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An Agent-Based Model to Investigate how Citizens' Opinion Dynamics Impact on Countries' Accessions and Withdrawals from an Economic Union

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Abstract

Economic unions are the type of international agreement that displays the highest level of economic integration. They exist due to historical, economical and cultural reasons.

At some point in time, it might happen that some countries become new members, as well as others withdraw from them.

This dissertation's main interest is that of simulating socio-economic dynamics around and inside an economic union in order to identify what are the factors that can actually make a withdrawal happen.

In principle we introduce Brexit, as it represents the most relevant withdrawal event in recent history, and we make a brief recap of how opinion about the EU has evolved across UK throughout the years, basing on existing data and surveys.

We then expose some previous research results about what the main driving factors of Brexit were.

We remark that in socio-economic systems (and in particular in economic unions), dynamics occur at many different levels: there can be interactions among citizens or among countries, and such interactions might significantly differ according to their nature, and to the nature of the agents involved.

We therefore need a model which can replicate with a good level of detail the way countries earn and then redistribute taxation revenues, how countries' citizens make their profits, the way they interact with each other, and the way their opinion about joining or leaving the Union gets shaped.

In fact, in recent literature there are not many examples of such omnicomprehensive models. However, one recent attempt to replicate all the above mentioned dynamics is that of [11], where our problem gets tackled by means of an agent-based model. We therefore take that as our main reference and we make some modifications to it.

We find out that countries are generally attracted by the perspective of economic union membership, but also that inefficiencies in the Union's economic and redistribution poli-

cies can be bad for these international agreements' survival in the long run.

Keywords: Agent-Based Model, Complex Systems, Opinion Dynamics, Social Status.

Abstract in Italian

Il massimo livello di integrazione economica che vi può essere fra un gruppo di Stati si ha nelle unioni economiche, le cui origini non sono limitate alla sola convenienza dell'accordo, ma hanno profonde radici storico-culturali.

Da un punto di vista socioeconomico, è interessante osservare quali dinamiche si instaurano tra i Paesi che fanno parte dell'accordo (i cosiddetti Stati-membri), tra quelli che non ne fanno parte ma ne avrebbero la possibilità, così come le relazioni che si instaurano tra Stati-membri e non membri. Gli Stati che non fanno parte dell'accordo possono accedervi se certe condizioni sono soddisfatte, così come possono ritirarsi dall'accordo in particolari circostanze. Questo è quanto successo ad esempio nel caso della Brexit, della quale viene presentato un breve excursus storico, seguito da alcuni risultati di sondaggi e ricerche che hanno provato ad identificarne le cause.

Anche alla luce di questi risultati, lo scopo della tesi è quello di provare a simulare numericamente le dinamiche socioeconomiche sopra citate per cercare di analizzare meglio da un punto di vista empirico gli episodi di uscita di un Paese-membro da una unione economica. Tali interazioni possono chiaramente essere di varia natura e vanno dagli scambi economici, alle migrazioni, agli scambi di opinione. A volte esse coinvolgono solamente gli Stati e l'Unione come se si trattasse di singoli agenti (ad esempio quando l'Unione redistribuisce fra gli Stati-membri quanto prelevato da essi tramite tassazione, o quando avviene la definitiva adesione/uscita di una nazione dall'unione economica in oggetto). Nella gran parte dei casi tuttavia le interazioni avvengono tra i cittadini: quando partecipano all'attività economica, quando interagiscono tra loro sul piano sociale, e soprattutto quando manifestano il loro livello di gradimento riguardo l'accordo economico internazionale.

Per effettuare delle simulazioni verosimili abbiamo quindi bisogno di un modello versatile e in grado di tenere traccia di un gran numero di fattori contemporaneamente, e a tale proposito si propone un modello ad agente. In letteratura non vi sono molti esempi di modelli ad agente in relazione a unioni economiche, ma un interessante tentativo di sviluppo in tal senso è quello che si trova in [11], che usiamo quindi come modello di

riferimento. La sua struttura non viene alterata, sebbene vengano introdotti alcuni nuovi parametri (come il fattore moltiplicativo pre-redistribuzione dell'Unione), e sebbene la dinamica dell'opinione sia rivista, ponendo ulteriore enfasi sulla condizione sociale dei cittadini-agenti.

Alcuni risultati fondamentali ottenuti dal vecchio modello sono stati replicati fedelmente e non vengono alterati in modo significativo dalle modifiche (la tendenza dell'opinione rimane in generale favorevole all'accordo economico).

Vediamo inoltre come in generale l'efficienza delle politiche economiche e redistributive dell'Unione sia fondamentale per la sopravvivenza dell'Unione stessa.

Parole chiave: Modello ad Agente, Sistema Complesso, Dinamica di Opinione, Posizione Sociale.

Contents

Introduction

When it comes to international trade, countries can enter various kinds of agreements. Some of them display a relatively low level of economic integration while others display a higher one [7]. Some examples are respectively represented by common trade areas in which countries remove trading boundaries between each other and by customs unions, where they also share a common trade policy towards other countries not belonging to the agreement. The highest level of economic integration is observed in economic unions. Economic unions indeed represent a type of agreement where belonging countries not only share a common trade area and a common market policy towards countries not belonging to the agreement itself, but they also pursue common fiscal and monetary policies [7]. Research results show that economic unions can actually provide many benefits (in particular in terms of economic growth) to member countries [9].

The most relevant example of economic union that we can consider is of course that of European Union [13], while other examples are represented by the CARICOM Single Market and Economy [6], the Central American Common Market [8], the Eurasian Economic Union [15] and the Gulf Cooperation Council [1].

This dissertation's goal is that of analyzing opinion dynamics around and within such unions (in other words we seek to find out how agents develop the idea of of becoming anti or pro-Union and why they can actually change their mind), and also that of finding out which factors are the most relevant for such dynamics: are the economic ones the most influential ones or are social connections and information systems' structures the most relevant elements when it comes to opinion shaping? Are some individuals' opinions more relevant and influential than those of others in these processes? And if the answer is yes, why? We are going to try to formulate some answers for questions that are similar to these and to do so, we make use of a computational approach.

So far, only a few authors have explored the possibility to analyze socio-economic systems involving an economic union by means of a mathematical/computational approach, and they have focused on agent-based dynamical models. This turns out to be extremely helpful since we can actually look for the existence of a steady state in which agents'

opinion regarding their actual (or hypothetical) belonging to an economic union is shaped in a specific way. We can then change initial conditions and parameters' values in order to check how robust our results are: we pay attention to oscillations, different outcomes for long periods, sensitivity of opinion changing to parameters modifications, etc.

This thesis takes [11] as its main reference, and therefore most of that original structure is here maintained.

Dynamics concerning interactions in the agent-based model are firstly simulated and subsequently analyzed at many different levels: at macro level we consider interactions between countries, at mesoscale level we account for interactions between agents and their country, whereas at micro level we consider interactions among agents (countries' citizens) that mainly consist in trading and opinion exchanging. We make use of a computational model that includes individual-specific probabilities (whose value is based on wealth status) to change opinion at the end of every time step.

1 | Motivations and Background

The main event that we consider as a benchmark for our analysis is of course Brexit.

Many saw it as a watershed event in the history of globalization and European integration, even though it's not been longtime since its occurrence and therefore a full display of its consequences both on the UK and on the European Union itself still remains to be seen. There are many data available suggesting us that the opinion of UK citizens about remaining or leaving the European Union (or the European Economic Community, as it was called when UK joined in 1973), has been far from constant from 1970s up until the well known Brexit referendum held in June 2016 [14].

In the years that followed after UK became an official member of the European Economic Community, there was a growing feeling that the majority of UK citizens would have voted to leave the Union, if they had been asked via referendum. But in March 1975 UK membership's terms were renegotiated and at the subsequent referendum in June the "remain" prevailed. Then, there was another decrease of EU's popularity across UK, which reached its bottom level in early 1980s, when M.Thatcher managed to substantially reduce the level of the country's net funding to the Union. From then on, despite some ups and downs (especially during 1990s, where European issues were often at the center of UK political debates), public opinion seemed to be slightly in favour of remaining in the EU, despite being wary of joining the Union currency, and indeed this ultimately never happened.

Here we have two representations of how opinion has fluctuated between 1970s and 2016 (figure 1) and of how it has behaved more in detail throughout months right before the Brexit referendum (figure 2), whose result was not forecasted by many (but could still be expected due to some of the latest polls' results).

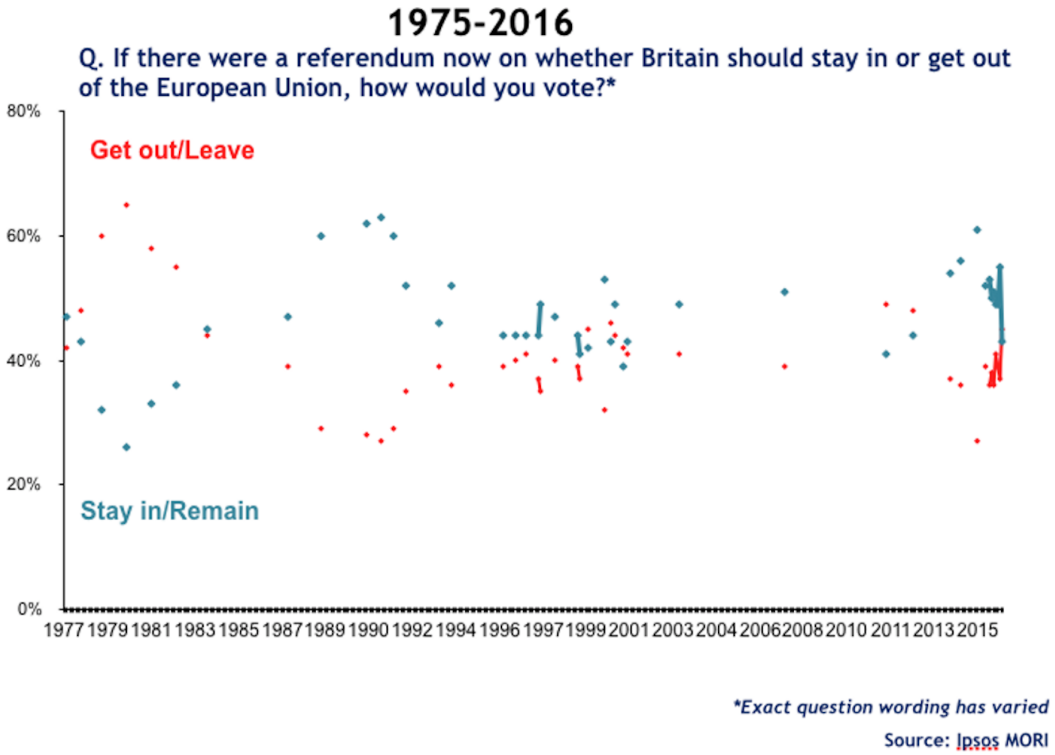


Figure 1.1: A brief graphical recap reproduced from [14] of how opinion about the EU has evolved across UK throughout the years.

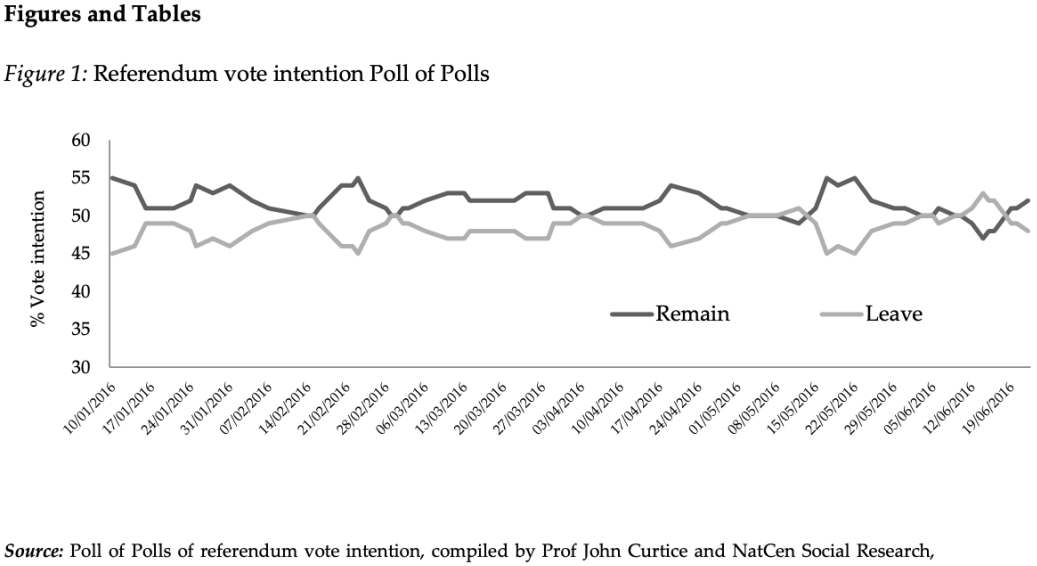


Figure 1.2: A figure reproduced from [12] ("The poll of the polls"), which shows that while in 2016 the slight majority of people seemed to have a "remain" opinion, a few polls in the final month before the Brexit referendum suggested there could be a "leave" majority.

We have also tried to collect some academic researches' results, along with some newspapers' analyses, about what the main driving factors that ultimately led UK to Brexit actually were.

Analyses suggest there was a wide range of factors that had an impact on the way UK citizens voted at the referendum. A consistent fraction of them were economic-related [5], some among the most important being :

- The relative size of British budgetary contributions to the European Union.
- The quality of public service provision, that seemed positively correlated with eagerness to remain in the EU. We can consider healthcare service to make an example: district-level analyses [5] suggest that people in areas where waits for treatments were shorter were more incline to vote "remain". Another interesting result from the same source is that areas displaying a relatively higher number of workers employed in the public sector were more reluctant to leave the EU.
- Fiscal cuts, i.e. the austerity measures adopted by the EU in response to 2008 financial crisis. Areas that were significantly hit by fiscal cuts were more likely to vote "leave".
- the debate about international trade being an additional opportunity or a threat to local businesses and jobs. Clearly, those who saw it as a threat were more eager to vote "leave".
- pressure on the housing market.
- socio-economic implications of migrations.

There are also many non economic-related impactful factors [3–5, 10], some of them being:

- education, with those with a lower level of education being more Union-averse.
- age, since younger people were more likely to vote "remain" and older ones were more in favour of "leave".
- well-being, in fact many of those belonging to a wealthier social group (or to a wealthier area in general) seemed to be more likely to vote "remain".

Here it is another interesting image concerning voters' ideas:

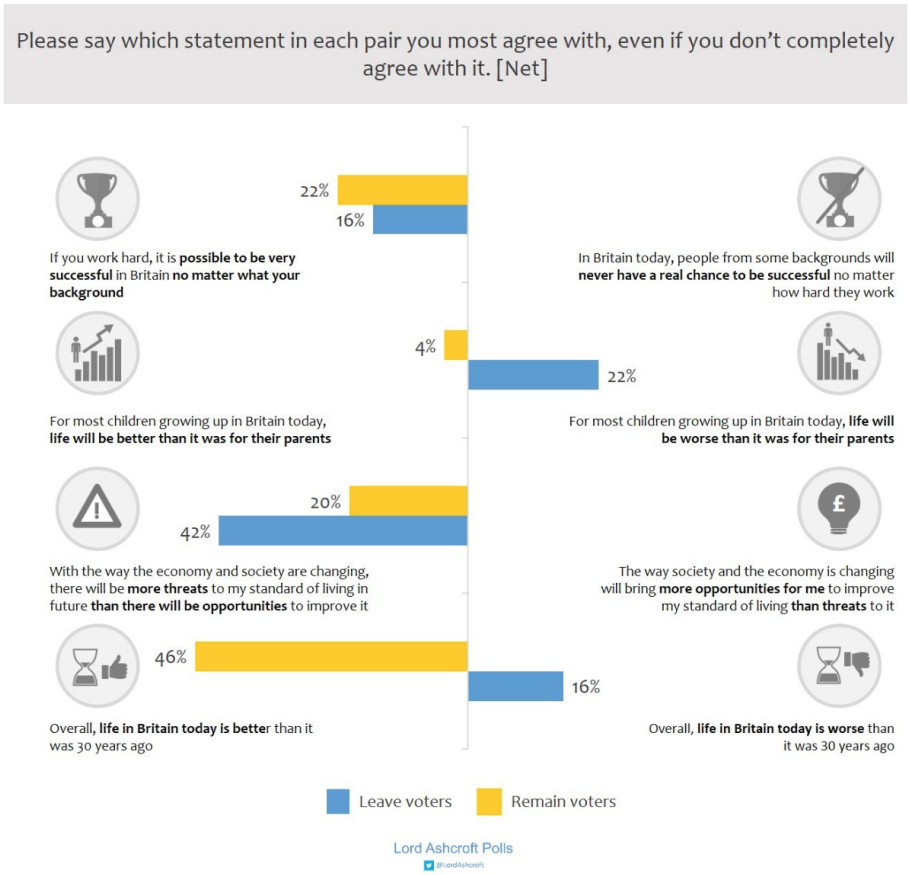


Figure 1.3: This figure, reproduced from [4], displays how the majority (expressed as net percentage) of those who voted "leave" and of those who voted "remain" answered to some questions about the quality of life and of the opportunities they could have in UK. A lesser well-being seems to be correlated with a "leave" vote.

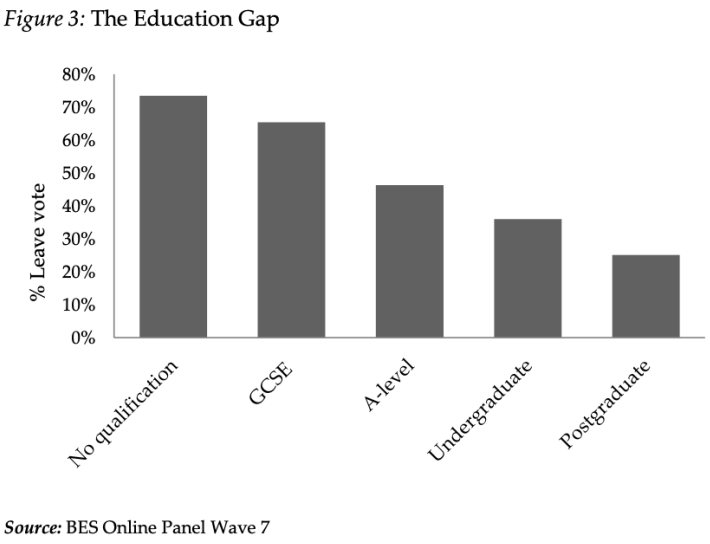


Figure 1.4: A graphical display reproduced from [12], showcasing an estimate of the percentage of "leave" votes for many education levels. The higher the education level is, the more votes in favour of "remain".

2 | The Model

This chapter is about how the model works, including formulas and the main elements' features.

2.1. Citizens and Countries

Countries

Countries are uniquely identified by an ID, by a fixed population that does not change its composition throughout simulations (migrations don't occur in our model) and by a value $U_i = \{0,1\}$ indicating whether that specific i-th country belongs to the Union or not. Other important features that we are going to use later on are:

- an overall approximation of the average country's taxation rate,
- the country's common pool (revenues coming from taxation, or from Union redistribution),
- the country's latest economic contribution to the Union,
- the country's latest earning from Union redistribution,
- the country's citizens' average wealth,
- the sum of all country's citizens' wealths
- a pair of indexes indicating whether the country is in the process of becoming a member or to withdraw from the Union,
- the ID of another existing country which is the "closest" (in terms of per-capita wealth) to the one we are considering.

Citizens

Citizens represent agents and therefore the core of our model. We identify each one of them by a_i^j, where j represents their unique ID and i is the ID of the country where they are from. To each agent we associate a value $\sigma_i^j = \{0,1\}$. Such a value states whether that agent's opinion is pro-Union ($\sigma_i^j = 1$) or anti-Union ($\sigma_i^j = 0$). Every citizen has an accumulated payoff earned by participating to businesses. We emphasize the fact that at the beginning of every simulation such a payoff has an initial value that differs from zero in most of the cases. Its starting value can also differ from agent to agent (heterogeneous scenario).

2.2. Businesses

Business is a general term that we are going to use in order to describe income sources for citizens.

The word "businesses" embodies firms, capital assets, stocks and anything else that can actually generate profits and contribute to the dynamics of the economics within and between countries. We consider three possible kinds of businesses:

1. Local Businesses,
2. Union Businesses,
3. Global Businesses.

Every business has its own ID (for identification) and a time-varying number of participants. The level of income it guarantees to its participants depends on such number and also on the business type. We will return to this point later on.

It is clear that we must impose constraints on businesses' compositions according to their type: Union Businesses can only be accessed by Union-member agents, Local Businesses can only include citizens from a certain country (and that country's ID represents itself a time-invariant unique feature for every Local Business), while Global Businesses are open to anyone.

Differences in computing incomes also occur, and are going to be described in a further section.

2.3. Union

The Union is the most abstract subject belonging to our field.

We might just consider it as a set of subjects and of both time-varying and time-invariant conditions that include the set of Union Businesses (actually its only subset that has time-invariant dimension), the Union's common pool (for now we can simply address to it as to the Union's "level of wealth"), countries (along with their citizens) belonging to Union, and citizens' opinions about it.

The Union is also deeply connected to citizens and countries that are not part of it, since their will to enter is measured at every time step throughout our simulation. It is also connected to Global Businesses and Local Businesses by means of Union Businesses, since interactions between businesses of different kinds will occur very frequently.

2.4. World

The World includes everything we've been describing so far. In particular, it includes all citizens, countries and businesses.

2.5. Revenues and Taxes

2.5.1. Revenues

At every round of our simulation, each citizen makes profits out of all the businesses he/she is involved in. Every business has its own profit function, depending on the category it belongs. For Local Businesses we have:

$$Y_{L,i} = \Gamma_{L,i} f(n) - 1 \tag{2.1}$$

where $\Gamma_{L,i}$ refers to the local profitability for Local Businesses of the i-th country, and the function f corresponds to the following expression:

$$f(n) = \frac{1}{1 + (\chi/n)^{\beta}} \tag{2.2}$$

where n is the number of participants of that specific business, χ is the average number of participants that any business is expected to have at the beginning of the simulation

process (we will be back on this point when we'll discuss initial conditions) and β is called the Hill Coefficient. In fact, if we increase β we have that the yield curve becomes steeper i.e. adding one more participant to a business becomes less effective in terms of profit increasing. Indeed, if n becomes greater yields increase, but not always by the same amount: due to our function's shape, at some point such increase will start to slow down. This effect's principle lies within the fact that having more people cooperating in a business activity will make it become more profitable, but this increase becomes significantly slower when there are already a lot of participants involved. Considering such a shape for the gross income function, it's also easy to see why we have decided to use χ as the average number of participants for every business at the beginning of our simulation: since f(χ) is equal to 0.5 whenever n = χ, and since the profitability coefficient is always chosen between 2 and 3, the average gross income for every business at the beginning of the simulation will be slightly above zero.

From an economic perspective this means that in average, at our initial conditions, all businesses are at a stage in which either they have just started to produce wealth, or they are close to that point.

Net local profits are:

$$\begin{cases} Y_{L,i}(1-T_i) & \text{if } \mathrm{Y}_{L,i} > 0 \\ Y_{L,i} & \text{otherwise} \end{cases} \tag{2.3}$$

where T_i refers to the i-th country's average taxation rate.

For Union Businesses we have:

$$Y_U = \Gamma_U f(n) - 1 \tag{2.4}$$

where Γ_U refers to the Union Businesses' profitability, and f is the same as in 2.2:

Net Union profits are:

$$\begin{cases} Y_U(1-T_i) & \text{if } \mathrm{Y}_U > 0 \\ Y_U & \text{otherwise} \end{cases} \tag{2.5}$$

where T_i refers to the i-th country's average taxation rate.

For Global Businesses we have:

$$Y_G = \Gamma_G f(n) - 1 \tag{2.6}$$

where Γ_G refers to the Global Businesses' profitability, and f is still the same as in 2.2:

Net global profits are:

$$\begin{cases} Y_G(1 - T_i - C_i S_{i,u}) & \text{if } \mathrm{Y}_G > 0 \\ Y_G & \text{otherwise} \end{cases} \tag{2.7}$$

where T_i refers to the i-th country's average taxation rate, and C_i refers to customs fees coefficient, i.e. a coefficient for an additional tax that agents have to pay at every round to their own country for every Global Business they participate to. Such an additional amount depends also on the coefficient $\mathrm{S}_{i,u}$, whose meaning depends on the agent we are referring to: if he/she is from a Union country, $\mathrm{S}_{i,u}$ will stand for the fraction of non Union partners belonging to the business; otherwise $\mathrm{S}_{i,u}$ will represent the fraction of citizens (still belonging to that business) that are from foreign countries. Actually, this is a relevant detail because it implies that people belonging to the Union will be subject (in average) to an overall lower level of taxation.

2.5.2. Redistribution

In this subsection we are going to explain how taxation and redistribution of taxation revenues work.

First of all, we need to consider that all institutions, including the Union itself, have their own mantainance costs. As we mentioned in Section 1, in the Brexit case a high quality of public service provision often seemed to be associated with a "remain" vote [5]. In [11], it is explicitly mentioned that the process of taxation and redistribution is not merely about wealth redistribution, but represents also the way institutions spend their money to purchase public goods, provide public services such as infrastructures, healthcare etc. In [11], the enhancement-factor parameter α (that here we will call α_C from now on) was supposed to embody how efficient such policies were at country-level. Indeed, every country's common pool gets multiplied by such a parameter after Union redistribution, right before the final redistribution to citizens. If its value is smaller than 1, this means that institutions' costs are probably too high.

Here we introduce an additional parameter to evaluate public policies' quality also at Union level. We call it Union enhancement factor and denote it by α_U. Its interpretation is totally similar to that of α_C. As seen with previous equations, every country earns money by imposing taxes on its citizens' earnings (that come from business participations). Then, for countries that are part of the Union, a portion of their common pool earned after taxation is given to the Union itself, which thereby collects its own common pool as well. The amount the Union gains from a country's common pool is proportional to

the sum of all that country's citizens' individual gross payoffs. The Union common pool is then multiplied by the enhancement factor α_U and then redistributed among Union members (countries) in an inversely proportional way, so that poorer countries earn more from the redistribution with respect to the wealthier ones. Proportionality coefficients are computed by means of the simple following formula:

$$\gamma_j = \frac{1/G_j}{\sum_{i=1}^{C} 1/G_i} \tag{2.8}$$

where we have that G_j is the sum of all individual gross payoffs for the j-th country at the ongoing round of our simulation.

At this stage, every country redistributes its own common pool (now consisting in the final result of the following algebraic sum: ordinary taxes + custom fees - Union taxes + Union redistribution), multiplying it by the enhancement factor α_C to all of its citizens in an even way, i.e. guaranteeing the same amount to all of them (that is, the Country's common pool divided by the country's population).

2.6. Agents' Actions: Profit Making, Opinion Dynamics and Business Migration

Agents play a major role in the model since not only they participate to businesses and try to leave the least profitable ones in order to take part into the more appealing ones, but especially since they can change their opinion from pro-Union to anti-Union and vice-versa.

Opinion changes occur following two main paths:

1. economic ones, in particular according to citizens' level of satisfaction/dissatisfaction towards the balance of payments between the Union and their country;
2. social ones: agents tend to imitate the best performing citizens' opinion while they tend to ignore that of poorer citizens.

These two dynamics, along with their interactions, are going to be developed more in detail throughout the next sections.

2.6.1. Individual Payoffs

In section 2.5 we described the way income is generated from the perspective of businesses, and more in general of countries and Union. In the following section we describe wealth dynamics from a generic agent's perspective.

1. Dynamics have rounds as their time reference: rounds may symbolize a specific time window in the real world: as in [11], we consider the equivalence one round = one week.

2. At round 1, agents are algorithmically "generated" with a certain amount of income that can be the same for all of them (homogeneous scenario) or that might follow a random distribution (heterogeneous and more realistic scenario).

3. Agents participate to businesses, and obtain gross profits according to equations 2.1, 2.4, and 2.6.

   Then, for every business an agent takes part into, a fraction of his/her gross income is subtracted (according to that agent's country's average taxation coefficient) and collected in his/her own belonging country's common pool.

4. For every Global Business he/she takes part into, an additional fraction of his/her gross profit (depending on the customs coefficient, and on that specific business' composition) is discounted and still collected in the country's common pool.

5. Countries belonging to the Union award a part of their own common pool to the Union itself, according to the Union taxation rate. The Union collects such revenues in its own common pool, possibly making them become greater or smaller (enhancement factor α_U), and then redistributes them in the way we described before.

2.6.2. Taxation and Redistribution

1. As we mentioned earlier on, the Union redistributes its own common pool to member countries in an inversely proportional way (coefficients are computed by means of formula 2.8) i.e. worse-performing countries receive more than better-performing ones.

2. For every country in the system the now updated common pool is multiplied by a second enhancement factor α_C and then shared among citizens according to an equality principle (every citizen receives an equal share from his/her own country's common pool).

3. The remaining individual wealth after taxation and redistribution is then considered and such a quantity will be called instantpayoff for the rest of the round.

4. We now consider that part of agents' payoff which was collected during former rounds or that they were born endowed with, (the so-called accumulated payoff, denoted by $W_i^j(t)$) and their instantpayoff Π_i^j.

   At every round, each citizen's accumulated payoff evolves in the following way:

$$W_i^j(t+1) = \Pi_i^j + (\rho W_i^j(t) - 1)^+ \tag{2.9}$$

   where the coefficient ρ embodies inflation/deflation dynamics: if it's below one, then we are in a situation of inflation, whereas if it is above one we are in a situation of deflation. Actually, the multiplication of the accumulated payoff by a quantity that is below or above one represents the loss or gain of agents' purchasing power at a certain round. Then, the subtraction inside the equation stands for consumption dynamics. For computational reasons we assume that any wealth accumulated at former rounds cannot go below zero due to inflation and consumption (and goes to zero if it was negative before them).

2.6.3. Opinion Dynamics - Economics

As we have already mentioned, agents' opinions determine whether a country is willing to remain in the Union or to step out of it. It's also quite reasonable to state that the process of opinion changing is mainly ruled by individual wealth status and overall economic convenience. Following Game Theory's main theoretical assumptions [2] we assume that individuals are selfish: they are not willing to cooperate unless cooperating guarantees themselves a greater individual utility than not cooperating (in other words, agents will always prioritize individual well-being over social well-being).

In addition, we suppose that agents' probabilities to change their mind about Union belonging depends also on their status within their country, a feature that is depicted by their relative level of wealth. In order to catch such an effect, we define a coefficient K (greater than 1) to help us describe the level of wealth (for every country) that will affect the way people change their opinion. We now list all possible ways for individuals to change their mind:

1. We introduce the symbol μ_i to identify the average level of wealth for the i-th country. Agents in Union countries whose level of wealth is $W_i^j > \mathrm{K}\mu_i$ will have their

own opinion dynamics. Since we made the assumption that agents are completely selfish, if those citizens already display a pro-Union idea, then they will likely stick to it, since they are likely to believe that being part of the Union will help them to keep their status of privilege. We assume they won't change their mind throughout the simulation process, unless of course their level of wealth goes down below the selected threshold.

2. For the generic a_i^j j-th pro-Union citizen coming from the i-th country (σ_i^j = U_i = 1) that belongs to the Union and whose level of wealth is below $K\mu_i$, there is a probability $P_i(\text{pro} \rightarrow \text{anti})$ that he/she will become anti-Union (σ_i^j: 1 $\rightarrow$ 0) that is directly proportional to the difference between the latest subsidies his/her country received from the Union's redistribution and the most recent amount his/her country paid to the Union itself.

   In fact, in compliance with historical data and analyses' results that we mentioned in Chapter 1, it is rational to assume that that a strong misalignment between subsidies paid to the Union and those earned from it might directly influence citizens' mind, since such an amount will be evenly redistributed among them. For such a reason, pro-Union citizens belonging to Union that don't have a level of wealth highly above the average one in their country (and are therefore not "too afraid" of losing any status of privilege, should their country leave the Union) have the following probability to become anti-Union:

$$P_i(pro \rightarrow anti) = \begin{cases} \epsilon\Delta_i(1+g_i) & \text{if } \Delta_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.10}$$

   where:

$$\Delta_i = \frac{C_i - R_i}{C_i} \tag{2.11}$$

   and

$$g_i = \frac{\sum_j [(1-\sigma_i^j)(W_i^j)^\eta]}{\sum_j (W_i^j)^\eta} \tag{2.12}$$

   where C_i is the i-th country's latest contribution to Union and R_i is the amount it received from the Union's common pool's redistribution at the same round. The higher this difference is, the more i-th country's citizens will be eager to leave the Union, knowing that a significant portion of their earning will get transferred to

poorer countries (we remind we made the assumption of selfish agents).

In this framework, the probability of changing opinion gets also influenced by a coefficient ϵ, which determines how impactful the economic factor is on opinion dynamics. It's reasonable to assume that not all individuals' opinions have the same influential weight (differences in opinions' relevance are captured by equation 2.2). The wealthier one is, the more his/her fellow citizens will be likely to get influenced by his/her own beliefs and to change their mind accordingly. In other words, if a country displays a high number of wealthy anti-Union citizens, it will be more likely that its overall opinion trend will become anti-Union as well. The relative level of relevance of wealthy people is captured by coefficient η. For larger values of η, the relative weight of anti-Union wealthier citizens' becomes greater in the overall dynamics.

3. Anti-Union citizens whose level of wealth is above Kμ_i will change their opinion proportionally to the difference between their wealth and μ_i. The logic behind this mechanism is that selfishness might make them develop a sense of "fear" that by getting out of Union they might actually lose their status. The richer they are, the more willing to stick to Union they are likely to become. Therefore, for them we have:

$$P_i(anti \to pro) = \frac{W_i^j - \mu_i}{W_i^j} \tag{2.13}$$

4. Then, symmetrically to the pro-Union case, for anti-Union people in Union (a_i^j: $\sigma_i^j = 0$, $U_i = 1$), whose level of wealth is below Kμ_i, there is a probability to become pro-Union P_i(anti $\to$ pro):

$$P_i(anti \to pro) = \begin{cases} -\epsilon\Delta_i(1+g_i') & \text{if } \Delta_i < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.14}$$

where Δ_i is computed as before, and g_i' is computed in such a way that:

$$g_i' = \frac{\sum_j [\sigma_i^j (W_i^j)^\eta]}{\sum_j (W_i^j)^\eta} \tag{2.15}$$

Here the situation is at the exact opposite of before: we need to take into account how influential pro-Union agents are. Then, in the negative Δ_i scenario, anti-Union

people from a certain country will be more likely to become pro-Union as well if pro-Union people from the same country are relatively rich. Parameters ϵ and η have exactly the same interpretation as before, with the second one now ruling the relative influence of wealthier pro-Union citizens.

5. The former formulas also apply to those countries that do not belong to the Union. Indeed, for every nation that is not in Union, we simply compute them over the "closest" Union country, i.e. over the one with the most similar per-capita wealth to the one we are analyzing, still with the exception of those agents whose level of wealth is above $\mathrm{K}\mu_i$. Logic behind this for less well-off citizens is that a nation not belonging to the Union will try to figure out what its own economic position in an inside-Union scenario would be, and to behave accordingly.

6. In addition, anti-Union citizens whose level of wealth is above threshold $\mathrm{K}\mu_i$ will be more likely to change their opinion if the average wealth inside Union is greater than theirs, and this is since many of them will be likely to see in Union membership an additional opportunity to increase their well-being even further (we consider this effect as dominant with respect to their fear that a sudden change in their country, such as accessing an Economic Union would be, could potentially modify their privileged status in a substantial way). Probability to become pro-Union in this case becomes:

$$P_i(anti \to pro) = \begin{cases} \dfrac{\mu_U - W_i^j}{\mu_U} & \text{if } \mu_U > \mathrm{W}_i^j \\ 0 & \text{otherwise} \end{cases} \tag{2.16}$$

where μ_U is the average level of countries' wealth in Union.

7. Ultimately, about those who are out of Union and still have a level of wealth above mentioned thresholds but are pro-Union, they might become against Union if and only if their level of wealth is highly above the average wealth in Union (to model this information, we introduce the multiplier K_2). Such a multiplier emphasizes the fact that in our framework people with a higher level of wealth have the tendency to stay more conservative when it comes to changing their mind, and for those being out of Union but pro-Union at the same time, it won't be enough to have a level of wealth slightly above the average Union wealth in order to change their opinion. Probability to become against Union in this case is given by:

$$P_i(pro \to anti) = \begin{cases} \dfrac{W_i^j - \mu_U K_2}{W_i^j} & \text{if } W_i^j > \mu_U K_2 \\ 0 & \text{otherwise} \end{cases} \tag{2.17}$$

2.6.4. Opinion Dynamics - Imitation

We then consider a second chance for citizens that have not updated their strategy at the current round to change their opinion: in fact it is well known that not every decision is made by basing on completely rational profit-loss evaluations. We also need to account for connections and communication between agents.

It is therefore reasonable to assume that some citizens may be less informed about the economic advantages/disadvantages that being part of the Union can actually bring to them. Such citizens, that almost surely did not update their strategy at the step before, may change their opinion by imitation. In fact, they can choose (more or less randomly) another individual in the system and subsequently copy his/her own strategy. It might occur that also some other individuals (not necessarily those who are less informed) did not change their opinion at the previous step and therefore get involved in the imitation mechanism.

We introduce a new parameter ψ taking values in [0,1], which stands for the probability for a generic citizen a_i^j to choose an agent to imitate a_z^k in such a way that z = i.

In other words, ψ represents the probability for an agent to choose another individual from his/her own country, and 1 - ψ is the probability that such an individual comes from abroad. From a social point of view, ψ can be interpreted as the power of existing information systems. Indeed, in order to imitate the opinion of someone from a foreign country, an agent needs to rely on some powerful tools (such as social networks or social media) that allow him/her to connect with someone that lives faraway. Therefore, a group of countries displaying a high ψ will have less agents connecting (and then potentially imitating) with others from abroad, having a weak information/social connection system.

Once the citizen to imitate is chosen, we use the following formula in order to compute the probability that opinion imitation actually occurs:

$$P(\sigma_i^j \to \sigma_z^k) = \begin{cases} \dfrac{W_z^k - W_i^j}{W_z^k} & \text{if } W_i^j < W_z^k \\ 0 & \text{otherwise} \end{cases} \tag{2.18}$$

2.7. Businesses and Union - Entering and Withdrawing

2.7.1. Business Migration

At the end of each round, every agent not involved in any business tries to enter one of them.

Every agent already involved in one or more businesses checks his/her businesses' payoff and applies to one whose actual payoff is superior to that of his/her worst one. We assume information here is incomplete and thereby agents will actually check gross business revenues also while applying to Global Businesses.

More in detail, the partial information assumption about about Global Businesses' compositions means that every citizen will know how many people are involved in each one of them, but not the exact composition of their participants (and therefore the exact additional amount they will have to pay due to the presence of customs fees). Therefore, considering also that international business involvement can be seen as an advantage for social reasons, and that citizens will always be aware that they are allowed to make businesses changes at every round (and therefore to fix wrong business choices), they will make a partial gambling here as they will choose to enter Global Businesses by applying the same criterion that holds for Local and Union Businesses.

2.7.2. Accessions and Withdrawals

A country that is not in Union becomes a member only if at some point in time its number of pro-Union agents becomes 55% of the overall population or more, and remains above 50% for 20 consecutive steps. If that happens, that country's citizens will start to have access to Union Businesses.

A country that belongs to Union withdraws from it only if at some point in time its number of pro-Union agents becomes 45% of the overall population or less, and remains below 50% for 20 consecutive steps. If that happens, that country's citizens will have to leave Union Businesses and they will try to replace them by immediately applying to a number of Global and/or Local Businesses that equals the number of Union Businesses they were forced to leave.

2.8. Initial Conditions

First of all, we set the probability for the i-th country to be part of the Union at the beginning of our simulation. We set it as $\mathrm{P}(U_i = 1) = 1/2$ and $\mathrm{P}(U_i = 0) = 1/2\ \forall i$.

The initial probability for an agent to be pro-Union is denoted by p_0 if he/she is a member of a Union country, and by p_0' if he/she is not a member of a Union country. In our development we consider $p_0 = 0.6$ and $p_0' = 0.4$. Setting initial conditions also means assigning types to businesses (citizens can access or leave businesses throughout the simulation process whereas businesses' types, once set, stay the same all the way through). For every business generated by our algorithm, we set a probability ζ_l that it is a Local Business, a probability ζ_u that it is a Union Business, and a probability ζ_g that it is a Global one.

In our setting we have $\zeta_l = 1/2$, $\zeta_u = 1/6$, $\zeta_g = 1/3$.

The probability for a Local Business to get assigned to a specific i-th country is proportional to that country's population N_i.

Agents, on their side, take part into different kinds of businesses according to their own preferences: anti-Union agents will of course be less eager to get involved into Union Businesses, whereas pro-Union citizens in Union will prefer to be part of Union Businesses rather than of Global Businesses.

Nevertheless, all businesses will have an average amount of participants that is the same for every business type. We call such amount "critical mass" and denote it by χ.

- Local Businesses' memberships are distributed among all citizens belonging to the country to which the business is associated. Every agent a_i^j from the i-th country has a probability χ/N_i to participate to any of that country's Local Businesses (we remind that by assumption every LB has χ partners in average).
- Similarly, Union Businesses' memberships are distributed among all citizens from countries belonging to the Union. For every pro-Union agent in Union a_i^j ($\sigma_i^j = 1$, $U_i = 1$) the probability to take part into any Union Business is 2τ while for an anti-Union agent in Union ($\sigma_i^j = 0$, $U_i = 1$), such a value drops to τ. Writing down the equations we get:

$$\begin{cases} 2N_U p_0 \tau + N_U(1 - p_0)\tau = \chi, \\ N_U = \sum_{i=1}^{C} N_i U_i. \end{cases} \tag{2.19}$$

and rearranging we obtain:

$$\tau = \frac{\chi}{N_U(p_0 + 1)} \tag{2.20}$$

- Any citizen can take part into a Global Business. We let ω_o be the initial probability of participating to a Global Business for citizens out of the Union (regardless of what their opinion about the Union is), ω_p that same probability for pro-Union citizens in Union and ω_a still the same probability for anti-Union citizens in Union. Then, we impose as a constraint that the expected number of participants to a Global Business (considering all the cases above) equalizes χ.

  Then:

$$\begin{cases} N_O\omega_o + N_U p_0\omega_p + N_U(1-p_0)\omega_a = \chi \\ N_o = \sum_{i=1}^{C} N_i(1-U_i) \\ \zeta_G\omega_o = \zeta_G\omega_p + 2\tau\zeta_U = \zeta_G\omega_a + \zeta_U\tau \end{cases} \tag{2.21}$$

  and the set of equations solving the former system is:

$$\begin{cases} \omega_p = \dfrac{\chi + N_U\phi\tau(p_0+1)}{N} \\ \omega_p = \omega_o - 2\phi\tau \\ \omega_a = \omega_o - \phi\tau \end{cases} \tag{2.22}$$

  where $\phi = \zeta_U/\zeta_G$ and N represents the total number of agents (respectively the sum of those inside and outside of the Union). $\mathrm{N} = \mathrm{N}_U + \mathrm{N}_O$

There are two different possibilities that we are going to consider for the initial distribution of wealth:

- The first one is an homogeneous (and kinda unrealistic) scenario in which all agents are endowed with the same initial starting payoff: $\mathrm{W}_i^j(\mathrm{t}{=}0) = 10\ \forall i, \forall j$.
- The second one is an heterogeneous and more realistic scenario where initial agents' wealths $\mathrm{W}_i^j(\mathrm{t}{=}0)$ within any generic i-th country are distributed as a Poisson distribution $\mathrm{P}(\lambda_i)$ with mean λ_i. The λ_i values are independently simulated from a normal distribution with mean $\mu = 10$ and variance $\sigma^2 = 4$.

3 | Algorithms

This brief chapter is devoted to the explanation of the more technical details of our simulation. We use some pseudocode tables to summarize the most important passages of our original code.

3.1. Pseudocode for the Main Computational Methods

The following is a pseudocode version of our main algorithm. It is then followed by some other pseudocodes that summarize how some among the main simulation steps were implemented (original code is written in C++).

Algorithm 3.1 Simulation process description

```
Set initial conditions
for 1<i<numberofrounds do
    Citizens participate to businesses and make profits
    Every citizen pays taxes to his/her own country
    Countries pay taxes to the Union
    Union's redistribution
    Countries' redistribution
    Agents' wealths get updated due to inflation/deflation and consumption factors
    Opinion dynamics - economic reasons
    Opinion dynamics - imitation (for agents that did not update their opinion at the
    previous step)
    Business migration
    if some countries belonging to the Union satisfy withdrawal conditions then
        Those countries leave the Union
        Citizens leaving the Union leave Union Businesses and try to replace them
    end if
    if some countries outside Union satisfy conditions to obtain Union membership
    then
        Those countries become Union members
    end if
    Additional technical instructions
end for
Print results (using Matlab)
```

Initial conditions are set according to the rules described in section 2.8. However, since many of their features are exposed to randomness, we need to set an initial random seeder's value as well, in order to distinguish among them (every initial condition is associated to a specific value of the initial random seeder).

In order to build features such as the World itself and its subsets, some appropriate constructors have been implemented.

"Additional technical instructions" stands for our simulation's mere technicalities: they include all those methods that have to be performed at every round (mainly at the end of it) in order to set up conditions for the next one. A clear example of "additional technical instruction" is the one that resets all booleans indicating whether citizens have already

updated their opinion throughout the last round or not. We remind that keeping track of this is necessary in order to select those citizens who will take part in the next imitation step. In the tables that follow it's possible to observe more in detail how opinion dynamics methods are implemented.

Algorithm 3.2 Opinion dynamics - economics

```
for 1<i<numberofcountries do
  if considered i-th country belongs to the Union then
    for 1<j<numberofcitizens in the i-th country do
      update j-th citizen's opinion with probabilities computed as in the set of formulas that goes from 2.10 to 2.15
      keep track of those citizens who are not going to be included in the imitation dynamics at this round
    end for
  end if
  if considered i-th country does not belong to the Union then
    select the Union member country that is the closest to i in terms of per-capita wealth
    for 1<j<numberofcitizens in the i-th country do
      update citizens' opinions with probabilities computed as in the set of formulas that goes from 2.10 to 2.17 (excluding 2.13)
      keep track of those citizens who are not going to be included in the imitation dynamics at this round
    end for
  end if
end for
```

Algorithm 3.3 Opinion dynamics - imitation

```
for 1<i<numberofcountries do
  for 1<j<numberofcitizens in the i-th country do
    if j-th citizen did not update his/her own opinion at the previous step then
      choose another citizen from the system (probability ψ that he/she still belongs to the i-th country, probability 1 - ψ that he/she doesn't)
      update j-th citizen's opinion with probability as in 2.18
    end if
  end for
end for
```

3.2. Parameters' Values used for Simulating

The following tables describe input parameters' values that have been used in our simulations.

We collect their default values, including also their range of variation in case they were modified to perform sensitivity analyses that we described in the previous section.

The first table includes all those parameters that are involved in the setting of initial conditions, while the second one shows those whose values play a more active role throughout simulations' rounds.

Symbol	Description	Default value
C	Number of Countries	20
B	Number of Businesses	800
N_i	Agents per Country	400
p_0	Initial probability to be pro-Union in Union	0.6
p_0'	Initial probability to be pro-Union out Union	0.4
χ	Critical mass	20
ζ_L	Probability that a business is a LB	1/2
ζ_U	Probability that a business is a UB	1/6
ζ_G	Probability that a business is a GB	1/3

Table 3.1: Table of parameters - Initial conditions

Symbol	Description	Default value (range)
Γ_L	Local Businesses' profitability	2.25
Γ_U	Union Businesses' profitability	2.25
Γ_G	Global Businesses' profitability	2.25
T_i	Average local taxes	0.20 (0.1 - 0.3)
T_U	Union Taxes	0.15
C_i	Customs fees	0.10
α_C	Countries' common pool enhancement factor	1.01
α_U	Union's common pool enhancement factor	1.00 (0.9 - 1.1)
η	Influence factor	1.00 (0.00 - 2.00)
ϵ	Influence of economics on opinion dynamics	0.10
ρ	Inflation/deflation	0.95
ψ	Power of the information systems	0.75(0.5 - 1.00)
K	Threshold1	1.70
K2	Threshold2	1.30

Table 3.2: Table of parameters - Dynamics

4 | Results and Discussion

This chapter is devoted to the commentary of our simulations' results. To this aim, we firstly plot the proportion of Union participants in the World and of citizens' opinion about it as a function of time, in order to analyze their shape in the long term. We then look at the trend of average countries' wealth in order to identify why some countries are more likely to withdraw from the Union, and why such countries might possibly regret their withdrawal decision once they are out.

Then we focus on some parameters, we change their values and we observe the effects of such changes, still relying on graphical tools to help ourselves.

4.1. Opinion Trend and Average Wealth

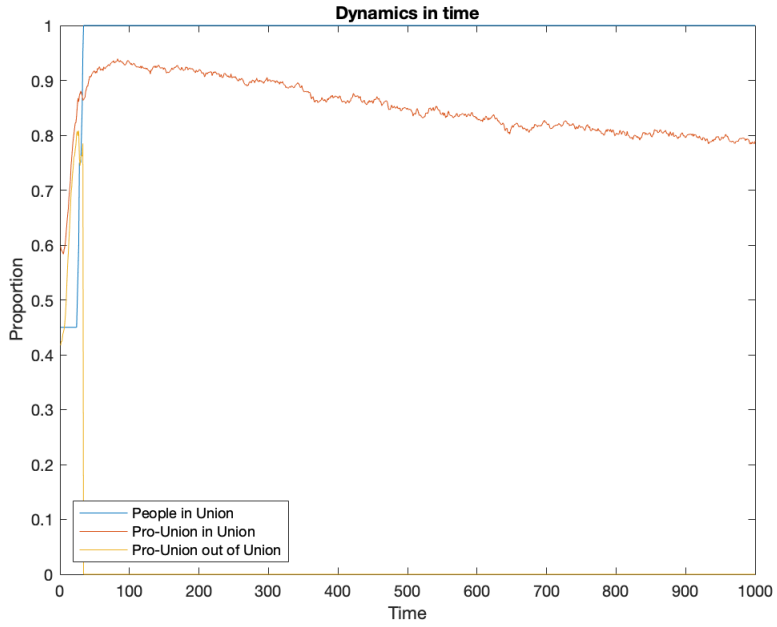
We start to analyze accession and withdrawal dynamics by means of some graphical displays of how the fraction of agents in Union and that of agents pro-Union, both in Union and in non Union countries, evolves over time; each display is paired up with another figure showing how average citizens' wealth in every country is evolving in the meantime.

We choose some initial conditions and plot the resulting dynamics. Each one among the chosen pairs of figures represents the realization of one initial condition.

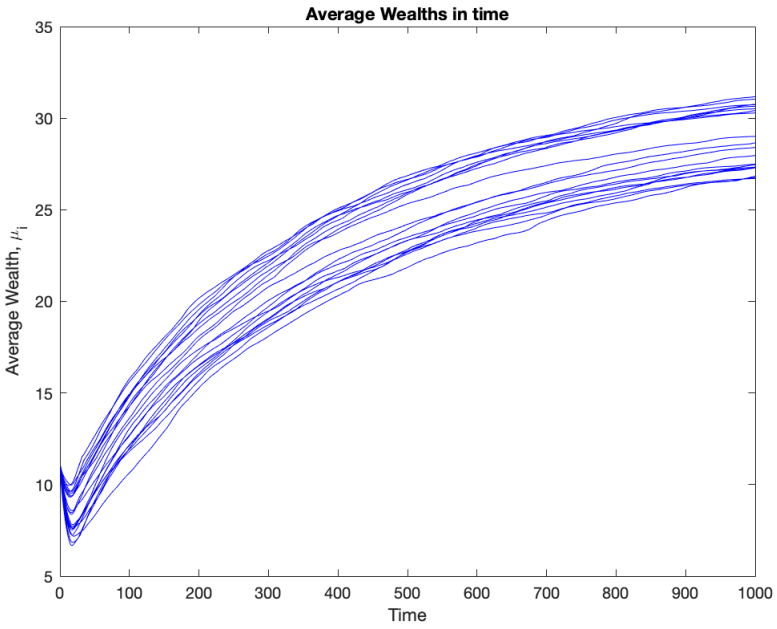
We only include outcomes that occur quite frequently for many different initial conditions. These representations are indeed used to show how the described dynamics are more likely to look like.

Countries are all set to have the same number of agents. Therefore, the fraction of agents inside the Union is also a proxy for the number of countries belonging to it.

More specifically, since we always have 20 countries and all agents within a country must be inside or outside of Union all at once, a country that withdraws from the Union (or that enters the Union) is equivalent to the withdrawal from it (or to the accession to it) of the 5% of the overall World population.

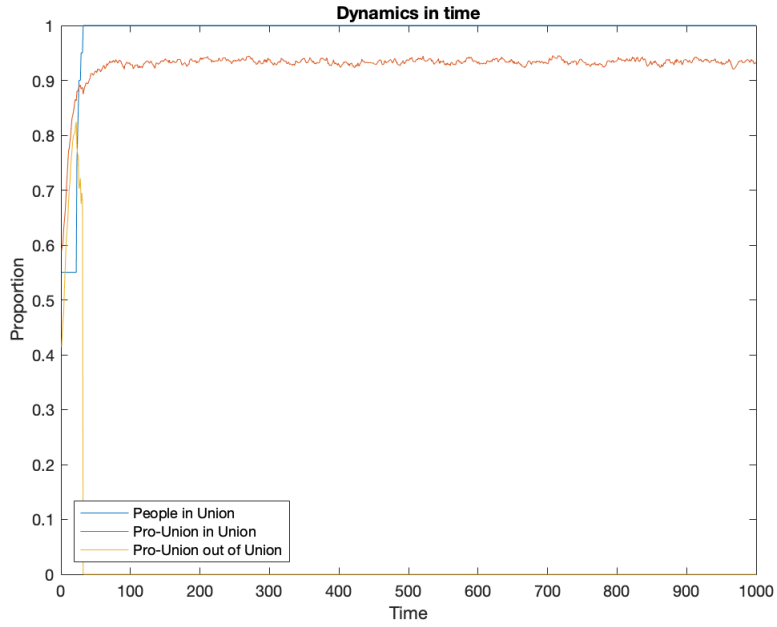


(a) Homogeneous scenario I - no withdrawals

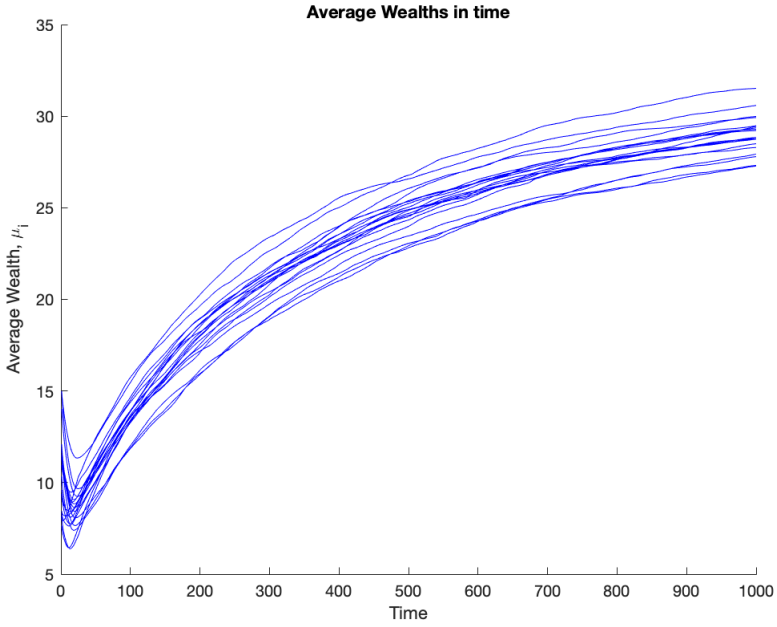


(b) Homogeneous scenario I - average wealth in countries

Figure 4.1: After a short transitory, all countries become Union members. No withdrawals occur later on. The blue curves in figure b stand for average countries' wealth.



(a) Heterogeneous scenario I - no withdrawals



(b) Heterogeneous scenario I - average wealth in countries

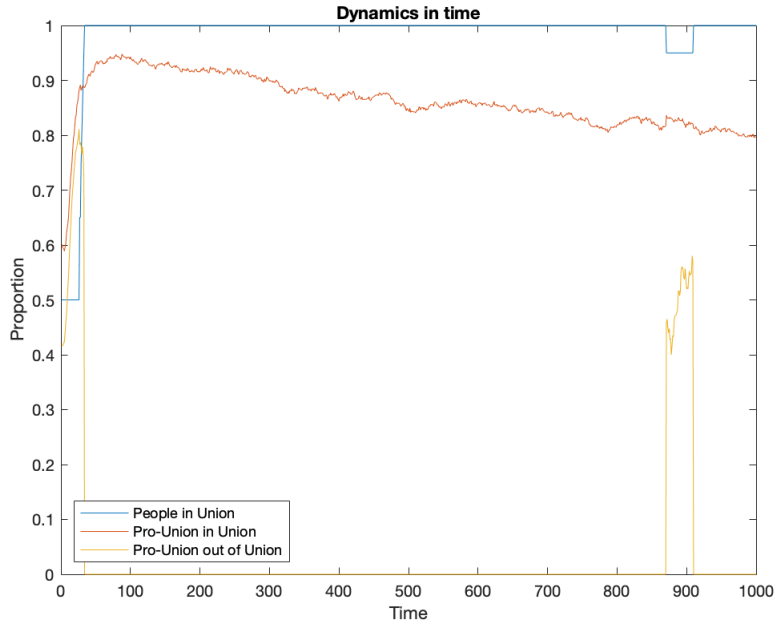
Figure 4.2: After a short transitory, all countries become Union members. No withdrawals occur later on. The blue curves in figure b stand for average countries' wealth.

It is easily seen that the results displayed in section a of figures 4.1 and 4.2 are similar to those obtained by [11], with a short transitory in which Union popularity reaches its peak

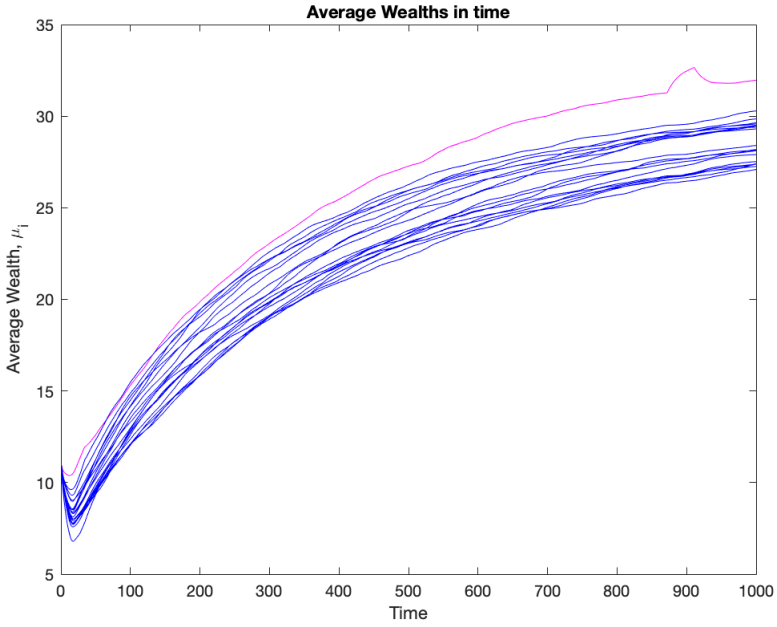
and all the countries in the system become official members (regardless of whether we are in an heterogeneous or homogeneous wealth scenario). In these first cases, no withdrawals occur after the transitory.

Average wealths are increasing for all countries throughout the whole simulation process, and their trends seem to be somewhat similar. However, it is worthy putting into evidence that it occurs quite often that curves cross each other. This means that countries that were wealthier (in average) than some others at the beginning of our simulation might have been surpassed by them before the end, and vice-versa.

We now want to investigate situations where withdrawals actually occur. More specifically, we start by displaying an homogeneous and an heterogeneous scenario with only one exit.

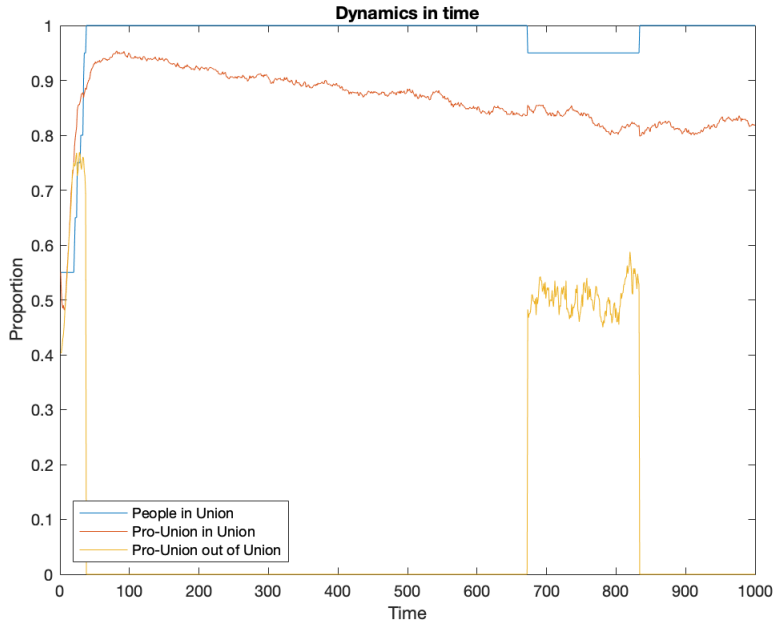


(a) Homogeneous scenario II - one withdrawal

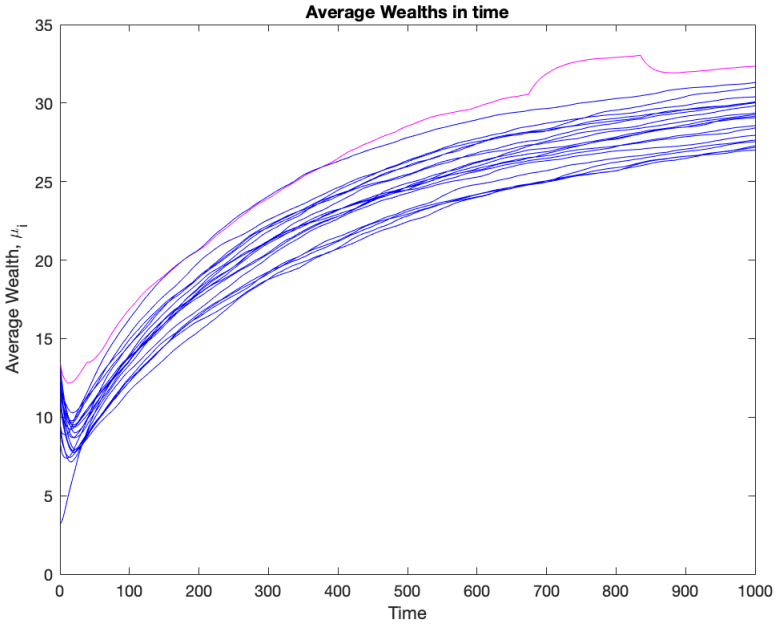


(b) Homogeneous scenario II - average wealth in countries

Figure 4.3: We now add the pink colour that stands for average wealth of those countries who leave Union but regret their decision. The wealthiest (in average) leaves between round 800 and round 900. At first the level of its average wealth goes up, but such an increase suddenly stops and the trend gets reversed. The country then decides to move back to Union.



(a) Heterogeneous scenario II - one withdrawal

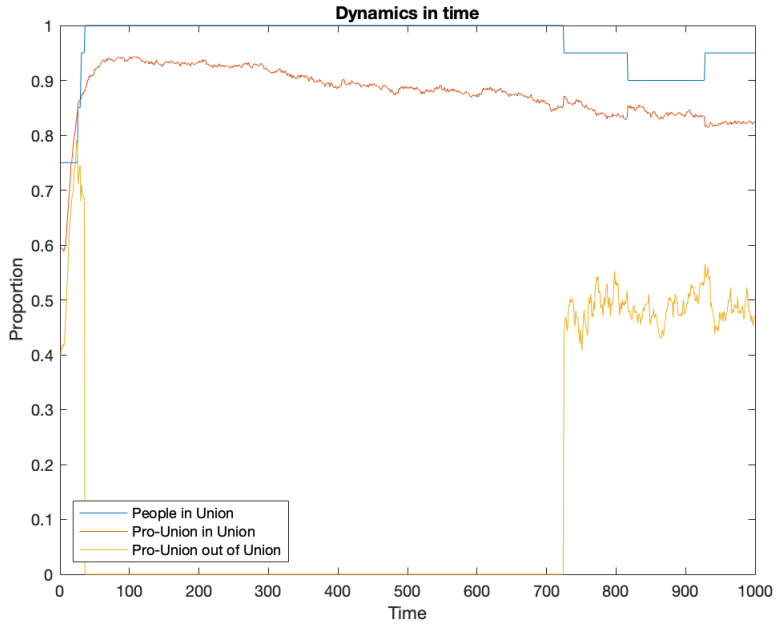


(b) Heterogeneous scenario II - one withdrawal

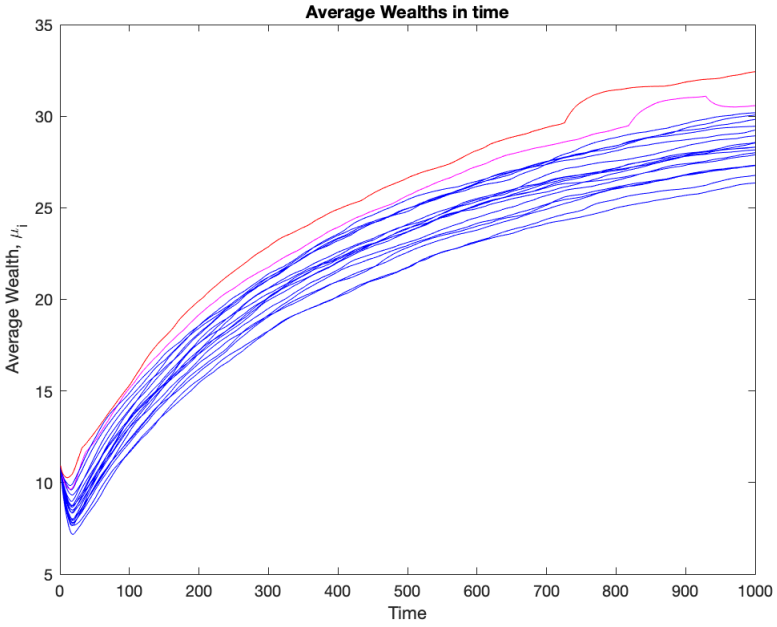
Figure 4.4: We now add the pink colour that stands for average wealth of those countries who leave Union but regret their decision. The wealthiest (in average) leaves between round 600 and round 700. At first the level of its average wealth goes up, but such an increase suddenly stops and the trend gets reversed. The country then decides to move back to Union.

Trends above seem to suggest that if a country leaves the Union, there will be a period in which its citizens' average level of wealth will (more or less sharply) increase. However, such a favourable trend might not last long, and in this case the country will regret its own decision of leaving and will step back into Union.

In order to check if our last statement is correct, we analyze more complex scenarios. To this aim we consider some cases in which multiple withdrawals occur.

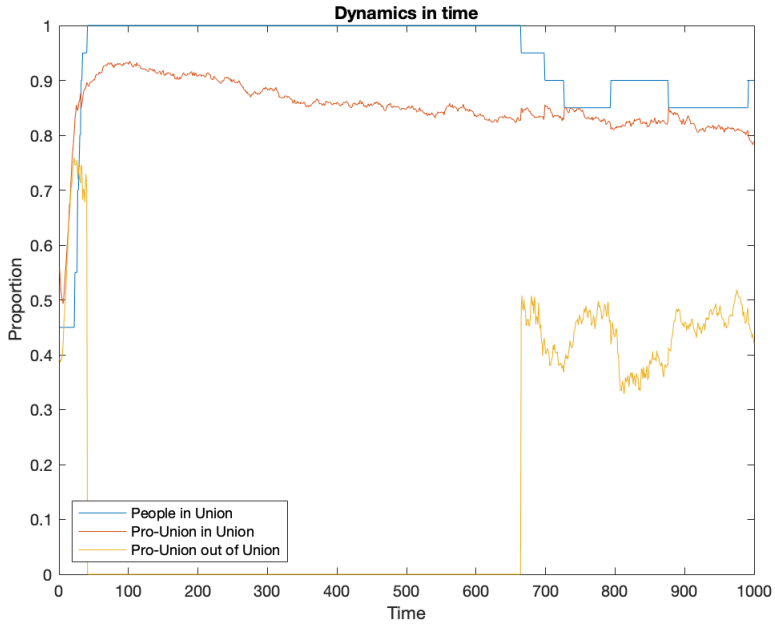


(a) Homogeneous scenario III - multiple withdrawals

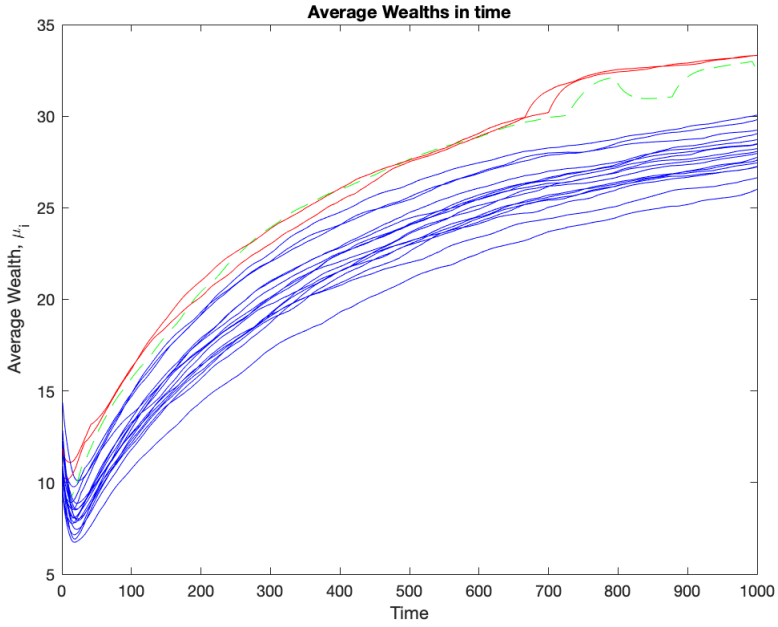


(b) Homogeneous scenario III - average wealth in countries

Figure 4.5: Curves' colours in b have the same interpretation as before, with the only addition in red which represents average wealths of countries that leave the Union without regretting their decision. Here we have two countries leaving. One of them regrets, while the other one does not.

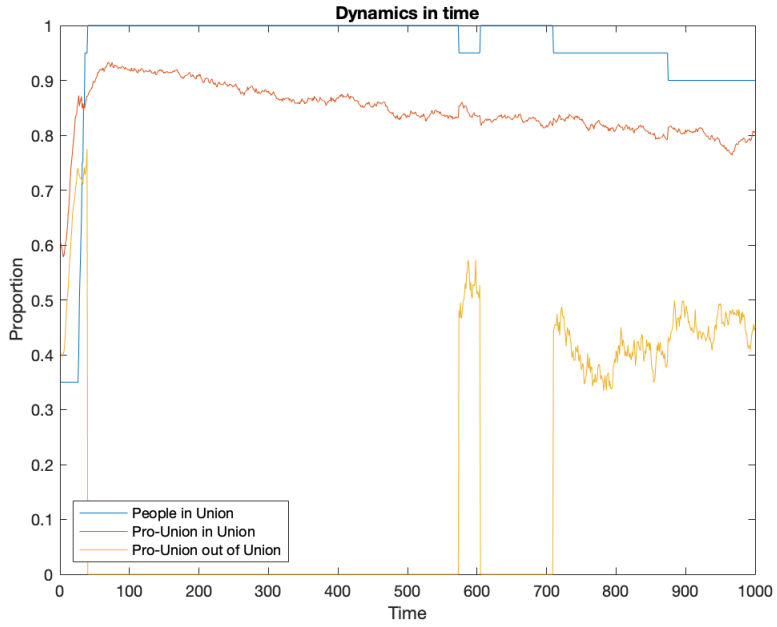


(a) Heterogeneous scenario III - multiple withdrawals

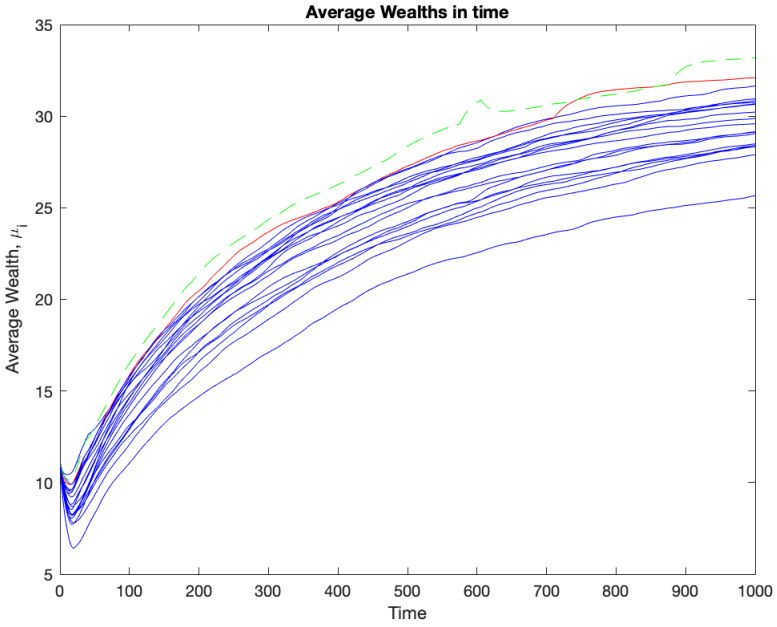


(b) Heterogeneous scenario III - average wealth in countries

Figure 4.6: Curves' colours in b have the same interpretation as before. We also add green for those countries that leave Union, get back into it and then leave it again. One country displays this "oscillating" behaviour.

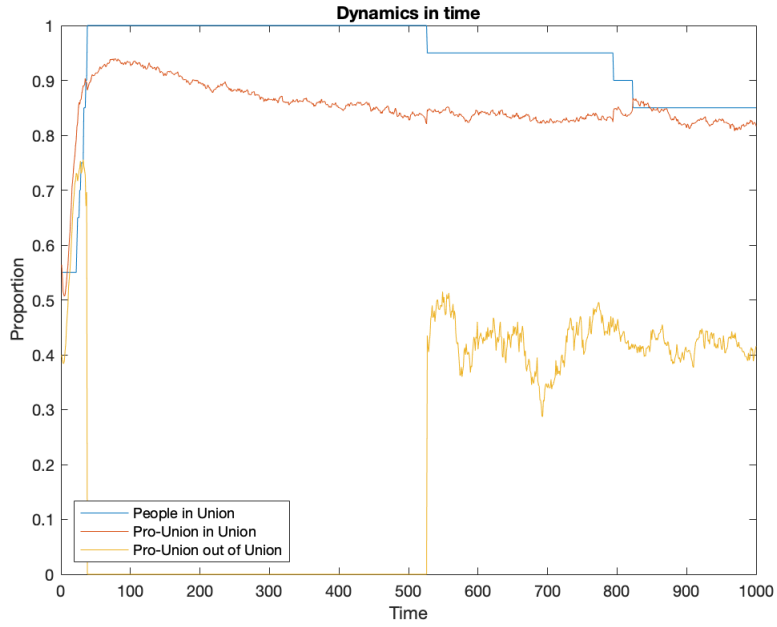


(a) Homogeneous scenario IV - multiple withdrawals

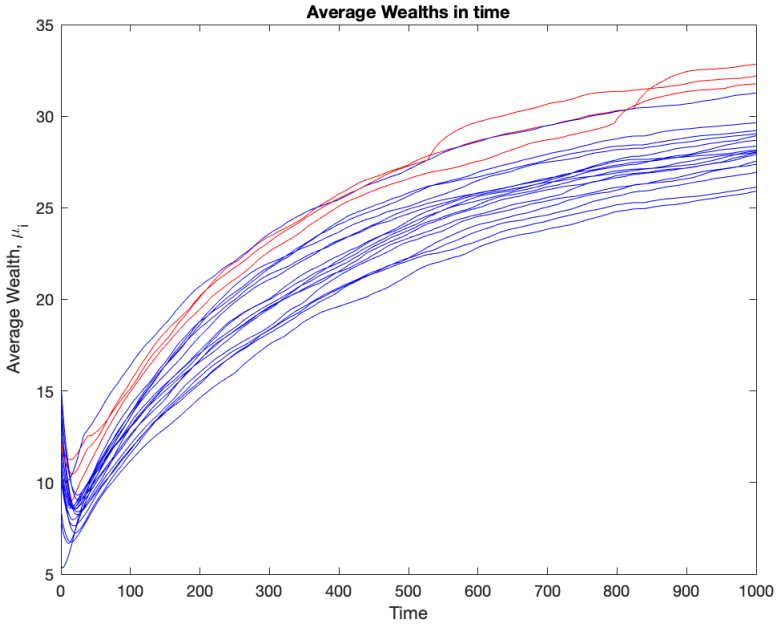


(b) Homogeneous scenario IV - average wealth in countries

Figure 4.7: Curves' colours in b have the same interpretation as before. Here we have one country leaving with no regrets and another one displaying the "oscillating" behaviour.



(a) Heterogeneous scenario IV - multiple withdrawals



(b) Heterogeneous scenario IV - average wealth in countries

Figure 4.8: Curves' colours in b have the same interpretation as before. Here we have three countries leaving with no regrets.

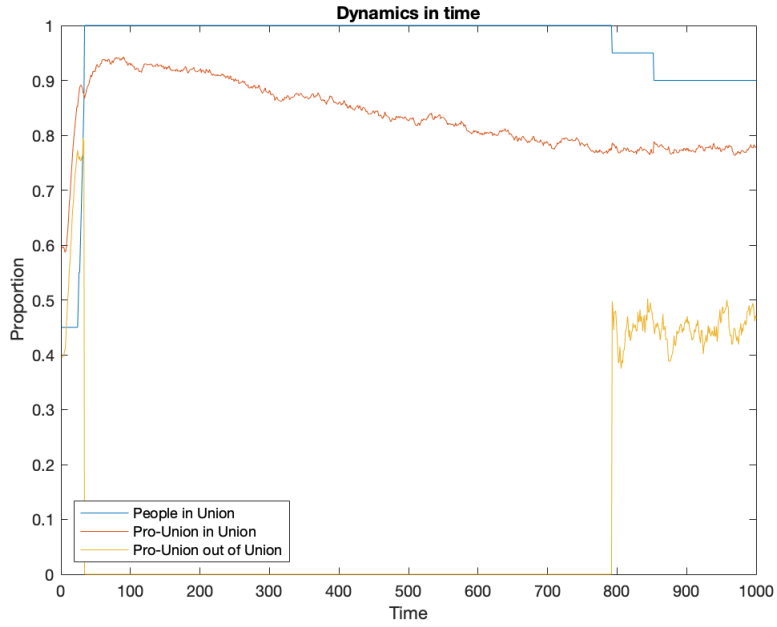
Our former statement seems to be correct: when a country leaves the Union, an immediate increase of its citizens' average wealth occurs. If this improvement is consistent in time

(i.e. either it becomes even greater in the subsequent rounds or at least it is not followed by an immediate decrease), the country is likely to remain out. Otherwise, it will be back on its decision and it will rejoin Union.

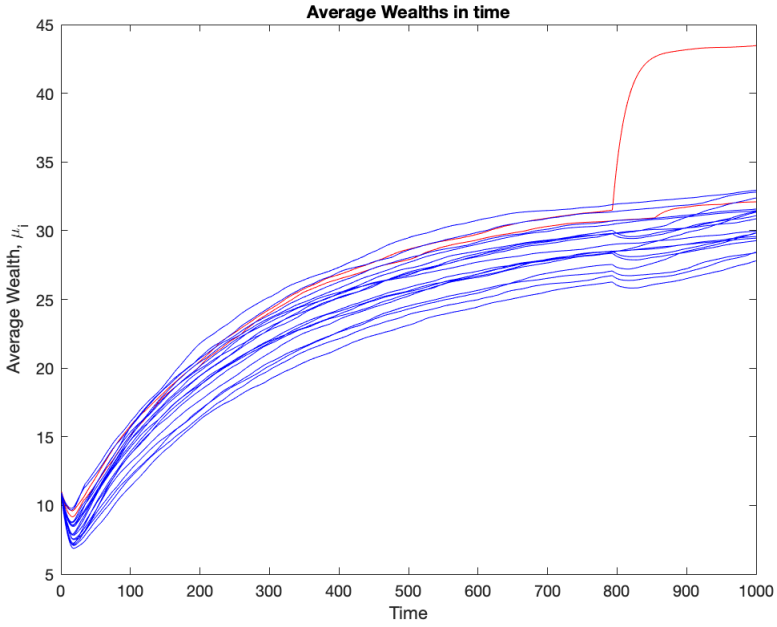
At a computational level, there is also the possibility that a country leaves, rejoins, and then leaves again (figures 4.6 and 4.7, green line), which is useful in order to check our statement once more. However, in the real world it is quite unrealistic that a country will be allowed to leave and join an economic union more than once in a few years. As the Brexit case suggests some years might actually be required simply in order to find an exit agreement between an economic union and a leaving country.

As expected, countries that are more likely to leave will be those with the highest levels of average wealth (since their contribution to the Union will be probably greater than the amount they will earn by redistribution).

We now show an example in which leaving countries are not the wealthiest of all (at least in average), in order to remark that also other factors (such as imitation dynamics) can play an important role in withdrawals (figure 4.9).



(a) Homogeneous scenario V - multiple withdrawals



(b) Homogeneous scenario V - average wealth in countries

Figure 4.9: Colours' scale is the same used in previous figures. Two countries leave, but they are not the wealthiest (in average) in the simulated world.

We come to the conclusion that countries leaving Union are (obviously) rich countries. After their exit, their average level of wealth grows faster than inside Union for a while,

then we can have three different possibilities:

1. The decision of leaving seems to provide clear benefits: average wealth keeps growing faster than inside Union. The country remains out of Union.
2. The decision of leaving seems to provide some benefits: after a short period of fast growth average wealth's growth trend goes back to Union level. The country still remains out of Union.
3. Despite providing some initial benefits (the initial fast growth of average wealth), average wealth in the country decreases sharply. The country regrets its decision of leaving the Union.

It will be interesting to see in the upcoming years which of the 3 scenarios above will take place in UK after Brexit.

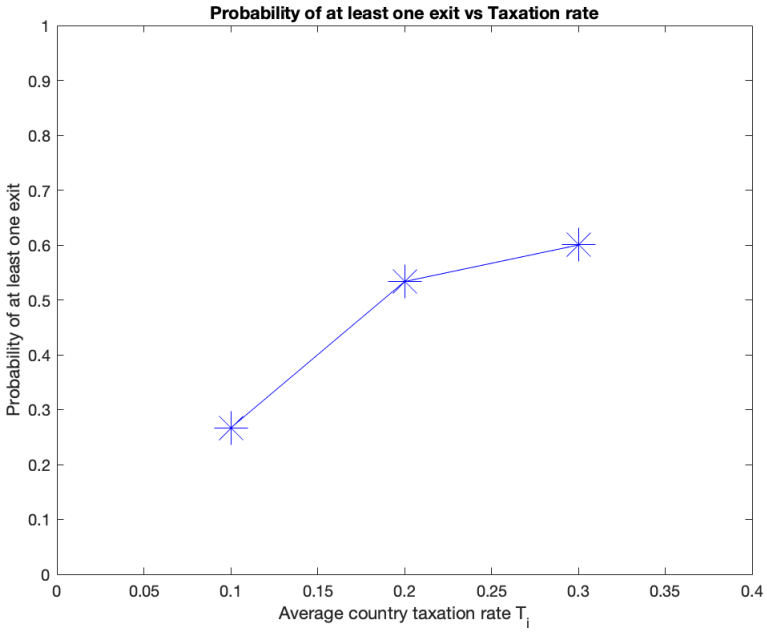
4.2. Probability of Withdrawal and Parameters' Values

We are now interested in giving some further insights about what factors cause withdrawals by estimating the probability that at least a withdrawal occurs after a transitory of 500 rounds for different parameters' values.

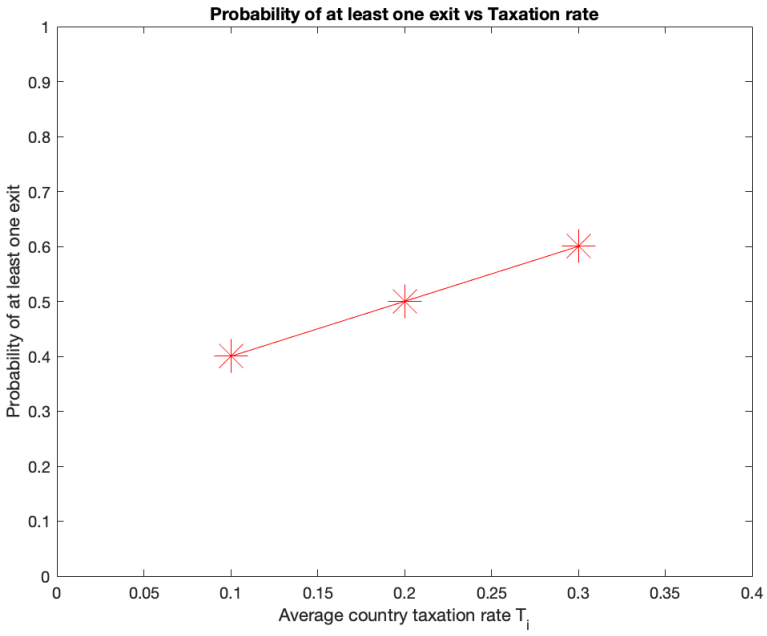
In order to perform probability estimations, we set a number of initial conditions and observe (on every single realization of the dynamics) if at least one country leaves the Union after the transitory. We then simply divide the number of simulations in which at least one withdrawal has occurred after 500 rounds by the overall number of initial conditions.

In most of the cases we have performed the estimation for about 20 different initial starting points. Adding more of them generally does not seem to change trends and requires a lot of time to perform the simulations.

As a first instance, we can check how impactful countries' average taxation coefficients can be. Coherently with results of [11], we see that the higher average taxation rate is, the more likely withdrawals are to occur (as captured by figure 4.10).



(a)



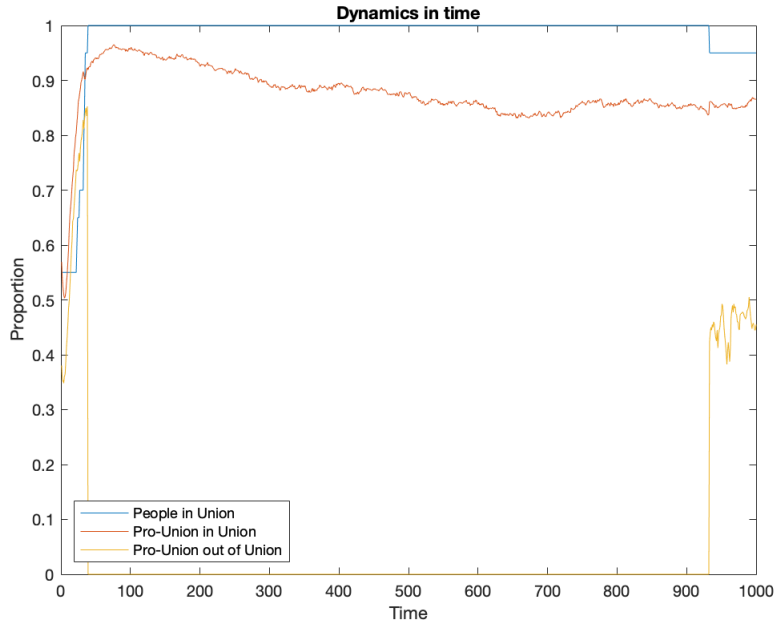
(b)

Figure 4.10: Both in the homogeneous (above) and the heterogeneous scenario (below), a lower level of average country taxation rate is associated to a lower withdrawal probability.

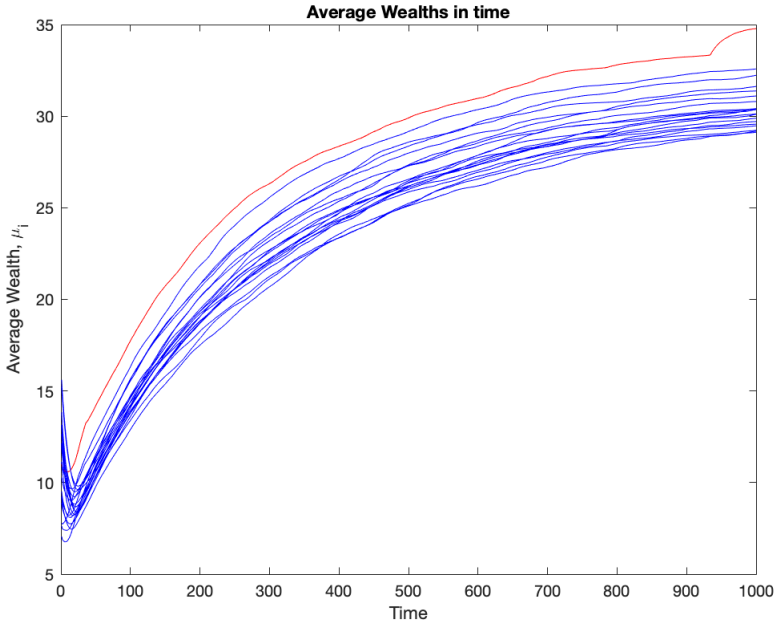
We then study more in depth how significantly a higher (or lower) level of efficiency in Union's economic policies affects withdrawal probability. More specifically it is of our

interest to check if, under our redistribution hypotheses, better efficiency will turn out to please only those citizens who are already pro-Union or if it will get translated into an overall increase of the Union's popularity. Before the discussion of the withdrawal probability, we show an example of how an increase in α_U can actually impact on the opinion dynamics.

In fact, in the scenario represented in figure 4.11 and 4.12, to an increase in α_U corresponds one less exit in the simulated period. It's also interesting to notice that the leaving country in figure 4.11 would actually have more convenience in withdrawing, since in the observable period his average wealth grows more outside than inside Union (figure 4.12), even in spite of a higher amount gained from Union redistribution. However, if the Union is more efficient, opinion within that country stays in favour of the "remain" option and the more convenient scenario does not take place.

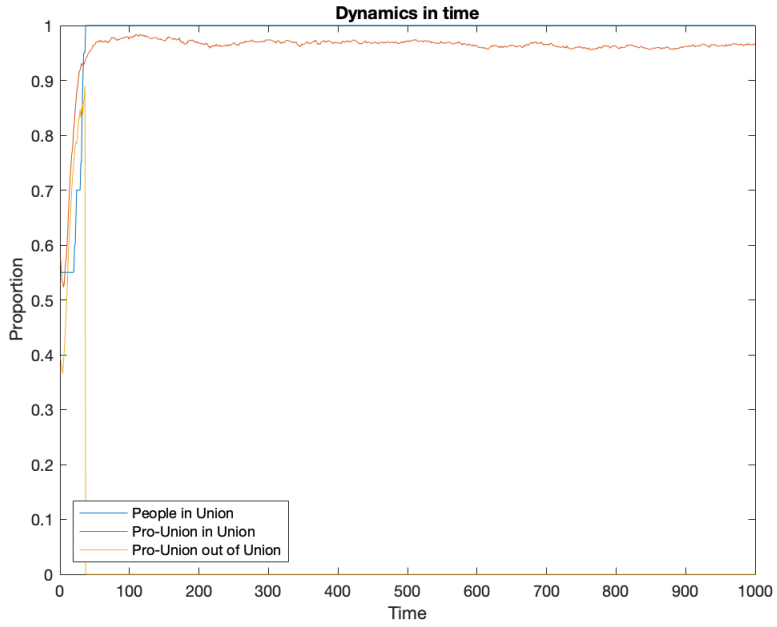


(a) Heterogeneous scenario V - $\alpha_U = 1$

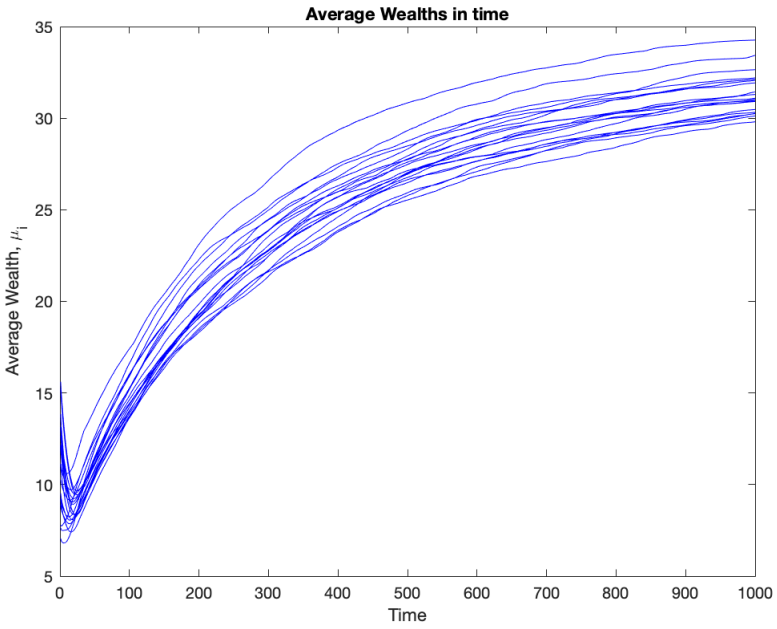


(b) Heterogeneous scenario V - average wealth in countries

Figure 4.11: With an enhancement Union factor equal to 1, the wealthiest country leaves the Union in the final rounds.



(a) Heterogeneous scenario V - $\alpha_U = 1.1$

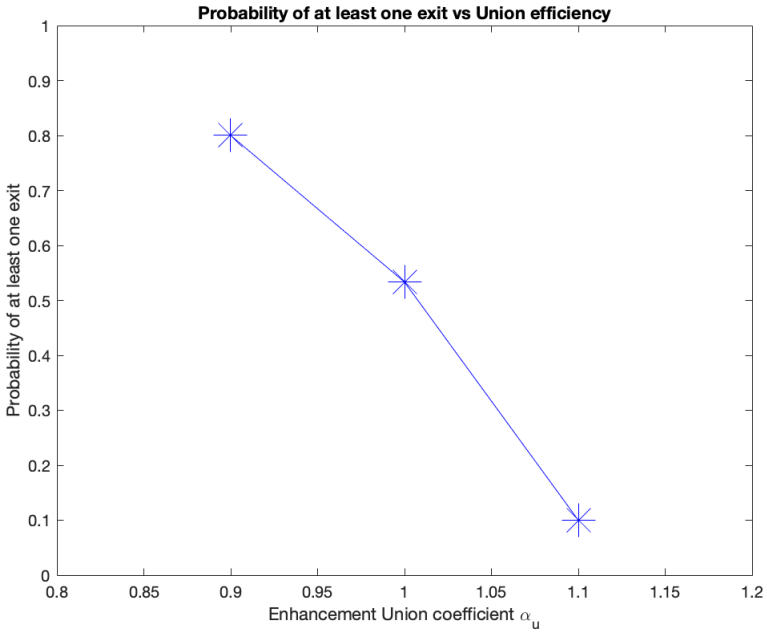


(b) Heterogeneous scenario V - average wealth in countries

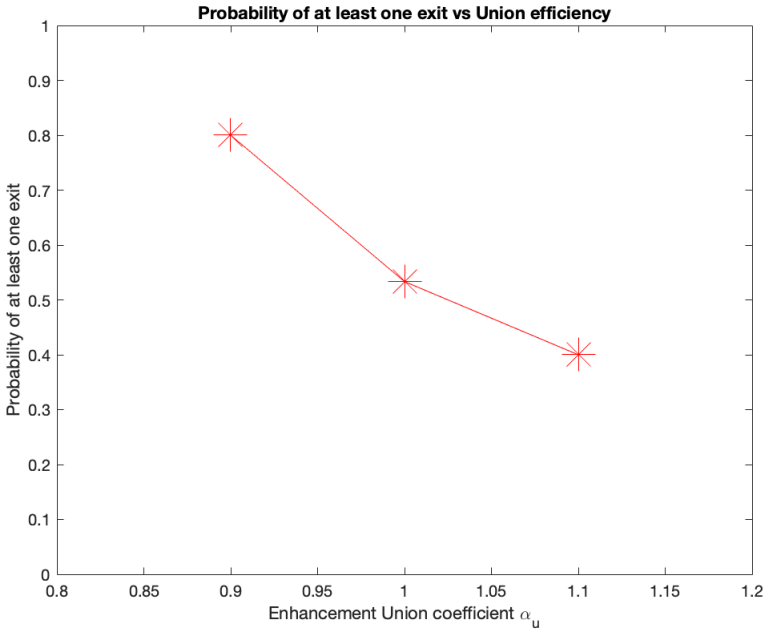
Figure 4.12: With an enhancement Union factor equal to 1.1, no countries leave the Union before round 1000.

Figure 4.13 clearly shows that for greater values of the booster coefficient α_U, the probability to have an exit goes down. Therefore, all Union citizens seem to have benefits

from more efficient institutions at Union level, and more efficiency translates into less withdrawals.



(a)



(b)

Figure 4.13: Both in the homogeneous (above) and the heterogeneous scenario (below), a relatively less efficient Union redistribution/economic policy is associated to a higher withdrawal probability.

We are now interested in how influence factor's variations might be associated to a different withdrawal probability. We make our computations in an heterogeneous scenario, since this parameter is associated to wealth inequality. Since in our model wealthy people inside Union are often not eager to become anti Union, there is no surprise in our result: if the influence factor becomes greater, the withdrawal probability goes down.

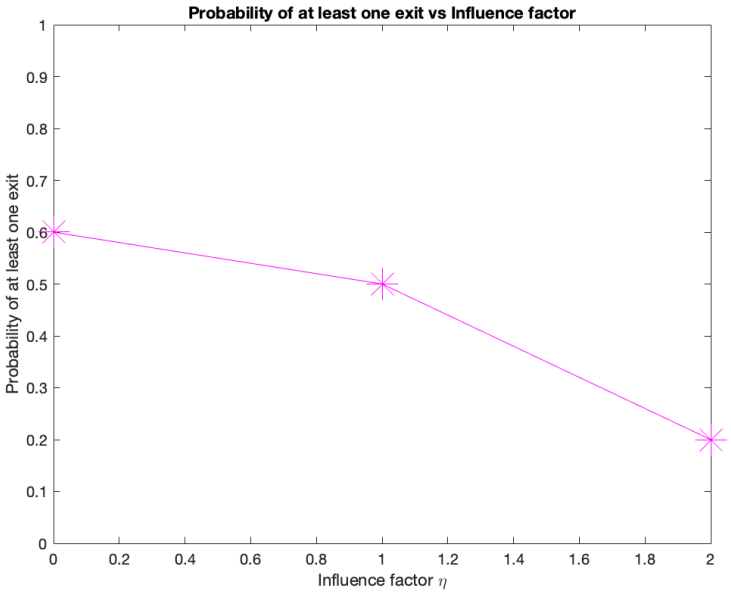


Figure 4.14: Sensitivity of exit probability to variations in the influence factor.

It is also interesting to check whether the information system's power is relevant for opinion changing or not.

In order to do this, we consider a scenario where initial level of wealths are simulated from a uniform distribution on [0,1]. We set this initial condition in order to minimize any potential effect that a high initial level of wealth could have on the imitation dynamics.

We observe (figure 4.15) that to a higher value of ψ (and therefore to a weaker information system) a greater withdrawal probability is associated. This seems to suggest us that if agents' imitations occur only (or mainly) within their own country, this might translate into an acceleration of the process of unsatisfaction spreading, once unsatisfaction towards the Union reaches certain levels.

However, we can also tell that the effect is not that significant, since a variation of 0.25 in ψ translates into a variation of the estimated probability of only 0.1 (figure 4.15).

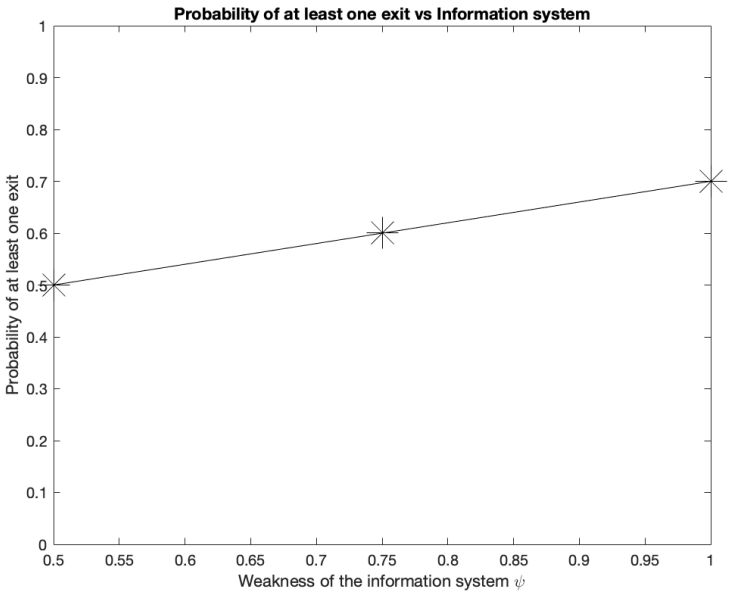


Figure 4.15: Sensitivity of exit probability to the weakness of the information system.

5 | Conclusions and Future Developments

We tried to improve an already existing agent-based model that replicates interactions between countries and citizens both inside and outside an economic union by introducing some new parameters and slightly modifying some conditions for opinion changing. In particular we redefined some probabilities for an agent to change his/her own opinion, emphasizing their dependence from that citizen's level of wealth, compared to the average one in his/her own country of provenience. Our analyses have been performed both in homogeneous and heterogeneous wealth scenarios and we had the confirmation (for both scenarios) that while countries are all likely to become Union members at an early stage in the simulations, occasional withdrawals after long periods can actually occur.

We analyzed how countries' average wealth evolves in time inside and outside of Union: some countries seem to gain consistent benefits from withdrawals, while for others benefits are just temporary. Therefore, some countries regret their decision of leaving the Union and step back into it again. A few countries might leave and join Union more than once throughout the simulation process (even though this is not likely to happen with ease in the real world).

Countries that leave Union tend to display a higher level of average wealth (this is due to their unsatisfaction about the difference between the amount they pay to Union and the amount they receive from redistribution). However, sometimes those which leave are not the richest (in average) in the World, and this puts into evidence that also other mechanisms such as opinion imitation influence withdrawal dynamics.

On the other side, countries that regret their exit decision seem to do it mainly for economic-related reasons (their average wealth experiences a fast decrease).

We then performed some analyses of how different values of some parameters affect withdrawal probability: in particular, higher taxation burdens imposed by countries and less efficient Union economic policies increase the probability of withdrawals by a significant amount. As predictable, a higher influence factor translates into a lower exit probability.

Furthermore, a less powerful information system is associated to a higher likelihood to have at least a withdrawal, but such an effect seems to be less significant.

All of our simulations were performed by means of C++. More specifically, we used an object-oriented approach devoting to businesses, citizens, countries and to the World itself their own classes. The Union does not have its own class, but its composition is instead (along with all of its features) an outcome of the interactions among previous class members throughout the simulation process. All simulations' results were saved on txt. files and then moved to a Matlab environment to be graphically represented.

Considering that research literature about agent-based models to represent economic unions' opinion dynamics is still at an early stage, we feel that there are a lot of possible developments that could actually be explored in the future.

1. Since data and research results have proven that opinion about migrations is a driving factor in withdrawal dynamics, migrations could be modeled by moving to an environment where the number of agents for country is not constant anymore, and where for every agent the opinion-changing mechanism takes into account the level of ongoing migration towards that agent's country.

2. To simulate imitation dynamics, a more sophisticated model could be implemented: for example such dynamics can be analyzed by means of a social network model where connections among agents occur if they belong to the same businesses or to some countries with some similar features, etc.

3. Considering the EU example, the Union's unique currency and the exchange rate to compare its value with those of non-member countries' currencies (along with all the effects that such a comparison directly implies) could also be added to the model.

Bibliography

[1] F. Alqahtani and D. G. Mayes. Financial stability of Islamic banking and the global financial crisis: Evidence from the Gulf Cooperation Council. *Economic Systems*, 42 (2):346–360, 2018.

[2] G. Arioli and R. Lucchetti. Basics of Game Theory, Politecnico di Milano. 2020.

[3] A. Arnorsson and G. Zoega. On the causes of Brexit. *European Journal of Political Economy*, 55:301–323, 2018.

[4] M. Ashcroft. How the United Kingdom voted on Thursday... and why. *Lord Ashcroft Polls*.

[5] S. Becker, T. Fetzer, and D. Novy. Who voted for Brexit? A comprehensive district-level analysis. *Economic Policy*, 33:601–650, 2017.

[6] D. Berry. *Caribbean Integration Law*. Oxford University Press, 2014.

[7] E. Borghi. Politiche Commerciali, Lecture Notes in International Economics, Università Commerciale Luigi Bocconi. 2017.

[8] V. Bulmer-Thomas. The central American Common Market: From closed to open regionalism. *World Development*, 26(2):313–322, 1998.

[9] N. F. Campos, F. Coricelli, and L. Moretti. Institutional integration and economic growth in Europe. *Journal of Monetary Economics*, 103:88–104, 2019.

[10] L. Dijkstra, H. Poelman, F. Coricelli, and A. Rodriguez-Pose. The geography of EU discontent. *Journal of Monetary Economics*, 54:737–753, 2020.

[11] C. Gracia-Làzaro, F. Dercole, and Y. Moreno. Dynamics of economic unions: An agent-based model to investigate the economic and social drivers of withdrawals. *Chaos, Solitons and Fractals*, 160, 2022.

[12] S. B. Hobolt. The Brexit vote: a divided nation, a divided continent. *Journal of European Public Policy*, 23:1259–1277, 2016.

[13] K. Kiljunen. *The European Constitution in the Making*. CEPS, 2004.

[14] R. Mortimore. Polling history: 40 years of British views on 'in or out' of Europe. *The Conversation*, 2016.

[15] E. Vinokurov. Eurasian Economic Union: Current state and preliminary results. *Russian Journal of Economics*, 3(1):54–70, 2017.

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