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SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

Method of characteristics to design symmetric and asymmetric converging-diverging nozzles for real gas applications

Laurea Magistrale in Mechanical Engineering - Ingegneria Meccanica

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1. Introduction

The increasing urgency to combat climate change has encouraged a growing interest in electrification and the generation of electric energy from low-carbon sources. Current trends suggest a significant increase in electricity production from solar PV and wind turbines [1].
However, the inherent drawbacks of these technologies, such as low power density and non-scheduled operation, must be effectively addressed to ensure the affordability and reliability of electric energy production. This is crucial for a smooth transition from a fossil-based society to a low-carbon one. One promising approach involves developing technologies that lack these drawbacks while also having low or no emissions. Supersonic turbines, with their diverse low-carbon applications, particularly stand out. An intriguing application is ORC, as highlighted by Anand [2].
The appeal of the ORC lies in its ability to efficiently exploit low enthalpy sources, allowing for the effective utilization of renewable resources like geothermal energy or the heat discarded by various processes, including industrial ones.
A significant challenge associated with turbines in an ORC is the occurrence of supersonic flows. To prevent the formation of shocks and a subsequent drastic decrease in efficiency, specialized design methods must be employed for the blades. The MoC, addressed in this master thesis, is one such established method that enables the design of shock-free supersonic nozzles [3] and nozzle cascades [2].
This work presents the development and verification of two MoC approaches using non-ideal gas models. These methods are applied in the design of both symmetric and asymmetric planar nozzles. Subsequently, the two codes are employed for the design of converging-diverging vanes.

2. MoC for the design of a planar wind tunnel nozzle

The MoC is a technique that enables the transformation of one or more quasi-linear nonhomogeneous partial differential equations into total differential equations along specific curves, known as characteristics.
To elaborate, consider a steady, two-dimensional, irrotational, planar, or axial flow. This flow is characterized by a system comprising two equations representing the gas dynamic and irrotational conditions, along with

an additional equation linking the flow velocity to the speed of sound. The MoC provides an efficient method for solving this system.
The gas dynamic equation and the irrotational condition are reformulated into two equations. The first equation, (1), describes the characteristic curves, where α is the Mach angle and θ denotes the flow direction. The second equation, (2), called compatibility equation, expresses the variation of the two unknown, horizontal and vertical velocities u and v, along a characteristic.

$$\lambda_{\pm} = \tan(\theta \pm \alpha) \tag{1}$$

$$\left(u^2 - c^2\right) du_{\pm} + \left[2uv - \left(u^2 - c^2\right)\lambda_{\pm}\right] dv_{\pm} - \left(\frac{\delta c^2 v}{y}\right) dx_{\pm} = 0 \tag{2}$$

It is worth noting that from every point, two characteristics can be drawn, referred to as C_+ and C_-. For each characteristic only one compatibility equation can be written, implying that is not possible to independently calculate the values of u and v.
Hence, to fully characterize the flow at a solution point, two already-known points are required. From these points, two characteristics, one for each point, can be drawn. The position of the solution point is calculated as the intersection of these two characteristics. Subsequently, it is possible to compute the two velocity components of the solution point using a system composed of two compatibility equations, one for each characteristic. This procedure results in the creation of a network or grid of points.
To determine the solution in each point, the Eulerian predictor-corrector method, an iterative and numerical technique, needs to be employed. This method is executed in a processing unit, that is the basic computational unit of a MoC. The implementation of this unit follows the approach presented by Hoffman and Zucrow [3], offering the significant advantage of decoupling the resolution of this processing unit from the equation of state used. This is possible because the only thermodynamic quantity that is necessary for the algorithm is the speed of sound c, which is computed by an external function.
In certain specific cases, a processing unit may require only one input point. This occurs when the position and velocity of the solution point are constrained by a condition external to the flow. For instance, if the solution point is located along a solid wall, its position is along the wall, and its flow direction must be tangent to it.
The decision to employ a planar wind tunnel nozzle is driven by the need to utilize the results obtained through the MoC for the design of a turbine vane. In this context, a rectangular passage area is consistent with the blade geometry, and ensuring the uniformity of the exit flow becomes a desirable condition.

2.1. Symmetric case

This nozzle, as the name suggests, consists of two symmetric walls. To mitigate computational costs, the flow computed by the MoC is confined by an axis of symmetry and a solid wall. This solid wall is one of the MoC outputs, along with the thermodynamic information for every point in the computed network. The wall is divided into two successive sections. The first, referred to as the kernel region, assumes the shape of a circular arc that starts from the throat and extends to the point where the departing expansion wave reaches the desired outlet Mach at the axis of symmetry of the nozzle. The second, called reflex region or turning contour, is defined by maintaining mass flow conservation between the end of the kernel region and the reflex region itself.
The method employed for designing this nozzle follows the approach presented by Zucrow and Hoffman [3] with slight modifications. A non-ideal gas model has been implemented by computing the speed of sound with a non-ideal equation of state.
Since the MoC is applicable only in supersonic flow conditions, a mathematical model is required to initialize the flow in the vicinity of the throat, forming the initial-value line, which is a locus of points sharing a common property, such as $v = 0$. To extend the applicability of the MoC to a fully non-ideal flow, the mathematical model used must be adapted to a non-ideal model. Further explanation on this adaptation is provided in Section 2.3.
The designed nozzle has undergone verification through inviscid CFD simulations conducted using *Ansys*® Fluent under varying outlet Mach and initial total conditions. The code has demonstrated its capability to design a shock-free nozzle and accurately model the supersonic

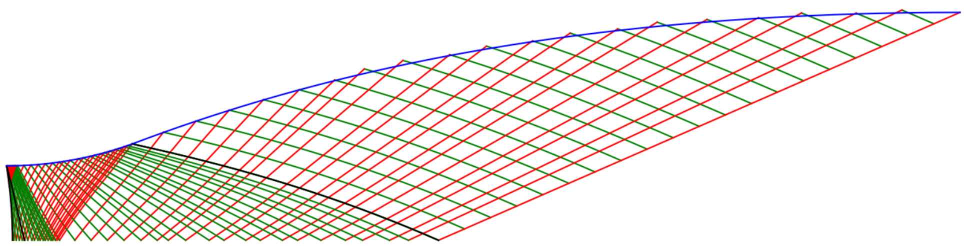

Figure 1: Example of the network of points generated by a symmetric MoC. The red lines are C_+ while the green ones are C_-.

flow inside it.
An illustrative example of the MoC capability is presented in Fig. 2. This showcases a nozzle expanding MDM starting from total quantities $T_t = 269$°C and $P_t = 9.02$ bar, with a total compressibility factor $Z_t = 0.63$, up to a design Mach $M_d = 2$. This case is denominated as SH2. This proves that, even in the presence of strong non-ideal gas behavior, the MoC can model the flow inside a shock-free nozzle with acceptable accuracy.

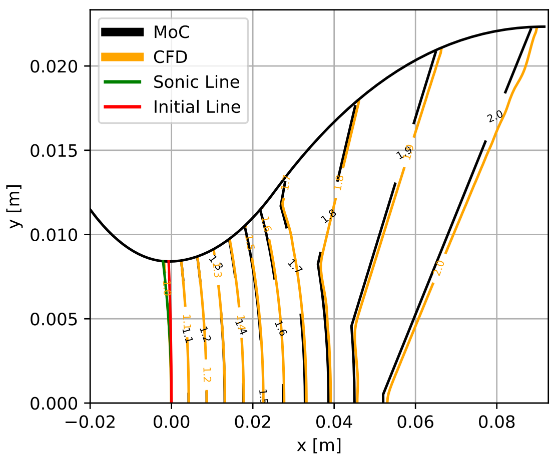

Figure 2: Comparison between the Mach iso-lines computed through MoC and CFD for the SH2 symmetric case.

2.2. Asymmetric nozzle with opposite curvature

This specific nozzle configuration has been derived from the previously mentioned symmetric one by replacing the axis of symmetry with a prescribed wall, called the lower wall. In this case, this prescribed wall takes the form of an arc of a circumference with opposite curvature to that of the upper wall.
This variant of the MoC has been recently introduced by Zocca et al. [4], who applied it to design the vane of a radial turbine, and by Anand for the design of an axial turbine vane [2]. To elaborate further, Anand's version represents a middle ground between a symmetric and an asymmetric MoC, with the lower wall being a straight line.
In both cases, the objective is to develop a design method for the semi-vaned part of a blade. Currently, this region is typically designed by connecting the end of the diverging bladed part of the suction side with the trailing edge while keeping the exit angle constant.
The implemented code closely resembles the one developed for the symmetric case. However, some modifications are necessary to accommodate the presence of a lower curved wall in substitution of the axis of symmetry, an outlet flow direction different from the horizontal one, and a different model for the computation of the initial-value line.
In contrast with the symmetric case, here the MoC is employed to design two walls and not one, because of the substitution of the axis of symmetry with the lower wall. It is worth noting that now only the upper wall is divided into two consecutive regions, called, as before, kernel and reflex, while the lower is composed of only the kernel region.
The kernel region for the upper wall is composed of a prescribed circular arc that links the throat with the point where the departing expansion wave reaches the design Mach number at the lower wall. The lower wall is composed of a circular arc, with a radius higher than the upper one, that connects the point where the designed Mach number is reached with the throat. The reflex region of the upper wall is computed as the symmetric case (see Section 2.1).
This MoC version is verified through CFD sim-

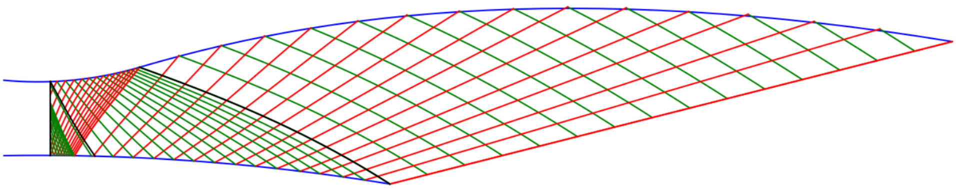

Figure 3: Example of the network of points generated by an asymmetric MoC. The red lines represent the C_+ while the green ones represent the C_-.

ulations. An example of the obtained results is shown in Fig. 4. The case presented has the same thermodynamic properties, but different geometrical properties, compared to the case presented in the verification of the symmetric MoC (see Section 2.1). From the figure, it is evident that the asymmetric MoC is capable of designing a shock-free nozzle and predicting the supersonic flow inside it.

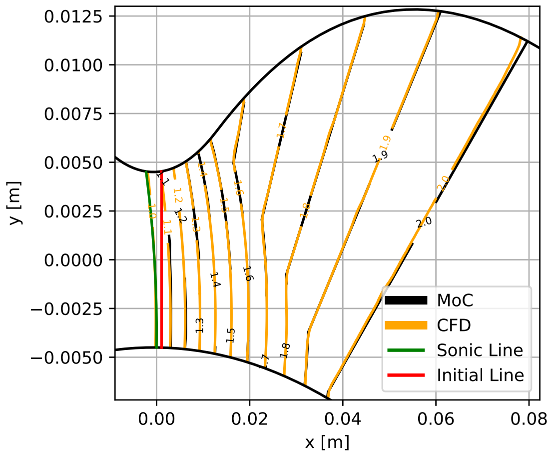

Figure 4: Comparison between the Mach isolines computed through MoC and CFD for the SH2 asymmetric case.

2.3. Transonic flow modeled in the vicinity of the throat with k-model

As explained in Sections 2.1 and 2.2, the MoC operates exclusively with supersonic flows, necessitating a mathematical model to initialize the flow near the throat. Since the primary focus of this work is to characterize non-ideal flows, a novel model has been developed specifically for these types of flows.
This model relies on the perturbation equation commonly employed in the literature to describe transonic flow under the assumption of two-dimensional, planar, or axisymmetric and irrotational flow. It has been adapted for non-ideal flow by incorporating the isentropic exponent k, defined as:

$$k = \frac{\rho}{P}\left(\frac{\partial P}{\partial \rho}\right)_s \tag{3}$$

Because, at given total inlet conditions, the thermodynamic state in the vicinity of the throat is a function of the flow velocity only, which can be assumed near constant in the throat due to the hypothesis of small perturbation, it is possible to assume the value of k constant and equal to its value in sonic condition $\bar{k}$.
The perturbation equation for an ideal flow, equation (4), is derived from the general perturbation equation, valid for two-dimensional planar or axisymmetric irrotational flow, by expressing the speed of sound as a function of γ, which is constant, and of the flow field velocity.

$$\begin{aligned}&\left(1-M_\infty^2\right)\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\\&\delta\frac{v}{y}=M_\infty^2\left(\gamma+1\right)\left(\frac{u}{U_\infty}\right)\frac{\partial u}{\partial x}\end{aligned} \tag{4}$$

Because both $\bar{k}$ and γ describe the relative change of density and pressure over an isentropic expansion for a non-ideal and a perfect flow respectively and are both constant, is possible to substitute γ in equation (4) with $\bar{k}$ without substantial modification, obtaining the perturbation velocity potential equation for a non-ideal flow as:

$$\begin{aligned}&\left(1-M_\infty^2\right)\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\\&\delta\frac{v}{y}=M_\infty^2\left(\bar{k}+1\right)\left(\frac{u}{U_\infty}\right)\frac{\partial u}{\partial x}\end{aligned} \tag{5}$$

The velocity of the flow field is computed, because γ and $\bar{k}$ are of the same order, by assuming a power series solution, a method outlined

by Zucrow and Hoffman [3] for the symmetric case.
For the asymmetric case with opposite curvature, the method derived by Zocca et al. [4] for a non-ideal flow is recast.
The solution computed in the aforementioned paper is based on a perturbation velocity potential for non-ideal flow computed from the general one using the fundamental derivative of gas dynamic $\Gamma = 1 + \frac{\rho}{c}\left(\frac{\partial c}{\partial \rho}\right)_s$ [5], under the hypothesis that this quantity is constant and equal to its value computed in the sonic state $\bar{\Gamma}$, assumed as a $O(1)$. Because $\bar{\Gamma}$ is of the same order of γ [4], which is of the same order of $\bar{k}$, is possible to resolve equation (5) with the same method proposed by Zocca et al [4]. without substantial modification, obtaining the equations that describe the flow in the vicinity of the throat for an asymmetric nozzle with opposite curvature.
The verification of the proposed models is considered successful as the computed sonic line, the iso-line where $M = 1$, aligns with the one computed by the CFD, as depicted in Fig. 2 and 4 for both symmetric and asymmetric cases.

3. MoC application: converging-diverging axial turbine nozzles

The two MoCs presented are applied to the design of converging-diverging blades. For the symmetric MoC, the method introduced by Anand [2] is implemented with minor modifications on the converging part. Meanwhile, for the asymmetric MoC, a new method has been developed and implemented.
The latter method has exhibited a drawback: due to the selection of an opposite curvature nozzle, the section where the blade has the minimum thickness is not the trailing edge, but rather the interface between the converging and diverging parts on the pressure side.
A comparison between the two implemented codes has been conducted with three cases. The first involves CO2 at $T_t = 500^\circ\ C$, $P_t = 200\ bar$, and an outlet metal angle $\varphi_a = 70^\circ$, referred to as CO2-70. The other two cases utilize MDM at $T_t = 272^\circ\ C$, $P_t = 8\ bar$, with $\varphi_a = 70^\circ$ and $\varphi_a = 65^\circ$, labeled as MDM-70 and MDM-65, respectively. In all three cases, the outlet Mach number is set to 2.

Both methods are capable of designing a vane with shock-free blade passage and an outlet flow with similar properties to the design ones.
However, the comparison made using a series of quantities of interest (refer to Table 1) at one axial chord from the trailing edge reveals that the design of the semibladed region with an asymmetric MoC did not yield the expected results. For the asymmetric blade, higher losses are observed compared to the symmetric case. In addition, Mach numbers and outlet flow angles deviate more from the design values compared to a symmetric blade.
It is essential to note that no boundary layer correction has been applied to the diverging part, which explains why the Mach number at one axial chord downstream of the blades is always lower than the design outlet Mach number.

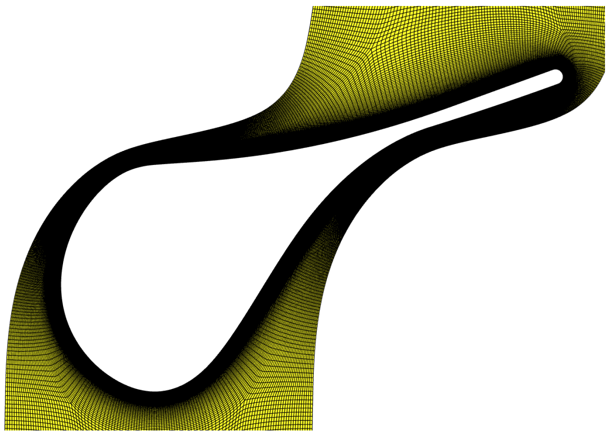

Figure 5: Particular of the mesh used to simulate an asymmetric blade. Note that the actual inlet and outlet are positioned at half chord and three chords from the profile respectively.

	Symmetric			Asymmetric		
Case	Y [%]	M	φ_a [°]	Y [%]	M	φ_a [°]
CO2-70	17	1.9	71.96	17	1.88	73.98
MDM-70	19.3	1.87	73.2	24.3	1.83	75.73
MDM-65	15.8	1.88	68.80	18.7	1.87	72.00

Table 1: Losses computed as $Y = \frac{P_{t,in} - P_{t,out}}{P_{t,in} - P_{out}}$, mass-weighted value of Mach number and outlet flow angle for the analyzed cases at one axial chord downstream of the profile, where the quantities P_t and $P_{t,out}$ are also computed.

4. Conclusions

The two MoCs implemented in this work have successfully passed the verification, proving their

ability to design a free-shock nozzle that respects the assigned outlet Mach number, as well as to predict with acceptable accuracy the flow inside the designed nozzle, even in the presence of strong non-ideal effects.
The verification of the k-model, derived for the description of the flow in the vicinity of the throat and thus for the initialization of the MoCs, has been successfully passed, showing its capability to model the transonic flow, even if strong non-ideal gas effects are present.
For what concern the method presented here for the design of converging-diverging blades, both can deflect and expand a flow up to the design values without the formation of shocks inside the blade passage. The fish-tail shocks are present on the trailing edge, but this is a common problem for which a solution does not currently exist. The comparison between the vanes designed with symmetric and asymmetric MoC has demonstrated that the former remains the preferable choice, thanks to lower losses and an outlet flow with properties closer to the desired ones. This conclusion is in agreement with Anand [2].
It is important to note that the code for the blade design with asymmetric MoC developed here is still in its preliminary stages, and more refined studies may be necessary to determine if the increased flexibility inherent in the asymmetric method could be exploited to design more efficient blades.

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