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Executive Summary of the Thesis

Towards Automated Planetary Moon Tour Design: Beam Search for Optimal Sequence Generation

Laurea Magistrale in Space Engineering - Ingegneria Spaziale

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1. Introduction

In recent decades, there's been increased interest in planetary moon missions to the Jovian and Saturnian systems, driven by discoveries from missions like NASA's Galileo and Cassini, that identified Moons as Europa or Enceladus as possible hosts for microbial life. Ambitious missions like ESA's JUICE and NASA's Europa Clipper, along with concepts like Europa Lander and Orbilander, have therefore been designed. To successfully reach the Moon of interest with a feasible ΔV cost, all these missions include a Moon Tour, a long sequence of fly-bys around the moons of the planetary system.

However, designing a Moon Tour is a complex astrodynamics problem due to the vast design space and, as for now, no state of the art automated strategy exists to solve it. This thesis aims to make the first steps towards the automation of Moon Tour design, answering to the following two research questions:

1. *Can the selection of the sequence of encounters of a Moon Tour be automated?*
2. *Which is a suitable optimization technique to compute such a sequence?*

Basically, the goal is to find a suitable way to tackle a complex mixed integer combinatory optimization problem, in which integer variables as the sequence of Moon and the total number of fly-bys need to be optimized together with continuous variables as the v_∞ vector at each fly-by and the ΔV values of the manoeuvres.

Firstly a trade off between models and transfer techniques from the literature is presented, to select the most suitable for the objectives of this research.

The selected tools are then utilized to create a convenient fully discrete representation of the design space. Then, a multi-objective beam search algorithm is presented, to find optimal sequences of ballistic and powered transfers, linked together by fly-bys.

The thesis aims to propose an approach to obtain continuous feasible tours with limited ΔV and time of flight ToF. The obtained sequences aim to serve as initial guesses for full-ephemeris solvers, to obtain high-fidelity Tours.

2. Review and Trade Off

To answer to the research questions, it is needed to select the model in which the design space will be represented, together with the type of transfers to build the Tours. The literature review is organised as a trade-off, for two separate sets of

instruments: representation tools and Endgame Techniques. A brief trade-off on optimization techniques has been also carried out, focused on tree search techniques.
The representation tools considered are:

- Tisserand Poincare graph: extension in the planar circular restriced 3-body problem (PCR3BP) of the well known Tisserand graphs, useful to identify sequences of encounters between different moons.
- Fly-By Map: Numerically integrated map in (PC3TBP) to assess the effect of a close encounter with a moon on the primary centred osculating orbital elements of a spacecraft.
- v-inf globe: Maps the v-inf vector expressed in v_∞, α, κ (v_∞ value, pump and crank angle) into the corresponding primary centred keplerian elements and vice versa, in the patched conics 0-Sphere Of Influence (0-SOI) model. Families of orbits and possible sequences can be easily identified on the globe.

A brief overview on the endgame techniques analised is given in table 1.

Table 1: Comparison between the studied Endgame techniques

Strategy	Pros	Cons
V-Infinite Leveraging Transfers (VILTs)	- Simple - Light - Linearisable	- Simplified model - Only High energy endgames
Tisserand Leveraging Transfers	- Low-energy endgames	- Fly-by Map needed
Multibody Technique	- Low ΔV achievable	- PCR3BP integration
Patched Periodic Orbits	- Almost ballistic transfers - Versatile design	- Transfers to be pre-selected - Possible num. issues - PCR3BP integration

The trade-off allowed to select non-tangent V-Infinite Leveraging Transfers (VILTs) [1] as an Endgame technique with the v-inf globe [2] selected as a representation tool, in the Keplerian 0-SOI model. This choice is done to be able to quickly and conveniently compute large sets of trajectories, obtaining a discretised design space, ready to be pruned by an optimiser.
The v-inf globe brings further advantages: constraints on the transfers and available trajectory opportunities can be easily incorporated in the design of a sequence of VILTs, using the mapping in terms of fly-by parameters v_∞, α, κ (v_∞ value, pump and crank angle, fig. 1). In addition, knowing the allowed maximum turning angle of a fly-by δ_{max}, tours can be designed on the globe, moving on the contour of the sphere with steps of max amplitude δ_{max}.

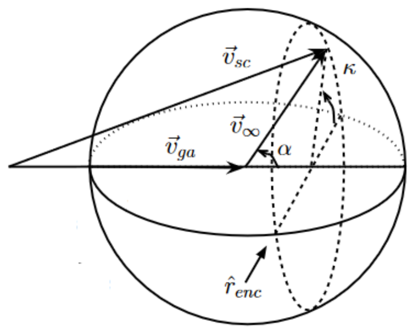

Figure 1: Model of a ballistic fly-by on the v-inf globe, [3]

For what concerns optimization, it has been chosen to focus on Pareto-Pruning beam search [4]. Indeed, classic tree search algorithms are unpractical for the huge design space of Moon Tours. Non-deterministic versions of the Beam Search as beam P-ACO have not been selected for simplicity.

3. Design Space Study

3.1. Building Blocks

As stated in the the literature review, the selected endgame technique is the VILT, in the Keplerian 0-SOI models. The v-inf globe is then exploited to represent and connect together sequences of VILTs, with ballistic transfers. Consequently, a Tour computed using the set of tools of this Thesis is made by the following building blocks:

- Resonant Transfers: they begin and end with an encounter with the moon, happening in the same point of the s/c's orbit around the primary. Resonant transfers are defined by the ratio $N : M$, in which N is the number of Moon revolutions and M the number of s/c revolutions.

- Non-Resonant Transfers [3]: They are transfers linking two moon encounters, happening at opposite true anomalies of the s/c's orbit around the primary. They are identified by the ratio $N : M$ and by the nomenclature OI for outbound to inbound transfers and IO for inbound to outbound transfers.
- VILTs: They are powered transfers, in which a tangent burn is performed after K s/c revolutions around the primary. The burn can happen either at the apocentre (ext) or at the pericentre (int). The following notation is used to define a VILT:

  $$\{\text{int or ext}\} - \{\text{IO, II, OI or OO}\}N : M(K)$$

  The nomenclature specifies also the characteristic of the departure and arrival encounters of the VILTs.
- Moon Changes: In the context of this Thesis, phasing between moons is not considered. Consequently, a moon change transfer is any primary centred orbit that intersects the orbits of two moons.

The motion has been restricted to the plane, as this choice grants far more possibilities of VILTs and Non-resonant transfers, facilitating the design of an Endgame. Indeed, out of plane Non Resonant fly-bys (and therefore also VILTs) would be constrained to have both encounters on the line of nodes of the orbit.

An example sequence using the building blocks selected is shown in fig. 2.

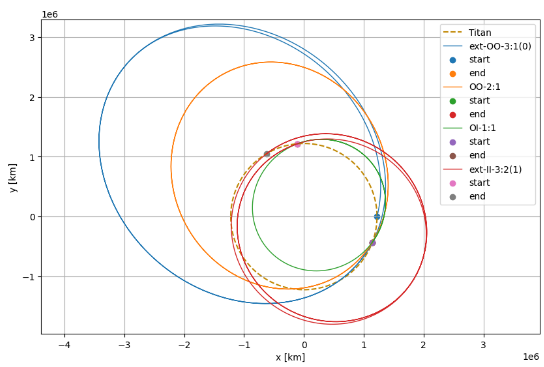

Figure 2: Example sequence, Saturn Centred Inertial Frame

3.2. Discretization

The design space is fully discretised with the v-inf globe, using the presented set of building blocks. Basically, for each moon of the planetary system of interest, a v_∞ grid is defined, identifying concentric v-inf circles (2D views of v-inf globes). On each v_∞ level, relevant sets of ballistic transfers and VILTs are then computed and stored, filling the circles, as it can be seen in fig. 3 for a set of ballistic transfers.

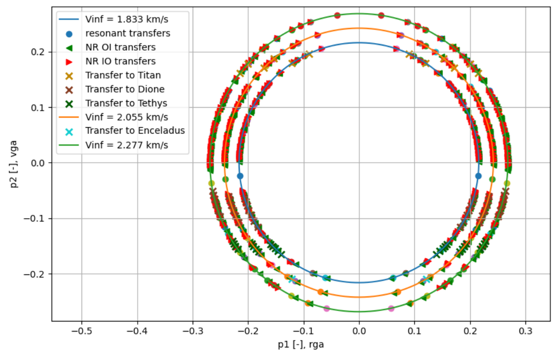

Figure 3: Example of the sets of transfers of 3 v_∞ levels at Rhea, 2D view of the v-inf sphere

All the data is stored in an database of objects, organised as in fig. 4. With this strategy, the Moon Tour design space is fully discretised, ready to be fed to a combinatory optimiser.

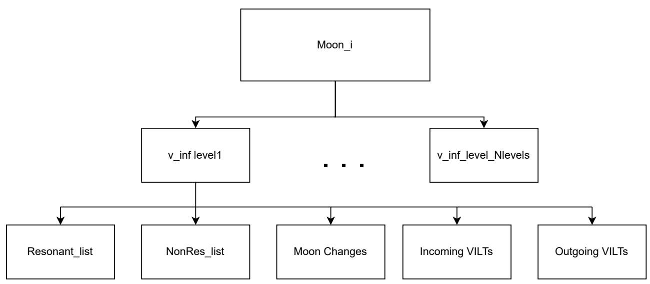

Figure 4: Scheme of the Design Space representation

4. Optimization Strategy

4.1. Overview

A multiobjective forward-backward Pareto pruning strategy is developed ad-hoc for the presented design space organization. VILTs sequences are generated, forward from a given departure Moon and v_∞ and backwards from a desired v_∞ at the arrival Moon. Once the beams

reach the same Moon with close v_∞ values, the found sequence is saved as a candidate Tour.
For this optimization problem a node is defined by the following array of elements, describing the incoming conditions of the encounter ending a sequence of transfers:

$$\mathbf{x} = [Moon, v_\infty, \alpha, \kappa] \tag{1}$$

The optimization is designed to exploit the structure of the design space: at each branching step, only physically reasonable sequences are tried, as each node is branched using only the VILTs lists of its v_∞ level. This way, the number of combinations to try is vastly reduced. In addition, only continuous trajectories are considered, continuity constraints are automatically satisfied.
Ballistic transfers and VILTs have different roles: new branches are generated considering the VILTs in the lists, while ballistic transfer sequences are used to connect the conditions at the node (α, κ) with the ones of each new VILT.
The ballistic sequences are automatically chosen among the ones available in the lists of the level of the node, in a pseudo-time optimal VILT-Patching strategy. The procedure connects VILTs by performing resonance hopping and non-resonant transfers, selecting after each fly-by the transfer with the minimum time of flight N. Useless transfers are then automatically removed from the sequences, keeping the turning angle of the fly-bys as close as possible to the maximum allowed δ_{max}.
This way, the process is sped up: if for each node all the possible combinations of ballistic transfers and VILTs were to be tried, the computational cost would considerably increase.
In this framework, therefore, a new branch is a sequence of ballistic transfers ending with a VILT, starting from the conditions of a node. Branches are pruned checking their Pareto optimality with respect to sets of cost functions, detailed in section 4.2.
The algorithm gives as output ΔV-ToF Pareto Optimal batches of Tours, each built by a Forward (FW) and a Backward (BW) leg. The computed sequences do not consider phasing between the moons. Therefore, a refined algorithm using the found Tours as initial guesses will have to also include phasing, in addition to patching the FW and BW legs.

4.2. Cost Functions

Three sets of couples of objective functions have been defined. In each couple, one of the objective functions is related to the ΔV, while the other to the ToF.
For a sequence of N VILTs, the first set of functions, $\mathbf{J_{tot}}$, reads:

$$\mathbf{J_{tot}} = \begin{bmatrix} \sum_{i=1}^{N} \Delta V_i \\ \sum_{i=1}^{N} ToF_i \end{bmatrix} \tag{2}$$

This choice of objective functions is simple: at the N-th pruning step, the overall ΔV and ToF cost of each sequence up to the last branch is considered.
The second couple of functions used is instead based on the concept of VILT efficiency (the ratio between the variation of v_∞ and the cost of the VILT). Consequently, the objective function set, named $\mathbf{J_{eff}}$, reads:

$$\mathbf{J_{eff}} = \begin{bmatrix} Eff_{\Delta V} = K_{FB} \dfrac{v_{\infty 2} - v_{\infty 1}}{\Delta V} \\ Eff_{ToF} = K_{FB} \dfrac{v_{\infty 2} - v_{\infty 1}}{ToF} \end{bmatrix} \tag{3}$$

In the same fashion, another set has been introduced, defining an energy efficiency of the VILTs:

$$\mathbf{J_{eng}} = \tilde{\mu}_P K_{FB} \begin{bmatrix} Eff_{\Delta V} = \dfrac{1/\tilde{a}_1 - 1/\tilde{a}_2}{\tilde{\Delta V}} \\ Eff_{ToF} = \dfrac{1/\tilde{a}_1 - 1/\tilde{a}_2}{\tilde{ToF}} \end{bmatrix} \tag{4}$$

$$\text{in which } K_{FB} = \begin{cases} 1 & \text{for the FW leg} \\ -1 & \text{for the BW leg} \end{cases}$$

The equations refer to an outer Moon to inner Moon Tour, for the opposite case the signs of K_{FB} are switched. The tilde refers to dimensional quantities.
With $\mathbf{J_{eff}}$ and $\mathbf{J_{eng}}$, at each pruning step, the ΔV considered in the computation is the one of the last VILT added to the sequence, while the ToF is the sum of the one of the VILT plus the one of its patching sequence to link it to the previous node (if present). This way, the sequences that are saved during the optimization are built by Pareto optimal chunks.
The three sets of objective functions have some substantial differences. Firstly $\mathbf{J_{tot}}$ considers

the contribution of the whole sequence, while $\mathbf{J_{eff}}$ and $\mathbf{J_{eng}}$ take into account only the last branch. Then, $\mathbf{J_{tot}}$ uses only the ΔV and ToF cost as it is, without considering the effect of the manoeuvres on the v_∞. This is instead taken into account into the other two sets. It can be said that the second and third set consider the mission goal of the Moon Tour at each step of the optimization, while the first is concerned only with the overall costs.

5. Case Studies

5.1. Overview

Case studies of interest have been analysed, computing batches of sequences and comparing the results with the literature.
In addition, the sets of cost functions are compared in all the case studies. The aim is to understand if there is a set that is always better than the others, or if the nature of the system of Moons influences the performance of the cost functions.
The case studies analised are the following:

1. Enceladus Endgame, briefly summarised in the next paragraph, to benchmark the results with the Tour in [1].
2. Europa Endgame, to test the tools in a different planetary system.
3. Europa Constrained Endgame, to test the tools with a limited set of available transfers, with only OO and II VILTs.

5.2. Enceladus Endgame

As it can be seen in fig. 5 only $\mathbf{J_{eff}}$ and $\mathbf{J_{tot}}$ gave useful results for this case study.

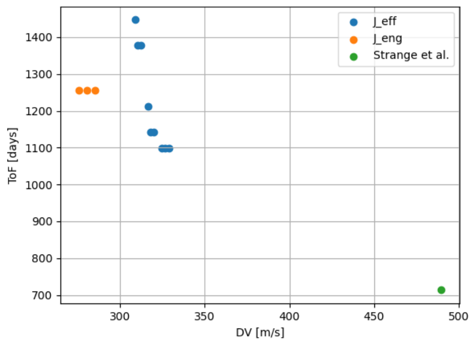

Figure 5: Reference vs Non Dominated Tours found by $\mathbf{J_{eff}}$ and $\mathbf{J_{eng}}$

Compared with the reference, the found tour can allow for significant ΔV savings,up to 44%, at the expense of a longer time of flight.
The fuel optimal Tour in fig. 6 costs $275.4 m/s$ and takes $1255 days$ to be completed.

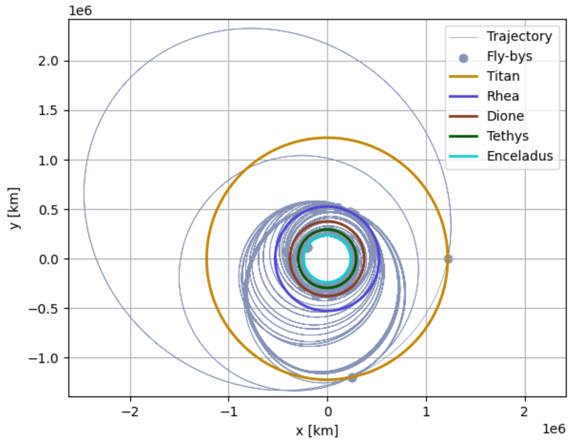

Figure 6: Min. ΔV Enceladus Tour, Saturn Centred Inertial Frame

Built by 63 fly-bys, it exploits both VILTs and ballistic sequences around each moon to reduce the energy of the s/c, as visible in fig. 7 and in fig. 8 for the Tehtys leg of the Tour.
Once Enceladus is reached, the v_∞ is finally reduced to the target value of $0.3 km/s$.
Looking at the computational times, the whole process was relatively fast: the generation of the design space took $7'58''$, while the beam search was completed in $1'01''$, on a laptop with a 9th generation Intel Core i7 CPU, with 16 GB of RAM.

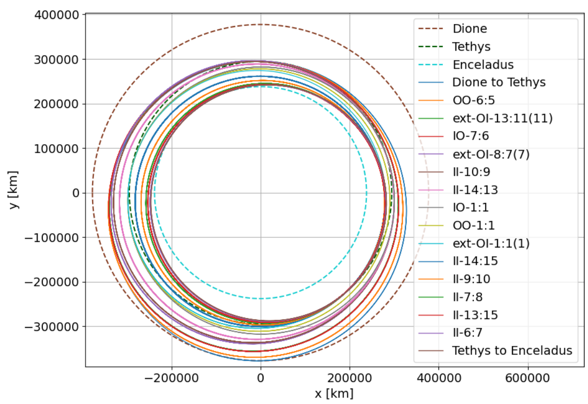

Figure 7: Tethys leg of the min. ΔV Tour, Saturn Centred Inertial Frame

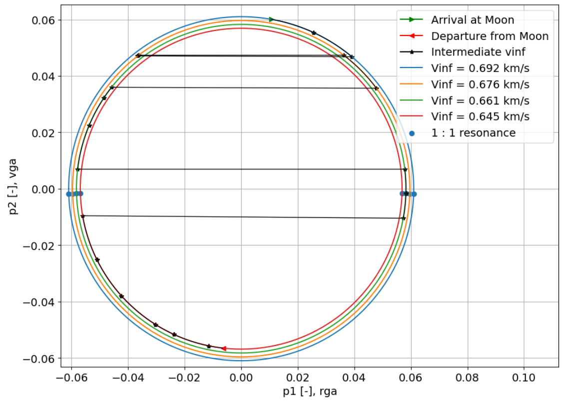

Figure 8: Tethys leg of the min. ΔV Tour, 2D view of the v-inf sphere

In general, relevant solutions have been obtained in all the case studies, especially using $\mathbf{J_{eff}}$ and $\mathbf{J_{eng}}$. Indeed, the solutions found by the efficiency-based cost function dominate the sets of Tours found by $\mathbf{J_{tot}}$. At each pruning round $\mathbf{J_{tot}}$ tries to minimise the costs of the sequences, without looking at the progress achieved by the manoeuvres.
Comparing $\mathbf{J_{eng}}$ and $\mathbf{J_{eff}}$, instead, the former finds solutions with far lower ΔV costs, but with longer ToF.

6. Conclusions

The findings of the work and the main takeaways drawn from the case studies are briefly summarised in the following list.

- The presented discretization of the design space allows to quickly compute relevant sets of possible transfers. The obtained discretised spaces are large enough to include possible optimal combinations, but limited enough to be computed in a matter of minutes on a normal laptop.
- Pareto pruning Beam Search shows promising results and can be used to automate the sequence selection process for Moon Tour design: indeed, the case studies show that the computed sequence compare well with the Tours present in the state of the art, with significant ΔV savings.
- The optimization is very sensitive to the parameters chosen in the generation of the design space. Too coarse v_∞ grids can lead to costly sequences, while too fine grids can give convergence problems and computational load issues. In addition, the best tuning of the design space parameters is cost function dependant.

Comparing these results with the research questions of this work:

1. *Can the selection of the sequence of encounters of a Moon Tour be automated?*
   The preliminary findings of this work allow to positively answer to this research question: the set of tools developed allows to automatically compute batches of possible Tours.
   However, further work on the cost functions is needed.
2. *Which is a suitable optimization technique to compute such a sequence?*
   This Thesis allows to conclude that Multi-objective Beam Search is a suitable technique to tackle the Moon Tour problem. However, it can be useful to try using different techniques to solve the problem, to see if the results or the computational speed of the algorithm can be improved.

References

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