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Executive Summary of the Thesis

Physics-Informed Neural Network for damage localization using Lamb waves

Laurea Magistrale in Aeronautical Engineering - Ingegneria Aeronautica

Author: Jacopo Ghellero

Advisor: Prof. Francesco Cadini

Co-advisor: Dr. Luca Lomazzi, Eng. Lucio Pinello

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1. Introduction

This study focuses on condition-based maintenance, particularly in the realm of Structural Health Monitoring (SHM), which relies on real-time data and sensor information. The primary emphasis is on Lamb wave-based algorithms for diagnosing damage in thin-walled structures, utilizing piezoelectric devices for wave generation and detection. Traditional approaches involve tomographic algorithms and diagnostic signals, but they face limitations such as uneven sensing network density, subjective parameter selection, and difficulty in quantifying damage. Machine learning methods, including feed-forward neural networks and convolutional neural networks, have been explored for damage diagnosis [1]. However, supervised learning schemes are identified as costly [2], prompting the introduction of Physics-Informed Neural Networks (PINNs) as an unsupervised alternative. PINNs aim to solve partial differential equations by approximating the solution field with a neural network. While potentially less efficient than classical solvers in some contexts, PINNs are considered promising, offering a more comprehensible model that integrates both data and theoretical elements. The current research trend in SHM centres on developing models capable of overcoming the aforementioned limitations and eliminating the need for pre-processing or extracting damage indices from sensor signals. The objective of this thesis is to develop tools and methodologies using PINNs to diagnose damage using Lamb waves. Additionally, it aims to evaluate the practicality of employing PINNs for SHM applications, with the intention of addressing challenges encountered by traditional methods.

2. Methodology

2.1. Theoretical Background

Lamb waves refer to ultrasonic-guided waves that move through slender structures like plates and shells described by equation (1). In contrast to bulk waves, Lamb waves are restricted to the structure's thickness. This confinement enables them to engage with defects, boundaries, and other features within the material. This characteristic enhances their effectiveness in identifying concealed flaws and evaluating the structural soundness of various engineering components, spanning from aircraft wings to pipelines and bridges.

$$\rho \boldsymbol{u}_{tt} = \boldsymbol{\nabla} \cdot (\boldsymbol{\sigma}) + \boldsymbol{f} \tag{1}$$

Where material properties of the domain are considered trough elastic modulus E and Poisson ratio ν inside the constitutive relationship for elastic material hence relating stress tensor $\boldsymbol{\sigma}$ and displacement vector $\boldsymbol{u}$. The density is represented by ρ, forcing vector by $\boldsymbol{f}$ and $\boldsymbol{\nabla}\cdot$ is the divergence operator applied to stress tensor $\boldsymbol{\sigma}$. Three different models can be derived from the general equation (1): one-dimensional, two-dimensional and three-dimensional. The equations presented assume an homogeneous material, making it impractical to accurately describe domains where material properties vary. The introduction of variable material properties becomes essential for representing real scenarios such as damages and holes within the domain.
The mono-dimensional wave equation is solved, which describes an axial wave that propagates along the coordinate x through a one-dimensional string with the two ends fully constrained. Parametrization used to solve full inversion problem for the one-dimensional wave equation is the one written in equation (2):

$$c(x)^2 = \gamma(x)c_0^2 = \gamma(x)\frac{E}{\rho} \tag{2}$$

where c_0 is the constant wave speed defined as a function of density ρ and elastic modulus E, instead $\gamma(x)$ is the parameter that is a function of the position x and multiplied with the material property c_0, gives the actual material property of the domain in that position. Introducing this parametrization general wave equation (1) in one-dimensional form with variable material property is obtained:

$$u_{tt} = c_0^2\frac{\partial}{\partial x}(\gamma(x)\frac{\partial u}{\partial x}) + \frac{f}{\rho} \tag{3}$$

where u is the displacement in the x direction and f is the force applied in the same direction. In order to solve the aformentioned inversion problem for the wave equation and, thus, to perform damage identification, the paper discusses a recently developed class of machine learning techniques, specifically PINNs. In a PINN, neural networks approximate the solution to a physical system while adhering to the governing laws of physics. This is achieved by incorporating the equations describing the underlying physical phenomena as constraints during the neural network training process. PINNs are advantageous for efficiently solving partial differential equations, modelling complex systems, and adapting to various data types. The neural network structure typically follows a Feed-Forward Neural Network (FFNN) format, where inputs are the coordinates of the partial differential equation to be solved, and outputs represent the solution for each coordinate. The optimization algorithm adjusts the network weights during training based on both the available data and the physics in the form of the partial differential equation residual, ultimately optimizing the fit of the solution to the partial differential equation being solved.

2.2. Workflow

The first step is to develop tools able to solve the wave equation, with the aim of being implemented in the PINN methods developed. In this context is useful to present the procedure to realize the finite difference solver for the mono-dimensional wave equation (3). The domain selected is a one-dimensional string along dimension x with the edges fully constrained and thus u at those points is set to zero. The wave equation is then solved in the finite difference framework using the following stencils for the partial differential equation, and initial and boundary conditions:

$$\begin{aligned} u_i^{n+1} &= 2u_i^n - u_i^{n-1} \\ &+c^2\frac{dt^2}{2dx^2}[(\gamma_i + \gamma_{i+1})(u_{i+1}^n - u_i^n)] \\ &-c^2\frac{dt^2}{2dx^2}[(\gamma i + \gamma_{i-1})(u_i^n - u_{i-1}^n)] + \frac{dt^2}{\rho}f_i^n \end{aligned} \tag{4}$$

$$BC: \quad u_0^n = 0, \;\; u_{N_x}^n = 0 \tag{5}$$

$$IC: \quad u_i^0 = 0 \tag{6}$$

where i is the index for the space position node and can assume values between $i = 0$ and $i = N_x$, while n is the index for time, varying across $n = 0$ to $n = N_t$. Consider that $N_t + 1$ is the number of time nodes and $N_x + 1$ is the number of spatial nodes of the grid, as a consequence

dx and dt are defined as the step in respectively space and time, computed as follows:

$$dx = \frac{L_x}{N_x}, \quad dt = \frac{L_t}{N_t} \tag{7}$$

where L_x is the length of the domain in x direction, while L_t is the end time of the simulation. The stencil of the initial condition on the derivative is not important because the derivative of a null quantity is already zero. Consider that the mono-dimensional wave equation is too simple to model Lamb wave behavior, hence during thesis work more complex solver has been developed and in order to be clear and concise are not reported in this summary. Those solvers have also been validated using simulation coming from the Abaqus software package.

Methods developed make use of a neural network that predicts the distribution of the material on the domain, they differ on how the loss function is defined. The prediction of material distribution is the focus of a Feed-Forward Neural Network (FFNN) characterized by a straightforward architecture. In this network, the spatial position is the input, and the output is the material distribution, denoted as $\boldsymbol{\gamma}$. This distribution is a function that, when multiplied by the material property, yields the material property value at that specific point. The simplified physics-informed method uses displacement measurement $\boldsymbol{u_m}$ coming from sensors placed on the domain and the material distribution prediction $\boldsymbol{\gamma}$ from the neural network A_γ to compute the residual of the partial differential equation. This residual represents the loss function that is minimized by the optimization algorithm modifying neural network parameters $\boldsymbol{\theta}$. Steps made for each epoch are represented in the Scheme 1.

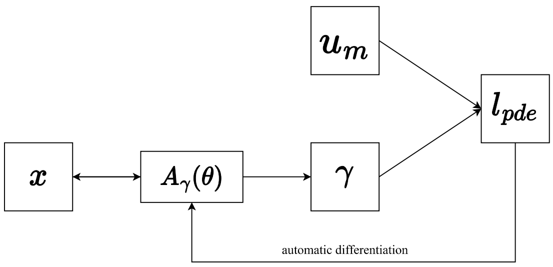

Figure 1: Simplified physics-informed method

In order to overcome the limitations presented in the previous method the Physics-informed method coupled with a numerical solver is developed starting from the work done in this paper [3]. The original method was modified so it could deal with Lamb waves. Such a method is selected due to the reduction in complexity of the optimization process compared to a more traditional PINN approach that uses a neural network to solve both forward and inverse problems as in [4]. This method implements a finite difference solver (Forward Solver) that computes the wave field u using material distribution $\boldsymbol{\gamma}$ predicted from the neural network, in this way the partial differential equation is always satisfied and therefore the loss function is the mean of the square difference between predicted displacement in the sensors points $\boldsymbol{u}(\boldsymbol{x_m})$ and the actual displacement in those points $\boldsymbol{u_m}$.

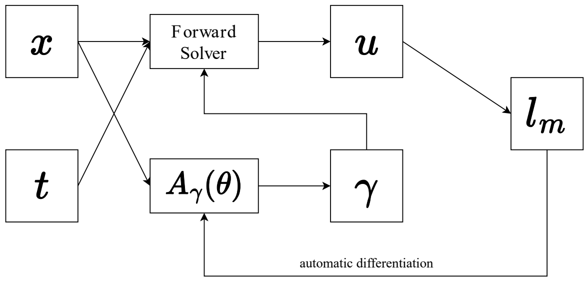

Figure 2: Physics-informed coupled with finite difference solver method

3. Case studies

3.1. 1D

The first model developed and analyzed is the one-dimensional model with two variants studied in order to assess the capabilities and performance of the methods presented in this paper. Simulating a wave in a one-dimensional domain is comparably simple to propagating Lamb waves in a plate and thus it makes the one-dimensional case a perfect environment to compare the two methods. The selected domain is a string clamped at both ends, with a length of $L_x = 10 \quad cm$, composed of aluminum and exhibiting damage. A force is applied in the middle using a sine wave modulated with a Hanning window with a specified number of cycles, $n_{cycles} = 3$, to reproduce the lamb wave excitation. There are 9 sensors, equally spaced along the string, and no sensor is placed on the clamped ends as shown in Illustration 3.

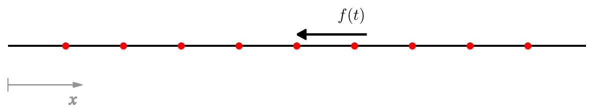

Figure 3: Domain configuration, red dots are the sensors

The neural network selected is an FFNN, with $\boldsymbol{x}$ as input and $\boldsymbol{\gamma}$ output, it has 4 hidden layers of 5,10,10,5 neurons per layer. The results obtained with the simplified physics-informed method are reported in Figure 4, for reasons of numerical stability the frequency selected in this case is $f = 25 \ \ kHz$.

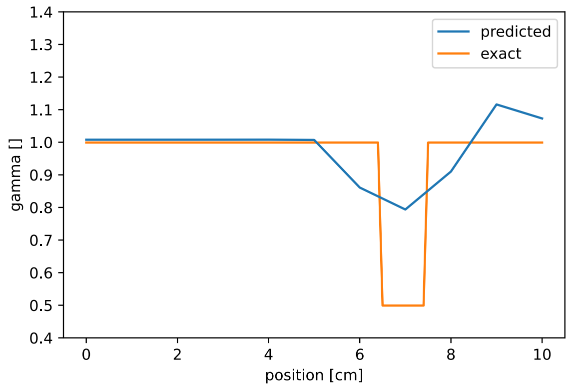

Figure 4: Simplified physics-informed method results compared to actual material distribution

Then, the Physics-informed neural network coupled with a finite difference solver method is tested, this time the frequency of the forcing term is similar to the one of typical Lamb waves propagation i.e. $f = 300 \ \ kHz$.

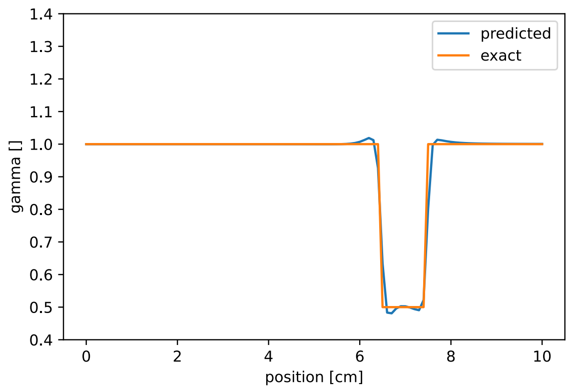

Figure 5: Physics-informed coupled with finite difference method results compared to actual material distribution

Both methods are capable of detecting the damage, however accuracy of the Simplified method is lower since it is not capable of estimating correctly the change in material property and has strong limitations in the type of forcing term for numerical stability reasons. Physics-informed coupled with finite difference solver method is comparably more accurate in both position and material property estimation. Nevertheless, the computational effort is increased in comparison with the other method tested.

3.2. 2D

Physics-informed coupled with finite difference solver is tested on a two-dimensional case study to correctly model lamb waves, which is not feasible in the mono-dimensional model. The domain selected is a plate section with two sides clamped and the other two free to move, the two dimensions are the thickness and length of the plate, the other dimension is not considered for the reason that is under plane strain condition. The plate section is made of aluminum and has the following dimensions $5 \times 0.1 \ \ cm$, the force is placed at the middle with frequency $f = 400 \ \ kHz$ with a configuration that excites the A0 mode. Four sensors are placed on the top free surface equally spaced one between the other, no sensor is placed at the clamped sides as represented in Illustration 6. The neural network selected is the same as the one for the mono-dimensional case and material distribution is considered constant along the thickness, and thus the algorithm has to perform material prediction only along the length direction.

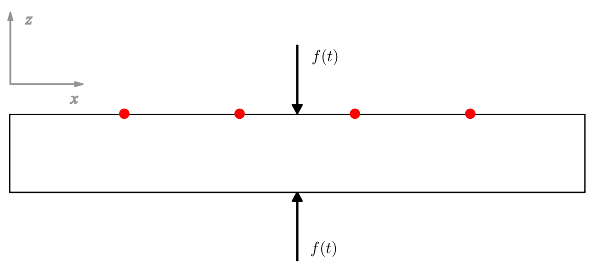

Figure 6: Domain configuration, red dots are the sensors

Results obtained running the algorithm are reported in Figure 7, the implemented neural network is able to correctly identify the position and quantify material properties change due to damage presence.

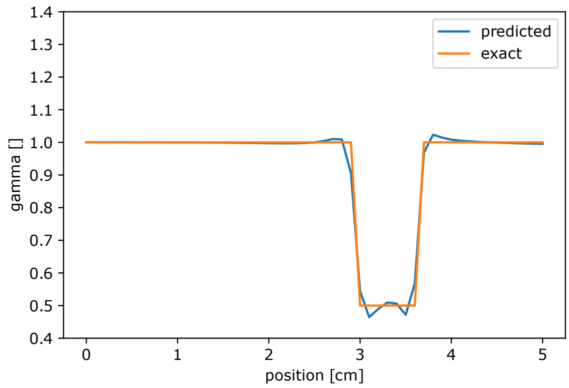

Figure 7: Physics-informed coupled with finite difference method results compared to actual material distribution

3.3. 3D

The mono-dimensional and two-dimensional studies prove the capability of the Physics-informed coupled with finite difference solver method using actual Lamb waves, and then the three-dimensional domain is tested to extensively prove this method. In the latter case, the domain is a plate clamped at the edges and the upper and lower surfaces are free to move, with dimensions $5 \times 5 \times 0.1 \quad cm$. This time the wave propagating is an actual Lamb wave, the excitation configuration produces a pure A0 mode Lamb wave that propagates from the middle of the plate with frequency $f = 400 \quad kHz$. The force selection is driven by some problem related to the dispersion behaviour of the Lamb waves that appeared during the formulation of the finite difference solvers, i.e. certain modes, in particular the A0, do not show the correct dispersion. A detailed analysis of this topic is carried out in the paper. The sensors configuration is a 5×5 equally spaced grid placed on the free upper surface with the middle sensor removed due to the presence of the force, sensors are not placed at the clamped sides as shown in Illustration 8.

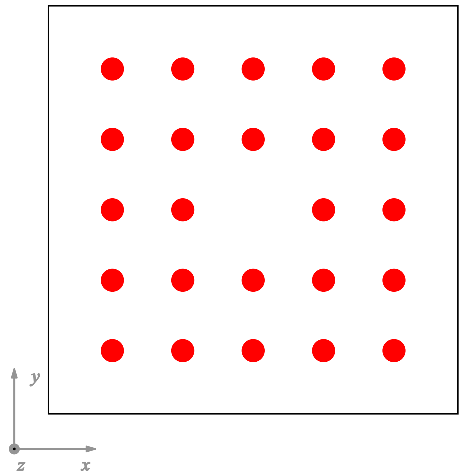

Figure 8: Sensors position reprensented by red dots

In Figure 9 the material distribution is shown, note that along the thickness of the plate material properties remain constant.

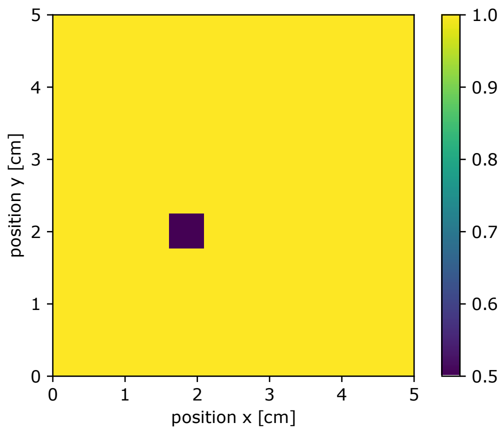

Figure 9: Material distribution

The network selected has the same structure as the one used in the mono-dimensional case, the only difference is the number of inputs that becomes $\boldsymbol{x}$ and $\boldsymbol{y}$, the position in space in the respective direction. The results obtained by running the algorithm are represented in Figure 10. The method has been capable of indicating the damage position, however not as precisely as in the other case due to the increased complexity of the domain.

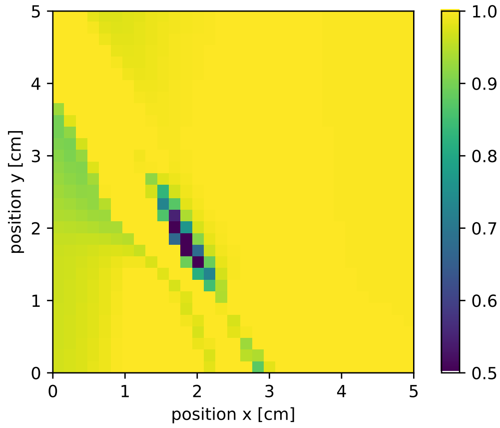

Figure 10: Physics-informed coupled with finite difference solver method results

4. Conclusions

In this study, two physics-informed data-driven methods for localizing damage in thin-walled structures using Lamb waves are presented. These methods aim to perform damage diagnosis without relying on black-box neural networks or feature extraction processes. The two methods differ in how they implement known physical laws. The first method incorporates the physics of the problem directly into the loss function, while the second method involves a neural network predicting material distribution, coupled with an in-house finite difference solver for solving elastodynamic simulations involving Lamb waves in 1D, 2D, and 3D. Three case studies were conducted: in the mono-dimensional case, both methods effectively detected damage position in a string domain. However, the first method had limitations in accurately quantifying material properties in the damaged region and suffered from numerical stability problems. The second method overcame these limitations but had increased computational demands. Regarding the two-dimensional case, only the second method was tested on a plate under plane strain, demonstrating its ability to detect the damage location and evaluate wave speed changes in the damaged area. Finally, in the three-dimensional case, the second method was tested on a plate, successfully detecting damage position but with less accuracy than in the 1D and 2D cases. The computational cost of the second method was noted to be high due to the embedded finite difference solver. Recommendations to improve efficiency included implementing convolutional neural networks, using gradient clippings, loss function weighting, and GPU implementation of finite difference solvers. The formulation of the finite difference method for the wave equation is intricate, and establishing accurate stress-free boundary conditions has posed a challenge in simulating Lamb wave modes. This complexity has resulted in an incomplete representation of dispersion phenomena in specific configurations of the forcing term, thereby restricting the available frequency range. As a prospective expansion, evaluating the method's capacity for damage detection using sensor data obtained from an actual plate equipped with piezoelectric sensors could enhance its assessment in a real-world context.

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