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A Markov-Switching dynamic approach to non-linear hedge fund risk exposures

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Abstract

Our study wants to assist investors in having a better knowledge of the determinants of Hedge Fund risk and performance during different cycles of the market, since we found a covering gap of HF performance evaluation during last few years. We first want to ascertain the well-known non-linear returns of HF through cross sectional multiple linear regressions with linear risk factors using Hedge Fund Research (HFR) strategies indexes. Then, we compare the results obtained carrying out a second multiple linear regression adding non-linear derivatives risk factors. In the end they guarantee a slight better explanatory power improvement of our model but they don't contribute to overcome the limitations of a linear model and, at the same time, complicate the understanding for a retail investor. With a view of binding market performance to HFs, we have decided to adopt a dynamic regime-switching model regression according to hidden market states. In this way we move from outdated quantile analysis and we let the model determine endogenously the market states which we assume to follow a Markovian process as in the framework of Billio et al. in 2010 and Stafylas et al. in 2018.

Keywords: Hedge Funds, investment strategies, Regime switching model, non-linear returns

Abstract in lingua italiana

L’obiettivo principale di questo studio è di assistere gli investitori nel comprendere quali siano i fattori di rischio che definiscono i ritorni derivanti da investimenti in strategie di fondi Hedge. Il punto di partenza è l’analisi dei ritorni degli HF, caratterizzati da una non linearità, attraverso regressioni multilineari di tipo cross-sectional con fattori di rischio lineari, usando indici di performance aggregati divulgati dalla piattaforma Hedge Fund Research (HFR). I risultati ottenuti verranno comparati con un’ulteriore regressione multilineare, questa volta aggiungendo fattori di rischio non lineari provenienti da strumenti derivati. Questa ultima modifica migliora solo parzialmente il modello statico di valutazione delle performance e lo complica eccessivamente per un investitore, contribuendo però ad evidenziare ulteriormente l’uso di strategie non-lineari dei fondi Hedge. Successivamente, interessati a evidenziare le relazioni tra i vari stati del mercato azionario ed i ritorni Hedge, abbiamo utilizzato un modello dinamico di regressione a cambio regime con stati di mercato nascosti. In questo modo ci stacchiamo da un’analisi statica e obsoleta per quantili e in modo endogeno tramite il modello siamo in grado di determinare i regimi di stato che assumiamo seguano un processo Markoviano come deciso da Billio et al. nel 2010 e Stafylas et al. nel 2018.

Parole chiave: Hedge funds, strategie, regime dinamico Markoviano, ritorni non lineari

Contents

1 Introduction

1.1 Characteristics of the hedge fund industry

A hedge fund can be defined as an actively managed, pooled investment vehicle that is open to only a limited group of investors and whose performance is measured in absolute return units. However, this simple definition excludes some hedge funds and includes some funds that are clearly not hedge funds. There is no simple and all-encompassing definition. The nomenclature “hedge fund” provides insight into its original definition. To “hedge” is to lower overall risk by taking on an asset position that offsets an existing source of risk. For example, an investor holding a large position in foreign equities can hedge the portfolio’s currency risk by going short on currency futures. A trader with a large inventory position in an individual stock can hedge the market component of the stock’s risk by going short on equity index futures. One might define a hedge fund as an information motivated fund that hedges away all or most sources of risk not related to the price-relevant information available for speculation. Note that short positions are intrinsic to hedging and are critical in the original definition of hedge funds.

Hedge fund managers are usually motivated to maximize absolute returns under any market condition. Most of them receive asymmetric incentive fees based on positive absolute returns and are not measured against the performance of passive benchmarks that represent the overall market. Hedge fund management is fundamentally skill-based, relying on the talents of active investment management to exceed the returns of passive indexing.

Hedge fund managers have the flexibility to choose from a wide range of investment techniques and assets, including long and short positions in stocks, bonds, and commodities. Leverage is commonly used (83% of global investment funds) to magnify the effect of investment decisions [Liang, 1999]. Fund managers may trade in foreign currencies and derivatives (options or futures) and concentrate, rather than diversify, their investments in chosen countries or industry sectors. Hedge fund managers commonly invest their own money in the fund, which aligns their personal motivation with outside investors. Some hedge funds do not hedge at all; they simply take advantage of the legal and compensatory structures of hedge funds to pursue desired trading strategies. In practice, a particular legal structure lets hedge funds avoid certain regulatory constraints common to the industry. Hence it is possible to use their legal status as an alternative means of defining a hedge fund.

Many people think that hedge funds are completely unregulated, but it is more accurate to say that hedge funds are structured to take advantage of exemptions in regulations. Fung and Hsieh (1999) explain the justification for these exemptions is that the regulations are meant for the general public and that hedge funds are intended for well-informed, well-financed, private investors. The legal structure of hedge funds is intrinsic to their nature. Flexibility, opaqueness, and aggressive incentive compensation are fundamental to the highly speculative, information-motivated trading strategies of hedge funds. These features conflict with a highly regulated legal environment. Hedge funds are almost always organized as limited partnerships or limited liability companies to provide pass-through tax treatment. The fund itself does not pay taxes on investment returns, but returns are passed through so that individual investors pay the taxes on their personal tax bills (if hedge funds were set up as corporations, profits would be taxed twice.)

Hedge funds are usually more secretive than other pooled investment vehicles, such as mutual funds. A hedge fund manager may want to acquire her positions quietly, so as not to tip off other investors of her intentions. For example, a fund manager may use proprietary trading models without wanting to reveal clues to her systematic approach. With so much flexibility and privacy conferred to managers, investors must heavily rely upon managers' judgment in investment selection, asset allocation, and risk management.

1.2 History of hedge funds

In 1949, Alfred Winslow Jones started an investment partnership that is regarded as the first hedge fund. Remarkably many of the ideas that he introduced over fifty years ago remain fundamental to today's hedge fund industry. Jones structured his fund to be exempt from the SEC regulations described in the Investment Company Act of 1940. This enabled Jones' fund to use a wider variety of investment techniques, including short selling, leverage, and concentration (rather than diversification) of his portfolio. Jones committed his own money to the partnership and based his remuneration on a performance incentive fee, of 20% of profits. Both practices encourage interest alignment between managers and outside investors and continue to be used today by most hedge funds. Jones pioneered combining shorting and leverage, techniques that generally increase risk, and used them to hedge against market movements and reduce his risk exposure.

During the mid-1960s, Jones' fund was still active and began to be considered a benchmark within the industry. A stock market boom started in the late 60s, led by a group of stocks dubbed the Nifty Fifty, and hedge funds that followed the Jones' long-short style appeared to underperform the overall market. To capture the potential upside coming from the rising market, hedge fund managers changed and improved their investing strategy. Their funds became directional, abandoned the risk reduction afforded by long-short hedging, and opted for investments favoring leveraged long-bias exposure. During the subsequent bear market of 1972-1974, many hedge funds went out of business, and hedge funds decreased in popularity for the next 10 years. A mid-80s revival of hedge funds is generally related to the publicity surrounding Julian Robertson's Tiger Fund (and its offshore sibling, the Jaguar Fund). The Tiger Fund was one of several so-called global macro funds that leveraged investments in securities and currencies based on assessments of global macroeconomic and political conditions.

In the late 90s, hedge funds made the headlines once more, but for their large losses. In 1998, Soros' Quantum Fund lost $2 billion during the Russian debt crisis. Robertson's Tiger Fund incorrectly bet upon the depreciation of the yen versus the dollar and lost more than $2 billion. During the dot-com boom, Quantum lost almost $3 billion more from first shorting high-tech stocks and then reversing its strategy and purchasing stocks near the market top.

As already mention, hedge funds don't have to register with the U.S. Securities and Exchange Commission (SEC). The funds and their managers also aren't required to register with the National

Association of Securities Dealers (NASD) or the Commodity Futures Trading Commission, the major self-regulatory bodies in the investment business. However, many funds register with these bodies anyway, choosing to give investors peace of mind and many protections otherwise not afforded to them. Whether registered or not, hedge funds can't commit fraud, engage in insider trading, or otherwise violate the laws of the land. Since hedge funds are structured to avoid regulation, even disclosure of the existence of a hedge fund is not mandatory. There is no regulatory agency that maintains official hedge fund data. Private firms gather data that are voluntarily reported by the hedge funds themselves. This gives an obvious source of self-selection bias, since only successful funds may choose to report. Some databases combine hedge funds with commodity trading advisers (CTAs) and some separate them into two categories. Also, different hedge funds define leverage inconsistently, which affects the determination of assets under management (AUM), so aggregate hedge fund data are best viewed as estimates.

Even with the caveat about data reliability and the usefulness of AUM, the growth of the hedge fund industry is apparent. In 1990, Lhabitant (2002) estimates about 600 active hedge funds with an aggregate AUM of less than $20 billion; Agarwal and Naik (2000) cite an aggregate AUM of $39 billion. By 2000, Lhabitant report between 4000 and 6000 hedge funds in existence, with aggregate AUM between $400-600 billion. Agarwal and Naik quote an aggregate AUM of $487 billion. De Brouwer (2002) summarizes a wide range of end of 90's estimates: between 1082 to 5830 hedge funds and $139-400 billion in aggregate AUM. Lhabitant's reported data imply averaging at least 20% annualized growth in several hedge funds and 35% in AUM. These estimates seem to be out of this world but it must be said that in the end of 90's was also a period of tremendous growth in the overall equity markets.

1.3 Hedge Fund fee structure

Hedge fund managers are compensated by two types of fees: a management fee, usually a percentage of the size of the fund (measured by AUM), and a performance-based incentive fee, like the 20% of the profit that Alfred Winslow Jones collected on the very first hedge fund. Fung and Hsieh (1999) determine that the median management fee is between 1-2% of AUM and the median incentive fee is 15-20% of profits. Ackermann et al. (1999) cite similar median figures: a management fee of 1% of assets and an incentive fee of 20% (a so-called "1 and 20 fund"). The incentive fee is a crucial feature for the success of hedge funds. A pay-for-profits compensation causes the manager's aim to be absolute returns, not merely beating a benchmark. To achieve absolute returns regularly, the hedge fund manager must pursue investment strategies that generate returns regardless of market conditions; that is, strategies with low correlation to the market. However, a hedge fund incentive fee is asymmetric; it rewards positive absolute returns without a corresponding penalty for negative returns. Empirical studies provide evidence for the effectiveness of incentive fees. Liang (1999) reports that a 1% increase in incentive fees is coupled with an average 1.3% increase in monthly return. Ackermann et al. (1999) determine that the presence of a 20% incentive fee results in an average 66% increase in the Sharpe ratio, as opposed to having no incentive fee. The performance fee enables a hedge fund manager to earn the same money as running a mutual fund 10 times larger (Tremont, 2002). However, there is the possibility that managers will be tempted to take excessive risks in pursuit of (asymmetric) incentive fees. This is one reason why, in many jurisdictions, asymmetric incentive fees are not permitted.

Another important characteristic of hedge funds is their lock-up periods. A lock-up period is a window of time when investors are not allowed to redeem or sell shares of a particular investment. The lock-up period is intended to give the hedge fund manager time to exit investments that may be illiquid or otherwise unbalance their portfolio of investments too rapidly. Hedge fund lockups can vary from 30 days to almost 3 years, based on the chosen strategy and illiquidity of the investment portfolio.

To ensure profits are determined fairly, high-water marks and hurdle rates are sometimes included in the calculation of incentive fees. A high-water mark is an absolute minimum level of performance over the life of an investment that must be reached before incentive fees are paid. A high-water mark ensures that a fund manager does not receive incentive fees for gains that merely recover losses in previous time periods. A hurdle rate is another minimum level of performance (typically the return of a risk-free investment, such as a short-term government bond) that must be achieved before profits are determined. Unlike a high-watermark, a hurdle rate is only for a single time period. Liang (1999) determined that funds with high water marks have significantly better performance (0.2% monthly) and are widespread (79% of funds). Hurdle rates are only used by 16% of funds and have a statistically insignificant effect on performance.

Minimum investment levels for hedge funds are usually high, implicitly dictated by legal limits on the number of investors who are not high net worth individuals and restrictions on promotion and advertising. To stay free of strict regulation, hedge funds agree to accept investments just from accredited or qualified investors. Accredited investors are individuals with a net worth of at least $1 million or an annual income of $200,000 ($300,000 for a married couple). Qualified investors are individuals, trust accounts, or institutional funds with at least $5 million in investable assets. The SEC & FSA requirement of the private placement for hedge funds means that hedge funds tend to be exclusive with a comparatively small number of investors. $250,000 is a common minimum initial investment, and $100,000 is common for subsequent investments [Ackermann et al., 1999; Liang, 1999]. From the perspective of the fund manager, having a small number of clients with relatively large investments keeps client servicing costs low. This allows the hedge fund manager to concentrate more on trading and less on client servicing and fund promotion.

1.4 Hedge Fund investment strategies

To compare performance, risk, and other characteristics, it is helpful to categorize hedge funds by their investment strategies. Strategies may be designed to be market-neutral (very low correlation to the overall market) or directional (a “bet” anticipating a specific market movement). Selection decisions may be purely systematic (based on computer models) or discretionary (ultimately based on a person). A hedge fund may pursue several strategies at the same time, internally allocating its assets proportionately across different strategies.

Equity Hedge strategies maintain positions both long and short in primarily equity and equity derivative securities. A wide variety of investment processes can be employed to arrive at an investment decision, including both quantitative and fundamental techniques; strategies can be broadly diversified or narrowly focused on specific sectors and can range broadly in terms of levels of net exposure, leverage employed, holding period, concentrations of market capitalizations and valuation ranges of typical portfolios. EH managers would typically maintain at least 50% exposure to, and may in some cases be entirely invested in, equities, both long and short. Long-short hedge funds focus on security selection to achieve absolute returns while decreasing market risk exposure by offsetting short and long positions. Compared to a long-only portfolio, or simply following a passive strategy investing in the markets, short selling reduces correlation with the market, provides additional leverage, and allows the manager to take advantage of overvalued as well as undervalued securities. Derivatives may also be used for either hedging or leverage. Security selection decisions may incorporate industry long-short or regional long-short. The classic long-short position is to choose two closely related securities, short the perceived overvalued one and long the undervalued one. Long-short portfolios are rarely completely market-neutral. They typically exhibit a "direction" either a long bias or short bias, and so have a corresponding market exposure (positive or negative). They are also likely to be exposed to other market-wide sources of risk, such as style or industry risk factors. When the exposure is no greater than 10% long or short, we obtain the so-called Equity Market Neutral Strategies in which the investment thesis is predicated on exploiting pricing anomalies which may occur as a function of expected mean reversion inherent in security prices. High frequency techniques and trading strategies may be employed on the basis on technical analysis or opportunistically to exploit new information the investment manager believes has not been fully discounted into current security prices.

Emerging Markets funds invest, primarily long, in securities of companies or the sovereign debt of developing or 'emerging' countries. Emerging Markets regions include Africa, Asia ex-Japan, Latin America, the Middle East, and Russia/Eastern Europe. Emerging Markets - Global funds will shift their weightings among these regions according to market conditions and manager perspectives.

Relative value funds use market-neutral strategies that take advantage of perceived mispricing between related financial instruments. It includes strategies in which the investment core is made on the realization of a spread between instruments in which one or multiple components of the spread is a convertible or corporate or sovereign fixed income instrument. Convertible arbitrage profits from situations where convertible bonds are undervalued compared to the theoretical value of the underlying equity and pure bond. In these cases, the hedge fund manager takes long positions on the convertible bond and shorts the underlying stock. Corporate arbitrage strategies try to realize a return between multiple corporate bonds or between a corporate and risk-free government bond. Situations for corporate arbitrage often occur with illiquid assets in presence of low number of trades, so leverage is often used to increase total returns. Multi-Strategies employ an investment methodology that is based on the realization of a spread between related yield instruments in which one or multiple components of the spread contain a fixed income, derivative, equity, real estate, or combination of these or other instruments. To conclude, RV strategies are typically quantitatively driven to measure the existing relationship between fixed-income instruments and, in some cases, identify attractive positions in which the risk adjusted spread between these instruments represents an attractive opportunity for the investment manager. In many cases, RV strategies may exist as distinct strategies across which a vehicle allocates directly, or may exist as related strategies over which a single individual or decision-making process manages.

Event-driven strategies exploit perceived mispricing of securities by anticipating events such as corporate mergers or bankruptcies, and their effects. Merger (or risk) arbitrage is the investment in both companies (the acquirer and takeover candidate) after a merger has been announced. Until the merger is completed, there is usually a difference between the takeover bid price and the current price of the takeover candidate, which reflects uncertainty about whether the merger will actually happen. For instance, a fund manager may buy the takeover candidate, short stock of the acquirer, and expect the prices of the two companies to converge. In this case, there may be a substantial risk that the merger will fail to occur. Bankruptcy and financial distress are also hedge fund trading opportunities, because managers in traditional pooled vehicles (such as mutual funds and pension funds) may be forced to avoid distressed securities, which drive their values below their true worth. Certain hedge fund managers may also invest in Regulation D securities, which are privately placed by small companies seeking capital, and not accessible to traditionally managed funds. Distressed/Restructuring strategies employ an investment process focused on corporate fixed income instruments, primarily on corporate credit instruments of companies trading at significant discounts to their value at issuance or obliged (par value) at maturity as a result of either formal bankruptcy proceeding or financial market perception of near-term proceedings. Managers are typically actively involved with the management of these companies, frequently involved on creditors' committees in negotiating the exchange of securities for alternative obligations, either swaps of debt, equity or hybrid securities. Managers employ fundamental credit processes focused on the valuation and asset coverage of securities of distressed firms. Investing in distressed securities typically increases liquidity risks.

Macro funds trade a broad range of strategies in which the investment process is predicated on movements in underlying economic variables and the impact these have on equity, fixed income, hard currency, and commodity markets. Managers employ a variety of techniques, both discretionary and systematic analysis, combinations of top down and bottom-up methods, quantitative and fundamental approaches, and long and short-term holding periods. Although some strategies employ Relative Value techniques, Macro strategies are distinct from RV strategies in that the primary investment thesis is predicated on predicted or future movements in the underlying instruments, rather than the realization of a valuation discrepancy between securities. Similarly, while both Macro and Equity Hedge managers may hold equity securities, the overriding investment thesis is predicated on the impact movements in underlying macroeconomic variables may have on security prices, as opposed to EH, in which the fundamental characteristics of the company are the most significant and are integral to the investment thesis.

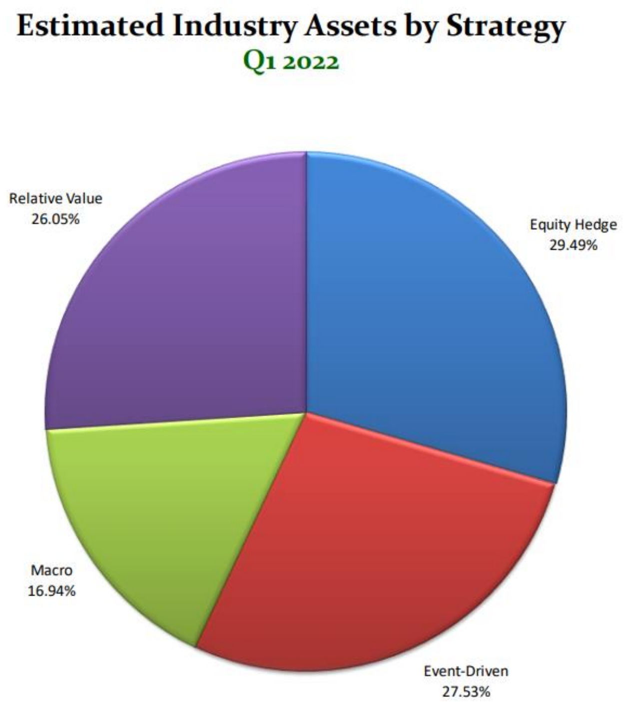


Figure 1 - Subdivision of strategies among HFs

As Figure 1 reports, the most dominant strategy is equity hedge, followed by Relative value, Event-driven and Macro. However, the magnitude of difference among the strategies is very little. This similar popularity can be explained due to the customized investment choices provided by HFs to investors.

1.5 Actual snapshot of the hedge fund industry and its evolution

Total hedge fund industry capital rose surpassing the $4 trillion threshold at the beginning of 2022, with managers navigating a volatile end of year 2021 driven by another wave of coronavirus variant; as well as rising interest rates and increased expectations for additional increases in 2022, as the US Federal Reserve reduces stimulus bond buying with inflationary pressures reaching the highest level in nearly 40 years. Total hedge fund capital had an increase of over $400 billion from the start of 2021, as reported by HFR, in the latest release of the HFR Global Hedge Fund Industry Report.

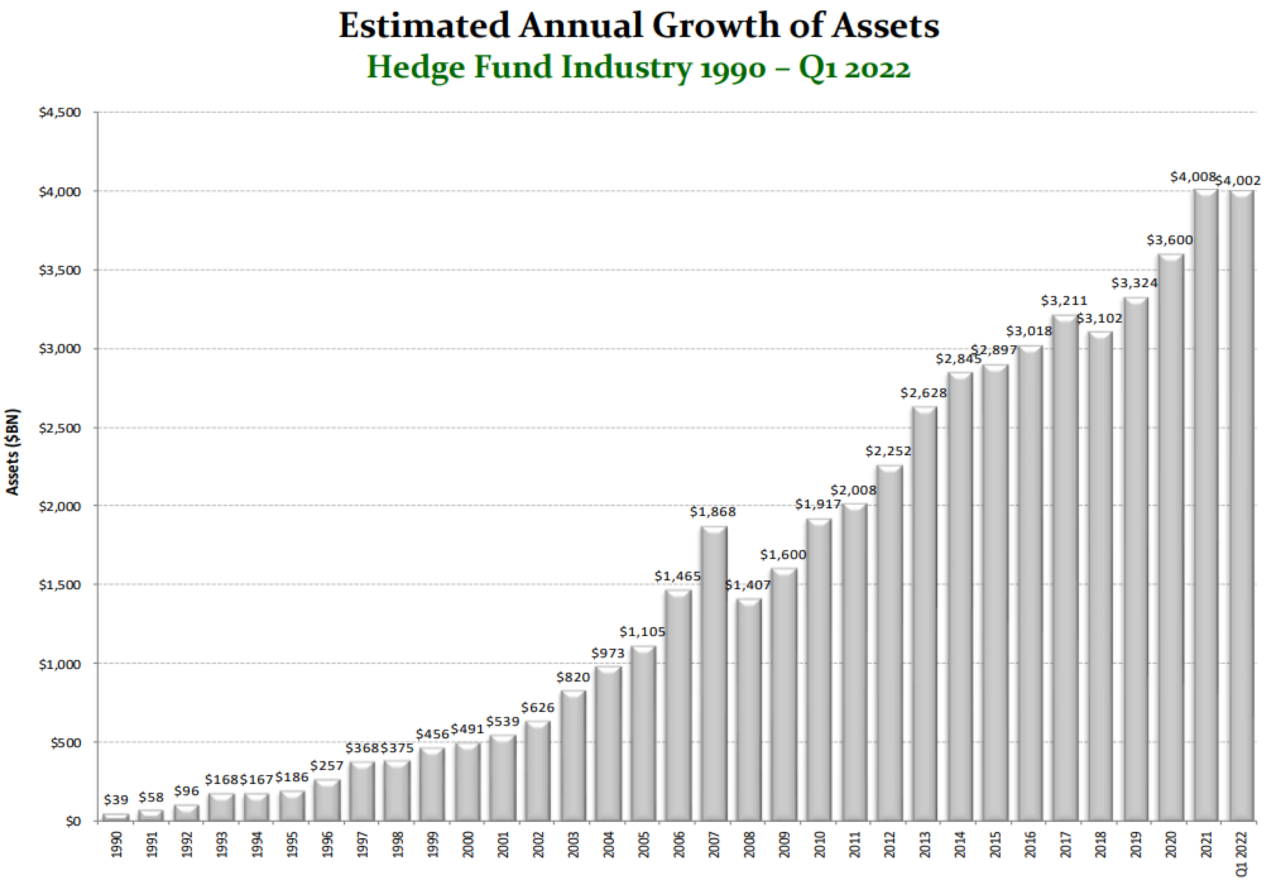


Figure 2 - Global annual growth of Asset Under Management

Total hedge fund industry capital has soared by over $1 trillion in the trailing seven quarters since falling below $3 trillion in first quarter 2020 as the global pandemic began. The most evident downfall of the industry is observed during the great financial crisis, within the time period between 2007 and 2008. Illiquidity of investments and shorter lock-up periods caused the death and liquidation of several hedge funds, allowing the full recovery of the industry only in 2010, when it went back to a total cumulative AUM of $1.9 trillion.

The HFRI Fund Weighted Composite Index (FWC) posted a gain of +0.5 percent for 4Q21, bringing the FY 2021 performance to +10.3 percent. The 2021 gain trails the prior two years as the strongest years of performance since 2009. Event-Driven (ED) strategies, which categorically focus on out of favor, often heavily-shorted, deep value equity and credit positions, extended asset increases through year end, with capital raising over $155 billion in 2021 to surpass $1.115 trillion, trailing only Equity Hedge as the largest strategy area of the capital. The investable HFRI 500 Event-Driven Index rose +2.1 percent in last quarter of 2021 and +14.5 percent for 2021. Total capital invested in Equity Hedge (EH) strategies experienced an increase of over $133 billion for 2021, bringing total EH capital to a record $1.227 trillion to begin 2022, as managers navigated intense volatility and rapidly evolving market cycles driven by a coronavirus, accelerating inflation and rising interest rates. The investable HFRI 500 Equity Hedge (Total) Index posted a +1.9 percent return in last quarter of 2021, bringing an annual performance to +11.5 percent. As interest rates rose to conclude 2021 as they did throughout second half of 2021, capital managed by credit- and interest rate-sensitive fixed income-based Relative Value Arbitrage (RVA) strategies increased by over $86 billion for FY 2021, to begin 2022 at $1.027 trillion. As investors positioned for higher interest rates, RVA led main strategy net inflows for FY with $15 billion of new allocations. The investable HFRI 500 Relative Value Index gained +6.6 percent for 2021, while the HFRI Relative Value (Total) Index returned +7.5 percent. Total capital invested in Macro strategies rose over $33 billion in 2021 to end the year at $637.1 billion AUM, led by increases in Systematic Diversified/CTA and Commodity strategies, with these rising $20.7 and $5.2 billion, respectively for 2021. Like RVA, Macro also experienced net inflows for 2021, with investors allocating an estimated $3.1 billion of new capital during the year, led by $2.6 billion of inflows to Discretionary Thematic funds. Driven by commodity gains, the HFRI Macro (Total) Index gained +7.6 percent for 2021, while the investable HFRI 500 Macro: Commodity Index led Macro sub-strategy performance with a +26.35 percent return. Following five consecutive quarters of the industry's largest firms leading mid- to small firms in inflows, investors reversed this trend in last quarter of 2021, with the largest firms experiencing an estimated net outflow of $7.4 billion during the quarter. Firms managing between $1 billion and $5 billion experienced a modest outflow of $113 million, while firms managing less than $1 billion experienced outflows of $1.3 billion over the quarter. For the full year 2021, firms managing greater than $5 billion received an estimated $5.7 billion, while mid-sized firms managing between $1 billion and $5 billion experienced net inflows of $3.94 billion, while firms managing less than $1 billion collectively saw estimated inflows of $5.5 billion over the year. Year-end capital flows also indicated institutions are actively and tactically rebalancing portfolios across strategies, sub-strategies, and firm sizes, focusing intently on portfolio duration, credit and interest rate sensitivity, strategic commodity and equity market exposures, as well as advanced metrics of defensive capital preservation. Funds which have demonstrated their strategy's robustness through the multiple market cycles since the beginning of the historic pandemic and which are effectively positioned to navigate these multi-asset trends are likely to lead industry performance and growth into the new year.

1.6 Literature review

1.6.1 Evolution of main thread HFs studies

Due to the increase popularity of HF and the presence of HF data collected by institutions, a plenty of studies have been performed during last 25 years with the aim to study HF industry and assess hedge funds performance. The first innovative study on HF performance saw its light in 1997 with Hsieh and Fung which extended Sharpe (1992) asset-class based style regression[1] (that in turn was an extension of CAPM of Sharpe 1964).

While Sharpe's focus was to mimic the performance of mutual funds that implement a strategy of buy and hold of asset classes, Fung and Hsieh (1997,2001) were oriented to replicate HF performance which used also dynamic trading strategies like short-selling, derivatives and leverage.

[1] An asset-based model is a univariate or multivariate regression where risk factors are securities; when class of securities are gathered together and proxied with market indices, it is called asset-class based model.

Sharpe's style regression could not be extended easily because of the infinite number of dynamic trading strategies existing, so Fung and Hsieh clustered individual funds into five different strategies of trading through common factor analysis. One of them, "trend-follower", exhibited returns that were *not linear* but large and positive during the best and worst performing months of the global equity market, mimicking lookback straddles payoffs. To replicate dynamic trading in the underlying assets, Fung and Hsieh (2002, 2004) included in their model five non-linear trend following factors represented by the monthly returns of lookback straddles on commodities, currencies, 3-months interest rates, stocks and government bonds.

Evidence of option-like payoffs in hedge funds returns can be found also with Mitchell and Pulvino (2001) and Agarwal and Naik (2004). Using this time Fama and French (1992) asset-based regression, Mitchell and Pulvino focus shifted on Merger arbitrage HFs which they discover couldn't detach completely from the market risk. In particular, HFs returns were positively correlated with market returns during bearish markets but uncorrelated with market returns in appreciating markets. The same outcome is to be found being short on put options. Adopting instead a Fung-Hsieh style regression, Agarwal and Naik (2004) accomplished a broader study discovering that non-linear short put option payoffs were found not only in risk arbitrage and trend follower HFs but also on a wider range of HF strategies. It seems that HFs behave as insurance sellers, suffering from huge losses during economic downturns.

Few years later, a new school of thought was emerging regarding the difficulty of replication of HFs returns through same financial risk factors used also for mutual funds assessment of performance because of the complexity of HFs investment vehicles. This new approach called "up-bottom" focused on assessing HFs returns through ad-hoc proper financial and macroeconomic risk factors together with advanced statistical techniques. Starting from Aragon (2006), he found that the performance of hedge funds can be largely affected by liquidity risk premium due to the lock-up period. Indeed, HF mangers can invest more easily than mutual funds in illiquid securities that allow to earn higher returns because they can dispose of the capital for a longer period of time. This view is coherent with Getmansky et al. (2004) that claimed HFs returns where serial correlated if compared to other alternative investments because they all share high risk of illiquidity exposure.

Moreover, Bali, Brown, & Caglayan (2011) claimed that there is a positive relation between hedge fund exposure to default risk premium and hedge fund future returns. Indeed, investors demand higher expected returns in recessions and lower expected returns in booms when holding risky assets. They also found that hedge funds with lower exposure to inflation derived higher returns in the future because of uncertainty in the economy. Bali et al. in 2014, extending 2011 work, found that default spread, term spread, aggregate dividend yield, inflation rate, and the growth rate of real gross domestic product per capita could describe a significant proportion of the cross-sectional returns among hedge funds.

Racicot & Theoret (2016) examined the behavior of the cross-sectional dispersions of hedge funds returns, market betas and alphas focusing on times of macroeconomic uncertainty. Macroeconomic uncertainty was modeled using the conditional variances of six macro and financial variables (growth on industrial production, interest rate, inflation, market return, growth of consumer credit, and the term spread). Using the Kalman filter technique they found that hedge fund market beta reduces with macroeconomic uncertainty, whereas the dispersion of hedge fund returns and alphas increase.

In 2010 Agarwal et al. introduced for the first time dynamic higher moment risk factors like Volatility of Aggregate Volatility of equity market return to better target the HFs non-linearity performance. With a style-by-style analysis they showed that higher-moment risks were more significant for directional HFs rather than for equity market neutral ones.

In later papers Racicot and Théoret (2018, 2019, 2021) and Gregoriou, Racicot, Theoret (2020) relying on nonlinear impulse response functions they found asymmetric behaviour of cross-sectional dispersion hedge funds returns depending on the phase of the business cycle. Indeed, HFs during uncertainty periods seemed to trail their higher moment risk by timing macroeconomic and financial risk and uncertainty: managers seem to trade co-skewness and co-kurtosis in order to build optimal portfolios, being less exposed to them during turmoil periods. Directional HFs strategies benefitted more from volatility during period of crisis mimicking lookback straddles. Moreover, they showed the HFs return smoothing effect derived from the more illiquid type of investment vehicles, especially during crisis. With robustness check they showed that this result may lead to a substantial underestimation of systematic risk, in particular fat-tail risk.

1.6.2 Alternative approaches

Up-bottom approaches and higher moment models are used to study and justify the performance of HFs but do not provide simple and useful insights for an investor. This is why a new current of relatively recent alternative studies has become established in the literature having as main objective the research of structural breaks and patterns in HFs returns not purely the replication of HFs returns. "Alternative approach" is the name given by Stafylas et al. in 2017 in their model review where they consider as common factor for these models the extensive use of advanced econometric techniques. In the end, we have decided to perform a research study aligned with this stream of studies because they are coherent with our final scope of providing an easy instruction manual to an investor willing to invest in successful hedge fund strategies.

Already with Fung and Hsieh in 2004 the perspective that linear models were inappropriate was making its way. In fact, the two proposed to test stability of the regressors trying to find sample breakpoints with a Kalman filter identifying March 2000 (the end of the Internet bubble) and September 1998 (the LTCM debacle) as HFs performance turning points. They found that with this approach they had a better fit of the model resulting in higher adjusted R^2.

Later in 2006, Bollen and Whaley introduced the use of Kalman Filter in addition to changepoint style regression like Fung-Hsieh (2004), to account for non-static risk exposures. Here, Kalman Filter used an AR(1) to model the time varying loadings of the risk-factors. However, Bollen and Wiley found that changepoint regression was superior in fitting HFs returns with respect to the stochastic beta model according to BIC criterion. Using Fama and French(1992) 3 factor model, Racicot (2007) argued that far from being a pure random walk process, the conditional alpha and beta might also react to conditioning financial market information as the interest rate variable, the market risk premium, and the squared market risk premium. Results showed that the alphas of the majority of the HFs index strategies were not very responsive to financial market variables, except for distressed securities. A pure recursive process seems preferable for the alpha than a conditional model. On the contrary, estimated beta followed a cycle related to financial market conditions.

Another approach was adopted by Billio et al. 2010 which implemented a Markov regime-switching beta model with 3 regimes to measure the dynamic exposures of hedge fund CSFB /Tremont indexes to risk factors during three different market regimes (according to mean and volatility of equity market risk factor they defined “tranquil”, “up”, “down”) from 1994 to 2009. They had two main results. First, hedge funds exhibited significant non-linear exposures not only to the equity market risk factor, but also to liquidity risk factor, commodities, volatility, credit and term spreads. Moreover, many risk factor exposures were zero during tranquil regimes while over the market downturns were statistically different from zero.

Related to the above study was one from Ozgur et al. (2011) who found evidence for a decline in risk adjusted returns (alpha) for most investment strategies, especially after the stock market crash in 2000, using a Markov-regime switching model with 3 regimes (crash state, low mean and high mean similarly to Billio et al. (2010)) and Dow Jones Credit Suisse Hedge Fund Indices database from 1994 to 2010. Moreover, they found that co-movement in hedge fund returns, after counting for common risk factors, was not only restricted to times of extreme financial turbulence. Last but not least, they linked the probability of observing the “crash state” to liquidity proxies and panic, measured by the VIX index and found that both played a significant role in leading to contagion.

Another study was from Jawadi & Khanniche (2012). They used the CSFB/Tremont database (hedge fund indices) over the period 1994 to 2009. They examined the adjustment dynamics of hedge fund returns and their exposures using a threshold-switching regime model. They confirmed asymmetry and non-linearity dynamics of hedge fund returns, showing that they differ asymmetrically with respect to different financial conditions especially after the global financial crisis (2008–2009).

A more recent study was the one from Stafylas et al. 2018 performed using BarclayHedge and Eurekahedge indexes. They used exogenous and endogenous break points based on business cycles and on a regime switching process conditional on different states of the market to investigate HFs performance. In addition, they perform a stepwise regression to offset not significant regime risk factors. They found that during difficult market conditions most hedge fund strategies do not provide significant alphas and reduce both the number of their exposures to different asset classes and their portfolio allocations, while some non-directional strategies even reverse their exposures. Moreover, they showed that factors related to commodity asset classes are more common during turmoil conditions whereas factors related to equity asset classes are most common during good market conditions.

2 Data

2.1 Hedge funds indexes

A standard method for modeling hedge fund strategies risk is to use broad-based index of performance over a period of time. However, as it was stated in Chapter 1, HFs don't have a legal obligation to publish neither their investment strategies nor performances. This entails the low availability of quality data because indexes constructed from averaging individual hedge funds can inherit errors present in the hedge fund databases. The problem of biases in the literature is well known since early 2000s with Brown, Goezmann and Ibbotson 1999. In the end, we can distinguish 3 types of biases.

Selection Bias

Hedge funds decision to share their data to a database is voluntary because they are not required to publicly disclose their activities. In addition, no hedge fund industry association exists (comparable to the Investment Company Institute for mutual funds) that could act as a central depository of fund information. Hedge fund data are generally collected by few data vendors and sold, with the consent of the hedge fund manager, to accredited investors. Therefore, selection bias can arise if the sample of funds in the database is not a representative sample of the universe of hedge funds.

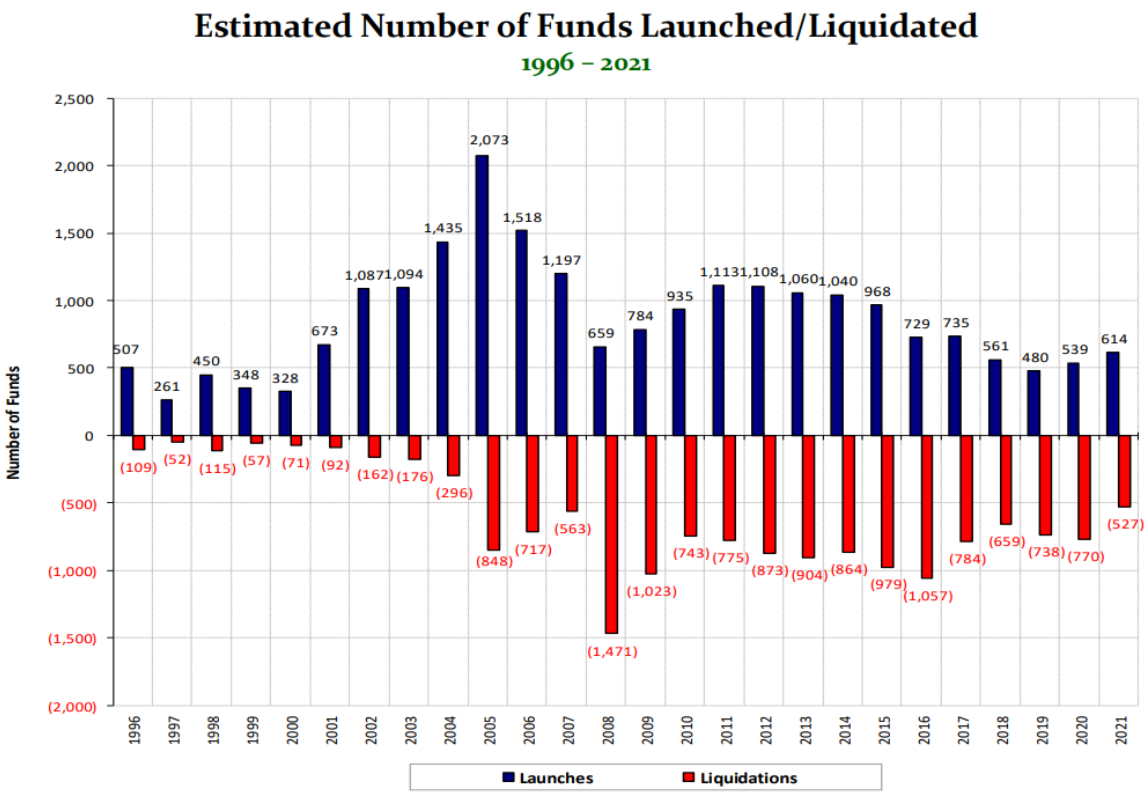


Figure 3 - Estimated number of funds Launched and Liquidated per year

Survivorship bias

Most hedge fund databases provide information only on operating funds. Funds that have stopped reporting information or ceased operation due to poor performance are regarded as uninteresting to investors and are purged from the database. The result is survivorship bias because the performance of disappearing funds is typically worse than the performance of surviving funds. Observing Figure 4, we can see how many HFs got liquidated or exited the industry. In particular, the HF industry had a very stressful period during the great financial crisis: almost 1500 funds got liquidated compared to an inflow of 659 new funds. This is clear evidence about how survivorship bias can affect database vendors.

Instant-history bias

When a fund enters a database, its past performance history (prior to the entry date) is appended to the database, which creates instant-history bias. Many new funds start with an incubation period to accumulate a track record. If the performance is "good enough" they enter a database to seek for new investors. If the performance is "bad," they cease operations. Thus, when a data vendor backfills the fund's performance, the average return in the database is biased (upward). When hedge fund indexes are created from hedge fund databases, they inherit all the instant history biases.

Although HFs industry nowadays has flourished, there are few options to recover data. Online it is possible to access to composite indices of pool of funds by HFR, Credit Swisse, Lipper TASS, GAI, BarclayHedge, Eurekahedge, Morningstar. All the recent and past studies used databases coming from one of these providers or other sources no longer available. In order to access to Lipper TASS database, you either have to be a student of Princeton university or pay a conspicuous amount of money, same for GAI database which doesn’t disclose indices for free. According to two studies performed by Juha et al. (2012-2019), Eurekahedge and Morningstar exhibited higher survivorship and backfilling bias in the construction of the databases compared to HFR and BarclayHedge. More specifically, they consider HFR to be the most suitable for research purposes using only one database. After performing due diligence between HFR and Credit Swisse strategy indexes, we relied on HFR’s database for two reasons. First, Credit Swisse’s strategies indexes seem to be over performant being similar in trend to Morningstar and Eurekahedge. Secondly, HFR is the most complete in terms of availability of strategy indices, plus it has a very clear index construction methodology, which gives both local and global views of the HFs ecosystem. Like all the providers, HFR discloses the methodology to construct the indices but does not reveal the individual components included, unless under payment. However, as a proof of the quality of the indexes, there exist studies of very well-known academics that have used HFR indices for their researches like Fung and Hsieh (2004), Racicot & Theoret (2007) and Dimitrios et al. (2009).

Below we have plotted HFR composite index together with Eurekahedge Hedge Fund Index, Barclay Hedge Fund Index and an equal-weighted index from a database entirely constructed by ourselves composed by more than 1100 active HFs at April 2022. We built our index including 1100 active funds, spread among all the most common strategies mentioned before. To have a hint about the effect of survivorship bias in terms of difference of performances, we show the difference on monthly performances and aggregate results of the indexes:

- Our index (equal-weighted): 0,76% per month and 632% aggregate results in the time period
- HFRI (equal-weighted) Composite Index: 0,47% and 238%
- EurekaHedge (equal-weighted): 0,70% and 530%
- Barclay HF Index (equal-weighted): 0,53% and 292%

Lower or upper performances are due to the amount of dead funds inside an index. As it can be seen in the graph, our index is the most performant, followed by EurekaHedge, Barclays and HFR Composite Index. HFR is the most underperforming index due to its relatively high presence of dead funds, further validating the hypothesis that provides one of the best representations of the HFs world.

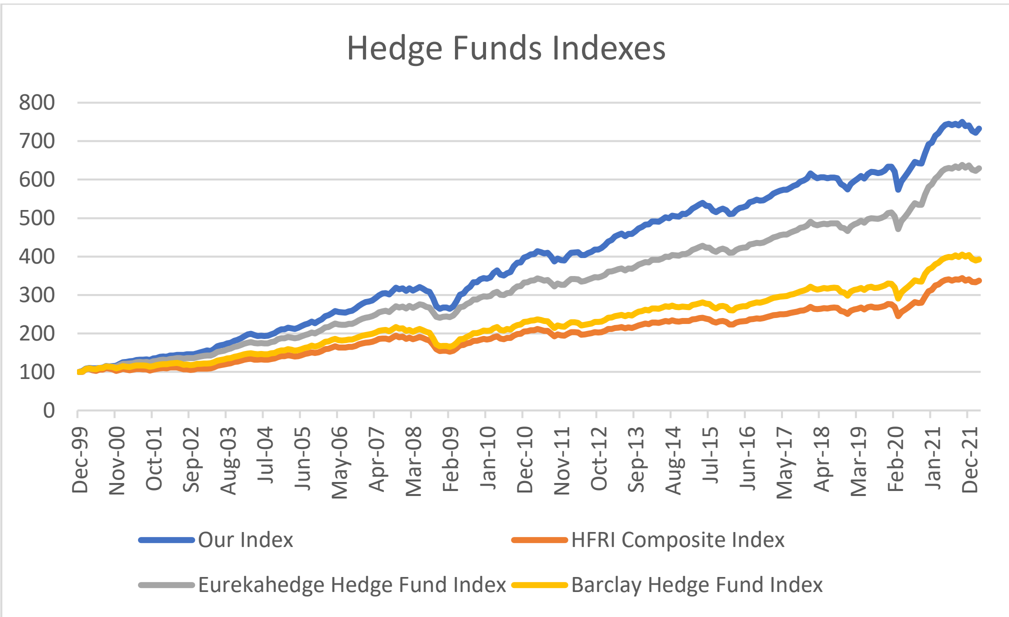


Figure 4 – Performance comparison of different hedge fund indexes

Following Dimitrios strategies choices (2009) we have considered in our analysis 10 HFs of the group named “HFRI” from the data vendor HFR. Each index report monthly returns starting from February 2001 to July 2022. We decided to choose the most common and used strategies in HF studies. We below report the names and explain how these indices are calculated.

- Global macro (GM)
- Emerging markets (EM)
- Equity hedge (EH)
- EH: Equity market neutral (EH-N)
- Event-Driven (ED)
- ED: Distressed securities (ED-D)

- ED: Merger arbitrage (ED-M)
- Relative value (RV)
- RV: Fixed Income-Convertible arbitrage (RV-CA)
- RV: Fixed Income-Corporate (ex-fixed income arbitrage) (RV-A)

The HFRI Indices are broadly constructed indices designed to capture the breadth of hedge fund performance trends across all strategies and regions. In order to be considered for inclusion in the HFRI, a hedge fund manager must submit a complete set of information to HFR Database. To be eligible for inclusion in the HFRI Indices a hedge fund must satisfy different criteria:

- Report monthly returns
- Report Net of All Fees Returns
- Report assets in USD
- Meet the AUM minimum eligibility criteria of:
  a) Having at least $50 Million USD under management on the last reported month prior to the annual rebalance
  b) Having at least $10 Million USD under management on the last reported month prior to the annual rebalance and have been actively trading for at least twelve (12) months.
- Open to new investment
- Available in a fund structure

The index itself is affected by changings in terms of both number of funds and Net Asset Value, due to the inflows and outflows of HFs from the database. If a fund is not able to satisfy the minimum requested criteria, it will be ejected from the index; in such a case, the weight of the constituent will be spread equally to the remaining funds inside the index, or it will be allocated to another fund entering the index for the first time or to a prospective constituent. Rebalancing the index is done on an annual basis and involves changings in NAV, weights and number of funds.

Funds eligible to be inside an HFRI index at the evaluation date are combined together and will form different families of indexes. HFRI offers different typologies of indexes:

- HFRI Equal-Weighted Composite Index: all qualified funds are included
- HFRI Asset Weighted Composite Index: all qualified funds are included
- HFRI Single Strategy Equal-Weighted Indices: all qualified funds in the specific strategy are included
- HFRI Single Strategy Asset Weighted Indices: all qualified funds in the specific strategy are included
- HFRI Single Substrategy Equal-Weighted Indices: all qualified funds in the specific substrategy are included

- HFRI Regional Equal-Weighted Indices: all qualified funds with the specific region investments focus are included

For the purpose of our study, we are using just HFRI EW Single Strategy Indices and HFRI EW Single Substrategy Indices.

2.2 Risk factors

Since we consider HF strategies with a global perspective, we included investable risk factors indices with a global view of the market as well as momentum and, at a later time, ad-hoc non-linear risk factors. Our candidate factors are selected according to specific criteria: availability, what other authors used based on their significance, the collinearity between them and correlation with strategies. In particular, we considered as a threshold a VIF (variance inflation factor) of 5 among risk factors and minimum risk factors correlation to HF strategies of an absolute value of 25%. The results are reported in table 1. Other factors not mentioned below, like Fama and French 1992 HML (high-minus-low) factor and Gold price index, resulted correlated less than 25% to the HFs strategies considered, and so excluded from our analysis.

2.2.1 Equity factors

S&P500 (SP500)

The S&P U.S. Indices are a family of equity indices designed to measure the market performance of U.S. domiciled stocks trading on U.S. exchanges. The family is composed of a wide range of indices based on size, sector, and style. The indices are weighted by float-adjusted market capitalization. In particular, the S&P500 measures the performance of the US large-cap segment of the market, composed of 500 constituent companies. It has been widely adopted in the HFs studies like Mitchell, Pulvino (2001), Fung, Hsieh (2004) Billio et al (2010), Metzger, Shenai (2019) Lambert, Platania (2020) and in general financial literature to represent the evolution of world developed countries equity.

MSCI ACWI EM IMI (EMMKT)

Following Dimitrios et al. (2009) and Jawadi, Khannike (2012), we included the MSCI Emerging Markets Investable Market Index (IMI) as it captures large, mid and small cap representation across 24 Emerging Markets countries. With 3,165 constituents, the index covers approximately 99% of the free float-adjusted market capitalization in each country.

Monthly Momentum Factor (MOM)

Momentum investing is a system of buying stocks or other securities that have had high returns over the past three to twelve months, and selling those that have had poor returns over the same period. In financial studies, it has been observed that securities that have risen in recent months tend to continue to do so for a few more months. Its use date back to Carhart 1997 four factor model where Carhart added this factor to 3-factor model of Fama and French to explain asset prices. In order to proxy this factor, we have taken the MOM provided by the online library of Kenneth R. French. Here, six value-weight portfolios formed on size and prior (2-12) returns are used to construct Mom. The portfolios, which are formed monthly, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on prior (2-12) return. The monthly size breakpoint is the median NYSE market equity. The monthly prior (2-12) return breakpoints are the 30th and 70th NYSE percentiles. MOM is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios. The six portfolios used to construct MOM each month include NYSE, AMEX, and NASDAQ stocks with prior return data. To be included in a portfolio for month t (formed at the end of month t-1), a stock must have a price for the end of month t-13 and a good return for t-2. In addition, any missing returns from t-12 to t-3 must be -99.0, CRSP's code for a missing price. Each included stock also must have ME for the end of month t-1. Many authors like Dimitrios et al. (2009), Billio et al. (2010), Jawadi, Khanniche (2012), Bali et al. (2014), Stafylas et al. (2018) have included momentum in their HFs performance attribution regressions.

Delta Volatility Index (DVIX)

The VIX Index is a financial benchmark designed to be an up-to-the-minute market estimate of expected volatility of the S&P 500 Index, and is calculated by using the midpoint of real-time S&P 500 Index (SPX) option bid/ask quotes. More specifically, the VIX Index is intended to provide a forward-looking measure of the forecasted evolution of S&P 500 Index in the 30 days from the time of each tick of the VIX Index. Thus, the VIX Index is in contrast to realized (or actual) volatility, which measures the variability of historical (or known) prices and has an opposite sign with respect to forecasted fluctuation of SP500. In our analysis following Billio et al. (2010) and Stafylas et al. (2018), we have used Delta VIX, which is the absolute variation of the index between 2 subsequent periods of time.

Small Minus Big (SMB)

Small minus big factor, also referred to as the small capitalized firm effect, considers the contribution of the small size of the firm compared to bigger ones in explaining the excess of performance. It was introduced for the first time in 1992 in the three factors Fama and French model to describe stocks return and has been widely used in HFs performance attribution models. Albeit there is heterogeneity in the construction of SMB examples of the use are to be found in Bollen and Whaley (2006), Racicot et al. (2007), Dimitrios et al. (2009), Billio et al. (2010), Jawadi, Khanniché (2012), Bali et al.(2014) Lambert, Platania (2020).

In order to give to SMB a global perspective we have modeled it as *MSCI ACWI Small Cap monthly return – MSCI ACWI IMI monthly return.*

- *MSCI ACWI Small Cap Index*

  The MSCI ACWI Small Cap Index captures small cap representation across 23 Developed Markets and 24 Emerging Markets countries. With 6,338 constituents, the index covers about 14% of the free float-adjusted market capitalization in each country.

- *MSCI ACWI Index*

  The MSCI ACWI Index is designed to represent performance of large- and mid-cap stocks across 23 developed and 24 emerging markets. It covers more than 2,933 constituents across 11 sectors and approximately 85% of the free float-adjusted market capitalization in each market.

2.2.2 Bond factors

Albeit modeled differently among academics, Billio et al.(2010), Jawadi, Khanniche (2012), Bali et al.(2014), Stafylas et al.(2018), Lambert, Platania(2020) have included these 2 factors in their analysis.

Credit Spread Factor (CRSPRD)

The credit spread factor is used to reflect the additional yield obtained investing in corporate bond with respect to treasury bonds with same maturities in order to highlight the riskiness of the first.

To guarantee a global perspective of the factor, we modeled CRSPRD as: *JPM Global bond index – Bloomberg Global Yield.*

- *JPM Global Bond Index*

  JPM GABI is an extension of JPM GABI US, a U.S. dollar denominated, investment-grade index spanning asset classes from developed to emerging markets, and the JPM GABI extends the U.S. index to also include multi-currency, investment-grade instruments. JPM GABI represents nine distinct asset classes: Developed Market Treasuries, Emerging Market Local Treasuries, Emerging Markets External Debt, Emerging Markets Credit, US Credit, Euro Credit, US Agencies, US MBS, Pfandbriefe – represented by well-established J.P. Morgan indices. The index is constructed from over 3,200 instruments issued from over 50 countries, and collectively represents US$8.6 trillion in market value. The JPM GABI is constructed from over 5,500 instruments issued from over 60 countries and denominated in over 25 currencies, collectively representing US$20 trillion in market value.

- *Bloomberg Global Yield*

  The Bloomberg Global High Yield Index is a multi-currency measure of the global high yield debt market. The index represents the union of the US High Yield, the Pan-European High Yield, and Emerging Markets Hard Currency High Yield Indices. The high yield and emerging markets sub-components are mutually exclusive. Until January 1, 2011, the index also included CMBS high yield securities.

Term spread (TRSPRD)

The spread is used to reflect the additional yield obtained in investing in treasury bonds with respect to risk-free benchmark.

In order to give a global view, we created TRSPRD as: *Bloomberg Global Yield – 1 month US Treasury bill*

- *Bloomberg Global Yield*

  See the description reported above.

- *1 month US-Treasury Bill*

  The 1 Month Treasury Rate is the yield received for investing in a US government issued treasury bill that has a maturity of 1 month. The 1-month treasury yield is included on the shorter end of the yield curve. It is used as a proxy for virtually risk-free investment.

2.2.3 Commodity factors

We took into consideration from the FRED site *Global metal index (METAL)*, *Global price of Agricultural Raw material index (RAWM)* and *Global price of energy index (ENERGY)* which consider respectively the global market of metal, energy and agriculture raw material according to the largest exporter of the commodity. Even in this case there is heterogeneity in the choice of the indexes but every author includes at least a commodity component in their studies. Stafylas et al. (2018) and Dimitrios et al. (2009) for instance include SPGSCI indexes.

2.2.4 Non-linear primitive trend following factors

Hedge fund managers typically employ dynamic trading strategies that result in option-like returns that can't be modeled using just linear-factor models with standard asset benchmarks. A remedy was suggested by Glosten and Jagannathan (1994), where they propose to use benchmark-style indices that have embedded option-like features. This has been done in Fung and Hsieh (2002) where they defined style factors from a broad sample of trend-following hedge fund strategies returns. By construction, these factors, called *"primitive trend-following"* because constructed on a single trade, captured much of the option-like features during extreme market periods while preserving the general lack of correlation with standard asset benchmarks.

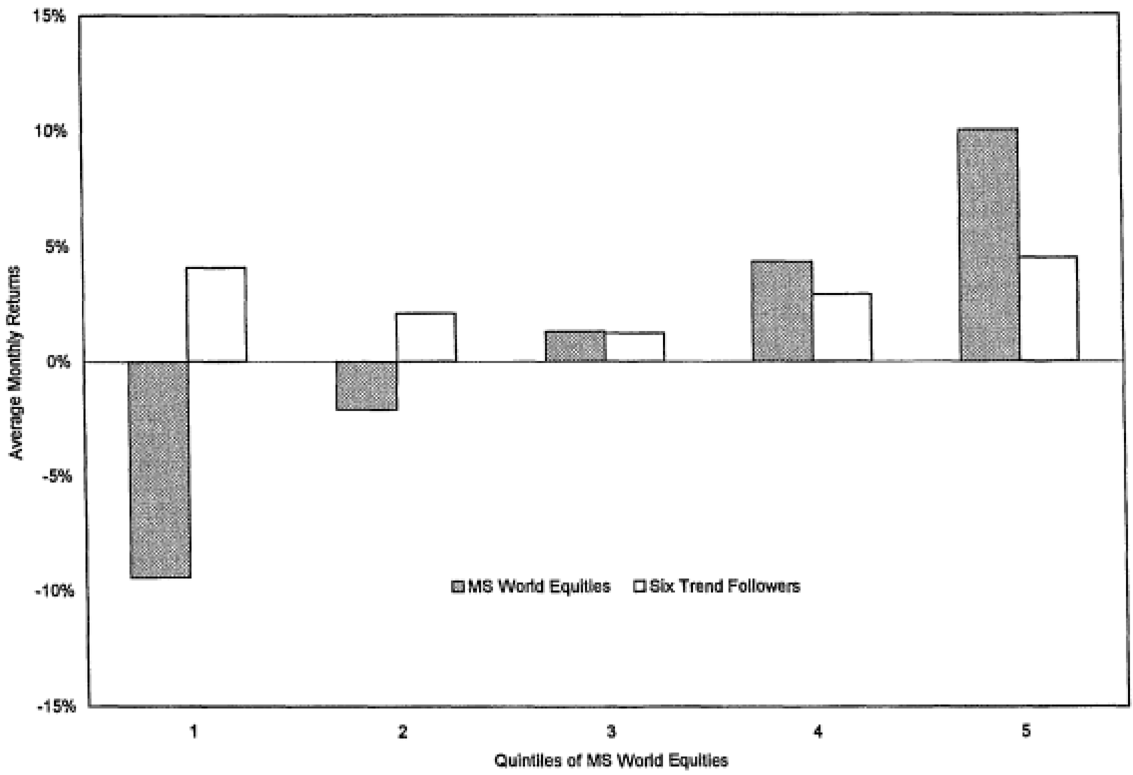


Figure 1
Average monthly returns of six large trend-following funds in five different MS world equity market states
Source: Fung and Hsieh (1997a).

Figure 5

Fung and Hsieh posit that the simplest trend-following strategies have the same payout of structured option known as "lookback straddle", which is the combination of 2 instruments. The first one is a lookback call option that gives the right to the owner to buy the underlying asset at the lowest price over the life of the option whereas the lookback put option gives the right to sell at the highest. Together, they provide the ex-post maximum payout for each PTFS over a fixed period of time. In the empirical implementation, Fung and Hsieh have used three-month options, which tend to be the most liquid options.

Moreover, since lookback options are not exchange-traded contracts, so prices could not be directly observed, they replicated the payout of a lookback straddle by rolling a pair of standard straddles, following the intuition of Goldman et al. (1979). Then, to verify if the PTFS can mimic the performance of HFs, Fung and Hsieh empirically generated the historical returns of the PTFS applied to the most 26 active markets in the world.

- To construct PTFS-STK (PTFS-stock) they used the futures contracts on the S&P 500 (CME), Nikkei 225 (Osaka), FTSE 100 (LIFFE), DAX 30 (DTB), and the Australian All Ordinary Index (SFE).
- For PTFSBD (PTFS-bond), they used futures contracts on the U.S. 30-year Treasury bonds (CBOT), UK Gilts (LIFFE), German Bunds (LIFFE), the French 10-year Government Bond (MATIF), and the Australian 10-year Government Bond (SFE).
- To construct PTFSFX (PTFS-forex) futures contracts on the British pound, Deutschemark, Japanese yen, and Swiss franc on the CME were used.
- PTFSIR (PTFS-interest rates) is obtained using futures contracts on the 3-month Eurodollar (CME), Euro-Deutsche Mark (LIFFE), Euro-Yen (TIFFE), the Paris Interbank Offer Rate (PIBOR) (MATIF), 3-month Sterling (LIFFE), and the Australian Bankers Acceptance-Rate (SFE).
- For PTFSCOM (PTFS-commodity) they used futures contracts on soybean, wheat, and corn futures traded on the CBOT and gold, silver, and crude oil traded on the NYMEX.

As a final remark, we must say that in reality trend followers make multiple entries and exits over a sufficiently long investment horizon smoothing or increasing PTFS payoffs. However, we consider PTFS as the best factors to proxy HFs behaviors and are widely used in the literature. Examples beside Fung and Hsieh are found in Platania et al. (2020), Bollen and Whaley (2006), Agarwal et al. (2010), Ozgur, Akay (2011).

Comparison between buy and hold, market timing and trend following strategies

In this section our scope is to provide a mathematical comprehension of the differences among different styles of profit strategies to let the reader understand the meaning of a "trend-following strategy". Let Z(t) denote the return per dollar invested in the stock market and R(t) the return per dollar invested in Treasury bills in period t-x. The standard buy-and-hold strategy buys at t-x and sells at the end of the period, generating the payout Z(t). The Primitive Market Timing Strategy instead, attempts to capture the price movement between t-x and t. If Z(t) is expected to be greater (lower) than 0, a long (short) position is initiated. For a perfect market timer, Merton (1981) showed that the return of the portfolio is given by R(t) + Max{0, Z(t) - R(t)} + Max {0, R(t) - Z(t)}, which is the return of a portfolio of bills and a straddle on stocks. The PTFS, on the other hand, attempts to capture the largest price movement during the time interval. Consequently, the return of a portfolio invested at time t-x generates a payout equal to R(t) + Max{0, Zmax(from t-x to t) - R(t)} + Max {0, R(t) - Zmin(from t-x to t)}. Generally, market timers enter into a trade in anticipation of a price move over a given time period, whereas trend followers trade only after they have observed certain patterns on price movements during a period.

Empirically, the payout of the PMTS equals that of the PTFS if and only if Smax and Smin occur at the beginning and end of the period in any order. Hence, as the investment horizon shrinks, the payouts of the two strategies converge. Instead, if the investment horizon lengthens, the payout of the two strategies will diverge, because the probability of Smax and Smin being interior points to the investment horizon increases.

Strategy	SP500	EMMKT	SMB	MOM	DVIX	TRSPRD	CRSPRD	PTFSBD	PTFSCOM	PTFSIR	PTFSFX	PTFSSTK	ENERGY	RAWM	METAL
GM	23.01%	36.23%	16.45%	4.57%	-19.36%	6.53%	14.88%	16.94%	18.40%	-2.60%	27.81%	-1.12%	16.97%	18.61%	23.81%
EM	71.33%	92.75%	43.03%	-36.16%	-55.82%	-19.80%	79.15%	-34.69%	-27.64%	-39.28%	-28.91%	-37.83%	32.78%	27.27%	36.03%
EH	85.41%	86.50%	51.29%	-38.45%	-68.07%	-25.85%	81.27%	-31.00%	-27.12%	-40.97%	-31.13%	-39.83%	33.41%	25.46%	31.17%
EH-N	39.91%	41.42%	31.50%	14.46%	-40.07%	-19.55%	39.08%	-22.21%	-26.28%	-27.61%	-14.16%	-24.59%	25.13%	17.96%	18.59%
ED	77.26%	77.58%	56.71%	-39.36%	-60.18%	-25.51%	83.53%	-36.22%	-31.84%	-44.14%	-35.40%	-42.55%	39.54%	28.94%	38.35%
ED-D	61.46%	63.23%	53.45%	-31.30%	-46.41%	-25.17%	77.33%	-37.95%	-28.92%	-39.88%	-32.52%	-36.26%	48.36%	37.18%	43.52%
ED-M	62.33%	64.61%	51.12%	-27.52%	-51.40%	-12.63%	62.73%	-28.08%	-26.42%	-34.72%	-31.75%	-40.14%	25.62%	15.05%	22.26%
RV	63.57%	69.59%	49.40%	-31.83%	-53.02%	-17.79%	83.00%	-38.37%	-34.79%	-44.36%	-40.73%	-45.55%	44.90%	33.81%	39.16%
RV-CA	54.17%	63.76%	42.86%	-32.95%	-46.10%	-11.92%	77.06%	-24.46%	-28.89%	-41.48%	-32.43%	-32.70%	32.31%	31.23%	35.43%
RV-A	64.29%	68.08%	49.55%	-35.79%	-49.31%	-16.19%	84.01%	-38.07%	-35.95%	-46.33%	-39.66%	-43.17%	41.99%	32.66%	41.86%

>70%

>25% V <-25%

Y	DVIX	SMB	EMMKT	SP500	TRSPRD	CRSPRD	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	ENERGY	RAWM	METAL	R^2	VIF
DVIX		•	•	•	•	•	•	•	•	•	•	•	•	•	•	0.68	3.12
SMB	•		•	•	•	•	•	•	•	•	•	•	•	•	•	0.26	1.34
EMMKT	•	•		•	•	•	•	•	•	•	•	•	•	•	•	0.67	3.00
SP500	•	•	•		•	•	•	•	•	•	•	•	•	•	•	0.78	4.45
TRSPRD	•	•	•	•		•	•	•	•	•	•	•	•	•	•	0.18	1.22
CRSPRD	•	•	•	•	•		•	•	•	•	•	•	•	•	•	0.76	4.13
MOM	•	•	•	•	•	•		•	•	•	•	•	•	•	•	0.36	1.57
PTFSBD	•	•	•	•	•	•	•		•	•	•	•	•	•	•	0.27	1.37
PTFSFX	•	•	•	•	•	•	•	•		•	•	•	•	•	•	0.34	1.50
PTFSCOM	•	•	•	•	•	•	•	•	•		•	•	•	•	•	0.23	1.29
PTFSIR	•	•	•	•	•	•	•	•	•	•		•	•	•	•	0.31	1.45
PTFSSTK	•	•	•	•	•	•	•	•	•	•	•		•	•	•	0.43	1.77
ENERGY	•	•	•	•	•	•	•	•	•	•	•	•		•	•	0.35	2.83
RAWM	•	•	•	•	•	•	•	•	•	•	•	•	•		•	0.32	3.09
METAL	•	•	•	•	•	•	•	•	•	•	•	•	•	•		0.35	2.83

Table 1 - The table on the top shows risk factors' correlations with HFs strategies.
The table on the bottom reports VIF factor calculated as 1/(1-R^2). Y defines the dependent variable of the regression while the marker is in correspondence of the regressor variables used.

2.3 Performance ratio

2.3.1 Sharpe Ratio

The Sharpe ratio compares the return of an investment with its risk. It's a mathematical expression of the insight that excess returns over a period of time may signify more volatility and risk, rather than investing skill. Sharpe ratio, also called reward-to-variability ratio, is presented as an outgrowth of Sharpe's previous work on the capital asset pricing model of 1964.

The Sharpe ratio's numerator is the difference over time between realized, or expected, returns and a benchmark such as the risk-free rate of return or the performance of a particular investment category. Its denominator is the standard deviation of returns over the same period of time. The Sharpe ratio is defined as:

$$Sharpe\ Ratio = \frac{E(Rp_i - Rf_i)}{\sigma p}$$

Where:

- Rp_i = return of portfolio at time i
- Rf_i = risk free rate at time i
- σp = standard deviation of the portfolio's excess return

The ratio is useful in determining to what degree historical returns measured compared to a benchmark were accompanied by excess volatility.

2.3.2 Sortino Ratio

The Sortino ratio measures the risk-adjusted return of an investment asset, portfolio, or strategy. It is a modification of the Sharpe ratio but penalizes only those returns falling below a required rate of return.

Sortino ratio divides the excess return on a portfolio, with respect to a minimum acceptable return (MAR), by a term called the downside deviation (deviation of returns below the MAR).

The Sortino ratio of a portfolio is defined as:

$$SORp = \frac{E(Rp_i\ -\ MAR)}{\sqrt{E(Min\ (Rp_i\ -\ MAR), 0)^2\ )}}$$

where Rp_i is the return of the portfolio at time i, MAR is the minimum acceptable return, and $\sqrt{E(Min\,(Rp_i\, -\, MAR), 0)^2\,)}$ is the Downside risk (DR).

The Sortino ratio is used to score a portfolio's risk-adjusted returns relative to an investment target using downside risk.

2.3.3 Upside potential ratio

The upside-potential ratio is a measure of a return of an investment asset relative to the minimal acceptable return. The measurement allows a firm or individual to choose investments which have had relatively good upside performance, per unit of downside risk.

The Upside Potential ratio of a portfolio is defined as:

$$UPRp = \frac{E(Max(Rp_i\, -\, MAR), 0)}{\sqrt{E(Min(Rp_i - \, MAR), 0)^2})}$$

The upside potential ratio divides the average of the excess positive returns, compared to the minimum acceptable return (MAR), by the downside risk (DR).

The ratio attempts to highlight the magnitude of excess positive returns over the portfolio's potential risk of loss.

3 Empirical Analysis

3.1 Models used

We decided to perform our study according to the styles put in place by many other authors accomplished before. One initial static model and one dynamic. Here below we describe the two models in details.

3.1.1 Asset-based factor model

The first statistical approach used in this paper is the cross-sectional multiple linear regression. With this method the explicit assumption is the existence of observable factors that explain more or less faithfully observed data. In finance we can have two different views of application:

- Pricing of an asset or a portfolio in the view of CAPM alpha and APT.
- Return-based analysis in which the aim is to evaluate a fund performance deconstructing the returns of investment strategies using a variety of regressors. This view is the one we will adopt.

In our study the observed data are monthly strategy indexes hedge fund returns, net of fees and risk free. Regarding regressors instead, in the first regression typology we include only simple investable financial regressors (for example, it's possible to invest in the exchange market through ETFs mimicking indexes' performance) while in second instance we include more complex and far from being easy investable derivatives factors. For both regressions, we adopt a stepwise approach to offset non significative risk factors.

The regression is written as:

$$HF_t = \alpha_i + \beta_{i,1}F_1 + \beta_{i,2}F_2 + \cdots + \beta_{i,K}F_K + \varepsilon_i \quad or\ equivalently\colon\ HF_i = \alpha_i + \sum_{J=1}^{K} \beta_{i,j}F_J + \varepsilon_i$$

Where HF_t denotes the HFs' strategy return at time t, K the total number of risk factors, $F_{1,t}, \dots, F_{K,t}$ are the values of the factors at time t, α_i is the excess return of strategy i, $\beta_1, \dots, \beta_K$ are the relevant sensitivities, and ε_t is a i.i.d zero mean random variable with ω^2 variance.

In order to leave out errors in the estimates, risk factors must be mutually exclusive and exhaustive avoiding collinearity. Moreover, the inclusion of risk factors should be parsimonious to have the advantages of dimensionality reduction of the model but at the same time keeping a certain level of accuracy. Usually, regressors are included according to previous evidence acquired in the past literature or with tools like factor model.

3.1.2 Regime-switching theta model

The second implemented statistical approach is a dynamic model named Markov *n* regime-switching theta regression. The aim of the model is to automatize the process of quantile analysis implemented by Fung and Hsieh (1997). While they exogenously imposed certain states at the market looking at the various business cycles, this model tries to identify the states assuming that the return of the market is a random variable which takes its value from an unknown Markov random variable st. The latter can assume as many values as the regimes we believe exist in the market. Once we can capture the state in which the market is in at time t, we can determine the exposure (defined by the theta vector) of the specific hedge fund strategy to the risk factors at time t. Consequently, we obtain *n* different theta vectors according to the *n* regimes.

Method in detail

Suppose that the random variable of interest R_t follows a stochastic process that depends on the value of an unobserved discrete state random variable s_t that follows a discrete Markovian stochastic process S_t. We assume there are 3 regimes for R_t, each characterized by a different mean and variance, and so 3 possible values for s_t: 0,1,2. Mean and variance of the R_t regimes are not known so they have to be estimated through 3 linear only-constant regressions.

$$R_t = \mu(s_t) + \sigma(s_t)\varepsilon_t \qquad \textit{or equivalently}: \begin{cases} R_t = \mu(0) + \sigma(0)\varepsilon_t \\ R_t = \mu(1) + \sigma(1)\varepsilon_t \\ R_t = \mu(2) + \sigma(2)\varepsilon_t \end{cases} \qquad \varepsilon \sim iid\ N(0,1)$$

$t = 1, 2, \dots T$ with T that indicates the total number of monthly observations

R_t = market return at time t

$\mu(s_t)$ = intercept at regime s_t

$\sigma(s_t)$ = standard deviation at regime s_t

Since S_t is a stochastic process, the change in regime should not be regarded as predictable but rather as a random event. The first-order Markov assumption requires that the probability of being in a regime depends only on the previous state so that $P(s_t = j | s_{t-1} = i) = p_{ij}$.

We may write these probabilities in a transition matrix P where the $ij-th$ element represents the probability of transitioning from regime i in period t-1 to regime j in period t.

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{bmatrix}$$

Using the ergodic (unconditional) probability π of each of the n different regimes to initialize the process, we are able to obtain a matrix of state probabilities with dimension *n* x number of observations of R_t.

Afterwards, through Viterbi algorithm we can estimate the hidden most likely state sequence of the Markovian process S_t and so attribute to each observation R_t a state s_t ranging from 0 to 2.

Since our final scope is to assess HFs performance during different states of the market, we divide each HF strategy pool of returns in three regimes performing three different regressions according to S_t. In this way we are able to infer how much the index strategy HF is exposed to each of risk factors belonging to the correspondent regime according to the market states. Below we show the formalization of the 3 regressions:

$$HF_t = \alpha(s_t) + \sum_{i=0}^{I} \vartheta_i(s_t) F_{it} + \omega(s_t)\varepsilon_t\ ,\ i.e.\ \begin{cases} HF_t = \alpha(0) + \sum_{i=0}^{I} \vartheta_i(0) F_{it} + \omega(0)\varepsilon_t \\ HF_t = \alpha(1) + \sum_{i=0}^{I} \vartheta_i(1) F_{it} + \omega(1)\varepsilon_t \\ HF_t = \alpha(2) + \sum_{i=0}^{I} \vartheta_i(2) F_{it} + \omega(2)\varepsilon_t \end{cases} \quad \varepsilon \sim iid\ N(0,1)$$

$i = 1, 2, \dots I$ with I that indicates the number of risk factors

$t = 1, 2, \dots T$ with T that indicates the total number of monthly observations

HF_t = hedge fund strategy return at period t

$\alpha(s_t)$ = intercept of the regime s_t

$\vartheta_i(s_t)$ = sensitivity factor i at regime s_t

F_{it} = risk factor i at time t

$\omega(s_t)$ = standard deviation of the regime s_t

Finally, we carry out a forward at 5% p value and backward at 10% p value stepwise regression analysis to underline the best possible set of risk factors for each HFs strategy during the regimes.

3.2 General analysis

Whole period										
Strategy	**Mean**	**Median**	**Maximum**	**Minimum**	**Std. dev**	**Sharpe**	**Skeweness**	**Kurtosis**	**Jarque bera**	**p value**
GM	0,30%	0,20%	5,58%	-3,81%	1,42%	21,37%	0,39	0,59	68,73	1,2E-15
EM	0,49%	0,80%	9,62%	-14,53%	3,11%	15,88%	-0,98	3,13	41,27	1,1E-09
EH	0,33%	0,58%	8,27%	-11,02%	2,45%	13,47%	-0,75	2,94	24,48	4,8E-06
EH-N	0,13%	0,23%	1,85%	-3,02%	0,75%	18,03%	-1,01	2,85	43,98	2,8E-10
ED	0,41%	0,65%	7,03%	-12,53%	1,97%	20,90%	-1,58	8,44	426,40	2,6E-93
ED-D	0,49%	0,64%	6,43%	-11,18%	1,91%	25,71%	-1,40	6,91	248,97	8,6E-55
ED-M	0,27%	0,39%	4,84%	-9,71%	1,16%	23,40%	-2,28	21,89	4059,17	0,0E+00
RV	0,35%	0,50%	3,93%	-9,90%	1,31%	27,09%	-3,37	22,50	4576,19	0,0E+00
RV-CA	0,35%	0,44%	9,74%	-16,09%	2,03%	17,50%	-2,61	23,83	4957,55	0,0E+00
RV-A	0,35%	0,56%	4,47%	-11,14%	1,68%	20,54%	-2,58	15,35	1924,12	0,0E+00
SP500	0,42%	0,86%	12,68%	-17,02%	4,41%	9,61%	-0,55	1,06	53,42	2,5E-12

Table 2 - The table reports a descriptive performance analysis together with higher moments of returns for HF strategy and SP500 index during the period from February 2001 to July 2022

Although every index strategy exhibits average positive returns for the period considered, as it can be seen in Mean column, each of them behaves quite differently. According to the most important principle of finance, returns and risk are negatively correlated and this is true also regarding hedge fund strategies. Strategies that have lower mean exhibit also a lower standard deviation and a reduced span of performance. However, Sharpe ratio rewards the first place to Relative Value strategy followed by Distressed and Merger Arbitrage. As testified by SR measure, there is not a clear preference among directional strategies like ED-D AND ED-M and absolute return strategies as RV-A and RV-CA even if their goal as written in the introduction is quite different.

At first glance, looking at the last 3 columns, it is possible to notice the non-normality of the HFs strategies returns. Kurtosis and skewness differ from 3 and 0 and the hypothesis of normality with Jarque-Bera test is rejected at 5% for all the indices. This is the first proof of non-normality due to heteroskedasticity of HFs returns derived by the active management put in place with the strong use of derivatives, short-selling and leverage. An interesting strategy to mention is Global Macro which is able to exhibit consistent small positive monthly returns and very few negative returns by being the only strategy with a positive skewness and little kurtosis.

Shifting the attention to SP500, it appears to have one of the best monthly mean returns, which places it in third position after Emerging markets and Distressed strategies. On the contrary analysing the standard deviation, SP500 returns exhibit the worst volatility. Evidence of this are consistent higher median and maximum and minimum returns at the extremes of the sample. As a consequence, SR value places SP500 at the end of the ranking.

In order to visualize the performance evolution of all strategies taken into account, below we provide a graph of the strategies indexed at 100 at the beginning of the series.

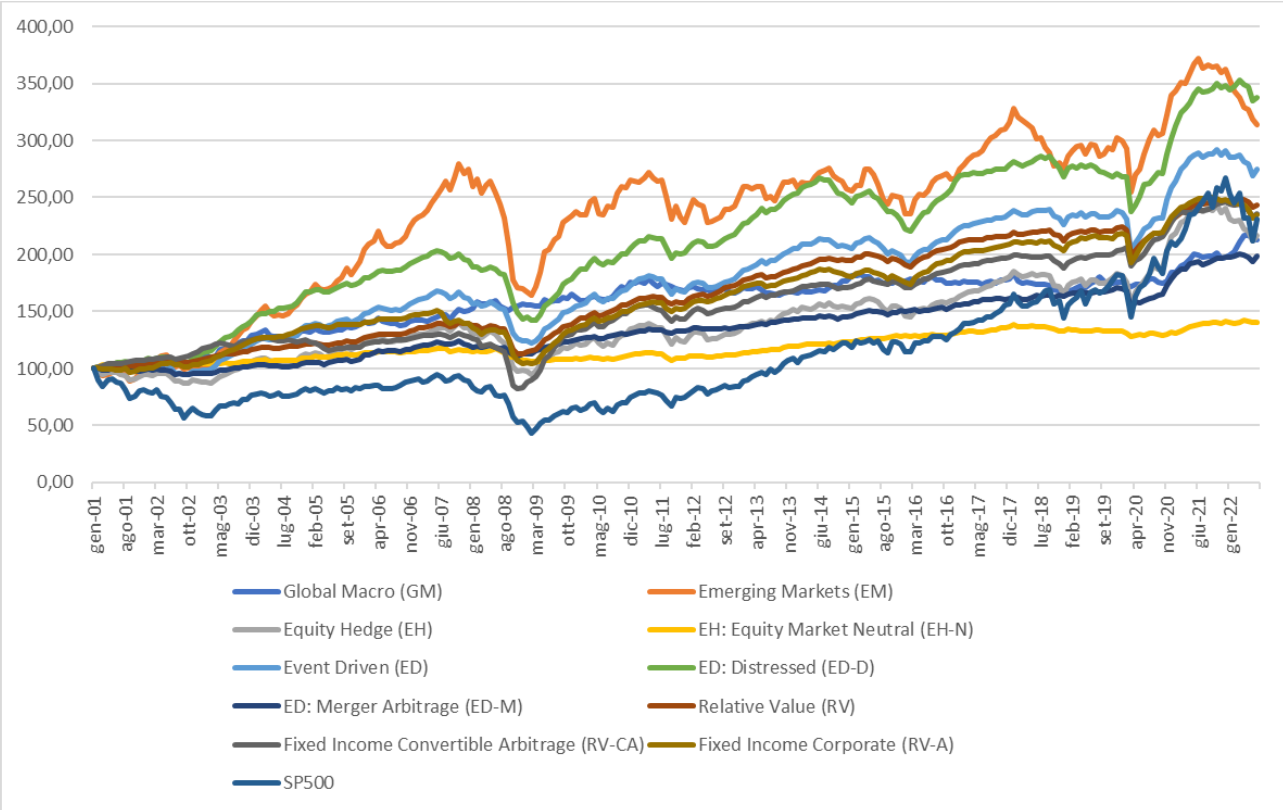


Figure 6 - HFs relative series performances over the period February 2001-July 2022

Emerging Markets, Distressed and slight below in third position Event Driven, are the three strategies more detached in terms of higher cumulative returns. It's not surprising to find the three in top positions due to their high monthly mean returns and variance. In particular, EM displays highest monthly returns and variance among the strategies and second highest drawdown and peak of monthly performance. However, the general trend can be defined as steady positive returns with not so rare great losses during downturns of the market.

In the table below we report more specific indicators of performance in excess of Rf with respect to SR in order to provide further insights among negative and positive periods of HFs strategies. Sharpe ratio is ranked in ascending order. Weight rank is calculated equal-weighting the 'strategy rank' of the 3 measures and 'final strategy rank' takes its value accordingly. Mean+ represents the average of positive monthly returns while sigma- represents the standard deviation of negative monthly returns.

Strategy	Mean	Mean +	Std. dev -	Sharpe	Rank	Sortino	Rank	Up. Potential	Rank	Weight rank	Final rank
RV	0,35%	0,63%	1,68%	27,09%	1	21,13%	6	37,31%	10	5,67	6
ED-D	0,49%	0,97%	1,74%	25,71%	2	28,12%	2	55,50%	6	3,33	2
ED-M	0,27%	0,54%	1,20%	23,40%	3	22,75%	3	45,33%	8	4,67	3
GM	0,30%	0,71%	0,74%	21,37%	4	41,10%	1	96,21%	1	2,00	1
ED	0,41%	0,94%	1,85%	20,90%	5	22,31%	4	50,98%	7	5,33	5
RV-A	0,35%	0,75%	1,89%	20,54%	6	18,31%	8	39,86%	9	7,67	8
EH-N	0,13%	0,35%	0,62%	18,03%	7	21,70%	5	56,89%	3	5,00	4
RV-CA	0,35%	0,79%	2,41%	17,50%	8	14,73%	10	32,62%	11	9,67	11
EM	0,49%	1,45%	2,51%	15,88%	9	19,65%	7	57,62%	2	6,00	7
EH	0,33%	1,09%	1,94%	13,47%	10	16,95%	9	56,16%	5	8,00	9
SP500	0,42%	1,89%	3,35%	9,61%	11	12,63%	11	56,40%	4	8,67	10

Table 3 – The table reports the calculation of performance rating measures on the period February 2001 to July 2022 with the classification according to their values.

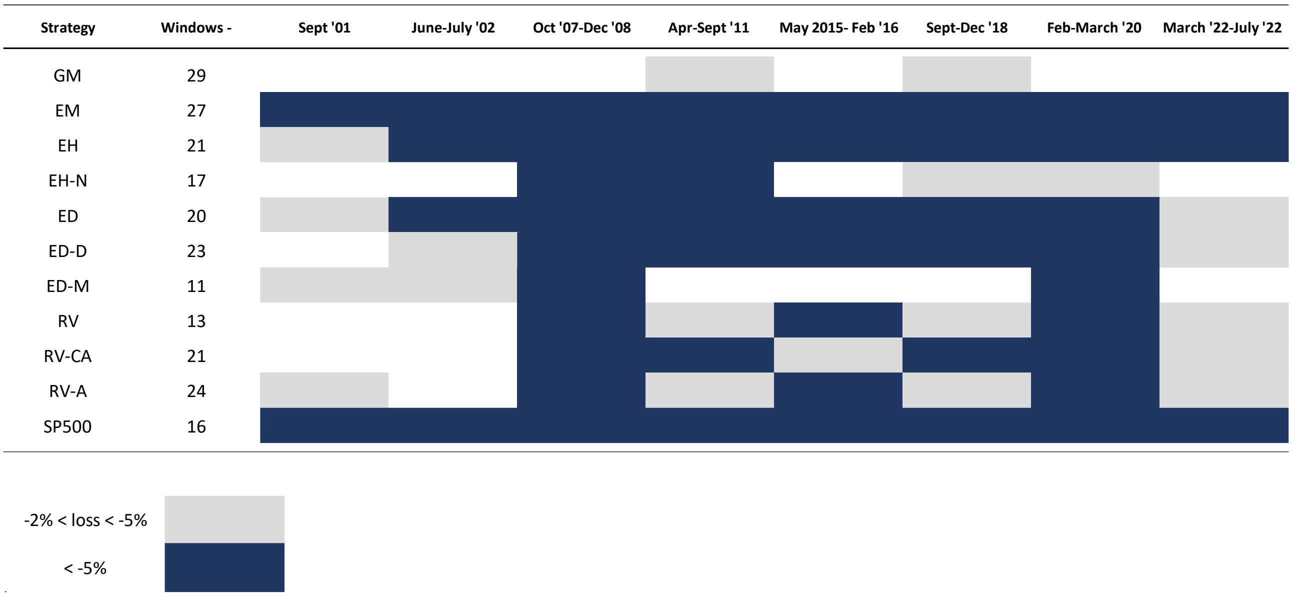
The very first thing that stands out is the fairly different arrangement of the final strategy rank with respect to SR rank if we consider in the analysis the other further two measures SORTINO and UPSIDE POTENTIAL. As previously mentioned, Global Macro can limit losses due to distressed periods better than the other strategies in terms of volatility of returns, beaten only by Equity Market Neutral, and guaranteeing respectable consistent positive returns with a low level of variance. On the other hand, Distressed strategy is the winner if we consider in the analysis cumulative performance and risk exposures management. As a proof its position in the rank doesn’t vary at all. Emerging markets succeeds in moving up to 7 position due to its significant positive returns during good months despite very high variance. Regarding SP500, its above average performance during positive monthly returns is not enough to balance the worst variance performance during negative periods resulting in second last place at the final ranking. Finally, the last place is occupied by Fixed income Convertible Arbitrage which despite average mean performance has been constantly highly exposed to volatility of its investments.

In order to find patterns in hedge funds returns we define the concept of “negative (positive) window” if for 3 months in a row the index strategy returns are negative (positive). We opted for 3 months following the outcomes of Agarwal, Naik (2000) HFs study in which the authors, using HFR net-of-fee returns, found that the extent of persistence is highest at the quarterly horizon for hedge funds and, whenever present, is unrelated to the type of strategy (directional or non-directional) followed by the fund.

Strategy	Windows +	Windows -	Mean	Mean +	Mean -	Std. dev	Std. dev +	Std. dev-
GM	227	29	1,15%	1,56%	-2,03%	2,49%	2,32%	1,08%
EM	229	27	1,84%	3,11%	-8,92%	6,66%	5,41%	6,61%
EH	235	21	1,27%	2,07%	-7,71%	4,90%	4,01%	5,12%
EH-N	239	17	0,60%	0,80%	-2,26%	1,39%	1,11%	1,82%
ED	236	20	1,50%	2,20%	-6,74%	4,24%	3,39%	4,63%
ED-D	233	23	1,75%	2,52%	-6,05%	4,39%	3,53%	4,63%
ED-M	245	11	1,03%	1,22%	-3,19%	2,17%	1,93%	2,99%
RV	243	13	1,29%	1,66%	-5,49%	2,83%	2,16%	4,87%
RV-CA	235	21	1,32%	2,02%	-6,54%	4,72%	3,67%	7,48%
RV-A	232	24	1,28%	1,83%	-4,02%	3,70%	3,12%	4,65%
SP500	240	16	1,57%	2,52%	-12,63%	7,64%	6,73%	6,49%
Strategy	**Sharpe**	**Rank**	**Sortino**	**Rank**	**Up. Potential**	**Rank**	**Weight rank**	**Final rank**
GM	0,46	2	1,07	1	1,44	1	1,33	1
EM	0,28	9	0,28	6	0,47	4	6,33	6
EH	0,26	10	0,25	9	0,40	7	8,67	9
EH-N	0,43	4	0,33	4	0,44	5	4,33	4
ED	0,35	6	0,32	5	0,47	3	4,67	5
ED-D	0,40	5	0,38	2	0,54	2	3,00	2
ED-M	0,47	1	0,34	3	0,41	6	3,33	3
RV	0,46	3	0,27	8	0,34	10	7,00	7
RV-CA	0,28	8	0,18	11	0,27	11	10,00	10
RV-A	0,35	7	0,28	7	0,39	8	7,33	8
SP500	0,21	11	0,24	10	0,39	9	10,00	10

Table 4 - The table reports descriptive statistics for HFs strategies and SP500 during 3 months rolling-windows during the period February 2001-July 2022 together with their correspondent performance ratio measures and their comparison in ranks.

Through table 4 and 5 we can claim with even more confidence that Global Macro can face crises better than everyone else while defending quite well in bull periods, RV-Distressed is the most performing but much riskier than GM, EH-Market Neutral is the less risky overall and RV-Convertible Arbitrage has the worst profile of profitability and risk.



Legenda

September 2001	Twin-Towers terrorist's attack.
June-July 2002	Accounting scandals of big US companies
Oct 2007-Dec 2008	Subprime crises
April-Sept 2011	European sovereign debt crises
May 2015-Feb 2016	Low interest rate environment & higher correlation of stocks & low volatility due to expansionary policies of central banks after the great recession
Sept-Dec 2018	US trade war with China, slowdown in global economic growth and concern of fast raising interest rates by Federal Reserve
Feb-March 2020	Pandemic crisis
March 2022-July 2022	(ongoing) Ukrainian war & shock on the offer side after the pandemic and consequent inflation & period of rising of interest rates

Table 5 – The table connected to the explanation in the legenda reports the most significative bad occurrences and economic-financial distress events over February 2001-July 2022 which have influenced HFs and SP500 performances.

HF Global Macro managers employ a variety of techniques, both discretionary and systematic analysis, quantitative and fundamental approaches during long or short-term holding periods. Their style of investment is predicated on the impact movements macroeconomic variables may have on security prices. Complex economic periods after the great recessions, characterized by low interest rates due to expansionary central banks maneuver to spur the economy, until reaching 2020 pandemic period and subsequent soaring inflation and Ukrainian war, have collectively given rise to geopolitical tensions and economic uncertainty, establishing a breeding ground for global macro investment style. Moreover, even RV-D took advantage from the flood of liquidity injected by central banks in the post 2009 until lately period, because struggling companies were able to get access to bank capital at a low price. For the same reason worse fate has befallen for RV-CA strategy which didn't benefit at all from the low level of interest rates. Indeed, long position in the convertible bond balanced by a short position in the underlying equity led to frequent losses due to the capacity of firms to get back on track with more ease.

3.3 First model

3.3.1 Regression with only linear risk factors

Risk factors	GM	EM	EH	EH-N	ED	ED-D	ED-M	RV	RV-CA	RV-A
alpha	0,0014*	0,0001	-0,0005	0,0004	0,0012**	0,0023**	0,0013**	0,0015**	0,0009	0,001*
SP500			0,2654**	0,0574**	0,1536**	0,0805**	0,0937**			
EMMKT	0,1685**	0,4954**	0,1857**	0,0229*	0,0706**		0,0621**	0,0509**	0,0580**	0,0413**
SMB		0,1284**	0,4084**	0,1074**	0,3973**	0,3565**	0,28501**	0,1371**	0,1438**	0,1744**
MOM	0,0589**	0,0390**	0,0402**	0,0768**				0,0216**		
DVIX										
TRSPRD	0,1820**							0,153**	0,3008**	0,2068**
CRSPRD	-0,0973**	0,1823**	0,0708**	0,0428**	0,1682**	0,2494**		0,2584**	0,4080**	0,3458**
ENERGY		0,0222**	0,0217**		0,0296**	0,0509**		0,026**		0,0355**
RAWM			0,0341**					0,0398**	0,0949**	
METAL	0,0566**	0,0409**			0,0255**	0,0432**				
std.dev	0,0125	0,0101	0,0071	0,0058	0,0077	0,0102	0,0078	0,0063	0,0122	0,0081
R^2	0,242	0,898	0,918	0,4	0,853	0,721	0,559	0,771	0,643	0,772
ADJ R^2	0,224	0,895	0,915	0,386	0,849	0,714	0,552	0,764	0,634	0,767
Loglikelihood	768,24	824,26	913,43	964,32	894,63	820,44	889,03	943,55	773,21	878,75
parameters	7	8	9	7	8	7	5	9	7	7
AIC	-1522,48	-1632,51	-1808,85	-1914,63	-1773,25	-1626,88	-1768,06	-1869,10	-1532,42	-1743,49
BIC	-1519,60	-1629,22	-1805,15	-1911,75	-1769,96	-1624,00	-1766,00	-1865,40	-1529,53	-1740,61

*Table 6 – Representation of the stepwise regressions using forward inclusion at 5% p value and backward offset at 10% p value of linear risk factors over dependent variable defined by HFs strategy indexes. **stands for significance at 5%, * stands for significance at 10% using t-test. Data used range from Feb 2001 to July 2022.*

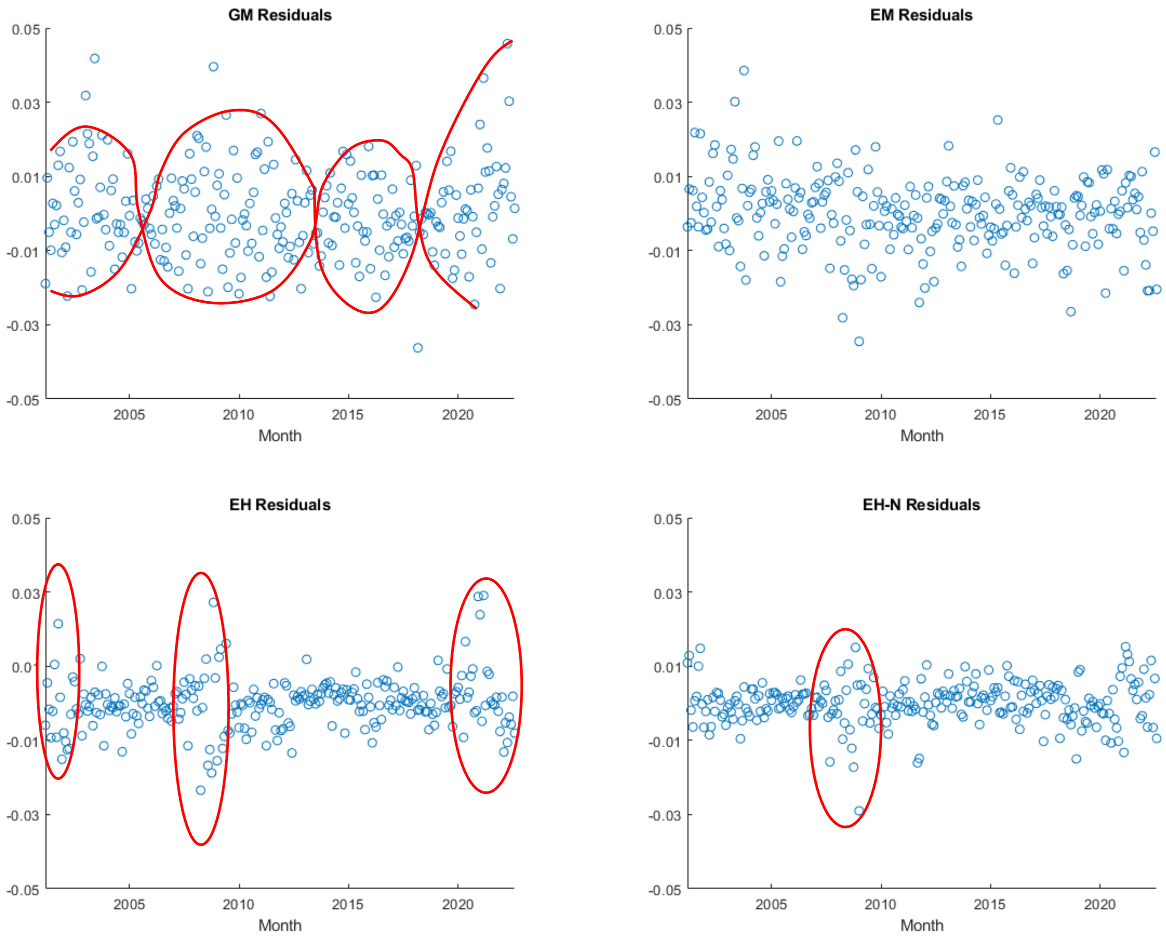
Globally the static regression implemented together with the risk factors chosen are able to explain quite well strong directional strategies like EM and EH where the Adj R^2 hits 90%, and are a good fit for ED strategies. In particular for Event Driven global strategy with the sub-category Distressed the risk factors movements explain respectively 85% and 72% of the correspondent index strategy. Different story regards Merger Arbitrage where the poor performance displayed after the financial global crises of 2008 can't be reproduced easily.

As far as RV strategies concern, the Adj R^2 factor average attests at 70% showing a quiet good fit for strategies with the aim to be detached from the risk of the market. They are exposed to the majority of equity factors we picked, demonstrating a lack of capacity to remain neutral to market risk and profit from market arbitrages during the period considered.

A final separate consideration is deserved for Global Macro and Equity Market neutral strategies where the adaptability of the static model is very weak. Indeed, the model is able to capture only partially GM and EH-N risks exposition. However, Global Macro has been more neutral with respect to EH-N strategies even if is considered to be part of directional strategies.

Analyzing more in-depth HFs exposures, as reported in Table 6, almost all the strategies are constantly exposed to EMMKT, SMB and CRDSPRD in a positive way with the exception of GM for CRSPRD. In addition, Directional strategies like EH and ED are exposed to SP500 but they are not the only ones. In fact, EH-N seems to be less neutral than imagined with significant positive values for all equity risk factors including SP500. Overall, commodities loading factors are significant for every strategy with the preference for energy factors followed by metal and raw materials indexes. Excess returns, i.e. alpha factors, are low and positive and significant for 6 out of 9 strategies, with ED, ED-D, GM and ED-M exhibiting the highest, seemingly reproducing the final rank of table 3. Finally, all the strategies reported are neutral to DVIX.

Since HFs due to their nature are characterized by a not common strategy of buying and hold but adopt dynamic strategies with leverage and use of derivatives and short-selling resulting in non-linear payoffs, it is normal that during extreme phases of the market replication might be very approximative. We have already pointed out the not normal kurtosis and skewness at the beginning of this study, here is graphically visible looking at the red markers in residual plots. Heteroskedasticity of HFs returns is clear if we compare tranquil regimes of the market with respect to bear or bull periods. Residuals deviate considerably during extreme events, having as result a significant enlargement of the tails of the distribution.



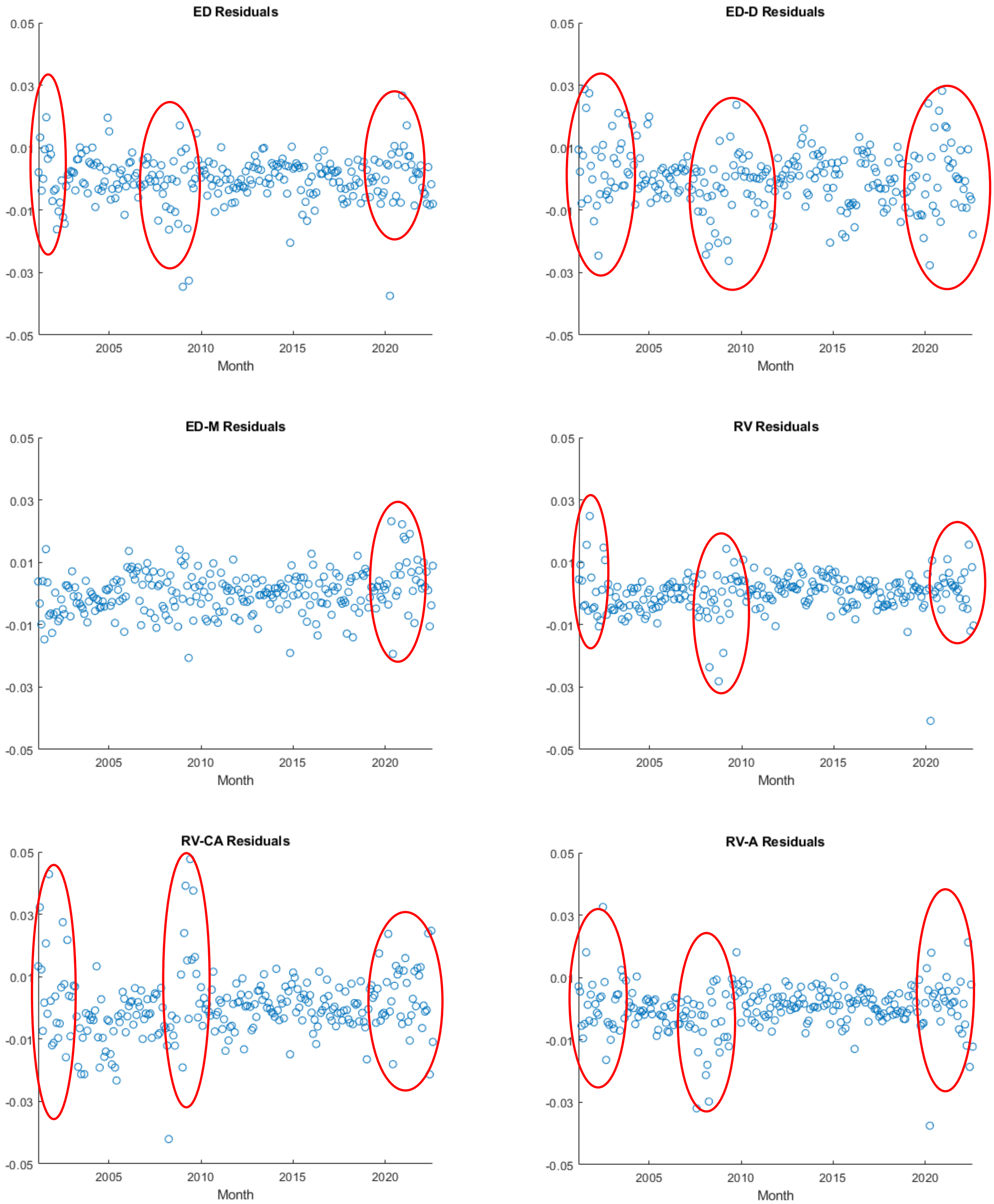


Figure 7 - The scatter plots above report the residuals of stepwise regressions associated to Table 6

The non-linearity is evident in particular around 2002, 2008-2009 and after 2020 when major changes influenced the economic landscape. Regarding the first period, the large span of performance is due to accounting scandals emerged in 2002 and 2003 from US companies Worldcom, Tyco, HealthSouth and Freddie Mac. The crisis of 2008-2009 is responsible for the behaviour near the 100th observation. Latest periods of rocketing inflation after the cumulated expansionary monetary policies operated by the CBs to counter 2008 recession and 2020 offer-side shock, together with energy crises after Russian-Ukrainian war, are to be attributable for the stretched residuals.

Almost each strategy shows the same residual pattern behaviour except for Global Macro strategy which exhibits a peculiar DNA-style scatter plot and EM with very low concentration of residuals. In the first case residuals are systematically greater than 0 with outliers more distributed towards the positive side, enabling the generation of a positive skewness. In the latter case there is not such a feature but instead residuals seem to be more stretched on the negative side. However, in both cases the residual plots are evidence of the poor explanatory power of the static model, which leaves out business cycle variables exposures and considers risk exposures to have the same statistical mean and variance during the whole period.

3.3.2 Regression with additional non-linear risk factors

Risk factors	GM	EM	EH	EH-N	ED	ED-D	ED-M	RV	RV-CA	RV-A
alpha	0,0011	0,0002	-0,0005	0,0006*	0,0012**	0,0024**	0,0011**	0,0014**	0,0009	0,0011**
SP500			0,2654**	0,0621**	0,1521**	0,0821**	0,0873**			
EMMKT	0,1413**	0,4946**	0,1857**	0,0309**	0,0686**		0,0575**	0,0485**	0,0580**	0,0384**
SMB		0,1103**	0,4084**	0,1078**	0,3929**	0,3595**	0,2732**	0,1251**	0,1438**	0,1570**
MOM	0,0521**	0,0472*	0,0402**	0,0764**				0,0208**		
DVIX	-0,0331*									
TRSPRD	0,2695**			-0,0902**				0,1463**	0,3008**	0,1808**
CRSPRD		0,1863**	0,0708**		0,1585**	0,2273**		0,2438**	0,4080**	0,3197**
PTFSBD	0,0228**		0,0217**		0,0301**	0,0506**		0,0236**		0,0252**
PTFSCOM	0,0566**	0,0505**	0,0341**					0,0420**	0,0949**	
PTFSIR	0,0336**	0,0397**			0,0244**	0,0420**				0,0361**
PTFSFX	0,0136**					-0,0085**				
PTFSSTK	0,0147**	-0,0097**		-0,0088**				-0,0067**		-0,0102**
ENERGY					-0,0037**					-0,0043**
RAWM	0,0218**									
METAL							-0,0081**	-0,0049*		
std.dev	0,0109	0,0999	0,0071	0,0057	0,0076	0,0101	0,0077	0,0062	0,0122	0,0079
R^2	0,436	0,9	0,918	0,430	0,855	0,727	0,569	0,781	0,643	0,785
ADJ R^2	0,411	0,897	0,915	0,414	0,850	0,719	0,560	0,772	0,634	0,777
Loglikelihood	806,26	826,41	913,43	970,95	896,72	823,49	892,08	949,11	773,21	886,28
parameters	12	9	9	8	9	8	6	11	7	10
AIC	-1588,53	-1634,81	-1808,85	-1925,89	-1775,45	-1630,97	-1772,15	-1876,22	-1532,42	-1752,55
BIC	-1583,59	-1631,11	-1805,15	-1922,60	-1771,74	-1627,68	-1769,68	-1871,69	-1529,53	-1748,44

*Table 7 – Representation of stepwise regressions using forward inclusion at 5% p value and backward offset at 10% p value of linear and non-linear risk factors over dependent variable defined by HFs strategy indexes. **stands for significance at 5%, * stands for significance at 10% using t-test. Data used range from Feb 2001 to July 2022.*

Looking at Table 7, it is interesting that new risk factors introduced in some cases haven't led to not even minimal change while in others have been slight perceptible, however the stepwise consider them significative in the explanation of the returns for almost every strategy considered: PTFSBD is significative for 6 strategies while PTFSCOM PTFSIR and PTFSSTK are for 5. This denotes the non-linear exposition of HFs toward different market asset classes well documented in the literature by many authors starting from Fung, Hsieh (1997), Mitchell, Pulvino (2001), Agarwal, Naik (2004), Bollen (2006), Billio (2010), Jawadi et al (2012), Bali (2014), Stafylas (2018) and many others.

Specifically, AIC and BIC reward the increase of Likelihood not penalizing too much the addition of PTFS factors whereas Adjusted R^2 seems to prefer the first more parsimonious model with the exceptions of EH and RV-CA where however the results are not so different. Considering Information criterion, they result lower for all the strategies with the exceptions of EH and RV-CA where they remain exact the same. In particular, Global Macro is the strategy which has benefitted the most from PTFS inclusion lowering AIC and BIC by nearly 70 points followed by EH-N with a reduction of more than 10. This result is more noticeable looking at the comparison of the two model residuals plot below. On the right they appear more concentrated meaning that the introduction of non-linear factors has helped to partially justify the systematic non-linear exposure to risk factors.

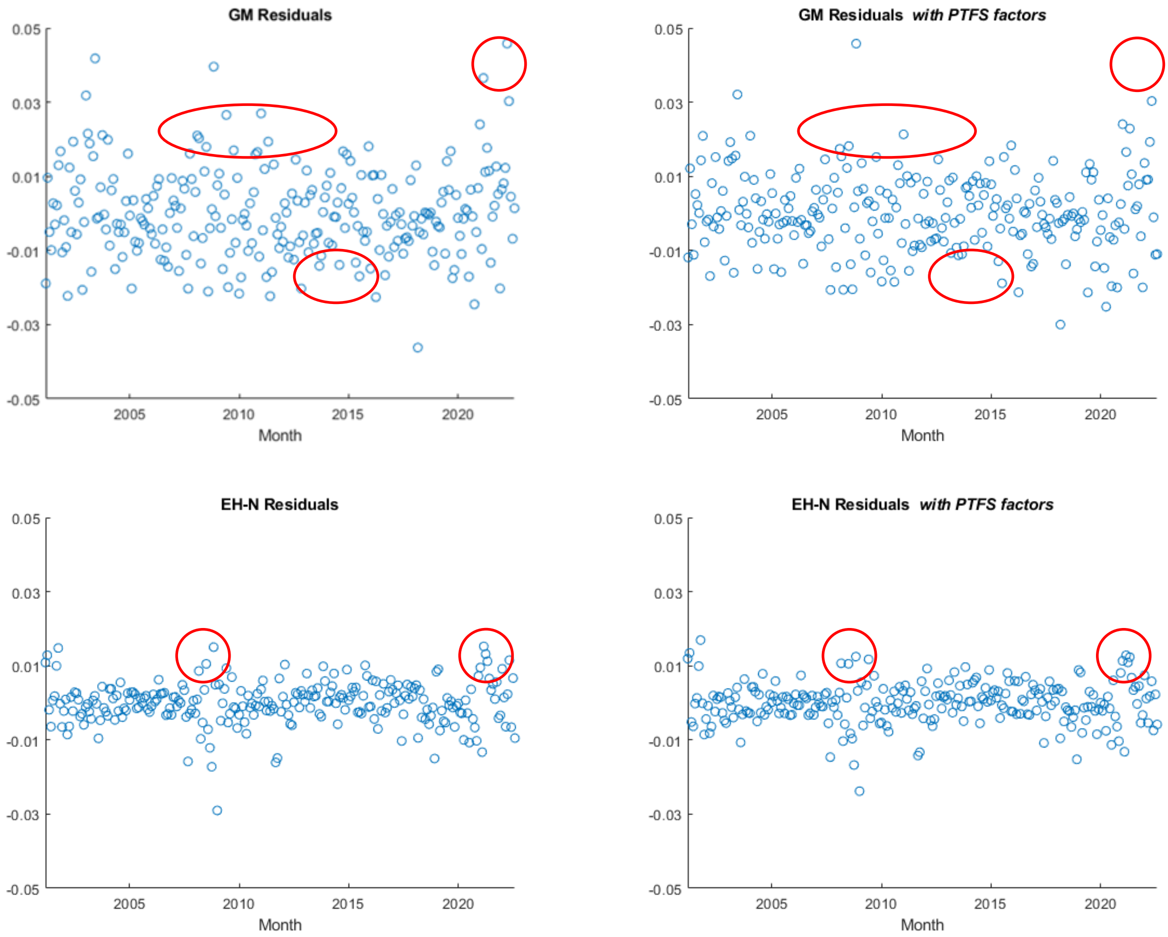


Figure 8 - The scatter plots above show comparison before and after the inclusion of non-linear risk factors in the stepwise regressions for Global Macro ed Equity-Market Neutral strategies.

Even Equity Hedge Market Neutral strategy has seen a substantial decrease of model criteria values. Looking above, EH-N exhibits a reduction of the variance of residuals, especially the ones correspondent to distressed periods as can be noticed through the red circles drawn above in Figure 8.

In conclusion, the addition of PTFS factors, while has helped to better assess the Global Macro trend-following strategy and infer on the non-linear behaviour of HFs towards common risk-factors, it hasn't still allowed us to discover some pattern behaviour according to business cycles which is the final scope of our research. Indeed, it has highlighted the partial result we can obtain treating HFs like normal investment funds.

This is why we have decided to opt for a dynamic model, with regime-switching regressors and variance, which attempts to better mimic the behaviour of HFs managers.

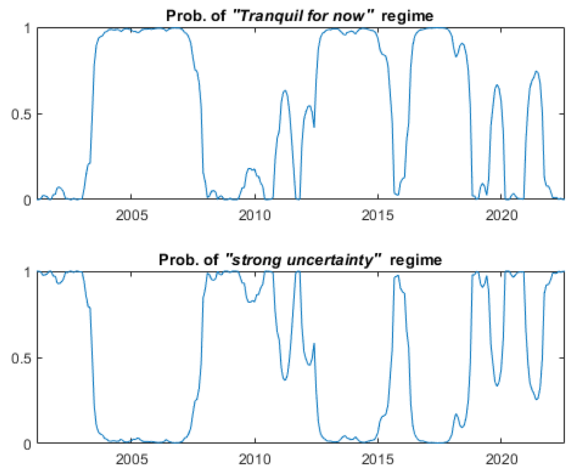
3.4 Second model

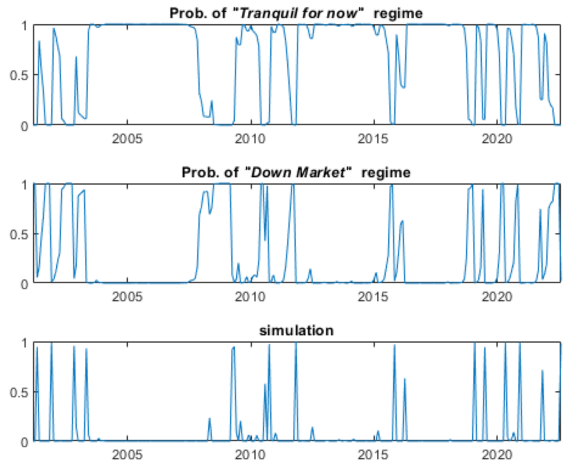
3.4.1 Analysis of results

We consider static model a very old-fashioned and inefficient way to assess HFs performance. Indeed, carrying out a static regression is an over-simplistic way to assess HFs exposures due to their dynamic nature. In addition, since we are targeting HFs behavior according to business cycles, we have decided to simplify the model avoiding the inclusion of options factors proposing a model similar to the one of Billio et al.(2010) and Stafylas et al. (2018). We are aware that in doing this we are sacrificing some HFs returns replication capacity but at the same time we provide a view of HFs non-linear exposures looking only at business cycles of the equity market and not complicating the model too much.

As first goal we used SP500 index in excess of risk-free to define the hidden market regimes. The literature is divided into two streams regarding the number of regimes to consider. Ozgur et al. (2011) and Billio et al. (2010) consider 3 regimes to better model the market while Stafylas et al (2018) assume 2 states in addition to exogenous imposed state. We have followed Billio et al. (2010) opting for the choice of three regimes to characterize the market states since AIC and BIC criterion are minimized and is more in accord with the literature that well recognizes the presence of up, down and tranquil states.

MS regime-probabilities with 2 regimes on SP500 *MS regime-probabilities with 3 regimes on SP500*





	2-regimes	3-regimes
alpha		
1	0,0111	0,0121
2	-0,0034	-0,0426
3	-	0,0831
std.dev		
1	0,0234	0,0254
2	0,0581	0,0467
3	-	0,0181
Loglikelihood	469,39	483,22
AIC	-934,78	-960,44
BIC	-927,67	-949,78
Transition matrix	$\begin{bmatrix} 0.949 & 0,051 \\ 0,058 & 0,942 \end{bmatrix}$	$\begin{bmatrix} 0,926 & 0,074 & 0 \\ 0 & 0,735 & 0,265 \\ 0,841 & 0,083 & 0,075 \end{bmatrix}$

Table 8 – This table associated with the respective two graphs report the results of Markov-Switching only-constant regressions on SP500 with 2 and 3 regimes considered. 1,2 stand for respectively "tranquil for now", "strong uncertainty" for 2-regimes MS while 1,2,3 define "tranquil for now", "down-market" and "up-market" for 3-regimes MS.

According to the sub-models estimates we can distinguish 3 different regimes. State 1 has a low mean with great stability, but returns are very volatile with a standard deviation being twice as much as the mean. For this reason, we have designated the latter as "tranquil for now" state. State 2 can be considered "down market" associated with financial distress, with negative mean and quite strong volatility while is relatively stable since it moves to state 3 almost 27% of the times. Finally, we have named state 3 as "up-market" condition due to very higher than 0 mean and low variance.

However, it is the less stable regime with the probability to keep the same state lower than 8 percent. From this analysis it emerges the huge discrepancy in the characteristics of the 3 states and the complexity that an investor may face in investing actively in the market.

Secondly, distinguishing from Billio's work, we have adopted Viterbi algorithm to define the most likely path for hidden states, rather than maximizing the probability of the single state, because we are more interested in the evolution of the regimes in order to find some hidden patterns. Examples of Viterbi usage can be found in Wang, Ling (2020) where they use the hidden Markov model (HMM) to identify different market regimes in the US stock market proposing an investment strategy customized on the current detected regime.

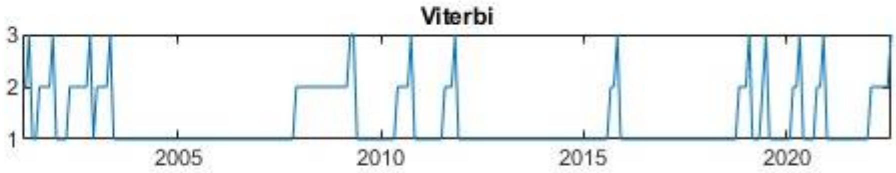


Figure 9 - This graph reports the estimated hidden states of SP500 obtained through Viterbi algorithm over the period February 2001-July 2022.

It is quiet peculiar the Viterbi reported trend of market regimes. It seems to follow the path 1-2-3-1, moving from "tranquil for now" to "down" to "up" and starting over again. It reflects the real world where the condition of strong up-market bounces after a huge fall caused by a macroeconomic or financial destabilization during a period of relatively tranquil market.

Deviating from Billio et al. (2010) work we think that to provide a more meaningful insight on each individual HFs strategy exposures we need to select proper ad-hoc risk factors from the whole set of regressors as done by Stafylas et al. (2018). In order to do that we have implemented a stepwise regime regression analysis on same risk factors used in the first static regression that adopts both forward selection at 5% p-value and back removing at 10% p-value.

	GM	EM	EH	EH-N	ED	ED-D	ED-M	RV	RV-CA	RV-A
alpha										
1	-0,0012	-0,0001	-0,0006	0,0009**	0,0017**	0,0034**	0,0017**	0,0024**	0,0011	0,0019**
2	0,002762	-0,0034	0,0053**	-0,0045**	-0,0035*	-0,006**	0,0003	0	0,0033	-0,0031
3	0,002586	-0,0008	0,0159**	0,0044	0,0043	-0,0024	-0,0557**	0,0113**	0,0119**	0,0057
SP500										
1	0,16115**		0,28**	0,0681**	0,179**		0,1074**			
2			0,1825**		0,1287**					
3							0,797**			
EMMKT										
1	0,20825**	0,4975**	0,1835**		0,0737**		0,0549**	0,0360**	0,0527**	
2		0,4879**	0,2081**				0,1459**			
3		0,5809**								
SMB										
1		0,1496**	0,3926**	0,0869**	0,381**	0,3337**	0,1894**	0,0852**		0,1076**
2			0,3494**		0,5034**	0,3787**	0,3945**	0,2998**		0,4279**
3			0,6544**		0,3587**	0,4112**		0,353**	0,3409**	0,4604**
MOM										
1				0,0403**						
2		0,1076**	0,0943**	0,1545**		0,0908**				
3									-0,0839**	-0,1118**
DVIX										
1		0,0552**		-0,0383**	0,0348**		0,04**		0,0486**	
2										
3			-0,207**		-0,2156**	-0,2044**				
TRSPRD										
1	0,361109**	0,2127**				0,1777**		0,1868**	0,3048**	0,3329**
2										
3							0,8537**			
CRSPRD										
1	-0,173643**	0,2893**	0,1083**		0,2113**	0,4059**		0,27**	0,4738**	0,443**
2		0,1787**	0,0823*	0,117**	0,2135**	0,28**		0,2497**	0,5445**	0,2803**
3										
ENERGY										
1		0,0267**	0,0215**		0,0153**	0,0393**		0,0156**		0,0183**
2	0,0519**	0,047**	0,0325**		0,0393**	0,071**		0,047**		0,0519**
3										
RAWM										
1	0,072140**	0,0651**	0,0348**		0,0322**	0,0518**		0,0395**	0,0747**	
2										
3					0,1264**	0,206**	0,1222**			
METAL										
1				0,0155**	0,0201**	0,0254*				0,0307**
2										
3										
std.dev										
1	0,0107	0,0085	0,0054	0,0044	0,0057	0,00799	0,00614	0,00403	0,00883	0,0051
2	0,0151	0,0128	0,00858	0,00771	0,0103	0,0129	0,0104	0,011	0,0213	0,0133
3	0,0127	0,0123	0,0108	0,00781	0,00741	0,0105	0,00586	0,00501	0,0072	0,00664
R^2	0,32	0,8724	0,897	0,413	0,8332	0,7153	0,5469	0,738	0,5794	0,7512
1	0,417	0,874	0,9	0,349	0,825	0,672	0,354	0,742	0,562	0,775
2	0,112	0,884	0,909	0,519	0,839	0,752	0,619	0,737	0,58	0,732
3	/	0,729	0,793	/	0,852	0,777	0,884	0,711	0,784	0,808
ADJ R^2	0,290	0,863	0,889	0,384	0,819	0,694	0,521	0,722	0,557	0,736
1	0,398	0,868	0,896	0,328	0,816	0,659	0,336	0,732	0,548	0,768
2	0,078	0,872	0,895	0,491	0,823	0,727	0,597	0,716	0,564	0,711
3	/	0,680	0,731	/	0,786	0,678	0,832	0,658	0,719	0,750
Loglikelihood	787,9	844,32	955,52	998,49	941,82	858,22	926,33	1005,06	814,22	945,03
parameters	12	18	20	13	21	19	15	16	14	16
AIC	-1551,8	-1652,64	-1871,04	-1970,98	-1841,64	-1678,44	-1822,66	-1978,12	-1600,44	-1858,06
BIC	-1509,16	-1588,69	-1799,98	-1924,79	-1767,03	-1610,93	-1769,37	-1921,27	-1550,70	-1801,21

Table 9 - Results of 3 stepwise regressions, *one for each regime defined by Markov-Switching model on SP500, using forward inclusion at 5% value and backward offset at 10% of risk factors over dependent variable defined by HFs strategy indexes. ** stands for significance at 5%, * stands for significance at 10% using t-test. Data used range from Feb 2001 to July 2022. 1,2,3 stand for respectively "tranquil for now", "down-market" and "up-market".*

Looking at the risk-return profiles of the strategies presented at the beginning of this paper, an investor would be willing to invest in Global Macro HFs in first place. In Table 9 we have further evidence to support the theory GM has a superior management of risk-return trade off compared to the other strategies as reported also by Stafylas et al. (2018). Indeed, GM seems to provide positive small returns being exposed to the equity market only during tranquil regimes while insuring with corporate bond market and remaining neutral in the other 2 scenarios. However, this model is preferred to static models only using AIC but not according to BIC and Adjusted R^2 to the static models with PTFS factors previously analyzed. This could be due to a lack of capacity, even with a dynamic model of this genre, to mimic the fast-adjusting trend-following strategy pursued by GM. Indeed, GM residuals exhibit a completely different magnitude of variance (0,017) with respect to the other strategies. We guess that GM strategy can exploit at the most hidden regimes within the 3 regimes obtained that this model can't capture because defined by other financial and macroeconomic variables.
Considering directional strategies, Equity Hedge, Emerging Market and Event driven strategies (ED, ED-D, ED-M) are constantly exposed to equity market factors in good, tranquil and crisis periods as showed by DVIX SP500 and EMMKT for EM sensitivities, benefitting of high returns but also suffering high losses (look at positive alphas in tranquil regime and negative alphas in down-market regime in particular for ED and ED-D) depending on the business cycle. Specifically, they prefer to invest in small capitalized listed firms as it can be seen with SMB factors, preferring to shift the investments to higher returns companies during down market (MOM is quite significative in second regime) as found by Stafylas et al(2018). In addition, all of them adopt a strategy of diversification investing in bonds and commodities during tranquil and bad periods. Furthermore, this model is preferred according to AIC while BIC penalizes more the additional factors preferring static models, with the only exception of ED-M which BIC considers equivalent to static. Adjusted R^2 is even more severe in the judgement preferring static models for all directional strategies.
Moreover, Relative Value strategies seem to be neutral to SP500 while being exposed to Emerging Market equity factor during calm periods and up-market with the exception of RV-A. SMB is quite significative for all of them with different values of sensitivities, meaning that they try to remain neutral to the market using convertible instruments or going long on growth stocks while keeping short positions on value stocks as reported by Mitchell, Pulvino (2001), Billio et al. (2010), Agarwal et al. (2004). This line of conduct is adopted especially during extreme phases of the market as further witnessed by negative sensitivity to MOM in good periods. In addition, they all invest in commodities and bonds during tranquil and bad periods. Considering AIC and BIC and Adjusting R2 they all evaluate MS regime switching model superior in the explanation power of static regressions. Finally, equity market neutral seems to behave not so neutrally with respect to the market, being exposed in tranquil periods with SP500, negative DVIX and significant momentum factor. AIC, BIC repute MS model better in the assessment of EH-N systematic risks while Adjusted R2 still prefers static models.

More in general we observe that for many strategies factor loadings change consistently when volatility increases or the market is in a less stable regime showing proof of HFs non-linear behavior. Examples are SMB, Credit spread and energy exposures for directional strategies and Relative value ones. The increase of SMB loading factors along tranquil, down and good is an evidence of liquidity risk of this particular investment vehicles determined by Getmanski et al. (2004) and Aragon et al.

(2006) and reported by Billio et al. (2010), because small stocks have greater sensitivity to market illiquidity than large stocks during distressed periods meaning that they hold greater liquidity risks.

Considering Credit spread factor, we have found HFs to exhibit positive exposures both during tranquil and bad market conditions. Since credit spread is a proxy for credit risk and funding liquidity risk, the rationale behind is that HFs have to face constantly the risk of sudden liquidation and margin calls. Indeed, while during bad states of the market the reason is quite straightforward, in case of “tranquil for now” condition HFs positive exposure could be attributable to the high variance and so high uncertainty that characterizes this regime. This is quite in contrast to what discovered by Billio et al. (2010) that found significance of CRSPRD only during distressed periods.

Finally, we report the common behavior of HFs to invest in commodities, especially energy, during distressed periods or high-variance ones demonstrated by Billio et al. (2010), Stafylas et al. (2018) and many others.

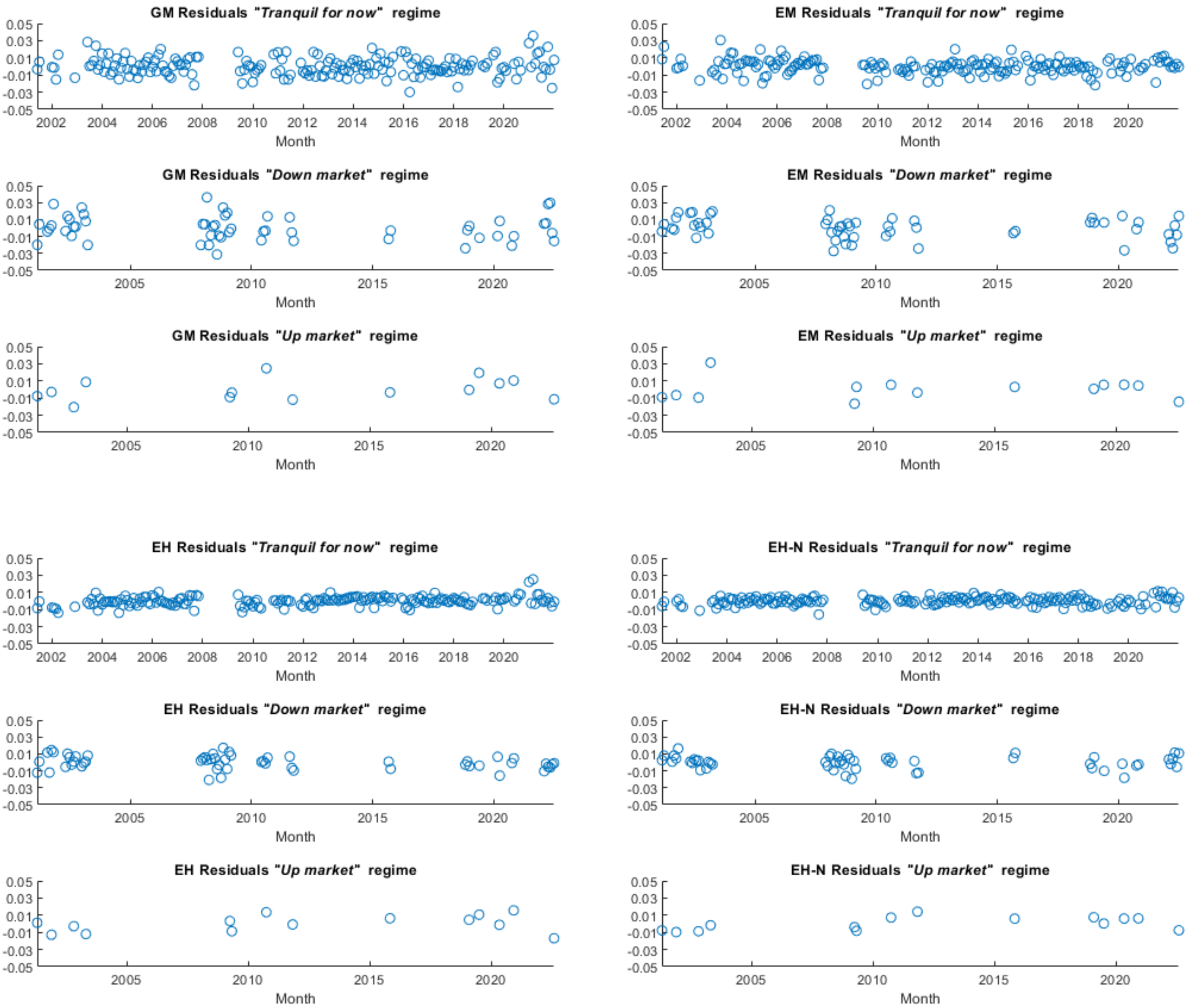


Figure 10 - The scatter plots above report the residuals of the 3 regimes defined by the MS model for all the HFs strategies.

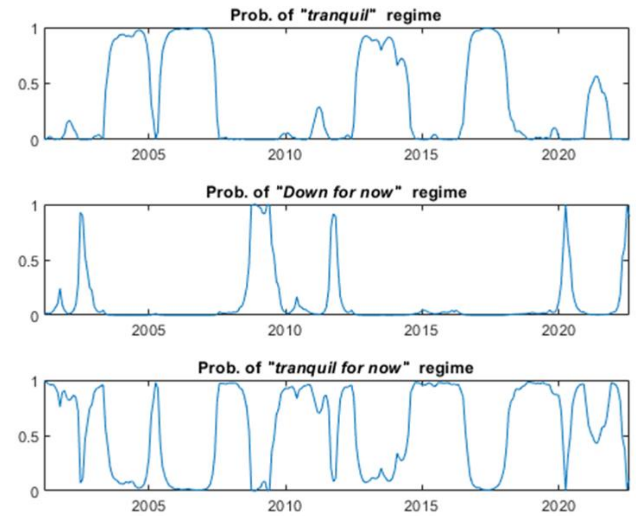
3.4.2 Limits of the Markov regime-switching model & Viterbi simulation

1. *We assume that HF_t reacts to three hidden states defined by SP500. It can't be excluded that HF_t reacts to more or less than three states derived by a combination of different macroeconomic and/or financial risk factors.*
2. *Since we don't know the hidden states but we are able to observe only the return of the market and define the state probabilities matrix, the inference of the state that Viterbi algorithm provides entails errors in the estimates. This issue is enlarged in case of similar mean and high variances and low stability among the regimes.*
3. *The different sizes of the samples belonging to the regimes found by Markov-switching model entail better estimates for tranquil regime represented by 189 observations with respect to the down market and up-market respectively defined by 55 and 14.*

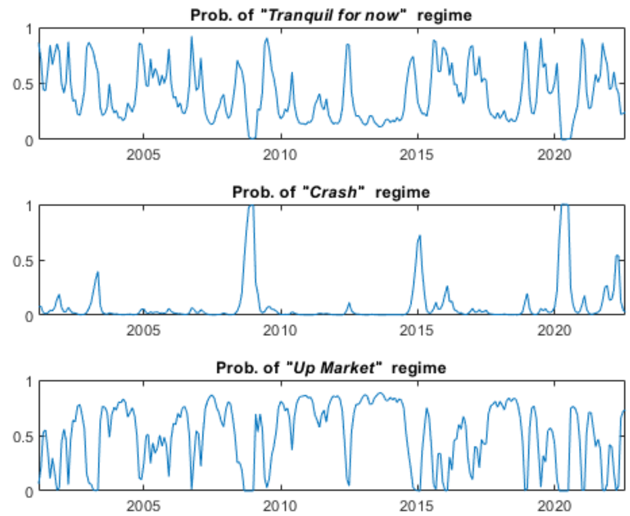
First limit further insight

We have implemented Markov-regime switching model using SP500 to define different market regimes but we think that also other risk factors can be used to define economic and financial states influencing HFs behaviour. For this purpose, we have provided Markov-switching dynamic regimes using credit spread index and energy price index. Both variables can be used to define specific regimes since are significative for all HFs strategies in the greater part of the regimes defined by SP500. As it can be seen in the figure, the 2 variables exhibit quite different behaviour. CRSPRD resembles SP500 regimes but with less spikes in the down-market regime and more stable tranquil periods while ENERGY displays very high instability, passing from tranquil with high variance to up-market very often, with fewer spikes in the crash regime.

Credit spread index regimes



Energy index regimes



	CRSPRD	ENERGY
alpha		
1	0,0085	0,0204
2	-0,0114	-0,0501
3	0,0028	0,0068
std.dev		
1	0,0102	0,0397
2	0,0708	0,1519
3	0,0249	0,0807
Loglikelihood	596,22	483,22
AIC	-1186,44	-641,75
BIC	-1175,79	-631,09
Transition matrix	$\begin{bmatrix} 0,924 & 0 & 0,076 \\ 0 & 0,791 & 0,209 \\ 0,047 & 0,048 & 0,905 \end{bmatrix}$	$\begin{bmatrix} 0,778 & 0 & 0,222 \\ 0,290 & 0,704 & 0,006 \\ 0,215 & 0,064 & 0,721 \end{bmatrix}$

Table 10 - This table associated with the respective two graphs reports the results of Markov-Switching only-constant regressions on Credit Spread index and Energy index considering 3 regimes for each. 1,2,3 stand for respectively "tranquil", "down-market" and "tranquil for now" regimes for CRSPRD while stand for "tranquil for now", "crash", "up-market" for ENERGY.

Quantitative view of second limit

As a proof of Viterbi limit, we have simulated 3,000 SP500 observations originated by known input real states, with same characteristics of our sample. We have then estimated the state probabilities on the simulated observations and through Viterbi we have derived the estimated states. We have then compared the real states with the estimated ones to understand the efficiency of Viterbi algorithm in capturing the real states.

		ESTIMATED		
		Tranquil for now	*Down-market*	*Up-market*
REAL	*Tranquil for now*	98,57%	1,38%	0,05%
	Down-market	17,29%	82,40%	0,31%
	Up-market	20,63%	12,17%	67%

Table 11 - The table reports the hidden state error estimates of the Viterbi algorithm.

As we can notice Viterbi is hardly ever wrong in defining state 1 due to the latter state great stability, while its efficiency decreases moving to the second and the third state as a consequence of great variance in second regime and low stability in case of the third.

4 Conclusion

Even with a dynamic model defining periods in the equity market, the capacity to detect HFs strategies of investment gives weak results, with adjusted R^2 associated which always prefer static models, and AIC and BIC criteria not improving substantially. However, we have successfully showed and proved HFs non-linear payoffs and underlined peculiar differences among distinct strategies. Directional strategies are constantly exposed to the market factors using diversification to mitigate systemic risks while RV strategies and Global Macro seem to adopt an insurance perspective. In particular the latter is able to follow punctually the market cycles, enabling a small but consistent positive return being exposed to equity factor only during tranquil regimes but insuring with CRSPRD. Evidences are to be found in positive skewness, higher Sortino and Upside potential, significative exposures to the greater part of non-linear PTFS factors and neutrality to market risk in more unstable regimes. Equity-neutral strategy appears to be less neutral than imagined being exposed to fluctuations during “tranquil for now” periods, where the mean is low but variance is quite high. Moreover, we have provided evidence that supports the presence of liquidity risks suffered by HFs during all regimes with the exception of Global Macro strategy. In contrast to Billio et al. (2010) we have found credit and funding liquidity risk exposures also during “tranquil for now” periods. In addition, we have proved the HFs behavior to flew to investments in commodity in periods of instability, with energy which is preferred to metal and raw materials in down-market periods.

5 List of Figures

6 List of Tables

7 Bibliography

Ackermann, Carl, Richard, McEnally, David, Ravenscraft, *1999, "The Performance of Hedge Funds: Risk, Return, and Incentives", Journal of Finance, vol. 54 833-874*

Agarwal, Vikas, Bakshi, Gurdip, Huij, Joop, *2010, Do higher-moment equity risks explain hedge fund returns?, CFR working paper, No. 10-07, University of Cologne, Centre for Financial Research (CFR), Cologne*

Agarwal, Vikas, Naik, Narayan Y. , *2000,* ***"****Multi-Period Performance Persistence Analysis of Hedge Fund", Journal of Financial and Quantitative Analysis 35(3), pp. 327-342*

Agarwal, Vikas, Naik, Narayan Y. , *2004, "Risks and Portfolio Decisions Involving Hedge Fund", Review of Financial Studies 17, 63–98*

Akay Ozgur (Ozzy), Senyuz Zeynep, Yoldas Emre, *2011,* ***"****Hedge fund contagion and risk-adjusted returns: A Markov-switching dynamic factor approach", Journal of Empirical Finance 22, pp. 16-29*

Aragon George O., Ferson Wayne E, *2006, "Portfolio performance evaluation" Foundations and Trends R in Finance, vol 2, no 2, pp 83– 190, 2006*

Bali, T., Brown, S., Caglayan, M. *, 2011, "Do hedge funds' exposures to risk factors predict their future returns?", Journal of Financial Economics, 101(1), pp.36- 68.*

Bali, T., Brown, S., Caglayan, M. *, 2014, "Macroeconomic risk and hedge fund returns", Journal of Financial Economics, 114, 1–19*

Billio, Monica, Getmansky, Mila, Pelizzon, Loriana , *2010, "Dynamic risk exposures in hedge funds", Comput Stat Data Anal 56:3517–3532*

Bollen, Nicolas P.B., Whaley, Robert E. *, 2006, "Hedge Fund Risk Dynamics: Implications for Performance Appraisal", The Journal of Finance, 66(2), pp.985-1035.*

Brown, Stephen J., Goezmann, William N., Ibbotson, Roger G*. , 1999, "Offshore Hedge funds: Survival and Performance 1989-1995", Journal of Business 91-117*

De Brouwer, Gordon, *2001,* ***"****Hedge Funds in Emerging Markets", Cambridge University Press*

Fama, Eugene F, French, Kenneth R*. , 1992, "The Cross-Section of Expected Stock Returns", . J Finance 47:427–465*

Fung, William, Hsieh, David A. *, 1999, "A primer on hedge funds", journal of Empirical Finance vol. 7 1-36*

Fung, William, Hsieh, David A. *, 2002,* ***"****Asset-Based Style Factors for Hedge Funds", Financ Anal J 58:16–27*

Fung, William, Hsieh, David A. *, 1997, "Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Fund", Rev Financ Stud 10:275–302*

Fung, William, Hsieh, David A. *, 2004, "Hedge fund benchmarks: A risk-based approach", Financ Anal J 60:65–80*

Fung, William, Hsieh, David A*. , 2001, "The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers", The Review of Financial Studies 14, 313–341.*

Getmansky Mila, Lo* Andrew W., Makarov Igor *, 2004, "An econometric model of serial correlation and illiquidity in hedge fund returns", J Financ Econ 74:529–609*

Giannikis Dimitrios, Vrontos Ioannis D. *, 2009, "A Bayesian approach to detecting nonlinear risk exposures in hedge fund strategies", Journal of Banking and Finance, 35(6), pp.1399-1414.*

Glosten L.R., Jagannathan Ravi , *1994, "A contingent claim approach to performance evaluation", J Empir Financ 1:133–160*

Goldman M., Sosin H., Gatto M*. , 1979, "Path dependent options: "buy at the low, sell at the high", The Journal of Finance 34(5), pp. 1111-1127*

Jawadi, Fredj, Khanniche, Sabrina *, 2012, "Modeling hedge fund exposure to risk factors", Economic Model-Ling 29:1003–1018*

Joenväärä, J., Kauppila, M., Kosowski, R., Tolonen, P., *2021, "Hedge fund performance: Are stylized facts sensitive to which database one uses?" , Critical Finance Review 10(2), pp. 271-327*

Juha Joenväärä, Kosowski Robert, Tolonen Pekka *, 2012, "New 'Stylised facts' about Hedge Funds and Database Selection Bias"*

Lhabitant, François-Serge*, 2002, "Hedge Funds: Myths and Limits", John Wiley & Sons*

Liang, Bing*, , 1999, "On the Performance of Hedge Funds," Financial Analysts Journal vol. 55 72-85*

Marie, Lambert, Federico Platania *, 2020, "The macroeconomic drivers in hedge fund beta management", Econ Model 91:65–80*

Merton, Robert C. *,1981, "On Market Timing and Investment Performance: An Equilibrium Theory of Value for Market Forecasts", Journal of Business 54, no. 3 (July): 363-406*

Metzger, Nicola, Shenai, Vijay *, 2019, "Hedge Fund Performance during and after the Crisis: A Comparative Analysis of Strategies 2007–2017", International Journal of Financial Studies, 7, 1–31.*

Mitchell, Mark, Pulvino, Todd *, 2001, "Characteristics of Risk and Return in Risk Arbitrage", The Journal of Finance 56 (6), 2135–2175*

Racicot, François-Eric *,2007, "A study of dynamic market strategies of hedge funds using the Kalman Filter"*

Racicot, François-Eric, Theoret, Raymond *, 2019, "Hedge fund return higher moments over the business cycle", Econ. Modell. 78, 73–97*

Racicot, François-Eric, Theoret, Raymond *, 2018, "Multi-moment risk, hedging strategies, & the business cycle", International Review of Economics & Finance, 58, 637–675.*

Racicot, François-Eric, Theoret, Raymond, Gregoriou Greg N. *, 2020, "The response of hedge fund tail risk to macroeconomic shocks: A nonlinear VAR approach", Economic Modelling 94, pp. 843-872*

Racicot, François-Eric, Theoret, Raymond, Gregoriou Greg N. , *2021,"The response of hedge fund higher moment risk to macroeconomic and illiquidity shock", Int Rev Econ Financ 72:289–318*

Racicot, F. E., & Theoret, R. , *2016, "Macroeconomic shocks, forward-looking dynamics, and the behaviour of hedge funds, Journal of Banking and Finance, 62(1), pp.41-61.*

René M., Stulz, *2007, Hedge Funds: Past, Present, and Future, Journal of Economic Perspectives*

Sharpe F. William *, 1992, "Asset allocation: management style and performance measurement", Journal of Portfolio Management Winter 7–19, J Portf Manag 18:7–19*

Sharpe F. William *, 1964, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", The Journal of Finance, 19, 425–442.*

Stafylas Dimitrios, Andersonb Keith, Uddin Moshfique *,2018, "Hedge fund performance attribution under various market conditions", Int Rev Financ Anal 56:221–237*

Stafylas Dimitrios, Andersonb Keith, Uddin Moshfique *, 2017, "Recent advances in explaining hedge fund returns: Implicit factors and exposures", Global Finance Journal, 33, 69–87.*

Tremont Advisers and TASS Investment Research*, 2002, The Handbook of Alternative Investments, John Wiley & Sons*

Wang Matthew, Lin Yi-Hong, Mikhelson Ilya*, 2020, "Regime-Switching Factor Investing with Hidden Markov Models", J. Risk Financial Manag., 13(12), 311*

8 Webliography

https://www.hfr.com/

https://www.investopedia.com/

https://www.statista.com/

https://www.eurekahedge.com/

https://lab.credit-suisse.com/#/en/home

https://portal.barclayhedge.com/

https://it.wikipedia.org/wiki/Pagina_principale

https://seekingalpha.com/

https://www.spglobal.com/ratings/en/

https://people.duke.edu/~dah7/HFData.htm

https://fred.stlouisfed.org

https://www.msci.com/

Kenneth R. French - Data Library (dartmouth.edu)

https://www.tradingview.com/

9 Appendix A – Matlab codes

```
% 3 regimes Markov-Switching regressions with only constant

clc
opts = spreadsheetImportOptions("NumVariables", 23);
opts.Sheet = "Sheet1";
opts.DataRange = "A3:W260";
opts.VariableNames = ["Date","OBS","SP500", "EMMKT", "SMB", "MOM", "DVIX",
"TRSPRD","CRSPRD","ENERGY","RAWM","METAL","GM","EM","EH","EH_N","ED","ED_D","ED_M","RV","RV_CA","
RV_A","RF"];
opts = setvartype(opts,2:23,"double");
opts = setvartype(opts,1,"datetime");
alldata = readtable("all_data.xlsx", opts, "UseExcel", false);
clear opts


RHF = alldata.RV_A; % To change with "GM","EM","EH","EH_N","ED","ED_D","ED_M","RV","RV_CA","RV_A"
Data = alldata.Date;
OBS = alldata.OBS;
SP500 = alldata.SP500-alldata.RF;
EMMKT = alldata.EMMKT-alldata.RF;
SMB = alldata.SMB;
MOM = alldata.MOM;
DVIX = alldata.DVIX;
TRSPRD = alldata.TRSPRD;
CRSPRD = alldata.CRSPRD;
ENERGY = alldata.ENERGY;
RAWM = alldata.RAWM;
METAL = alldata.METAL;


% Model with only constant on SP500 (to do MS regression on Credit spread or Metal replace
CRSPRD/METAL)

P = NaN(3);
mc = dtmc(P,'StateNames',["Up-market" "Tranquil" "Down-market"]);
mdl = arima(0,0,0);
Mdl = msVAR(mc,[mdl; mdl; mdl]);

% Initialization of MS

P0 = 0.5*ones(3);
mc0 = dtmc(P0,'StateNames',Mdl.StateNames);
mdl01 = arima('Constant',0.02,'Variance',.005);
mdl02 = arima('Constant',-0.01,'Variance',.005);
mdl03 = arima('Constant',0,'Variance',.005);
Mdl0 = msVAR(mc0,[mdl01; mdl02; mdl03]);
[EstMdl,SS,logL] = estimate(Mdl,Mdl0,SP500,'MaxIterations',1000);

% Viterbi algorithm

STATES = hmmviterbi(OBS, EstMdl.Switch.P, SS.');
A=dummyvar(STATES);

% Regression on each regime

i11=A(:,1).*SP500;
i11=i11(A(:,1)==1);
i12=A(:,2).*SP500;
i12=i12(A(:,2)==1);
i13=A(:,3).*SP500;
i13=i13(A(:,3)==1);
```

```
i21=A(:,1).*EMMKT;
i21=i21(A(:,1)==1);
i22=A(:,2).*EMMKT;
i22=i22(A(:,2)==1);
i23=A(:,3).*EMMKT;
i23=i23(A(:,3)==1);

i31=A(:,1).*SMB;
i31=i31(A(:,1)==1);
i32=A(:,2).*SMB;
i32=i32(A(:,2)==1);
i33=A(:,3).*SMB;
i33=i33(A(:,3)==1);

i41=A(:,1).*MOM;
i41=i41(A(:,1)==1);
i42=A(:,2).*MOM;
i42=i42(A(:,2)==1);
i43=A(:,3).*MOM;
i43=i43(A(:,3)==1);

i51=A(:,1).*DVIX;
i51=i51(A(:,1)==1);
i52=A(:,2).*DVIX;
i52=i52(A(:,2)==1);
i53=A(:,3).*DVIX;
i53=i53(A(:,3)==1);

i61=A(:,1).*TRSPRD;
i61=i61(A(:,1)==1);
i62=A(:,2).*TRSPRD;
i62=i62(A(:,2)==1);
i63=A(:,3).*TRSPRD;
i63=i63(A(:,3)==1);

i71=A(:,1).*CRSPRD;
i71=i71(A(:,1)==1);
i72=A(:,2).*CRSPRD;
i72=i72(A(:,2)==1);
i73=A(:,3).*CRSPRD;
i73=i73(A(:,3)==1);

i81=A(:,1).*ENERGY;
i81=i81(A(:,1)==1);
i82=A(:,2).*ENERGY;
i82=i82(A(:,2)==1);
i83=A(:,3).*ENERGY;
i83=i83(A(:,3)==1);

i91=A(:,1).*RAWM;
i91=i91(A(:,1)==1);
i92=A(:,2).*RAWM;
i92=i92(A(:,2)==1);
i93=A(:,3).*RAWM;
i93=i93(A(:,3)==1);

i101=A(:,1).*METAL;
i101=i101(A(:,1)==1);
i102=A(:,2).*METAL;
i102=i102(A(:,2)==1);

i103=A(:,3).*METAL;
i103=i103(A(:,3)==1);

RHF1=A(:,1).*RHF;
RHF1=RHF1(A(:,1)==1);
```

```
RHF2=A(:,2).*RHF;
RHF2=RHF2(A(:,2)==1);
RHF3=A(:,3).*RHF;
RHF3=RHF3(A(:,3)==1);


varnames = ["SP500", "EMMKT", "SMB", "MOM", "DVIX", "TRSPRD","CRSPRD","ENERGY","RAWM","METAL"];
x21=[i11,i21,i31,i41,i51,i61,i71,i81,i91,i101];
x22=[i12,i22,i32,i42,i52,i62,i72,i82,i92,i102];
x23=[i13,i23,i33,i43,i53,i63,i73,i83,i93,i103];


DATAtotable1 = array2table(x21,'VariableNames',varnames);
DATAtotable2 = array2table(x22,'VariableNames',varnames);
DATAtotable3 = array2table(x23,'VariableNames',varnames);

X21 = DATAtotable1{:,{'SP500', 'EMMKT', 'SMB', 'MOM', 'DVIX',
'TRSPRD','CRSPRD','ENERGY','RAWM','METAL'}};
X22 = DATAtotable2{:,{'SP500', 'EMMKT', 'SMB', 'MOM', 'DVIX',
'TRSPRD','CRSPRD','ENERGY','RAWM','METAL'}};
X23 = DATAtotable3{:,{'SP500', 'EMMKT', 'SMB', 'MOM', 'DVIX',
'TRSPRD','CRSPRD','ENERGY','RAWM','METAL'}};


% Stepwise regression on each regime

eq21 = stepwiselm(X21,RHF1,'constant','Upper','linear','PredictorVars',{'SP500', 'EMMKT', 'SMB',
'MOM', 'DVIX', 'TRSPRD','CRSPRD','ENERGY','RAWM','METAL'})
loglikelihood1 = eq21.LogLikelihood
eq22 = stepwiselm(X22,RHF2,'constant','Upper','linear','PredictorVars',{'SP500', 'EMMKT', 'SMB',
'MOM', 'DVIX', 'TRSPRD','CRSPRD','ENERGY','RAWM','METAL'})
loglikelihood2 = eq22.LogLikelihood
eq23 = stepwiselm(X23,RHF3,'constant','Upper','linear','PredictorVars',{'SP500', 'EMMKT', 'SMB',
'MOM', 'DVIX', 'TRSPRD','CRSPRD','ENERGY','RAWM','METAL'})
loglikelihood3 = eq23.LogLikelihood

rss=eq21.SSR+eq22.SSR+eq23.SSR;
tss=eq21.SST+eq22.SST+eq23.SST;
R2=rss/tss;

res21=table2array(eq21.Residuals(:,1));
res22=table2array(eq22.Residuals(:,1));
res23=table2array(eq23.Residuals(:,1));
res21_plus=A(:,1);
res22_plus=A(:,2);
res23_plus=A(:,3);
res21_plus(A(:,1)==1)=res21;
res22_plus(A(:,2)==1)=res22;
res23_plus(A(:,3)==1)=res23;
res21_plus(A(:,1)==0)=NaN;
res22_plus(A(:,2)==0)=NaN;
res23_plus(A(:,3)==0)=NaN;




% Residuals plots

tiledlayout(3,1)
nexttile

scatter(Data,res21_plus)
ylim([-0.05 0.05])
yticks([-0.05:0.02:0.05])
xlabel("Month")
ylabel("")
```

```
title('RV-A Residuals {\it"Tranquil for now"}  regime')   %To change with "GM","EM","EH","EH-N","ED","ED-D","ED-M","RV","RV-CA","RV-A"

nexttile
scatter(Data,res22_plus)
ylim([-0.05 0.05])
yticks([-0.05:0.02:0.05])
xlabel("Month")
ylabel("")
title('RV-A Residuals {\it"Down market"}  regime')   % To change with "GM","EM","EH","EH-N","ED","ED-D","ED-M","RV","RV-CA","RV-A"

nexttile
scatter(Data,res23_plus)
ylim([-0.05 0.05])
yticks([-0.05:0.02:0.05])
xlabel("Month")
ylabel("")
title('RV-A Residuals {\it"Up market"}  regime')    % To change with "GM","EM","EH","EH-N","ED","ED-D","ED-M","RV","RV-CA","RV-A"



% Simulation on Viterbi

rng(1); % For reproducibility
[y,~,sp] = simulate(EstMdl,3000);

% Estimates on the simulated data

[EstMdl1,SS,logL] = estimate(Mdl,Mdl0,y,'MaxIterations',1000);
obs=1:3000;
vit_STATES = hmmviterbi(obs, EstMdl.Switch.P, SS.');
```

```
% 2 regimes Markov-Switching regressions with only constant

clc
opts = spreadsheetImportOptions("NumVariables", 23);
opts.Sheet = "Sheet1";
opts.DataRange = "A3:W260";
opts.VariableNames = ["Date","OBS","SP500", "EMMKT", "SMB", "MOM", "DVIX",
"TRSPRD","CRSPRD","ENERGY","RAWM","METAL","GM","EM","EH","EH_N","ED","ED_D","ED_M","RV","RV_CA","
RV_A","RF"];
opts = setvartype(opts,2:23,"double");
opts = setvartype(opts,1,"datetime");
alldata = readtable("all_data.xlsx", opts, "UseExcel", false);
clear opts


RHF = alldata.RV_A; % To change with "GM","EM","EH","EH_N","ED","ED_D","ED_M","RV","RV_CA","RV_A"
Data = alldata.Date;
OBS = alldata.OBS;
SP500 = alldata.SP500-alldata.RF;
EMMKT = alldata.EMMKT-alldata.RF;
SMB = alldata.SMB;
MOM = alldata.MOM;
DVIX = alldata.DVIX;
TRSPRD = alldata.TRSPRD;
CRSPRD = alldata.CRSPRD;
ENERGY = alldata.ENERGY;
RAWM = alldata.RAWM;
METAL = alldata.METAL;

% model

P = NaN(2);
mc = dtmc(P,'StateNames',["Up-market" "Down-market"]);
mdl = arima(0,0,0);
Mdl = msVAR(mc,[mdl; mdl]);

% Initialization

P0 = 0.5*ones(2);
mc0 = dtmc(P0,'StateNames',Mdl.StateNames);
mdl01 = arima('Constant',0.02,'Variance',.005);
mdl02 = arima('Constant',-0.01,'Variance',.005);
Mdl0 = msVAR(mc0,[mdl01; mdl02]);
[EstMdl,SS,logL] = estimate(Mdl,Mdl0,SP500,'MaxIterations',1000);
```