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SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

EXECUTIVE SUMMARY OF THE THESIS

An application of the Unknown Input Observer to the estimation of railway track irregularities

TESI MAGISTRALE IN MECHANICAL ENGINEERING – INGEGNERIA MECCANICA

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1. Introduction

The issue of monitoring railway vehicles' track misalignment is fundamental for a series of reasons; to grant safety (both by planning maintenance and regulate maximum traverse speed of vehicles along particularly irregular sections), but also to allow passengers to enjoy a sufficient degree of comfort.

Many methods are nowadays employed to ensure monitoring of the track geometry (collectively described by Esveld, [1]), from laser-based chord measuring to additional wheels hydraulically actuated to maintain contact with the rails (and whose motion thus corresponds to their geometry). These ways of measuring irregularity are usually implemented on so-called Diagnostic Trains, such as the "Archimede" [2] currently operating on the RFI train lines; however, such diagnostic vehicles are nearly impossible to operate on urban lines – especially underground – for a number of issues, namely the reduced distance between stations, the short time interval between on-service trains and the relative scarcity of alternative paths for the trains to follow when the rails are occupied by one such vehicles. Thus the requirement to infer the tracks geometrical data from measurements directly obtained from on-service trains (mostly accelerations). Two broad categories of methods can be discerned:

- Signal-based methods only employ an algorithm to elaborate the measuration data; the simplest example is double integration of the wheelset vertical motion in order to derive the track vertical misalignment; although simple, double integration can result in drift issues in presence of non-white noise;
- Model-based methods instead utilize a (usually linear) model and employ measurements on different points of such model to reconstruct the irregularity which generated them.

Within this last category a number of possibilities exist, both in frequency domain [3] and in time domain; the latter, although less accurate, are to be preferred for several reasons: they can work on-line (rather than relying on data post-processing) and can detect localized irregularities.

Among time-based methods two are of particular relevance: the Kalman Filter-based estimator and the U.I.O.. Once again, although the first is more accurate and robust (as exemplified by De Rosa et al. [4]), the U.I.O. features its own set of advantages, in particular it is much simpler in its application and, being deterministic as opposed to the stochastic Kalman Filter, does not require prehemptive knowledge of statistical parameters of the to-be-estimated quantities.

The paper deals with the employment of the U.I.O. to reconstruct vertical and transversal geometry of the tracks through measurations conducted on board of the 900 Series Meneghino Underground Train, currently operating on Milan Metro line. Since most works found in literature deal with high-speed convoys, the reduced speed and the largely non-straight trajectory impose some adapting to be made on the algorithm in order for it to properly work.

2. U.I.O. Theory

The U.I.O. (Unknown Input Observer) is an estimator of non-deterministic inputs based on Luenberger Observer structured as depicted in the block diagram at Figure 1 and expressed as in the (1):

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + E\hat{d} + L(y - C\hat{x}) \\ \hat{d} = (CE)^{+}(\dot{y} - CAx - CBu) = \dots \\ \dots = M(\dot{y} - CAx - CBu) \end{cases} \qquad (1)$$

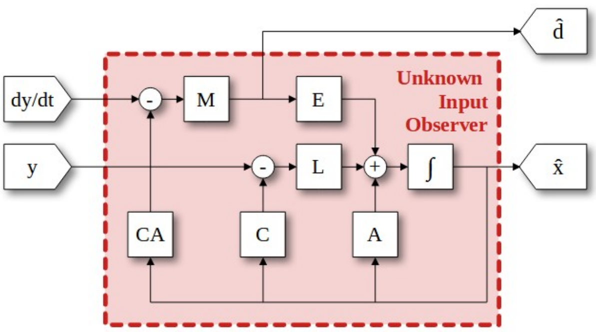

Figure 1: U.I.O. Algorithm Block Diagram

The U.I.O. requires both the measuration and its derivative as inputs; however, doing such on a noisy signal would result in the derived input to be mostly driven by said noise. Therefore, the possibility explored in this work is to *integrate* accelerational measurements (an operation which results in inherently continuous outputs) and use the obtained signal – that is, a velocity – as the input *y*, while the original acceleration as its derivative *dy/dt*.

This means that whatever the system, the output matrix *C* of the linear system (2) is structured like a selection of rows from an identity matrix, corresponding to the rows of the accelerations employed for the estimation.

$$\begin{cases} \dot{x} = Ax + Bu + Ed \\ y = Cx \end{cases} \qquad (2)$$

After some trials on a couple of simple dummy systems to deduce some general rules about which inputs are required for estimation, the U.I.O. has been applied on two separate systems, one for vertical and one for transversal dynamics.

3. Vertical Dynamics

First, the U.I.O. was used to estimate average vertical irregularity of the two rails. The model on which the U.I.O. was built is a 10 D.o.F. model (these D.o.F. being the vertical motion of carbody, bogies and wheelsets and the pitch of carbody and bogies).

Since the vertical contact between wheels and rails can be considered as anelastic, the vertical motion of the wheelsets is treated as an externally constrained variable, and the one the U.I.O. will try to estimate. That being said, the wheels are connected to the bogies through the primary suspensions, which include both springs and dampers; this means that both vertical displacement and velocity are inputs, and must be related through the augmentation of the state vector as depicted in the (3):

$$\begin{Bmatrix} \dot{x} \\ \dot{d} \end{Bmatrix} = \begin{bmatrix} A & E \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} x \\ d \end{Bmatrix} + \begin{bmatrix} E \\ I \end{bmatrix} \dot{d} \qquad (3)$$

Furthermore, another extension of the state vector must be made in order to take multiple mutually delayed inputs (that is, the vertical motions of the four wheelsets) into account; this operation is known as Padé Approximation and (for a simplified 2 inputs case) is expressed as in the (4):

$$\begin{Bmatrix} \dot{x} \\ \dot{x}_P \end{Bmatrix} = \begin{bmatrix} A & E_2 C_P \\ [0] & A_P \end{bmatrix} \begin{Bmatrix} x \\ x_P \end{Bmatrix} + \dots$$
$$\dots + \begin{bmatrix} E_1 + E_2 D_P \\ B_P \end{bmatrix} d_1 \qquad (4)$$

However, the dependency of the approximation on the value of the delay itself makes some further considerations necessary. The vehicle is supposed to travel between 10 and 100km/h, a relatively low speed which results in a large time delay between the third and fourth wheelsets and the first one. This means that the approximation is not valid, but should be limited at the rear wheelset of each bogie (which features a relatively small delay from the front one), resulting in a system featuring 2 separate (although not entirely independent) inputs.

Once the system has been entirely built, the poles of observer matrix L have to be positioned to regulate observer dynamics; in order to grant sufficient dampening for all poles (included the purely imaginary ones) it was decided to double their negative real part and furtherly subtract to it their imaginary part (Figure 2):

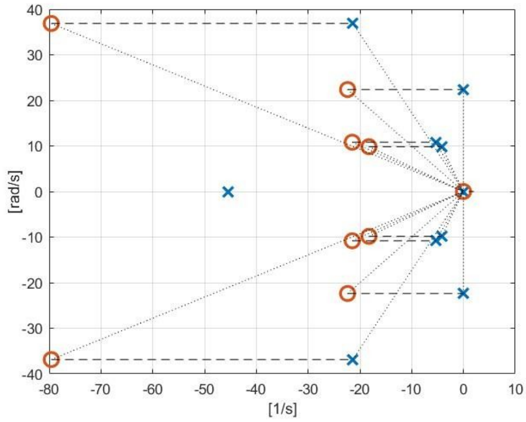

Figure 2: Example of Pole Placement for V = 50km/h.

Moreover, since it would be unpractical to say the least to recalculate gain matrix everytime the vehicle changes its velocity, the possibility of employing the same precalculated L matrix for a range of vehicular speeds was explored. The final result made it possible to employ the same matrix for any speed up to 5km/h lower than the one for which said matrix was calculated.

Now that the system has been assembled, the irregularity can be estimated and compared with the results of a multibody model provided by PoliMi, acting in this case as an almost perfect reflection of the real system. As observed states, the vertical and pitch motion of the bogies were utilized (neglecting the carbody motion) to estimate the first and third wheelsets vertical displacement. Results (Figure 3) for the straight case can be considered highly accurate (average absolute error lower than 0,25mm, largest error lower than 1mm); furthermore, the main source of this error seems to be related to slight temporal disalignment between real data and estimation, as demonstrated by the power spectrum density graph pictured in Figure 4:

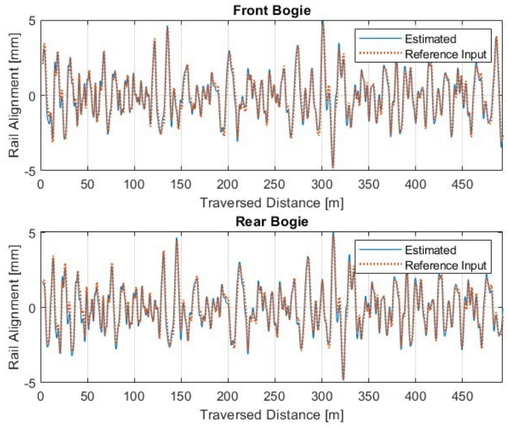

Figure 3: 10 D.o.F. Vertical Model - U.I.O. results for a straight track

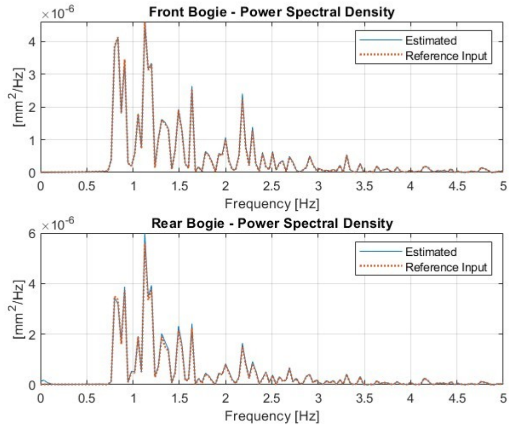

Figure 4: 10 D.o.F. Vertical Model – Straight Track estimation PSD

The same considerations can be made for a curved track, specifically a 135m turn (among the tightest the considered vehicle can traverse), although it can be inferred from the slightly worse results (Figure 5) that distancing from the linearized conditions has a negative effect on the estimation (average absolute error ~0,35mm at the front, ~0,55mm at the rear bogie):

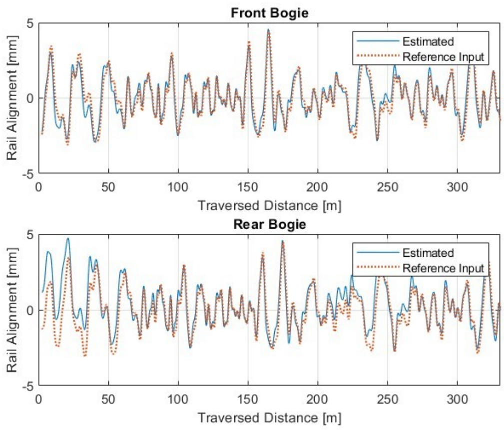

Figure 5: 10 D.o.F. Vertical Model – U.I.O. results for a 135m radius – curved track

4. Lateral Dynamics

For the estimation of track roll angle and transversal irregularity a U.I.O. based on a 21 D.o.F. system was employed, which accounts for the roll, yaw and transversal motion of carbody, bogies and wheelsets as degrees of freedom. In this case, only roll motion behaves as a constrained variable, as the lateral behaviour is regulated by friction dynamics. Such dynamics were modeled through the theory of creepages [5] (5), thus relating relative contact point velocity with tangential force through a linearized coefficient (6):

$$\varepsilon = \frac{V_{CP}}{V_{vehicle}} \tag{5}$$

$$\begin{cases} F_L = f_{LL}\varepsilon_L + f_{LT}\varepsilon_T \\ F_T = f_{TL}\varepsilon_L + f_{TT}\varepsilon_T \end{cases} \tag{6}$$

After some mathematical passages which won't be reported here for brevity, contact point velocity – and thus contact forces – can be expressed as a linear function of the entire set of variables (y_{track} included). This allows to introduce the track lateral disalignment as an additional input, and relate it to the state vector by accordingly modifying the stiffness and damping matrices.

Once again, Padé Approximation was employed to take multiple delayed inputs into account; however, due to higher frequency of the irregularity to be estimated compared with the vertical case, a quadratic approximation was found to be inadequate to correctly reconstruct all accelerations through the linearized system; for this reason, it was decided to increase the Approximation Order to 3 – for which the linearized system and the real one resulted in perfectly compatible accelerations when fed the same irregularity.

For the actual estimation, however, it was decided to first make a tentative trial reconstruction still with the second-order Approximation, taking into account that the aforementioned issues seem limited to the bogies' roll acceleration. The results, obtained by using all 17 unconstrained accelerations as inputs, are pictured in Figure 6 below:

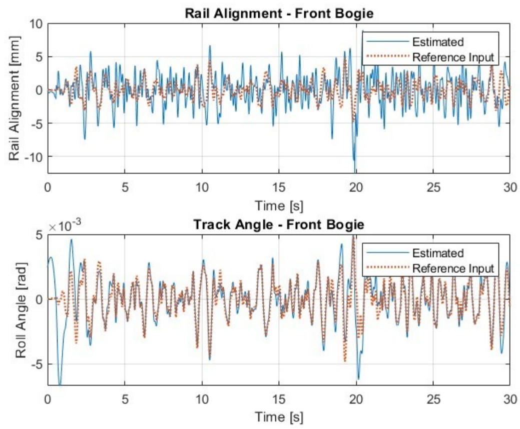

Figure 6: 21 D.o.F. Lateral Model - U.I.O. results using 17 Accelerations

As it can be easily deduced, the roll track angle has been overall correctly estimated, but the transversal alignment suffers from heavy distorsion – most likely depending on the partial incompatibility of the linear model with the actual behaviour. The same experiment can be tried by increasing Padé Approximation order to 3 as done in the previous paragraph.

This operation, however, proves to be highly detrimental to the possibility of proceeding with the estimation; increasing the number of states, in fact, makes the system much less observable. This, in turn, makes the matrix L required even to just place unstable poles within the real negative half of Gaussian Plane composed of such high values that the estimator would be, in fact, driven by noise and estimation error rather than by the model itself, as evident in the (7).

$$\dot{\hat{x}} = A\hat{x} + Bu + E\hat{d} + L(y - C\hat{x}) \tag{7}$$

This means that, even utilizing the full set of 17 non-constrained accelerations as inputs of the U.I.O., it is not possible to proceed with the estimation by using the third-order approximation, at least not with the current set of inputs. One way to bypass the issue is by extending the set of utilized inputs to some velocities as well, namely the 8 unconstrained ones of the four wheelsets (lateral and yaw); on a physical point of view, this is obtained through integration of the acceleration signals coming from corresponding accelerometers.

However, since these signals are now being processed through multiple integrative phases, a more adequate high-pass filter is required to filter out noise-induced drifts; a fourth-order Butterworth Filter was thus introduced to post-process estimation results, to obtain the following results in Figure 7 to Figure 9:

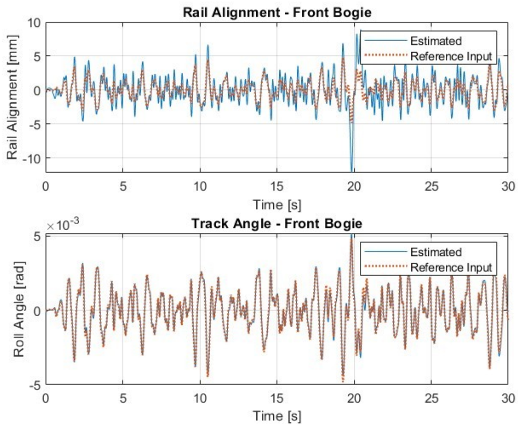

Figure 7: 21 D.o.F. Lateral Model - U.I.O. results using 17 Accelerations+8 Velocities

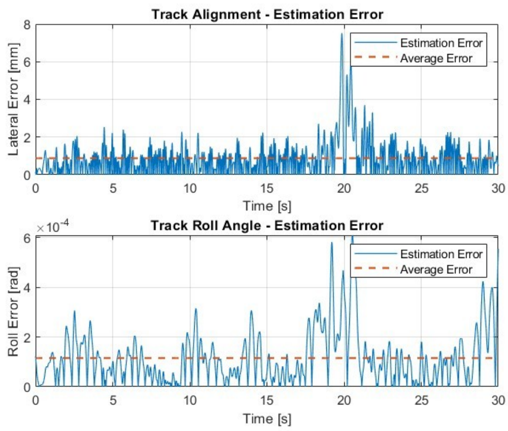

Figure 8: 21 D.o.F. Lateral Model – Estimation Error

As it can be seen, average absolute error is negligible for roll track angle, and lower than 1mm for track alignment; most of the contribution to the latter is due to a very noticeable spike at around 20s, most likely due to a localized irregularity on the track itself.

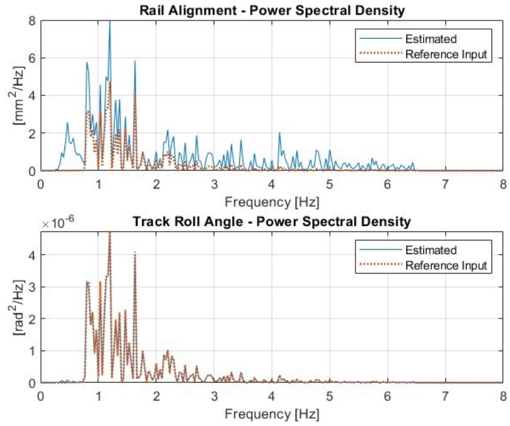

Figure 9: 21 D.o.F. Lateral Model – Estimation PSD

Even frequency-wise, both roll track angle and lateral alignment have been greatly improved, although the lateral estimation still suffers from disturbance over all the frequency spectrum; it is entirely possible that different order filtering or differentiated thresholds could be applied between the two results to better refine them, but this possibility was left inexplored within this study case.

5. Conclusions

The work tried to demonstrate whether it was possible to employ U.I.O. algorithm to reconstruct the time history of track irregularity by using accelerometers' measurements on a Metro vehicle during its service.

First, the vertical dynamics were analyzed through a 10 D.o.F. model of the vehicle; 2nd order Padé Approximation was employed to factor in multiple inputs' delay. Due to the relative simplicity of the system, it was decided to use it as testbed for a simple gain scheduling system, in order to account for variable velocity. Results were deemed more than satisfying, so it was possible to proceed with lateral dynamics.

Lateral dynamics were thus first analyzed from a kinematic point of view, in order to extract a linearized expression of contact forces through the theory of creepages; such model was thus validated through comparison of its accelerations with the ones from a multibody model when fed the same irregularity, only to find out that 2nd order Padé Approximation was too rough to work and 3rd order Approximation had to be selected. Estimation was thus conducted first with the 2nd order approximation, obtaining somewhat promising but ultimately insufficient results, while 3rd order one required an extension of the inputs to include some velocities as well to be employed. Results were highly accurate for the roll track angle, but plagued by distorsions for the lateral irregularity.

The experiment can be deemed as a success. The U.I.O. proved to be able to work in this environment, although as expectable fared much better when applied to simpler systems. For it to be applied to highly complex ones, both the inputs set and the filtering capabilities have to be upgraded accordingly, but ultimately not betraying the initially described advantages of the algorithm with respect to other similarly-focused competitors, namely the possibility to work with no previous knowledge of any characteristic of the to-be-estimated quantities.

On the topic of possible future developments, some of the considerations done for the simpler vertical case could be extended to the lateral one, in particular by considering a curved track, and trying to setup a variable speed gain scheduling as well; this last operation was deemed as not particularly interesting for the analyzed topic, due to it adding nothing particularly relevant to the aforementioned vertical example, and being mostly limited to the operation of assembling of a lookup table for steady-state values at various fixed-radius curve traverse velocities.

References

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