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SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

Real-time estimation of power system inertia

Laurea Magistrale in Electrical Engineering - Ingegneria Elettrica

Author: Marco Migliaretti

Advisor: Prof. Alberto Berizzi

Co-advisor: Andrea Vicario, Edoardo Daccò

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1. Introduction

In the last years, renewable energy has spread more and more inside the grids of every country. Their huge increase is expected to mitigates many important problems of our society like climate change, energy security and environmental sustainability. While the increase in the use of renewable energy contributes to reducing greenhouse gas emissions and thus mitigating climate change, the increasing penetration of renewable energy in the national electric systems has brought many challenges to be addressed in the future as:

- **Voltage Control**
  The RES can be installed only where the natural resources, exploited for energy production, are available. In this condition, the main demanding centre could be far from the renewable generating units and this could lead to the inversion of the power flow in some lines of the distribution network causing congestion and voltage control issues.
- **Flexibility of the plants**
  Most of the RES generation are characterized by uncertainty, unpredictability and variability; therefore, the flexibility of the plants becomes one of the most important features of the modern power plants.
- **System security**
  Any power imbalance in the grid involves dangerous frequency excursion that, without any security systems, could be catastrophic for the entire grid. Usually, the power imbalances are damped by the inertia of the rotary machines connected to the grid, but it decreases when wind and photovoltaic generators are connected to the grid. This is because RES are decoupled from the grid by converters and do not provide directly rotational inertia.

The main aim of this work is focused on this last problem related to the diffusion of the RES generation.

With a continuous monitoring of the inertia level of the system, it is possible to understand when the system is in a dangerous situation and to start compensate it.

Nowadays, for the measurements of the system we can exploit the Phasor Measurement Unit (PMU). PMUs are devices able to acquire measurements voltages and current phasors of a three-phase network in a synchronized way with a reporting maximum frequency of typically 30–60 samples per seconds [8]. About 10 years ago the commercial PMUs were very expensive (reported as costing \$40000 to \$180000

in 2014, [7]) and this aspect was very limiting for a real application, but the continuous development of this technology has lead to a sensible reduction of the PMUs's price. In the last years, low-cost PMU are being developed and the cost of developed prototypes is very low about 110€[5].
The new features in real-time measurements of the PMUs merged with the huge decrease in the price of this technology open the possibility of a true application of the methods exposed in this work.

2. Power system stability

A power system is an electrical network whose main function is to supply the loads through the energy that is transmitted and distributed from the generation units. Traditionally, the power was produced by big thermal or hydraulic generation units and then was converted and carried directly to the load sites. In this condition the power flow was unidirectional from the generator to the load and the system was defined *centralized*. In the last 15 year, the massive increase of the installed capacity of RES has modified the operation of the transmission and distribution grid. In particular, the distribution system is designed to distribute energy from upstream to downstream. With the advent of the RES, the distribution network reaches not only loads but also generators, which means that power flows in this new scenario are bidirectional and no longer unidirectional.
The bi-directionality of power flows has changed many of the logic initially used to operate the distribution networks. This change is due not only to the single big renewable generation units but also to the overall effect of all small renewable generation units of individual users. This new concept of the system is called *distributed* system. The traditional power systems were based on operating at synchronous speed. Any time in the system there is a power imbalance between the mechanical power of the generating unit's shaft and the electrical power absorbed by the loads the system experiences a perturbation of the synchronous speed. The relationship between the power imbalance and the changing of the synchronous speed can described by the *equation of motion* for a generator [6]

$$J\frac{d\omega_m}{dt} = T_a = T_m - T_e \qquad (N\cdot m) \qquad (1)$$

Another fundamental equation is the swing equation. To obtain the swing equation we have to start from the Equation 1. After some computation is possible to reach the Equation 2 commonly referred to as the *swing equation* because it represents swings in rotor angle δ_m during disturbances.

$$\frac{2H}{\omega_{sm}}\frac{d\omega_m}{dt} \cong (P_m - P_e - D(\omega_m - \omega_{sm}))\frac{1}{A_{nom}} \quad (W)$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} \cong p_m - p_e - \frac{D}{A_{nom}}(\omega - \omega_s) \quad (p.u.) \qquad (2)$$

The swing equation is the basic equation that drives the motion of the rotor of a generator and links its dynamic behaviour with the working frequency of a synchronous machine.
Systems in which many generators and loads are interconnected by tie-lines are called *multi-machine systems*. In these systems, the Equation 2 can be used to describe the dynamic of each one of the k machines of the system [1]. The general set of the k equation, neglecting the damping for the sake of simplicity, appears as

$$\begin{cases} \frac{2}{\omega_s}H_1A_{nom,1}\frac{d\omega_1}{dt} \cong P_{1,m} - P_{1,e} \quad (W) \\ \frac{2}{\omega_s}H_2A_{nom,2}\frac{d\omega_2}{dt} \cong P_{2,m} - P_{2,e} \quad (W) \\ \vdots \\ \frac{2}{\omega_s}H_kA_{nom,k}\frac{d\omega_k}{dt} \cong P_{k,m} - P_{k,e} \quad (W) \end{cases} \qquad (3)$$

A common transformation used to study the multi-machine dynamic models is the center-of-inertia (COI) reference. The equivalent motion equation of a multi-machine system expressed with respect to its COI reference is given by

$$\frac{2H_{COI}}{\omega_s}\frac{d\omega_{COI}}{dt} \cong p_{eq,m} - p_{eq,e} \quad (p.u.) \qquad (4)$$

3. Determination of the Power System Inertia

3.1. Sparse identification of nonlinear dynamics (SINDy)

The sparse identification of nonlinear dynamics (SINDy) algorithm is a flexible method which is used to discover linear and nonlinear dynamic system models from data.
The increase of the available data, due to the

lower cost of the sensor, together with the increase of computation power and data storage of our devices allow us to obtain a proper environment for the application of the SINDy algorithm.
The SINDy algorithm bypasses the intractable combinatorial search through all possible model structures, leveraging on the fact that many dynamic systems have dynamics with only a few active terms in the space of possible righthand side functions [2].

3.2. Application of SINDy to a generic nonlinear dynamic system

If we consider a generic nonlinear dynamic system [2]

$$\frac{d}{dt}\mathbf{v}^T(t) = \dot{\mathbf{v}}^T(t) = \mathbf{f}^T(\mathbf{v}(t)) \tag{5}$$

The column vector $\mathbf{v}(t)$ represents the n state of the system at time t, and the nonlinear function $\mathbf{f}(\mathbf{v}(t))$ represents the dynamic constraints that define the equations of motion of the system [3]. To determine the function $\mathbf{f}$ from data, we collect a time-history of the state $\mathbf{v}(t)$ and we either measure the derivative $\dot{\mathbf{v}}(t)$ or approximate it numerically from $\mathbf{v}(t)$. The data is sampled at several times t_1, t_2, $\ldots$,t_m and arranged into two large matrices $\mathbf{V}$ and $\dot{\mathbf{V}}$.
Starting from a vector of possible non linear functions $\boldsymbol{\theta}(\mathbf{v}(t))_{(m\times c)}$, where c defines the number of columns of $\boldsymbol{\theta}(\mathbf{v}(t))$ that cannot be uniquely defined, since they depends by the different nonlinear functions inserted in the vector. A matrix $\boldsymbol{\Theta}(\mathbf{V})$ may be constructed, considering the vector $\boldsymbol{\theta}(\mathbf{v}(t))$ in each one of the m instants of time:

$$\boldsymbol{\Theta}(\mathbf{V}) = \begin{bmatrix} \theta(\mathbf{v}(t_1)) \\ \theta(\mathbf{v}(t_2)) \\ \vdots \\ \theta(\mathbf{v}(t_m)) \end{bmatrix}_{(m\times c)} \tag{6}$$

Each column of $\boldsymbol{\theta}(\mathbf{v})$ represents a candidate function for the right hand side of Equation 5. Note that $\boldsymbol{\theta}(\mathbf{v}(t))$ is a vector of non linear functions of $\mathbf{v}(t)$, as opposed to $\boldsymbol{\Theta}(\mathbf{V})$, which is a data matrix.
The entries in the matrix of nonlinearities are built with great freedom. Since only a few of these nonlinearities are active in each row of $\mathbf{f}$, it is possible to set up a sparse regression problem to define the sparse vectors of the coefficients $\boldsymbol{\Xi} = [\boldsymbol{\xi}_1 \ \ \boldsymbol{\xi}_2 \ \ \cdots \ \ \boldsymbol{\xi}_n]$ that determine which nonlinearities are active.
The dynamic system in Equation 5 can now be represented in terms of data matrices as:

$$\dot{\mathbf{V}} = \boldsymbol{\Theta}(\mathbf{V})\boldsymbol{\Xi} \tag{7}$$

Each column $\boldsymbol{\xi}_k$ in $\boldsymbol{\Xi}$ is a vector of unknowns determining the active terms in the k-th equation of the Equation 5 and can be found using a convex l_1-regularized sparse regression algorithm as:

$$\boldsymbol{\xi}_k = \arg\min_{\boldsymbol{\xi}_k'} \|\dot{\mathbf{V}}_k - \boldsymbol{\Theta}(\mathbf{V})\boldsymbol{\xi}_k'\|_2 + \lambda\|\boldsymbol{\xi}_k'\|_1 \quad \forall k \tag{8}$$

where $\dot{\mathbf{V}}_k$ is the k-th column of $\dot{\mathbf{V}}$, and λ is called sparsity-promoting knob.
A parsimonious model will provide an accurate model fit in Equation 7 with as few terms as possible in $\boldsymbol{\Xi}$. The algorithm used in SINDy for the estimation of the few active terms present in the matrix $\boldsymbol{\Xi}$ is the Sequentially thresholded least-squares algorithm (STLS) analyzed more in depth in Subsection 3.3. Once $\boldsymbol{\Xi}$ has been determined, a model of each of the governing equations in the Equation 5 may be constructed as follows:

$$\dot{\mathbf{v}}_k = \theta(\mathbf{v}_k)\boldsymbol{\xi}_k \tag{9}$$

The result of the SINDy regression is a parsimonious model that includes only the most important terms required to explain the observed behaviour.

3.3. Sequentially thresholded least-squares algorithm (STLS)

The STLS, given a parameter λ that specifies the minimum magnitude for a coefficient in $\boldsymbol{\Xi}$, perform a least squares fit and then zero out all coefficients with magnitude below the threshold. [4]
In particular, if we want to solve the Equation 7 with respect to $\boldsymbol{\Xi}$, the result is an overfitted matrix in which it is difficult to identify the true active terms for the reconstruction of the dynamic behaviour of the system. For this reason, we start a sequential method that allows to increase the sparsity of the matrix $\boldsymbol{\Xi}$ repeating the same procedure for k times.
In every repetition the terms of the matrix $\boldsymbol{\Xi}$ that respect this relationship $|\xi_k| < \lambda$ are set

equal to zero.
The *sparsification knob* allows to identify a threshold under which the algorithm sets the term ξ_k equal to zero.
Then, we exclude from the matrix $\mathbf{\Theta}(\mathbf{V})$ the term whose results is a term ξ_k considered negligible and fixed equal to zero.
Finally, the matrix $\mathbf{\Xi}$ is recomputed with the new matrix $\mathbf{\Theta}(\mathbf{V})$.
This process of fitting and thresholding is performed until convergence.
The convergence is reached when the sparsity of the matrix doesn't change between two consecutive iterations. Usually, ten iterations are enough to achieve convergence.

3.4. Application of SINDy for the estimation of the power system inertia

If we consider to have the possibility of measure the value over time of system variables, we can define the matrices $\mathbf{X}_{(m\times n)}$,$\mathbf{Y}_{(m\times n)}$,$\mathbf{Z}_{(m\times n)}$ in which each column is related to a machine of the studied electric system.
Where:

- $\mathbf{X}$ = matrix of the electromagnetic power in MW over the time t in each machine
- $\mathbf{Y}$ = matrix of the angular velocity of the rotor in rad/s over the time t in each machine
- $\mathbf{Z}$ = matrix of the mechanical power in MW over the time t in each machine
- n = number of variables measured for that specific quantity
- m= number of measurement made for each specific variables

The relationship between these three matrices can be expressed using the Equation 2 and it results as

$$\frac{2H}{\omega_s} A_{nom} \frac{d\mathbf{Y}}{dt} \cong \mathbf{Z} - \mathbf{X} - D(\mathbf{Y} - \omega_s) \qquad (MW) \tag{10}$$

If we elaborate the Equation 10, in order to obtain an equation with the same shape of Equation 5, the result is

$$\begin{aligned} \frac{d\mathbf{Y}}{dt} &\cong \frac{\omega_s}{2H} \frac{1}{A_{nom}} \left(\mathbf{Z} - \mathbf{X} - D\left(\mathbf{Y} - \omega_s\right)\right) \;\; (rad/s^2) \\ &\cong \frac{\omega_s}{2H} \frac{1}{A_{nom}} \left(D\omega_s - \mathbf{X} - D\mathbf{Y} + \mathbf{Z}\right) \;\; (rad/s^2) \end{aligned} \tag{11}$$

Where the matrix $\dot{\mathbf{Y}}$ is the matrix which holds time derivative computed and obtained using the fourth-order centered difference approximation.
Starting from the vector $\boldsymbol{\theta}(t)$ of the functions present in the Equation 2, a matrix $\mathbf{\Theta}$ is built considering the vector $\boldsymbol{\theta}(t)$ in each one of the m instants of time:

$$\begin{aligned} \mathbf{\Theta}(\mathbf{W}) = \begin{bmatrix} \boldsymbol{\theta}(t_1) \\ \boldsymbol{\theta}(t_2) \\ \vdots \\ \boldsymbol{\theta}(t_m) \end{bmatrix}_{(m\times(3n+1))} &= \begin{bmatrix} \mathbf{1} & \mathbf{X} & \mathbf{Y} & \mathbf{Z} \end{bmatrix}_{(m\times(3n+1))} \\ = \begin{bmatrix} \mathbf{1} & \mathbf{W} \end{bmatrix}_{(m\times(3n+1))} \end{aligned} \tag{12}$$

where the symbol $\mathbf{W}$ defines the matrix of the measurements.
In matrix $\mathbf{\Theta}(\mathbf{W})$ the terms corresponding to polynomial terms with order higher than one and any other function is not considered, since they do not appear in the Equation 11.
The Equation 11 can now be represented in terms of data matrices as:

$$\dot{\mathbf{Y}}_{\mathbf{est}} = \mathbf{\Theta}(\mathbf{Y})\,\mathbf{\Xi} \tag{13}$$

The non-zero elements ξ of the matrix $\mathbf{\Xi}$ are corresponding to the active terms of the right side of the Equation 11.
Each column $\boldsymbol{\xi}_k$ in $\mathbf{\Xi}$ is a vector of coefficients determining the active terms in the k-th differential equation in Equation 11 and can be found using a a convex l_1-regularized sparse regression algorithm as:

$$\boldsymbol{\xi}_k = \arg\min_{\boldsymbol{\xi}'_k} \|\dot{\mathbf{W}}_k - \mathbf{\Theta}(\mathbf{Y})\boldsymbol{\xi}'_k\|_2 + \lambda\|\boldsymbol{\xi}'_k\|_1 \tag{14}$$

Ideally, once the sparse regression problem is solved, the matrix $\mathbf{\Xi}$ becomes very sparse and it can be expressed as

$$\dot{\mathbf{Y}}_{\mathbf{est}} = \begin{bmatrix} \mathbf{1} & \mathbf{X} & \mathbf{Y} & \mathbf{Z} \end{bmatrix}_{(m\times(3n+1))} \mathbf{\Xi}_{(3n+1)\times n} \tag{15}$$

However, is very common obtain higher number of non-zero terms in the $\mathbf{\Xi}$, due to errors during the sparse regression problem. Since the Equation 15 approximates the matrix $\dot{\mathbf{Y}}$, the following equality can be defined as

$$\dot{\mathbf{Y}} \cong \dot{\mathbf{Y}}_{\mathbf{est}} \tag{16}$$

Then, if we only consider the terms related to the 1-st machine for the sake of simplicity, the Equation 16 becomes

$$\frac{\omega_s}{2H_1}\frac{1}{A_{nom,1}}\,[D\omega_s - \mathbf{x_1}(t) - D\mathbf{y_1}(t) + \mathbf{z_1}(t)] \simeq \dot{\mathbf{Y}}_{1,\mathbf{est}} \tag{17}$$

where the matrix $\dot{\mathbf{Y}}_{1,\mathbf{est}}$ appear as:

$$\dot{\mathbf{Y}}_{1,\mathbf{est}} = [\mathbf{1}\quad \mathbf{X}\quad \mathbf{Y}\quad \mathbf{Z}]\begin{bmatrix} \xi_{1,k} \\ \xi_{1,x_1} \\ 0 \\ \vdots \\ 0 \\ \xi_{1,y_1} \\ 0 \\ \vdots \\ 0 \\ \xi_{1,z_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Then, it is possible to correlate the terms of the matrix $\mathbf{\Xi}$ with the constant terms that multiply the vectors $\mathbf{x_1}(t)$, $\mathbf{y_1}(t)$, $\mathbf{z_1}(t)$ in the Equation 17

$$\xi_{1,k} = \frac{\omega_s}{2H_1}\frac{1}{A_{nom,1}}\,D\,\omega_s \tag{18}$$

$$\xi_{1,x_1} = -\frac{\omega_s}{2H_1}\frac{1}{A_{nom,1}} \tag{19}$$

$$\xi_{1,y_1} = -\frac{\omega_s}{2H_1}\frac{1}{A_{nom,1}}\,D \tag{20}$$

$$\xi_{1,z_1} = \frac{\omega_s}{2H_1}\frac{1}{A_{nom,1}} \tag{21}$$

The same procedure can be repeated for each machine of the system.
The estimated inertia value can be extracted from the active terms of the vector $\boldsymbol{\xi}_k$ through these computations:

$$H_1 = \frac{\omega_{sm}}{2\xi_{1,k}}\frac{1}{A_{nom,1}}\,D\,\omega_{sm} \tag{22}$$

$$H_1 = -\frac{\omega_{sm}}{2\xi_{1,x_1}}\frac{1}{A_{nom,1}} \tag{23}$$

$$H_1 = -\frac{\omega_{sm}}{2\xi_{1,y_1}}\frac{1}{A_{nom,1}}\,D \tag{24}$$

$$H_1 = \frac{\omega_{sm}}{2\xi_{1,z_1}}\frac{1}{A_{nom,1}} \tag{25}$$

3.5. Division of the machine dynamics

From the Equation 11 and the Equation 12 previously defined in the Section 3.4 it is possible to delineate a different method in which the dynamic of a single machine is studied individually. Considering in the vector $\boldsymbol{\theta}(t)$ the functions related to a single machine, it is possible to solve many smaller problems, instead of one large problem.
The vector $\boldsymbol{\theta}_j(t)$ of a generic j-th machine result as

$$\boldsymbol{\theta}_j(t) = [1 \quad x_j(t) \quad y_j(t) \quad z_j(t)]_{(1\times 4)} \tag{26}$$

In the same way done in previous sections, a matrix $\mathbf{\Theta}$ is built considering the vector $\boldsymbol{\theta}_j(t)$ in each one of the m instants of time:

$$\mathbf{\Theta}_j(\mathbf{w}) = \begin{bmatrix} \boldsymbol{\theta}_j(t_1) \\ \boldsymbol{\theta}_j(t_2) \\ \vdots \\ \boldsymbol{\theta}_j(t_m) \end{bmatrix}_{(m\times 4)} = [\mathbf{1} \quad \mathbf{x}_j(t) \quad \mathbf{y}_j(t) \quad \mathbf{z}_j(t)]_{(m\times 4)}$$

$$= [\mathbf{1} \quad \mathbf{w}]_{(m\times 4)} \tag{27}$$

where the symbol $\mathbf{w}$ defines the matrix of the measurements related to a single machine.
Instead, the time derivative vector of a generic machine j-th can be represented considering the j-th column of in matrix $\dot{\mathbf{Y}}$. The dynamics of a j-th machine can be represented in term of vector product as

$$\dot{\mathbf{y}}_{j,est}(t) = \mathbf{\Theta}_j\boldsymbol{\xi}_j \tag{28}$$

The elements of the vector $\boldsymbol{\xi}_j$ are the active terms corresponding to the terms of the right side of the Equation 11 take into account only the equation of the j-th machine .
In this case we know that each term of the vector $\boldsymbol{\xi}_j$ must be different from zero, since all are

present in the Equation 11 related to a generic j-th machine.
The SINDy algorithm exploit a sequential sparse regression and optimization process performing the STLS.
This process is used to increase the sparsity of the matrix $\boldsymbol{\Xi}$ providing a better estimation of active terms related to the matrix $\boldsymbol{\Theta}$ and fixing to zero the elements not present in each one of the equations studied.
Nevertheless, in this case is possible to reduce the computation burden avoiding the use of a sequential method, since we don't need to increase the sparsity of the vector $\boldsymbol{\xi}_j$.
Therefore, the elements of the vector $\boldsymbol{\xi}_j$ can be computed using a simpler algorithm. In this work we test some algorithm already provided by the software MATLAB such as:

- mldivide:
- Moore-Penrose pseudoinverse
- Fit robust linear regression
- Singular value decomposition
- Least Absolute Shrinkage and Selection Operator (LASSO)

Once the elements of the vector $\boldsymbol{\xi}_j$ are determined, taking into account that
Equation 28 approximates the j-th column of the matrix $\dot{\mathbf{Y}}$, the following equality can be defined

$$\dot{\mathbf{y}}_j(t) \cong \dot{\mathbf{y}}_{j,est}(t) \tag{29}$$

Then, the Equation 29 can be represented as

$$\frac{\omega_s}{2H_j}\frac{1}{A_{nom,j}}\left[D\omega_s - \mathbf{x_j}(t) - D\mathbf{y_j}(t)+\mathbf{z_j}(t)\right] \cong \dot{\mathbf{y}}_{j,est}(t) \tag{30}$$

Finally, it is possible to correlate the terms of the vector $\boldsymbol{\xi}_j$ with the constant terms that multiply the vectors $\mathbf{x_j}(t)$, $\mathbf{y_j}(t)$, $\mathbf{z_j}(t)$ in right side of the Equation 30

$$\xi_{j,k} = \frac{\omega_s}{2H_j}\frac{1}{A_{nom,j}}\,D\,\omega_{sm} \tag{31}$$

$$\xi_{j,x} = -\frac{\omega_s}{2H_j}\frac{1}{A_{nom,j}} \tag{32}$$

$$\xi_{j,y} = -\frac{\omega_s}{2H_j}\frac{1}{A_{nom,j}}\,D \tag{33}$$

$$\xi_{j,z} = \frac{\omega_s}{2H_j}\frac{1}{A_{nom,j}} \tag{34}$$

The estimated inertia value can be extracted from the active terms of the vector $\boldsymbol{\xi}_j$ through these computations:

$$H_j = \frac{\omega_s}{2\xi_{j,k}}\frac{1}{A_{nom,j}}\,D\,\omega_s \tag{35}$$

$$H_j = -\frac{\omega_s}{2\xi_{j,x}}\frac{1}{A_{nom,j}} \tag{36}$$

$$H_j = -\frac{\omega_s}{2\xi_{j,y}}\frac{1}{A_{nom,j}}\,D \tag{37}$$

$$H_j = \frac{\omega_s}{2\xi_{j,z}}\frac{1}{A_{nom,j}} \tag{38}$$

The same procedure can be repeated for each machine of the system.

3.6. Moving window method for a real time estimation

The application of this method requires the measurements of electric power and frequency and the computation of the frequency derivative. The following step is the identification of a time window that defines a small part of the variable trends where to apply the algorithm studied in Section 3.5 and Section 3.4. In this work, each time windows is assumed to be of 5 s large.
Then, iterating this process moving the time window one second ahead, is possible to obtain the trend of the inertia in real-time estimated every second. The obtained values of inertia are assigned to the end of the time window considered. For example, the inertia value estimated using the time window 0 s - 5 s is assigned to the instant 5 s.

4. Performed tests and Numerical results

In this chapter are reported the application of the method exposed in the previous chapters and the numerical results obtained.
Theoretically, the value of inertia can be computed starting form one of these variables: ξ_k, ξ_x, ξ_z, ξ_z.
However, in every test of this chapter only the variable ξ_x were used for the estimation. Due to the fact that the computation using the variable ξ_z is not feasible in a real application since is impossible measure the mechanical power (z), while the calculation using the variables ξ_k and ξ_y are affected by the variable D.
The value of D is difficult to be identified in a

precise way, therefore also the value of the variables ξ_k and ξ_y will not be precise.
The different tests performed and the most significant result obtained are listed below:

- **Test 1**: Application of SINDy
  - Simulation 1:
    SINDy algorithm is applied in an ideal condition with a single time windows of 30 s to estimate the inertia of each of machine of the system.
  - Simulation 2:
    SINDy algorithm is applied for the inertia estimation in the moving window method to estimate the equivalent inertia of two areas.

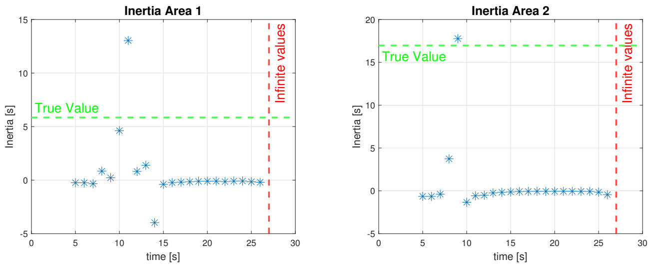

Figure 1: Test 1 - Simulation 2 - Inertia trend of Area 1 and Area 2

- **Test 2**: Removal of mechanical power
  - Simulation 1:
    Comparison of the application of the moving window method considering or not the mechanical power variables in the inertia estimation, during the opening of the line 1.

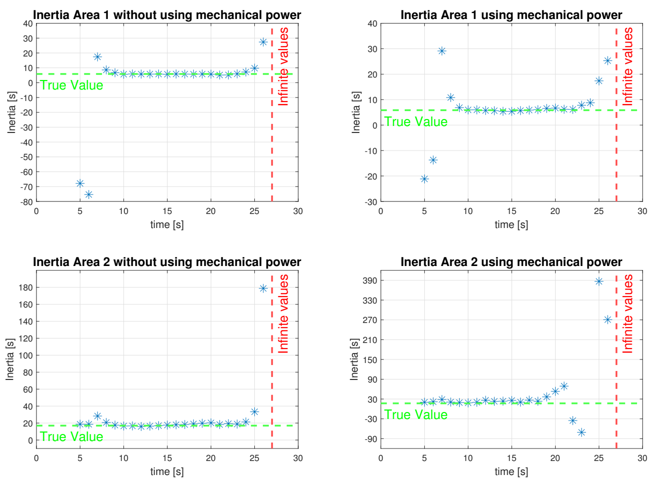

Figure 2: Test 2 - Simulation 1 - Inertia trend of Area 1 and Area 2

  - Simulation 2:
    Comparison of the application of the moving window method considering or not the mechanical power variables in the inertia estimation, during the opening of the line 8.

- **Test 3**: MATLAB algorithms comparison
  - Simulation 1:
    The algorithms listed in Section 3.5 are applied to estimate the inertia of each machine of the system, considering an ideal condition with a single time windows of 30 s.
  - Simulation 2:
    The algorithms listed in Section 3.5 are applied for the inertia estimation in the moving window method to estimate the equivalent inertia of two areas.
- **Test 4**: Opening of a line
  - Simulation 1:
    Moving window method to estimate the inertia trend during the opening of the line 1 is applied.

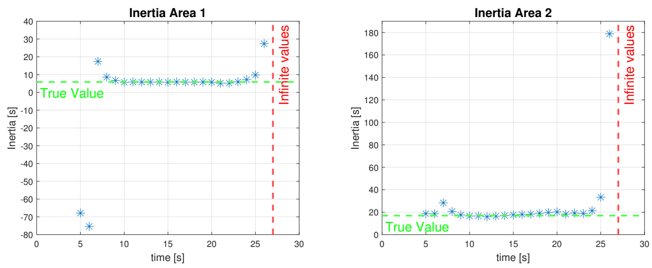

Figure 3: Test 4 - Simulation 1 - Inertia trend of Area 1 and Area 2

  - Simulation 2:
    Moving window method to estimate the inertia trend during the opening of the line 8 is applied.
- **Test 5**: Trig of a load
  - Simulation 1:
    Moving window method to estimate the inertia trend during the trig and reclosure of load 7 is applied.
  - Simulation 2:
    Moving window method to estimate the inertia trend during the trig and reclosure of load 9 is applied.
- **Test 6**: Short-circuit at a bus
  - Simulation 1:
    Moving window method to estimate the inertia trend during a short-circuit at the bus 5 is applied.
  - Simulation 2:
    Moving window method to estimate the

inertia trend during a short-circuit at the bus 10 is applied.

- **Test 7**: Increased sensitivity of the PMUs
  - In this test, we perform a single simulation in which the same computation of Test 4 are performed increasing the sensitivity of the PMUs.

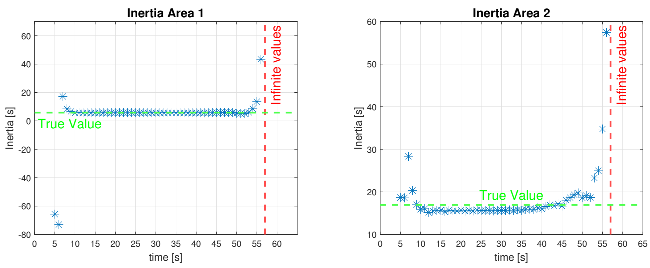

Figure 4: Test 7 - Inertia trend of Area 1 and Area 2

5. Conclusions and future developments

This thesis proposed a moving windows method exploiting different algorithms to estimate the inertia of the system, to increase system awareness. The conclusion highlighted in these tests is that is possible to identify the inertia of the system immediately after a perturbation, for a few times, before the estimation tends to infinite. Moreover, we prove that a possibility to increase the time in which the inertia converges to the theoretical value is to increase the PMUs sensitivity.
This thesis studies a possible solution of a small part of the great goal to be able to estimate the inertia of the system in every instant of time. To reach this goal there are many aspects still to study such as:

- **Real measurement**:
  The measurements used for the application of the exposed method are ideal. The real measurement is characterized by measurement errors and noise that can negatively impact the estimation.
- **Increase of the equivalent area considered**:
  The increase of the equivalent area considered could be problematic. Since, as the dimension of the equivalent area increases the frequency oscillation after a perturbation decreases. When the oscillation of frequency becomes less than the sensitivity of the PMUs the changes in the system are not detected by the PMUs.
- **Synthetic inertia monitoring**:
  The estimating system should consider also the presence of the synthetic inertia
- **Better algorithm for the estimation**:
  The study around new data-driven algorithms are several and many of these are not been tested for the inertia estimation. An algorithm more robust and with a less computational burden always improves the real-time estimation performance.

The reaching of all these goals opens a serious possibility to permit the estimation of the inertia of the system during every perturbation of the system.

6. Acknowledgements

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