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Executive Summary of the Thesis

Relative motion control of cluster formation in a geostationary orbit with the J_{22} perturbation

Laurea Magistrale in Space Engineering - Ingegneria Spaziale

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1. Introduction

Nowadays Formation Flight finds application in different fields: from communication to Earth observation. With the advent of CubeSats, the Earth and Space observation missions, once carried out by large spacecraft, are now undertaken by multiple satellites configurations pursuing the same objective. The benefits of clusters of micro-spacecraft improve capabilities that would not be achievable by single large spacecraft, in terms of time and spatial resolution.
In this direction, interferometry in Geosynchronous Earth Orbit (GEO) is a promising technology. Passive microwave interferometric radiometry is a feasible instrument to be implemented considering a geostationary orbit [1]. This setup requires precise modelling of the relative motion to grant accurate imaging.
The present study aims at enhancing the precise relative motion in GEO. The description of the dynamics is based on Relative Orbital Elements (ROEs) and the analytical evolution is computed through a State Transition Matrix (STM). The geodetic effect up to J_{22} is introduced and controlled. A continuous feedback control assures the precision to comply with the requirements imposed by the scientific payload.

2. Relative motion

The quasi-nonsingular ROEs are selected for the model formulation and reported in Eq. (1) [2].

$$\delta\alpha = \begin{pmatrix} \delta a \\ \delta\lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{pmatrix} = \begin{pmatrix} \Delta a / a_c \\ \Delta u + \Delta\Omega \cos(i_c) \\ \Delta e_x \\ \Delta e_y \\ \Delta i \\ \Delta\Omega \sin(i_c) \end{pmatrix} \tag{1}$$

Historically, the relative motion is described by the Hill-Clohessy-Wiltshire equations. This formulation is represented in Hill's frame, which in GEO corresponds to the Radial-Tangential-Normal (RTN). Starting from the general closed-form solution of these equations, it is possible to re-wright the formulation in terms of ROEs [2]. The Gauss' variational equations are exploited to implement the relative state into that configuration.
Once retrieved a convenient mapping, the analytical model can be introduced. Brouwer's transformation to Mean Relative Orbital Elements allows to focus only on the secular ROEs variations, without accounting for periodical oscillations. The behaviour of the new set can be

linearised and the relative dynamics can be the result of an algebraic problem, Eq. (2).

$$\delta\dot{\alpha} = A(\alpha_c)\delta\alpha + Bu \tag{2}$$

The plant matrix A is composed of the partial derivatives of the evolution of the chief's Keplerian elements α_c, that have been evaluated. The STM, reported in Eq. (3), is assembled with $p = 2$, taking into account the non-spherical symmetry of Earth's mass distribution [3]. α_c is updated at each time step, this is possible numerically propagating its trajectory throughout the two-body problem in Cartesian coordinates and transformed in Keplerian.
The linear system needs to be controlled. The control logic optimises the required acceleration u in Hill's frame. The control matrix B performs the mapping, interfacing the actuation with the ROEs.

3. Feedback control

The chief satellite is allowed to drift and the control only seeks formation-keeping. A closed-loop algorithm takes as input the reference orbit and compares it with the actual state, updated with the analytic model. The continuous feedback control is implemented only on the deputies, the analysed time scale is below the station-keeping impulsive manoeuvres interval and therefore it is not implemented.
The selected controller is based on the *Optimal Control* theory. It solves problems described by a linear set of differential equations by minimising a quadratic cost function (LQ problems). The Linear Quadratic Regulator is set for a limited time, therefore, the *Finite Horizon* formulation is chosen. It requires the numerical integration of the *Differential Riccati's Equation* to compute the control[4].
To evaluate the optimal actuation, the desired ROE state is introduced in the algorithm. The final controlled state should match the target, so $\delta\alpha(t = t_f) = \delta\alpha_d$. To grant this assumption the numerical integration is performed backwards, taking $\delta\alpha_d$ as the initial condition. A better and faster convergence is obtained by tuning the *Weighting Matrices* with one of the deputies.
The implemented control algorithm is schematised hereafter.

Algorithm 1 Control Algorithm

```
Load: input % Problem set-up
Propagate: odeCart % Chief perturbed evolution
for t ∈ [t0, tf] do
    Compute A(αc(t)) & B(αc(t)) % System plant definition
end for
for t ∈ [tf, t0] do
    Integrate: odeRiccati % Backward
    Compute M(t)
end for
for t ∈ [t0, tf] do
    Flip: M(t) % For time coherence
    Compute K(t) % Gain matrix
    Compute u(t) % Optimal control
    if u(t) < u_min then
        u(t)=0
    else if u(t) > u_min then
        u(t)=u
    end if
    Compute α(t) % State update
end for
Store: (α, u) % Result report
```

The operations describe a low-thrust control mission. The adequate thrusters selection falls on micro-thrusters. Electric and cold gas propulsion are proposed. A lower limit on the thrust generated is set to keep a realistic simulation: $T_{min} = 10^{-7}\ N$. Finally, the total acceleration is computed and, consequently, by trapezoidal integration Δv is obtained as well.

$$\vec{A}(\delta\vec{\alpha_c}) \approx \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ a\sum_p g_a^{(p)} & 0 & \sum_p g_{e_x}^{(p)} & \sum_p g_{e_y}^{(p)} & \sum_p g_i^{(p)} & 0 \\ aA_1\sum_p \dot{\omega}_a^{(p)} & 0 & C + A_1\sum_p \dot{\omega}_{e_x}^{(p)} & -S + A_1\sum_p \dot{\omega}_{e_y}^{(p)} & A_1\sum_p \dot{\omega}_i^{(p)} & 0 \\ aA_2\sum_p \dot{\omega}_a^{(p)} & 0 & S + A_2\sum_p \dot{\omega}_{e_x}^{(p)} & C + A_2\sum_p \dot{\omega}_{e_y}^{(p)} & A_2\sum_p \dot{\omega}_i^{(p)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ a\sin(i)\sum_p \dot{\Omega}_a^{(p)} & 0 & \sin(i)\sum_p \dot{\Omega}_{e_x}^{(p)} & \sin(i)\sum_p \dot{\Omega}_{e_y}^{(p)} & \sin(i)\sum_p \dot{\Omega}_i^{(p)} & 0 \end{bmatrix} \tag{3}$$

4. Mission simulation

The analysed test case is a cluster formation of seven satellites for a remote sensing mission. A central chief is propagated for ten days with six deputies forming a hexagon around it Fig. 1.

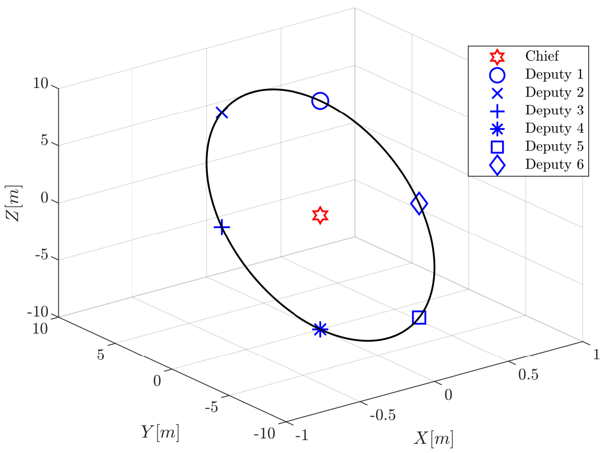

Figure 1: Initial cluster formation.

The selected weighting matrices for the LQR control are reported in Eqs. (4) and (5). The tuning is performed on deputy 1.

$$Q = 10^{-3}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix} \tag{4}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The analysis focuses on three different aspects of the simulated scenario, for each of them, the results are here displayed only for deputy 1. The perturbing geodetic effect depends on the longitude, therefore the results are slightly different for each couple of satellites (1-4, 2-3, 5-6). However, the disturbing accelerations are always effectively counteracted by the control.

4.1. Controlled dynamic

In Fig. 2, the natural, desired and controlled evolution is portrayed. As expected, all the Relative Orbital Elements oscillate around the prescribed reference value. The component related to the semi-major axis δa is bounded around 1 mm from the target. The variation in eccentricity seems to be above the desired threshold but once reported in dimensionless form the difference is of the order of 10^{-10}. The inclination is less affected by the Geodetic effect due to the equatorial orbit.

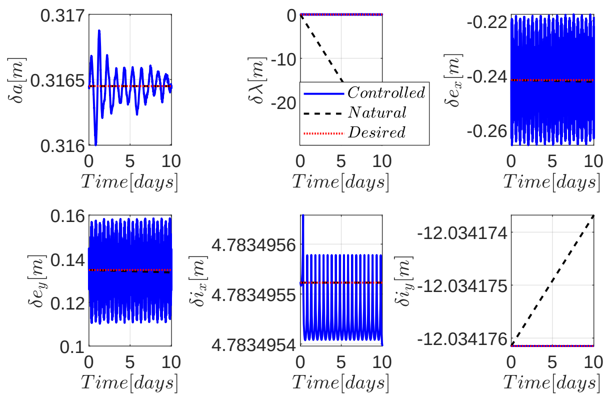

Figure 2: ROE controlled evolution.

4.2. Deviation from target

The scientific instrument requires a rigid formation. Fig. 3 highlights how the difference between the desired relative position of the first deputy with respect to the chief is in the range of 10^{-3} m. This specific satellite benefits from the tuning performed on it. Nevertheless, the control is effective and below the required precision for all the spacecraft.

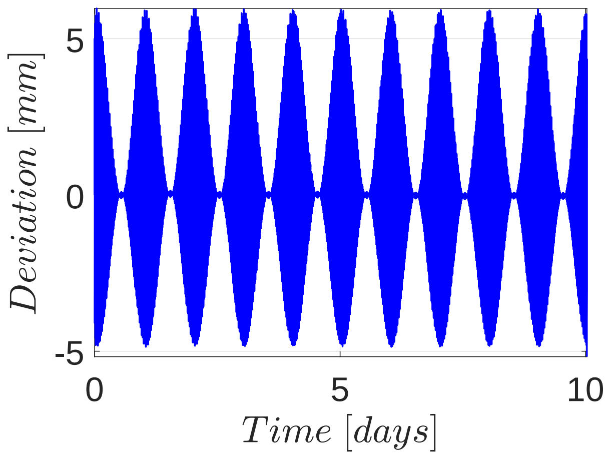

Figure 3: Deviations from desired positions.

4.3. Control effort

The provided accelerations are also reported. The radial, tangential and normal directions are aligned with the Hill chief's frame. The main contribution is the one tangentially direct, the one that affects the elements that need more control δa and $\delta\lambda$. Some spikes are present in Fig. 4, they can be explained by the overshoot provoked when the control is resumed after the imposed limitation in minimum available thrust. The overall *delta-v* required from each deputy for the ten days interval is around 7 m/s.

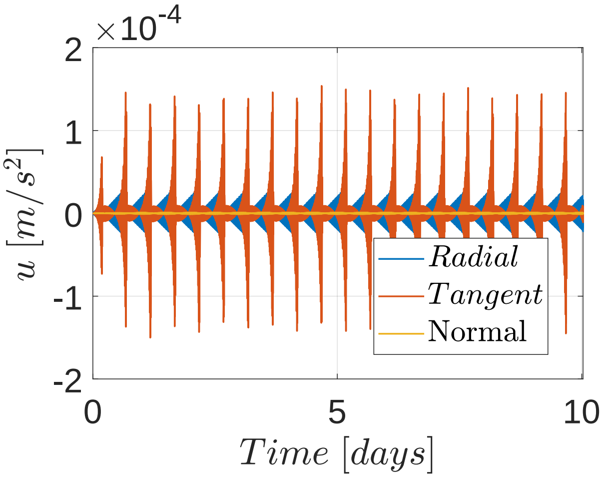

Figure 4: Control action in Hill's frame.

4.4. Inter-satellite distances

Concerning safety, to assure the lack of collisions, a check on minimum distance is performed. The lower set limit of 7 m is never violated. The nature of the mission tends to keep all the inter-satellite distances around the desired ones. Comparing the relative position, it is possible to appreciate a similar pattern for deputies laying at the same longitude. The couples 2-3 and 5-6 are perturbed and controlled in the same way, leading to an inter-satellite distance very rigid and accurate, the same that happens for the first and the fourth satellites concerning the chief.

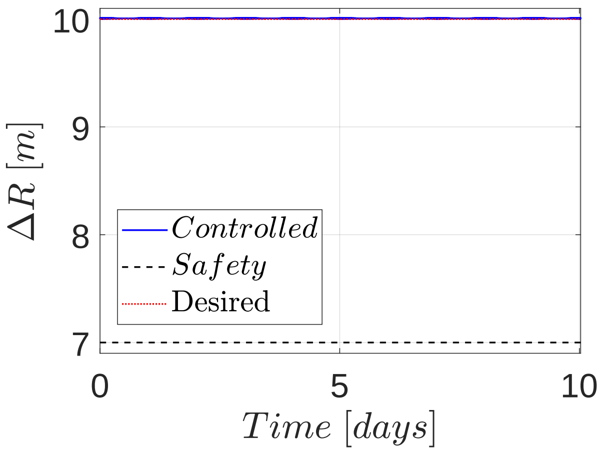

Figure 5: Chief-Deputy 1 distance.

5. Conclusions

Tab. 1 resumes the results of the simulations, highlighting the difference between the propagated and the desired formation configuration.

S/C	Target [m]	Final [m]	Dispersion [mm]
1	10	10.017	3.904
2	10	10.064	57.869
3	10	10.151	63.263
4	10	10.002	4.067
5	10	10.049	61.153
6	10	10.019	61.950

Table 1: Model and target comparison.

The implemented model represents with accuracy the real environment. The chief's natural evolution has been successfully validated with the NASA solver GMAT. The implementation of Mean Relative Orbital Elements allows an efficient and precise formulation of the relative motion. The State Transition Matrix effectively introduces the perturbation due to J_{22}, by computing the partial derivatives of the Chief's orbital elements. The Linear Quadratic Regulator assures a control accuracy in the centimetres order for all the deputies, even with a tuning performed through a qualitative sensitivity analysis on the first one.

The presented model represents a novelty for formation flying in geostationary orbit. The implementation of second-order contributions enhances the formulations already described in the literature. Linearity is its main feature, consequently, robust and accurate control is implemented. Thanks to Relative Orbital Elements computational efficiency is obtained, aiming at granting onboard relative position determination and formation-keeping. It has been proved that low thrust continuous control is a feasible choice for this kind of mission, laying the foundations for future space tests.

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