are created by the producer at a unit cost of 'c', and are then sold to the market by the retailer at a unit cost 'p'. Market demand 'D' follows a cumulative distribution function $F(\cdot)$ and is stochastic. In a Stackelberg game, where the cost, price, and market demand information are presumed to be public and known to both parties, the producer negotiates deals with the retailer under a buyback agreement. The provider may choose to set the buyback pricing ex-ante or ex-post.

Ex-ante buyback pricing

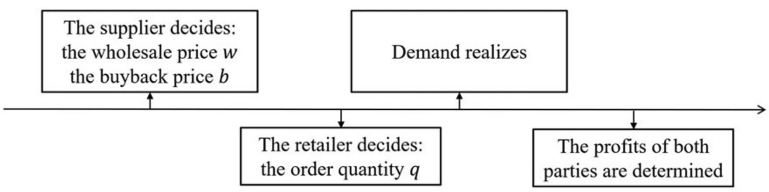

Figure 4.1: Ex-ante buyback pricing sequence of events within the strategic interaction between producer and retailer

Under ex-ante buyback pricing, the sequence of events is shown in Figure 4.1 for the game between the producer and the retailer. First, the producer offers a wholesale price 'w' and a buyback price 'b' (naturally, $0 \leq b \leq w \leq p$). Second, the retailer places an order of quantity 'q' and pays the producer 'w_q'.

Third, market demand realizes; if the realized demand 'd' is higher than 'q', all products are sold to the market; otherwise, the retailer has leftover inventory

$$(q-d)^{+} \equiv max(q-d, 0)$$

Equation 4.4

and the producer buys back it by paying the retailer

$$b(q\ -\ d)$$

Equation 4.5

The retailer's profit is as follows:

$$\pi_r(q,d) = p\ min(q,d) - wq + b(q-d)^{+}$$

Equation 4.6

where the first term is the sales revenue from the market, the second is the ordering cost paying to the producer, and the last is the buyback payment from the producer.

The retailer chooses a quantity to maximize the expected profit.

The optimal order quantity is given by:

$$q_A^{*}(w,b) = \begin{cases} F^{-1}\left(\dfrac{p-w}{p-b}\right), & if\ b < w \leq p \\ +\infty, & if\ w = b < p \\ arbitrary\ value, & if\ w = b = p \end{cases}$$

Equation 4.7

Given a retailer's order quantity 'q', the producer's profit is as follows:

$$\pi_s(w,b,q,d) = q(w-c) -\ b(q-d)^{+}$$

Equation 4.8

where the first term represents wholesale income and the second is the cost of buying back unused inventory. The shop chooses the best order quantity by using backward induction.

The producer sets the wholesale price and the ex-ante buyback price close to the market price, as the first player in the game, to get the biggest share of the supply chain profit. As a result, the retailer's order quantity is the same as the one that maximizes the supply chain profit because the double marginalization is eliminated. However, at this equilibrium solution, the retailer receives zero, which is the smallest percentage of the supply chain profit.

Ex-post buyback pricing

Ex-post buyback pricing follows the same course of events as ex-ante buyback pricing, with the exception that the producer determines the repurchase price 'b' after the market demand has been satisfied and the remaining inventory has been identified. As a result, there are three decision stages in the game: the wholesale price 'w' is decided by the producer, the order quantity 'q' is decided by the retailer, and the ex-post buyback price 'b' is decided by the producer. By using backward induction, the equilibrium of this game is once more determined.

The third stage's decision problem is for the producer to select the best ex-post pricing, which is as follows:

$$\max_{b \geq 0}\{(w-c)q - b(q-d)^{+}\}$$

Equation 4.9

where $(w - c)q$ is the wholesale income and $b(q-d)^{+}$ is the buyback payment to the retailer after observing demand realization 'd'.

The equilibrium means that when the producer and the retailer maximize the profit, the ex-post buyback price is zero. Consequently, the game is similar to one under the wholesale price contract. Under the equilibrium, the wholesale price ‘w^*’ is greater than ‘c ’ and smaller than ‘p ’. Hence, there is double marginalization in the supply chain. The order quantity in equilibrium is smaller than the quantity that maximizes the supply chain profit.

In conclusion, the order quantity, supply chain efficiency and producer profit are all theoretically higher under ex-ante buyback pricing than they are under ex-post buyback pricing; but retailer profit is lower under ex-ante buyback pricing.

Ex-post buyback price offers the store additional advantages, such as improved producer confidence, lessened concern over fairness and more rational decision-making with lower rejection. Compared to ex-ante buyback price, ex-post buyback pricing has a significantly higher supply chain efficiency, increasing the profitability of both the retailer and the producer.

4.2.2. Future evolution

A future study might look into ex-post buyback pricing based on random matching as the experiment is based on fixed matching. Another study may develop preference-dependent contracts in the future to increase the effectiveness of the supply chain, and, secondly, consider the possibility that contract specifications are not altered in accordance with behavioural preferences.

4.3. Revenues – sharing contracts

In the realm of revenue-sharing contracts, the retailer pays a reduced wholesale price, denoted as ‘w_r’ per unit, to the producer (Govindan et al., 2013). In return, a pivotal agreement is struck: the retailer commits to sharing the profit generated from sales with the producer, with the retailer retaining a fraction ‘φ’ of the revenue, and the remainder $(1 - \varphi)$ going to the producer. This complex yet intriguing transfer payment mechanism is mathematically represented as follows, assuming an equal division of all revenue:

$$T_r(q, w, \varphi) = (w_r + (q - \varphi)v)q + (1 - \varphi)(p - v)S(q)$$

Equation 4.10

Numerous scholarly references delve into the intricate nuances of revenue-sharing contracts, exploring various dimensions such as inventory management, competitiveness, scenarios involving risk-averse retailers, and even the application of fuzzy variables. Notable contributors in this field include Cachon and Lariviere (2005), Li and He (2008), and Dong and Li (2009)

A dive into practical scenario is proposed to better grasp the dynamics of revenue-sharing contracts. Consider a supply chain with two risk-neutral companies: a downstream entity, referred to as the retailer, and an upstream player known as the producer. The retailer places an order for 'q' units of a particular asset from the producer. These 'q' units are utilized by the retailer to generate sales over a single selling season, while the producer manufactures these units at a fixed marginal cost.

It can be assumed that the quantity of units bought by the retailer, denoted as 'q ', directly influences the predicted revenue of the retailer throughout the selling season, represented as $R(q)$. It's crucial to note that, for all values of 'q ' greater than or equal to zero, the function $R(q)$ is strictly concave and differentiable. In simpler terms, $R(q)$ has a downward-curving shape and it changes smoothly with variations in 'q '.

Moreover, to ensure the product's viability in the market, it is assumed that $R'(0)$, which represents the derivative of $R(q)$ at 'q' equals zero, is greater than the marginal production cost 'c'. Additionally, another assumption is that there exists an optimal finite production quantity, where $R'(\infty)$, the derivative of $R(q)$ as 'q' approaches infinity, is less than the marginal cost 'c'.

Now, it brings the revenue-sharing contract into the picture. This contract involves two critical parameters: 'f ' and 'w'.

'f ' represents the "share of retail revenue the retailer keeps", essentially determining how the retailer and producer split the revenue generated from retail sales. 'f ' varies between 0 and 1, reflecting the proportion of revenue retained by the retailer.

The second parameter, 'w', signifies the "wholesale price" per unit that the retailer pays to the producer. It's worth noting that a standard wholesale price contract is a specific case of a revenue-sharing contract where 'f ' equals 1.

The producer takes the lead in establishing and disclosing the terms of the revenue-sharing contract. The retailer then proceeds to place an order for 'q' units and pays the producer 'wq'. Subsequently, the producer manufactures and delivers the requested 'q' units, and the retailer generates revenue amounting to $R(q)$. The retailer is obliged to transfer $(1 - f)R(q)$ to the producer while retaining the remaining $f_R(q)$. Given that the other firm does the same, each firm maximizes its expected profit. Both companies are well-informed about every aspect of this contractual arrangement.

Here's where the beauty of revenue-sharing comes into play: with every unit purchased, the retailer not only pays the producer a wholesale price but also shares a portion of the sales proceeds. When the producer sets a wholesale price above the marginal cost, the profit within the supply chain falls short of its optimal potential because the retailer tends to order fewer units than what would be ideal (a concept discussed by Spengler in 1950). The power of revenue-sharing lies in its ability to incentivize the retailer to order the optimal quantity for the supply chain, effectively aligning the interests of all parties involved.

In essence, revenue-sharing contracts offer a solution to the challenge of coordinating supply chains by ensuring that production and ordering decisions lead to optimal outcomes.

4.3.1. Limitation

Revenue-sharing agreements are quite helpful in many different supply chain scenarios. However, there must be some limitations placed on revenue-sharing agreements, otherwise, every sector of the economy would be expected to adopt them.

- Administrative costs

  The producer's capacity to ex-post validates the retailer's income is crucial for adopting revenue-sharing. The cost of tying the retailer's and producer's information systems together would be borne by the channel, at the very least. Any rise in the producer's earnings could rapidly be overwhelmed by covering such administrative charges. The income from coordination may not always be enough to pay for these costs. If the supply chain performance under the wholesale price contract is very close to perfection and if the producer obtains a large percentage of the supply chain profit, administrative costs may limit the adoption of revenue-sharing contracts (Supply Chain Coordination With Revenue-Sharing Contracts: Strengths and Limitations on JSTOR, n.d.).
- Retailer effort, moral hazard and revenue-sharing

  It has previously been presumed that revenue is influenced by the retailer's order volume and the sale price.

  In truth, the retailer affects sales through a variety of different activities, such as marketing, customer service, and merchandising.

  The influence of those potential judgments on the viability of revenue-sharing agreements has now been taken into account.

Particularly, store effort has an impact on such demand. Better service and cleaner establishments do not come for free; effort is naturally expensive. Quantity effects predominate when retail effort has little or no effect on demand. The producer prefers to add a significant portion of the supply chain earnings through revenue-sharing. If retail activity has a substantial impact on demand, it is preferable for the producer to adopt a wholesale-price contract as opposed to a revenue-sharing structure (Supply Chain Coordination With Revenue-Sharing Contracts: Strengths and Limitations on JSTOR, n.d.).

4.3.2. Future evolution

As it is now primarily qualitative due to the lack of statistical significance, a quantitative study could be performed in the future to statistically verify the critical characteristics revealed. In addition, more research is needed to fully understand the impacts of revenue-sharing agreements, particularly how well they can lower risk and foster cooperation among supply chain actors.

4.3.3. Conclusion

According to the data, a revenue-sharing arrangement is particularly tempting. It synchronizes the supply chain and distributes the earnings in a random manner. The producer's cost of production is lower than his wholesale price, but the money he pays the retailer more than makes up for this. The following conclusions were drawn from comparing this type of contract with other contract typologies:

– The best wholesale price should be set above marginal cost under a wholesale-price contract, but with revenue-sharing, it should be set below marginal cost (Supply Chain Coordination With Revenue-Sharing Contracts: Strengths and Limitations on JSTOR, n.d.);

- The contract can solve issues that buybacks cannot coordinate. Since it is independent of the retail price, a coordinating revenue-sharing agreement can coordinate a Newsvendor problem with price-dependent demand (Supply Chain Coordination With Revenue-Sharing Contracts: Strengths and Limitations on JSTOR, n.d.).

The conclusion is that revenue-sharing contracts are effective at coordinating the retailer's choice of the quantity of purchases, but they are ineffective at coordinating the retailer's choice of the level of effort. When retail activity has an impact on demand that is significant enough, revenue-sharing arrangements ought to be avoided.

With the correct balance of the wholesale price discount rate and revenue-share ratio, both the producer and the retailer can increase their revenue-sharing contract earnings (Ryu, 2022). The numerical analysis's findings indicate that while revenue-sharing unquestionably improves the effectiveness of the supply chain, it does not fully synchronize the system. By controlling the wholesale pricing and revenue share ratio, each supply chain participant can benefit from this supply chain contract.

Therefore, although buyback and revenue-sharing mechanisms produce superior results when compared to wholesale price contracts, experimental examinations on supply chain contracts show that these mechanisms do not lead to the system performing at its best overall (Katok & Wu, 2009).

4.4. Other types of minor contracts

According to Tsay (1999), a quantity flexibility contract that allows the retailer to select the order quantity within a given range helps synchronize a supply chain.

Barnes-Schuster, Bassok, and Anupindi (2002) demonstrate the possibility of coordination with option implementation enabling a buyer to reserve the quantity and exercise those options at a subsequent time using an option contract.

Cachon (2004) proves how a supply chain can be coordinated utilizing an advance purchase discount contract by offering discounts for the early buy orders in comparison to the emergency orders placed during the selling season.

4.5. Future evolution

In light of the conducted analysis, potential future advancements shared by various considered publications revolve around enhancing transparency and authenticity, risk management, and the implementation of more dynamic pricing strategies. Below are some suggestions identified in the analysed sources:

- A deeper analysis of the significance of demand management and sales channels in the luxury sector, aimed at developing more sophisticated models for demand forecasting and optimizing the customer experience (Govindan et al., 2013);
- Further research on structuring contracts in situations of irregular or fluctuating demand, with the intention of creating more intricate models for optimal contract management (Govindan et al., 2013);

- Delving into the design of contracts for supply chain partners with incomplete information, with the goal of improving information management models and reducing information asymmetry;
- Integrating interactive elements into digital contracts, allowing producers to monitor sales trends, product placement in stores, and all aspects related to the store that influence brand perception for customers;
- Implementing mechanisms of dynamic pricing that adjust contract terms and prices in real-time based on market conditions and customer demand.

These improvements can serve as a useful guide to steer the parties involved in contract usage through the evolving landscape of the luxury sector.

4.6. Pros & Cons

When weighing the benefits and drawbacks of the contracts discussed in the earlier parts, it becomes clear that:

- Revenue-sharing agreements promote improved coordination and communication between the various supply chain partners. The agreement encourages the maximization of overall profits for all parties involved in the supply chain as well as for individual actors. The biggest drawback of this kind of contract is how challenging it is to determine each actor's profit in the supply chain with accuracy. In addition, reaching an agreement on the profit split between the parties and anticipating every scenario that can arise throughout the supply chain could be difficult (Supply Chain Coordination With Revenue-Sharing Contracts: Strengths and Limitations on JSTOR, n.d.);
- The flexibility and reduced risk for the buyer, who has the chance to see the product before making the actual purchase, as well as the producer's capacity to plan production and procurement more skilfully because of

better knowledge of future demand, are the key benefits of option contracts. This sort of contract entails the risk that the producer may not have sufficient incentive to produce sizable quantities of goods because there is no assurance that the option will be exercised (Zhao, 2010);

- On the other side, the main advantage of buyback contracts is to make sure that the producer can repurchase goods that the retailer doesn't sell, hence lowering the risk of overproduction. Furthermore, knowing that unsold merchandise might be returned to the producer may encourage the retailer to place greater initial orders for the goods. However, it's possible that the producer will reduce the buyback price to offset its own losses, which could not be in the retailer's advantage (Zhu et al., 2022);
- Ultimately, the wholesale pricing contract often performs better than a buyback contract. It gives retailers the ability to adjust pricing in response to changes in demand and market trends and is simple to apply and comprehend. The wholesale pricing contract has a shortcoming, nevertheless, in terms of efficiently coordinating a supply chain, as shown by classical modelling. Wholesale price contracts are yet widely used in practical applications despite this drawback. However, they are not without their own difficulties, which can result in double marginalization and a situation where the retailer's order amount is insufficient to maximize supply chain earnings (Spengler, 1950).

  Further complexity is introduced by the tendency for retailers to underestimate demand. Both the retailer and the producer may experience challenges with inventory management as a result.

In principle, all of these contractual arrangements can help to coordinate the supply chain and lower the possibility of over- or under-production. The sort of contract to use will be determined by the particular needs of the supply chain in question as well as the connection between the parties. To choose the choice that best suits a person's needs, it is crucial to thoroughly weigh the advantages and disadvantages of each.

4.7. Conclusions

It can be stated that the buyback contract is the one that has been most extensively investigated and analysed in the literature, followed by the revenue-sharing contract type. Despite many current studies being primarily theoretical in nature, it is possible to conclude that these two contracts exhibit characteristics particularly aligned with the sector under examination. This allows other contract types to be excluded, favouring a focus on their various nuances that may emerge. The aid of game theory, which has been widely studied and examined in the modern contractual landscape, could be an excellent tool for analysing them from an additional prospective.

5. Game theory literature overview

A supply chain can be described as *"a system of suppliers, manufacturers, distributors, retailers, and customers where materials flow downstream from suppliers to customers and information flows in both directions"* (Ganeshan et al. [57]). This intricate interplay of resources and information forms the backbone of modern commerce.

Supply chain management, on the other hand, is often defined by scholars as *"a set of multifaceted management processes"* (Fiala, 2015).

Everything is included in it, from distribution and customer service to production and procurement. Essentially, it involves orchestrating the various components of the supply chain to ensure smooth and efficient operations.

One key insight that has emerged with the evolution of supply chain management is the recognition that the operational decisions, made by different entities within the supply chain, can have a profound impact, not only on their profitability but also on the overall profitability of the entire supply chain. In other words, what may seem like an optimal decision for one part of the supply chain can have unintended consequences when viewed from a global perspective (Fiala, 2015).

Each entity within the supply chain naturally strives to optimize its preferences and objectives. However, herein lies a critical challenge. What may appear as a locally efficient decision, one that serves the immediate interests of a single player can sometimes lead to inefficiencies at the broader, global level of the supply chain. This paradox underscores the need for effective coordination and collaboration among all stakeholders.

In the highly competitive landscape of modern business, companies are acutely aware of the imperative to enhance supply chain coordination. The goal is twofold: to reduce operational costs and to maintain a competitive edge in a dynamic business environment. To achieve this, researchers and practitioners have explored various mechanisms aimed at ensuring seamless coordination within supply chains (Yao et al., 2007).

Supply chain contracts emerge as pivotal tools in mitigating uncertainty and fostering coordination among these mechanisms. Contracts play a central role in aligning the interests of different supply chain partners and guiding their actions toward mutually beneficial outcomes. Effective contract design is, therefore, crucial for enhancing supply chain performance and profitability.

In the context of producer–retailer supply chains, a common practice has been the adoption of wholesale price mechanisms. However, this seemingly straightforward approach has often led to conflicts of interest between producers and retailers. It is considered this example: owing to the unpredictable nature of market demand, retailers often prefer the flexibility to place orders with producers as needed. This helps them avoid the costs of maintaining excess inventory and enables them to promptly adapt to shifting market dynamics (Zhao, 2010).

On the flip side, producers are inclined to encourage retailers to place comprehensive orders as early as possible. This strategy serves to mitigate the risks associated with overproduction and underproduction, promoting a more stable supply chain.

Such conflicts of interest between retailers and producers can, unfortunately, lead to inefficiencies in the supply chain, hampering overall performance. Recognizing the significance of this challenge, this paper adopts a cooperative game theory approach to tackle the issue of coordination within producer-retailer supply chains. By fostering collaboration and aligning the interests of all involved parties, the aim is to unlock the full potential of these supply chains, ultimately leading to improved efficiency and profitability.

5.1. Introduction

In this section, the focus is shifted towards a detailed examination of contract characteristics, with an emphasis on game theory, and in particular on Nash equilibrium. This concept plays a crucial role in understanding the dynamics of supply chain contracts.

Game theory, initially introduced by Von Neumann and Morgenstern in 1944, is a branch of applied mathematics that finds applications in a wide array of domains, including economics, politics, management, and organization. It provides a powerful framework for analysing strategic interactions among rational decision-makers (Barari et al., 2012; Hernández et al., 2014; Sadeghi & Zandieh, 2011; Tsai et al., 2011; Xiaohui et al., 2014).

A recent study by Kundu, Jain, Kumar, and Chandra (2014) underscore the significance of game theory as the preferred method for analysing relational and behavioural aspects within the context of supply chains. Researchers in supply chain management increasingly leverage game theory and economic principles to gain insights into, predict, and guide strategic operational decisions in the complex, multi-agent environments that characterize modern supply chains. Notably, these studies have shown that employing game theory can lead to more equitable profit sharing among the actors within the supply chain (Köse & Canbulut, 2023).

Basically, game theory provides a formal framework for modelling situations where decision-makers, often referred to as players, make choices to maximize their own utility (Nagarajan & Sošić, 2008). Crucially, they do so while recognizing that other players are also pursuing strategies to maximize their own gains. The decisions made by one player can significantly impact the utilities or outcomes of others.

This framework is built upon several fundamental assumptions (Canbulut et al., 2020):

- Logical Decision-Making: both producers and retailers are assumed to make logical decisions. In other words, their choices are rational and aimed at increasing their profitability;
- Risk Neutrality: each member of the supply chain is considered risk-neutral. This means that their primary motivation is to enhance their profitability, and they are not inherently risk averse;
- Diverse Strategies: players have access to various strategies that can be employed to achieve their profitability objectives. Different strategy combinations yield different outcomes;
- Mutual Benefit: in cases where there exists a strategy or set of strategies that maximize the profits of both the producer and the retailer simultaneously, rational parties will not unilaterally deviate from these strategies. This concept aligns with the notion of a Nash equilibrium, a fundamental concept in game theory.

In essence, the study of game theory equips with the tools to analyse and understand the strategic interactions within supply chains, shedding light on how rational actors navigate these complex systems while considering their own interests and those of their counterparts.

5.2. Non-cooperative vs cooperative games

Game theoretic models can be broadly categorized into two main types: non-cooperative and cooperative, each offering a unique perspective on how players interact within a given context. Despite their differences in theoretical content and analytical methodologies, these two approaches essentially address the same fundamental issue-strategic decision-making (Fiala, 2015).

Non-cooperative game theory primarily focuses on strategies, delving into what actions one can reasonably anticipate from the players and the intricate details of how they arrive at these actions. It delves into the tactical aspects of individual decision-making within the broader context of a game. In contrast, cooperative game theory takes a different route. Instead of scrutinizing individual strategies, it directly examines the set of potential outcomes that can be achieved when players collaborate. It explores questions such as what players can collectively attain, how coalitions may form with the goal of achieving more collectively than individually, how these coalitions distribute the outcome, and whether these outcomes are stable and robust over time (Nagarajan & Sošić, 2008).

One pivotal theme in game theory analysis is that of feasible outcomes. Feasible outcomes encompass the entire spectrum of potential results that players may achieve, even if some of these outcomes may not align with their individual incentives. Once this set is established, the focus shifts to how players navigate and select an outcome from this feasible set (Nagarajan & Sošić, 2008).

Another significant theme revolves around stability. As players decide on allocations from the set of feasible outcomes, they may consider options such as forming coalitions and agreeing on joint courses of action. This dynamic process involves assessing whether these coalitions and outcomes will remain stable and sustainable in the face of potential deviations.

Game theory analysis plays a critical role in the decision-making process, aiding participants in evaluating whether cooperation or non-cooperation is the most favourable strategy. In the realm of cooperative games, participants must decide on the type of contract worth implementing and design it to ensure the satisfaction of both parties with the contractual terms (Guardiola et al. 2007). However, if the parties fail to reach a consensus on these terms through bargaining, they may resort to non-cooperation, rendering them rivals in the supply chain game (

Figure 5.1) (Govindan et al., 2013).

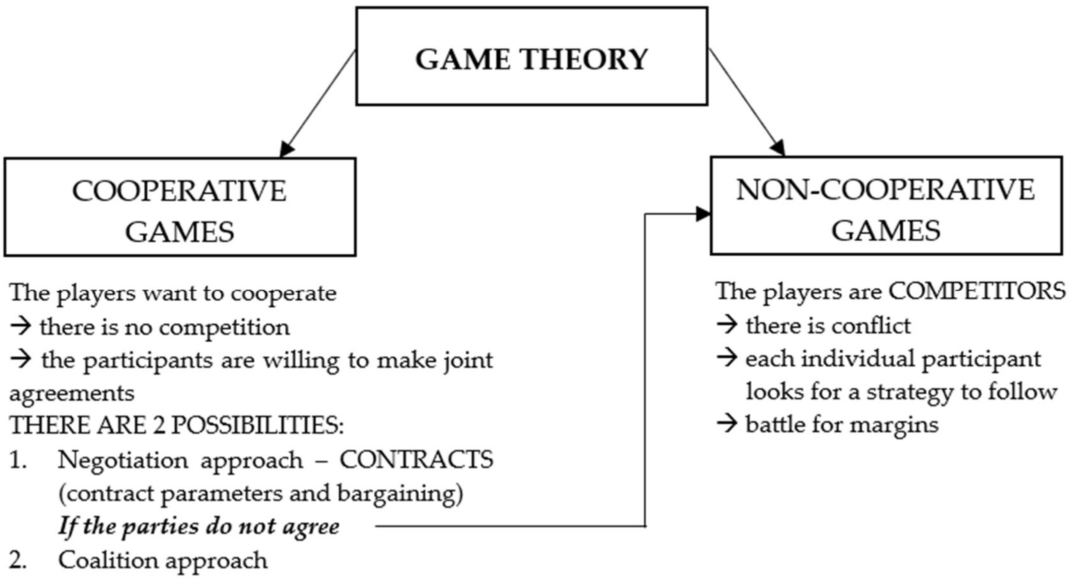

Figure 5.1: Game theory approach for coordination
Source(s): Govindan, K., Popiuc, M. N., & Diabat, A. (2013). Overview of coordination contracts within forward and reverse supply chains. Journal of Cleaner Production, 47, 319–334. https://doi.org/10.1016/j.jclepro.2013.02.001

A notable study by Li et al. (2007) explores the dynamics of cooperation in a buyer-seller, starting from the premise of a monopolistic market. They employ a game theoretical model, drawing parallels between cooperative and non-cooperative games. By considering quantity discounts as a mechanism for achieving cooperation, their research demonstrates several key findings:

- Total system profit is higher in a cooperative setting than in a non-cooperative one;
- The optimal order quantity for the buyer is greater under cooperation than under non-cooperation;
- The wholesale price from the seller to the buyer is lower in a cooperative arrangement than in a non-cooperative one.

For a more comprehensive exploration of the applicability of game theory within supply chain management, interested readers can delve into the detailed analyses provided by Cachon and Netessine (2005), Nagarajan (2005), and Nagarajan and Sosic (2008). These sources offer extensive insights into the strategic intricacies of supply chain coordination.

5.2.1. Fundamental concept of the games

As embarking on this journey into the realm of contract analysis within the framework of game theory, it's essential to build a solid foundation by acquiring some of the fundamental concepts that underpin this discipline. Game theory, a multifaceted field of study, offers invaluable insights into strategic interactions, decision-making, and cooperation among rational actors. At the heart of game theory lies the concept of strategic interactions. This notion is akin to a chess match where each move by one player directly affects the options and strategies available to the opponent. Game theory equips with the tools to dissect and comprehend these intricate dynamics.

Definition 1

A game is a situation that can be described by:

- *A set of players (with more than one element);*
- *An initial situation;*
- *The way the players must act (rules) and all their available moves;*
- *All possible final situations (outcome): they can be clustered in 3 groups*
  - *Each player wants to win;*
  - *Each player wants to lose;*
  - *Each player wants to tie;*
- *The preferences of all agents on the set of final situations.*

(*A Primer in Game Theory - Roberto Lucchetti - Libro - Esculapio - | IBS*, 2011)

Definition 2

A strategy encompasses a set of actions or decisions that a player can undertake. A dominant strategy is a strategic choice that allows each player to achieve, regardless of the other's options, the best possible outcome (Che Significa Strategia Dominante - Dizionari Simone Online, n.d.).

Definition 3

In game theory, a payoff is defined as a player's winnings that result from a function defined over an appropriate set of players and strategies, or a matrix evaluated at an equilibrium point deduced by the minimax method (Payoff, n.d.).

Definition 4

Consider a two-person game, where Player 1 assumes the role of the leader (P1) and Player 2 (P2) is the follower.

Each player has their respective objective functions[14] *(*$f_1(x_1, x_2)$ *for P1 and* $f_2(x_1, x_2)$ *for P2) that depend on their strategy choices (*x_1 *for P1 and* x_2 *for P2*[15]*). We assume that each player's objective is to maximize his objective function. Suppose P2 chooses the strategy as* $x_2 = \hat{x}_2$ *and announces it to P1. The best response* $x_1^R(\hat{x}_2)$ *of P1 is obtained as the solution of the optimization problem* $x_1^R(\hat{x}_2) = arg\, max_{x_1 \epsilon X_1} f_1(x_1, \hat{x}_2)$ *. Performing this optimization for all* $x_2 \epsilon X_2$ *we obtain the best response* $x_1^R(x_2)$ *given as a function of* x_2*. Similarly, the best response* $x_2^R(x_1)$ *of P2 can be found as a function of* x_1 (Nagarajan & Sošić, 2008).

Definition 5

An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player (Shor, n.d.).

Non-cooperative games

Definition 6

A non-cooperative game in strategic form with n players is

$$(X_i, u_i : X = \times_{i=1}^{n} X_i (X_i, u_i : X = \times_{i=1}^{n} X_i \to R)$$

Equation 5.1

- X_i *represents the set of strategies of Player i;*
- $u_i : X \to R$ *is the utility function of Player i.*

---

[14] The function that it is desired to maximize or minimize (Objective Function - Quick Search Results | Oxford English Dictionary, n.d.).

[15] $x_1 \epsilon X_1$ *and* $x_2 \epsilon X_2$, with X_1 and X_2 their respective feasible regions.

In the case of two players:

$$(X, Y, f : X \times Y \rightarrow R, g : X \times Y \rightarrow R)$$

Equation 5.2

Observe: the utility of Player i also depends on the strategies of the other players

(*A Primer in Game Theory - Roberto Lucchetti - Libro - Esculapio - | IBS, 2011*)

In the context of a non-cooperative game, the actions available to the players are referred to as their strategies. Each player has a unique strategy set, encompassing all the possible set of actions they can take. Within this strategy set, players make choices, selecting a feasible strategy that may not necessarily be the optimal one. The outcomes of these choices are represented as payoffs, which are contingent on the strategies adopted by the other players (Nagarajan & Sošić, 2008).

Nash and Stackelberg equilibria are two important solution concepts used in many non-cooperative games, whose applicability varies depending on the nature of the game and the sequencing of player decisions. A Nash equilibrium emerges as a set of decisions where no player can enhance the value of their payoff function by unilaterally deviating from their chosen strategy (Fiala, 2015). This concept is particularly relevant when players make their strategy choices simultaneously, with no communication or coordination among them. It's akin to situations like the classic children's game "rock, paper, scissors," where players simultaneously reveal their choices without prior knowledge of their opponents' selections (Kreps, 1990). It also comes into play when players cannot communicate, as exemplified by the famous "prisoner's dilemma" game (Weiss & Shubik, 1984). The formal definition of Nash equilibrium, as introduced by Nash, encapsulates this idea (Nash, 1950).

Definition 7

A pair of strategies (x_1^N, x_2^N) *is said to constitute a Nash equilibrium if the following pair of inequalities is satisfied for all* $x_1 \epsilon X_1$ *and for all* $x_2 \epsilon X_2$*:*

$$f_1(x_1^N, x_2^N) \geq f_1(x_1, x_2^N) \quad and \quad f_2(x_1^N, x_2^N) \geq f_1(x_1^N, x_2)$$

Equation 5.3

This is, x_1^N *and* x_2^N *solve* $max_{x_1 \epsilon X_1} f_1(x_1, x_2^N)$ *and* $max_{x_2 \epsilon X_2} f_2(x_1^N, x_2)$*, respectively* (See, for example, Başar and Olsder (1982) and Gibbons (1992)).

Assuming continuity, differentiability and $(x_1, x_2) \in \mathbb{R}^2$*, this definition implies that if the pair* (x_1^N, x_2^N) *is to be a Nash equilibrium, the players' decisions must satisfy*

$$\left.\frac{\partial f_1(x_1, x_2^N)}{\partial x_1}\right|_{x_1 = x_1^N} = 0 \quad and \quad \left.\frac{\partial f_2(x_1^N, x_2)}{\partial x_2}\right|_{x_2 = x_2^N} = 0$$

Equation 5.4

Equivalently, the Nash equilibrium is obtained by solving the (nonlinear) system of equations $x_1 = x_1^R(x_2)$ and $x_2 = x_2^R(x_1)$ (Nagarajan & Sošić, 2008).

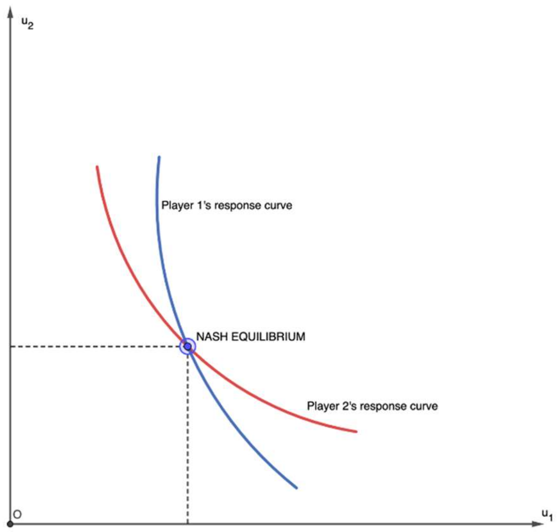

Figure 5.2: Graphical depiction of the leader-follower dynamic and the Nash equilibrium

However, in scenarios where one player has the advantage of making their move before others, we shift our focus to the Stackelberg equilibrium. This leader-follower dynamic involves the leader making the first move, anticipating the follower's response, and the follower subsequently aligning his actions optimally with what the leader anticipated. Basically, the follower's strategy choice is driven by her best response to the leader's decision. The leader, fully aware of the follower's response to his decision (assuming complete information), optimizes his objective function while taking into account the follower's anticipated actions (Fiala, 2015).

Definition 8

In a two-person game with P1 as the leader and P2 as the follower, the strategy $x_1^S \epsilon X_1$ *is called a Stackelberg equilibrium for the leader if, for all* x_1,

$$f_1\left(x_1^S, x_2 |\, x_2 = x_2^R(\, x_1^S)\right) \geq \, f_1\left(x_1 \,, x_2 |\, x_2 = x_2^R(\, x_1 \,)\right)$$

Equation 5.5

where $x_2^R(\, x_1 \,)$ *is the best response function of the follower* (See Başar and Olsder [11]).

Both Nash and Stackelberg’s strategies hinge on analysing "best response functions," which guide players in selecting their strategies.

Cooperative games

Definition 9

A cooperative game (assignment of utility for every possible coalition) is (N, V : $2^N \rightarrow \mathbb{R}^n$) where

$$V(A) \subseteq \mathbb{R}$$

Equation 5.6

A with the convention V(∅) = ∅.

- $V(A)$, *for a given* $A \in 2^N$, *is the set of the aggregate utilities of the players in coalition* A: $x = (x_i)_{i \in A} \in V(A)$ *if the players in A, acting by themselves in the game, can guarantee utility* x_i *to every* $i \in A$;
- *Sometimes* $V(A)$ *represents costs rather than utilities.*

(*A Primer in Game Theory - Roberto Lucchetti - Libro - Esculapio - | IBS*, 2011)

Within the domain of game theory, cooperative games represent a category where players have the ability to enter into binding agreements and establish coalitions aimed at maximizing the combined payoffs of the participating members. This approach acknowledges the significance of collaboration among players and its potential to enhance overall outcomes.

The Nash Bargaining Game, originally introduced by John Nash in 1950, embodies a cooperative approach to addressing bargaining situations. In the context of cooperative games, agents engage in pre-game negotiations, striving to reach mutually beneficial agreements (Nash, 1950). If an agreement is successfully negotiated, agents commit to implementing the terms of the agreement during the actual game. In cases where no agreement is reached, agents default to non-cooperative behaviour. The primary objective of the Nash Bargaining game is to explore how agents should collaborate, particularly when non-cooperation would lead to outcomes that are Pareto-inefficient. Pareto inefficiency implies that there are alternative outcomes that would benefit at least one agent without detriment to the others (Arsenyan et al., 2015).

The Nash Bargaining Game typically involves two rational agents, both possessing comprehensive information about available agreement options. Within this framework, they have the opportunity to cooperate in multiple ways, choosing from a range of possible alternatives. The ultimate aim is to identify a solution that garners agreement from both agents. It is important to note that bargaining theory presumes the resulting solution to be unbiased and equitable.

Definition 10

The payoff functions of the agents are presented as $u_1(a)$ and $u_2(a)$ for a given $a \in A$. Let $(u_1(\bar{a}), u_2(\bar{a}))$ be the vector of disagreement payoff, or the payoff corresponding to the case $\bar{a} \in A$ that two agents are in disagreement. It is assumed that $\bar{u}$ is fixed, in other words, $u_1(\bar{a})$ and $u_2(\bar{a})$ are determined by the rules of the game. Then, the bargaining problem is finding the solution $a^* \in A$ that the agents will eventually reach given $\bar{a} \in A$.

Nash Bargaining solution can be obtained by solving the following maximization problem:

$$\left(u_1(a^*) - u_1(\bar{a})\right)\left(u_2(a^*) - u_2(\bar{a})\right) = max_{a \in A}[(u_1(a) - u_1(\bar{a}))(u_2(a) - u_2(\bar{a}))]$$

Equation 5.7

(Arsenyan et al., 2015)

Nash's arbitration approach relies on five fundamental axioms (Arsenyan et al., 2015):

- Individual rationality: this axiom posits that no agent will willingly accept an agreement that provides them with a lower payoff than what they could achieve by not reaching an agreement (i.e., under disagreement);
- Linear invariance: when one agent's payoff function undergoes an affine transformation (while leaving the other agent's payoff function unchanged), the new bargaining solution mirrors the previous one under the same transformation;
- Symmetry: when the agents are identical, they should receive identical payoffs from the agreement, fostering fairness;
- Independence of irrelevant alternatives: if the set of available alternatives is reduced but still encompasses both the Nash Bargaining solution and the disagreement alternative, the solution remains unaltered;
- Pareto optimality: the agreement is reached when there is no feasible solution that allows one agent to enhance their payoff without diminishing the payoff of the other agent, ensuring an equitable outcome;
- By adhering to these axioms, Nash establishes that a unique arbitration solution can be determined through optimization, providing a robust foundation for cooperative decision-making.

Assumption 1 - *Collaboration and Added Value* (Bhaskaran & Krishnan, 2009):

One critical assumption in this context is that collaboration between firms generates added value in the product development process. It is evident that the synergistic benefits arising from collaboration serve as a natural incentive for firms to seek out partners with complementary capabilities. This assumption underscores the significance of cooperation and the potential advantages it offers to firms involved in collaborative endeavours, especially within the realm of product development.

5.3. Negotiation models

Over the past few years, significant advancements have been made in the field of negotiation and contracting within supply chain management (Nagarajan & Sošić, 2008). The following discussion will explore these developments in the context of various supply chains, analysing and deliberating on these outcomes from the perspective of game theory, which offers a powerful framework for gaining deeper insights into the multifaceted aspects of supply chain management. By examining these results through the lens of game theory, this paper aims to gain deeper insights into the strategic interactions, decision-making processes and cooperative or non-cooperative behaviours that influence supply chain performance. By doing so, it wants to offer a comprehensive understanding of the evolving landscape of supply chain management in the face of these recent developments.

These results represent a crucial turning point, which this document intends to use to introduce new dimensions of analysis aimed at managing the supply chain more efficiently, thus bridging the gap between theoretical advances in game theory and their practical applications.

5.3.1. Sequence of events within the producer-retailer relationship

When it delves into the realm of game theory and its application to the dynamics between producers and retailers, it uncovers a fascinating sequence of events that unfold when both parties find common ground and the retailer gives its nod of approval to the producer's contract offer. This sequence, which plays out over a specific period of time, provides valuable insights into the intricate mechanism that is the producer-retailer relationship. For a clearer grasp of this dynamic, reference can be made to the visual representation presented in Figure 5.3, which illustrates the chronological progression of events (Govindan et al., 2013).

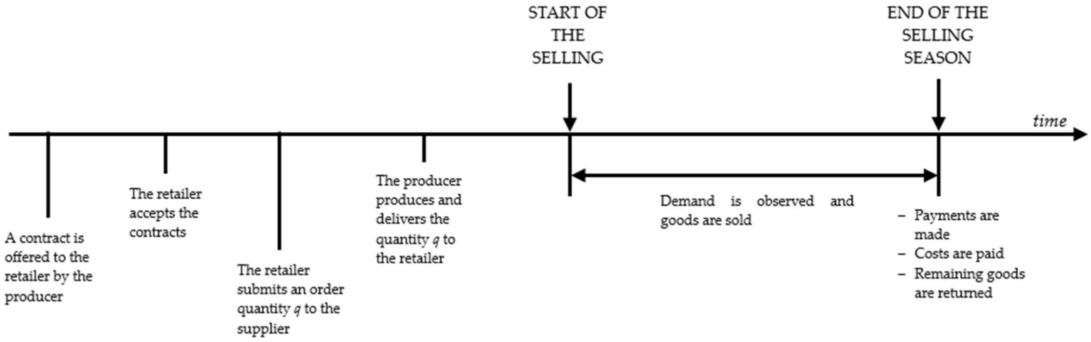

Figure 5.3: Chronological delineation of events within the producer-retailer relationship
Source(s): Govindan, K., Popiuc, M. N., & Diabat, A. (2013). Overview of coordination contracts within forward and reverse supply chains. Journal of Cleaner Production, 47, 319–334. https://doi.org/10.1016/j.jclepro.2013.02.001

Particularly in the context of selecting the most suitable contract for execution, can be useful a step-by-step process that should be meticulously adhered to. This procedural sequence, crucial for effective implementation, is thoughtfully elucidated in Figure 5.4 (Govindan et al., 2013).

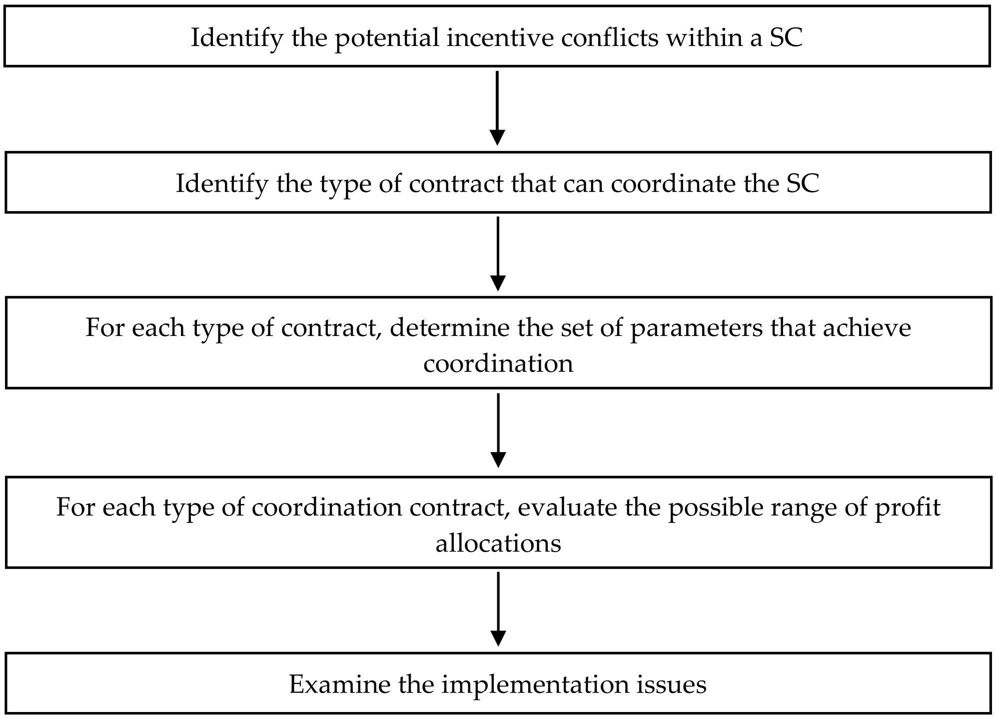

Figure 5.4: Sequence of steps regarding the decision process of contract implementation
Source(s): Govindan, K., Popiuc, M. N., & Diabat, A. (2013). Overview of coordination contracts within forward and reverse supply chains. Journal of Cleaner Production, 47, 319–334. https://doi.org/10.1016/j.jclepro.2013.02.001

This paper will now delve into the intricate concepts that underpin strategic decision-making and explore their potential for fostering collaboration. Its objective is to unlock the optimization of outcomes, encourage cooperative endeavours, and facilitate well-informed decision-making within the realm of contractual agreements. To achieve this, it will embark on an exploration of three distinct contract types, employing the analytical lens of game theory, with a specific focus on one driver: the optimal quantity.

5.4. Buyback

In the context of a buyback contract (Pasternack, 1985), the producer adopts a pricing strategy wherein they charge the retailer a price denoted as 'w' per unit purchased. However, at the end of the season, the producer pays the retailer 'b' per unit remaining as inventory. It's worth noting that in this arrangement, the underlying principle is to ensure that the retailer does not gain any profit from holding on to excess inventory. Therefore, it's assumed that 'b' (the sum paid back to the retailer) is less than or equal to 'w' (the initial purchase price) (Fiala, 2015).

$$b \leq w$$

Equation 5.8

The values of both the wholesale price denoted as 'w' and the buyback price 'b' are explicitly defined and predetermined:

$$w = \lambda\,(p - c) + c$$

Equation 5.9

$$b = \lambda\, p$$

Equation 5.10

$$\text{where } 0 \leq \lambda \leq 1$$

Equation 5.11

In this context, the variable *'p'* is used to denote the retail price, which represents the price at which a product or service is offered to the end consumers. On the other hand, *'c'* represents the production cost incurred by the producer, reflecting the expenses and investments involved in manufacturing or procuring the goods or services in question.

When the values of the parameters *'w'* and *'b'* are established, it is essentially determining the profits that accrue to both the retailer and the producer at any given combination of output quantity *'q'* and retail price *'p'*. These price settings play a pivotal role in shaping the financial outcomes for both parties involved in the transaction. The retailer's profit and the producer's profit are intricately linked to the interplay of these chosen prices and the quantities of goods exchanged (Fiala, 2015).

$$\begin{aligned}\pi_R &= E\,\{p\,[\min(q, D(\,p, u)] - wq + bmax\,(0; q - D(\,p, u))\} \\ &= E\,\{(\,p - w - c\,)q - (p\,-\,b)\max(0; q - D(\,p, u)\} \\ &= (1 - \lambda)E\big\{(\,p - c)q - p\,max\,\big(0; q - D(\,p, u)\big)\big\} = (1 - \lambda)\pi\end{aligned}$$

Equation 5.12

$$\begin{aligned}\pi_P &= E\,\{(w - c)q\; - bmax\,(0; q - D(\,p, u))\} \\ &= E\,\{\lambda(\,p - c\,)q - \lambda p\max(0; q - D(\,p, u)\} = \lambda\pi\end{aligned}$$

Equation 5.13

Focusing on the earlier expressions representing the profit of the retailer and the profit of the producer, it becomes evident that the retailer and the producer are effectively addressing the same fundamental challenge posed by the centralized supply chain. What's particularly noteworthy is that the combined value of the retailer's profit and the producer's profit equates to the total profit realized within the centralized supply chain framework. This relationship underscores the interdependency between the retailer and the producer in achieving an outcome that mirrors the centralized approach. The parameter *'λ'* signifies the manner in which the total profit is distributed or shared between the retailer and the producer, essentially characterizing their respective stakes in the collaborative endeavour (Fiala, 2015).

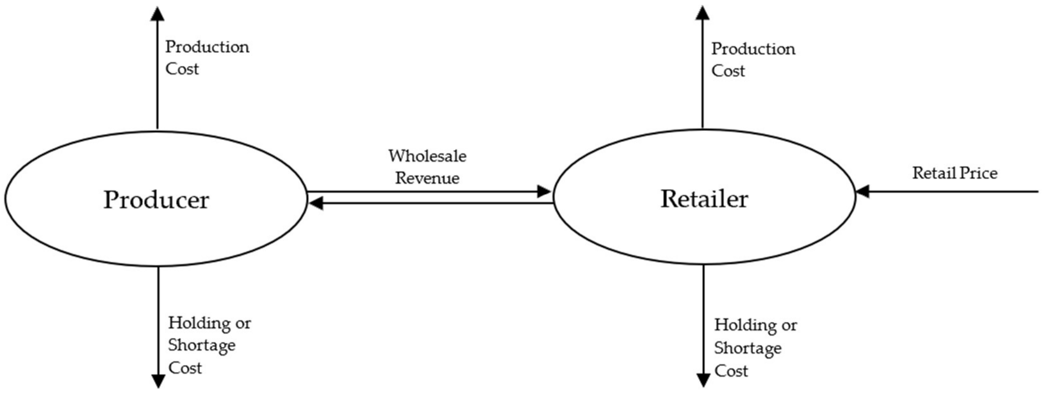

Figure 5.5: Flow of costs and revenues between the retailer and the producer

The concept underpinning the development of supply chain contracts is rooted in the pursuit of enhancing the collective profitability of the supply chain as a whole. It revolves around the notion of offering supply chain participants the prospect of earning more together than they would individually. To achieve this, members within the supply chain employ various strategies aimed at optimizing the processes in which they are engaged (Köse & Canbulut, 2023b).
In the forthcoming analysis[16], a pivotal driver of this optimization effort is the order quantity at the retail level. This analysis aims to elucidate how the choice

[16] The analysis that follows has been extracted from a comprehensive study conducted by Köse, E., and Canbulut, G. in 2023, titled '*Game Theory Solution to Buyback Contracts,*' which specifically examines the dynamics of such contracts but declined in a scenario characterized by the presence of only two distinct agents.

of this particular parameter can wield significant influence over the ultimate profit outcomes, thereby exerting a substantial impact on the negotiation and bargaining dynamics within the supply chain. In essence, it underscores the critical role that the retail order quantity plays in shaping the profitability and, by extension, the bargaining power of the involved parties.

p	Retail price
c_r	The retailer's marginal cost per unit
c_p	The producer's marginal cost per unit
g_r	Goodwill penalty cost for the retailer
g_p	Goodwill penalty cost for the producer
v_r	The salvage value for the retailer
w_{pr}	The wholesale price from producer to retailer
R_p	The buyback rate for the producer
k_p	The buyback price for the producer
Q	The order quantity
D	The demand of end customer
$L(Q)$	The demand variance, $L(Q) = D - S(Q)$
$S(Q)$	The expected sales, min $\{Q, D\}$
$I(Q)$	The expected leftover inventory, $I(Q) = Q - S(Q) = Q - min\{Q, D\}$

Table 5.1: Notation system for buyback contract analysis

In the following analysis, there will be two distinct scenarios, each representing a unique context within the supply chain. The first scenario involves a lack of collaboration, where entities operate independently. The second scenario, on the other hand, introduces collaboration between the producer and the retailer. The primary objective is to conduct a comparative evaluation, discerning the tangible advantages and gains in terms of profitability that emerge when collaboration is introduced. This comparative approach allows to assess how collaboration impacts the profit outcomes for both parties involved within the context of the supply chain.

5.4.1. Case I (No Coordination)

In this particular scenario, an absence of any formalized contract or agreement exists to facilitate coordination among the various members of the supply chain. Consequently, all entities within the supply chain engage in transactions at the wholesale price, devoid of any overarching coordination mechanism. The visual depiction of this supply chain structure, characterized by this non-coordination, is illustrated in Figure 5.6 (Köse & Canbulut, 2023b).

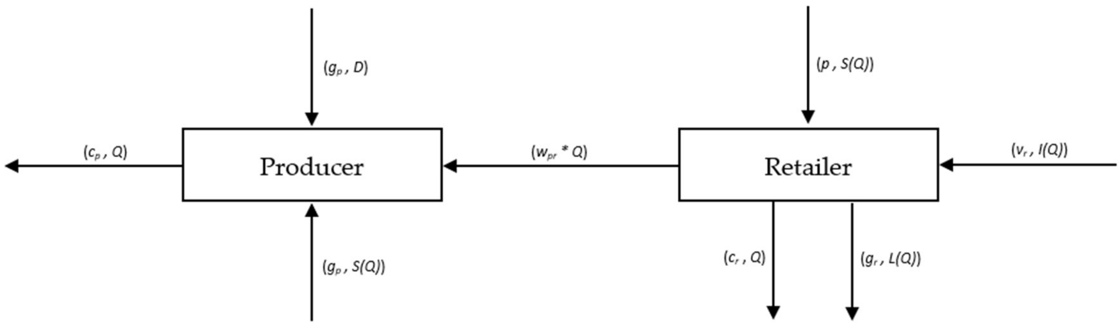

Figure 5.6: Visual representation of a non-coordination supply chain structure

In this context of non-coordination, several key variables come into play. First and foremost is the determination of the optimal order quantity. The calculations for the profit functions for both the retailer and producer are carried out through specific formulations, all while recognizing that the demand variable is subject to stochastic fluctuations, thus introducing an element of uncertainty into the equations.

The profit function of the retailer is defined in Equation 5.14

$$\pi_r(Q) = (p - v_r + g_r)S(Q) + (v_r - c_r - w_{pr})Q - g_r D$$

Equation 5.14

The profit function of the producer is defined in Equation 5.15

$$\pi_p(Q) = g_p S(Q) + (w_{pr} - c_p)Q - g_p D$$

Equation 5.15

The responsibility for deciding the order quantity falls upon the retailer. To identify the order quantity that maximizes their benefit, it involves setting the first derivative of the retailer's function, in relation to the order quantity, equal to zero. This mathematical procedure is essential in pinpointing the most advantageous order quantity for the retailer, as it identifies the point where the retailer's profit function reaches its peak (Köse & Canbulut, 2023b).

$$\frac{\partial \pi_r'(Q)}{\partial Q} = (p - v_r + g_r)S'(Q) + (v_r - c_r - w_{pr}) = 0$$

Equation 5.16

Then the optimum order quantity is obtained as in Equation 5.17

$$S'(Q) = \frac{w_{pr} + c_r - v_r}{p - v_r + g_r}$$

Equation 5.17

5.4.2. Case II (Producer – Retailer Cooperation)

In this particular scenario, the concept of 'total profit' is construed as the combined sum of both the producer's and the retailer's profit values. It's important to note that this approach considers the holistic financial outcome of the entire supply chain, incorporating the interests and profitability of both key stakeholders.

As the season concludes, the producer initiates a repayment process, whereby a buyback price, denoted as 'k_p', is offered for each remaining unit of the product to the producer. This buyback arrangement carries implications for the overall profitability dynamics within the supply chain, as it influences the financial interactions between the producer and the retailer (Köse & Canbulut, 2023b).

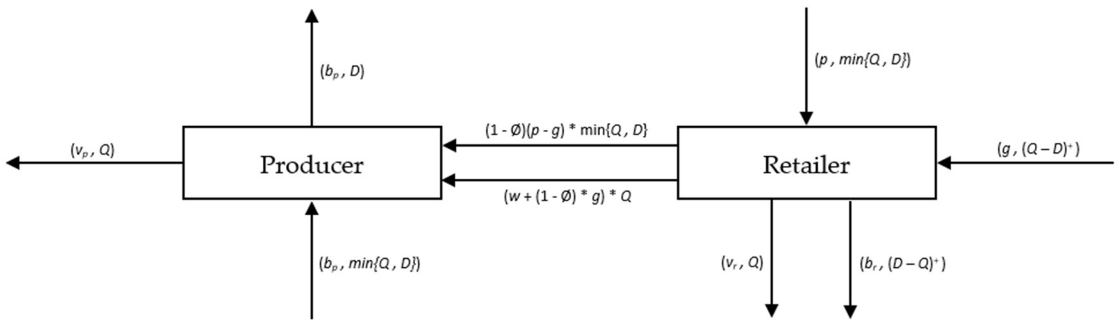

Figure 5.7: Implications of the buyback mechanism on the profitability dynamics within the supply chain

In this particular scenario, it's important to recognize that demand is treated as a stochastic variable, implying that it fluctuates with a degree of uncertainty. Within this context, the profit functions of both the retailer and the producer are computed as follows:

The profit function of the retailer is defined in Equation 5.18

$$\pi_r(Q) = \left(p - v_r + g_r - R_p k_p\right)S(Q) + \left(v_r - c_r - w_{pr} + R_p k_p\right)Q - g_r D$$

Equation 5.18

The profit function of the producer is defined in Equation 5.19

$$\pi_p(Q) = \left(g_p - v_p + R_p k_p\right)S(Q) + \left(w_{pr} + v_p - c_p - R_p k_p\right)Q$$

Equation 5.19

In this situation, supply chain members collectively determine the optimal order quantity. This means that the determination of the most advantageous order quantity involves a joint effort by all parties involved.
To ascertain this optimal order quantity, must be put the first derivative of the total profit function for the entire supply chain, concerning the order quantity, equal to zero. This mathematical process is pivotal in pinpointing the quantity that maximizes the collective profitability of the supply chain. It's noteworthy that within this collaborative framework, the total profit of the supply chain encompasses and is equivalent to the sum of the individual profits realized by each of its members (Köse & Canbulut, 2023b). This approach underscores the shared nature of the supply chain's financial outcomes and the collaborative nature of decision-making in optimizing operations.

$$\pi\ (Q) = \pi_r(Q) + \pi_p(Q) = (p - v_r - v_p + g_r + g_p)S(Q) + (v_r + v_p - c_r - c_p)Q - g_r D$$

Equation 5.20

$$\frac{\partial \pi'(Q)}{\partial Q} = (p - v_r - v_p + g_r + g_p)S'(Q) + (v_r + v_p - c_r - c_p) = 0$$

Equation 5.21

The optimum order quantity is obtained as in Equation 5.22

$$S'(Q) = \frac{c_r + c_p - v_r - v_p}{p - v_r - v_p + g_r + g_p}$$

Equation 5.22

5.4.3. Conclusions

This analysis presents a novel approach to the allocation of profits within supply chains. The proposed mechanism is rooted in the recognition that activities within a supply chain often involve a combination of cooperative and non-cooperative behaviours among its participants. At the heart of this approach is the pivotal role played by the order quantity, serving as the primary driver for optimizing supply chain operations.

For this concern, the buyback contract is a versatile and effective means of achieving comprehensive coordination within the supply chain. This contract not only facilitates smooth cooperation but also provides the flexibility needed to accommodate various profit-sharing arrangements among supply chain participants. In essence, it offers a solution that can adapt to different scenarios while fostering a harmonious and efficient supply chain environment.

5.5. Option mechanism

In the upcoming section, it is analysed a different breed of contract within the context of game theory models: the option contract. Option contracts, as introduced by Barnes-Schuster and colleagues in 2002, introduce an intriguing dimension to the contractual landscape. They stipulate that, alongside placing firm orders at regular prices, retailers also have the opportunity to acquire options at a predetermined option price at the onset of the selling season.

The option mechanism is characterized by two fundamental parameters — namely, the option price, denoted as *'o'*, and the exercise price, designated as *'e'*. The option price effectively serves as a reservation fee paid by the retailer to the producer, securing a unit of production capacity. Conversely, the exercise price is the fee the retailer must pay to the producer when they decide to exercise the option and purchase one unit of the product (Zhao et al., 2010).

In the realm of coordination contracts, the fundamental objective is to ensure that each party involved is at least as well off as they would be without such contractual arrangements. Here, it is demonstrated that option contracts possess the inherent capacity to coordinate the supply chain dynamics and yield a Pareto improvement, benefiting all parties involved. Consequently, both the producer and the retailer find compelling incentives to embrace the option mechanism as part of their collaboration.

However, the final outcome hinges significantly upon the bargaining power wielded by individual supply chain members and their respective risk preferences. By taking these factors into account, it is explored the specific type of option contract that the retailer and the producer opt to implement, employing both the Nash bargaining model and the Eliashberg model to shed light on this choice and conduct a detailed analytical investigation into how these factors influence the ultimate outcome (Zhao et al., 2010).

5.5.1. Model description

It is examined a two-tier supply chain, consisting of a single producer and a single retailer. In this setup, the producer's products find their way to end consumers through the retailer. The market demand for these products is described by a stochastic variable 'X', which follows a strictly increasing distribution function $F(x)$ with $x \geq 0$.

To facilitate production and procurement within this supply chain, it is introduced an option mechanism characterized by two key parameters: the option price 'o' and the exercise price 'e'.

In addition to this, it must consider the current landscape of the industry, where many retailers wield substantial influence, at times even surpassing that of their producers. As a result, it is assumed the producer operates under a 'make-to-order' production policy. Here's how the sequence of events unfolds in this model: at the onset of the production season, the retailer secures a predetermined quantity of production capacity from the producer, denoted as 'Q', at a unit price of 'o'. Following this reservation, the producer employs a 'make-to-order' approach and produces 'Q' units of the product, adhering to the quantity reserved by the retailer. During the selling season, the retailer acquires a quantity of the product from the producer, up to 'Q' units, based on the actual market demand. This is done at the exercise price 'e', in an effort to fulfil demand. Any unsold product does not incur a penalty cost but is simply lost (Zhao et al., 2010).

The analysis assumes that demand information is symmetric between the producer and the retailer. For both parties, the marginal production cost is 'c' and the salvage value per unsold unit is 'v'. Additionally, the retailer sets a retail price denoted as 'p'. The focus lies in the practical and non-trivial scenario where

$$p > c > v$$

Equation 5.23

$$0 \leq o < c - v$$

Equation 5.24

$$e > v$$

Equation 5.25

p	The retail price
c	The marginal production cost
v	The salvage value
o	The option price
e	The exercise price

Table 5.2: Notation system for option contract analysis

These assumptions are made to ensure a reasonable balance in risk and avoid extreme situations, such as risk-free production for the producer or the retailer always exercising all their options. The central aim of this model is to delve into the implementation aspects of coordinating option contract forms using a game theory approach. In doing so, it takes into account the risk preferences and negotiation power of the supply chain members, offering insights into the intricate dynamics of such contracts within this context (Zhao et al., 2010).

5.5.2. Basic option contract model

Given that the wholesale price mechanism is a widely prevalent practice in producer-retailer supply chains, it serves as the foundational reference point against which it will be assessed and compared the newly introduced option mechanism in this study (Zhao et al., 2010).

To initiate this comparative analysis, let's begin by examining the expected profit function for the retailer when operating under the conventional wholesale price mechanism. This profit function essentially quantifies the retailer's expected financial gains within the established wholesale pricing framework.

$$E\pi_{wr}(Q_{wr}) = E[p \, min \, \{Q_{wr}, x\} - wQ_{wr} + v \, max \, \{Q_{wr} - x, 0\}]$$

Equation 5.26

where 'Q_{wr}' is the retailer's order quantity and 'w' represents the wholesale price. It's important to note that to ensure practicality and reasonability, it is established the condition

$$p > w > c$$

Equation 5.27

This set of conditions is essential to maintain a realistic pricing structure. Breaking down Equation 5.26, it can be observed that the first term represents the revenue generated from the retailer's sales, the second term corresponds to the order cost incurred by the retailer and the third term signifies the salvage value of any unsold products. In essence, these components encapsulate the financial aspects relevant to the retailer's operations when employing the wholesale pricing mechanism (Zhao et al., 2010).

Under the wholesale pricing system, the primary challenge facing the retailer is to optimize their expected profit function. The outcome of this optimization effort leads to the subsequent proposition.

Proposition 1 (Zhao et al., 2010)

With the wholesale price mechanism, the retailer will earn an expected profit of:

$$\pi_{wr} = (p - w)\bar{Q}_{wr} - (p - v)\int_0^{\bar{Q}_{wr}} F(x)dx$$

Equation 5.28

and the producer will earn an expected profit of

$$\pi_{wp} = (w - c)\bar{Q}_{wr}$$

Equation 5.29

where

$$\bar{Q}_{wr} = F^{-1}\left(\frac{p - w}{p - v}\right)$$

Equation 5.30

When the producer does not employ a 'make-to-order' production policy and instead strategizes its production in alignment with its own interests, the objective becomes the maximization of the following expected profit function with respect to the production quantity 'Q_{wm}'.

$$E\pi_{wp}\,(Q_{wm}) = E[w\,min\{Q_{wp}, x\} - cQ_{wp} + v\,max\,\{Q_{wr} - x, 0\}]$$

Equation 5.31

Much like the approach used to establish Proposition 1, it is undertaken a similar procedure to calculate the producer's optimal production quantity. This involves deriving the quantity of production that maximizes the producer's expected profit, and the result is expressed as follows

$$\overline{Q}_{wp} = F^{-1}\left(\frac{w-c}{w-v}\right)$$

Equation 5.32

All of the outcomes and findings that have been derived using the wholesale price mechanism serve as essential benchmarks. These benchmarks will be utilized to evaluate the performance of the option mechanism that has been developed in the course of this analysis (Zhao et al., 2010).

Moving forward, now shift the focus to the option model. Within this framework, proceeding to analyse the expected profit function for the retailer, which is represented as follows:

$$E\pi_{or}(Q_{or}) = E[(p-e)\,min\,\{Q_{or}, x\} - oQ_{or}]$$

Equation 5.33

where 'Q_{or}' is the retailer's reserved quantity under the option contract mechanism. In Equation 5.33, the components can be broken down as follows:

the first term represents the retailer's sales profit, capturing the earnings from product sales, while the second term accounts for the allowance payout made for the reserved production capacity.

Similarly, when it is examined the producer's approach to determining its production quantity, not guided by the 'make-to-order' production policy but rather driven by its own interests, it is evaluated its expected profit function.

This function is expressed as follows:

$$E\pi_{op}(Q_{op}) = E[oQ_{op} + e\, min\{Q_{op}, x\} + v\, max(Q_{op} - x, 0\} - cQ_{op}]$$

Equation 5.34

Thus, within the framework of the option mechanism, the retailer faces the task of optimizing their expected profit function, as denoted by Equation 5.33. Simultaneously, the producer undertakes the challenge of maximizing its own expected profit function, expressed in Equation 5.34. These optimization endeavours for both the retailer and the producer ultimately culminate in the following proposition (Zhao et al., 2010).

Proposition 2 (Zhao et al., 2010)

(i) Given (o, e), the producer's optimal production quantity is

$$\overline{Q}_{op} = F^{-1}\left(\frac{e + o - c}{e - v}\right)$$

Equation 5.35

And the retailer's optimal reserved quantity is

$$\overline{Q}_{or} = F^{-1}\left(\frac{p - o - e}{p - e}\right)$$

Equation 5.36

(ii) *Given* $o + e$, $E\pi_{or}(Q_{or})$ *is decreasing in 'o' or increasing in 'e', whereas* $E\pi_{op}(Q_{op})$ *is increasing in 'o' or decreasing in 'e'.*

(iii) *Only if the option contract satisfies*

$$e = p - \frac{p - v}{c - v}\, o$$

Equation 5.37

$$(o < c - v)$$

Equation 5.38

will the retailer's optimal reserved quantity be just consistent with the producer's optimal production quantity.

Through the utilization of the option mechanism, it is established that $o + e$ yields the unit price for the quantity of the product purchased by the retailer. Building on the insights from Proposition 2, discern that, given a fixed $o + e$, the retailer's willingness to pay a higher option price directly correlates with an increase in the optimal production quantity determined by the producer. This suggests that, when afforded the flexibility to pay a lower option price initially and a higher exercise price later, the retailer is inclined to reserve a larger quantity of the product. Interestingly, this preference aligns with typical industry practices (Zhao et al., 2010).

Conversely, the producer exhibits a contrasting preference. They favour scenarios where the option price is higher, and the exercise price is lower. This divergence in preferences between the retailer and producer mirrors real-world dynamics observed in practice.

Furthermore, when it comes to configuring the option contract in a way that aligns the retailer's optimal reserved quantity with the producer's optimal production quantity, it's possible to discover that the exercise price behaves as a negative linear function of the option price.

Now, when these findings are compared to those under the wholesale price mechanism, it is arrived at the following proposition.

Proposition 3 (Zhao et al., 2010)

(i) $\bar{Q}_{wr} < \bar{Q}_{or}\ iff\ o < \frac{(p-e)(w-v)}{p-v}$

and

(ii) $\bar{Q}_{wp} < \bar{Q}_{op}\ iff\ o > \frac{(c-v)(w-e)}{w-v}$

Proposition 3 provides an important insight when we contrast the option contract mechanism with the conventional wholesale price approach. It indicates that when the option price is set at a level below a certain threshold, denoted as $\frac{(p-e)(w-v)}{p-v}$, it incentivizes the retailer to reserve a larger quantity of the product. In essence, a lower option price entices the retailer to secure a greater amount of production capacity.

Conversely, when the option price surpasses another threshold, marked as $\frac{(c-v)(w-e)}{w-v}$, it exerts pressure on the producer to ramp up production. In this scenario, a higher option price prompts the producer to produce a greater quantity of the product.

These observations highlight how the option contract mechanism can influence the actions of both the retailer and the producer, pushing them toward different decisions based on the chosen option price. It underscores the dynamic nature of this contractual approach in contrast to the traditional wholesale pricing mechanism (Zhao et al., 2010).

5.5.3. Supply chain coordination with option contracts

To determine the optimal expected profit for the entire supply chain system, it is adopted the perspective of treating the supply chain as a centralized entity. It is used the notation '$E\pi_s(Q_s)$' to represent the expected profit of this centralized entity, considering a specific production quantity, 'Q'.

$$E\pi_s\,(Q_s\,) = \;E[p\,min\{Q_s, x\} + \;v\,max\,\{Q_s - \;x, 0\} \;-\; cQ_s]$$
$$= (\,p - c)Q_s - (\,p - v)\int_0^{Q_s} F(x)dx$$

Equation 5.39

It can be also established that the expected profit function '$E\pi_s\,(Q_s\,)$' exhibits strict concavity concerning 'Q_s'. This characteristic is crucial, as it allows to apply the first-order optimality condition in the analysis (Zhao et al., 2010). Consequently, it can be determined the production quantity that leads to the best possible outcome for the supply chain, often referred to as the 'first-best production quantity', is represented as

$$Q_s = \;F^{-1}\left(\frac{p\; - \;c}{p\; - \;v}\right)$$

Equation 5.40

In accordance with this, it is calculated the system-wide optimal expected profit, denoted as 'π_c', which is expressed as follows:

$$\pi_c = E\pi_s(\bar{Q}_s) = (\,p - c)\bar{Q}_s - (\,p - v)\int_0^{\bar{Q}_s} F(x)dx$$

Equation 5.41

It's quite evident that an option contract structured in a way that encourages the retailer to reserve a quantity equal to or approaching $Q_s = \;F^{-1}\left(\frac{p - c}{p - v}\right)$ will have the remarkable effect of aligning the supply chain system with its optimal state.

In other words, when the option contract incentivizes the retailer to secure a production capacity close to the maximum possible $Q_s = F^{-1}\left(\frac{p-c}{p-v}\right)$, the supply chain system is poised to attain its pinnacle of performance in terms of system-wide optimal expected profit.

Proposition 4 (Zhao et al., 2010)

(i) The system-wide optimal expected profit of the supply chain can be achieved under any option contract (o, e) in the following set M:

$$M = \{(o, e) : o = \lambda(c - v), e = (1 - \lambda)p + \lambda v, \quad where\ \lambda \in [0,1)\}$$

Equation 5.42

(ii) Under any option contract (o, e) in M, we have $\bar{Q}_{or} = \bar{Q}_{op} = \bar{Q}_s$. Furthermore, with the option contract associated with 'λ', the maximum expected profit received by the retailer, denoted by $\pi_{op}(\lambda)$, is given by

$$\pi_{op}(\lambda) = (1 - \lambda)\pi_c$$

Equation 5.43

where π_c given by (5.41), is the system-wide optimal expected profit of the supply chain.

The relationship between $'o'$ and $'e'$ can be derived from Equation 5.42 as

$$e = p - \frac{p - v}{c - v} o$$

Equation 5.44

These findings bear significant implications for the dynamics of option contracts within the supply chain. Firstly, they reveal a noteworthy relationship between the exercise price and the option price. Specifically:

- The exercise price exhibits a negative correlation with the option price;
- An increase in the option price by one unit leads to a decrease in the exercise price by more than one unit.

This observation aligns with the intuitive understanding observed in practical scenarios, where early payments for products often come with lower prices.

From the perspective of equilibrium conditions where $\bar{Q}_{or} = \bar{Q}_{op} = \bar{Q}_s$, it can be deduced that for any option contract present in 'M', the producer's production quantity within this model corresponds to the optimal production quantity. This holds true even when considering the 'make-to-order' production policy. This robustness enhances the practical feasibility of implementing option contracts (Zhao et al., 2010). Furthermore, Proposition 4 underscores the flexibility and adaptability of option contracts within 'M' for profit allocation within the supply chain system. It suggests that the supply chain's profit can be allocated according to varying proportions, utilizing different values of 'λ' within the range of [0,1). In essence, 'λ' represents the fractional division of optimal joint profit within the supply chain system associated with the specific option contract. Higher values of 'λ' in an option contract favour the retailer in terms of profit sharing, while lower values lean towards the producer.

Under the wholesale price mechanism, retailers often tend to order less than the system-wide optimal quantity due to the impact of double marginalization, unless the producer is willing to offer a wholesale price equivalent to the unit production cost. Proposition 3 highlights that only option contracts meeting the criteria of $\frac{(c-v)(w-e)}{w-v} < o < \frac{(p-e)(w-v)}{p-v}$ possess the potential to motivate both the retailer and the producer to reserve and produce quantities closely aligned with the system-wide optimal quantity (Zhao et al., 2010).

In practice, while some contracts might theoretically enable the achievement of the supply chain's system-wide optimal profit, they may not always align with the individual interests of each member. Therefore, effective supply chain contracts should be thoughtfully designed to cater to the interests of every member while ensuring the attainment of the system-wide optimal profit.

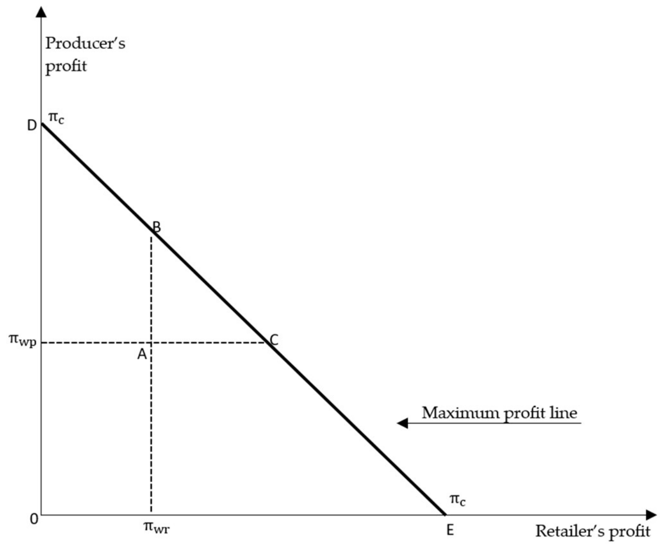

Figure 5.8: Illustrative representation of the intricate matter of profit allocation between the retailer and the producer within the context of the option mechanism

Figure 5.8 serves as an illustrative representation of the intricate matter of profit allocation between the retailer and the producer within the context of the option mechanism. When there's a lack of coordination in the supply chain, the retailer stands to gain a profit indicated by 'π_{wr}', while the producer's profit is denoted as 'π_{wp}'. This scenario is visually depicted in Figure 5.8 as 'point A'. Now, it is considered the line segment $\overline{DE}$. Here, all the points along this segment correspond to various profit allocations under the option contracts established as part of 'M'. However, a critical observation arises: neither the retailer nor the producer is inclined to accept a situation where their post-coordination profit falls below what they were earning before coordination. This practical constraint effectively narrows down the possible profit allocations to points along the line segment $\overline{BC}$. At 'point B', the reader witnesses a coordinating option contract where the producer captures all the additional profit derived from the coordination effort, leaving the retailer's earnings unchanged from their pre-coordination levels. In contrast, 'point C' mirrors the converse scenario, where the retailer gains the lion's share of the benefits from coordination while the producer maintains earnings at pre-coordination levels. The remaining points along the $\overline{BC}$ line segment represent those option contracts where both the retailer and the producer derive some degree of benefit from the coordination, ensuring that no party suffers a decline in profits post-coordination (Zhao et al., 2010).

5.5.4. Conclusion

This study brings forth compelling evidence that, in comparison to the conventional wholesale pricing mechanism, any option contract that respects the pre-established conditions for being in the 'M' set has the potential to effectively coordinate the complex dynamics of a producer-retailer supply chain while achieving Pareto improvement (Zhao et al., 2010). Additionally, the analysis delves into the intricacies surrounding the practical implementation of these coordinating option contracts. This comprehensive exploration takes into account the factors of supply chain members' risk preferences and their respective negotiating capabilities. Within the domain of option mechanisms, several significant findings emerge:

- Individual risk preferences emerge as a critical determinant in shaping the outcome of coordination efforts. Specifically, when both the retailer and the producer exhibit risk-averse tendencies, the degree of risk aversion significantly impacts the portion of extra profit each party obtains. A more risk-averse member tends to secure a smaller share of the incremental profit. In cases where both the retailer and producer share similar risk profiles, either both being risk-averse or both risk-neutral, they tend to divide the additional profit equally;

- Beyond risk preferences, the influence of an individual's negotiating power also looms large in determining the ultimate coordination outcome. Members wielding greater negotiating power are positioned to extract higher compensation fees from their counterparts, while those with lesser negotiating clout may find themselves receiving smaller compensation.

In summary, this research provides a robust and relatively comprehensive understanding of how option contracts can effectively serve as coordination mechanisms in the intricate realm of producer-retailer supply chains. The inherent flexibility in order quantity offered by these contracts empowers retailers to better respond to the flow of demand, making them a valuable tool for optimizing supply chain performance, even in dynamic and uncertain environments.

5.6. Revenues sharing

Revenue-sharing contracts have become a prevalent and widely employed strategy for achieving coordination within supply chains. These contracts come into play in scenarios where a product traverses a supply chain comprised of two key entities: a producer and a retailer. In the context of revenue-sharing contracts, the producer adopts a pricing strategy aimed at keeping the selling price of the product relatively low. This strategic pricing encourages the retailer to place larger orders for the product before the commencement of the selling season. In return for this cooperative approach, the retailer commits to sharing a portion of the revenue he generates from product sales with the producer upon the season's conclusion.

In this supply chain, the producer is responsible for distributing products to the retailer, who, in turn, is tasked with selling these products to end customers. It's crucial to note that, in this context, the demand originating from end customers is treated as a stochastic variable (Canbulut et al., 2020).
Before the selling season kicks off, the retailer faces a pivotal decision, choosing an order quantity based on the wholesale price proposed by the producer. For ease of reference and clarity in this study, it is employed a specific notation system, which is summarized in Table 5.3.

p	The retail price
v_p	The producer's production cost per unit
v_r	The retailer's marginal cost per unit
b_p	Goodwill penalty cost for the producer
b_r	Goodwill penalty cost for the retailer
g	The salvage value
Q	The order quantity
T	The transfer cost at the end of the season (from retailer to producer)

Table 5.3: Notation system for revenue-sharing contract analysis

To ensure that this model aligns with real-life scenarios, certain conditions must be met. Specifically, the sum of production cost per unit incurred by the producer (v_p) and the retailer's marginal cost per unit (v_r) must not exceed the retail price.

$$v_p + v_r < p$$

Equation 5.45

Additionally, the retailer must factor in a salvage value (g) for each unsold unit of products, and this value must be lower than 'v'.

$$g < v$$

Equation 5.46

For the sake of convenience, the notation is simplified by letting 'b' represent the sum of 'b_p' and 'b_r', and '$v$signifies the sum of ' v_p' and ' v_r'.

$$b = b_p + b_r$$

Equation 5.47

$$v = v_p + v_r$$

Equation 5.48

In this foundational model, one assumption is that each member within the supply chain is risk-neutral, a pivotal assumption within the realm of game theory. This means that both the producer and the retailer are motivated to maximize their expected profit without factoring in any risk considerations.

Now, let's delve into the mechanics of the revenue-sharing contract. At the conclusion of the selling season, the retailer commits to sharing a designated portion of the revenue ($\emptyset$) generated from product sales with the producer (Canbulut et al., 2020). Additionally, the retailer determines the cost of transfer from himself to the producer based on this contract's provisions. The transfer cost, as per this contract, is defined by Equation 5.49:

$$T = (1 - \emptyset)(p - g)\, min\,\{Q, D\} + (w + (1 - \emptyset)\, g)\, Q$$

Equation 5.49

The schematic illustration of the supply chain structure operating under the framework of the revenue-sharing contract is thoughtfully presented in Figure 5.9. This visual representation helps elucidate how the various components and entities within the supply chain interact and collaborate under the specific terms and conditions of the revenue-sharing agreement.

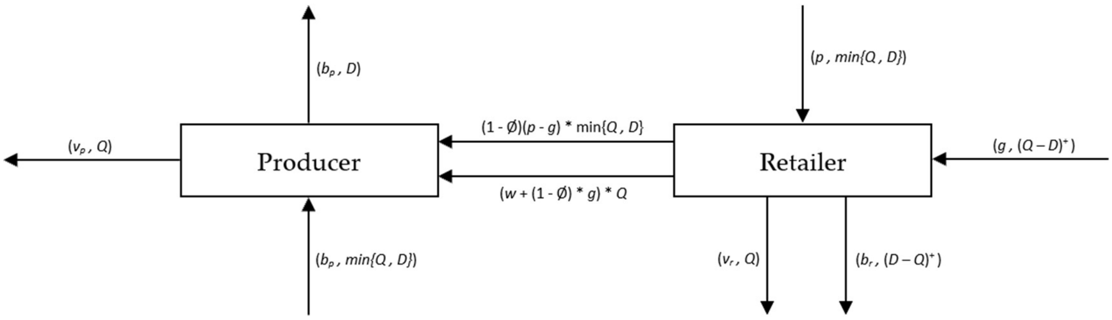

Figure 5.9: Implications of the revenue-sharing mechanism on the profitability dynamics within the supply chain

In the context of revenue-sharing contracts, we can express the profit functions of both the retailer and the producer using the following equations: Equation 5.50 captures the retailer's profit function, while Equation 5.51 details the profit function for the producer.

$$\pi_r(Q) = \begin{cases} pD + g(Q - D) - v_r Q - [(1 - \emptyset)(p - g)D + (w + (1 - \emptyset)g)Q] & D \leq Q \\ pQ - b_r(D - Q) - v_r Q - [(1 - \emptyset)(p - g)Q + (w + (1 - \emptyset)g)Q] & D \geq Q \end{cases}$$

Equation 5.50

$$\pi_p(Q) = \begin{cases} - v_s Q + [(1 - \emptyset)(p - g)D + (w + (1 - \emptyset)g)Q] & D \leq Q \\ b_p Q - v_p Q - b_p D + [(1 - \emptyset)(p - g)Q + (w + (1 - \emptyset)g)Q] & D \geq Q \end{cases}$$

Equation 5.51

Given that the total profit of the supply chain is essentially the summation of the profits earned by each member of the supply chain. The total profit of the supply chain can be calculated through the following expression:

$$\pi = \pi_r(Q) + \pi_p(Q)$$

Equation 5.52

$$\pi = \begin{cases} pD + g(Q - D) - Qv & D \leq Q \\ pQ - b(D - Q) - Qv & D \geq Q \end{cases}$$

Equation 5.53

To find the optimal order quantity that maximizes the total profit of the supply chain, must be taken the first derivative of the total profit function with respect to the order quantity and set it equal to zero. This process yields the following equation (Sluisand and Giovanni 2016):

$$F(Q^{*}) = \frac{v-g}{p-g+b}$$

Equation 5.54

As the selling season draws to a close, the retailer engages in revenue-sharing with the producer, and this involves determining the cost of transfer from the retailer to the producer. If the retailer decides to place an order quantity as per the calculation derived in Equation 5.54, the retailer's profit can be computed using the following equation:

$$\pi_r(Q) = \begin{cases} (p-g)\emptyset D + (-v_r - w + g\emptyset)F^{-1}\left(\frac{v-g}{p-g+b}\right) & D \leq Q \\ (-b_r - v_r + p\emptyset - w)F^{-1}\left(\frac{v-g}{p-g+b}\right) - b_r D & D \geq Q \end{cases}$$

Equation 5.55

The equation that signifies the profit share attributed to the producer at the conclusion of the season can be expressed in the following manner:

$$\pi_p(Q) = \begin{cases} \left(-v_p + w + (1-\emptyset)g\right)F^{-1}\left(\frac{v-g}{p-g+b}\right) + (p-g)(1-\emptyset)D & D \leq Q \\ \left(b_p - v_p + p - p\emptyset + w\right)F^{-1}\left(\frac{v-g}{p-g+b}\right) - b_p D & D \geq Q \end{cases}$$

Equation 5.56

The interaction dynamics between the retailer and the producer in this scenario bear a resemblance to the characteristics often observed in two-person non-constant sum games. In both scenarios, the involved players strategically manoeuvre to enhance their respective earnings. At the commencement of the season, the producer initiates the contract proposal process with the retailer (Canbulut et al., 2020).

Once the retailer agrees to the contract, the producer proceeds to present a wholesale price to the retailer. Following this, it is the retailer's responsibility to determine the revenue-sharing rate. Notably, various combinations of wholesale prices and revenue-sharing rates lead to distinct revenue outcomes for both the retailer and the producer. When the resulting gains of both the retailer and the producer are recorded, factoring in the various wholesale prices and revenue-sharing rates, they can be transferred to a structured tabular format (as demonstrated in Table 5.4), which precisely mirrors the characteristics of a two-person non-constant sum game matrix.

Retailer strategies	Producer strategies			
Revenue-sharing rate	Wholesale price			
	A	B	C	…
X	($\prod_{rX}$, $\prod_{pA}$)*	($\prod_{rX}$, $\prod_{pB}$)	($\prod_{rX}$, $\prod_{pC}$)	…
Y	($\prod_{rY}$, $\prod_{pA}$)	($\prod_{rY}$, $\prod_{pB}$)	($\prod_{rY}$, $\prod_{pC}$)	…
Z	($\prod_{rZ}$, $\prod_{pA}$)	($\prod_{rZ}$, $\prod_{pB}$)	($\prod_{rZ}$, $\prod_{pC}$)	…
…	…	…	…	…

*($\prod_{rX}$, $\prod_{pA}$) = (profits of retailer and producer, respectively, if the profit-sharing rate is X and the wholesale price is A)

Table 5.4: Brief overview of the profit matrix in a two-person non-constant sum game

Typically, a two-person non-constant sum game is mathematically represented as an $(n \, x \, m)$ matrix, where each entry comprises an ordered pair denoted as $'g_1'$ and $'g_2'$. Specifically, $g_1 = (x_i, y_j)$ and $g_2 = (x_i, y_j)$ represent the payoffs that accrue to Player I and Player II, respectively, when they opt for their respective i_{th} and j_{th} pure strategies. When these payoff values are put into a single matrix, we create what is known as a bimatrix, exemplified in Table 5.5.

	y_1	y_2	…	y_m
x_1	$g_1(x_1, y_1); g_2(x_1, y_1)$	$g_1(x_1, y_2); g_2(x_1, y_2)$	…	$g_1(x_1, y_m); g_2(x_1, y_m)$
x_2	$g_1(x_2, y_1); g_2(x_2, y_1)$	$g_1(x_2, y_2); g_2(x_2, y_2)$	…	$g_1(x_2, y_m); g_2(x_2, y_m)$
…	…	…	…	…
x_n	$g_1(x_n, y_1); g_2(x_n, y_1)$	$g_1(x_n, y_2); g_2(x_n, y_2)$	…	$g_1(x_n, y_m); g_2(x_n, y_m)$

Table 5.5: Bimatrix resulting from joining the payoff values into a single matrix.

In this context, the variables $x \in X_n$ and $y \in Y_m$ are referred to as the sets of mixed strategies for the first and second players, respectively. The overarching objective for Player I in this game is to maximize his individual gains by strategically selecting mixed strategies within the set $x \in X_n$, while Player II endeavors to optimize his own gains by choosing from the set of mixed strategies $y \in Y_m$.

$$X_n = \left\{ x = (x_1, x_2, \dots, x_n) : x_i \geq 0 \ and \ \sum_{i=1}^{n} x_i = 1 \ for \ i = 1,2, \dots, n \right\}$$

Equation 5.57

$$Y_m = \left\{ y = (y_1, y_2, \dots, y_m) : y_j \geq 0 \ and \ \sum_{j=1}^{m} y_j = 1 \ for \ j = 1,2, \dots, m \right\}$$

Equation 5.58

When embarking on the solution for two-person non-constant sum games, the initial inquiry revolves around the presence of an equilibrium point. This equilibrium point denotes the intersection within the matrix where players can secure the highest possible returns, taking into account the strategic choices made by their counterpart (Canbulut et al., 2020).

The presence of an equilibrium point in the game signifies that both players are inclined to employ their respective pure strategies. In such a scenario, the optimal gains for both players are determined by the game values corresponding to these pure strategies.

However, it's important to note that not all two-person non-constant sum games exhibit just a single equilibrium point. In cases where there is no equilibrium point in the game, or multiple equilibrium points exist, it implies that players will opt for mixed strategies during gameplay. When this is the case, employing linear programming or more sophisticated computational techniques becomes necessary to calculate the probabilities of players utilizing each strategy and to determine the expected value of the game. The establishment of such a procedure becomes crucial for resolving the intricate dynamics of these games.

It is possible to establish a systematic procedure that serves as a reliable method for determining both the value and optimal strategies for any two-person non-constant sum games. This procedure consists of the following steps:

1. Begin by checking for the presence of an equilibrium point in the game. If the game lacks an equilibrium point, proceed to the next step.
2. In this step, identify and eliminate any strategies that are dominated by others within the row player's set of strategies. Once this is accomplished, move on to the next step.
3. If, at this stage, the game matrix has been reduced to a 2×2 format, it can be solved graphically. However, if the game matrix is more complex, it'll need to resort to using linear programming or more advanced computational methods for a solution.

Now, let's delve deeper into the concept of an equilibrium point. An equilibrium point signifies a situation where neither player can improve their outcome by unilaterally changing their strategy. Essentially, it implies that when each player selects a strategy from the equilibrium set, neither can gain further advantage by unilaterally altering their choice (Canbulut et al., 2020).

This brings to the fundamental question: do two-person non-constant sum games always possess equilibrium strategies? The answer, as provided by Nash's theorem in 1951, is affirmative. Nash's theorem establishes that every two-person non-constant sum games has at least one equilibrium pair of strategies.

5.6.1. Conclusions

The revenue-sharing contract bears significant resemblance to the fundamental characteristics of two-person non-constant sum games. In both scenarios, the primary motivation driving the players revolves around enhancing their individual profitability. Furthermore, they confront a multitude of strategies at their disposal, each potentially leading to distinct outcomes in terms of gains. Initially, in the context of revenue-sharing contracts, the process entails the identification of various strategies that the producer and retailer can employ. Following this, the profitability of each party is meticulously calculated based on different combinations of these strategies. When the outcomes of these calculations are consolidated into a single matrix, it becomes evident that the resultant structure mirrors the bimatrix format often observed in two-person non-constant sum games. In the final phase of this analysis, the objective revolves around identifying the optimal strategies for both producers and retailers. To achieve this, the established solution methods commonly associated with two-person non-constant sum games are employed.

A central challenge, that remains open, faced by both producers and retailers within this type of contractual arrangement, pertains to the determination of two critical parameters: the wholesale price and the profit-sharing rates. These parameters are pivotal in shaping the dynamics of their collaborative endeavour.

5.7. Final remarks

In the previous sections, an in-depth analysis of the three most common contract forms has been conducted from a game theory perspective. This exploration has encompassed a range of critical factors, including the risk preferences of the various stakeholders within the supply chain, their negotiation capabilities, and, most notably, the pivotal order quantity. The latter has served as the focal point and driving force of this study, being the primary tool for optimizing the overall performance of the supply chain.

From this new analytical perspective, multiple advantages that account for the widespread adoption of these contracts in today's business landscape have been identified. For instance, the inherent flexibility of these agreements has been highlighted, enabling retailers to adapt more efficiently to demand fluctuations. Furthermore, their effectiveness in coordinating the intricate dynamics within a supply chain, while simultaneously achieving Pareto improvements, has been appreciated.

However, during this analysis, it was impossible to overlook the unresolved challenges related to the practical implementation of these contracts. In particular, the complexity of defining critical parameters that significantly influence contractual dynamics cannot be denied. Moreover, it has been recognized that, although many of these contracts can, at least in theory, lead to the maximization of total chain profit, they often do not align with the individual interests of each member.

Building on these premises, the aim is to develop a new contracting model that can capitalize on the strengths of existing models while at the same time vigorously addressing outstanding challenges. This innovative model introduces a revolutionary approach to profit allocation within supply chains, as it focuses on a completely new driver than in the past: time to order.

6. Research Methodology and Literature Gaps

6.1. Objective and Research Methodology

6.1.1. Literature analysis

In the luxury jewellery business, contractual arrangements are extremely important, as the choice of contract type has a significant impact on the relationships between the different actors in the supply chain. This sector involves high-value transactions, where the goods themselves are valuable and often unique, so it is essential to establish clear and well-defined contractual arrangements to ensure that the interests of all parties involved are protected. The lack of an all-encompassing contractual form in the luxury jewellery sector is a significant shortcoming. In many cases, contractual practices are based on standardised contracts or generic agreements that may not perfectly fit the specific needs of each transaction. This can lead to problems such as ambiguities, conflicting interpretations or disputes between buyers and sellers.

To address this challenge, we began our work by conducting a literature review to examine in depth a significant sample of articles published since 2001 to nowadays. In particular, the theoretical frameworks underpinning our research and on which the tools used to develop the discussion are the publications Cachon and Lariviere (2005) and Brun and Moretto (2012).

This review helped to identify trends, best practices, and common issues in the luxury jewellery industry in relation to contractual arrangements. The articles reviewed span a broad spectrum of techniques, from game theory to various mathematical programming methodologies, thus offering diverse approaches to addressing contractual and management issues. The decision to examine a broad spectrum of techniques is crucial because the luxury jewellery sector has many facets and complexities in transactions and operations, which it is essential to know in depth.

In the process of searching for these articles, we used two selection criteria: source of origin and keywords.

In particular, the selected research journals are in the field of operations research and management science (OR/MS/OM), including INFORMS publications such as Management Science, Operations Research, Manufacturing and Service Operations Management, Interfaces, Information Systems Research, Marketing Science, Service Science, Transportation Science, Mathematics of Operations Research and INFORMS Journal of Computing. The selection of these journals is significant, as they are known for publishing high-quality research and for their relevance to business issues. The choice to focus on publications such as Management Science, Operations Research and other INFORMS journals is an indicator of our attention to quality and credible sources. This gives our work a high level of trustworthiness, as their publications are generally subject to rigorous peer review. The reference to management science is relevant because contractual issues in the luxury jewellery industry are strongly linked to business operations and management. The application of operational research techniques and management science can help optimise contracting processes, improve efficiency and risk management, and guide strategic decisions in the sector of interest. This diversity of sources contributes to the comprehensiveness and in-depth understanding of industry dynamics and possible contractual solutions, as well as providing a solid basis for future developments.

When selecting articles, we also adopted a targeted methodology using keywords such as 'fashion', 'supply chain', 'retail', 'game theory', 'luxury', and 'contract' in order to narrow the search by identifying contributions relevant to our area of study. This helped save time and resources. The selected keywords are directly related to key themes in the luxury jewellery sector. 'Fashion', 'luxury' and 'contract', for example, are central concepts in our research, while 'supply chain' and 'retail' reflect important operational and commercial aspects. Without the use of targeted keywords, we could have been overwhelmed by too many articles, with the risk that not all of them would have been relevant. The scattering of the search could have made it harder to find high-quality articles and identify significant trends or results. Instead, this made it possible to build a relevant corpus of literature.

Subsequently, a very useful practice for research was the distinction of the selected articles into three macro-categories, based on the proposed content and the objectives pursued by the authors. This made it possible to organise and structure the literature review in a clearer and more coherent way. A more detailed explanation of each category follows:

- Articles dealing with the luxury industry: this category contributes to an in-depth understanding of the luxury industry itself. It elucidates concepts such as the definition of what constitutes a product a luxury product, the historical evolution of the concept of luxury itself, the history of the luxury jewellery industry and the critical success factors that have helped shape the industry over time. This background knowledge is fundamental, as it provides an overview of the context we are entering, and in particular, makes explicit its specific needs and dynamics.
- Articles concerning the various existing contractual forms: this category offers a review of the contractual forms developed and implemented over the last few years in the luxury jewellery sector. In particular, the collection is divided into:
    - Introduction of existing contract types;
    - Identification of their strengths and weaknesses;

- Assessment of the level of alignment between the characteristics of contracts and the characteristics of the luxury sector;
- Identification of existing gaps;
- Analysis of possible improvements and future developments;

This phase is crucial to understanding existing contractual practices and the challenges that still need to be resolved. These can be the turning point from which a new hybrid contractual form can be elaborated and subsequently implemented, one that can not only resolve open issues but also take their strengths and elevate them.

– Articles related to the application of game theory in supply chain management: this section underlines how game theory is a powerful analytical approach to studying contracting contexts, especially in terms of cooperation and possible conflicts between parties. This provided a broad overview of the strategic dynamics and possible interactions between buyers and sellers in the context of luxury jewellery transactions. Among them, it was possible to identify optimal solutions and winning strategies in the context under investigation, which are worth considering for the development of a new contractual model.

Each category served a specific purpose and together they helped to form a comprehensive picture of the current state and lay the foundation for the development of the new hybrid contractual form.

Based on the information and knowledge gained from the literature review, our goal is to develop a hybrid contract form that improves the efficiency and effectiveness of transactions in this area while minimising risk and litigation. This contract form is designed to address the problems and gaps identified in modern contracting. This results in a flexible and adaptable contract that takes into account the specific needs of each transaction, but at the same time is based on established principles and guidelines to ensure clarity, fairness and security for all parties involved. In particular, the new contractual model is able to combine the characteristics of the most in use current contracts in the luxury jewellery sector. Their study emphasised the need to introduce a model that can provide a balance between the flexibility offered by contracts such as the revenue-sharing and the risk management guaranteed by the buyback contract.

6.1.2. Model development

In the process of developing the new contract form, key variables for optimisation were carefully considered. In particular, in addition to the quantity of articles ordered by the retailer, the timing of reordering was included as a key variable. This represents a significant turning point compared to previously conducted analyses.

At the beginning of the season, when the retailer places the order according to the current contracts, he is operating on a significantly limited range of information about market demand. This implies that the initial order is placed with approximate knowledge of the actual demand, resulting in an order process characterised by uncertainty. Inaccurate forecasting leads to greater difficulties in optimally managing inventory levels, resulting in high risks and costs associated with out-of-stock or overstocking. The new contract model responds to this issue by introducing the possibility of more diluted orders over time. This approach results in a better alignment with demand fluctuations over the season and contributes to a significant reduction in risks and costs. This differs substantially from traditional contract models, such as the Newsvendor model, which allow for a single order at the beginning of the season, neglecting the importance of the evolving dynamics of demand forecasting during the course of sales activities. The consideration of time plays a crucial role, since market dynamics may change during the selling season, as well as consumer behaviour. Therefore, the new contract model is designed to reflect the reality of changing markets more accurately. The introduction of the possibility to reorder during the selling period is a significant advantage, allowing for greater flexibility and adaptability to changing market needs.

Diligently, we also considered concepts rediscovered during the analysis of articles relevant to game theory. This methodological approach allowed us to outline a set of scenarios aimed at testing our model under different conditions, with the aim of identifying the circumstances in which it might or might not prove particularly efficient. To further deepen our understanding of the results and to assess the practicality and effectiveness of our model, we proceeded with the development of a detailed simulation. In this context, we carefully compared the results obtained from our model with those that a hypothetical retailer or producer would have achieved by applying the existing models in the industry. This carefully and analytically conducted comparison provided a solid basis for assessing the validity of our proposals and revealed the strengths of our model as well as possible areas for improvement.

6.1.3. Model testing: the Simulation

After having formulated the new contract model in an analytical way, it is imperative to proceed with a quantitative verification of its effectiveness. For this purpose, a simulation was implemented using Microsoft Excel software, a well-known tool widely used for quantitative analysis and modelling. This simulation was designed to generate realistic data for 100 different articles over an observation period of 100 years. For each article, a demand distribution characterised by a mean of 100 and a standard deviation of 10 was assumed. In order to discriminate between high and low-demand products, a threshold was set, below which it would not be appropriate for the retailer to take the risk of placing a large order, and a probability of occurrence of this event was associated.

The simulation covers a wide range of scenarios, reflecting all possible contexts envisaged in the model development phase. This approach makes it possible to evaluate the behaviour of the new contractual model under different circumstances, thus providing a complete picture of its effectiveness. The primary objective of this simulation is to conduct a detailed comparative analysis between the results obtained through the application of the new contractual model and those resulting from the implementation of the traditional Newsvendor model, examining a variety of scenarios.

The inherent benefits of this approach are multifaceted. First, the simulation offers the possibility to evaluate the new model under controlled conditions, analysing its behaviour in a wide range of potential situations and scenarios. This includes variations in demand, seasonal influences, changes in consumer tastes and market dynamics. Simulation provides realistic and reliable data, constituting an essential element for robust evidence-based decision-making. The provision of numerical data highlighting profit maximisation benefits is an effective means of positively influencing business decisions. In this sense, such data provides a solid basis to promote the adoption of the new approach within business dynamics.

The comparison with existing models using consistent and plausible data was useful for assessing the validity of our proposals. This paper constitutes a fundamental empirical source for future business decisions in the context of the supply chain.

6.2. Gaps of the Literature and Research Questions

Contemporary literature shows a marked focus on the analysis of existing contractual forms and the evolution of new models and paradigms of contract interpretation (Brun & Moretto, 2012). The predominant objective of this research is to improve the efficiency and effectiveness of the use of contracts in the market. However, the core problem that emerges from this approach is the lack of a systematic effort to reconcile different contract types with the intent of developing a hybrid contract form. Such a contractual form would be aimed at synthesising the advantages of the different contract types that are widely used. The synergy resulting from such integration could constitute a considerable advance in optimising contractual relations between the parties involved (Bhaskaran & Krishnan, 2009).

Within the broad landscape of contractual studies, game theory has frequently played a prominent role. However, the focus has historically been mainly on the quantity variable. A more comprehensive and inclusive analysis, contemplating the consideration of different variables, may prove crucial in filling the significant gaps found in the current corpus of literature and opening up new paths of research. The gaps identified emphasise the urgent need to keep the following key factors in mind (Fisher et al., 2000):

- Price changes, discounts and pricing policies in the luxury industry. These factors require in-depth analysis as they greatly influence consumer behaviour and business profitability;
- Exclusivity clauses and their impact on competition in the luxury industry. They constitute a critical aspect of contractual transactions in the luxury industry. A thorough evaluation of these elements is crucial to fully understand the competitive and distribution dynamics in this field;

- Supply management in the luxury sector. Supply management, including optimisation of production schedules, material quality and producer relationship management, plays a key role in ensuring consistency and excellence of the production process in the luxury sector. A detailed investigation of these aspects is imperative to guarantee superior quality standards;
- Distribution in the luxury industry and partnerships with retailers. A rigorous analysis of these dynamics is essential to ensure effective distribution in line with the brand image in the luxury market segment;
- Guarantees, after-sales service and returns management in the luxury industry. These elements are crucial to guarantee customer satisfaction and to preserve a positive reputation.

Although an extensive corpus of studies has been devoted to the analysis of traditional contracts, a substantial gap emerges in the understanding of innovative contractual models. The latter could embrace forms such as profit-sharing agreements, performance-based contracts or more open modes of collaboration that go beyond simple product transactions. Such new contractual configurations present an opportunity to enhance flexibility and adaptability in producer-retailer relationships.

Another consideration of substantial importance is the asymmetry of information that characterises relationships between producers and retailers, a phenomenon that frequently generates opportunistic behaviour and contractual disputes (Govindan et al., 2013). In this context, the use of game theory plays a central role, as it contributes to the development of contracts that mitigate these issues and foster broader cooperation between the parties involved.

The need to develop a simulation based on realistic data at the end of the elaboration phase of the new contractual form arises in response to a significant gap in the existing contract literature. Frequently, new contractual models and interpretations are proposed without being subjected to adequate quantitative verification by scholars. The implementation of this simulation, using real data, aims to quantitatively confirm the effectiveness and validity of the new contractual model.

Summarising, the existing literature on coordination agreements between producers and retailers looks like an area of study full of opportunities. The knowledge gaps identified offer an intriguing challenge, providing us with an opportunity to contribute to filling them and promote more effective management of supply chains and coordination agreements, especially in a field as complex and competitive as the luxury industry. Future research perspectives should aim to consolidate theoretical contributions through valid empirical validation, thus contributing to enhancing the understanding and effectiveness of contractual practices in the luxury sphere.

The gaps identified in the literature, together with the initial insights we had about the contractual landscape, triggered a series of questions that subsequently shaped the guiding structure for the drafting of this paper:

1. *What are the predominant challenges and peculiarities associated with Demand Management (DM)? How can a mid-season product reorder model in the luxury jewellery sector impact the resolution of these challenges? In this regard, are the dynamics of the new model more efficient compared to those of conventional procurement models?*
2. *How can game theory be applied to optimise contracts, taking into account variables such as price, quantity, and timing of orders?*
3. *How can a new contractual model maximise profits for both sides of the supply chain, considering the unique dynamics of the industry under consideration?*

4. *What could be the possible application scenarios and how could these be affected? Which stochastic and non-stochastic variables need to be considered besides seasonal changes, demand fluctuations and price variations?*

7. Model formulation

7.1. Introduction

In the current business landscape, companies are grappling with increasingly dynamic and unpredictable conditions within volatile markets. Fluctuating customer demands, shorter lifecycles for luxury products, growing diversity of product offerings, and the ever-changing economic environment have imposed a heightened level of adaptability compared to previous years (Daxböck et al., 2011, p. 7; Schuh et al., 2012, p. 3; Stich et al., 2012, p. 123).

In this fast-paced and turbulent market setting, the fickle customer demand has become a significant challenge. Procurement departments are now grappling with uncertainties related to demand and other stochastic variables, making procurement planning and risk management more and more complex (Martinez-de-Albeniz and Simchi-Levi, 2005; Fu et al., 2009). Procurement risks materialize when substantial disparities emerge between supply and customer demand, resulting in inventory shortages or excess, and sometimes even obsolescence. The untimely acquisition of inbound components or raw materials within a dynamic environment can severely impact profitability, potentially leading to unexpected and substantial losses. Effective procurement strategies are, therefore, essential to safeguard a company's financial performance in the face of unpredictable demand (Huang and Qu, 2008; Qu et al., 2010).

As a response, a range of flexible supply contracts have been proposed to mitigate the risks associated with procurement. An effective procurement approach seeks to devise a procurement plan that minimizes risk exposure while optimizing procurement costs. Traditionally, procurement problems assume that both parties involved are risk-neutral, but in reality, it is common for participants to have differing risk preferences. Furthermore, their choices, in uncertain conditions, are often influenced by their individual risk attitudes.

Therefore, to achieve the best possible outcomes, producers must carefully evaluate the optimal order quantities from various supply contracts and market dynamics, with the dual goal of minimizing procurement costs and managing risks effectively. This involves recognizing and accounting for the different risk preferences of the parties involved in the decision-making process.

The "Newsvendor model", as proposed by Cachon, aims to assist businesses in determining the optimal inventory level in situations where forecasting future demand and deciding on the quantity of inventory to purchase or produce is required. This model seeks to strike a delicate balance between the costs associated with maintaining excessive inventory, including storage costs and the risk of obsolescence, and the costs resulting from sales loss due to insufficient inventory, which would result in missed profits. Ultimately, the goal is to maximize the so-called "expected profit" or minimize "expected losses", factors that depend on the distribution of demand and the quantity of inventory held (DeMarle, 2021).
The key characteristics of this model can be summarized as follows:

- Determine the optimal order quantity when facing uncertain demand;
- Resolve the dilemma between ordering too much, resulting in unsold items, or ordering too little, causing the loss of sales opportunities;
- Consider a single-period time horizon, meaning a retailer can place only one order during the selling season (DeMarle, 2021).

However, in a dynamic and ever-evolving market, this type of contract may not effectively support the retailer's side. The model does not allow for the possibility of additional reordering to adjust inventory to market demand fluctuations. In this situation, it appears that the advantage lies with the producer, who can benefit from:

- Increased sales, as the retailer may be inclined to place larger orders, even at the risk of accumulating excessive inventory;
- Reduced production costs by capitalizing on economies of scale, optimizing production, and lowering operational costs;
- Improved production planning due to the increased predictability of the quantities to be produced, no longer subject to unpredictable demand fluctuations.

In conclusion, while the Newsvendor model is a valuable tool for balancing storage costs and losses due to insufficient inventory, in an evolving market, the producer seems to derive more benefits compared to the retailer, who may desire greater flexibility in inventory management.

On the other hand, over the past few decades, it has been witnessed the rise of retailers. This transformative trend highlights a significant shift in bargaining power towards the downstream end of the supply chain. As a consequence, vulnerable producers often find themselves pressured into making concessions, such as reducing prices and shortening delivery times for these dominant retailers (Wang, 2007). The transfer of this leadership position significantly impacts the decision-making processes involved, a situation distinct from when producers have contractual power (Ertek & Griffin, 2002; J. Wang et al., 2013).

In addition, this phenomenon underscores that decision-making challenges often manifest within a multi-period framework. Unlike the single-period model, the interplay between successive decisions cannot be disregarded in multi-period models. As the planning horizon is divided into numerous interconnected time periods, the optimization process becomes increasingly intricate.

In recent years, the multi-period inventory problem has gained significant traction as a hot topic, capturing the widespread interest of researchers (Federgruen & Heching, 1999; (Wan & Chen, 2018). Nevertheless, very few studies, such as Li et al. (2011), have delved into the related issues within the context of the entire supply chain. Until now, discussions on retailer-led supply chain management have predominantly revolved around single-period settings, with the multi-period scenario remaining unexplored. On the other hand, the latter setting strikes a balance between realism and the mathematical model, rendering it more representative of real-world complexities.

Considering this scenario, where the capricious nature of market demand creates a dilemma for retailers, they find it advantageous to maintain flexibility in their orders from producers, both to evade the encumbrance of excess inventory costs and to stay agile in their responses to market shifts. In contrast, producers lean towards the idea of retailers placing comprehensive orders as early as possible. This preference is underpinned by their need to mitigate the risks associated with production excesses and shortfalls. Such discord between retailers and producers can cast a shadow over the efficiency of the supply chain.

In light of this challenge, this chapter delves into the problem, taking a cooperative game approach, to tackle the coordination issues within a producer-retailer supply chain. It introduces a new model of contract, striving to bridge the gap between the two key stakeholders in the supply chain.

Traditionally, producer-retailer supply chains adopt a wholesale price mechanism, a model that has, in practice, frequently driven producers and retailers into the throes of conflicting interests. This discord is further exacerbated by the volatile nature of market demand. As previously mentioned, retailers find adaptability in their orders an asset for sidestepping inventory-related expenses and accommodating the oscillations in market demand.

Conversely, producers' interests align with retailers placing comprehensive orders early in the process. This early commitment allows producers to mitigate risks associated with production excesses and shortages.
This clash of preferences between retailers and producers can lead to decisions that, while serving the interests of one party, may not be optimally beneficial for the overall efficiency of the supply chain. The resulting inefficiencies can ripple through the entire system.

7.2. Key concepts of the Newsvendor model

The Newsvendor model is a fundamental framework in inventory management that is used to determine the optimal order level in situations where future demand is uncertain. This model is of particular relevance and utility in industries such as retail, manufacturing, and distribution, where companies must balance the costs associated with overproduction (having too much inventory) with the costs associated with underproduction (not having enough inventory to meet demand). The Newsvendor model helps to determine the optimal reorder point and the desired service level, i.e. the desired probability of not running out of inventory, by carefully evaluating the issue of the right balance between profit maximization and total cost minimization (Newsvendor Problem – the Tale of the First Formula in the Textbook | Stitch Fix Technology – Multithreaded, 2019).

To use the model, it is necessary to define a marginal cost function representing the additional costs associated with purchasing or producing additional units of inventory. This marginal cost function takes into account factors such as purchase costs, storage costs, and costs associated with the lack of inventory.

The model is based on some key variables, which are essential to introduce before delving into its mathematical aspects and operation (Tang et al., 2018):

- $'D'$ (Expected demand): this indicates the expected quantity of a product that is projected to be requested during a specific period, such as a sales season. This estimation of future demand is based on historical data, forecasts, or other factors;
- $'C'$ (Product cost): this represents the cost of acquisition or production for each unit of the product;
- $'h'$ (Inventory holding cost): this cost refers to the expenses associated with the preservation and storage of inventory. It includes costs like warehousing, insurance, obsolescence, and the cost of tied-up capital. It signifies how much it costs to maintain one unit of inventory for a specific period;
- $'p'$ (Cost of lost profit or penalty for underproduction): this cost indicates the losses or penalties due to underproduction, i.e., when the company does not have enough inventory to meet customer demand. This cost can include lost sales, customer dissatisfaction, or missed opportunities;
- $'\alpha'$ (Stockout probability): it refers to the probability that the actual demand exceeds the available inventory quantity. Often, this probability is based on a probability distribution of demand; thus, it represents the risk associated with underproduction;
- $'Q'$ (Order quantity): this indicates the quantity to be ordered, which is the unknown to be solved in the model. It represents the amount of product that should be ordered to maximize the net expected value, i.e., the expected profit minus the expected costs. The optimal order level depends on all the other variables and considerations mentioned above (Rekik et al., 2006).

Considering these factors, the Newsvendor model determines how much inventory should actually be ordered in a given situation to maximize profits or minimize costs. The variables $'D'$, $'C'$, $'h'$, $'p'$, and $'\alpha'$ allow modeling the dynamics between uncertain demand, inventory costs, and the risk associated with underproduction.

The ultimate goal of the Newsvendor model is to determine the optimal order level represented by $'Q'$, which minimises the total cost (Newsvendor Problem – the Tale of the First Formula in the Textbook | Stitch Fix Technology – Multithreaded, 2019). This total cost is composed of the sum of the inventory holding costs (hQ) and the sub-production costs (αpQ), from which the expected gain (CQ) is subtracted if demand exceeds the order. In other words, the model aims to find the point at which the costs of maintaining inventory and the costs associated with underproduction are balanced by the potential loss of profit due to excess demand. This means that if one orders too little inventory (underproduction), one may lose sales or incur costs due to the lack of inventory. On the other hand, if one orders too much inventory (overproduction), one faces storage and other costs associated with excess inventory.

The problem can be formalised in mathematical terms, and the exact formula for total cost is as follows:

$$\begin{aligned} Minimize\ Total\ costs \\ &= i\text{nventory holding costs } + \text{ subproduction costs} - \text{expected gain} \\ &= hQ + \alpha pQ - CQ \end{aligned}$$

Equation 7.1

By actually evaluating the impact of each term on the ultimate outcome, it is possible to identify the value of $'Q'$ that makes the total cost as low as possible, thus maximizing profit or minimizing total costs (Newsvendor Problem – the

Tale of the First Formula in the Textbook | Stitch Fix Technology – Multithreaded, 2019).

The objective function of the retailer can be simplified as follows:

$$Mazimise\ \pi_R = min\ (h + \alpha p - C)Q$$

Equation 7.2

To determine the optimal value of $'Q'$ in the Newsvendor model, it is essential to calculate the "reinforcement point" or "Safety Stock" (SS). This concept is of fundamental importance because it refers to the difference between the expected value of demand (D) and the optimal order level (Q). More precisely, safety stock is the additional amount of inventory a firm maintains to deal with uncertainty in demand. It serves as a "safety net" to avoid underproduction in situations where demand exceeds expectations or is more volatile than expected, and thus to avoid loss of sales or customer dissatisfaction (Jeong & Leon, 2012).

The determination of how much safety stock a business needs depends on various factors and requires careful consideration regarding:

- Demand analysis: this may involve the use of historical sales data, the identification of seasonal trends and the application of sales forecasts;
- Customer demand variability analysis: this assesses how much customer demand may vary over time. It applies statistical measures such as standard deviation to quantify this variability. More variability will require more safety stock;
- Supply variability analysis: this examines the variability in the delivery of materials or products by producers. This variability may be caused by

delays in delivery, fluctuations in material prices or quality problems. Greater variability in supply will also require a greater share of safety stock;

- Plan the service and the desired level of service one wants to offer one's customers. For example, one might plan to have enough safety stock to cover 95% of possible customer demands. This means that one is able to meet demand 95% of the time without running out of stock;
- Calculation of the reorder point, which represents the inventory level below which a new order should be placed in order to avoid running out of stock. The reorder point takes into account the delivery time and the expected consumption during that period.

The safety stock quota is not a fixed quantity and should be periodically reviewed and updated according to changes in demand, supply variability and service targets (Jeong & Leon, 2012). Furthermore, the calculation of the safety stock quota may vary from business to business and from product to product. It is important to tailor the approach to the specific circumstances of the business.

Exactly because it takes into account these fluctuations in actual versus expected demand, the calculation of safety stock is essential in determining the optimal value of $'Q'$.

$$SS = expected\ value\ of\ demand - \ optimal\ order\ level = D - Q$$

Equation 7.3

After calculating safety stock, the probability of stockout (α) can be determined. The stockout probability is the chance that the actual demand exceeds the quantity available in the warehouse, taking into account the safety stock. It can be calculated using a probability distribution that models the variability in future demand (C¸ akanyıldırım, s.d.). The mean and standard deviation of the distribution reflect forecasts of demand and its variability. The normal, or

Gaussian, distribution is the most often used probability distribution. The normal distribution is a probability distribution that is often used to model casual events in which the data are symmetrically distributed around a known mean and a specified standard deviation. With this information and the value of the safety stock, probability α can be computed. The exact formula may vary depending on the distribution used and the specific conditions of the problem. For example, if the demand follows a normal distribution and orders are placed based on a given reorder point $'R'$, the stockout probability can be calculated using the cumulative distribution function of the normal distribution. In light of this, it is clear that the calculation of stockout probability is critical to understanding the risk associated with inventory management and ensuring that the level of safety stock is adequate to face the selling season without underproduction problems (C¸ akanyıldırım, s.d.). As a result, inventory planning is more accurate and reliable.

$$\alpha = 1 - \Phi((D - R)/\sigma)$$

Equation 7.4

Where:

- ‘Φ' represents the cumulative distribution function of a normal distribution. It gives the probability that a casual variable from a normal distribution is less than or equal to a certain value, in this context, ‘Φ' is used to calculate the probability α that the actual demand exceeds the reorder point $'R'$;
- ‘σ' stands for the standard deviation of demand, which measures how variable future demand is. A high value of standard deviation indicates a higher variability of the measure;
- $'R'$ is the reorder point, which is the inventory level at which the decision to place a new order to replenish the stock is made. In mathematical terms, the

reorder point $'R'$ is equal to the sum of the optimal order level $'Q'$ and the Safety Stock (SS).

Thus,

$$R = optimal\ order\ level + safety\ stock = Q + SS$$

Equation 7.5

In the Equation 7.4, $(D - R)$ denotes the difference between the expected demand (D) and the reorder point (R). This value represents how much demand is expected to exceed the reorder point. Dividing this difference by the standard deviation (σ), one normalises the variable with respect to its variability. Then, $'\Phi'$ computes the probability that demand exceeds the reorder point, considering the normal distribution of demand and safety stock (C̦ akanyıldırım, s.d.). This equation is a key part of the Newsvendor model.

Thus, the optimal quantity $'Q^{*}'$ can be calculated. The optimal quantity $'Q^{*}'$ is the quantity of product that minimises the expected total cost, allowing the company to achieve maximum profit or minimise total costs. In order to find $'Q^{*}'$, the derivative of the total cost formula with respect to $'Q'$ must be determined. The calculation of this derivative returns the rate of change of total cost as $'Q'$ changes. Once the derivative is obtained, it is set equal to zero. This method is known as "derivation and equality to zero" and is a common technique for determining the maximum or minimum points of a function, i.e. those points at which the rate of change of total cost is flat, and hence the point at which total cost stops decreasing or increasing and settles at a minimum. This point is the optimal $'Q'$ (Q^{*}). Solving the equation can be done using mathematical methods such as solving differential equations or numerical methods. Once the value of $'Q^{*}'$ has been obtained, it is used in the company as the optimal quantity to order to maximise profit or minimise total costs. In this way, the calculation of $'Q^{*}'$ enables the optimisation of the trade-off between the costs of maintaining inventory and the costs associated with sub-production, taking into account the expected profit. This is a critical step in inventory management, as it allows the company to make informed procurement decisions (Rekik et al., 2006).

$$\partial(\text{expected total cost})/\partial Q = 0$$

Equation 7.6

In addition, the importance of making periodic order decisions based on new information on demand and market conditions is emphasised. Market conditions and customer demand can change over time. Therefore, it is crucial that companies are able to adjust inventory levels and order decisions according to this new information. For example, during a selling season, demand may be affected by unforeseen events, market trends, or other variables. Periodically, companies should review and update their reorder plans in response to these changes. This concept reflects the dynamic nature of inventory management and the Newsvendor model.

In conclusion, the Newsvendor model is a valuable tool that helps companies make informed inventory management decisions, taking demand uncertainty into account. This model supports regular planning and review of orders, ensuring that the company is able to adapt to changing market conditions and maximise overall value while avoiding excessive costs. Inventory management thus becomes a dynamic process that is optimised over time.

7.3. Overview of the new contracting model

The aim and added value of this research work described here is to develop a decision model for the area of procurement using solutions concepts of game theory. Especially in times of high volatility, such a decision model can support material requirements planners better than today's common selective planning logic.

Consider a two-party supply chain where one producer and one retailer are the members. The producer manufactures the goods and distributes them via the retailer to the end customers. The retailer is more powerful and acts as the leader of the market. The whole time horizon is divided into two different and connected periods. The producer needs to decide his production quantity for each period to obtain the highest profitability over multiple periods. The retailer needs to decide his order quantity for each period to maximize his expected total discounted profit across different periods.

When the sales period T_0 begins, the initial firm order is distributed to the retailer immediately. After stochastic demand in period T_0 has been observed, the options are exercised by the retailer based on the actual realized demand. When the sales in period T_0 end, unsatisfied demand is backlogged, and surplus goods are used for the next period. When the last period ends, part of the leftovers owned by the retailer can be sold at a discounted price or returned to the producer.

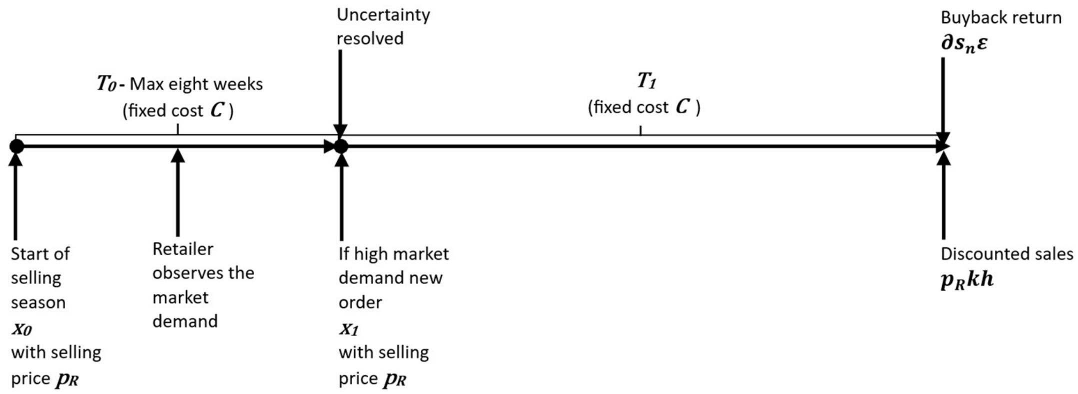

Figure 7.1: Graphical representation of the events occurring within the selling season in the Mid-Season Reorder model

This new model considers a supplementary supply-order system in which a retailer has an opportunity to place an additional order with a producer after the demand is realized during the selling period T_0, besides his first order according to the prediction and observation of demand. This multiperiod problem is formulated as an inventory game between the producer and the retailer, in which it is possible to derive the optimal decision policies for both parties and prove the existence of Nash Equilibrium. This study also investigates the benefits of the supplementary supply-order system and the parameter's influence by a numerical study.

This can be a very simple and applicable method to allocate the risk of demand uncertainty between the retailer and the producer thus improving the supply chain’s performance. More importantly is that the difference between this model and the previous works is in the formulation of a multi-period game and the derivation of the Nash Equilibrium solution, considering not only the producer's motivation but also the retailer’s interest.

The motivation behind developing this model stems from a gap in the existing literature on this topic. Many of the articles reviewed previous to the preparation of this paper address the problem of supply chain coordination, considering two key aspects:

– A single-period order time horizon, implying without considering the possibility of reordering for the retailer once demand has materialized;
– Optimizing the model using 'quantity' as the variable of interest.

However, in a dynamic and hardly predictable market, like the luxury goods industry, these assumptions do not necessarily lead to achieving the maximum efficiency of the supply chain. Therefore, given these challenges, the model proposed takes into consideration two pivotal factors:

– A multi-period time horizon. This means that considering a sales season of approximately 16 to 20 weeks, the retailer will have the opportunity to plan procurement in two distinct moments: one before the start of the sales season or simultaneously with it (before market demand for all items materializes), and a second moment, approximately between the fourth and the eighth weeks from the start of the sales season (when market demand becomes more predictable and the trend of the season has been observed for four weeks);
– The number of items the retailer can order.

In this way, the objective is to overcome the limitations of previous approaches and develop a more suitable model to optimize the supply chain in a dynamic market like that of luxury.

Concerning the reordering option, the retailer can structure his procurement as follows:

– At the first moment of purchasing the goods (t_0), the retailer acquires, according to the Newsvendor model approach, in order to cover the request of items for the entire selling season. This percentage covers the demand for products with low demand during the season and temporarily satisfies the demand for products with higher demand;

- At the second moment (t_1), the retailer has the option to purchase another percentage of products, but only for those for which they have evidence there is strong market demand.

The expectation is that products with low market demand will follow the logic of the Newsvendor model, as the application of this model is sufficient to optimize the supply chain and maximize profits for these items. For those products influenced by high market demand, this new model is superior as it enables both parties to manage their inventories better and avoids the problem of overstock from the retailer's perspective. On the producer's side, it allows him to extend the production over a longer time horizon.

To ensure the availability of products to the retailer in period T_1, the retailer has the option to reserve a maximum quantity of a specific product portfolio at the time of purchase in T_0, without the obligation to purchase the entire quantity. This will be underwritten by paying a guaranteed fee to ensure that the product quantity is indeed available. Subsequently, the retailer can purchase the chosen product quantity at a pre-established price, 'p_P'. The fee serves as a deposit to incentivise the retailer to actually purchase the products and to avoid excessive overstocking by the producer. This deposit will be deducted from the total economic amount of the reorder in t_1 based on the quantity purchased by the retailer. To provide motivation for the retailer to submit a second purchase, they will benefit from several advantages, including a larger discount on the product, a higher buyback percentage at the end of the sales season, as well as greater product availability and variety. On the other hand, to protect the producer, retailer who reorders in advance of the eight-week[17] deadline will be

[17] During the first four weeks the retailer observes the real demand of the season and, if necessary, in the second four weeks put a second order to the producer.

given preferential treatment. Conversely, who waits until the eight-week deadline will have fewer advantages.

Upon completing the development of the model, it will be tested and validated through simulation. The dataset includes estimates for approximately 100 realistic data points related to products purchased by different companies in the industry. Some products are considered to exhibit high market demand, and for these, the new model is expected to be more efficient than the conventional Newsvendor model. For others, there is an expected low probability of sale. For the latter, the Newsvendor model is presumed to respond more effectively to the needs of the supply chain members. This allows a comparison between the two types of contracts, highlighting the circumstances under which one type prevails over the other.

Additionally, the profit margins for both the producer and retailer will be highlighted, but notably for the entire supply chain. Additionally, it's important to remark, that the retailer's profit will benefit from a percentage derived from discounted sales made at the end of the season for those unsold products for which a buyback was not possible, in fact only for those items bought for the selling period T_1 the retailer can benefit from the buyback option.

7.4. Problem formulation

The following section introduces the new model that is solved using procurement quantity calculation methods. To provide a comprehensive understanding of the model, it will outline some key assumptions for the context:

– It is considered two distinct product types within the supply chain, namely, the range of the products with high market demand (referred to as 'i') and those products with low market demand (referred to as 'j'). These products are procured independently of each other;

- The time frame being examined is divided into two successive periods, T_0 and T_1, which are closely connected to each other. This connection is due to the fact that decisions made in the first period have a major influence on the development of the second period;
- The distribution of these products is handled by a single distributor;
- The demand for the products is known and provided as input for each individual period, starting with t_0. Across all individual periods within the entire time horizon, the demand can either remain constant (representing stable, static demand) or fluctuate (indicating dynamic demand);
- Assuming the retailer has a storage space large enough to accommodate the inventory for the entire selling season, as it is expected that the quantity of seasonal stock to not deviate significantly from previous seasons, unless unforeseen stochastic events occur during the period under consideration.

Additional assumptions within the model framework include:

- The possibility of stock-outs from previous periods;
- Prompt activation of the order and subsequent order fulfilment at the conclusion of the eight-week interval from the commencement of the sales season, thereby exerting a direct influence on the beginning of the succeeding period;
- A constant procurement price set by the producer (denoted as 'p_P');
- Consistent storage costs as a percentage (denoted as 'L');
- The initial inventory, so the leftover of the previous selling season (R_0) at the start of the first period T_0 (denoted as t_0) under review is set at 0;
- At the end of the selling season (denoted as moment t_n) the leftovers (R_n), which comprehends the remaining inventories of both periods T_0 and T_1, are equal to 0;

- The selling season begins at t_0, when a certain quantity of 'x_0', purchased by the retailer in anticipation of the season's beginning, starts to be sold. The quantity 'x_0' has been chosen in accordance with the sales forecast for the entire selling season.

In order to improve the understanding and accuracy of the seasonal demand forecast, this was assessed in accordance with different time periods. Specifically, in t_0, an initial forecast was conducted on the entire sales interval (b) and an analysis of its trend referring only to the first eight weeks (b_0). Subsequently, in t_1, a further forecast was made for the following twelve weeks (b_1). The latter value reflects the actual trend in market demand and underlines the importance of considering the demand progress when deciding on further reorders.

Let's use 'π' to represent the objective functions of the two agents and 'x_t' to represent the quantity to be ready to be sold from the moment t_0. The demand for period T is indicated by 'b_T', and is introduced a large arbitrary value denoted as 'M', which stands for the maximum quantity the retailer can reserve to the producer for the reorder. Furthermore, it is introduced a binary variable, 'μ', which equals 1 if the retailer chooses to place a new order in the eight-week period, and 0 otherwise. In this scenario, although the retailer does not issue a second order, which might suggest that the selected product range is not in high demand and should be handled via the Newsvendor model, it is nevertheless considered that the right conditions are in place to apply the new model. This is due to the fact that the absence of a second order is not the result of incorrect product clustering, but rather a casual event that occurred during the reorder interval and could not be predicted at the time t_0. This event is outside the retailer's decision-making control.

To make the understanding of the model clearer, a table summarizing both the previously used symbology and the one that will be presented next is provided below.

Symbol	Definition
i	Range of products with high market demand
j	Range of products with low market demand
T_0	Eight-week period from the beginning of the selling season to the end of the interval to carry out reordering
T_1	Period of 12 weeks starting from the end of the reorder interval and ending with the end of the selling season
t_0	Commencement of period T_0, corresponding to the start of the new selling season for products of both types i and j
t_1	Time of beginning of the period T_1
t_n	Moment that marks the end of the selling season
x_T	Quantity of products purchased by the retailer for the period T
b	Demand forecast referring to the all selling season
b_T	Demand forecast for the period T
p_P	Procurement price set by the producer
p_R	Selling price set by the retailer
L	Storage costs (as a percentage) incurred by the retailer
s_t	Inventories present in the warehouse at the moment t subjected to buyback
s_n	Inventories at the end of the selling season subjected to buyback
k_T	Inventories referred to the period T not subjected to buyback
$R_{t,T}$	End-of-period inventory for period T, calculated at time t, encompassing both types of stocks present at that particular moment for that specific period
π	Profit function

M	Quantity locked in t_0 for possible later reorder, i.e., maximum quantity that can be ordered by the retailer in t_1
μ	Binary variable indicating the occurrence of a second-order
h	Percentage discount applied at the end of the selling season to all products for which the buyback option is not available
ε	Mutually agreed-upon value between the retailer and the producer for the return of unsold goods at the end of the season
∂	Percentage of product s_n, for which the buyback option is available, that the producer reclaims at the season's end
w	Percentage reduction from the selling price for the goods reordered at the beginning of the period T_1
F	Guaranteed fee to reserve M for the following period
Δk	Number of pieces of k_0 sold in the period T_1
C	Unitary costs incurred by the producer regarding all the selling period
U	Unitary revenue for the producer associated with the disposal of the unsold product units at the end of the selling season

Table 7.1: Notation system for Mid-Season Reorder model analysis

Furthermore, a graphical representation of the time interval considered for the selling season discussed in this document is provided. This illustration is valuable as it offers a clear insight into how inventory has been analysed, categorized, and utilized in the formulation of the proposed model.

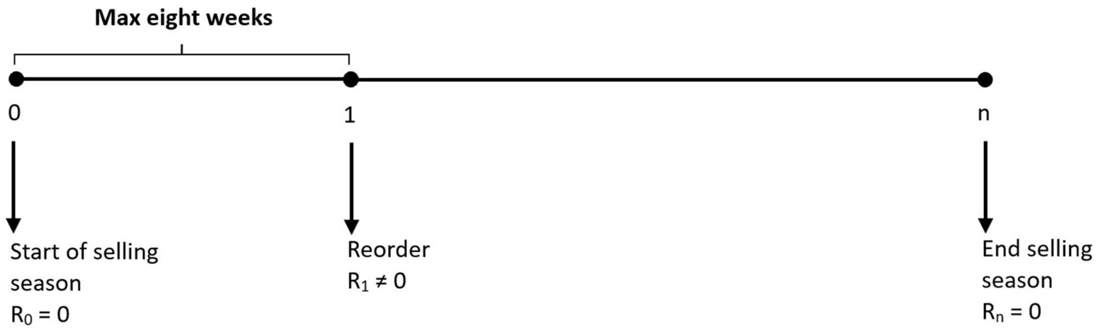

Figure 7.2: Chronological exemplification of the reordering moments within the Mid-Season Reorder model

In the process of analysing this model, inventories are referred to in a variety of ways, and it is essential to distinguish them according to their nature and when they are referred to. This distinction is essential for a better understanding of the context that the model is intended to examine. Regarding the type of inventories, two main categories can be distinguished. The first category includes inventories that are intended to be sold to customers at a discount price at the end of the season or even the following year through an affiliated outlet sales channel. These stocks cannot be returned to the producer and include all products that were ordered at the beginning of the season but remained unsold. This category is referred to as 'k' stock. The second category of stocks includes those that can be returned to the producer through a buyback agreement. These stocks are referred to as 's'. Regarding the time dimension, inventories can be tied to the end of the first period, or to the end of the second period, thus concurrent with the end of the selling season, which coincides with the end of the typical 20 weeks of a sales season. Stocks at the end of the first period are represented exclusively by 'k_0', with no further complications. In the second period, the situation is more complex. It is necessary to take into account not only the stock advanced through the order placed in 't_1' (s), but also the part of the initial stock (k_0) that was not sold in the first period and remains unsold even in the second period. This part of the stock is referred to as 'k_1'.

$$k_1 = k_0 - \Delta k$$

Equation 7.7

Where 'Δk' indicates the amount of inventories 'k_0' that was sold during the second period, T_1. In short, total inventories at the end of the season result from the combination of stocks remaining in the first period and unsold in the next, and stocks remaining at the end of the second period and purchased in the same period.

$$R_n = k_1 + s_n = (k_0 - \Delta k) + s_n$$

Equation 7.8

It is important to note that the simultaneous presence of $'k_1'$ and $'s_n'$ inventories in the Equation 7.8 is due to the fact that they refer to a set of products of type i and not to an individual product. Furthermore, the fact that the calculation of inventories is expressed as a function of time and not of the specific products grouped in category i, allows them to be calculated independently of individual products, but only in relation to the sales window considered. Thus, it is possible that a specific product i that remains unsold in the first period remains the same in the second period if it is not further demanded by the market. This explains the presence of unsold stock from the first period to the end of the season. But, given that the range of repurchased products in the second period consists of a mix of various items such as rings, necklaces, and bracelets, allowing for the coverage of the entire inventory of offered items, this scenario is unlikely, and for this reason it has been considered as an extreme case. Thus, in principle, end-of-season inventories align with inventories at the conclusion of the second period T_1. However, it is worth noting that regardless of the nature of the inventories, all of them are assumed to leave the retailer's warehouse at the end of the selling season. Consequently, the total value of inventories 'R_n' is zero.

$$R_n = k_1 + s_n = (k_0 - \Delta k) + s_n = 0$$

Equation 7.9

Before introducing the contract agents' objective functions, it is conventional to emphasize the model's constraints that determine their proper use:

$$R_{t,T} = \begin{cases} if\ t = 0, & R_{0,T=-1} = \emptyset \\ if\ t = 1, & R_{1,T=0} = k_0 = x_0 - b_0 \\ if\ t = n, & R_{n,T=1} = k_1 + s_n = k_0 - \Delta k + x_1\mu\ - b_1 \end{cases}$$

Equation 7.10

$$x_t - M\mu\ \leq 0$$

Equation 7.11

$$x_t \geq 0$$

Equation 7.12

$$R_{t,T} \geq 0$$

Equation 7.13

$$s_t, \quad s_0 = 0$$

Equation 7.14

$$b_T\ \geq 0$$

Equation 7.15

$$k_T\ \geq 0$$

Equation 7.16

with

$$\mu \in \{0,1\}$$

Equation 7.17

$$t \in \{0, 1, \dots, n\}$$

Equation 7.18

$$T = 0, 1$$

Equation 7.19

$$t_0 \in T_0\, ;\ t_{1,}\, t_n \in T_1$$

Equation 7.20

For a comprehensive understanding of the problem at hand, one can refer to sources such as Tompelmeier (2006, p.138) or Neumann (2004, p. 594 - 595). These sources have provided useful and interesting insights for the detailed definition of this problem.

The first condition, known as the storage equilibrium equation (Equation 7.10), establishes that the specific formula for calculating the leftovers level directly depends on the values of the variables T and t. By closely examining the different scenarios that can be articulated in relation to the values they assume, some important conclusions can be drawn:

– At the beginning of the season, when $t = 0$, the leftovers are zero, in line with the assumption that the storage is empty, as if the retailer is facing his first selling season. In this context, the variable T assumes a negative value because any inventory, which is emphasized to be zero, should refer to a period before the one under consideration, which, however, is not within the scope of the research;

- When $t = 1$, the stock levels consist of the quantity of products ordered to meet the selling season but remained unsold during the time interval from t_0 to t_1, which corresponds to the period T_0. To clearly distinguish between different types of inventories that undergo different disposal treatments, one refers to these stocks as 'k_0', namely type 'k' stocks that remained unsold in the period T_0;
- When t takes the value n, the stock levels are made up of what remains from the previous period and was not sold during the second period, as well as any inventory resulting from a possible second order that was not completely used. In this case, as well, it is proposed a distinction between the stock at the end of the period, emphasizing the future disposal methods.

This binary variable 'μ' is a crucial part of Equation 7.11 and plays a role in the maximization aspect of the objective function (as discussed in Tempelmeier, 2006, p. 139). In this context, 'M' is introduced as an arbitrary value, chosen in a way that it is sufficiently large to cover quantity demands in line with the market demand 'b_1', via a potential second order 'x_1', without specific restrictions (as explained in Tempelmeier, 2006, p. 139). Equation 7.12 ensures that there are no negative quantities to be ordered, while Equation 7.13 ensures that inventory levels do not go negative, which would not be logically correct. This model provides a reference framework for calculating order quantities, and in particular, the portion of the model that includes Equation 7.11 through Equation 7.20 is commonly referred to as the Single-Level Uncapacitated Lot Sizing Problem (SLULSP) (as defined in Tempelmeier, 2006, p. 138). This model proves to be a valuable resource for optimizing procurement in various contexts.

In the following paragraphs, it will be examined in detail the target-setting steps for both the retailer and the producer, with the aim of maximizing profits for both parties involved in the supply chain. This will be done using a model that departs from the traditional Newsvendor model approach, which has been widely adopted in recent years.

7.4.1. Retailer side

The main purpose of the proposed model is to define the target functions for the retailer and the producer, which essentially represent their profit functions. To calculate the profits of both parties involved in the upcoming contract, it is essential to highlight the different sources of revenues and cost items involved.

According to the CFI (the Corporate Finance Institute), the concept of profit is defined as follows:

Definition 11

"Economic profit (or loss) refers to the difference between the total revenues, less costs, and the opportunity cost associated with the revenue generated. Opportunity cost is the cost of an opportunity foregone, i.e., given up in order to pursue another one" (Team, 2022b).

$$\pi = Total\ revenues - Total\ costs$$

Equation 7.21

7.4.1.1. Revenues

During the selling season, there are several aspects to be considered to ensure effective inventory control and a satisfactory response to market needs, which have a critical impact on the total revenues.

Firstly, the sale of products in response to market demand, occurs in two distinct periods: the first period, referred to as T_0, and the second period, known as T_1. During T_0 and T_1, the retailer's objective is to supply customers with what they need in a timely and efficient manner.

However, it is not certain that the sales forecast is exactly in line with what actually occurs during the 20-week selling season, so it is not certain that all of the product stock during these two periods will be sold. The remaining items, instead of being neglected, are carefully considered. It is decided to offer it again to customers at a later time, once the sales season under consideration has ended at the moment t_n. To stimulate customers' interest, special discounts are offered to them on these unsold products. Finally, since there is a buyback agreement with the producer for certain goods remaining in stock at the end of the season, those unsold goods can be returned to the producer in exchange for a previously agreed value.

In summary, this approach to seasonal sales management involves optimising sales in two main periods, focusing on the recovery of sales through special offers and the judicious management of unsold goods through buyback agreements with the producer.

$$
\begin{aligned}
&Total\ revenues_R \\
&\quad = sales\ in\ T_0 + sales\ in\ T_1 + buyback\ value \\
&\quad + discounted\ sales\ in\ t_n + discount\ on\ the\ second\ order \\
&\quad = p_R(x_0 + x_1\mu - R) + \partial s_n\varepsilon + p_R(1-h)[k_1 + (1-\partial)s_n] \\
&\quad + \frac{F}{M}x_1\mu \\
&\quad = p_R\{(x_0 + x_1\mu - R) + (1-h)\,[k_1 + (1-\partial)s_n]\} + \partial s_n\varepsilon \\
&\quad + \frac{F}{M}x_1\mu
\end{aligned}
$$

Equation 7.22

In this formula, there are various terms indicating the different aspects of the retailer's earnings during the selling season.

The first two terms pertain to sales conducted during the peak season. This values is determined by the product of the price at which the retailer sells the goods to the end consumer (p_R) and the quantity actually sold. The quantity sold is the sum of the initially ordered quantities, x_0, and, if applicable, x_1 if the retailer decides to place an additional order, minus the quantity remaining in stock at the end of the entire selling season. The combination of these terms denotes the profit generated from the core sales.

The third addend concerns sales with a buyback agreement, where 'ε', symbolizes the value agreed with the retailer for the return of unsold goods for which this option is available at the end of the season and '∂' is the share of the product that is actually taken by the producer, based on when the retailer decided to place the second order.

$$\partial \in \{0, 1\}$$

Equation 7.23

The fourth addend in the formula reflects the discounted sales of the goods that remained unsold at the end of the season. 'k_1' represents the number of products subject to this particular discount, and 'h' is the percentage discount applied to these items.

$$h \in \{0, 1\}$$

Equation 7.24

The formula also includes the portion of type 's' stock for which buyback is possible, but which is not recovered by the producer. So, overall, this term reflects the importance of the income generated by the sale of the goods remaining at reduced prices.

Subsequently, an additional term not previously accounted for in the formula is introduced, encompassing a discount applied to the overall sum of the second order. This term serves as both reimbursement and incentive for the retailer to initiate an additional order. The reimbursement value is determined by three factors: the initially paid fee to the producer (F), the quantity reserved in the initial period T_0 (M), and the quantity of merchandise effectively purchased in t_1. It is essential to note that the reimbursement value is variable and is calculated exclusively based on the number of units acquired. Therefore, it is imperative to emphasize its independence from the economic value of each individual item. As these items collectively impact the discount value in relation to the total amount of the second order, they are considered of equal worth. In the event that the retailer opts not to place the second order, this value does not contribute to their profit margins. This decision is explicitly indicated through the utilization of the binary variable 'μ'. If 'μ' equals 0, it signifies that the retailer has chosen to abstain from placing any additional orders. Under such circumstances, the entire fee 'F' remains with the producer.

In summary, this complex formula helps to calculate the total revenues during the selling season, taking into account standard sales, sales with discounts, buyback and a discount based on the delay in reordering goods.

7.4.1.2. Costs

In the context of a retailer's operations, there are various factors that contribute to the costs of acquiring and handling goods from an external source. With regard to the costs to be borne by the retailer, it is important to keep in mind:

- The costs incurred for the purchase of goods from the producer during the period T_0, and potentially also for the subsequent period, T_1, should the retailer choose to re-supply;
- The charge that ensures the availability of a specific quantity of goods 'M' until the end of the replenishment interval, which extends over eight weeks;
- The costs associated with storing the goods in the warehouse during the first period, T_0, and in the second period, T_1.

$$\begin{aligned} Total\ costs_R = &\ purchase\ cost\ in\ T_0 + purchase\ cost\ at\ the\ beginning\ of\ T_1 \\ &+ Fee + storage\ cost \\ &= x_0 p_P + x_1 \mu (1-w) p_P + F + L p_P (x_o + \mu x_1) \\ &= p_P[(1+L)x_0 + x_1 \mu (1 - w + L)] + F \end{aligned}$$

Equation 7.25

The initial two terms in the formula delineate the expenses incurred through the tangible procurement of goods, encompassing factors such as shipping and transportation costs. In contrast to the computation of the revenues, this equation involves the multiplication of the quantity of goods acquired at a specific moment t by 'p_P', denoting the price at which the retailer procures the goods from the producer. In the eventuality that the retailer places a second order (μ = 1), the ordered quantity 'x_1' is multiplied by $(1 - w)$. In the context of seasonal sales management, the parameter 'w' plays a significant role. It represents the discount rate applied to the goods the retailer decides to purchase for the period T_1. Since 'w' is expressed as a percentage, it is essential to understand the range of possible values. This range reflects the extent to which the retailer can apply discounts to goods purchased in t_1. In other words, 'w' represents the percentage reduction from the selling price of the goods.

$$w \in \{0,1\}$$

Equation 7.26

This range of values allows the retailer to adjust the degree of discount according to his sales strategies and market conditions, offering greater flexibility in managing prices during the selling season. This discount rate, however, is not static but varies according to how much time has elapsed since the start of the sales season t_0. In other words, the value of 'w' decreases as the end of the reorder interval approaches, which is set at eight weeks after the start of the season. The variation of 'w' is a measure that takes into account the importance of the time at which the retailer decides to re-order goods. A value of 'w' of 0 could represent the absence of discounts, while a value of 1 would indicate the application of a maximum discount. In essence, this dynamic implies that the retailer can obtain more favourable conditions in terms of discounts if he can reorder goods within a certain time period. This incentive to place timely orders can significantly influence supply management and cost optimisation during the selling season.

The variable 'F', as mentioned above, represents the initial fee that the retailer pays to the producer at the beginning of the sales season. This fee is a guarantee for the retailer to have a specific quantity of products, called 'M', available for use at a later date. The value of 'F' is considered fixed.

Finally, the last addend refers to the storage costs for the two specific periods, T_0 and T_1. The coefficient 'L'[18] represents the percentage of storage costs associated with the value of the selling price set by the producer. This

[18] Detailed analyses to determine the exact calculation will not follow as it is not the purpose of this research.

percentage, multiplied by the producer's selling price and the total quantity of ordered products, determines the overall amount of storage costs incurred by the retailer. The calculation has been done considering the entire ordered quantity, including the second order. This calculation is based on two fundamental assumptions:

- The first is that the storage cost is calculated at the beginning of the period, i.e., before any product from the just-delivered order has been sold, reason why it is calculated on the basis of the order quantity and not on the inventories;
- The second assumption is that the retailer has a sufficiently large warehouse to accommodate the entire quantity of products ordered for the current selling season.

7.4.1.3. Profit function

Taking into account the considerations outlined above, it is straightforward to derive the fundamental profit function for the retailer. This function represents the primary objective of the retailer's business activity: maximizing its profit. To achieve this, the retailer deducts the costs it must incur from the revenues generated over a 20-week period of business.

In this way, the profit function takes into account all financial inflows and outflows during the considered period. In other words, this function reflects the core objective of the retailer, which is to ensure that revenues exceed costs to achieve optimal profitability. To reach this goal, the retailer must carefully consider all financial components, not just sales revenues but also all costs associated with running the sales season.

The profit function is a crucial tool for assessing the retailer's financial performance during the specified period and for making informed business decisions to maximize its objective function. This evaluation process is vital for the long-term success of the retailer's business in the market.

In light of this, the expression indicating the objective function of the retailer is as follows.

$$
\begin{aligned}
Maximise\ \pi_R &= Total\ revenues_R - Total\ costs_R \\
&= p_R(x_0 + x_1\mu - R) + \partial s_n\varepsilon + p_R(1-h)[k_1 + (1-\partial)s_n] + \frac{F}{M}x_1\mu \\
&\quad - [x_0 p_P + x_1\mu(1-w)p_P + F + Lp_P(x_o + \mu x_1)]
\end{aligned}
$$

Equation 7.27

7.4.2. Producer side

Now it is time for the calculation of the objective function, similar to what was done previously for the retailer, but this time referring to the producer. This equation allows one to assess how much one has actually gained or lost from the business activity. Previously, it has been discussed which factors influence the retailer's profit function, to assess which actions are best for the retailer to implement in order to maximise its objective function. Now, it is stated that the same approach is to be applied to the producer, whose main objective is also to maximise his profit.

In this profit calculation process for the producer, all financial inputs and outputs are taken into account, including operating costs, disposal costs, revenues from sales to the retailer, and any other relevant financial components mentioned in the previous two paragraphs. The ultimate goal is to maximise the producer's profit in the same way as the retailer seeks to maximise his own profit in their business.

7.4.2.1. Revenues

Looking at the factors that can impact the revenues of the producer's period, it is important to consider three key elements. First and foremost, it needs to be taken into account the revenue generated from the sale of goods to the retailer in both periods, namely T_0 and T_1. Furthermore, it is essential to consider how much is received in advance in the form of a deposit guarantee for the merchandise 'M' intended for the T_1 period. This deposit guarantee represents a significant component of the producer's revenues and should be taken into account when evaluating the overall financial performance of the producer in this context. The final factor to take into consideration pertains to the revenues generated from the disposal, at the end of the season, of the unsold merchandise accumulated throughout the course of the same period.

$$Total\ revenues_P = sales\ in\ T_0 + sales\ in\ T_1 + Fee + disposal\ cost^{19}$$
$$= [x_0 + x_1\mu(1-w)]p_P + F + U(M - x_1 + \partial s_n)$$

Equation 7.28

The first two terms refer to sales made in two distinct periods during the selling season. The term 'p_P' represents the price at which the producer sells their products to the retailer, while 'x_T' denotes the quantity of products sold, in line with the market demand forecasted by the retailer for that period. Furthermore, the variable 'w' is introduced, signifying the discount rate applied to the second order, its value being contingent upon the week in which the reorder is executed. These two combined terms represent the profit generated from the primary sales.

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[19] Although the disposal of unsold merchandise can pose a challenge and, in many cases, incur consequential costs for the producer, in this instance, it presents an opportunity for profit. This opportunity arises from the possibility to reintroduce the same products to the market through alternative channels at a lower price.

The third term pertains to the profits obtained through the payment of an advance, referred to as 'F', made by the retailer to secure an inventory of products up to a maximum limit of 'M' units, intended for sale in period T_1. This agreement allows the retailer to avoid holding the entire inventory in stock for the entire 20-week duration. Furthermore, it provides the retailer with the opportunity to conduct further analyses on demand forecasting, as it is closer to the moment when actual demand occurs, enabling him to purchase a quantity of products in line with the actual demand.

The third component of the formula focuses on analysing the revenues associated with the disposal of products that remained unsold during the specific period. These disposal costs not only concern the products that remain unsold in the producer's warehouses, but they also encompass other scenarios. First at all, such as when the retailer, at the beginning of the period T_0 decided to reserve a certain quantity of products but did not purchase them subsequently for period T_1. These products, despite being initially requested, have become unsold. Furthermore, this component of the formula takes into account the entire quantity of merchandise purchased by the retailer in period T_1, for which the producer has committed to offering the option of a buyback, so in which he agrees to reacquire the unsold merchandise from the retailer at the end of the period. To quantify these disposal costs, the formula utilizes a variable denoted as 'U', representing the unitary revenue for the producer associated with the disposal of the unsold product units at the end of the selling season. The primary objective of this component of the formula is to ensure efficient management of unsold products at the end of the considered period. This means minimizing the disposal costs associated with these unsold products seeking to maximize the recovery or redistribution of such products to optimize the overall management of unsold merchandise.

7.4.2.2. Costs

In terms of costs, it's important to consider the following aspects:

- Production and storage costs for the period under consideration, which include all expenses related to manufacturing the products;
- The reimbursement of a portion of the advance payment made by the retailer in t_0 to secure a stock of a certain number of products, denoted as 'M'.

To streamline the development of the new model, it has been chosen to treat the production costs and storage costs incurred by the producer as variable data according to the units of product ordered, not necessarily purchased and constant throughout the entire period being considered. Consistent with the approach used for the retailer portion, one will refrain from conducting a more detailed analysis on the calculation of these costs, as they are not the primary focus of this study.

$$\begin{aligned} Total\ costs_P &= fixed\ costs + storage\ costs + share\ of\ F\ as\ reimbursement \\ &= C(\,x_0 + max\{M,\, x_1\}) + \frac{Fx_1}{M} \end{aligned}$$

Equation 7.29

A more in-depth analysis of the costs borne by the producer allows to draw some relevant conclusions. The first component, denoted as 'C', represents the set of unitary costs related to:

- The design, production, and finishing of products in preparation for the upcoming selling season;
- The unity storage costs incurred by the producer.

This variable multiplies the maximum value between the quantity reserved by the retailer in the initial period and the quantity actually purchased, 'x_1'. This observation will be significant in setting up the simulation in the following chapter, as it will be crucial for the development of potential scenarios in which the model could be applied.

The last term reflects the share of reimbursement of a portion of the deposit to the retailer at the end of the season following the reorder. However, because this cash outflow is calculated based on an advance previously paid by the retailer and not directly from the resources of the producer, it would be more accurate to define this term in the formula not as an actual 'cost' but rather as a missed opportunity for gain. Reflecting on this aspect, a high value of $\frac{Fx_1}{M}$ shouldn't necessarily be viewed negatively, as it implies a greater sale of 'M' type products.

7.4.2.3. Profit function

As discussed above, it is quite clear to deduce that the producer's fundamental objective is to maximise his profit, which is the beating heart of the business. To achieve this goal, the producer takes all financial components into account by subtracting the costs it has to incur from the revenues generated during the 20-week business period. In other words, the profit function takes into account every significant financial input and output during the reporting period. Elements influencing profit include revenues from the sale of goods to the retailer, operating and disposal costs, and warehousing guarantees for future goods. This function is a crucial indicator that helps the producer assess the financial success of his business and make strategic decisions to maximise overall earnings.

In the mathematical expression below, various terms describing the different components of the producer's objective function are included.

$$Maximise\ \pi_P = Total\ revenues_P - Total\ costs_P$$
$$= [x_0 + x_1\mu(1-w)]p_P + F + U(M - x_1 + \partial s_n)$$
$$- C(x_0 + max\{M, x_1\}) - \frac{Fx_1}{M}$$

Equation 7.30

7.5. Game theory application

At the end of the complex process of analysis and development of the proposed new model, and after providing a detailed exposition of the so-called Newsvendor model, widely discussed in the specialized literature and regularly used in practice as a coordination tool between the producer and the retailer, arises the opportunity to exploit the principles of game theory for an in-depth comparison between these two models.

Game theory offers an analytical tool for exploring the dynamics and interactions between the two members of the supply chain, the producer and the retailer, in greater depth, enabling a more accurate assessment of the choices and strategies available in this context. In this way, it aims to gain a more comprehensive view of the potential implications and advantages of using one model over the other.

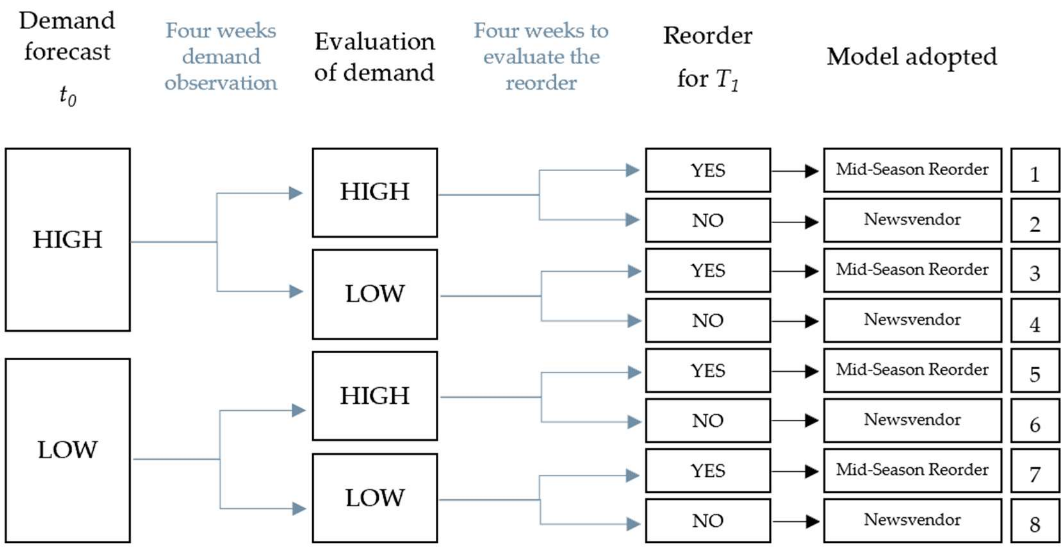

Table 7.2: Game theoretic scenarios for the Mid-Season Reorder model

The aforementioned graphical model serves as a representation wherein the retailer wields decision-making authority regarding the choice of contract to be employed for the sales season. The variables encompassed within the initial phase comprise categories denoting High Market Demand and Low Market Demand. These categories are indicative of the product types that the producer has developed and those foreseen and identified by the retailer in t_0.

Subsequent to the commencement of the sales season, the retailer leverages the initial four weeks to assess the true trajectory of items by closely monitoring sales data. During the selling season, due to unforeseen casual factors events occurring in the four weeks following the start of the selling season, the forecast may turn out to be wrong and may be an increase or decrease in market demand. This approach enables the retailer to make a more informed determination as to whether to initiate a new order by the conclusion of the eight-week period, denoting the replenishment of products from the producer. The model employed evolves in accordance with the decision taken in this regard.

In essence, this representation prospects the time when the producer and the retailer must make critical decisions regarding supply and product management. The producer must decide how many products in each category (High Market Demand or Low Market Demand) to produce and make available to the retailer. On the other hand, the retailer must decide whether to place a new order with the producer to supply additional products or to do without. These decisions directly affect inventory management, customer satisfaction, and the financial performance of both the producer and retailer, making this representation a key tool for analysing and optimizing decision making in this supply and demand dynamic.

In particular, if the retailer decides not to place a new order, it will automatically follow the Newsvendor model. As a result, the newly developed model would not be involved in the profit calculation. However, in the event that the retailer chooses to take advantage of the opportunity to place a second order, profits would be calculated according to the procedure outlined by the new contract model, as described in detail above. It is important to consider the impact of this choice in relation to the type of product in question. If the product results of high demand after the four weeks of observation and the retailer opts for a second order, this decision will have a very positive impact on its objective function, especially because the order it will place is based on a more informed and accurate decision, enabling the retailer to understand the right quantity to order. This allows the retailer to save on stocking costs and to better align with market demand, avoiding situations of out-of-stock or overstocking. On the other hand, if the product is recognized as a high demand item but the retailer does not place a second order, this configuration directly resembles current contract practice, i.e. the use of the Newsvendor model. However, such a choice implies a significant financial commitment at the beginning of the season and the need to have a big enough storage space for inventory. In this scenario, it is imperative to discern whether the initial forecast at t_0 for the item in question indicates high or low demand. If the item has been classified as low demand, the decision to place another order depends on the context under consideration: if a sufficient quantity of products was initially purchased to cover demand for the entire season, the decision to reorder may lead to overstock situations; conversely, if a quantity was initially placed with a view to a future reorder, it is necessary to place it in order to avoid stockout situations that do not allow for sufficient products to cover demand for the entire sales period. This choice directly affects the cost and profit management of the retailer and the producer, so following the newly proposed model will remain more advantageous for the retailer.

In the case of products recognized as low demand items, it would not be convenient for the retailer to place a second order and follow the proposed new model. This is because this could lead to overstocking, as the quantity ordered would exceed the market demand for those specific products. In this situation, the Newsvendor model is the best choice, thus excluding the possibility of a second order. In particular, the remote option $high\ demand - low\ demand - yes\ reorder$ concerns products that were originally clustered as having high market demand, which led to the decision to mistakenly adopt the new contract model and make a reorder. As a result, due to the wrong initial forecast, there incurred a build-up of products in the producer's warehouses, even though the retailer did not actually need them, which are now in danger of not being sold. In addition, the retailer, despite the decline in demand, is still on the receiving end of these products that cannot be easily placed on the market. In this situation, choosing to place a new order, despite the decrease in demand, can lead to overproduction and overstocking problems. This highlights the importance of carefully considering the accuracy of demand forecasts and having flexibility in reordering decisions in response to changes in market conditions.

It is imperative to make a distinction if a product that was recognized to have low demand does not receive a second order. The implementation of the Newsvendor model would be a wise choice in an ideal scenario where the product was first categorized as having low demand. On the other hand, if the product was initially deemed to be in high demand, delaying a second order could lessen the chance of excessive hoarding towards the end of the season, which would make it easier to get rid of inventory at the conclusion of the T_0 period.

The outputs of the representation correspond, respectively, to the producer's and retailer's profits in each of the eight possible scenarios that are considered. The objective behind this analysis is to determine which combination of decisions leads to maximised profits for both parties involved. These profits are the result of choices made by both the producer and the retailer in the context of supply management. The matrix helps to visualise and systematically calculate how these decisions affect the financial results of both parties, helping to identify the optimal strategy.

As will later be confirmed through targeted simulations involving a sample of 100 items with different demand requirements, the two Pareto-optimal solutions are expected to be the $high\ demand - high\ demand - yes\ reorder$ and $low\ demand - low\ demand - no\ reorder$ combinations. This prediction is based on the consideration that products characterised by low demand, as outlined above, do not require the need for a second replenishment, as the amount of units required to the retailer is not high, making the Newsvendor model sufficient and efficient for handling this product category. In other words, the analysis predicts that the two optimal solutions, i.e. those that maximise profits for both parties involved, are represented by two specific scenarios.

On the other hand, for products with a high market demand, it is observed that the use of the newly developed model is more advantageous, both for the producer and the retailer, as illustrated above. This is because the contract model offers a number of advantages that can help to improve supply management and increase profitability, such as greater accuracy in aligning supply with demand, reduced warehousing costs and better overall stock management.

In addition, the use of the new model could allow the producer and retailer to gain competitive advantages over other competitors in the market. For example, they might be able to serve customers better, avoid out-of-stock situations, improve delivery times and optimise overall costs. This once again underlines how the choice between the Newsvendor model and the new model should be guided by the nature of demand and the specific characteristics of the product.

7.6. Model comparison

The Newsvendor model, although being a powerful tool for inventory management, has some inherent limitations (DeMarle, 2021). These limitations stem from the fact that the model assumes certain rigid conditions that may not always be realistic or suitable for a dynamic and changing business environment. Some of the main limitations include:

– Orders at the beginning of the season: the Newsvendor model assumes that all orders must be placed at the beginning of the selling season. This static decision-making process don't take into account unforeseen fluctuations in demand or market opportunities that may arise during the season (DeMarle, 2021);

- Irrevocable orders: the model assumes that orders cannot be changed or adjusted once placed. However, in business reality, situations may arise where adjustments need to be made to orders to better align with changes in demand, changes in delivery times or other unforeseen circumstances.

To address these limitations and provide more flexibility in inventory optimization, a new model can be developed. This new model could be called "Mid-Season Reorder model" and should take into account several considerations:

- Forecast updating: in this new model, retailers have the opportunity to update demand forecasts based on the most recent information, reducing uncertainty in future demand. This allows for more informed decision-making;
- Second reorder: the model allows retailers to place a second order during the season based on market dynamics. This means they don't have to commit to large orders upfront but can gradually adapt to demand fluctuations;
- Reorder costs: costs associated with mid-season reordering are included in the overall calculation. This takes into account transaction costs, contract penalties, or other administrative expenses related to subsequent reorders;
- Waste and obsolescence reduction: this model offers greater control over excess inventory and obsolete items. Parties can reduce losses from the need to dispose of unsold or expired products at the end of the season. This is particularly relevant for industries where products quickly become obsolete, such as luxury;
- Adaptability to demand variability: the new model allows producers and retailers to better adapt to unexpected demand fluctuations. In a world

where volatility has become the norm, the ability to reorder mid-season enables both the actors to manage variability more efficiently. This can reduce the risk of stockouts when demand exceeds expectations and minimize waste when demand is lower than anticipated;

- Better utilization of financial resources: with the traditional Newsvendor model, retailers often have to commit significant financial resources upfront for large orders. However, in many cases, these resources could be put to more advantageous use in other areas of the business. The new model allows for a more efficient distribution of inventory and reorder-related costs, improving corporate liquidity;
- Improved customer satisfaction: the flexibility offered by the new model can result in higher customer satisfaction. Retailers can respond more quickly to end customer requests and ensure they have the right products available when needed. The same for the producer from the retailer's point of view. This can lead to greater customer loyalty and cross-selling opportunities;
- Market competitiveness: adopting the new model can provide a competitive advantage to retailers willing to innovate and adapt in an ever-dynamic market. The ability to respond promptly to changing conditions can enable them to gain a larger share of the market or maintain a leadership position.

In summary, the introduction of a new, more flexible inventory management model represents a significant step forward for the agents of the supply chain. This model addresses some of the limitations of traditional approaches, such as the Newsvendor model, and offers several advantages for modern supply chain management. In today's fast-paced business environment, where consumer preferences, demand patterns, and external factors can shift rapidly, the flexibility of the Mid-Season Reorder model is crucial. However, it's important to acknowledge that implementing this new model comes with some challenges. Decision-making becomes more complex, and businesses need to develop more sophisticated inventory management and forecasting systems to make mid-season adjustments effectively. This entails investing in technology and data analysis capabilities.

The choice between the two models, whether traditional or flexible, should be based on a careful evaluation of a business's specific needs and the market context. Businesses should consider the costs and benefits associated with each approach, recognizing that the Mid-Season Reorder model offers agility and adaptability but requires a more advanced infrastructure to support it.

In conclusion, the Mid-Season Reorder model is a valuable evolution in inventory management, aligning better with the demands of today's dynamic business environment. It empowers retailers and producers to optimize their resources, reduce waste, and maintain better control over their inventories, ultimately leading to improved performance and customer satisfaction.

7.7. Conclusions

Concluding this chapter, it becomes abundantly clear that there is a compelling need for the introduction of a new contractual model that can adeptly address the complex challenges associated with modern supply chain management. This is particularly essential in contexts marked by pronounced market demand peaks, where traditional approaches may fall short. While the traditional Newsvendor model has been a valuable tool in inventory management across various sectors, its limitations become evident when applied to products subject to highly variable demand, especially those found in the luxury sector, whose managing is multifaceted. Consumer preferences for luxury items can change rapidly, and the market for such products is sensitive to external factors like economic conditions, fashion trends, and even social influences. Consequently, predicting demand for luxury goods is a hard task. The traditional Newsvendor model, designed for situations with more stable and predictable demand, may struggle to accommodate the volatility inherent to the luxury sector. This lack of flexibility can lead to significant missed opportunities and challenges for businesses, especially when dealing with products subject to highly variable demand patterns. Furthermore, luxury products are often associated with premium pricing and brand image. Having excess inventory or facing stockouts can be particularly damaging in this context. Overstocking luxury products can lead to deep discounts and brand dilution, while stockouts can result in missed sales opportunities and dissatisfied customers who expect exclusivity and immediate availability. The Mid-Season Reorder model introduced in this chapter offers a promising solution to these challenges. By allowing almost-mid-season adjustments and a more flexible approach to inventory management, it empowers luxury brands to respond more effectively to the capricious nature of their market. This flexibility enables them to align their supply with actual demand, reducing the risk of stockouts and excess inventory. While the traditional Newsvendor model remains valuable in many contexts, recognizing its limitations and adopting innovative models like the Mid-Season Reorder model can be a strategic advantage for businesses operating in dynamic and unpredictable markets. It enables them to uphold their brand reputation and seize opportunities in an industry where exclusivity and timeliness are of utmost importance.

Thus, a contractual model that allows reordering introduces a range of significant advantages for the parties of the supply chain, especially those operating in dynamic and unpredictable markets. Firstly, it substantially improves customer satisfaction by ensuring a constant availability of products, even in the presence of unexpected demand variations. This level of reliability and availability is crucial in retaining and satisfying customers. It reduces the chances of customers experiencing stockouts and, consequently, prevents their dissatisfaction. Additionally, it substantially contributes to optimizing business revenues by avoiding waste due to unnecessary inventory or missed sales due to insufficient inventory. In traditional models, businesses may overstock to ensure they don't run out of products, but this ties up capital. Therefore, the advantages of a contractual model that allows reordering extend beyond mere flexibility. They encompass improved customer satisfaction, revenue optimization, and a stronger competitive edge. Such models empower businesses to navigate the complexities of dynamic markets more effectively, ensuring they are well-equipped to meet changing customer demands and adapt to unexpected challenges.

Consequently, it is imperative to consider the introduction of a new contractual model that permits dynamic reordering. This new model should enable the adjustment of product quantities available based on the actual market demand, offering greater flexibility and adaptability to the operating context. This transition towards a more flexible approach is essential to ensure that business operations can adequately respond to market demands and maintain their competitiveness in an ever-evolving commercial landscape. The research and adoption of these innovations represent a fundamental step toward the long-term success of businesses and the optimization of supply chain management strategies.

However, it should be noted that the introduction of a contractual model that allows for dynamic reordering may involve some additional challenges and complexities in supply chain management, so it is also crucial to understand how to mitigate potential challenges that this transition may entail. First at all the primary obstacles is the need for effective communication and collaboration with producers, but also the data analysis capabilities and organizational flexibility to rapidly adapt to changes in market conditions. Finally, it should be noted that not all products or industries are suitable for the implementation of a contractual model that allows for dynamic reordering. Its effectiveness largely depends on demand variability and supply chain complexity. The introduction of a contractual model that allows for dynamic reordering represents a concrete response to the inherent challenges of supply chain management, especially in cases where market demand exhibits high variability. So, it is important to carefully evaluate in which contexts this model can be successfully applied.

Summarizing, the rigidity of the Newsvendor model, with its reliance on static forecasts and initial decisions, can hinder businesses from fully capitalizing on dynamic market conditions (DeMarle, 2021). It underscores the need for more flexible inventory management models, particularly in industries where demand is highly variable. The ability to adjust inventory levels in response to real-time data can be a significant advantage, enabling businesses to better meet market needs, minimize missed opportunities, and enhance overall performance. Hence, the introduction of a contractual model that allows for dynamic reordering can be a significant step towards more efficient and adaptable supply chain management in response to evolving market needs.

8. Simulation

8.1. Introduction

In the previous chapter, the development of a new contract model was examined in detail, and conducted an in-depth analysis of the so-called Newsvendor model. It turns out that the Newsvendor model is very efficient when it comes to managing products with easily predictable and constant demand, but in a dynamic and fluctuating reality such as the luxury industry, this model is often ineffective, so it would be better to use more flexible models, which enable more aware and informed decision-making. This inefficiency is intensified when dealing with products with highly varying demand forecasts: in fact, when it comes to handling products with a very uniform demand among them, especially if this demand is at a relatively low level, the Newsvendor model is extremely effective. However, when dealing with a number of products with a considerable variation in market demand, especially if this variation is significant, this model becomes drastically inadequate. In other words, the Newsvendor model works well in situations where the market demand for products is constant and predictable, otherwise, it loses its effectiveness.

The above observations highlight how crucial it is to adopt an approach that is open to new and alternative strategies to address the challenges of managing product demand in scenarios that deviate from ideal conditions. The Mid-Season Reorder model may represent an alternative approach to this problem. It offers a perspective to manage in a much more flexible and time-distributed manner, the demand for products subject to considerable market fluctuations and with much more complex demand forecasting, often characterized by seasonal peaks or unpredictable fluctuations. In other words, this model enables challenges beyond simply forecasting demand, as it also enables supply chain agents to adapt quickly to market changes and make smarter decisions in real-time. It is a step forward in supply chain management as it reflects the complexity of the real world.

The introduction of the Mid-Season Reorder model is important for the evolution of supply chain management. This approach is key to strengthening a company's ability to profitably manage a wide range of products and adapt to changing market conditions. Supply chain management challenges are no longer limited to constant and predictable demand. In this mutual context, the Mid-Season Reorder model emerges as a solution that not only improves the efficiency of operations but also responds more comprehensively and effectively to customer needs. This model stands as an example of the constant search for innovative and flexible solutions to meet the increasingly complex challenges in the field of supply chain management and product demand. In a world where flexibility is essential for business success, the Mid-Season Reorder model represents a necessary evolution in optimizing operations and meeting customer needs by proactively addressing market variability and complexity.

The discussion between the two models, the Newsvendor model and the Mid-Season Reorder model, has been subject to a rigorous evaluation process through the application of game theory, in which the interactions between the models have been tested in a competitive context. This approach allowed for an examination, at first essentially theoretical and then practical, of the dynamics involved in choosing between the two models, with the aim of identifying situations in which one of the two models may prove more advantageous than the other. The analysis focused on assessing the strategies and decision-making dynamics implicated in the use of each model. A number of parameters, variables and conditions that may influence the optimal choice between the models have been considered. These may include the nature of demand, the variability of the market, the availability of resources, the costs associated with each model and the individual's preferences in terms of supply chain management. In summary, the application of game theory has allowed for a thorough and rational analysis of situations in which the Newsvendor or Mid-Season Reorder model may be more advantageous, enabling individual businesses to make informed decisions based on a detailed understanding of the dynamics involved.

The process of comparing the Newsvendor model and the Mid-Season Reorder model was crucial to assessing the capabilities and limitations of each approach. Through this analysis, it was possible to identify strengths, i.e. areas in which each model excels, and weaknesses, i.e. scenarios in which they may be less effective. A crucial aspect of this analysis was to determine in which specific scenarios or application contexts each model is best suited. The fundamental objective of this analysis process is to provide a solid and rational basis for business decisions. Businesses can now make an informed choice between the two models according to their specific requirements and the characteristics of the context in which they will be applied. This approach maximises operational efficiency and adapts supply chain management to real market challenges, ensuring an agile response to changing conditions in the business environment. The application of game theory has enriched the decision-making process by providing an analytical framework and tools to better understand the dynamics of decisions, predict outcomes and identify optimal choices, thus making it possible to make a wiser choice between the two models under consideration.

Therefore, in order to subject the theoretical assertions concerning demand management strategies advanced above to rigorous practical and empirical examination, a sophisticated simulation experiment is conducted in a context involving a large sample of 100 diversified high-end products, using Microsoft Excel software. The simulation is structured in such a way to explore and assess in depth the market demand, the reordering strategies adopted and the financial results associated with them.

A simulation experiment is advantageous as it allows to explore how different demand management strategies might affect business results. This is particularly useful when an actor in the supply chain wants to test different theoretical hypotheses in a controlled environment before making decisions in a real environment. This reduces risk, avoids costly errors and provides a better understanding of the consequences of own decisions. The simulation also allows one to control and manipulate specific variables to study their effects, such as market demand, reorder interval, discount rate and buyback rate. By modifying these variables within the Excel software, it is possible to examine how they influence decision-making and business results. The fact that it offers numerical proof of the hypotheses and strategies tested makes it possible to derive concrete and objective data that support or refute the theoretical claims made previously. Finally, its scalability and repeatability make it easy to explore further scenarios or evaluate the effectiveness of strategies over time. This allows for a more comprehensive analysis and broader application of conclusions.

In the first stage of the simulation experiment, the demand levels for each of the 100 items included in the simulation are randomly generated by the software. This stage is critical in a simulation experiment, as it forms the basis for the model input variables and is the starting point for subsequent development and testing. This casual generation is conducted on the basis of statistical parameters, including mean and standard deviation. These parameters indicate the desired statistical behaviour for product demand under examination. In particular, an average of 100 products is used, indicating that the demand should be at that level. Simultaneously, the standard deviation, which denotes the dispersion of data around this mean, is set at 10 products. This can be a realistic representation of demand situations in many business contexts. It is important to note that the casual generation process is carefully designed to ensure that demand can never take on negative values. This is critical because negative demand would not make sense in a business context and could lead to problems in the simulation. Random generation allows for a wide range of demand scenarios with different levels, both high and low. This variety is important because it allows one to examine how demand management strategies behave in different demand situations. Each randomly generated demand scenario is assigned its associated probability of occurrence. This is useful for assessing the uncertainty in the simulation results and for weighing the importance of different scenarios according to their probability of occurrence.

The analysis was undertaken from three unique perspectives in order to empirically validate the correctness of the model outlined in the preceding chapter. This method allowed for the collection of a wide range of data, which served as a solid basis for the formulation of comprehensive conclusions on the dynamics of the model and its relative effectiveness. The three suggested analytical vantage points are as follows:

- A preliminary comparative analysis of the financial performance was conducted between the proposed new model and two other feasible policies, in order to draw a broad picture of its cost-effectiveness. The policies considered are:
  - Newsvendor model
  - Mid-Season Reorder model
  - Mid-Season Reorder model with the introduction of a second mid-season demand forecast

  The distinction between the two scenarios concerning the application of the Mid-Season Reorder lies in the fact that, in one of the cases, the decision to reorder is made in advance at the beginning of the selling season, irrespective of the actual demand trend in the second period. In the second case, on the other hand, the reordering decision is the result of a careful examination of the market situation in the first weeks.

- The next critical analysis conducted relates to the previous game theory case study. In this phase, the focus is on examining real-life situations with which a retailer might deal, with the aim of identifying the best practices to be adopted and assessing the associated financial impacts.
- The last stage of analysis outlines a more in-depth and specific examination of the proposed new model, focusing on its effectiveness in relation to increasing the disparity in demand between high and low demand products. Once again, in order to enrich the evaluation, a second demand forecast was integrated during the initial eight weeks of the selling season.

The algorithm at the heart of this simulation takes inspiration from game theory, which means that it is designed to represent and evaluate the decisions of the various actors involved in the supply chain according to principles of rationality and strategy. The simulation focuses on the analysis of different supply chain management strategies, which may include procurement policies, storage policies, production policies, and more. The algorithm used assesses the effectiveness of each strategy based on various parameters and variables, including demand, inventory, costs, and revenue.

The 100-year simulation period is significant because it allows one to observe and evaluate financial performance over the long term. This approach makes it possible to assess the effect of supply chain management strategies over time and see how dynamics change and evolve. This is particularly important in business scenarios where long-term decisions can have a significant impact on profitability. The main objective of simulation is to observe and compare financial performance. This may include measuring revenues, costs and profit for both the individual actors involved in the supply chain and the entire supply chain. Analysing this financial performance helps evaluate the effectiveness of supply chain management strategies and identify best practices.

A crucial element of the simulation is the creation of different scenarios based on the desired spread percentages. The concept of the spread is related to the variability in demand between products, namely the difference in sales demand within a given product range. This concept is decisive because it emphasises the heterogeneity of demand, which can differ considerably across products. For example, some products might have a constant, low demand, while others might have a seasonal or highly variable demand. The variability of the spread makes it possible to explore the dynamics of demand management strategies under different scenarios, taking into account the specific requirements of the market and products involved. This is critical for many businesses, as it requires differentiated storage, procurement and production strategies. Demand management strategies must take these factors into account to maximise profitability and meet market demand efficiently. Simulation allows testing how different strategies adapt to these needs. For example, in a situation with a high spread, more sophisticated storage strategies may be required to manage products with different demands. This provides a more realistic view of the challenges and opportunities that companies may face in managing a portfolio of products with different demand requirements.

In conclusion, simulation conducted through the use of Microsoft Excel software represents an advanced analysis methodology that makes a significant contribution to the understanding and optimization of demand management strategies. The systematic approach based on game theory allows the financial performance of different strategies to be evaluated in realistic contexts, enabling wiser and more informed business decisions. Simulation is an important tool for addressing changing market challenges and ensuring more effective and profitable supply chain management.

8.2. Assumptions and simulation description

When crafting a simulation, the objective is to portray a complex situation in a controlled and more manageable environment. Before going into the technical details of the simulation, it is necessary to establish a number of basic assumptions or presuppositions, which constitute the fundamentals upon which the simulation will be constructed. This part refers to the technical and mathematical details that constitute the simulation. The correctness of the assumptions is crucial because they will greatly influence the simulation results: if they are incorrect or do not accurately represent the real-world situation, the simulation results will be unreliable. Therefore, before proceeding with the formulas and calculations, it is essential to carefully evaluate and, if necessary, question the basic assumptions to ensure that they adequately reflect reality and the problem that one is trying to solve.

The simulation considers a time interval of 100 years. A simulation covering a 100-year period makes it possible to assess the evolution and impact of choices and events over the long term. This is particularly important when dealing with complex phenomena or decisions that may have long-term consequences. In many situations, decisions made at a given time may have little or no impact in the short term but only become significant in the long term. Short-term events or trends may be overridden by random fluctuations or short-term behaviour. In addition, there may be cumulative phenomena of decisions that require a longer period of time to manifest their full impact. Therefore, using a shorter time interval could have led to results that do not adequately reflect the impact of long-term choices. Therefore, a 100-year interval allows these dynamics to be captured and provides a more complete perspective on the effects of decisions.

Product demand follows a normal distribution. The mean of the distribution is given as 100, which suggests that on average 100 products are expected to be sold. The standard deviation of 10 provides a measure of the variability of the data in the normal distribution. In this case, a standard deviation of 10 suggests that the variability around the mean is relatively low, which indicates that demand is expected to be fairly consistent around the mean. Using a normal distribution with a mean of 100 and a standard deviation of 10 provides a forecast range that is considered feasible and plausible for demand for these products. This is important for inventory planning and management strategies because it helps identify which demand levels are most likely and realistic.

In addition, choosing a normal distribution allows one to develop different demand trend scenarios. For example, it is possible to use this distribution to calculate the probability of selling a specific number of products in a given period. This allows retailers to examine hypothetical scenarios, such as "What is the probability of selling at least 110 products per month?" or "What is the probability of selling less than 90 products per month?" These scenarios can be useful for planning and resource management, and for making informed decisions about variability in demand.

The sample consists of 100 different items of high-end jewellery. This diversification is important because it can include a variety of jewellery types such as rings, necklaces, wristbands, earrings, etc. This diversity in the sample can be useful to better portray the product portfolio of a high luxury jewellery retailer. The reference is to a retailer operating in a high-end jewellery context. This suggests that the sample is selected and researched in relation to a specific market, and thus may be of interest to an individual working in the field. This type of jewellery is known to be associated with precious materials, sophisticated design and often with a high-end shopping experience. However, it is important to emphasise that the luxury referred to is not too extreme but accessible to part of the population. This indicates that the market segment considered could include consumers with some purchasing power who wish to buy luxury products without reaching the exorbitant figures often associated with high luxury goods. Indicatively, the sales price to the end customer referred to is around 1200 € for an item. The calculation of the sales price is made from a hypothetical producer price of 600 €. However, the sales price may fluctuate, which is why a standard deviation of 100 € is taken into account.

To guarantee the retailer a certain profit margin, the sales price is increased by multiplying it by a factor of 2 (which equals a 100% increase). This means that the actual sales price will be 2 times the base price of 600 €, i.e. 1200 €. The selling price to the end customer is a key aspect when it comes to luxury products. The value of 1200 € suggests that the items in the sample can be considered affordable for a wealthy customer segment, but not so exclusive as to exclude most people. This type of segmentation may be important to understand the target market and the positioning of the retailer. This description provides an insight into the context in which the analysis was carried out.

The initial order calculation for the entire season is a process that seeks to strike a balance between satisfying customer demand for the entire season and managing the risks associated with fluctuations in demand. The purchase process of the initial order is made at the beginning of the selling season and must be sufficient to cover the expected demand for the entire season, with the exception of the case of products with a high demand, because in such a case the quantity $'x_0'$ is given by the expected demand for the period T_0 only plus a number of standard deviations equal to the 10% of the inverse of the normal demand distribution of the critical ratio value. In all the other cases, the initial order is equal to the entire forecast for the selling season plus a number of standard deviations equal to the inverse of the normal demand distribution of the critical ratio value. This approach is intended to protect the retailer from potential situations where problems occur during the first few weeks that could prevent him from placing a reorder and subsequent out-of-stock problems that damage the company's reputation. The initial order quantity calculation is based on the demand forecast for the product in question. This forecast is an indication of how many products customers are expected to purchase during the entire selling season. The addition of a certain number of standard deviations to the demand forecasting is aimed at creating a margin of safety or buffer to handle unforeseen fluctuations in demand. In other words, it seeks to ensure that even under conditions of higher than forecast demand or unforeseen fluctuations, there are still products available to satisfy customers without interruption.

Classifying demand into high or low is an important step in inventory management and sales activity planning. To do this, it is necessary to define a threshold or cut-off point that allows decisions to be made based on the amount of demand. This makes it possible to focus sales and inventory management efforts on products with high demand and consider different strategies for those with low demand. The choice of threshold may vary depending on business needs and strategies. Some businesses may prefer a higher or lower threshold based on their specific circumstances and business objectives. For example, in high-risk or low-margin businesses, it may be preferable to adopt a higher threshold to provide greater certainty in labelling a specific product of “high demand". This threshold thus helps set a clear guideline for product classification, but it is important to customize that according to the company's specific requirements and market dynamics.

In this context, the simulation has been organized into several mirrors each of which refers to a set of data or specific assumptions which contain information relevant to the management of a given year's selling season. Each mirror represents a reference point for decision-making and planning. Each annual mirror is divided into three distinct parts, each indicating a specific time in the selling season. The division into three separate moments allows for more flexible and responsive management of sales season planning. The first part, in particular, represents the opening point where decisions must be made based on historical estimates and forecasts, since current data may not yet be available. Subsequent parts refer to later moments in the selling season, which allow planning to be adjusted and updated based on changing market conditions and actual data as they become available during the selling season. For example, changes could be made to forecasts and sales strategies mid-season or at major seasonal events such as holidays or promotions.

- Part one - t_0 (beginning of the selling season): this section is devoted to the variables and assumptions made at the beginning of the selling season (t_0). Since t_0 represents the beginning of the season, current information or actual sales data for the current year may not yet be available. Therefore, decisions at this time are based primarily on estimates and sales forecasts made based on experience and previous selling seasons.
- Part two - variables and assumptions regarding the first 8 weeks: this section of the annual mirror focuses on variables and assumptions regarding the interval of the first 8 weeks after the start of the selling season. As the selling season progresses, market dynamics tend to stabilise and become more reliable and predictable. Agents can observe how consumers respond to prices, gaining a more accurate understanding of what works best and how the market is evolving. This knowledge allows them to compare actual demand data with initial forecasts and consequently make more informed decisions. Decisions made during this period are better informed than decisions made at the beginning of the season and this is also reflected in

more targeted strategy adoption. This may include stock adjustments, changes in pricing and promotion strategies, or other actions based on actual data rather than just historical forecasts. Decisions made during this period are often more adapted to the actual conditions of the moment. This makes it possible to optimise strategies and resources to maximise sales opportunities.

- Last part – beginning of week 8: this section of the annual mirror covers the last part of the selling season, that is, the last weeks. During this period, the agent has already had the opportunity to observe demand trends, acquire real sales data and evaluate product performance. In this phase, therefore, decisions are influenced by the real information gathered during the selling season, which makes it possible to make more informed and targeted decisions. At this point, the agent must make a key decision: reorder or not to reorder. The retailer must decide whether to place further reorders of products to maintain stock or to stop buying new units. This decision is crucial to avoid overstocking or out-of-stocks at the end of the season. In the event of a reorder, the agent follows the new supply management model, the Mid-Season Reorder model. This includes a series of implications and advantages on stock optimisation, the use of more up-to-date forecasts, careful monitoring of residual demand and optimisation of reorder costs. Of course, the optimal version of the Mid-Season Reorder model is being considered here, which implies a careful analysis of the actual market demand in the first weeks of sales before making the reorder decision. Should this decision be taken earlier, i.e. at the beginning of the season, the retailer would not be able to take full advantage of the benefits of better alignment with actual market demand. Therefore, it is important to emphasise that the assumptions concerning the variables in this section should only be considered for the former model. Otherwise, the model followed is the current model, i.e. the Newsvendor model.

The mirror rows represent various information and decisions made during the selling season. From the producer's point of view, these rows may have little impact on its objective function. Decisions made by the retailer, on the other hand, in particular the choice to reorder or not to reorder, is crucial for the producer as it directly influences sales and product demand. The producer can influence the retailer's behaviour by offering discounts and incentives to encourage reordering. This practice is a common strategy to stimulate further purchases by the retailer. Discounts and incentives may include volume discounts, special promotions, favourable payment terms or other concessions linked to the time when the retailer decides to reorder. The producer has a direct interest in encouraging the retailer to place a second order. This is because, as mentioned, the initial fee paid by the retailer at the beginning of the season may not fully cover the production, storage and disposal costs for the additional units. Therefore, selling more units through a second order may help cover costs and generate additional profits for the producer. The producer wishes to maintain a positive relationship with the retailer, as this collaboration can be beneficial to both parties. The producer is motivated to meet the retailer's demands without incurring losses, and the retailer is incentivised to reorder when there are advantageous offers. This helps to ensure sales continuity and maximise the overall gain in the production and distribution ecosystem. In summary, the producer has a strategic role in managing the relationship with the retailer and encouraging re-ordering. By offering incentives and facilities for follow-up orders, the producer aims to maximise sales, cover costs and make profits, while maintaining a collaborative and profitable partnership with the retailer.

Continuing with the overview of the variables used in the simulation and the values assigned to them, it is worth mentioning the storage costs, the expenses a retailer incurs to maintain and store products in his warehouse or storage space during the selling season. These costs may include a number of aspects, such as rental payments for storage space, operating expenses associated with managing the warehouse, insurance costs to protect products from possible damage or loss, and maintenance costs to ensure that products are kept in proper condition. The storage cost coefficient (denoted as 'L') is a key parameter in their computation. It is the percentage of storage costs in relation to the total quantity of products ordered by the retailer over the 20-week sales season. In this specific case, the coefficient 'L' is set at 10%. This means that 10% of the total amount of the order placed by the retailer represents the storage costs he will have to bear. To concretely calculate the storage costs, the retailer takes into account the quantity of product ordered over the entire sales season, i.e. the number of units that will be stored. He then applies the coefficient 'L' to this total amount of the order and multiplied for the producer cost 'p_P'. In this way, he is able to determine the value of the storage costs associated with that specific order. Storage costs are a direct component of the total costs that the retailer will face during the selling season. These costs may vary depending on the quantity of products the retailer decides to order and stock. Reducing these costs can be an important strategy to improve overall profitability. This can be achieved either by reducing the amount of products kept in stock or by negotiating more advantageous conditions with the producer.

'ε' is the value agreed between the producer and the retailer for the return of unsold goods at the end of the selling season, i.e. it is the price at which the producer agrees to buyback part of the unsold goods from the retailer. This value is important because it determines how the retailer will handle any surplus of unsold goods at the end of the season. Its value coincides with the 80% of the price at which the producer originally sold the goods to the retailer at the beginning of the season. This arrangement may have important implications for the retailer. For example, if the retailer manages to sell all products during the season, it will not need to return unsold goods. However, if consumer demand is lower than expected and unsold products remain, the retailer has the option of returning those products to the producer and obtaining a refund equal to the 80% of the original purchase price. The possibility of returning unsold goods at a guaranteed price can be seen as a form of risk management for the retailer. This arrangement enables the retailer to reduce the risk of unwanted stockpiling and to avoid potential financial losses due to unsold products.

The fee (F) that the retailer pays to the producer at the beginning of the selling season is proportional to the selling price set by the producer and it is calculated as the product of that price times the quantity booked by a coefficient of 20%. This means that the higher the selling price of the products, the higher the amount of tax paid by the retailer. This direct link between price and charge is a logical basis, as it reflects the fact that the cost of keeping a quantity of products in stock should be proportional to their value. The 20% coefficient is chosen because it is a reasonable proportion for both parties involved. On the one hand, for the producer, this fee helps to cover the costs of keeping products in stock and to guarantee an initial income. On the other hand, for the retailer, it represents a manageable expense that allows access to a quantity of products to be sold without having to face the full purchase cost at the beginning of the season. This balanced fee system succeeds in positively engaging the interests of both the retailer and the producer. The producer can generate initial revenue through the fee and shares the risk of keeping products in stock with the retailer. At the same time, the retailer can gain access to the products without an excessive initial financial commitment. In summary, the fee calculation based on the economic value of the locked quantity and the 20% coefficient is designed to be a balanced system that reflects the interests and needs of both the parties.

The unit costs $'C'$ incurred by the producer related to the activities of designing, producing, finishing and storing products for the next selling season are also calculated in relation to the producer's selling price. In this case, however, the coefficient used is 60%, reflecting the significant increase in importance, effort and resources committed to these activities. Indeed, the production of luxury and high-value products requires high-quality materials, specialised labour, and careful and often complex finishing processes. Managing stocks of higher value products may also require more attention and resources. So, this higher coefficient reflects the complexity and importance of these activities in the production and distribution process of high-value products.

The unit cost of disposing of the goods $'U'$ at the producer's premises at the end of the season depends on various factors, including the value of the activities carried out during the production and handling of the products. The unit cost of disposal is directly related to the unit cost $'C'$. Specifically, the cost of disposal is equivalent to 70% of the production value of the products. This reflects the fact that the disposal of unsold goods can be a significant expense for the producer, particularly if the products are of high value. Disposal costs may include the physical removal of the products, their disposal or liquidation through specialised sales channels.

The factors $'\partial'$ and $'w'$ represent the benefits and discounts available to the retailer, if he decides to reorder during the first 8 weeks after the start of the selling season. These two factors influence reordering decisions and reflect the benefits of such decisions:

– $'\partial'$ (Percentage of product reclaimed by producer): this factor indicates the percentage of product $'s_n'$ for which the producer has the option to buyback at the end of the selling season. The amount of product reclaimed by the producer is influenced by the time at which the retailer decides to reorder. The values that $'\partial'$ can take are as follows:

- 0,30 if the reorder takes place in weeks 3 and 4;
- 0,25 if the reorder takes place in weeks 5 and 6;
- 0,20 if the reorder takes place in weeks 7 or 8.

In essence, the earlier the retailer reorders, the greater the amount of goods the producer can buyback at the end of the season. This can have a significant impact on inventory management and overall profitability, as it influences potential losses or profits from remaining stock.

– 'w' (Percentage reduction from selling price): this factor denotes the percentage reduction from the selling price applicable to products reordered at the commencement of period T_1. This numerical value is subsequently applied to the fee originally remitted by the retailer for maintaining a quantity 'M' of products in stock, constituting the proportion of said fee subject to reimbursement to the retailer. The values that w can assume are as follows:

- 0,30 if the reorder takes place in weeks 3 and 4;
- 0,20 if the reorder takes place in weeks 5 and 6;
- 0,10 if the reorder takes place in weeks 7 or 8.

In practice, the earlier the retailer decides to reorder, the higher the percentage of the initial charge that will be returned to him. This may influence his reordering decision, as it is a financial incentive for early reordering.

Follows a table explaining the various scenarios and the values that the two variables can assume.

WEEKS VARIABLES	3-4	5-6	7-8
∂	0,30	0,25	0,20
w	0,30	0,20	0,10

Table 8.1:Schedule of possible values of ∂ and w, according to the reorder week

The process for calculating the retailer's and producer's profit applies the formulas derived from the two models described previously, the Newsvendor model and the Mid-Season Reorder model. These expressions take into account the various parameters analysed in detail above. Based on these parameters, the formulas derive the profit for each agent. The formulations governing the profits of the two agents are entered meticulously into designated cells within the spreadsheets integrated into the simulation system. This procedural step facilitates the automated calculation of the agents' profits based on specific data and decisions for each simulation period. To impart variability to the simulation profits and outcomes, a function is employed to assign values to the variable μ, representing the retailer's decision to reorder. This computational process is executed independently for the retailer and the producer. The function assigns an integer value to 'μ' within the range of 0 to 1 for a given simulation period. Upon computation of individual profits for the retailer and producer during each simulation period for 100 items, the aggregate supply chain profit is calculated by summating the profits of both agents. This comprehensive metric encapsulates the overall outcome of the simulation, acknowledging the interactive dynamics between the two agents within the supply chain. This methodological approach facilitates an examination of how the decisions made by individual agents exert influence on the overarching results of the supply chain across diverse scenarios.

When the simulation was set up, a special cell was included in the spreadsheet. A specialized function is implemented in this cell with the explicit purpose of accurately and comprehensively discerning the specific scenario among the eight arising from the application of game theory within the simulation. The number entered in this cell is automatically generated by the function set in the spreadsheet. These numbers follow the numbering logic previously illustrated in Table 7.2.

In essence, this cell performs a dual function:

- It indicates the type of result or scenario obtained from the simulation according to the predefined numbering. This helps to categorise and label the results in a clear, descriptive and easily understandable way;
- It directly links the simulation result to the figure explaining the application of game theory theorems, facilitating cross-referencing between results and associated visual representations.

In details the simulation has been conducted following precise steps.

The simulation conducted had as its starting point the determination of a random number between 0 and 1. Thanks to this value, it is possible to forecast demand for the period T_0, based on a normal distribution. This process involved rounding up the inverse of a randomly generated value, considering an average of 40 products and a standard deviation of 8 products if the previously generated number is greater than 0.7; otherwise, the demand forecast is generated using an average of 20 products and a standard deviation of 4 products.

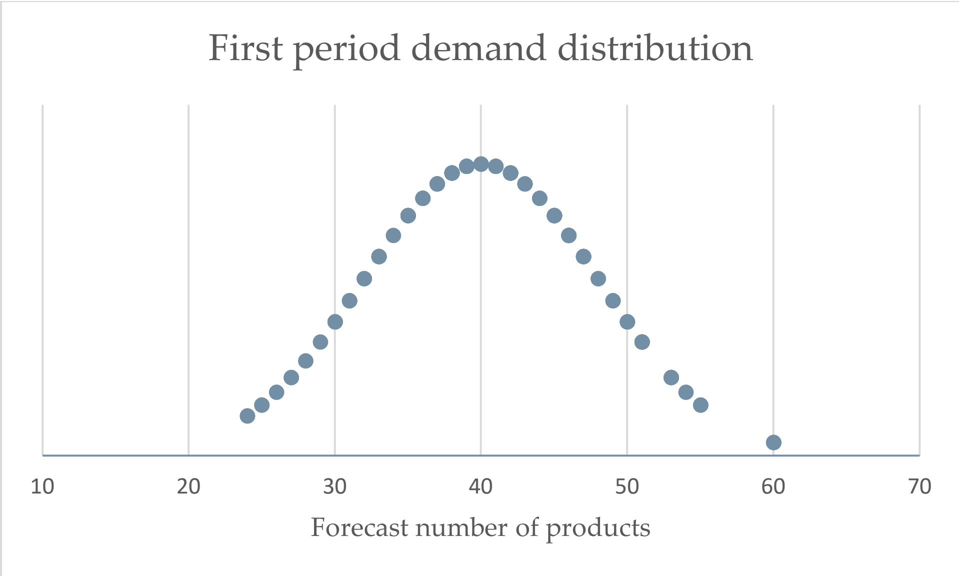

Figure 8.1: Demand forecasts' normal distribution scatter plot for the first period in case of high demand

After obtaining the demand forecast, it was categorized as high or low by applying a threshold value set at 36 product units (equivalent to 90% of the mean of 40). Specifically, the demand was deemed high if the value exceeded this threshold, and low otherwise. Subsequently, the cumulative probability of high demand was determined using a normal distribution, with the demand forecast for period T_0 as the reference point, maintaining a mean of 40 or 20 and a standard deviation of 8 or 4, based on the randomly generated number at the beginning of the simulation process. The obtained result represents the cumulative probability value associated with the specific demand forecast.

After dealing with the demand forecast phase, the producer's price was determined, defined as the cost at which the producer sells its products to the retailer. This price was generated randomly, based on an average of 600 € and a standard deviation of 100 €. Furthermore, the retailer's price was obtained by multiplying the producer's price by two in order to determine the latter's revenue.

The detailed analysis was subsequently enriched by the evaluation of overstock and stockout costs, potential situations in which the retailer might find itself.

The overstock cost was defined as the difference between the purchase price from the producer and 80% of the latter, representing the agreed buyback value with the producer and indicated with the symbol 'ε'. The stockout cost, on the other hand, represents the lost earnings in the event of a stock shortage and it was calculated as the difference between the retailer's price and the producer's price, divided by 2 in order to consider every possible cost incurred. The incorporation of these costs facilitated the derivation of the Critical Ratio, a valuable metric for ascertaining the optimal quantity of standard deviations to be procured, thereby mitigating the risk of stock outs or overstock scenarios.

$$CR = \frac{stockout\ cost}{stockout\ cost + overstock\ cost}$$

Equation 8.1

The quantity 'x_0' to be purchased at time t_0 break down into two straightforward scenarios:

- If the initial demand forecast was high, only the products needed to cover the time horizon of the first 8 weeks of sales were purchased. This quantity was determined by adding the demand forecast to a number of standard deviations equal to the 10% of the inverse of the normal demand distribution of the critical ratio value;
- If the previous condition was not verified, the products were purchased for the entire season. The demand forecast and the standard deviation were adjusted to 20 weeks, and the quantity purchased was equal to this demand forecast for the entire sales period, plus a number of standard deviations equal to the inverse of the normal demand distribution of the critical ratio value.

Subsequently, the percentage value of storage costs incurred by the retailer was introduced, set at 10% of the order for the selling season and denoted by the letter 'L'.

If the condition for the purchase of 'x_0' was fulfilled, this indicated that the retailer had decided to adopt the reorder model from the beginning of the selling season. In this case, the retailer had the option to stock a quantity 'M' of products, determined on the 12-week mean demand adjustment with the addition of a number of standard deviations (also adjusted to 12 weeks) equal to the inverse of the normal demand distribution of the critical ratio value. This figure was rounded upward, as it identifies the discrete product count.

Next, a constant representing the discount applied to unsold products at the end of the selling season was introduced, set at 40% of the retailer's price and indicated as 'h'. This discount was implemented in order to ensure the sale of products and their disposal in a period after the considered interval.

The deposit 'F' paid by the retailer to reserve 'M' items with the producer was defined as the result between 'M' and 20% of the producer's price.

Subsequently, the production and disposal costs to which the producer is subject were determined. The production costs represent 60% of the producer's selling price, contributing to ensuring a profit margin. Disposal costs, despite being indicated as a cost, turn out to be a source of profit for the producer, since the 'U' value represents the unit selling price at the end of the season of products that remain unsold or are returned to the producer via buyback. This 'U' value is given by 70% of the production costs 'C'.

The number of 'k_0' pieces remaining in stock at the end of the first sales period and not subject to buyback were determined by taking the maximum value between 0 and the difference between 'x_0' and the demand forecast for the first period. This decision ensures the attainment of a positive value for the inventory in question.

At this point, for the advanced version of the model, the need emerged to generate a number of additional parameters, first and foremost the demand forecast for the second period. To achieve this, the same approach was adopted as in the first period. For products with a high demand, an average of 60 units and a standard deviation of 18 units was used. In contrast, for products with lower demand, the average used was 30 pieces, with a standard deviation of 9. These values, calculated together with a predefined threshold set at 54 units, were used to determine whether demand should be classified as high or low, thus confirming or rejecting the initial forecast.

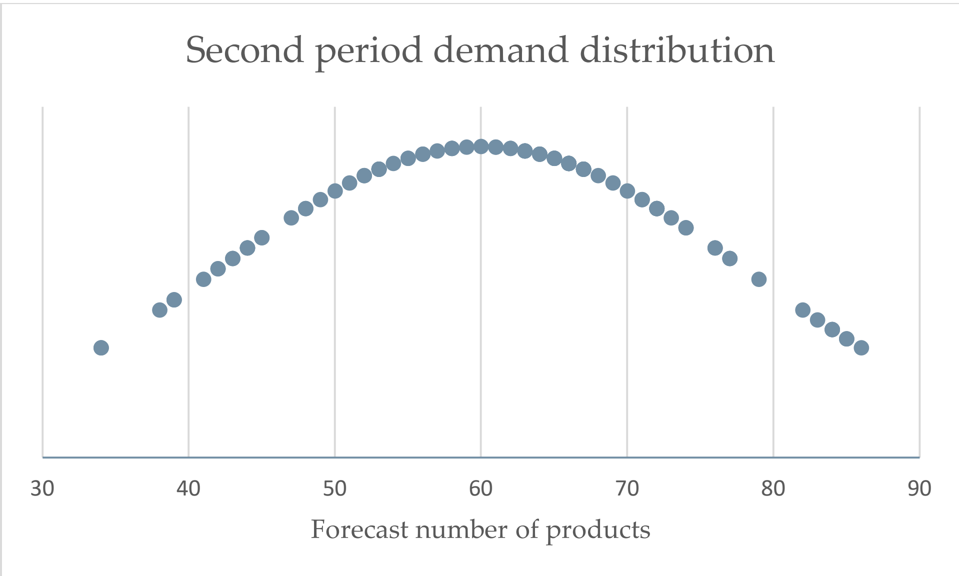

Figure 8.2: Demand forecasts' normal distribution scatter plot for the second period in case of high demand

Subsequently, it was possible to estimate the 'x_1' quantity of pieces to be reordered for the second 12-week period. Two scenarios were considered:

- If the quantity remaining in stock was greater than the quantity needed for the T_1 period, no purchase was made;

- Otherwise, if the quantity in stock was insufficient, the number of missing units, considering the mean value of 60 or 30, was purchased, plus a number of standard deviations equal to the inverse of the normal demand distribution of the critical ratio value.

The value of 'μ' was then defined as follows:

- μ = 1 if reordering was carried out, indicating the choice of the Mid-Season Reorder model;
- μ = 0 if no reorder was made, following the adoption of the Newsvendor model.

It was possible to determine the week in which to place the order (the order would still be delivered at the end of the eighth week), giving advantages to those who placed the order early. The variables '∂' and 'w' indicate the percentage of products subject to buyback and the discount applied to reordered products for the period T_1, respectively. These values vary, becoming less advantageous for the retailer, as the weeks pass.

The last variable to be identified before calculating profits was 'R', representing the quantity left in stock at the end of the selling season. This value was determined by the difference between the quantities 'x_0' and 'x_1' purchased and the expected demand for the two respective periods.

Finally, it was possible to determine profits 'π' using the formulas described in detail in the previous chapter by selecting the profit value for the selected model and the retailer's marginal contribution given by the ratio between profits and revenues. The complexity of the model, based on a number of variables and conditions, highlights the need for judicious management of resources to maximise profits and minimise waste.

8.3. A preliminary comparative analysis

The initial stage of analysis of the models considered offers a key opportunity to explore and compare the financial implications of the strategic choices made by the retailer, and consequently by the producer and the entire supply chain. Such approach allows to carefully scrutinise the impact of each model on economic performance, enabling the identification of strengths and weaknesses within the financial dynamics at both individual and systemic levels. In this way, one is able to gain a deeper understanding of the economic implications and formulate more informed conclusions about the validity of the models under examination.

In order to ensure a fair comparison between the three models, it was essential to establish the same initial value for the random variable in the three contexts. This methodological choice aims to create a homogeneous comparison ground, eliminating potential external influences linked to random variables. Homogeneity in initial values is a crucial element in achieving a neutral and accurate assessment of model performance. An initial identity in the random variables makes it possible to isolate the specific impact of each model on system dynamics more precisely, without initial differences affecting the results disproportionately. This methodological approach contributes to a more robust and reliable comparison.

The simulation procedures were undertaken using the same formulae and assumptions that were examined and described in detail in the previous paragraphs.

After running the simulation, it was possible to collect a series of results that provide a detailed overview of the dynamics that emerged in the examined context. These results are a key resource for understanding the performance of the models, allowing to explore how each approach affected various aspects of the supply chain and actors. The analysis of the simulation results allows the identification of trends, weaknesses and strengths in the different models, thus contributing to an in-depth evaluation of the strategies adopted. This process of collecting results plays a crucial role in providing concrete data and detailed observations, which will be essential for formulating robust conclusions on the relative performance of the models under investigation. The results are provided below.

	Advanced Mid-Season	Newsvendor	Mid-Season
Average producer	17.925 €	16.441 €	13.096 €
Average retailer	43.699 €	33.905 €	35.092 €
Average margin	43 %	43 %	45 %

Table 8.2: Summary of the three approaches comparison

The analysis in the table provides a clear perspective on which model is most beneficial to implement. The financial performance resulting from the implementation of the advanced Mid-Season Reorder model significantly exceeds that of the other models, highlighting benefits for both the retailer and the producer, as well as for the entire supply chain. In particular, it is noteworthy that, even in situations where market demand is classified as low, the new model proves to be even more effective than the Newsvendor model. The latter is traditionally known for its ability to successfully manage demand for products with low demand and low variability. This result underlines the inherent effectiveness of the advanced Mid-Season Reorder model in optimally managing a diverse range of market conditions, giving this approach an edge in supply chain management strategies.

8.4. Average scenarios

In the first period of the simulation, which covers a total of 8 weeks, the algorithm tests the two demand management models, Newsvendor model and Mid-Season Reorder model, in order to determine which one of them is able to most effectively minimize the probability of stock out or overstocking. This initial phase plays a crucial role because it establishes the conditions under which one of the two demand management models may be preferable to the other. For example, if the new model succeeds in avoiding stock-out situations more effectively, it may be preferable in circumstances where depleted stock leads to significant losses. Initial sales performance plays a crucial role at this stage, as it is a significant indicator for predicting future demand and making informed decisions on managing reorder strategies in the long term. If initial sales are going well, it could be indicative of stable or growing demand and that could influence the choice of demand management model. Conversely, if there are overstocking problems at the beginning, it may be necessary to adopt the model that best suits these circumstances, in particular the choice of placing a second order would no longer be so cost-effective. Based on sales in the first period, the algorithm selects the demand management strategy that best fits the specific characteristics of each product. The selection of a specific strategy has a direct impact on the overall financial performance of both supply chain actors and the chain as a whole. For example, a strategy that is well tailored to product needs can reduce storage costs and minimize stock-outs, helping to maximize profit. So, this approach makes it possible to determine which demand management model is most appropriate in a variety of situations and contexts, thus contributing significantly to optimizing the profitability of the entire supply chain.

After executing the simulation for 100 products spanning a century, a comprehensive dataset comprising 10,000 data points pertaining to the profits of the retailer, the producer, and the overall supply chain was amassed. To facilitate a more accessible and comprehensible interpretation of these results, a function was incorporated to provide a summarized perspective on the profit outcomes. The pivotal function introduced is AVERAGE. This function yields the arithmetic mean of profits. Notably, 16 mirrors were instantiated: eight pertain to the retailer's profits, categorized based on the adopted game theory scenario, while the remaining eight replicate the identical scenarios from the producer's perspective. The results obtained from this function are compiled into a summary report, which lists the different scenarios for each possible hypothesis, regarding the reorder week of reference. In other words, for each possible combination of agent (retailer or producer or supply chain), case and reorder week, the average profit among all products belong to that category was recorded. Thus, the value shown in these tables represents the result achieved among all product types for both the producer, the retailer and the entire supply chain in relation to the specific case selected. In summary, the use of the AVERAGE function allowed to identify and recap the average results obtained from the simulation, offering a clear perspective on the profits obtainable for each agent and scenario considered. This greatly facilitates the understanding of the results and the identification of the most profitable situations within the simulation.

Subsequently, the eight scenarios developed in relation to the retailer's decision to make or forgo additional reorder were compared in pairs. These comparisons aim to highlight how the decision to reorder can affect the final profit and whether the implementation of the new model is more advantageous than the application of the traditional Newsvendor model.

- Case 1 vs. Case 2: in this comparison, it is possible to analyse the impact of the reordering decision when dealing with high-demand products. The new model may demonstrate advantages and prevent stockout situations;
- Case 3 vs. Case 4: this comparison emphasizes how an incorrect and overestimated demand forecast can affect the agents' objective function. In this context, the reordering decision must be carefully considered;
- Case 5 vs. Case 6: this comparison highlights how an incorrect and underestimated demand forecast can affect the agents' objective function. Here, one could examine how strategic reordering during the sales season can improve results compared to a single initial reorder. The new model may prove to be more flexible in adapting inventory management to real circumstances and avoiding stockout problems;
- Case 7 vs. Case 8: in this comparison, it is possible to analyse the impact of the reordering decision when dealing with low-demand products. In a general situation, reordering may not be very suitable or helpful for inventory management, given the low market demand.

Case 1:

In Case 1, a scenario is contemplated wherein a particular product category was initially categorized as a high-demand item. Following four weeks of sales, this forecast was validated, prompting the retailer to initiate a reorder for the product. In this instance, the retailer exercises an optimal decision-making process grounded in the precise and validated prediction of heightened demand for the product.

Case 2:

In Case 2, an alternative scenario is being examined in which the retailer decides not to reorder the product, despite the initial forecast of high demand and actual good sales performance. In this situation, the retailer fails to fully meet market demand and risks running into stock out situations, i.e., running out of product inventory.

Analysis Case 1 vs. Case 2:

The absence of data pertaining to Case 2 is attributable to the procurement of a specific quantity of products in anticipation of high demand during the initial eight weeks of the season. Because of the later confirmed high demand, the stock that is left in the warehouse at the end of the eight-week period is not enough to cover the needs for the entire season. It is therefore important that a rearrangement be started. In the event when a precise projection of high demand is made, Case 1 reveals be the best option because Case 2 is not feasible. This emphasizes how effective the Mid-Season approach is. The case study highlights the danger of lost sales opportunities and merger earnings when real demand exceeds preliminary estimates, and the retailer is unable to fulfil it comprehensively. The simulation results clearly indicate that the retailer would be wise to place a reorder in the event that the initial estimate of high demand turns out to be accurate. By doing this, the possibility of stockouts is reduced, guaranteeing that both the producer and the retailer will make the highest amount of income possible.

Case 3:

In this particular scenario, the retailer erred by overestimating the demand for a specific product, anticipating robust sales; however, the realized market demand was, in fact, low. Despite the overestimation, the retailer opted to reorder the product.

Case 4:

In Case 4, an error in demand forecasting manifests through an initial overestimation of product demand. Nevertheless, unlike Case 3, the retailer opted against initiating a reorder in this scenario.

Analysis Case 3 vs. Case 4:

Similarly to the previous comparison, it can be seen that there is no data to support Case 4. This is justified by the fact that, even if the forecast was wrong and demand was initially overestimated, the quantity x_o purchased was necessary in order to cover only the first eight weeks of sales, so, although demand is low, a reorder is necessary in order to be able to end the season without a stockout situation.

Case 5:

Case 5 describes a situation where the retailer initially predicted a low demand for a certain product. However, it turned out that this initial forecast was underestimated, as the market has shown a higher actual demand than expected. In order to deal with this situation, the retailer decides to place a reorder, which allows him to fill the gap of product shortages that might occur as a result of an order placed at the beginning of the season based on a low demand forecast.

Case 6:

The situation described here concerns the case where the retailer, having initially wrongly predicted a low demand for a certain product, decides not to place further orders in order to fill any supply gaps. This choice entails the risk of slipping into an unpleasant situation, characterised by possible stock-outs, lost sales and potentially lower revenues than the retailer could have achieved if he had responded promptly to growing market demand.

Analysis Case 5 vs. Case 6:

In these specific cases, the retailer had to make a critical decision regarding the supply and demand management of a particular product. After initially anticipating a low demand, the market manifested a higher actual demand. Despite this favourable demand scenario, in case 6 it is assumed that the retailer does not place any further orders to fill the supply gap. A retailer's decision not to respond effectively to increased market demand will certainly result in lost opportunities for both parties involved. These results highlight the importance of accurate demand forecasting and supply chain management in dynamic situations. The ability to adapt promptly to changes in market demand is crucial to ensure a stronger partnership between retailer and producer. This case is a valuable lesson in optimising procurement and stock management decisions.

Case 7:

This particular case could be considered the most unlikely case in the examined case history applied to the real world. The retailer, initially forecasting low demand for a certain product and having ordered accordingly at the beginning of the season, later, when his forecast proves to be correct, reacts unexpectedly to the market situation. In fact, despite the initial low demand being in line with his forecast, the retailer decides to place a new order for the product. This decision is unusual as it entails the risk of stockpiling and leading to an overstock situation.

Case 8:

Case 8 reproposes the situation described above, but in this case the retailer decides not to reorder at all. This choice is in line with the actual market demand trend for that particular product type. This choice is expected to lead to higher profit values than in the previous case, where the retailer decided to place a new order.

Analysis Case 7 vs. Case 8:

Maintaining a balance between product supply and market demand is crucial for any business. This is true even in unusual situations. Managing this balance plays a critical role in ensuring the sustainable profitability of business operations. These two specific cases highlight the importance of carefully weighing stock management decisions. The first case concerns the purchase and maintenance of stocks in excess of actual market demand. When accumulating excess stock, the retailer exposes himself to a number of risks. First, there are the costs associated with storing unused goods, which can become burdensome over time. In addition, there is the risk that excess products become obsolete, losing their value in the market. In order to get rid of excess stock, he may be forced to offer significant discounts, thus reducing profit margins.
The second case concerns the decision not to re-order. In situations where the retailer can look forward to a future increase in demand, it might seem a good idea to place new orders to meet this growing demand. However, it is essential to consider that, given the initial assumptions, there is no guarantee that the market will be willing to accept all additional units. This leads to uncertainty regarding the profitability of such new orders. In summary, it is important to base procurement decisions on accurate information and reliable forecasts. This can help avoid the accumulation of unnecessary stockpiles and costly management errors, thereby maintaining company profitability.

In general, inventory management and reordering decisions are of crucial importance for the retailer and the producer. The key to success in this context depends largely on the ability to respond effectively to fluctuations in demand and real-time market dynamics. When market demand is high, the best choice for the retailer seems to be using the Mid-Season Reorder model, which allows him to respond promptly to demand. This model offers greater flexibility, which is particularly useful when initial forecasts are wrong. In these situations, adapting quickly to changes in demand and market dynamics is essential. On the other hand, when demand is low, the Newsvendor model remains a convenient choice for retailers. Under these circumstances, placing additional orders may only increase costs and entail unnecessary risks.

As far as the producer is concerned, his profit remains relatively constant in all possible cases. However, it is important to note that the retailer's decision to place new orders can significantly influence the producer's earnings. The application of the new stock management model can also allow for better organisation and planning of production, optimising time and resources.

All these thoughts on optimising inventory and reordering decisions are reflected in the profits of the entire supply chain, from production to retail. Collaboration and synchronisation between the various actors in the supply chain become crucial to maximise the overall efficiency and profitability of the system.

Subsequently, summaries obtained from the simulation for each actor (retailer, producer, and supply chain) and for each scenario are provided. In particular, the comparisons previously examined refer to the context in which the replenishment process occurs in week eight. Interesting for the future would be integrated situations in which the moment of replenishment does not coincide with the actual delivery phase of these products. In this context, it would be appropriate to consider the introduction of an additional demand forecast phase, which would investigate demand trends in the time gap between the moment when the replenishment decision occurs and the eighth week, when the items are actually delivered. This is intended to provide a more complete and reliable analysis of the dynamics involved.

WEEK 8				
GT case	Producer profits	Retailer profits	Retailer margin	Supply chain profits
Average case 1	22.487 €	63.725 €	44%	86.212 €
Average case 2	-	-	-	-
Average case 3	23.601 €	50.703 €	39%	74.304 €
Average case 4	-	-	-	-
Average case 5	18.456 €	52.337 €	45%	70.793 €
Average case 6	13.311 €	26.671 €	40%	39.982 €
Average case 7	12.178 €	26.567 €	42%	38.744 €
Average case 8	14.251 €	33.529 €	47%	47.780 €

Table 8.3: Average profits and contribution margin in relation to the reorder week 8

It is noteworthy that through simulation, it was possible to identify the probability density associated with the various scenarios under consideration. Cases 1, 7, and 8 emerge as the most probable. In contrast, Cases 2, 4 and 6 exhibits the lowest probability of occurrence, as previously anticipated. The graphical representation of this reflection on the probability density of the different game theory scenarios is included in the Appendix at the end of the document.

8.5. Spread scenarios

After a meticulous assessment of the outcomes derived from the AVERAGE analysis, it is deduced that a shift in perspective was imperative to attain a more thorough and inclusive comprehension of the examined context. This novel approach is characterized by its orientation toward a more intricate and nuanced understanding of the market demand distribution, surpassing the analysis concentrated on the specificity of individual cases. Central to this revitalized perspective is the notion of 'spread', a pivotal indicator assuming a critical role in delineating the percentage disparity between the quantities of two distinct product types subjected to a discerning comparison. To undertake a meticulous and all-encompassing analysis, a total of five distinct scenarios were formulated, each distinguished by a spectrum of values assigned to the spread. This configuration of scenarios is designed to intricately examine the tangible repercussions of demand diversification on the decision models to be employed. The assigned spread values encompass a broad range, ranging from a minimum of 50% to a maximum of 400%.

In the course of analysing the results, an interesting trend emerges. Initially, in the first cases examined, there is substantial parity in the profits recorded between the different scenarios. In particular, in the first scenario in which the spread is set at 50%, a clear preference for adopting the Newsvendor model is found, in line with the initial assumptions. This preference could be attributed to the correspondence between this specific scenario and the initial expectations of the analysis.

Nevertheless, upon exploring subsequent scenarios, a distinct dynamic becomes evident. The Mid-Season Reorder model emerges as the most lucrative option, and this disparity in profit expands with an escalation in the spread percentage. As the spread value incrementally rises, a discernible correlation is observed with the magnitude of the profit differential among the considered models. This phenomenon underscores the substantial influence of demand distribution variation on the optimal model selection.

In the analysis phase, the primary focus remains on the profit obtained by the different parties involved in the process: the retailer, the producer and the entire supply chain. The approaches and formulations used in this context remain constant with respect to the initial analysis, ensuring methodological consistency. The broad scope of the range of values considered allows a comprehensive representation of the market demand distribution by means of a scatter plot. Figure 8.3 offers a clear and detailed view of the complex underlying dynamics, thus providing a solid basis for business and strategic decisions in an ever-changing business environment. The depth of this analysis contributes to a better understanding of the impacts of changing demand distribution and offers valuable guidance for business decision-making.

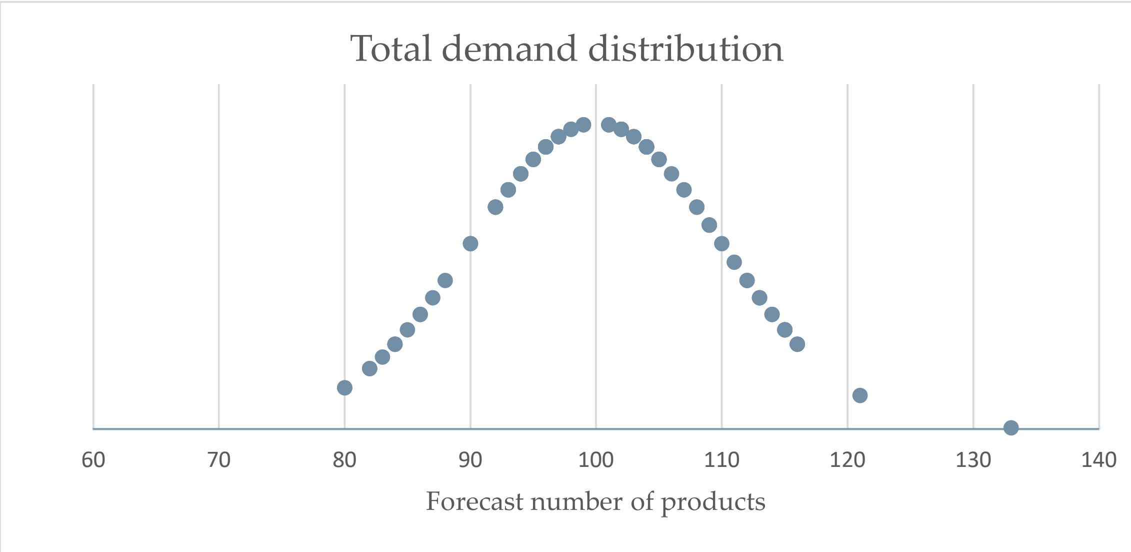

Figure 8.3:Demand forecasts' normal distribution scatter plot for the entire selling season

The analysis commences with the initial scenario, considering a spread value of 50% (Table 8.4). Intriguingly, the demand for the two products under consideration does not exhibit a significant disparity; nonetheless, this apparent similarity conceals a pivotal distinction. One of the products is initially categorized as having low demand, while the other is characterized as having high demand. This preliminary classification prompts disparate management approaches for the two products, bearing substantial implications. In light of distinct strategies employed for the two products, it becomes evident within this specific context that the optimal choice for the comprehensive management of low demand product is the Newsvendor model, for the high demand, instead, the Mid-Season Reorder model is preferrable. This outcome not only substantiates the soundness of the initial assumptions underpinning the model but also underscores the efficacy of the model when addressing products characterized by closely aligned demand patterns. Specifically, this model manifests notable suitability in the management of products categorized under low-demand scenarios. The decision to adopt the Newsvendor model is the direct consequence of a series of strategic considerations that take into account sales trends, the quantity of product to be purchased, and possible inventories. This demonstrates that the management decision is not one-sided, but requires in-depth analysis and adaptation to specific market and product conditions. The adaptability of the Newsvendor model to a wide range of scenarios confirms its usefulness in optimising supply chain management. The choice of management model must always be based on the characteristics of the product and its context. For example, in the case of products with highly variable demand or different market characteristics, other more advantageous options may emerge. Furthermore, this analysis demonstrates the importance of making detailed assessments before making strategic decisions related to supply chain management. An exhaustive understanding of demand trends and the appropriate adaptation of management models are crucial to maximise operational efficiency and profit optimisation. In this context, data analysis and consideration of variations in demand distribution play a crucial role in guiding business decisions and ensuring optimal product management.

SPREAD 50%		
DEMAND in T_0	**30**	**45**
HIGH/LOW in T_0	LOW	HIGH
x_0	81	46
DEMAND in T_1	24	46
HIGH/LOW in T_1	LOW	LOW
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	17.010 €	11.776 €
π_P Mid-Season	18.203 €	25.359 €
π_P	**17.010 €**	**25.359 €**
π_R Newsvendor	42.525 €	17.912 €
π_R Mid-Season	26.799 €	57.780 €
π_R	**42.525 €**	**57.780 €**
π_S	**59.535 €**	**83.139 €**
Margin	**50 %**	**40 %**

Table 8.4: Summary of a hypothetical scenario with a 50% spread between high and low demand products

The analysis of the second scenario (Table 8.5), depicting a 200% spread, yields noteworthy observations. Substantive evidence indicates that the Newsvendor model adeptly accommodates scenarios marked by an initial prediction of low demand. Conversely, where the initial forecast denotes high demand, the implementation of the new model demonstrates a propensity for yielding more favourable profits. This discernment is derived from a meticulous examination of the data, revealing profit augmentation for high demand product types upon the adoption of the new model. It is therefore important to note that, as the difference between low and high demand increases, the new model stands out as generating higher profits for all parties involved in the case of high-demand products, whereas the Newsvendor model allows higher profits for the retailer for low-demand products, but is not favourable for the producer. Consequently, in the case of products with high demand, the Mid-Season Reorder model continues to confirm its validity as the optimal choice, whereas the Newsvendor for products with low demand.

The analysis underlines the remarkable adaptability of the new model, demonstrating its effectiveness in different scenarios. Particularly noteworthy is its ability to produce remarkably high profits when initial forecasts align with high demand projections, specifically when the demand forecast is confirmed for the second period of the selling season.

SPREAD 200%		
DEMAND in T_0	**18**	**54**
HIGH/LOW in T_0	LOW	HIGH
x_0	51	55
DEMAND in T_1	13	55
HIGH/LOW in T_1	LOW	HIGH
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	13.719 €	11.110 €
π_P Mid-Season	14.855 €	21.455 €
π_P	**13.719 €**	**21.455 €**
π_R Newsvendor	34.981 €	16.817 €
π_R Mid-Season	19.074 €	53.540 €
π_R	**34.981 €**	**53.540 €**
π_S	**48.700 €**	**74.995 €**
Margin	**51 %**	**42 %**

Table 8.5: Summary of a hypothetical scenario with a 200% spread between high and low demand products

The gradual increase in the value of the spread, reaching a value of 400%, implying a significant disparity between low and high demand, clearly shows the effectiveness of the new model compared to the one currently in use (Table 8.6). As far as the producer is concerned, in the case of low demand the Newsvendor model emerges equally profitable, while in the case of high demand the Mid-Season Reorder model proves to be the most cost-effective.

This follows from the fact that, when the retailer decides to reorder, the new model offers considerable advantages. However, it is crucial to note that, in situations of high demand, the retailer's profit obtained by applying the new model significantly exceeds that which would be obtained by using the Newsvendor model, in contrast to the case with low demand, in which the profits with the Mid-Season Reorder model are higher than the Newsvendor by a very irrisory percentage. In conclusion, this analysis affirms the efficacy of the new model, particularly in environments characterized by high demand, attributed to its capacity to optimize profits and ensure superior supply chain management. Nonetheless, it is imperative to underscore that the selection between the two models is contingent upon variables such as the precision of forecasts, the acceptable level of risk, and the idiosyncrasies of the product. These factors necessitate meticulous evaluation to attain optimal results within the framework of real-world business dynamics.

SPREAD 400%		
DEMAND in T_0	**19**	**95**
HIGH/LOW in T_0	LOW	HIGH
x_0	54	96
DEMAND in T_1	42	81
HIGH/LOW in T_1	LOW	HIGH
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	13.176 €	23.424 €
π_P Mid-Season	13.176 €	35.097 €
π_P	**13.176 €**	**35.097 €**
π_R Newsvendor	28.365 €	38.064 €
π_R Mid-Season	29.646 €	95.770 €
π_R	**28.365 €**	**95.770 €**
π_S	**41.541 €**	**130.867 €**
Margin	**43 %**	**45 %**

Table 8.6: Summary of a hypothetical scenario with a 400% spread between high and low demand products

To conclude, although the three cases presented above received particular emphasis for their significance, it should be noted that further analyses with various spread evaluations confirm the results presented above (see Appendix for further details).

It is clear from these analyses that the Newsvendor model is confirmed as the optimal option in situations characterised by constant, low and easily predictable demand. In addition, the model is particularly suitable in contexts where an assortment of products with similar average demands is managed. However, in all other circumstances, the new model proves to be more functional and elastic in dealing with variations in demand, and this flexibility translates into better economic performance for the entire supply chain.

These results confirm the importance of selecting the appropriate model according to the specific dynamics of the market and the characteristics of the products involved. The choice between the Newsvendor model and the Mid-Season Reorder model must be carefully weighed, taking into account the variability of demand, the accuracy of forecasts and the need to maximise profits in supply chain management. Understanding these dynamics is key to optimising business decisions and ensuring optimal economic performance within the supply chain.

8.6. Conclusion

The simulation conducted within this chapter denotes a significant step forward in the understanding and validation of supply chain management models, focusing on the comparison between two crucial approaches: the well-established Newsvendor model and the more recent Mid-Season Reorder model. While the former has wide support in the literature, the latter is a new development that still requires valid empirical corroboration to confirm its effectiveness in real operational contexts. The simulation, based on computer-generated but realistic data, allowed to examine and evaluate the performance of these models under a diversified range of demand conditions, making an important contribution to filling the gap in the supply chain management literature.

The chapter comprised multiple stages of analysis, each of which contributed to a more profound comprehension of the examined models and the contexts in which they perform well.

The initial phase of the simulation focused on comparing the two models and their variants. By introducing assumptions that made the initial conditions fair, it was possible to obtain truthful results on the outcome of the analysis. This showed a predominant growth in terms of profit for the agents involved and the supply chain in general, as a result of the implementation of the new model with the reorder decision taken at week eight.

The second simulation phase focused on the assessment of mean values derived from the implementation of both the Newsvendor and Mid-Season Reorder models. This evaluation was conducted using a simulated dataset spanning a century and encompassing 100 products with the most varied characteristics. The results of this analysis showed that the Mid-Season Reorder model demonstrates greater effectiveness in cases where the data reflect the model's predictability assumptions. In these situations, the Mid-Season Reorder model clearly demonstrates its advantage in situations where flexibility is required, unlike the Newsvendor model which is best suited to handle constant and predictable demands. This analysis confirms the previously mentioned theoretical assumption on Pareto optimality of cases one and eight.

However, as mentioned above, more complex and realistic situations require more in-depth analysis, and the third phase of the simulation addressed this need. Detailed studies were conducted on five different scenarios based on the percentage spread between low and high demand products. This analysis was crucial to further explore the performance of the two models under more changeable and volatile demand conditions, so in a more realistic representation of the challenges that supply chains often face. The results that emerged from this phase of the analysis clearly confirm the advantage of the Mid-Season Reorder model in high demand contexts, especially when the spread percentage is significant. In these scenarios, the model demonstrates its ability to maximise profits and ensure efficient supply chain management. Its flexibility in dealing with deviations in demand results in superior performance in dynamic contexts. On the other hand, the Newsvendor model continues to prove to be the optimal choice in cases of low demand and when product requirements are more stable. This result is in agreement with the theories underlying the model, which suggest that it is best suited for situations where demand is constant and predictable.

One of the key conclusions that emerged from this simulation is the importance of adapting the model according to the specific characteristics of the market and products involved. The choice between the Newsvendor model and the Mid-Season Reorder model must be carefully weighed, taking into account key variables such as demand variability, forecast accuracy and the goal of maximising profits in supply chain management.

This simulation constitutes an important step for several reasons. Firstly, it allows a practical test of the Mid-Season Reorder model, filling a gap in the supply chain management literature. Many of the theoretical proposals in this field have never been validated empirically or through simulations based on realistic data. This simulation provides concrete evidence of the performance of the Mid-Season Reorder model in realistic situations, contributing greatly to the understanding of how this model can be successfully applied. Furthermore, this simulation provides a solid starting point for future research and development in supply chain management. The scenarios explored and results obtained provide a reliable basis for further studies on the validity and applicability of supply chain management models in different business situations. This may help to develop new strategies and approaches better suited to the specific challenges companies may face in supply chain management.

In conclusion, this simulation embodies an important contribution to the supply chain management literature by providing an empirical evaluation of the examined models under real operating conditions. The results obtained confirm the importance of carefully considering market and product characteristics when choosing the most suitable supply chain management model. This knowledge can be of great value to companies engaged in planning and optimising their supply chain operations, enabling them to make more informed decisions and maximise profits within the supply chain. It also highlighted the effectiveness of the new model compared to the current one in certain contexts and how it is able to significantly enhance the financial performance of all involved participants and the supply chain as a whole. Therefore, the need to thoroughly evaluate its implementation in the market is empirically confirmed. The simulation provides a solid empirical basis for addressing the practical challenges of supply chain management and is an important step towards a better understanding of the complex business dynamics in this field.

9. Discussion and conclusions

9.1. Comments on the Findings

A careful examination of the results that emerged from the simulation conducted in Microsoft Excel, following the theoretical modelling of the new type of contract, unequivocally highlights the effectiveness of the Mid-Season Reorder model compared to the consolidated Newsvendor model, especially in situations of high demand in the luxury sector. This chapter aims to analyse in detail the data that emerged, highlighting the evidence that clearly indicates the added value of the new contractual model.

The Newsvendor model, known for its validity in a variety of contexts, revealed clear limitations during the simulation when applied to luxury products subject to highly variable demand. Its inability to adapt to market fluctuations in a timely manner generated missed opportunities and significant challenges for companies in the sector. The simulation clearly demonstrated that the rigidity of the Newsvendor model can lead to negative consequences, compromising brand image and causing significant economic losses.

On the other hand, the Mid-Season Reorder model has excelled in situations of high demand in the luxury sector. Its ability to allow for adjustments around mid-season has proved crucial in adapting inventory management to changing market dynamics. The flexibility offered by this innovative contract model allowed luxury brands to precisely align supply with actual demand, mitigating the risks of stock outs or overstocking.

An exhaustive examination of the data, involving the investigation of various spread percentages applied to market demand, revealed that the Mid-Season Reorder model surpassed the Newsvendor model in effectively handling inventory, particularly in scenarios characterized by substantial demand spikes and associated high spread values. Its capacity to dynamically respond to market fluctuations ensured a more streamlined inventory management process, mitigating adverse effects on brand reputation and corporate financial statements.

Inventory management in the luxury sector, where products are often associated with high prices and brand image, requires a careful and targeted approach. The Mid-Season Reorder proved to be not only a concrete answer to this need, but also an efficient solution for companies wishing to maintain a competitive advantage.

In conclusion, the analysis of the results unequivocally underlines that the adoption of the Mid-Season Reorder model represents a significant step in the optimisation of supply chain management in the luxury sector; the profits calculated in the simulation phase on the basis of the newly developed formulas and the new parameters identified are a clear example of this. The innovation introduced by this new contract model offers a strategic advantage, allowing companies in the sector to distinguish themselves in inventory management and more effectively meet the evolving needs of consumers. The effectiveness of the Mid-Season Reorder model, highlighted by the simulation results, confirms its relevance and suitability as a key tool to address the challenges of modern supply chain management in the dynamic context of the luxury sector.

9.2. Answer to the Research Questions

The in-depth understanding of the dynamics of mid-season reorder management in the luxury jewellery sector, as revealed by the initial interrogations of this research, offers a clear insight into the challenges and opportunities associated with this innovative approach. In this context, it will be examined how the findings confirm or confute existing knowledge and add new insights essential to supply chain management in the luxury sector.

Primarily, precise demand forecasting emerges as a pivotal component within the management framework of Mid-Season Reorder in the luxury jewellery sector. A meticulous analysis of detailed data affirms that the dynamic nature of trends necessitates a profound comprehension of customer preferences and market influences. The simulation underscores the critical significance of accurately predicting seasonal demand to mitigate adverse scenarios such as overstocking or product shortages. Equally crucial is the adept handling of returns and refunds. Effective pre-treatment of unsold merchandise returns can yield substantial financial performance benefits for the entire supply chain.

Furthermore, judicious compromises and incentives wield notable influence over sales, guiding procurement activities during sales periods and positively impacting the overall profit function of the involved parties. Such bilateral advantages contribute positively to the financial standing of the parties and foster long-term, mutually beneficial relationships devoid of conflicts. The ongoing positive interaction between parties creates added value for both, minimizing communication gaps and potential misalignments between the interests of the retailer and the producer. Enhanced communication also enables the swift implementation of corrective actions in response to demand fluctuations, ensuring continuous alignment with end consumer interests. Effective collaboration with producers also ensures the availability of materials when needed and minimising production times and delivery time. The flexible sourcing dynamics of the Mid-Season Reorder model proves to be a crucial element in meeting the needs of an ever-changing market.

The positive outcomes derived from the implementation of a more flexible contractual framework aligned with the perspectives of both agents reflect in superior performance compared to competitors, thereby securing a substantial market share.

Game theory, introduced as an analytical tool in supply chain management, has been validated through simulation as a central and influential element. The negotiation dynamics between the retailer and the producer, particularly in the context of mid-season reorder options, emerge as an effective strategic approach. The optimal game solution, as determined by game theory, holds the potential to maximize the overall value of the supply chain, fostering mutually advantageous contractual terms. Moreover, the focus on variables such as quantity, price, and reorder time has not only allowed for the identification of the optimal batch size to be ordered but has also unveiled intricacies and challenges beyond this aspect. Detailed examination and appropriate adjustment of these facets enable the realization of significant positive impacts on the profit functions of the involved parties.

The results confirm that the Mid-Season Reorder model offers greater flexibility, allowing to avoid excesses or shortages of inventory, and to maximise overall profitability. The ability of retailers to adjust the quantity of mid-season orders according to actual demand dynamics emerges as a winning strategy in inventory management.

In conclusion, the simulation unequivocally affirms that the adoption of the Mid-Season Reorder model represents a significant advancement in optimizing supply chain management within the luxury sector. The outcomes attained offer crucial new insights, underscoring the imperative to adapt to the evolving needs of the industry. Furthermore, it has presented a clear spectrum of scenarios in which a hypothetical retailer might find itself, elucidating prudent actions to be implemented in light of such circumstances. The simulation has also underscored the essential need of continuously adjusting initial forecasts in tandem with the evolution of market demands, emphasizing the significance of establishing a threshold to discriminate between high and low-demand products for assessing the most effective course of action. An additional element to consider for demand deviation is the critical ratio, as it allows for the identification of an optimal quantity of products to cover almost entirely the fluctuations in demand without incurring issues related to product availability.

Future developments and ongoing research endeavours could delve deeper into the potential applications of game theory in supply chain management. This exploration aims to identify progressively sophisticated strategies to navigate complex market dynamics and ensure optimal supply chain management within the luxury sector, particularly in instances where there may have been inaccuracies in forecasting demand at the onset of the sales period.

9.3. Implications and Limitations of the study

The introduction of the contractual model for dynamic reordering in the luxury jewellery sector offers a significant theoretical contribution to the dynamics of supply chain management. However, it is crucial to examine the implications and limitations arising from this strategic transition in order to fully understand the context in which the answers provided can be relied upon.

Initially, the recognition of the imperative for efficient communication and collaboration with manufacturers underscores the pivotal role of partnership between the retailer and producers. Close integration and timely exchange of information become imperative for the success of dynamic reordering. This theoretical insight posits that transparency and open communication constitute essential pillars for agile supply chain management.

Equally pertinent is the significance of data analysis capabilities. Prioritizing investment in advanced analytical tools implies that companies should regard staff training in analytical skills as an essential component. The theoretical contribution underscores the criticality of accurately interpreting market and real demand information to optimize the advantages derived from mid-season reorganization.

One of the study's most important practical implications is the difficulty of organizational adaptability. An organizational structure that is both flexible and agile is necessary to enable quick adaptation to changing market conditions. In order to maximize the effectiveness of the new contract model, businesses should give top priority to creating an adaptable working environment that allows them to quickly adapt to changes in demand. Companies can revaluate production levels, modify inventory management plans, and improve channels of communication with suppliers and merchants by implementing a flexible framework. This flexibility is especially important when it comes to managing mid-season reorders, as the supply chain's overall performance can be greatly impacted by the supply chain's ability to respond to market changes. To achieve persistent flexibility, the organization must cultivate a culture of adaptability and continual improvement. This could entail continuing education initiatives to improve staff members' proficiencies in data analysis, communication, and decision-making. Businesses that embrace organizational flexibility put themselves in a better position to handle ambiguity, seize new possibilities, and improve their general competitiveness in the market.

However, it is essential to recognise the limitations of this research. The universality of the implementation of the contractual model for dynamic reorganisation is questioned, indicating that not all products or sectors may benefit equally from this approach. The theoretical contribution posits that the efficacy of the proposed model is intricately tied to demand variability and the intricacies of the supply chain. It underscores the necessity for a meticulous assessment of the contexts in which the model can be effectively applied.

When it comes to operational advice, it becomes clear that careful strategic planning, focused investments in necessary capabilities, and cooperative producer relationships are essential. These useful suggestions can operate as a compass for businesses, helping them to maximize the advantages and skilfully handle the difficulties that come with implementing this novel contractual strategy.

In conclusion, while this research contributes significantly to the theoretical understanding of supply chain management in the luxury jewellery sector, it is important to recognise that its scope of reliability is limited to the specific characteristics of the context examined. The limitations of the study indicate the need for further research exploring other sectors and contexts in order to generalise and refine the conclusions reached here.

9.4. Conclusion and Future Evolutions of the work

This study explored in depth the dynamics and challenges of implementing a contractual model for dynamic reordering in the luxury jewellery sector. Through the detailed analysis of initial queries and the critical examination of traditional models, new insights have emerged that make significant contributions to the understanding of supply chain management in contexts of variable and changing demand.

First, the importance of accurate demand forecasting became clear. In the luxury context, where trends can change rapidly, the ability to accurately anticipate customer tastes and market influences becomes crucial. The new contractual model for dynamic reordering is a strategic enabler, allowing supply chain actors to adapt to changes in demand in a timely manner, avoiding overstocking or shortages.

Supply chain flexibility has been identified as a key element. The need to optimise production and delivery times, together with effective collaboration with suppliers, proved essential. Comparison with the traditional Newsvendor model highlighted the limitations of the latter, while the dynamic reordering model offers a more agile and responsive solution, particularly suited to an industry where speed of response to trends is critical.

Game theory has emerged as a remarkably useful tool for optimising contracts in supply chain management. Its application in the context of mid-season reordering has introduced a more complex negotiation dynamic, allowing retailers to adjust inventory according to actual demand. This approach opened up new possibilities for collaboration between retailers and suppliers, helping to maximise the overall value of the supply chain.

The results of the comparison between the Newsvendor model and the new contract model clearly highlight the effectiveness of the latter in the context of luxury jewellery stores. Its flexibility and ability to adapt to the changing needs of the market place it as a practical response to the inherent challenges of supply chain management in this industry.

However, it is crucial to recognise that the implementation of this model is not without its challenges. Effective communication and collaboration with producers, the need for advanced data analysis skills and the demand for organisational flexibility are aspects that require careful management.

The practical implications of the study suggest that companies in the luxury jewellery industry should wisely consider the transition to dynamic reordering as a strategic decision that requires targeted investments and a change of mindset. The creation of an agile supply chain and the enhancement of analytical skills are key aspects to maximise the benefits of this innovative contractual approach.

In conclusion, this study provides a clear and thorough overview of supply chain management in the luxury jewellery industry, introducing an innovative contractual model as a response to market challenges. The new insights provided not only enrich existing theory, but also provide practical guidelines for industry players wishing to improve the flexibility, adaptability and efficiency of their supply chain.

The development of the model revealed considerable challenges with regard to effective communication between the parties and an evident weakness in accurately predicting the development of market demand. In consideration of this, possible future developments could be the introduction of mechanisms and incentives that could help the alignment between the parties, allowing for better communication and a more efficient reaction in the market, as well as an analysis of what could be effective tools to introduce in order to facilitate forecasting analysis. Furthermore, since the model is efficient in contexts where demand is highly variable, an interesting and further development could involve conducting a benchmark analysis to identify sectors that manifest characteristics aligning with the guidelines of the model, thereby facilitating its advantageous implementation.

In order to optimize the efficiency and adaptability of the Mid-Season Reorder model to the variability of the considered market, future research could focus on restructuring the model to provide greater reliability in case of inaccurate initial demand forecasts. This enhancement aims to strengthen effectiveness and efficiency for all actors in the supply chain. Furthermore, it would be interesting to conduct additional research to deepen the assessment of the most opportune week for reordering, perhaps by introducing an additional demand forecast to generate even more precise solutions and scenarios. Finally, another future development could involve examining the financial impact resulting from aligning the reordering week with the delivery week, as such alignment would have a direct impact on inventories in the current period.

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Appendix

Simulation

a. Macro text

```
Sub Macro3()

  'Variables definition
  Dim riga As Integer
  Dim colonna As Integer
  Dim caso_gt As Integer
  Dim check As Integer


        For riga = 4 To 103
            For colonna = 3 To 102


            'Disable automatic calculation
            Application.Calculation = xlCalculationManual

            'Define case GT
            caso_gt = Sheets("Calcoli").Cells(30, 3).Value


            ' Copy pigreco_P to sheet "Profitto producer"
            Sheets("Profitto producer").Cells(riga + 105 * (caso_gt - 1),
            colonna).Value =    Sheets("Calcoli").Range("C33").Value

            'Copy pigreco_R to sheet "Profitto retailer"
            Sheets("Profitto retailer").Cells(riga + 105 * (caso_gt - 1),
            colonna).Value = Sheets("Calcoli").Range("C36").Value

            'Copy margin_R to sheet "Margine retailer"
            Sheets("Margine retailer").Cells(riga + 105 * (caso_gt - 1),
            colonna).Value = Sheets("Calcoli").Range("C51").Value
```

```
'Copy profit_P_Newsvendor to sheet "Only producer"
 Sheets("Only producer").Cells(riga, colonna).Value =
 Sheets("Calcoli").Range("D44").Value

'Copy profit_P_MidSeason to sheet "Only producer"
 Sheets("Secco producer").Cells(riga + 105, colonna).Value =
Sheets("Calcoli").Range("E44").Value

'Copy profit_R_Newsvendor to sheet "Only retailer"
 Sheets("Secco retailer").Cells(riga, colonna).Value =
Sheets("Calcoli").Range("D47").Value

'Copy profit_R_MidSeason to sheet "Only retailer"
 Sheets("Secco retailer").Cells(riga + 105, colonna).Value =
Sheets("Calcoli").Range("E47").Value

'Copy margin_R_Newsvendor to sheet "Only margin"
 Sheets("Secco margine").Cells(riga, colonna).Value =
Sheets("Calcoli").Range("D51").Value

'Copy margin_R_MidSeason to sheet "Only margin"
 Sheets("Secco margine").Cells(riga + 105, colonna).Value =
Sheets("Calcoli").Range("E51").Value



'Reactivation of automatic calculation
 Application.Calculation = xlCalculationAutomatic

 Next colonna
Next riga


'Reactivation of screen update
Application.ScreenUpdating = True

End Sub
```

b. Excel calculations

	A	B	C	D	E
1		**BOTH MODELS**		**ONLY NEWSVENDOR**	**ONLY MID-SEASON**
2		Casual	0,57		
3		Mean High T0	40		
4		St. dev. High T0	8		
5		Mean Low T0	20		
6		St. dev. Low T0	4		
7		Demand in T0	24		
8		REAL	LOW		
9		High/Low D0	LOW		
10		% D0	84%		
11		pP	529 €		
12		pR	1.058 €		
13		Overstock	106 €		
14		Stockout	265 €		
15		CR	0,57		
16		X0	66	56	25
17		L	0,1		
18		M	0		34
19		h	0,4		
20		ε	423 €		
21		F	- €		
22		C	318 €		
23		U	223 €		
24		k0	42		1
25		Mean High T1	60		
26		St. dev. High T1	18		
27		Mean Low T1	30		
28		St. dev. Low T1	9		
29		Demand in T1	28		
30		High/Low D1	LOW		
31		X1	0		29
32		µ	0		
33		Reorder week	8		
34		∂	0,2		
35		w	0,1		
36		Δk	42		
37		k1	0		
38		R	14		2
39		Model	NEWSVENDOR		
40		GT calculation	LOW-LOW-0		
41		Case GT	8		
42		π P (Newsvendor)	13.926 €		
43		π P (Mid-season)	14.550 €		
44		π P	**13.926 €**	**11.816 €**	**9.474 €**
45		π R (Newsvendor)	33.644 €		
46		π R (Mid-season)	24.905 €		
47		π R	**33.644 €**	**29.941 €**	**26.312 €**
48		π S	**47.570 €**	**41.757 €**	**35.787 €**
49		Revenues R (Newsvendor)	69.828 €	59.248 €	
50		Revenues R (Mid-Season)	63.311 €		56.201 €
51		Margin	**48%**	**51%**	**47%**

Cell	Formula
C2	=CASUALE()
C3	=40
C4	=8
C5	=20
C6	=4
C7	=SE(C2>0,7;ARROTONDA.DIFETTO(INV.NORM(CASUALE();C3;C4);1);ARROTONDA.DIFETTO(INV.NORM(CASUALE();C5;C6);1))
C8	=SE(C7>=36;"HIGH";"LOW")
C9	=SE(C2>0,7;"HIGH";"LOW")
C10	=SE(C2>0,7;DISTRIB.NORM(C7;C3;C4;VERO);DISTRIB.NORM(C7;C5;C6;VERO))
C11	=ARROTONDA.DIFETTO(INV.NORM(CASUALE();600;100);1)
C12	=2*C11
C13	=C11-C20
C14	=(C12-C11)/2
C15	=INV.NORM.S(C14/(C14+C13))
C16	=ARROTONDA.ECCESSO(SE(C8="HIGH";C7+0,1*C4*C15;C7/8*20+C6/8*20*C15);1)
C17	=0,1
C18	=ARROTONDA.ECCESSO(SE(C8="HIGH";C3/8*12+C4/8*12*C15;0);1)
C19	=0,4
C20	=C11*0,8
C21	=MAX(C11*0,2*C18;0)
C22	=ARROTONDA.ECCESSO(C11*0,6;1)
C23	=ARROTONDA.ECCESSO(C22*0,7;1)
C24	=MAX(C16-C7;0)
C25	=60
C26	=18
C27	=30
C28	=9
C29	=SE(C2>0,7;ARROTONDA.ECCESSO(INV.NORM(CASUALE();C25;C26);1);ARROTONDA.ECCESSO(INV.NORM(CASUALE();C27;C28);1))
C30	=SE(C29>=54;"HIGH";"LOW")
C31	=MAX(ARROTONDA.ECCESSO(SE(C2>0,7;SE(C25-C24>0;C25-C24+C26*C15;0);C27-C24);1);0)
C32	=SE(C31>0;1;0)

Cell	Formula
C33	=8
C34	=0,2
C35	=0,1
C36	=C24
C37	=C24-C36
C38	=MAX(C16+C31-C7-C29;0)
C39	=SE(C32=1;"MID-SEASON REORDER";"NEWSVENDOR")
C40	=C8&"-"&C30&"-"&C32
C41	=CERCA.VERT(C40;Foglio2!A1:B8;2;FALSO)
C42	=C16*(C11-C22)
C43	=SE.ERRORE(SE(C18>0;C11*(C16+C31*C32*(1-C35))+C21-C22*(C16+MAX(C18;C31))+C23*(C16+C18-C16-C31+C38*C34)-C21/C18*C31;C11*(C16+C31*C32*(1-C35))+C21-C22*(C16+MAX(C18;C31))+C23*(C16+C18-C16-C31+C38*C34));"ERRORE")
C44	=SE(C32=1;C43;C42)
C45	=(C12-C11)*C16-C13*(C16-C7-C29)-C14*(C7+C29-C16)-C17*C16*C11
C46	=SE.ERRORE(SE(C18>0;C12*(C16+C31*C32-C38)+C34*C38*C20+C12*(1-C19)*(C37+(1-C34)*C38)+C21/C18*C31-(C17*C11*(C16+C31*C32)+C11*(C16+C31*C32*(1-C35))+C21);C12*(C16+C31*C32-C38)+C34*C38*C20+C12*(1-C19)*(C37+(1-C34)*C38)-(C17*C11*(C16+C31*C32)+C11*(C16+C31*C32*(1-C35))+C21));"ERRORE")
C47	=SE(C32=1;C46;C45)
C48	=C44+C47
C49	=C12*C16
C50	=SE(C18>0;C12*(C16+C31-C38)+C34*C38*C20+C12*(1-C19)*(C37+(1-C34)*C38)+C21/C18*C31;C12*(C16+C31-C38)+C34*C38*C20+C12*(1-C19)*(C37+(1-C34)*C38))
C51	=SE(C32=1;C47/C50;C47/C49)
D16	=ARROTONDA.ECCESSO(SE(C8=”HIGH”;C3/8*20+C4/8*20*C15;C5/8*20+C6/8*20*C15);1)
D44	=D16*(C11-C22)
D47	=(C12-C11)*D16-C13*(D16-C7-C29)-C14*(C7+C29-C16)-C17*D16*C11
D48	=D44+D47
D49	=C12*D16
D51	=D47/D49
E16	=ARROTONDA.ECCESSO(SE(C8=”HIGH”;C7+0,1*C4*C15;C7+C6*0,1*C15);1)

Cell	Formula
E18	=ARROTONDA.ECCESSO(SE(C8=”HIGH”;C3/8*12+C4/8*12*C15;C5/8*12+C6 /8*12*C15);1)
E24	=MAX(E16-C7;0)
E31	=MAX(ARROTONDA.ECCESSO(SE(C8=”HIGH”;SE(C25-E24>0;C25-E24+C26*C15;0);C27-E24);1);0)
E38	=MAX(E16+E31-C7-C29;0)
E44	=SE.ERRORE(SE(E18>0;C11*(E16+E31*(1-C35))+C21-C22*(E16+MAX(E18;E31))+C23*(E16+E18-E16-E31+E38*C34)-C21/E18*E31;C11*(E16+E31*(1-C35))+C21-C22*(E16+MAX(E18;E31))+C23*(E16+E18-E16-E31+E38*E34));"ERRORE")
E47	=SE.ERRORE(SE(E18>0;C12*(E16+E31-E38)+C34*E38*C20+C12*(1-C19)*(C37+(1-C34)*E38)+C21/E18*E31-(C17*C11*(E16+E31)+C11*(E16+E31*(1-C35))+C21);C12*(E16+E31-E38)+C34*E38*C20+C12*(1-C19)*(C37+(1-C34)*E38)-(C17*C11*(E16+E31)+C11*(E16+E31*(1-C35))+C21));"ERRORE")
E48	=E44+E47
E50	=SE(E18>0;C12*(E16+E31-E38)+C34*E38*C20+C12*(1-C19)*(C37+(1-C34)*E38)+C21/E18*E31;C12*(E16+E31-E38)+C34*E38*C20+C12*(1-C19)*(C37+(1-C34)*E38))
E51	=E47/E50

	A	B
1	HIGH-HIGH-1	1
2	HIGH-HIGH-0	2
3	HIGH-LOW-1	3
4	HIGH-LOW-0	4
5	LOW-HIGH-1	5
6	LOW-HIGH-0	6
7	LOW-LOW-1	7
8	LOW-LOW-0	8

Foglio 2

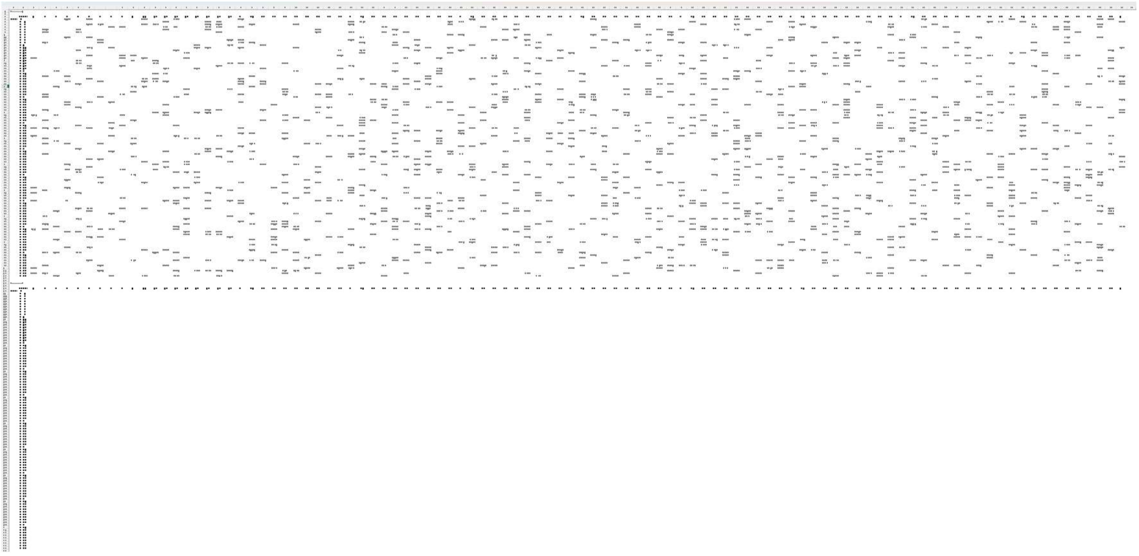
c. Spread scenarios

SPREAD 100%		
DEMAND in T_0	**28**	**56**
HIGH/LOW in T_0	LOW	HIGH
x_0	76	57
DEMAND in T_1	25	81
HIGH/LOW in T_1	LOW	HIGH
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	20.368 €	15.105 €
π_P Mid-Season	21.670 €	27.779 €
π_P	**20.368 €**	**27.779 €**
π_R Newsvendor	50.602 €	18.100 €
π_R Mid-Season	32.364 €	80.820 €
π_R	**50.602 €**	**80.820 €**
π_S	**70.970 €**	**108.599 €**
Margin	**50 %**	**45 %**

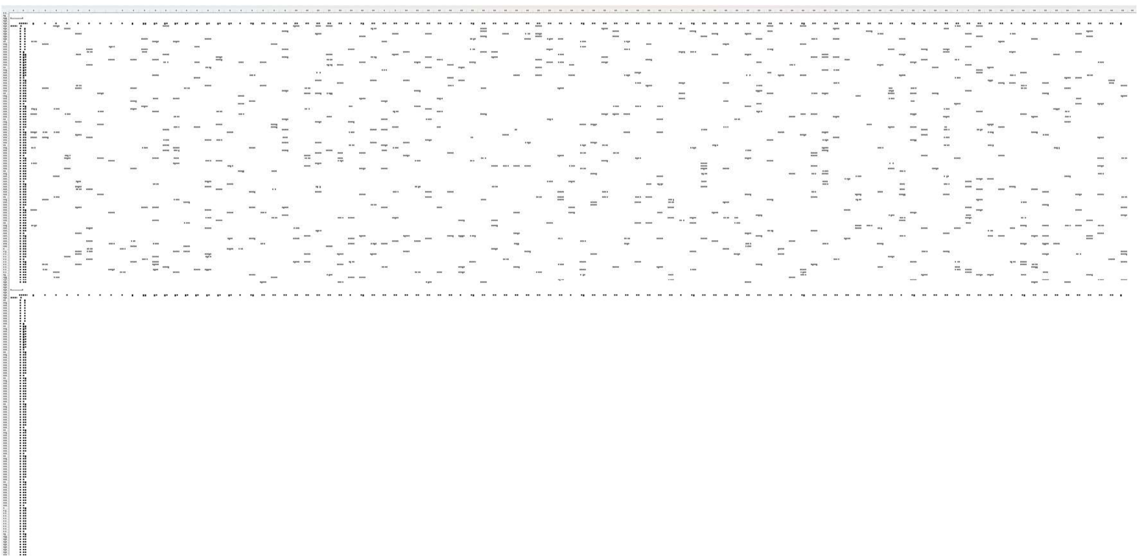
SPREAD 300%		
DEMAND in T_0	**16**	**64**
HIGH/LOW in T_0	LOW	HIGH
x_0	46	65
DEMAND in T_1	35	80
HIGH/LOW in T_1	LOW	HIGH
x_1	0	70
MODEL	**NEWSVENDOR**	**MID-SEASON REORDER**
π_P Newsvendor	15.042 €	16.770 €
π_P Mid-Season	15.042 €	29.104 €
π_P	**15.042 €**	**29.104 €**
π_R Newsvendor	32.638 €	22.481 €
π_R Mid-Season	33.865 €	83.399 €
π_R	**32.638 €**	**83.399 €**
π_S	**47.680 €**	**112.503 €**
Margin	**43 %**	**45 %**

d. Probability density for the different game theoretic scenarios

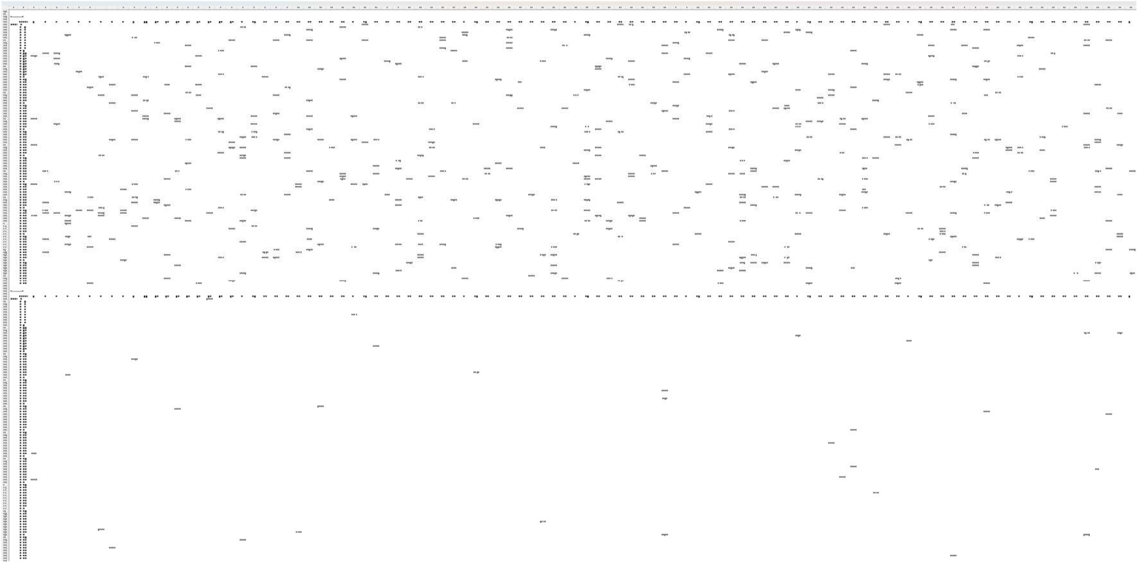
Case 1 vs Case 2



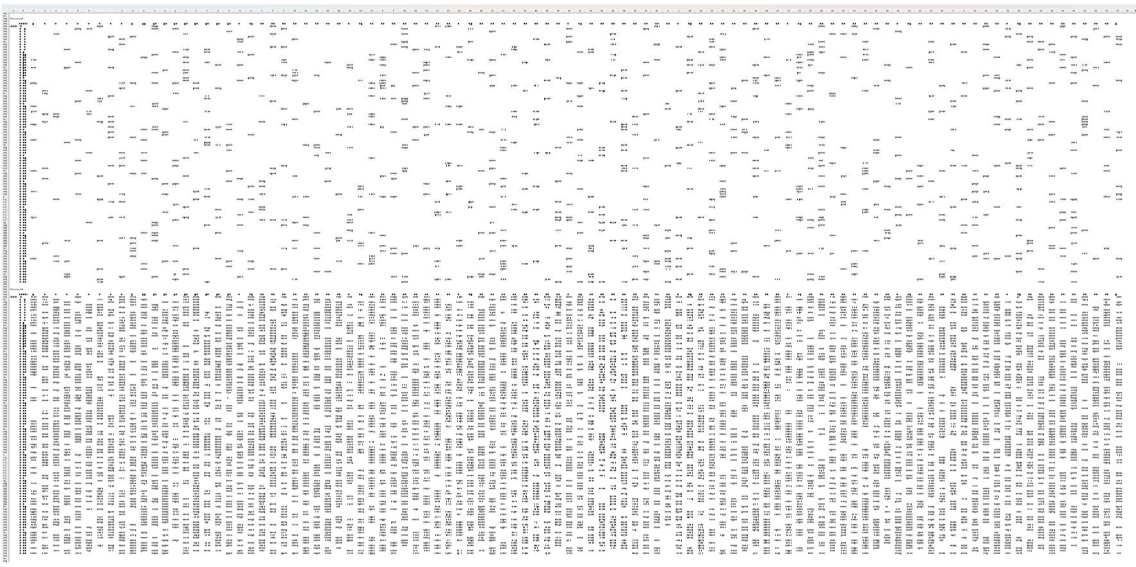
Case 3 vs Case 4



Case 5 vs Case 6



Case 7 vs Case 8



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