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Executive Summary of the Thesis

# Digital twins of structural systems using Dynamic Bayesian Networks and Reduced Order Models

Laurea Magistrale in Mathematical Engineering - Ingegneria Matematica

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## 1. Introduction

The failure or non-optimized maintenance planning of structural systems might yield extremely high safety, economic and social costs. To this aim, the digital twin (DT) concept represents the most appealing opportunity to move forward condition-based and predictive maintenance practices. The digital twin perspective aims to build a virtual representation of a physical object or process capable of collecting information from the real environment to represent, validate and simulate the physical twin's present and future behavior, [6]. For the purpose of closely characterizing the operations of the original physical asset, the digital twin must be kept synchronized through the assimilation of observational data and updating of the parameters comprising the digital state, which characterize the variability in the physical asset. The updated digital state thus enables to predict the expected evolution of the digital state and the associated uncertainty, as well as to inform an optimal planning of control inputs feeding back to the physical system. Despite their recent methodological formalization, Digital twins have a wide range of applications; among the others, they have been proposed as a feasible solution to monitor the health state of a structure, [7]. This encompasses the framework of structural health monitoring (SHM), which refers to strategies aimed at detecting changes and damages in structures from sensed data, thus allowing to promptly implement maintenance actions before the occurrence of major failures. In this thesis, I consider the application of a probabilistic graphical model, proposed in [3] and relying on a dynamic Bayesian network (DBN), to enable digital twins for SHM purposes. Several numerical analysis have been carried out starting from the code available at [1, 2].

## 2. Methodology

DBNs are probabilistic graphical models describing a set of variables and their conditional dependencies using a directed acyclic graph. Each node of the graph represents a random variable, while the edges of the graph represent the dependence relationship between them. The main feature of a DBN is that the involved variables are also characterized by dependence relationships over adjacent time steps. Such a general and flexible framework allows us to conveniently describe the evolution of the asset-twin system and of their interactions over a time interval $(0, T)$. In particular, the involved random variables are the following:

- $S_t$ represents the physical state at time $t$, which is only indirectly and partially observable via observational data $O_t$;
- $D_t$ represent the digital state at time $t$;
- $U_t$ represents the actions and decisions which influence the physical state at time $t$;
- $Q_t$ represents the quantities of interest ($QoI$) computed from the updated digital state at time $t$;
- $R_t$ represents the reward quantifying the performance of the asset-twin system at time $t$.

At each time $t$, new data $o_t$ are acquired and new nodes representing the variables referring to time $t$ are added to the graph. Once the graph topology has been established, the belief about unobserved variables is updated and propagated using the Bayesian update sum-product algorithm, [4]. A snapshot of the adopted DBN is reported in Figure 1.

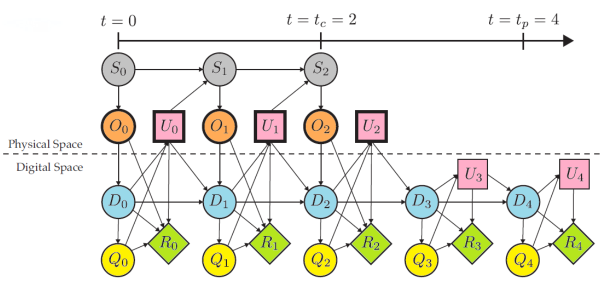


Figure 1: Adopted DBN (figure taken from [3]): nodes with bold outline represent the observed quantities (sensed data and enacted actions), while nodes with thin outline represent estimated quantities. Edges represent the conditional dependences between random variables.

The digital twin simulation is carried out in two different phases. In a first calibration phase the parameters describing the digital state are calibrated, to closely reflect the physical asset. Then in a second operational phase, the digital state is continuously updated through the assimilation of observational data, and adopted to compute quantities of interest and to derive the most appropriate control input. While performing these two phases in our current framework, the observational data collected from a sensing system deployed on the structure are assumed to be simulated using a high-fidelity full-order model (FOM) of the monitored structure describing a ground truth representation of the physical asset. On the other hand, the dataset provided to the DBN to perform the digital state updating is instead generated by exploiting a faster reduced-order model (ROM), relying on the reduced basis method [5], to employ an efficient low-fidelity model for the sake of efficiency. Whenever aiming at informing a real-time data assimilation in a physical based model involving a set of partial differential equations to solve relying on a ROM is indeed essential to overcome the computational cost usually entailed by a FOM.

Figure 2 reports a graphical abstraction outlining the main computational procedures involved in the considered digital twin framework. Observational data from the physical asset are computed with the FOM, which is also adopted to construct the ROM. This latter is instead adopted to form the dataset provided to the DBN, after the preliminary calibration phase. Finally, during the operational phase, the DBN enables the online SHM of the considered structure.

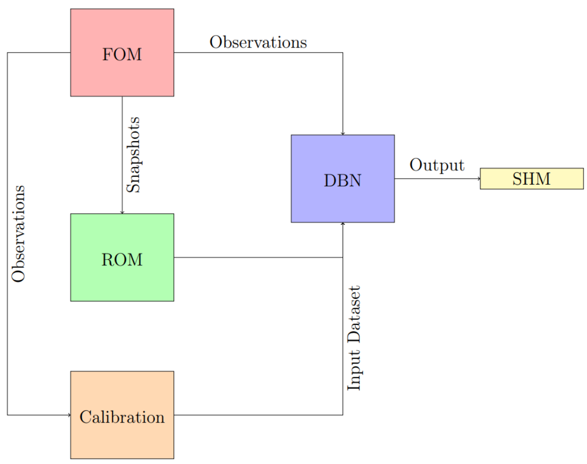


Figure 2: A schematic representation of the main methodologies involved in the considered DT framework.

## 3. SHM of a concrete beam

Starting from the test case described in [3],the considered DBN framework is adapted and tested on different problems. A first test case deals with the SHM of a bridge-like structure depicted in Figure 3. The beam has height $h = 1\,\mathrm{m}$, length $l = 6\,\mathrm{m}$ and width $w = 1\,\mathrm{m}$. The beam is assumed to be made of concrete, with mechanical properties given as follows: Young modulus $E = 47.25\,\mathrm{Gpa}$, Poisson ratio equal to 0.1, density equal to $2400\,\mathrm{kg/m^3}$.

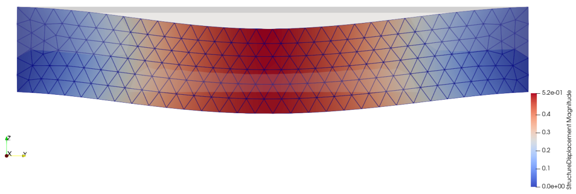

Figure 3: Beam displacement obtained for a 200 kg load applied on its mid span.

At each simulation timestep, the beam can be loaded by two different force loads located at its mid span, representing a very simplified description of a traffic situation occurring over the bridge. High or low force loads (equal to 300 kg, 200 kg respectively) are introduced to model the case of high or low traffic on the bridge. As in [3], time is discretized in such a way that the transient period between the two admissible loading conditions is neglected, and only the related static configuration is accounted for. The digital state is described by the following vector of parameters:

$$\mathbf{d} = \left[ \; l, \quad h, \quad w, \quad e, \quad \mathbf{z} \; \right]^T . \tag{1}$$

Where $l, h, w$ are geometrical parameters describing the sizes of the beam, $e$ is a material parameter adopted to rescale the concrete Young's modulus, and $\mathbf{z}$ is a vector of structural health parameters describing the health state of the structure as explained below.
The calibration phase is carried out assuming that the geometrical parameters are certain and accurate, so that only the Young's modulus scaling factor needs to be calibrated. This is done by simulating a series of load-displacement tests, whose outcome is exploited to update the prior belief about $e$ through a particle filter algorithm.

Figure 4 reports the obtained posterior probability distribution, which is centered at 0.9919 (corresponding to a 0.81% reduction) and features a reduced standard deviation compared to the prior distribution initially selected of $e$.

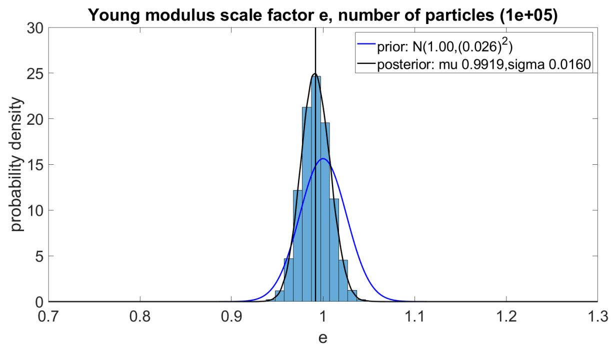


Figure 4: Prior and posterior distributions of the Young's modulus scaling factor $e$ for the beam test case.

Once the calibration phase is completed, the input dataset for the DBN is assembled by simulating a set of potential damage scenarios affecting the structure. This is done for each possible damage state $\mathbf{z}$ with reference to the updated distribution of parameter $e$, by taking 30 samples from the posterior distribution of $e$ and computing the relevant quantities of interest for each sample, for instance in terms of displacements or strains.

### 3.1. Three cracks test case to describe the damage state z

In this first test case, we consider three structural health parameters, each one describing the crack length in a predefined region of the beam. The location of the cracks is assumed to be known, and their width is kept fixed at 0.02 m. Observational data, in terms of Von Mises stress, are simulated assuming that the severe load is always applied at the mid span, except from the case when any of the $\mathbf{z}$ components is greater than 10% of the height of the beam; in this case, the digital twin is supposed to suggest a control action, for which only the low traffic condition is allowed over the bridge. Figure 5 reports the result about the estimation and the future forecast of the three structural health parameters and the relative 95% confidence intervals at time $t = 40$. Note how the digital twin is capable of accurately tracking the evolution of the structural health with relatively low uncertainty. The value of Von Mieses stress at sensor locations computed from the updated digital state is instead displayed in Figure 6. Also in this case, we can observe a clear correspondence between computed and measured values.

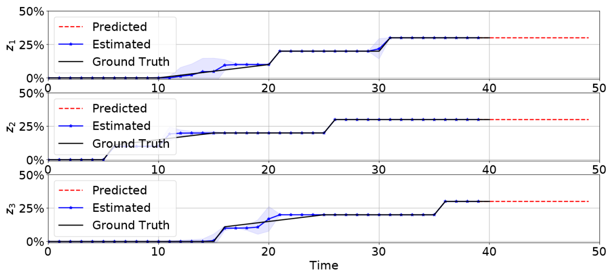


Figure 5: Health state prediction results for the three crack beam test case at $t_c = 40$.

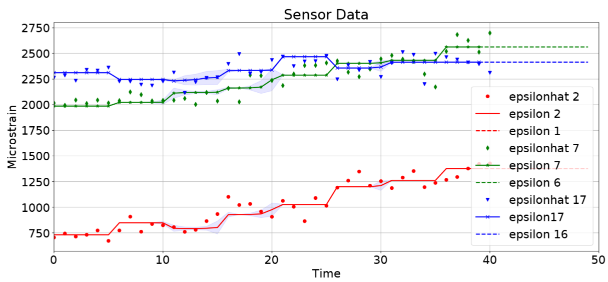


Figure 6: Prediction of the stress for the three crack beam case at $t_c = 40$.

This test case has shown that the DBN framework proposed in [4] can be suitably generalized to problems characterized by more than two structural health parameters, which is the only case previously considered in [3]. Another aspect of novelty consists in the modeling of damage patterns as variable size cracks, instead of a localized stiffness reduction as done in [3].

## 3.2. Data augmentation using a ROM

In this second test case, the possible damage scenarios affecting the beam are modeled as localized reductions of the material stiffness. In particular, we consider ten possible zones in which the stiffness reduction can take place, and we assume that the vector $\mathbf{z}$ is made by two components, ruling the stiffness reduction in two of the ten subdomains. We first provide the DBN with an input dataset generated with a FOM, and accounting for stress measurements in damage scenarios characterized by $\mathbf{z} = \{0\%, 20\%, 40\%, 60\%\} \times \{0\%, 20\%, 40\%, 60\%\}$. The control policy is kept as in the previous case. Figure 7 reports the outcome of the estimation and future evolution prediction of the structural health parameters.

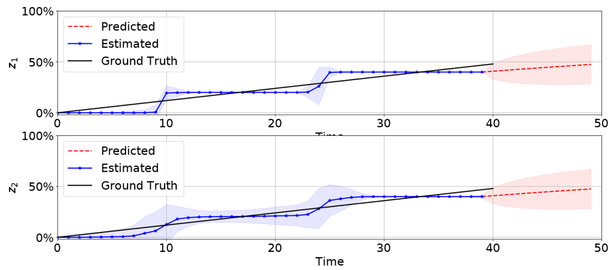


Figure 7: Health state prediction results for the beam test case with localized stiffness reductions and input dataset build built using a FOM at $t_c = 40$.

As it can be seen from Figure 7, when a linear model is introduced to describe the evolution of the structural health, the DT accuracy worsens when the health state $\mathbf{z}$ takes values far from those contained in the input dataset. To improve the estimation and prediction capabilities of the DT, we chose to perform a data augmentation of the dataset provided to the DBN. This is done by refining the possible states that can be assumed by the components of the structural health vector $\mathbf{z}$ as $\{0\%, 10\%, 20\%, 30\%, 40\%, 50\%, 60\%\}$. In this case, the value of Von Mises stress is computed with a computationally convenient ROM relying on the reduced basis method. After providing the augmented dataset to the DBN, the obtained results in terms of estimation and prediction of the digital state evolution are reported in Figure 8. Comparing the latter results with those shown in Figure 7, it is possible conclude that using a ROM to perform data augmentation on the input dataset allows us to obtain more accurate health state estimation and prediction results. The obtained improvements also comes with a low computational cost thanks to the adoption of a ROM.

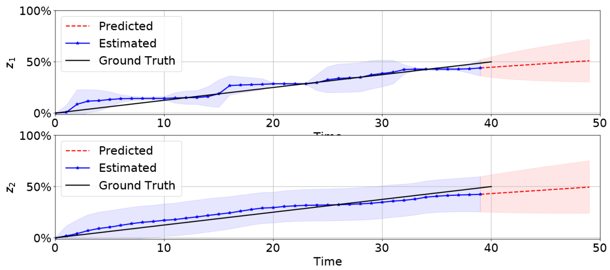


Figure 8: Health state prediction results for the beam test case with localized stiffness reductions and augmented input dataset at $t_c = 40$.

## 4. Vibration-based SHM of a four-story frame

In this second experiment we consider the SHM of the four-story frame structure depicted in Figure 9.

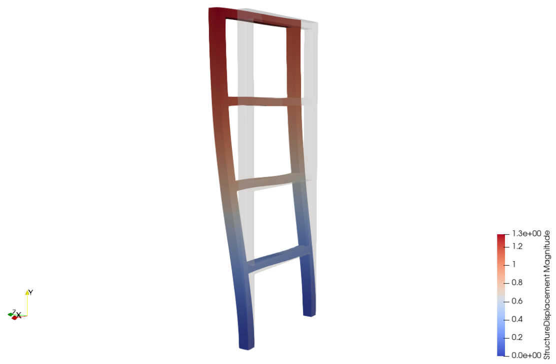

Figure 9: Four-stories frame displacement at $t = 5\,\mathrm{s}$.

This structure has a total height $h = 6\,\mathrm{m}$, length $l = 0.3\,\mathrm{m}$ and width $w = 0.4\,\mathrm{m}$. The frame is assumed to be made of concrete, with mechanical proprieties: Young modulus $E = 30\,\mathrm{Gpa}$, Poisson ratio equal to 0.2, density equal to $2500\,\mathrm{kg/m^3}$. The frame is excited with a random lateral force, applied to mimic low-intensity seismic loads such as ambient vibrations. Damage is modeled by means of a vector $\mathbf{z}$ of structural health parameters, with two components, describing the amount of stiffness reduction applied within two subdomains located at the two basis of the frame. In this case, the observed quantities of interest are not direct measurements, as displacements or stress recordings; instead, by means of the frequency domain decomposition method, we compute the first four eigenfrequencies of the structure from horizontal and vertical displacement recordings acquired at eight sensor locations over a time interval of $5s$. Accordingly, each time step in the digital twin data assimilation is assumed to cover a time interval of $5s$. Moreover, we consider a control policy for which a perfect maintenance action should be performed on the structure if any of the components of $\mathbf{z}$ is greater than 30%. Similarly to the previous experiment, the digital state is given by:

$$\mathbf{d} = [\ l, \quad h, \quad w, \quad n, \quad e, \quad \mathbf{z}\ ]^T, \tag{2}$$

where $l, h, w$ are the dimensions of the frame, $n$ is the number of stories, $e$ is the Young's modulus scaling factor and $\mathbf{z}$ is the health state vector previously described. The only parameter which need to be calibrated is the Young's modulus scaling factor $e$; this is done following the same strategy adopted in the previous experiment. Figure 10 reports the obtained posterior probability distribution, which is centered at 0.9927 (corresponding to a 0.73% reduction) and features a reduced standard deviation compared to the prior distribution initially selected of $e$.

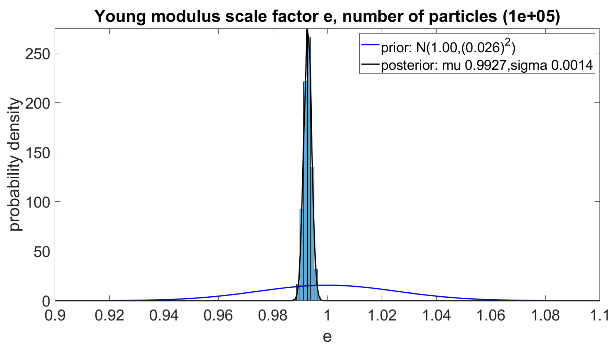


Figure 10: Prior and posterior distributions of the Young's modulus scaling factor for the frame test case.

Figure 11 reports the results of the post-calibration operational phase, in terms of estimation and prediction of the evolution of the structural health parameters at time $t = 40$. From the latter observed results, we can see that the DBN is able to promptly suggest when a repairing action is needed.

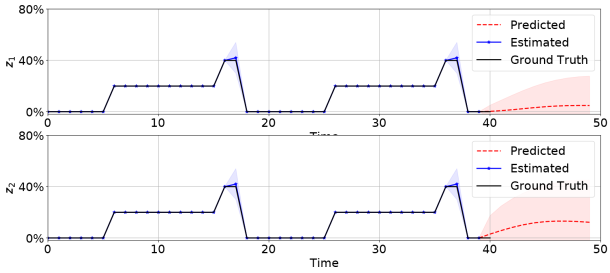


Figure 11: Health state prediction results for the frame test case at $t_c = 40$.

Figure 12 reports the corresponding prediction of the first three eigenfrequencies of the structure, computed from the updated digital state, with respect to their ground truth value. The uncertainty bands are much wider than in previous cases, and this is due to the fact that in this new test case indirect measurements are considered as quantities of interest.

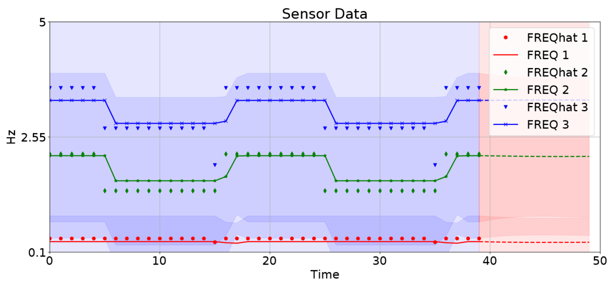


Figure 12: Prediction of the first three natural frequencies for the frame test case at $t_c = 40$.

These results prove that a DBN-based digital twin framework can be also used in very complex situations where we do not neglect the transient period between the application of two actions and the force acting on the structure is not known. This test also shows that the performance of the DBN framework does not depend on the observed quantity of interest; indeed, in this case we considered an indirect quantities such as the natural frequencies.

## 5. Conclusions

This thesis shows how the digital twin paradigm proposed in [3], combined with the use of projection based reduced order models for the creation of the required input dataset, can be successfully employed for the sake of structural health monitoring. The primary goal was to extend the range of applications of the method to more realistic cases than the UAV case described in [3]. In particular, it has been shown that the DBN performance is not influenced by the definition of the health state $\mathbf{z}$ and by the quantity of interest considered. The framework proposed in [3] has been also extended to the more difficult case in which the forces applied on the structure are not known, and in which the transient period between the application of two actions is not neglected. According to the test cases performed, the joint use of dynamic Bayesian networks and reduced order models for parameterised systems represents an extremely promising framework to enable digital twins for structural health monitoring purposes.

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